

Computer algebra independent integration tests

1-Algebraic-functions/1.2-Trinomial-products/1.2.2-Quartic/1.2.2.3-d+e-x^2-
^m-a+b-x^2+c-x^4-^p

Nasser M. Abbasi

May 13, 2020

Compiled on May 13, 2020 at 10:56pm

Contents

| | | |
|----------|---|-----------|
| 1 | Introduction | 3 |
| 1.1 | Listing of CAS systems tested | 3 |
| 1.2 | Results | 3 |
| 1.3 | Performance | 5 |
| 1.4 | list of integrals that has no closed form antiderivative | 6 |
| 1.5 | list of integrals solved by CAS but has no known antiderivative | 6 |
| 1.6 | list of integrals solved by CAS but failed verification | 6 |
| 1.7 | Timing | 7 |
| 1.8 | Verification | 7 |
| 1.9 | Important notes about some of the results | 7 |
| 1.10 | Design of the test system | 8 |
| 2 | detailed summary tables of results | 11 |
| 2.1 | List of integrals sorted by grade for each CAS | 11 |
| 2.2 | Detailed conclusion table per each integral for all CAS systems | 14 |
| 2.3 | Detailed conclusion table specific for Rubi results | 73 |
| 3 | Listing of integrals | 85 |
| 3.1 | $\int \frac{c+dx^2}{a+bx^4} dx$ | 85 |
| 3.2 | $\int \frac{c-dx^2}{a+bx^4} dx$ | 89 |
| 3.3 | $\int \frac{c+dx^2}{a-bx^4} dx$ | 93 |
| 3.4 | $\int \frac{c-dx^2}{a-bx^4} dx$ | 96 |
| 3.5 | $\int \frac{2+3x^2}{4+9x^4} dx$ | 99 |
| 3.6 | $\int \frac{2-3x^2}{4+9x^4} dx$ | 102 |
| 3.7 | $\int \frac{2+3x^2}{4-9x^4} dx$ | 105 |
| 3.8 | $\int \frac{2-3x^2}{4-9x^4} dx$ | 108 |
| 3.9 | $\int \frac{\sqrt{a}\sqrt{b+bx^2}}{a+bx^4} dx$ | 111 |
| 3.10 | $\int \frac{\sqrt{a}\sqrt{b-bx^2}}{a+bx^4} dx$ | 114 |
| 3.11 | $\int \frac{d+ex^2}{d^2+e^2x^4} dx$ | 117 |
| 3.12 | $\int \frac{d-ex^2}{d^2+e^2x^4} dx$ | 120 |
| 3.13 | $\int \frac{5+2x^2}{-1+x^4} dx$ | 123 |

| | | |
|------|---|-----|
| 3.14 | $\int \frac{1+bx^2}{\sqrt{1-b^2x^4}} dx$ | 126 |
| 3.15 | $\int \frac{1-bx^2}{\sqrt{1-b^2x^4}} dx$ | 129 |
| 3.16 | $\int \frac{1+bx^2}{\sqrt{-1+b^2x^4}} dx$ | 132 |
| 3.17 | $\int \frac{1-bx^2}{\sqrt{-1+b^2x^4}} dx$ | 135 |
| 3.18 | $\int \frac{1-bx^2}{\sqrt{1+b^2x^4}} dx$ | 138 |
| 3.19 | $\int \frac{1+bx^2}{\sqrt{1+b^2x^4}} dx$ | 141 |
| 3.20 | $\int \frac{1-bx^2}{\sqrt{-1-b^2x^4}} dx$ | 144 |
| 3.21 | $\int \frac{1+bx^2}{\sqrt{-1-b^2x^4}} dx$ | 147 |
| 3.22 | $\int \frac{\sqrt{1+c^2x^2}}{\sqrt{1-c^2x^2}} dx$ | 150 |
| 3.23 | $\int \frac{1+c^2x^2}{\sqrt{1-c^4x^4}} dx$ | 152 |
| 3.24 | $\int \frac{\sqrt{1-c^2x^2}}{\sqrt{1+c^2x^2}} dx$ | 155 |
| 3.25 | $\int \frac{1-c^2x^2}{\sqrt{1-c^4x^4}} dx$ | 158 |
| 3.26 | $\int \frac{d+ex^2}{d^2+bx^2+e^2x^4} dx$ | 161 |
| 3.27 | $\int \frac{d+ex^2}{d^2+fx^2+e^2x^4} dx$ | 167 |
| 3.28 | $\int \frac{d+ex^2}{d^2-bx^2+e^2x^4} dx$ | 173 |
| 3.29 | $\int \frac{d+ex^2}{d^2-fx^2+e^2x^4} dx$ | 179 |
| 3.30 | $\int \frac{d-ex^2}{d^2+bx^2+e^2x^4} dx$ | 185 |
| 3.31 | $\int \frac{d-ex^2}{d^2+fx^2+e^2x^4} dx$ | 190 |
| 3.32 | $\int \frac{d-ex^2}{d^2-bx^2+e^2x^4} dx$ | 195 |
| 3.33 | $\int \frac{d-ex^2}{d^2-fx^2+e^2x^4} dx$ | 200 |
| 3.34 | $\int \frac{d-ex^2}{\frac{cd^2}{e^2}+bx^2+cx^4} dx$ | 205 |
| 3.35 | $\int \frac{d+ex^2}{\frac{cd^2}{e^2}+bx^2+cx^4} dx$ | 211 |
| 3.36 | $\int \frac{d+ex^2}{bx^2+c\left(\frac{d^2}{e^2}+x^4\right)} dx$ | 217 |
| 3.37 | $\int \frac{a-bx^2}{a^2+(-1+2ab)x^2+b^2x^4} dx$ | 223 |
| 3.38 | $\int \frac{a+bx^2}{a^2+(-1+2ab)x^2+b^2x^4} dx$ | 226 |
| 3.39 | $\int \frac{1+2x^2}{1+bx^2+4x^4} dx$ | 229 |
| 3.40 | $\int \frac{1+2x^2}{1-bx^2+4x^4} dx$ | 234 |
| 3.41 | $\int \frac{1+2x^2}{1+6x^2+4x^4} dx$ | 239 |
| 3.42 | $\int \frac{1+2x^2}{1+5x^2+4x^4} dx$ | 242 |
| 3.43 | $\int \frac{1+2x^2}{1+4x^2+4x^4} dx$ | 245 |
| 3.44 | $\int \frac{1+2x^2}{1+3x^2+4x^4} dx$ | 248 |
| 3.45 | $\int \frac{1+2x^2}{1+2x^2+4x^4} dx$ | 251 |
| 3.46 | $\int \frac{1+2x^2}{1+x^2+4x^4} dx$ | 254 |
| 3.47 | $\int \frac{1+2x^2}{1+4x^4} dx$ | 257 |
| 3.48 | $\int \frac{1+2x^2}{1-x^2+4x^4} dx$ | 260 |
| 3.49 | $\int \frac{1+2x^2}{1-2x^2+4x^4} dx$ | 263 |
| 3.50 | $\int \frac{1+2x^2}{1-3x^2+4x^4} dx$ | 266 |

| | | |
|------|--------------------------------------|-----|
| 3.51 | $\int \frac{1+2x^2}{1-4x^2+4x^4} dx$ | 269 |
| 3.52 | $\int \frac{1+2x^2}{1-5x^2+4x^4} dx$ | 271 |
| 3.53 | $\int \frac{1+2x^2}{1-6x^2+4x^4} dx$ | 274 |
| 3.54 | $\int \frac{1-2x^2}{1+bx^2+4x^4} dx$ | 277 |
| 3.55 | $\int \frac{1-2x^2}{1+6x^2+4x^4} dx$ | 282 |
| 3.56 | $\int \frac{1-2x^2}{1+5x^2+4x^4} dx$ | 285 |
| 3.57 | $\int \frac{1-2x^2}{1+4x^2+4x^4} dx$ | 287 |
| 3.58 | $\int \frac{1-2x^2}{1+3x^2+4x^4} dx$ | 289 |
| 3.59 | $\int \frac{1-2x^2}{1+2x^2+4x^4} dx$ | 292 |
| 3.60 | $\int \frac{1-2x^2}{1+x^2+4x^4} dx$ | 295 |
| 3.61 | $\int \frac{1-2x^2}{1+4x^4} dx$ | 298 |
| 3.62 | $\int \frac{1-2x^2}{1-x^2+4x^4} dx$ | 301 |
| 3.63 | $\int \frac{1-2x^2}{1-2x^2+4x^4} dx$ | 304 |
| 3.64 | $\int \frac{1-2x^2}{1-3x^2+4x^4} dx$ | 307 |
| 3.65 | $\int \frac{1-2x^2}{1-4x^2+4x^4} dx$ | 310 |
| 3.66 | $\int \frac{1-2x^2}{1-5x^2+4x^4} dx$ | 313 |
| 3.67 | $\int \frac{1-2x^2}{1-6x^2+4x^4} dx$ | 316 |
| 3.68 | $\int \frac{1+x^2}{1+bx^2+x^4} dx$ | 319 |
| 3.69 | $\int \frac{1+x^2}{1+5x^2+x^4} dx$ | 322 |
| 3.70 | $\int \frac{1+x^2}{1+4x^2+x^4} dx$ | 325 |
| 3.71 | $\int \frac{1+x^2}{1+3x^2+x^4} dx$ | 328 |
| 3.72 | $\int \frac{1+x^2}{1+2x^2+x^4} dx$ | 331 |
| 3.73 | $\int \frac{1+x^2}{1+x^2+x^4} dx$ | 333 |
| 3.74 | $\int \frac{1+x^2}{1+x^4} dx$ | 336 |
| 3.75 | $\int \frac{1+x^2}{1-x^2+x^4} dx$ | 339 |
| 3.76 | $\int \frac{1+x^2}{1-2x^2+x^4} dx$ | 342 |
| 3.77 | $\int \frac{1+x^2}{1-3x^2+x^4} dx$ | 345 |
| 3.78 | $\int \frac{1+x^2}{1-4x^2+x^4} dx$ | 348 |
| 3.79 | $\int \frac{1+x^2}{1-5x^2+x^4} dx$ | 351 |
| 3.80 | $\int \frac{1-x^2}{1+bx^2+x^4} dx$ | 354 |
| 3.81 | $\int \frac{1-x^2}{1+5x^2+x^4} dx$ | 357 |
| 3.82 | $\int \frac{1-x^2}{1+4x^2+x^4} dx$ | 360 |
| 3.83 | $\int \frac{1-x^2}{1+3x^2+x^4} dx$ | 363 |
| 3.84 | $\int \frac{1-x^2}{1+2x^2+x^4} dx$ | 366 |
| 3.85 | $\int \frac{1-x^2}{1+x^2+x^4} dx$ | 368 |
| 3.86 | $\int \frac{1-x^2}{1+x^4} dx$ | 371 |
| 3.87 | $\int \frac{1-x^2}{1-x^2+x^4} dx$ | 374 |
| 3.88 | $\int \frac{1-x^2}{1-2x^2+x^4} dx$ | 377 |
| 3.89 | $\int \frac{1-x^2}{1-3x^2+x^4} dx$ | 380 |
| 3.90 | $\int \frac{1-x^2}{1-4x^2+x^4} dx$ | 383 |
| 3.91 | $\int \frac{1-x^2}{1-5x^2+x^4} dx$ | 386 |

| | | |
|-------|---|-----|
| 3.92 | $\int -\frac{1+3x^2}{1+2x^2+9x^4} dx$ | 389 |
| 3.93 | $\int \frac{1+3x^2}{-1-2x^2-9x^4} dx$ | 392 |
| 3.94 | $\int \frac{3+2x^2}{1-2x^2+x^4} dx$ | 395 |
| 3.95 | $\int \frac{2+3x^2}{5-8x^2+3x^4} dx$ | 398 |
| 3.96 | $\int \frac{d+ex^2}{5-8x^2+3x^4} dx$ | 401 |
| 3.97 | $\int \frac{3+x^2}{1+3x^2+x^4} dx$ | 404 |
| 3.98 | $\int \frac{a+bx^2}{1+x^2+x^4} dx$ | 407 |
| 3.99 | $\int \frac{a+bx^2}{(1+x^2+x^4)^2} dx$ | 411 |
| 3.100 | $\int \frac{a+bx^2}{2+x^2+x^4} dx$ | 415 |
| 3.101 | $\int \frac{a+bx^2}{(2+x^2+x^4)^2} dx$ | 420 |
| 3.102 | $\int \frac{\sqrt{2-x^2}}{1-\sqrt{2}x^2+x^4} dx$ | 426 |
| 3.103 | $\int \frac{\sqrt{2+x^2}}{1+\sqrt{2}x^2+x^4} dx$ | 429 |
| 3.104 | $\int \frac{\sqrt{2-x^2}}{1+bx^2+x^4} dx$ | 432 |
| 3.105 | $\int \frac{\sqrt{2+x^2}}{1+bx^2+x^4} dx$ | 437 |
| 3.106 | $\int \frac{2a-x^2}{a^2-ax^2+x^4} dx$ | 442 |
| 3.107 | $\int \frac{2\sqrt{a-x^2}}{a-\sqrt{a}x^2+x^4} dx$ | 445 |
| 3.108 | $\int \frac{2b^{2/3}+x^2}{b^{4/3}+b^{2/3}x^2+x^4} dx$ | 448 |
| 3.109 | $\int \frac{A+Bx^2}{a^2-ax^2+x^4} dx$ | 452 |
| 3.110 | $\int \frac{A+Bx^2}{a-\sqrt{a}x^2+x^4} dx$ | 457 |
| 3.111 | $\int \frac{A+Bx^2}{a-\sqrt{ac}x^2+cx^4} dx$ | 461 |
| 3.112 | $\int \frac{A+Bx^2}{a-\sqrt{a}\sqrt{c}x^2+cx^4} dx$ | 465 |
| 3.113 | $\int \frac{3-x^2}{\sqrt{3+x^2-x^4}} dx$ | 469 |
| 3.114 | $\int \frac{3-x^2}{\sqrt{3+2x^2-x^4}} dx$ | 472 |
| 3.115 | $\int \frac{3-x^2}{\sqrt{3+3x^2-x^4}} dx$ | 475 |
| 3.116 | $\int \frac{3-x^2}{\sqrt{3-x^2-x^4}} dx$ | 478 |
| 3.117 | $\int \frac{3-x^2}{\sqrt{3-2x^2-x^4}} dx$ | 481 |
| 3.118 | $\int \frac{3-x^2}{\sqrt{3-3x^2-x^4}} dx$ | 484 |
| 3.119 | $\int \frac{b-\sqrt{b^2-4ac}+2cx^2}{\sqrt{a+bx^2+cx^4}} dx$ | 487 |
| 3.120 | $\int (d+ex^2)^4 (a+cx^4) dx$ | 490 |
| 3.121 | $\int (d+ex^2)^3 (a+cx^4) dx$ | 493 |
| 3.122 | $\int (d+ex^2)^2 (a+cx^4) dx$ | 496 |
| 3.123 | $\int (d+ex^2) (a+cx^4) dx$ | 499 |
| 3.124 | $\int \frac{a+cx^4}{d+ex^2} dx$ | 501 |
| 3.125 | $\int \frac{a+cx^4}{(d+ex^2)^2} dx$ | 504 |
| 3.126 | $\int \frac{a+cx^4}{(d+ex^2)^3} dx$ | 507 |
| 3.127 | $\int \frac{a+cx^4}{(d+ex^2)^4} dx$ | 510 |
| 3.128 | $\int (d+ex^2)^3 (a+cx^4)^2 dx$ | 514 |
| 3.129 | $\int (d+ex^2)^2 (a+cx^4)^2 dx$ | 517 |

| | | |
|-------|---|-----|
| 3.130 | $\int (d + ex^2)(a + cx^4)^2 dx$ | 520 |
| 3.131 | $\int (a + cx^4)^2 dx$ | 523 |
| 3.132 | $\int \frac{(a+cx^4)^2}{d+ex^2} dx$ | 525 |
| 3.133 | $\int \frac{(a+cx^4)^2}{(d+ex^2)^2} dx$ | 528 |
| 3.134 | $\int \frac{(a+cx^4)^2}{(d+ex^2)^3} dx$ | 532 |
| 3.135 | $\int \frac{(a+cx^4)^2}{(d+ex^2)^4} dx$ | 536 |
| 3.136 | $\int \frac{(a+cx^4)^2}{(d+ex^2)^5} dx$ | 540 |
| 3.137 | $\int \frac{(d+ex^2)^4}{a+cx^4} dx$ | 544 |
| 3.138 | $\int \frac{(d+ex^2)^3}{a+cx^4} dx$ | 550 |
| 3.139 | $\int \frac{(d+ex^2)^2}{a+cx^4} dx$ | 555 |
| 3.140 | $\int \frac{d+ex^2}{a+cx^4} dx$ | 560 |
| 3.141 | $\int \frac{1}{a+cx^4} dx$ | 564 |
| 3.142 | $\int \frac{1}{(d+ex^2)(a+cx^4)} dx$ | 568 |
| 3.143 | $\int \frac{1}{(d+ex^2)^2(a+cx^4)} dx$ | 573 |
| 3.144 | $\int \frac{(d+ex^2)^3}{(a+cx^4)^2} dx$ | 581 |
| 3.145 | $\int \frac{(d+ex^2)^2}{(a+cx^4)^2} dx$ | 587 |
| 3.146 | $\int \frac{d+ex^2}{(a+cx^4)^2} dx$ | 592 |
| 3.147 | $\int \frac{1}{(a+cx^4)^2} dx$ | 596 |
| 3.148 | $\int \frac{1}{(d+ex^2)(a+cx^4)^2} dx$ | 600 |
| 3.149 | $\int \frac{1}{(d+ex^2)^2(a+cx^4)^2} dx$ | 608 |
| 3.150 | $\int \frac{(d+ex^2)^4}{\sqrt{a+cx^4}} dx$ | 613 |
| 3.151 | $\int \frac{(d+ex^2)^3}{\sqrt{a+cx^4}} dx$ | 617 |
| 3.152 | $\int \frac{(d+ex^2)^2}{\sqrt{a+cx^4}} dx$ | 621 |
| 3.153 | $\int \frac{d+ex^2}{\sqrt{a+cx^4}} dx$ | 625 |
| 3.154 | $\int \frac{1}{(d+ex^2)\sqrt{a+cx^4}} dx$ | 628 |
| 3.155 | $\int \frac{1}{(d+ex^2)^2\sqrt{a+cx^4}} dx$ | 631 |
| 3.156 | $\int \frac{1}{(d+ex^2)^3\sqrt{a+cx^4}} dx$ | 636 |
| 3.157 | $\int \frac{(d+ex^2)^3}{\sqrt{a-cx^4}} dx$ | 641 |
| 3.158 | $\int \frac{(d+ex^2)^2}{\sqrt{a-cx^4}} dx$ | 645 |
| 3.159 | $\int \frac{d+ex^2}{\sqrt{a-cx^4}} dx$ | 649 |
| 3.160 | $\int \frac{1}{(d+ex^2)\sqrt{a-cx^4}} dx$ | 653 |
| 3.161 | $\int \frac{1}{(d+ex^2)^2\sqrt{a-cx^4}} dx$ | 656 |

| | | |
|-------|--|-----|
| 3.162 | $\int \frac{1}{(d+ex^2)^3 \sqrt{a-cx^4}} dx$ | 661 |
| 3.163 | $\int \frac{1}{(d+ex^2)^4 \sqrt{a-cx^4}} dx$ | 666 |
| 3.164 | $\int \frac{d+ex^2}{\sqrt{-a+cx^4}} dx$ | 671 |
| 3.165 | $\int \frac{1}{(d+ex^2)\sqrt{-a+cx^4}} dx$ | 675 |
| 3.166 | $\int \frac{\sqrt{a+\sqrt{c}x^2}}{\sqrt{-a+cx^4}} dx$ | 678 |
| 3.167 | $\int \frac{1+\sqrt{\frac{c}{a}}x^2}{\sqrt{-a+cx^4}} dx$ | 681 |
| 3.168 | $\int \frac{d+ex^2}{\sqrt{-a-cx^4}} dx$ | 684 |
| 3.169 | $\int \frac{1}{(d+ex^2)\sqrt{-a-cx^4}} dx$ | 687 |
| 3.170 | $\int \frac{1}{(a+bx^2)\sqrt{4-5x^4}} dx$ | 690 |
| 3.171 | $\int \frac{1}{(a+bx^2)\sqrt{4+5x^4}} dx$ | 693 |
| 3.172 | $\int \frac{1}{(a+bx^2)\sqrt{4-dx^4}} dx$ | 696 |
| 3.173 | $\int \frac{1}{(a+bx^2)\sqrt{4+dx^4}} dx$ | 699 |
| 3.174 | $\int \frac{\sqrt{a+bx^2}}{\sqrt{1-x^4}} dx$ | 702 |
| 3.175 | $\int (c+ex^2)^q (a+bx^4)^p dx$ | 704 |
| 3.176 | $\int (c+ex^2)^3 (a+bx^4)^p dx$ | 706 |
| 3.177 | $\int (c+ex^2)^2 (a+bx^4)^p dx$ | 709 |
| 3.178 | $\int (c+ex^2) (a+bx^4)^p dx$ | 712 |
| 3.179 | $\int (a+bx^4)^p dx$ | 715 |
| 3.180 | $\int \frac{(a+bx^4)^p}{c+ex^2} dx$ | 718 |
| 3.181 | $\int \frac{(a+bx^4)^p}{(c+ex^2)^2} dx$ | 721 |
| 3.182 | $\int (1-x^2)^3 (1+bx^4)^p dx$ | 724 |
| 3.183 | $\int (1-x^2)^2 (1+bx^4)^p dx$ | 727 |
| 3.184 | $\int (1-x^2) (1+bx^4)^p dx$ | 730 |
| 3.185 | $\int (1+bx^4)^p dx$ | 733 |
| 3.186 | $\int \frac{(1+bx^4)^p}{1-x^2} dx$ | 735 |
| 3.187 | $\int \frac{(1+bx^4)^p}{(1-x^2)^2} dx$ | 738 |
| 3.188 | $\int \frac{(1+bx^4)^p}{(1-x^2)^3} dx$ | 741 |
| 3.189 | $\int \frac{(d+ex^2)^4}{d^2-e^2x^4} dx$ | 744 |
| 3.190 | $\int \frac{(d+ex^2)^3}{d^2-e^2x^4} dx$ | 747 |
| 3.191 | $\int \frac{(d+ex^2)^2}{d^2-e^2x^4} dx$ | 750 |
| 3.192 | $\int \frac{d+ex^2}{d^2-e^2x^4} dx$ | 753 |
| 3.193 | $\int \frac{1}{(d+ex^2)(d^2-e^2x^4)} dx$ | 756 |
| 3.194 | $\int \frac{1}{(d+ex^2)^2 (d^2-e^2x^4)} dx$ | 759 |
| 3.195 | $\int \frac{(d+ex^2)^{3/2}}{d^2-e^2x^4} dx$ | 763 |
| 3.196 | $\int \frac{\sqrt{d+ex^2}}{d^2-e^2x^4} dx$ | 767 |

| | | |
|-------|---|-----|
| 3.197 | $\int \frac{1}{\sqrt{d+ex^2}(d^2-e^2x^4)} dx$ | 770 |
| 3.198 | $\int \frac{1}{(d+ex^2)^{3/2}(d^2-e^2x^4)} dx$ | 773 |
| 3.199 | $\int \frac{(a+bx^2)^{5/2}}{\sqrt{a^2-b^2x^4}} dx$ | 777 |
| 3.200 | $\int \frac{(a+bx^2)^{3/2}}{\sqrt{a^2-b^2x^4}} dx$ | 781 |
| 3.201 | $\int \frac{\sqrt{a+bx^2}}{\sqrt{a^2-b^2x^4}} dx$ | 784 |
| 3.202 | $\int \frac{1}{\sqrt{a+bx^2}\sqrt{a^2-b^2x^4}} dx$ | 787 |
| 3.203 | $\int \frac{1}{(a+bx^2)^{3/2}\sqrt{a^2-b^2x^4}} dx$ | 790 |
| 3.204 | $\int \frac{1}{(a+bx^2)^{5/2}\sqrt{a^2-b^2x^4}} dx$ | 794 |
| 3.205 | $\int \frac{(a-bx^2)^{5/2}}{\sqrt{a^2-b^2x^4}} dx$ | 798 |
| 3.206 | $\int \frac{(a-bx^2)^{3/2}}{\sqrt{a^2-b^2x^4}} dx$ | 802 |
| 3.207 | $\int \frac{\sqrt{a-bx^2}}{\sqrt{a^2-b^2x^4}} dx$ | 805 |
| 3.208 | $\int \frac{1}{\sqrt{a-bx^2}\sqrt{a^2-b^2x^4}} dx$ | 808 |
| 3.209 | $\int \frac{1}{(a-bx^2)^{3/2}\sqrt{a^2-b^2x^4}} dx$ | 811 |
| 3.210 | $\int \frac{1}{(a-bx^2)^{5/2}\sqrt{a^2-b^2x^4}} dx$ | 815 |
| 3.211 | $\int \frac{\sqrt{-1+x^2}}{\sqrt{-1+x^4}} dx$ | 819 |
| 3.212 | $\int \frac{\sqrt{1+x^2}}{\sqrt{-1+x^4}} dx$ | 822 |
| 3.213 | $\int \frac{-\sqrt{-1+x^2}+\sqrt{1+x^2}}{\sqrt{-1+x^4}} dx$ | 825 |
| 3.214 | $\int \frac{(d+ex^2)^4}{-cd^2+bde+be^2x^2+ce^2x^4} dx$ | 828 |
| 3.215 | $\int \frac{(d+ex^2)^3}{-cd^2+bde+be^2x^2+ce^2x^4} dx$ | 831 |
| 3.216 | $\int \frac{(d+ex^2)^2}{-cd^2+bde+be^2x^2+ce^2x^4} dx$ | 834 |
| 3.217 | $\int \frac{d+ex^2}{-cd^2+bde+be^2x^2+ce^2x^4} dx$ | 837 |
| 3.218 | $\int \frac{1}{(d+ex^2)(-cd^2+bde+be^2x^2+ce^2x^4)} dx$ | 840 |
| 3.219 | $\int \frac{1}{(d+ex^2)^2(-cd^2+bde+be^2x^2+ce^2x^4)} dx$ | 845 |
| 3.220 | $\int \frac{(d+ex^2)^{5/2}}{-cd^2+bde+be^2x^2+ce^2x^4} dx$ | 849 |
| 3.221 | $\int \frac{(d+ex^2)^{3/2}}{-cd^2+bde+be^2x^2+ce^2x^4} dx$ | 853 |
| 3.222 | $\int \frac{\sqrt{d+ex^2}}{-cd^2+bde+be^2x^2+ce^2x^4} dx$ | 858 |
| 3.223 | $\int \frac{1}{\sqrt{d+ex^2}(-cd^2+bde+be^2x^2+ce^2x^4)} dx$ | 862 |
| 3.224 | $\int \frac{1}{(d+ex^2)^{3/2}(-cd^2+bde+be^2x^2+ce^2x^4)} dx$ | 866 |
| 3.225 | $\int (1+x^2)^3 \sqrt{1+x^2+x^4} dx$ | 871 |
| 3.226 | $\int (1+x^2)^2 \sqrt{1+x^2+x^4} dx$ | 875 |
| 3.227 | $\int (1+x^2) \sqrt{1+x^2+x^4} dx$ | 879 |
| 3.228 | $\int \frac{\sqrt{1+x^2+x^4}}{1+x^2} dx$ | 882 |
| 3.229 | $\int \frac{\sqrt{1+x^2+x^4}}{(1+x^2)^2} dx$ | 886 |
| 3.230 | $\int \frac{\sqrt{1+x^2+x^4}}{(1+x^2)^3} dx$ | 889 |

| | | |
|-------|--|------|
| 3.231 | $\int \frac{\sqrt{1+x^2+x^4}}{(1+x^2)^4} dx$ | 895 |
| 3.232 | $\int \frac{(1+x^2)^3}{\sqrt{1+x^2+x^4}} dx$ | 901 |
| 3.233 | $\int \frac{(1+x^2)^2}{\sqrt{1+x^2+x^4}} dx$ | 905 |
| 3.234 | $\int \frac{1+x^2}{\sqrt{1+x^2+x^4}} dx$ | 908 |
| 3.235 | $\int \frac{1}{(1+x^2)\sqrt{1+x^2+x^4}} dx$ | 911 |
| 3.236 | $\int \frac{1}{(1+x^2)^2\sqrt{1+x^2+x^4}} dx$ | 914 |
| 3.237 | $\int \frac{1}{(1+x^2)^3\sqrt{1+x^2+x^4}} dx$ | 919 |
| 3.238 | $\int \frac{(1+x^2)^3}{(1+x^2+x^4)^{3/2}} dx$ | 924 |
| 3.239 | $\int \frac{(1+x^2)^2}{(1+x^2+x^4)^{3/2}} dx$ | 927 |
| 3.240 | $\int \frac{1+x^2}{(1+x^2+x^4)^{3/2}} dx$ | 930 |
| 3.241 | $\int \frac{1}{(1+x^2)(1+x^2+x^4)^{3/2}} dx$ | 933 |
| 3.242 | $\int \frac{1}{(1+x^2)^2(1+x^2+x^4)^{3/2}} dx$ | 937 |
| 3.243 | $\int \frac{1}{(1+x^2)^3(1+x^2+x^4)^{3/2}} dx$ | 942 |
| 3.244 | $\int (d+ex^2)^4 (a+bx^2+cx^4) dx$ | 948 |
| 3.245 | $\int (d+ex^2)^3 (a+bx^2+cx^4) dx$ | 951 |
| 3.246 | $\int (d+ex^2)^2 (a+bx^2+cx^4) dx$ | 954 |
| 3.247 | $\int (d+ex^2) (a+bx^2+cx^4) dx$ | 957 |
| 3.248 | $\int \frac{a+bx^2+cx^4}{d+ex^2} dx$ | 959 |
| 3.249 | $\int \frac{a+bx^2+cx^4}{(d+ex^2)^2} dx$ | 962 |
| 3.250 | $\int \frac{a+bx^2+cx^4}{(d+ex^2)^3} dx$ | 965 |
| 3.251 | $\int \frac{a+bx^2+cx^4}{(d+ex^2)^4} dx$ | 968 |
| 3.252 | $\int (d+ex^2)^3 (a+bx^2+cx^4)^2 dx$ | 972 |
| 3.253 | $\int (d+ex^2)^2 (a+bx^2+cx^4)^2 dx$ | 975 |
| 3.254 | $\int (d+ex^2) (a+bx^2+cx^4)^2 dx$ | 978 |
| 3.255 | $\int (a+bx^2+cx^4)^2 dx$ | 981 |
| 3.256 | $\int \frac{(a+bx^2+cx^4)^2}{d+ex^2} dx$ | 983 |
| 3.257 | $\int \frac{(a+bx^2+cx^4)^2}{(d+ex^2)^2} dx$ | 986 |
| 3.258 | $\int \frac{(a+bx^2+cx^4)^2}{(d+ex^2)^3} dx$ | 990 |
| 3.259 | $\int \frac{(a+bx^2+cx^4)^2}{(d+ex^2)^4} dx$ | 994 |
| 3.260 | $\int \frac{(a+bx^2+cx^4)^2}{(d+ex^2)^5} dx$ | 998 |
| 3.261 | $\int \frac{a+bx^2+cx^4}{(d+ex^2)^2} dx$ | 1002 |
| 3.262 | $\int \frac{a+x^2(b+cx^2)}{(d+ex^2)^2} dx$ | 1005 |

| | | |
|-------|--|------|
| 3.263 | $\int \frac{(d+ex^2)^4}{a+bx^2+cx^4} dx$ | 1008 |
| 3.264 | $\int \frac{(d+ex^2)^3}{a+bx^2+cx^4} dx$ | 1012 |
| 3.265 | $\int \frac{(d+ex^2)^2}{a+bx^2+cx^4} dx$ | 1019 |
| 3.266 | $\int \frac{d+ex^2}{a+bx^2+cx^4} dx$ | 1024 |
| 3.267 | $\int \frac{1}{a+bx^2+cx^4} dx$ | 1030 |
| 3.268 | $\int \frac{1}{(d+ex^2)(a+bx^2+cx^4)} dx$ | 1034 |
| 3.269 | $\int \frac{1}{(d+ex^2)^2(a+bx^2+cx^4)} dx$ | 1037 |
| 3.270 | $\int \frac{(d+ex^2)^3}{(a+bx^2+cx^4)^2} dx$ | 1041 |
| 3.271 | $\int \frac{(d+ex^2)^2}{(a+bx^2+cx^4)^2} dx$ | 1045 |
| 3.272 | $\int \frac{d+ex^2}{(a+bx^2+cx^4)^2} dx$ | 1051 |
| 3.273 | $\int \frac{1}{(a+bx^2+cx^4)^2} dx$ | 1057 |
| 3.274 | $\int \frac{1}{(d+ex^2)(a+bx^2+cx^4)^2} dx$ | 1061 |
| 3.275 | $\int \frac{1}{(d+ex^2)^2(a+bx^2+cx^4)^2} dx$ | 1066 |
| 3.276 | $\int (d+ex^2)^{5/2} (a+bx^2+cx^4) dx$ | 1070 |
| 3.277 | $\int (d+ex^2)^{3/2} (a+bx^2+cx^4) dx$ | 1074 |
| 3.278 | $\int \sqrt{d+ex^2} (a+bx^2+cx^4) dx$ | 1078 |
| 3.279 | $\int \frac{a+bx^2+cx^4}{\sqrt{d+ex^2}} dx$ | 1082 |
| 3.280 | $\int \frac{a+bx^2+cx^4}{(d+ex^2)^{3/2}} dx$ | 1085 |
| 3.281 | $\int \frac{a+bx^2+cx^4}{(d+ex^2)^{5/2}} dx$ | 1088 |
| 3.282 | $\int \frac{a+bx^2+cx^4}{(d+ex^2)^{7/2}} dx$ | 1092 |
| 3.283 | $\int \frac{a+bx^2+cx^4}{(d+ex^2)^{9/2}} dx$ | 1096 |
| 3.284 | $\int \frac{a+bx^2+cx^4}{(d+ex^2)^{11/2}} dx$ | 1100 |
| 3.285 | $\int \frac{a+bx^2+cx^4}{(d+ex^2)^{13/2}} dx$ | 1104 |
| 3.286 | $\int (7+5x^2)^3 \sqrt{2+3x^2+x^4} dx$ | 1108 |
| 3.287 | $\int (7+5x^2)^2 \sqrt{2+3x^2+x^4} dx$ | 1112 |
| 3.288 | $\int (7+5x^2) \sqrt{2+3x^2+x^4} dx$ | 1115 |
| 3.289 | $\int \sqrt{2+3x^2+x^4} dx$ | 1118 |
| 3.290 | $\int \frac{\sqrt{2+3x^2+x^4}}{7+5x^2} dx$ | 1121 |
| 3.291 | $\int \frac{\sqrt{2+3x^2+x^4}}{(7+5x^2)^2} dx$ | 1125 |
| 3.292 | $\int \frac{\sqrt{2+3x^2+x^4}}{(7+5x^2)^3} dx$ | 1129 |
| 3.293 | $\int (7+5x^2)^3 (2+3x^2+x^4)^{3/2} dx$ | 1134 |
| 3.294 | $\int (7+5x^2)^2 (2+3x^2+x^4)^{3/2} dx$ | 1138 |
| 3.295 | $\int (7+5x^2) (2+3x^2+x^4)^{3/2} dx$ | 1142 |
| 3.296 | $\int (2+3x^2+x^4)^{3/2} dx$ | 1145 |
| 3.297 | $\int \frac{(2+3x^2+x^4)^{3/2}}{7+5x^2} dx$ | 1148 |

| | | |
|-------|--|------|
| 3.298 | $\int \frac{(2+3x^2+x^4)^{3/2}}{(7+5x^2)^2} dx$ | 1152 |
| 3.299 | $\int \frac{(2+3x^2+x^4)^{3/2}}{(7+5x^2)^3} dx$ | 1157 |
| 3.300 | $\int \frac{(7+5x^2)^3}{\sqrt{2+3x^2+x^4}} dx$ | 1162 |
| 3.301 | $\int \frac{(7+5x^2)^2}{\sqrt{2+3x^2+x^4}} dx$ | 1166 |
| 3.302 | $\int \frac{7+5x^2}{\sqrt{2+3x^2+x^4}} dx$ | 1169 |
| 3.303 | $\int \frac{1}{\sqrt{2+3x^2+x^4}} dx$ | 1172 |
| 3.304 | $\int \frac{1}{(7+5x^2)\sqrt{2+3x^2+x^4}} dx$ | 1175 |
| 3.305 | $\int \frac{1}{(7+5x^2)^2\sqrt{2+3x^2+x^4}} dx$ | 1178 |
| 3.306 | $\int \frac{1}{(7+5x^2)^3\sqrt{2+3x^2+x^4}} dx$ | 1182 |
| 3.307 | $\int \frac{(7+5x^2)^5}{(2+3x^2+x^4)^{3/2}} dx$ | 1187 |
| 3.308 | $\int \frac{(7+5x^2)^4}{(2+3x^2+x^4)^{3/2}} dx$ | 1191 |
| 3.309 | $\int \frac{(7+5x^2)^3}{(2+3x^2+x^4)^{3/2}} dx$ | 1195 |
| 3.310 | $\int \frac{(7+5x^2)^2}{(2+3x^2+x^4)^{3/2}} dx$ | 1198 |
| 3.311 | $\int \frac{7+5x^2}{(2+3x^2+x^4)^{3/2}} dx$ | 1201 |
| 3.312 | $\int \frac{1}{(2+3x^2+x^4)^{3/2}} dx$ | 1204 |
| 3.313 | $\int \frac{1}{(7+5x^2)(2+3x^2+x^4)^{3/2}} dx$ | 1207 |
| 3.314 | $\int \frac{1}{(7+5x^2)^2(2+3x^2+x^4)^{3/2}} dx$ | 1211 |
| 3.315 | $\int \frac{1}{(7+5x^2)^3(2+3x^2+x^4)^{3/2}} dx$ | 1216 |
| 3.316 | $\int (7+5x^2)^4 \sqrt{2+x^2-x^4} dx$ | 1222 |
| 3.317 | $\int (7+5x^2)^3 \sqrt{2+x^2-x^4} dx$ | 1226 |
| 3.318 | $\int (7+5x^2)^2 \sqrt{2+x^2-x^4} dx$ | 1230 |
| 3.319 | $\int (7+5x^2) \sqrt{2+x^2-x^4} dx$ | 1233 |
| 3.320 | $\int \sqrt{2+x^2-x^4} dx$ | 1236 |
| 3.321 | $\int \frac{\sqrt{2+x^2-x^4}}{7+5x^2} dx$ | 1239 |
| 3.322 | $\int \frac{\sqrt{2+x^2-x^4}}{(7+5x^2)^2} dx$ | 1242 |
| 3.323 | $\int \frac{\sqrt{2+x^2-x^4}}{(7+5x^2)^3} dx$ | 1246 |
| 3.324 | $\int (7+5x^2)^4 (2+x^2-x^4)^{3/2} dx$ | 1251 |
| 3.325 | $\int (7+5x^2)^3 (2+x^2-x^4)^{3/2} dx$ | 1255 |
| 3.326 | $\int (7+5x^2)^2 (2+x^2-x^4)^{3/2} dx$ | 1259 |
| 3.327 | $\int (7+5x^2) (2+x^2-x^4)^{3/2} dx$ | 1262 |
| 3.328 | $\int (2+x^2-x^4)^{3/2} dx$ | 1265 |
| 3.329 | $\int \frac{(2+x^2-x^4)^{3/2}}{7+5x^2} dx$ | 1268 |
| 3.330 | $\int \frac{(2+x^2-x^4)^{3/2}}{(7+5x^2)^2} dx$ | 1272 |

| | | |
|-------|---|------|
| 3.331 | $\int \frac{(2+x^2-x^4)^{3/2}}{(7+5x^2)^3} dx$ | 1277 |
| 3.332 | $\int \frac{(7+5x^2)^3}{\sqrt{2+x^2-x^4}} dx$ | 1282 |
| 3.333 | $\int \frac{(7+5x^2)^2}{\sqrt{2+x^2-x^4}} dx$ | 1286 |
| 3.334 | $\int \frac{7+5x^2}{\sqrt{2+x^2-x^4}} dx$ | 1289 |
| 3.335 | $\int \frac{1}{\sqrt{2+x^2-x^4}} dx$ | 1292 |
| 3.336 | $\int \frac{1}{(7+5x^2)\sqrt{2+x^2-x^4}} dx$ | 1295 |
| 3.337 | $\int \frac{1}{(7+5x^2)^2\sqrt{2+x^2-x^4}} dx$ | 1298 |
| 3.338 | $\int \frac{1}{(7+5x^2)^3\sqrt{2+x^2-x^4}} dx$ | 1302 |
| 3.339 | $\int \frac{(7+5x^2)^5}{(2+x^2-x^4)^{3/2}} dx$ | 1306 |
| 3.340 | $\int \frac{(7+5x^2)^4}{(2+x^2-x^4)^{3/2}} dx$ | 1310 |
| 3.341 | $\int \frac{(7+5x^2)^3}{(2+x^2-x^4)^{3/2}} dx$ | 1314 |
| 3.342 | $\int \frac{(7+5x^2)^2}{(2+x^2-x^4)^{3/2}} dx$ | 1317 |
| 3.343 | $\int \frac{7+5x^2}{(2+x^2-x^4)^{3/2}} dx$ | 1320 |
| 3.344 | $\int \frac{1}{(2+x^2-x^4)^{3/2}} dx$ | 1323 |
| 3.345 | $\int \frac{1}{(7+5x^2)(2+x^2-x^4)^{3/2}} dx$ | 1326 |
| 3.346 | $\int \frac{1}{(7+5x^2)^2(2+x^2-x^4)^{3/2}} dx$ | 1330 |
| 3.347 | $\int \frac{1}{(7+5x^2)^3(2+x^2-x^4)^{3/2}} dx$ | 1334 |
| 3.348 | $\int (7+5x^2)^4 \sqrt{4+3x^2+x^4} dx$ | 1339 |
| 3.349 | $\int (7+5x^2)^3 \sqrt{4+3x^2+x^4} dx$ | 1343 |
| 3.350 | $\int (7+5x^2)^2 \sqrt{4+3x^2+x^4} dx$ | 1347 |
| 3.351 | $\int (7+5x^2) \sqrt{4+3x^2+x^4} dx$ | 1351 |
| 3.352 | $\int \sqrt{4+3x^2+x^4} dx$ | 1354 |
| 3.353 | $\int \frac{\sqrt{4+3x^2+x^4}}{7+5x^2} dx$ | 1357 |
| 3.354 | $\int \frac{\sqrt{4+3x^2+x^4}}{(7+5x^2)^2} dx$ | 1361 |
| 3.355 | $\int \frac{\sqrt{4+3x^2+x^4}}{(7+5x^2)^3} dx$ | 1365 |
| 3.356 | $\int (7+5x^2)^4 (4+3x^2+x^4)^{3/2} dx$ | 1370 |
| 3.357 | $\int (7+5x^2)^3 (4+3x^2+x^4)^{3/2} dx$ | 1374 |
| 3.358 | $\int (7+5x^2)^2 (4+3x^2+x^4)^{3/2} dx$ | 1378 |
| 3.359 | $\int (7+5x^2) (4+3x^2+x^4)^{3/2} dx$ | 1382 |
| 3.360 | $\int (4+3x^2+x^4)^{3/2} dx$ | 1385 |
| 3.361 | $\int \frac{(4+3x^2+x^4)^{3/2}}{7+5x^2} dx$ | 1389 |
| 3.362 | $\int \frac{(4+3x^2+x^4)^{3/2}}{(7+5x^2)^2} dx$ | 1394 |
| 3.363 | $\int \frac{(4+3x^2+x^4)^{3/2}}{(7+5x^2)^3} dx$ | 1399 |

| | | |
|-------|--|------|
| 3.364 | $\int \frac{(7+5x^2)^3}{\sqrt{4+3x^2+x^4}} dx$ | 1405 |
| 3.365 | $\int \frac{(7+5x^2)^2}{\sqrt{4+3x^2+x^4}} dx$ | 1409 |
| 3.366 | $\int \frac{7+5x^2}{\sqrt{4+3x^2+x^4}} dx$ | 1413 |
| 3.367 | $\int \frac{1}{\sqrt{4+3x^2+x^4}} dx$ | 1416 |
| 3.368 | $\int \frac{1}{(7+5x^2)\sqrt{4+3x^2+x^4}} dx$ | 1419 |
| 3.369 | $\int \frac{1}{(7+5x^2)^2\sqrt{4+3x^2+x^4}} dx$ | 1422 |
| 3.370 | $\int \frac{1}{(7+5x^2)^3\sqrt{4+3x^2+x^4}} dx$ | 1427 |
| 3.371 | $\int \frac{(7+5x^2)^5}{(4+3x^2+x^4)^{3/2}} dx$ | 1432 |
| 3.372 | $\int \frac{(7+5x^2)^4}{(4+3x^2+x^4)^{3/2}} dx$ | 1436 |
| 3.373 | $\int \frac{(7+5x^2)^3}{(4+3x^2+x^4)^{3/2}} dx$ | 1440 |
| 3.374 | $\int \frac{(7+5x^2)^2}{(4+3x^2+x^4)^{3/2}} dx$ | 1444 |
| 3.375 | $\int \frac{7+5x^2}{(4+3x^2+x^4)^{3/2}} dx$ | 1448 |
| 3.376 | $\int \frac{1}{(4+3x^2+x^4)^{3/2}} dx$ | 1451 |
| 3.377 | $\int \frac{1}{(7+5x^2)(4+3x^2+x^4)^{3/2}} dx$ | 1455 |
| 3.378 | $\int \frac{1}{(7+5x^2)^2(4+3x^2+x^4)^{3/2}} dx$ | 1460 |
| 3.379 | $\int \frac{1}{(7+5x^2)^3(4+3x^2+x^4)^{3/2}} dx$ | 1465 |
| 3.380 | $\int \frac{(d+ex^2)^3}{\sqrt{a+bx^2+cx^4}} dx$ | 1471 |
| 3.381 | $\int \frac{(d+ex^2)^2}{\sqrt{a+bx^2+cx^4}} dx$ | 1475 |
| 3.382 | $\int \frac{d+ex^2}{\sqrt{a+bx^2+cx^4}} dx$ | 1479 |
| 3.383 | $\int \frac{1}{(d+ex^2)\sqrt{a+bx^2+cx^4}} dx$ | 1482 |
| 3.384 | $\int \frac{1}{(d+ex^2)^2\sqrt{a+bx^2+cx^4}} dx$ | 1485 |
| 3.385 | $\int \frac{(d+ex^2)^3}{\sqrt{a+bx^2-cx^4}} dx$ | 1490 |
| 3.386 | $\int \frac{(d+ex^2)^2}{\sqrt{a+bx^2-cx^4}} dx$ | 1495 |
| 3.387 | $\int \frac{d+ex^2}{\sqrt{a+bx^2-cx^4}} dx$ | 1499 |
| 3.388 | $\int \frac{1}{(d+ex^2)\sqrt{a+bx^2-cx^4}} dx$ | 1503 |
| 3.389 | $\int \frac{1}{(d+ex^2)^2\sqrt{a+bx^2-cx^4}} dx$ | 1506 |
| 3.390 | $\int \frac{d+ex^2}{\sqrt{-a+bx^2+cx^4}} dx$ | 1511 |
| 3.391 | $\int \frac{1}{(d+ex^2)\sqrt{-a+bx^2+cx^4}} dx$ | 1515 |
| 3.392 | $\int \frac{d+ex^2}{\sqrt{-a+bx^2-cx^4}} dx$ | 1518 |
| 3.393 | $\int \frac{1}{(d+ex^2)\sqrt{-a+bx^2-cx^4}} dx$ | 1521 |
| 3.394 | $\int \frac{(d+ex^2)^3}{\sqrt{2+3x^2+x^4}} dx$ | 1524 |

| | | |
|-------|--|------|
| 3.395 | $\int \frac{(d+ex^2)^2}{\sqrt{2+3x^2+x^4}} dx$ | 1528 |
| 3.396 | $\int \frac{d+ex^2}{\sqrt{2+3x^2+x^4}} dx$ | 1531 |
| 3.397 | $\int \frac{1}{(d+ex^2)\sqrt{2+3x^2+x^4}} dx$ | 1534 |
| 3.398 | $\int \frac{1}{(d+ex^2)^2\sqrt{2+3x^2+x^4}} dx$ | 1537 |
| 3.399 | $\int (c+ex^2)^q (a+cx^2+bx^4)^p dx$ | 1541 |
| 3.400 | $\int (c+ex^2)^3 (a+cx^2+bx^4)^p dx$ | 1543 |
| 3.401 | $\int (c+ex^2)^2 (a+cx^2+bx^4)^p dx$ | 1547 |
| 3.402 | $\int (c+ex^2) (a+cx^2+bx^4)^p dx$ | 1551 |
| 3.403 | $\int (a+cx^2+bx^4)^p dx$ | 1554 |
| 3.404 | $\int \frac{(a+cx^2+bx^4)^p}{c+ex^2} dx$ | 1557 |
| 3.405 | $\int \frac{(a+cx^2+bx^4)^p}{(c+ex^2)^2} dx$ | 1559 |
| 3.406 | $\int \frac{f+gx}{(d+ex)\sqrt{a+cx^4}} dx$ | 1561 |
| 3.407 | $\int \frac{f+gx}{(d+ex)\sqrt{-a+cx^4}} dx$ | 1565 |
| 3.408 | $\int \frac{1-\sqrt{3}+x}{(1+\sqrt{3}+x)\sqrt{-4+4\sqrt{3}x^2+x^4}} dx$ | 1570 |
| 3.409 | $\int \frac{1+\sqrt{3}+x}{(1-\sqrt{3}+x)\sqrt{-4-4\sqrt{3}x^2+x^4}} dx$ | 1574 |
| 3.410 | $\int \frac{1-\sqrt{3}+2x}{(1+\sqrt{3}+2x)\sqrt{-1+4\sqrt{3}x^2+4x^4}} dx$ | 1578 |
| 3.411 | $\int \frac{1+\sqrt{3}+2x}{(1-\sqrt{3}+2x)\sqrt{-1-4\sqrt{3}x^2+4x^4}} dx$ | 1582 |
| 3.412 | $\int \frac{f+gx}{(d+ex)\sqrt{a+bx^2+cx^4}} dx$ | 1586 |
| 3.413 | $\int \frac{f+gx}{(d+ex)\sqrt{-a+bx^2+cx^4}} dx$ | 1591 |

4 Listing of Grading functions

1597

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from <https://rulebasedintegration.org>

The number of integrals in this report is [413]. This is test number [40].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.1 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12 on windows 10.
3. Maple 2020 (64 bit) on windows 10.
4. Maxima 5.43 on Linux. (via sagemath 8.9)
5. Fricas 1.3.6 on Linux (via sagemath 9.0)
6. Sympy 1.5 under Python 3.7.3 using Anaconda distribution.
7. Giac/Xcas 1.5 on Linux. (via sagemath 8.9)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or Hypergeometric2F1 functions. RootSum and RootOf are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

| System | solved | Failed |
|-------------|-----------------|-----------------|
| Rubi | % 99.76 (412) | % 0.24 (1) |
| Mathematica | % 91.53 (378) | % 8.47 (35) |
| Maple | % 96.61 (399) | % 3.39 (14) |
| Maxima | % 14.53 (60) | % 85.47 (353) |
| Fricas | % 50.61 (209) | % 49.39 (204) |
| Sympy | % 46. (190) | % 54. (223) |
| Giac | % 35.84 (148) | % 64.16 (265) |

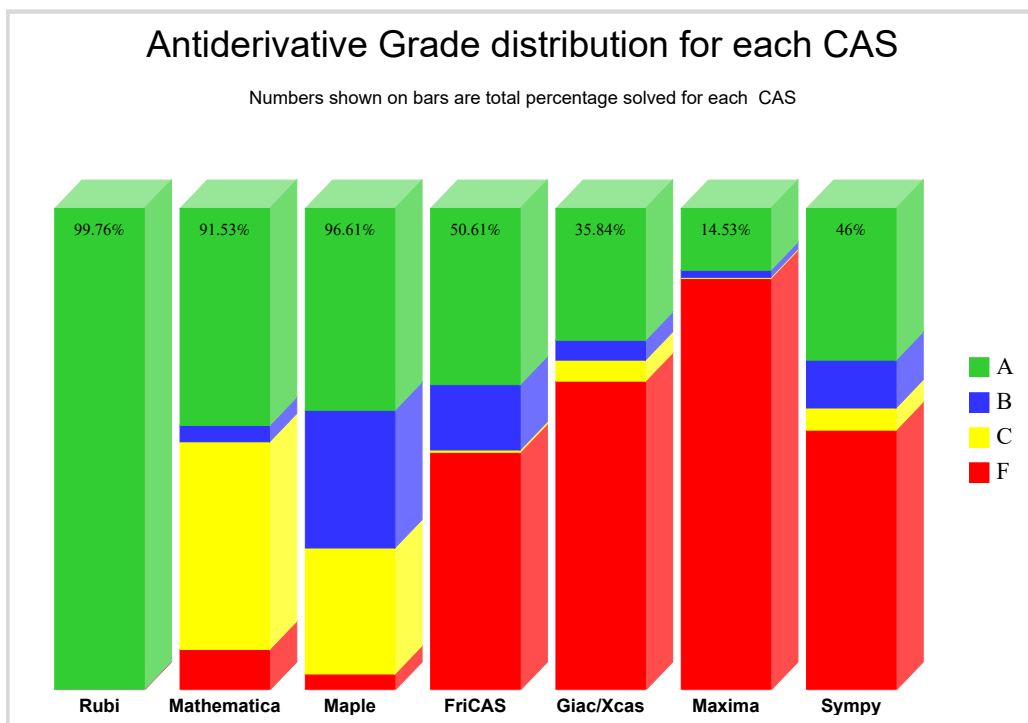
The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

| grade | description |
|-------|---|
| A | Integral was solved and antiderivative is optimal in quality and leaf size. |
| B | Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size. |
| C | Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons <ol style="list-style-type: none"> 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not. |
| F | Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised. |

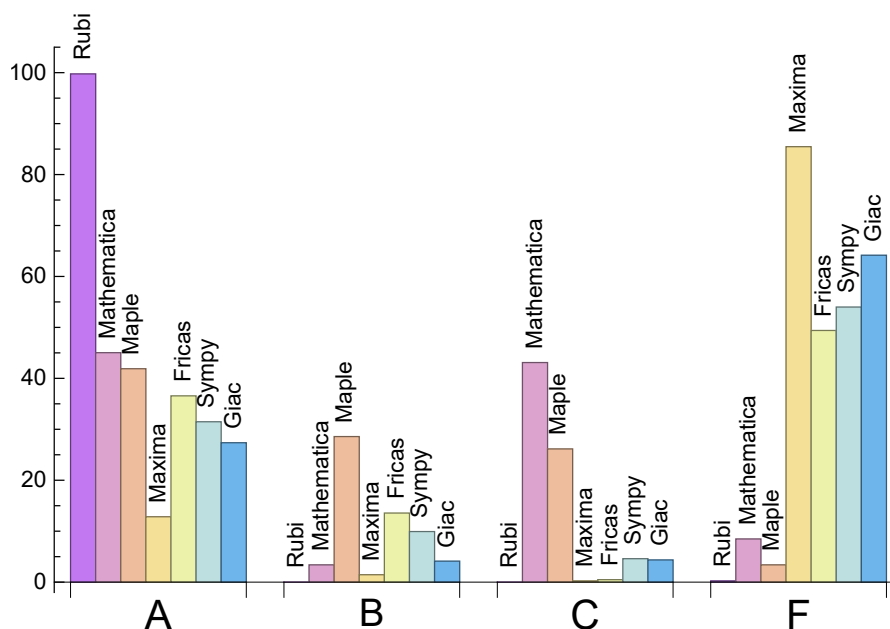
Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

| System | % A grade | % B grade | % C grade | % F grade |
|-------------|-----------|-----------|-----------|-----------|
| Rubi | 99.76 | 0. | 0. | 0.24 |
| Mathematica | 45.04 | 3.39 | 43.1 | 8.47 |
| Maple | 41.89 | 28.57 | 26.15 | 3.39 |
| Maxima | 12.83 | 1.45 | 0.24 | 85.47 |
| Fricas | 36.56 | 13.56 | 0.48 | 49.39 |
| Sympy | 31.48 | 9.93 | 4.6 | 54. |
| Giac | 27.36 | 4.12 | 4.36 | 64.16 |

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



1.3 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

| System | Mean time (sec) | Mean size | Normalized mean | Median size | Normalized median |
|-------------|-----------------|-----------|-----------------|-------------|-------------------|
| Rubi | 0.21 | 154.91 | 1. | 112.5 | 1. |
| Mathematica | 0.33 | 182.54 | 1.33 | 111. | 0.99 |
| Maple | 0.05 | 303.9 | 2.16 | 169. | 1.35 |
| Maxima | 1.1 | 78.73 | 1.4 | 45. | 1.23 |
| Fricas | 4.22 | 1225.75 | 7.02 | 359. | 4.57 |
| Sympy | 6.54 | 162.48 | 1.69 | 83. | 1.1 |
| Giac | 1.19 | 733.36 | 7.89 | 105.5 | 1.31 |

1.4 list of integrals that has no closed form antiderivative

{175, 399, 404, 405}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {98, 99, 113, 114, 115, 116, 118, 198, 223, 224, 400, 401, 402, 403, 408, 409, 410, 411, 412}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime')
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using `sagemath` (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 30 such integrals out of total 705, or about 4 percent. This pecentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. If the output was an exception `ValueError` then this is most likely due to this reason.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See <https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/> for reference.

1.9.2 Important note about FriCAS and Giac/X-CAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error Exception raised: NotImplementedError

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the builtin system function LeafSize is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size is determined as follows.

For Fricas, Giac and Maxima (all called via sagemath) the following code is used

#see <https://stackoverflow.com/questions/25202346/how-to-obtain-leaf-count-expression-size-in->

```
def tree(expr):
    if expr.operator() is None:
        return expr
    else:
        return [expr.operator()+map(tree, expr.operands())
```

```
try:
    # 1.35 is a fudge factor since this estimate of leaf count is bit lower than
    #what it should be compared to Mathematica's
    leafCount = round(1.35*len(flatten(tree(anti))))
except Exception as ee:
    leafCount =1
```

For Sympy, called directly from Python, the following code is used

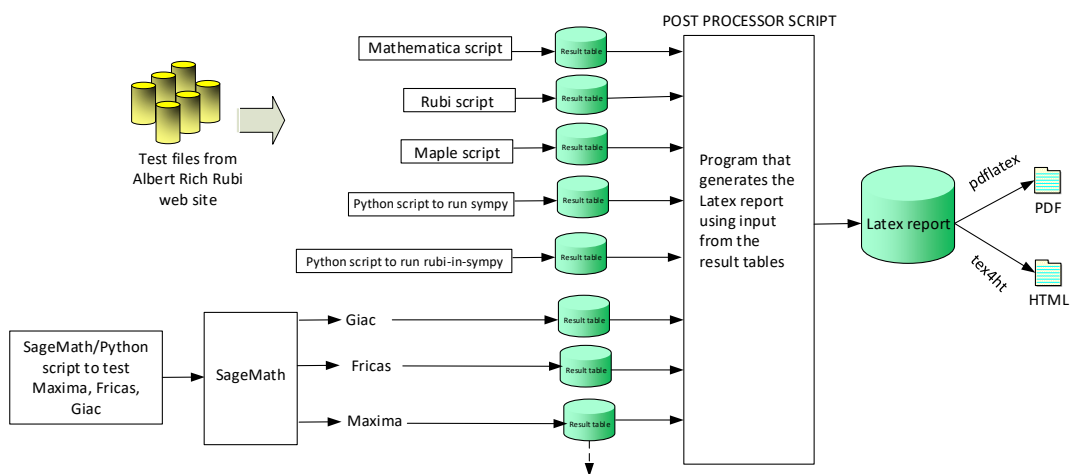
```
try:
    # 1.7 is a fudge factor since it is low side from actual leaf count
    leafCount = round(1.7*count_ops(anti))

except Exception as ee:
    leafCount =1
```

When these cas systems have a builtin function to find the leaf size of expressions, it will be used instead, and these tests run again.

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. It contains 13 fields. This is description of each record (line)

1. integer, the problem number.
2. integer. 0 or 1 for failed or passed. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be "A", "B", "C", or "F"
13. String. The optimal antiderivative in CAS own syntax.

High level overview of the CAS independent integration test build system

Naser M. Abbasi
June 22, 2018

Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413 }

B grade: { }

C grade: { }

F grade: { 174 }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 22, 24, 34, 35, 36, 37, 41, 42, 43, 47, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 66, 67, 68, 69, 70, 71, 72, 74, 75, 76, 77, 78, 79, 81, 82, 83, 84, 85, 86, 87, 89, 90, 91, 94, 95, 96, 97, 104, 105, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 170, 175, 176, 177, 178, 179, 182, 183, 184, 185, 189, 190, 191, 192, 193, 194, 195, 196, 197, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 399, 400, 401, 402, 403, 404, 405 }

B grade: { 26, 27, 28, 29, 30, 31, 32, 33, 38, 39, 40, 65, 80, 88 }

C grade: { 14, 15, 16, 17, 18, 19, 20, 21, 23, 25, 44, 45, 46, 48, 49, 73, 92, 93, 98, 99, 100, 101, 102, 103, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 171, 172, 173, 198, 199, 200, 201, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 241, 242, 243, 286, 287, 288, 289, 290, 291, 292, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 406, 407, 408, 409, 410, 411, 412, 413 }

F grade: { 174, 180, 181, 186, 187, 188, 238, 239, 240, 293, 294, 295, 307, 308, 309, 310, 311, 324, 325, 326, 327, 339, 340, 341, 342, 343, 344, 356, 357, 358, 359, 372, 373, 374, 375 }

2.1.3 Maple

A grade: { 1, 2, 7, 8, 13, 17, 26, 27, 28, 29, 30, 31, 32, 33, 37, 38, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 72, 73, 75, 76, 77, 84, 85, 86, 87, 88, 89, 90, 92, 93, 94, 95, 98, 99, 102, 103, 106, 107, 108, 109, 110, 111, 112, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 138, 139, 140, 141, 142, 143, 145, 146, 147, 148, 149, 158, 159, 160, 164, 165, 175, 182, 183, 184, 185, 189, 190, 191, 192, 193, 194, 199, 200, 201, 205, 206, 207, 211, 212, 213, 215, 216, 217, 218, 219, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 260, 261, 262, 267, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 316, 323, 324, 325, 331, 338, 347, 382, 383, 384, 386, 387, 388, 390, 391, 392, 393, 399, 404, 405, 407, 412, 413 }

B grade: { 3, 4, 5, 6, 9, 10, 11, 12, 14, 15, 16, 23, 25, 34, 35, 36, 39, 40, 41, 53, 54, 55, 67, 68, 69, 70, 71, 74, 78, 79, 80, 81, 82, 83, 91, 96, 97, 100, 101, 104, 105, 113, 114, 115, 116, 117, 118, 137, 144, 157, 161, 162, 163, 166, 167, 170, 172, 195, 196, 197, 198, 202, 203, 204, 208, 209, 210, 214, 220, 221, 222, 223, 224, 256, 257, 258, 259, 263, 264, 265, 266, 268, 269, 270, 271, 272, 273, 274, 275, 317, 318, 319, 320, 321, 322, 326, 327, 328, 329, 330, 332, 333, 334, 335, 336, 337, 339, 340, 341, 342, 343, 344, 345, 346, 380, 381, 385, 389 }

C grade: { 18, 19, 20, 21, 22, 24, 150, 151, 152, 153, 154, 155, 156, 168, 169, 171, 173, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 394, 395, 396, 397, 398, 406, 408, 409, 410, 411 }

F grade: { 174, 176, 177, 178, 179, 180, 181, 186, 187, 188, 400, 401, 402, 403 }

2.1.4 Maxima

A grade: { 5, 6, 8, 13, 37, 42, 43, 44, 47, 51, 52, 56, 57, 58, 61, 66, 72, 73, 74, 76, 77, 84, 85, 86, 89, 92, 93, 94, 96, 98, 99, 120, 121, 122, 123, 128, 129, 130, 131, 175, 244, 245, 246, 247, 252, 253, 254, 255, 284, 285, 399, 404, 405 }

B grade: { 7, 65, 88, 95, 282, 283 }

C grade: { 108 }

F grade: { 1, 2, 3, 4, 9, 10, 11, 12, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 38, 39, 40, 41, 45, 46, 48, 49, 50, 53, 54, 55, 59, 60, 62, 63, 64, 67, 68, 69, 70, 71, 75, 78, 79, 80, 81, 82, 83, 87, 90, 91, 97, 100, 101, 102, 103, 104, 105, 106, 107, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 124, 125, 126, 127, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 248, 249, 250, 251, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322,

323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 400, 401, 402, 403, 406, 407, 408, 409, 410, 411, 412, 413 }

2.1.5 FriCAS

A grade: { 5, 6, 8, 9, 10, 11, 12, 13, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 89, 90, 91, 92, 93, 98, 99, 108, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 141, 147, 175, 189, 190, 191, 192, 193, 195, 196, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 215, 216, 217, 218, 220, 221, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 261, 262, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 399, 404, 405 }

B grade: { 1, 2, 3, 4, 7, 65, 88, 94, 95, 96, 97, 100, 101, 104, 105, 106, 107, 109, 110, 111, 112, 137, 138, 139, 140, 142, 143, 144, 145, 146, 148, 194, 197, 198, 211, 212, 213, 214, 219, 222, 223, 224, 258, 259, 260, 264, 265, 266, 267, 271, 272, 273, 408, 409, 410, 411 }

C grade: { 102, 103 }

F grade: { 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 113, 114, 115, 116, 117, 118, 119, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 263, 268, 269, 270, 274, 275, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 400, 401, 402, 403, 406, 407, 412, 413 }

2.1.6 Sympy

A grade: { 1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 13, 16, 17, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 39, 40, 41, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 89, 90, 91, 92, 93, 94, 97, 100, 101, 106, 109, 120, 121, 122, 123, 127, 128, 129, 130, 131, 134, 135, 136, 137, 138, 139, 140, 141, 144, 145, 146, 147, 157, 158, 159, 164, 166, 189, 190, 191, 244, 245, 246, 247, 250, 251, 252, 253, 254, 255, 258, 259, 266, 267, 273, 279, 280 }

B grade: { 7, 14, 15, 23, 25, 38, 42, 65, 88, 95, 96, 104, 105, 124, 125, 126, 132, 133, 167, 192, 193, 194, 214, 215, 216, 217, 218, 248, 249, 256, 257, 261, 262, 265, 272, 276, 277, 278, 281, 282, 283 }

C grade: { 18, 19, 20, 21, 98, 99, 108, 150, 151, 152, 153, 168, 177, 178, 179, 182, 183, 184, 185 }

F grade: { 22, 24, 102, 103, 107, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 142, 143, 148, 149, 154, 155, 156, 160, 161, 162, 163, 165, 169, 170, 171, 172, 173, 174, 175, 176, 180, 181, 186, 187, 188, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 260, 263, 264, 268, 269, 270, 271, 274, 275, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413 }

2.1.7 Giac

A grade: { 1, 2, 5, 6, 8, 37, 38, 41, 42, 43, 44, 51, 52, 55, 56, 57, 58, 61, 66, 67, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 81, 82, 83, 84, 85, 86, 87, 89, 90, 91, 92, 93, 94, 97, 98, 99, 108, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 175, 195, 197, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 399, 404, 405 }

B grade: { 3, 4, 7, 11, 12, 13, 47, 53, 65, 88, 95, 96, 189, 190, 191, 192, 196 }

C grade: { 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 39, 40, 54, 104, 105, 266, 267 }

F grade: { 9, 10, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 45, 46, 48, 49, 50, 59, 60, 62, 63, 64, 68, 80, 100, 101, 102, 103, 106, 107, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 193, 194, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 263, 264, 265, 268, 269, 270, 271, 272, 273, 274, 275, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 400, 401, 402, 403, 406, 407, 408, 409, 410, 411, 412, 413 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table, the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

| Problem 1 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|-------|
| grade | A | A | A | A | F(-2) | B | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 247 | 247 | 183 | 260 | 0 | 1544 | 109 | 325 |
| normalized size | 1 | 1. | 0.74 | 1.05 | 0. | 6.25 | 0.44 | 1.32 |
| time (sec) | N/A | 0.151 | 0.075 | 0.044 | 0. | 1.782 | 0.598 | 1.184 |

| Problem 2 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|-------|
| grade | A | A | A | A | F(-2) | B | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 247 | 247 | 184 | 260 | 0 | 1544 | 110 | 325 |
| normalized size | 1 | 1. | 0.74 | 1.05 | 0. | 6.25 | 0.45 | 1.32 |
| time (sec) | N/A | 0.138 | 0.044 | 0.045 | 0. | 1.647 | 0.764 | 1.153 |

| Problem 3 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|-------|
| grade | A | A | A | B | F(-2) | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 86 | 86 | 95 | 122 | 0 | 1527 | 110 | 347 |
| normalized size | 1 | 1. | 1.1 | 1.42 | 0. | 17.76 | 1.28 | 4.03 |
| time (sec) | N/A | 0.045 | 0.028 | 0.044 | 0. | 1.497 | 0.631 | 1.136 |

| Problem 4 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|------|-------------|-------|--------|--------|-------|-------|
| grade | A | A | A | B | F(-2) | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 86 | 86 | 95 | 122 | 0 | 1527 | 110 | 347 |
| normalized size | 1 | 1. | 1.1 | 1.42 | 0. | 17.76 | 1.28 | 4.03 |
| time (sec) | N/A | 0.04 | 0.022 | 0.043 | 0. | 1.44 | 0.79 | 1.138 |

| Problem 5 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|------|-------------|-------|--------|--------|-------|-------|
| grade | A | A | A | B | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 40 | 40 | 33 | 122 | 53 | 112 | 41 | 70 |
| normalized size | 1 | 1. | 0.82 | 3.05 | 1.32 | 2.8 | 1.02 | 1.75 |
| time (sec) | N/A | 0.02 | 0.012 | 0.045 | 1.498 | 1.309 | 0.106 | 1.145 |

| Problem 6 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|-------|
| grade | A | A | A | B | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 51 | 51 | 44 | 82 | 53 | 105 | 49 | 54 |
| normalized size | 1 | 1. | 0.86 | 1.61 | 1.04 | 2.06 | 0.96 | 1.06 |
| time (sec) | N/A | 0.021 | 0.013 | 0.043 | 1.493 | 1.439 | 0.099 | 1.137 |

| Problem 7 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|-------|
| grade | A | A | A | A | B | B | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 16 | 16 | 32 | 13 | 34 | 77 | 32 | 39 |
| normalized size | 1 | 1. | 2. | 0.81 | 2.12 | 4.81 | 2. | 2.44 |
| time (sec) | N/A | 0.003 | 0.014 | 0.043 | 1.493 | 1.405 | 0.092 | 1.156 |

| Problem 8 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|-------|
| grade | A | A | A | A | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 16 | 16 | 16 | 13 | 16 | 47 | 15 | 16 |
| normalized size | 1 | 1. | 1. | 0.81 | 1. | 2.94 | 0.94 | 1. |
| time (sec) | N/A | 0.003 | 0.005 | 0.041 | 1.429 | 1.257 | 0.094 | 1.125 |

| Problem 9 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|-------|
| grade | A | A | A | B | F(-2) | A | A | F(-2) |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 75 | 75 | 60 | 254 | 0 | 458 | 138 | 0 |
| normalized size | 1 | 1. | 0.8 | 3.39 | 0. | 6.11 | 1.84 | 0. |
| time (sec) | N/A | 0.037 | 0.019 | 0.047 | 0. | 1.506 | 0.525 | 0. |

| Problem 10 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|-------|
| grade | A | A | A | B | F(-2) | A | A | F(-2) |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 106 | 106 | 91 | 254 | 0 | 462 | 131 | 0 |
| normalized size | 1 | 1. | 0.86 | 2.4 | 0. | 4.36 | 1.24 | 0. |
| time (sec) | N/A | 0.047 | 0.022 | 0.048 | 0. | 1.487 | 0.534 | 0. |

| Problem 11 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|------|-------------|-------|--------|--------|-------|-------|
| grade | A | A | A | B | F(-2) | A | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 75 | 75 | 60 | 290 | 0 | 343 | 87 | 300 |
| normalized size | 1 | 1. | 0.8 | 3.87 | 0. | 4.57 | 1.16 | 4. |
| time (sec) | N/A | 0.05 | 0.032 | 0.07 | 0. | 1.363 | 0.176 | 1.168 |

| Problem 12 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|-------|
| grade | A | A | A | B | F(-2) | A | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 90 | 90 | 75 | 290 | 0 | 346 | 80 | 300 |
| normalized size | 1 | 1. | 0.83 | 3.22 | 0. | 3.84 | 0.89 | 3.33 |
| time (sec) | N/A | 0.047 | 0.022 | 0.045 | 0. | 1.293 | 0.331 | 1.158 |

| Problem 13 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|-------|
| grade | A | A | A | A | A | A | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 13 | 13 | 25 | 18 | 23 | 68 | 22 | 26 |
| normalized size | 1 | 1. | 1.92 | 1.38 | 1.77 | 5.23 | 1.69 | 2. |
| time (sec) | N/A | 0.006 | 0.006 | 0.044 | 1.486 | 1.35 | 0.133 | 1.128 |

| Problem 14 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | C | B | F | F | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 16 | 16 | 45 | 100 | 0 | 0 | 70 | 0 |
| normalized size | 1 | 1. | 2.81 | 6.25 | 0. | 0. | 4.38 | 0. |
| time (sec) | N/A | 0.016 | 0.012 | 0.052 | 0. | 0. | 1.529 | 0. |

| Problem 15 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | C | B | F | F | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 35 | 35 | 45 | 99 | 0 | 0 | 70 | 0 |
| normalized size | 1 | 1. | 1.29 | 2.83 | 0. | 0. | 2. | 0. |
| time (sec) | N/A | 0.033 | 0.012 | 0.048 | 0. | 0. | 1.878 | 0. |

| Problem 16 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | C | B | F | F | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 43 | 43 | 74 | 107 | 0 | 0 | 61 | 0 |
| normalized size | 1 | 1. | 1.72 | 2.49 | 0. | 0. | 1.42 | 0. |
| time (sec) | N/A | 0.025 | 0.023 | 0.054 | 0. | 0. | 1.505 | 0. |

| Problem 17 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | C | A | F | F | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 89 | 89 | 74 | 108 | 0 | 0 | 60 | 0 |
| normalized size | 1 | 1. | 0.83 | 1.21 | 0. | 0. | 0.67 | 0. |
| time (sec) | N/A | 0.046 | 0.021 | 0.046 | 0. | 0. | 1.867 | 0. |

| Problem 18 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | C | C | F | F | C | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 89 | 89 | 47 | 120 | 0 | 0 | 66 | 0 |
| normalized size | 1 | 1. | 0.53 | 1.35 | 0. | 0. | 0.74 | 0. |
| time (sec) | N/A | 0.014 | 0.012 | 0.153 | 0. | 0. | 1.839 | 0. |

| Problem 19 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | C | C | F | F | C | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 152 | 152 | 47 | 120 | 0 | 0 | 66 | 0 |
| normalized size | 1 | 1. | 0.31 | 0.79 | 0. | 0. | 0.43 | 0. |
| time (sec) | N/A | 0.031 | 0.01 | 0.046 | 0. | 0. | 1.495 | 0. |

| Problem 20 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | C | C | F | F | C | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 90 | 90 | 76 | 122 | 0 | 0 | 70 | 0 |
| normalized size | 1 | 1. | 0.84 | 1.36 | 0. | 0. | 0.78 | 0. |
| time (sec) | N/A | 0.015 | 0.026 | 0.131 | 0. | 0. | 1.906 | 0. |

| Problem 21 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | C | C | F | F | C | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 156 | 156 | 76 | 122 | 0 | 0 | 71 | 0 |
| normalized size | 1 | 1. | 0.49 | 0.78 | 0. | 0. | 0.46 | 0. |
| time (sec) | N/A | 0.032 | 0.019 | 0.046 | 0. | 0. | 1.52 | 0. |

| Problem 22 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | A | C | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 10 | 10 | 10 | 15 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 1. | 1.5 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.009 | 0.007 | 0.063 | 0. | 0. | 0. | 0. |

| Problem 23 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | C | B | F | F | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 10 | 10 | 47 | 118 | 0 | 0 | 71 | 0 |
| normalized size | 1 | 1. | 4.7 | 11.8 | 0. | 0. | 7.1 | 0. |
| time (sec) | N/A | 0.016 | 0.013 | 0.052 | 0. | 0. | 1.549 | 0. |

| Problem 24 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | A | C | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 23 | 23 | 24 | 28 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 1.04 | 1.22 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.026 | 0.008 | 0.049 | 0. | 0. | 0. | 0. |

| Problem 25 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | C | B | F | F | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 23 | 23 | 47 | 117 | 0 | 0 | 71 | 0 |
| normalized size | 1 | 1. | 2.04 | 5.09 | 0. | 0. | 3.09 | 0. |
| time (sec) | N/A | 0.033 | 0.012 | 0.048 | 0. | 0. | 1.934 | 0. |

| Problem 26 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|------|-------------|-------|--------|--------|-------|-------|
| grade | A | A | B | A | F | A | A | C |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 82 | 82 | 181 | 71 | 0 | 375 | 122 | 5779 |
| normalized size | 1 | 1. | 2.21 | 0.87 | 0. | 4.57 | 1.49 | 70.48 |
| time (sec) | N/A | 0.1 | 0.109 | 0.201 | 0. | 1.373 | 0.422 | 1.621 |

| Problem 27 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|------|-------------|-------|--------|--------|-------|-------|
| grade | A | A | B | A | F | A | A | C |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 82 | 82 | 181 | 71 | 0 | 375 | 122 | 5779 |
| normalized size | 1 | 1. | 2.21 | 0.87 | 0. | 4.57 | 1.49 | 70.48 |
| time (sec) | N/A | 0.11 | 0.111 | 0.204 | 0. | 1.349 | 0.422 | 1.61 |

| Problem 28 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|-------|
| grade | A | A | B | A | F | A | A | C |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 78 | 78 | 189 | 75 | 0 | 375 | 110 | 5387 |
| normalized size | 1 | 1. | 2.42 | 0.96 | 0. | 4.81 | 1.41 | 69.06 |
| time (sec) | N/A | 0.098 | 0.109 | 0.179 | 0. | 1.348 | 0.455 | 1.624 |

| Problem 29 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|-------|
| grade | A | A | B | A | F | A | A | C |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 86 | 86 | 189 | 75 | 0 | 379 | 121 | 5387 |
| normalized size | 1 | 1. | 2.2 | 0.87 | 0. | 4.41 | 1.41 | 62.64 |
| time (sec) | N/A | 0.104 | 0.107 | 0.175 | 0. | 1.39 | 0.424 | 1.598 |

| Problem 30 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|-------|
| grade | A | A | B | A | F | A | A | C |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 78 | 78 | 182 | 88 | 0 | 378 | 121 | 5779 |
| normalized size | 1 | 1. | 2.33 | 1.13 | 0. | 4.85 | 1.55 | 74.09 |
| time (sec) | N/A | 0.052 | 0.12 | 0.173 | 0. | 1.328 | 0.575 | 1.64 |

| Problem 31 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|-------|
| grade | A | A | B | A | F | A | A | C |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 78 | 78 | 182 | 69 | 0 | 379 | 110 | 5779 |
| normalized size | 1 | 1. | 2.33 | 0.88 | 0. | 4.86 | 1.41 | 74.09 |
| time (sec) | N/A | 0.048 | 0.129 | 0.173 | 0. | 1.285 | 0.599 | 1.634 |

| Problem 32 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|-------|
| grade | A | A | B | A | F | A | A | C |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 70 | 70 | 190 | 61 | 0 | 378 | 112 | 5387 |
| normalized size | 1 | 1. | 2.71 | 0.87 | 0. | 5.4 | 1.6 | 76.96 |
| time (sec) | N/A | 0.044 | 0.134 | 0.18 | 0. | 1.409 | 0.606 | 1.609 |

| Problem 33 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|-------|
| grade | A | A | B | A | F | A | A | C |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 70 | 70 | 190 | 61 | 0 | 378 | 112 | 5387 |
| normalized size | 1 | 1. | 2.71 | 0.87 | 0. | 5.4 | 1.6 | 76.96 |
| time (sec) | N/A | 0.047 | 0.133 | 0.171 | 0. | 1.289 | 0.6 | 1.584 |

| Problem 34 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|-------|
| grade | A | A | A | B | F | A | A | C |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 134 | 134 | 250 | 582 | 0 | 494 | 158 | 6884 |
| normalized size | 1 | 1. | 1.87 | 4.34 | 0. | 3.69 | 1.18 | 51.37 |
| time (sec) | N/A | 0.101 | 0.164 | 0.275 | 0. | 1.406 | 0.796 | 2.444 |

| Problem 35 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|-------|
| grade | A | A | A | B | F | A | A | C |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 130 | 130 | 248 | 582 | 0 | 490 | 160 | 6884 |
| normalized size | 1 | 1. | 1.91 | 4.48 | 0. | 3.77 | 1.23 | 52.95 |
| time (sec) | N/A | 0.166 | 0.122 | 0.223 | 0. | 1.323 | 0.777 | 2.415 |

| Problem 36 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|-------|
| grade | A | A | A | B | F | A | A | C |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 130 | 130 | 248 | 582 | 0 | 490 | 160 | 6884 |
| normalized size | 1 | 1. | 1.91 | 4.48 | 0. | 3.77 | 1.23 | 52.95 |
| time (sec) | N/A | 0.131 | 0.044 | 0.212 | 0. | 1.351 | 0.784 | 2.442 |

| Problem 37 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|-------|
| grade | A | A | A | A | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 29 | 29 | 29 | 26 | 34 | 66 | 26 | 34 |
| normalized size | 1 | 1. | 1. | 0.9 | 1.17 | 2.28 | 0.9 | 1.17 |
| time (sec) | N/A | 0.026 | 0.018 | 0.049 | 0.962 | 1.282 | 0.506 | 1.227 |

| Problem 38 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|-------|
| grade | A | A | B | A | F(-2) | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 60 | 60 | 138 | 52 | 0 | 389 | 117 | 69 |
| normalized size | 1 | 1. | 2.3 | 0.87 | 0. | 6.48 | 1.95 | 1.15 |
| time (sec) | N/A | 0.069 | 0.2 | 0.123 | 0. | 1.307 | 0.38 | 1.263 |

| Problem 39 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|-------|
| grade | A | A | B | B | F | A | A | C |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 62 | 62 | 126 | 277 | 0 | 286 | 95 | 3352 |
| normalized size | 1 | 1. | 2.03 | 4.47 | 0. | 4.61 | 1.53 | 54.06 |
| time (sec) | N/A | 0.058 | 0.059 | 0.137 | 0. | 1.353 | 0.27 | 1.448 |

| Problem 40 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|-------|
| grade | A | A | B | B | F | A | A | C |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 66 | 66 | 134 | 277 | 0 | 298 | 83 | 3069 |
| normalized size | 1 | 1. | 2.03 | 4.2 | 0. | 4.52 | 1.26 | 46.5 |
| time (sec) | N/A | 0.058 | 0.058 | 0.106 | 0. | 1.269 | 0.286 | 1.272 |

| Problem 41 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|-------|
| grade | A | A | A | B | F | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 45 | 45 | 83 | 136 | 0 | 117 | 42 | 53 |
| normalized size | 1 | 1. | 1.84 | 3.02 | 0. | 2.6 | 0.93 | 1.18 |
| time (sec) | N/A | 0.059 | 0.074 | 0.075 | 0. | 1.316 | 0.115 | 1.176 |

| Problem 42 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|-------|
| grade | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 15 | 15 | 17 | 12 | 15 | 66 | 22 | 15 |
| normalized size | 1 | 1. | 1.13 | 0.8 | 1. | 4.4 | 1.47 | 1. |
| time (sec) | N/A | 0.009 | 0.008 | 0.05 | 1.457 | 1.275 | 0.104 | 1.136 |

| Problem 43 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|-------|
| grade | A | A | A | A | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 14 | 14 | 14 | 12 | 15 | 42 | 14 | 15 |
| normalized size | 1 | 1. | 1. | 0.86 | 1.07 | 3. | 1. | 1.07 |
| time (sec) | N/A | 0.007 | 0.004 | 0.041 | 1.445 | 1.358 | 0.09 | 1.135 |

| Problem 44 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|-------|
| grade | A | A | C | A | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 38 | 38 | 97 | 34 | 45 | 112 | 44 | 45 |
| normalized size | 1 | 1. | 2.55 | 0.89 | 1.18 | 2.95 | 1.16 | 1.18 |
| time (sec) | N/A | 0.035 | 0.185 | 0.043 | 1.44 | 1.307 | 0.115 | 1.136 |

| Problem 45 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|------|-------------|-------|--------|--------|-------|------|
| grade | A | A | C | A | F | A | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 48 | 48 | 99 | 40 | 0 | 107 | 42 | 0 |
| normalized size | 1 | 1. | 2.06 | 0.83 | 0. | 2.23 | 0.88 | 0. |
| time (sec) | N/A | 0.04 | 0.104 | 0.06 | 0. | 1.24 | 0.111 | 0. |

| Problem 46 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | C | A | F | A | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 46 | 46 | 97 | 40 | 0 | 112 | 44 | 0 |
| normalized size | 1 | 1. | 2.11 | 0.87 | 0. | 2.43 | 0.96 | 0. |
| time (sec) | N/A | 0.043 | 0.225 | 0.061 | 0. | 1.343 | 0.113 | 0. |

| Problem 47 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | A | A | A | A | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 21 | 21 | 17 | 18 | 23 | 53 | 14 | 62 |
| normalized size | 1 | 1. | 0.81 | 0.86 | 1.1 | 2.52 | 0.67 | 2.95 |
| time (sec) | N/A | 0.013 | 0.006 | 0.043 | 1.459 | 1.306 | 0.096 | 1.12 |

| Problem 48 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | C | A | F | A | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 46 | 46 | 101 | 40 | 0 | 109 | 42 | 0 |
| normalized size | 1 | 1. | 2.2 | 0.87 | 0. | 2.37 | 0.91 | 0. |
| time (sec) | N/A | 0.041 | 0.278 | 0.062 | 0. | 1.399 | 0.116 | 0. |

| Problem 49 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | C | A | F | A | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 44 | 44 | 99 | 40 | 0 | 90 | 29 | 0 |
| normalized size | 1 | 1. | 2.25 | 0.91 | 0. | 2.05 | 0.66 | 0. |
| time (sec) | N/A | 0.034 | 0.103 | 0.063 | 0. | 1.326 | 0.107 | 0. |

| Problem 50 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | A | A | F | A | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 23 | 23 | 14 | 20 | 0 | 45 | 12 | 0 |
| normalized size | 1 | 1. | 0.61 | 0.87 | 0. | 1.96 | 0.52 | 0. |
| time (sec) | N/A | 0.026 | 0.007 | 0.061 | 0. | 1.339 | 0.1 | 0. |

| Problem 51 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|-------|
| grade | A | A | A | A | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 11 | 11 | 12 | 11 | 16 | 22 | 8 | 16 |
| normalized size | 1 | 1. | 1.09 | 1. | 1.45 | 2. | 0.73 | 1.45 |
| time (sec) | N/A | 0.005 | 0.006 | 0.046 | 0.956 | 1.315 | 0.081 | 1.131 |

| Problem 52 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|-------|
| grade | A | A | A | A | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 39 | 39 | 29 | 30 | 39 | 68 | 26 | 45 |
| normalized size | 1 | 1. | 0.74 | 0.77 | 1. | 1.74 | 0.67 | 1.15 |
| time (sec) | N/A | 0.018 | 0.006 | 0.047 | 0.968 | 1.318 | 0.098 | 1.152 |

| Problem 53 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|-------|
| grade | A | A | A | B | F | A | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 44 | 44 | 42 | 82 | 0 | 111 | 46 | 104 |
| normalized size | 1 | 1. | 0.95 | 1.86 | 0. | 2.52 | 1.05 | 2.36 |
| time (sec) | N/A | 0.035 | 0.013 | 0.073 | 0. | 1.38 | 0.102 | 1.294 |

| Problem 54 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|-------|
| grade | A | A | A | B | F | A | A | C |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 66 | 66 | 127 | 279 | 0 | 286 | 94 | 3352 |
| normalized size | 1 | 1. | 1.92 | 4.23 | 0. | 4.33 | 1.42 | 50.79 |
| time (sec) | N/A | 0.029 | 0.07 | 0.102 | 0. | 1.305 | 0.267 | 1.296 |

| Problem 55 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|-------|
| grade | A | A | A | B | F | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 46 | 46 | 84 | 136 | 0 | 99 | 39 | 53 |
| normalized size | 1 | 1. | 1.83 | 2.96 | 0. | 2.15 | 0.85 | 1.15 |
| time (sec) | N/A | 0.031 | 0.071 | 0.054 | 0. | 1.387 | 0.114 | 1.168 |

| Problem 56 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|-------|
| grade | A | A | A | A | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 9 | 9 | 12 | 10 | 12 | 47 | 14 | 12 |
| normalized size | 1 | 1. | 1.33 | 1.11 | 1.33 | 5.22 | 1.56 | 1.33 |
| time (sec) | N/A | 0.009 | 0.008 | 0.048 | 1.443 | 1.313 | 0.102 | 1.117 |

| Problem 57 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|-------|
| grade | A | A | A | A | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 11 | 11 | 11 | 11 | 15 | 20 | 7 | 15 |
| normalized size | 1 | 1. | 1. | 1. | 1.36 | 1.82 | 0.64 | 1.36 |
| time (sec) | N/A | 0.005 | 0.005 | 0.046 | 0.975 | 1.238 | 0.085 | 1.142 |

| Problem 58 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|-------|
| grade | A | A | A | A | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 29 | 29 | 29 | 26 | 34 | 66 | 26 | 34 |
| normalized size | 1 | 1. | 1. | 0.9 | 1.17 | 2.28 | 0.9 | 1.17 |
| time (sec) | N/A | 0.016 | 0.006 | 0.045 | 0.964 | 1.291 | 0.1 | 1.126 |

| Problem 59 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | A | A | F | A | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 50 | 50 | 42 | 39 | 0 | 111 | 46 | 0 |
| normalized size | 1 | 1. | 0.84 | 0.78 | 0. | 2.22 | 0.92 | 0. |
| time (sec) | N/A | 0.023 | 0.013 | 0.05 | 0. | 1.351 | 0.101 | 0. |

| Problem 60 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | A | A | F | A | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 50 | 50 | 42 | 39 | 0 | 108 | 46 | 0 |
| normalized size | 1 | 1. | 0.84 | 0.78 | 0. | 2.16 | 0.92 | 0. |
| time (sec) | N/A | 0.023 | 0.013 | 0.051 | 0. | 1.266 | 0.101 | 0. |

| Problem 61 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|-------|
| grade | A | A | A | A | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 31 | 31 | 31 | 28 | 36 | 72 | 22 | 46 |
| normalized size | 1 | 1. | 1. | 0.9 | 1.16 | 2.32 | 0.71 | 1.48 |
| time (sec) | N/A | 0.015 | 0.005 | 0.046 | 0.978 | 1.395 | 0.099 | 1.106 |

| Problem 62 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | A | A | F | A | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 50 | 50 | 42 | 39 | 0 | 109 | 46 | 0 |
| normalized size | 1 | 1. | 0.84 | 0.78 | 0. | 2.18 | 0.92 | 0. |
| time (sec) | N/A | 0.023 | 0.014 | 0.052 | 0. | 1.422 | 0.102 | 0. |

| Problem 63 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | A | A | F | A | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 50 | 50 | 42 | 39 | 0 | 113 | 46 | 0 |
| normalized size | 1 | 1. | 0.84 | 0.78 | 0. | 2.26 | 0.92 | 0. |
| time (sec) | N/A | 0.024 | 0.019 | 0.052 | 0. | 1.38 | 0.1 | 0. |

| Problem 64 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | A | A | F | A | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 50 | 50 | 42 | 39 | 0 | 113 | 46 | 0 |
| normalized size | 1 | 1. | 0.84 | 0.78 | 0. | 2.26 | 0.92 | 0. |
| time (sec) | N/A | 0.022 | 0.015 | 0.051 | 0. | 1.358 | 0.103 | 0. |

| Problem 65 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|-------|
| grade | A | A | B | A | B | B | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 14 | 14 | 32 | 12 | 34 | 76 | 32 | 39 |
| normalized size | 1 | 1. | 2.29 | 0.86 | 2.43 | 5.43 | 2.29 | 2.79 |
| time (sec) | N/A | 0.006 | 0.007 | 0.04 | 1.493 | 1.282 | 0.092 | 1.136 |

| Problem 66 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|-------|
| grade | A | A | A | A | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 39 | 39 | 31 | 30 | 39 | 72 | 29 | 45 |
| normalized size | 1 | 1. | 0.79 | 0.77 | 1. | 1.85 | 0.74 | 1.15 |
| time (sec) | N/A | 0.017 | 0.006 | 0.052 | 0.961 | 1.372 | 0.102 | 1.136 |

| Problem 67 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|-------|
| grade | A | A | A | B | F | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 48 | 48 | 42 | 82 | 0 | 116 | 46 | 104 |
| normalized size | 1 | 1. | 0.88 | 1.71 | 0. | 2.42 | 0.96 | 2.17 |
| time (sec) | N/A | 0.039 | 0.019 | 0.055 | 0. | 1.274 | 0.104 | 1.183 |

| Problem 68 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|-------|
| grade | A | A | A | B | F | A | A | F(-2) |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 62 | 62 | 124 | 277 | 0 | 273 | 88 | 0 |
| normalized size | 1 | 1. | 2. | 4.47 | 0. | 4.4 | 1.42 | 0. |
| time (sec) | N/A | 0.056 | 0.056 | 0.138 | 0. | 1.328 | 0.258 | 0. |

| Problem 69 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|-------|
| grade | A | A | A | B | F | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 49 | 49 | 83 | 136 | 0 | 109 | 41 | 35 |
| normalized size | 1 | 1. | 1.69 | 2.78 | 0. | 2.22 | 0.84 | 0.71 |
| time (sec) | N/A | 0.088 | 0.137 | 0.079 | 0. | 1.311 | 0.108 | 1.137 |

| Problem 70 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|------|-------------|-------|--------|--------|-------|-------|
| grade | A | A | A | B | F | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 43 | 43 | 81 | 110 | 0 | 109 | 41 | 35 |
| normalized size | 1 | 1. | 1.88 | 2.56 | 0. | 2.53 | 0.95 | 0.81 |
| time (sec) | N/A | 0.05 | 0.067 | 0.065 | 0. | 1.348 | 0.108 | 1.115 |

| Problem 71 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|-------|
| grade | A | A | A | B | F | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 49 | 49 | 83 | 104 | 0 | 109 | 41 | 35 |
| normalized size | 1 | 1. | 1.69 | 2.12 | 0. | 2.22 | 0.84 | 0.71 |
| time (sec) | N/A | 0.062 | 0.096 | 0.069 | 0. | 1.327 | 0.109 | 1.136 |

| Problem 72 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|-------|
| grade | A | A | A | A | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 2 | 2 | 2 | 3 | 3 | 15 | 2 | 3 |
| normalized size | 1 | 1. | 1. | 1.5 | 1.5 | 7.5 | 1. | 1.5 |
| time (sec) | N/A | 0.002 | 0.003 | 0.044 | 1.435 | 1.335 | 0.083 | 1.105 |

| Problem 73 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|-------|
| grade | A | A | C | A | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 38 | 38 | 99 | 34 | 45 | 109 | 41 | 35 |
| normalized size | 1 | 1. | 2.61 | 0.89 | 1.18 | 2.87 | 1.08 | 0.92 |
| time (sec) | N/A | 0.027 | 0.195 | 0.045 | 1.488 | 1.283 | 0.105 | 1.124 |

| Problem 74 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|-------|
| grade | A | A | A | B | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 35 | 35 | 30 | 88 | 53 | 107 | 39 | 53 |
| normalized size | 1 | 1. | 0.86 | 2.51 | 1.51 | 3.06 | 1.11 | 1.51 |
| time (sec) | N/A | 0.018 | 0.013 | 0.044 | 1.453 | 1.375 | 0.101 | 1.137 |

| Problem 75 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|------|-------------|-------|--------|--------|-------|-------|
| grade | A | A | A | A | F | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 23 | 23 | 12 | 20 | 0 | 34 | 7 | 41 |
| normalized size | 1 | 1. | 0.52 | 0.87 | 0. | 1.48 | 0.3 | 1.78 |
| time (sec) | N/A | 0.02 | 0.007 | 0.055 | 0. | 1.355 | 0.102 | 1.133 |

| Problem 76 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|-------|
| grade | A | A | A | A | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 11 | 11 | 10 | 16 | 14 | 19 | 7 | 15 |
| normalized size | 1 | 1. | 0.91 | 1.45 | 1.27 | 1.73 | 0.64 | 1.36 |
| time (sec) | N/A | 0.003 | 0.004 | 0.043 | 1.008 | 1.275 | 0.079 | 1.142 |

| Problem 77 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|-------|
| grade | A | A | A | A | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 65 | 65 | 29 | 22 | 28 | 62 | 19 | 58 |
| normalized size | 1 | 1. | 0.45 | 0.34 | 0.43 | 0.95 | 0.29 | 0.89 |
| time (sec) | N/A | 0.032 | 0.006 | 0.044 | 0.954 | 1.28 | 0.097 | 1.138 |

| Problem 78 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | A | B | F | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 43 | 43 | 40 | 70 | 0 | 92 | 39 | 53 |
| normalized size | 1 | 1. | 0.93 | 1.63 | 0. | 2.14 | 0.91 | 1.23 |
| time (sec) | N/A | 0.033 | 0.012 | 0.067 | 0. | 1.351 | 0.098 | 1.16 |

| Problem 79 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|-------|
| grade | A | A | A | B | F | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 46 | 46 | 40 | 82 | 0 | 100 | 39 | 53 |
| normalized size | 1 | 1. | 0.87 | 1.78 | 0. | 2.17 | 0.85 | 1.15 |
| time (sec) | N/A | 0.036 | 0.012 | 0.073 | 0. | 1.33 | 0.102 | 1.155 |

| Problem 80 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|-------|
| grade | A | A | B | B | F | A | A | F(-2) |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 62 | 62 | 125 | 279 | 0 | 273 | 87 | 0 |
| normalized size | 1 | 1. | 2.02 | 4.5 | 0. | 4.4 | 1.4 | 0. |
| time (sec) | N/A | 0.029 | 0.071 | 0.105 | 0. | 1.356 | 0.252 | 0. |

| Problem 81 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|------|-------------|-------|--------|--------|-------|-------|
| grade | A | A | A | B | F | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 50 | 50 | 87 | 136 | 0 | 109 | 42 | 35 |
| normalized size | 1 | 1. | 1.74 | 2.72 | 0. | 2.18 | 0.84 | 0.7 |
| time (sec) | N/A | 0.04 | 0.13 | 0.058 | 0. | 1.411 | 0.112 | 1.139 |

| Problem 82 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|-------|
| grade | A | A | A | B | F | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 44 | 44 | 82 | 111 | 0 | 109 | 42 | 35 |
| normalized size | 1 | 1. | 1.86 | 2.52 | 0. | 2.48 | 0.95 | 0.8 |
| time (sec) | N/A | 0.029 | 0.065 | 0.053 | 0. | 1.313 | 0.111 | 1.143 |

| Problem 83 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|-------|
| grade | A | A | A | B | F | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 39 | 39 | 10 | 104 | 0 | 42 | 10 | 35 |
| normalized size | 1 | 1. | 0.26 | 2.67 | 0. | 1.08 | 0.26 | 0.9 |
| time (sec) | N/A | 0.033 | 0.007 | 0.054 | 0. | 1.24 | 0.098 | 1.154 |

| Problem 84 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|-------|
| grade | A | A | A | A | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 9 | 9 | 9 | 10 | 12 | 18 | 5 | 9 |
| normalized size | 1 | 1. | 1. | 1.11 | 1.33 | 2. | 0.56 | 1. |
| time (sec) | N/A | 0.004 | 0.004 | 0.045 | 0.98 | 1.334 | 0.079 | 1.113 |

| Problem 85 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|-------|
| grade | A | A | A | A | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 25 | 25 | 25 | 22 | 28 | 61 | 19 | 47 |
| normalized size | 1 | 1. | 1. | 0.88 | 1.12 | 2.44 | 0.76 | 1.88 |
| time (sec) | N/A | 0.013 | 0.006 | 0.043 | 0.986 | 1.35 | 0.097 | 1.128 |

| Problem 86 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | A | A | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 46 | 46 | 40 | 62 | 46 | 92 | 39 | 46 |
| normalized size | 1 | 1. | 0.87 | 1.35 | 1. | 2. | 0.85 | 1. |
| time (sec) | N/A | 0.019 | 0.011 | 0.042 | 1.454 | 1.37 | 0.097 | 1.1 |

| Problem 87 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|-------|
| grade | A | A | A | A | F | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 46 | 46 | 40 | 35 | 0 | 100 | 39 | 53 |
| normalized size | 1 | 1. | 0.87 | 0.76 | 0. | 2.17 | 0.85 | 1.15 |
| time (sec) | N/A | 0.021 | 0.013 | 0.049 | 0. | 1.326 | 0.101 | 1.126 |

| Problem 88 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|-------|
| grade | A | A | B | A | B | B | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 2 | 2 | 19 | 3 | 18 | 45 | 12 | 20 |
| normalized size | 1 | 1. | 9.5 | 1.5 | 9. | 22.5 | 6. | 10. |
| time (sec) | N/A | 0.002 | 0.002 | 0.039 | 0.969 | 1.325 | 0.087 | 1.144 |

| Problem 89 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|-------|
| grade | A | A | A | A | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 38 | 38 | 40 | 34 | 74 | 104 | 39 | 53 |
| normalized size | 1 | 1. | 1.05 | 0.89 | 1.95 | 2.74 | 1.03 | 1.39 |
| time (sec) | N/A | 0.029 | 0.014 | 0.043 | 1.444 | 1.36 | 0.103 | 1.159 |

| Problem 90 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|-------|
| grade | A | A | A | A | F | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 47 | 47 | 40 | 70 | 0 | 104 | 39 | 53 |
| normalized size | 1 | 1. | 0.85 | 1.49 | 0. | 2.21 | 0.83 | 1.13 |
| time (sec) | N/A | 0.036 | 0.017 | 0.053 | 0. | 1.343 | 0.104 | 1.159 |

| Problem 91 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|-------|
| grade | A | A | A | B | F | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 46 | 46 | 40 | 82 | 0 | 104 | 39 | 53 |
| normalized size | 1 | 1. | 0.87 | 1.78 | 0. | 2.26 | 0.85 | 1.15 |
| time (sec) | N/A | 0.035 | 0.014 | 0.055 | 0. | 1.392 | 0.101 | 1.146 |

| Problem 92 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|-------|
| grade | A | A | C | A | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 43 | 43 | 99 | 34 | 45 | 113 | 46 | 45 |
| normalized size | 1 | 1. | 2.3 | 0.79 | 1.05 | 2.63 | 1.07 | 1.05 |
| time (sec) | N/A | 0.035 | 0.1 | 0.045 | 1.458 | 1.417 | 0.12 | 1.125 |

| Problem 93 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|-------|
| grade | A | A | C | A | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 43 | 43 | 99 | 34 | 45 | 113 | 46 | 45 |
| normalized size | 1 | 1. | 2.3 | 0.79 | 1.05 | 2.63 | 1.07 | 1.05 |
| time (sec) | N/A | 0.032 | 0.033 | 0.043 | 1.452 | 1.459 | 0.124 | 1.166 |

| Problem 94 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|-------|
| grade | A | A | A | A | A | B | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 21 | 21 | 27 | 28 | 31 | 92 | 22 | 34 |
| normalized size | 1 | 1. | 1.29 | 1.33 | 1.48 | 4.38 | 1.05 | 1.62 |
| time (sec) | N/A | 0.005 | 0.01 | 0.05 | 0.988 | 1.292 | 0.096 | 1.162 |

| Problem 95 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | A | A | B | B | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 28 | 28 | 53 | 26 | 51 | 146 | 53 | 59 |
| normalized size | 1 | 1. | 1.89 | 0.93 | 1.82 | 5.21 | 1.89 | 2.11 |
| time (sec) | N/A | 0.013 | 0.02 | 0.045 | 1.448 | 1.381 | 0.49 | 1.1 |

| Problem 96 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|------|-------------|-------|--------|--------|-------|-------|
| grade | A | A | A | B | A | B | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 36 | 36 | 72 | 56 | 69 | 163 | 474 | 81 |
| normalized size | 1 | 1. | 2. | 1.56 | 1.92 | 4.53 | 13.17 | 2.25 |
| time (sec) | N/A | 0.04 | 0.04 | 0.049 | 1.481 | 1.467 | 1.05 | 1.106 |

| Problem 97 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | A | B | F | B | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 74 | 74 | 73 | 104 | 0 | 421 | 46 | 55 |
| normalized size | 1 | 1. | 0.99 | 1.41 | 0. | 5.69 | 0.62 | 0.74 |
| time (sec) | N/A | 0.045 | 0.1 | 0.064 | 0. | 1.548 | 0.166 | 1.12 |

| Problem 98 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | C | A | A | A | C | A |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD |
| size | 83 | 83 | 97 | 114 | 93 | 223 | 740 | 93 |
| normalized size | 1 | 1. | 1.17 | 1.37 | 1.12 | 2.69 | 8.92 | 1.12 |
| time (sec) | N/A | 0.055 | 0.125 | 0.047 | 1.476 | 1.399 | 0.886 | 1.13 |

| Problem 99 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|-------|
| grade | A | A | C | A | A | A | C | A |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD |
| size | 119 | 119 | 147 | 168 | 142 | 474 | 876 | 147 |
| normalized size | 1 | 1. | 1.24 | 1.41 | 1.19 | 3.98 | 7.36 | 1.24 |
| time (sec) | N/A | 0.091 | 0.244 | 0.056 | 1.461 | 1.388 | 1.412 | 1.138 |

| Problem 100 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|------|-------------|-------|--------|--------|-------|------|
| grade | A | A | C | B | F | B | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 234 | 234 | 111 | 710 | 0 | 7710 | 122 | 0 |
| normalized size | 1 | 1. | 0.47 | 3.03 | 0. | 32.95 | 0.52 | 0. |
| time (sec) | N/A | 0.23 | 0.116 | 0.08 | 0. | 2.28 | 0.899 | 0. |

| Problem 101 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | C | B | F | B | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 316 | 316 | 165 | 756 | 0 | 11750 | 167 | 0 |
| normalized size | 1 | 1. | 0.52 | 2.39 | 0. | 37.18 | 0.53 | 0. |
| time (sec) | N/A | 0.289 | 0.209 | 0.403 | 0. | 2.601 | 1.389 | 0. |

| Problem 102 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | C | A | F | C | F(-2) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 160 | 160 | 53 | 199 | 0 | 387 | 0 | 0 |
| normalized size | 1 | 1. | 0.33 | 1.24 | 0. | 2.42 | 0. | 0. |
| time (sec) | N/A | 0.146 | 0.045 | 0.104 | 0. | 1.465 | 0. | 0. |

| Problem 103 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | C | A | F | C | F(-2) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 172 | 172 | 53 | 199 | 0 | 387 | 0 | 0 |
| normalized size | 1 | 1. | 0.31 | 1.16 | 0. | 2.25 | 0. | 0. |
| time (sec) | N/A | 0.137 | 0.035 | 0.151 | 0. | 1.673 | 0. | 0. |

| Problem 104 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|-------|
| grade | A | A | A | B | F | B | B | C |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 160 | 160 | 137 | 285 | 0 | 1226 | 332 | 2847 |
| normalized size | 1 | 1. | 0.86 | 1.78 | 0. | 7.66 | 2.08 | 17.79 |
| time (sec) | N/A | 0.119 | 0.091 | 0.106 | 0. | 2.085 | 1.81 | 1.258 |

| Problem 105 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|-------|
| grade | A | A | A | B | F | B | B | C |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 160 | 160 | 136 | 283 | 0 | 1224 | 330 | 2847 |
| normalized size | 1 | 1. | 0.85 | 1.77 | 0. | 7.65 | 2.06 | 17.79 |
| time (sec) | N/A | 0.103 | 0.057 | 0.103 | 0. | 2.061 | 1.761 | 1.258 |

| Problem 106 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | C | A | F | B | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 114 | 114 | 115 | 92 | 0 | 1616 | 27 | 0 |
| normalized size | 1 | 1. | 1.01 | 0.81 | 0. | 14.18 | 0.24 | 0. |
| time (sec) | N/A | 0.076 | 0.176 | 0.067 | 0. | 1.817 | 0.37 | 0. |

| Problem 107 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|------|-------------|-------|--------|--------|-------|-------|
| grade | A | A | C | A | F | B | F(-2) | F(-2) |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 122 | 122 | 115 | 96 | 0 | 682 | 0 | 0 |
| normalized size | 1 | 1. | 0.94 | 0.79 | 0. | 5.59 | 0. | 0. |
| time (sec) | N/A | 0.08 | 0.157 | 0.073 | 0. | 1.671 | 0. | 0. |

| Problem 108 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|-------|
| grade | A | A | C | A | C | A | C | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 124 | 124 | 115 | 89 | 170 | 768 | 143 | 124 |
| normalized size | 1 | 1. | 0.93 | 0.72 | 1.37 | 6.19 | 1.15 | 1. |
| time (sec) | N/A | 0.077 | 0.135 | 0.055 | 1.511 | 1.847 | 0.398 | 1.158 |

| Problem 109 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | C | A | F | B | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 136 | 136 | 130 | 190 | 0 | 9567 | 172 | 0 |
| normalized size | 1 | 1. | 0.96 | 1.4 | 0. | 70.35 | 1.26 | 0. |
| time (sec) | N/A | 0.104 | 0.148 | 0.061 | 0. | 6.157 | 1.203 | 0. |

| Problem 110 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|-------|
| grade | A | A | C | A | F | B | F(-2) | F(-2) |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 160 | 160 | 138 | 194 | 0 | 2531 | 0 | 0 |
| normalized size | 1 | 1. | 0.86 | 1.21 | 0. | 15.82 | 0. | 0. |
| time (sec) | N/A | 0.116 | 0.137 | 0.097 | 0. | 2.411 | 0. | 0. |

| Problem 111 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|-------|
| grade | A | A | C | A | F | B | F(-2) | F(-2) |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 414 | 414 | 247 | 404 | 0 | 3148 | 0 | 0 |
| normalized size | 1 | 1. | 0.6 | 0.98 | 0. | 7.6 | 0. | 0. |
| time (sec) | N/A | 0.453 | 0.197 | 0.194 | 0. | 3.61 | 0. | 0. |

| Problem 112 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|-------|
| grade | A | A | C | A | F | B | F(-2) | F(-2) |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 234 | 234 | 163 | 318 | 0 | 3245 | 0 | 0 |
| normalized size | 1 | 1. | 0.7 | 1.36 | 0. | 13.87 | 0. | 0. |
| time (sec) | N/A | 0.172 | 0.185 | 0.092 | 0. | 7.487 | 0. | 0. |

| Problem 113 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | C | B | F | F | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD |
| size | 96 | 96 | 103 | 200 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 1.07 | 2.08 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.123 | 0.136 | 0.31 | 0. | 0. | 0. | 0. |

| Problem 114 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | C | B | F | F | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD |
| size | 25 | 25 | 19 | 113 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0.76 | 4.52 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.032 | 0.053 | 0.054 | 0. | 0. | 0. | 0. |

| Problem 115 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | C | B | F | F | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD |
| size | 96 | 96 | 103 | 204 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 1.07 | 2.12 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.178 | 0.165 | 0.296 | 0. | 0. | 0. | 0. |

| Problem 116 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|------|-------------|-------|--------|--------|-------|------|
| grade | A | A | C | B | F | F | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD |
| size | 92 | 92 | 107 | 204 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 1.16 | 2.22 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.12 | 0.132 | 0.403 | 0. | 0. | 0. | 0. |

| Problem 117 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | C | B | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 27 | 27 | 35 | 95 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 1.3 | 3.52 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.039 | 0.064 | 0.05 | 0. | 0. | 0. | 0. |

| Problem 118 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | C | B | F | F | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD |
| size | 92 | 92 | 107 | 204 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 1.16 | 2.22 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.165 | 0.166 | 0.291 | 0. | 0. | 0. | 0. |

| Problem 119 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|-------|
| grade | A | A | C | A | F | F | F | F(-2) |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 296 | 296 | 187 | 515 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0.63 | 1.74 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.119 | 0.309 | 0.075 | 0. | 0. | 0. | 0. |

| Problem 120 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|-------|
| grade | A | A | A | A | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 106 | 106 | 106 | 97 | 127 | 234 | 110 | 127 |
| normalized size | 1 | 1. | 1. | 0.92 | 1.2 | 2.21 | 1.04 | 1.2 |
| time (sec) | N/A | 0.083 | 0.021 | 0.042 | 0.996 | 1.347 | 0.078 | 1.117 |

| Problem 121 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|-------|
| grade | A | A | A | A | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 79 | 79 | 79 | 72 | 96 | 171 | 78 | 96 |
| normalized size | 1 | 1. | 1. | 0.91 | 1.22 | 2.16 | 0.99 | 1.22 |
| time (sec) | N/A | 0.056 | 0.016 | 0.042 | 0.977 | 1.417 | 0.073 | 1.141 |

| Problem 122 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|-------|
| grade | A | A | A | A | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 56 | 56 | 56 | 49 | 65 | 120 | 56 | 68 |
| normalized size | 1 | 1. | 1. | 0.88 | 1.16 | 2.14 | 1. | 1.21 |
| time (sec) | N/A | 0.032 | 0.011 | 0.042 | 0.976 | 1.367 | 0.066 | 1.125 |

| Problem 123 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|-------|
| grade | A | A | A | A | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 32 | 32 | 32 | 27 | 35 | 66 | 29 | 38 |
| normalized size | 1 | 1. | 1. | 0.84 | 1.09 | 2.06 | 0.91 | 1.19 |
| time (sec) | N/A | 0.014 | 0.002 | 0.041 | 0.958 | 1.554 | 0.057 | 1.125 |

| Problem 124 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|-------|
| grade | A | A | A | A | F(-2) | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 55 | 55 | 55 | 57 | 0 | 293 | 104 | 59 |
| normalized size | 1 | 1. | 1. | 1.04 | 0. | 5.33 | 1.89 | 1.07 |
| time (sec) | N/A | 0.035 | 0.036 | 0.048 | 0. | 1.939 | 0.427 | 1.124 |

| Problem 125 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|-------|
| grade | A | A | A | A | F(-2) | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 74 | 74 | 78 | 82 | 0 | 455 | 138 | 84 |
| normalized size | 1 | 1. | 1.05 | 1.11 | 0. | 6.15 | 1.86 | 1.14 |
| time (sec) | N/A | 0.052 | 0.051 | 0.052 | 0. | 1.876 | 0.64 | 1.136 |

| Problem 126 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|-------|
| grade | A | A | A | A | F(-2) | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 93 | 93 | 92 | 99 | 0 | 635 | 219 | 104 |
| normalized size | 1 | 1. | 0.99 | 1.06 | 0. | 6.83 | 2.35 | 1.12 |
| time (sec) | N/A | 0.067 | 0.063 | 0.053 | 0. | 1.992 | 0.856 | 1.122 |

| Problem 127 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|-------|
| grade | A | A | A | A | F(-2) | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 123 | 123 | 113 | 122 | 0 | 875 | 204 | 135 |
| normalized size | 1 | 1. | 0.92 | 0.99 | 0. | 7.11 | 1.66 | 1.1 |
| time (sec) | N/A | 0.114 | 0.082 | 0.053 | 0. | 1.992 | 1.09 | 1.131 |

| Problem 128 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | A | A | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 133 | 133 | 133 | 130 | 174 | 304 | 144 | 173 |
| normalized size | 1 | 1. | 1. | 0.98 | 1.31 | 2.29 | 1.08 | 1.3 |
| time (sec) | N/A | 0.107 | 0.022 | 0.042 | 1.008 | 1.642 | 0.084 | 1.12 |

| Problem 129 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|-------|
| grade | A | A | A | A | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 97 | 97 | 97 | 90 | 120 | 215 | 104 | 123 |
| normalized size | 1 | 1. | 1. | 0.93 | 1.24 | 2.22 | 1.07 | 1.27 |
| time (sec) | N/A | 0.068 | 0.018 | 0.041 | 1.027 | 1.485 | 0.079 | 1.165 |

| Problem 130 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|------|-------------|-------|--------|--------|-------|-------|
| grade | A | A | A | A | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 60 | 60 | 60 | 51 | 68 | 123 | 60 | 72 |
| normalized size | 1 | 1. | 1. | 0.85 | 1.13 | 2.05 | 1. | 1.2 |
| time (sec) | N/A | 0.03 | 0.002 | 0.041 | 0.932 | 1.647 | 0.067 | 1.176 |

| Problem 131 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|-------|
| grade | A | A | A | A | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 25 | 25 | 25 | 22 | 28 | 47 | 22 | 28 |
| normalized size | 1 | 1. | 1. | 0.88 | 1.12 | 1.88 | 0.88 | 1.12 |
| time (sec) | N/A | 0.008 | 0.001 | 0.043 | 1. | 1.679 | 0.06 | 1.131 |

| Problem 132 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|-------|
| grade | A | A | A | A | F(-2) | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 108 | 108 | 97 | 136 | 0 | 585 | 235 | 142 |
| normalized size | 1 | 1. | 0.9 | 1.26 | 0. | 5.42 | 2.18 | 1.31 |
| time (sec) | N/A | 0.077 | 0.08 | 0.047 | 0. | 1.862 | 0.594 | 1.105 |

| Problem 133 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|-------|
| grade | A | A | A | A | F(-2) | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 131 | 131 | 134 | 170 | 0 | 807 | 314 | 173 |
| normalized size | 1 | 1. | 1.02 | 1.3 | 0. | 6.16 | 2.4 | 1.32 |
| time (sec) | N/A | 0.188 | 0.109 | 0.053 | 0. | 1.865 | 1.021 | 1.156 |

| Problem 134 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|-------|
| grade | A | A | A | A | F(-2) | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 155 | 155 | 154 | 211 | 0 | 1071 | 257 | 196 |
| normalized size | 1 | 1. | 0.99 | 1.36 | 0. | 6.91 | 1.66 | 1.26 |
| time (sec) | N/A | 0.252 | 0.111 | 0.053 | 0. | 1.936 | 1.857 | 1.195 |

| Problem 135 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|-------|
| grade | A | A | A | A | F(-2) | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 184 | 184 | 174 | 262 | 0 | 1366 | 292 | 225 |
| normalized size | 1 | 1. | 0.95 | 1.42 | 0. | 7.42 | 1.59 | 1.22 |
| time (sec) | N/A | 0.297 | 0.143 | 0.054 | 0. | 2.004 | 3.058 | 1.158 |

| Problem 136 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|-------|
| grade | A | A | A | A | F(-2) | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 223 | 223 | 200 | 231 | 0 | 1697 | 335 | 267 |
| normalized size | 1 | 1. | 0.9 | 1.04 | 0. | 7.61 | 1.5 | 1.2 |
| time (sec) | N/A | 0.339 | 0.191 | 0.056 | 0. | 1.939 | 5.932 | 1.126 |

| Problem 137 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|-------|
| grade | A | A | A | B | F(-2) | B | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 437 | 437 | 444 | 741 | 0 | 6198 | 500 | 672 |
| normalized size | 1 | 1. | 1.02 | 1.7 | 0. | 14.18 | 1.14 | 1.54 |
| time (sec) | N/A | 0.453 | 0.341 | 0.048 | 0. | 28.782 | 4.097 | 1.168 |

| Problem 138 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|-------|
| grade | A | A | A | A | F(-2) | B | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 370 | 370 | 360 | 572 | 0 | 4462 | 350 | 547 |
| normalized size | 1 | 1. | 0.97 | 1.55 | 0. | 12.06 | 0.95 | 1.48 |
| time (sec) | N/A | 0.501 | 0.281 | 0.049 | 0. | 8.33 | 2.368 | 1.171 |

| Problem 139 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|-------|
| grade | A | A | A | A | F(-2) | B | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 297 | 297 | 269 | 412 | 0 | 3004 | 238 | 454 |
| normalized size | 1 | 1. | 0.91 | 1.39 | 0. | 10.11 | 0.8 | 1.53 |
| time (sec) | N/A | 0.293 | 0.262 | 0.046 | 0. | 3.22 | 1.501 | 1.148 |

| Problem 140 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|-------|
| grade | A | A | A | A | F(-2) | B | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 247 | 247 | 183 | 260 | 0 | 1544 | 109 | 331 |
| normalized size | 1 | 1. | 0.74 | 1.05 | 0. | 6.25 | 0.44 | 1.34 |
| time (sec) | N/A | 0.152 | 0.058 | 0.046 | 0. | 1.971 | 0.641 | 1.151 |

| Problem 141 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|-------|
| grade | A | A | A | A | F(-2) | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 185 | 185 | 134 | 128 | 0 | 306 | 20 | 242 |
| normalized size | 1 | 1. | 0.72 | 0.69 | 0. | 1.65 | 0.11 | 1.31 |
| time (sec) | N/A | 0.111 | 0.019 | 0.043 | 0. | 1.999 | 0.157 | 1.128 |

| Problem 142 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|------|-------------|-------|--------|--------|-------|-------|
| grade | A | A | A | A | F(-2) | B | F(-1) | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 336 | 336 | 234 | 363 | 0 | 7792 | 0 | 458 |
| normalized size | 1 | 1. | 0.7 | 1.08 | 0. | 23.19 | 0. | 1.36 |
| time (sec) | N/A | 0.27 | 0.155 | 0.054 | 0. | 6.77 | 0. | 1.131 |

| Problem 143 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|---------|-------|-------|
| grade | A | A | A | A | F(-2) | B | F(-1) | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 453 | 453 | 362 | 650 | 0 | 16741 | 0 | 698 |
| normalized size | 1 | 1. | 0.8 | 1.43 | 0. | 36.96 | 0. | 1.54 |
| time (sec) | N/A | 0.384 | 0.494 | 0.056 | 0. | 121.687 | 0. | 1.259 |

| Problem 144 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|------|-------------|-------|--------|--------|-------|-------|
| grade | A | A | A | B | F(-2) | B | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 363 | 363 | 371 | 624 | 0 | 4196 | 352 | 574 |
| normalized size | 1 | 1. | 1.02 | 1.72 | 0. | 11.56 | 0.97 | 1.58 |
| time (sec) | N/A | 0.41 | 0.269 | 0.058 | 0. | 2.412 | 3.678 | 1.183 |

| Problem 145 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|-------|
| grade | A | A | A | A | F(-2) | B | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 349 | 349 | 295 | 464 | 0 | 3308 | 275 | 497 |
| normalized size | 1 | 1. | 0.85 | 1.33 | 0. | 9.48 | 0.79 | 1.42 |
| time (sec) | N/A | 0.313 | 0.172 | 0.052 | 0. | 2.928 | 2.122 | 1.137 |

| Problem 146 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|-------|
| grade | A | A | A | A | F(-2) | B | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 275 | 275 | 267 | 303 | 0 | 1825 | 136 | 369 |
| normalized size | 1 | 1. | 0.97 | 1.1 | 0. | 6.64 | 0.49 | 1.34 |
| time (sec) | N/A | 0.203 | 0.288 | 0.05 | 0. | 1.787 | 1.165 | 1.126 |

| Problem 147 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | A | A | F(-2) | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 202 | 202 | 183 | 143 | 0 | 414 | 39 | 262 |
| normalized size | 1 | 1. | 0.91 | 0.71 | 0. | 2.05 | 0.19 | 1.3 |
| time (sec) | N/A | 0.133 | 0.112 | 0.049 | 0. | 1.697 | 0.503 | 1.11 |

| Problem 148 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|---------|-------|-------|
| grade | A | A | A | A | F(-2) | B | F(-1) | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 689 | 689 | 429 | 873 | 0 | 20650 | 0 | 814 |
| normalized size | 1 | 1. | 0.62 | 1.27 | 0. | 29.97 | 0. | 1.18 |
| time (sec) | N/A | 0.623 | 0.314 | 0.061 | 0. | 114.867 | 0. | 1.173 |

| Problem 149 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|-------|
| grade | A | A | A | A | F(-2) | F(-1) | F(-1) | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 864 | 864 | 540 | 1169 | 0 | 0 | 0 | 1146 |
| normalized size | 1 | 1. | 0.62 | 1.35 | 0. | 0. | 0. | 1.33 |
| time (sec) | N/A | 0.906 | 0.624 | 0.066 | 0. | 0. | 0. | 1.154 |

| Problem 150 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | C | C | F | F | C | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 388 | 386 | 203 | 506 | 0 | 0 | 214 | 0 |
| normalized size | 1 | 0.99 | 0.52 | 1.3 | 0. | 0. | 0.55 | 0. |
| time (sec) | N/A | 0.415 | 0.216 | 0.192 | 0. | 0. | 4.703 | 0. |

| Problem 151 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | C | C | F | F | C | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 326 | 326 | 140 | 388 | 0 | 0 | 173 | 0 |
| normalized size | 1 | 1. | 0.43 | 1.19 | 0. | 0. | 0.53 | 0. |
| time (sec) | N/A | 0.287 | 0.144 | 0.052 | 0. | 0. | 3.583 | 0. |

| Problem 152 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | C | C | F | F | C | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 264 | 264 | 120 | 266 | 0 | 0 | 124 | 0 |
| normalized size | 1 | 1. | 0.45 | 1.01 | 0. | 0. | 0.47 | 0. |
| time (sec) | N/A | 0.129 | 0.095 | 0.051 | 0. | 0. | 2.729 | 0. |

| Problem 153 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | C | C | F | F | C | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 226 | 226 | 77 | 169 | 0 | 0 | 78 | 0 |
| normalized size | 1 | 1. | 0.34 | 0.75 | 0. | 0. | 0.35 | 0. |
| time (sec) | N/A | 0.069 | 0.03 | 0.046 | 0. | 0. | 1.651 | 0. |

| Problem 154 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | C | C | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 334 | 334 | 95 | 107 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0.28 | 0.32 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.266 | 0.154 | 0.191 | 0. | 0. | 0. | 0. |

| Problem 155 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | C | C | F | F(-1) | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 581 | 581 | 522 | 556 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0.9 | 0.96 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.762 | 0.779 | 0.197 | 0. | 0. | 0. | 0. |

| Problem 156 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|------|-------------|-------|--------|--------|-------|------|
| grade | A | A | C | C | F | F(-1) | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 729 | 729 | 332 | 1018 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0.46 | 1.4 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 1.25 | 1.121 | 0.195 | 0. | 0. | 0. | 0. |

| Problem 157 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | C | B | F | F | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 213 | 213 | 141 | 360 | 0 | 0 | 180 | 0 |
| normalized size | 1 | 1. | 0.66 | 1.69 | 0. | 0. | 0.85 | 0. |
| time (sec) | N/A | 0.282 | 0.167 | 0.291 | 0. | 0. | 3.681 | 0. |

| Problem 158 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | C | A | F | F | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 162 | 162 | 121 | 246 | 0 | 0 | 129 | 0 |
| normalized size | 1 | 1. | 0.75 | 1.52 | 0. | 0. | 0.8 | 0. |
| time (sec) | N/A | 0.145 | 0.105 | 0.055 | 0. | 0. | 2.819 | 0. |

| Problem 159 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | C | A | F | F | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 124 | 124 | 77 | 154 | 0 | 0 | 82 | 0 |
| normalized size | 1 | 1. | 0.62 | 1.24 | 0. | 0. | 0.66 | 0. |
| time (sec) | N/A | 0.089 | 0.031 | 0.05 | 0. | 0. | 1.708 | 0. |

| Problem 160 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | C | A | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 72 | 72 | 91 | 97 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 1.26 | 1.35 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.041 | 0.152 | 0.193 | 0. | 0. | 0. | 0. |

| Problem 161 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | C | B | F | F(-1) | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 299 | 299 | 508 | 523 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 1.7 | 1.75 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.356 | 0.974 | 0.368 | 0. | 0. | 0. | 0. |

| Problem 162 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | C | B | F | F(-1) | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 425 | 425 | 321 | 961 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0.76 | 2.26 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.751 | 1.274 | 0.286 | 0. | 0. | 0. | 0. |

| Problem 163 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | C | B | F | F(-1) | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 563 | 563 | 458 | 1420 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0.81 | 2.52 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 1.206 | 1.997 | 0.195 | 0. | 0. | 0. | 0. |

| Problem 164 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | C | A | F | F | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 126 | 126 | 78 | 160 | 0 | 0 | 73 | 0 |
| normalized size | 1 | 1. | 0.62 | 1.27 | 0. | 0. | 0.58 | 0. |
| time (sec) | N/A | 0.083 | 0.035 | 0.181 | 0. | 0. | 1.662 | 0. |

| Problem 165 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | C | A | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 73 | 73 | 92 | 99 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 1.26 | 1.36 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.042 | 0.15 | 0.18 | 0. | 0. | 0. | 0. |

| Problem 166 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|-------|
| grade | A | A | C | B | F | F | A | F(-2) |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 54 | 54 | 86 | 158 | 0 | 0 | 70 | 0 |
| normalized size | 1 | 1. | 1.59 | 2.93 | 0. | 0. | 1.3 | 0. |
| time (sec) | N/A | 0.049 | 0.037 | 0.072 | 0. | 0. | 1.796 | 0. |

| Problem 167 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | C | B | F | F | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 52 | 52 | 85 | 165 | 0 | 0 | 76 | 0 |
| normalized size | 1 | 1. | 1.63 | 3.17 | 0. | 0. | 1.46 | 0. |
| time (sec) | N/A | 0.046 | 0.031 | 0.071 | 0. | 0. | 1.753 | 0. |

| Problem 168 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | C | C | F | F | C | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 236 | 236 | 80 | 175 | 0 | 0 | 83 | 0 |
| normalized size | 1 | 1. | 0.34 | 0.74 | 0. | 0. | 0.35 | 0. |
| time (sec) | N/A | 0.069 | 0.037 | 0.175 | 0. | 0. | 1.636 | 0. |

| Problem 169 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | C | C | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 347 | 347 | 98 | 110 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0.28 | 0.32 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.295 | 0.147 | 0.178 | 0. | 0. | 0. | 0. |

| Problem 170 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | A | B | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 40 | 40 | 43 | 79 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 1.08 | 1.98 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.063 | 0.123 | 0.191 | 0. | 0. | 0. | 0. |

| Problem 171 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | C | C | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 310 | 310 | 50 | 86 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0.16 | 0.28 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.273 | 0.103 | 0.382 | 0. | 0. | 0. | 0. |

| Problem 172 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | C | B | F | F(-1) | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 40 | 40 | 59 | 78 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 1.48 | 1.95 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.018 | 0.125 | 0.095 | 0. | 0. | 0. | 0. |

| Problem 173 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | C | C | F | F(-1) | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 300 | 300 | 65 | 86 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0.22 | 0.29 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.223 | 0.112 | 0.023 | 0. | 0. | 0. | 0. |

| Problem 174 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | F | F | F | F | F | F | F |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 112 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 0. | 0. | 0. | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.009 | 0.06 | 0.144 | 0. | 0. | 0. | 0. |

| Problem 175 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | N/A | A | A | A | A | A | F(-1) | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 21 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 0. | 0. | 0. | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.008 | 0.076 | 0.102 | 0. | 0. | 0. | 0. |

| Problem 176 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|------|-------------|-------|--------|--------|-------|------|
| grade | A | A | A | F | F | F | F(-1) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 204 | 196 | 136 | 0 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 0.96 | 0.67 | 0. | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.23 | 0.067 | 0.051 | 0. | 0. | 0. | 0. |

| Problem 177 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|--------|------|
| grade | A | A | A | F | F | F | C | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 150 | 142 | 106 | 0 | 0 | 0 | 119 | 0 |
| normalized size | 1 | 0.95 | 0.71 | 0. | 0. | 0. | 0.79 | 0. |
| time (sec) | N/A | 0.132 | 0.041 | 0.04 | 0. | 0. | 118.77 | 0. |

| Problem 178 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|--------|------|
| grade | A | A | A | F | F | F | C | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 96 | 96 | 75 | 0 | 0 | 0 | 75 | 0 |
| normalized size | 1 | 1. | 0.78 | 0. | 0. | 0. | 0.78 | 0. |
| time (sec) | N/A | 0.051 | 0.023 | 0.028 | 0. | 0. | 56.897 | 0. |

| Problem 179 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|------|-------------|-------|--------|--------|--------|------|
| grade | A | A | A | F | F | F | C | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 44 | 44 | 44 | 0 | 0 | 0 | 34 | 0 |
| normalized size | 1 | 1. | 1. | 0. | 0. | 0. | 0.77 | 0. |
| time (sec) | N/A | 0.01 | 0.003 | 0.021 | 0. | 0. | 12.409 | 0. |

| Problem 180 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | F | F | F | F | F(-1) | F |
| verified | N/A | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 123 | 123 | 0 | 0 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0. | 0. | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.127 | 0.128 | 0.042 | 0. | 0. | 0. | 0. |

| Problem 181 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | F | F | F | F | F(-1) | F |
| verified | N/A | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 189 | 189 | 0 | 0 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0. | 0. | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.194 | 0.241 | 0.053 | 0. | 0. | 0. | 0. |

| Problem 182 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|---------|------|
| grade | A | A | A | A | F | F | C | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 108 | 103 | 86 | 75 | 0 | 0 | 129 | 0 |
| normalized size | 1 | 0.95 | 0.8 | 0.69 | 0. | 0. | 1.19 | 0. |
| time (sec) | N/A | 0.115 | 0.017 | 0.105 | 0. | 0. | 176.712 | 0. |

| Problem 183 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|--------|------|
| grade | A | A | A | A | F | F | C | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 86 | 79 | 65 | 56 | 0 | 0 | 94 | 0 |
| normalized size | 1 | 0.92 | 0.76 | 0.65 | 0. | 0. | 1.09 | 0. |
| time (sec) | N/A | 0.068 | 0.011 | 0.045 | 0. | 0. | 99.188 | 0. |

| Problem 184 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|--------|------|
| grade | A | A | A | A | F | F | C | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 42 | 42 | 42 | 37 | 0 | 0 | 61 | 0 |
| normalized size | 1 | 1. | 1. | 0.88 | 0. | 0. | 1.45 | 0. |
| time (sec) | N/A | 0.021 | 0.007 | 0.027 | 0. | 0. | 45.966 | 0. |

| Problem 185 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | A | A | F | F | C | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 18 | 18 | 18 | 17 | 0 | 0 | 29 | 0 |
| normalized size | 1 | 1. | 1. | 0.94 | 0. | 0. | 1.61 | 0. |
| time (sec) | N/A | 0.004 | 0.002 | 0.021 | 0. | 0. | 9.933 | 0. |

| Problem 186 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | F | F | F | F | F(-1) | F |
| verified | N/A | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 50 | 50 | 0 | 0 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0. | 0. | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.054 | 0.08 | 0.043 | 0. | 0. | 0. | 0. |

| Problem 187 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | F | F | F | F | F(-1) | F |
| verified | N/A | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 77 | 77 | 0 | 0 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0. | 0. | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.082 | 0.104 | 0.056 | 0. | 0. | 0. | 0. |

| Problem 188 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | F | F | F | F | F(-1) | F |
| verified | N/A | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 101 | 101 | 0 | 0 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0. | 0. | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.114 | 0.148 | 0.072 | 0. | 0. | 0. | 0. |

| Problem 189 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|-------|
| grade | A | A | A | A | F(-2) | A | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 51 | 51 | 51 | 42 | 0 | 255 | 75 | 194 |
| normalized size | 1 | 1. | 1. | 0.82 | 0. | 5. | 1.47 | 3.8 |
| time (sec) | N/A | 0.041 | 0.023 | 0.006 | 0. | 1.848 | 0.475 | 1.183 |

| Problem 190 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|-------|
| grade | A | A | A | A | F(-2) | A | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 38 | 38 | 38 | 31 | 0 | 201 | 58 | 166 |
| normalized size | 1 | 1. | 1. | 0.82 | 0. | 5.29 | 1.53 | 4.37 |
| time (sec) | N/A | 0.034 | 0.017 | 0.002 | 0. | 1.85 | 0.405 | 1.176 |

| Problem 191 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|-------|
| grade | A | A | A | A | F(-2) | A | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 29 | 29 | 29 | 22 | 0 | 149 | 34 | 159 |
| normalized size | 1 | 1. | 1. | 0.76 | 0. | 5.14 | 1.17 | 5.48 |
| time (sec) | N/A | 0.023 | 0.009 | 0.004 | 0. | 1.879 | 0.359 | 1.155 |

| Problem 192 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|-------|
| grade | A | A | A | A | F(-2) | A | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 24 | 16 | 0 | 151 | 46 | 157 |
| normalized size | 1 | 1. | 1. | 0.67 | 0. | 6.29 | 1.92 | 6.54 |
| time (sec) | N/A | 0.012 | 0.004 | 0.003 | 0. | 1.904 | 0.187 | 1.166 |

| Problem 193 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|-------|
| grade | A | A | A | A | F(-2) | A | B | F(-2) |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 72 | 72 | 65 | 55 | 0 | 421 | 226 | 0 |
| normalized size | 1 | 1. | 0.9 | 0.76 | 0. | 5.85 | 3.14 | 0. |
| time (sec) | N/A | 0.057 | 0.034 | 0.016 | 0. | 1.854 | 0.654 | 0. |

| Problem 194 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|-------|
| grade | A | A | A | A | F(-2) | B | B | F(-2) |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 89 | 89 | 76 | 73 | 0 | 608 | 255 | 0 |
| normalized size | 1 | 1. | 0.85 | 0.82 | 0. | 6.83 | 2.87 | 0. |
| time (sec) | N/A | 0.083 | 0.058 | 0.01 | 0. | 1.828 | 0.907 | 0. |

| Problem 195 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|-------|
| grade | A | A | A | B | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 62 | 62 | 61 | 1442 | 0 | 491 | 0 | 32 |
| normalized size | 1 | 1. | 0.98 | 23.26 | 0. | 7.92 | 0. | 0.52 |
| time (sec) | N/A | 0.044 | 0.028 | 0.054 | 0. | 2.056 | 0. | 1.175 |

| Problem 196 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|-------|
| grade | A | A | A | B | F | A | F | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 38 | 38 | 38 | 986 | 0 | 340 | 0 | 177 |
| normalized size | 1 | 1. | 1. | 25.95 | 0. | 8.95 | 0. | 4.66 |
| time (sec) | N/A | 0.026 | 0.157 | 0.027 | 0. | 2.156 | 0. | 1.454 |

| Problem 197 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|-------|
| grade | A | A | A | B | F | B | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 61 | 61 | 108 | 441 | 0 | 490 | 0 | 1 |
| normalized size | 1 | 1. | 1.77 | 7.23 | 0. | 8.03 | 0. | 0.02 |
| time (sec) | N/A | 0.039 | 0.127 | 0.023 | 0. | 2.017 | 0. | 1.228 |

| Problem 198 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | C | B | F | B | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD |
| size | 80 | 80 | 345 | 911 | 0 | 632 | 0 | 0 |
| normalized size | 1 | 1. | 4.31 | 11.39 | 0. | 7.9 | 0. | 0. |
| time (sec) | N/A | 0.069 | 1.682 | 0.022 | 0. | 2.102 | 0. | 0. |

| Problem 199 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | C | A | F | A | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 153 | 153 | 98 | 132 | 0 | 549 | 0 | 0 |
| normalized size | 1 | 1. | 0.64 | 0.86 | 0. | 3.59 | 0. | 0. |
| time (sec) | N/A | 0.055 | 0.165 | 0.089 | 0. | 2.086 | 0. | 0. |

| Problem 200 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | C | A | F | A | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 110 | 110 | 86 | 107 | 0 | 489 | 0 | 0 |
| normalized size | 1 | 1. | 0.78 | 0.97 | 0. | 4.45 | 0. | 0. |
| time (sec) | N/A | 0.035 | 0.074 | 0.02 | 0. | 2.102 | 0. | 0. |

| Problem 201 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | C | A | F | A | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 65 | 65 | 50 | 69 | 0 | 270 | 0 | 0 |
| normalized size | 1 | 1. | 0.77 | 1.06 | 0. | 4.15 | 0. | 0. |
| time (sec) | N/A | 0.022 | 0.043 | 0.02 | 0. | 1.989 | 0. | 0. |

| Problem 202 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | A | B | F | A | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 78 | 78 | 78 | 249 | 0 | 359 | 0 | 0 |
| normalized size | 1 | 1. | 1. | 3.19 | 0. | 4.6 | 0. | 0. |
| time (sec) | N/A | 0.036 | 0.054 | 0.043 | 0. | 2.165 | 0. | 0. |

| Problem 203 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | A | B | F | A | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 125 | 125 | 111 | 488 | 0 | 655 | 0 | 0 |
| normalized size | 1 | 1. | 0.89 | 3.9 | 0. | 5.24 | 0. | 0. |
| time (sec) | N/A | 0.051 | 0.084 | 0.063 | 0. | 2.238 | 0. | 0. |

| Problem 204 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | A | B | F | A | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 168 | 168 | 123 | 711 | 0 | 801 | 0 | 0 |
| normalized size | 1 | 1. | 0.73 | 4.23 | 0. | 4.77 | 0. | 0. |
| time (sec) | N/A | 0.086 | 0.108 | 0.054 | 0. | 1.873 | 0. | 0. |

| Problem 205 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | A | A | F | A | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 152 | 152 | 123 | 105 | 0 | 551 | 0 | 0 |
| normalized size | 1 | 1. | 0.81 | 0.69 | 0. | 3.62 | 0. | 0. |
| time (sec) | N/A | 0.055 | 0.201 | 0.017 | 0. | 1.995 | 0. | 0. |

| Problem 206 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | A | A | F | A | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 109 | 109 | 110 | 85 | 0 | 490 | 0 | 0 |
| normalized size | 1 | 1. | 1.01 | 0.78 | 0. | 4.5 | 0. | 0. |
| time (sec) | N/A | 0.035 | 0.115 | 0.016 | 0. | 1.974 | 0. | 0. |

| Problem 207 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | A | A | F | A | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 64 | 64 | 67 | 54 | 0 | 269 | 0 | 0 |
| normalized size | 1 | 1. | 1.05 | 0.84 | 0. | 4.2 | 0. | 0. |
| time (sec) | N/A | 0.025 | 0.041 | 0.014 | 0. | 1.941 | 0. | 0. |

| Problem 208 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | A | B | F | A | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 77 | 77 | 77 | 267 | 0 | 359 | 0 | 0 |
| normalized size | 1 | 1. | 1. | 3.47 | 0. | 4.66 | 0. | 0. |
| time (sec) | N/A | 0.036 | 0.051 | 0.057 | 0. | 2. | 0. | 0. |

| Problem 209 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | A | B | F | A | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 124 | 124 | 110 | 510 | 0 | 660 | 0 | 0 |
| normalized size | 1 | 1. | 0.89 | 4.11 | 0. | 5.32 | 0. | 0. |
| time (sec) | N/A | 0.054 | 0.083 | 0.041 | 0. | 2.353 | 0. | 0. |

| Problem 210 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | A | B | F | A | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 167 | 167 | 122 | 739 | 0 | 803 | 0 | 0 |
| normalized size | 1 | 1. | 0.73 | 4.43 | 0. | 4.81 | 0. | 0. |
| time (sec) | N/A | 0.087 | 0.107 | 0.05 | 0. | 2.392 | 0. | 0. |

| Problem 211 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|------|-------------|-------|--------|--------|-------|------|
| grade | A | A | A | A | F | B | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 30 | 30 | 38 | 25 | 0 | 165 | 0 | 0 |
| normalized size | 1 | 1. | 1.27 | 0.83 | 0. | 5.5 | 0. | 0. |
| time (sec) | N/A | 0.01 | 0.023 | 0.01 | 0. | 2.219 | 0. | 0. |

| Problem 212 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | A | A | F | B | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 40 | 34 | 33 | 0 | 165 | 0 | 0 |
| normalized size | 1 | 1.67 | 1.42 | 1.38 | 0. | 6.88 | 0. | 0. |
| time (sec) | N/A | 0.011 | 0.02 | 0.01 | 0. | 2.084 | 0. | 0. |

| Problem 213 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | A | A | F | B | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 73 | 72 | 71 | 59 | 0 | 331 | 0 | 0 |
| normalized size | 1 | 0.99 | 0.97 | 0.81 | 0. | 4.53 | 0. | 0. |
| time (sec) | N/A | 0.118 | 0.057 | 0.004 | 0. | 1.922 | 0. | 0. |

| Problem 214 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|------|-------------|-------|--------|--------|-------|-------|
| grade | A | A | A | B | F(-2) | B | B | F(-1) |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 121 | 121 | 121 | 226 | 0 | 903 | 343 | 0 |
| normalized size | 1 | 1. | 1. | 1.87 | 0. | 7.46 | 2.83 | 0. |
| time (sec) | N/A | 0.16 | 0.077 | 0.01 | 0. | 1.941 | 0.955 | 0. |

| Problem 215 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|-------|
| grade | A | A | A | A | F(-2) | A | B | F(-1) |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 86 | 86 | 84 | 142 | 0 | 632 | 275 | 0 |
| normalized size | 1 | 1. | 0.98 | 1.65 | 0. | 7.35 | 3.2 | 0. |
| time (sec) | N/A | 0.107 | 0.046 | 0.004 | 0. | 1.954 | 0.74 | 0. |

| Problem 216 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|-------|
| grade | A | A | A | A | F(-2) | A | B | F(-1) |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 64 | 64 | 63 | 79 | 0 | 424 | 212 | 0 |
| normalized size | 1 | 1. | 0.98 | 1.23 | 0. | 6.62 | 3.31 | 0. |
| time (sec) | N/A | 0.078 | 0.055 | 0.003 | 0. | 2.008 | 0.568 | 0. |

| Problem 217 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|-------|
| grade | A | A | A | A | F(-2) | A | B | F(-1) |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 49 | 49 | 48 | 33 | 0 | 279 | 124 | 0 |
| normalized size | 1 | 1. | 0.98 | 0.67 | 0. | 5.69 | 2.53 | 0. |
| time (sec) | N/A | 0.029 | 0.013 | 0.003 | 0. | 1.603 | 0.218 | 0. |

| Problem 218 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|--------|-------|
| grade | A | A | A | A | F(-2) | A | B | F(-1) |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 136 | 136 | 133 | 155 | 0 | 1829 | 2664 | 0 |
| normalized size | 1 | 1. | 0.98 | 1.14 | 0. | 13.45 | 19.59 | 0. |
| time (sec) | N/A | 0.178 | 0.208 | 0.02 | 0. | 2.546 | 22.464 | 0. |

| Problem 219 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|-------|
| grade | A | A | A | A | F(-2) | B | F(-1) | F(-1) |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 187 | 187 | 177 | 319 | 0 | 3611 | 0 | 0 |
| normalized size | 1 | 1. | 0.95 | 1.71 | 0. | 19.31 | 0. | 0. |
| time (sec) | N/A | 0.278 | 0.425 | 0.013 | 0. | 7.545 | 0. | 0. |

| Problem 220 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|-------|
| grade | A | A | A | B | F | A | F | F(-1) |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 139 | 139 | 134 | 7043 | 0 | 2303 | 0 | 0 |
| normalized size | 1 | 1. | 0.96 | 50.67 | 0. | 16.57 | 0. | 0. |
| time (sec) | N/A | 0.276 | 0.264 | 0.05 | 0. | 4.713 | 0. | 0. |

| Problem 221 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|-------|
| grade | A | A | A | B | F | A | F | F(-1) |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 108 | 108 | 103 | 4308 | 0 | 1985 | 0 | 0 |
| normalized size | 1 | 1. | 0.95 | 39.89 | 0. | 18.38 | 0. | 0. |
| time (sec) | N/A | 0.126 | 0.088 | 0.02 | 0. | 2.559 | 0. | 0. |

| Problem 222 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|------|-------------|-------|--------|--------|-------|-------|
| grade | A | A | A | B | F | B | F | F(-1) |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 76 | 76 | 76 | 2252 | 0 | 907 | 0 | 0 |
| normalized size | 1 | 1. | 1. | 29.63 | 0. | 11.93 | 0. | 0. |
| time (sec) | N/A | 0.07 | 0.068 | 0.02 | 0. | 2.316 | 0. | 0. |

| Problem 223 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|-------|
| grade | A | A | C | B | F | B | F | F(-1) |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD |
| size | 106 | 106 | 418 | 771 | 0 | 1440 | 0 | 0 |
| normalized size | 1 | 1. | 3.94 | 7.27 | 0. | 13.58 | 0. | 0. |
| time (sec) | N/A | 0.117 | 0.8 | 0.018 | 0. | 2.782 | 0. | 0. |

| Problem 224 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|-------|
| grade | A | A | C | B | F | B | F | F(-1) |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD |
| size | 149 | 149 | 1058 | 1637 | 0 | 2169 | 0 | 0 |
| normalized size | 1 | 1. | 7.1 | 10.99 | 0. | 14.56 | 0. | 0. |
| time (sec) | N/A | 0.269 | 3.492 | 0.02 | 0. | 5.156 | 0. | 0. |

| Problem 225 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | C | C | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 183 | 183 | 169 | 263 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0.92 | 1.44 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.088 | 0.313 | 0.174 | 0. | 0. | 0. | 0. |

| Problem 226 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | C | C | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 164 | 164 | 162 | 248 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0.99 | 1.51 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.061 | 0.154 | 0.006 | 0. | 0. | 0. | 0. |

| Problem 227 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | C | C | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 145 | 145 | 168 | 233 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 1.16 | 1.61 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.043 | 0.183 | 0.006 | 0. | 0. | 0. | 0. |

| Problem 228 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | C | C | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 137 | 137 | 118 | 293 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0.86 | 2.14 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.086 | 0.095 | 0.052 | 0. | 0. | 0. | 0. |

| Problem 229 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | C | C | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 49 | 49 | 164 | 224 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 3.35 | 4.57 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.011 | 0.353 | 0.017 | 0. | 0. | 0. | 0. |

| Problem 230 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | C | C | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 93 | 93 | 176 | 333 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 1.89 | 3.58 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.508 | 0.302 | 0.022 | 0. | 0. | 0. | 0. |

| Problem 231 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | C | C | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 166 | 166 | 240 | 438 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 1.45 | 2.64 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.615 | 0.419 | 0.024 | 0. | 0. | 0. | 0. |

| Problem 232 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | C | C | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 159 | 159 | 157 | 233 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0.99 | 1.47 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.071 | 0.173 | 0.027 | 0. | 0. | 0. | 0. |

| Problem 233 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | C | C | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 137 | 137 | 143 | 218 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 1.04 | 1.59 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.045 | 0.137 | 0.007 | 0. | 0. | 0. | 0. |

| Problem 234 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | C | C | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 115 | 115 | 94 | 205 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0.82 | 1.78 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.022 | 0.07 | 0.005 | 0. | 0. | 0. | 0. |

| Problem 235 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | C | C | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 69 | 69 | 73 | 104 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 1.06 | 1.51 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.059 | 0.069 | 0.016 | 0. | 0. | 0. | 0. |

| Problem 236 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | C | C | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 118 | 118 | 226 | 397 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 1.92 | 3.36 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.135 | 0.407 | 0.019 | 0. | 0. | 0. | 0. |

| Problem 237 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | C | C | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 142 | 142 | 235 | 418 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 1.65 | 2.94 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.277 | 0.332 | 0.021 | 0. | 0. | 0. | 0. |

| Problem 238 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | F | C | F | F | F | F |
| verified | N/A | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 144 | 144 | 0 | 268 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0. | 1.86 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.045 | 0. | 0.033 | 0. | 0. | 0. | 0. |

| Problem 239 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | F | C | F | F | F | F |
| verified | N/A | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 98 | 98 | 0 | 268 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0. | 2.73 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.025 | 0. | 0.007 | 0. | 0. | 0. | 0. |

| Problem 240 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | F | C | F | F | F | F |
| verified | N/A | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 96 | 96 | 0 | 247 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0. | 2.57 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.021 | 0. | 0.007 | 0. | 0. | 0. | 0. |

| Problem 241 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | C | C | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 166 | 166 | 204 | 398 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 1.23 | 2.4 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.104 | 0.213 | 0.017 | 0. | 0. | 0. | 0. |

| Problem 242 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | C | C | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 111 | 111 | 168 | 419 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 1.51 | 3.77 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.278 | 0.379 | 0.025 | 0. | 0. | 0. | 0. |

| Problem 243 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | C | C | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 190 | 190 | 192 | 439 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 1.01 | 2.31 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.571 | 0.336 | 0.024 | 0. | 0. | 0. | 0. |

| Problem 244 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|-------|
| grade | A | A | A | A | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 135 | 135 | 135 | 136 | 182 | 355 | 156 | 192 |
| normalized size | 1 | 1. | 1. | 1.01 | 1.35 | 2.63 | 1.16 | 1.42 |
| time (sec) | N/A | 0.126 | 0.038 | 0. | 0.973 | 1.349 | 0.093 | 1.203 |

| Problem 245 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|-------|
| grade | A | A | A | A | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 103 | 103 | 104 | 103 | 138 | 263 | 112 | 146 |
| normalized size | 1 | 1. | 1.01 | 1. | 1.34 | 2.55 | 1.09 | 1.42 |
| time (sec) | N/A | 0.095 | 0.029 | 0. | 0.982 | 1.388 | 0.084 | 1.136 |

| Problem 246 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|------|-------------|-------|--------|--------|-------|-------|
| grade | A | A | A | A | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 73 | 73 | 73 | 70 | 93 | 185 | 78 | 103 |
| normalized size | 1 | 1. | 1. | 0.96 | 1.27 | 2.53 | 1.07 | 1.41 |
| time (sec) | N/A | 0.06 | 0.02 | 0. | 0.969 | 1.368 | 0.078 | 1.157 |

| Problem 247 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|-------|
| grade | A | A | A | A | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 42 | 42 | 42 | 37 | 49 | 104 | 39 | 58 |
| normalized size | 1 | 1. | 1. | 0.88 | 1.17 | 2.48 | 0.93 | 1.38 |
| time (sec) | N/A | 0.027 | 0.008 | 0. | 0.95 | 1.382 | 0.063 | 1.138 |

| Problem 248 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|-------|
| grade | A | A | A | A | F(-2) | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 66 | 66 | 65 | 84 | 0 | 347 | 117 | 76 |
| normalized size | 1 | 1. | 0.98 | 1.27 | 0. | 5.26 | 1.77 | 1.15 |
| time (sec) | N/A | 0.045 | 0.051 | 0.003 | 0. | 1.637 | 0.64 | 1.262 |

| Problem 249 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|-------|
| grade | A | A | A | A | F(-2) | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 83 | 83 | 88 | 118 | 0 | 541 | 153 | 101 |
| normalized size | 1 | 1. | 1.06 | 1.42 | 0. | 6.52 | 1.84 | 1.22 |
| time (sec) | N/A | 0.093 | 0.056 | 0.008 | 0. | 1.628 | 1.063 | 1.155 |

| Problem 250 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|-------|
| grade | A | A | A | A | F(-2) | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 115 | 115 | 110 | 131 | 0 | 813 | 196 | 136 |
| normalized size | 1 | 1. | 0.96 | 1.14 | 0. | 7.07 | 1.7 | 1.18 |
| time (sec) | N/A | 0.107 | 0.097 | 0.008 | 0. | 1.573 | 1.808 | 1.137 |

| Problem 251 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|-------|
| grade | A | A | A | A | F(-2) | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 150 | 150 | 142 | 158 | 0 | 1091 | 241 | 181 |
| normalized size | 1 | 1. | 0.95 | 1.05 | 0. | 7.27 | 1.61 | 1.21 |
| time (sec) | N/A | 0.205 | 0.132 | 0.01 | 0. | 1.674 | 3.32 | 1.172 |

| Problem 252 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | A | A | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 223 | 223 | 223 | 219 | 294 | 620 | 272 | 344 |
| normalized size | 1 | 1. | 1. | 0.98 | 1.32 | 2.78 | 1.22 | 1.54 |
| time (sec) | N/A | 0.199 | 0.09 | 0.002 | 0.954 | 1.372 | 0.107 | 1.14 |

| Problem 253 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|-------|
| grade | A | A | A | A | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 155 | 155 | 156 | 155 | 198 | 436 | 192 | 244 |
| normalized size | 1 | 1. | 1.01 | 1. | 1.28 | 2.81 | 1.24 | 1.57 |
| time (sec) | N/A | 0.141 | 0.054 | 0.001 | 0.975 | 1.433 | 0.095 | 1.135 |

| Problem 254 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|-------|
| grade | A | A | A | A | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 96 | 96 | 96 | 91 | 122 | 252 | 107 | 143 |
| normalized size | 1 | 1. | 1. | 0.95 | 1.27 | 2.62 | 1.11 | 1.49 |
| time (sec) | N/A | 0.068 | 0.024 | 0. | 0.966 | 1.336 | 0.081 | 1.187 |

| Problem 255 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|-------|
| grade | A | A | A | A | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 49 | 49 | 49 | 42 | 61 | 104 | 48 | 58 |
| normalized size | 1 | 1. | 1. | 0.86 | 1.24 | 2.12 | 0.98 | 1.18 |
| time (sec) | N/A | 0.025 | 0.006 | 0. | 0.964 | 1.347 | 0.068 | 1.107 |

| Problem 256 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|------|-------------|-------|--------|--------|-------|-------|
| grade | A | A | A | B | F(-2) | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 143 | 143 | 144 | 267 | 0 | 892 | 366 | 250 |
| normalized size | 1 | 1. | 1.01 | 1.87 | 0. | 6.24 | 2.56 | 1.75 |
| time (sec) | N/A | 0.14 | 0.065 | 0.005 | 0. | 1.619 | 1.268 | 1.144 |

| Problem 257 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|-------|
| grade | A | A | A | B | F(-2) | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 166 | 166 | 183 | 320 | 0 | 1272 | 479 | 279 |
| normalized size | 1 | 1. | 1.1 | 1.93 | 0. | 7.66 | 2.89 | 1.68 |
| time (sec) | N/A | 0.298 | 0.103 | 0.013 | 0. | 1.715 | 3.274 | 1.123 |

| Problem 258 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|--------|-------|
| grade | A | A | A | B | F(-2) | B | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 201 | 201 | 217 | 402 | 0 | 1694 | 396 | 329 |
| normalized size | 1 | 1. | 1.08 | 2. | 0. | 8.43 | 1.97 | 1.64 |
| time (sec) | N/A | 0.419 | 0.115 | 0.012 | 0. | 1.724 | 17.105 | 1.132 |

| Problem 259 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|---------|-------|
| grade | A | A | A | B | F(-2) | B | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 250 | 250 | 267 | 506 | 0 | 2130 | 457 | 400 |
| normalized size | 1 | 1. | 1.07 | 2.02 | 0. | 8.52 | 1.83 | 1.6 |
| time (sec) | N/A | 0.543 | 0.15 | 0.011 | 0. | 1.69 | 110.179 | 1.114 |

| Problem 260 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|------|-------------|-------|--------|--------|-------|-------|
| grade | A | A | A | A | F(-2) | B | F(-1) | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 317 | 317 | 345 | 412 | 0 | 2766 | 0 | 491 |
| normalized size | 1 | 1. | 1.09 | 1.3 | 0. | 8.73 | 0. | 1.55 |
| time (sec) | N/A | 0.65 | 0.229 | 0.012 | 0. | 1.777 | 0. | 1.169 |

| Problem 261 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|-------|
| grade | A | A | A | A | F(-2) | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 83 | 83 | 88 | 118 | 0 | 541 | 153 | 101 |
| normalized size | 1 | 1. | 1.06 | 1.42 | 0. | 6.52 | 1.84 | 1.22 |
| time (sec) | N/A | 0.093 | 0.047 | 0. | 0. | 1.653 | 1.036 | 1.155 |

| Problem 262 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|-------|
| grade | A | A | A | A | F(-2) | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 83 | 83 | 88 | 118 | 0 | 541 | 153 | 101 |
| normalized size | 1 | 1. | 1.06 | 1.42 | 0. | 6.52 | 1.84 | 1.22 |
| time (sec) | N/A | 0.084 | 0.017 | 0.007 | 0. | 1.585 | 1.07 | 1.193 |

| Problem 263 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|-------|
| grade | A | A | A | B | F(-1) | F(-1) | F(-1) | F(-2) |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 459 | 459 | 570 | 1888 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 1.24 | 4.11 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 1.537 | 0.711 | 0.062 | 0. | 0. | 0. | 0. |

| Problem 264 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|---------|-------|-------|
| grade | A | A | A | B | F | B | F(-1) | F(-2) |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 316 | 316 | 402 | 1211 | 0 | 19539 | 0 | 0 |
| normalized size | 1 | 1. | 1.27 | 3.83 | 0. | 61.83 | 0. | 0. |
| time (sec) | N/A | 0.786 | 0.575 | 0.036 | 0. | 152.596 | 0. | 0. |

| Problem 265 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|-------|
| grade | A | A | A | B | F | B | B | F(-2) |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 238 | 238 | 269 | 695 | 0 | 9230 | 920 | 0 |
| normalized size | 1 | 1. | 1.13 | 2.92 | 0. | 38.78 | 3.87 | 0. |
| time (sec) | N/A | 0.635 | 0.332 | 0.027 | 0. | 13.603 | 45.21 | 0. |

| Problem 266 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|-------|
| grade | A | A | A | B | F | B | A | C |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 174 | 174 | 172 | 328 | 0 | 3055 | 314 | 6479 |
| normalized size | 1 | 1. | 0.99 | 1.89 | 0. | 17.56 | 1.8 | 37.24 |
| time (sec) | N/A | 0.202 | 0.145 | 0.019 | 0. | 2.288 | 4.925 | 2.491 |

| Problem 267 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|-------|
| grade | A | A | A | A | F | B | A | C |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 150 | 150 | 129 | 116 | 0 | 1323 | 87 | 1365 |
| normalized size | 1 | 1. | 0.86 | 0.77 | 0. | 8.82 | 0.58 | 9.1 |
| time (sec) | N/A | 0.098 | 0.086 | 0.013 | 0. | 2.065 | 0.906 | 1.418 |

| Problem 268 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|-------|
| grade | A | A | A | B | F(-2) | F(-1) | F(-1) | F(-2) |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 254 | 254 | 274 | 480 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 1.08 | 1.89 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.586 | 0.287 | 0.023 | 0. | 0. | 0. | 0. |

| Problem 269 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|-------|
| grade | A | A | A | B | F(-2) | F(-1) | F(-1) | F(-2) |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 429 | 429 | 354 | 1141 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0.83 | 2.66 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 1.415 | 0.825 | 0.03 | 0. | 0. | 0. | 0. |

| Problem 270 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|-------|
| grade | A | A | A | B | F | F(-1) | F(-1) | F(-2) |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 563 | 563 | 540 | 1846 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0.96 | 3.28 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 3.519 | 1.712 | 0.047 | 0. | 0. | 0. | 0. |

| Problem 271 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|-------|
| grade | A | A | A | B | F(-1) | B | F(-1) | F(-2) |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 386 | 386 | 415 | 1223 | 0 | 14976 | 0 | 0 |
| normalized size | 1 | 1. | 1.08 | 3.17 | 0. | 38.8 | 0. | 0. |
| time (sec) | N/A | 2.079 | 1.187 | 0.039 | 0. | 40.587 | 0. | 0. |

| Problem 272 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|-------|
| grade | A | A | A | B | F | B | B | F(-2) |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 293 | 293 | 310 | 1761 | 0 | 9543 | 1180 | 0 |
| normalized size | 1 | 1. | 1.06 | 6.01 | 0. | 32.57 | 4.03 | 0. |
| time (sec) | N/A | 0.789 | 0.806 | 0.08 | 0. | 6.946 | 60.3 | 0. |

| Problem 273 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|-------|
| grade | A | A | A | B | F | B | A | F(-2) |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 252 | 252 | 243 | 733 | 0 | 4918 | 394 | 0 |
| normalized size | 1 | 1. | 0.96 | 2.91 | 0. | 19.52 | 1.56 | 0. |
| time (sec) | N/A | 0.517 | 0.443 | 0.054 | 0. | 2.163 | 4.414 | 0. |

| Problem 274 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|-------|
| grade | A | A | A | B | F(-2) | F(-1) | F(-1) | F(-2) |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 660 | 660 | 708 | 3841 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 1.07 | 5.82 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 2.873 | 3.043 | 0.074 | 0. | 0. | 0. | 0. |

| Problem 275 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|--------|-------------|-------|--------|--------|-------|-------|
| grade | A | A | A | B | F(-2) | F(-1) | F(-1) | F(-2) |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 1077 | 1077 | 1020 | 5709 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0.95 | 5.3 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 12.639 | 6.289 | 0.082 | 0. | 0. | 0. | 0. |

| Problem 276 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|--------|-------|
| grade | A | A | A | A | F(-2) | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 215 | 215 | 190 | 283 | 0 | 882 | 505 | 243 |
| normalized size | 1 | 1. | 0.88 | 1.32 | 0. | 4.1 | 2.35 | 1.13 |
| time (sec) | N/A | 0.161 | 0.402 | 0.008 | 0. | 5.982 | 55.099 | 1.214 |

| Problem 277 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|--------|-------|
| grade | A | A | A | A | F(-2) | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 175 | 175 | 157 | 229 | 0 | 701 | 413 | 196 |
| normalized size | 1 | 1. | 0.9 | 1.31 | 0. | 4.01 | 2.36 | 1.12 |
| time (sec) | N/A | 0.122 | 0.33 | 0.008 | 0. | 4.948 | 28.138 | 1.135 |

| Problem 278 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|--------|-------|
| grade | A | A | A | A | F(-2) | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 132 | 132 | 121 | 175 | 0 | 533 | 272 | 143 |
| normalized size | 1 | 1. | 0.92 | 1.33 | 0. | 4.04 | 2.06 | 1.08 |
| time (sec) | N/A | 0.109 | 0.238 | 0.01 | 0. | 4.963 | 11.074 | 1.133 |

| Problem 279 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|-------|
| grade | A | A | A | A | F(-2) | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 97 | 97 | 82 | 122 | 0 | 408 | 230 | 107 |
| normalized size | 1 | 1. | 0.85 | 1.26 | 0. | 4.21 | 2.37 | 1.1 |
| time (sec) | N/A | 0.061 | 0.063 | 0.007 | 0. | 4.697 | 6.374 | 1.176 |

| Problem 280 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|-------|
| grade | A | A | A | A | F(-2) | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 89 | 89 | 98 | 112 | 0 | 552 | 134 | 108 |
| normalized size | 1 | 1. | 1.1 | 1.26 | 0. | 6.2 | 1.51 | 1.21 |
| time (sec) | N/A | 0.073 | 0.107 | 0.007 | 0. | 4.298 | 7.825 | 1.211 |

| Problem 281 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|--------|-------|
| grade | A | A | A | A | F(-2) | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 101 | 101 | 112 | 124 | 0 | 612 | 450 | 119 |
| normalized size | 1 | 1. | 1.11 | 1.23 | 0. | 6.06 | 4.46 | 1.18 |
| time (sec) | N/A | 0.071 | 0.194 | 0.009 | 0. | 4.888 | 18.452 | 1.124 |

| Problem 282 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|--------|-------|
| grade | A | A | A | A | B | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 86 | 86 | 67 | 66 | 234 | 198 | 639 | 101 |
| normalized size | 1 | 1. | 0.78 | 0.77 | 2.72 | 2.3 | 7.43 | 1.17 |
| time (sec) | N/A | 0.107 | 0.051 | 0.006 | 0.952 | 4.72 | 56.603 | 1.162 |

| Problem 283 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|---------|-------|
| grade | A | A | A | A | B | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 126 | 126 | 101 | 100 | 306 | 294 | 1989 | 153 |
| normalized size | 1 | 1. | 0.8 | 0.79 | 2.43 | 2.33 | 15.79 | 1.21 |
| time (sec) | N/A | 0.146 | 0.095 | 0.006 | 0.996 | 5.136 | 145.542 | 1.176 |

| Problem 284 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|------|-------------|-------|--------|--------|-------|-------|
| grade | A | A | A | A | A | A | F(-1) | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 165 | 164 | 132 | 136 | 379 | 386 | 0 | 200 |
| normalized size | 1 | 0.99 | 0.8 | 0.82 | 2.3 | 2.34 | 0. | 1.21 |
| time (sec) | N/A | 0.21 | 0.119 | 0.005 | 0.99 | 6.216 | 0. | 1.143 |

| Problem 285 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|-------|
| grade | A | A | A | A | A | A | F(-1) | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 210 | 210 | 167 | 172 | 452 | 502 | 0 | 255 |
| normalized size | 1 | 1. | 0.8 | 0.82 | 2.15 | 2.39 | 0. | 1.21 |
| time (sec) | N/A | 0.222 | 0.145 | 0.006 | 0.989 | 7.455 | 0. | 1.198 |

| Problem 286 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | C | C | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 193 | 193 | 119 | 172 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0.62 | 0.89 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.099 | 0.101 | 0.027 | 0. | 0. | 0. | 0. |

| Problem 287 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | C | C | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 168 | 168 | 114 | 155 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0.68 | 0.92 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.068 | 0.079 | 0.007 | 0. | 0. | 0. | 0. |

| Problem 288 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | C | C | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 149 | 149 | 109 | 137 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0.73 | 0.92 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.048 | 0.062 | 0.007 | 0. | 0. | 0. | 0. |

| Problem 289 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | C | C | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 141 | 141 | 102 | 121 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0.72 | 0.86 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.042 | 0.034 | 0.004 | 0. | 0. | 0. | 0. |

| Problem 290 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | C | C | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 178 | 232 | 90 | 138 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1.3 | 0.51 | 0.78 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.124 | 0.141 | 0.026 | 0. | 0. | 0. | 0. |

| Problem 291 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | C | C | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 209 | 209 | 208 | 162 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 1. | 0.78 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.123 | 0.268 | 0.02 | 0. | 0. | 0. | 0. |

| Problem 292 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | C | C | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 237 | 237 | 174 | 186 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0.73 | 0.78 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.595 | 0.342 | 0.018 | 0. | 0. | 0. | 0. |

| Problem 293 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | F | C | F | F | F | F |
| verified | N/A | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 219 | 219 | 0 | 206 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0. | 0.94 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.124 | 0. | 0.017 | 0. | 0. | 0. | 0. |

| Problem 294 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | F | C | F | F | F | F |
| verified | N/A | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 198 | 198 | 0 | 189 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0. | 0.95 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.086 | 0. | 0.008 | 0. | 0. | 0. | 0. |

| Problem 295 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | F | C | F | F | F | F |
| verified | N/A | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 179 | 179 | 0 | 172 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0. | 0.96 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.064 | 0. | 0.005 | 0. | 0. | 0. | 0. |

| Problem 296 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | C | C | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 172 | 172 | 114 | 155 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0.66 | 0.9 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.058 | 0.042 | 0.003 | 0. | 0. | 0. | 0. |

| Problem 297 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | C | C | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 207 | 207 | 148 | 170 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0.71 | 0.82 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.199 | 0.176 | 0.014 | 0. | 0. | 0. | 0. |

| Problem 298 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | C | C | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 222 | 333 | 213 | 177 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1.5 | 0.96 | 0.8 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.444 | 0.291 | 0.019 | 0. | 0. | 0. | 0. |

| Problem 299 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | C | C | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 231 | 288 | 174 | 186 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1.25 | 0.75 | 0.81 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.668 | 0.375 | 0.02 | 0. | 0. | 0. | 0. |

| Problem 300 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | C | C | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 157 | 157 | 106 | 138 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0.68 | 0.88 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.084 | 0.102 | 0.016 | 0. | 0. | 0. | 0. |

| Problem 301 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | C | C | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 142 | 142 | 104 | 121 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0.73 | 0.85 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.054 | 0.088 | 0.007 | 0. | 0. | 0. | 0. |

| Problem 302 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | C | C | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 121 | 121 | 69 | 106 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0.57 | 0.88 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.032 | 0.061 | 0.004 | 0. | 0. | 0. | 0. |

| Problem 303 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | C | C | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 48 | 48 | 50 | 46 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 1.04 | 0.96 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.006 | 0.015 | 0.003 | 0. | 0. | 0. | 0. |

| Problem 304 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | C | C | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 106 | 106 | 55 | 47 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0.52 | 0.44 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.072 | 0.094 | 0.012 | 0. | 0. | 0. | 0. |

| Problem 305 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | C | C | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 209 | 209 | 208 | 162 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 1. | 0.78 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.188 | 0.258 | 0.016 | 0. | 0. | 0. | 0. |

| Problem 306 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|------|-------------|-------|--------|--------|-------|------|
| grade | A | A | C | C | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 237 | 237 | 186 | 186 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0.78 | 0.78 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.25 | 0.334 | 0.017 | 0. | 0. | 0. | 0. |

| Problem 307 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | F | C | F | F | F | F |
| verified | N/A | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 189 | 189 | 0 | 274 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0. | 1.45 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.113 | 0. | 0.031 | 0. | 0. | 0. | 0. |

| Problem 308 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | F | C | F | F | F | F |
| verified | N/A | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 170 | 170 | 0 | 234 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0. | 1.38 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.083 | 0. | 0.007 | 0. | 0. | 0. | 0. |

| Problem 309 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | F | C | F | F | F | F |
| verified | N/A | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 149 | 149 | 0 | 196 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0. | 1.32 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.052 | 0. | 0.006 | 0. | 0. | 0. | 0. |

| Problem 310 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | F | C | F | F | F | F |
| verified | N/A | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 149 | 149 | 0 | 173 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0. | 1.16 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.051 | 0. | 0.007 | 0. | 0. | 0. | 0. |

| Problem 311 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | F | C | F | F | F | F |
| verified | N/A | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 145 | 145 | 0 | 150 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0. | 1.03 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.041 | 0. | 0.004 | 0. | 0. | 0. | 0. |

| Problem 312 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | C | C | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 149 | 149 | 99 | 129 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0.66 | 0.87 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.042 | 0.04 | 0.003 | 0. | 0. | 0. | 0. |

| Problem 313 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | C | C | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 173 | 207 | 138 | 161 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1.2 | 0.8 | 0.93 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.141 | 0.17 | 0.015 | 0. | 0. | 0. | 0. |

| Problem 314 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | C | C | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 235 | 235 | 208 | 185 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0.89 | 0.79 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.429 | 0.274 | 0.019 | 0. | 0. | 0. | 0. |

| Problem 315 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|------|-------------|-------|--------|--------|-------|------|
| grade | A | A | C | C | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 263 | 263 | 159 | 209 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0.6 | 0.79 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.76 | 0.489 | 0.023 | 0. | 0. | 0. | 0. |

| Problem 316 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | C | A | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 116 | 116 | 112 | 193 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0.97 | 1.66 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.112 | 0.126 | 0.032 | 0. | 0. | 0. | 0. |

| Problem 317 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | C | B | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 95 | 95 | 107 | 176 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 1.13 | 1.85 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.087 | 0.102 | 0.008 | 0. | 0. | 0. | 0. |

| Problem 318 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | C | B | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 74 | 74 | 102 | 159 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 1.38 | 2.15 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.061 | 0.089 | 0.009 | 0. | 0. | 0. | 0. |

| Problem 319 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | C | B | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 46 | 46 | 94 | 141 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 2.04 | 3.07 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.046 | 0.081 | 0.006 | 0. | 0. | 0. | 0. |

| Problem 320 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|------|-------------|-------|--------|--------|-------|------|
| grade | A | A | C | B | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 44 | 44 | 90 | 125 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 2.05 | 2.84 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.04 | 0.045 | 0.003 | 0. | 0. | 0. | 0. |

| Problem 321 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | C | B | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 46 | 46 | 51 | 141 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 1.11 | 3.07 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.079 | 0.13 | 0.013 | 0. | 0. | 0. | 0. |

| Problem 322 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|------|-------------|-------|--------|--------|-------|------|
| grade | A | A | C | B | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 74 | 74 | 196 | 165 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 2.65 | 2.23 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.08 | 0.277 | 0.02 | 0. | 0. | 0. | 0. |

| Problem 323 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | C | A | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 102 | 102 | 244 | 189 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 2.39 | 1.85 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.414 | 0.354 | 0.021 | 0. | 0. | 0. | 0. |

| Problem 324 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | F | A | F | F | F | F |
| verified | N/A | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 142 | 142 | 0 | 227 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0. | 1.6 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.132 | 0. | 0.023 | 0. | 0. | 0. | 0. |

| Problem 325 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | F | A | F | F | F | F |
| verified | N/A | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 121 | 121 | 0 | 210 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0. | 1.74 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.101 | 0. | 0.007 | 0. | 0. | 0. | 0. |

| Problem 326 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | F | B | F | F | F | F |
| verified | N/A | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 100 | 100 | 0 | 193 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0. | 1.93 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.074 | 0. | 0.008 | 0. | 0. | 0. | 0. |

| Problem 327 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | F | B | F | F | F | F |
| verified | N/A | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 81 | 81 | 0 | 176 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0. | 2.17 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.055 | 0. | 0.006 | 0. | 0. | 0. | 0. |

| Problem 328 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | C | B | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 74 | 74 | 102 | 159 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 1.38 | 2.15 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.052 | 0.053 | 0.003 | 0. | 0. | 0. | 0. |

| Problem 329 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | C | B | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 72 | 72 | 130 | 173 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 1.81 | 2.4 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.136 | 0.196 | 0.015 | 0. | 0. | 0. | 0. |

| Problem 330 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|------|-------------|-------|--------|--------|-------|------|
| grade | A | A | C | B | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 93 | 93 | 201 | 180 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 2.16 | 1.94 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.32 | 0.314 | 0.021 | 0. | 0. | 0. | 0. |

| Problem 331 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | C | A | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 102 | 102 | 244 | 189 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 2.39 | 1.85 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.497 | 0.423 | 0.018 | 0. | 0. | 0. | 0. |

| Problem 332 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | C | B | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 65 | 65 | 97 | 142 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 1.49 | 2.18 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.076 | 0.113 | 0.018 | 0. | 0. | 0. | 0. |

| Problem 333 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | C | B | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 46 | 46 | 92 | 125 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 2. | 2.72 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.051 | 0.097 | 0.007 | 0. | 0. | 0. | 0. |

| Problem 334 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | C | B | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 25 | 25 | 34 | 110 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 1.36 | 4.4 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.038 | 0.059 | 0.005 | 0. | 0. | 0. | 0. |

| Problem 335 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | C | B | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 10 | 10 | 19 | 47 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 1.9 | 4.7 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.011 | 0.015 | 0.004 | 0. | 0. | 0. | 0. |

| Problem 336 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | C | B | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 17 | 17 | 24 | 48 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 1.41 | 2.82 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.033 | 0.097 | 0.01 | 0. | 0. | 0. | 0. |

| Problem 337 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | C | B | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 74 | 74 | 196 | 165 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 2.65 | 2.23 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.134 | 0.279 | 0.016 | 0. | 0. | 0. | 0. |

| Problem 338 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | C | A | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 102 | 102 | 108 | 189 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 1.06 | 1.85 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.192 | 0.405 | 0.019 | 0. | 0. | 0. | 0. |

| Problem 339 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | F | B | F | F | F | F |
| verified | N/A | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 93 | 93 | 0 | 280 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0. | 3.01 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.101 | 0. | 0.033 | 0. | 0. | 0. | 0. |

| Problem 340 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | F | B | F | F | F | F |
| verified | N/A | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 74 | 74 | 0 | 240 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0. | 3.24 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.076 | 0. | 0.008 | 0. | 0. | 0. | 0. |

| Problem 341 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | F | B | F | F | F | F |
| verified | N/A | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 55 | 55 | 0 | 202 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0. | 3.67 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.051 | 0. | 0.008 | 0. | 0. | 0. | 0. |

| Problem 342 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | F | B | F | F | F | F |
| verified | N/A | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 55 | 55 | 0 | 179 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0. | 3.25 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.051 | 0. | 0.007 | 0. | 0. | 0. | 0. |

| Problem 343 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | F | B | F | F | F | F |
| verified | N/A | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 55 | 55 | 0 | 156 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0. | 2.84 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.045 | 0. | 0.006 | 0. | 0. | 0. | 0. |

| Problem 344 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | F | B | F | F | F | F |
| verified | N/A | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 55 | 55 | 0 | 133 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0. | 2.42 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.041 | 0. | 0.004 | 0. | 0. | 0. | 0. |

| Problem 345 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|------|-------------|-------|--------|--------|-------|------|
| grade | A | A | C | B | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 72 | 72 | 101 | 164 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 1.4 | 2.28 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.09 | 0.212 | 0.014 | 0. | 0. | 0. | 0. |

| Problem 346 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | C | B | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 100 | 100 | 196 | 188 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 1.96 | 1.88 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.297 | 0.325 | 0.02 | 0. | 0. | 0. | 0. |

| Problem 347 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|------|-------------|-------|--------|--------|-------|------|
| grade | A | A | C | A | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 128 | 128 | 244 | 212 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 1.91 | 1.66 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.57 | 0.402 | 0.023 | 0. | 0. | 0. | 0. |

| Problem 348 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|------|-------------|-------|--------|--------|-------|------|
| grade | A | A | C | C | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 242 | 242 | 354 | 292 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 1.46 | 1.21 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.15 | 0.588 | 0.148 | 0. | 0. | 0. | 0. |

| Problem 349 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | C | C | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 221 | 221 | 349 | 275 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 1.58 | 1.24 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.112 | 0.515 | 0.011 | 0. | 0. | 0. | 0. |

| Problem 350 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | C | C | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 198 | 198 | 343 | 258 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 1.73 | 1.3 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.077 | 0.473 | 0.009 | 0. | 0. | 0. | 0. |

| Problem 351 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | C | C | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 177 | 177 | 338 | 240 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 1.91 | 1.36 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.054 | 0.437 | 0.007 | 0. | 0. | 0. | 0. |

| Problem 352 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | C | C | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 169 | 169 | 331 | 224 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 1.96 | 1.33 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.051 | 0.354 | 0.003 | 0. | 0. | 0. | 0. |

| Problem 353 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | C | C | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 322 | 322 | 283 | 386 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0.88 | 1.2 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.152 | 0.251 | 0.051 | 0. | 0. | 0. | 0. |

| Problem 354 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | C | C | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 284 | 284 | 481 | 410 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 1.69 | 1.44 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.151 | 0.771 | 0.023 | 0. | 0. | 0. | 0. |

| Problem 355 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | C | C | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 312 | 312 | 308 | 434 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0.99 | 1.39 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.711 | 0.671 | 0.024 | 0. | 0. | 0. | 0. |

| Problem 356 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | F | C | F | F | F | F |
| verified | N/A | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 268 | 268 | 0 | 326 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0. | 1.22 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.174 | 0. | 0.039 | 0. | 0. | 0. | 0. |

| Problem 357 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|------|-------------|-------|--------|--------|-------|------|
| grade | A | A | F | C | F | F | F | F |
| verified | N/A | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 247 | 247 | 0 | 309 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0. | 1.25 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.13 | 0. | 0.007 | 0. | 0. | 0. | 0. |

| Problem 358 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | F | C | F | F | F | F |
| verified | N/A | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 226 | 226 | 0 | 292 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0. | 1.29 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.098 | 0. | 0.007 | 0. | 0. | 0. | 0. |

| Problem 359 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | F | C | F | F | F | F |
| verified | N/A | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 207 | 207 | 0 | 275 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0. | 1.33 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.071 | 0. | 0.007 | 0. | 0. | 0. | 0. |

| Problem 360 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | C | C | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 198 | 198 | 343 | 258 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 1.73 | 1.3 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.069 | 0.418 | 0.003 | 0. | 0. | 0. | 0. |

| Problem 361 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | C | C | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 284 | 284 | 477 | 418 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 1.68 | 1.47 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.236 | 0.718 | 0.019 | 0. | 0. | 0. | 0. |

| Problem 362 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | C | C | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 305 | 372 | 309 | 425 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1.22 | 1.01 | 1.39 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.534 | 0.585 | 0.023 | 0. | 0. | 0. | 0. |

| Problem 363 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|------|-------------|-------|--------|--------|-------|------|
| grade | A | A | C | C | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 440 | 440 | 309 | 434 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0.7 | 0.99 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.8 | 0.702 | 0.023 | 0. | 0. | 0. | 0. |

| Problem 364 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | C | C | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 187 | 187 | 337 | 241 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 1.8 | 1.29 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.093 | 0.484 | 0.029 | 0. | 0. | 0. | 0. |

| Problem 365 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | C | C | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 170 | 170 | 331 | 224 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 1.95 | 1.32 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.059 | 0.438 | 0.007 | 0. | 0. | 0. | 0. |

| Problem 366 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | C | C | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 151 | 151 | 214 | 209 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 1.42 | 1.38 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.036 | 0.177 | 0.007 | 0. | 0. | 0. | 0. |

| Problem 367 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | C | C | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 64 | 64 | 142 | 85 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 2.22 | 1.33 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.007 | 0.055 | 0.003 | 0. | 0. | 0. | 0. |

| Problem 368 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | C | C | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 168 | 168 | 159 | 107 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0.95 | 0.64 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.082 | 0.135 | 0.016 | 0. | 0. | 0. | 0. |

| Problem 369 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | C | C | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 286 | 286 | 481 | 410 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 1.68 | 1.43 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.225 | 0.783 | 0.02 | 0. | 0. | 0. | 0. |

| Problem 370 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|------|-------------|-------|--------|--------|-------|------|
| grade | A | A | C | C | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 314 | 314 | 308 | 434 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0.98 | 1.38 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.29 | 0.894 | 0.025 | 0. | 0. | 0. | 0. |

| Problem 371 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | C | C | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 219 | 219 | 339 | 379 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 1.55 | 1.73 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.121 | 0.521 | 0.048 | 0. | 0. | 0. | 0. |

| Problem 372 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | F | C | F | F | F | F |
| verified | N/A | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 200 | 200 | 0 | 339 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0. | 1.7 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.089 | 0. | 0.007 | 0. | 0. | 0. | 0. |

| Problem 373 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | F | C | F | F | F | F |
| verified | N/A | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 181 | 181 | 0 | 301 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0. | 1.66 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.059 | 0. | 0.006 | 0. | 0. | 0. | 0. |

| Problem 374 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | F | C | F | F | F | F |
| verified | N/A | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 181 | 181 | 0 | 278 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0. | 1.54 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.059 | 0. | 0.006 | 0. | 0. | 0. | 0. |

| Problem 375 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | F | C | F | F | F | F |
| verified | N/A | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 181 | 181 | 0 | 255 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0. | 1.41 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.052 | 0. | 0.004 | 0. | 0. | 0. | 0. |

| Problem 376 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|------|-------------|-------|--------|--------|-------|------|
| grade | A | A | C | C | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 181 | 181 | 328 | 232 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 1.81 | 1.28 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.05 | 0.357 | 0.004 | 0. | 0. | 0. | 0. |

| Problem 377 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | C | C | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 284 | 284 | 483 | 409 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 1.7 | 1.44 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.163 | 0.54 | 0.018 | 0. | 0. | 0. | 0. |

| Problem 378 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | C | C | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 312 | 312 | 311 | 433 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 1. | 1.39 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.505 | 0.593 | 0.025 | 0. | 0. | 0. | 0. |

| Problem 379 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | C | C | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 340 | 340 | 320 | 457 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0.94 | 1.34 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.872 | 0.739 | 0.025 | 0. | 0. | 0. | 0. |

| Problem 380 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | C | B | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 467 | 467 | 584 | 1186 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 1.25 | 2.54 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.425 | 2.904 | 0.066 | 0. | 0. | 0. | 0. |

| Problem 381 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | C | B | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 356 | 356 | 488 | 756 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 1.37 | 2.12 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.191 | 1.634 | 0.009 | 0. | 0. | 0. | 0. |

| Problem 382 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | C | A | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 283 | 283 | 302 | 362 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 1.07 | 1.28 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.083 | 0.261 | 0.003 | 0. | 0. | 0. | 0. |

| Problem 383 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | C | A | F | F(-1) | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 401 | 401 | 214 | 200 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0.53 | 0.5 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.346 | 0.228 | 0.03 | 0. | 0. | 0. | 0. |

| Problem 384 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | C | A | F | F(-1) | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 718 | 718 | 1069 | 1279 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 1.49 | 1.78 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 1.081 | 1.888 | 0.027 | 0. | 0. | 0. | 0. |

| Problem 385 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|------|-------------|-------|--------|--------|-------|------|
| grade | A | A | C | B | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 553 | 553 | 596 | 1195 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 1.08 | 2.16 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 1.28 | 2.474 | 0.058 | 0. | 0. | 0. | 0. |

| Problem 386 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | C | A | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 454 | 454 | 503 | 761 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 1.11 | 1.68 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.794 | 1.396 | 0.01 | 0. | 0. | 0. | 0. |

| Problem 387 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | C | A | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 385 | 385 | 293 | 364 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0.76 | 0.95 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.337 | 0.261 | 0.006 | 0. | 0. | 0. | 0. |

| Problem 388 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | C | A | F | F(-1) | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 197 | 197 | 205 | 201 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 1.04 | 1.02 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.157 | 0.23 | 0.029 | 0. | 0. | 0. | 0. |

| Problem 389 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | C | B | F | F(-1) | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 718 | 718 | 464 | 1293 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0.65 | 1.8 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 1.015 | 5.593 | 0.025 | 0. | 0. | 0. | 0. |

| Problem 390 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | C | A | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 479 | 479 | 304 | 355 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0.63 | 0.74 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.475 | 0.301 | 0.028 | 0. | 0. | 0. | 0. |

| Problem 391 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | C | A | F | F(-1) | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 204 | 204 | 216 | 198 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 1.06 | 0.97 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.192 | 0.223 | 0.024 | 0. | 0. | 0. | 0. |

| Problem 392 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|------|-------------|-------|--------|--------|-------|------|
| grade | A | A | C | A | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 293 | 293 | 295 | 357 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 1.01 | 1.22 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.09 | 0.311 | 0.03 | 0. | 0. | 0. | 0. |

| Problem 393 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | C | A | F | F(-1) | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 412 | 412 | 207 | 199 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0.5 | 0.48 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.363 | 0.23 | 0.023 | 0. | 0. | 0. | 0. |

| Problem 394 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | C | C | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 229 | 229 | 154 | 380 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0.67 | 1.66 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.154 | 0.229 | 0.01 | 0. | 0. | 0. | 0. |

| Problem 395 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | C | C | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 168 | 168 | 127 | 235 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0.76 | 1.4 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.074 | 0.154 | 0.009 | 0. | 0. | 0. | 0. |

| Problem 396 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | C | C | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 122 | 122 | 73 | 108 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0.6 | 0.89 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.034 | 0.075 | 0.006 | 0. | 0. | 0. | 0. |

| Problem 397 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | C | C | F(-2) | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 124 | 124 | 59 | 55 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0.48 | 0.44 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.091 | 0.11 | 0.016 | 0. | 0. | 0. | 0. |

| Problem 398 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | C | C | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 316 | 399 | 175 | 443 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1.26 | 0.55 | 1.4 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.327 | 0.583 | 0.026 | 0. | 0. | 0. | 0. |

| Problem 399 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | N/A | A | A | A | A | A | F(-1) | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 0. | 0. | 0. | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.011 | 0.099 | 0.074 | 0. | 0. | 0. | 0. |

| Problem 400 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | A | F | F | F | F(-1) | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD |
| size | 498 | 498 | 373 | 0 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0.75 | 0. | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.814 | 0.518 | 0.051 | 0. | 0. | 0. | 0. |

| Problem 401 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | A | F | F | F | F(-1) | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD |
| size | 358 | 345 | 303 | 0 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 0.96 | 0.85 | 0. | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.356 | 0.354 | 0.041 | 0. | 0. | 0. | 0. |

| Problem 402 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | A | F | F | F | F(-1) | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD |
| size | 274 | 274 | 232 | 0 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0.85 | 0. | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.222 | 0.244 | 0.025 | 0. | 0. | 0. | 0. |

| Problem 403 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | A | F | F | F | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD |
| size | 133 | 133 | 161 | 0 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 1.21 | 0. | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.058 | 0.181 | 0.013 | 0. | 0. | 0. | 0. |

| Problem 404 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | N/A | A | A | A | A | A | F(-1) | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 0. | 0. | 0. | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.011 | 0.131 | 0.037 | 0. | 0. | 0. | 0. |

| Problem 405 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | N/A | A | A | A | A | A | F(-1) | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 0. | 0. | 0. | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.011 | 0.267 | 0.053 | 0. | 0. | 0. | 0. |

| Problem 406 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | C | C | F | F(-1) | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 446 | 446 | 258 | 251 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0.58 | 0.56 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.497 | 0.695 | 0.052 | 0. | 0. | 0. | 0. |

| Problem 407 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | C | A | F | F(-1) | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 218 | 218 | 719 | 247 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 3.3 | 1.13 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.295 | 1.227 | 0.029 | 0. | 0. | 0. | 0. |

| Problem 408 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | C | C | F | B | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD |
| size | 65 | 65 | 685 | 327 | 0 | 953 | 0 | 0 |
| normalized size | 1 | 1. | 10.54 | 5.03 | 0. | 14.66 | 0. | 0. |
| time (sec) | N/A | 0.145 | 1.627 | 0.153 | 0. | 3.567 | 0. | 0. |

| Problem 409 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | C | C | F | B | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD |
| size | 63 | 63 | 876 | 311 | 0 | 301 | 0 | 0 |
| normalized size | 1 | 1. | 13.9 | 4.94 | 0. | 4.78 | 0. | 0. |
| time (sec) | N/A | 0.141 | 5.614 | 0.138 | 0. | 2.665 | 0. | 0. |

| Problem 410 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | C | C | F | B | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD |
| size | 72 | 72 | 623 | 336 | 0 | 950 | 0 | 0 |
| normalized size | 1 | 1. | 8.65 | 4.67 | 0. | 13.19 | 0. | 0. |
| time (sec) | N/A | 0.133 | 1.632 | 0.143 | 0. | 2.705 | 0. | 0. |

| Problem 411 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | C | C | F | B | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD |
| size | 70 | 70 | 881 | 337 | 0 | 302 | 0 | 0 |
| normalized size | 1 | 1. | 12.59 | 4.81 | 0. | 4.31 | 0. | 0. |
| time (sec) | N/A | 0.129 | 5.818 | 0.134 | 0. | 2.683 | 0. | 0. |

| Problem 412 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | C | A | F | F(-1) | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD |
| size | 560 | 560 | 3652 | 437 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 6.52 | 0.78 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.617 | 7.824 | 0.023 | 0. | 0. | 0. | 0. |

| Problem 413 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|-----------------|---------|-------|-------------|-------|--------|--------|-------|------|
| grade | A | A | C | A | F | F(-1) | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 527 | 527 | 3658 | 439 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 6.94 | 0.83 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.717 | 7.863 | 0.021 | 0. | 0. | 0. | 0. |

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [147] had the largest ratio of [0.7778]

Table 2.1: Rubi specific breakdown of results for each integral

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|----|-------|----------------------|------------------------|-------------------------------------|---------------------|---|
| 1 | A | 9 | 6 | 1. | 17 | 0.353 |
| 2 | A | 9 | 6 | 1. | 18 | 0.333 |
| 3 | A | 3 | 3 | 1. | 18 | 0.167 |
| 4 | A | 3 | 3 | 1. | 19 | 0.158 |
| 5 | A | 5 | 3 | 1. | 17 | 0.176 |
| 6 | A | 3 | 2 | 1. | 17 | 0.118 |
| 7 | A | 2 | 2 | 1. | 17 | 0.118 |
| 8 | A | 2 | 2 | 1. | 17 | 0.118 |
| 9 | A | 5 | 3 | 1. | 27 | 0.111 |
| 10 | A | 3 | 2 | 1. | 28 | 0.071 |
| 11 | A | 5 | 3 | 1. | 21 | 0.143 |
| 12 | A | 3 | 2 | 1. | 22 | 0.091 |
| 13 | A | 3 | 3 | 1. | 15 | 0.2 |
| 14 | A | 2 | 2 | 1. | 22 | 0.091 |
| 15 | A | 5 | 5 | 1. | 23 | 0.217 |
| 16 | A | 3 | 3 | 1. | 21 | 0.143 |
| 17 | A | 6 | 6 | 1. | 22 | 0.273 |
| 18 | A | 1 | 1 | 1. | 22 | 0.045 |
| 19 | A | 3 | 3 | 1. | 21 | 0.143 |
| 20 | A | 1 | 1 | 1. | 23 | 0.043 |
| 21 | A | 3 | 3 | 1. | 22 | 0.136 |
| 22 | A | 1 | 1 | 1. | 28 | 0.036 |
| 23 | A | 2 | 2 | 1. | 24 | 0.083 |
| 24 | A | 4 | 4 | 1. | 28 | 0.143 |
| 25 | A | 5 | 5 | 1. | 25 | 0.2 |
| 26 | A | 5 | 3 | 1. | 26 | 0.115 |
| 27 | A | 5 | 3 | 1. | 26 | 0.115 |
| 28 | A | 5 | 3 | 1. | 27 | 0.111 |
| 29 | A | 5 | 3 | 1. | 27 | 0.111 |
| 30 | A | 3 | 2 | 1. | 27 | 0.074 |
| 31 | A | 3 | 2 | 1. | 27 | 0.074 |
| 32 | A | 3 | 2 | 1. | 28 | 0.071 |
| 33 | A | 3 | 2 | 1. | 28 | 0.071 |
| 34 | A | 3 | 2 | 1. | 30 | 0.067 |
| 35 | A | 5 | 3 | 1. | 29 | 0.103 |
| 36 | A | 6 | 4 | 1. | 29 | 0.138 |
| 37 | A | 3 | 2 | 1. | 32 | 0.062 |
| 38 | A | 5 | 3 | 1. | 31 | 0.097 |
| 39 | A | 5 | 3 | 1. | 22 | 0.136 |
| 40 | A | 5 | 3 | 1. | 23 | 0.13 |
| 41 | A | 3 | 2 | 1. | 22 | 0.091 |
| 42 | A | 3 | 2 | 1. | 22 | 0.091 |
| 43 | A | 3 | 3 | 1. | 22 | 0.136 |

Continued on next page

Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|----|-------|----------------------|------------------------|-------------------------------------|---------------------|---|
| 44 | A | 5 | 3 | 1. | 22 | 0.136 |
| 45 | A | 5 | 3 | 1. | 22 | 0.136 |
| 46 | A | 5 | 3 | 1. | 20 | 0.15 |
| 47 | A | 5 | 3 | 1. | 17 | 0.176 |
| 48 | A | 5 | 3 | 1. | 22 | 0.136 |
| 49 | A | 5 | 3 | 1. | 22 | 0.136 |
| 50 | A | 5 | 3 | 1. | 22 | 0.136 |
| 51 | A | 2 | 2 | 1. | 22 | 0.091 |
| 52 | A | 7 | 3 | 1. | 22 | 0.136 |
| 53 | A | 5 | 3 | 1. | 22 | 0.136 |
| 54 | A | 3 | 2 | 1. | 22 | 0.091 |
| 55 | A | 3 | 2 | 1. | 22 | 0.091 |
| 56 | A | 3 | 2 | 1. | 22 | 0.091 |
| 57 | A | 2 | 2 | 1. | 22 | 0.091 |
| 58 | A | 3 | 2 | 1. | 22 | 0.091 |
| 59 | A | 3 | 2 | 1. | 22 | 0.091 |
| 60 | A | 3 | 2 | 1. | 20 | 0.1 |
| 61 | A | 3 | 2 | 1. | 17 | 0.118 |
| 62 | A | 3 | 2 | 1. | 22 | 0.091 |
| 63 | A | 3 | 2 | 1. | 22 | 0.091 |
| 64 | A | 3 | 2 | 1. | 22 | 0.091 |
| 65 | A | 3 | 3 | 1. | 22 | 0.136 |
| 66 | A | 7 | 3 | 1. | 22 | 0.136 |
| 67 | A | 5 | 3 | 1. | 22 | 0.136 |
| 68 | A | 5 | 3 | 1. | 18 | 0.167 |
| 69 | A | 3 | 2 | 1. | 18 | 0.111 |
| 70 | A | 3 | 2 | 1. | 18 | 0.111 |
| 71 | A | 3 | 2 | 1. | 18 | 0.111 |
| 72 | A | 2 | 2 | 1. | 18 | 0.111 |
| 73 | A | 5 | 3 | 1. | 16 | 0.188 |
| 74 | A | 5 | 3 | 1. | 13 | 0.231 |
| 75 | A | 5 | 3 | 1. | 18 | 0.167 |
| 76 | A | 2 | 2 | 1. | 18 | 0.111 |
| 77 | A | 7 | 3 | 1. | 18 | 0.167 |
| 78 | A | 5 | 3 | 1. | 18 | 0.167 |
| 79 | A | 5 | 3 | 1. | 18 | 0.167 |
| 80 | A | 3 | 2 | 1. | 20 | 0.1 |
| 81 | A | 3 | 2 | 1. | 20 | 0.1 |
| 82 | A | 3 | 2 | 1. | 20 | 0.1 |
| 83 | A | 3 | 2 | 1. | 20 | 0.1 |
| 84 | A | 2 | 2 | 1. | 20 | 0.1 |
| 85 | A | 3 | 2 | 1. | 18 | 0.111 |
| 86 | A | 3 | 2 | 1. | 15 | 0.133 |
| 87 | A | 3 | 2 | 1. | 20 | 0.1 |

Continued on next page

Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|-----|-------|----------------------|------------------------|-------------------------------------|---------------------|---|
| 88 | A | 3 | 3 | 1. | 20 | 0.15 |
| 89 | A | 5 | 3 | 1. | 20 | 0.15 |
| 90 | A | 5 | 3 | 1. | 20 | 0.15 |
| 91 | A | 5 | 3 | 1. | 20 | 0.15 |
| 92 | A | 5 | 3 | 1. | 23 | 0.13 |
| 93 | A | 5 | 3 | 1. | 22 | 0.136 |
| 94 | A | 3 | 3 | 1. | 20 | 0.15 |
| 95 | A | 3 | 2 | 1. | 22 | 0.091 |
| 96 | A | 3 | 2 | 1. | 22 | 0.091 |
| 97 | A | 3 | 2 | 1. | 18 | 0.111 |
| 98 | A | 9 | 5 | 1. | 18 | 0.278 |
| 99 | A | 10 | 6 | 1. | 18 | 0.333 |
| 100 | A | 9 | 5 | 1. | 18 | 0.278 |
| 101 | A | 10 | 6 | 1. | 18 | 0.333 |
| 102 | A | 9 | 5 | 1. | 29 | 0.172 |
| 103 | A | 9 | 5 | 1. | 26 | 0.192 |
| 104 | A | 9 | 5 | 1. | 24 | 0.208 |
| 105 | A | 9 | 5 | 1. | 22 | 0.227 |
| 106 | A | 9 | 5 | 1. | 25 | 0.2 |
| 107 | A | 9 | 5 | 1. | 31 | 0.161 |
| 108 | A | 9 | 5 | 1. | 32 | 0.156 |
| 109 | A | 9 | 5 | 1. | 23 | 0.217 |
| 110 | A | 9 | 5 | 1. | 25 | 0.2 |
| 111 | A | 9 | 5 | 1. | 29 | 0.172 |
| 112 | A | 9 | 5 | 1. | 32 | 0.156 |
| 113 | A | 4 | 4 | 1. | 22 | 0.182 |
| 114 | A | 5 | 5 | 1. | 24 | 0.208 |
| 115 | A | 4 | 4 | 1. | 24 | 0.167 |
| 116 | A | 4 | 4 | 1. | 24 | 0.167 |
| 117 | A | 4 | 4 | 1. | 24 | 0.167 |
| 118 | A | 4 | 4 | 1. | 24 | 0.167 |
| 119 | A | 3 | 3 | 1. | 39 | 0.077 |
| 120 | A | 2 | 1 | 1. | 17 | 0.059 |
| 121 | A | 2 | 1 | 1. | 17 | 0.059 |
| 122 | A | 2 | 1 | 1. | 17 | 0.059 |
| 123 | A | 2 | 1 | 1. | 15 | 0.067 |
| 124 | A | 3 | 2 | 1. | 17 | 0.118 |
| 125 | A | 3 | 3 | 1. | 17 | 0.176 |
| 126 | A | 3 | 3 | 1. | 17 | 0.176 |
| 127 | A | 4 | 4 | 1. | 17 | 0.235 |
| 128 | A | 2 | 1 | 1. | 19 | 0.053 |
| 129 | A | 2 | 1 | 1. | 19 | 0.053 |
| 130 | A | 2 | 1 | 1. | 17 | 0.059 |
| 131 | A | 2 | 1 | 1. | 9 | 0.111 |

Continued on next page

Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|-----|-------|----------------------|------------------------|-------------------------------------|---------------------|---|
| 132 | A | 3 | 2 | 1. | 19 | 0.105 |
| 133 | A | 4 | 3 | 1. | 19 | 0.158 |
| 134 | A | 5 | 4 | 1. | 19 | 0.21 |
| 135 | A | 5 | 5 | 1. | 19 | 0.263 |
| 136 | A | 5 | 5 | 1. | 19 | 0.263 |
| 137 | A | 11 | 7 | 1. | 19 | 0.368 |
| 138 | A | 11 | 7 | 1. | 19 | 0.368 |
| 139 | A | 11 | 7 | 1. | 19 | 0.368 |
| 140 | A | 9 | 6 | 1. | 17 | 0.353 |
| 141 | A | 9 | 6 | 1. | 9 | 0.667 |
| 142 | A | 12 | 8 | 1. | 19 | 0.421 |
| 143 | A | 14 | 9 | 1. | 19 | 0.474 |
| 144 | A | 11 | 8 | 1. | 19 | 0.421 |
| 145 | A | 11 | 8 | 1. | 19 | 0.421 |
| 146 | A | 10 | 7 | 1. | 17 | 0.412 |
| 147 | A | 10 | 7 | 1. | 9 | 0.778 |
| 148 | A | 22 | 9 | 1. | 19 | 0.474 |
| 149 | A | 24 | 10 | 1. | 19 | 0.526 |
| 150 | A | 6 | 5 | 0.99 | 21 | 0.238 |
| 151 | A | 5 | 5 | 1. | 21 | 0.238 |
| 152 | A | 4 | 4 | 1. | 21 | 0.19 |
| 153 | A | 3 | 3 | 1. | 19 | 0.158 |
| 154 | A | 3 | 3 | 1. | 21 | 0.143 |
| 155 | A | 6 | 6 | 1. | 21 | 0.286 |
| 156 | A | 7 | 7 | 1. | 21 | 0.333 |
| 157 | A | 8 | 8 | 1. | 22 | 0.364 |
| 158 | A | 7 | 7 | 1. | 22 | 0.318 |
| 159 | A | 6 | 6 | 1. | 20 | 0.3 |
| 160 | A | 2 | 2 | 1. | 22 | 0.091 |
| 161 | A | 10 | 10 | 1. | 22 | 0.454 |
| 162 | A | 11 | 11 | 1. | 22 | 0.5 |
| 163 | A | 12 | 11 | 1. | 22 | 0.5 |
| 164 | A | 6 | 6 | 1. | 21 | 0.286 |
| 165 | A | 2 | 2 | 1. | 23 | 0.087 |
| 166 | A | 3 | 3 | 1. | 29 | 0.103 |
| 167 | A | 3 | 3 | 1. | 29 | 0.103 |
| 168 | A | 3 | 3 | 1. | 22 | 0.136 |
| 169 | A | 3 | 3 | 1. | 24 | 0.125 |
| 170 | A | 2 | 2 | 1. | 21 | 0.095 |
| 171 | A | 3 | 3 | 1. | 21 | 0.143 |
| 172 | A | 1 | 1 | 1. | 22 | 0.045 |
| 173 | A | 3 | 3 | 1. | 21 | 0.143 |
| 174 | F | 0 | 0 | N/A | 0 | N/A |
| 175 | A | 0 | 0 | 0. | 0 | 0. |

Continued on next page

Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|-----|-------|----------------------|------------------------|-------------------------------------|---------------------|---|
| 176 | A | 9 | 6 | 0.96 | 19 | 0.316 |
| 177 | A | 7 | 6 | 0.95 | 19 | 0.316 |
| 178 | A | 6 | 5 | 1. | 17 | 0.294 |
| 179 | A | 2 | 2 | 1. | 9 | 0.222 |
| 180 | A | 6 | 5 | 1. | 19 | 0.263 |
| 181 | A | 8 | 5 | 1. | 19 | 0.263 |
| 182 | A | 6 | 4 | 0.95 | 19 | 0.21 |
| 183 | A | 5 | 4 | 0.92 | 19 | 0.21 |
| 184 | A | 4 | 3 | 1. | 17 | 0.176 |
| 185 | A | 1 | 1 | 1. | 9 | 0.111 |
| 186 | A | 4 | 3 | 1. | 19 | 0.158 |
| 187 | A | 5 | 3 | 1. | 19 | 0.158 |
| 188 | A | 6 | 3 | 1. | 19 | 0.158 |
| 189 | A | 4 | 3 | 1. | 24 | 0.125 |
| 190 | A | 4 | 3 | 1. | 24 | 0.125 |
| 191 | A | 3 | 3 | 1. | 24 | 0.125 |
| 192 | A | 2 | 2 | 1. | 22 | 0.091 |
| 193 | A | 5 | 5 | 1. | 24 | 0.208 |
| 194 | A | 6 | 6 | 1. | 24 | 0.25 |
| 195 | A | 6 | 6 | 1. | 26 | 0.231 |
| 196 | A | 3 | 3 | 1. | 26 | 0.115 |
| 197 | A | 4 | 4 | 1. | 26 | 0.154 |
| 198 | A | 6 | 6 | 1. | 26 | 0.231 |
| 199 | A | 5 | 5 | 1. | 28 | 0.179 |
| 200 | A | 4 | 4 | 1. | 28 | 0.143 |
| 201 | A | 3 | 3 | 1. | 28 | 0.107 |
| 202 | A | 3 | 3 | 1. | 28 | 0.107 |
| 203 | A | 4 | 4 | 1. | 28 | 0.143 |
| 204 | A | 6 | 6 | 1. | 28 | 0.214 |
| 205 | A | 5 | 5 | 1. | 29 | 0.172 |
| 206 | A | 4 | 4 | 1. | 29 | 0.138 |
| 207 | A | 3 | 3 | 1. | 29 | 0.103 |
| 208 | A | 3 | 3 | 1. | 29 | 0.103 |
| 209 | A | 4 | 4 | 1. | 29 | 0.138 |
| 210 | A | 6 | 6 | 1. | 29 | 0.207 |
| 211 | A | 2 | 2 | 1. | 19 | 0.105 |
| 212 | A | 3 | 3 | 1.67 | 19 | 0.158 |
| 213 | A | 7 | 5 | 0.99 | 31 | 0.161 |
| 214 | A | 4 | 3 | 1. | 39 | 0.077 |
| 215 | A | 4 | 3 | 1. | 39 | 0.077 |
| 216 | A | 3 | 3 | 1. | 39 | 0.077 |
| 217 | A | 2 | 2 | 1. | 37 | 0.054 |
| 218 | A | 5 | 5 | 1. | 39 | 0.128 |
| 219 | A | 6 | 6 | 1. | 39 | 0.154 |

Continued on next page

Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|-----|-------|----------------------|------------------------|-------------------------------------|---------------------|---|
| 220 | A | 7 | 7 | 1. | 41 | 0.171 |
| 221 | A | 6 | 6 | 1. | 41 | 0.146 |
| 222 | A | 3 | 3 | 1. | 41 | 0.073 |
| 223 | A | 4 | 4 | 1. | 41 | 0.098 |
| 224 | A | 6 | 6 | 1. | 41 | 0.146 |
| 225 | A | 6 | 6 | 1. | 20 | 0.3 |
| 226 | A | 5 | 5 | 1. | 20 | 0.25 |
| 227 | A | 4 | 4 | 1. | 18 | 0.222 |
| 228 | A | 8 | 7 | 1. | 20 | 0.35 |
| 229 | A | 1 | 1 | 1. | 20 | 0.05 |
| 230 | A | 23 | 13 | 1. | 20 | 0.65 |
| 231 | A | 26 | 14 | 1. | 20 | 0.7 |
| 232 | A | 5 | 5 | 1. | 20 | 0.25 |
| 233 | A | 4 | 4 | 1. | 20 | 0.2 |
| 234 | A | 3 | 3 | 1. | 18 | 0.167 |
| 235 | A | 4 | 4 | 1. | 20 | 0.2 |
| 236 | A | 8 | 8 | 1. | 20 | 0.4 |
| 237 | A | 9 | 9 | 1. | 20 | 0.45 |
| 238 | A | 4 | 4 | 1. | 20 | 0.2 |
| 239 | A | 2 | 2 | 1. | 20 | 0.1 |
| 240 | A | 2 | 2 | 1. | 18 | 0.111 |
| 241 | A | 9 | 8 | 1. | 20 | 0.4 |
| 242 | A | 16 | 11 | 1. | 20 | 0.55 |
| 243 | A | 23 | 14 | 1. | 20 | 0.7 |
| 244 | A | 2 | 1 | 1. | 22 | 0.045 |
| 245 | A | 2 | 1 | 1. | 22 | 0.045 |
| 246 | A | 2 | 1 | 1. | 22 | 0.045 |
| 247 | A | 2 | 1 | 1. | 20 | 0.05 |
| 248 | A | 3 | 2 | 1. | 22 | 0.091 |
| 249 | A | 3 | 3 | 1. | 22 | 0.136 |
| 250 | A | 3 | 3 | 1. | 22 | 0.136 |
| 251 | A | 4 | 4 | 1. | 22 | 0.182 |
| 252 | A | 2 | 1 | 1. | 24 | 0.042 |
| 253 | A | 2 | 1 | 1. | 24 | 0.042 |
| 254 | A | 2 | 1 | 1. | 22 | 0.045 |
| 255 | A | 2 | 1 | 1. | 14 | 0.071 |
| 256 | A | 3 | 2 | 1. | 24 | 0.083 |
| 257 | A | 4 | 3 | 1. | 24 | 0.125 |
| 258 | A | 5 | 4 | 1. | 24 | 0.167 |
| 259 | A | 5 | 4 | 1. | 24 | 0.167 |
| 260 | A | 5 | 4 | 1. | 24 | 0.167 |
| 261 | A | 3 | 3 | 1. | 22 | 0.136 |
| 262 | A | 3 | 3 | 1. | 23 | 0.13 |
| 263 | A | 5 | 3 | 1. | 24 | 0.125 |

Continued on next page

Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|-----|-------|----------------------|------------------------|-------------------------------------|---------------------|---|
| 264 | A | 5 | 3 | 1. | 24 | 0.125 |
| 265 | A | 5 | 3 | 1. | 24 | 0.125 |
| 266 | A | 3 | 2 | 1. | 22 | 0.091 |
| 267 | A | 3 | 2 | 1. | 14 | 0.143 |
| 268 | A | 6 | 3 | 1. | 24 | 0.125 |
| 269 | A | 8 | 4 | 1. | 24 | 0.167 |
| 270 | A | 4 | 3 | 1. | 24 | 0.125 |
| 271 | A | 4 | 3 | 1. | 24 | 0.125 |
| 272 | A | 4 | 3 | 1. | 22 | 0.136 |
| 273 | A | 4 | 3 | 1. | 14 | 0.214 |
| 274 | A | 10 | 4 | 1. | 24 | 0.167 |
| 275 | A | 12 | 5 | 1. | 24 | 0.208 |
| 276 | A | 7 | 5 | 1. | 24 | 0.208 |
| 277 | A | 6 | 5 | 1. | 24 | 0.208 |
| 278 | A | 5 | 5 | 1. | 24 | 0.208 |
| 279 | A | 4 | 4 | 1. | 24 | 0.167 |
| 280 | A | 4 | 4 | 1. | 24 | 0.167 |
| 281 | A | 4 | 4 | 1. | 24 | 0.167 |
| 282 | A | 4 | 4 | 1. | 24 | 0.167 |
| 283 | A | 5 | 5 | 1. | 24 | 0.208 |
| 284 | A | 6 | 5 | 0.99 | 24 | 0.208 |
| 285 | A | 7 | 5 | 1. | 24 | 0.208 |
| 286 | A | 6 | 6 | 1. | 24 | 0.25 |
| 287 | A | 5 | 5 | 1. | 24 | 0.208 |
| 288 | A | 4 | 4 | 1. | 22 | 0.182 |
| 289 | A | 4 | 4 | 1. | 14 | 0.286 |
| 290 | A | 8 | 7 | 1.3 | 24 | 0.292 |
| 291 | A | 8 | 7 | 1. | 24 | 0.292 |
| 292 | A | 25 | 10 | 1. | 24 | 0.417 |
| 293 | A | 7 | 6 | 1. | 24 | 0.25 |
| 294 | A | 6 | 5 | 1. | 24 | 0.208 |
| 295 | A | 5 | 4 | 1. | 22 | 0.182 |
| 296 | A | 5 | 5 | 1. | 14 | 0.357 |
| 297 | A | 13 | 8 | 1. | 24 | 0.333 |
| 298 | A | 21 | 10 | 1.5 | 24 | 0.417 |
| 299 | A | 27 | 10 | 1.25 | 24 | 0.417 |
| 300 | A | 5 | 5 | 1. | 24 | 0.208 |
| 301 | A | 4 | 4 | 1. | 24 | 0.167 |
| 302 | A | 3 | 3 | 1. | 22 | 0.136 |
| 303 | A | 1 | 1 | 1. | 14 | 0.071 |
| 304 | A | 4 | 4 | 1. | 24 | 0.167 |
| 305 | A | 9 | 8 | 1. | 24 | 0.333 |
| 306 | A | 10 | 9 | 1. | 24 | 0.375 |
| 307 | A | 6 | 5 | 1. | 24 | 0.208 |

Continued on next page

Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|-----|-------|----------------------|------------------------|-------------------------------------|---------------------|---|
| 308 | A | 5 | 5 | 1. | 24 | 0.208 |
| 309 | A | 4 | 4 | 1. | 24 | 0.167 |
| 310 | A | 4 | 4 | 1. | 24 | 0.167 |
| 311 | A | 4 | 4 | 1. | 22 | 0.182 |
| 312 | A | 4 | 4 | 1. | 14 | 0.286 |
| 313 | A | 9 | 8 | 1.2 | 24 | 0.333 |
| 314 | A | 19 | 10 | 1. | 24 | 0.417 |
| 315 | A | 29 | 11 | 1. | 24 | 0.458 |
| 316 | A | 8 | 7 | 1. | 24 | 0.292 |
| 317 | A | 7 | 7 | 1. | 24 | 0.292 |
| 318 | A | 6 | 6 | 1. | 24 | 0.25 |
| 319 | A | 5 | 5 | 1. | 22 | 0.227 |
| 320 | A | 5 | 5 | 1. | 14 | 0.357 |
| 321 | A | 7 | 7 | 1. | 24 | 0.292 |
| 322 | A | 7 | 7 | 1. | 24 | 0.292 |
| 323 | A | 21 | 10 | 1. | 24 | 0.417 |
| 324 | A | 9 | 7 | 1. | 24 | 0.292 |
| 325 | A | 8 | 7 | 1. | 24 | 0.292 |
| 326 | A | 7 | 6 | 1. | 24 | 0.25 |
| 327 | A | 6 | 5 | 1. | 22 | 0.227 |
| 328 | A | 6 | 6 | 1. | 14 | 0.429 |
| 329 | A | 13 | 8 | 1. | 24 | 0.333 |
| 330 | A | 21 | 13 | 1. | 24 | 0.542 |
| 331 | A | 27 | 13 | 1. | 24 | 0.542 |
| 332 | A | 6 | 6 | 1. | 24 | 0.25 |
| 333 | A | 5 | 5 | 1. | 24 | 0.208 |
| 334 | A | 4 | 4 | 1. | 22 | 0.182 |
| 335 | A | 2 | 2 | 1. | 14 | 0.143 |
| 336 | A | 2 | 2 | 1. | 24 | 0.083 |
| 337 | A | 8 | 8 | 1. | 24 | 0.333 |
| 338 | A | 9 | 9 | 1. | 24 | 0.375 |
| 339 | A | 7 | 6 | 1. | 24 | 0.25 |
| 340 | A | 6 | 6 | 1. | 24 | 0.25 |
| 341 | A | 5 | 5 | 1. | 24 | 0.208 |
| 342 | A | 5 | 5 | 1. | 24 | 0.208 |
| 343 | A | 5 | 5 | 1. | 22 | 0.227 |
| 344 | A | 5 | 5 | 1. | 14 | 0.357 |
| 345 | A | 8 | 8 | 1. | 24 | 0.333 |
| 346 | A | 17 | 10 | 1. | 24 | 0.417 |
| 347 | A | 26 | 11 | 1. | 24 | 0.458 |
| 348 | A | 7 | 6 | 1. | 24 | 0.25 |
| 349 | A | 6 | 6 | 1. | 24 | 0.25 |
| 350 | A | 5 | 5 | 1. | 24 | 0.208 |
| 351 | A | 4 | 4 | 1. | 22 | 0.182 |

Continued on next page

Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|-----|-------|----------------------|------------------------|-------------------------------------|---------------------|---|
| 352 | A | 4 | 4 | 1. | 14 | 0.286 |
| 353 | A | 7 | 6 | 1. | 24 | 0.25 |
| 354 | A | 7 | 6 | 1. | 24 | 0.25 |
| 355 | A | 18 | 9 | 1. | 24 | 0.375 |
| 356 | A | 8 | 6 | 1. | 24 | 0.25 |
| 357 | A | 7 | 6 | 1. | 24 | 0.25 |
| 358 | A | 6 | 5 | 1. | 24 | 0.208 |
| 359 | A | 5 | 4 | 1. | 22 | 0.182 |
| 360 | A | 5 | 5 | 1. | 14 | 0.357 |
| 361 | A | 12 | 7 | 1. | 24 | 0.292 |
| 362 | A | 19 | 11 | 1.22 | 24 | 0.458 |
| 363 | A | 22 | 10 | 1. | 24 | 0.417 |
| 364 | A | 5 | 5 | 1. | 24 | 0.208 |
| 365 | A | 4 | 4 | 1. | 24 | 0.167 |
| 366 | A | 3 | 3 | 1. | 22 | 0.136 |
| 367 | A | 1 | 1 | 1. | 14 | 0.071 |
| 368 | A | 3 | 3 | 1. | 24 | 0.125 |
| 369 | A | 6 | 6 | 1. | 24 | 0.25 |
| 370 | A | 7 | 7 | 1. | 24 | 0.292 |
| 371 | A | 6 | 5 | 1. | 24 | 0.208 |
| 372 | A | 5 | 5 | 1. | 24 | 0.208 |
| 373 | A | 4 | 4 | 1. | 24 | 0.167 |
| 374 | A | 4 | 4 | 1. | 24 | 0.167 |
| 375 | A | 4 | 4 | 1. | 22 | 0.182 |
| 376 | A | 4 | 4 | 1. | 14 | 0.286 |
| 377 | A | 8 | 7 | 1. | 24 | 0.292 |
| 378 | A | 15 | 10 | 1. | 24 | 0.417 |
| 379 | A | 22 | 11 | 1. | 24 | 0.458 |
| 380 | A | 5 | 5 | 1. | 26 | 0.192 |
| 381 | A | 4 | 4 | 1. | 26 | 0.154 |
| 382 | A | 3 | 3 | 1. | 24 | 0.125 |
| 383 | A | 3 | 3 | 1. | 26 | 0.115 |
| 384 | A | 6 | 6 | 1. | 26 | 0.231 |
| 385 | A | 6 | 6 | 1. | 27 | 0.222 |
| 386 | A | 5 | 5 | 1. | 27 | 0.185 |
| 387 | A | 4 | 4 | 1. | 25 | 0.16 |
| 388 | A | 2 | 2 | 1. | 27 | 0.074 |
| 389 | A | 8 | 8 | 1. | 27 | 0.296 |
| 390 | A | 5 | 5 | 1. | 26 | 0.192 |
| 391 | A | 2 | 2 | 1. | 28 | 0.071 |
| 392 | A | 3 | 3 | 1. | 27 | 0.111 |
| 393 | A | 3 | 3 | 1. | 29 | 0.103 |
| 394 | A | 5 | 5 | 1. | 24 | 0.208 |
| 395 | A | 4 | 4 | 1. | 24 | 0.167 |

Continued on next page

Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|-----|-------|----------------------|------------------------|-------------------------------------|---------------------|---|
| 396 | A | 3 | 3 | 1. | 22 | 0.136 |
| 397 | A | 4 | 4 | 1. | 24 | 0.167 |
| 398 | A | 9 | 8 | 1.26 | 24 | 0.333 |
| 399 | A | 0 | 0 | 0. | 0 | 0. |
| 400 | A | 8 | 7 | 1. | 24 | 0.292 |
| 401 | A | 7 | 6 | 0.96 | 24 | 0.25 |
| 402 | A | 6 | 5 | 1. | 22 | 0.227 |
| 403 | A | 2 | 2 | 1. | 14 | 0.143 |
| 404 | A | 0 | 0 | 0. | 0 | 0. |
| 405 | A | 0 | 0 | 0. | 0 | 0. |
| 406 | A | 8 | 8 | 1. | 24 | 0.333 |
| 407 | A | 10 | 10 | 1. | 26 | 0.385 |
| 408 | A | 2 | 2 | 1. | 40 | 0.05 |
| 409 | A | 2 | 2 | 1. | 40 | 0.05 |
| 410 | A | 2 | 2 | 1. | 46 | 0.043 |
| 411 | A | 2 | 2 | 1. | 46 | 0.043 |
| 412 | A | 8 | 8 | 1. | 29 | 0.276 |
| 413 | A | 10 | 10 | 1. | 31 | 0.323 |

Chapter 3

Listing of integrals

3.1 $\int \frac{c+dx^2}{a+bx^4} dx$

Optimal. Leaf size=247

$$\frac{(\sqrt{bc} - \sqrt{ad}) \log(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2})}{4\sqrt{2}a^{3/4}b^{3/4}} + \frac{(\sqrt{bc} - \sqrt{ad}) \log(\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2})}{4\sqrt{2}a^{3/4}b^{3/4}} - \frac{(\sqrt{ad} + \sqrt{bc}) \tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2}}{2\sqrt{2}a^{3/4}b^{3/4}}\right)}{2\sqrt{2}a^{3/4}b^{3/4}}$$

```
[Out] -((Sqrt[b]*c + Sqrt[a]*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(3/4)) + ((Sqrt[b]*c + Sqrt[a]*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(3/4)) - ((Sqrt[b]*c - Sqrt[a]*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(3/4)) + ((Sqrt[b]*c - Sqrt[a]*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(3/4))
```

Rubi [A] time = 0.150941, antiderivative size = 247, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {1168, 1162, 617, 204, 1165, 628}

$$\frac{(\sqrt{bc} - \sqrt{ad}) \log(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2})}{4\sqrt{2}a^{3/4}b^{3/4}} + \frac{(\sqrt{bc} - \sqrt{ad}) \log(\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2})}{4\sqrt{2}a^{3/4}b^{3/4}} - \frac{(\sqrt{ad} + \sqrt{bc}) \tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2}}{2\sqrt{2}a^{3/4}b^{3/4}}\right)}{2\sqrt{2}a^{3/4}b^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)/(a + b*x^4), x]
```

```
[Out] -((Sqrt[b]*c + Sqrt[a]*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(3/4)) + ((Sqrt[b]*c + Sqrt[a]*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(3/4)) - ((Sqrt[b]*c - Sqrt[a]*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(3/4)) + ((Sqrt[b]*c - Sqrt[a]*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(3/4))
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + Dist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
```

c)]

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{c + dx^2}{a + bx^4} dx &= \frac{\left(\frac{\sqrt{bc}}{\sqrt{a}} - d\right) \int \frac{\sqrt{a}\sqrt{b-bx^2}}{a+bx^4} dx}{2b} + \frac{\left(\frac{\sqrt{bc}}{\sqrt{a}} + d\right) \int \frac{\sqrt{a}\sqrt{b+bx^2}}{a+bx^4} dx}{2b} \\ &= \frac{\left(\frac{\sqrt{bc}}{\sqrt{a}} + d\right) \int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}} - \frac{\sqrt{2}\sqrt[4]{ax}}{\sqrt[4]{b}} + x^2} dx}{4b} + \frac{\left(\frac{\sqrt{bc}}{\sqrt{a}} + d\right) \int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}} + \frac{\sqrt{2}\sqrt[4]{ax}}{\sqrt[4]{b}} + x^2} dx}{4b} - \frac{(\sqrt{bc} - \sqrt{ad}) \int \frac{\frac{\sqrt{2}\sqrt[4]{a} + 2x}{\sqrt[4]{b}}}{-\frac{\sqrt{a}}{\sqrt{b}} - \frac{\sqrt{2}\sqrt[4]{ax}}{\sqrt[4]{b}} - x^2} dx}{4\sqrt{2}a^{3/4}b^{3/4}}}{4\sqrt{2}a^{3/4}b^{3/4}} - \frac{(\sqrt{bc} - \sqrt{ad}) \int \frac{\frac{\sqrt{2}\sqrt[4]{a} - 2x}{\sqrt[4]{b}}}{-\frac{\sqrt{a}}{\sqrt{b}} - \frac{\sqrt{2}\sqrt[4]{ax}}{\sqrt[4]{b}} - x^2} dx}{4\sqrt{2}a^{3/4}b^{3/4}}}{4\sqrt{2}a^{3/4}b^{3/4}} \\ &= -\frac{(\sqrt{bc} - \sqrt{ad}) \log(\sqrt{a} - \sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{bx^2})}{4\sqrt{2}a^{3/4}b^{3/4}} + \frac{(\sqrt{bc} - \sqrt{ad}) \log(\sqrt{a} + \sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{bx^2})}{4\sqrt{2}a^{3/4}b^{3/4}} + \frac{(\sqrt{bc} + \sqrt{ad}) \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{bx}}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{3/4}} + \frac{(\sqrt{bc} + \sqrt{ad}) \tan^{-1}\left(1 + \frac{\sqrt{2}\sqrt[4]{bx}}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{3/4}} - \frac{(\sqrt{bc} - \sqrt{ad}) \log(\sqrt{a} - \sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{bx^2})}{4\sqrt{2}a^{3/4}b^{3/4}} \end{aligned}$$

Mathematica [A] time = 0.074574, size = 183, normalized size = 0.74

$$\frac{-\left(\sqrt{bc} - \sqrt{ad}\right) \left(\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2}\right) - \log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2}\right)\right) - 2\left(\sqrt{ad} + \sqrt{bc}\right) \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{bx}}{\sqrt[4]{a}}\right) + \left(\sqrt{bc} + \sqrt{ad}\right) \tan^{-1}\left(1 + \frac{\sqrt{2}\sqrt[4]{bx}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{3/4}b^{3/4}}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x^2)/(a + b*x^4), x]

[Out] $(-2*(\text{Sqrt}[b]*c + \text{Sqrt}[a]*d)*\text{ArcTan}[1 - (\text{Sqrt}[2]*b^{(1/4)}*x)/a^{(1/4)}] + 2*(\text{Sqrt}[b]*c + \text{Sqrt}[a]*d)*\text{ArcTan}[1 + (\text{Sqrt}[2]*b^{(1/4)}*x)/a^{(1/4)}] - (\text{Sqrt}[b]*c - \text{Sqrt}[a]*d)*(\text{Log}[\text{Sqrt}[a] - \text{Sqrt}[2]*a^{(1/4)}*b^{(1/4)}*x + \text{Sqrt}[b]*x^2] - \text{Log}[\text{Sqrt}[a] + \text{Sqrt}[2]*a^{(1/4)}*b^{(1/4)}*x + \text{Sqrt}[b]*x^2]))/(4*\text{Sqrt}[2]*a^{(3/4)}*b^{(3/4)})$

Maple [A] time = 0.044, size = 260, normalized size = 1.1

$$\frac{c\sqrt{2}}{8a} \frac{\sqrt[4]{a}}{\sqrt{b}} \ln\left(\left(x^2 + \sqrt[4]{\frac{a}{b}}x\sqrt{2} + \sqrt{\frac{a}{b}}\right)\left(x^2 - \sqrt[4]{\frac{a}{b}}x\sqrt{2} + \sqrt{\frac{a}{b}}\right)^{-1}\right) + \frac{c\sqrt{2}}{4a} \frac{\sqrt[4]{a}}{\sqrt{b}} \arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{b}}} + 1\right) + \frac{c\sqrt{2}}{4a} \frac{\sqrt[4]{a}}{\sqrt{b}} \arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{b}}} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x^2+c)/(b*x^4+a), x)

[Out] $\frac{1}{8}*c*(1/b*a)^{(1/4)}/a*2^{(1/2)}*\ln((x^2+(1/b*a)^{(1/4)}*x*2^{(1/2)}+(1/b*a)^{(1/2)})/(x^2-(1/b*a)^{(1/4)}*x*2^{(1/2)}+(1/b*a)^{(1/2)}))+1/4*c*(1/b*a)^{(1/4)}/a*2^{(1/2)}*\arctan(2^{(1/2)}/(1/b*a)^{(1/4)}*x+1)+1/4*c*(1/b*a)^{(1/4)}/a*2^{(1/2)}*\arctan(2^{(1/2)}/(1/b*a)^{(1/4)}*x-1)+1/8*d/b/(1/b*a)^{(1/4)}*2^{(1/2)}*\ln((x^2-(1/b*a)^{(1/4)}*x*2^{(1/2)}+(1/b*a)^{(1/2)})/(x^2+(1/b*a)^{(1/4)}*x*2^{(1/2)}+(1/b*a)^{(1/2)}))+1/4*d/b/(1/b*a)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(1/b*a)^{(1/4)}*x+1)+1/4*d/b/(1/b*a)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(1/b*a)^{(1/4)}*x-1)$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x^2+c)/(b*x^4+a), x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [B] time = 1.78205, size = 1544, normalized size = 6.25

$$-\frac{1}{4} \sqrt{\frac{ab\sqrt{-\frac{b^2c^4-2abc^2d^2+a^2d^4}{a^3b^3}} + 2cd}{ab}} \log\left(-\left(b^2c^4 - a^2d^4\right)x + \left(a^3b^2d\sqrt{-\frac{b^2c^4-2abc^2d^2+a^2d^4}{a^3b^3}} + ab^2c^3 - a^2bcd^2\right)\sqrt{-\frac{b^2c^4-2abc^2d^2+a^2d^4}{a^3b^3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x^2+c)/(b*x^4+a), x, algorithm="fricas")

[Out] $-1/4*\text{sqrt}(-(a*b*\text{sqrt}(-(b^2*c^4 - 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3))) + 2*c*d)/(a*b))*\log(-(b^2*c^4 - a^2*d^4)*x + (a^3*b^2*d*\text{sqrt}(-(b^2*c^4 - 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3))) + a*b^2*c^3 - a^2*b*c*d^2)*\text{sqrt}(-(a*b*\text{sqrt}(-(b^2*c^4 - 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3))) + 2*c*d)/(a*b))$

$$2*c^4 - 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)) + 2*c*d)/(a*b))) + 1/4*\sqrt{-(a*b*\sqrt{-(b^2*c^4 - 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)} + 2*c*d)/(a*b))*\log(-(b^2*c^4 - a^2*d^4)*x - (a^3*b^2*d*\sqrt{-(b^2*c^4 - 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)} + a*b^2*c^3 - a^2*b*c*d^2)*\sqrt{-(a*b*\sqrt{-(b^2*c^4 - 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)} + 2*c*d)/(a*b))) + 1/4*\sqrt{((a*b*\sqrt{-(b^2*c^4 - 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)} - 2*c*d)/(a*b))*\log(-(b^2*c^4 - a^2*d^4)*x + (a^3*b^2*d*\sqrt{-(b^2*c^4 - 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)} - a*b^2*c^3 + a^2*b*c*d^2)*\sqrt{(a*b*\sqrt{-(b^2*c^4 - 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)} - 2*c*d)/(a*b))) - 1/4*\sqrt{((a*b*\sqrt{-(b^2*c^4 - 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)} - 2*c*d)/(a*b))*\log(-(b^2*c^4 - a^2*d^4)*x - (a^3*b^2*d*\sqrt{-(b^2*c^4 - 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)} - a*b^2*c^3 + a^2*b*c*d^2)*\sqrt{(a*b*\sqrt{-(b^2*c^4 - 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)} - 2*c*d)/(a*b)))}$$

Sympy [A] time = 0.59846, size = 109, normalized size = 0.44

$$\text{RootSum}\left(256t^4a^3b^3 + 64t^2a^2b^2cd + a^2d^4 + 2abc^2d^2 + b^2c^4, \left(t \mapsto t \log\left(x + \frac{64t^3a^3b^2d + 12ta^2bcd^2 - 4tab^2c^3}{a^2d^4 - b^2c^4}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x**2+c)/(b*x**4+a),x)

[Out] RootSum(256*_t**4*a**3*b**3 + 64*_t**2*a**2*b**2*c*d + a**2*d**4 + 2*a*b*c**2*d**2 + b**2*c**4, Lambda(_t, _t*log(x + (64*_t**3*a**3*b**2*d + 12*_t*a**2*b*c*d**2 - 4*_t*a*b**2*c**3)/(a**2*d**4 - b**2*c**4))))

Giac [A] time = 1.18441, size = 325, normalized size = 1.32

$$\frac{\sqrt{2}\left((ab^3)^{\frac{1}{4}}b^2c + (ab^3)^{\frac{3}{4}}d\right)\arctan\left(\frac{\sqrt{2}\left(2x + \sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4ab^3} + \frac{\sqrt{2}\left((ab^3)^{\frac{1}{4}}b^2c + (ab^3)^{\frac{3}{4}}d\right)\arctan\left(\frac{\sqrt{2}\left(2x - \sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4ab^3} + \frac{\sqrt{2}\left((ab^3)^{\frac{1}{4}}b^2c + (ab^3)^{\frac{3}{4}}d\right)\arctan\left(\frac{\sqrt{2}\left(2x + \sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4ab^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x^2+c)/(b*x^4+a),x, algorithm="giac")

[Out] 1/4*\sqrt{2}*((a*b^3)^(1/4)*b^2*c + (a*b^3)^(3/4)*d)*arctan(1/2*\sqrt{2}*(2*x + \sqrt{2}*(a/b)^(1/4))/(a/b)^(1/4))/(a*b^3) + 1/4*\sqrt{2}*((a*b^3)^(1/4)*b^2*c + (a*b^3)^(3/4)*d)*arctan(1/2*\sqrt{2}*(2*x - \sqrt{2}*(a/b)^(1/4))/(a/b)^(1/4))/(a*b^3) + 1/8*\sqrt{2}*((a*b^3)^(1/4)*b^2*c - (a*b^3)^(3/4)*d)*\log(x^2 + \sqrt{2}*x*(a/b)^(1/4) + \sqrt{a/b))/(a*b^3) - 1/8*\sqrt{2}*((a*b^3)^(1/4)*b^2*c - (a*b^3)^(3/4)*d)*\log(x^2 - \sqrt{2}*x*(a/b)^(1/4) + \sqrt{a/b))/(a*b^3)

3.2 $\int \frac{c-dx^2}{a+bx^4} dx$

Optimal. Leaf size=247

$$\frac{(\sqrt{ad} + \sqrt{bc}) \log(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2})}{4\sqrt{2}a^{3/4}b^{3/4}} + \frac{(\sqrt{ad} + \sqrt{bc}) \log(\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2})}{4\sqrt{2}a^{3/4}b^{3/4}} - \frac{(\sqrt{bc} - \sqrt{ad}) \tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2}}{2\sqrt{2}a^{3/4}b^{3/4}}\right)}{2\sqrt{2}a^{3/4}b^{3/4}}$$

```
[Out] -((Sqrt[b]*c - Sqrt[a]*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(3/4)) + ((Sqrt[b]*c - Sqrt[a]*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(3/4)) - ((Sqrt[b]*c + Sqrt[a]*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(3/4)) + ((Sqrt[b]*c + Sqrt[a]*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(3/4))
```

Rubi [A] time = 0.137699, antiderivative size = 247, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1168, 1162, 617, 204, 1165, 628}

$$\frac{(\sqrt{ad} + \sqrt{bc}) \log(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2})}{4\sqrt{2}a^{3/4}b^{3/4}} + \frac{(\sqrt{ad} + \sqrt{bc}) \log(\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2})}{4\sqrt{2}a^{3/4}b^{3/4}} - \frac{(\sqrt{bc} - \sqrt{ad}) \tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2}}{2\sqrt{2}a^{3/4}b^{3/4}}\right)}{2\sqrt{2}a^{3/4}b^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - d*x^2)/(a + b*x^4), x]
```

```
[Out] -((Sqrt[b]*c - Sqrt[a]*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(3/4)) + ((Sqrt[b]*c - Sqrt[a]*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(3/4)) - ((Sqrt[b]*c + Sqrt[a]*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(3/4)) + ((Sqrt[b]*c + Sqrt[a]*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(3/4))
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + Dist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*c)]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 1165

Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rubi steps

$$\begin{aligned} \int \frac{c - dx^2}{a + bx^4} dx &= \frac{\left(\frac{\sqrt{bc}}{\sqrt{a}} - d\right) \int \frac{\sqrt{a}\sqrt{b+bx^2}}{a+bx^4} dx}{2b} + \frac{\left(\frac{\sqrt{bc}}{\sqrt{a}} + d\right) \int \frac{\sqrt{a}\sqrt{b-bx^2}}{a+bx^4} dx}{2b} \\ &= \frac{\left(\frac{\sqrt{bc}}{\sqrt{a}} - d\right) \int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}} - \frac{\sqrt{2}\sqrt[4]{ax}}{\sqrt{b}} + x^2} dx}{4b} + \frac{\left(\frac{\sqrt{bc}}{\sqrt{a}} - d\right) \int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}} + \frac{\sqrt{2}\sqrt[4]{ax}}{\sqrt{b}} + x^2} dx}{4b} - \frac{(\sqrt{bc} + \sqrt{ad}) \int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt{b}} + 2x}{-\frac{\sqrt{a}}{\sqrt{b}} - \frac{\sqrt{2}\sqrt[4]{ax}}{\sqrt{b}} - x^2} dx}{4\sqrt{2}a^{3/4}b^{3/4}}}{4\sqrt{2}a^{3/4}b^{3/4}} - \frac{(\sqrt{bc} + \sqrt{ad}) \log(\sqrt{a} - \sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{bx^2})}{4\sqrt{2}a^{3/4}b^{3/4}} + \frac{(\sqrt{bc} + \sqrt{ad}) \log(\sqrt{a} + \sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{bx^2})}{4\sqrt{2}a^{3/4}b^{3/4}} + \frac{(\sqrt{bc} - \sqrt{ad}) \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{bx}}{\sqrt{a}}\right)}{2\sqrt{2}a^{3/4}b^{3/4}} + \frac{(\sqrt{bc} - \sqrt{ad}) \tan^{-1}\left(1 + \frac{\sqrt{2}\sqrt[4]{bx}}{\sqrt{a}}\right)}{2\sqrt{2}a^{3/4}b^{3/4}} - \frac{(\sqrt{bc} + \sqrt{ad}) \log(\sqrt{a} - \sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{bx^2})}{4\sqrt{2}a^{3/4}b^{3/4}} \end{aligned}$$

Mathematica [A] time = 0.0437469, size = 184, normalized size = 0.74

$$\frac{-\left(\sqrt{ad} + \sqrt{bc}\right) \left(\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2}\right) - \log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2}\right)\right) + \left(2\sqrt{ad} - 2\sqrt{bc}\right) \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{bx}}{\sqrt{a}}\right) - \left(2\sqrt{ad} + 2\sqrt{bc}\right) \tan^{-1}\left(1 + \frac{\sqrt{2}\sqrt[4]{bx}}{\sqrt{a}}\right)}{4\sqrt{2}a^{3/4}b^{3/4}}$$

Antiderivative was successfully verified.

[In] Integrate[(c - d*x^2)/(a + b*x^4), x]

[Out] ((-2*Sqrt[b]*c + 2*Sqrt[a]*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 2*(Sqrt[b]*c - Sqrt[a]*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)] - (Sqrt[b]*c + Sqrt[a]*d)*(Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] - Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2]))/(4*Sqrt[2]*a^(3/4)*b^(3/4))

Maple [A] time = 0.045, size = 260, normalized size = 1.1

$$\frac{c\sqrt{2}}{8a}\sqrt[4]{\frac{a}{b}}\ln\left(\left(x^2+\sqrt[4]{\frac{a}{b}}x\sqrt{2}+\sqrt{\frac{a}{b}}\right)\left(x^2-\sqrt[4]{\frac{a}{b}}x\sqrt{2}+\sqrt{\frac{a}{b}}\right)^{-1}\right)+\frac{c\sqrt{2}}{4a}\sqrt[4]{\frac{a}{b}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{b}}}+1\right)+\frac{c\sqrt{2}}{4a}\sqrt[4]{\frac{a}{b}}\arctan\left(\dots\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-d*x^2+c)/(b*x^4+a),x)

[Out] 1/8*c*(1/b*a)^(1/4)/a*2^(1/2)*ln((x^2+(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2))/(x^2-(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2)))+1/4*c*(1/b*a)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x+1)+1/4*c*(1/b*a)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x-1)-1/8*d/b/(1/b*a)^(1/4)*2^(1/2)*ln((x^2-(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2))/(x^2+(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2)))-1/4*d/b/(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x+1)-1/4*d/b/(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x-1)

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-d*x^2+c)/(b*x^4+a),x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [B] time = 1.64726, size = 1544, normalized size = 6.25

$$-\frac{1}{4}\sqrt{\frac{ab\sqrt{-\frac{b^2c^4-2abc^2d^2+a^2d^4}{a^3b^3}}+2cd}{ab}}\log\left(-\left(b^2c^4-a^2d^4\right)x+\left(a^3b^2d\sqrt{-\frac{b^2c^4-2abc^2d^2+a^2d^4}{a^3b^3}}+ab^2c^3-a^2bcd^2\right)\sqrt{\frac{ab\sqrt{-\frac{b^2c^4-2abc^2d^2+a^2d^4}{a^3b^3}}+2cd}{ab}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-d*x^2+c)/(b*x^4+a),x, algorithm="fricas")

[Out] -1/4*sqrt((a*b*sqrt(-(b^2*c^4 - 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)) + 2*c*d)/(a*b))*log(-(b^2*c^4 - a^2*d^4)*x + (a^3*b^2*d*sqrt(-(b^2*c^4 - 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)) + a*b^2*c^3 - a^2*b*c*d^2)*sqrt((a*b*sqrt(-(b^2*c^4 - 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)) + 2*c*d)/(a*b))) + 1/4*sqrt((a*b*sqrt(-(b^2*c^4 - 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)) + 2*c*d)/(a*b))*log(-(b^2*c^4 - a^2*d^4)*x - (a^3*b^2*d*sqrt(-(b^2*c^4 - 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)) + a*b^2*c^3 - a^2*b*c*d^2)*sqrt((a*b*sqrt(-(b^2*c^4 - 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)) + 2*c*d)/(a*b))) + 1/4*sqrt(-(a*b*sqrt(-(b^2*c^4 - 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)) - 2*c*d)/(a*b))*log(-(b^2*c^4 - a^2*d^4)*x + (a^3*b^2*d*sqrt(-(b^2*c^4 - 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)) - a*b^2*c^3 + a^2*b*c*d^2)*sqrt(-(a*b*sqrt(-(b^2*c^4 - 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)) - 2*c*d)/(a*b))) - 1/4*sqrt(-(a*b*sqrt(-(b^2*c^4 - 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)) - 2*c*d)/(a*b))*log(-(b^2*c^4 - a^2*d^4)*x - (a^3*b^2*d*sqrt(-(b^2*c^4 - 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)) - a*b^2*c^3 + a^2*b*c*d^2)*sqrt(-(a*b*sqrt(-(b^2*c^4 - 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)) - 2*c*d)/(a*b)))

- 2*c*d)/(a*b))

Sympy [A] time = 0.764273, size = 110, normalized size = 0.45

-RootSum $\left(256t^4a^3b^3 - 64t^2a^2b^2cd + a^2d^4 + 2abc^2d^2 + b^2c^4, \left(t \mapsto t \log\left(x + \frac{64t^3a^3b^2d - 12ta^2bcd^2 + 4tab^2c^3}{a^2d^4 - b^2c^4}\right)\right)\right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-d*x**2+c)/(b*x**4+a),x)

[Out] -RootSum(256*_t**4*a**3*b**3 - 64*_t**2*a**2*b**2*c*d + a**2*d**4 + 2*a*b*c**2*d**2 + b**2*c**4, Lambda(_t, _t*log(x + (64*_t**3*a**3*b**2*d - 12*_t*a**2*b*c*d**2 + 4*_t*a*b**2*c**3)/(a**2*d**4 - b**2*c**4))))

Giac [A] time = 1.15306, size = 325, normalized size = 1.32

$$\frac{\sqrt{2}\left((ab^3)^{\frac{1}{4}}b^2c - (ab^3)^{\frac{3}{4}}d\right) \arctan\left(\frac{\sqrt{2}\left(2x + \sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4ab^3} + \frac{\sqrt{2}\left((ab^3)^{\frac{1}{4}}b^2c - (ab^3)^{\frac{3}{4}}d\right) \arctan\left(\frac{\sqrt{2}\left(2x - \sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4ab^3} + \frac{\sqrt{2}\left((ab^3)^{\frac{1}{4}}b^2c - (ab^3)^{\frac{3}{4}}d\right) \log\left(x + \sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{4ab^3} - \frac{\sqrt{2}\left((ab^3)^{\frac{1}{4}}b^2c - (ab^3)^{\frac{3}{4}}d\right) \log\left(x - \sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{4ab^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-d*x^2+c)/(b*x^4+a),x, algorithm="giac")

[Out] 1/4*sqrt(2)*((a*b^3)^(1/4)*b^2*c - (a*b^3)^(3/4)*d)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a*b^3) + 1/4*sqrt(2)*((a*b^3)^(1/4)*b^2*c - (a*b^3)^(3/4)*d)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a*b^3) + 1/8*sqrt(2)*((a*b^3)^(1/4)*b^2*c + (a*b^3)^(3/4)*d)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a*b^3) - 1/8*sqrt(2)*((a*b^3)^(1/4)*b^2*c + (a*b^3)^(3/4)*d)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a*b^3)

3.3 $\int \frac{c+dx^2}{a-bx^4} dx$

Optimal. Leaf size=86

$$\frac{(\sqrt{bc} - \sqrt{ad}) \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}} + \frac{(\sqrt{ad} + \sqrt{bc}) \tanh^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}}$$

[Out] ((Sqrt[b]*c - Sqrt[a]*d)*ArcTan[(b^(1/4)*x)/a^(1/4)]/(2*a^(3/4)*b^(3/4)) + ((Sqrt[b]*c + Sqrt[a]*d)*ArcTanh[(b^(1/4)*x)/a^(1/4)]/(2*a^(3/4)*b^(3/4)))

Rubi [A] time = 0.0446384, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1167, 205, 208}

$$\frac{(\sqrt{bc} - \sqrt{ad}) \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}} + \frac{(\sqrt{ad} + \sqrt{bc}) \tanh^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x^2)/(a - b*x^4), x]

[Out] ((Sqrt[b]*c - Sqrt[a]*d)*ArcTan[(b^(1/4)*x)/a^(1/4)]/(2*a^(3/4)*b^(3/4)) + ((Sqrt[b]*c + Sqrt[a]*d)*ArcTanh[(b^(1/4)*x)/a^(1/4)]/(2*a^(3/4)*b^(3/4)))

Rule 1167

Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[-(a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x^2), x], x] + Dist[e/2 - (c*d)/(2*q), Int[1/(q + c*x^2), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 - a*e^2, 0] && PosQ[-(a*c)]

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rubi steps

$$\begin{aligned} \int \frac{c+dx^2}{a-bx^4} dx &= \frac{1}{2} \left(-\frac{\sqrt{bc}}{\sqrt{a}} + d \right) \int \frac{1}{-\sqrt{a}\sqrt{b}-bx^2} dx + \frac{1}{2} \left(\frac{\sqrt{bc}}{\sqrt{a}} + d \right) \int \frac{1}{\sqrt{a}\sqrt{b}-bx^2} dx \\ &= \frac{(\sqrt{bc} - \sqrt{ad}) \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}} + \frac{(\sqrt{bc} + \sqrt{ad}) \tanh^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}} \end{aligned}$$

Mathematica [A] time = 0.0285, size = 95, normalized size = 1.1

$$\frac{2(\sqrt{bc} - \sqrt{ad}) \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right) - (\sqrt{ad} + \sqrt{bc}) (\log(\sqrt[4]{a} - \sqrt[4]{bx}) - \log(\sqrt[4]{a} + \sqrt[4]{bx}))}{4a^{3/4}b^{3/4}}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x^2)/(a - b*x^4), x]

[Out] (2*(Sqrt[b]*c - Sqrt[a]*d)*ArcTan[(b^(1/4)*x)/a^(1/4)] - (Sqrt[b]*c + Sqrt[a]*d)*(Log[a^(1/4) - b^(1/4)*x] - Log[a^(1/4) + b^(1/4)*x]))/(4*a^(3/4)*b^(3/4))

Maple [B] time = 0.044, size = 122, normalized size = 1.4

$$\frac{c}{4a} \sqrt[4]{\frac{a}{b}} \ln\left(\left(x + \sqrt[4]{\frac{a}{b}}\right)\left(x - \sqrt[4]{\frac{a}{b}}\right)^{-1}\right) + \frac{c}{2a} \sqrt[4]{\frac{a}{b}} \arctan\left(x \frac{1}{\sqrt[4]{\frac{a}{b}}}\right) - \frac{d}{2b} \arctan\left(x \frac{1}{\sqrt[4]{\frac{a}{b}}}\right) \frac{1}{\sqrt[4]{\frac{a}{b}}} + \frac{d}{4b} \ln\left(\left(x + \sqrt[4]{\frac{a}{b}}\right)\left(x - \sqrt[4]{\frac{a}{b}}\right)^{-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x^2+c)/(-b*x^4+a), x)

[Out] 1/4*c*(1/b*a)^(1/4)/a*ln((x+(1/b*a)^(1/4))/(x-(1/b*a)^(1/4)))+1/2*c*(1/b*a)^(1/4)/a*arctan(x/(1/b*a)^(1/4))-1/2*d/b/(1/b*a)^(1/4)*arctan(x/(1/b*a)^(1/4))+1/4*d/b/(1/b*a)^(1/4)*ln((x+(1/b*a)^(1/4))/(x-(1/b*a)^(1/4)))

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x^2+c)/(-b*x^4+a), x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [B] time = 1.49715, size = 1527, normalized size = 17.76

$$\frac{1}{4} \sqrt{\frac{ab \sqrt{\frac{b^2c^4 + 2abc^2d^2 + a^2d^4}{a^3b^3}} + 2cd}{ab}} \log\left(- (b^2c^4 - a^2d^4)x + \left(a^3b^2d \sqrt{\frac{b^2c^4 + 2abc^2d^2 + a^2d^4}{a^3b^3}} - ab^2c^3 - a^2bcd^2\right) \sqrt{\frac{ab \sqrt{\frac{b^2c^4 + 2abc^2d^2 + a^2d^4}{a^3b^3}}}{a}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x^2+c)/(-b*x^4+a), x, algorithm="fricas")

[Out] 1/4*sqrt((a*b*sqrt((b^2*c^4 + 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)) + 2*c*d)/(a*b))*log(-(b^2*c^4 - a^2*d^4)*x + (a^3*b^2*d*sqrt((b^2*c^4 + 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)) - a*b^2*c^3 - a^2*b*c*d^2)*sqrt((a*b*sqrt((b^2*c^4

$$+ 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)) + 2*c*d)/(a*b))) - 1/4*sqrt((a*b*sqrt((b^2*c^4 + 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)) + 2*c*d)/(a*b))*log(-(b^2*c^4 - a^2*d^4)*x - (a^3*b^2*d*sqrt((b^2*c^4 + 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)) - a*b^2*c^3 - a^2*b*c*d^2)*sqrt((a*b*sqrt((b^2*c^4 + 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)) + 2*c*d)/(a*b))) - 1/4*sqrt(-(a*b*sqrt((b^2*c^4 + 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)) - 2*c*d)/(a*b))*log(-(b^2*c^4 - a^2*d^4)*x + (a^3*b^2*d*sqrt((b^2*c^4 + 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)) + a*b^2*c^3 + a^2*b*c*d^2)*sqrt(-(a*b*sqrt((b^2*c^4 + 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)) - 2*c*d)/(a*b))) + 1/4*sqrt(-(a*b*sqrt((b^2*c^4 + 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)) - 2*c*d)/(a*b))*log(-(b^2*c^4 - a^2*d^4)*x - (a^3*b^2*d*sqrt((b^2*c^4 + 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)) + a*b^2*c^3 + a^2*b*c*d^2)*sqrt(-(a*b*sqrt((b^2*c^4 + 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)) - 2*c*d)/(a*b)))$$

Sympy [A] time = 0.631181, size = 110, normalized size = 1.28

$$- \text{RootSum}\left(256t^4a^3b^3 - 64t^2a^2b^2cd - a^2d^4 + 2abc^2d^2 - b^2c^4, \left(t \mapsto t \log\left(x + \frac{-64t^3a^3b^2d + 12ta^2bcd^2 + 4tab^2c^3}{a^2d^4 - b^2c^4}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x**2+c)/(-b*x**4+a), x)

[Out] -RootSum(256*_t**4*a**3*b**3 - 64*_t**2*a**2*b**2*c*d - a**2*d**4 + 2*a*b*c**2*d**2 - b**2*c**4, Lambda(_t, _t*log(x + (-64*_t**3*a**3*b**2*d + 12*_t*a**2*b*c*d**2 + 4*_t*a*b**2*c**3)/(a**2*d**4 - b**2*c**4))))

Giac [B] time = 1.13599, size = 347, normalized size = 4.03

$$\frac{\sqrt{2}\left((-ab^3)^{\frac{1}{4}}b^2c + (-ab^3)^{\frac{3}{4}}d\right) \arctan\left(\frac{\sqrt{2}\left(2x + \sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4ab^3} + \frac{\sqrt{2}\left((-ab^3)^{\frac{1}{4}}b^2c + (-ab^3)^{\frac{3}{4}}d\right) \arctan\left(\frac{\sqrt{2}\left(2x - \sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4ab^3} + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x^2+c)/(-b*x^4+a), x, algorithm="giac")

[Out] 1/4*sqrt(2)*((-a*b^3)^(1/4)*b^2*c + (-a*b^3)^(3/4)*d)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(a*b^3) + 1/4*sqrt(2)*((-a*b^3)^(1/4)*b^2*c + (-a*b^3)^(3/4)*d)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(a*b^3) + 1/8*sqrt(2)*((-a*b^3)^(1/4)*b^2*c - (-a*b^3)^(3/4)*d)*log(x^2 + sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/(a*b^3) - 1/8*sqrt(2)*((-a*b^3)^(1/4)*b^2*c - (-a*b^3)^(3/4)*d)*log(x^2 - sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/(a*b^3)

3.4 $\int \frac{c-dx^2}{a-bx^4} dx$

Optimal. Leaf size=86

$$\frac{(\sqrt{ad} + \sqrt{bc}) \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}} + \frac{(\sqrt{bc} - \sqrt{ad}) \tanh^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}}$$

[Out] ((Sqrt[b]*c + Sqrt[a]*d)*ArcTan[(b^(1/4)*x)/a^(1/4)]/(2*a^(3/4)*b^(3/4)) + ((Sqrt[b]*c - Sqrt[a]*d)*ArcTanh[(b^(1/4)*x)/a^(1/4)]/(2*a^(3/4)*b^(3/4)))

Rubi [A] time = 0.0401515, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {1167, 205, 208}

$$\frac{(\sqrt{ad} + \sqrt{bc}) \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}} + \frac{(\sqrt{bc} - \sqrt{ad}) \tanh^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}}$$

Antiderivative was successfully verified.

[In] Int[(c - d*x^2)/(a - b*x^4), x]

[Out] ((Sqrt[b]*c + Sqrt[a]*d)*ArcTan[(b^(1/4)*x)/a^(1/4)]/(2*a^(3/4)*b^(3/4)) + ((Sqrt[b]*c - Sqrt[a]*d)*ArcTanh[(b^(1/4)*x)/a^(1/4)]/(2*a^(3/4)*b^(3/4)))

Rule 1167

Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[-(a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x^2), x], x] + Dist[e/2 - (c*d)/(2*q), Int[1/(q + c*x^2), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 - a*e^2, 0] && PosQ[-(a*c)]

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rubi steps

$$\begin{aligned} \int \frac{c-dx^2}{a-bx^4} dx &= \frac{1}{2} \left(-\frac{\sqrt{bc}}{\sqrt{a}} - d \right) \int \frac{1}{-\sqrt{a}\sqrt{b}-bx^2} dx + \frac{1}{2} \left(\frac{\sqrt{bc}}{\sqrt{a}} - d \right) \int \frac{1}{\sqrt{a}\sqrt{b}-bx^2} dx \\ &= \frac{(\sqrt{bc} + \sqrt{ad}) \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}} + \frac{(\sqrt{bc} - \sqrt{ad}) \tanh^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}} \end{aligned}$$

Mathematica [A] time = 0.0224051, size = 95, normalized size = 1.1

$$\frac{2(\sqrt{ad} + \sqrt{bc}) \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right) - (\sqrt{bc} - \sqrt{ad}) \left(\log(\sqrt[4]{a} - \sqrt[4]{bx}) - \log(\sqrt[4]{a} + \sqrt[4]{bx})\right)}{4a^{3/4}b^{3/4}}$$

Antiderivative was successfully verified.

[In] Integrate[(c - d*x^2)/(a - b*x^4), x]

[Out] (2*(Sqrt[b]*c + Sqrt[a]*d)*ArcTan[(b^(1/4)*x)/a^(1/4)] - (Sqrt[b]*c - Sqrt[a]*d)*(Log[a^(1/4) - b^(1/4)*x] - Log[a^(1/4) + b^(1/4)*x]))/(4*a^(3/4)*b^(3/4))

Maple [B] time = 0.043, size = 122, normalized size = 1.4

$$\frac{c}{4a} \sqrt[4]{\frac{a}{b}} \ln\left(\left(x + \sqrt[4]{\frac{a}{b}}\right)\left(x - \sqrt[4]{\frac{a}{b}}\right)^{-1}\right) + \frac{c}{2a} \sqrt[4]{\frac{a}{b}} \arctan\left(x \frac{1}{\sqrt[4]{\frac{a}{b}}}\right) + \frac{d}{2b} \arctan\left(x \frac{1}{\sqrt[4]{\frac{a}{b}}}\right) \frac{1}{\sqrt[4]{\frac{a}{b}}} - \frac{d}{4b} \ln\left(\left(x + \sqrt[4]{\frac{a}{b}}\right)\left(x - \sqrt[4]{\frac{a}{b}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-d*x^2+c)/(-b*x^4+a), x)

[Out] 1/4*c*(1/b*a)^(1/4)/a*ln((x+(1/b*a)^(1/4))/(x-(1/b*a)^(1/4)))+1/2*c*(1/b*a)^(1/4)/a*arctan(x/(1/b*a)^(1/4))+1/2*d/b/(1/b*a)^(1/4)*arctan(x/(1/b*a)^(1/4))-1/4*d/b/(1/b*a)^(1/4)*ln((x+(1/b*a)^(1/4))/(x-(1/b*a)^(1/4)))

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-d*x^2+c)/(-b*x^4+a), x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [B] time = 1.44033, size = 1527, normalized size = 17.76

$$\frac{1}{4} \sqrt{-\frac{ab\sqrt{\frac{b^2c^4+2abc^2d^2+a^2d^4}{a^3b^3}} + 2cd}{ab}} \log\left[-(b^2c^4 - a^2d^4)x + \left(a^3b^2d\sqrt{\frac{b^2c^4 + 2abc^2d^2 + a^2d^4}{a^3b^3}} - ab^2c^3 - a^2bcd^2\right)\sqrt{-\frac{ab\sqrt{\frac{b^2c^4+2abc^2d^2+a^2d^4}{a^3b^3}}}{ab}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-d*x^2+c)/(-b*x^4+a), x, algorithm="fricas")

[Out] 1/4*sqrt(-(a*b*sqrt((b^2*c^4 + 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)) + 2*c*d)/(a*b))*log(-(b^2*c^4 - a^2*d^4)*x + (a^3*b^2*d*sqrt((b^2*c^4 + 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)) - a*b^2*c^3 - a^2*b*c*d^2)*sqrt(-(a*b*sqrt((b^2*c^4 + 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)) + 2*c*d)/(a*b)))

$$4 + 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3) + 2*c*d)/(a*b))) - 1/4*\sqrt{-(a*b*\sqrt{(b^2*c^4 + 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)} + 2*c*d)/(a*b))*\log(-(b^2*c^4 - a^2*d^4)*x - (a^3*b^2*d*\sqrt{(b^2*c^4 + 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)} - a*b^2*c^3 - a^2*b*c*d^2)*\sqrt{-(a*b*\sqrt{(b^2*c^4 + 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)} + 2*c*d)/(a*b))) - 1/4*\sqrt{(a*b*\sqrt{(b^2*c^4 + 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)} - 2*c*d)/(a*b))*\log(-(b^2*c^4 - a^2*d^4)*x + (a^3*b^2*d*\sqrt{(b^2*c^4 + 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)} + a*b^2*c^3 + a^2*b*c*d^2)*\sqrt{(a*b*\sqrt{(b^2*c^4 + 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)} - 2*c*d)/(a*b))) + 1/4*\sqrt{(a*b*\sqrt{(b^2*c^4 + 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)} - 2*c*d)/(a*b))*\log(-(b^2*c^4 - a^2*d^4)*x - (a^3*b^2*d*\sqrt{(b^2*c^4 + 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)} + a*b^2*c^3 + a^2*b*c*d^2)*\sqrt{(a*b*\sqrt{(b^2*c^4 + 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)} - 2*c*d)/(a*b)))}$$

Sympy [A] time = 0.789761, size = 110, normalized size = 1.28

$$\text{RootSum}\left(256t^4a^3b^3 + 64t^2a^2b^2cd - a^2d^4 + 2abc^2d^2 - b^2c^4, \left(t \mapsto t \log\left(x + \frac{-64t^3a^3b^2d - 12ta^2bcd^2 - 4tab^2c^3}{a^2d^4 - b^2c^4}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-d*x**2+c)/(-b*x**4+a),x)

[Out] RootSum(256*_t**4*a**3*b**3 + 64*_t**2*a**2*b**2*c*d - a**2*d**4 + 2*a*b*c**2*d**2 - b**2*c**4, Lambda(_t, _t*log(x + (-64*_t**3*a**3*b**2*d - 12*_t*a**2*b*c*d**2 - 4*_t*a*b**2*c**3)/(a**2*d**4 - b**2*c**4))))

Giac [B] time = 1.13848, size = 347, normalized size = 4.03

$$\frac{\sqrt{2}\left((-ab^3)^{\frac{1}{4}}b^2c - (-ab^3)^{\frac{3}{4}}d\right)\arctan\left(\frac{\sqrt{2}\left(2x + \sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4ab^3} + \frac{\sqrt{2}\left((-ab^3)^{\frac{1}{4}}b^2c - (-ab^3)^{\frac{3}{4}}d\right)\arctan\left(\frac{\sqrt{2}\left(2x - \sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4ab^3} + \frac{\sqrt{2}\left((-ab^3)^{\frac{1}{4}}b^2c - (-ab^3)^{\frac{3}{4}}d\right)\arctan\left(\frac{\sqrt{2}\left(2x + \sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4ab^3} + \frac{\sqrt{2}\left((-ab^3)^{\frac{1}{4}}b^2c - (-ab^3)^{\frac{3}{4}}d\right)\arctan\left(\frac{\sqrt{2}\left(2x - \sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4ab^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-d*x^2+c)/(-b*x^4+a),x, algorithm="giac")

[Out] 1/4*\sqrt{2}*((-a*b^3)^(1/4)*b^2*c - (-a*b^3)^(3/4)*d)*arctan(1/2*\sqrt{2}*(2*x + \sqrt{2}*(-a/b)^(1/4))/(-a/b)^(1/4))/(a*b^3) + 1/4*\sqrt{2}*((-a*b^3)^(1/4)*b^2*c - (-a*b^3)^(3/4)*d)*arctan(1/2*\sqrt{2}*(2*x - \sqrt{2}*(-a/b)^(1/4))/(-a/b)^(1/4))/(a*b^3) + 1/8*\sqrt{2}*((-a*b^3)^(1/4)*b^2*c + (-a*b^3)^(3/4)*d)*\log(x^2 + \sqrt{2}*x*(-a/b)^(1/4) + \sqrt{-a/b})/(a*b^3) - 1/8*\sqrt{2}*((-a*b^3)^(1/4)*b^2*c + (-a*b^3)^(3/4)*d)*\log(x^2 - \sqrt{2}*x*(-a/b)^(1/4) + \sqrt{-a/b})/(a*b^3)

3.5 $\int \frac{2+3x^2}{4+9x^4} dx$

Optimal. Leaf size=40

$$\frac{\tan^{-1}(\sqrt{3}x+1)}{2\sqrt{3}} - \frac{\tan^{-1}(1-\sqrt{3}x)}{2\sqrt{3}}$$

[Out] -ArcTan[1 - Sqrt[3]*x]/(2*Sqrt[3]) + ArcTan[1 + Sqrt[3]*x]/(2*Sqrt[3])

Rubi [A] time = 0.0198282, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {1162, 617, 204}

$$\frac{\tan^{-1}(\sqrt{3}x+1)}{2\sqrt{3}} - \frac{\tan^{-1}(1-\sqrt{3}x)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[(2 + 3*x^2)/(4 + 9*x^4), x]

[Out] -ArcTan[1 - Sqrt[3]*x]/(2*Sqrt[3]) + ArcTan[1 + Sqrt[3]*x]/(2*Sqrt[3])

Rule 1162

Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]

Rule 617

Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned} \int \frac{2+3x^2}{4+9x^4} dx &= \frac{1}{6} \int \frac{1}{\frac{2}{3} - \frac{2x}{\sqrt{3}} + x^2} dx + \frac{1}{6} \int \frac{1}{\frac{2}{3} + \frac{2x}{\sqrt{3}} + x^2} dx \\ &= \frac{\text{Subst}\left(\int \frac{1}{-1-x^2} dx, x, 1-\sqrt{3}x\right)}{2\sqrt{3}} - \frac{\text{Subst}\left(\int \frac{1}{-1-x^2} dx, x, 1+\sqrt{3}x\right)}{2\sqrt{3}} \\ &= -\frac{\tan^{-1}(1-\sqrt{3}x)}{2\sqrt{3}} + \frac{\tan^{-1}(1+\sqrt{3}x)}{2\sqrt{3}} \end{aligned}$$

Mathematica [A] time = 0.011595, size = 33, normalized size = 0.82

$$\frac{\tan^{-1}(\sqrt{3}x+1) - \tan^{-1}(1-\sqrt{3}x)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Integrate[(2 + 3*x^2)/(4 + 9*x^4), x]

[Out] (-ArcTan[1 - Sqrt[3]*x] + ArcTan[1 + Sqrt[3]*x])/(2*Sqrt[3])

Maple [B] time = 0.045, size = 122, normalized size = 3.1

$$\frac{\sqrt{6}\sqrt{2}}{12} \arctan\left(\frac{\sqrt{6}x\sqrt{2}}{2} - 1\right) + \frac{\sqrt{6}\sqrt{2}}{48} \ln\left(\left(x^2 + \frac{\sqrt{6}x\sqrt{2}}{3} + \frac{2}{3}\right)\left(x^2 - \frac{\sqrt{6}x\sqrt{2}}{3} + \frac{2}{3}\right)^{-1}\right) + \frac{\sqrt{6}\sqrt{2}}{12} \arctan\left(\frac{\sqrt{6}x\sqrt{2}}{2} + 1\right) + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3*x^2+2)/(9*x^4+4), x)

[Out] 1/12*6^(1/2)*2^(1/2)*arctan(1/2*6^(1/2)*x*2^(1/2)-1)+1/48*6^(1/2)*2^(1/2)*ln((x^2+1/3*6^(1/2)*x*2^(1/2)+2/3)/(x^2-1/3*6^(1/2)*x*2^(1/2)+2/3))+1/12*6^(1/2)*2^(1/2)*arctan(1/2*6^(1/2)*x*2^(1/2)+1)+1/48*6^(1/2)*2^(1/2)*ln((x^2-1/3*6^(1/2)*x*2^(1/2)+2/3)/(x^2+1/3*6^(1/2)*x*2^(1/2)+2/3))

Maxima [A] time = 1.49751, size = 53, normalized size = 1.32

$$\frac{1}{6} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3}(3x + \sqrt{3})\right) + \frac{1}{6} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3}(3x - \sqrt{3})\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x^2+2)/(9*x^4+4), x, algorithm="maxima")

[Out] 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(3*x + sqrt(3))) + 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(3*x - sqrt(3)))

Fricas [A] time = 1.30891, size = 112, normalized size = 2.8

$$\frac{1}{6} \sqrt{3} \arctan\left(\frac{1}{4} \sqrt{3}(3x^3 + 2x)\right) + \frac{1}{6} \sqrt{3} \arctan\left(\frac{1}{2} \sqrt{3}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x^2+2)/(9*x^4+4), x, algorithm="fricas")

[Out] 1/6*sqrt(3)*arctan(1/4*sqrt(3)*(3*x^3 + 2*x)) + 1/6*sqrt(3)*arctan(1/2*sqrt(3)*x)

Sympy [A] time = 0.106144, size = 41, normalized size = 1.02

$$\frac{\sqrt{3} \left(2 \operatorname{atan} \left(\frac{\sqrt{3}x}{2} \right) + 2 \operatorname{atan} \left(\frac{3\sqrt{3}x^3}{4} + \frac{\sqrt{3}x}{2} \right) \right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x**2+2)/(9*x**4+4),x)

[Out] sqrt(3)*(2*atan(sqrt(3)*x/2) + 2*atan(3*sqrt(3)*x**3/4 + sqrt(3)*x/2))/12

Giac [A] time = 1.14543, size = 70, normalized size = 1.75

$$\frac{1}{6} \sqrt{3} \arctan \left(\frac{9}{8} \sqrt{2} \left(\frac{4}{9} \right)^{\frac{3}{4}} \left(2x + \sqrt{2} \left(\frac{4}{9} \right)^{\frac{1}{4}} \right) \right) + \frac{1}{6} \sqrt{3} \arctan \left(\frac{9}{8} \sqrt{2} \left(\frac{4}{9} \right)^{\frac{3}{4}} \left(2x - \sqrt{2} \left(\frac{4}{9} \right)^{\frac{1}{4}} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x^2+2)/(9*x^4+4),x, algorithm="giac")

[Out] 1/6*sqrt(3)*arctan(9/8*sqrt(2)*(4/9)^(3/4)*(2*x + sqrt(2)*(4/9)^(1/4))) + 1/6*sqrt(3)*arctan(9/8*sqrt(2)*(4/9)^(3/4)*(2*x - sqrt(2)*(4/9)^(1/4)))

3.6 $\int \frac{2-3x^2}{4+9x^4} dx$

Optimal. Leaf size=51

$$\frac{\log(3x^2 + 2\sqrt{3}x + 2)}{4\sqrt{3}} - \frac{\log(3x^2 - 2\sqrt{3}x + 2)}{4\sqrt{3}}$$

[Out] -Log[2 - 2*Sqrt[3]*x + 3*x^2]/(4*Sqrt[3]) + Log[2 + 2*Sqrt[3]*x + 3*x^2]/(4*Sqrt[3])

Rubi [A] time = 0.0213201, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {1165, 628}

$$\frac{\log(3x^2 + 2\sqrt{3}x + 2)}{4\sqrt{3}} - \frac{\log(3x^2 - 2\sqrt{3}x + 2)}{4\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[(2 - 3*x^2)/(4 + 9*x^4), x]

[Out] -Log[2 - 2*Sqrt[3]*x + 3*x^2]/(4*Sqrt[3]) + Log[2 + 2*Sqrt[3]*x + 3*x^2]/(4*Sqrt[3])

Rule 1165

Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rubi steps

$$\begin{aligned} \int \frac{2-3x^2}{4+9x^4} dx &= -\frac{\int \frac{\frac{2}{\sqrt{3}}+2x}{-\frac{2}{3}-\frac{2x}{\sqrt{3}}-x^2} dx}{4\sqrt{3}} - \frac{\int \frac{\frac{2}{\sqrt{3}}-2x}{-\frac{2}{3}+\frac{2x}{\sqrt{3}}-x^2} dx}{4\sqrt{3}} \\ &= -\frac{\log(2-2\sqrt{3}x+3x^2)}{4\sqrt{3}} + \frac{\log(2+2\sqrt{3}x+3x^2)}{4\sqrt{3}} \end{aligned}$$

Mathematica [A] time = 0.0127092, size = 44, normalized size = 0.86

$$\frac{\log(3x^2 + 2\sqrt{3}x + 2) - \log(-3x^2 + 2\sqrt{3}x - 2)}{4\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Integrate[(2 - 3*x^2)/(4 + 9*x^4), x]

[Out] (-Log[-2 + 2*Sqrt[3]*x - 3*x^2] + Log[2 + 2*Sqrt[3]*x + 3*x^2])/(4*Sqrt[3])

Maple [B] time = 0.043, size = 82, normalized size = 1.6

$$\frac{\sqrt{6}\sqrt{2}}{48} \ln\left(\left(x^2 + \frac{\sqrt{6}x\sqrt{2}}{3} + \frac{2}{3}\right)\left(x^2 - \frac{\sqrt{6}x\sqrt{2}}{3} + \frac{2}{3}\right)^{-1}\right) - \frac{\sqrt{6}\sqrt{2}}{48} \ln\left(\left(x^2 - \frac{\sqrt{6}x\sqrt{2}}{3} + \frac{2}{3}\right)\left(x^2 + \frac{\sqrt{6}x\sqrt{2}}{3} + \frac{2}{3}\right)^{-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-3*x^2+2)/(9*x^4+4), x)

[Out] 1/48*6^(1/2)*2^(1/2)*ln((x^2+1/3*6^(1/2)*x*2^(1/2)+2/3)/(x^2-1/3*6^(1/2)*x*2^(1/2)+2/3))-1/48*6^(1/2)*2^(1/2)*ln((x^2-1/3*6^(1/2)*x*2^(1/2)+2/3)/(x^2+1/3*6^(1/2)*x*2^(1/2)+2/3))

Maxima [A] time = 1.49291, size = 53, normalized size = 1.04

$$\frac{1}{12} \sqrt{3} \log(3x^2 + 2\sqrt{3}x + 2) - \frac{1}{12} \sqrt{3} \log(3x^2 - 2\sqrt{3}x + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-3*x^2+2)/(9*x^4+4), x, algorithm="maxima")

[Out] 1/12*sqrt(3)*log(3*x^2 + 2*sqrt(3)*x + 2) - 1/12*sqrt(3)*log(3*x^2 - 2*sqrt(3)*x + 2)

Fricas [A] time = 1.43909, size = 105, normalized size = 2.06

$$\frac{1}{12} \sqrt{3} \log\left(\frac{9x^4 + 24x^2 + 4\sqrt{3}(3x^3 + 2x) + 4}{9x^4 + 4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-3*x^2+2)/(9*x^4+4), x, algorithm="fricas")

[Out] 1/12*sqrt(3)*log((9*x^4 + 24*x^2 + 4*sqrt(3)*(3*x^3 + 2*x) + 4)/(9*x^4 + 4))

Sympy [A] time = 0.099225, size = 49, normalized size = 0.96

$$-\frac{\sqrt{3} \log\left(x^2 - \frac{2\sqrt{3}x}{3} + \frac{2}{3}\right)}{12} + \frac{\sqrt{3} \log\left(x^2 + \frac{2\sqrt{3}x}{3} + \frac{2}{3}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-3*x**2+2)/(9*x**4+4),x)

[Out] -sqrt(3)*log(x**2 - 2*sqrt(3)*x/3 + 2/3)/12 + sqrt(3)*log(x**2 + 2*sqrt(3)*x/3 + 2/3)/12

Giac [A] time = 1.13689, size = 54, normalized size = 1.06

$$\frac{1}{12} \sqrt{3} \log \left(x^2 + \sqrt{2} \left(\frac{4}{9} \right)^{\frac{1}{4}} x + \frac{2}{3} \right) - \frac{1}{12} \sqrt{3} \log \left(x^2 - \sqrt{2} \left(\frac{4}{9} \right)^{\frac{1}{4}} x + \frac{2}{3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-3*x^2+2)/(9*x^4+4),x, algorithm="giac")

[Out] 1/12*sqrt(3)*log(x^2 + sqrt(2)*(4/9)^(1/4)*x + 2/3) - 1/12*sqrt(3)*log(x^2 - sqrt(2)*(4/9)^(1/4)*x + 2/3)

$$3.7 \quad \int \frac{2+3x^2}{4-9x^4} dx$$

Optimal. Leaf size=16

$$\frac{\tanh^{-1}\left(\sqrt{\frac{3}{2}}x\right)}{\sqrt{6}}$$

[Out] ArcTanh[Sqrt[3/2]*x]/Sqrt[6]

Rubi [A] time = 0.0029153, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {26, 206}

$$\frac{\tanh^{-1}\left(\sqrt{\frac{3}{2}}x\right)}{\sqrt{6}}$$

Antiderivative was successfully verified.

[In] Int[(2 + 3*x^2)/(4 - 9*x^4), x]

[Out] ArcTanh[Sqrt[3/2]*x]/Sqrt[6]

Rule 26

Int[(u_.)*((a_.) + (b_.)*(x_)^(n_.))^(m_.)*((c_.) + (d_.)*(x_)^(j_.))^(p_.), x_Symbol] :> Dist[(-b^2/d)^m, Int[u/(a - b*x^n)^m, x], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[j, 2*n] && EqQ[p, -m] && EqQ[b^2*c + a^2*d, 0] && GtQ[a, 0] && LtQ[d, 0]

Rule 206

Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned} \int \frac{2+3x^2}{4-9x^4} dx &= \int \frac{1}{2-3x^2} dx \\ &= \frac{\tanh^{-1}\left(\sqrt{\frac{3}{2}}x\right)}{\sqrt{6}} \end{aligned}$$

Mathematica [A] time = 0.0140431, size = 32, normalized size = 2.

$$\frac{\log(3x + \sqrt{6}) - \log(\sqrt{6} - 3x)}{2\sqrt{6}}$$

Antiderivative was successfully verified.

[In] Integrate[(2 + 3*x^2)/(4 - 9*x^4), x]

[Out] $(-\text{Log}[\text{Sqrt}[6] - 3*x] + \text{Log}[\text{Sqrt}[6] + 3*x])/(2*\text{Sqrt}[6])$

Maple [A] time = 0.043, size = 13, normalized size = 0.8

$$\frac{\sqrt{6}}{6} \text{Artanh}\left(\frac{x\sqrt{6}}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((3*x^2+2)/(-9*x^4+4),x)`

[Out] `1/6*arctanh(1/2*x*6^(1/2))*6^(1/2)`

Maxima [B] time = 1.49265, size = 34, normalized size = 2.12

$$-\frac{1}{12} \sqrt{6} \log\left(\frac{3x - \sqrt{6}}{3x + \sqrt{6}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((3*x^2+2)/(-9*x^4+4),x, algorithm="maxima")`

[Out] `-1/12*sqrt(6)*log((3*x - sqrt(6))/(3*x + sqrt(6)))`

Fricas [B] time = 1.40545, size = 77, normalized size = 4.81

$$\frac{1}{12} \sqrt{6} \log\left(\frac{3x^2 + 2\sqrt{6}x + 2}{3x^2 - 2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((3*x^2+2)/(-9*x^4+4),x, algorithm="fricas")`

[Out] `1/12*sqrt(6)*log((3*x^2 + 2*sqrt(6)*x + 2)/(3*x^2 - 2))`

Sympy [B] time = 0.091549, size = 32, normalized size = 2.

$$-\frac{\sqrt{6} \log\left(x - \frac{\sqrt{6}}{3}\right)}{12} + \frac{\sqrt{6} \log\left(x + \frac{\sqrt{6}}{3}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((3*x**2+2)/(-9*x**4+4),x)`

[Out] `-sqrt(6)*log(x - sqrt(6)/3)/12 + sqrt(6)*log(x + sqrt(6)/3)/12`

Giac [B] time = 1.15612, size = 39, normalized size = 2.44

$$\frac{1}{12} \sqrt{6} \log\left(\left|x + \frac{1}{3} \sqrt{6}\right|\right) - \frac{1}{12} \sqrt{6} \log\left(\left|x - \frac{1}{3} \sqrt{6}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/(-9*x^4+4),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(6)*log(abs(x + 1/3*sqrt(6))) - 1/12*sqrt(6)*log(abs(x - 1/3*sqrt(6)))
```

$$3.8 \quad \int \frac{2-3x^2}{4-9x^4} dx$$

Optimal. Leaf size=16

$$\frac{\tan^{-1}\left(\sqrt{\frac{3}{2}}x\right)}{\sqrt{6}}$$

[Out] ArcTan[Sqrt[3/2]*x]/Sqrt[6]

Rubi [A] time = 0.0025036, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {26, 203}

$$\frac{\tan^{-1}\left(\sqrt{\frac{3}{2}}x\right)}{\sqrt{6}}$$

Antiderivative was successfully verified.

[In] Int[(2 - 3*x^2)/(4 - 9*x^4), x]

[Out] ArcTan[Sqrt[3/2]*x]/Sqrt[6]

Rule 26

Int[(u_.)*((a_.) + (b_.)*(x_)^(n_.))^(m_.)*((c_.) + (d_.)*(x_)^(j_.))^(p_.), x_Symbol] :> Dist[(-b^2/d)^m, Int[u/(a - b*x^n)^m, x], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[j, 2*n] && EqQ[p, -m] && EqQ[b^2*c + a^2*d, 0] && GtQ[a, 0] && LtQ[d, 0]

Rule 203

Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rubi steps

$$\begin{aligned} \int \frac{2-3x^2}{4-9x^4} dx &= \int \frac{1}{2+3x^2} dx \\ &= \frac{\tan^{-1}\left(\sqrt{\frac{3}{2}}x\right)}{\sqrt{6}} \end{aligned}$$

Mathematica [A] time = 0.0050491, size = 16, normalized size = 1.

$$\frac{\tan^{-1}\left(\sqrt{\frac{3}{2}}x\right)}{\sqrt{6}}$$

Antiderivative was successfully verified.

[In] Integrate[(2 - 3*x^2)/(4 - 9*x^4), x]

[Out] ArcTan[Sqrt[3/2]*x]/Sqrt[6]

Maple [A] time = 0.041, size = 13, normalized size = 0.8

$$\frac{\sqrt{6}}{6} \arctan\left(\frac{x\sqrt{6}}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-3*x^2+2)/(-9*x^4+4),x)

[Out] 1/6*arctan(1/2*x*6^(1/2))*6^(1/2)

Maxima [A] time = 1.42857, size = 16, normalized size = 1.

$$\frac{1}{6} \sqrt{6} \arctan\left(\frac{1}{2} \sqrt{6}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-3*x^2+2)/(-9*x^4+4),x, algorithm="maxima")

[Out] 1/6*sqrt(6)*arctan(1/2*sqrt(6)*x)

Fricas [A] time = 1.25673, size = 47, normalized size = 2.94

$$\frac{1}{6} \sqrt{6} \arctan\left(\frac{1}{2} \sqrt{6}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-3*x^2+2)/(-9*x^4+4),x, algorithm="fricas")

[Out] 1/6*sqrt(6)*arctan(1/2*sqrt(6)*x)

Sympy [A] time = 0.093686, size = 15, normalized size = 0.94

$$\frac{\sqrt{6} \operatorname{atan}\left(\frac{\sqrt{6}x}{2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-3*x**2+2)/(-9*x**4+4),x)

[Out] sqrt(6)*atan(sqrt(6)*x/2)/6

Giac [A] time = 1.12524, size = 16, normalized size = 1.

$$\frac{1}{6} \sqrt{6} \arctan\left(\frac{1}{2} \sqrt{6} x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*x^2+2)/(-9*x^4+4),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(6)*arctan(1/2*sqrt(6)*x)
```

3.9 $\int \frac{\sqrt{a}\sqrt{b+bx^2}}{a+bx^4} dx$

Optimal. Leaf size=75

$$\frac{\sqrt[4]{b} \tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{bx}}{\sqrt[4]{a}} + 1\right)}{\sqrt{2}\sqrt[4]{a}} - \frac{\sqrt[4]{b} \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{bx}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}}$$

[Out] $-\left((b^{1/4} \operatorname{ArcTan}\left[1 - \left(\operatorname{Sqrt}[2] b^{1/4} x\right) / a^{1/4}\right]\right) / \left(\operatorname{Sqrt}[2] a^{1/4}\right)\right) + \left(b^{1/4} \operatorname{ArcTan}\left[1 + \left(\operatorname{Sqrt}[2] b^{1/4} x\right) / a^{1/4}\right]\right) / \left(\operatorname{Sqrt}[2] a^{1/4}\right)$

Rubi [A] time = 0.0367142, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1162, 617, 204}

$$\frac{\sqrt[4]{b} \tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{bx}}{\sqrt[4]{a}} + 1\right)}{\sqrt{2}\sqrt[4]{a}} - \frac{\sqrt[4]{b} \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{bx}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}\left[\left(\operatorname{Sqrt}[a] \operatorname{Sqrt}[b] + b x^2\right) / \left(a + b x^4\right), x\right]$

[Out] $-\left((b^{1/4} \operatorname{ArcTan}\left[1 - \left(\operatorname{Sqrt}[2] b^{1/4} x\right) / a^{1/4}\right]\right) / \left(\operatorname{Sqrt}[2] a^{1/4}\right)\right) + \left(b^{1/4} \operatorname{ArcTan}\left[1 + \left(\operatorname{Sqrt}[2] b^{1/4} x\right) / a^{1/4}\right]\right) / \left(\operatorname{Sqrt}[2] a^{1/4}\right)$

Rule 1162

$\operatorname{Int}\left[\left((d_) + (e_)(x_)^2\right) / \left((a_) + (c_)(x_)^4\right), x_Symbol\right] \rightarrow \operatorname{With}\left[\{q = \operatorname{Rt}\left[(2*d)/e, 2\right]\}, \operatorname{Dist}\left[e/(2*c), \operatorname{Int}\left[1/\operatorname{Simp}\left[d/e + q*x + x^2, x\right], x\right], x\right] + \operatorname{Dist}\left[e/(2*c), \operatorname{Int}\left[1/\operatorname{Simp}\left[d/e - q*x + x^2, x\right], x\right], x\right] \right] /;$ $\operatorname{FreeQ}\left[\{a, c, d, e\}, x\right] \& \& \operatorname{EqQ}\left[c*d^2 - a*e^2, 0\right] \& \& \operatorname{PosQ}\left[d*e\right]$

Rule 617

$\operatorname{Int}\left[\left((a_) + (b_)(x_) + (c_)(x_)^2\right)^{-1}, x_Symbol\right] \rightarrow \operatorname{With}\left[\{q = 1 - 4*\operatorname{Simplify}\left[(a*c)/b^2\right]\}, \operatorname{Dist}\left[-2/b, \operatorname{Subst}\left[\operatorname{Int}\left[1/(q - x^2), x\right], x, 1 + (2*c*x)/b\right], x\right] /;$ $\operatorname{RationalQ}\left[q\right] \& \& \left(\operatorname{EqQ}\left[q^2, 1\right] \mid \mid \operatorname{!RationalQ}\left[b^2 - 4*a*c\right]\right) /;$ $\operatorname{FreeQ}\left[\{a, b, c\}, x\right] \& \& \operatorname{NeQ}\left[b^2 - 4*a*c, 0\right]$

Rule 204

$\operatorname{Int}\left[\left((a_) + (b_)(x_)^2\right)^{-1}, x_Symbol\right] \rightarrow -\operatorname{Simp}\left[\operatorname{ArcTan}\left[\left(\operatorname{Rt}\left[-b, 2\right]*x\right) / \operatorname{Rt}\left[-a, 2\right]\right] / \left(\operatorname{Rt}\left[-a, 2\right] * \operatorname{Rt}\left[-b, 2\right]\right), x\right] /;$ $\operatorname{FreeQ}\left[\{a, b\}, x\right] \& \& \operatorname{PosQ}\left[a/b\right] \& \& \left(\operatorname{LtQ}\left[a, 0\right] \mid \mid \operatorname{LtQ}\left[b, 0\right]\right)$

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a}\sqrt{b} + bx^2}{a + bx^4} dx &= \frac{1}{2} \int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}} - \frac{\sqrt{2}\sqrt[4]{ax}}{\sqrt[4]{b}} + x^2} dx + \frac{1}{2} \int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}} + \frac{\sqrt{2}\sqrt[4]{ax}}{\sqrt[4]{b}} + x^2} dx \\
&= \frac{\sqrt[4]{b} \operatorname{Subst}\left(\int \frac{1}{-1-x^2} dx, x, 1 - \frac{\sqrt{2}\sqrt[4]{bx}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}} - \frac{\sqrt[4]{b} \operatorname{Subst}\left(\int \frac{1}{-1-x^2} dx, x, 1 + \frac{\sqrt{2}\sqrt[4]{bx}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}} \\
&= -\frac{\sqrt[4]{b} \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{bx}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}} + \frac{\sqrt[4]{b} \tan^{-1}\left(1 + \frac{\sqrt{2}\sqrt[4]{bx}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}}
\end{aligned}$$

Mathematica [A] time = 0.0188539, size = 60, normalized size = 0.8

$$\frac{\sqrt[4]{b} \left(\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{bx}}{\sqrt[4]{a}} + 1\right) - \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{bx}}{\sqrt[4]{a}}\right) \right)}{\sqrt{2}\sqrt[4]{a}}$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[a]*Sqrt[b] + b*x^2)/(a + b*x^4), x]

[Out] (b^(1/4)*(-ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)]))/(Sqrt[2]*a^(1/4))

Maple [B] time = 0.047, size = 254, normalized size = 3.4

$$\frac{\sqrt{2}\sqrt{b}\sqrt[4]{a}}{8}\sqrt{\frac{a}{b}} \ln\left(\left(x^2 + \sqrt{\frac{a}{b}}x\sqrt{2} + \sqrt{\frac{a}{b}}\right)\left(x^2 - \sqrt{\frac{a}{b}}x\sqrt{2} + \sqrt{\frac{a}{b}}\right)^{-1}\right) \frac{1}{\sqrt{a}} + \frac{\sqrt{2}\sqrt{b}\sqrt[4]{a}}{4} \arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{b}}} + 1\right) \frac{1}{\sqrt{a}} + \frac{\sqrt{2}\sqrt{b}\sqrt[4]{a}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^2+a^(1/2)*b^(1/2))/(b*x^4+a), x)

[Out] 1/8/a^(1/2)*b^(1/2)*(1/b*a)^(1/4)*2^(1/2)*ln((x^2+(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2))/(x^2-(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2)))+1/4/a^(1/2)*b^(1/2)*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x+1)+1/4/a^(1/2)*b^(1/2)*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x-1)+1/8/(1/b*a)^(1/4)*2^(1/2)*ln((x^2-(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2))/(x^2+(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2)))+1/4/(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x+1)+1/4/(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x-1)

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^2+a^(1/2)*b^(1/2))/(b*x^4+a), x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [A] time = 1.50566, size = 458, normalized size = 6.11

$$\left[\frac{1}{2} \sqrt{\frac{1}{2}} \sqrt{\frac{\sqrt{b}}{\sqrt{a}}} \log \left(\frac{bx^4 - 4\sqrt{a}\sqrt{b}x^2 + 4\sqrt{\frac{1}{2}}(\sqrt{a}\sqrt{b}x^3 - ax)\sqrt{-\frac{\sqrt{b}}{\sqrt{a}}} + a}{bx^4 + a} \right), \sqrt{\frac{1}{2}} \sqrt{\frac{\sqrt{b}}{\sqrt{a}}} \arctan \left(\sqrt{\frac{1}{2}} x \sqrt{\frac{\sqrt{b}}{\sqrt{a}}} \right) + \sqrt{\frac{1}{2}} \sqrt{\frac{\sqrt{b}}{\sqrt{a}}} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^2+a^(1/2)*b^(1/2))/(b*x^4+a),x, algorithm="fricas")

[Out] [1/2*sqrt(1/2)*sqrt(-sqrt(b)/sqrt(a))*log((b*x^4 - 4*sqrt(a)*sqrt(b)*x^2 + 4*sqrt(1/2)*(sqrt(a)*sqrt(b)*x^3 - a*x)*sqrt(-sqrt(b)/sqrt(a)) + a)/(b*x^4 + a), sqrt(1/2)*sqrt(sqrt(b)/sqrt(a))*arctan(sqrt(1/2)*x*sqrt(sqrt(b)/sqrt(a))) + sqrt(1/2)*sqrt(sqrt(b)/sqrt(a))*arctan(sqrt(1/2)*(sqrt(a)*sqrt(b)*x^3 + a*x)*sqrt(sqrt(b)/sqrt(a))/a)]

Sympy [A] time = 0.524981, size = 138, normalized size = 1.84

$$-\frac{\sqrt{2}\sqrt{-\frac{\sqrt{b}}{\sqrt{a}}}\log\left(-\frac{\sqrt{2}\sqrt{ax}\sqrt{-\frac{\sqrt{b}}{\sqrt{a}}}}{\sqrt{b}}-\frac{\sqrt{a}}{\sqrt{b}}+x^2\right)}{4}+\frac{\sqrt{2}\sqrt{-\frac{\sqrt{b}}{\sqrt{a}}}\log\left(\frac{\sqrt{2}\sqrt{ax}\sqrt{-\frac{\sqrt{b}}{\sqrt{a}}}}{\sqrt{b}}-\frac{\sqrt{a}}{\sqrt{b}}+x^2\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**2+a**(1/2)*b**(1/2))/(b*x**4+a),x)

[Out] -sqrt(2)*sqrt(-sqrt(b)/sqrt(a))*log(-sqrt(2)*sqrt(a)*x*sqrt(-sqrt(b)/sqrt(a)))/sqrt(b) - sqrt(a)/sqrt(b) + x**2)/4 + sqrt(2)*sqrt(-sqrt(b)/sqrt(a))*log(sqrt(2)*sqrt(a)*x*sqrt(-sqrt(b)/sqrt(a))/sqrt(b) - sqrt(a)/sqrt(b) + x**2)/4

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^2+a^(1/2)*b^(1/2))/(b*x^4+a),x, algorithm="giac")

[Out] Exception raised: TypeError

3.10 $\int \frac{\sqrt{a}\sqrt{b-bx^2}}{a+bx^4} dx$

Optimal. Leaf size=106

$$\frac{\sqrt[4]{b} \log(\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2})}{2\sqrt{2}\sqrt[4]{a}} - \frac{\sqrt[4]{b} \log(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2})}{2\sqrt{2}\sqrt[4]{a}}$$

[Out] $-(b^{(1/4)}*\text{Log}[\text{Sqrt}[a] - \text{Sqrt}[2]*a^{(1/4)}*b^{(1/4)}*x + \text{Sqrt}[b]*x^2])/(2*\text{Sqrt}[2]*a^{(1/4)}) + (b^{(1/4)}*\text{Log}[\text{Sqrt}[a] + \text{Sqrt}[2]*a^{(1/4)}*b^{(1/4)}*x + \text{Sqrt}[b]*x^2])/(2*\text{Sqrt}[2]*a^{(1/4)})$

Rubi [A] time = 0.0472239, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {1165, 628}

$$\frac{\sqrt[4]{b} \log(\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2})}{2\sqrt{2}\sqrt[4]{a}} - \frac{\sqrt[4]{b} \log(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2})}{2\sqrt{2}\sqrt[4]{a}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(\text{Sqrt}[a]*\text{Sqrt}[b] - b*x^2)/(a + b*x^4), x]$

[Out] $-(b^{(1/4)}*\text{Log}[\text{Sqrt}[a] - \text{Sqrt}[2]*a^{(1/4)}*b^{(1/4)}*x + \text{Sqrt}[b]*x^2])/(2*\text{Sqrt}[2]*a^{(1/4)}) + (b^{(1/4)}*\text{Log}[\text{Sqrt}[a] + \text{Sqrt}[2]*a^{(1/4)}*b^{(1/4)}*x + \text{Sqrt}[b]*x^2])/(2*\text{Sqrt}[2]*a^{(1/4)})$

Rule 1165

$\text{Int}[(d + (e \cdot x^2)/(a + (c \cdot x^4)), x_Symbol] \rightarrow \text{With}[\{q = \text{Rt}[-(2 \cdot d)/e, 2]\}, \text{Dist}[e/(2 \cdot c \cdot q), \text{Int}[(q - 2 \cdot x)/\text{Simp}[d/e + q \cdot x - x^2, x], x], x] + \text{Dist}[e/(2 \cdot c \cdot q), \text{Int}[(q + 2 \cdot x)/\text{Simp}[d/e - q \cdot x - x^2, x], x], x] /; \text{FreeQ}[\{a, c, d, e\}, x] \&\& \text{EqQ}[c \cdot d^2 - a \cdot e^2, 0] \&\& \text{NegQ}[d \cdot e]$

Rule 628

$\text{Int}[(d + (e \cdot x))/(a + (b \cdot x) + (c \cdot x^2)), x_Symbol] \rightarrow \text{Simp}[(d \cdot \text{Log}[\text{RemoveContent}[a + b \cdot x + c \cdot x^2, x]])/b, x] /; \text{FreeQ}[\{a, b, c, d, e\}, x] \&\& \text{EqQ}[2 \cdot c \cdot d - b \cdot e, 0]$

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{a}\sqrt{b-bx^2}}{a+bx^4} dx &= -\frac{\sqrt[4]{b} \int \frac{\frac{\sqrt{2}\sqrt[4]{a}+2x}{\sqrt[4]{b}}}{-\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{ax}}{\sqrt[4]{b}}-x^2} dx}{2\sqrt{2}\sqrt[4]{a}} - \frac{\sqrt[4]{b} \int \frac{\frac{\sqrt{2}\sqrt[4]{a}-2x}{\sqrt[4]{b}}}{-\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{ax}}{\sqrt[4]{b}}-x^2} dx}{2\sqrt{2}\sqrt[4]{a}} \\ &= -\frac{\sqrt[4]{b} \log(\sqrt{a} - \sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{bx^2})}{2\sqrt{2}\sqrt[4]{a}} + \frac{\sqrt[4]{b} \log(\sqrt{a} + \sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{bx^2})}{2\sqrt{2}\sqrt[4]{a}} \end{aligned}$$

Mathematica [A] time = 0.0223496, size = 91, normalized size = 0.86

$$\frac{\sqrt[4]{b} (\log(\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2}) - \log(\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} - \sqrt{a} - \sqrt{bx^2}))}{2\sqrt{2}\sqrt[4]{a}}$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[a]*Sqrt[b] - b*x^2)/(a + b*x^4), x]

[Out] (b^(1/4)*(-Log[-Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x - Sqrt[b]*x^2] + Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(2*Sqrt[2]*a^(1/4))

Maple [B] time = 0.048, size = 254, normalized size = 2.4

$$\frac{\sqrt{2}}{8} \sqrt{b} \sqrt[4]{a} \ln \left(\left(x^2 + \sqrt[4]{\frac{a}{b}} x \sqrt{2} + \sqrt{\frac{a}{b}} \right) \left(x^2 - \sqrt[4]{\frac{a}{b}} x \sqrt{2} + \sqrt{\frac{a}{b}} \right)^{-1} \right) \frac{1}{\sqrt{a}} + \frac{\sqrt{2}}{4} \sqrt{b} \sqrt[4]{a} \arctan \left(x \sqrt{2} \frac{1}{\sqrt[4]{\frac{a}{b}}} + 1 \right) \frac{1}{\sqrt{a}} + \frac{\sqrt{2}}{4} \sqrt{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-b*x^2+a^(1/2)*b^(1/2))/(b*x^4+a), x)

[Out] 1/8/a^(1/2)*b^(1/2)*(1/b*a)^(1/4)*2^(1/2)*ln((x^2+(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2))/(x^2-(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2)))+1/4/a^(1/2)*b^(1/2)*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x+1)+1/4/a^(1/2)*b^(1/2)*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x-1)-1/8/(1/b*a)^(1/4)*2^(1/2)*ln((x^2-(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2))/(x^2+(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2)))-1/4/(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x+1)-1/4/(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x-1)

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b*x^2+a^(1/2)*b^(1/2))/(b*x^4+a), x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [A] time = 1.48736, size = 462, normalized size = 4.36

$$\left[\frac{1}{2} \sqrt{\frac{1}{2}} \sqrt{\frac{\sqrt{b}}{\sqrt{a}}} \log \left(\frac{bx^4 + 4\sqrt{a}\sqrt{b}x^2 + 4\sqrt{\frac{1}{2}}(\sqrt{a}\sqrt{b}x^3 + ax)\sqrt{\frac{\sqrt{b}}{\sqrt{a}}} + a}{bx^4 + a} \right), -\sqrt{\frac{1}{2}} \sqrt{\frac{\sqrt{b}}{\sqrt{a}}} \arctan \left(\sqrt{\frac{1}{2}} x \sqrt{\frac{\sqrt{b}}{\sqrt{a}}} \right) + \sqrt{\frac{1}{2}} \sqrt{\frac{\sqrt{b}}{\sqrt{a}}} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b*x^2+a^(1/2)*b^(1/2))/(b*x^4+a), x, algorithm="fricas")

[Out] [1/2*sqrt(1/2)*sqrt(sqrt(b)/sqrt(a))*log((b*x^4 + 4*sqrt(a)*sqrt(b)*x^2 + 4*sqrt(1/2)*(sqrt(a)*sqrt(b)*x^3 + a*x)*sqrt(sqrt(b)/sqrt(a)) + a)/(b*x^4 + a)), -sqrt(1/2)*sqrt(-sqrt(b)/sqrt(a))*arctan(sqrt(1/2)*x*sqrt(-sqrt(b)/sqrt(a))) + sqrt(1/2)*sqrt(-sqrt(b)/sqrt(a))*arctan(sqrt(1/2)*(sqrt(a)*sqrt(b)*x^3 - a*x)*sqrt(-sqrt(b)/sqrt(a)))/a]

Sympy [A] time = 0.534222, size = 131, normalized size = 1.24

$$-\frac{\sqrt{2}\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\log\left(-\frac{\sqrt{2}\sqrt{ax}\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}}{\sqrt{b}}+\frac{\sqrt{a}}{\sqrt{b}}+x^2\right)}{4}+\frac{\sqrt{2}\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\log\left(\frac{\sqrt{2}\sqrt{ax}\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}}{\sqrt{b}}+\frac{\sqrt{a}}{\sqrt{b}}+x^2\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b*x**2+a**(1/2)*b**(1/2))/(b*x**4+a),x)

[Out] -sqrt(2)*sqrt(sqrt(b)/sqrt(a))*log(-sqrt(2)*sqrt(a)*x*sqrt(sqrt(b)/sqrt(a))
/sqrt(b) + sqrt(a)/sqrt(b) + x**2)/4 + sqrt(2)*sqrt(sqrt(b)/sqrt(a))*log(sq
rt(2)*sqrt(a)*x*sqrt(sqrt(b)/sqrt(a))/sqrt(b) + sqrt(a)/sqrt(b) + x**2)/4

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b*x^2+a^(1/2)*b^(1/2))/(b*x^4+a),x, algorithm="giac")

[Out] Exception raised: TypeError

3.11 $\int \frac{d+ex^2}{d^2+e^2x^4} dx$

Optimal. Leaf size=75

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{ex}}{\sqrt{d}} + 1\right)}{\sqrt{2}\sqrt{d}\sqrt{e}} - \frac{\tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt{ex}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}\sqrt{e}}$$

[Out] -(ArcTan[1 - (Sqrt[2]*Sqrt[e]*x)/Sqrt[d]]/(Sqrt[2]*Sqrt[d]*Sqrt[e])) + ArcTan[1 + (Sqrt[2]*Sqrt[e]*x)/Sqrt[d]]/(Sqrt[2]*Sqrt[d]*Sqrt[e])

Rubi [A] time = 0.0498835, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1162, 617, 204}

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{ex}}{\sqrt{d}} + 1\right)}{\sqrt{2}\sqrt{d}\sqrt{e}} - \frac{\tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt{ex}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}\sqrt{e}}$$

Antiderivative was successfully verified.

[In] Int[(d + e*x^2)/(d^2 + e^2*x^4), x]

[Out] -(ArcTan[1 - (Sqrt[2]*Sqrt[e]*x)/Sqrt[d]]/(Sqrt[2]*Sqrt[d]*Sqrt[e])) + ArcTan[1 + (Sqrt[2]*Sqrt[e]*x)/Sqrt[d]]/(Sqrt[2]*Sqrt[d]*Sqrt[e])

Rule 1162

Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x] /; FreeQ[{a, c, d, e}, x] & EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]

Rule 617

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c]) /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 204

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned} \int \frac{d+ex^2}{d^2+e^2x^4} dx &= \frac{\int \frac{1}{\frac{d}{e}-\frac{\sqrt{2}\sqrt{dx}}{\sqrt{e}}+x^2} dx}{2e} + \frac{\int \frac{1}{\frac{d}{e}+\frac{\sqrt{2}\sqrt{dx}}{\sqrt{e}}+x^2} dx}{2e} \\ &= \frac{\text{Subst}\left(\int \frac{1}{-1-x^2} dx, x, 1-\frac{\sqrt{2}\sqrt{ex}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}\sqrt{e}} - \frac{\text{Subst}\left(\int \frac{1}{-1-x^2} dx, x, 1+\frac{\sqrt{2}\sqrt{ex}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}\sqrt{e}} \\ &= -\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{ex}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}\sqrt{e}} + \frac{\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{ex}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}\sqrt{e}} \end{aligned}$$

Mathematica [A] time = 0.0321727, size = 60, normalized size = 0.8

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{ex}}{\sqrt{d}}+1\right)-\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{ex}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}\sqrt{e}}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e*x^2)/(d^2 + e^2*x^4), x]

[Out] (-ArcTan[1 - (Sqrt[2]*Sqrt[e]*x)/Sqrt[d]] + ArcTan[1 + (Sqrt[2]*Sqrt[e]*x)/Sqrt[d]])/(Sqrt[2]*Sqrt[d]*Sqrt[e])

Maple [B] time = 0.07, size = 290, normalized size = 3.9

$$\frac{\sqrt{2}}{8d} \sqrt[4]{\frac{d^2}{e^2}} \ln\left(\left(x^2 + \sqrt[4]{\frac{d^2}{e^2}} x \sqrt{2} + \sqrt{\frac{d^2}{e^2}}\right)\left(x^2 - \sqrt[4]{\frac{d^2}{e^2}} x \sqrt{2} + \sqrt{\frac{d^2}{e^2}}\right)^{-1}\right) + \frac{\sqrt{2}}{4d} \sqrt[4]{\frac{d^2}{e^2}} \arctan\left(x \sqrt{2} \frac{1}{\sqrt[4]{\frac{d^2}{e^2}}} + 1\right) + \frac{\sqrt{2}}{4d} \sqrt[4]{\frac{d^2}{e^2}} \arctan\left(x \sqrt{2} \frac{1}{\sqrt[4]{\frac{d^2}{e^2}}} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x^2+d)/(e^2*x^4+d^2), x)

[Out] 1/8/d*(d^2/e^2)^(1/4)*2^(1/2)*ln((x^2+(d^2/e^2)^(1/4)*x*2^(1/2)+(d^2/e^2)^(1/2))/(x^2-(d^2/e^2)^(1/4)*x*2^(1/2)+(d^2/e^2)^(1/2)))+1/4/d*(d^2/e^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(d^2/e^2)^(1/4)*x+1)+1/4/d*(d^2/e^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(d^2/e^2)^(1/4)*x-1)+1/8/e/(d^2/e^2)^(1/4)*2^(1/2)*ln((x^2-(d^2/e^2)^(1/4)*x*2^(1/2)+(d^2/e^2)^(1/2))/(x^2+(d^2/e^2)^(1/4)*x*2^(1/2)+(d^2/e^2)^(1/2)))+1/4/e/(d^2/e^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(d^2/e^2)^(1/4)*x+1)+1/4/e/(d^2/e^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(d^2/e^2)^(1/4)*x-1)

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)/(e^2*x^4+d^2), x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [A] time = 1.36301, size = 343, normalized size = 4.57

$$\left[\frac{\sqrt{2}\sqrt{-de} \log\left(\frac{e^2x^4 - 4dex^2 - 2\sqrt{2}(ex^3 - dx)\sqrt{-de+d^2}}{e^2x^4 + d^2}\right)}{4de}, \frac{\sqrt{2}\sqrt{de} \arctan\left(\frac{\sqrt{2}\sqrt{dex}}{2d}\right) + \sqrt{2}\sqrt{de} \arctan\left(\frac{\sqrt{2}(ex^3 + dx)\sqrt{de}}{2d^2}\right)}{2de} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)/(e^2*x^4+d^2),x, algorithm="fricas")

[Out] [-1/4*sqrt(2)*sqrt(-d*e)*log((e^2*x^4 - 4*d*e*x^2 - 2*sqrt(2)*(e*x^3 - d*x)*sqrt(-d*e) + d^2)/(e^2*x^4 + d^2))/(d*e), 1/2*(sqrt(2)*sqrt(d*e)*arctan(1/2*sqrt(2)*sqrt(d*e)*x/d) + sqrt(2)*sqrt(d*e)*arctan(1/2*sqrt(2)*(e*x^3 + d*x)*sqrt(d*e)/d^2))/(d*e)]

Sympy [A] time = 0.176202, size = 87, normalized size = 1.16

$$-\frac{\sqrt{2}\sqrt{-\frac{1}{de}} \log\left(-\sqrt{2}dx\sqrt{-\frac{1}{de}} - \frac{d}{e} + x^2\right)}{4} + \frac{\sqrt{2}\sqrt{-\frac{1}{de}} \log\left(\sqrt{2}dx\sqrt{-\frac{1}{de}} - \frac{d}{e} + x^2\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x**2+d)/(e**2*x**4+d**2),x)

[Out] -sqrt(2)*sqrt(-1/(d*e))*log(-sqrt(2)*d*x*sqrt(-1/(d*e)) - d/e + x**2)/4 + sqrt(2)*sqrt(-1/(d*e))*log(sqrt(2)*d*x*sqrt(-1/(d*e)) - d/e + x**2)/4

Giac [B] time = 1.16751, size = 300, normalized size = 4.

$$\frac{\sqrt{2}\left((d^2)^{\frac{1}{4}}de^{\frac{11}{2}} + (d^2)^{\frac{3}{4}}e^{\frac{11}{2}}\right) \arctan\left(\frac{\sqrt{2}\left(\sqrt{2}(d^2)^{\frac{1}{4}}e^{\left(-\frac{1}{2}\right)+2x}\right)e^{\frac{1}{2}}}{2(d^2)^{\frac{1}{4}}}\right)e^{(-6)}}{4d^2} + \frac{\sqrt{2}\left((d^2)^{\frac{1}{4}}de^{\frac{11}{2}} + (d^2)^{\frac{3}{4}}e^{\frac{11}{2}}\right) \arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}(d^2)^{\frac{1}{4}}e^{\left(-\frac{1}{2}\right)+2x}\right)e^{\frac{1}{2}}}{2(d^2)^{\frac{1}{4}}}\right)}{4d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)/(e^2*x^4+d^2),x, algorithm="giac")

[Out] 1/4*sqrt(2)*((d^2)^(1/4)*d*e^(11/2) + (d^2)^(3/4)*e^(11/2))*arctan(1/2*sqrt(2)*(sqrt(2)*(d^2)^(1/4)*e^(-1/2) + 2*x)*e^(1/2)/(d^2)^(1/4))*e^(-6)/d^2 + 1/4*sqrt(2)*((d^2)^(1/4)*d*e^(11/2) + (d^2)^(3/4)*e^(11/2))*arctan(-1/2*sqrt(2)*(sqrt(2)*(d^2)^(1/4)*e^(-1/2) - 2*x)*e^(1/2)/(d^2)^(1/4))*e^(-6)/d^2 + 1/8*sqrt(2)*((d^2)^(1/4)*d*e^(11/2) - (d^2)^(3/4)*e^(11/2))*e^(-6)*log(sqrt(2)*(d^2)^(1/4)*x*e^(-1/2) + x^2 + sqrt(d^2)*e^(-1))/d^2 - 1/8*sqrt(2)*((d^2)^(1/4)*d*e^(11/2) - (d^2)^(3/4)*e^(11/2))*e^(-6)*log(-sqrt(2)*(d^2)^(1/4)*x*e^(-1/2) + x^2 + sqrt(d^2)*e^(-1))/d^2

$$3.12 \quad \int \frac{d-ex^2}{d^2+e^2x^4} dx$$

Optimal. Leaf size=90

$$\frac{\log(\sqrt{2}\sqrt{d}\sqrt{ex+d+ex^2})}{2\sqrt{2}\sqrt{d}\sqrt{e}} - \frac{\log(-\sqrt{2}\sqrt{d}\sqrt{ex+d+ex^2})}{2\sqrt{2}\sqrt{d}\sqrt{e}}$$

[Out] -Log[d - Sqrt[2]*Sqrt[d]*Sqrt[e]*x + e*x^2]/(2*Sqrt[2]*Sqrt[d]*Sqrt[e]) + Log[d + Sqrt[2]*Sqrt[d]*Sqrt[e]*x + e*x^2]/(2*Sqrt[2]*Sqrt[d]*Sqrt[e])

Rubi [A] time = 0.0466638, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1165, 628}

$$\frac{\log(\sqrt{2}\sqrt{d}\sqrt{ex+d+ex^2})}{2\sqrt{2}\sqrt{d}\sqrt{e}} - \frac{\log(-\sqrt{2}\sqrt{d}\sqrt{ex+d+ex^2})}{2\sqrt{2}\sqrt{d}\sqrt{e}}$$

Antiderivative was successfully verified.

[In] Int[(d - e*x^2)/(d^2 + e^2*x^4), x]

[Out] -Log[d - Sqrt[2]*Sqrt[d]*Sqrt[e]*x + e*x^2]/(2*Sqrt[2]*Sqrt[d]*Sqrt[e]) + Log[d + Sqrt[2]*Sqrt[d]*Sqrt[e]*x + e*x^2]/(2*Sqrt[2]*Sqrt[d]*Sqrt[e])

Rule 1165

Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rubi steps

$$\begin{aligned} \int \frac{d-ex^2}{d^2+e^2x^4} dx &= -\frac{\int \frac{\frac{\sqrt{2}\sqrt{d}+2x}{\sqrt{e}}}{-\frac{d}{e}-\frac{\sqrt{2}\sqrt{d}x}{\sqrt{e}}-x^2} dx}{2\sqrt{2}\sqrt{d}\sqrt{e}} - \frac{\int \frac{\frac{\sqrt{2}\sqrt{d}-2x}{\sqrt{e}}}{-\frac{d}{e}+\frac{\sqrt{2}\sqrt{d}x}{\sqrt{e}}-x^2} dx}{2\sqrt{2}\sqrt{d}\sqrt{e}} \\ &= -\frac{\log(d - \sqrt{2}\sqrt{d}\sqrt{ex+ex^2})}{2\sqrt{2}\sqrt{d}\sqrt{e}} + \frac{\log(d + \sqrt{2}\sqrt{d}\sqrt{ex+ex^2})}{2\sqrt{2}\sqrt{d}\sqrt{e}} \end{aligned}$$

Mathematica [A] time = 0.0223228, size = 75, normalized size = 0.83

$$\frac{\log(\sqrt{2}\sqrt{d}\sqrt{ex+d+ex^2}) - \log(\sqrt{2}\sqrt{d}\sqrt{ex-d-ex^2})}{2\sqrt{2}\sqrt{d}\sqrt{e}}$$

Antiderivative was successfully verified.

[In] Integrate[(d - e*x^2)/(d^2 + e^2*x^4), x]

[Out] (-Log[-d + Sqrt[2]*Sqrt[d]*Sqrt[e]*x - e*x^2] + Log[d + Sqrt[2]*Sqrt[d]*Sqrt[e]*x + e*x^2])/(2*Sqrt[2]*Sqrt[d]*Sqrt[e])

Maple [B] time = 0.045, size = 290, normalized size = 3.2

$$\frac{\sqrt{2}}{8d} \sqrt[4]{\frac{d^2}{e^2}} \ln \left(\left(x^2 + \sqrt[4]{\frac{d^2}{e^2}} x \sqrt{2} + \sqrt{\frac{d^2}{e^2}} \right) \left(x^2 - \sqrt[4]{\frac{d^2}{e^2}} x \sqrt{2} + \sqrt{\frac{d^2}{e^2}} \right)^{-1} \right) + \frac{\sqrt{2}}{4d} \sqrt[4]{\frac{d^2}{e^2}} \arctan \left(x \sqrt{2} \frac{1}{\sqrt[4]{\frac{d^2}{e^2}}} + 1 \right) + \frac{\sqrt{2}}{4d} \sqrt[4]{\frac{d^2}{e^2}} \arctan \left(x \sqrt{2} \frac{1}{\sqrt[4]{\frac{d^2}{e^2}}} - 1 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-e*x^2+d)/(e^2*x^4+d^2), x)

[Out] 1/8/d*(d^2/e^2)^(1/4)*2^(1/2)*ln((x^2+(d^2/e^2)^(1/4)*x*2^(1/2)+(d^2/e^2)^(1/2))/(x^2-(d^2/e^2)^(1/4)*x*2^(1/2)+(d^2/e^2)^(1/2)))+1/4/d*(d^2/e^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(d^2/e^2)^(1/4)*x+1)+1/4/d*(d^2/e^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(d^2/e^2)^(1/4)*x-1)-1/8/e/(d^2/e^2)^(1/4)*2^(1/2)*ln((x^2-(d^2/e^2)^(1/4)*x*2^(1/2)+(d^2/e^2)^(1/2))/(x^2+(d^2/e^2)^(1/4)*x*2^(1/2)+(d^2/e^2)^(1/2)))-1/4/e/(d^2/e^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(d^2/e^2)^(1/4)*x+1)-1/4/e/(d^2/e^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(d^2/e^2)^(1/4)*x-1)

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e*x^2+d)/(e^2*x^4+d^2), x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [A] time = 1.29279, size = 346, normalized size = 3.84

$$\left[\frac{\sqrt{2}\sqrt{de} \log\left(\frac{e^2x^4+4dex^2+2\sqrt{2}(ex^3+dx)\sqrt{de+d^2}}{e^2x^4+d^2}\right)}{4de}, -\frac{\sqrt{2}\sqrt{-de} \arctan\left(\frac{\sqrt{2}\sqrt{-dex}}{2d}\right) - \sqrt{2}\sqrt{-de} \arctan\left(\frac{\sqrt{2}(ex^3-dx)\sqrt{-de}}{2d^2}\right)}{2de} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e*x^2+d)/(e^2*x^4+d^2), x, algorithm="fricas")

[Out] [1/4*sqrt(2)*sqrt(d*e)*log((e^2*x^4 + 4*d*e*x^2 + 2*sqrt(2)*(e*x^3 + d*x)*sqrt(d*e) + d^2)/(e^2*x^4 + d^2))/(d*e), -1/2*(sqrt(2)*sqrt(-d*e)*arctan(1/2*sqrt(2)*sqrt(-d*e)*x/d) - sqrt(2)*sqrt(-d*e)*arctan(1/2*sqrt(2)*(e*x^3 - d*x)*sqrt(-d*e)/d^2))/(d*e)]

Sympy [A] time = 0.330698, size = 80, normalized size = 0.89

$$-\frac{\sqrt{2}\sqrt{\frac{1}{de}}\log\left(-\sqrt{2}dx\sqrt{\frac{1}{de}}+\frac{d}{e}+x^2\right)}{4}+\frac{\sqrt{2}\sqrt{\frac{1}{de}}\log\left(\sqrt{2}dx\sqrt{\frac{1}{de}}+\frac{d}{e}+x^2\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e*x**2+d)/(e**2*x**4+d**2),x)

[Out] -sqrt(2)*sqrt(1/(d*e))*log(-sqrt(2)*d*x*sqrt(1/(d*e)) + d/e + x**2)/4 + sqrt(2)*sqrt(1/(d*e))*log(sqrt(2)*d*x*sqrt(1/(d*e)) + d/e + x**2)/4

Giac [B] time = 1.15793, size = 300, normalized size = 3.33

$$\frac{\sqrt{2}\left((d^2)^{\frac{1}{4}}de^{\frac{11}{2}}-(d^2)^{\frac{3}{4}}e^{\frac{11}{2}}\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}(d^2)^{\frac{1}{4}}e^{\left(-\frac{1}{2}\right)+2x}\right)e^{\frac{1}{2}}}{2(d^2)^{\frac{1}{4}}}\right)e^{(-6)}}{4d^2}+\frac{\sqrt{2}\left((d^2)^{\frac{1}{4}}de^{\frac{11}{2}}-(d^2)^{\frac{3}{4}}e^{\frac{11}{2}}\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}(d^2)^{\frac{1}{4}}e^{\left(-\frac{1}{2}\right)-2x}\right)}{2(d^2)^{\frac{1}{4}}}\right)}{4d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e*x^2+d)/(e^2*x^4+d^2),x, algorithm="giac")

[Out] 1/4*sqrt(2)*((d^2)^(1/4)*d*e^(11/2) - (d^2)^(3/4)*e^(11/2))*arctan(1/2*sqrt(2)*(sqrt(2)*(d^2)^(1/4)*e^(-1/2) + 2*x)*e^(1/2)/(d^2)^(1/4))*e^(-6)/d^2 + 1/4*sqrt(2)*((d^2)^(1/4)*d*e^(11/2) - (d^2)^(3/4)*e^(11/2))*arctan(-1/2*sqrt(2)*(sqrt(2)*(d^2)^(1/4)*e^(-1/2) - 2*x)*e^(1/2)/(d^2)^(1/4))*e^(-6)/d^2 + 1/8*sqrt(2)*((d^2)^(1/4)*d*e^(11/2) + (d^2)^(3/4)*e^(11/2))*e^(-6)*log(sqrt(2)*(d^2)^(1/4)*x*e^(-1/2) + x^2 + sqrt(d^2)*e^(-1))/d^2 - 1/8*sqrt(2)*((d^2)^(1/4)*d*e^(11/2) + (d^2)^(3/4)*e^(11/2))*e^(-6)*log(-sqrt(2)*(d^2)^(1/4)*x*e^(-1/2) + x^2 + sqrt(d^2)*e^(-1))/d^2

$$3.13 \quad \int \frac{5+2x^2}{-1+x^4} dx$$

Optimal. Leaf size=13

$$-\frac{3}{2} \tan^{-1}(x) - \frac{7}{2} \tanh^{-1}(x)$$

[Out] (-3*ArcTan[x])/2 - (7*ArcTanh[x])/2

Rubi [A] time = 0.0062671, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$, Rules used = {1167, 207, 203}

$$-\frac{3}{2} \tan^{-1}(x) - \frac{7}{2} \tanh^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[(5 + 2*x^2)/(-1 + x^4), x]

[Out] (-3*ArcTan[x])/2 - (7*ArcTanh[x])/2

Rule 1167

Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] :> With[{q = Rt[-(a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x^2), x], x] + Dist[e/2 - (c*d)/(2*q), Int[1/(q + c*x^2), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 - a*e^2, 0] && PosQ[-(a*c)]

Rule 207

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 203

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rubi steps

$$\begin{aligned} \int \frac{5+2x^2}{-1+x^4} dx &= -\left(\frac{3}{2} \int \frac{1}{1+x^2} dx\right) + \frac{7}{2} \int \frac{1}{-1+x^2} dx \\ &= -\frac{3}{2} \tan^{-1}(x) - \frac{7}{2} \tanh^{-1}(x) \end{aligned}$$

Mathematica [A] time = 0.00597, size = 25, normalized size = 1.92

$$\frac{7}{4} \log(1-x) - \frac{7}{4} \log(x+1) - \frac{3}{2} \tan^{-1}(x)$$

Antiderivative was successfully verified.

[In] Integrate[(5 + 2*x^2)/(-1 + x^4),x]

[Out] (-3*ArcTan[x])/2 + (7*Log[1 - x])/4 - (7*Log[1 + x])/4

Maple [A] time = 0.044, size = 18, normalized size = 1.4

$$\frac{7 \ln(-1+x)}{4} - \frac{7 \ln(1+x)}{4} - \frac{3 \arctan(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*x^2+5)/(x^4-1),x)

[Out] 7/4*ln(-1+x)-7/4*ln(1+x)-3/2*arctan(x)

Maxima [A] time = 1.48577, size = 23, normalized size = 1.77

$$-\frac{3}{2} \arctan(x) - \frac{7}{4} \log(x+1) + \frac{7}{4} \log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2+5)/(x^4-1),x, algorithm="maxima")

[Out] -3/2*arctan(x) - 7/4*log(x + 1) + 7/4*log(x - 1)

Fricas [A] time = 1.34957, size = 68, normalized size = 5.23

$$-\frac{3}{2} \arctan(x) - \frac{7}{4} \log(x+1) + \frac{7}{4} \log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2+5)/(x^4-1),x, algorithm="fricas")

[Out] -3/2*arctan(x) - 7/4*log(x + 1) + 7/4*log(x - 1)

Sympy [A] time = 0.132927, size = 22, normalized size = 1.69

$$\frac{7 \log(x-1)}{4} - \frac{7 \log(x+1)}{4} - \frac{3 \operatorname{atan}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x**2+5)/(x**4-1),x)

[Out] 7*log(x - 1)/4 - 7*log(x + 1)/4 - 3*atan(x)/2

Giac [B] time = 1.12775, size = 26, normalized size = 2.

$$-\frac{3}{2} \arctan(x) - \frac{7}{4} \log(|x+1|) + \frac{7}{4} \log(|x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+5)/(x^4-1),x, algorithm="giac")
```

```
[Out] -3/2*arctan(x) - 7/4*log(abs(x + 1)) + 7/4*log(abs(x - 1))
```

$$3.14 \quad \int \frac{1+bx^2}{\sqrt{1-b^2x^4}} dx$$

Optimal. Leaf size=16

$$\frac{E\left(\sin^{-1}(\sqrt{bx})\middle| -1\right)}{\sqrt{b}}$$

[Out] EllipticE[ArcSin[Sqrt[b]*x], -1]/Sqrt[b]

Rubi [A] time = 0.0163202, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1199, 424}

$$\frac{E\left(\sin^{-1}(\sqrt{bx})\middle| -1\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

[In] Int[(1 + b*x^2)/Sqrt[1 - b^2*x^4], x]

[Out] EllipticE[ArcSin[Sqrt[b]*x], -1]/Sqrt[b]

Rule 1199

Int[((d_) + (e_)*(x_)^2)/Sqrt[(a_) + (c_)*(x_)^4], x_Symbol] :> Dist[d/Sqrt[a], Int[Sqrt[1 + (e*x^2)/d]/Sqrt[1 - (e*x^2)/d], x, x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]

Rule 424

Int[Sqrt[(a_) + (b_)*(x_)^2]/Sqrt[(c_) + (d_)*(x_)^2], x_Symbol] :> Simp[(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]

Rubi steps

$$\begin{aligned} \int \frac{1+bx^2}{\sqrt{1-b^2x^4}} dx &= \int \frac{\sqrt{1+bx^2}}{\sqrt{1-bx^2}} dx \\ &= \frac{E\left(\sin^{-1}(\sqrt{bx})\middle| -1\right)}{\sqrt{b}} \end{aligned}$$

Mathematica [C] time = 0.0117658, size = 45, normalized size = 2.81

$$\frac{1}{3}bx^3 {}_2F_1\left(\frac{1}{2}, \frac{3}{4}; \frac{7}{4}; b^2x^4\right) + x {}_2F_1\left(\frac{1}{4}, \frac{1}{2}; \frac{5}{4}; b^2x^4\right)$$

Antiderivative was successfully verified.

[In] Integrate[(1 + b*x^2)/Sqrt[1 - b^2*x^4], x]

[Out] $x \cdot \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, b^2 x^4\right] + (b x^3 \cdot \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, b^2 x^4\right]) / 3$

Maple [B] time = 0.052, size = 100, normalized size = 6.3

$$-\sqrt{-bx^2+1}\sqrt{bx^2+1}\left(\text{EllipticF}\left(x\sqrt{b},i\right)-\text{EllipticE}\left(x\sqrt{b},i\right)\right)\frac{1}{\sqrt{b}\sqrt{-b^2x^4+1}}+\sqrt{-bx^2+1}\sqrt{bx^2+1}\text{EllipticF}\left(x\sqrt{b},i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^2+1)/(-b^2*x^4+1)^(1/2),x)`

[Out] $-1/b^{1/2}*(-b*x^2+1)^{(1/2)}*(b*x^2+1)^{(1/2)}/(-b^2*x^4+1)^{(1/2)}*(\text{EllipticF}(x*b^{1/2},I)-\text{EllipticE}(x*b^{1/2},I))+1/b^{1/2}*(-b*x^2+1)^{(1/2)}*(b*x^2+1)^{(1/2)}/(-b^2*x^4+1)^{(1/2)}*\text{EllipticF}(x*b^{1/2},I)$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{bx^2+1}{\sqrt{-b^2x^4+1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2+1)/(-b^2*x^4+1)^(1/2),x, algorithm="maxima")`

[Out] `integrate((b*x^2 + 1)/sqrt(-b^2*x^4 + 1), x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-b^2x^4+1}}{bx^2-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2+1)/(-b^2*x^4+1)^(1/2),x, algorithm="fricas")`

[Out] `integral(-sqrt(-b^2*x^4 + 1)/(b*x^2 - 1), x)`

Sympy [B] time = 1.52931, size = 70, normalized size = 4.38

$$\frac{bx^3\Gamma\left(\frac{3}{4}\right) {}_2F_1\left(\frac{1}{2}, \frac{3}{4} \middle| \frac{1}{4}; b^2x^4e^{2i\pi}\right)}{4\Gamma\left(\frac{7}{4}\right)} + \frac{x\Gamma\left(\frac{1}{4}\right) {}_2F_1\left(\frac{1}{4}, \frac{1}{2} \middle| \frac{1}{4}; b^2x^4e^{2i\pi}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**2+1)/(-b**2*x**4+1)**(1/2),x)`

```
[Out] b*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), b**2*x**4*exp_polar(2*I*pi))/(4
*gamma(7/4)) + x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), b**2*x**4*exp_polar(2
*I*pi))/(4*gamma(5/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{bx^2 + 1}{\sqrt{-b^2x^4 + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+1)/(-b^2*x^4+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + 1)/sqrt(-b^2*x^4 + 1), x)
```

$$3.15 \quad \int \frac{1-bx^2}{\sqrt{1-b^2x^4}} dx$$

Optimal. Leaf size=35

$$\frac{2\text{EllipticF}(\sin^{-1}(\sqrt{bx}), -1)}{\sqrt{b}} - \frac{E(\sin^{-1}(\sqrt{bx})|-1)}{\sqrt{b}}$$

[Out] -(EllipticE[ArcSin[Sqrt[b]*x], -1]/Sqrt[b]) + (2*EllipticF[ArcSin[Sqrt[b]*x], -1])/Sqrt[b]

Rubi [A] time = 0.0327558, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {1199, 423, 424, 248, 221}

$$\frac{2F(\sin^{-1}(\sqrt{bx})|-1)}{\sqrt{b}} - \frac{E(\sin^{-1}(\sqrt{bx})|-1)}{\sqrt{b}}$$

Antiderivative was successfully verified.

[In] Int[(1 - b*x^2)/Sqrt[1 - b^2*x^4], x]

[Out] -(EllipticE[ArcSin[Sqrt[b]*x], -1]/Sqrt[b]) + (2*EllipticF[ArcSin[Sqrt[b]*x], -1])/Sqrt[b]

Rule 1199

Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sqrt[a], Int[Sqrt[1 + (e*x^2)/d]/Sqrt[1 - (e*x^2)/d], x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]

Rule 423

Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[b/d, Int[Sqrt[c + d*x^2]/Sqrt[a + b*x^2], x], x] - Dist[(b*c - a*d)/d, Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c] && NegQ[b/a]

Rule 424

Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]

Rule 248

Int[((a1_.) + (b1_.)*(x_)^(n_))^(p_.)*((a2_.) + (b2_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[(a1*a2 + b1*b2*x^(2*n))^p, x] /; FreeQ[{a1, b1, a2, b2, n, p}, x] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[p] || (GtQ[a1, 0] && GtQ[a2, 0]))

Rule 221

Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b, 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[

b/a] && GtQ[a, 0]

Rubi steps

$$\begin{aligned}
 \int \frac{1-bx^2}{\sqrt{1-b^2x^4}} dx &= \int \frac{\sqrt{1-bx^2}}{\sqrt{1+bx^2}} dx \\
 &= 2 \int \frac{1}{\sqrt{1-bx^2}\sqrt{1+bx^2}} dx - \int \frac{\sqrt{1+bx^2}}{\sqrt{1-bx^2}} dx \\
 &= -\frac{E(\sin^{-1}(\sqrt{bx})|-1)}{\sqrt{b}} + 2 \int \frac{1}{\sqrt{1-b^2x^4}} dx \\
 &= -\frac{E(\sin^{-1}(\sqrt{bx})|-1)}{\sqrt{b}} + \frac{2F(\sin^{-1}(\sqrt{bx})|-1)}{\sqrt{b}}
 \end{aligned}$$

Mathematica [C] time = 0.0115553, size = 45, normalized size = 1.29

$$x {}_2F_1\left(\frac{1}{4}, \frac{1}{2}; \frac{5}{4}; b^2x^4\right) - \frac{1}{3}bx^3 {}_2F_1\left(\frac{1}{2}, \frac{3}{4}; \frac{7}{4}; b^2x^4\right)$$

Antiderivative was successfully verified.

[In] Integrate[(1 - b*x^2)/Sqrt[1 - b^2*x^4],x]

[Out] x*Hypergeometric2F1[1/4, 1/2, 5/4, b^2*x^4] - (b*x^3*Hypergeometric2F1[1/2, 3/4, 7/4, b^2*x^4])/3

Maple [B] time = 0.048, size = 99, normalized size = 2.8

$$\sqrt{-bx^2+1}\sqrt{bx^2+1}\left(\text{EllipticF}\left(x\sqrt{b},i\right)-\text{EllipticE}\left(x\sqrt{b},i\right)\right)\frac{1}{\sqrt{b}\sqrt{-b^2x^4+1}}+\sqrt{-bx^2+1}\sqrt{bx^2+1}\text{EllipticF}\left(x\sqrt{b},i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-b*x^2+1)/(-b^2*x^4+1)^(1/2),x)

[Out] 1/b^(1/2)*(-b*x^2+1)^(1/2)*(b*x^2+1)^(1/2)/(-b^2*x^4+1)^(1/2)*(EllipticF(x*b^(1/2),I)-EllipticE(x*b^(1/2),I))+1/b^(1/2)*(-b*x^2+1)^(1/2)*(b*x^2+1)^(1/2)/(-b^2*x^4+1)^(1/2)*EllipticF(x*b^(1/2),I)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{bx^2-1}{\sqrt{-b^2x^4+1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b*x^2+1)/(-b^2*x^4+1)^(1/2),x, algorithm="maxima")

[Out] -integrate((b*x^2 - 1)/sqrt(-b^2*x^4 + 1), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-b^2x^4+1}}{bx^2+1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b*x^2+1)/(-b^2*x^4+1)^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(-b^2*x^4 + 1)/(b*x^2 + 1), x)

Sympy [B] time = 1.8776, size = 70, normalized size = 2.

$$\frac{bx^3\Gamma\left(\frac{3}{4}\right) {}_2F_1\left(\frac{1}{2}, \frac{3}{4} \middle| \frac{7}{4} \right) b^2x^4 e^{2i\pi}}{4\Gamma\left(\frac{7}{4}\right)} + \frac{x\Gamma\left(\frac{1}{4}\right) {}_2F_1\left(\frac{1}{4}, \frac{1}{2} \middle| \frac{5}{4} \right) b^2x^4 e^{2i\pi}}{4\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b*x**2+1)/(-b**2*x**4+1)**(1/2), x)

[Out] -b*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), b**2*x**4*exp_polar(2*I*pi))/(4*gamma(7/4)) + x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), b**2*x**4*exp_polar(2*I*pi))/(4*gamma(5/4))

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{bx^2-1}{\sqrt{-b^2x^4+1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b*x^2+1)/(-b^2*x^4+1)^(1/2),x, algorithm="giac")

[Out] integrate(-(b*x^2 - 1)/sqrt(-b^2*x^4 + 1), x)

$$3.16 \quad \int \frac{1+bx^2}{\sqrt{-1+b^2x^4}} dx$$

Optimal. Leaf size=43

$$\frac{\sqrt{1-b^2x^4}E(\sin^{-1}(\sqrt{bx})|-1)}{\sqrt{b}\sqrt{b^2x^4-1}}$$

[Out] (Sqrt[1 - b^2*x^4]*EllipticE[ArcSin[Sqrt[b]*x], -1])/(Sqrt[b]*Sqrt[-1 + b^2*x^4])

Rubi [A] time = 0.0254657, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1200, 1199, 424}

$$\frac{\sqrt{1-b^2x^4}E(\sin^{-1}(\sqrt{bx})|-1)}{\sqrt{b}\sqrt{b^2x^4-1}}$$

Antiderivative was successfully verified.

[In] Int[(1 + b*x^2)/Sqrt[-1 + b^2*x^4], x]

[Out] (Sqrt[1 - b^2*x^4]*EllipticE[ArcSin[Sqrt[b]*x], -1])/(Sqrt[b]*Sqrt[-1 + b^2*x^4])

Rule 1200

Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + (c*x^4)/a], x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && !GtQ[a, 0]

Rule 1199

Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sqrt[a], Int[Sqrt[1 + (e*x^2)/d]/Sqrt[1 - (e*x^2)/d], x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]

Rule 424

Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]

Rubi steps

$$\begin{aligned} \int \frac{1+bx^2}{\sqrt{-1+b^2x^4}} dx &= \frac{\sqrt{1-b^2x^4} \int \frac{1+bx^2}{\sqrt{1-b^2x^4}} dx}{\sqrt{-1+b^2x^4}} \\ &= \frac{\sqrt{1-b^2x^4} \int \frac{\sqrt{1+bx^2}}{\sqrt{1-bx^2}} dx}{\sqrt{-1+b^2x^4}} \\ &= \frac{\sqrt{1-b^2x^4}E(\sin^{-1}(\sqrt{bx})|-1)}{\sqrt{b}\sqrt{-1+b^2x^4}} \end{aligned}$$

Mathematica [C] time = 0.0225834, size = 74, normalized size = 1.72

$$\frac{\sqrt{1-b^2x^4}\left(bx^3{}_2F_1\left(\frac{1}{2}, \frac{3}{4}; \frac{7}{4}; b^2x^4\right) + 3x{}_2F_1\left(\frac{1}{4}, \frac{1}{2}; \frac{5}{4}; b^2x^4\right)\right)}{3\sqrt{b^2x^4-1}}$$

Antiderivative was successfully verified.

[In] Integrate[(1 + b*x^2)/Sqrt[-1 + b^2*x^4], x]

[Out] (Sqrt[1 - b^2*x^4]*(3*x*Hypergeometric2F1[1/4, 1/2, 5/4, b^2*x^4] + b*x^3*Hypergeometric2F1[1/2, 3/4, 7/4, b^2*x^4]))/(3*Sqrt[-1 + b^2*x^4])

Maple [B] time = 0.054, size = 107, normalized size = 2.5

$$\sqrt{bx^2+1}\sqrt{-bx^2+1}\left(\text{EllipticF}\left(x\sqrt{-b}, i\right) - \text{EllipticE}\left(x\sqrt{-b}, i\right)\right) \frac{1}{\sqrt{-b}} \frac{1}{\sqrt{b^2x^4-1}} + \sqrt{bx^2+1}\sqrt{-bx^2+1}\text{EllipticF}\left(x\sqrt{-b}, i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^2+1)/(b^2*x^4-1)^(1/2), x)

[Out] 1/(-b)^(1/2)*(b*x^2+1)^(1/2)*(-b*x^2+1)^(1/2)/(b^2*x^4-1)^(1/2)*(EllipticF(x*(-b)^(1/2), I)-EllipticE(x*(-b)^(1/2), I))+1/(-b)^(1/2)*(b*x^2+1)^(1/2)*(-b*x^2+1)^(1/2)/(b^2*x^4-1)^(1/2)*EllipticF(x*(-b)^(1/2), I)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{bx^2+1}{\sqrt{b^2x^4-1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^2+1)/(b^2*x^4-1)^(1/2), x, algorithm="maxima")

[Out] integrate((b*x^2 + 1)/sqrt(b^2*x^4 - 1), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b^2x^4-1}}{bx^2-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^2+1)/(b^2*x^4-1)^(1/2), x, algorithm="fricas")

[Out] integral(sqrt(b^2*x^4 - 1)/(b*x^2 - 1), x)

Sympy [A] time = 1.50467, size = 61, normalized size = 1.42

$$-\frac{ibx^3\Gamma\left(\frac{3}{4}\right) {}_2F_1\left(\frac{1}{2}, \frac{3}{4} \middle| \frac{7}{4}; b^2x^4\right)}{4\Gamma\left(\frac{7}{4}\right)} - \frac{ix\Gamma\left(\frac{1}{4}\right) {}_2F_1\left(\frac{1}{4}, \frac{1}{2} \middle| \frac{5}{4}; b^2x^4\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**2+1)/(b**2*x**4-1)**(1/2),x)

[Out] -I*b*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), b**2*x**4)/(4*gamma(7/4)) - I*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), b**2*x**4)/(4*gamma(5/4))

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{bx^2 + 1}{\sqrt{b^2x^4 - 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^2+1)/(b^2*x^4-1)^(1/2),x, algorithm="giac")

[Out] integrate((b*x^2 + 1)/sqrt(b^2*x^4 - 1), x)

$$3.17 \quad \int \frac{1-bx^2}{\sqrt{-1+b^2x^4}} dx$$

Optimal. Leaf size=89

$$\frac{2\sqrt{1-b^2x^4}\text{EllipticF}(\sin^{-1}(\sqrt{bx}), -1)}{\sqrt{b}\sqrt{b^2x^4-1}} - \frac{\sqrt{1-b^2x^4}E(\sin^{-1}(\sqrt{bx})|-1)}{\sqrt{b}\sqrt{b^2x^4-1}}$$

[Out] -((Sqrt[1 - b^2*x^4]*EllipticE[ArcSin[Sqrt[b]*x], -1])/(Sqrt[b]*Sqrt[-1 + b^2*x^4])) + (2*Sqrt[1 - b^2*x^4]*EllipticF[ArcSin[Sqrt[b]*x], -1])/(Sqrt[b]*Sqrt[-1 + b^2*x^4])

Rubi [A] time = 0.0455572, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {1200, 1199, 423, 424, 248, 221}

$$\frac{2\sqrt{1-b^2x^4}F(\sin^{-1}(\sqrt{bx})|-1)}{\sqrt{b}\sqrt{b^2x^4-1}} - \frac{\sqrt{1-b^2x^4}E(\sin^{-1}(\sqrt{bx})|-1)}{\sqrt{b}\sqrt{b^2x^4-1}}$$

Antiderivative was successfully verified.

[In] Int[(1 - b*x^2)/Sqrt[-1 + b^2*x^4], x]

[Out] -((Sqrt[1 - b^2*x^4]*EllipticE[ArcSin[Sqrt[b]*x], -1])/(Sqrt[b]*Sqrt[-1 + b^2*x^4])) + (2*Sqrt[1 - b^2*x^4]*EllipticF[ArcSin[Sqrt[b]*x], -1])/(Sqrt[b]*Sqrt[-1 + b^2*x^4])

Rule 1200

Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + (c*x^4)/a], x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && !GtQ[a, 0]

Rule 1199

Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sqrt[a], Int[Sqrt[1 + (e*x^2)/d]/Sqrt[1 - (e*x^2)/d], x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]

Rule 423

Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[b/d, Int[Sqrt[c + d*x^2]/Sqrt[a + b*x^2], x], x] - Dist[(b*c - a*d)/d, Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c] && NegQ[b/a]

Rule 424

Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]

Rule 248

Int[((a1_.) + (b1_.)*(x_)^(n_))^(p_.)*((a2_.) + (b2_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[(a1*a2 + b1*b2*x^(2*n))^p, x] /; FreeQ[{a1, b1, a2, b2, n, p}

```
}, x] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[p] || (GtQ[a1, 0] && GtQ[a2, 0]
))
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_)*(x_)^4], x_Symbol] := Simp[EllipticF[ArcSin[(Rt[-b,
4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{1-bx^2}{\sqrt{-1+b^2x^4}} dx &= \frac{\sqrt{1-b^2x^4} \int \frac{1-bx^2}{\sqrt{1-b^2x^4}} dx}{\sqrt{-1+b^2x^4}} \\ &= \frac{\sqrt{1-b^2x^4} \int \frac{\sqrt{1-bx^2}}{\sqrt{1+bx^2}} dx}{\sqrt{-1+b^2x^4}} \\ &= -\frac{\sqrt{1-b^2x^4} \int \frac{\sqrt{1+bx^2}}{\sqrt{1-bx^2}} dx}{\sqrt{-1+b^2x^4}} + \frac{(2\sqrt{1-b^2x^4}) \int \frac{1}{\sqrt{1-bx^2}\sqrt{1+bx^2}} dx}{\sqrt{-1+b^2x^4}} \\ &= -\frac{\sqrt{1-b^2x^4} E(\sin^{-1}(\sqrt{bx})| -1)}{\sqrt{b}\sqrt{-1+b^2x^4}} + \frac{(2\sqrt{1-b^2x^4}) \int \frac{1}{\sqrt{1-b^2x^4}} dx}{\sqrt{-1+b^2x^4}} \\ &= -\frac{\sqrt{1-b^2x^4} E(\sin^{-1}(\sqrt{bx})| -1)}{\sqrt{b}\sqrt{-1+b^2x^4}} + \frac{2\sqrt{1-b^2x^4} F(\sin^{-1}(\sqrt{bx})| -1)}{\sqrt{b}\sqrt{-1+b^2x^4}} \end{aligned}$$

Mathematica [C] time = 0.0209353, size = 74, normalized size = 0.83

$$-\frac{\sqrt{1-b^2x^4} \left(bx^3 {}_2F_1\left(\frac{1}{2}, \frac{3}{4}; \frac{7}{4}; b^2x^4\right) - 3x {}_2F_1\left(\frac{1}{4}, \frac{1}{2}; \frac{5}{4}; b^2x^4\right) \right)}{3\sqrt{b^2x^4-1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - b*x^2)/Sqrt[-1 + b^2*x^4], x]
```

```
[Out] -(Sqrt[1 - b^2*x^4]*(-3*x*Hypergeometric2F1[1/4, 1/2, 5/4, b^2*x^4] + b*x^3
*Hypergeometric2F1[1/2, 3/4, 7/4, b^2*x^4]))/(3*Sqrt[-1 + b^2*x^4])
```

Maple [A] time = 0.046, size = 108, normalized size = 1.2

$$-\sqrt{bx^2+1}\sqrt{-bx^2+1} \left(\text{EllipticF}\left(x\sqrt{-b}, i\right) - \text{EllipticE}\left(x\sqrt{-b}, i\right) \right) \frac{1}{\sqrt{-b}} \frac{1}{\sqrt{b^2x^4-1}} + \sqrt{bx^2+1}\sqrt{-bx^2+1} \text{EllipticF}\left(x\sqrt{-b}, i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^2+1)/(b^2*x^4-1)^(1/2), x)
```

```
[Out] -1/(-b)^(1/2)*(b*x^2+1)^(1/2)*(-b*x^2+1)^(1/2)/(b^2*x^4-1)^(1/2)*(EllipticF
(x*(-b)^(1/2), I)-EllipticE(x*(-b)^(1/2), I))+1/(-b)^(1/2)*(b*x^2+1)^(1/2)*(-
b*x^2+1)^(1/2)/(b^2*x^4-1)^(1/2)*EllipticF(x*(-b)^(1/2), I)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{bx^2 - 1}{\sqrt{b^2x^4 - 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b*x^2+1)/(b^2*x^4-1)^(1/2),x, algorithm="maxima")

[Out] -integrate((b*x^2 - 1)/sqrt(b^2*x^4 - 1), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{b^2x^4 - 1}}{bx^2 + 1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b*x^2+1)/(b^2*x^4-1)^(1/2),x, algorithm="fricas")

[Out] integral(-sqrt(b^2*x^4 - 1)/(b*x^2 + 1), x)

Sympy [A] time = 1.86707, size = 60, normalized size = 0.67

$$\frac{ibx^3\Gamma\left(\frac{3}{4}\right) {}_2F_1\left(\frac{1}{2}, \frac{3}{4} \middle| \frac{7}{4}; b^2x^4\right)}{4\Gamma\left(\frac{7}{4}\right)} - \frac{ix\Gamma\left(\frac{1}{4}\right) {}_2F_1\left(\frac{1}{4}, \frac{1}{2} \middle| \frac{5}{4}; b^2x^4\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b*x**2+1)/(b**2*x**4-1)**(1/2),x)

[Out] I*b*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), b**2*x**4)/(4*gamma(7/4)) - I*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), b**2*x**4)/(4*gamma(5/4))

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{bx^2 - 1}{\sqrt{b^2x^4 - 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b*x^2+1)/(b^2*x^4-1)^(1/2),x, algorithm="giac")

[Out] integrate(-(b*x^2 - 1)/sqrt(b^2*x^4 - 1), x)

3.18 $\int \frac{1-bx^2}{\sqrt{1+b^2x^4}} dx$

Optimal. Leaf size=89

$$\frac{(bx^2 + 1) \sqrt{\frac{b^2x^4+1}{(bx^2+1)^2}} E\left(2 \tan^{-1}(\sqrt{bx}) \middle| \frac{1}{2}\right)}{\sqrt{b}\sqrt{b^2x^4+1}} - \frac{x\sqrt{b^2x^4+1}}{bx^2+1}$$

[Out] $-\left(\frac{x\sqrt{1+b^2x^4}}{1+b^2x^2}\right) + \left(\frac{(1+b^2x^2)\sqrt{(1+b^2x^4)/(1+b^2x^2)^2} \text{EllipticE}[2\text{ArcTan}[\text{Sqrt}[b]*x], 1/2]}{\text{Sqrt}[b]*\text{Sqrt}[1+b^2x^4]}\right)$

Rubi [A] time = 0.0135721, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {1196}

$$\frac{(bx^2 + 1) \sqrt{\frac{b^2x^4+1}{(bx^2+1)^2}} E\left(2 \tan^{-1}(\sqrt{bx}) \middle| \frac{1}{2}\right)}{\sqrt{b}\sqrt{b^2x^4+1}} - \frac{x\sqrt{b^2x^4+1}}{bx^2+1}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(1 - b*x^2)/\text{Sqrt}[1 + b^2*x^4], x]$

[Out] $-\left(\frac{x\sqrt{1+b^2x^4}}{1+b^2x^2}\right) + \left(\frac{(1+b^2x^2)\sqrt{(1+b^2x^4)/(1+b^2x^2)^2} \text{EllipticE}[2\text{ArcTan}[\text{Sqrt}[b]*x], 1/2]}{\text{Sqrt}[b]*\text{Sqrt}[1+b^2x^4]}\right)$

Rule 1196

$\text{Int}[\frac{(d_+ + (e_+)*(x_+)^2)/\text{Sqrt}[(a_+ + (c_+)*(x_+)^4], x_Symbol]}{> \text{With}[\{q = \text{Rt}[c/a, 4]\}, -\text{Simp}[(d*x*\text{Sqrt}[a + c*x^4])/(a*(1 + q^2*x^2)), x] + \text{Simp}[(d*(1 + q^2*x^2)*\text{Sqrt}[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*\text{EllipticE}[2*\text{ArcTan}[q*x], 1/2)]/(q*\text{Sqrt}[a + c*x^4]), x] /; \text{EqQ}[e + d*q^2, 0]] /; \text{FreeQ}[\{a, c, d, e\}, x] \&\& \text{PosQ}[c/a]$

Rubi steps

$$\int \frac{1-bx^2}{\sqrt{1+b^2x^4}} dx = -\frac{x\sqrt{1+b^2x^4}}{1+b^2x^2} + \frac{(1+b^2x^2) \sqrt{\frac{1+b^2x^4}{(1+b^2x^2)^2}} E\left(2 \tan^{-1}(\sqrt{bx}) \middle| \frac{1}{2}\right)}{\sqrt{b}\sqrt{1+b^2x^4}}$$

Mathematica [C] time = 0.0116996, size = 47, normalized size = 0.53

$$x {}_2F_1\left(\frac{1}{4}, \frac{1}{2}; \frac{5}{4}; -b^2x^4\right) - \frac{1}{3}bx^3 {}_2F_1\left(\frac{1}{2}, \frac{3}{4}; \frac{7}{4}; -b^2x^4\right)$$

Antiderivative was successfully verified.

[In] $\text{Integrate}[(1 - b*x^2)/\text{Sqrt}[1 + b^2*x^4], x]$

[Out] $x \cdot \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, -(b^2 x^4)\right] - (b x^3 \cdot \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -(b^2 x^4)\right]) / 3$

Maple [C] time = 0.153, size = 120, normalized size = 1.4

$$-i\sqrt{1-ibx^2}\sqrt{1+ibx^2}\left(\text{EllipticF}\left(x\sqrt{ib},i\right)-\text{EllipticE}\left(x\sqrt{ib},i\right)\right)\frac{1}{\sqrt{ib}}\frac{1}{\sqrt{b^2x^4+1}}+\sqrt{1-ibx^2}\sqrt{1+ibx^2}\text{EllipticF}\left(x\sqrt{ib},i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((-b*x^2+1)/(b^2*x^4+1)^(1/2),x)`

[Out] $-I/(I*b)^{(1/2)}*(1-I*b*x^2)^{(1/2)}*(1+I*b*x^2)^{(1/2)}/(b^2*x^4+1)^{(1/2)}*(\text{EllipticF}(x*(I*b)^{(1/2)},I)-\text{EllipticE}(x*(I*b)^{(1/2)},I))+1/(I*b)^{(1/2)}*(1-I*b*x^2)^{(1/2)}*(1+I*b*x^2)^{(1/2)}/(b^2*x^4+1)^{(1/2)}*\text{EllipticF}(x*(I*b)^{(1/2)},I)$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{bx^2 - 1}{\sqrt{b^2x^4 + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-b*x^2+1)/(b^2*x^4+1)^(1/2),x, algorithm="maxima")`

[Out] `-integrate((b*x^2 - 1)/sqrt(b^2*x^4 + 1), x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{bx^2 - 1}{\sqrt{b^2x^4 + 1}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-b*x^2+1)/(b^2*x^4+1)^(1/2),x, algorithm="fricas")`

[Out] `integral(-(b*x^2 - 1)/sqrt(b^2*x^4 + 1), x)`

Sympy [C] time = 1.83919, size = 66, normalized size = 0.74

$$-\frac{bx^3\Gamma\left(\frac{3}{4}\right) {}_2F_1\left(\frac{1}{2}, \frac{3}{4} \middle| \frac{1}{4} \right) b^2 x^4 e^{i\pi}}{4\Gamma\left(\frac{7}{4}\right)} + \frac{x\Gamma\left(\frac{1}{4}\right) {}_2F_1\left(\frac{1}{4}, \frac{1}{2} \middle| \frac{5}{4} \right) b^2 x^4 e^{i\pi}}{4\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-b*x**2+1)/(b**2*x**4+1)**(1/2),x)`

```
[Out] -b*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), b**2*x**4*exp_polar(I*pi))/(4*
gamma(7/4)) + x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), b**2*x**4*exp_polar(I*
pi))/(4*gamma(5/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{bx^2 - 1}{\sqrt{b^2x^4 + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+1)/(b^2*x^4+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-(b*x^2 - 1)/sqrt(b^2*x^4 + 1), x)
```


$$3.19 \quad \int \frac{1+bx^2}{\sqrt{1+b^2x^4}} dx$$

Optimal. Leaf size=152

$$\frac{(bx^2 + 1) \sqrt{\frac{b^2x^4+1}{(bx^2+1)^2}} \text{EllipticF}\left(2 \tan^{-1}(\sqrt{bx}), \frac{1}{2}\right)}{\sqrt{b}\sqrt{b^2x^4+1}} + \frac{x\sqrt{b^2x^4+1}}{bx^2+1} - \frac{(bx^2 + 1) \sqrt{\frac{b^2x^4+1}{(bx^2+1)^2}} E\left(2 \tan^{-1}(\sqrt{bx}) \middle| \frac{1}{2}\right)}{\sqrt{b}\sqrt{b^2x^4+1}}$$

[Out] (x*Sqrt[1 + b^2*x^4])/(1 + b*x^2) - ((1 + b*x^2)*Sqrt[(1 + b^2*x^4)/(1 + b*x^2)^2]*EllipticE[2*ArcTan[Sqrt[b]*x], 1/2])/(Sqrt[b]*Sqrt[1 + b^2*x^4]) + ((1 + b*x^2)*Sqrt[(1 + b^2*x^4)/(1 + b*x^2)^2]*EllipticF[2*ArcTan[Sqrt[b]*x], 1/2])/(Sqrt[b]*Sqrt[1 + b^2*x^4])

Rubi [A] time = 0.0309574, antiderivative size = 152, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1198, 220, 1196}

$$\frac{x\sqrt{b^2x^4+1}}{bx^2+1} + \frac{(bx^2 + 1) \sqrt{\frac{b^2x^4+1}{(bx^2+1)^2}} F\left(2 \tan^{-1}(\sqrt{bx}) \middle| \frac{1}{2}\right)}{\sqrt{b}\sqrt{b^2x^4+1}} - \frac{(bx^2 + 1) \sqrt{\frac{b^2x^4+1}{(bx^2+1)^2}} E\left(2 \tan^{-1}(\sqrt{bx}) \middle| \frac{1}{2}\right)}{\sqrt{b}\sqrt{b^2x^4+1}}$$

Antiderivative was successfully verified.

[In] Int[(1 + b*x^2)/Sqrt[1 + b^2*x^4], x]

[Out] (x*Sqrt[1 + b^2*x^4])/(1 + b*x^2) - ((1 + b*x^2)*Sqrt[(1 + b^2*x^4)/(1 + b*x^2)^2]*EllipticE[2*ArcTan[Sqrt[b]*x], 1/2])/(Sqrt[b]*Sqrt[1 + b^2*x^4]) + ((1 + b*x^2)*Sqrt[(1 + b^2*x^4)/(1 + b*x^2)^2]*EllipticF[2*ArcTan[Sqrt[b]*x], 1/2])/(Sqrt[b]*Sqrt[1 + b^2*x^4])

Rule 1198

Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0] /; FreeQ[{a, c, d, e}, x] && PosQ[c/a]

Rule 220

Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2]]*EllipticF[2*ArcTan[q*x], 1/2])/(2*q*Sqrt[a + b*x^4]), x] /; FreeQ[{a, b}, x] && PosQ[b/a]

Rule 1196

Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2]]*EllipticE[2*ArcTan[q*x], 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0] /; FreeQ[{a, c, d, e}, x] && PosQ[c/a]

Rubi steps

$$\int \frac{1+bx^2}{\sqrt{1+b^2x^4}} dx = 2 \int \frac{1}{\sqrt{1+b^2x^4}} dx - \int \frac{1-bx^2}{\sqrt{1+b^2x^4}} dx$$

$$= \frac{x\sqrt{1+b^2x^4}}{1+bx^2} - \frac{(1+bx^2) \sqrt{\frac{1+b^2x^4}{(1+bx^2)^2}} E\left(2 \tan^{-1}(\sqrt{bx}) \middle| \frac{1}{2}\right)}{\sqrt{b}\sqrt{1+b^2x^4}} + \frac{(1+bx^2) \sqrt{\frac{1+b^2x^4}{(1+bx^2)^2}} F\left(2 \tan^{-1}(\sqrt{bx}) \middle| \frac{1}{2}\right)}{\sqrt{b}\sqrt{1+b^2x^4}}$$

Mathematica [C] time = 0.0104511, size = 47, normalized size = 0.31

$$\frac{1}{3}bx^3 {}_2F_1\left(\frac{1}{2}, \frac{3}{4}; \frac{7}{4}; -b^2x^4\right) + x {}_2F_1\left(\frac{1}{4}, \frac{1}{2}; \frac{5}{4}; -b^2x^4\right)$$

Antiderivative was successfully verified.

[In] Integrate[(1 + b*x^2)/Sqrt[1 + b^2*x^4], x]

[Out] x*Hypergeometric2F1[1/4, 1/2, 5/4, -(b^2*x^4)] + (b*x^3*Hypergeometric2F1[1/2, 3/4, 7/4, -(b^2*x^4)])/3

Maple [C] time = 0.046, size = 120, normalized size = 0.8

$$i\sqrt{1-ibx^2}\sqrt{1+ibx^2}\left(\text{EllipticF}\left(x\sqrt{ib}, i\right) - \text{EllipticE}\left(x\sqrt{ib}, i\right)\right) \frac{1}{\sqrt{ib}} \frac{1}{\sqrt{b^2x^4+1}} + \sqrt{1-ibx^2}\sqrt{1+ibx^2}\text{EllipticF}\left(x\sqrt{ib}, i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^2+1)/(b^2*x^4+1)^(1/2), x)

[Out] I/(I*b)^(1/2)*(1-I*b*x^2)^(1/2)*(1+I*b*x^2)^(1/2)/(b^2*x^4+1)^(1/2)*(EllipticF(x*(I*b)^(1/2), I)-EllipticE(x*(I*b)^(1/2), I))+1/(I*b)^(1/2)*(1-I*b*x^2)^(1/2)*(1+I*b*x^2)^(1/2)/(b^2*x^4+1)^(1/2)*EllipticF(x*(I*b)^(1/2), I)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{bx^2+1}{\sqrt{b^2x^4+1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^2+1)/(b^2*x^4+1)^(1/2), x, algorithm="maxima")

[Out] integrate((b*x^2 + 1)/sqrt(b^2*x^4 + 1), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{bx^2+1}{\sqrt{b^2x^4+1}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^2+1)/(b^2*x^4+1)^(1/2),x, algorithm="fricas")

[Out] integral((b*x^2 + 1)/sqrt(b^2*x^4 + 1), x)

Sympy [C] time = 1.49538, size = 66, normalized size = 0.43

$$\frac{bx^3\Gamma\left(\frac{3}{4}\right) {}_2F_1\left(\frac{1}{2}, \frac{3}{4} \middle| \frac{7}{4}; b^2x^4e^{i\pi}\right)}{4\Gamma\left(\frac{7}{4}\right)} + \frac{x\Gamma\left(\frac{1}{4}\right) {}_2F_1\left(\frac{1}{4}, \frac{1}{2} \middle| \frac{5}{4}; b^2x^4e^{i\pi}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**2+1)/(b**2*x**4+1)**(1/2),x)

[Out] b*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), b**2*x**4*exp_polar(I*pi))/(4*gamma(7/4)) + x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), b**2*x**4*exp_polar(I*pi))/(4*gamma(5/4))

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{bx^2 + 1}{\sqrt{b^2x^4 + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^2+1)/(b^2*x^4+1)^(1/2),x, algorithm="giac")

[Out] integrate((b*x^2 + 1)/sqrt(b^2*x^4 + 1), x)

$$3.20 \quad \int \frac{1-bx^2}{\sqrt{-1-b^2x^4}} dx$$

Optimal. Leaf size=90

$$\frac{x\sqrt{-b^2x^4-1}}{bx^2+1} + \frac{(bx^2+1)\sqrt{\frac{b^2x^4+1}{(bx^2+1)^2}}E\left(2\tan^{-1}(\sqrt{bx})\middle|\frac{1}{2}\right)}{\sqrt{b}\sqrt{-b^2x^4-1}}$$

[Out] (x*Sqrt[-1 - b^2*x^4])/(1 + b*x^2) + ((1 + b*x^2)*Sqrt[(1 + b^2*x^4)/(1 + b*x^2)^2]*EllipticE[2*ArcTan[Sqrt[b]*x], 1/2])/(Sqrt[b]*Sqrt[-1 - b^2*x^4])

Rubi [A] time = 0.0148451, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {1196}

$$\frac{x\sqrt{-b^2x^4-1}}{bx^2+1} + \frac{(bx^2+1)\sqrt{\frac{b^2x^4+1}{(bx^2+1)^2}}E\left(2\tan^{-1}(\sqrt{bx})\middle|\frac{1}{2}\right)}{\sqrt{b}\sqrt{-b^2x^4-1}}$$

Antiderivative was successfully verified.

[In] Int[(1 - b*x^2)/Sqrt[-1 - b^2*x^4], x]

[Out] (x*Sqrt[-1 - b^2*x^4])/(1 + b*x^2) + ((1 + b*x^2)*Sqrt[(1 + b^2*x^4)/(1 + b*x^2)^2]*EllipticE[2*ArcTan[Sqrt[b]*x], 1/2])/(Sqrt[b]*Sqrt[-1 - b^2*x^4])

Rule 1196

Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] := With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x], 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e}, x] && PosQ[c/a]

Rubi steps

$$\int \frac{1-bx^2}{\sqrt{-1-b^2x^4}} dx = \frac{x\sqrt{-1-b^2x^4}}{1+bx^2} + \frac{(1+bx^2)\sqrt{\frac{1+b^2x^4}{(1+bx^2)^2}}E\left(2\tan^{-1}(\sqrt{bx})\middle|\frac{1}{2}\right)}{\sqrt{b}\sqrt{-1-b^2x^4}}$$

Mathematica [C] time = 0.0259272, size = 76, normalized size = 0.84

$$\frac{\sqrt{b^2x^4+1}\left(bx^3{}_2F_1\left(\frac{1}{2}, \frac{3}{4}; \frac{7}{4}; -b^2x^4\right) - 3x{}_2F_1\left(\frac{1}{4}, \frac{1}{2}; \frac{5}{4}; -b^2x^4\right)\right)}{3\sqrt{-b^2x^4-1}}$$

Antiderivative was successfully verified.

[In] Integrate[(1 - b*x^2)/Sqrt[-1 - b^2*x^4], x]

[Out] $-(\text{Sqrt}[1 + b^2x^4] * (-3x * \text{Hypergeometric2F1}[1/4, 1/2, 5/4, -(b^2x^4)] + b * x^3 * \text{Hypergeometric2F1}[1/2, 3/4, 7/4, -(b^2x^4)])) / (3 * \text{Sqrt}[-1 - b^2x^4])$

Maple [C] time = 0.131, size = 122, normalized size = 1.4

$i\sqrt{1+ibx^2}\sqrt{1-ibx^2}\left(\text{EllipticF}\left(x\sqrt{-ib},i\right)-\text{EllipticE}\left(x\sqrt{-ib},i\right)\right)\frac{1}{\sqrt{-ib}}\frac{1}{\sqrt{-b^2x^4-1}}+\sqrt{1+ibx^2}\sqrt{1-ibx^2}\text{EllipticF}$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((-b*x^2+1)/(-b^2*x^4-1)^{(1/2)},x)$

[Out] $I/(-I*b)^{(1/2)}*(1+I*b*x^2)^{(1/2)}*(1-I*b*x^2)^{(1/2)}/(-b^2*x^4-1)^{(1/2)}*(\text{EllipticF}(x*(-I*b)^{(1/2)},I)-\text{EllipticE}(x*(-I*b)^{(1/2)},I))+1/(-I*b)^{(1/2)}*(1+I*b*x^2)^{(1/2)}*(1-I*b*x^2)^{(1/2)}/(-b^2*x^4-1)^{(1/2)}*\text{EllipticF}(x*(-I*b)^{(1/2)},I)$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{bx^2 - 1}{\sqrt{-b^2x^4 - 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}((-b*x^2+1)/(-b^2*x^4-1)^{(1/2)},x, \text{algorithm}="maxima")$

[Out] $-\text{integrate}((b*x^2 - 1)/\text{sqrt}(-b^2*x^4 - 1), x)$

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{bx \text{integral}\left(-\frac{\sqrt{-b^2x^4-1}(bx^2-1)}{b^3x^6+bx^2}, x\right) + \sqrt{-b^2x^4-1}}{bx}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}((-b*x^2+1)/(-b^2*x^4-1)^{(1/2)},x, \text{algorithm}="fricas")$

[Out] $(b*x*\text{integral}(-\text{sqrt}(-b^2*x^4 - 1)*(b*x^2 - 1)/(b^3*x^6 + b*x^2), x) + \text{sqrt}(-b^2*x^4 - 1))/(b*x)$

Sympy [C] time = 1.9063, size = 70, normalized size = 0.78

$$\frac{ibx^3\Gamma\left(\frac{3}{4}\right)_2F_1\left(\frac{1}{2}, \frac{3}{4} \middle| \frac{1}{4} \right) b^2x^4 e^{i\pi}}{4\Gamma\left(\frac{7}{4}\right)} - \frac{ix\Gamma\left(\frac{1}{4}\right)_2F_1\left(\frac{1}{4}, \frac{1}{2} \middle| \frac{1}{4} \right) b^2x^4 e^{i\pi}}{4\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**2+1)/(-b**2*x**4-1)**(1/2),x)
```

```
[Out] I*b*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), b**2*x**4*exp_polar(I*pi))/(4
*gamma(7/4)) - I*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), b**2*x**4*exp_polar
(I*pi))/(4*gamma(5/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{bx^2 - 1}{\sqrt{-b^2x^4 - 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+1)/(-b^2*x^4-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-(b*x^2 - 1)/sqrt(-b^2*x^4 - 1), x)
```

$$3.21 \quad \int \frac{1+bx^2}{\sqrt{-1-b^2x^4}} dx$$

Optimal. Leaf size=156

$$\frac{(bx^2 + 1) \sqrt{\frac{b^2x^4+1}{(bx^2+1)^2}} \text{EllipticF}\left(2 \tan^{-1}(\sqrt{bx}), \frac{1}{2}\right)}{\sqrt{b}\sqrt{-b^2x^4-1}} - \frac{x\sqrt{-b^2x^4-1}}{bx^2+1} - \frac{(bx^2 + 1) \sqrt{\frac{b^2x^4+1}{(bx^2+1)^2}} E\left(2 \tan^{-1}(\sqrt{bx}) \middle| \frac{1}{2}\right)}{\sqrt{b}\sqrt{-b^2x^4-1}}$$

[Out] $-\left(\frac{x\sqrt{-1-b^2x^4}}{1+b^2x^2}\right) - \left(\frac{(1+b^2x^2)\sqrt{(1+b^2x^4)/(1+b^2x^2)^2} \text{EllipticE}[2\text{ArcTan}[\sqrt{b}x], 1/2]}{\sqrt{b}\sqrt{-1-b^2x^4}}\right) + \left(\frac{(1+b^2x^2)\sqrt{(1+b^2x^4)/(1+b^2x^2)^2} \text{EllipticF}[2\text{ArcTan}[\sqrt{b}x], 1/2]}{\sqrt{b}\sqrt{-1-b^2x^4}}\right)$

Rubi [A] time = 0.03229, antiderivative size = 156, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {1198, 220, 1196}

$$-\frac{x\sqrt{-b^2x^4-1}}{bx^2+1} + \frac{(bx^2+1) \sqrt{\frac{b^2x^4+1}{(bx^2+1)^2}} F\left(2 \tan^{-1}(\sqrt{bx}) \middle| \frac{1}{2}\right)}{\sqrt{b}\sqrt{-b^2x^4-1}} - \frac{(bx^2+1) \sqrt{\frac{b^2x^4+1}{(bx^2+1)^2}} E\left(2 \tan^{-1}(\sqrt{bx}) \middle| \frac{1}{2}\right)}{\sqrt{b}\sqrt{-b^2x^4-1}}$$

Antiderivative was successfully verified.

[In] Int[(1 + b*x^2)/Sqrt[-1 - b^2*x^4], x]

[Out] $-\left(\frac{x\sqrt{-1-b^2x^4}}{1+b^2x^2}\right) - \left(\frac{(1+b^2x^2)\sqrt{(1+b^2x^4)/(1+b^2x^2)^2} \text{EllipticE}[2\text{ArcTan}[\sqrt{b}x], 1/2]}{\sqrt{b}\sqrt{-1-b^2x^4}}\right) + \left(\frac{(1+b^2x^2)\sqrt{(1+b^2x^4)/(1+b^2x^2)^2} \text{EllipticF}[2\text{ArcTan}[\sqrt{b}x], 1/2]}{\sqrt{b}\sqrt{-1-b^2x^4}}\right)$

Rule 1198

Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] := With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0] /; FreeQ[{a, c, d, e}, x] && PosQ[c/a]

Rule 220

Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] := With[{q = Rt[b/a, 4]}, Simp[(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x], 1/2]/(2*q*Sqrt[a + b*x^4]), x] /; FreeQ[{a, b}, x] && PosQ[b/a]

Rule 1196

Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] := With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x], 1/2]/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0] /; FreeQ[{a, c, d, e}, x] && PosQ[c/a]

Rubi steps

$$\int \frac{1+bx^2}{\sqrt{-1-b^2x^4}} dx = 2 \int \frac{1}{\sqrt{-1-b^2x^4}} dx - \int \frac{1-bx^2}{\sqrt{-1-b^2x^4}} dx$$

$$= \frac{x\sqrt{-1-b^2x^4}}{1+bx^2} - \frac{(1+bx^2) \sqrt{\frac{1+b^2x^4}{(1+bx^2)^2}} E\left(2 \tan^{-1}(\sqrt{bx}) \middle| \frac{1}{2}\right)}{\sqrt{b}\sqrt{-1-b^2x^4}} + \frac{(1+bx^2) \sqrt{\frac{1+b^2x^4}{(1+bx^2)^2}} F\left(2 \tan^{-1}(\sqrt{bx}) \middle| \frac{1}{2}\right)}{\sqrt{b}\sqrt{-1-b^2x^4}}$$

Mathematica [C] time = 0.0186291, size = 76, normalized size = 0.49

$$\frac{\sqrt{b^2x^4+1} \left(bx^3 {}_2F_1\left(\frac{1}{2}, \frac{3}{4}; \frac{7}{4}; -b^2x^4\right) + 3x {}_2F_1\left(\frac{1}{4}, \frac{1}{2}; \frac{5}{4}; -b^2x^4\right) \right)}{3\sqrt{-b^2x^4-1}}$$

Antiderivative was successfully verified.

[In] Integrate[(1 + b*x^2)/Sqrt[-1 - b^2*x^4], x]

[Out] (Sqrt[1 + b^2*x^4]*(3*x*Hypergeometric2F1[1/4, 1/2, 5/4, -(b^2*x^4)] + b*x^3*Hypergeometric2F1[1/2, 3/4, 7/4, -(b^2*x^4)]))/(3*Sqrt[-1 - b^2*x^4])

Maple [C] time = 0.046, size = 122, normalized size = 0.8

$$-i\sqrt{1+ibx^2}\sqrt{1-ibx^2} \left(\text{EllipticF}\left(x\sqrt{-ib}, i\right) - \text{EllipticE}\left(x\sqrt{-ib}, i\right) \right) \frac{1}{\sqrt{-ib}} \frac{1}{\sqrt{-b^2x^4-1}} + \sqrt{1+ibx^2}\sqrt{1-ibx^2} \text{EllipticF}\left(x\sqrt{-ib}, i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^2+1)/(-b^2*x^4-1)^(1/2), x)

[Out] -I/(-I*b)^(1/2)*(1+I*b*x^2)^(1/2)*(1-I*b*x^2)^(1/2)/(-b^2*x^4-1)^(1/2)*(EllipticF(x*(-I*b)^(1/2), I)-EllipticE(x*(-I*b)^(1/2), I))+1/(-I*b)^(1/2)*(1+I*b*x^2)^(1/2)*(1-I*b*x^2)^(1/2)/(-b^2*x^4-1)^(1/2)*EllipticF(x*(-I*b)^(1/2), I)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{bx^2+1}{\sqrt{-b^2x^4-1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^2+1)/(-b^2*x^4-1)^(1/2), x, algorithm="maxima")

[Out] integrate((b*x^2 + 1)/sqrt(-b^2*x^4 - 1), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{bx \int \left(-\frac{\sqrt{-b^2x^4-1}(bx^2+1)}{b^3x^6+bx^2}, x \right) - \sqrt{-b^2x^4-1}}{bx}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^2+1)/(-b^2*x^4-1)^(1/2),x, algorithm="fricas")

[Out] (b*x*integral(-sqrt(-b^2*x^4 - 1)*(b*x^2 + 1)/(b^3*x^6 + b*x^2), x) - sqrt(-b^2*x^4 - 1))/(b*x)

Sympy [C] time = 1.51971, size = 71, normalized size = 0.46

$$\frac{ibx^3\Gamma\left(\frac{3}{4}\right) {}_2F_1\left(\frac{1}{2}, \frac{3}{4} \middle| \frac{7}{4}; b^2x^4e^{i\pi}\right)}{4\Gamma\left(\frac{7}{4}\right)} - \frac{ix\Gamma\left(\frac{1}{4}\right) {}_2F_1\left(\frac{1}{4}, \frac{1}{2} \middle| \frac{5}{4}; b^2x^4e^{i\pi}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**2+1)/(-b**2*x**4-1)**(1/2),x)

[Out] -I*b*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), b**2*x**4*exp_polar(I*pi))/(4*gamma(7/4)) - I*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), b**2*x**4*exp_polar(I*pi))/(4*gamma(5/4))

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{bx^2 + 1}{\sqrt{-b^2x^4 - 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^2+1)/(-b^2*x^4-1)^(1/2),x, algorithm="giac")

[Out] integrate((b*x^2 + 1)/sqrt(-b^2*x^4 - 1), x)

$$3.22 \quad \int \frac{\sqrt{1+c^2x^2}}{\sqrt{1-c^2x^2}} dx$$

Optimal. Leaf size=10

$$\frac{E(\sin^{-1}(cx)|-1)}{c}$$

[Out] EllipticE[ArcSin[c*x], -1]/c

Rubi [A] time = 0.0093696, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.036$, Rules used = {424}

$$\frac{E(\sin^{-1}(cx)|-1)}{c}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[1 + c^2*x^2]/Sqrt[1 - c^2*x^2],x]

[Out] EllipticE[ArcSin[c*x], -1]/c

Rule 424

Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]

Rubi steps

$$\int \frac{\sqrt{1+c^2x^2}}{\sqrt{1-c^2x^2}} dx = \frac{E(\sin^{-1}(cx)|-1)}{c}$$

Mathematica [A] time = 0.0071131, size = 10, normalized size = 1.

$$\frac{E(\sin^{-1}(cx)|-1)}{c}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[1 + c^2*x^2]/Sqrt[1 - c^2*x^2],x]

[Out] EllipticE[ArcSin[c*x], -1]/c

Maple [C] time = 0.063, size = 15, normalized size = 1.5

$$\frac{\text{EllipticE}(xc\text{sgn}(c), i) c\text{sgn}(c)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c^2*x^2+1)^(1/2)/(-c^2*x^2+1)^(1/2),x)`

[Out] `EllipticE(x*csgn(c)*c,I)*csgn(c)/c`

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c^2x^2+1}}{\sqrt{-c^2x^2+1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c^2*x^2+1)^(1/2)/(-c^2*x^2+1)^(1/2),x, algorithm="maxima")`

[Out] `integrate(sqrt(c^2*x^2 + 1)/sqrt(-c^2*x^2 + 1), x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{c^2x^2+1}\sqrt{-c^2x^2+1}}{c^2x^2-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c^2*x^2+1)^(1/2)/(-c^2*x^2+1)^(1/2),x, algorithm="fricas")`

[Out] `integral(-sqrt(c^2*x^2 + 1)*sqrt(-c^2*x^2 + 1)/(c^2*x^2 - 1), x)`

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c^2x^2+1}}{\sqrt{-(cx-1)(cx+1)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c**2*x**2+1)**(1/2)/(-c**2*x**2+1)**(1/2),x)`

[Out] `Integral(sqrt(c**2*x**2 + 1)/sqrt(-(c*x - 1)*(c*x + 1)), x)`

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c^2x^2+1}}{\sqrt{-c^2x^2+1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c^2*x^2+1)^(1/2)/(-c^2*x^2+1)^(1/2),x, algorithm="giac")`

[Out] `integrate(sqrt(c^2*x^2 + 1)/sqrt(-c^2*x^2 + 1), x)`

$$3.23 \quad \int \frac{1+c^2x^2}{\sqrt{1-c^4x^4}} dx$$

Optimal. Leaf size=10

$$\frac{E(\sin^{-1}(cx)|-1)}{c}$$

[Out] EllipticE[ArcSin[c*x], -1]/c

Rubi [A] time = 0.015723, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1199, 424}

$$\frac{E(\sin^{-1}(cx)|-1)}{c}$$

Antiderivative was successfully verified.

[In] Int[(1 + c^2*x^2)/Sqrt[1 - c^4*x^4],x]

[Out] EllipticE[ArcSin[c*x], -1]/c

Rule 1199

Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] := Dist[d/Sqrt[a], Int[Sqrt[1 + (e*x^2)/d]/Sqrt[1 - (e*x^2)/d], x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]

Rule 424

Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] := Simp[(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)]/(Sqrt[c]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]

Rubi steps

$$\begin{aligned} \int \frac{1+c^2x^2}{\sqrt{1-c^4x^4}} dx &= \int \frac{\sqrt{1+c^2x^2}}{\sqrt{1-c^2x^2}} dx \\ &= \frac{E(\sin^{-1}(cx)|-1)}{c} \end{aligned}$$

Mathematica [C] time = 0.0130183, size = 47, normalized size = 4.7

$$\frac{1}{3}c^2x^3 {}_2F_1\left(\frac{1}{2}, \frac{3}{4}; \frac{7}{4}; c^4x^4\right) + x {}_2F_1\left(\frac{1}{4}, \frac{1}{2}; \frac{5}{4}; c^4x^4\right)$$

Antiderivative was successfully verified.

[In] Integrate[(1 + c^2*x^2)/Sqrt[1 - c^4*x^4],x]

[Out] x*Hypergeometric2F1[1/4, 1/2, 5/4, c^4*x^4] + (c^2*x^3*Hypergeometric2F1[1/2, 3/4, 7/4, c^4*x^4])/3

Maple [B] time = 0.052, size = 118, normalized size = 11.8

$$\sqrt{-c^2x^2+1}\sqrt{c^2x^2+1}\text{EllipticF}\left(x\sqrt{c^2},i\right)\frac{1}{\sqrt{c^2}}\frac{1}{\sqrt{-c^4x^4+1}}-\sqrt{-c^2x^2+1}\sqrt{c^2x^2+1}\left(\text{EllipticF}\left(x\sqrt{c^2},i\right)-\text{EllipticE}\left(x\sqrt{c^2},i\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c^2*x^2+1)/(-c^4*x^4+1)^(1/2),x)

[Out] 1/(c^2)^(1/2)*(-c^2*x^2+1)^(1/2)*(c^2*x^2+1)^(1/2)/(-c^4*x^4+1)^(1/2)*EllipticF(x*(c^2)^(1/2),I)-1/(c^2)^(1/2)*(-c^2*x^2+1)^(1/2)*(c^2*x^2+1)^(1/2)/(-c^4*x^4+1)^(1/2)*(EllipticF(x*(c^2)^(1/2),I)-EllipticE(x*(c^2)^(1/2),I))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{c^2x^2+1}{\sqrt{-c^4x^4+1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c^2*x^2+1)/(-c^4*x^4+1)^(1/2),x, algorithm="maxima")

[Out] integrate((c^2*x^2+1)/sqrt(-c^4*x^4+1),x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-c^4x^4+1}}{c^2x^2-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c^2*x^2+1)/(-c^4*x^4+1)^(1/2),x, algorithm="fricas")

[Out] integral(-sqrt(-c^4*x^4+1)/(c^2*x^2-1),x)

Sympy [B] time = 1.54896, size = 71, normalized size = 7.1

$$\frac{c^2x^3\Gamma\left(\frac{3}{4}\right) {}_2F_1\left(\frac{1}{2}, \frac{3}{4}; \frac{7}{4}; c^4x^4e^{2i\pi}\right)}{4\Gamma\left(\frac{7}{4}\right)} + \frac{x\Gamma\left(\frac{1}{4}\right) {}_2F_1\left(\frac{1}{4}, \frac{1}{2}; \frac{5}{4}; c^4x^4e^{2i\pi}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c**2*x**2+1)/(-c**4*x**4+1)**(1/2),x)

[Out] c**2*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), c**4*x**4*exp_polar(2*I*pi))/(4*gamma(7/4)) + x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), c**4*x**4*exp_polar(2*I*pi))

$r(2*I*pi)/(4*gamma(5/4))$

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{c^2x^2 + 1}{\sqrt{-c^4x^4 + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c^2*x^2+1)/(-c^4*x^4+1)^(1/2),x, algorithm="giac")

[Out] integrate((c^2*x^2 + 1)/sqrt(-c^4*x^4 + 1), x)

$$3.24 \quad \int \frac{\sqrt{1-c^2x^2}}{\sqrt{1+c^2x^2}} dx$$

Optimal. Leaf size=23

$$\frac{2\text{EllipticF}(\sin^{-1}(cx), -1)}{c} - \frac{E(\sin^{-1}(cx)|-1)}{c}$$

[Out] -(EllipticE[ArcSin[c*x], -1]/c) + (2*EllipticF[ArcSin[c*x], -1])/c

Rubi [A] time = 0.0260898, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {423, 424, 248, 221}

$$\frac{2F(\sin^{-1}(cx)|-1)}{c} - \frac{E(\sin^{-1}(cx)|-1)}{c}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[1 - c^2*x^2]/Sqrt[1 + c^2*x^2], x]

[Out] -(EllipticE[ArcSin[c*x], -1]/c) + (2*EllipticF[ArcSin[c*x], -1])/c

Rule 423

Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[b/d, Int[Sqrt[c + d*x^2]/Sqrt[a + b*x^2], x], x] - Dist[(b*c - a*d)/d, Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c] && NegQ[b/a]

Rule 424

Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]

Rule 248

Int[((a1_.) + (b1_.)*(x_)^(n_))^(p_.)*((a2_.) + (b2_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[(a1*a2 + b1*b2*x^(2*n))^p, x] /; FreeQ[{a1, b1, a2, b2, n, p}, x] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[p] || (GtQ[a1, 0] && GtQ[a2, 0]))

Rule 221

Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b, 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[b/a] && GtQ[a, 0]

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{1-c^2x^2}}{\sqrt{1+c^2x^2}} dx &= 2 \int \frac{1}{\sqrt{1-c^2x^2}\sqrt{1+c^2x^2}} dx - \int \frac{\sqrt{1+c^2x^2}}{\sqrt{1-c^2x^2}} dx \\ &= -\frac{E(\sin^{-1}(cx)|-1)}{c} + 2 \int \frac{1}{\sqrt{1-c^4x^4}} dx \\ &= -\frac{E(\sin^{-1}(cx)|-1)}{c} + \frac{2F(\sin^{-1}(cx)|-1)}{c} \end{aligned}$$

Mathematica [A] time = 0.0075945, size = 24, normalized size = 1.04

$$\frac{E\left(\sin^{-1}\left(\sqrt{-c^2}x\right)\middle| -1\right)}{\sqrt{-c^2}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[1 - c^2*x^2]/Sqrt[1 + c^2*x^2], x]

[Out] EllipticE[ArcSin[Sqrt[-c^2]*x], -1]/Sqrt[-c^2]

Maple [C] time = 0.049, size = 28, normalized size = 1.2

$$\frac{(2 \operatorname{EllipticF}(x \operatorname{csgn}(c), c, i) - \operatorname{EllipticE}(x \operatorname{csgn}(c), c, i)) \operatorname{csgn}(c)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-c^2*x^2+1)^(1/2)/(c^2*x^2+1)^(1/2), x)

[Out] (2*EllipticF(x*csgn(c)*c, I)-EllipticE(x*csgn(c)*c, I))*csgn(c)/c

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-c^2x^2+1}}{\sqrt{c^2x^2+1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-c^2*x^2+1)^(1/2)/(c^2*x^2+1)^(1/2), x, algorithm="maxima")

[Out] integrate(sqrt(-c^2*x^2 + 1)/sqrt(c^2*x^2 + 1), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\sqrt{-c^2x^2+1}}{\sqrt{c^2x^2+1}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-c^2*x^2+1)^(1/2)/(c^2*x^2+1)^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(-c^2*x^2 + 1)/sqrt(c^2*x^2 + 1), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-(cx-1)(cx+1)}}{\sqrt{c^2x^2+1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-c**2*x**2+1)**(1/2)/(c**2*x**2+1)**(1/2),x)

[Out] Integral(sqrt(-(c*x - 1)*(c*x + 1))/sqrt(c**2*x**2 + 1), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-c^2x^2+1}}{\sqrt{c^2x^2+1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-c^2*x^2+1)^(1/2)/(c^2*x^2+1)^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(-c^2*x^2 + 1)/sqrt(c^2*x^2 + 1), x)

$$3.25 \quad \int \frac{1-c^2x^2}{\sqrt{1-c^4x^4}} dx$$

Optimal. Leaf size=23

$$\frac{2\text{EllipticF}(\sin^{-1}(cx), -1)}{c} - \frac{E(\sin^{-1}(cx)|-1)}{c}$$

[Out] -(EllipticE[ArcSin[c*x], -1]/c) + (2*EllipticF[ArcSin[c*x], -1])/c

Rubi [A] time = 0.0334165, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$, Rules used = {1199, 423, 424, 248, 221}

$$\frac{2F(\sin^{-1}(cx)|-1)}{c} - \frac{E(\sin^{-1}(cx)|-1)}{c}$$

Antiderivative was successfully verified.

[In] Int[(1 - c^2*x^2)/Sqrt[1 - c^4*x^4], x]

[Out] -(EllipticE[ArcSin[c*x], -1]/c) + (2*EllipticF[ArcSin[c*x], -1])/c

Rule 1199

Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] := Dist[d/Sqrt[a], Int[Sqrt[1 + (e*x^2)/d]/Sqrt[1 - (e*x^2)/d], x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]

Rule 423

Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] := Dist[b/d, Int[Sqrt[c + d*x^2]/Sqrt[a + b*x^2], x], x] - Dist[(b*c - a*d)/d, Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c] && NegQ[b/a]

Rule 424

Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] := Simp[(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]

Rule 248

Int[((a1_.) + (b1_.)*(x_)^(n_))^(p_.)*((a2_.) + (b2_.)*(x_)^(n_))^(p_.), x_Symbol] := Int[(a1*a2 + b1*b2*x^(2*n))^p, x] /; FreeQ[{a1, b1, a2, b2, n, p}, x] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[p] || (GtQ[a1, 0] && GtQ[a2, 0]))

Rule 221

Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] := Simp[EllipticF[ArcSin[(Rt[-b, 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[b/a] && GtQ[a, 0]

Rubi steps

$$\begin{aligned}
\int \frac{1-c^2x^2}{\sqrt{1-c^4x^4}} dx &= \int \frac{\sqrt{1-c^2x^2}}{\sqrt{1+c^2x^2}} dx \\
&= 2 \int \frac{1}{\sqrt{1-c^2x^2}\sqrt{1+c^2x^2}} dx - \int \frac{\sqrt{1+c^2x^2}}{\sqrt{1-c^2x^2}} dx \\
&= -\frac{E(\sin^{-1}(cx)|-1)}{c} + 2 \int \frac{1}{\sqrt{1-c^4x^4}} dx \\
&= -\frac{E(\sin^{-1}(cx)|-1)}{c} + \frac{2F(\sin^{-1}(cx)|-1)}{c}
\end{aligned}$$

Mathematica [C] time = 0.0117478, size = 47, normalized size = 2.04

$$x {}_2F_1\left(\frac{1}{4}, \frac{1}{2}; \frac{5}{4}; c^4x^4\right) - \frac{1}{3}c^2x^3 {}_2F_1\left(\frac{1}{2}, \frac{3}{4}; \frac{7}{4}; c^4x^4\right)$$

Antiderivative was successfully verified.

[In] Integrate[(1 - c^2*x^2)/Sqrt[1 - c^4*x^4], x]

[Out] x*Hypergeometric2F1[1/4, 1/2, 5/4, c^4*x^4] - (c^2*x^3*Hypergeometric2F1[1/2, 3/4, 7/4, c^4*x^4])/3

Maple [B] time = 0.048, size = 117, normalized size = 5.1

$$\sqrt{-c^2x^2+1}\sqrt{c^2x^2+1}\text{EllipticF}\left(x\sqrt{c^2}, i\right) \frac{1}{\sqrt{c^2}} \frac{1}{\sqrt{-c^4x^4+1}} + \sqrt{-c^2x^2+1}\sqrt{c^2x^2+1} \left(\text{EllipticF}\left(x\sqrt{c^2}, i\right) - \text{EllipticE}\left(x\sqrt{c^2}, i\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-c^2*x^2+1)/(-c^4*x^4+1)^(1/2), x)

[Out] 1/(c^2)^(1/2)*(-c^2*x^2+1)^(1/2)*(c^2*x^2+1)^(1/2)/(-c^4*x^4+1)^(1/2)*EllipticF(x*(c^2)^(1/2), I)+1/(c^2)^(1/2)*(-c^2*x^2+1)^(1/2)*(c^2*x^2+1)^(1/2)/(-c^4*x^4+1)^(1/2)*(EllipticF(x*(c^2)^(1/2), I)-EllipticE(x*(c^2)^(1/2), I))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{c^2x^2-1}{\sqrt{-c^4x^4+1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-c^2*x^2+1)/(-c^4*x^4+1)^(1/2), x, algorithm="maxima")

[Out] -integrate((c^2*x^2 - 1)/sqrt(-c^4*x^4 + 1), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-c^4x^4+1}}{c^2x^2+1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-c^2*x^2+1)/(-c^4*x^4+1)^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(-c^4*x^4 + 1)/(c^2*x^2 + 1), x)

Sympy [B] time = 1.93362, size = 71, normalized size = 3.09

$$-\frac{c^2 x^3 \Gamma\left(\frac{3}{4}\right) {}_2F_1\left(\frac{1}{2}, \frac{3}{4} \middle| \frac{7}{4}; c^4 x^4 e^{2i\pi}\right)}{4 \Gamma\left(\frac{7}{4}\right)} + \frac{x \Gamma\left(\frac{1}{4}\right) {}_2F_1\left(\frac{1}{4}, \frac{1}{2} \middle| \frac{5}{4}; c^4 x^4 e^{2i\pi}\right)}{4 \Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-c**2*x**2+1)/(-c**4*x**4+1)**(1/2),x)

[Out] -c**2*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), c**4*x**4*exp_polar(2*I*pi))/(4*gamma(7/4)) + x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), c**4*x**4*exp_polar(2*I*pi))/(4*gamma(5/4))

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{c^2 x^2 - 1}{\sqrt{-c^4 x^4 + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-c^2*x^2+1)/(-c^4*x^4+1)^(1/2),x, algorithm="giac")

[Out] integrate(-(c^2*x^2 - 1)/sqrt(-c^4*x^4 + 1), x)

$$3.26 \quad \int \frac{d+ex^2}{d^2+bx^2+e^2x^4} dx$$

Optimal. Leaf size=82

$$\frac{\tan^{-1}\left(\frac{\sqrt{2de-b+2ex}}{\sqrt{b+2de}}\right)}{\sqrt{b+2de}} - \frac{\tan^{-1}\left(\frac{\sqrt{2de-b-2ex}}{\sqrt{b+2de}}\right)}{\sqrt{b+2de}}$$

[Out] $-(\text{ArcTan}[(\text{Sqrt}[-b + 2*d*e] - 2*e*x)/\text{Sqrt}[b + 2*d*e]]/\text{Sqrt}[b + 2*d*e]) + \text{ArcTan}[(\text{Sqrt}[-b + 2*d*e] + 2*e*x)/\text{Sqrt}[b + 2*d*e]]/\text{Sqrt}[b + 2*d*e]$

Rubi [A] time = 0.0999168, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1161, 618, 204}

$$\frac{\tan^{-1}\left(\frac{\sqrt{2de-b+2ex}}{\sqrt{b+2de}}\right)}{\sqrt{b+2de}} - \frac{\tan^{-1}\left(\frac{\sqrt{2de-b-2ex}}{\sqrt{b+2de}}\right)}{\sqrt{b+2de}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(d + e*x^2)/(d^2 + b*x^2 + e^2*x^4), x]$

[Out] $-(\text{ArcTan}[(\text{Sqrt}[-b + 2*d*e] - 2*e*x)/\text{Sqrt}[b + 2*d*e]]/\text{Sqrt}[b + 2*d*e]) + \text{ArcTan}[(\text{Sqrt}[-b + 2*d*e] + 2*e*x)/\text{Sqrt}[b + 2*d*e]]/\text{Sqrt}[b + 2*d*e]$

Rule 1161

$\text{Int}[(d + e*x^2)/(d^2 + b*x^2 + e^2*x^4), x_Symbol] :$
 $> \text{With}\{q = \text{Rt}[(2*d)/e - b/c, 2]\}, \text{Dist}[e/(2*c), \text{Int}[1/\text{Simp}[d/e + q*x + x^2, x], x], x] + \text{Dist}[e/(2*c), \text{Int}[1/\text{Simp}[d/e - q*x + x^2, x], x], x] /;$
 $\text{FreeQ}\{a, b, c, d, e\}, x \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0] \ \&\& \ \text{EqQ}[c*d^2 - a*e^2, 0] \ \&\& \ (\text{GtQ}[(2*d)/e - b/c, 0] \ || \ (\text{!LtQ}[(2*d)/e - b/c, 0] \ \&\& \ \text{EqQ}[d - e*\text{Rt}[a/c, 2], 0]))$

Rule 618

$\text{Int}[(a + b*x + c*x^2)^{-1}, x_Symbol] := \text{Dist}[-2, \text{Subst}[\text{Int}[1/\text{Simp}[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /;$
 $\text{FreeQ}\{a, b, c, x\} \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0]$

Rule 204

$\text{Int}[(a + b*x)^{-1}, x_Symbol] := -\text{Simp}[\text{ArcTan}[(\text{Rt}[-b, 2]*x)/\text{Rt}[-a, 2]]/(\text{Rt}[-a, 2]*\text{Rt}[-b, 2]), x] /;$
 $\text{FreeQ}\{a, b\}, x \ \&\& \ \text{PosQ}[a/b] \ \&\& \ (\text{LtQ}[a, 0] \ || \ \text{LtQ}[b, 0])$

Rubi steps

$$\begin{aligned} \int \frac{d + ex^2}{d^2 + bx^2 + e^2x^4} dx &= \frac{\int \frac{1}{\frac{d}{e} - \frac{\sqrt{-b+2de}}{e} + x^2} dx}{2e} + \frac{\int \frac{1}{\frac{d}{e} + \frac{\sqrt{-b+2de}}{e} + x^2} dx}{2e} \\ &= \frac{\text{Subst}\left(\int \frac{1}{\frac{-b+2de}{e^2} - x^2} dx, x, -\frac{\sqrt{-b+2de}}{e} + 2x\right)}{e} - \frac{\text{Subst}\left(\int \frac{1}{\frac{-b+2de}{e^2} - x^2} dx, x, \frac{\sqrt{-b+2de}}{e} + 2x\right)}{e} \\ &= -\frac{\tan^{-1}\left(\frac{\sqrt{-b+2de}-2ex}{\sqrt{b+2de}}\right)}{\sqrt{b+2de}} + \frac{\tan^{-1}\left(\frac{\sqrt{-b+2de}+2ex}{\sqrt{b+2de}}\right)}{\sqrt{b+2de}} \end{aligned}$$

Mathematica [B] time = 0.108621, size = 181, normalized size = 2.21

$$\frac{\left(\sqrt{b^2-4d^2e^2}-b+2de\right)\tan^{-1}\left(\frac{\sqrt{2ex}}{\sqrt{b-\sqrt{b^2-4d^2e^2}}}\right)}{\sqrt{b-\sqrt{b^2-4d^2e^2}}} + \frac{\left(\sqrt{b^2-4d^2e^2}+b-2de\right)\tan^{-1}\left(\frac{\sqrt{2ex}}{\sqrt{\sqrt{b^2-4d^2e^2}+b}}\right)}{\sqrt{\sqrt{b^2-4d^2e^2}+b}}}{\sqrt{2}\sqrt{b^2-4d^2e^2}}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e*x^2)/(d^2 + b*x^2 + e^2*x^4),x]

[Out] (((-b + 2*d*e + Sqrt[b^2 - 4*d^2*e^2])*ArcTan[(Sqrt[2]*e*x)/Sqrt[b - Sqrt[b^2 - 4*d^2*e^2]]])/Sqrt[b - Sqrt[b^2 - 4*d^2*e^2]] + ((b - 2*d*e + Sqrt[b^2 - 4*d^2*e^2])*ArcTan[(Sqrt[2]*e*x)/Sqrt[b + Sqrt[b^2 - 4*d^2*e^2]]])/Sqrt[b + Sqrt[b^2 - 4*d^2*e^2]]/(Sqrt[2]*Sqrt[b^2 - 4*d^2*e^2])

Maple [A] time = 0.201, size = 71, normalized size = 0.9

$$-\arctan\left(\left(-2ex + \sqrt{2de - b}\right)\frac{1}{\sqrt{2de + b}}\right)\frac{1}{\sqrt{2de + b}} + \arctan\left(\left(2ex + \sqrt{2de - b}\right)\frac{1}{\sqrt{2de + b}}\right)\frac{1}{\sqrt{2de + b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x^2+d)/(e^2*x^4+b*x^2+d^2),x)

[Out] -arctan((-2*e*x+(2*d*e-b)^(1/2))/(2*d*e+b)^(1/2))/(2*d*e+b)^(1/2)+arctan((2*e*x+(2*d*e-b)^(1/2))/(2*d*e+b)^(1/2))/(2*d*e+b)^(1/2)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex^2 + d}{e^2x^4 + bx^2 + d^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)/(e^2*x^4+b*x^2+d^2),x, algorithm="maxima")

[Out] integrate((e*x^2 + d)/(e^2*x^4 + b*x^2 + d^2), x)

Fricas [A] time = 1.37312, size = 375, normalized size = 4.57

$$\left[\frac{\sqrt{-2de-b} \log\left(\frac{e^2x^4 - (4de+b)x^2 + d^2 - 2(ex^3 - dx)\sqrt{-2de-b}}{e^2x^4 + bx^2 + d^2}\right)}{2(2de+b)}, \frac{\sqrt{2de+b} \arctan\left(\frac{ex}{\sqrt{2de+b}}\right) + \sqrt{2de+b} \arctan\left(\frac{(e^2x^3 + (de+b)x)\sqrt{2de+b}}{2d^2e + bd}\right)}{2de+b} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)/(e^2*x^4+b*x^2+d^2),x, algorithm="fricas")

[Out] [-1/2*sqrt(-2*d*e - b)*log((e^2*x^4 - (4*d*e + b)*x^2 + d^2 - 2*(e*x^3 - d*x)*sqrt(-2*d*e - b))/(e^2*x^4 + b*x^2 + d^2))/(2*d*e + b), (sqrt(2*d*e + b)*arctan(e*x/sqrt(2*d*e + b)) + sqrt(2*d*e + b)*arctan((e^2*x^3 + (d*e + b)*x)*sqrt(2*d*e + b)/(2*d^2*e + b*d)))/(2*d*e + b)]

Sympy [A] time = 0.421942, size = 122, normalized size = 1.49

$$\frac{\sqrt{-\frac{1}{b+2de}} \log\left(-\frac{d}{e} + x^2 + \frac{x\left(-b\sqrt{-\frac{1}{b+2de}} - 2de\sqrt{-\frac{1}{b+2de}}\right)}{e}\right)}{2} + \frac{\sqrt{-\frac{1}{b+2de}} \log\left(-\frac{d}{e} + x^2 + \frac{x\left(b\sqrt{-\frac{1}{b+2de}} + 2de\sqrt{-\frac{1}{b+2de}}\right)}{e}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x**2+d)/(e**2*x**4+b*x**2+d**2),x)

[Out] -sqrt(-1/(b + 2*d*e))*log(-d/e + x**2 + x*(-b*sqrt(-1/(b + 2*d*e)) - 2*d*e*sqrt(-1/(b + 2*d*e)))/e)/2 + sqrt(-1/(b + 2*d*e))*log(-d/e + x**2 + x*(b*sqrt(-1/(b + 2*d*e)) + 2*d*e*sqrt(-1/(b + 2*d*e)))/e)/2

Giac [C] time = 1.62124, size = 5779, normalized size = 70.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)/(e^2*x^4+b*x^2+d^2),x, algorithm="giac")

[Out] 1/2*(3*(4*(d^2)^(3/4)*d^2*e^(13/2) - b^2*(d^2)^(3/4)*e^(9/2) - sqrt(-4*d^2*e^2 + b^2)*b*(d^2)^(3/4)*e^(9/2))*cos(5/4*pi + 1/2*real_part(arcsin(1/2*b*e^(-1)/abs(d))))^2*cosh(1/2*imag_part(arcsin(1/2*b*e^(-1)/abs(d))))^3*e*sin(5/4*pi + 1/2*real_part(arcsin(1/2*b*e^(-1)/abs(d)))) - (4*(d^2)^(3/4)*d^2*e^(13/2) - b^2*(d^2)^(3/4)*e^(9/2) - sqrt(-4*d^2*e^2 + b^2)*b*(d^2)^(3/4)*e^(9/2))*cosh(1/2*imag_part(arcsin(1/2*b*e^(-1)/abs(d))))^3*e*sin(5/4*pi + 1/2*real_part(arcsin(1/2*b*e^(-1)/abs(d))))^3 - 9*(4*(d^2)^(3/4)*d^2*e^(13/2) - b^2*(d^2)^(3/4)*e^(9/2) - sqrt(-4*d^2*e^2 + b^2)*b*(d^2)^(3/4)*e^(9/2))*cos(5/4*pi + 1/2*real_part(arcsin(1/2*b*e^(-1)/abs(d))))^2*cosh(1/2*imag_part(arcsin(1/2*b*e^(-1)/abs(d))))^2*e*sin(5/4*pi + 1/2*real_part(arcsin(1/2*b*e^(-1)/abs(d))))*sinh(1/2*imag_part(arcsin(1/2*b*e^(-1)/abs(d)))) + 3*(4*(d^2)^(3/4)*d^2*e^(13/2) - b^2*(d^2)^(3/4)*e^(9/2) - sqrt(-4*d^2*e^2 + b^2)*b*(d^2)^(3/4)*e^(9/2))*cosh(1/2*imag_part(arcsin(1/2*b*e^(-1)/abs(d))))^2*e*sin(5/4*pi + 1/2*real_part(arcsin(1/2*b*e^(-1)/abs(d))))^3*sinh(1/2*imag_part(arcsin(1/2*b*e^(-1)/abs(d)))) + 9*(4*(d^2)^(3/4)*d^2*e^(13/2) - b^2*(d

$$\begin{aligned}
& ^2)^{(3/4)}e^{(9/2)} - \sqrt{-4d^2e^2 + b^2} * b * (d^2)^{(3/4)}e^{(9/2)} * \cos(5/4 * \pi \\
& + 1/2 * \text{real_part}(\arcsin(1/2 * b * e^{-1} / \text{abs}(d))))^2 * \cosh(1/2 * \text{imag_part}(\arcsin \\
& (1/2 * b * e^{-1} / \text{abs}(d)))) * e * \sin(5/4 * \pi + 1/2 * \text{real_part}(\arcsin(1/2 * b * e^{-1} / \text{abs} \\
& (d)))) * \sinh(1/2 * \text{imag_part}(\arcsin(1/2 * b * e^{-1} / \text{abs}(d))))^2 - 3 * (4 * (d^2)^{(3/4)} \\
& * d^2 * e^{(13/2)} - b^2 * (d^2)^{(3/4)} * e^{(9/2)} - \sqrt{-4d^2e^2 + b^2} * b * (d^2)^{(3/4)} \\
& * e^{(9/2)} * \cosh(1/2 * \text{imag_part}(\arcsin(1/2 * b * e^{-1} / \text{abs}(d)))) * e * \sin(5/4 * \pi \\
& + 1/2 * \text{real_part}(\arcsin(1/2 * b * e^{-1} / \text{abs}(d))))^3 * \sinh(1/2 * \text{imag_part}(\arcsin \\
& (1/2 * b * e^{-1} / \text{abs}(d))))^2 - 3 * (4 * (d^2)^{(3/4)} * d^2 * e^{(13/2)} - b^2 * (d^2)^{(3/4)} \\
& * e^{(9/2)} - \sqrt{-4d^2e^2 + b^2} * b * (d^2)^{(3/4)} * e^{(9/2)} * \cos(5/4 * \pi + 1/2 * \text{r} \\
& \text{eal_part}(\arcsin(1/2 * b * e^{-1} / \text{abs}(d))))^2 * e * \sin(5/4 * \pi + 1/2 * \text{real_part}(\arcsi \\
& n(1/2 * b * e^{-1} / \text{abs}(d)))) * \sinh(1/2 * \text{imag_part}(\arcsin(1/2 * b * e^{-1} / \text{abs}(d))))^3 \\
& + (4 * (d^2)^{(3/4)} * d^2 * e^{(13/2)} - b^2 * (d^2)^{(3/4)} * e^{(9/2)} - \sqrt{-4d^2e^2 \\
& + b^2} * b * (d^2)^{(3/4)} * e^{(9/2)} * e * \sin(5/4 * \pi + 1/2 * \text{real_part}(\arcsin(1/2 * b * e^{(\\
& -1) / \text{abs}(d))))^3 * \sinh(1/2 * \text{imag_part}(\arcsin(1/2 * b * e^{-1} / \text{abs}(d))))^3 + (4 * (d^ \\
& 2)^{(1/4)} * d^3 * e^{(15/2)} - b^2 * (d^2)^{(1/4)} * d * e^{(11/2)} - \sqrt{-4d^2e^2 + b^2} \\
& * b * (d^2)^{(1/4)} * d * e^{(11/2)} * \cosh(1/2 * \text{imag_part}(\arcsin(1/2 * b * e^{-1} / \text{abs}(d)))) \\
& * \sin(5/4 * \pi + 1/2 * \text{real_part}(\arcsin(1/2 * b * e^{-1} / \text{abs}(d)))) - (4 * (d^2)^{(1/4)} * \\
& d^3 * e^{(15/2)} - b^2 * (d^2)^{(1/4)} * d * e^{(11/2)} - \sqrt{-4d^2e^2 + b^2} * b * (d^2)^{(1/4)} \\
& * d * e^{(11/2)} * \sin(5/4 * \pi + 1/2 * \text{real_part}(\arcsin(1/2 * b * e^{-1} / \text{abs}(d)))) * \\
& \sinh(1/2 * \text{imag_part}(\arcsin(1/2 * b * e^{-1} / \text{abs}(d)))) * \arctan(-((d^2)^{(1/4)} * \cos(\\
& 5/4 * \pi + 1/2 * \arcsin(1/2 * b * e^{-1} / \text{abs}(d))) * e^{(-1/2)} - x) * e^{(1/2)} / ((d^2)^{(1/4)} \\
&) * \sin(5/4 * \pi + 1/2 * \arcsin(1/2 * b * e^{-1} / \text{abs}(d)))) / (4 * d^4 * e^8 - b^2 * d^2 * e^6) \\
& + 1/2 * (3 * (4 * (d^2)^{(3/4)} * d^2 * e^{(13/2)} - b^2 * (d^2)^{(3/4)} * e^{(9/2)} - \sqrt{-4d \\
& ^2 * e^2 + b^2} * b * (d^2)^{(3/4)} * e^{(9/2)} * \cos(1/4 * \pi + 1/2 * \text{real_part}(\arcsin(1/2 * \\
& b * e^{-1} / \text{abs}(d))))^2 * \cosh(1/2 * \text{imag_part}(\arcsin(1/2 * b * e^{-1} / \text{abs}(d))))^3 * e * s \\
& in(1/4 * \pi + 1/2 * \text{real_part}(\arcsin(1/2 * b * e^{-1} / \text{abs}(d)))) - (4 * (d^2)^{(3/4)} * d^ \\
& 2 * e^{(13/2)} - b^2 * (d^2)^{(3/4)} * e^{(9/2)} - \sqrt{-4d^2e^2 + b^2} * b * (d^2)^{(3/4)} \\
& * e^{(9/2)} * \cosh(1/2 * \text{imag_part}(\arcsin(1/2 * b * e^{-1} / \text{abs}(d))))^3 * e * \sin(1/4 * \pi + \\
& 1/2 * \text{real_part}(\arcsin(1/2 * b * e^{-1} / \text{abs}(d))))^3 - 9 * (4 * (d^2)^{(3/4)} * d^2 * e^{(13 \\
& /2)} - b^2 * (d^2)^{(3/4)} * e^{(9/2)} - \sqrt{-4d^2e^2 + b^2} * b * (d^2)^{(3/4)} * e^{(9/2)} \\
&) * \cos(1/4 * \pi + 1/2 * \text{real_part}(\arcsin(1/2 * b * e^{-1} / \text{abs}(d))))^2 * \cosh(1/2 * \text{imag} \\
& _part(\arcsin(1/2 * b * e^{-1} / \text{abs}(d))))^2 * e * \sin(1/4 * \pi + 1/2 * \text{real_part}(\arcsin(1 \\
& /2 * b * e^{-1} / \text{abs}(d)))) * \sinh(1/2 * \text{imag_part}(\arcsin(1/2 * b * e^{-1} / \text{abs}(d)))) + 3 * \\
& (4 * (d^2)^{(3/4)} * d^2 * e^{(13/2)} - b^2 * (d^2)^{(3/4)} * e^{(9/2)} - \sqrt{-4d^2e^2 + b \\
& ^2} * b * (d^2)^{(3/4)} * e^{(9/2)} * \cosh(1/2 * \text{imag_part}(\arcsin(1/2 * b * e^{-1} / \text{abs}(d)))) \\
& ^2 * e * \sin(1/4 * \pi + 1/2 * \text{real_part}(\arcsin(1/2 * b * e^{-1} / \text{abs}(d))))^3 * \sinh(1/2 * \text{im} \\
& ag_part(\arcsin(1/2 * b * e^{-1} / \text{abs}(d)))) + 9 * (4 * (d^2)^{(3/4)} * d^2 * e^{(13/2)} - b^2 \\
& * (d^2)^{(3/4)} * e^{(9/2)} - \sqrt{-4d^2e^2 + b^2} * b * (d^2)^{(3/4)} * e^{(9/2)} * \cos(1/ \\
& 4 * \pi + 1/2 * \text{real_part}(\arcsin(1/2 * b * e^{-1} / \text{abs}(d))))^2 * \cosh(1/2 * \text{imag_part}(\arc \\
& sin(1/2 * b * e^{-1} / \text{abs}(d)))) * e * \sin(1/4 * \pi + 1/2 * \text{real_part}(\arcsin(1/2 * b * e^{-1} \\
& / \text{abs}(d)))) * \sinh(1/2 * \text{imag_part}(\arcsin(1/2 * b * e^{-1} / \text{abs}(d))))^2 - 3 * (4 * (d^2)^{(\\
& 3/4)} * d^2 * e^{(13/2)} - b^2 * (d^2)^{(3/4)} * e^{(9/2)} - \sqrt{-4d^2e^2 + b^2} * b * (d^ \\
& 2)^{(3/4)} * e^{(9/2)} * \cosh(1/2 * \text{imag_part}(\arcsin(1/2 * b * e^{-1} / \text{abs}(d)))) * e * \sin(1/ \\
& 4 * \pi + 1/2 * \text{real_part}(\arcsin(1/2 * b * e^{-1} / \text{abs}(d))))^3 * \sinh(1/2 * \text{imag_part}(\arc \\
& sin(1/2 * b * e^{-1} / \text{abs}(d))))^2 - 3 * (4 * (d^2)^{(3/4)} * d^2 * e^{(13/2)} - b^2 * (d^2)^{(3 \\
& /4)} * e^{(9/2)} - \sqrt{-4d^2e^2 + b^2} * b * (d^2)^{(3/4)} * e^{(9/2)} * \cos(1/4 * \pi + 1/ \\
& 2 * \text{real_part}(\arcsin(1/2 * b * e^{-1} / \text{abs}(d))))^2 * e * \sin(1/4 * \pi + 1/2 * \text{real_part}(\ar \\
& csin(1/2 * b * e^{-1} / \text{abs}(d)))) * \sinh(1/2 * \text{imag_part}(\arcsin(1/2 * b * e^{-1} / \text{abs}(d))) \\
&)^3 + (4 * (d^2)^{(3/4)} * d^2 * e^{(13/2)} - b^2 * (d^2)^{(3/4)} * e^{(9/2)} - \sqrt{-4d^2 * e \\
& ^2 + b^2} * b * (d^2)^{(3/4)} * e^{(9/2)} * e * \sin(1/4 * \pi + 1/2 * \text{real_part}(\arcsin(1/2 * b * \\
& e^{-1} / \text{abs}(d))))^3 * \sinh(1/2 * \text{imag_part}(\arcsin(1/2 * b * e^{-1} / \text{abs}(d))))^3 + (4 * \\
& (d^2)^{(1/4)} * d^3 * e^{(15/2)} - b^2 * (d^2)^{(1/4)} * d * e^{(11/2)} - \sqrt{-4d^2 * e^2 + b \\
& ^2} * b * (d^2)^{(1/4)} * d * e^{(11/2)} * \cosh(1/2 * \text{imag_part}(\arcsin(1/2 * b * e^{-1} / \text{abs}(d) \\
&))) * \sin(1/4 * \pi + 1/2 * \text{real_part}(\arcsin(1/2 * b * e^{-1} / \text{abs}(d)))) - (4 * (d^2)^{(1/ \\
& 4)} * d^3 * e^{(15/2)} - b^2 * (d^2)^{(1/4)} * d * e^{(11/2)} - \sqrt{-4d^2 * e^2 + b^2} * b * (d^ \\
& 2)^{(1/4)} * d * e^{(11/2)} * \sin(1/4 * \pi + 1/2 * \text{real_part}(\arcsin(1/2 * b * e^{-1} / \text{abs}(d) \\
&))) * \sinh(1/2 * \text{imag_part}(\arcsin(1/2 * b * e^{-1} / \text{abs}(d)))) * \arctan(-((d^2)^{(1/4)} * c \\
& os(1/4 * \pi + 1/2 * \arcsin(1/2 * b * e^{-1} / \text{abs}(d))) * e^{(-1/2)} - x) * e^{(1/2)} / ((d^2)^{(\\
& 1/4)} * \sin(1/4 * \pi + 1/2 * \arcsin(1/2 * b * e^{-1} / \text{abs}(d)))) / (4 * d^4 * e^8 - b^2 * d^2 * e
\end{aligned}$$

$$\begin{aligned}
& /4)*d^2*e^{(13/2)} - b^2*(d^2)^{(3/4)}*e^{(9/2)} - \sqrt{-4*d^2*e^2 + b^2}*b*(d^2)^{(3/4)}*e^{(9/2)}*\cos(1/4*\pi + 1/2*\text{real_part}(\arcsin(1/2*b*e^{(-1)}/\text{abs}(d))))^3* \\
& e*\sinh(1/2*\text{imag_part}(\arcsin(1/2*b*e^{(-1)}/\text{abs}(d))))^3 + 3*(4*(d^2)^{(3/4)}*d^2 \\
& *e^{(13/2)} - b^2*(d^2)^{(3/4)}*e^{(9/2)} - \sqrt{-4*d^2*e^2 + b^2}*b*(d^2)^{(3/4)}* \\
& e^{(9/2)})*\cos(1/4*\pi + 1/2*\text{real_part}(\arcsin(1/2*b*e^{(-1)}/\text{abs}(d))))*e*\sin(1/4 \\
& *\pi + 1/2*\text{real_part}(\arcsin(1/2*b*e^{(-1)}/\text{abs}(d))))^2*\sinh(1/2*\text{imag_part}(\arcs \\
& \text{in}(1/2*b*e^{(-1)}/\text{abs}(d))))^3 + (4*(d^2)^{(1/4)}*d^3*e^{(15/2)} - b^2*(d^2)^{(1/4)} \\
& *d*e^{(11/2)} - \sqrt{-4*d^2*e^2 + b^2}*b*(d^2)^{(1/4)}*d*e^{(11/2)})*\cos(1/4*\pi + \\
& 1/2*\text{real_part}(\arcsin(1/2*b*e^{(-1)}/\text{abs}(d))))*\cosh(1/2*\text{imag_part}(\arcsin(1/2* \\
& b*e^{(-1)}/\text{abs}(d)))) - (4*(d^2)^{(1/4)}*d^3*e^{(15/2)} - b^2*(d^2)^{(1/4)}*d*e^{(11/ \\
& 2)} - \sqrt{-4*d^2*e^2 + b^2}*b*(d^2)^{(1/4)}*d*e^{(11/2)})*\cos(1/4*\pi + 1/2*\text{real} \\
& _part(\arcsin(1/2*b*e^{(-1)}/\text{abs}(d))))*\sinh(1/2*\text{imag_part}(\arcsin(1/2*b*e^{(-1)}/ \\
& \text{abs}(d))))*\log(-2*(d^2)^{(1/4)}*x*\cos(1/4*\pi + 1/2*\arcsin(1/2*b*e^{(-1)}/\text{abs}(d) \\
&))*e^{(-1/2)} + x^2 + \sqrt{d^2}*e^{(-1)})/(4*d^4*e^8 - b^2*d^2*e^6)
\end{aligned}$$

$$3.27 \quad \int \frac{d+ex^2}{d^2+fx^2+e^2x^4} dx$$

Optimal. Leaf size=82

$$\frac{\tan^{-1}\left(\frac{\sqrt{2de-f+2ex}}{\sqrt{2de+f}}\right)}{\sqrt{2de+f}} - \frac{\tan^{-1}\left(\frac{\sqrt{2de-f-2ex}}{\sqrt{2de+f}}\right)}{\sqrt{2de+f}}$$

[Out] -(ArcTan[(Sqrt[2*d*e - f] - 2*e*x)/Sqrt[2*d*e + f]]/Sqrt[2*d*e + f]) + ArcTan[(Sqrt[2*d*e - f] + 2*e*x)/Sqrt[2*d*e + f]]/Sqrt[2*d*e + f]

Rubi [A] time = 0.109659, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1161, 618, 204}

$$\frac{\tan^{-1}\left(\frac{\sqrt{2de-f+2ex}}{\sqrt{2de+f}}\right)}{\sqrt{2de+f}} - \frac{\tan^{-1}\left(\frac{\sqrt{2de-f-2ex}}{\sqrt{2de+f}}\right)}{\sqrt{2de+f}}$$

Antiderivative was successfully verified.

[In] Int[(d + e*x^2)/(d^2 + f*x^2 + e^2*x^4), x]

[Out] -(ArcTan[(Sqrt[2*d*e - f] - 2*e*x)/Sqrt[2*d*e + f]]/Sqrt[2*d*e + f]) + ArcTan[(Sqrt[2*d*e - f] + 2*e*x)/Sqrt[2*d*e + f]]/Sqrt[2*d*e + f]

Rule 1161

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2,
x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[
{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[(2*d)/e - b/c, 0] || (!LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[
Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{d + ex^2}{d^2 + fx^2 + e^2x^4} dx = \frac{\int \frac{1}{\frac{d - \sqrt{2de-fx}}{e} + x^2} dx}{2e} + \frac{\int \frac{1}{\frac{d + \sqrt{2de-fx}}{e} + x^2} dx}{2e}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{-\frac{2de+f}{e^2} - x^2} dx, x, -\frac{\sqrt{2de-f}}{e} + 2x\right)}{e} - \frac{\text{Subst}\left(\int \frac{1}{-\frac{2de+f}{e^2} - x^2} dx, x, \frac{\sqrt{2de-f}}{e} + 2x\right)}{e}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{2de-f}-2ex}{\sqrt{2de+f}}\right)}{\sqrt{2de+f}} + \frac{\tan^{-1}\left(\frac{\sqrt{2de-f}+2ex}{\sqrt{2de+f}}\right)}{\sqrt{2de+f}}$$

Mathematica [B] time = 0.111037, size = 181, normalized size = 2.21

$$\frac{(\sqrt{f^2-4d^2e^2+2de-f}) \tan^{-1}\left(\frac{\sqrt{2ex}}{\sqrt{f-\sqrt{f^2-4d^2e^2}}}\right) + (\sqrt{f^2-4d^2e^2-2de+f}) \tan^{-1}\left(\frac{\sqrt{2ex}}{\sqrt{\sqrt{f^2-4d^2e^2}+f}}\right)}{\sqrt{f-\sqrt{f^2-4d^2e^2}} + \sqrt{\sqrt{f^2-4d^2e^2}+f}}$$

$$\frac{\sqrt{2}\sqrt{f^2-4d^2e^2}}{\sqrt{f-\sqrt{f^2-4d^2e^2}} + \sqrt{\sqrt{f^2-4d^2e^2}+f}}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e*x^2)/(d^2 + f*x^2 + e^2*x^4), x]

[Out] (((2*d*e - f + Sqrt[-4*d^2*e^2 + f^2])*ArcTan[(Sqrt[2]*e*x)/Sqrt[f - Sqrt[-4*d^2*e^2 + f^2]]])/Sqrt[f - Sqrt[-4*d^2*e^2 + f^2]] + ((-2*d*e + f + Sqrt[-4*d^2*e^2 + f^2])*ArcTan[(Sqrt[2]*e*x)/Sqrt[f + Sqrt[-4*d^2*e^2 + f^2]]])/Sqrt[f + Sqrt[-4*d^2*e^2 + f^2]])/(Sqrt[2]*Sqrt[-4*d^2*e^2 + f^2])

Maple [A] time = 0.204, size = 71, normalized size = 0.9

$$-\arctan\left(\left(-2ex + \sqrt{2de-f}\right) \frac{1}{\sqrt{2de+f}}\right) \frac{1}{\sqrt{2de+f}} + \arctan\left(\left(2ex + \sqrt{2de-f}\right) \frac{1}{\sqrt{2de+f}}\right) \frac{1}{\sqrt{2de+f}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x^2+d)/(e^2*x^4+f*x^2+d^2), x)

[Out] -arctan((-2*e*x+(2*d*e-f)^(1/2))/(2*d*e+f)^(1/2))/(2*d*e+f)^(1/2)+arctan((2*e*x+(2*d*e-f)^(1/2))/(2*d*e+f)^(1/2))/(2*d*e+f)^(1/2)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex^2 + d}{e^2x^4 + fx^2 + d^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)/(e^2*x^4+f*x^2+d^2), x, algorithm="maxima")

[Out] integrate((e*x^2 + d)/(e^2*x^4 + f*x^2 + d^2), x)

Fricas [A] time = 1.34911, size = 375, normalized size = 4.57

$$\left[\frac{\sqrt{-2de-f} \log\left(\frac{e^2x^4 - (4de+f)x^2 + d^2 - 2(ex^3 - dx)\sqrt{-2de-f}}{e^2x^4 + fx^2 + d^2}\right)}{2(2de+f)}, \frac{\sqrt{2de+f} \arctan\left(\frac{ex}{\sqrt{2de+f}}\right) + \sqrt{2de+f} \arctan\left(\frac{(e^2x^3 + (de+f)x)\sqrt{2de+f}}{2d^2e + df}\right)}{2de+f} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)/(e^2*x^4+f*x^2+d^2),x, algorithm="fricas")

[Out] [-1/2*sqrt(-2*d*e - f)*log((e^2*x^4 - (4*d*e + f)*x^2 + d^2 - 2*(e*x^3 - d*x)*sqrt(-2*d*e - f))/(e^2*x^4 + f*x^2 + d^2))/(2*d*e + f), (sqrt(2*d*e + f)*arctan(e*x/sqrt(2*d*e + f)) + sqrt(2*d*e + f)*arctan((e^2*x^3 + (d*e + f)*x)*sqrt(2*d*e + f)/(2*d^2*e + d*f)))/(2*d*e + f)]

Sympy [A] time = 0.42151, size = 122, normalized size = 1.49

$$\frac{\sqrt{-\frac{1}{2de+f}} \log\left(-\frac{d}{e} + x^2 + \frac{x\left(-2de\sqrt{-\frac{1}{2de+f}} - f\sqrt{-\frac{1}{2de+f}}\right)}{e}\right)}{2} + \frac{\sqrt{-\frac{1}{2de+f}} \log\left(-\frac{d}{e} + x^2 + \frac{x\left(2de\sqrt{-\frac{1}{2de+f}} + f\sqrt{-\frac{1}{2de+f}}\right)}{e}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x**2+d)/(e**2*x**4+f*x**2+d**2),x)

[Out] -sqrt(-1/(2*d*e + f))*log(-d/e + x**2 + x*(-2*d*e*sqrt(-1/(2*d*e + f)) - f*sqrt(-1/(2*d*e + f)))/e)/2 + sqrt(-1/(2*d*e + f))*log(-d/e + x**2 + x*(2*d*e*sqrt(-1/(2*d*e + f)) + f*sqrt(-1/(2*d*e + f)))/e)/2

Giac [C] time = 1.61013, size = 5779, normalized size = 70.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)/(e^2*x^4+f*x^2+d^2),x, algorithm="giac")

[Out] 1/2*(3*(4*(d^2)^(3/4)*d^2*e^(13/2) - (d^2)^(3/4)*f^2*e^(9/2) - sqrt(-4*d^2*e^2 + f^2)*(d^2)^(3/4)*f*e^(9/2))*cos(5/4*pi + 1/2*real_part(arcsin(1/2*f*e^(-1)/abs(d))))^2*cosh(1/2*imag_part(arcsin(1/2*f*e^(-1)/abs(d))))^3*e*sin(5/4*pi + 1/2*real_part(arcsin(1/2*f*e^(-1)/abs(d)))) - (4*(d^2)^(3/4)*d^2*e^(13/2) - (d^2)^(3/4)*f^2*e^(9/2) - sqrt(-4*d^2*e^2 + f^2)*(d^2)^(3/4)*f*e^(9/2))*cosh(1/2*imag_part(arcsin(1/2*f*e^(-1)/abs(d))))^3*e*sin(5/4*pi + 1/2*real_part(arcsin(1/2*f*e^(-1)/abs(d))))^3 - 9*(4*(d^2)^(3/4)*d^2*e^(13/2) - (d^2)^(3/4)*f^2*e^(9/2) - sqrt(-4*d^2*e^2 + f^2)*(d^2)^(3/4)*f*e^(9/2))*cos(5/4*pi + 1/2*real_part(arcsin(1/2*f*e^(-1)/abs(d))))^2*cosh(1/2*imag_part(arcsin(1/2*f*e^(-1)/abs(d))))^2*e*sin(5/4*pi + 1/2*real_part(arcsin(1/2*f*e^(-1)/abs(d))))*sinh(1/2*imag_part(arcsin(1/2*f*e^(-1)/abs(d)))) + 3*(4*(d^2)^(3/4)*d^2*e^(13/2) - (d^2)^(3/4)*f^2*e^(9/2) - sqrt(-4*d^2*e^2 + f^2)*(d^2)^(3/4)*f*e^(9/2))*cosh(1/2*imag_part(arcsin(1/2*f*e^(-1)/abs(d))))^2*

$$\begin{aligned}
& n(1/2*f*e^{(-1)/abs(d)})) * \sin(1/4*\pi + 1/2*real_part(\arcsin(1/2*f*e^{(-1)/abs(d)})))^2 * \sinh(1/2*imag_part(\arcsin(1/2*f*e^{(-1)/abs(d)})))^2 - (4*(d^2)^{(3/4)} * d^2 * e^{(13/2)} - (d^2)^{(3/4)} * f^2 * e^{(9/2)} - \sqrt{-4*d^2*e^2 + f^2} * (d^2)^{(3/4)} * f * e^{(9/2)}) * \cos(1/4*\pi + 1/2*real_part(\arcsin(1/2*f*e^{(-1)/abs(d)})))^3 * \\
& e * \sinh(1/2*imag_part(\arcsin(1/2*f*e^{(-1)/abs(d)})))^3 + 3*(4*(d^2)^{(3/4)} * d^2 * e^{(13/2)} - (d^2)^{(3/4)} * f^2 * e^{(9/2)} - \sqrt{-4*d^2*e^2 + f^2} * (d^2)^{(3/4)} * f * e^{(9/2)}) * \cos(1/4*\pi + 1/2*real_part(\arcsin(1/2*f*e^{(-1)/abs(d)}))) * e * \sin(1/4 * \\
& \pi + 1/2*real_part(\arcsin(1/2*f*e^{(-1)/abs(d)})))^2 * \sinh(1/2*imag_part(\arcsin(1/2*f*e^{(-1)/abs(d)})))^3 + (4*(d^2)^{(1/4)} * d^3 * e^{(15/2)} - (d^2)^{(1/4)} * d * f^2 * e^{(11/2)} - \sqrt{-4*d^2*e^2 + f^2} * (d^2)^{(1/4)} * d * f * e^{(11/2)}) * \cos(1/4*\pi + \\
& 1/2*real_part(\arcsin(1/2*f*e^{(-1)/abs(d)}))) * \cosh(1/2*imag_part(\arcsin(1/2*f*e^{(-1)/abs(d)}))) - (4*(d^2)^{(1/4)} * d^3 * e^{(15/2)} - (d^2)^{(1/4)} * d * f^2 * e^{(11/2)} - \sqrt{-4*d^2*e^2 + f^2} * (d^2)^{(1/4)} * d * f * e^{(11/2)}) * \cos(1/4*\pi + 1/2*real \\
& _part(\arcsin(1/2*f*e^{(-1)/abs(d)}))) * \sinh(1/2*imag_part(\arcsin(1/2*f*e^{(-1)/abs(d)}))) * \log(-2*(d^2)^{(1/4)} * x * \cos(1/4*\pi + 1/2*\arcsin(1/2*f*e^{(-1)/abs(d)}))) * e^{(-1/2)} + x^2 + \sqrt{d^2} * e^{(-1)}) / (4*d^4*e^8 - d^2*f^2*e^6)
\end{aligned}$$

$$3.28 \quad \int \frac{d+ex^2}{d^2-bx^2+e^2x^4} dx$$

Optimal. Leaf size=78

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b+2de}-2ex}{\sqrt{b-2de}}\right)}{\sqrt{b-2de}} - \frac{\tanh^{-1}\left(\frac{\sqrt{b+2de}+2ex}{\sqrt{b-2de}}\right)}{\sqrt{b-2de}}$$

[Out] ArcTanh[(Sqrt[b + 2*d*e] - 2*e*x)/Sqrt[b - 2*d*e]]/Sqrt[b - 2*d*e] - ArcTanh[(Sqrt[b + 2*d*e] + 2*e*x)/Sqrt[b - 2*d*e]]/Sqrt[b - 2*d*e]

Rubi [A] time = 0.0980389, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1161, 618, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b+2de}-2ex}{\sqrt{b-2de}}\right)}{\sqrt{b-2de}} - \frac{\tanh^{-1}\left(\frac{\sqrt{b+2de}+2ex}{\sqrt{b-2de}}\right)}{\sqrt{b-2de}}$$

Antiderivative was successfully verified.

[In] Int[(d + e*x^2)/(d^2 - b*x^2 + e^2*x^4), x]

[Out] ArcTanh[(Sqrt[b + 2*d*e] - 2*e*x)/Sqrt[b - 2*d*e]]/Sqrt[b - 2*d*e] - ArcTanh[(Sqrt[b + 2*d*e] + 2*e*x)/Sqrt[b - 2*d*e]]/Sqrt[b - 2*d*e]

Rule 1161

Int[((d_) + (e_)*(x_)^2)/((a_) + (b_)*(x_)^2 + (c_)*(x_)^4), x_Symbol] :> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (GtQ[(2*d)/e - b/c, 0] || (!LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2], 0]))

Rule 618

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned} \int \frac{d + ex^2}{d^2 - bx^2 + e^2x^4} dx &= \frac{\int \frac{1}{\frac{d}{e} - \frac{\sqrt{b+2de}}{e} + x^2} dx}{2e} + \frac{\int \frac{1}{\frac{d}{e} + \frac{\sqrt{b+2de}}{e} + x^2} dx}{2e} \\ &= \frac{\text{Subst}\left(\int \frac{1}{\frac{b-2de}{e^2} - x^2} dx, x, -\frac{\sqrt{b+2de}}{e} + 2x\right)}{e} - \frac{\text{Subst}\left(\int \frac{1}{\frac{b-2de}{e^2} - x^2} dx, x, \frac{\sqrt{b+2de}}{e} + 2x\right)}{e} \\ &= \frac{\tanh^{-1}\left(\frac{\sqrt{b+2de}-2ex}{\sqrt{b-2de}}\right)}{\sqrt{b-2de}} - \frac{\tanh^{-1}\left(\frac{\sqrt{b+2de}+2ex}{\sqrt{b-2de}}\right)}{\sqrt{b-2de}} \end{aligned}$$

Mathematica [B] time = 0.109107, size = 189, normalized size = 2.42

$$\frac{\left(\sqrt{b^2-4d^2e^2+b+2de}\right)\tan^{-1}\left(\frac{\sqrt{2ex}}{\sqrt{-\sqrt{b^2-4d^2e^2}-b}}\right) + \left(\sqrt{b^2-4d^2e^2-b-2de}\right)\tan^{-1}\left(\frac{\sqrt{2ex}}{\sqrt{\sqrt{b^2-4d^2e^2}-b}}\right)}{\sqrt{2}\sqrt{b^2-4d^2e^2}}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e*x^2)/(d^2 - b*x^2 + e^2*x^4),x]

[Out] (((b + 2*d*e + Sqrt[b^2 - 4*d^2*e^2])*ArcTan[(Sqrt[2]*e*x)/Sqrt[-b - Sqrt[b^2 - 4*d^2*e^2]]])/Sqrt[-b - Sqrt[b^2 - 4*d^2*e^2]] + ((-b - 2*d*e + Sqrt[b^2 - 4*d^2*e^2])*ArcTan[(Sqrt[2]*e*x)/Sqrt[-b + Sqrt[b^2 - 4*d^2*e^2]]])/Sqrt[-b + Sqrt[b^2 - 4*d^2*e^2]])/(Sqrt[2]*Sqrt[b^2 - 4*d^2*e^2])

Maple [A] time = 0.179, size = 75, normalized size = 1.

$$\arctan\left(\left(2ex + \sqrt{2de + b}\right)\frac{1}{\sqrt{2de - b}}\right)\frac{1}{\sqrt{2de - b}} - \arctan\left(\left(-2ex + \sqrt{2de + b}\right)\frac{1}{\sqrt{2de - b}}\right)\frac{1}{\sqrt{2de - b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x^2+d)/(e^2*x^4-b*x^2+d^2),x)

[Out] 1/(2*d*e-b)^(1/2)*arctan((2*e*x+(2*d*e+b)^(1/2))/(2*d*e-b)^(1/2))-1/(2*d*e-b)^(1/2)*arctan((-2*e*x+(2*d*e+b)^(1/2))/(2*d*e-b)^(1/2))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex^2 + d}{e^2x^4 - bx^2 + d^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)/(e^2*x^4-b*x^2+d^2),x, algorithm="maxima")

[Out] integrate((e*x^2 + d)/(e^2*x^4 - b*x^2 + d^2), x)

Fricas [A] time = 1.34758, size = 375, normalized size = 4.81

$$\left[\frac{\sqrt{-2de+b} \log\left(\frac{e^2x^4-(4de-b)x^2+d^2-2(ex^3-dx)\sqrt{-2de+b}}{e^2x^4-bx^2+d^2}\right)}{2(2de-b)}, \frac{\sqrt{2de-b} \arctan\left(\frac{ex}{\sqrt{2de-b}}\right) + \sqrt{2de-b} \arctan\left(\frac{(e^2x^3+(de-b)x)\sqrt{2de-b}}{2d^2e-bd}\right)}{2de-b} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)/(e^2*x^4-b*x^2+d^2),x, algorithm="fricas")

[Out] [-1/2*sqrt(-2*d*e + b)*log((e^2*x^4 - (4*d*e - b)*x^2 + d^2 - 2*(e*x^3 - d*x)*sqrt(-2*d*e + b))/(e^2*x^4 - b*x^2 + d^2))/(2*d*e - b), (sqrt(2*d*e - b)*arctan(e*x/sqrt(2*d*e - b)) + sqrt(2*d*e - b)*arctan((e^2*x^3 + (d*e - b)*x)*sqrt(2*d*e - b)/(2*d^2*e - b*d)))/(2*d*e - b)]

Sympy [A] time = 0.45524, size = 110, normalized size = 1.41

$$\frac{\sqrt{\frac{1}{b-2de}} \log\left(-\frac{d}{e} + x^2 + \frac{x\left(-b\sqrt{\frac{1}{b-2de}} + 2de\sqrt{\frac{1}{b-2de}}\right)}{e}\right)}{2} - \frac{\sqrt{\frac{1}{b-2de}} \log\left(-\frac{d}{e} + x^2 + \frac{x\left(b\sqrt{\frac{1}{b-2de}} - 2de\sqrt{\frac{1}{b-2de}}\right)}{e}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x**2+d)/(e**2*x**4-b*x**2+d**2),x)

[Out] sqrt(1/(b - 2*d*e))*log(-d/e + x**2 + x*(-b*sqrt(1/(b - 2*d*e)) + 2*d*e*sqrt(1/(b - 2*d*e)))/e)/2 - sqrt(1/(b - 2*d*e))*log(-d/e + x**2 + x*(b*sqrt(1/(b - 2*d*e)) - 2*d*e*sqrt(1/(b - 2*d*e)))/e)/2

Giac [C] time = 1.62429, size = 5387, normalized size = 69.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)/(e^2*x^4-b*x^2+d^2),x, algorithm="giac")

[Out] 1/2*(3*(4*(d^2)^(3/4)*d^2*e^(13/2) - b^2*(d^2)^(3/4)*e^(9/2) + sqrt(-4*d^2*e^2 + b^2)*b*(d^2)^(3/4)*e^(9/2))*cos(1/2*real_part(arccos(1/2*b*e^(-1)/abs(d))))^2*cosh(1/2*imag_part(arccos(1/2*b*e^(-1)/abs(d))))^3*e*sin(1/2*real_part(arccos(1/2*b*e^(-1)/abs(d)))) - (4*(d^2)^(3/4)*d^2*e^(13/2) - b^2*(d^2)^(3/4)*e^(9/2) + sqrt(-4*d^2*e^2 + b^2)*b*(d^2)^(3/4)*e^(9/2))*cosh(1/2*imag_part(arccos(1/2*b*e^(-1)/abs(d))))^3*e*sin(1/2*real_part(arccos(1/2*b*e^(-1)/abs(d))))^3 - 9*(4*(d^2)^(3/4)*d^2*e^(13/2) - b^2*(d^2)^(3/4)*e^(9/2) + sqrt(-4*d^2*e^2 + b^2)*b*(d^2)^(3/4)*e^(9/2))*cos(1/2*real_part(arccos(1/2*b*e^(-1)/abs(d))))^2*cosh(1/2*imag_part(arccos(1/2*b*e^(-1)/abs(d))))^2*e*sin(1/2*real_part(arccos(1/2*b*e^(-1)/abs(d))))*sinh(1/2*imag_part(arccos(1/2*b*e^(-1)/abs(d)))) + 3*(4*(d^2)^(3/4)*d^2*e^(13/2) - b^2*(d^2)^(3/4)*e^(9/2) + sqrt(-4*d^2*e^2 + b^2)*b*(d^2)^(3/4)*e^(9/2))*cosh(1/2*imag_part(arccos(1/2*b*e^(-1)/abs(d))))^2*e*sin(1/2*real_part(arccos(1/2*b*e^(-1)/abs(d))))^3*sinh(1/2*imag_part(arccos(1/2*b*e^(-1)/abs(d)))) + 9*(4*(d^2)^(3/4)*d^2*e^(13/2) - b^2*(d^2)^(3/4)*e^(9/2) + sqrt(-4*d^2*e^2 + b^2)*b*(d^2)^(3/4)*e^(9/2)

$$\begin{aligned} & /2) + \sqrt{-4*d^2*e^2 + b^2} * b * (d^2)^{1/4} * d * e^{11/2}) * \cos(1/2 * \text{real_part}(\arccos(1/2 * b * e^{-1} / \text{abs}(d)))) * \cosh(1/2 * \text{imag_part}(\arccos(1/2 * b * e^{-1} / \text{abs}(d)))) \\ &) - (4 * (d^2)^{1/4} * d^3 * e^{15/2} - b^2 * (d^2)^{1/4} * d * e^{11/2} + \sqrt{-4*d^2 * e^2 + b^2} * b * (d^2)^{1/4} * d * e^{11/2}) * \cos(1/2 * \text{real_part}(\arccos(1/2 * b * e^{-1} / \text{abs}(d)))) * \sinh(1/2 * \text{imag_part}(\arccos(1/2 * b * e^{-1} / \text{abs}(d)))) * \log(-2 * (d^2)^{1/4} * x * \cos(1/2 * \arccos(1/2 * b * e^{-1} / \text{abs}(d)))) * e^{-1/2} + x^2 + \sqrt{d^2} * e^{-1}) / (4 * d^4 * e^8 - b^2 * d^2 * e^6) \end{aligned}$$

$$3.29 \quad \int \frac{d+ex^2}{d^2-fx^2+e^2x^4} dx$$

Optimal. Leaf size=86

$$\frac{\tan^{-1}\left(\frac{\sqrt{2de+f+2ex}}{\sqrt{2de-f}}\right)}{\sqrt{2de-f}} - \frac{\tan^{-1}\left(\frac{\sqrt{2de+f-2ex}}{\sqrt{2de-f}}\right)}{\sqrt{2de-f}}$$

[Out] -(ArcTan[(Sqrt[2*d*e + f] - 2*e*x)/Sqrt[2*d*e - f]]/Sqrt[2*d*e - f]) + ArcTan[(Sqrt[2*d*e + f] + 2*e*x)/Sqrt[2*d*e - f]]/Sqrt[2*d*e - f]

Rubi [A] time = 0.103915, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1161, 618, 204}

$$\frac{\tan^{-1}\left(\frac{\sqrt{2de+f+2ex}}{\sqrt{2de-f}}\right)}{\sqrt{2de-f}} - \frac{\tan^{-1}\left(\frac{\sqrt{2de+f-2ex}}{\sqrt{2de-f}}\right)}{\sqrt{2de-f}}$$

Antiderivative was successfully verified.

[In] Int[(d + e*x^2)/(d^2 - f*x^2 + e^2*x^4), x]

[Out] -(ArcTan[(Sqrt[2*d*e + f] - 2*e*x)/Sqrt[2*d*e - f]]/Sqrt[2*d*e - f]) + ArcTan[(Sqrt[2*d*e + f] + 2*e*x)/Sqrt[2*d*e - f]]/Sqrt[2*d*e - f]

Rule 1161

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (b_)*(x_)^2 + (c_)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2,
x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[
{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[(2*d)/e - b/c, 0] || (!LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 618

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{d + ex^2}{d^2 - fx^2 + e^2x^4} dx = \frac{\int \frac{1}{\frac{d - \sqrt{2de+fx}}{e} + x^2} dx}{2e} + \frac{\int \frac{1}{\frac{d + \sqrt{2de+fx}}{e} + x^2} dx}{2e}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{-\frac{2de-f}{e^2} - x^2} dx, x, -\frac{\sqrt{2de+f}}{e} + 2x\right)}{e} - \frac{\text{Subst}\left(\int \frac{1}{-\frac{2de-f}{e^2} - x^2} dx, x, \frac{\sqrt{2de+f}}{e} + 2x\right)}{e}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{2de+f}-2ex}{\sqrt{2de-f}}\right)}{\sqrt{2de-f}} + \frac{\tan^{-1}\left(\frac{\sqrt{2de+f}+2ex}{\sqrt{2de-f}}\right)}{\sqrt{2de-f}}$$

Mathematica [B] time = 0.107449, size = 189, normalized size = 2.2

$$\frac{(\sqrt{f^2-4d^2e^2+2de+f}) \tan^{-1}\left(\frac{\sqrt{2ex}}{\sqrt{-\sqrt{f^2-4d^2e^2}-f}}\right) + (\sqrt{f^2-4d^2e^2-2de-f}) \tan^{-1}\left(\frac{\sqrt{2ex}}{\sqrt{\sqrt{f^2-4d^2e^2}-f}}\right)}{\sqrt{-\sqrt{f^2-4d^2e^2}-f} + \sqrt{\sqrt{f^2-4d^2e^2}-f}}$$

$$\frac{\sqrt{2}\sqrt{f^2-4d^2e^2}}{\sqrt{-\sqrt{f^2-4d^2e^2}-f} + \sqrt{\sqrt{f^2-4d^2e^2}-f}}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e*x^2)/(d^2 - f*x^2 + e^2*x^4), x]

[Out] (((2*d*e + f + Sqrt[-4*d^2*e^2 + f^2])*ArcTan[(Sqrt[2]*e*x)/Sqrt[-f - Sqrt[-4*d^2*e^2 + f^2]]])/Sqrt[-f - Sqrt[-4*d^2*e^2 + f^2]] + ((-2*d*e - f + Sqrt[-4*d^2*e^2 + f^2])*ArcTan[(Sqrt[2]*e*x)/Sqrt[-f + Sqrt[-4*d^2*e^2 + f^2]]])/Sqrt[-f + Sqrt[-4*d^2*e^2 + f^2]])/(Sqrt[2]*Sqrt[-4*d^2*e^2 + f^2])

Maple [A] time = 0.175, size = 75, normalized size = 0.9

$$-\arctan\left(\left(-2ex + \sqrt{2de+f}\right) \frac{1}{\sqrt{2de-f}}\right) \frac{1}{\sqrt{2de-f}} + \arctan\left(\left(2ex + \sqrt{2de+f}\right) \frac{1}{\sqrt{2de-f}}\right) \frac{1}{\sqrt{2de-f}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x^2+d)/(e^2*x^4-f*x^2+d^2), x)

[Out] -arctan((-2*e*x+(2*d*e+f)^(1/2))/(2*d*e-f)^(1/2))/(2*d*e-f)^(1/2)+arctan((2*e*x+(2*d*e+f)^(1/2))/(2*d*e-f)^(1/2))/(2*d*e-f)^(1/2)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex^2 + d}{e^2x^4 - fx^2 + d^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)/(e^2*x^4-f*x^2+d^2), x, algorithm="maxima")

[Out] integrate((e*x^2 + d)/(e^2*x^4 - f*x^2 + d^2), x)

Fricas [A] time = 1.39025, size = 379, normalized size = 4.41

$$\left[\frac{\sqrt{-2de+f} \log\left(\frac{e^2x^4 - (4de-f)x^2 + d^2 - 2(ex^3 - dx)\sqrt{-2de+f}}{e^2x^4 - fx^2 + d^2}\right)}{2(2de-f)}, \frac{\sqrt{2de-f} \arctan\left(-\frac{ex}{\sqrt{2de-f}}\right) + \sqrt{2de-f} \arctan\left(-\frac{(e^2x^3 + (de-f)x^2 + d^2x)}{2d^2e}\right)}{2de-f} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)/(e^2*x^4-f*x^2+d^2),x, algorithm="fricas")

[Out] [-1/2*sqrt(-2*d*e + f)*log((e^2*x^4 - (4*d*e - f)*x^2 + d^2 - 2*(e*x^3 - d*x)*sqrt(-2*d*e + f))/(e^2*x^4 - f*x^2 + d^2))/(2*d*e - f), -(sqrt(2*d*e - f)*arctan(-e*x/sqrt(2*d*e - f)) + sqrt(2*d*e - f)*arctan(-(e^2*x^3 + (d*e - f)*x)*sqrt(2*d*e - f)/(2*d^2*e - d*f)))/(2*d*e - f)]

Sympy [A] time = 0.424422, size = 121, normalized size = 1.41

$$\frac{\sqrt{-\frac{1}{2de-f}} \log\left(-\frac{d}{e} + x^2 + \frac{x\left(-2de\sqrt{-\frac{1}{2de-f}} + f\sqrt{-\frac{1}{2de-f}}\right)}{e}\right)}{2} + \frac{\sqrt{-\frac{1}{2de-f}} \log\left(-\frac{d}{e} + x^2 + \frac{x\left(2de\sqrt{-\frac{1}{2de-f}} - f\sqrt{-\frac{1}{2de-f}}\right)}{e}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x**2+d)/(e**2*x**4-f*x**2+d**2),x)

[Out] -sqrt(-1/(2*d*e - f))*log(-d/e + x**2 + x*(-2*d*e*sqrt(-1/(2*d*e - f)) + f*sqrt(-1/(2*d*e - f)))/e)/2 + sqrt(-1/(2*d*e - f))*log(-d/e + x**2 + x*(2*d*e*sqrt(-1/(2*d*e - f)) - f*sqrt(-1/(2*d*e - f)))/e)/2

Giac [C] time = 1.59795, size = 5387, normalized size = 62.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)/(e^2*x^4-f*x^2+d^2),x, algorithm="giac")

[Out] 1/2*(3*(4*(d^2)^(3/4)*d^2*e^(13/2) - (d^2)^(3/4)*f^2*e^(9/2) + sqrt(-4*d^2*e^2 + f^2)*(d^2)^(3/4)*f*e^(9/2))*cos(1/2*real_part(arccos(1/2*f*e^(-1)/abs(d))))^2*cosh(1/2*imag_part(arccos(1/2*f*e^(-1)/abs(d))))^3*e*sin(1/2*real_part(arccos(1/2*f*e^(-1)/abs(d)))) - (4*(d^2)^(3/4)*d^2*e^(13/2) - (d^2)^(3/4)*f^2*e^(9/2) + sqrt(-4*d^2*e^2 + f^2)*(d^2)^(3/4)*f*e^(9/2))*cosh(1/2*imag_part(arccos(1/2*f*e^(-1)/abs(d))))^3*e*sin(1/2*real_part(arccos(1/2*f*e^(-1)/abs(d))))^3 - 9*(4*(d^2)^(3/4)*d^2*e^(13/2) - (d^2)^(3/4)*f^2*e^(9/2) + sqrt(-4*d^2*e^2 + f^2)*(d^2)^(3/4)*f*e^(9/2))*cos(1/2*real_part(arccos(1/2*f*e^(-1)/abs(d))))^2*cosh(1/2*imag_part(arccos(1/2*f*e^(-1)/abs(d))))^2*e*sin(1/2*real_part(arccos(1/2*f*e^(-1)/abs(d))))*sinh(1/2*imag_part(arccos(1/2*f*e^(-1)/abs(d)))) + 3*(4*(d^2)^(3/4)*d^2*e^(13/2) - (d^2)^(3/4)*f^2*e^(9/2) + sqrt(-4*d^2*e^2 + f^2)*(d^2)^(3/4)*f*e^(9/2))*cosh(1/2*imag_part(arccos(1/2*f*e^(-1)/abs(d))))^2*e*sin(1/2*real_part(arccos(1/2*f*e^(-1)/abs(d))))

$$\begin{aligned}
& /2*f*e^{(-1)/abs(d)})^3*cosh(1/2*imag_part(arccos(1/2*f*e^{(-1)/abs(d)}))\wedge 3* \\
& e - 3*(4*(d^2)\wedge(3/4)*d^2*e^{(13/2)} - (d^2)\wedge(3/4)*f^2*e^{(9/2)} + \sqrt{-4*d^2*e \\
& ^2 + f^2}*(d^2)\wedge(3/4)*f*e^{(9/2)})*\cos(1/2*real_part(arccos(1/2*f*e^{(-1)/abs(\\
& d)})))*cosh(1/2*imag_part(arccos(1/2*f*e^{(-1)/abs(d)}))\wedge 3*e*\sin(1/2*real_par \\
& t(arccos(1/2*f*e^{(-1)/abs(d)}))\wedge 2 - 3*(4*(d^2)\wedge(3/4)*d^2*e^{(13/2)} - (d^2)\wedge(\\
& 3/4)*f^2*e^{(9/2)} + \sqrt{-4*d^2*e^2 + f^2}*(d^2)\wedge(3/4)*f*e^{(9/2)})*\cos(1/2*re \\
& al_part(arccos(1/2*f*e^{(-1)/abs(d)}))\wedge 3*cosh(1/2*imag_part(arccos(1/2*f*e^{(\\
& -1)/abs(d)}))\wedge 2*e*\sinh(1/2*imag_part(arccos(1/2*f*e^{(-1)/abs(d)})) + 9*(4*(\\
& d^2)\wedge(3/4)*d^2*e^{(13/2)} - (d^2)\wedge(3/4)*f^2*e^{(9/2)} + \sqrt{-4*d^2*e^2 + f^2}* \\
& (d^2)\wedge(3/4)*f*e^{(9/2)})*\cos(1/2*real_part(arccos(1/2*f*e^{(-1)/abs(d)})))*cosh \\
& (1/2*imag_part(arccos(1/2*f*e^{(-1)/abs(d)}))\wedge 2*e*\sin(1/2*real_part(arccos(1 \\
& /2*f*e^{(-1)/abs(d)}))\wedge 2*\sinh(1/2*imag_part(arccos(1/2*f*e^{(-1)/abs(d)})) + \\
& 3*(4*(d^2)\wedge(3/4)*d^2*e^{(13/2)} - (d^2)\wedge(3/4)*f^2*e^{(9/2)} + \sqrt{-4*d^2*e^2 + \\
& f^2}*(d^2)\wedge(3/4)*f*e^{(9/2)})*\cos(1/2*real_part(arccos(1/2*f*e^{(-1)/abs(d)})) \\
&)\wedge 3*cosh(1/2*imag_part(arccos(1/2*f*e^{(-1)/abs(d)})))*e*\sinh(1/2*imag_part(a \\
& rccos(1/2*f*e^{(-1)/abs(d)}))\wedge 2 - 9*(4*(d^2)\wedge(3/4)*d^2*e^{(13/2)} - (d^2)\wedge(3/4 \\
&)*f^2*e^{(9/2)} + \sqrt{-4*d^2*e^2 + f^2}*(d^2)\wedge(3/4)*f*e^{(9/2)})*\cos(1/2*real_ \\
& part(arccos(1/2*f*e^{(-1)/abs(d)})))*cosh(1/2*imag_part(arccos(1/2*f*e^{(-1)/a \\
& bs(d)})))*e*\sin(1/2*real_part(arccos(1/2*f*e^{(-1)/abs(d)}))\wedge 2*\sinh(1/2*imag_ \\
& part(arccos(1/2*f*e^{(-1)/abs(d)}))\wedge 2 - (4*(d^2)\wedge(3/4)*d^2*e^{(13/2)} - (d^2)\wedge \\
& (3/4)*f^2*e^{(9/2)} + \sqrt{-4*d^2*e^2 + f^2}*(d^2)\wedge(3/4)*f*e^{(9/2)})*\cos(1/2*r \\
& eal_part(arccos(1/2*f*e^{(-1)/abs(d)}))\wedge 3*e*\sinh(1/2*imag_part(arccos(1/2*f* \\
& e^{(-1)/abs(d)}))\wedge 3 + 3*(4*(d^2)\wedge(3/4)*d^2*e^{(13/2)} - (d^2)\wedge(3/4)*f^2*e^{(9/2)} \\
&) + \sqrt{-4*d^2*e^2 + f^2}*(d^2)\wedge(3/4)*f*e^{(9/2)})*\cos(1/2*real_part(arccos(\\
& 1/2*f*e^{(-1)/abs(d)})))*e*\sin(1/2*real_part(arccos(1/2*f*e^{(-1)/abs(d)}))\wedge 2* \\
& \sinh(1/2*imag_part(arccos(1/2*f*e^{(-1)/abs(d)}))\wedge 3 + (4*(d^2)\wedge(1/4)*d^3*e^{(\\
& 15/2)} - (d^2)\wedge(1/4)*d*f^2*e^{(11/2)} + \sqrt{-4*d^2*e^2 + f^2}*(d^2)\wedge(1/4)*d*f \\
& *e^{(11/2)})*\cos(1/2*real_part(arccos(1/2*f*e^{(-1)/abs(d)})))*cosh(1/2*imag_pa \\
& rt(arccos(1/2*f*e^{(-1)/abs(d)})) - (4*(d^2)\wedge(1/4)*d^3*e^{(15/2)} - (d^2)\wedge(1/4 \\
&)*d*f^2*e^{(11/2)} + \sqrt{-4*d^2*e^2 + f^2}*(d^2)\wedge(1/4)*d*f*e^{(11/2)})*\cos(1/2 \\
& *real_part(arccos(1/2*f*e^{(-1)/abs(d)})))*sinh(1/2*imag_part(arccos(1/2*f*e^{ \\
& (-1)/abs(d)})))*\log(2*(d^2)\wedge(1/4)*x*\cos(1/2*arccos(1/2*f*e^{(-1)/abs(d)}))*e^{ \\
& (-1/2)} + x^2 + \sqrt{d^2}*e^{(-1)})/(4*d^4*e^8 - d^2*f^2*e^6) - 1/4*((4*(d^2)\wedge \\
& (3/4)*d^2*e^{(13/2)} - (d^2)\wedge(3/4)*f^2*e^{(9/2)} + \sqrt{-4*d^2*e^2 + f^2}*(d^2) \\
& ^{(3/4)*f*e^{(9/2)})*\cos(1/2*real_part(arccos(1/2*f*e^{(-1)/abs(d)}))\wedge 3*cosh(1/ \\
& 2*imag_part(arccos(1/2*f*e^{(-1)/abs(d)}))\wedge 3*e - 3*(4*(d^2)\wedge(3/4)*d^2*e^{(13/ \\
& 2)} - (d^2)\wedge(3/4)*f^2*e^{(9/2)} + \sqrt{-4*d^2*e^2 + f^2}*(d^2)\wedge(3/4)*f*e^{(9/2)} \\
&)*\cos(1/2*real_part(arccos(1/2*f*e^{(-1)/abs(d)})))*cosh(1/2*imag_part(arccos \\
& (1/2*f*e^{(-1)/abs(d)}))\wedge 3*e*\sin(1/2*real_part(arccos(1/2*f*e^{(-1)/abs(d)})) \\
&)\wedge 2 - 3*(4*(d^2)\wedge(3/4)*d^2*e^{(13/2)} - (d^2)\wedge(3/4)*f^2*e^{(9/2)} + \sqrt{-4*d^2* \\
& e^2 + f^2}*(d^2)\wedge(3/4)*f*e^{(9/2)})*\cos(1/2*real_part(arccos(1/2*f*e^{(-1)/abs \\
& (d)}))\wedge 3*cosh(1/2*imag_part(arccos(1/2*f*e^{(-1)/abs(d)}))\wedge 2*e*\sinh(1/2*imag \\
& _part(arccos(1/2*f*e^{(-1)/abs(d)})) + 9*(4*(d^2)\wedge(3/4)*d^2*e^{(13/2)} - (d^2) \\
& ^{(3/4)*f^2*e^{(9/2)} + \sqrt{-4*d^2*e^2 + f^2}*(d^2)\wedge(3/4)*f*e^{(9/2)})*\cos(1/2* \\
& real_part(arccos(1/2*f*e^{(-1)/abs(d)})))*cosh(1/2*imag_part(arccos(1/2*f*e^{(\\
& -1)/abs(d)}))\wedge 2*e*\sin(1/2*real_part(arccos(1/2*f*e^{(-1)/abs(d)}))\wedge 2*\sinh(1/ \\
& 2*imag_part(arccos(1/2*f*e^{(-1)/abs(d)})) + 3*(4*(d^2)\wedge(3/4)*d^2*e^{(13/2)} - \\
& (d^2)\wedge(3/4)*f^2*e^{(9/2)} + \sqrt{-4*d^2*e^2 + f^2}*(d^2)\wedge(3/4)*f*e^{(9/2)})*\co \\
& s(1/2*real_part(arccos(1/2*f*e^{(-1)/abs(d)}))\wedge 3*cosh(1/2*imag_part(arccos(1 \\
& /2*f*e^{(-1)/abs(d)})))*e*\sinh(1/2*imag_part(arccos(1/2*f*e^{(-1)/abs(d)}))\wedge 2 \\
& - 9*(4*(d^2)\wedge(3/4)*d^2*e^{(13/2)} - (d^2)\wedge(3/4)*f^2*e^{(9/2)} + \sqrt{-4*d^2*e^2 \\
& + f^2}*(d^2)\wedge(3/4)*f*e^{(9/2)})*\cos(1/2*real_part(arccos(1/2*f*e^{(-1)/abs(d) \\
&)))*cosh(1/2*imag_part(arccos(1/2*f*e^{(-1)/abs(d)})))*e*\sin(1/2*real_part(ar \\
& ccos(1/2*f*e^{(-1)/abs(d)}))\wedge 2*\sinh(1/2*imag_part(arccos(1/2*f*e^{(-1)/abs(d) \\
&))\wedge 2 - (4*(d^2)\wedge(3/4)*d^2*e^{(13/2)} - (d^2)\wedge(3/4)*f^2*e^{(9/2)} + \sqrt{-4*d^2 \\
& *e^2 + f^2}*(d^2)\wedge(3/4)*f*e^{(9/2)})*\cos(1/2*real_part(arccos(1/2*f*e^{(-1)/ab \\
& s(d)}))\wedge 3*e*\sinh(1/2*imag_part(arccos(1/2*f*e^{(-1)/abs(d)}))\wedge 3 + 3*(4*(d^2) \\
& ^{(3/4)*d^2*e^{(13/2)} - (d^2)\wedge(3/4)*f^2*e^{(9/2)} + \sqrt{-4*d^2*e^2 + f^2}*(d^2) \\
&)\wedge(3/4)*f*e^{(9/2)})*\cos(1/2*real_part(arccos(1/2*f*e^{(-1)/abs(d)})))*e*\sin(1/
\end{aligned}$$

$$\begin{aligned}
& 2*\text{real_part}(\arccos(1/2*f*e^{(-1)}/\text{abs}(d)))^2*\sinh(1/2*\text{imag_part}(\arccos(1/2*f* \\
& *e^{(-1)}/\text{abs}(d))))^3 + (4*(d^2)^{(1/4)}*d^3*e^{(15/2)} - (d^2)^{(1/4)}*d*f^2*e^{(11 \\
& /2)} + \text{sqrt}(-4*d^2*e^2 + f^2)*(d^2)^{(1/4)}*d*f*e^{(11/2)})*\cos(1/2*\text{real_part}(\ar \\
& \text{ccos}(1/2*f*e^{(-1)}/\text{abs}(d))))*\cosh(1/2*\text{imag_part}(\arccos(1/2*f*e^{(-1)}/\text{abs}(d))) \\
&) - (4*(d^2)^{(1/4)}*d^3*e^{(15/2)} - (d^2)^{(1/4)}*d*f^2*e^{(11/2)} + \text{sqrt}(-4*d^2* \\
& e^2 + f^2)*(d^2)^{(1/4)}*d*f*e^{(11/2)})*\cos(1/2*\text{real_part}(\arccos(1/2*f*e^{(-1)}/ \\
& \text{abs}(d))))*\sinh(1/2*\text{imag_part}(\arccos(1/2*f*e^{(-1)}/\text{abs}(d))))*\log(-2*(d^2)^{(1 \\
& /4)}*x*\cos(1/2*\arccos(1/2*f*e^{(-1)}/\text{abs}(d)))*e^{(-1/2)} + x^2 + \text{sqrt}(d^2)*e^{(-1 \\
&)})/(4*d^4*e^8 - d^2*f^2*e^6)
\end{aligned}$$

$$3.30 \quad \int \frac{d-ex^2}{d^2+bx^2+e^2x^4} dx$$

Optimal. Leaf size=78

$$\frac{\log(x\sqrt{2de-b}+d+ex^2)}{2\sqrt{2de-b}} - \frac{\log(-x\sqrt{2de-b}+d+ex^2)}{2\sqrt{2de-b}}$$

[Out] -Log[d - Sqrt[-b + 2*d*e]*x + e*x^2]/(2*Sqrt[-b + 2*d*e]) + Log[d + Sqrt[-b + 2*d*e]*x + e*x^2]/(2*Sqrt[-b + 2*d*e])

Rubi [A] time = 0.0515393, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {1164, 628}

$$\frac{\log(x\sqrt{2de-b}+d+ex^2)}{2\sqrt{2de-b}} - \frac{\log(-x\sqrt{2de-b}+d+ex^2)}{2\sqrt{2de-b}}$$

Antiderivative was successfully verified.

[In] Int[(d - e*x^2)/(d^2 + b*x^2 + e^2*x^4), x]

[Out] -Log[d - Sqrt[-b + 2*d*e]*x + e*x^2]/(2*Sqrt[-b + 2*d*e]) + Log[d + Sqrt[-b + 2*d*e]*x + e*x^2]/(2*Sqrt[-b + 2*d*e])

Rule 1164

Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[(-2*d)/e - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && !GtQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rubi steps

$$\begin{aligned} \int \frac{d-ex^2}{d^2+bx^2+e^2x^4} dx &= -\frac{\int \frac{\frac{\sqrt{-b+2de}+2x}{e}}{-\frac{d}{e}-\frac{\sqrt{-b+2de}x}{e}-x^2} dx}{2\sqrt{-b+2de}} - \frac{\int \frac{\frac{\sqrt{-b+2de}-2x}{e}}{-\frac{d}{e}+\frac{\sqrt{-b+2de}x}{e}-x^2} dx}{2\sqrt{-b+2de}} \\ &= -\frac{\log(d-\sqrt{-b+2de}+ex^2)}{2\sqrt{-b+2de}} + \frac{\log(d+\sqrt{-b+2de}+ex^2)}{2\sqrt{-b+2de}} \end{aligned}$$

Mathematica [B] time = 0.120087, size = 182, normalized size = 2.33

$$\frac{(-\sqrt{b^2-4d^2e^2+b+2de})\tan^{-1}\left(\frac{\sqrt{2ex}}{\sqrt{b-\sqrt{b^2-4d^2e^2}}}\right) - (\sqrt{b^2-4d^2e^2+b+2de})\tan^{-1}\left(\frac{\sqrt{2ex}}{\sqrt{\sqrt{b^2-4d^2e^2}+b}}\right)}{\sqrt{b-\sqrt{b^2-4d^2e^2}} - \sqrt{\sqrt{b^2-4d^2e^2}+b}}$$

$$\frac{\sqrt{2}\sqrt{b^2-4d^2e^2}}{\sqrt{b-\sqrt{b^2-4d^2e^2}} - \sqrt{\sqrt{b^2-4d^2e^2}+b}}$$

Antiderivative was successfully verified.

[In] Integrate[(d - e*x^2)/(d^2 + b*x^2 + e^2*x^4),x]

[Out] (((b + 2*d*e - Sqrt[b^2 - 4*d^2*e^2])*ArcTan[(Sqrt[2]*e*x)/Sqrt[b - Sqrt[b^2 - 4*d^2*e^2]]])/Sqrt[b - Sqrt[b^2 - 4*d^2*e^2]] - ((b + 2*d*e + Sqrt[b^2 - 4*d^2*e^2])*ArcTan[(Sqrt[2]*e*x)/Sqrt[b + Sqrt[b^2 - 4*d^2*e^2]]])/Sqrt[b + Sqrt[b^2 - 4*d^2*e^2]])/(Sqrt[2]*Sqrt[b^2 - 4*d^2*e^2])

Maple [A] time = 0.173, size = 88, normalized size = 1.1

$$\frac{1}{-4de+2b}\sqrt{2de-b}\ln\left(-ex^2+x\sqrt{2de-b}-d\right)-\frac{1}{-4de+2b}\sqrt{2de-b}\ln\left(d+ex^2+x\sqrt{2de-b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-e*x^2+d)/(e^2*x^4+b*x^2+d^2),x)

[Out] 1/(-4*d*e+2*b)*(2*d*e-b)^(1/2)*ln(-e*x^2+x*(2*d*e-b)^(1/2)-d)-1/(-4*d*e+2*b)*(2*d*e-b)^(1/2)*ln(d+e*x^2+x*(2*d*e-b)^(1/2))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{ex^2 - d}{e^2x^4 + bx^2 + d^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e*x^2+d)/(e^2*x^4+b*x^2+d^2),x, algorithm="maxima")

[Out] -integrate((e*x^2 - d)/(e^2*x^4 + b*x^2 + d^2), x)

Fricas [A] time = 1.32831, size = 378, normalized size = 4.85

$$\left[\frac{\log\left(\frac{e^2x^4+(4de-b)x^2+d^2+2(ex^3+dx)\sqrt{2de-b}}{e^2x^4+bx^2+d^2}\right)}{2\sqrt{2de-b}}, -\frac{\sqrt{-2de+b}\arctan\left(\frac{\sqrt{-2de+bx}}{2de-b}\right)-\sqrt{-2de+b}\arctan\left(\frac{(e^2x^3-(de-b)x)\sqrt{-2de+b}}{2d^2e-bd}\right)}{2de-b} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e*x^2+d)/(e^2*x^4+b*x^2+d^2),x, algorithm="fricas")

[Out] [1/2*log((e^2*x^4 + (4*d*e - b)*x^2 + d^2 + 2*(e*x^3 + d*x)*sqrt(2*d*e - b))/(e^2*x^4 + b*x^2 + d^2))/sqrt(2*d*e - b), -(sqrt(-2*d*e + b)*arctan(sqrt(-2*d*e + b)*e*x/(2*d*e - b)) - sqrt(-2*d*e + b)*arctan((e^2*x^3 - (d*e - b)*x)*sqrt(-2*d*e + b)/(2*d^2*e - b*d)))/(2*d*e - b)]

Sympy [A] time = 0.574788, size = 121, normalized size = 1.55

$$\frac{\sqrt{-\frac{1}{b-2de}} \log\left(\frac{d}{e} + x^2 + \frac{x\left(-b\sqrt{-\frac{1}{b-2de}} + 2de\sqrt{-\frac{1}{b-2de}}\right)}{e}\right)}{2} - \frac{\sqrt{-\frac{1}{b-2de}} \log\left(\frac{d}{e} + x^2 + \frac{x\left(b\sqrt{-\frac{1}{b-2de}} - 2de\sqrt{-\frac{1}{b-2de}}\right)}{e}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e*x**2+d)/(e**2*x**4+b*x**2+d**2),x)

[Out] sqrt(-1/(b - 2*d*e))*log(d/e + x**2 + x*(-b*sqrt(-1/(b - 2*d*e)) + 2*d*e*sqrt(-1/(b - 2*d*e)))/e)/2 - sqrt(-1/(b - 2*d*e))*log(d/e + x**2 + x*(b*sqrt(-1/(b - 2*d*e)) - 2*d*e*sqrt(-1/(b - 2*d*e)))/e)/2

Giac [C] time = 1.63989, size = 5779, normalized size = 74.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e*x^2+d)/(e^2*x^4+b*x^2+d^2),x, algorithm="giac")

[Out]
$$\begin{aligned} & -1/2*(3*(4*(d^2)^{(3/4)}*d^2*e^{(13/2)} - b^2*(d^2)^{(3/4)}*e^{(9/2)} - \sqrt{-4*d^2*e^2 + b^2}*b*(d^2)^{(3/4)}*e^{(9/2)})*\cos(5/4*\pi + 1/2*\text{real_part}(\arcsin(1/2*b*e^{(-1)/\text{abs}(d)})))^2*\cosh(1/2*\text{imag_part}(\arcsin(1/2*b*e^{(-1)/\text{abs}(d)})))^3*e*\sin(5/4*\pi + 1/2*\text{real_part}(\arcsin(1/2*b*e^{(-1)/\text{abs}(d)}))) - (4*(d^2)^{(3/4)}*d^2*e^{(13/2)} - b^2*(d^2)^{(3/4)}*e^{(9/2)} - \sqrt{-4*d^2*e^2 + b^2}*b*(d^2)^{(3/4)}*e^{(9/2)})*\cosh(1/2*\text{imag_part}(\arcsin(1/2*b*e^{(-1)/\text{abs}(d)})))^3*e*\sin(5/4*\pi + 1/2*\text{real_part}(\arcsin(1/2*b*e^{(-1)/\text{abs}(d)})))^3 - 9*(4*(d^2)^{(3/4)}*d^2*e^{(13/2)} - b^2*(d^2)^{(3/4)}*e^{(9/2)} - \sqrt{-4*d^2*e^2 + b^2}*b*(d^2)^{(3/4)}*e^{(9/2)})*\cos(5/4*\pi + 1/2*\text{real_part}(\arcsin(1/2*b*e^{(-1)/\text{abs}(d)})))^2*\cosh(1/2*\text{imag_part}(\arcsin(1/2*b*e^{(-1)/\text{abs}(d)})))^2*e*\sin(5/4*\pi + 1/2*\text{real_part}(\arcsin(1/2*b*e^{(-1)/\text{abs}(d)})))*\sinh(1/2*\text{imag_part}(\arcsin(1/2*b*e^{(-1)/\text{abs}(d)}))) + 3*(4*(d^2)^{(3/4)}*d^2*e^{(13/2)} - b^2*(d^2)^{(3/4)}*e^{(9/2)} - \sqrt{-4*d^2*e^2 + b^2}*b*(d^2)^{(3/4)}*e^{(9/2)})*\cosh(1/2*\text{imag_part}(\arcsin(1/2*b*e^{(-1)/\text{abs}(d)})))^2*e*\sin(5/4*\pi + 1/2*\text{real_part}(\arcsin(1/2*b*e^{(-1)/\text{abs}(d)})))^3*\sinh(1/2*\text{imag_part}(\arcsin(1/2*b*e^{(-1)/\text{abs}(d)}))) + 9*(4*(d^2)^{(3/4)}*d^2*e^{(13/2)} - b^2*(d^2)^{(3/4)}*e^{(9/2)} - \sqrt{-4*d^2*e^2 + b^2}*b*(d^2)^{(3/4)}*e^{(9/2)})*\cos(5/4*\pi + 1/2*\text{real_part}(\arcsin(1/2*b*e^{(-1)/\text{abs}(d)})))^2*\cosh(1/2*\text{imag_part}(\arcsin(1/2*b*e^{(-1)/\text{abs}(d)})))*e*\sin(5/4*\pi + 1/2*\text{real_part}(\arcsin(1/2*b*e^{(-1)/\text{abs}(d)})))*\sinh(1/2*\text{imag_part}(\arcsin(1/2*b*e^{(-1)/\text{abs}(d)})))^2 - 3*(4*(d^2)^{(3/4)}*d^2*e^{(13/2)} - b^2*(d^2)^{(3/4)}*e^{(9/2)} - \sqrt{-4*d^2*e^2 + b^2}*b*(d^2)^{(3/4)}*e^{(9/2)})*\cosh(1/2*\text{imag_part}(\arcsin(1/2*b*e^{(-1)/\text{abs}(d)})))^3*\sinh(1/2*\text{imag_part}(\arcsin(1/2*b*e^{(-1)/\text{abs}(d)})))^2 - 3*(4*(d^2)^{(3/4)}*d^2*e^{(13/2)} - b^2*(d^2)^{(3/4)}*e^{(9/2)} - \sqrt{-4*d^2*e^2 + b^2}*b*(d^2)^{(3/4)}*e^{(9/2)})*\cos(5/4*\pi + 1/2*\text{real_part}(\arcsin(1/2*b*e^{(-1)/\text{abs}(d)})))^2*e*\sin(5/4*\pi + 1/2*\text{real_part}(\arcsin(1/2*b*e^{(-1)/\text{abs}(d)})))*\sinh(1/2*\text{imag_part}(\arcsin(1/2*b*e^{(-1)/\text{abs}(d)})))^3 + (4*(d^2)^{(3/4)}*d^2*e^{(13/2)} - b^2*(d^2)^{(3/4)}*e^{(9/2)} - \sqrt{-4*d^2*e^2 + b^2}*b*(d^2)^{(3/4)}*e^{(9/2)})*e*\sin(5/4*\pi + 1/2*\text{real_part}(\arcsin(1/2*b*e^{(-1)/\text{abs}(d)})))^3*\sinh(1/2*\text{imag_part}(\arcsin(1/2*b*e^{(-1)/\text{abs}(d)})))^3 - (4*(d^2)^{(1/4)}*d^3*e^{(15/2)} - b^2*(d^2)^{(1/4)}*d*e^{(11/2)} - \sqrt{-4*d^2*e^2 + b^2}*b*(d^2)^{(1/4)}*d*e^{(11/2)})*\cosh(1/2*\text{imag_part}(\arcsin(1/2*b*e^{(-1)/\text{abs}(d)})))*\sin(5/4*\pi + 1/2*\text{real_part}(\arcsin(1/2*b*e^{(-1)/\text{abs}(d)}))) + (4*(d^2)^{(1/4)}*d^3*e^{(15/2)} - b^2*(d^2)^{(1/4)}*d*e^{(11/2)} - \sqrt{-4*d^2*e^2 + b^2}*b*(d^2)^{(1/4)}*d*e^{(11/2)})*\sin(5/4*\pi + 1/2*\text{real_part}(\arcsin(1/2*b*e^{(-1)/\text{abs}(d)}))) \end{aligned}$$

```

*sinh(1/2*imag_part(arcsin(1/2*b*e^(-1)/abs(d))))*arctan(-((d^2)^(1/4)*cos
(5/4*pi + 1/2*arcsin(1/2*b*e^(-1)/abs(d)))e^(-1/2) - x)*e^(1/2)/((d^2)^(1/
4)*sin(5/4*pi + 1/2*arcsin(1/2*b*e^(-1)/abs(d))))/(4*d^4*e^8 - b^2*d^2*e^6
) - 1/2*(3*(4*(d^2)^(3/4)*d^2*e^(13/2) - b^2*(d^2)^(3/4)*e^(9/2) - sqrt(-4*
d^2*e^2 + b^2)*b*(d^2)^(3/4)*e^(9/2))*cos(1/4*pi + 1/2*real_part(arcsin(1/2
*b*e^(-1)/abs(d))))^2*cosh(1/2*imag_part(arcsin(1/2*b*e^(-1)/abs(d))))^3*e*
sin(1/4*pi + 1/2*real_part(arcsin(1/2*b*e^(-1)/abs(d)))) - (4*(d^2)^(3/4)*d
^2*e^(13/2) - b^2*(d^2)^(3/4)*e^(9/2) - sqrt(-4*d^2*e^2 + b^2)*b*(d^2)^(3/4
)*e^(9/2))*cosh(1/2*imag_part(arcsin(1/2*b*e^(-1)/abs(d))))^3*e*sin(1/4*pi
+ 1/2*real_part(arcsin(1/2*b*e^(-1)/abs(d))))^3 - 9*(4*(d^2)^(3/4)*d^2*e^(1
3/2) - b^2*(d^2)^(3/4)*e^(9/2) - sqrt(-4*d^2*e^2 + b^2)*b*(d^2)^(3/4)*e^(9/
2))*cos(1/4*pi + 1/2*real_part(arcsin(1/2*b*e^(-1)/abs(d))))^2*cosh(1/2*ima
g_part(arcsin(1/2*b*e^(-1)/abs(d))))^2*e*sin(1/4*pi + 1/2*real_part(arcsin(
1/2*b*e^(-1)/abs(d))))*sinh(1/2*imag_part(arcsin(1/2*b*e^(-1)/abs(d)))) + 3
*(4*(d^2)^(3/4)*d^2*e^(13/2) - b^2*(d^2)^(3/4)*e^(9/2) - sqrt(-4*d^2*e^2 +
b^2)*b*(d^2)^(3/4)*e^(9/2))*cosh(1/2*imag_part(arcsin(1/2*b*e^(-1)/abs(d)))
)^2*e*sin(1/4*pi + 1/2*real_part(arcsin(1/2*b*e^(-1)/abs(d))))^3*sinh(1/2*i
mag_part(arcsin(1/2*b*e^(-1)/abs(d)))) + 9*(4*(d^2)^(3/4)*d^2*e^(13/2) - b^
2*(d^2)^(3/4)*e^(9/2) - sqrt(-4*d^2*e^2 + b^2)*b*(d^2)^(3/4)*e^(9/2))*cos(1
/4*pi + 1/2*real_part(arcsin(1/2*b*e^(-1)/abs(d))))^2*cosh(1/2*imag_part(ar
csin(1/2*b*e^(-1)/abs(d))))*e*sin(1/4*pi + 1/2*real_part(arcsin(1/2*b*e^(-1)
)/abs(d))))*sinh(1/2*imag_part(arcsin(1/2*b*e^(-1)/abs(d))))^2 - 3*(4*(d^2)
^(3/4)*d^2*e^(13/2) - b^2*(d^2)^(3/4)*e^(9/2) - sqrt(-4*d^2*e^2 + b^2)*b*(d
^2)^(3/4)*e^(9/2))*cosh(1/2*imag_part(arcsin(1/2*b*e^(-1)/abs(d))))*e*sin(1
/4*pi + 1/2*real_part(arcsin(1/2*b*e^(-1)/abs(d))))^3*sinh(1/2*imag_part(ar
csin(1/2*b*e^(-1)/abs(d))))^2 - 3*(4*(d^2)^(3/4)*d^2*e^(13/2) - b^2*(d^2)^(
3/4)*e^(9/2) - sqrt(-4*d^2*e^2 + b^2)*b*(d^2)^(3/4)*e^(9/2))*cos(1/4*pi + 1
/2*real_part(arcsin(1/2*b*e^(-1)/abs(d))))^2*e*sin(1/4*pi + 1/2*real_part(a
rcsin(1/2*b*e^(-1)/abs(d))))*sinh(1/2*imag_part(arcsin(1/2*b*e^(-1)/abs(d))
))^3 + (4*(d^2)^(3/4)*d^2*e^(13/2) - b^2*(d^2)^(3/4)*e^(9/2) - sqrt(-4*d^2*
e^2 + b^2)*b*(d^2)^(3/4)*e^(9/2))*e*sin(1/4*pi + 1/2*real_part(arcsin(1/2*b
*e^(-1)/abs(d))))^3*sinh(1/2*imag_part(arcsin(1/2*b*e^(-1)/abs(d))))^3 - (4
*(d^2)^(1/4)*d^3*e^(15/2) - b^2*(d^2)^(1/4)*d*e^(11/2) - sqrt(-4*d^2*e^2 +
b^2)*b*(d^2)^(1/4)*d*e^(11/2))*cosh(1/2*imag_part(arcsin(1/2*b*e^(-1)/abs(d
))))*sin(1/4*pi + 1/2*real_part(arcsin(1/2*b*e^(-1)/abs(d)))) + (4*(d^2)^(1
/4)*d^3*e^(15/2) - b^2*(d^2)^(1/4)*d*e^(11/2) - sqrt(-4*d^2*e^2 + b^2)*b*(d
^2)^(1/4)*d*e^(11/2))*sin(1/4*pi + 1/2*real_part(arcsin(1/2*b*e^(-1)/abs(d)
))) *sinh(1/2*imag_part(arcsin(1/2*b*e^(-1)/abs(d)))) *arctan(-((d^2)^(1/4)*
cos(1/4*pi + 1/2*arcsin(1/2*b*e^(-1)/abs(d)))e^(-1/2) - x)*e^(1/2)/((d^2)^(
1/4)*sin(1/4*pi + 1/2*arcsin(1/2*b*e^(-1)/abs(d))))/(4*d^4*e^8 - b^2*d^2*
e^6) + 1/4*((4*(d^2)^(3/4)*d^2*e^(13/2) - b^2*(d^2)^(3/4)*e^(9/2) - sqrt(-4
*d^2*e^2 + b^2)*b*(d^2)^(3/4)*e^(9/2))*cos(5/4*pi + 1/2*real_part(arcsin(1/
2*b*e^(-1)/abs(d))))^3*cosh(1/2*imag_part(arcsin(1/2*b*e^(-1)/abs(d))))^3*e
- 3*(4*(d^2)^(3/4)*d^2*e^(13/2) - b^2*(d^2)^(3/4)*e^(9/2) - sqrt(-4*d^2*e^
2 + b^2)*b*(d^2)^(3/4)*e^(9/2))*cos(5/4*pi + 1/2*real_part(arcsin(1/2*b*e^(-
1)/abs(d))))*cosh(1/2*imag_part(arcsin(1/2*b*e^(-1)/abs(d))))^3*e*sin(5/4*
pi + 1/2*real_part(arcsin(1/2*b*e^(-1)/abs(d))))^2 - 3*(4*(d^2)^(3/4)*d^2*e
^(13/2) - b^2*(d^2)^(3/4)*e^(9/2) - sqrt(-4*d^2*e^2 + b^2)*b*(d^2)^(3/4)*e
^(9/2))*cos(5/4*pi + 1/2*real_part(arcsin(1/2*b*e^(-1)/abs(d))))^3*cosh(1/2*
imag_part(arcsin(1/2*b*e^(-1)/abs(d))))^2*e*sinh(1/2*imag_part(arcsin(1/2*b
*e^(-1)/abs(d)))) + 9*(4*(d^2)^(3/4)*d^2*e^(13/2) - b^2*(d^2)^(3/4)*e^(9/2)
- sqrt(-4*d^2*e^2 + b^2)*b*(d^2)^(3/4)*e^(9/2))*cos(5/4*pi + 1/2*real_part
(arcsin(1/2*b*e^(-1)/abs(d))))*cosh(1/2*imag_part(arcsin(1/2*b*e^(-1)/abs(d)
)))^2*e*sin(5/4*pi + 1/2*real_part(arcsin(1/2*b*e^(-1)/abs(d))))^2*sinh(1/
2*imag_part(arcsin(1/2*b*e^(-1)/abs(d)))) + 3*(4*(d^2)^(3/4)*d^2*e^(13/2) -
b^2*(d^2)^(3/4)*e^(9/2) - sqrt(-4*d^2*e^2 + b^2)*b*(d^2)^(3/4)*e^(9/2))*co
s(5/4*pi + 1/2*real_part(arcsin(1/2*b*e^(-1)/abs(d))))^3*cosh(1/2*imag_part
(arcsin(1/2*b*e^(-1)/abs(d))))*e*sinh(1/2*imag_part(arcsin(1/2*b*e^(-1)/abs
(d))))^2 - 9*(4*(d^2)^(3/4)*d^2*e^(13/2) - b^2*(d^2)^(3/4)*e^(9/2) - sqrt(-

```


$$3.31 \quad \int \frac{d-ex^2}{d^2+fx^2+e^2x^4} dx$$

Optimal. Leaf size=78

$$\frac{\log(x\sqrt{2de-f}+d+ex^2)}{2\sqrt{2de-f}} - \frac{\log(-x\sqrt{2de-f}+d+ex^2)}{2\sqrt{2de-f}}$$

[Out] -Log[d - Sqrt[2*d*e - f]*x + e*x^2]/(2*Sqrt[2*d*e - f]) + Log[d + Sqrt[2*d*e - f]*x + e*x^2]/(2*Sqrt[2*d*e - f])

Rubi [A] time = 0.0480425, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {1164, 628}

$$\frac{\log(x\sqrt{2de-f}+d+ex^2)}{2\sqrt{2de-f}} - \frac{\log(-x\sqrt{2de-f}+d+ex^2)}{2\sqrt{2de-f}}$$

Antiderivative was successfully verified.

[In] Int[(d - e*x^2)/(d^2 + f*x^2 + e^2*x^4),x]

[Out] -Log[d - Sqrt[2*d*e - f]*x + e*x^2]/(2*Sqrt[2*d*e - f]) + Log[d + Sqrt[2*d*e - f]*x + e*x^2]/(2*Sqrt[2*d*e - f])

Rule 1164

Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[(-2*d)/e - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && !GtQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rubi steps

$$\begin{aligned} \int \frac{d-ex^2}{d^2+fx^2+e^2x^4} dx &= -\frac{\int \frac{\frac{\sqrt{2de-f}+2x}{e}}{-\frac{d}{e}-\frac{\sqrt{2de-f}x}{e}-x^2} dx}{2\sqrt{2de-f}} - \frac{\int \frac{\frac{\sqrt{2de-f}-2x}{e}}{-\frac{d}{e}+\frac{\sqrt{2de-f}x}{e}-x^2} dx}{2\sqrt{2de-f}} \\ &= -\frac{\log(d-\sqrt{2de-f}x+ex^2)}{2\sqrt{2de-f}} + \frac{\log(d+\sqrt{2de-f}x+ex^2)}{2\sqrt{2de-f}} \end{aligned}$$

Mathematica [B] time = 0.129162, size = 182, normalized size = 2.33

$$\frac{(-\sqrt{f^2-4d^2e^2+2de+f}) \tan^{-1}\left(\frac{\sqrt{2ex}}{\sqrt{f-\sqrt{f^2-4d^2e^2}}}\right) + (\sqrt{f^2-4d^2e^2+2de+f}) \tan^{-1}\left(\frac{\sqrt{2ex}}{\sqrt{\sqrt{f^2-4d^2e^2}+f}}\right)}{\sqrt{f-\sqrt{f^2-4d^2e^2}} \sqrt{\sqrt{f^2-4d^2e^2}+f}}$$

Antiderivative was successfully verified.

[In] Integrate[(d - e*x^2)/(d^2 + f*x^2 + e^2*x^4),x]

[Out] (((2*d*e + f - Sqrt[-4*d^2*e^2 + f^2])*ArcTan[(Sqrt[2]*e*x)/Sqrt[f - Sqrt[-4*d^2*e^2 + f^2]]])/Sqrt[f - Sqrt[-4*d^2*e^2 + f^2]] - ((2*d*e + f + Sqrt[-4*d^2*e^2 + f^2])*ArcTan[(Sqrt[2]*e*x)/Sqrt[f + Sqrt[-4*d^2*e^2 + f^2]]])/Sqrt[f + Sqrt[-4*d^2*e^2 + f^2]])/(Sqrt[2]*Sqrt[-4*d^2*e^2 + f^2])

Maple [A] time = 0.173, size = 69, normalized size = 0.9

$$\frac{1}{2} \ln(d + ex^2 + x\sqrt{2de-f}) \frac{1}{\sqrt{2de-f}} - \frac{1}{2} \ln(-ex^2 + x\sqrt{2de-f} - d) \frac{1}{\sqrt{2de-f}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-e*x^2+d)/(e^2*x^4+f*x^2+d^2),x)

[Out] 1/2*ln(d+e*x^2+x*(2*d*e-f)^(1/2))/(2*d*e-f)^(1/2)-1/2/(2*d*e-f)^(1/2)*ln(-e*x^2+x*(2*d*e-f)^(1/2)-d)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{ex^2 - d}{e^2x^4 + fx^2 + d^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e*x^2+d)/(e^2*x^4+f*x^2+d^2),x, algorithm="maxima")

[Out] -integrate((e*x^2 - d)/(e^2*x^4 + f*x^2 + d^2), x)

Fricas [A] time = 1.28469, size = 379, normalized size = 4.86

$$\left[\frac{\log\left(\frac{e^2x^4+(4de-f)x^2+d^2+2(ex^3+dx)\sqrt{2de-f}}{e^2x^4+fx^2+d^2}\right)}{2\sqrt{2de-f}}, \frac{\sqrt{-2de+f} \arctan\left(-\frac{\sqrt{-2de+fx}}{2de-f}\right) - \sqrt{-2de+f} \arctan\left(-\frac{(e^2x^3-(de-f)x)\sqrt{-2de+f}}{2d^2e-df}\right)}{2de-f} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e*x^2+d)/(e^2*x^4+f*x^2+d^2),x, algorithm="fricas")

[Out] [1/2*log((e^2*x^4 + (4*d*e - f)*x^2 + d^2 + 2*(e*x^3 + d*x)*sqrt(2*d*e - f))/(e^2*x^4 + f*x^2 + d^2))/sqrt(2*d*e - f), (sqrt(-2*d*e + f)*arctan(-sqrt(-2*d*e + f)*e*x/(2*d*e - f)) - sqrt(-2*d*e + f)*arctan(-(e^2*x^3 - (d*e - f)*x)*sqrt(-2*d*e + f)/(2*d^2*e - d*f)))/(2*d*e - f)]

Sympy [A] time = 0.599116, size = 110, normalized size = 1.41

$$\frac{\sqrt{\frac{1}{2de-f}} \log\left(\frac{d}{e} + x^2 + \frac{x\left(-2de\sqrt{\frac{1}{2de-f}} + f\sqrt{\frac{1}{2de-f}}\right)}{e}\right)}{2} + \frac{\sqrt{\frac{1}{2de-f}} \log\left(\frac{d}{e} + x^2 + \frac{x\left(2de\sqrt{\frac{1}{2de-f}} - f\sqrt{\frac{1}{2de-f}}\right)}{e}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e*x**2+d)/(e**2*x**4+f*x**2+d**2),x)

[Out] -sqrt(1/(2*d*e - f))*log(d/e + x**2 + x*(-2*d*e*sqrt(1/(2*d*e - f)) + f*sqrt(1/(2*d*e - f)))/e)/2 + sqrt(1/(2*d*e - f))*log(d/e + x**2 + x*(2*d*e*sqrt(1/(2*d*e - f)) - f*sqrt(1/(2*d*e - f)))/e)/2

Giac [C] time = 1.63378, size = 5779, normalized size = 74.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e*x^2+d)/(e^2*x^4+f*x^2+d^2),x, algorithm="giac")

[Out]
$$\begin{aligned} & -1/2*(3*(4*(d^2)^{(3/4)}*d^2*e^{(13/2)} - (d^2)^{(3/4)}*f^2*e^{(9/2)} - \sqrt{-4*d^2 \\ & *e^2 + f^2}*(d^2)^{(3/4)}*f*e^{(9/2)})*\cos(5/4*\pi + 1/2*\text{real_part}(\arcsin(1/2*f* \\ & e^{(-1)/\text{abs}(d)})))^2*\cosh(1/2*\text{imag_part}(\arcsin(1/2*f*e^{(-1)/\text{abs}(d)})))^3*e*\sin \\ & (5/4*\pi + 1/2*\text{real_part}(\arcsin(1/2*f*e^{(-1)/\text{abs}(d)}))) - (4*(d^2)^{(3/4)}*d^2* \\ & e^{(13/2)} - (d^2)^{(3/4)}*f^2*e^{(9/2)} - \sqrt{-4*d^2*e^2 + f^2}*(d^2)^{(3/4)}*f*e \\ & ^{(9/2)})*\cosh(1/2*\text{imag_part}(\arcsin(1/2*f*e^{(-1)/\text{abs}(d)})))^3*e*\sin(5/4*\pi + 1 \\ & /2*\text{real_part}(\arcsin(1/2*f*e^{(-1)/\text{abs}(d)})))^3 - 9*(4*(d^2)^{(3/4)}*d^2*e^{(13/2)} \\ &) - (d^2)^{(3/4)}*f^2*e^{(9/2)} - \sqrt{-4*d^2*e^2 + f^2}*(d^2)^{(3/4)}*f*e^{(9/2)}) \\ & *\cos(5/4*\pi + 1/2*\text{real_part}(\arcsin(1/2*f*e^{(-1)/\text{abs}(d)})))^2*\cosh(1/2*\text{imag_p} \\ & \text{art}(\arcsin(1/2*f*e^{(-1)/\text{abs}(d)})))^2*e*\sin(5/4*\pi + 1/2*\text{real_part}(\arcsin(1/2 \\ & *f*e^{(-1)/\text{abs}(d)})))*\sinh(1/2*\text{imag_part}(\arcsin(1/2*f*e^{(-1)/\text{abs}(d)}))) + 3*(4 \\ & *(d^2)^{(3/4)}*d^2*e^{(13/2)} - (d^2)^{(3/4)}*f^2*e^{(9/2)} - \sqrt{-4*d^2*e^2 + f^2} \\ &)*(d^2)^{(3/4)}*f*e^{(9/2)})*\cosh(1/2*\text{imag_part}(\arcsin(1/2*f*e^{(-1)/\text{abs}(d)})))^2 \\ & *e*\sin(5/4*\pi + 1/2*\text{real_part}(\arcsin(1/2*f*e^{(-1)/\text{abs}(d)})))^3*\sinh(1/2*\text{imag} \\ & _part(\arcsin(1/2*f*e^{(-1)/\text{abs}(d)}))) + 9*(4*(d^2)^{(3/4)}*d^2*e^{(13/2)} - (d^2) \\ & ^{(3/4)}*f^2*e^{(9/2)} - \sqrt{-4*d^2*e^2 + f^2}*(d^2)^{(3/4)}*f*e^{(9/2)})*\cos(5/4* \\ & \pi + 1/2*\text{real_part}(\arcsin(1/2*f*e^{(-1)/\text{abs}(d)})))^2*\cosh(1/2*\text{imag_part}(\arcsi \\ & \text{in}(1/2*f*e^{(-1)/\text{abs}(d)})))*e*\sin(5/4*\pi + 1/2*\text{real_part}(\arcsin(1/2*f*e^{(-1)/a} \\ & \text{bs}(d))))*\sinh(1/2*\text{imag_part}(\arcsin(1/2*f*e^{(-1)/\text{abs}(d)})))^2 - 3*(4*(d^2)^{(3} \\ & /4)*d^2*e^{(13/2)} - (d^2)^{(3/4)}*f^2*e^{(9/2)} - \sqrt{-4*d^2*e^2 + f^2}*(d^2)^{(\\ & 3/4)}*f*e^{(9/2)})*\cosh(1/2*\text{imag_part}(\arcsin(1/2*f*e^{(-1)/\text{abs}(d)})))*e*\sin(5/4* \\ & \pi + 1/2*\text{real_part}(\arcsin(1/2*f*e^{(-1)/\text{abs}(d)})))^3*\sinh(1/2*\text{imag_part}(\arcsi \\ & \text{in}(1/2*f*e^{(-1)/\text{abs}(d)})))^2 - 3*(4*(d^2)^{(3/4)}*d^2*e^{(13/2)} - (d^2)^{(3/4)}*f^ \\ & 2*e^{(9/2)} - \sqrt{-4*d^2*e^2 + f^2}*(d^2)^{(3/4)}*f*e^{(9/2)})*\cos(5/4*\pi + 1/2* \\ & \text{real_part}(\arcsin(1/2*f*e^{(-1)/\text{abs}(d)})))^2*e*\sin(5/4*\pi + 1/2*\text{real_part}(\arcs \\ & \text{in}(1/2*f*e^{(-1)/\text{abs}(d)})))*\sinh(1/2*\text{imag_part}(\arcsin(1/2*f*e^{(-1)/\text{abs}(d)})))^ \\ & 3 + (4*(d^2)^{(3/4)}*d^2*e^{(13/2)} - (d^2)^{(3/4)}*f^2*e^{(9/2)} - \sqrt{-4*d^2*e^2 \\ & + f^2}*(d^2)^{(3/4)}*f*e^{(9/2)})*e*\sin(5/4*\pi + 1/2*\text{real_part}(\arcsin(1/2*f*e^ \\ & ^{(-1)/\text{abs}(d)})))^3*\sinh(1/2*\text{imag_part}(\arcsin(1/2*f*e^{(-1)/\text{abs}(d)})))^3 - (4*(d \\ & ^2)^{(1/4)}*d^3*e^{(15/2)} - (d^2)^{(1/4)}*d*f^2*e^{(11/2)} - \sqrt{-4*d^2*e^2 + f^2} \\ &)*(d^2)^{(1/4)}*d*f*e^{(11/2)})*\cosh(1/2*\text{imag_part}(\arcsin(1/2*f*e^{(-1)/\text{abs}(d)})) \\ &)*\sin(5/4*\pi + 1/2*\text{real_part}(\arcsin(1/2*f*e^{(-1)/\text{abs}(d)}))) + (4*(d^2)^{(1/4)} \\ & *d^3*e^{(15/2)} - (d^2)^{(1/4)}*d*f^2*e^{(11/2)} - \sqrt{-4*d^2*e^2 + f^2}*(d^2)^{(\\ & 1/4)}*d*f*e^{(11/2)})*\sin(5/4*\pi + 1/2*\text{real_part}(\arcsin(1/2*f*e^{(-1)/\text{abs}(d)}))) \end{aligned}$$

```

*sinh(1/2*imag_part(arcsin(1/2*f*e^(-1)/abs(d))))*arctan(-((d^2)^(1/4)*cos
(5/4*pi + 1/2*arcsin(1/2*f*e^(-1)/abs(d)))e^(-1/2) - x)*e^(1/2)/((d^2)^(1/
4)*sin(5/4*pi + 1/2*arcsin(1/2*f*e^(-1)/abs(d))))/(4*d^4*e^8 - d^2*f^2*e^6
) - 1/2*(3*(4*(d^2)^(3/4)*d^2*e^(13/2) - (d^2)^(3/4)*f^2*e^(9/2) - sqrt(-4*
d^2*e^2 + f^2)*(d^2)^(3/4)*f*e^(9/2))*cos(1/4*pi + 1/2*real_part(arcsin(1/2
*f*e^(-1)/abs(d))))^2*cosh(1/2*imag_part(arcsin(1/2*f*e^(-1)/abs(d))))^3*e*
sin(1/4*pi + 1/2*real_part(arcsin(1/2*f*e^(-1)/abs(d)))) - (4*(d^2)^(3/4)*d
^2*e^(13/2) - (d^2)^(3/4)*f^2*e^(9/2) - sqrt(-4*d^2*e^2 + f^2)*(d^2)^(3/4)*
f*e^(9/2))*cosh(1/2*imag_part(arcsin(1/2*f*e^(-1)/abs(d))))^3*e*sin(1/4*pi
+ 1/2*real_part(arcsin(1/2*f*e^(-1)/abs(d))))^3 - 9*(4*(d^2)^(3/4)*d^2*e^(1
3/2) - (d^2)^(3/4)*f^2*e^(9/2) - sqrt(-4*d^2*e^2 + f^2)*(d^2)^(3/4)*f*e^(9/
2))*cos(1/4*pi + 1/2*real_part(arcsin(1/2*f*e^(-1)/abs(d))))^2*cosh(1/2*ima
g_part(arcsin(1/2*f*e^(-1)/abs(d))))^2*e*sin(1/4*pi + 1/2*real_part(arcsin(
1/2*f*e^(-1)/abs(d))))*sinh(1/2*imag_part(arcsin(1/2*f*e^(-1)/abs(d)))) + 3
*(4*(d^2)^(3/4)*d^2*e^(13/2) - (d^2)^(3/4)*f^2*e^(9/2) - sqrt(-4*d^2*e^2 +
f^2)*(d^2)^(3/4)*f*e^(9/2))*cosh(1/2*imag_part(arcsin(1/2*f*e^(-1)/abs(d)))
)^2*e*sin(1/4*pi + 1/2*real_part(arcsin(1/2*f*e^(-1)/abs(d))))^3*sinh(1/2*i
mag_part(arcsin(1/2*f*e^(-1)/abs(d)))) + 9*(4*(d^2)^(3/4)*d^2*e^(13/2) - (d
^2)^(3/4)*f^2*e^(9/2) - sqrt(-4*d^2*e^2 + f^2)*(d^2)^(3/4)*f*e^(9/2))*cos(1
/4*pi + 1/2*real_part(arcsin(1/2*f*e^(-1)/abs(d))))^2*cosh(1/2*imag_part(ar
csin(1/2*f*e^(-1)/abs(d))))*e*sin(1/4*pi + 1/2*real_part(arcsin(1/2*f*e^(-1)
)/abs(d))))*sinh(1/2*imag_part(arcsin(1/2*f*e^(-1)/abs(d))))^2 - 3*(4*(d^2)
^(3/4)*d^2*e^(13/2) - (d^2)^(3/4)*f^2*e^(9/2) - sqrt(-4*d^2*e^2 + f^2)*(d^2
)^(3/4)*f*e^(9/2))*cosh(1/2*imag_part(arcsin(1/2*f*e^(-1)/abs(d))))*e*sin(1
/4*pi + 1/2*real_part(arcsin(1/2*f*e^(-1)/abs(d))))^3*sinh(1/2*imag_part(ar
csin(1/2*f*e^(-1)/abs(d))))^2 - 3*(4*(d^2)^(3/4)*d^2*e^(13/2) - (d^2)^(3/4)
*f^2*e^(9/2) - sqrt(-4*d^2*e^2 + f^2)*(d^2)^(3/4)*f*e^(9/2))*cos(1/4*pi + 1
/2*real_part(arcsin(1/2*f*e^(-1)/abs(d))))^2*e*sin(1/4*pi + 1/2*real_part(a
rcsin(1/2*f*e^(-1)/abs(d))))*sinh(1/2*imag_part(arcsin(1/2*f*e^(-1)/abs(d))
))^3 + (4*(d^2)^(3/4)*d^2*e^(13/2) - (d^2)^(3/4)*f^2*e^(9/2) - sqrt(-4*d^2*
e^2 + f^2)*(d^2)^(3/4)*f*e^(9/2))*e*sin(1/4*pi + 1/2*real_part(arcsin(1/2*f
*e^(-1)/abs(d))))^3*sinh(1/2*imag_part(arcsin(1/2*f*e^(-1)/abs(d))))^3 - (4
*(d^2)^(1/4)*d^3*e^(15/2) - (d^2)^(1/4)*d*f^2*e^(11/2) - sqrt(-4*d^2*e^2 +
f^2)*(d^2)^(1/4)*d*f*e^(11/2))*cosh(1/2*imag_part(arcsin(1/2*f*e^(-1)/abs(d)
))) *sin(1/4*pi + 1/2*real_part(arcsin(1/2*f*e^(-1)/abs(d)))) + (4*(d^2)^(1
/4)*d^3*e^(15/2) - (d^2)^(1/4)*d*f^2*e^(11/2) - sqrt(-4*d^2*e^2 + f^2)*(d^2
)^(1/4)*d*f*e^(11/2))*sin(1/4*pi + 1/2*real_part(arcsin(1/2*f*e^(-1)/abs(d)
))) *sinh(1/2*imag_part(arcsin(1/2*f*e^(-1)/abs(d)))) *arctan(-((d^2)^(1/4)*
cos(1/4*pi + 1/2*arcsin(1/2*f*e^(-1)/abs(d)))e^(-1/2) - x)*e^(1/2)/((d^2)^(
1/4)*sin(1/4*pi + 1/2*arcsin(1/2*f*e^(-1)/abs(d))))/(4*d^4*e^8 - d^2*f^2*
e^6) + 1/4*((4*(d^2)^(3/4)*d^2*e^(13/2) - (d^2)^(3/4)*f^2*e^(9/2) - sqrt(-4
*d^2*e^2 + f^2)*(d^2)^(3/4)*f*e^(9/2))*cos(5/4*pi + 1/2*real_part(arcsin(1/
2*f*e^(-1)/abs(d))))^3*cosh(1/2*imag_part(arcsin(1/2*f*e^(-1)/abs(d))))^3*e
- 3*(4*(d^2)^(3/4)*d^2*e^(13/2) - (d^2)^(3/4)*f^2*e^(9/2) - sqrt(-4*d^2*e^
2 + f^2)*(d^2)^(3/4)*f*e^(9/2))*cos(5/4*pi + 1/2*real_part(arcsin(1/2*f*e^(-
1)/abs(d))))*cosh(1/2*imag_part(arcsin(1/2*f*e^(-1)/abs(d))))^3*e*sin(5/4*
pi + 1/2*real_part(arcsin(1/2*f*e^(-1)/abs(d))))^2 - 3*(4*(d^2)^(3/4)*d^2*e
^(13/2) - (d^2)^(3/4)*f^2*e^(9/2) - sqrt(-4*d^2*e^2 + f^2)*(d^2)^(3/4)*f*e^
(9/2))*cos(5/4*pi + 1/2*real_part(arcsin(1/2*f*e^(-1)/abs(d))))^3*cosh(1/2*
imag_part(arcsin(1/2*f*e^(-1)/abs(d))))^2*e*sinh(1/2*imag_part(arcsin(1/2*f
*e^(-1)/abs(d)))) + 9*(4*(d^2)^(3/4)*d^2*e^(13/2) - (d^2)^(3/4)*f^2*e^(9/2)
- sqrt(-4*d^2*e^2 + f^2)*(d^2)^(3/4)*f*e^(9/2))*cos(5/4*pi + 1/2*real_part
(arcsin(1/2*f*e^(-1)/abs(d))))*cosh(1/2*imag_part(arcsin(1/2*f*e^(-1)/abs(d)
))))^2*e*sin(5/4*pi + 1/2*real_part(arcsin(1/2*f*e^(-1)/abs(d))))^2*sinh(1/
2*imag_part(arcsin(1/2*f*e^(-1)/abs(d)))) + 3*(4*(d^2)^(3/4)*d^2*e^(13/2) -
(d^2)^(3/4)*f^2*e^(9/2) - sqrt(-4*d^2*e^2 + f^2)*(d^2)^(3/4)*f*e^(9/2))*co
s(5/4*pi + 1/2*real_part(arcsin(1/2*f*e^(-1)/abs(d))))^3*cosh(1/2*imag_part
(arcsin(1/2*f*e^(-1)/abs(d))))*e*sinh(1/2*imag_part(arcsin(1/2*f*e^(-1)/abs
(d))))^2 - 9*(4*(d^2)^(3/4)*d^2*e^(13/2) - (d^2)^(3/4)*f^2*e^(9/2) - sqrt(-

```


$$3.32 \quad \int \frac{d-ex^2}{d^2-bx^2+e^2x^4} dx$$

Optimal. Leaf size=70

$$\frac{\log(x\sqrt{b+2de}+d+ex^2)}{2\sqrt{b+2de}} - \frac{\log(-x\sqrt{b+2de}+d+ex^2)}{2\sqrt{b+2de}}$$

[Out] -Log[d - Sqrt[b + 2*d*e]*x + e*x^2]/(2*Sqrt[b + 2*d*e]) + Log[d + Sqrt[b + 2*d*e]*x + e*x^2]/(2*Sqrt[b + 2*d*e])

Rubi [A] time = 0.0436809, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {1164, 628}

$$\frac{\log(x\sqrt{b+2de}+d+ex^2)}{2\sqrt{b+2de}} - \frac{\log(-x\sqrt{b+2de}+d+ex^2)}{2\sqrt{b+2de}}$$

Antiderivative was successfully verified.

[In] Int[(d - e*x^2)/(d^2 - b*x^2 + e^2*x^4), x]

[Out] -Log[d - Sqrt[b + 2*d*e]*x + e*x^2]/(2*Sqrt[b + 2*d*e]) + Log[d + Sqrt[b + 2*d*e]*x + e*x^2]/(2*Sqrt[b + 2*d*e])

Rule 1164

Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[(-2*d)/e - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && !GtQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rubi steps

$$\begin{aligned} \int \frac{d-ex^2}{d^2-bx^2+e^2x^4} dx &= -\frac{\int \frac{\frac{\sqrt{b+2de}}{e}+2x}{-\frac{d}{e}-\frac{\sqrt{b+2de}x}{e}-x^2} dx}{2\sqrt{b+2de}} - \frac{\int \frac{\frac{\sqrt{b+2de}}{e}-2x}{-\frac{d}{e}+\frac{\sqrt{b+2de}x}{e}-x^2} dx}{2\sqrt{b+2de}} \\ &= -\frac{\log(d-\sqrt{b+2de}+ex^2)}{2\sqrt{b+2de}} + \frac{\log(d+\sqrt{b+2de}+ex^2)}{2\sqrt{b+2de}} \end{aligned}$$

Mathematica [B] time = 0.133923, size = 190, normalized size = 2.71

$$\frac{(-\sqrt{b^2-4d^2e^2+b-2de}) \tan^{-1}\left(\frac{\sqrt{2ex}}{\sqrt{b^2-4d^2e^2-b}}\right) - (\sqrt{b^2-4d^2e^2+b-2de}) \tan^{-1}\left(\frac{\sqrt{2ex}}{\sqrt{-\sqrt{b^2-4d^2e^2-b}}}\right)}{\sqrt{\sqrt{b^2-4d^2e^2-b}} \sqrt{-\sqrt{b^2-4d^2e^2-b}}}$$

$$\frac{\sqrt{2}\sqrt{b^2-4d^2e^2}}{\sqrt{\sqrt{b^2-4d^2e^2-b}} \sqrt{-\sqrt{b^2-4d^2e^2-b}}}$$

Antiderivative was successfully verified.

[In] Integrate[(d - e*x^2)/(d^2 - b*x^2 + e^2*x^4),x]

[Out]
$$\frac{-\left(\left(b - 2de + \sqrt{b^2 - 4d^2e^2}\right) \operatorname{ArcTan}\left[\frac{\sqrt{2}ex}{\sqrt{-b - \sqrt{b^2 - 4d^2e^2}}}\right]\right) + \left(\left(b - 2de - \sqrt{b^2 - 4d^2e^2}\right) \operatorname{ArcTan}\left[\frac{\sqrt{2}ex}{\sqrt{-b + \sqrt{b^2 - 4d^2e^2}}}\right]\right)}{\sqrt{-b + \sqrt{b^2 - 4d^2e^2}}}$$

Maple [A] time = 0.18, size = 61, normalized size = 0.9

$$\frac{1}{2} \ln\left(d + ex^2 + x\sqrt{2de + b}\right) \frac{1}{\sqrt{2de + b}} - \frac{1}{2} \ln\left(-ex^2 + x\sqrt{2de + b} - d\right) \frac{1}{\sqrt{2de + b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-e*x^2+d)/(e^2*x^4-b*x^2+d^2),x)

[Out] $\frac{1}{2} \ln(d + ex^2 + x(2de + b)^{1/2}) / (2de + b)^{1/2} - \frac{1}{2} \ln(-ex^2 + x(2de + b)^{1/2} - d) / (2de + b)^{1/2}$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{ex^2 - d}{e^2x^4 - bx^2 + d^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e*x^2+d)/(e^2*x^4-b*x^2+d^2),x, algorithm="maxima")

[Out] -integrate((e*x^2 - d)/(e^2*x^4 - b*x^2 + d^2), x)

Fricas [A] time = 1.40936, size = 378, normalized size = 5.4

$$\left[\frac{\log\left(\frac{e^2x^4 + (4de+b)x^2 + d^2 + 2(ex^3 + dx)\sqrt{2de+b}}{e^2x^4 - bx^2 + d^2}\right)}{2\sqrt{2de+b}}, -\frac{\sqrt{-2de-b} \arctan\left(\frac{\sqrt{-2de-b}ex}{2de+b}\right) - \sqrt{-2de-b} \arctan\left(\frac{(e^2x^3 - (de+b)x)\sqrt{-2de-b}}{2d^2e + bd}\right)}{2de+b} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e*x^2+d)/(e^2*x^4-b*x^2+d^2),x, algorithm="fricas")

[Out] $\frac{1}{2} \log((e^2x^4 + (4de + b)x^2 + d^2 + 2(ex^3 + dx)\sqrt{2de + b}) / (e^2x^4 - bx^2 + d^2)) / \sqrt{2de + b}, -(\sqrt{-2de - b}) \arctan(\sqrt{-2de - b}ex / (2de + b)) - \sqrt{-2de - b} \arctan((e^2x^3 - (de + b)x)\sqrt{-2de - b} / (2d^2e + bd)) / (2de + b)$

Sympy [A] time = 0.606013, size = 112, normalized size = 1.6

$$\frac{\sqrt{\frac{1}{b+2de}} \log\left(\frac{d}{e} + x^2 + \frac{x\left(-b\sqrt{\frac{1}{b+2de}} - 2de\sqrt{\frac{1}{b+2de}}\right)}{e}\right)}{2} + \frac{\sqrt{\frac{1}{b+2de}} \log\left(\frac{d}{e} + x^2 + \frac{x\left(b\sqrt{\frac{1}{b+2de}} + 2de\sqrt{\frac{1}{b+2de}}\right)}{e}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e*x**2+d)/(e**2*x**4-b*x**2+d**2),x)

[Out] -sqrt(1/(b + 2*d*e))*log(d/e + x**2 + x*(-b*sqrt(1/(b + 2*d*e)) - 2*d*e*sqrt(1/(b + 2*d*e)))/e)/2 + sqrt(1/(b + 2*d*e))*log(d/e + x**2 + x*(b*sqrt(1/(b + 2*d*e)) + 2*d*e*sqrt(1/(b + 2*d*e)))/e)/2

Giac [C] time = 1.60893, size = 5387, normalized size = 76.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e*x^2+d)/(e^2*x^4-b*x^2+d^2),x, algorithm="giac")

[Out]
$$\begin{aligned} & -1/2*(3*(4*(d^2)^{(3/4)}*d^2*e^{(13/2)} - b^2*(d^2)^{(3/4)}*e^{(9/2)} + \sqrt{-4*d^2 \\ & *e^2 + b^2}*b*(d^2)^{(3/4)}*e^{(9/2)})*\cos(1/2*\text{real_part}(\arccos(1/2*b*e^{(-1)}/\text{abs}(d))))^2 \\ & *\cosh(1/2*\text{imag_part}(\arccos(1/2*b*e^{(-1)}/\text{abs}(d))))^3*e*\sin(1/2*\text{real_part} \\ & (\arccos(1/2*b*e^{(-1)}/\text{abs}(d)))) - (4*(d^2)^{(3/4)}*d^2*e^{(13/2)} - b^2*(d^2)^{(3/4)}*e^{(9/2)} \\ & + \sqrt{-4*d^2*e^2 + b^2}*b*(d^2)^{(3/4)}*e^{(9/2)})*\cosh(1/2*\text{imag_part}(\arccos(1/2*b*e^{(-1)}/\text{abs}(d))))^3 \\ & *e*\sin(1/2*\text{real_part}(\arccos(1/2*b*e^{(-1)}/\text{abs}(d))))^3 - 9*(4*(d^2)^{(3/4)}*d^2*e^{(13/2)} - b^2*(d^2)^{(3/4)}*e^{(9/2)} \\ & + \sqrt{-4*d^2*e^2 + b^2}*b*(d^2)^{(3/4)}*e^{(9/2)})*\cos(1/2*\text{real_part}(\arccos(1/2*b*e^{(-1)}/\text{abs}(d))))^2 \\ & *\cosh(1/2*\text{imag_part}(\arccos(1/2*b*e^{(-1)}/\text{abs}(d))))^2*e*\sin(1/2*\text{real_part}(\arccos(1/2*b*e^{(-1)}/\text{abs}(d)))) \\ & *\sinh(1/2*\text{imag_part}(\arccos(1/2*b*e^{(-1)}/\text{abs}(d)))) + 3*(4*(d^2)^{(3/4)}*d^2*e^{(13/2)} - b^2*(d^2)^{(3/4)}*e^{(9/2)} \\ & + \sqrt{-4*d^2*e^2 + b^2}*b*(d^2)^{(3/4)}*e^{(9/2)})*\cosh(1/2*\text{imag_part}(\arccos(1/2*b*e^{(-1)}/\text{abs}(d))))^2 \\ & *e*\sin(1/2*\text{real_part}(\arccos(1/2*b*e^{(-1)}/\text{abs}(d))))^3*\sinh(1/2*\text{imag_part}(\arccos(1/2*b*e^{(-1)}/\text{abs}(d)))) \\ & + 9*(4*(d^2)^{(3/4)}*d^2*e^{(13/2)} - b^2*(d^2)^{(3/4)}*e^{(9/2)} + \sqrt{-4*d^2*e^2 + b^2}*b*(d^2)^{(3/4)}*e^{(9/2)}) \\ & *\cos(1/2*\text{real_part}(\arccos(1/2*b*e^{(-1)}/\text{abs}(d))))^2*\cosh(1/2*\text{imag_part}(\arccos(1/2*b*e^{(-1)}/\text{abs}(d)))) \\ & *e*\sin(1/2*\text{real_part}(\arccos(1/2*b*e^{(-1)}/\text{abs}(d))))^3*\sinh(1/2*\text{imag_part}(\arccos(1/2*b*e^{(-1)}/\text{abs}(d))))^2 \\ & - 3*(4*(d^2)^{(3/4)}*d^2*e^{(13/2)} - b^2*(d^2)^{(3/4)}*e^{(9/2)} + \sqrt{-4*d^2*e^2 + b^2}*b*(d^2)^{(3/4)}*e^{(9/2)}) \\ & *\cos(1/2*\text{real_part}(\arccos(1/2*b*e^{(-1)}/\text{abs}(d))))^2*e*\sin(1/2*\text{real_part}(\arccos(1/2*b*e^{(-1)}/\text{abs}(d)))) \\ & *\sinh(1/2*\text{imag_part}(\arccos(1/2*b*e^{(-1)}/\text{abs}(d))))^3 + (4*(d^2)^{(3/4)}*d^2*e^{(13/2)} - b^2*(d^2)^{(3/4)}*e^{(9/2)} \\ & + \sqrt{-4*d^2*e^2 + b^2}*b*(d^2)^{(3/4)}*e^{(9/2)})*e*\sin(1/2*\text{real_part}(\arccos(1/2*b*e^{(-1)}/\text{abs}(d))))^3 \\ & *\sinh(1/2*\text{imag_part}(\arccos(1/2*b*e^{(-1)}/\text{abs}(d))))^3 - (4*(d^2)^{(1/4)}*d^3*e^{(15/2)} - b^2*(d^2)^{(1/4)}*d*e^{(11/2)} \\ & + \sqrt{-4*d^2*e^2 + b^2}*b*(d^2)^{(1/4)}*d*e^{(11/2)})*\cosh(1/2*\text{imag_part}(\arccos(1/2*b*e^{(-1)}/\text{abs}(d)))) \\ & *\sin(1/2*\text{real_part}(\arccos(1/2*b*e^{(-1)}/\text{abs}(d)))) + (4*(d^2)^{(1/4)}*d^3*e^{(15/2)} - b^2*(d^2)^{(1/4)}*d*e^{(11/2)} \\ & + \sqrt{-4*d^2*e^2 + b^2}*b*(d^2)^{(1/4)}*d*e^{(11/2)})*\sin(1/2*\text{real_part}(\arccos(1/2*b*e^{(-1)}/\text{abs}(d)))) \\ & *\sinh(1/2*\text{imag_part}(\arccos(1/2*b*e^{(-1)}/\text{abs}(d)))))*\arctan(((d^2)^{(1/4)}*\cos(1/2*\arccos(1/2*b*e^{(-1)}/\text{abs}(d))))*e^{(-1/2)} + x)*e^{(1} \end{aligned}$$

$$\begin{aligned}
& \text{real_part}(\arccos(1/2*b*e^{(-1)/\text{abs}(d)}))^{3}*e*\sinh(1/2*\text{imag_part}(\arccos(1/2*b \\
& *e^{(-1)/\text{abs}(d)}))^{3} + 3*(4*(d^2)^{(3/4)}*d^2*e^{(13/2)} - b^2*(d^2)^{(3/4)}*e^{(9/ \\
& 2) + \sqrt{-4*d^2*e^2 + b^2}*b*(d^2)^{(3/4)}*e^{(9/2)})*\cos(1/2*\text{real_part}(\arccos \\
& (1/2*b*e^{(-1)/\text{abs}(d)})))*e*\sin(1/2*\text{real_part}(\arccos(1/2*b*e^{(-1)/\text{abs}(d)}))^{2} \\
& *\sinh(1/2*\text{imag_part}(\arccos(1/2*b*e^{(-1)/\text{abs}(d)}))^{3} - (4*(d^2)^{(1/4)}*d^3*e^{ \\
& (15/2) - b^2*(d^2)^{(1/4)}*d*e^{(11/2)} + \sqrt{-4*d^2*e^2 + b^2}*b*(d^2)^{(1/4)}* \\
& d*e^{(11/2)})*\cos(1/2*\text{real_part}(\arccos(1/2*b*e^{(-1)/\text{abs}(d)})))*\cosh(1/2*\text{imag_p} \\
& \text{art}(\arccos(1/2*b*e^{(-1)/\text{abs}(d)})) + (4*(d^2)^{(1/4)}*d^3*e^{(15/2)} - b^2*(d^2) \\
& ^{(1/4)}*d*e^{(11/2)} + \sqrt{-4*d^2*e^2 + b^2}*b*(d^2)^{(1/4)}*d*e^{(11/2)})*\cos(1/ \\
& 2*\text{real_part}(\arccos(1/2*b*e^{(-1)/\text{abs}(d)})))*\sinh(1/2*\text{imag_part}(\arccos(1/2*b*e \\
& ^{(-1)/\text{abs}(d)})))*\log(2*(d^2)^{(1/4)}*x*\cos(1/2*\arccos(1/2*b*e^{(-1)/\text{abs}(d)}))*e \\
& ^{(-1/2)} + x^2 + \sqrt{d^2}*e^{(-1)})/(4*d^4*e^8 - b^2*d^2*e^6) + 1/4*((4*(d^2) \\
& ^{(3/4)}*d^2*e^{(13/2)} - b^2*(d^2)^{(3/4)}*e^{(9/2)} + \sqrt{-4*d^2*e^2 + b^2}*b*(d \\
& ^2)^{(3/4)}*e^{(9/2)})*\cos(1/2*\text{real_part}(\arccos(1/2*b*e^{(-1)/\text{abs}(d)}))^{3}*\cosh(1 \\
& /2*\text{imag_part}(\arccos(1/2*b*e^{(-1)/\text{abs}(d)}))^{3}*e - 3*(4*(d^2)^{(3/4)}*d^2*e^{(13 \\
& /2) - b^2*(d^2)^{(3/4)}*e^{(9/2)} + \sqrt{-4*d^2*e^2 + b^2}*b*(d^2)^{(3/4)}*e^{(9/2) \\
& })*\cos(1/2*\text{real_part}(\arccos(1/2*b*e^{(-1)/\text{abs}(d)})))*\cosh(1/2*\text{imag_part}(\arcco \\
& s(1/2*b*e^{(-1)/\text{abs}(d)}))^{3}*e*\sin(1/2*\text{real_part}(\arccos(1/2*b*e^{(-1)/\text{abs}(d)})) \\
&)^{2} - 3*(4*(d^2)^{(3/4)}*d^2*e^{(13/2)} - b^2*(d^2)^{(3/4)}*e^{(9/2)} + \sqrt{-4*d^2 \\
& *e^2 + b^2}*b*(d^2)^{(3/4)}*e^{(9/2)})*\cos(1/2*\text{real_part}(\arccos(1/2*b*e^{(-1)/\text{ab} \\
& s(d)}))^{3}*\cosh(1/2*\text{imag_part}(\arccos(1/2*b*e^{(-1)/\text{abs}(d)}))^{2}*e*\sinh(1/2*\text{ima} \\
& g_part}(\arccos(1/2*b*e^{(-1)/\text{abs}(d)})) + 9*(4*(d^2)^{(3/4)}*d^2*e^{(13/2)} - b^2* \\
& (d^2)^{(3/4)}*e^{(9/2)} + \sqrt{-4*d^2*e^2 + b^2}*b*(d^2)^{(3/4)}*e^{(9/2)})*\cos(1/2 \\
& *\text{real_part}(\arccos(1/2*b*e^{(-1)/\text{abs}(d)})))*\cosh(1/2*\text{imag_part}(\arccos(1/2*b*e \\
& ^{(-1)/\text{abs}(d)}))^{2}*e*\sin(1/2*\text{real_part}(\arccos(1/2*b*e^{(-1)/\text{abs}(d)}))^{2}*\sinh(1 \\
& /2*\text{imag_part}(\arccos(1/2*b*e^{(-1)/\text{abs}(d)})) + 3*(4*(d^2)^{(3/4)}*d^2*e^{(13/2)} \\
& - b^2*(d^2)^{(3/4)}*e^{(9/2)} + \sqrt{-4*d^2*e^2 + b^2}*b*(d^2)^{(3/4)}*e^{(9/2)})*\text{c} \\
& \text{os}(1/2*\text{real_part}(\arccos(1/2*b*e^{(-1)/\text{abs}(d)}))^{3}*\cosh(1/2*\text{imag_part}(\arccos(\\
& 1/2*b*e^{(-1)/\text{abs}(d)})))*e*\sinh(1/2*\text{imag_part}(\arccos(1/2*b*e^{(-1)/\text{abs}(d)}))^{2} \\
& - 9*(4*(d^2)^{(3/4)}*d^2*e^{(13/2)} - b^2*(d^2)^{(3/4)}*e^{(9/2)} + \sqrt{-4*d^2*e^ \\
& 2 + b^2}*b*(d^2)^{(3/4)}*e^{(9/2)})*\cos(1/2*\text{real_part}(\arccos(1/2*b*e^{(-1)/\text{abs}(d) \\
&))))*\cosh(1/2*\text{imag_part}(\arccos(1/2*b*e^{(-1)/\text{abs}(d)})))*e*\sin(1/2*\text{real_part}(a \\
& rccos(1/2*b*e^{(-1)/\text{abs}(d)}))^{2}*\sinh(1/2*\text{imag_part}(\arccos(1/2*b*e^{(-1)/\text{abs}(d) \\
&)))^{2} - (4*(d^2)^{(3/4)}*d^2*e^{(13/2)} - b^2*(d^2)^{(3/4)}*e^{(9/2)} + \sqrt{-4*d^ \\
& 2*e^2 + b^2}*b*(d^2)^{(3/4)}*e^{(9/2)})*\cos(1/2*\text{real_part}(\arccos(1/2*b*e^{(-1)/\text{a} \\
& bs(d)}))^{3}*e*\sinh(1/2*\text{imag_part}(\arccos(1/2*b*e^{(-1)/\text{abs}(d)}))^{3} + 3*(4*(d^2 \\
&)^{(3/4)}*d^2*e^{(13/2)} - b^2*(d^2)^{(3/4)}*e^{(9/2)} + \sqrt{-4*d^2*e^2 + b^2}*b*(\\
& d^2)^{(3/4)}*e^{(9/2)})*\cos(1/2*\text{real_part}(\arccos(1/2*b*e^{(-1)/\text{abs}(d)})))*e*\sin(1 \\
& /2*\text{real_part}(\arccos(1/2*b*e^{(-1)/\text{abs}(d)}))^{2}*\sinh(1/2*\text{imag_part}(\arccos(1/2* \\
& b*e^{(-1)/\text{abs}(d)}))^{3} - (4*(d^2)^{(1/4)}*d^3*e^{(15/2)} - b^2*(d^2)^{(1/4)}*d*e^{(1 \\
& 1/2) + \sqrt{-4*d^2*e^2 + b^2}*b*(d^2)^{(1/4)}*d*e^{(11/2)})*\cos(1/2*\text{real_part}(a \\
& rccos(1/2*b*e^{(-1)/\text{abs}(d)})))*\cosh(1/2*\text{imag_part}(\arccos(1/2*b*e^{(-1)/\text{abs}(d) \\
&)) + (4*(d^2)^{(1/4)}*d^3*e^{(15/2)} - b^2*(d^2)^{(1/4)}*d*e^{(11/2)} + \sqrt{-4*d^2 \\
& *e^2 + b^2}*b*(d^2)^{(1/4)}*d*e^{(11/2)})*\cos(1/2*\text{real_part}(\arccos(1/2*b*e^{(-1) \\
& /\text{abs}(d)})))*\sinh(1/2*\text{imag_part}(\arccos(1/2*b*e^{(-1)/\text{abs}(d)})))*\log(-2*(d^2)^{(\\
& 1/4)}*x*\cos(1/2*\arccos(1/2*b*e^{(-1)/\text{abs}(d)}))*e^{(-1/2)} + x^2 + \sqrt{d^2}*e^{(- \\
& 1)})/(4*d^4*e^8 - b^2*d^2*e^6)
\end{aligned}$$

$$3.33 \quad \int \frac{d-ex^2}{d^2-fx^2+e^2x^4} dx$$

Optimal. Leaf size=70

$$\frac{\log(x\sqrt{2de+f}+d+ex^2)}{2\sqrt{2de+f}} - \frac{\log(-x\sqrt{2de+f}+d+ex^2)}{2\sqrt{2de+f}}$$

[Out] -Log[d - Sqrt[2*d*e + f]*x + e*x^2]/(2*Sqrt[2*d*e + f]) + Log[d + Sqrt[2*d*e + f]*x + e*x^2]/(2*Sqrt[2*d*e + f])

Rubi [A] time = 0.0472136, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {1164, 628}

$$\frac{\log(x\sqrt{2de+f}+d+ex^2)}{2\sqrt{2de+f}} - \frac{\log(-x\sqrt{2de+f}+d+ex^2)}{2\sqrt{2de+f}}$$

Antiderivative was successfully verified.

[In] Int[(d - e*x^2)/(d^2 - f*x^2 + e^2*x^4),x]

[Out] -Log[d - Sqrt[2*d*e + f]*x + e*x^2]/(2*Sqrt[2*d*e + f]) + Log[d + Sqrt[2*d*e + f]*x + e*x^2]/(2*Sqrt[2*d*e + f])

Rule 1164

Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[(-2*d)/e - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && !GtQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rubi steps

$$\begin{aligned} \int \frac{d-ex^2}{d^2-fx^2+e^2x^4} dx &= -\frac{\int \frac{\frac{\sqrt{2de+f}+2x}{e}}{\frac{d}{e}-\frac{\sqrt{2de+fx}}{e}-x^2} dx}{2\sqrt{2de+f}} - \frac{\int \frac{\frac{\sqrt{2de+f}-2x}{e}}{\frac{d}{e}+\frac{\sqrt{2de+fx}}{e}-x^2} dx}{2\sqrt{2de+f}} \\ &= -\frac{\log(d-\sqrt{2de+fx}+ex^2)}{2\sqrt{2de+f}} + \frac{\log(d+\sqrt{2de+fx}+ex^2)}{2\sqrt{2de+f}} \end{aligned}$$

Mathematica [B] time = 0.133033, size = 190, normalized size = 2.71

$$\frac{(-\sqrt{f^2-4d^2e^2-2de+f}) \tan^{-1}\left(\frac{\sqrt{2ex}}{\sqrt{\sqrt{f^2-4d^2e^2-f}}}\right)}{\sqrt{\sqrt{f^2-4d^2e^2-f}}} - \frac{(\sqrt{f^2-4d^2e^2-2de+f}) \tan^{-1}\left(\frac{\sqrt{2ex}}{\sqrt{-\sqrt{f^2-4d^2e^2-f}}}\right)}{\sqrt{-\sqrt{f^2-4d^2e^2-f}}}}{\sqrt{2}\sqrt{f^2-4d^2e^2}}$$

Antiderivative was successfully verified.

[In] Integrate[(d - e*x^2)/(d^2 - f*x^2 + e^2*x^4),x]

[Out]
$$\frac{-\left(\left(-2de + f + \sqrt{-4d^2e^2 + f^2}\right) \operatorname{ArcTan}\left[\frac{\sqrt{2}ex}{\sqrt{-f - \sqrt{-4d^2e^2 + f^2}}}\right] + \left(-2de + f - \sqrt{-4d^2e^2 + f^2}\right) \operatorname{ArcTan}\left[\frac{\sqrt{2}ex}{\sqrt{-f + \sqrt{-4d^2e^2 + f^2}}}\right]\right)}{\sqrt{-f + \sqrt{-4d^2e^2 + f^2}} \sqrt{2} \sqrt{-4d^2e^2 + f^2}}$$

Maple [A] time = 0.171, size = 61, normalized size = 0.9

$$-\frac{1}{2} \ln\left(-ex^2 + x\sqrt{2de + f} - d\right) \frac{1}{\sqrt{2de + f}} + \frac{1}{2} \ln\left(d + ex^2 + x\sqrt{2de + f}\right) \frac{1}{\sqrt{2de + f}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-e*x^2+d)/(e^2*x^4-f*x^2+d^2),x)

[Out]
$$-1/2/(2de+f)^{1/2}*\ln(-e*x^2+x*(2de+f)^{1/2}-d)+1/2*\ln(d+e*x^2+x*(2de+f)^{1/2})/(2de+f)^{1/2}$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{ex^2 - d}{e^2x^4 - fx^2 + d^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e*x^2+d)/(e^2*x^4-f*x^2+d^2),x, algorithm="maxima")

[Out] -integrate((e*x^2 - d)/(e^2*x^4 - f*x^2 + d^2), x)

Fricas [A] time = 1.28894, size = 378, normalized size = 5.4

$$\left[\frac{\log\left(\frac{e^2x^4 + (4de+f)x^2 + d^2 + 2(ex^3 + dx)\sqrt{2de+f}}{e^2x^4 - fx^2 + d^2}\right)}{2\sqrt{2de+f}}, -\frac{\sqrt{-2de-f} \arctan\left(\frac{\sqrt{-2de-f}ex}{2de+f}\right) - \sqrt{-2de-f} \arctan\left(\frac{(e^2x^3 - (de+f)x)\sqrt{-2de-f}}{2d^2e + df}\right)}{2de+f} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e*x^2+d)/(e^2*x^4-f*x^2+d^2),x, algorithm="fricas")

[Out]
$$\left[\frac{1}{2} \log\left(\frac{e^2x^4 + (4de+f)x^2 + d^2 + 2(ex^3 + dx)\sqrt{2de+f}}{e^2x^4 - fx^2 + d^2}\right) / \sqrt{2de+f}, -\left(\sqrt{-2de-f} \arctan\left(\frac{\sqrt{-2de-f}ex}{2de+f}\right) - \sqrt{-2de-f} \arctan\left(\frac{(e^2x^3 - (de+f)x)\sqrt{-2de-f}}{2d^2e + df}\right)\right) / (2de+f) \right]$$

Sympy [A] time = 0.600291, size = 112, normalized size = 1.6

$$\frac{\sqrt{\frac{1}{2de+f}} \log\left(\frac{d}{e} + x^2 + \frac{x\left(-2de\sqrt{\frac{1}{2de+f}} - f\sqrt{\frac{1}{2de+f}}\right)}{e}\right)}{2} + \frac{\sqrt{\frac{1}{2de+f}} \log\left(\frac{d}{e} + x^2 + \frac{x\left(2de\sqrt{\frac{1}{2de+f}} + f\sqrt{\frac{1}{2de+f}}\right)}{e}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e*x**2+d)/(e**2*x**4-f*x**2+d**2),x)

[Out] -sqrt(1/(2*d*e + f))*log(d/e + x**2 + x*(-2*d*e*sqrt(1/(2*d*e + f)) - f*sqrt(1/(2*d*e + f)))/e)/2 + sqrt(1/(2*d*e + f))*log(d/e + x**2 + x*(2*d*e*sqrt(1/(2*d*e + f)) + f*sqrt(1/(2*d*e + f)))/e)/2

Giac [C] time = 1.58361, size = 5387, normalized size = 76.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e*x^2+d)/(e^2*x^4-f*x^2+d^2),x, algorithm="giac")

[Out]
$$\begin{aligned} & -1/2*(3*(4*(d^2)^{(3/4)}*d^2*e^{(13/2)} - (d^2)^{(3/4)}*f^2*e^{(9/2)} + \sqrt{-4*d^2*e^2 + f^2}*(d^2)^{(3/4)}*f*e^{(9/2)})*\cos(1/2*\text{real_part}(\arccos(1/2*f*e^{(-1)}/\text{abs}(d))))^2*\cosh(1/2*\text{imag_part}(\arccos(1/2*f*e^{(-1)}/\text{abs}(d))))^3*e*\sin(1/2*\text{real_part}(\arccos(1/2*f*e^{(-1)}/\text{abs}(d)))) - (4*(d^2)^{(3/4)}*d^2*e^{(13/2)} - (d^2)^{(3/4)}*f^2*e^{(9/2)} + \sqrt{-4*d^2*e^2 + f^2}*(d^2)^{(3/4)}*f*e^{(9/2)})*\cosh(1/2*\text{imag_part}(\arccos(1/2*f*e^{(-1)}/\text{abs}(d))))^3*e*\sin(1/2*\text{real_part}(\arccos(1/2*f*e^{(-1)}/\text{abs}(d))))^3 - 9*(4*(d^2)^{(3/4)}*d^2*e^{(13/2)} - (d^2)^{(3/4)}*f^2*e^{(9/2)} + \sqrt{-4*d^2*e^2 + f^2}*(d^2)^{(3/4)}*f*e^{(9/2)})*\cos(1/2*\text{real_part}(\arccos(1/2*f*e^{(-1)}/\text{abs}(d))))^2*\cosh(1/2*\text{imag_part}(\arccos(1/2*f*e^{(-1)}/\text{abs}(d))))^2*e*\sin(1/2*\text{real_part}(\arccos(1/2*f*e^{(-1)}/\text{abs}(d))))*\sinh(1/2*\text{imag_part}(\arccos(1/2*f*e^{(-1)}/\text{abs}(d)))) + 3*(4*(d^2)^{(3/4)}*d^2*e^{(13/2)} - (d^2)^{(3/4)}*f^2*e^{(9/2)} + \sqrt{-4*d^2*e^2 + f^2}*(d^2)^{(3/4)}*f*e^{(9/2)})*\cosh(1/2*\text{imag_part}(\arccos(1/2*f*e^{(-1)}/\text{abs}(d))))^2*e*\sin(1/2*\text{real_part}(\arccos(1/2*f*e^{(-1)}/\text{abs}(d))))^3*\sinh(1/2*\text{imag_part}(\arccos(1/2*f*e^{(-1)}/\text{abs}(d)))) + 9*(4*(d^2)^{(3/4)}*d^2*e^{(13/2)} - (d^2)^{(3/4)}*f^2*e^{(9/2)} + \sqrt{-4*d^2*e^2 + f^2}*(d^2)^{(3/4)}*f*e^{(9/2)})*\cos(1/2*\text{real_part}(\arccos(1/2*f*e^{(-1)}/\text{abs}(d))))^2*\cosh(1/2*\text{imag_part}(\arccos(1/2*f*e^{(-1)}/\text{abs}(d))))*e*\sin(1/2*\text{real_part}(\arccos(1/2*f*e^{(-1)}/\text{abs}(d))))*\sinh(1/2*\text{imag_part}(\arccos(1/2*f*e^{(-1)}/\text{abs}(d))))^2 - 3*(4*(d^2)^{(3/4)}*d^2*e^{(13/2)} - (d^2)^{(3/4)}*f^2*e^{(9/2)} + \sqrt{-4*d^2*e^2 + f^2}*(d^2)^{(3/4)}*f*e^{(9/2)})*\cosh(1/2*\text{imag_part}(\arccos(1/2*f*e^{(-1)}/\text{abs}(d))))*e*\sin(1/2*\text{real_part}(\arccos(1/2*f*e^{(-1)}/\text{abs}(d))))^3*\sinh(1/2*\text{imag_part}(\arccos(1/2*f*e^{(-1)}/\text{abs}(d))))^2 - 3*(4*(d^2)^{(3/4)}*d^2*e^{(13/2)} - (d^2)^{(3/4)}*f^2*e^{(9/2)} + \sqrt{-4*d^2*e^2 + f^2}*(d^2)^{(3/4)}*f*e^{(9/2)})*\cos(1/2*\text{real_part}(\arccos(1/2*f*e^{(-1)}/\text{abs}(d))))^2*e*\sin(1/2*\text{real_part}(\arccos(1/2*f*e^{(-1)}/\text{abs}(d))))*\sinh(1/2*\text{imag_part}(\arccos(1/2*f*e^{(-1)}/\text{abs}(d))))^3 + (4*(d^2)^{(3/4)}*d^2*e^{(13/2)} - (d^2)^{(3/4)}*f^2*e^{(9/2)} + \sqrt{-4*d^2*e^2 + f^2}*(d^2)^{(3/4)}*f*e^{(9/2)})*e*\sin(1/2*\text{real_part}(\arccos(1/2*f*e^{(-1)}/\text{abs}(d))))^3*\sinh(1/2*\text{imag_part}(\arccos(1/2*f*e^{(-1)}/\text{abs}(d))))^3 - (4*(d^2)^{(1/4)}*d^3*e^{(15/2)} - (d^2)^{(1/4)}*d*f^2*e^{(11/2)} + \sqrt{-4*d^2*e^2 + f^2}*(d^2)^{(1/4)}*d*f*e^{(11/2)})*\cosh(1/2*\text{imag_part}(\arccos(1/2*f*e^{(-1)}/\text{abs}(d))))*\sin(1/2*\text{real_part}(\arccos(1/2*f*e^{(-1)}/\text{abs}(d)))) + (4*(d^2)^{(1/4)}*d^3*e^{(15/2)} - (d^2)^{(1/4)}*d*f^2*e^{(11/2)} + \sqrt{-4*d^2*e^2 + f^2}*(d^2)^{(1/4)}*d*f*e^{(11/2)})*\sin(1/2*\text{real_part}(\arccos(1/2*f*e^{(-1)}/\text{abs}(d))))*\sinh(1/2*\text{imag_part}(\arccos(1/2*f*e^{(-1)}/\text{abs}(d)))))*\arctan(((d^2)^{(1/4)}*\cos(1/2*\arccos(1/2*f*e^{(-1)}/\text{abs}(d))))*e^{(-1/2)} + x)*e^{(1} \end{aligned}$$

$$\begin{aligned} & \text{real_part}(\arccos(1/2*f*e^{(-1)/\text{abs}(d)}))^{3}* \sinh(1/2*\text{imag_part}(\arccos(1/2*f \\ & *e^{(-1)/\text{abs}(d)}))^{3} + 3*(4*(d^2)^{(3/4)}*d^2*e^{(13/2)} - (d^2)^{(3/4)}*f^2*e^{(9/ \\ & 2)} + \sqrt{-4*d^2*e^2 + f^2}*(d^2)^{(3/4)}*f*e^{(9/2)})*\cos(1/2*\text{real_part}(\arccos \\ & (1/2*f*e^{(-1)/\text{abs}(d)})))*e*\sin(1/2*\text{real_part}(\arccos(1/2*f*e^{(-1)/\text{abs}(d)}))^{2} \\ & *\sinh(1/2*\text{imag_part}(\arccos(1/2*f*e^{(-1)/\text{abs}(d)}))^{3} - (4*(d^2)^{(1/4)}*d^3*e^{ \\ & (15/2)} - (d^2)^{(1/4)}*d*f^2*e^{(11/2)} + \sqrt{-4*d^2*e^2 + f^2}*(d^2)^{(1/4)}*d* \\ & f*e^{(11/2)})*\cos(1/2*\text{real_part}(\arccos(1/2*f*e^{(-1)/\text{abs}(d)})))*\cosh(1/2*\text{imag_p} \\ & \text{art}(\arccos(1/2*f*e^{(-1)/\text{abs}(d)})) + (4*(d^2)^{(1/4)}*d^3*e^{(15/2)} - (d^2)^{(1/ \\ & 4)}*d*f^2*e^{(11/2)} + \sqrt{-4*d^2*e^2 + f^2}*(d^2)^{(1/4)}*d*f*e^{(11/2)})*\cos(1/ \\ & 2*\text{real_part}(\arccos(1/2*f*e^{(-1)/\text{abs}(d)})))*\sinh(1/2*\text{imag_part}(\arccos(1/2*f*e \\ & ^{(-1)/\text{abs}(d)})))*\log(2*(d^2)^{(1/4)}*x*\cos(1/2*\arccos(1/2*f*e^{(-1)/\text{abs}(d)}))*e \\ & ^{(-1/2)} + x^2 + \sqrt{d^2}*e^{(-1)})/(4*d^4*e^8 - d^2*f^2*e^6) + 1/4*((4*(d^2) \\ & ^{(3/4)}*d^2*e^{(13/2)} - (d^2)^{(3/4)}*f^2*e^{(9/2)} + \sqrt{-4*d^2*e^2 + f^2}*(d^2 \\ &)^{(3/4)}*f*e^{(9/2)})*\cos(1/2*\text{real_part}(\arccos(1/2*f*e^{(-1)/\text{abs}(d)}))^{3}*\cosh(1 \\ & /2*\text{imag_part}(\arccos(1/2*f*e^{(-1)/\text{abs}(d)}))^{3}*e - 3*(4*(d^2)^{(3/4)}*d^2*e^{(13 \\ & /2)} - (d^2)^{(3/4)}*f^2*e^{(9/2)} + \sqrt{-4*d^2*e^2 + f^2}*(d^2)^{(3/4)}*f*e^{(9/2 \\ &)})*\cos(1/2*\text{real_part}(\arccos(1/2*f*e^{(-1)/\text{abs}(d)})))*\cosh(1/2*\text{imag_part}(\arcco \\ & s(1/2*f*e^{(-1)/\text{abs}(d)}))^{3}*e*\sin(1/2*\text{real_part}(\arccos(1/2*f*e^{(-1)/\text{abs}(d)})) \\ &)^{2} - 3*(4*(d^2)^{(3/4)}*d^2*e^{(13/2)} - (d^2)^{(3/4)}*f^2*e^{(9/2)} + \sqrt{-4*d^2 \\ & *e^2 + f^2}*(d^2)^{(3/4)}*f*e^{(9/2)})*\cos(1/2*\text{real_part}(\arccos(1/2*f*e^{(-1)/\text{ab} \\ & s(d)}))^{3}*\cosh(1/2*\text{imag_part}(\arccos(1/2*f*e^{(-1)/\text{abs}(d)}))^{2}*e*\sinh(1/2*\text{ima} \\ & \text{g_part}(\arccos(1/2*f*e^{(-1)/\text{abs}(d)})) + 9*(4*(d^2)^{(3/4)}*d^2*e^{(13/2)} - (d^2 \\ &)^{(3/4)}*f^2*e^{(9/2)} + \sqrt{-4*d^2*e^2 + f^2}*(d^2)^{(3/4)}*f*e^{(9/2)})*\cos(1/2 \\ & *\text{real_part}(\arccos(1/2*f*e^{(-1)/\text{abs}(d)})))*\cosh(1/2*\text{imag_part}(\arccos(1/2*f*e^{ \\ & (-1)/\text{abs}(d)}))^{2}*e*\sin(1/2*\text{real_part}(\arccos(1/2*f*e^{(-1)/\text{abs}(d)}))^{2}*\sinh(1 \\ & /2*\text{imag_part}(\arccos(1/2*f*e^{(-1)/\text{abs}(d)})) + 3*(4*(d^2)^{(3/4)}*d^2*e^{(13/2)} \\ & - (d^2)^{(3/4)}*f^2*e^{(9/2)} + \sqrt{-4*d^2*e^2 + f^2}*(d^2)^{(3/4)}*f*e^{(9/2)})*\c \\ & \cos(1/2*\text{real_part}(\arccos(1/2*f*e^{(-1)/\text{abs}(d)}))^{3}*\cosh(1/2*\text{imag_part}(\arccos(\\ & 1/2*f*e^{(-1)/\text{abs}(d)})))*e*\sinh(1/2*\text{imag_part}(\arccos(1/2*f*e^{(-1)/\text{abs}(d)}))^{2} \\ & - 9*(4*(d^2)^{(3/4)}*d^2*e^{(13/2)} - (d^2)^{(3/4)}*f^2*e^{(9/2)} + \sqrt{-4*d^2*e^ \\ & 2 + f^2}*(d^2)^{(3/4)}*f*e^{(9/2)})*\cos(1/2*\text{real_part}(\arccos(1/2*f*e^{(-1)/\text{abs}(d \\ &)))*)*\cosh(1/2*\text{imag_part}(\arccos(1/2*f*e^{(-1)/\text{abs}(d)})))*e*\sin(1/2*\text{real_part}(a \\ & rccos(1/2*f*e^{(-1)/\text{abs}(d)}))^{2}*\sinh(1/2*\text{imag_part}(\arccos(1/2*f*e^{(-1)/\text{abs}(d \\ &)))^{2} - (4*(d^2)^{(3/4)}*d^2*e^{(13/2)} - (d^2)^{(3/4)}*f^2*e^{(9/2)} + \sqrt{-4*d^ \\ & 2*e^2 + f^2}*(d^2)^{(3/4)}*f*e^{(9/2)})*\cos(1/2*\text{real_part}(\arccos(1/2*f*e^{(-1)/a \\ & bs(d)}))^{3}*e*\sinh(1/2*\text{imag_part}(\arccos(1/2*f*e^{(-1)/\text{abs}(d)}))^{3} + 3*(4*(d^2 \\ &)^{(3/4)}*d^2*e^{(13/2)} - (d^2)^{(3/4)}*f^2*e^{(9/2)} + \sqrt{-4*d^2*e^2 + f^2}*(d^ \\ & 2)^{(3/4)}*f*e^{(9/2)})*\cos(1/2*\text{real_part}(\arccos(1/2*f*e^{(-1)/\text{abs}(d)})))*e*\sin(1 \\ & /2*\text{real_part}(\arccos(1/2*f*e^{(-1)/\text{abs}(d)}))^{2}*\sinh(1/2*\text{imag_part}(\arccos(1/2* \\ & f*e^{(-1)/\text{abs}(d)}))^{3} - (4*(d^2)^{(1/4)}*d^3*e^{(15/2)} - (d^2)^{(1/4)}*d*f^2*e^{(1 \\ & 1/2)} + \sqrt{-4*d^2*e^2 + f^2}*(d^2)^{(1/4)}*d*f*e^{(11/2)})*\cos(1/2*\text{real_part}(a \\ & rccos(1/2*f*e^{(-1)/\text{abs}(d)})))*\cosh(1/2*\text{imag_part}(\arccos(1/2*f*e^{(-1)/\text{abs}(d) \\ &)) + (4*(d^2)^{(1/4)}*d^3*e^{(15/2)} - (d^2)^{(1/4)}*d*f^2*e^{(11/2)} + \sqrt{-4*d^2 \\ & *e^2 + f^2}*(d^2)^{(1/4)}*d*f*e^{(11/2)})*\cos(1/2*\text{real_part}(\arccos(1/2*f*e^{(-1) \\ & /\text{abs}(d)})))*\sinh(1/2*\text{imag_part}(\arccos(1/2*f*e^{(-1)/\text{abs}(d)})))*\log(-2*(d^2)^{(\\ & 1/4)}*x*\cos(1/2*\arccos(1/2*f*e^{(-1)/\text{abs}(d)}))*e^{(-1/2)} + x^2 + \sqrt{d^2}*e^{(- \\ & 1)})/(4*d^4*e^8 - d^2*f^2*e^6) \end{aligned}$$

$$3.34 \quad \int \frac{d-ex^2}{\frac{cd^2}{e^2}+bx^2+cx^4} dx$$

Optimal. Leaf size=134

$$\frac{e^{3/2} \log(\sqrt{ex}\sqrt{2cd-be} + \sqrt{cd} + \sqrt{cex^2})}{2\sqrt{c}\sqrt{2cd-be}} - \frac{e^{3/2} \log(-\sqrt{ex}\sqrt{2cd-be} + \sqrt{cd} + \sqrt{cex^2})}{2\sqrt{c}\sqrt{2cd-be}}$$

[Out] $-(e^{(3/2)}*\text{Log}[\text{Sqrt}[c]*d - \text{Sqrt}[e]*\text{Sqrt}[2*c*d - b*e]*x + \text{Sqrt}[c]*e*x^2])/(2*\text{Sqrt}[c]*\text{Sqrt}[2*c*d - b*e]) + (e^{(3/2)}*\text{Log}[\text{Sqrt}[c]*d + \text{Sqrt}[e]*\text{Sqrt}[2*c*d - b*e]*x + \text{Sqrt}[c]*e*x^2])/(2*\text{Sqrt}[c]*\text{Sqrt}[2*c*d - b*e])$

Rubi [A] time = 0.100717, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {1164, 628}

$$\frac{e^{3/2} \log(\sqrt{ex}\sqrt{2cd-be} + \sqrt{cd} + \sqrt{cex^2})}{2\sqrt{c}\sqrt{2cd-be}} - \frac{e^{3/2} \log(-\sqrt{ex}\sqrt{2cd-be} + \sqrt{cd} + \sqrt{cex^2})}{2\sqrt{c}\sqrt{2cd-be}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(d - e*x^2)/((c*d^2)/e^2 + b*x^2 + c*x^4), x]$

[Out] $-(e^{(3/2)}*\text{Log}[\text{Sqrt}[c]*d - \text{Sqrt}[e]*\text{Sqrt}[2*c*d - b*e]*x + \text{Sqrt}[c]*e*x^2])/(2*\text{Sqrt}[c]*\text{Sqrt}[2*c*d - b*e]) + (e^{(3/2)}*\text{Log}[\text{Sqrt}[c]*d + \text{Sqrt}[e]*\text{Sqrt}[2*c*d - b*e]*x + \text{Sqrt}[c]*e*x^2])/(2*\text{Sqrt}[c]*\text{Sqrt}[2*c*d - b*e])$

Rule 1164

$\text{Int}(((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :$
 $> \text{With}\{q = \text{Rt}[(-2*d)/e - b/c, 2]\}, \text{Dist}[e/(2*c*q), \text{Int}[(q - 2*x)/\text{Simp}[d/e + q*x - x^2, x], x], x] + \text{Dist}[e/(2*c*q), \text{Int}[(q + 2*x)/\text{Simp}[d/e - q*x - x^2, x], x], x] /;$
 $\text{FreeQ}\{a, b, c, d, e\}, x] \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0] \ \&\& \ \text{EqQ}[c*d^2 - a*e^2, 0] \ \&\& \ !\text{GtQ}[b^2 - 4*a*c, 0]$

Rule 628

$\text{Int}(((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := \text{Simp}[(d*\text{Log}[\text{RemoveContent}[a + b*x + c*x^2, x]])/b, x] /;$
 $\text{FreeQ}\{a, b, c, d, e\}, x] \ \&\& \ \text{EqQ}[2*c*d - b*e, 0]$

Rubi steps

$$\int \frac{d-ex^2}{\frac{cd^2}{e^2}+bx^2+cx^4} dx = -\frac{e^{3/2} \int \frac{\frac{\sqrt{2cd-be}}{\sqrt{c}\sqrt{e}}+2x}{-\frac{d}{e}-\frac{\sqrt{2cd-bex}}{\sqrt{c}\sqrt{e}}-x^2} dx}{2\sqrt{c}\sqrt{2cd-be}} - \frac{e^{3/2} \int \frac{\frac{\sqrt{2cd-be}}{\sqrt{c}\sqrt{e}}-2x}{-\frac{d}{e}+\frac{\sqrt{2cd-bex}}{\sqrt{c}\sqrt{e}}-x^2} dx}{2\sqrt{c}\sqrt{2cd-be}}$$

$$= -\frac{e^{3/2} \log(\sqrt{cd} - \sqrt{e}\sqrt{2cd-bex} + \sqrt{cex^2})}{2\sqrt{c}\sqrt{2cd-be}} + \frac{e^{3/2} \log(\sqrt{cd} + \sqrt{e}\sqrt{2cd-bex} + \sqrt{cex^2})}{2\sqrt{c}\sqrt{2cd-be}}$$

Mathematica [A] time = 0.164182, size = 250, normalized size = 1.87

$$e^{3/2} \frac{\left(\frac{\left(\sqrt{b^2 e^2 - 4c^2 d^2} - be - 2cd \right) \tan^{-1} \left(\frac{\sqrt{2} \sqrt{c} \sqrt{ex}}{\sqrt{be - \sqrt{b^2 e^2 - 4c^2 d^2}}} \right)}{\sqrt{be - \sqrt{b^2 e^2 - 4c^2 d^2}}} - \frac{\left(\sqrt{b^2 e^2 - 4c^2 d^2} + be + 2cd \right) \tan^{-1} \left(\frac{\sqrt{2} \sqrt{c} \sqrt{ex}}{\sqrt{\sqrt{b^2 e^2 - 4c^2 d^2} + be}} \right)}{\sqrt{\sqrt{b^2 e^2 - 4c^2 d^2} + be}} \right)}{\sqrt{2} \sqrt{c} \sqrt{b^2 e^2 - 4c^2 d^2}}$$

Antiderivative was successfully verified.

[In] Integrate[(d - e*x^2)/((c*d^2)/e^2 + b*x^2 + c*x^4), x]

[Out] (e^(3/2)*(-(((-2*c*d - b*e + Sqrt[-4*c^2*d^2 + b^2*e^2])*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[e]*x)/Sqrt[b*e - Sqrt[-4*c^2*d^2 + b^2*e^2]]])/Sqrt[b*e - Sqrt[-4*c^2*d^2 + b^2*e^2]] - ((2*c*d + b*e + Sqrt[-4*c^2*d^2 + b^2*e^2])*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[e]*x)/Sqrt[b*e + Sqrt[-4*c^2*d^2 + b^2*e^2]]])/Sqrt[b*e + Sqrt[-4*c^2*d^2 + b^2*e^2]]))/(Sqrt[2]*Sqrt[c]*Sqrt[-4*c^2*d^2 + b^2*e^2])

Maple [B] time = 0.275, size = 582, normalized size = 4.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-e*x^2+d)/(c*d^2/e^2+b*x^2+c*x^4), x)

[Out] -1/2*e^4/(e^2*(b*e-2*c*d)*(b*e+2*c*d))^(1/2)*2^(1/2)/((-b*e^2+(e^2*(b*e-2*c*d)*(b*e+2*c*d))^(1/2))*c)^(1/2)*arctanh(c*e*x*2^(1/2)/((-b*e^2+(e^2*(b*e-2*c*d)*(b*e+2*c*d))^(1/2))*c)^(1/2))*b-e^3*c/(e^2*(b*e-2*c*d)*(b*e+2*c*d))^(1/2)*2^(1/2)/((-b*e^2+(e^2*(b*e-2*c*d)*(b*e+2*c*d))^(1/2))*c)^(1/2)*arctanh(c*e*x*2^(1/2)/((-b*e^2+(e^2*(b*e-2*c*d)*(b*e+2*c*d))^(1/2))*c)^(1/2))*d+1/2*e^2*2^(1/2)/((-b*e^2+(e^2*(b*e-2*c*d)*(b*e+2*c*d))^(1/2))*c)^(1/2)*arctanh(c*e*x*2^(1/2)/((-b*e^2+(e^2*(b*e-2*c*d)*(b*e+2*c*d))^(1/2))*c)^(1/2))-1/2*e^4/(e^2*(b*e-2*c*d)*(b*e+2*c*d))^(1/2)*2^(1/2)/((b*e^2+(e^2*(b*e-2*c*d)*(b*e+2*c*d))^(1/2))*c)^(1/2)*arctan(c*e*x*2^(1/2)/((b*e^2+(e^2*(b*e-2*c*d)*(b*e+2*c*d))^(1/2))*c)^(1/2))*b-e^3*c/(e^2*(b*e-2*c*d)*(b*e+2*c*d))^(1/2)*2^(1/2)/((b*e^2+(e^2*(b*e-2*c*d)*(b*e+2*c*d))^(1/2))*c)^(1/2)*arctan(c*e*x*2^(1/2)/((b*e^2+(e^2*(b*e-2*c*d)*(b*e+2*c*d))^(1/2))*c)^(1/2))*d-1/2*e^2*2^(1/2)/((b*e^2+(e^2*(b*e-2*c*d)*(b*e+2*c*d))^(1/2))*c)^(1/2)*arctan(c*e*x*2^(1/2)/((b*e^2+(e^2*(b*e-2*c*d)*(b*e+2*c*d))^(1/2))*c)^(1/2))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$- \int \frac{ex^2 - d}{cx^4 + bx^2 + \frac{cd^2}{e^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e*x^2+d)/(c*d^2/e^2+b*x^2+c*x^4), x, algorithm="maxima")

[Out] -integrate((e*x^2 - d)/(c*x^4 + b*x^2 + c*d^2/e^2), x)

Fricas [A] time = 1.40643, size = 494, normalized size = 3.69

$$\left[\frac{1}{2} e \sqrt{\frac{e}{2c^2d - bce}} \log \left(\frac{ce^2x^4 + cd^2 + (4cde - be^2)x^2 + 2((2c^2de - bce^2)x^3 + (2c^2d^2 - bcde)x) \sqrt{\frac{e}{2c^2d - bce}}}{ce^2x^4 + be^2x^2 + cd^2} \right), -e \sqrt{-\frac{e}{2c^2d - bce}} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e*x^2+d)/(c*d^2/e^2+b*x^2+c*x^4),x, algorithm="fricas")

[Out] [1/2*e*sqrt(e/(2*c^2*d - b*c*e))*log((c*e^2*x^4 + c*d^2 + (4*c*d*e - b*e^2)*x^2 + 2*((2*c^2*d*e - b*c*e^2)*x^3 + (2*c^2*d^2 - b*c*d*e)*x)*sqrt(e/(2*c^2*d - b*c*e)))/(c*e^2*x^4 + b*e^2*x^2 + c*d^2)), -e*sqrt(-e/(2*c^2*d - b*c*e))*arctan(c*x*sqrt(-e/(2*c^2*d - b*c*e))) + e*sqrt(-e/(2*c^2*d - b*c*e))*arctan((c*e*x^3 - (c*d - b*e)*x)*sqrt(-e/(2*c^2*d - b*c*e))/d)]

Sympy [A] time = 0.795899, size = 158, normalized size = 1.18

$$\frac{\sqrt{-\frac{e^3}{c(be-2cd)}} \log \left(\frac{d}{e} + x^2 + \frac{x \left(-be \sqrt{-\frac{e^3}{c(be-2cd)}} + 2cd \sqrt{-\frac{e^3}{c(be-2cd)}} \right)}{e^2} \right)}{2} - \frac{\sqrt{-\frac{e^3}{c(be-2cd)}} \log \left(\frac{d}{e} + x^2 + \frac{x \left(be \sqrt{-\frac{e^3}{c(be-2cd)}} - 2cd \sqrt{-\frac{e^3}{c(be-2cd)}} \right)}{e^2} \right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e*x**2+d)/(c*d**2/e**2+b*x**2+c*x**4),x)

[Out] sqrt(-e**3/(c*(b*e - 2*c*d)))*log(d/e + x**2 + x*(-b*e*sqrt(-e**3/(c*(b*e - 2*c*d))) + 2*c*d*sqrt(-e**3/(c*(b*e - 2*c*d))))/e**2)/2 - sqrt(-e**3/(c*(b*e - 2*c*d)))*log(d/e + x**2 + x*(b*e*sqrt(-e**3/(c*(b*e - 2*c*d))) - 2*c*d*sqrt(-e**3/(c*(b*e - 2*c*d))))/e**2)/2

Giac [C] time = 2.44361, size = 6884, normalized size = 51.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e*x^2+d)/(c*d^2/e^2+b*x^2+c*x^4),x, algorithm="giac")

[Out] -1/2*(3*(4*c^3*(d^2)^(3/4)*d^2*e^(9/2) - b^2*c*(d^2)^(3/4)*e^(13/2) - sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*b*c*(d^2)^(3/4)*e^(9/2))*cos(5/4*pi + 1/2*real_part(arcsin(1/2*b*e/(c*abs(d))))))^2*cosh(1/2*imag_part(arcsin(1/2*b*e/(c*abs(d))))))^3*e*sin(5/4*pi + 1/2*real_part(arcsin(1/2*b*e/(c*abs(d)))))) - (4*c^3*(d^2)^(3/4)*d^2*e^(9/2) - b^2*c*(d^2)^(3/4)*e^(13/2) - sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*b*c*(d^2)^(3/4)*e^(9/2))*cosh(1/2*imag_part(arcsin(1/2*b*e/(c*abs(d))))))^3*e*sin(5/4*pi + 1/2*real_part(arcsin(1/2*b*e/(c*abs(d))))))^3 - 9*(4*c^3*(d^2)^(3/4)*d^2*e^(9/2) - b^2*c*(d^2)^(3/4)*e^(13/2) - sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*b*c*(d^2)^(3/4)*e^(9/2))*cos(5/4*pi + 1/2*real_part(arcsin(1/2*b*e/(c*abs(d))))))^2*cosh(1/2*imag_part(arcsin(1/2*b*e/(c*abs(d))))))^2*e*sin(5/4*pi + 1/2*real_part(arcsin(1/2*b*e/(c*abs(d))))))*sinh(1/2*imag_part(arcsin(1/2*b*e/(c*abs(d)))))) + 3*(4*c^3*(d^2)^(3/4)*d^2*e^(9/2) - b^2*c

$$\begin{aligned}
& \text{ag_part}(\arcsin(1/2*b*e/(c*abs(d))))*\sin(1/4*\pi + 1/2*\text{real_part}(\arcsin(1/2* \\
& b*e/(c*abs(d)))) + (4*c^3*(d^2)^{(1/4)}*d^3*e^{(11/2)} - b^2*c*(d^2)^{(1/4)}*d*e \\
& ^{(15/2)} - \text{sqrt}(-4*c^2*d^2*e^2 + b^2*e^4)*b*c*(d^2)^{(1/4)}*d*e^{(11/2)})*\sin(1/ \\
& 4*\pi + 1/2*\text{real_part}(\arcsin(1/2*b*e/(c*abs(d)))))*\sinh(1/2*\text{imag_part}(\arcsin \\
& (1/2*b*e/(c*abs(d)))))*\arctan(-((d^2)^{(1/4)}*\cos(1/4*\pi + 1/2*\arcsin(1/2*b* \\
& e/(c*abs(d))))*e^{(-1/2)} - x)*e^{(1/2)}/((d^2)^{(1/4)}*\sin(1/4*\pi + 1/2*\arcsin(1 \\
& /2*b*e/(c*abs(d)))))/(4*c^4*d^4*e^4 - b^2*c^2*d^2*e^6) + 1/4*((4*c^3*(d^2) \\
& ^{(3/4)}*d^2*e^{(9/2)} - b^2*c*(d^2)^{(3/4)}*e^{(13/2)} - \text{sqrt}(-4*c^2*d^2*e^2 + b^2 \\
& *e^4)*b*c*(d^2)^{(3/4)}*e^{(9/2)})*\cos(5/4*\pi + 1/2*\text{real_part}(\arcsin(1/2*b*e/(c \\
& *abs(d))))^3*\cosh(1/2*\text{imag_part}(\arcsin(1/2*b*e/(c*abs(d))))^3*e - 3*(4*c^ \\
& 3*(d^2)^{(3/4)}*d^2*e^{(9/2)} - b^2*c*(d^2)^{(3/4)}*e^{(13/2)} - \text{sqrt}(-4*c^2*d^2*e^ \\
& 2 + b^2*e^4)*b*c*(d^2)^{(3/4)}*e^{(9/2)})*\cos(5/4*\pi + 1/2*\text{real_part}(\arcsin(1/2 \\
& *b*e/(c*abs(d))))*\cosh(1/2*\text{imag_part}(\arcsin(1/2*b*e/(c*abs(d))))^3*e*\sin(\\
& 5/4*\pi + 1/2*\text{real_part}(\arcsin(1/2*b*e/(c*abs(d))))^2 - 3*(4*c^3*(d^2)^{(3/4) \\
&)*d^2*e^{(9/2)} - b^2*c*(d^2)^{(3/4)}*e^{(13/2)} - \text{sqrt}(-4*c^2*d^2*e^2 + b^2*e^4) \\
& *b*c*(d^2)^{(3/4)}*e^{(9/2)})*\cos(5/4*\pi + 1/2*\text{real_part}(\arcsin(1/2*b*e/(c*abs(\\
& d))))^3*\cosh(1/2*\text{imag_part}(\arcsin(1/2*b*e/(c*abs(d))))^2*e*\sinh(1/2*\text{imag_} \\
& \text{part}(\arcsin(1/2*b*e/(c*abs(d)))) + 9*(4*c^3*(d^2)^{(3/4)}*d^2*e^{(9/2)} - b^2*c \\
& *(d^2)^{(3/4)}*e^{(13/2)} - \text{sqrt}(-4*c^2*d^2*e^2 + b^2*e^4)*b*c*(d^2)^{(3/4)}*e^{(\\
& 9/2)})*\cos(5/4*\pi + 1/2*\text{real_part}(\arcsin(1/2*b*e/(c*abs(d))))*\cosh(1/2*\text{imag} \\
& _part(\arcsin(1/2*b*e/(c*abs(d))))^2*e*\sin(5/4*\pi + 1/2*\text{real_part}(\arcsin(1/ \\
& 2*b*e/(c*abs(d))))^2*\sinh(1/2*\text{imag_part}(\arcsin(1/2*b*e/(c*abs(d)))) + 3*(\\
& 4*c^3*(d^2)^{(3/4)}*d^2*e^{(9/2)} - b^2*c*(d^2)^{(3/4)}*e^{(13/2)} - \text{sqrt}(-4*c^2*d^ \\
& 2*e^2 + b^2*e^4)*b*c*(d^2)^{(3/4)}*e^{(9/2)})*\cos(5/4*\pi + 1/2*\text{real_part}(\arcsin \\
& (1/2*b*e/(c*abs(d))))^3*\cosh(1/2*\text{imag_part}(\arcsin(1/2*b*e/(c*abs(d))))*e* \\
& \sinh(1/2*\text{imag_part}(\arcsin(1/2*b*e/(c*abs(d))))^2 - 9*(4*c^3*(d^2)^{(3/4)}*d^ \\
& 2*e^{(9/2)} - b^2*c*(d^2)^{(3/4)}*e^{(13/2)} - \text{sqrt}(-4*c^2*d^2*e^2 + b^2*e^4)*b*c \\
& *(d^2)^{(3/4)}*e^{(9/2)})*\cos(5/4*\pi + 1/2*\text{real_part}(\arcsin(1/2*b*e/(c*abs(d))) \\
&))*\cosh(1/2*\text{imag_part}(\arcsin(1/2*b*e/(c*abs(d))))*e*\sin(5/4*\pi + 1/2*\text{real_} \\
& \text{part}(\arcsin(1/2*b*e/(c*abs(d))))^2*\sinh(1/2*\text{imag_part}(\arcsin(1/2*b*e/(c*ab \\
& s(d))))^2 - (4*c^3*(d^2)^{(3/4)}*d^2*e^{(9/2)} - b^2*c*(d^2)^{(3/4)}*e^{(13/2)} - \\
& \text{sqrt}(-4*c^2*d^2*e^2 + b^2*e^4)*b*c*(d^2)^{(3/4)}*e^{(9/2)})*\cos(5/4*\pi + 1/2*\text{rea} \\
& \text{al_part}(\arcsin(1/2*b*e/(c*abs(d))))^3*e*\sinh(1/2*\text{imag_part}(\arcsin(1/2*b*e/ \\
& (c*abs(d))))^3 + 3*(4*c^3*(d^2)^{(3/4)}*d^2*e^{(9/2)} - b^2*c*(d^2)^{(3/4)}*e^{(1 \\
& 3/2)} - \text{sqrt}(-4*c^2*d^2*e^2 + b^2*e^4)*b*c*(d^2)^{(3/4)}*e^{(9/2)})*\cos(5/4*\pi + \\
& 1/2*\text{real_part}(\arcsin(1/2*b*e/(c*abs(d))))*e*\sin(5/4*\pi + 1/2*\text{real_part}(\ar \\
& csin(1/2*b*e/(c*abs(d))))^2*\sinh(1/2*\text{imag_part}(\arcsin(1/2*b*e/(c*abs(d)))) \\
&)^3 - (4*c^3*(d^2)^{(1/4)}*d^3*e^{(11/2)} - b^2*c*(d^2)^{(1/4)}*d*e^{(15/2)} - \text{sqrt} \\
& (-4*c^2*d^2*e^2 + b^2*e^4)*b*c*(d^2)^{(1/4)}*d*e^{(11/2)})*\cos(5/4*\pi + 1/2*\text{rea} \\
& \text{l_part}(\arcsin(1/2*b*e/(c*abs(d))))*\cosh(1/2*\text{imag_part}(\arcsin(1/2*b*e/(c*ab \\
& s(d)))) + (4*c^3*(d^2)^{(1/4)}*d^3*e^{(11/2)} - b^2*c*(d^2)^{(1/4)}*d*e^{(15/2)} - \\
& \text{sqrt}(-4*c^2*d^2*e^2 + b^2*e^4)*b*c*(d^2)^{(1/4)}*d*e^{(11/2)})*\cos(5/4*\pi + 1/ \\
& 2*\text{real_part}(\arcsin(1/2*b*e/(c*abs(d))))*\sinh(1/2*\text{imag_part}(\arcsin(1/2*b*e/ \\
& (c*abs(d)))))*\log(-2*(d^2)^{(1/4)}*x*\cos(5/4*\pi + 1/2*\arcsin(1/2*b*e/(c*abs(\\
& d))))*e^{(-1/2)} + x^2 + \text{sqrt}(d^2)*e^{(-1)})/(4*c^4*d^4*e^4 - b^2*c^2*d^2*e^6) \\
& + 1/4*((4*c^3*(d^2)^{(3/4)}*d^2*e^{(9/2)} - b^2*c*(d^2)^{(3/4)}*e^{(13/2)} - \text{sqrt}(- \\
& 4*c^2*d^2*e^2 + b^2*e^4)*b*c*(d^2)^{(3/4)}*e^{(9/2)})*\cos(1/4*\pi + 1/2*\text{real_par} \\
& \text{t}(\arcsin(1/2*b*e/(c*abs(d))))^3*\cosh(1/2*\text{imag_part}(\arcsin(1/2*b*e/(c*abs(d) \\
&))))^3*e - 3*(4*c^3*(d^2)^{(3/4)}*d^2*e^{(9/2)} - b^2*c*(d^2)^{(3/4)}*e^{(13/2)} - \\
& \text{sqrt}(-4*c^2*d^2*e^2 + b^2*e^4)*b*c*(d^2)^{(3/4)}*e^{(9/2)})*\cos(1/4*\pi + 1/2*\text{r} \\
& \text{eal_part}(\arcsin(1/2*b*e/(c*abs(d))))*\cosh(1/2*\text{imag_part}(\arcsin(1/2*b*e/(c* \\
& abs(d))))^3*e*\sin(1/4*\pi + 1/2*\text{real_part}(\arcsin(1/2*b*e/(c*abs(d))))^2 - \\
& 3*(4*c^3*(d^2)^{(3/4)}*d^2*e^{(9/2)} - b^2*c*(d^2)^{(3/4)}*e^{(13/2)} - \text{sqrt}(-4*c^2 \\
& *d^2*e^2 + b^2*e^4)*b*c*(d^2)^{(3/4)}*e^{(9/2)})*\cos(1/4*\pi + 1/2*\text{real_part}(\ar \\
& csin(1/2*b*e/(c*abs(d))))^3*\cosh(1/2*\text{imag_part}(\arcsin(1/2*b*e/(c*abs(d)))) \\
&)^2*e*\sinh(1/2*\text{imag_part}(\arcsin(1/2*b*e/(c*abs(d)))) + 9*(4*c^3*(d^2)^{(3/4) \\
&)*d^2*e^{(9/2)} - b^2*c*(d^2)^{(3/4)}*e^{(13/2)} - \text{sqrt}(-4*c^2*d^2*e^2 + b^2*e^4)* \\
& b*c*(d^2)^{(3/4)}*e^{(9/2)})*\cos(1/4*\pi + 1/2*\text{real_part}(\arcsin(1/2*b*e/(c*abs(d) \\
&))))^3*\cosh(1/2*\text{imag_part}(\arcsin(1/2*b*e/(c*abs(d))))^2*e*\sin(1/4*\pi + 1/2*
\end{aligned}$$

$$\begin{aligned}
& \text{real_part}(\arcsin(1/2*b*e/(c*abs(d))))^2*\sinh(1/2*imag_part(\arcsin(1/2*b*e/ \\
& (c*abs(d)))) + 3*(4*c^3*(d^2)^{(3/4)}*d^2*e^{(9/2)} - b^2*c*(d^2)^{(3/4)}*e^{(13/ \\
& 2)} - \sqrt{-4*c^2*d^2*e^2 + b^2*e^4}*b*c*(d^2)^{(3/4)}*e^{(9/2)})*\cos(1/4*\pi + 1 \\
& /2*\text{real_part}(\arcsin(1/2*b*e/(c*abs(d))))^3*\cosh(1/2*imag_part(\arcsin(1/2*b \\
& *e/(c*abs(d)))))*e*\sinh(1/2*imag_part(\arcsin(1/2*b*e/(c*abs(d))))^2 - 9*(4 \\
& *c^3*(d^2)^{(3/4)}*d^2*e^{(9/2)} - b^2*c*(d^2)^{(3/4)}*e^{(13/2)} - \sqrt{-4*c^2*d^2 \\
& *e^2 + b^2*e^4}*b*c*(d^2)^{(3/4)}*e^{(9/2)})*\cos(1/4*\pi + 1/2*\text{real_part}(\arcsin(\\
& 1/2*b*e/(c*abs(d))))*\cosh(1/2*imag_part(\arcsin(1/2*b*e/(c*abs(d)))))*e*\sin \\
& (1/4*\pi + 1/2*\text{real_part}(\arcsin(1/2*b*e/(c*abs(d))))^2*\sinh(1/2*imag_part(a \\
& rcsin(1/2*b*e/(c*abs(d))))^2 - (4*c^3*(d^2)^{(3/4)}*d^2*e^{(9/2)} - b^2*c*(d^2 \\
&)^{(3/4)}*e^{(13/2)} - \sqrt{-4*c^2*d^2*e^2 + b^2*e^4}*b*c*(d^2)^{(3/4)}*e^{(9/2)})* \\
& \cos(1/4*\pi + 1/2*\text{real_part}(\arcsin(1/2*b*e/(c*abs(d))))^3*e*\sinh(1/2*imag_p \\
& art(\arcsin(1/2*b*e/(c*abs(d))))^3 + 3*(4*c^3*(d^2)^{(3/4)}*d^2*e^{(9/2)} - b^2 \\
& *c*(d^2)^{(3/4)}*e^{(13/2)} - \sqrt{-4*c^2*d^2*e^2 + b^2*e^4}*b*c*(d^2)^{(3/4)}*e^{ \\
& (9/2)})*\cos(1/4*\pi + 1/2*\text{real_part}(\arcsin(1/2*b*e/(c*abs(d)))))*e*\sin(1/4*\pi \\
& + 1/2*\text{real_part}(\arcsin(1/2*b*e/(c*abs(d))))^2*\sinh(1/2*imag_part(\arcsin(1 \\
& /2*b*e/(c*abs(d))))^3 - (4*c^3*(d^2)^{(1/4)}*d^3*e^{(11/2)} - b^2*c*(d^2)^{(1/4 \\
&)}*d*e^{(15/2)} - \sqrt{-4*c^2*d^2*e^2 + b^2*e^4}*b*c*(d^2)^{(1/4)}*d*e^{(11/2)})*c \\
& \cos(1/4*\pi + 1/2*\text{real_part}(\arcsin(1/2*b*e/(c*abs(d))))*\cosh(1/2*imag_part(a \\
& rcsin(1/2*b*e/(c*abs(d)))) + (4*c^3*(d^2)^{(1/4)}*d^3*e^{(11/2)} - b^2*c*(d^2) \\
& ^{(1/4)}*d*e^{(15/2)} - \sqrt{-4*c^2*d^2*e^2 + b^2*e^4}*b*c*(d^2)^{(1/4)}*d*e^{(11/ \\
& 2)})*\cos(1/4*\pi + 1/2*\text{real_part}(\arcsin(1/2*b*e/(c*abs(d))))*\sinh(1/2*imag_p \\
& art(\arcsin(1/2*b*e/(c*abs(d)))))*\log(-2*(d^2)^{(1/4)}*x*\cos(1/4*\pi + 1/2*\text{arc} \\
& \sin(1/2*b*e/(c*abs(d))))*e^{(-1/2)} + x^2 + \sqrt{d^2}*e^{(-1)})/(4*c^4*d^4*e^4 \\
& - b^2*c^2*d^2*e^6)
\end{aligned}$$

$$3.35 \quad \int \frac{d+ex^2}{\frac{cd^2}{e^2}+bx^2+cx^4} dx$$

Optimal. Leaf size=130

$$\frac{e^{3/2} \tan^{-1}\left(\frac{\sqrt{2cd-be+2}\sqrt{c}\sqrt{ex}}{\sqrt{be+2cd}}\right)}{\sqrt{c}\sqrt{be+2cd}} - \frac{e^{3/2} \tan^{-1}\left(\frac{\sqrt{2cd-be-2}\sqrt{c}\sqrt{ex}}{\sqrt{be+2cd}}\right)}{\sqrt{c}\sqrt{be+2cd}}$$

[Out] $-\left(\frac{e^{3/2} \operatorname{ArcTan}\left[\frac{\sqrt{2cd-be}-2\sqrt{c}\sqrt{e}x}{\sqrt{2cd+be}}\right]}{\sqrt{c}\sqrt{be+2cd}}\right) + \left(\frac{e^{3/2} \operatorname{ArcTan}\left[\frac{\sqrt{2cd-be}+2\sqrt{c}\sqrt{e}x}{\sqrt{2cd+be}}\right]}{\sqrt{c}\sqrt{be+2cd}}\right)$

Rubi [A] time = 0.166098, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {1161, 618, 204}

$$\frac{e^{3/2} \tan^{-1}\left(\frac{\sqrt{2cd-be+2}\sqrt{c}\sqrt{ex}}{\sqrt{be+2cd}}\right)}{\sqrt{c}\sqrt{be+2cd}} - \frac{e^{3/2} \tan^{-1}\left(\frac{\sqrt{2cd-be-2}\sqrt{c}\sqrt{ex}}{\sqrt{be+2cd}}\right)}{\sqrt{c}\sqrt{be+2cd}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(d + e*x^2)/((c*d^2)/e^2 + b*x^2 + c*x^4), x]$

[Out] $-\left(\frac{e^{3/2} \operatorname{ArcTan}\left[\frac{\sqrt{2cd-be}-2\sqrt{c}\sqrt{e}x}{\sqrt{2cd+be}}\right]}{\sqrt{c}\sqrt{be+2cd}}\right) + \left(\frac{e^{3/2} \operatorname{ArcTan}\left[\frac{\sqrt{2cd-be}+2\sqrt{c}\sqrt{e}x}{\sqrt{2cd+be}}\right]}{\sqrt{c}\sqrt{be+2cd}}\right)$

Rule 1161

$\text{Int}[\frac{(d + e*x^2)}{(c*d^2)/e^2 + b*x^2 + c*x^4}, x_Symbol] :$
 $> \text{With}\{q = \text{Rt}[(2*d)/e - b/c, 2]\}, \text{Dist}[e/(2*c), \text{Int}[1/\text{Simp}[d/e + q*x + x^2, x], x], x] + \text{Dist}[e/(2*c), \text{Int}[1/\text{Simp}[d/e - q*x + x^2, x], x], x] /;$
 $\text{FreeQ}\{a, b, c, d, e\}, x \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0] \ \&\& \ \text{EqQ}[c*d^2 - a*e^2, 0] \ \&\& \ (\text{GtQ}[(2*d)/e - b/c, 0] \ || \ (\text{!LtQ}[(2*d)/e - b/c, 0] \ \&\& \ \text{EqQ}[d - e*\text{Rt}[a/c, 2], 0]))$

Rule 618

$\text{Int}[\frac{(a + b*x + c*x^2)^{-1}}{b^2 - 4*a*c - x^2}, x_Symbol] := \text{Dist}[-2, \text{Subst}[\text{Int}[1/\text{Simp}[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /;$
 $\text{FreeQ}\{a, b, c, x\} \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0]$

Rule 204

$\text{Int}[\frac{(a + b*x)^{-1}}{b^2 - 4*a*c - x^2}, x_Symbol] := -\text{Simp}[\text{ArcTan}[\text{Rt}[-b, 2]*x]/\text{Rt}[-a, 2]]/(\text{Rt}[-a, 2]*\text{Rt}[-b, 2]), x] /;$
 $\text{FreeQ}\{a, b\}, x \ \&\& \ \text{PosQ}[a/b] \ \&\& \ (\text{LtQ}[a, 0] \ || \ \text{LtQ}[b, 0])$

Rubi steps

$$\int \frac{d + ex^2}{\frac{cd^2}{e^2} + bx^2 + cx^4} dx = \frac{e \int \frac{1}{\frac{d}{e} - \frac{\sqrt{2cd-be}}{\sqrt{c}\sqrt{e}} + x^2} dx}{2c} + \frac{e \int \frac{1}{\frac{d}{e} + \frac{\sqrt{2cd-be}}{\sqrt{c}\sqrt{e}} + x^2} dx}{2c}$$

$$= \frac{e \operatorname{Subst}\left(\int \frac{1}{\frac{b}{c} - \frac{2d}{e} - x^2} dx, x, -\frac{\sqrt{2cd-be}}{\sqrt{c}\sqrt{e}} + 2x\right)}{c} - \frac{e \operatorname{Subst}\left(\int \frac{1}{\frac{b}{c} - \frac{2d}{e} - x^2} dx, x, \frac{\sqrt{2cd-be}}{\sqrt{c}\sqrt{e}} + 2x\right)}{c}$$

$$= -\frac{e^{3/2} \tan^{-1}\left(\frac{\sqrt{2cd-be-2\sqrt{c}\sqrt{e}x}}{\sqrt{2cd+be}}\right)}{\sqrt{c}\sqrt{2cd+be}} + \frac{e^{3/2} \tan^{-1}\left(\frac{\sqrt{2cd-be+2\sqrt{c}\sqrt{e}x}}{\sqrt{2cd+be}}\right)}{\sqrt{c}\sqrt{2cd+be}}$$

Mathematica [A] time = 0.121889, size = 248, normalized size = 1.91

$$e^{3/2} \left(\frac{\left(\sqrt{b^2e^2 - 4c^2d^2 - be + 2cd} \right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{e}x}{\sqrt{be - \sqrt{b^2e^2 - 4c^2d^2}}} \right)}{\sqrt{be - \sqrt{b^2e^2 - 4c^2d^2}}} + \frac{\left(\sqrt{b^2e^2 - 4c^2d^2 + be - 2cd} \right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{e}x}{\sqrt{\sqrt{b^2e^2 - 4c^2d^2} + be}} \right)}{\sqrt{\sqrt{b^2e^2 - 4c^2d^2} + be}} \right) \frac{1}{\sqrt{2}\sqrt{c}\sqrt{b^2e^2 - 4c^2d^2}}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e*x^2)/((c*d^2)/e^2 + b*x^2 + c*x^4), x]

[Out] (e^(3/2)*(((2*c*d - b*e + Sqrt[-4*c^2*d^2 + b^2*e^2])*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[e]*x)/Sqrt[b*e - Sqrt[-4*c^2*d^2 + b^2*e^2]]])/Sqrt[b*e - Sqrt[-4*c^2*d^2 + b^2*e^2]] + ((-2*c*d + b*e + Sqrt[-4*c^2*d^2 + b^2*e^2])*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[e]*x)/Sqrt[b*e + Sqrt[-4*c^2*d^2 + b^2*e^2]]])/Sqrt[b*e + Sqrt[-4*c^2*d^2 + b^2*e^2]]))/(Sqrt[2]*Sqrt[c]*Sqrt[-4*c^2*d^2 + b^2*e^2])

Maple [B] time = 0.223, size = 582, normalized size = 4.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x^2+d)/(c*d^2/e^2+b*x^2+c*x^4), x)

[Out] 1/2*e^4/(e^2*(b*e-2*c*d)*(b*e+2*c*d))^(1/2)*2^(1/2)/((-b*e^2+(e^2*(b*e-2*c*d)*(b*e+2*c*d))^(1/2))*c)^(1/2)*arctanh(c*e*x^2^(1/2)/((-b*e^2+(e^2*(b*e-2*c*d)*(b*e+2*c*d))^(1/2))*c)^(1/2))*b-e^3*c/(e^2*(b*e-2*c*d)*(b*e+2*c*d))^(1/2)*2^(1/2)/((-b*e^2+(e^2*(b*e-2*c*d)*(b*e+2*c*d))^(1/2))*c)^(1/2)*arctanh(c*e*x^2^(1/2)/((-b*e^2+(e^2*(b*e-2*c*d)*(b*e+2*c*d))^(1/2))*c)^(1/2))*d-1/2*e^2*2^(1/2)/((-b*e^2+(e^2*(b*e-2*c*d)*(b*e+2*c*d))^(1/2))*c)^(1/2)*arctanh(c*e*x^2^(1/2)/((-b*e^2+(e^2*(b*e-2*c*d)*(b*e+2*c*d))^(1/2))*c)^(1/2))+1/2*e^4/(e^2*(b*e-2*c*d)*(b*e+2*c*d))^(1/2)*2^(1/2)/((b*e^2+(e^2*(b*e-2*c*d)*(b*e+2*c*d))^(1/2))*c)^(1/2)*arctan(c*e*x^2^(1/2)/((b*e^2+(e^2*(b*e-2*c*d)*(b*e+2*c*d))^(1/2))*c)^(1/2))*b-e^3*c/(e^2*(b*e-2*c*d)*(b*e+2*c*d))^(1/2)*2^(1/2)/((b*e^2+(e^2*(b*e-2*c*d)*(b*e+2*c*d))^(1/2))*c)^(1/2)*arctan(c*e*x^2^(1/2)/((b*e^2+(e^2*(b*e-2*c*d)*(b*e+2*c*d))^(1/2))*c)^(1/2))*d+1/2*e^2*2^(1/2)/((b*e^2+(e^2*(b*e-2*c*d)*(b*e+2*c*d))^(1/2))*c)^(1/2)*arctan(c*e*x^2^(1/2)/((b*e^2+(e^2*(b*e-2*c*d)*(b*e+2*c*d))^(1/2))*c)^(1/2))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex^2 + d}{cx^4 + bx^2 + \frac{cd^2}{e^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)/(c*d^2/e^2+b*x^2+c*x^4),x, algorithm="maxima")

[Out] integrate((e*x^2 + d)/(c*x^4 + b*x^2 + c*d^2/e^2), x)

Fricas [A] time = 1.3234, size = 490, normalized size = 3.77

$$\left[\frac{1}{2} e \sqrt{-\frac{e}{2c^2d + bce}} \log \left(\frac{ce^2x^4 + cd^2 - (4cde + be^2)x^2 + 2((2c^2de + bce^2)x^3 - (2c^2d^2 + bcde)x) \sqrt{-\frac{e}{2c^2d + bce}}}{ce^2x^4 + be^2x^2 + cd^2} \right), e \sqrt{\frac{e}{2c^2d + bce}} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)/(c*d^2/e^2+b*x^2+c*x^4),x, algorithm="fricas")

[Out] [1/2*e*sqrt(-e/(2*c^2*d + b*c*e))*log((c*e^2*x^4 + c*d^2 - (4*c*d*e + b*e^2)*x^2 + 2*((2*c^2*d*e + b*c*e^2)*x^3 - (2*c^2*d^2 + b*c*d*e)*x)*sqrt(-e/(2*c^2*d + b*c*e)))/(c*e^2*x^4 + b*e^2*x^2 + c*d^2), e*sqrt(e/(2*c^2*d + b*c*e))*arctan(c*x*sqrt(e/(2*c^2*d + b*c*e))) + e*sqrt(e/(2*c^2*d + b*c*e))*arctan((c*e*x^3 + (c*d + b*e)*x)*sqrt(e/(2*c^2*d + b*c*e))/d)]

Sympy [A] time = 0.777016, size = 160, normalized size = 1.23

$$\frac{\sqrt{-\frac{e^3}{c(be+2cd)}} \log \left(-\frac{d}{e} + x^2 + \frac{x \left(-be \sqrt{-\frac{e^3}{c(be+2cd)}} - 2cd \sqrt{-\frac{e^3}{c(be+2cd)}} \right)}{e^2} \right)}{2} + \frac{\sqrt{-\frac{e^3}{c(be+2cd)}} \log \left(-\frac{d}{e} + x^2 + \frac{x \left(be \sqrt{-\frac{e^3}{c(be+2cd)}} + 2cd \sqrt{-\frac{e^3}{c(be+2cd)}} \right)}{e^2} \right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x**2+d)/(c*d**2/e**2+b*x**2+c*x**4),x)

[Out] -sqrt(-e**3/(c*(b*e + 2*c*d)))*log(-d/e + x**2 + x*(-b*e*sqrt(-e**3/(c*(b*e + 2*c*d))) - 2*c*d*sqrt(-e**3/(c*(b*e + 2*c*d))))/e**2)/2 + sqrt(-e**3/(c*(b*e + 2*c*d)))*log(-d/e + x**2 + x*(b*e*sqrt(-e**3/(c*(b*e + 2*c*d))) + 2*c*d*sqrt(-e**3/(c*(b*e + 2*c*d))))/e**2)/2

Giac [C] time = 2.41465, size = 6884, normalized size = 52.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(c*d^2/e^2+b*x^2+c*x^4),x, algorithm="giac")
```

```
[Out] 1/2*(3*(4*c^3*(d^2)^(3/4)*d^2*e^(9/2) - b^2*c*(d^2)^(3/4)*e^(13/2) - sqrt(-
4*c^2*d^2*e^2 + b^2*e^4)*b*c*(d^2)^(3/4)*e^(9/2))*cos(5/4*pi + 1/2*real_par
t(arcsin(1/2*b*e/(c*abs(d)))))^2*cosh(1/2*imag_part(arcsin(1/2*b*e/(c*abs(d)
))))^3*e*sin(5/4*pi + 1/2*real_part(arcsin(1/2*b*e/(c*abs(d))))) - (4*c^3*
(d^2)^(3/4)*d^2*e^(9/2) - b^2*c*(d^2)^(3/4)*e^(13/2) - sqrt(-4*c^2*d^2*e^2
+ b^2*e^4)*b*c*(d^2)^(3/4)*e^(9/2))*cosh(1/2*imag_part(arcsin(1/2*b*e/(c*ab
s(d)))))^3*e*sin(5/4*pi + 1/2*real_part(arcsin(1/2*b*e/(c*abs(d)))))^3 - 9*
(4*c^3*(d^2)^(3/4)*d^2*e^(9/2) - b^2*c*(d^2)^(3/4)*e^(13/2) - sqrt(-4*c^2*d
^2*e^2 + b^2*e^4)*b*c*(d^2)^(3/4)*e^(9/2))*cos(5/4*pi + 1/2*real_part(arcsi
n(1/2*b*e/(c*abs(d)))))^2*cosh(1/2*imag_part(arcsin(1/2*b*e/(c*abs(d)))))^2
*e*sin(5/4*pi + 1/2*real_part(arcsin(1/2*b*e/(c*abs(d)))))*sinh(1/2*imag_pa
rt(arcsin(1/2*b*e/(c*abs(d))))) + 3*(4*c^3*(d^2)^(3/4)*d^2*e^(9/2) - b^2*c*
(d^2)^(3/4)*e^(13/2) - sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*b*c*(d^2)^(3/4)*e^(9/
2))*cosh(1/2*imag_part(arcsin(1/2*b*e/(c*abs(d)))))^2*e*sin(5/4*pi + 1/2*re
al_part(arcsin(1/2*b*e/(c*abs(d)))))^3*sinh(1/2*imag_part(arcsin(1/2*b*e/(c
*abs(d))))) + 9*(4*c^3*(d^2)^(3/4)*d^2*e^(9/2) - b^2*c*(d^2)^(3/4)*e^(13/2)
- sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*b*c*(d^2)^(3/4)*e^(9/2))*cos(5/4*pi + 1/2
*real_part(arcsin(1/2*b*e/(c*abs(d)))))^2*cosh(1/2*imag_part(arcsin(1/2*b*e
/(c*abs(d)))))*e*sin(5/4*pi + 1/2*real_part(arcsin(1/2*b*e/(c*abs(d)))))*si
nh(1/2*imag_part(arcsin(1/2*b*e/(c*abs(d)))))^2 - 3*(4*c^3*(d^2)^(3/4)*d^2*
e^(9/2) - b^2*c*(d^2)^(3/4)*e^(13/2) - sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*b*c*(
d^2)^(3/4)*e^(9/2))*cosh(1/2*imag_part(arcsin(1/2*b*e/(c*abs(d)))))*e*sin(5
/4*pi + 1/2*real_part(arcsin(1/2*b*e/(c*abs(d)))))^3*sinh(1/2*imag_part(arc
sin(1/2*b*e/(c*abs(d)))))^2 - 3*(4*c^3*(d^2)^(3/4)*d^2*e^(9/2) - b^2*c*(d^2
)^(3/4)*e^(13/2) - sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*b*c*(d^2)^(3/4)*e^(9/2))*
cos(5/4*pi + 1/2*real_part(arcsin(1/2*b*e/(c*abs(d)))))^2*e*sin(5/4*pi + 1/
2*real_part(arcsin(1/2*b*e/(c*abs(d)))))*sinh(1/2*imag_part(arcsin(1/2*b*e/
(c*abs(d)))))^3 + (4*c^3*(d^2)^(3/4)*d^2*e^(9/2) - b^2*c*(d^2)^(3/4)*e^(13/
2) - sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*b*c*(d^2)^(3/4)*e^(9/2))*e*sin(5/4*pi +
1/2*real_part(arcsin(1/2*b*e/(c*abs(d)))))^3*sinh(1/2*imag_part(arcsin(1/2
*b*e/(c*abs(d)))))^3 + (4*c^3*(d^2)^(1/4)*d^3*e^(11/2) - b^2*c*(d^2)^(1/4)*
d*e^(15/2) - sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*b*c*(d^2)^(1/4)*d*e^(11/2))*cos
h(1/2*imag_part(arcsin(1/2*b*e/(c*abs(d)))))*sin(5/4*pi + 1/2*real_part(arc
sin(1/2*b*e/(c*abs(d))))) - (4*c^3*(d^2)^(1/4)*d^3*e^(11/2) - b^2*c*(d^2)^(
1/4)*d*e^(15/2) - sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*b*c*(d^2)^(1/4)*d*e^(11/2)
)*sin(5/4*pi + 1/2*real_part(arcsin(1/2*b*e/(c*abs(d)))))*sinh(1/2*imag_par
t(arcsin(1/2*b*e/(c*abs(d)))))*arctan(-((d^2)^(1/4)*cos(5/4*pi + 1/2*arcsi
n(1/2*b*e/(c*abs(d)))))e^(-1/2) - x)*e^(1/2)/((d^2)^(1/4)*sin(5/4*pi + 1/2*
arcsin(1/2*b*e/(c*abs(d)))))/(4*c^4*d^4*e^4 - b^2*c^2*d^2*e^6) + 1/2*(3*(4
*c^3*(d^2)^(3/4)*d^2*e^(9/2) - b^2*c*(d^2)^(3/4)*e^(13/2) - sqrt(-4*c^2*d^2
*e^2 + b^2*e^4)*b*c*(d^2)^(3/4)*e^(9/2))*cos(1/4*pi + 1/2*real_part(arcsin(
1/2*b*e/(c*abs(d)))))^2*cosh(1/2*imag_part(arcsin(1/2*b*e/(c*abs(d)))))^3*e
*sin(1/4*pi + 1/2*real_part(arcsin(1/2*b*e/(c*abs(d))))) - (4*c^3*(d^2)^(3/
4)*d^2*e^(9/2) - b^2*c*(d^2)^(3/4)*e^(13/2) - sqrt(-4*c^2*d^2*e^2 + b^2*e^4
)*b*c*(d^2)^(3/4)*e^(9/2))*cosh(1/2*imag_part(arcsin(1/2*b*e/(c*abs(d)))))^
3*e*sin(1/4*pi + 1/2*real_part(arcsin(1/2*b*e/(c*abs(d)))))^3 - 9*(4*c^3*(d
^2)^(3/4)*d^2*e^(9/2) - b^2*c*(d^2)^(3/4)*e^(13/2) - sqrt(-4*c^2*d^2*e^2 +
b^2*e^4)*b*c*(d^2)^(3/4)*e^(9/2))*cos(1/4*pi + 1/2*real_part(arcsin(1/2*b*e
/(c*abs(d)))))^2*cosh(1/2*imag_part(arcsin(1/2*b*e/(c*abs(d)))))^2*e*sin(1/
4*pi + 1/2*real_part(arcsin(1/2*b*e/(c*abs(d)))))*sinh(1/2*imag_part(arcsin
(1/2*b*e/(c*abs(d))))) + 3*(4*c^3*(d^2)^(3/4)*d^2*e^(9/2) - b^2*c*(d^2)^(3/
4)*e^(13/2) - sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*b*c*(d^2)^(3/4)*e^(9/2))*cosh(
1/2*imag_part(arcsin(1/2*b*e/(c*abs(d)))))^2*e*sin(1/4*pi + 1/2*real_part(a
rcsin(1/2*b*e/(c*abs(d)))))^3*sinh(1/2*imag_part(arcsin(1/2*b*e/(c*abs(d))
))) + 9*(4*c^3*(d^2)^(3/4)*d^2*e^(9/2) - b^2*c*(d^2)^(3/4)*e^(13/2) - sqrt(-
4*c^2*d^2*e^2 + b^2*e^4)*b*c*(d^2)^(3/4)*e^(9/2))*cos(1/4*pi + 1/2*real_par
t(arcsin(1/2*b*e/(c*abs(d)))))^2*cosh(1/2*imag_part(arcsin(1/2*b*e/(c*abs(d)
```


$$3.36 \quad \int \frac{d+ex^2}{bx^2+c\left(\frac{d^2}{e^2}+x^4\right)} dx$$

Optimal. Leaf size=130

$$\frac{e^{3/2} \tan^{-1}\left(\frac{\sqrt{2cd-be+2\sqrt{c}\sqrt{ex}}}{\sqrt{be+2cd}}\right)}{\sqrt{c}\sqrt{be+2cd}} - \frac{e^{3/2} \tan^{-1}\left(\frac{\sqrt{2cd-be-2\sqrt{c}\sqrt{ex}}}{\sqrt{be+2cd}}\right)}{\sqrt{c}\sqrt{be+2cd}}$$

[Out] $-\left(\frac{e^{3/2} \operatorname{ArcTan}\left[\left(\frac{\sqrt{2cd-be+2\sqrt{c}\sqrt{ex}}}{\sqrt{be+2cd}}\right) - 2\sqrt{c}\sqrt{e}x\right]}{\sqrt{2cd+be}}\right) + \left(\frac{e^{3/2} \operatorname{ArcTan}\left[\left(\frac{\sqrt{2cd-be-2\sqrt{c}\sqrt{ex}}}{\sqrt{be+2cd}}\right) + 2\sqrt{c}\sqrt{e}x\right]}{\sqrt{2cd+be}}\right)$

Rubi [A] time = 0.130766, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {1990, 1161, 618, 204}

$$\frac{e^{3/2} \tan^{-1}\left(\frac{\sqrt{2cd-be+2\sqrt{c}\sqrt{ex}}}{\sqrt{be+2cd}}\right)}{\sqrt{c}\sqrt{be+2cd}} - \frac{e^{3/2} \tan^{-1}\left(\frac{\sqrt{2cd-be-2\sqrt{c}\sqrt{ex}}}{\sqrt{be+2cd}}\right)}{\sqrt{c}\sqrt{be+2cd}}$$

Antiderivative was successfully verified.

[In] `Int[(d + e*x^2)/(b*x^2 + c*(d^2/e^2 + x^4)),x]`

[Out] $-\left(\frac{e^{3/2} \operatorname{ArcTan}\left[\left(\frac{\sqrt{2cd-be+2\sqrt{c}\sqrt{ex}}}{\sqrt{be+2cd}}\right) - 2\sqrt{c}\sqrt{e}x\right]}{\sqrt{2cd+be}}\right) + \left(\frac{e^{3/2} \operatorname{ArcTan}\left[\left(\frac{\sqrt{2cd-be-2\sqrt{c}\sqrt{ex}}}{\sqrt{be+2cd}}\right) + 2\sqrt{c}\sqrt{e}x\right]}{\sqrt{2cd+be}}\right)$

Rule 1990

`Int[(u_)^(q_.)*(v_)^(p_.), x_Symbol] := Int[ExpandToSum[u, x]^q*ExpandToSum[v, x]^p, x] /; FreeQ[{p, q}, x] && BinomialQ[u, x] && TrinomialQ[v, x] && !(BinomialMatchQ[u, x] && TrinomialMatchQ[v, x])`

Rule 1161

`Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (GtQ[(2*d)/e - b/c, 0] || (!LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2], 0]))`

Rule 618

`Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]`

Rule 204

`Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])`

Rubi steps

$$\begin{aligned}
 \int \frac{d + ex^2}{bx^2 + c\left(\frac{d^2}{e^2} + x^4\right)} dx &= \int \frac{d + ex^2}{\frac{cd^2}{e^2} + bx^2 + cx^4} dx \\
 &= \frac{e \int \frac{1}{\frac{d}{e} - \frac{\sqrt{2cd-bex}}{\sqrt{c}\sqrt{e}} + x^2} dx}{2c} + \frac{e \int \frac{1}{\frac{d}{e} + \frac{\sqrt{2cd-bex}}{\sqrt{c}\sqrt{e}} + x^2} dx}{2c} \\
 &= -\frac{e \operatorname{Subst}\left(\int \frac{1}{-\frac{b}{c} - \frac{2d}{e} - x^2} dx, x, -\frac{\sqrt{2cd-be}}{\sqrt{c}\sqrt{e}} + 2x\right)}{c} - \frac{e \operatorname{Subst}\left(\int \frac{1}{-\frac{b}{c} - \frac{2d}{e} - x^2} dx, x, \frac{\sqrt{2cd-be}}{\sqrt{c}\sqrt{e}} + 2x\right)}{c} \\
 &= -\frac{e^{3/2} \tan^{-1}\left(\frac{\sqrt{2cd-be}-2\sqrt{c}\sqrt{ex}}{\sqrt{2cd+be}}\right)}{\sqrt{c}\sqrt{2cd+be}} + \frac{e^{3/2} \tan^{-1}\left(\frac{\sqrt{2cd-be}+2\sqrt{c}\sqrt{ex}}{\sqrt{2cd+be}}\right)}{\sqrt{c}\sqrt{2cd+be}}
 \end{aligned}$$

Mathematica [A] time = 0.0444802, size = 248, normalized size = 1.91

$$\frac{e^{3/2} \left(\frac{\left(\sqrt{b^2e^2-4c^2d^2}-be+2cd\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{ex}}{\sqrt{be-\sqrt{b^2e^2-4c^2d^2}}}\right)}{\sqrt{be-\sqrt{b^2e^2-4c^2d^2}}} + \frac{\left(\sqrt{b^2e^2-4c^2d^2}+be-2cd\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{ex}}{\sqrt{\sqrt{b^2e^2-4c^2d^2}+be}}\right)}{\sqrt{\sqrt{b^2e^2-4c^2d^2}+be}} \right)}{\sqrt{2}\sqrt{c}\sqrt{b^2e^2-4c^2d^2}}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e*x^2)/(b*x^2 + c*(d^2/e^2 + x^4)), x]

[Out] (e^(3/2)*(((2*c*d - b*e + Sqrt[-4*c^2*d^2 + b^2*e^2])*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[e]*x)/Sqrt[b*e - Sqrt[-4*c^2*d^2 + b^2*e^2]]])/Sqrt[b*e - Sqrt[-4*c^2*d^2 + b^2*e^2]] + ((-2*c*d + b*e + Sqrt[-4*c^2*d^2 + b^2*e^2])*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[e]*x)/Sqrt[b*e + Sqrt[-4*c^2*d^2 + b^2*e^2]]])/Sqrt[b*e + Sqrt[-4*c^2*d^2 + b^2*e^2]])/(Sqrt[2]*Sqrt[c]*Sqrt[-4*c^2*d^2 + b^2*e^2])

Maple [B] time = 0.212, size = 582, normalized size = 4.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x^2+d)/(b*x^2+c*(d^2/e^2+x^4)), x)

[Out] 1/2*e^4/(e^2*(b*e-2*c*d)*(b*e+2*c*d))^(1/2)*2^(1/2)/((-b*e^2+(e^2*(b*e-2*c*d)*(b*e+2*c*d))^(1/2))*c)^(1/2)*arctanh(c*e*x^2^(1/2)/((-b*e^2+(e^2*(b*e-2*c*d)*(b*e+2*c*d))^(1/2))*c)^(1/2))*b-e^3*c/(e^2*(b*e-2*c*d)*(b*e+2*c*d))^(1/2)*2^(1/2)/((-b*e^2+(e^2*(b*e-2*c*d)*(b*e+2*c*d))^(1/2))*c)^(1/2)*arctanh(c*e*x^2^(1/2)/((-b*e^2+(e^2*(b*e-2*c*d)*(b*e+2*c*d))^(1/2))*c)^(1/2))*d-1/2*e^2*2^(1/2)/((-b*e^2+(e^2*(b*e-2*c*d)*(b*e+2*c*d))^(1/2))*c)^(1/2)*arctanh(c*e*x^2^(1/2)/((-b*e^2+(e^2*(b*e-2*c*d)*(b*e+2*c*d))^(1/2))*c)^(1/2))+1/2*e^4/(e^2*(b*e-2*c*d)*(b*e+2*c*d))^(1/2)*2^(1/2)/((b*e^2+(e^2*(b*e-2*c*d)*(b*e+2*c*d))^(1/2))*c)^(1/2)*arctan(c*e*x^2^(1/2)/((b*e^2+(e^2*(b*e-2*c*d)*(b*e+2*c*d))^(1/2))*c)^(1/2))*b-e^3*c/(e^2*(b*e-2*c*d)*(b*e+2*c*d))^(1/2)*2^(1/2)/((b*e^2+(e^2*(b*e-2*c*d)*(b*e+2*c*d))^(1/2))*c)^(1/2)*arctan(c*e*x^2^(1/2)/((b*e^2+(e^2*(b*e-2*c*d)*(b*e+2*c*d))^(1/2))*c)^(1/2))

$$\frac{1/2}{((b*e^2+(e^2*(b*e-2*c*d)*(b*e+2*c*d))^(1/2))*c)^(1/2)}*d+1/2*e^2*2^(1/2)/((b*e^2+(e^2*(b*e-2*c*d)*(b*e+2*c*d))^(1/2))*c)^(1/2)*\arctan(c*e*x^2^(1/2)/((b*e^2+(e^2*(b*e-2*c*d)*(b*e+2*c*d))^(1/2))*c)^(1/2))$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex^2 + d}{bx^2 + \left(x^4 + \frac{d^2}{e^2}\right)c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)/(b*x^2+c*(d^2/e^2+x^4)),x, algorithm="maxima")

[Out] integrate((e*x^2 + d)/(b*x^2 + (x^4 + d^2/e^2)*c), x)

Fricas [A] time = 1.35055, size = 490, normalized size = 3.77

$$\left[\frac{1}{2} e \sqrt{-\frac{e}{2c^2d + bce}} \log \left(\frac{ce^2x^4 + cd^2 - (4cde + be^2)x^2 + 2((2c^2de + bce^2)x^3 - (2c^2d^2 + bcde)x) \sqrt{-\frac{e}{2c^2d + bce}}}{ce^2x^4 + be^2x^2 + cd^2} \right), e \sqrt{\frac{e}{2c^2d + bce}} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)/(b*x^2+c*(d^2/e^2+x^4)),x, algorithm="fricas")

[Out] [1/2*e*sqrt(-e/(2*c^2*d + b*c*e))*log((c*e^2*x^4 + c*d^2 - (4*c*d*e + b*e^2)*x^2 + 2*((2*c^2*d*e + b*c*e^2)*x^3 - (2*c^2*d^2 + b*c*d*e)*x)*sqrt(-e/(2*c^2*d + b*c*e)))/(c*e^2*x^4 + b*e^2*x^2 + c*d^2)), e*sqrt(e/(2*c^2*d + b*c*e))*arctan(c*x*sqrt(e/(2*c^2*d + b*c*e))) + e*sqrt(e/(2*c^2*d + b*c*e))*arctan((c*e*x^3 + (c*d + b*e)*x)*sqrt(e/(2*c^2*d + b*c*e))/d)]

Sympy [A] time = 0.783604, size = 160, normalized size = 1.23

$$\frac{\sqrt{-\frac{e^3}{c(be+2cd)}} \log \left(-\frac{d}{e} + x^2 + \frac{x \left(-be \sqrt{-\frac{e^3}{c(be+2cd)}} - 2cd \sqrt{-\frac{e^3}{c(be+2cd)}} \right)}{e^2} \right)}{2} + \frac{\sqrt{-\frac{e^3}{c(be+2cd)}} \log \left(-\frac{d}{e} + x^2 + \frac{x \left(be \sqrt{-\frac{e^3}{c(be+2cd)}} + 2cd \sqrt{-\frac{e^3}{c(be+2cd)}} \right)}{e^2} \right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x**2+d)/(b*x**2+c*(d**2/e**2+x**4)),x)

[Out] -sqrt(-e**3/(c*(b*e + 2*c*d)))*log(-d/e + x**2 + x*(-b*e*sqrt(-e**3/(c*(b*e + 2*c*d))) - 2*c*d*sqrt(-e**3/(c*(b*e + 2*c*d))))/e**2)/2 + sqrt(-e**3/(c*(b*e + 2*c*d)))*log(-d/e + x**2 + x*(b*e*sqrt(-e**3/(c*(b*e + 2*c*d))) + 2*c*d*sqrt(-e**3/(c*(b*e + 2*c*d))))/e**2)/2

Giac [C] time = 2.44217, size = 6884, normalized size = 52.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(b*x^2+c*(d^2/e^2+x^4)),x, algorithm="giac")
```

```
[Out] 1/2*(3*(4*c^3*(d^2)^(3/4)*d^2*e^(9/2) - b^2*c*(d^2)^(3/4)*e^(13/2) - sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*b*c*(d^2)^(3/4)*e^(9/2))*cos(5/4*pi + 1/2*real_part(arcsin(1/2*b*e/(c*abs(d))))))^2*cosh(1/2*imag_part(arcsin(1/2*b*e/(c*abs(d))))))^3*e*sin(5/4*pi + 1/2*real_part(arcsin(1/2*b*e/(c*abs(d)))))) - (4*c^3*(d^2)^(3/4)*d^2*e^(9/2) - b^2*c*(d^2)^(3/4)*e^(13/2) - sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*b*c*(d^2)^(3/4)*e^(9/2))*cosh(1/2*imag_part(arcsin(1/2*b*e/(c*abs(d))))))^3*e*sin(5/4*pi + 1/2*real_part(arcsin(1/2*b*e/(c*abs(d)))))) - 9*(4*c^3*(d^2)^(3/4)*d^2*e^(9/2) - b^2*c*(d^2)^(3/4)*e^(13/2) - sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*b*c*(d^2)^(3/4)*e^(9/2))*cos(5/4*pi + 1/2*real_part(arcsin(1/2*b*e/(c*abs(d))))))^2*cosh(1/2*imag_part(arcsin(1/2*b*e/(c*abs(d))))))^2*e*sin(5/4*pi + 1/2*real_part(arcsin(1/2*b*e/(c*abs(d))))))*sinh(1/2*imag_part(arcsin(1/2*b*e/(c*abs(d)))))) + 3*(4*c^3*(d^2)^(3/4)*d^2*e^(9/2) - b^2*c*(d^2)^(3/4)*e^(13/2) - sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*b*c*(d^2)^(3/4)*e^(9/2))*cosh(1/2*imag_part(arcsin(1/2*b*e/(c*abs(d))))))^2*e*sin(5/4*pi + 1/2*real_part(arcsin(1/2*b*e/(c*abs(d))))))^3*sinh(1/2*imag_part(arcsin(1/2*b*e/(c*abs(d)))))) + 9*(4*c^3*(d^2)^(3/4)*d^2*e^(9/2) - b^2*c*(d^2)^(3/4)*e^(13/2) - sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*b*c*(d^2)^(3/4)*e^(9/2))*cos(5/4*pi + 1/2*real_part(arcsin(1/2*b*e/(c*abs(d))))))^2*cosh(1/2*imag_part(arcsin(1/2*b*e/(c*abs(d))))))*e*sin(5/4*pi + 1/2*real_part(arcsin(1/2*b*e/(c*abs(d))))))*sinh(1/2*imag_part(arcsin(1/2*b*e/(c*abs(d))))))^2 - 3*(4*c^3*(d^2)^(3/4)*d^2*e^(9/2) - b^2*c*(d^2)^(3/4)*e^(13/2) - sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*b*c*(d^2)^(3/4)*e^(9/2))*cosh(1/2*imag_part(arcsin(1/2*b*e/(c*abs(d))))))*e*sin(5/4*pi + 1/2*real_part(arcsin(1/2*b*e/(c*abs(d))))))^3*sinh(1/2*imag_part(arcsin(1/2*b*e/(c*abs(d))))))^2 - 3*(4*c^3*(d^2)^(3/4)*d^2*e^(9/2) - b^2*c*(d^2)^(3/4)*e^(13/2) - sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*b*c*(d^2)^(3/4)*e^(9/2))*cos(5/4*pi + 1/2*real_part(arcsin(1/2*b*e/(c*abs(d))))))^2*e*sin(5/4*pi + 1/2*real_part(arcsin(1/2*b*e/(c*abs(d))))))*sinh(1/2*imag_part(arcsin(1/2*b*e/(c*abs(d))))))^3 + (4*c^3*(d^2)^(3/4)*d^2*e^(9/2) - b^2*c*(d^2)^(3/4)*e^(13/2) - sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*b*c*(d^2)^(3/4)*e^(9/2))*e*sin(5/4*pi + 1/2*real_part(arcsin(1/2*b*e/(c*abs(d))))))^3 + (4*c^3*(d^2)^(1/4)*d^3*e^(11/2) - b^2*c*(d^2)^(1/4)*d*e^(15/2) - sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*b*c*(d^2)^(1/4)*d*e^(11/2))*cosh(1/2*imag_part(arcsin(1/2*b*e/(c*abs(d))))))*sin(5/4*pi + 1/2*real_part(arcsin(1/2*b*e/(c*abs(d)))))) - (4*c^3*(d^2)^(1/4)*d^3*e^(11/2) - b^2*c*(d^2)^(1/4)*d*e^(15/2) - sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*b*c*(d^2)^(1/4)*d*e^(11/2))*sin(5/4*pi + 1/2*real_part(arcsin(1/2*b*e/(c*abs(d))))))*sinh(1/2*imag_part(arcsin(1/2*b*e/(c*abs(d))))))*arctan(-((d^2)^(1/4)*cos(5/4*pi + 1/2*arcsin(1/2*b*e/(c*abs(d)))))*e^(-1/2) - x)*e^(1/2)/((d^2)^(1/4)*sin(5/4*pi + 1/2*arcsin(1/2*b*e/(c*abs(d))))))/(4*c^4*d^4*e^4 - b^2*c^2*d^2*e^6) + 1/2*(3*(4*c^3*(d^2)^(3/4)*d^2*e^(9/2) - b^2*c*(d^2)^(3/4)*e^(13/2) - sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*b*c*(d^2)^(3/4)*e^(9/2))*cos(1/4*pi + 1/2*real_part(arcsin(1/2*b*e/(c*abs(d))))))^2*cosh(1/2*imag_part(arcsin(1/2*b*e/(c*abs(d))))))^3*e*sin(1/4*pi + 1/2*real_part(arcsin(1/2*b*e/(c*abs(d)))))) - (4*c^3*(d^2)^(3/4)*d^2*e^(9/2) - b^2*c*(d^2)^(3/4)*e^(13/2) - sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*b*c*(d^2)^(3/4)*e^(9/2))*cosh(1/2*imag_part(arcsin(1/2*b*e/(c*abs(d))))))^3*e*sin(1/4*pi + 1/2*real_part(arcsin(1/2*b*e/(c*abs(d))))))^3 - 9*(4*c^3*(d^2)^(3/4)*d^2*e^(9/2) - b^2*c*(d^2)^(3/4)*e^(13/2) - sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*b*c*(d^2)^(3/4)*e^(9/2))*cos(1/4*pi + 1/2*real_part(arcsin(1/2*b*e/(c*abs(d))))))^2*cosh(1/2*imag_part(arcsin(1/2*b*e/(c*abs(d))))))^2*e*sin(1/4*pi + 1/2*real_part(arcsin(1/2*b*e/(c*abs(d))))))*sinh(1/2*imag_part(arcsin(1/2*b*e/(c*abs(d)))))) + 3*(4*c^3*(d^2)^(3/4)*d^2*e^(9/2) - b^2*c*(d^2)^(3/4)*e^(13/2) - sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*b*c*(d^2)^(3/4)*e^(9/2))*cosh(1/2*imag_part(arcsin(1/2*b*e/(c*abs(d))))))^2*e*sin(1/4*pi + 1/2*real_part(arcsin(1/2*b*e/(c*abs(d))))))^3*sinh(1/2*imag_part(arcsin(1/2*b*e/(c*abs(d)))))) + 9*(4*c^3*(d^2)^(3/4)*d^2*e^(9/2) - b^2*c*(d^2)^(3/4)*e^(13/2) - sqrt(-
```


$$\begin{aligned}
& 4*c^2*d^2*e^2 + b^2*e^4)*b*c*(d^2)^{(3/4)}*e^{(9/2)}*\cos(1/4*\pi + 1/2*\text{real_part} \\
& (\arcsin(1/2*b*e/(c*\text{abs}(d))))^2*\cosh(1/2*\text{imag_part}(\arcsin(1/2*b*e/(c*\text{abs}(d)))) \\
&)))*e*\sin(1/4*\pi + 1/2*\text{real_part}(\arcsin(1/2*b*e/(c*\text{abs}(d))))*\sinh(1/2*\text{imag_part} \\
& (\arcsin(1/2*b*e/(c*\text{abs}(d))))^2 - 3*(4*c^3*(d^2)^{(3/4)}*d^2*e^{(9/2)} - \\
& b^2*c*(d^2)^{(3/4)}*e^{(13/2)} - \sqrt{-4*c^2*d^2*e^2 + b^2*e^4}*b*c*(d^2)^{(3/4)} \\
&)*e^{(9/2)}*\cosh(1/2*\text{imag_part}(\arcsin(1/2*b*e/(c*\text{abs}(d)))))*e*\sin(1/4*\pi + 1 \\
& /2*\text{real_part}(\arcsin(1/2*b*e/(c*\text{abs}(d))))^3*\sinh(1/2*\text{imag_part}(\arcsin(1/2*b \\
& *e/(c*\text{abs}(d))))^2 - 3*(4*c^3*(d^2)^{(3/4)}*d^2*e^{(9/2)} - b^2*c*(d^2)^{(3/4)}*e \\
& ^{(13/2)} - \sqrt{-4*c^2*d^2*e^2 + b^2*e^4}*b*c*(d^2)^{(3/4)}*e^{(9/2)}*\cos(1/4*\pi \\
& + 1/2*\text{real_part}(\arcsin(1/2*b*e/(c*\text{abs}(d))))^2*e*\sin(1/4*\pi + 1/2*\text{real_part} \\
& (\arcsin(1/2*b*e/(c*\text{abs}(d))))*\sinh(1/2*\text{imag_part}(\arcsin(1/2*b*e/(c*\text{abs}(d) \\
&))))^3 + (4*c^3*(d^2)^{(3/4)}*d^2*e^{(9/2)} - b^2*c*(d^2)^{(3/4)}*e^{(13/2)} - \sqrt{ \\
& (-4*c^2*d^2*e^2 + b^2*e^4)*b*c*(d^2)^{(3/4)}*e^{(9/2)})*e*\sin(1/4*\pi + 1/2*\text{real} \\
& _part(\arcsin(1/2*b*e/(c*\text{abs}(d))))^3*\sinh(1/2*\text{imag_part}(\arcsin(1/2*b*e/(c*a \\
& bs(d))))^3 + (4*c^3*(d^2)^{(1/4)}*d^3*e^{(11/2)} - b^2*c*(d^2)^{(1/4)}*d*e^{(15/2)} \\
&) - \sqrt{-4*c^2*d^2*e^2 + b^2*e^4}*b*c*(d^2)^{(1/4)}*d*e^{(11/2)}*\cosh(1/2*\text{ima} \\
& g_part(\arcsin(1/2*b*e/(c*\text{abs}(d))))*\sin(1/4*\pi + 1/2*\text{real_part}(\arcsin(1/2*b \\
& *e/(c*\text{abs}(d)))) - (4*c^3*(d^2)^{(1/4)}*d^3*e^{(11/2)} - b^2*c*(d^2)^{(1/4)}*d*e^ \\
& ^{(15/2)} - \sqrt{-4*c^2*d^2*e^2 + b^2*e^4}*b*c*(d^2)^{(1/4)}*d*e^{(11/2)})*\sin(1/4 \\
& *\pi + 1/2*\text{real_part}(\arcsin(1/2*b*e/(c*\text{abs}(d))))*\sinh(1/2*\text{imag_part}(\arcsin(\\
& 1/2*b*e/(c*\text{abs}(d)))))*\arctan(-((d^2)^{(1/4)}*\cos(1/4*\pi + 1/2*\arcsin(1/2*b*e \\
& /(c*\text{abs}(d))))*e^{(-1/2)} - x)*e^{(1/2)}/((d^2)^{(1/4)}*\sin(1/4*\pi + 1/2*\arcsin(1/ \\
& 2*b*e/(c*\text{abs}(d)))))/(4*c^4*d^4*e^4 - b^2*c^2*d^2*e^6) - 1/4*((4*c^3*(d^2)^ \\
& (3/4)*d^2*e^{(9/2)} - b^2*c*(d^2)^{(3/4)}*e^{(13/2)} - \sqrt{-4*c^2*d^2*e^2 + b^2* \\
& e^4}*b*c*(d^2)^{(3/4)}*e^{(9/2)})*\cos(5/4*\pi + 1/2*\text{real_part}(\arcsin(1/2*b*e/(c* \\
& \text{abs}(d))))^3*\cosh(1/2*\text{imag_part}(\arcsin(1/2*b*e/(c*\text{abs}(d))))^3*e - 3*(4*c^3 \\
& *(d^2)^{(3/4)}*d^2*e^{(9/2)} - b^2*c*(d^2)^{(3/4)}*e^{(13/2)} - \sqrt{-4*c^2*d^2*e^2 \\
& + b^2*e^4}*b*c*(d^2)^{(3/4)}*e^{(9/2)})*\cos(5/4*\pi + 1/2*\text{real_part}(\arcsin(1/2* \\
& b*e/(c*\text{abs}(d))))*\cosh(1/2*\text{imag_part}(\arcsin(1/2*b*e/(c*\text{abs}(d))))^3*e*\sin(5 \\
& /4*\pi + 1/2*\text{real_part}(\arcsin(1/2*b*e/(c*\text{abs}(d))))^2 - 3*(4*c^3*(d^2)^{(3/4)} \\
& *d^2*e^{(9/2)} - b^2*c*(d^2)^{(3/4)}*e^{(13/2)} - \sqrt{-4*c^2*d^2*e^2 + b^2*e^4}* \\
& b*c*(d^2)^{(3/4)}*e^{(9/2)})*\cos(5/4*\pi + 1/2*\text{real_part}(\arcsin(1/2*b*e/(c*\text{abs}(d) \\
&))))^3*\cosh(1/2*\text{imag_part}(\arcsin(1/2*b*e/(c*\text{abs}(d))))^2*e*\sinh(1/2*\text{imag_p} \\
& art(\arcsin(1/2*b*e/(c*\text{abs}(d)))) + 9*(4*c^3*(d^2)^{(3/4)}*d^2*e^{(9/2)} - b^2*c \\
& *(d^2)^{(3/4)}*e^{(13/2)} - \sqrt{-4*c^2*d^2*e^2 + b^2*e^4}*b*c*(d^2)^{(3/4)}*e^{(9 \\
& /2)})*\cos(5/4*\pi + 1/2*\text{real_part}(\arcsin(1/2*b*e/(c*\text{abs}(d))))*\cosh(1/2*\text{imag_} \\
& part(\arcsin(1/2*b*e/(c*\text{abs}(d))))^2*e*\sin(5/4*\pi + 1/2*\text{real_part}(\arcsin(1/2 \\
& *b*e/(c*\text{abs}(d))))^2*\sinh(1/2*\text{imag_part}(\arcsin(1/2*b*e/(c*\text{abs}(d)))) + 3*(4 \\
& *c^3*(d^2)^{(3/4)}*d^2*e^{(9/2)} - b^2*c*(d^2)^{(3/4)}*e^{(13/2)} - \sqrt{-4*c^2*d^2 \\
& *e^2 + b^2*e^4}*b*c*(d^2)^{(3/4)}*e^{(9/2)})*\cos(5/4*\pi + 1/2*\text{real_part}(\arcsin(\\
& 1/2*b*e/(c*\text{abs}(d))))^3*\cosh(1/2*\text{imag_part}(\arcsin(1/2*b*e/(c*\text{abs}(d)))))*e*s \\
& inh(1/2*\text{imag_part}(\arcsin(1/2*b*e/(c*\text{abs}(d))))^2 - 9*(4*c^3*(d^2)^{(3/4)}*d^2 \\
& *e^{(9/2)} - b^2*c*(d^2)^{(3/4)}*e^{(13/2)} - \sqrt{-4*c^2*d^2*e^2 + b^2*e^4}*b*c* \\
& (d^2)^{(3/4)}*e^{(9/2)})*\cos(5/4*\pi + 1/2*\text{real_part}(\arcsin(1/2*b*e/(c*\text{abs}(d)))) \\
&)*\cosh(1/2*\text{imag_part}(\arcsin(1/2*b*e/(c*\text{abs}(d)))))*e*\sin(5/4*\pi + 1/2*\text{real_p} \\
& art(\arcsin(1/2*b*e/(c*\text{abs}(d))))^2*\sinh(1/2*\text{imag_part}(\arcsin(1/2*b*e/(c*\text{abs} \\
& (d))))^2 - (4*c^3*(d^2)^{(3/4)}*d^2*e^{(9/2)} - b^2*c*(d^2)^{(3/4)}*e^{(13/2)} - s \\
& qrt(-4*c^2*d^2*e^2 + b^2*e^4)*b*c*(d^2)^{(3/4)}*e^{(9/2)})*\cos(5/4*\pi + 1/2*\text{rea} \\
& l_part(\arcsin(1/2*b*e/(c*\text{abs}(d))))^3*e*\sinh(1/2*\text{imag_part}(\arcsin(1/2*b*e/(\\
& c*\text{abs}(d))))^3 + 3*(4*c^3*(d^2)^{(3/4)}*d^2*e^{(9/2)} - b^2*c*(d^2)^{(3/4)}*e^{(13 \\
& /2)} - \sqrt{-4*c^2*d^2*e^2 + b^2*e^4}*b*c*(d^2)^{(3/4)}*e^{(9/2)})*\cos(5/4*\pi + \\
& 1/2*\text{real_part}(\arcsin(1/2*b*e/(c*\text{abs}(d)))))*e*\sin(5/4*\pi + 1/2*\text{real_part}(\ar \\
& csin(1/2*b*e/(c*\text{abs}(d))))^2*\sinh(1/2*\text{imag_part}(\arcsin(1/2*b*e/(c*\text{abs}(d)))) \\
&)^3 + (4*c^3*(d^2)^{(1/4)}*d^3*e^{(11/2)} - b^2*c*(d^2)^{(1/4)}*d*e^{(15/2)} - \sqrt{ \\
& (-4*c^2*d^2*e^2 + b^2*e^4)*b*c*(d^2)^{(1/4)}*d*e^{(11/2)})*\cos(5/4*\pi + 1/2*\text{real} \\
& _part(\arcsin(1/2*b*e/(c*\text{abs}(d))))*\cosh(1/2*\text{imag_part}(\arcsin(1/2*b*e/(c*\text{abs} \\
& (d)))) - (4*c^3*(d^2)^{(1/4)}*d^3*e^{(11/2)} - b^2*c*(d^2)^{(1/4)}*d*e^{(15/2)} - \\
& \sqrt{-4*c^2*d^2*e^2 + b^2*e^4}*b*c*(d^2)^{(1/4)}*d*e^{(11/2)})*\cos(5/4*\pi + 1/2 \\
& *\text{real_part}(\arcsin(1/2*b*e/(c*\text{abs}(d))))*\sinh(1/2*\text{imag_part}(\arcsin(1/2*b*e/(
\end{aligned}$$

$$3.37 \quad \int \frac{a-bx^2}{a^2+(-1+2ab)x^2+b^2x^4} dx$$

Optimal. Leaf size=29

$$\frac{1}{2} \log(a+bx^2+x) - \frac{1}{2} \log(a+bx^2-x)$$

[Out] -Log[a - x + b*x^2]/2 + Log[a + x + b*x^2]/2

Rubi [A] time = 0.0255999, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {1164, 628}

$$\frac{1}{2} \log(a+bx^2+x) - \frac{1}{2} \log(a+bx^2-x)$$

Antiderivative was successfully verified.

[In] Int[(a - b*x^2)/(a^2 + (-1 + 2*a*b)*x^2 + b^2*x^4), x]

[Out] -Log[a - x + b*x^2]/2 + Log[a + x + b*x^2]/2

Rule 1164

Int[((d_) + (e_)*(x_)^2)/((a_) + (b_)*(x_)^2 + (c_)*(x_)^4), x_Symbol] :> With[{q = Rt[(-2*d)/e - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && !GtQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] :> Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rubi steps

$$\begin{aligned} \int \frac{a-bx^2}{a^2+(-1+2ab)x^2+b^2x^4} dx &= -\left(\frac{1}{2} \int \frac{\frac{1}{b}+2x}{-\frac{a}{b}-\frac{x}{b}-x^2} dx\right) - \frac{1}{2} \int \frac{\frac{1}{b}-2x}{-\frac{a}{b}+\frac{x}{b}-x^2} dx \\ &= -\frac{1}{2} \log(a-x+bx^2) + \frac{1}{2} \log(a+x+bx^2) \end{aligned}$$

Mathematica [A] time = 0.0177935, size = 29, normalized size = 1.

$$\frac{1}{2} \log(a+bx^2+x) - \frac{1}{2} \log(a+bx^2-x)$$

Antiderivative was successfully verified.

[In] Integrate[(a - b*x^2)/(a^2 + (-1 + 2*a*b)*x^2 + b^2*x^4), x]

[Out] $-\text{Log}[a - x + b*x^2]/2 + \text{Log}[a + x + b*x^2]/2$

Maple [A] time = 0.049, size = 26, normalized size = 0.9

$$-\frac{\ln(bx^2 + a - x)}{2} + \frac{\ln(bx^2 + a + x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((-b*x^2+a)/(a^2+(2*a*b-1)*x^2+b^2*x^4),x)`

[Out] $-1/2*\ln(b*x^2+a-x)+1/2*\ln(b*x^2+a+x)$

Maxima [A] time = 0.962186, size = 34, normalized size = 1.17

$$\frac{1}{2} \log(bx^2 + a + x) - \frac{1}{2} \log(bx^2 + a - x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-b*x^2+a)/(a^2+(2*a*b-1)*x^2+b^2*x^4),x, algorithm="maxima")`

[Out] $1/2*\log(b*x^2 + a + x) - 1/2*\log(b*x^2 + a - x)$

Fricas [A] time = 1.28171, size = 66, normalized size = 2.28

$$\frac{1}{2} \log(bx^2 + a + x) - \frac{1}{2} \log(bx^2 + a - x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-b*x^2+a)/(a^2+(2*a*b-1)*x^2+b^2*x^4),x, algorithm="fricas")`

[Out] $1/2*\log(b*x^2 + a + x) - 1/2*\log(b*x^2 + a - x)$

Sympy [A] time = 0.50626, size = 26, normalized size = 0.9

$$-\frac{\log\left(\frac{a}{b} + x^2 - \frac{x}{b}\right)}{2} + \frac{\log\left(\frac{a}{b} + x^2 + \frac{x}{b}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-b*x**2+a)/(a**2+(2*a*b-1)*x**2+b**2*x**4),x)`

[Out] $-\log(a/b + x**2 - x/b)/2 + \log(a/b + x**2 + x/b)/2$

Giac [A] time = 1.2268, size = 34, normalized size = 1.17

$$\frac{1}{2} \log(bx^2 + a + x) - \frac{1}{2} \log(bx^2 + a - x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)/(a^2+(2*a*b-1)*x^2+b^2*x^4),x, algorithm="giac")
```

```
[Out] 1/2*log(b*x^2 + a + x) - 1/2*log(b*x^2 + a - x)
```

$$3.38 \quad \int \frac{a+bx^2}{a^2+(-1+2ab)x^2+b^2x^4} dx$$

Optimal. Leaf size=60

$$\frac{\tanh^{-1}\left(\frac{1-2bx}{\sqrt{1-4ab}}\right)}{\sqrt{1-4ab}} - \frac{\tanh^{-1}\left(\frac{2bx+1}{\sqrt{1-4ab}}\right)}{\sqrt{1-4ab}}$$

[Out] ArcTanh[(1 - 2*b*x)/Sqrt[1 - 4*a*b]]/Sqrt[1 - 4*a*b] - ArcTanh[(1 + 2*b*x)/Sqrt[1 - 4*a*b]]/Sqrt[1 - 4*a*b]

Rubi [A] time = 0.0686734, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {1161, 618, 206}

$$\frac{\tanh^{-1}\left(\frac{1-2bx}{\sqrt{1-4ab}}\right)}{\sqrt{1-4ab}} - \frac{\tanh^{-1}\left(\frac{2bx+1}{\sqrt{1-4ab}}\right)}{\sqrt{1-4ab}}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x^2)/(a^2 + (-1 + 2*a*b)*x^2 + b^2*x^4), x]

[Out] ArcTanh[(1 - 2*b*x)/Sqrt[1 - 4*a*b]]/Sqrt[1 - 4*a*b] - ArcTanh[(1 + 2*b*x)/Sqrt[1 - 4*a*b]]/Sqrt[1 - 4*a*b]

Rule 1161

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[(2*d)/e - b/c, 0] || ( !LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[In
t[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{a + bx^2}{a^2 + (-1 + 2ab)x^2 + b^2x^4} dx = \frac{\int \frac{1}{\frac{a}{b} - \frac{x}{b} + x^2} dx}{2b} + \frac{\int \frac{1}{\frac{a}{b} + \frac{x}{b} + x^2} dx}{2b}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{\frac{1-4ab}{b^2} - x^2} dx, x, -\frac{1}{b} + 2x\right)}{b} - \frac{\text{Subst}\left(\int \frac{1}{\frac{1-4ab}{b^2} - x^2} dx, x, \frac{1}{b} + 2x\right)}{b}$$

$$= \frac{\tanh^{-1}\left(\frac{1-2bx}{\sqrt{1-4ab}}\right)}{\sqrt{1-4ab}} - \frac{\tanh^{-1}\left(\frac{1+2bx}{\sqrt{1-4ab}}\right)}{\sqrt{1-4ab}}$$

Mathematica [B] time = 0.199963, size = 138, normalized size = 2.3

$$\frac{(\sqrt{1-4ab}+1) \tan^{-1}\left(\frac{bx}{\sqrt{ab-\frac{1}{2}\sqrt{1-4ab}-\frac{1}{2}}}\right)}{\sqrt{2ab-\sqrt{1-4ab}-1}} + \frac{(\sqrt{1-4ab}-1) \tan^{-1}\left(\frac{\sqrt{2}bx}{\sqrt{2ab+\sqrt{1-4ab}-1}}\right)}{\sqrt{2ab+\sqrt{1-4ab}-1}}$$

$$\sqrt{2-8ab}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x^2)/(a^2 + (-1 + 2*a*b)*x^2 + b^2*x^4), x]

[Out] (((1 + Sqrt[1 - 4*a*b])*ArcTan[(b*x)/Sqrt[-1/2 + a*b - Sqrt[1 - 4*a*b]/2]])/Sqrt[-1 + 2*a*b - Sqrt[1 - 4*a*b]] + ((-1 + Sqrt[1 - 4*a*b])*ArcTan[(Sqrt[2]*b*x)/Sqrt[-1 + 2*a*b + Sqrt[1 - 4*a*b]]])/Sqrt[-1 + 2*a*b + Sqrt[1 - 4*a*b]])/Sqrt[2 - 8*a*b]

Maple [A] time = 0.123, size = 52, normalized size = 0.9

$$\arctan\left((2bx-1)\frac{1}{\sqrt{4ab-1}}\right)\frac{1}{\sqrt{4ab-1}} + \arctan\left((2bx+1)\frac{1}{\sqrt{4ab-1}}\right)\frac{1}{\sqrt{4ab-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^2+a)/(a^2+(2*a*b-1)*x^2+b^2*x^4), x)

[Out] 1/(4*a*b-1)^(1/2)*arctan((2*b*x-1)/(4*a*b-1)^(1/2))+1/(4*a*b-1)^(1/2)*arctan((2*b*x+1)/(4*a*b-1)^(1/2))

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^2+a)/(a^2+(2*a*b-1)*x^2+b^2*x^4), x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [A] time = 1.3072, size = 389, normalized size = 6.48

$$\left[\frac{\sqrt{-4ab+1} \log\left(\frac{b^2x^4 - (6ab-1)x^2 + a^2 - 2(bx^3 - ax)\sqrt{-4ab+1}}{b^2x^4 + (2ab-1)x^2 + a^2}\right)}{2(4ab-1)}, \frac{\sqrt{4ab-1} \arctan\left(\frac{bx}{\sqrt{4ab-1}}\right) + \sqrt{4ab-1} \arctan\left(\frac{(b^2x^3 + (3ab-1)x)\sqrt{4ab-1}}{4a^2b-a}\right)}{4ab-1} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^2+a)/(a^2+(2*a*b-1)*x^2+b^2*x^4),x, algorithm="fricas")

[Out] [-1/2*sqrt(-4*a*b + 1)*log((b^2*x^4 - (6*a*b - 1)*x^2 + a^2 - 2*(b*x^3 - a*x)*sqrt(-4*a*b + 1))/(b^2*x^4 + (2*a*b - 1)*x^2 + a^2))/(4*a*b - 1), (sqrt(4*a*b - 1)*arctan(b*x/sqrt(4*a*b - 1)) + sqrt(4*a*b - 1)*arctan((b^2*x^3 + (3*a*b - 1)*x)*sqrt(4*a*b - 1)/(4*a^2*b - a)))/(4*a*b - 1)]

Sympy [B] time = 0.379932, size = 117, normalized size = 1.95

$$\frac{\sqrt{-\frac{1}{4ab-1}} \log\left(-\frac{a}{b} + x^2 + \frac{x\left(-4ab\sqrt{-\frac{1}{4ab-1}} + \sqrt{-\frac{1}{4ab-1}}\right)}{b}\right)}{2} + \frac{\sqrt{-\frac{1}{4ab-1}} \log\left(-\frac{a}{b} + x^2 + \frac{x\left(4ab\sqrt{-\frac{1}{4ab-1}} - \sqrt{-\frac{1}{4ab-1}}\right)}{b}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**2+a)/(a**2+(2*a*b-1)*x**2+b**2*x**4),x)

[Out] -sqrt(-1/(4*a*b - 1))*log(-a/b + x**2 + x*(-4*a*b*sqrt(-1/(4*a*b - 1)) + sqrt(-1/(4*a*b - 1)))/b)/2 + sqrt(-1/(4*a*b - 1))*log(-a/b + x**2 + x*(4*a*b*sqrt(-1/(4*a*b - 1)) - sqrt(-1/(4*a*b - 1)))/b)/2

Giac [A] time = 1.26325, size = 69, normalized size = 1.15

$$\frac{\arctan\left(\frac{2bx+1}{\sqrt{4ab-1}}\right)}{\sqrt{4ab-1}} + \frac{\arctan\left(\frac{2bx-1}{\sqrt{4ab-1}}\right)}{\sqrt{4ab-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^2+a)/(a^2+(2*a*b-1)*x^2+b^2*x^4),x, algorithm="giac")

[Out] arctan((2*b*x + 1)/sqrt(4*a*b - 1))/sqrt(4*a*b - 1) + arctan((2*b*x - 1)/sqrt(4*a*b - 1))/sqrt(4*a*b - 1)

$$3.39 \quad \int \frac{1+2x^2}{1+bx^2+4x^4} dx$$

Optimal. Leaf size=62

$$\frac{\tan^{-1}\left(\frac{\sqrt{4-b+4x}}{\sqrt{b+4}}\right)}{\sqrt{b+4}} - \frac{\tan^{-1}\left(\frac{\sqrt{4-b-4x}}{\sqrt{b+4}}\right)}{\sqrt{b+4}}$$

[Out] -(ArcTan[(Sqrt[4 - b] - 4*x)/Sqrt[4 + b]]/Sqrt[4 + b]) + ArcTan[(Sqrt[4 - b] + 4*x)/Sqrt[4 + b]]/Sqrt[4 + b]

Rubi [A] time = 0.0579177, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {1161, 618, 204}

$$\frac{\tan^{-1}\left(\frac{\sqrt{4-b+4x}}{\sqrt{b+4}}\right)}{\sqrt{b+4}} - \frac{\tan^{-1}\left(\frac{\sqrt{4-b-4x}}{\sqrt{b+4}}\right)}{\sqrt{b+4}}$$

Antiderivative was successfully verified.

[In] Int[(1 + 2*x^2)/(1 + b*x^2 + 4*x^4), x]

[Out] -(ArcTan[(Sqrt[4 - b] - 4*x)/Sqrt[4 + b]]/Sqrt[4 + b]) + ArcTan[(Sqrt[4 - b] + 4*x)/Sqrt[4 + b]]/Sqrt[4 + b]

Rule 1161

Int[((d_) + (e_)*(x_)^2)/((a_) + (b_)*(x_)^2 + (c_)*(x_)^4), x_Symbol] :> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (GtQ[(2*d)/e - b/c, 0] || (!LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2], 0]))

Rule 618

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 204

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned} \int \frac{1+2x^2}{1+bx^2+4x^4} dx &= \frac{1}{4} \int \frac{1}{\frac{1}{2} - \frac{1}{2}\sqrt{4-bx} + x^2} dx + \frac{1}{4} \int \frac{1}{\frac{1}{2} + \frac{1}{2}\sqrt{4-bx} + x^2} dx \\ &= -\left(\frac{1}{2} \text{Subst}\left(\int \frac{1}{\frac{1}{4}(-4-b) - x^2} dx, x, -\frac{\sqrt{4-b}}{2} + 2x\right)\right) - \frac{1}{2} \text{Subst}\left(\int \frac{1}{\frac{1}{4}(-4-b) - x^2} dx, x, \frac{\sqrt{4-b}}{2}\right) \\ &= -\frac{\tan^{-1}\left(\frac{\sqrt{4-b}-4x}{\sqrt{4+b}}\right)}{\sqrt{4+b}} + \frac{\tan^{-1}\left(\frac{\sqrt{4-b}+4x}{\sqrt{4+b}}\right)}{\sqrt{4+b}} \end{aligned}$$

Mathematica [B] time = 0.0589532, size = 126, normalized size = 2.03

$$\frac{\left(\sqrt{b^2-16-b+4}\right) \tan^{-1}\left(\frac{2\sqrt{2}x}{\sqrt{b-\sqrt{b^2-16}}}\right) + \left(\sqrt{b^2-16+b-4}\right) \tan^{-1}\left(\frac{2\sqrt{2}x}{\sqrt{\sqrt{b^2-16}+b}}\right)}{\sqrt{b-\sqrt{b^2-16}} + \sqrt{\sqrt{b^2-16}+b}}$$

Antiderivative was successfully verified.

[In] Integrate[(1 + 2*x^2)/(1 + b*x^2 + 4*x^4), x]

[Out] (((4 - b + Sqrt[-16 + b^2])*ArcTan[(2*Sqrt[2]*x)/Sqrt[b - Sqrt[-16 + b^2]]])/Sqrt[b - Sqrt[-16 + b^2]] + ((-4 + b + Sqrt[-16 + b^2])*ArcTan[(2*Sqrt[2]*x)/Sqrt[b + Sqrt[-16 + b^2]]])/Sqrt[b + Sqrt[-16 + b^2]])/(Sqrt[2]*Sqrt[-16 + b^2])

Maple [B] time = 0.137, size = 277, normalized size = 4.5

$$-4 \frac{1}{\sqrt{(b-4)(4+b)}\sqrt{2\sqrt{(b-4)(4+b)}+2b}} \arctan\left(4 \frac{x}{\sqrt{2\sqrt{(b-4)(4+b)}+2b}}\right) + \arctan\left(4 \frac{x}{\sqrt{2\sqrt{(b-4)(4+b)}+2b}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*x^2+1)/(4*x^4+b*x^2+1), x)

[Out] -4/((b-4)*(4+b))^(1/2)/(2*((b-4)*(4+b))^(1/2)+2*b)^(1/2)*arctan(4*x/(2*((b-4)*(4+b))^(1/2)+2*b)^(1/2))+1/(2*((b-4)*(4+b))^(1/2)+2*b)^(1/2)*arctan(4*x/(2*((b-4)*(4+b))^(1/2)+2*b)^(1/2))+1/((b-4)*(4+b))^(1/2)/(2*((b-4)*(4+b))^(1/2)+2*b)^(1/2)*arctan(4*x/(2*((b-4)*(4+b))^(1/2)+2*b)^(1/2))*b+4/((b-4)*(4+b))^(1/2)/(-2*((b-4)*(4+b))^(1/2)+2*b)^(1/2)*arctan(4*x/(-2*((b-4)*(4+b))^(1/2)+2*b)^(1/2))+1/(-2*((b-4)*(4+b))^(1/2)+2*b)^(1/2)*arctan(4*x/(-2*((b-4)*(4+b))^(1/2)+2*b)^(1/2))-1/((b-4)*(4+b))^(1/2)/(-2*((b-4)*(4+b))^(1/2)+2*b)^(1/2)*arctan(4*x/(-2*((b-4)*(4+b))^(1/2)+2*b)^(1/2))*b

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{2x^2+1}{4x^4+bx^2+1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2+1)/(4*x^4+b*x^2+1),x, algorithm="maxima")

[Out] integrate((2*x^2 + 1)/(4*x^4 + b*x^2 + 1), x)

Fricas [A] time = 1.35349, size = 286, normalized size = 4.61

$$\left[\frac{\sqrt{-b-4} \log\left(\frac{4x^4-(b+8)x^2-2(2x^3-x)\sqrt{-b-4}+1}{4x^4+bx^2+1}\right)}{2(b+4)}, \frac{\sqrt{b+4} \arctan\left(\frac{4x^3+(b+2)x}{\sqrt{b+4}}\right) + \sqrt{b+4} \arctan\left(\frac{2x}{\sqrt{b+4}}\right)}{b+4} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2+1)/(4*x^4+b*x^2+1),x, algorithm="fricas")

[Out] [-1/2*sqrt(-b - 4)*log((4*x^4 - (b + 8)*x^2 - 2*(2*x^3 - x)*sqrt(-b - 4) + 1)/(4*x^4 + b*x^2 + 1))/(b + 4), (sqrt(b + 4)*arctan((4*x^3 + (b + 2)*x)/sqrt(b + 4)) + sqrt(b + 4)*arctan(2*x/sqrt(b + 4)))/(b + 4)]

Sympy [A] time = 0.27038, size = 95, normalized size = 1.53

$$\frac{\sqrt{-\frac{1}{b+4}} \log\left(x^2 + x\left(-\frac{b\sqrt{-\frac{1}{b+4}}}{2} - 2\sqrt{-\frac{1}{b+4}}\right) - \frac{1}{2}\right)}{2} + \frac{\sqrt{-\frac{1}{b+4}} \log\left(x^2 + x\left(\frac{b\sqrt{-\frac{1}{b+4}}}{2} + 2\sqrt{-\frac{1}{b+4}}\right) - \frac{1}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x**2+1)/(4*x**4+b*x**2+1),x)

[Out] -sqrt(-1/(b + 4))*log(x**2 + x*(-b*sqrt(-1/(b + 4))/2 - 2*sqrt(-1/(b + 4))) - 1/2)/2 + sqrt(-1/(b + 4))*log(x**2 + x*(b*sqrt(-1/(b + 4))/2 + 2*sqrt(-1/(b + 4))) - 1/2)/2

Giac [C] time = 1.44781, size = 3352, normalized size = 54.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2+1)/(4*x^4+b*x^2+1),x, algorithm="giac")

[Out] 1/4*(3*(sqrt(2)*b^2 + sqrt(2)*sqrt(b^2 - 16)*b - 16*sqrt(2))*cos(5/4*pi + 1/2*real_part(arcsin(1/4*b)))^2*cosh(1/2*imag_part(arcsin(1/4*b)))^3*sin(5/4*pi + 1/2*real_part(arcsin(1/4*b))) - (sqrt(2)*b^2 + sqrt(2)*sqrt(b^2 - 16)*b - 16*sqrt(2))*cosh(1/2*imag_part(arcsin(1/4*b)))^3*sin(5/4*pi + 1/2*real_part(arcsin(1/4*b)))^3 - 9*(sqrt(2)*b^2 + sqrt(2)*sqrt(b^2 - 16)*b - 16*sqrt(2))*cos(5/4*pi + 1/2*real_part(arcsin(1/4*b)))^2*cosh(1/2*imag_part(arcsin(1/4*b)))^2*sin(5/4*pi + 1/2*real_part(arcsin(1/4*b)))*sinh(1/2*imag_part(arcsin(1/4*b))) + 3*(sqrt(2)*b^2 + sqrt(2)*sqrt(b^2 - 16)*b - 16*sqrt(2))*cosh(1/2*imag_part(arcsin(1/4*b)))^2*sin(5/4*pi + 1/2*real_part(arcsin(1/4*b)))^3*sinh(1/2*imag_part(arcsin(1/4*b))) + 9*(sqrt(2)*b^2 + sqrt(2)*sqrt(b

$$\begin{aligned}
&^2 - 16)*b - 16*\sqrt{2})*\cos(5/4*\pi + 1/2*\text{real_part}(\arcsin(1/4*b)))^2*\cosh(\\
&1/2*\text{imag_part}(\arcsin(1/4*b)))*\sin(5/4*\pi + 1/2*\text{real_part}(\arcsin(1/4*b)))* \text{si} \\
&\text{nh}(1/2*\text{imag_part}(\arcsin(1/4*b)))^2 - 3*(\sqrt{2}*b^2 + \sqrt{2}*\sqrt{b^2 - 16} \\
&)*b - 16*\sqrt{2})*\cosh(1/2*\text{imag_part}(\arcsin(1/4*b)))*\sin(5/4*\pi + 1/2*\text{real_} \\
&\text{part}(\arcsin(1/4*b)))^3*\sinh(1/2*\text{imag_part}(\arcsin(1/4*b)))^2 - 3*(\sqrt{2}*b^2 \\
&+ \sqrt{2}*\sqrt{b^2 - 16})*b - 16*\sqrt{2})*\cos(5/4*\pi + 1/2*\text{real_part}(\arcsi \\
&n(1/4*b)))^2*\sin(5/4*\pi + 1/2*\text{real_part}(\arcsin(1/4*b)))*\sinh(1/2*\text{imag_part} \\
&(\arcsin(1/4*b)))^3 + (\sqrt{2}*b^2 + \sqrt{2}*\sqrt{b^2 - 16})*b - 16*\sqrt{2})*s \\
&\text{in}(5/4*\pi + 1/2*\text{real_part}(\arcsin(1/4*b)))^3*\sinh(1/2*\text{imag_part}(\arcsin(1/4*b \\
&)))^3 + (\sqrt{2}*b^2 + \sqrt{2}*\sqrt{b^2 - 16})*b - 16*\sqrt{2})*\cosh(1/2*\text{imag} \\
&\text{_part}(\arcsin(1/4*b)))*\sin(5/4*\pi + 1/2*\text{real_part}(\arcsin(1/4*b))) - (\sqrt{2} \\
&*b^2 + \sqrt{2}*\sqrt{b^2 - 16})*b - 16*\sqrt{2})*\sin(5/4*\pi + 1/2*\text{real_part}(\ar \\
&\text{csin}(1/4*b)))*\sinh(1/2*\text{imag_part}(\arcsin(1/4*b)))*\arctan(-4*(1/4)^(3/4)*((1 \\
&/4)^(1/4)*\cos(5/4*\pi + 1/2*\arcsin(1/4*b)) - x)/\sin(5/4*\pi + 1/2*\arcsin(1/4* \\
&b)))/(b^2 - 16) + 1/4*(3*(\sqrt{2}*b^2 + \sqrt{2}*\sqrt{b^2 - 16})*b - 16*\sqrt{2} \\
&)*\cos(1/4*\pi + 1/2*\text{real_part}(\arcsin(1/4*b)))^2*\cosh(1/2*\text{imag_part}(\arcsin \\
&1/4*b)))^3*\sin(1/4*\pi + 1/2*\text{real_part}(\arcsin(1/4*b))) - (\sqrt{2}*b^2 + \sqrt{2} \\
&)*\sqrt{b^2 - 16})*b - 16*\sqrt{2})*\cosh(1/2*\text{imag_part}(\arcsin(1/4*b)))^3*\sin \\
&(1/4*\pi + 1/2*\text{real_part}(\arcsin(1/4*b)))^3 - 9*(\sqrt{2}*b^2 + \sqrt{2}*\sqrt{b \\
&^2 - 16})*b - 16*\sqrt{2})*\cos(1/4*\pi + 1/2*\text{real_part}(\arcsin(1/4*b)))^2*\cosh(\\
&1/2*\text{imag_part}(\arcsin(1/4*b)))^2*\sin(1/4*\pi + 1/2*\text{real_part}(\arcsin(1/4*b)))* \\
&\sinh(1/2*\text{imag_part}(\arcsin(1/4*b))) + 3*(\sqrt{2}*b^2 + \sqrt{2}*\sqrt{b^2 - 16} \\
&)*b - 16*\sqrt{2})*\cosh(1/2*\text{imag_part}(\arcsin(1/4*b)))^2*\sin(1/4*\pi + 1/2*\text{rea} \\
&\text{l_part}(\arcsin(1/4*b)))^3*\sinh(1/2*\text{imag_part}(\arcsin(1/4*b))) + 9*(\sqrt{2}*b^2 \\
&+ \sqrt{2}*\sqrt{b^2 - 16})*b - 16*\sqrt{2})*\cos(1/4*\pi + 1/2*\text{real_part}(\arcsi \\
&n(1/4*b)))^2*\cosh(1/2*\text{imag_part}(\arcsin(1/4*b)))*\sin(1/4*\pi + 1/2*\text{real_part} \\
&(\arcsin(1/4*b)))*\sinh(1/2*\text{imag_part}(\arcsin(1/4*b)))^2 - 3*(\sqrt{2}*b^2 + \sqrt{2} \\
&)*\sqrt{b^2 - 16})*b - 16*\sqrt{2})*\cosh(1/2*\text{imag_part}(\arcsin(1/4*b)))*\sin(\\
&1/4*\pi + 1/2*\text{real_part}(\arcsin(1/4*b)))^3*\sinh(1/2*\text{imag_part}(\arcsin(1/4*b))) \\
&^2 - 3*(\sqrt{2}*b^2 + \sqrt{2}*\sqrt{b^2 - 16})*b - 16*\sqrt{2})*\cos(1/4*\pi + 1 \\
&/2*\text{real_part}(\arcsin(1/4*b)))^2*\sin(1/4*\pi + 1/2*\text{real_part}(\arcsin(1/4*b)))* \\
&\sinh(1/2*\text{imag_part}(\arcsin(1/4*b)))^3 + (\sqrt{2}*b^2 + \sqrt{2}*\sqrt{b^2 - 16} \\
&)*b - 16*\sqrt{2})*\sin(1/4*\pi + 1/2*\text{real_part}(\arcsin(1/4*b)))^3*\sinh(1/2*\text{imag} \\
&\text{_part}(\arcsin(1/4*b)))^3 + (\sqrt{2}*b^2 + \sqrt{2}*\sqrt{b^2 - 16})*b - 16*\sqrt{2} \\
&)*\cosh(1/2*\text{imag_part}(\arcsin(1/4*b)))*\sin(1/4*\pi + 1/2*\text{real_part}(\arcsin(1 \\
&/4*b))) - (\sqrt{2}*b^2 + \sqrt{2}*\sqrt{b^2 - 16})*b - 16*\sqrt{2})*\sin(1/4*\pi \\
&+ 1/2*\text{real_part}(\arcsin(1/4*b)))*\sinh(1/2*\text{imag_part}(\arcsin(1/4*b)))*\arctan(\\
&-4*(1/4)^(3/4)*((1/4)^(1/4)*\cos(1/4*\pi + 1/2*\arcsin(1/4*b)) - x)/\sin(1/4*\pi \\
&+ 1/2*\arcsin(1/4*b)))/(b^2 - 16) - 1/8*((\sqrt{2}*b^2 + \sqrt{2}*\sqrt{b^2 - \\
&16})*b - 16*\sqrt{2})*\cos(5/4*\pi + 1/2*\text{real_part}(\arcsin(1/4*b)))^3*\cosh(1/2*i \\
&\text{mag_part}(\arcsin(1/4*b)))^3 - 3*(\sqrt{2}*b^2 + \sqrt{2}*\sqrt{b^2 - 16})*b - 16 \\
&*\sqrt{2})*\cos(5/4*\pi + 1/2*\text{real_part}(\arcsin(1/4*b)))*\cosh(1/2*\text{imag_part}(\ar \\
&\text{csin}(1/4*b)))^3*\sin(5/4*\pi + 1/2*\text{real_part}(\arcsin(1/4*b)))^2 - 3*(\sqrt{2}*b^2 \\
&+ \sqrt{2}*\sqrt{b^2 - 16})*b - 16*\sqrt{2})*\cos(5/4*\pi + 1/2*\text{real_part}(\arcsi \\
&n(1/4*b)))^3*\cosh(1/2*\text{imag_part}(\arcsin(1/4*b)))^2*\sinh(1/2*\text{imag_part}(\arcsin \\
&(1/4*b))) + 9*(\sqrt{2}*b^2 + \sqrt{2}*\sqrt{b^2 - 16})*b - 16*\sqrt{2})*\cos(5/4 \\
&*\pi + 1/2*\text{real_part}(\arcsin(1/4*b)))*\cosh(1/2*\text{imag_part}(\arcsin(1/4*b)))^2*\si \\
&n(5/4*\pi + 1/2*\text{real_part}(\arcsin(1/4*b)))^2*\sinh(1/2*\text{imag_part}(\arcsin(1/4*b) \\
&)) + 3*(\sqrt{2}*b^2 + \sqrt{2}*\sqrt{b^2 - 16})*b - 16*\sqrt{2})*\cos(5/4*\pi + 1 \\
&/2*\text{real_part}(\arcsin(1/4*b)))^3*\cosh(1/2*\text{imag_part}(\arcsin(1/4*b)))*\sinh(1/2* \\
&\text{imag_part}(\arcsin(1/4*b)))^2 - 9*(\sqrt{2}*b^2 + \sqrt{2}*\sqrt{b^2 - 16})*b - 1 \\
&6*\sqrt{2})*\cos(5/4*\pi + 1/2*\text{real_part}(\arcsin(1/4*b)))*\cosh(1/2*\text{imag_part}(\ar \\
&\text{csin}(1/4*b)))*\sin(5/4*\pi + 1/2*\text{real_part}(\arcsin(1/4*b)))^2*\sinh(1/2*\text{imag_pa} \\
&\text{rt}(\arcsin(1/4*b)))^2 - (\sqrt{2}*b^2 + \sqrt{2}*\sqrt{b^2 - 16})*b - 16*\sqrt{2} \\
&)*\cos(5/4*\pi + 1/2*\text{real_part}(\arcsin(1/4*b)))^3*\sinh(1/2*\text{imag_part}(\arcsin(1/ \\
&4*b)))^3 + 3*(\sqrt{2}*b^2 + \sqrt{2}*\sqrt{b^2 - 16})*b - 16*\sqrt{2})*\cos(5/4* \\
&\pi + 1/2*\text{real_part}(\arcsin(1/4*b)))*\sin(5/4*\pi + 1/2*\text{real_part}(\arcsin(1/4*b) \\
&))^2*\sinh(1/2*\text{imag_part}(\arcsin(1/4*b)))^3 + (\sqrt{2}*b^2 + \sqrt{2}*\sqrt{b^2 \\
&- 16})*b - 16*\sqrt{2})*\cos(5/4*\pi + 1/2*\text{real_part}(\arcsin(1/4*b)))*\cosh(1/2*
\end{aligned}$$

$$\begin{aligned} & \text{imag_part}(\arcsin(1/4*b)) - (\sqrt{2}*b^2 + \sqrt{2}*\sqrt{b^2 - 16}*b - 16*\sqrt{2}) * \cos(5/4*\pi + 1/2*\text{real_part}(\arcsin(1/4*b))) * \sinh(1/2*\text{imag_part}(\arcsin(1/4*b))) \\ & * \log(-2*(1/4)^{(1/4)}*x*\cos(5/4*\pi + 1/2*\arcsin(1/4*b)) + x^2 + 1/2)/(b^2 - 16) - 1/8*((\sqrt{2}*b^2 + \sqrt{2}*\sqrt{b^2 - 16}*b - 16*\sqrt{2})*\cos(1/4*\pi + 1/2*\text{real_part}(\arcsin(1/4*b)))^3 * \cosh(1/2*\text{imag_part}(\arcsin(1/4*b)))^3 \\ & - 3*(\sqrt{2}*b^2 + \sqrt{2}*\sqrt{b^2 - 16}*b - 16*\sqrt{2})*\cos(1/4*\pi + 1/2*\text{real_part}(\arcsin(1/4*b))) * \cosh(1/2*\text{imag_part}(\arcsin(1/4*b)))^3 * \sin(1/4*\pi + 1/2*\text{real_part}(\arcsin(1/4*b)))^2 \\ & - 3*(\sqrt{2}*b^2 + \sqrt{2}*\sqrt{b^2 - 16}*b - 16*\sqrt{2})*\cos(1/4*\pi + 1/2*\text{real_part}(\arcsin(1/4*b)))^3 * \cosh(1/2*\text{imag_part}(\arcsin(1/4*b)))^2 * \sinh(1/2*\text{imag_part}(\arcsin(1/4*b))) \\ & + 9*(\sqrt{2}*b^2 + \sqrt{2}*\sqrt{b^2 - 16}*b - 16*\sqrt{2})*\cos(1/4*\pi + 1/2*\text{real_part}(\arcsin(1/4*b))) * \cosh(1/2*\text{imag_part}(\arcsin(1/4*b)))^2 * \sin(1/4*\pi + 1/2*\text{real_part}(\arcsin(1/4*b)))^2 * \sinh(1/2*\text{imag_part}(\arcsin(1/4*b))) \\ & + 3*(\sqrt{2}*b^2 + \sqrt{2}*\sqrt{b^2 - 16}*b - 16*\sqrt{2})*\cos(1/4*\pi + 1/2*\text{real_part}(\arcsin(1/4*b)))^3 * \cosh(1/2*\text{imag_part}(\arcsin(1/4*b))) * \sinh(1/2*\text{imag_part}(\arcsin(1/4*b)))^2 \\ & - 9*(\sqrt{2}*b^2 + \sqrt{2}*\sqrt{b^2 - 16}*b - 16*\sqrt{2})*\cos(1/4*\pi + 1/2*\text{real_part}(\arcsin(1/4*b))) * \cosh(1/2*\text{imag_part}(\arcsin(1/4*b))) * \sin(1/4*\pi + 1/2*\text{real_part}(\arcsin(1/4*b)))^2 * \sinh(1/2*\text{imag_part}(\arcsin(1/4*b)))^2 \\ & - (\sqrt{2}*b^2 + \sqrt{2}*\sqrt{b^2 - 16}*b - 16*\sqrt{2})*\cos(1/4*\pi + 1/2*\text{real_part}(\arcsin(1/4*b)))^3 * \sinh(1/2*\text{imag_part}(\arcsin(1/4*b)))^3 + 3*(\sqrt{2}*b^2 + \sqrt{2}*\sqrt{b^2 - 16}*b - 16*\sqrt{2})*\cos(1/4*\pi + 1/2*\text{real_part}(\arcsin(1/4*b))) * \sin(1/4*\pi + 1/2*\text{real_part}(\arcsin(1/4*b)))^2 * \sinh(1/2*\text{imag_part}(\arcsin(1/4*b)))^3 \\ & + (\sqrt{2}*b^2 + \sqrt{2}*\sqrt{b^2 - 16}*b - 16*\sqrt{2})*\cos(1/4*\pi + 1/2*\text{real_part}(\arcsin(1/4*b))) * \cosh(1/2*\text{imag_part}(\arcsin(1/4*b))) - (\sqrt{2}*b^2 + \sqrt{2}*\sqrt{b^2 - 16}*b - 16*\sqrt{2})*\cos(1/4*\pi + 1/2*\text{real_part}(\arcsin(1/4*b))) * \sinh(1/2*\text{imag_part}(\arcsin(1/4*b))) * \log(-2*(1/4)^{(1/4)}*x*\cos(1/4*\pi + 1/2*\arcsin(1/4*b)) + x^2 + 1/2)/(b^2 - 16) \end{aligned}$$

3.40 $\int \frac{1+2x^2}{1-bx^2+4x^4} dx$

Optimal. Leaf size=66

$$\frac{\tan^{-1}\left(\frac{\sqrt{b+4}+4x}{\sqrt{4-b}}\right)}{\sqrt{4-b}} - \frac{\tan^{-1}\left(\frac{\sqrt{b+4}-4x}{\sqrt{4-b}}\right)}{\sqrt{4-b}}$$

[Out] -(ArcTan[(Sqrt[4 + b] - 4*x)/Sqrt[4 - b]]/Sqrt[4 - b]) + ArcTan[(Sqrt[4 + b] + 4*x)/Sqrt[4 - b]]/Sqrt[4 - b]

Rubi [A] time = 0.0579648, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.13$, Rules used = {1161, 618, 204}

$$\frac{\tan^{-1}\left(\frac{\sqrt{b+4}+4x}{\sqrt{4-b}}\right)}{\sqrt{4-b}} - \frac{\tan^{-1}\left(\frac{\sqrt{b+4}-4x}{\sqrt{4-b}}\right)}{\sqrt{4-b}}$$

Antiderivative was successfully verified.

[In] Int[(1 + 2*x^2)/(1 - b*x^2 + 4*x^4),x]

[Out] -(ArcTan[(Sqrt[4 + b] - 4*x)/Sqrt[4 - b]]/Sqrt[4 - b]) + ArcTan[(Sqrt[4 + b] + 4*x)/Sqrt[4 - b]]/Sqrt[4 - b]

Rule 1161

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[(2*d)/e - b/c, 0] || (!LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{1+2x^2}{1-bx^2+4x^4} dx &= \frac{1}{4} \int \frac{1}{\frac{1}{2} - \frac{1}{2}\sqrt{4+bx} + x^2} dx + \frac{1}{4} \int \frac{1}{\frac{1}{2} + \frac{1}{2}\sqrt{4+bx} + x^2} dx \\ &= -\left(\frac{1}{2} \text{Subst}\left(\int \frac{1}{\frac{1}{4}(-4+b)-x^2} dx, x, -\frac{\sqrt{4+b}}{2} + 2x\right)\right) - \frac{1}{2} \text{Subst}\left(\int \frac{1}{\frac{1}{4}(-4+b)-x^2} dx, x, \frac{\sqrt{4+b}}{2} + 2x\right) \\ &= -\frac{\tan^{-1}\left(\frac{\sqrt{4+b}-4x}{\sqrt{4-b}}\right)}{\sqrt{4-b}} + \frac{\tan^{-1}\left(\frac{\sqrt{4+b}+4x}{\sqrt{4-b}}\right)}{\sqrt{4-b}} \end{aligned}$$

Mathematica [B] time = 0.0583946, size = 134, normalized size = 2.03

$$\frac{\left(\sqrt{b^2-16+b+4}\right) \tan^{-1}\left(\frac{2\sqrt{2}x}{\sqrt{-\sqrt{b^2-16}-b}}\right) + \left(\sqrt{b^2-16-b-4}\right) \tan^{-1}\left(\frac{2\sqrt{2}x}{\sqrt{\sqrt{b^2-16}-b}}\right)}{\sqrt{2}\sqrt{b^2-16}}$$

Antiderivative was successfully verified.

[In] Integrate[(1 + 2*x^2)/(1 - b*x^2 + 4*x^4), x]

[Out] (((4 + b + Sqrt[-16 + b^2])*ArcTan[(2*Sqrt[2]*x)/Sqrt[-b - Sqrt[-16 + b^2]]])/Sqrt[-b - Sqrt[-16 + b^2]] + ((-4 - b + Sqrt[-16 + b^2])*ArcTan[(2*Sqrt[2]*x)/Sqrt[-b + Sqrt[-16 + b^2]]])/Sqrt[-b + Sqrt[-16 + b^2]])/(Sqrt[2]*Sqrt[-16 + b^2])

Maple [B] time = 0.106, size = 277, normalized size = 4.2

$$-4 \frac{1}{\sqrt{(b-4)(4+b)}\sqrt{2\sqrt{(b-4)(4+b)}-2b}} \arctan\left(4 \frac{x}{\sqrt{2\sqrt{(b-4)(4+b)}-2b}}\right) + \arctan\left(4 \frac{x}{\sqrt{2\sqrt{(b-4)(4+b)}-2b}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*x^2+1)/(4*x^4-b*x^2+1), x)

[Out] -4/((b-4)*(4+b))^(1/2)/(2*((b-4)*(4+b))^(1/2)-2*b)^(1/2)*arctan(4*x/(2*((b-4)*(4+b))^(1/2)-2*b)^(1/2))+1/(2*((b-4)*(4+b))^(1/2)-2*b)^(1/2)*arctan(4*x/(2*((b-4)*(4+b))^(1/2)-2*b)^(1/2))-1/((b-4)*(4+b))^(1/2)/(2*((b-4)*(4+b))^(1/2)-2*b)^(1/2)*arctan(4*x/(2*((b-4)*(4+b))^(1/2)-2*b)^(1/2))*b+4/((b-4)*(4+b))^(1/2)/(-2*((b-4)*(4+b))^(1/2)-2*b)^(1/2)*arctan(4*x/(-2*((b-4)*(4+b))^(1/2)-2*b)^(1/2))+1/(-2*((b-4)*(4+b))^(1/2)-2*b)^(1/2)*arctan(4*x/(-2*((b-4)*(4+b))^(1/2)-2*b)^(1/2))+1/((b-4)*(4+b))^(1/2)/(-2*((b-4)*(4+b))^(1/2)-2*b)^(1/2)*arctan(4*x/(-2*((b-4)*(4+b))^(1/2)-2*b)^(1/2))*b

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{2x^2+1}{4x^4-bx^2+1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2+1)/(4*x^4-b*x^2+1),x, algorithm="maxima")

[Out] integrate((2*x^2 + 1)/(4*x^4 - b*x^2 + 1), x)

Fricas [A] time = 1.26934, size = 298, normalized size = 4.52

$$\left[\frac{\log\left(\frac{4x^4+(b-8)x^2-2(2x^3-x)\sqrt{b-4}+1}{4x^4-bx^2+1}\right)}{2\sqrt{b-4}}, \frac{\sqrt{-b+4}\arctan\left(\frac{(4x^3-(b-2)x)\sqrt{-b+4}}{b-4}\right) + \sqrt{-b+4}\arctan\left(\frac{2\sqrt{-b+4}x}{b-4}\right)}{b-4} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2+1)/(4*x^4-b*x^2+1),x, algorithm="fricas")

[Out] [1/2*log((4*x^4 + (b - 8)*x^2 - 2*(2*x^3 - x)*sqrt(b - 4) + 1)/(4*x^4 - b*x^2 + 1))/sqrt(b - 4), (sqrt(-b + 4)*arctan((4*x^3 - (b - 2)*x)*sqrt(-b + 4)/(b - 4)) + sqrt(-b + 4)*arctan(2*sqrt(-b + 4)*x/(b - 4)))/(b - 4)]

Sympy [A] time = 0.28598, size = 83, normalized size = 1.26

$$\frac{\sqrt{\frac{1}{b-4}} \log\left(x^2 + x\left(-\frac{b\sqrt{\frac{1}{b-4}}}{2} + 2\sqrt{\frac{1}{b-4}}\right) - \frac{1}{2}\right)}{2} - \frac{\sqrt{\frac{1}{b-4}} \log\left(x^2 + x\left(\frac{b\sqrt{\frac{1}{b-4}}}{2} - 2\sqrt{\frac{1}{b-4}}\right) - \frac{1}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x**2+1)/(4*x**4-b*x**2+1),x)

[Out] sqrt(1/(b - 4))*log(x**2 + x*(-b*sqrt(1/(b - 4))/2 + 2*sqrt(1/(b - 4))) - 1/2)/2 - sqrt(1/(b - 4))*log(x**2 + x*(b*sqrt(1/(b - 4))/2 - 2*sqrt(1/(b - 4)))) - 1/2)/2

Giac [C] time = 1.2722, size = 3069, normalized size = 46.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2+1)/(4*x^4-b*x^2+1),x, algorithm="giac")

[Out] 1/4*(3*(sqrt(2)*b^2 - sqrt(2)*sqrt(b^2 - 16)*b - 16*sqrt(2))*cos(1/2*real_part(arccos(1/4*b)))^2*cosh(1/2*imag_part(arccos(1/4*b)))^3*sin(1/2*real_part(arccos(1/4*b))) - (sqrt(2)*b^2 - sqrt(2)*sqrt(b^2 - 16)*b - 16*sqrt(2))*cosh(1/2*imag_part(arccos(1/4*b)))^3*sin(1/2*real_part(arccos(1/4*b)))^3 - 9*(sqrt(2)*b^2 - sqrt(2)*sqrt(b^2 - 16)*b - 16*sqrt(2))*cos(1/2*real_part(arccos(1/4*b)))^2*cosh(1/2*imag_part(arccos(1/4*b)))^2*sin(1/2*real_part(arccos(1/4*b)))*sinh(1/2*imag_part(arccos(1/4*b))) + 3*(sqrt(2)*b^2 - sqrt(2)*sqrt(b^2 - 16)*b - 16*sqrt(2))*cosh(1/2*imag_part(arccos(1/4*b)))^2*sin(1/2*real_part(arccos(1/4*b)))^3*sinh(1/2*imag_part(arccos(1/4*b))) + 9*(sqrt(2)*b^2 - sqrt(2)*sqrt(b^2 - 16)*b - 16*sqrt(2))*cos(1/2*real_part(arccos(1/4*


```

(2))*cos(1/2*real_part(arccos(1/4*b)))*cosh(1/2*imag_part(arccos(1/4*b)))^3
*sin(1/2*real_part(arccos(1/4*b)))^2 - 3*(sqrt(2)*b^2 - sqrt(2)*sqrt(b^2 -
16)*b - 16*sqrt(2))*cos(1/2*real_part(arccos(1/4*b)))^3*cosh(1/2*imag_part(
arccos(1/4*b)))^2*sinh(1/2*imag_part(arccos(1/4*b))) + 9*(sqrt(2)*b^2 - sqrt
(2)*sqrt(b^2 - 16)*b - 16*sqrt(2))*cos(1/2*real_part(arccos(1/4*b)))*cosh(
1/2*imag_part(arccos(1/4*b)))^2*sin(1/2*real_part(arccos(1/4*b)))^2*sinh(1/
2*imag_part(arccos(1/4*b))) + 3*(sqrt(2)*b^2 - sqrt(2)*sqrt(b^2 - 16)*b - 1
6*sqrt(2))*cos(1/2*real_part(arccos(1/4*b)))^3*cosh(1/2*imag_part(arccos(1/
4*b)))*sinh(1/2*imag_part(arccos(1/4*b)))^2 - 9*(sqrt(2)*b^2 - sqrt(2)*sqrt
(b^2 - 16)*b - 16*sqrt(2))*cos(1/2*real_part(arccos(1/4*b)))*cosh(1/2*imag_
part(arccos(1/4*b)))*sin(1/2*real_part(arccos(1/4*b)))^2*sinh(1/2*imag_part
(arccos(1/4*b)))^2 - (sqrt(2)*b^2 - sqrt(2)*sqrt(b^2 - 16)*b - 16*sqrt(2))*
cos(1/2*real_part(arccos(1/4*b)))^3*sinh(1/2*imag_part(arccos(1/4*b)))^3 +
3*(sqrt(2)*b^2 - sqrt(2)*sqrt(b^2 - 16)*b - 16*sqrt(2))*cos(1/2*real_part(a
rccos(1/4*b)))*sin(1/2*real_part(arccos(1/4*b)))^2*sinh(1/2*imag_part(arcco
s(1/4*b)))^3 + (sqrt(2)*b^2 - sqrt(2)*sqrt(b^2 - 16)*b - 16*sqrt(2))*cos(1/
2*real_part(arccos(1/4*b)))*cosh(1/2*imag_part(arccos(1/4*b))) - (sqrt(2)*b
^2 - sqrt(2)*sqrt(b^2 - 16)*b - 16*sqrt(2))*cos(1/2*real_part(arccos(1/4*b)
))*sinh(1/2*imag_part(arccos(1/4*b)))*log(-2*(1/4)^(1/4)*x*cos(1/2*arccos(
1/4*b)) + x^2 + 1/2)/(b^2 - 16)

```

$$3.41 \quad \int \frac{1+2x^2}{1+6x^2+4x^4} dx$$

Optimal. Leaf size=45

$$\frac{\tan^{-1}\left(\frac{2x}{\sqrt{3-\sqrt{5}}}\right)}{\sqrt{10}} + \frac{\tan^{-1}\left(\frac{2x}{\sqrt{3+\sqrt{5}}}\right)}{\sqrt{10}}$$

[Out] ArcTan[(2*x)/Sqrt[3 - Sqrt[5]]]/Sqrt[10] + ArcTan[(2*x)/Sqrt[3 + Sqrt[5]]]/Sqrt[10]

Rubi [A] time = 0.0593532, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1163, 203}

$$\frac{\tan^{-1}\left(\frac{2x}{\sqrt{3-\sqrt{5}}}\right)}{\sqrt{10}} + \frac{\tan^{-1}\left(\frac{2x}{\sqrt{3+\sqrt{5}}}\right)}{\sqrt{10}}$$

Antiderivative was successfully verified.

[In] Int[(1 + 2*x^2)/(1 + 6*x^2 + 4*x^4), x]

[Out] ArcTan[(2*x)/Sqrt[3 - Sqrt[5]]]/Sqrt[10] + ArcTan[(2*x)/Sqrt[3 + Sqrt[5]]]/Sqrt[10]

Rule 1163

Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && GtQ[b^2 - 4*a*c, 0]

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rubi steps

$$\begin{aligned} \int \frac{1+2x^2}{1+6x^2+4x^4} dx &= \frac{1}{5} (5-\sqrt{5}) \int \frac{1}{3-\sqrt{5}+4x^2} dx + \frac{1}{5} (5+\sqrt{5}) \int \frac{1}{3+\sqrt{5}+4x^2} dx \\ &= \frac{\tan^{-1}\left(\frac{2x}{\sqrt{3-\sqrt{5}}}\right)}{\sqrt{10}} + \frac{\tan^{-1}\left(\frac{2x}{\sqrt{3+\sqrt{5}}}\right)}{\sqrt{10}} \end{aligned}$$

Mathematica [A] time = 0.0743086, size = 83, normalized size = 1.84

$$\frac{(\sqrt{5}-1) \tan^{-1}\left(\frac{2x}{\sqrt{3-\sqrt{5}}}\right)}{2\sqrt{5}(3-\sqrt{5})} + \frac{(1+\sqrt{5}) \tan^{-1}\left(\frac{2x}{\sqrt{3+\sqrt{5}}}\right)}{2\sqrt{5}(3+\sqrt{5})}$$

Antiderivative was successfully verified.

[In] Integrate[(1 + 2*x^2)/(1 + 6*x^2 + 4*x^4),x]

[Out] ((-1 + Sqrt[5])*ArcTan[(2*x)/Sqrt[3 - Sqrt[5]]])/(2*Sqrt[5*(3 - Sqrt[5])]) + ((1 + Sqrt[5])*ArcTan[(2*x)/Sqrt[3 + Sqrt[5]]])/(2*Sqrt[5*(3 + Sqrt[5])])

Maple [B] time = 0.075, size = 136, normalized size = 3.

$$-\frac{2\sqrt{5}}{10\sqrt{10}-10\sqrt{2}} \arctan\left(8\frac{x}{2\sqrt{10}-2\sqrt{2}}\right) + 2\frac{1}{2\sqrt{10}-2\sqrt{2}} \arctan\left(8\frac{x}{2\sqrt{10}-2\sqrt{2}}\right) + \frac{2\sqrt{5}}{10\sqrt{10}+10\sqrt{2}} \arctan\left(8\frac{x}{2\sqrt{10}+2\sqrt{2}}\right) + 2\frac{1}{2\sqrt{10}+2\sqrt{2}} \arctan\left(8\frac{x}{2\sqrt{10}+2\sqrt{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*x^2+1)/(4*x^4+6*x^2+1),x)

[Out] -2/5*5^(1/2)/(2*10^(1/2)-2*2^(1/2))*arctan(8*x/(2*10^(1/2)-2*2^(1/2)))+2/(2*10^(1/2)-2*2^(1/2))*arctan(8*x/(2*10^(1/2)-2*2^(1/2)))+2/5*5^(1/2)/(2*10^(1/2)+2*2^(1/2))*arctan(8*x/(2*10^(1/2)+2*2^(1/2)))+2/(2*10^(1/2)+2*2^(1/2))*arctan(8*x/(2*10^(1/2)+2*2^(1/2)))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{2x^2 + 1}{4x^4 + 6x^2 + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2+1)/(4*x^4+6*x^2+1),x, algorithm="maxima")

[Out] integrate((2*x^2 + 1)/(4*x^4 + 6*x^2 + 1), x)

Fricas [A] time = 1.31568, size = 117, normalized size = 2.6

$$\frac{1}{10} \sqrt{10} \arctan\left(\frac{2}{5} \sqrt{10}(x^3 + 2x)\right) + \frac{1}{10} \sqrt{10} \arctan\left(\frac{1}{5} \sqrt{10}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2+1)/(4*x^4+6*x^2+1),x, algorithm="fricas")

[Out] 1/10*sqrt(10)*arctan(2/5*sqrt(10)*(x^3 + 2*x)) + 1/10*sqrt(10)*arctan(1/5*sqrt(10)*x)

Sympy [A] time = 0.115144, size = 42, normalized size = 0.93

$$\frac{\sqrt{10} \left(2 \operatorname{atan}\left(\frac{\sqrt{10}x}{5}\right) + 2 \operatorname{atan}\left(\frac{2\sqrt{10}x^3}{5} + \frac{4\sqrt{10}x}{5}\right) \right)}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x**2+1)/(4*x**4+6*x**2+1),x)

[Out] sqrt(10)*(2*atan(sqrt(10)*x/5) + 2*atan(2*sqrt(10)*x**3/5 + 4*sqrt(10)*x/5)/20

Giac [A] time = 1.17602, size = 53, normalized size = 1.18

$$\frac{1}{10} \sqrt{10} \arctan\left(\frac{4x}{\sqrt{10} + \sqrt{2}}\right) + \frac{1}{10} \sqrt{10} \arctan\left(\frac{4x}{\sqrt{10} - \sqrt{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2+1)/(4*x^4+6*x^2+1),x, algorithm="giac")

[Out] 1/10*sqrt(10)*arctan(4*x/(sqrt(10) + sqrt(2))) + 1/10*sqrt(10)*arctan(4*x/(sqrt(10) - sqrt(2)))

$$3.42 \quad \int \frac{1+2x^2}{1+5x^2+4x^4} dx$$

Optimal. Leaf size=15

$$\frac{1}{3} \tan^{-1}(x) + \frac{1}{3} \tan^{-1}(2x)$$

[Out] ArcTan[x]/3 + ArcTan[2*x]/3

Rubi [A] time = 0.0093769, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1163, 203}

$$\frac{1}{3} \tan^{-1}(x) + \frac{1}{3} \tan^{-1}(2x)$$

Antiderivative was successfully verified.

[In] Int[(1 + 2*x^2)/(1 + 5*x^2 + 4*x^4),x]

[Out] ArcTan[x]/3 + ArcTan[2*x]/3

Rule 1163

Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
 > With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Eq
 Q[c*d^2 - a*e^2, 0] && GtQ[b^2 - 4*a*c, 0]

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
 [a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
 , 0] || GtQ[b, 0])

Rubi steps

$$\begin{aligned} \int \frac{1+2x^2}{1+5x^2+4x^4} dx &= \frac{2}{3} \int \frac{1}{1+4x^2} dx + \frac{4}{3} \int \frac{1}{4+4x^2} dx \\ &= \frac{1}{3} \tan^{-1}(x) + \frac{1}{3} \tan^{-1}(2x) \end{aligned}$$

Mathematica [A] time = 0.0075067, size = 17, normalized size = 1.13

$$-\frac{1}{3} \tan^{-1}\left(\frac{3x}{2x^2-1}\right)$$

Antiderivative was successfully verified.

[In] Integrate[(1 + 2*x^2)/(1 + 5*x^2 + 4*x^4),x]

[Out] -ArcTan[(3*x)/(-1 + 2*x^2)]/3

Maple [A] time = 0.05, size = 12, normalized size = 0.8

$$\frac{\arctan(x)}{3} + \frac{\arctan(2x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*x^2+1)/(4*x^4+5*x^2+1),x)

[Out] 1/3*arctan(x)+1/3*arctan(2*x)

Maxima [A] time = 1.45741, size = 15, normalized size = 1.

$$\frac{1}{3} \arctan(2x) + \frac{1}{3} \arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2+1)/(4*x^4+5*x^2+1),x, algorithm="maxima")

[Out] 1/3*arctan(2*x) + 1/3*arctan(x)

Fricas [A] time = 1.27451, size = 66, normalized size = 4.4

$$\frac{1}{3} \arctan\left(\frac{4}{3}x^3 + \frac{7}{3}x\right) + \frac{1}{3} \arctan\left(\frac{2}{3}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2+1)/(4*x^4+5*x^2+1),x, algorithm="fricas")

[Out] 1/3*arctan(4/3*x^3 + 7/3*x) + 1/3*arctan(2/3*x)

Sympy [B] time = 0.104221, size = 22, normalized size = 1.47

$$\frac{\operatorname{atan}\left(\frac{2x}{3}\right)}{3} + \frac{\operatorname{atan}\left(\frac{4x^3}{3} + \frac{7x}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x**2+1)/(4*x**4+5*x**2+1),x)

[Out] atan(2*x/3)/3 + atan(4*x**3/3 + 7*x/3)/3

Giac [A] time = 1.13615, size = 15, normalized size = 1.

$$\frac{1}{3} \arctan(2x) + \frac{1}{3} \arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+1)/(4*x^4+5*x^2+1),x, algorithm="giac")
```

```
[Out] 1/3*arctan(2*x) + 1/3*arctan(x)
```


$$3.43 \quad \int \frac{1+2x^2}{1+4x^2+4x^4} dx$$

Optimal. Leaf size=14

$$\frac{\tan^{-1}(\sqrt{2}x)}{\sqrt{2}}$$

[Out] ArcTan[Sqrt[2]*x]/Sqrt[2]

Rubi [A] time = 0.0065785, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {28, 21, 203}

$$\frac{\tan^{-1}(\sqrt{2}x)}{\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[(1 + 2*x^2)/(1 + 4*x^2 + 4*x^4), x]

[Out] ArcTan[Sqrt[2]*x]/Sqrt[2]

Rule 28

Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]

Rule 21

Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
&& EqQ[b*c - a*d, 0] && IntegerQ[m] && (!IntegerQ[n] || SimplerQ[c + d*x,
a + b*x])

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rubi steps

$$\begin{aligned} \int \frac{1+2x^2}{1+4x^2+4x^4} dx &= 4 \int \frac{1+2x^2}{(2+4x^2)^2} dx \\ &= \int \frac{1}{1+2x^2} dx \\ &= \frac{\tan^{-1}(\sqrt{2}x)}{\sqrt{2}} \end{aligned}$$

Mathematica [A] time = 0.0036488, size = 14, normalized size = 1.

$$\frac{\tan^{-1}(\sqrt{2}x)}{\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Integrate[(1 + 2*x^2)/(1 + 4*x^2 + 4*x^4),x]

[Out] ArcTan[Sqrt[2]*x]/Sqrt[2]

Maple [A] time = 0.041, size = 12, normalized size = 0.9

$$\frac{\arctan(x\sqrt{2})\sqrt{2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*x^2+1)/(4*x^4+4*x^2+1),x)

[Out] 1/2*arctan(x*2^(1/2))*2^(1/2)

Maxima [A] time = 1.44544, size = 15, normalized size = 1.07

$$\frac{1}{2}\sqrt{2}\arctan(\sqrt{2}x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2+1)/(4*x^4+4*x^2+1),x, algorithm="maxima")

[Out] 1/2*sqrt(2)*arctan(sqrt(2)*x)

Fricas [A] time = 1.35752, size = 42, normalized size = 3.

$$\frac{1}{2}\sqrt{2}\arctan(\sqrt{2}x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2+1)/(4*x^4+4*x^2+1),x, algorithm="fricas")

[Out] 1/2*sqrt(2)*arctan(sqrt(2)*x)

Sympy [A] time = 0.089594, size = 14, normalized size = 1.

$$\frac{\sqrt{2}\operatorname{atan}(\sqrt{2}x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2+1)/(4*x**4+4*x**2+1),x)
```

```
[Out] sqrt(2)*atan(sqrt(2)*x)/2
```

Giac [A] time = 1.13536, size = 15, normalized size = 1.07

$$\frac{1}{2}\sqrt{2}\arctan(\sqrt{2}x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+1)/(4*x^4+4*x^2+1),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*arctan(sqrt(2)*x)
```

$$3.44 \quad \int \frac{1+2x^2}{1+3x^2+4x^4} dx$$

Optimal. Leaf size=38

$$\frac{\tan^{-1}\left(\frac{4x+1}{\sqrt{7}}\right)}{\sqrt{7}} - \frac{\tan^{-1}\left(\frac{1-4x}{\sqrt{7}}\right)}{\sqrt{7}}$$

[Out] -(ArcTan[(1 - 4*x)/Sqrt[7]]/Sqrt[7]) + ArcTan[(1 + 4*x)/Sqrt[7]]/Sqrt[7]

Rubi [A] time = 0.0352245, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {1161, 618, 204}

$$\frac{\tan^{-1}\left(\frac{4x+1}{\sqrt{7}}\right)}{\sqrt{7}} - \frac{\tan^{-1}\left(\frac{1-4x}{\sqrt{7}}\right)}{\sqrt{7}}$$

Antiderivative was successfully verified.

[In] Int[(1 + 2*x^2)/(1 + 3*x^2 + 4*x^4),x]

[Out] -(ArcTan[(1 - 4*x)/Sqrt[7]]/Sqrt[7]) + ArcTan[(1 + 4*x)/Sqrt[7]]/Sqrt[7]

Rule 1161

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[(2*d)/e - b/c, 0] || (!LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{1+2x^2}{1+3x^2+4x^4} dx &= \frac{1}{4} \int \frac{1}{\frac{1}{2} - \frac{x}{2} + x^2} dx + \frac{1}{4} \int \frac{1}{\frac{1}{2} + \frac{x}{2} + x^2} dx \\ &= -\left(\frac{1}{2} \text{Subst}\left(\int \frac{1}{-\frac{7}{4} - x^2} dx, x, -\frac{1}{2} + 2x\right)\right) - \frac{1}{2} \text{Subst}\left(\int \frac{1}{-\frac{7}{4} - x^2} dx, x, \frac{1}{2} + 2x\right) \\ &= -\frac{\tan^{-1}\left(\frac{1-4x}{\sqrt{7}}\right)}{\sqrt{7}} + \frac{\tan^{-1}\left(\frac{1+4x}{\sqrt{7}}\right)}{\sqrt{7}} \end{aligned}$$

Mathematica [C] time = 0.185003, size = 97, normalized size = 2.55

$$\frac{(\sqrt{7} - i) \tan^{-1}\left(\frac{2x}{\sqrt{\frac{1}{2}(3-i\sqrt{7})}}\right)}{\sqrt{42 - 14i\sqrt{7}}} + \frac{(\sqrt{7} + i) \tan^{-1}\left(\frac{2x}{\sqrt{\frac{1}{2}(3+i\sqrt{7})}}\right)}{\sqrt{42 + 14i\sqrt{7}}}$$

Antiderivative was successfully verified.

[In] Integrate[(1 + 2*x^2)/(1 + 3*x^2 + 4*x^4), x]

[Out] ((-I + Sqrt[7])*ArcTan[(2*x)/Sqrt[(3 - I*Sqrt[7])/2]])/Sqrt[42 - (14*I)*Sqrt[7]] + ((I + Sqrt[7])*ArcTan[(2*x)/Sqrt[(3 + I*Sqrt[7])/2]])/Sqrt[42 + (14*I)*Sqrt[7]]

Maple [A] time = 0.043, size = 34, normalized size = 0.9

$$\frac{\sqrt{7}}{7} \arctan\left(\frac{(-1 + 4x)\sqrt{7}}{7}\right) + \frac{\sqrt{7}}{7} \arctan\left(\frac{(4x + 1)\sqrt{7}}{7}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*x^2+1)/(4*x^4+3*x^2+1), x)

[Out] 1/7*7^(1/2)*arctan(1/7*(-1+4*x)*7^(1/2))+1/7*arctan(1/7*(4*x+1)*7^(1/2))*7^(1/2)

Maxima [A] time = 1.44026, size = 45, normalized size = 1.18

$$\frac{1}{7} \sqrt{7} \arctan\left(\frac{1}{7} \sqrt{7}(4x + 1)\right) + \frac{1}{7} \sqrt{7} \arctan\left(\frac{1}{7} \sqrt{7}(4x - 1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2+1)/(4*x^4+3*x^2+1), x, algorithm="maxima")

[Out] 1/7*sqrt(7)*arctan(1/7*sqrt(7)*(4*x + 1)) + 1/7*sqrt(7)*arctan(1/7*sqrt(7)*(4*x - 1))

Fricas [A] time = 1.30687, size = 112, normalized size = 2.95

$$\frac{1}{7} \sqrt{7} \arctan\left(\frac{1}{7} \sqrt{7}(4x^3 + 5x)\right) + \frac{1}{7} \sqrt{7} \arctan\left(\frac{2}{7} \sqrt{7}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2+1)/(4*x^4+3*x^2+1), x, algorithm="fricas")

[Out] 1/7*sqrt(7)*arctan(1/7*sqrt(7)*(4*x^3 + 5*x)) + 1/7*sqrt(7)*arctan(2/7*sqrt(7)*x)

Sympy [A] time = 0.115235, size = 44, normalized size = 1.16

$$\frac{\sqrt{7} \left(2 \operatorname{atan} \left(\frac{2\sqrt{7}x}{7} \right) + 2 \operatorname{atan} \left(\frac{4\sqrt{7}x^3}{7} + \frac{5\sqrt{7}x}{7} \right) \right)}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x**2+1)/(4*x**4+3*x**2+1),x)

[Out] sqrt(7)*(2*atan(2*sqrt(7)*x/7) + 2*atan(4*sqrt(7)*x**3/7 + 5*sqrt(7)*x/7))/14

Giac [A] time = 1.13606, size = 45, normalized size = 1.18

$$\frac{1}{7} \sqrt{7} \arctan \left(\frac{1}{7} \sqrt{7} (4x + 1) \right) + \frac{1}{7} \sqrt{7} \arctan \left(\frac{1}{7} \sqrt{7} (4x - 1) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2+1)/(4*x^4+3*x^2+1),x, algorithm="giac")

[Out] 1/7*sqrt(7)*arctan(1/7*sqrt(7)*(4*x + 1)) + 1/7*sqrt(7)*arctan(1/7*sqrt(7)*(4*x - 1))

$$3.45 \quad \int \frac{1+2x^2}{1+2x^2+4x^4} dx$$

Optimal. Leaf size=48

$$\frac{\tan^{-1}\left(\frac{2\sqrt{2x+1}}{\sqrt{3}}\right)}{\sqrt{6}} - \frac{\tan^{-1}\left(\frac{1-2\sqrt{2x}}{\sqrt{3}}\right)}{\sqrt{6}}$$

[Out] $-(\text{ArcTan}[(1 - 2*\text{Sqrt}[2]*x)/\text{Sqrt}[3]]/\text{Sqrt}[6]) + \text{ArcTan}[(1 + 2*\text{Sqrt}[2]*x)/\text{Sqrt}[3]]/\text{Sqrt}[6]$

Rubi [A] time = 0.0395263, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {1161, 618, 204}

$$\frac{\tan^{-1}\left(\frac{2\sqrt{2x+1}}{\sqrt{3}}\right)}{\sqrt{6}} - \frac{\tan^{-1}\left(\frac{1-2\sqrt{2x}}{\sqrt{3}}\right)}{\sqrt{6}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(1 + 2*x^2)/(1 + 2*x^2 + 4*x^4), x]$

[Out] $-(\text{ArcTan}[(1 - 2*\text{Sqrt}[2]*x)/\text{Sqrt}[3]]/\text{Sqrt}[6]) + \text{ArcTan}[(1 + 2*\text{Sqrt}[2]*x)/\text{Sqrt}[3]]/\text{Sqrt}[6]$

Rule 1161

$\text{Int}[(d + (e_*)*(x_)^2)/((a_) + (b_*)*(x_)^2 + (c_*)*(x_)^4), x_Symbol] :$
 $> \text{With}\{q = \text{Rt}[(2*d)/e - b/c, 2]\}, \text{Dist}[e/(2*c), \text{Int}[1/\text{Simp}[d/e + q*x + x^2, x], x], x] + \text{Dist}[e/(2*c), \text{Int}[1/\text{Simp}[d/e - q*x + x^2, x], x], x] /;$
 $\text{FreeQ}\{a, b, c, d, e\}, x \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0] \ \&\& \ \text{EqQ}[c*d^2 - a*e^2, 0] \ \&\& \ (\text{GtQ}[(2*d)/e - b/c, 0] \ || \ (\text{!LtQ}[(2*d)/e - b/c, 0] \ \&\& \ \text{EqQ}[d - e*\text{Rt}[a/c, 2], 0]))$

Rule 618

$\text{Int}[(a_) + (b_*)*(x_) + (c_*)*(x_)^2]^{-1}, x_Symbol] :$
 $> \text{Dist}[-2, \text{Subst}[\text{Int}[1/\text{Simp}[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /;$
 $\text{FreeQ}\{a, b, c, x\} \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0]$

Rule 204

$\text{Int}[(a_) + (b_*)*(x_)^2]^{-1}, x_Symbol] :$
 $> -\text{Simp}[\text{ArcTan}[(\text{Rt}[-b, 2]*x)/\text{Rt}[-a, 2]]/(\text{Rt}[-a, 2]*\text{Rt}[-b, 2]), x] /;$
 $\text{FreeQ}\{a, b, x\} \ \&\& \ \text{PosQ}[a/b] \ \&\& \ (\text{LtQ}[a, 0] \ || \ \text{LtQ}[b, 0])$

Rubi steps

$$\begin{aligned}
\int \frac{1+2x^2}{1+2x^2+4x^4} dx &= \frac{1}{4} \int \frac{1}{\frac{1}{2} - \frac{x}{\sqrt{2}} + x^2} dx + \frac{1}{4} \int \frac{1}{\frac{1}{2} + \frac{x}{\sqrt{2}} + x^2} dx \\
&= -\left(\frac{1}{2} \text{Subst}\left(\int \frac{1}{-\frac{3}{2} - x^2} dx, x, -\frac{1}{\sqrt{2}} + 2x\right)\right) - \frac{1}{2} \text{Subst}\left(\int \frac{1}{-\frac{3}{2} - x^2} dx, x, \frac{1}{\sqrt{2}} + 2x\right) \\
&= -\frac{\tan^{-1}\left(\frac{1-2\sqrt{2}x}{\sqrt{3}}\right)}{\sqrt{6}} + \frac{\tan^{-1}\left(\frac{1+2\sqrt{2}x}{\sqrt{3}}\right)}{\sqrt{6}}
\end{aligned}$$

Mathematica [C] time = 0.104063, size = 99, normalized size = 2.06

$$\frac{(\sqrt{3}-i)\tan^{-1}\left(\frac{2x}{\sqrt{1-i\sqrt{3}}}\right)}{2\sqrt{3}(1-i\sqrt{3})} + \frac{(\sqrt{3}+i)\tan^{-1}\left(\frac{2x}{\sqrt{1+i\sqrt{3}}}\right)}{2\sqrt{3}(1+i\sqrt{3})}$$

Antiderivative was successfully verified.

[In] Integrate[(1 + 2*x^2)/(1 + 2*x^2 + 4*x^4), x]

[Out] ((-I + Sqrt[3])*ArcTan[(2*x)/Sqrt[1 - I*Sqrt[3]]])/(2*Sqrt[3]*(1 - I*Sqrt[3])) + ((I + Sqrt[3])*ArcTan[(2*x)/Sqrt[1 + I*Sqrt[3]]])/(2*Sqrt[3]*(1 + I*Sqrt[3]))

Maple [A] time = 0.06, size = 40, normalized size = 0.8

$$\frac{\sqrt{6}}{6} \arctan\left(\frac{(4x + \sqrt{2})\sqrt{6}}{6}\right) + \frac{\sqrt{6}}{6} \arctan\left(\frac{(4x - \sqrt{2})\sqrt{6}}{6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*x^2+1)/(4*x^4+2*x^2+1), x)

[Out] 1/6*6^(1/2)*arctan(1/6*(4*x+2^(1/2))*6^(1/2))+1/6*6^(1/2)*arctan(1/6*(4*x-2^(1/2))*6^(1/2))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{2x^2 + 1}{4x^4 + 2x^2 + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2+1)/(4*x^4+2*x^2+1), x, algorithm="maxima")

[Out] integrate((2*x^2 + 1)/(4*x^4 + 2*x^2 + 1), x)

Fricas [A] time = 1.24012, size = 107, normalized size = 2.23

$$\frac{1}{6} \sqrt{6} \arctan\left(\frac{2}{3} \sqrt{6}(x^3 + x)\right) + \frac{1}{6} \sqrt{6} \arctan\left(\frac{1}{3} \sqrt{6}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2+1)/(4*x^4+2*x^2+1),x, algorithm="fricas")

[Out] 1/6*sqrt(6)*arctan(2/3*sqrt(6)*(x^3 + x)) + 1/6*sqrt(6)*arctan(1/3*sqrt(6)*x)

Sympy [A] time = 0.111128, size = 42, normalized size = 0.88

$$\frac{\sqrt{6} \left(2 \operatorname{atan}\left(\frac{\sqrt{6}x}{3}\right) + 2 \operatorname{atan}\left(\frac{2\sqrt{6}x^3}{3} + \frac{2\sqrt{6}x}{3}\right) \right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x**2+1)/(4*x**4+2*x**2+1),x)

[Out] sqrt(6)*(2*atan(sqrt(6)*x/3) + 2*atan(2*sqrt(6)*x**3/3 + 2*sqrt(6)*x/3))/12

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{2x^2 + 1}{4x^4 + 2x^2 + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2+1)/(4*x^4+2*x^2+1),x, algorithm="giac")

[Out] integrate((2*x^2 + 1)/(4*x^4 + 2*x^2 + 1), x)

$$3.46 \quad \int \frac{1+2x^2}{1+x^2+4x^4} dx$$

Optimal. Leaf size=46

$$\frac{\tan^{-1}\left(\frac{4x+\sqrt{3}}{\sqrt{5}}\right)}{\sqrt{5}} - \frac{\tan^{-1}\left(\frac{\sqrt{3}-4x}{\sqrt{5}}\right)}{\sqrt{5}}$$

[Out] -(ArcTan[(Sqrt[3] - 4*x)/Sqrt[5]]/Sqrt[5]) + ArcTan[(Sqrt[3] + 4*x)/Sqrt[5]]/Sqrt[5]

Rubi [A] time = 0.0431827, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.15$, Rules used = {1161, 618, 204}

$$\frac{\tan^{-1}\left(\frac{4x+\sqrt{3}}{\sqrt{5}}\right)}{\sqrt{5}} - \frac{\tan^{-1}\left(\frac{\sqrt{3}-4x}{\sqrt{5}}\right)}{\sqrt{5}}$$

Antiderivative was successfully verified.

[In] Int[(1 + 2*x^2)/(1 + x^2 + 4*x^4), x]

[Out] -(ArcTan[(Sqrt[3] - 4*x)/Sqrt[5]]/Sqrt[5]) + ArcTan[(Sqrt[3] + 4*x)/Sqrt[5]]/Sqrt[5]

Rule 1161

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[(2*d)/e - b/c, 0] || (!LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[In
t[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1+2x^2}{1+x^2+4x^4} dx &= \frac{1}{4} \int \frac{1}{\frac{1}{2} - \frac{\sqrt{3}x}{2} + x^2} dx + \frac{1}{4} \int \frac{1}{\frac{1}{2} + \frac{\sqrt{3}x}{2} + x^2} dx \\
&= -\left(\frac{1}{2} \text{Subst}\left(\int \frac{1}{-\frac{5}{4} - x^2} dx, x, -\frac{\sqrt{3}}{2} + 2x\right)\right) - \frac{1}{2} \text{Subst}\left(\int \frac{1}{-\frac{5}{4} - x^2} dx, x, \frac{\sqrt{3}}{2} + 2x\right) \\
&= -\frac{\tan^{-1}\left(\frac{\sqrt{3}-4x}{\sqrt{5}}\right)}{\sqrt{5}} + \frac{\tan^{-1}\left(\frac{\sqrt{3}+4x}{\sqrt{5}}\right)}{\sqrt{5}}
\end{aligned}$$

Mathematica [C] time = 0.225133, size = 97, normalized size = 2.11

$$\frac{(\sqrt{15}-3i)\tan^{-1}\left(\frac{2x}{\sqrt{\frac{1}{2}(1-i\sqrt{15})}}\right)}{\sqrt{30-30i\sqrt{15}}} + \frac{(\sqrt{15}+3i)\tan^{-1}\left(\frac{2x}{\sqrt{\frac{1}{2}(1+i\sqrt{15})}}\right)}{\sqrt{30+30i\sqrt{15}}}$$

Antiderivative was successfully verified.

[In] Integrate[(1 + 2*x^2)/(1 + x^2 + 4*x^4), x]

[Out] ((-3*I + Sqrt[15])*ArcTan[(2*x)/Sqrt[(1 - I*Sqrt[15])/2]])/Sqrt[30 - (30*I)*Sqrt[15]] + ((3*I + Sqrt[15])*ArcTan[(2*x)/Sqrt[(1 + I*Sqrt[15])/2]])/Sqrt[30 + (30*I)*Sqrt[15]]

Maple [A] time = 0.061, size = 40, normalized size = 0.9

$$\frac{\sqrt{5}}{5} \arctan\left(\frac{(4x - \sqrt{3})\sqrt{5}}{5}\right) + \frac{\sqrt{5}}{5} \arctan\left(\frac{(4x + \sqrt{3})\sqrt{5}}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*x^2+1)/(4*x^4+x^2+1), x)

[Out] 1/5*5^(1/2)*arctan(1/5*(4*x-3^(1/2))*5^(1/2))+1/5*arctan(1/5*(4*x+3^(1/2))*5^(1/2))*5^(1/2)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{2x^2+1}{4x^4+x^2+1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2+1)/(4*x^4+x^2+1), x, algorithm="maxima")

[Out] integrate((2*x^2 + 1)/(4*x^4 + x^2 + 1), x)

Fricas [A] time = 1.34345, size = 112, normalized size = 2.43

$$\frac{1}{5} \sqrt{5} \arctan\left(\frac{1}{5} \sqrt{5}(4x^3 + 3x)\right) + \frac{1}{5} \sqrt{5} \arctan\left(\frac{2}{5} \sqrt{5}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2+1)/(4*x^4+x^2+1),x, algorithm="fricas")

[Out] 1/5*sqrt(5)*arctan(1/5*sqrt(5)*(4*x^3 + 3*x)) + 1/5*sqrt(5)*arctan(2/5*sqrt(5)*x)

Sympy [A] time = 0.112677, size = 44, normalized size = 0.96

$$\frac{\sqrt{5} \left(2 \operatorname{atan}\left(\frac{2\sqrt{5}x}{5}\right) + 2 \operatorname{atan}\left(\frac{4\sqrt{5}x^3}{5} + \frac{3\sqrt{5}x}{5}\right) \right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x**2+1)/(4*x**4+x**2+1),x)

[Out] sqrt(5)*(2*atan(2*sqrt(5)*x/5) + 2*atan(4*sqrt(5)*x**3/5 + 3*sqrt(5)*x/5))/10

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{2x^2 + 1}{4x^4 + x^2 + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2+1)/(4*x^4+x^2+1),x, algorithm="giac")

[Out] integrate((2*x^2 + 1)/(4*x^4 + x^2 + 1), x)

$$3.47 \quad \int \frac{1+2x^2}{1+4x^4} dx$$

Optimal. Leaf size=21

$$\frac{1}{2} \tan^{-1}(2x+1) - \frac{1}{2} \tan^{-1}(1-2x)$$

[Out] -ArcTan[1 - 2*x]/2 + ArcTan[1 + 2*x]/2

Rubi [A] time = 0.0129114, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {1162, 617, 204}

$$\frac{1}{2} \tan^{-1}(2x+1) - \frac{1}{2} \tan^{-1}(1-2x)$$

Antiderivative was successfully verified.

[In] Int[(1 + 2*x^2)/(1 + 4*x^4), x]

[Out] -ArcTan[1 - 2*x]/2 + ArcTan[1 + 2*x]/2

Rule 1162

Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]

Rule 617

Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned} \int \frac{1+2x^2}{1+4x^4} dx &= \frac{1}{4} \int \frac{1}{\frac{1}{2}-x+x^2} dx + \frac{1}{4} \int \frac{1}{\frac{1}{2}+x+x^2} dx \\ &= \frac{1}{2} \text{Subst} \left(\int \frac{1}{-1-x^2} dx, x, 1-2x \right) - \frac{1}{2} \text{Subst} \left(\int \frac{1}{-1-x^2} dx, x, 1+2x \right) \\ &= -\frac{1}{2} \tan^{-1}(1-2x) + \frac{1}{2} \tan^{-1}(1+2x) \end{aligned}$$

Mathematica [A] time = 0.0061624, size = 17, normalized size = 0.81

$$-\frac{1}{2} \tan^{-1} \left(\frac{2x}{2x^2-1} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(1 + 2*x^2)/(1 + 4*x^4),x]

[Out] -ArcTan[(2*x)/(-1 + 2*x^2)]/2

Maple [A] time = 0.043, size = 18, normalized size = 0.9

$$\frac{\arctan(2x-1)}{2} + \frac{\arctan(1+2x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*x^2+1)/(4*x^4+1),x)

[Out] 1/2*arctan(2*x-1)+1/2*arctan(1+2*x)

Maxima [A] time = 1.45928, size = 23, normalized size = 1.1

$$\frac{1}{2} \arctan(2x+1) + \frac{1}{2} \arctan(2x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2+1)/(4*x^4+1),x, algorithm="maxima")

[Out] 1/2*arctan(2*x + 1) + 1/2*arctan(2*x - 1)

Fricas [A] time = 1.30594, size = 53, normalized size = 2.52

$$\frac{1}{2} \arctan(2x^3+x) + \frac{1}{2} \arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2+1)/(4*x^4+1),x, algorithm="fricas")

[Out] 1/2*arctan(2*x^3 + x) + 1/2*arctan(x)

Sympy [A] time = 0.095898, size = 14, normalized size = 0.67

$$\frac{\operatorname{atan}(x)}{2} + \frac{\operatorname{atan}(2x^3+x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x**2+1)/(4*x**4+1),x)

[Out] atan(x)/2 + atan(2*x**3 + x)/2

Giac [B] time = 1.11962, size = 62, normalized size = 2.95

$$\frac{1}{2} \arctan \left(2\sqrt{2} \left(\frac{1}{4} \right)^{\frac{3}{4}} \left(2x + \sqrt{2} \left(\frac{1}{4} \right)^{\frac{1}{4}} \right) \right) + \frac{1}{2} \arctan \left(2\sqrt{2} \left(\frac{1}{4} \right)^{\frac{3}{4}} \left(2x - \sqrt{2} \left(\frac{1}{4} \right)^{\frac{1}{4}} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2+1)/(4*x^4+1),x, algorithm="giac")

[Out] 1/2*arctan(2*sqrt(2)*(1/4)^(3/4)*(2*x + sqrt(2)*(1/4)^(1/4))) + 1/2*arctan(2*sqrt(2)*(1/4)^(3/4)*(2*x - sqrt(2)*(1/4)^(1/4)))

$$3.48 \quad \int \frac{1+2x^2}{1-x^2+4x^4} dx$$

Optimal. Leaf size=46

$$\frac{\tan^{-1}\left(\frac{4x+\sqrt{5}}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{\tan^{-1}\left(\frac{\sqrt{5}-4x}{\sqrt{3}}\right)}{\sqrt{3}}$$

[Out] -(ArcTan[(Sqrt[5] - 4*x)/Sqrt[3]]/Sqrt[3]) + ArcTan[(Sqrt[5] + 4*x)/Sqrt[3]]/Sqrt[3]

Rubi [A] time = 0.0414338, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {1161, 618, 204}

$$\frac{\tan^{-1}\left(\frac{4x+\sqrt{5}}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{\tan^{-1}\left(\frac{\sqrt{5}-4x}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[(1 + 2*x^2)/(1 - x^2 + 4*x^4), x]

[Out] -(ArcTan[(Sqrt[5] - 4*x)/Sqrt[3]]/Sqrt[3]) + ArcTan[(Sqrt[5] + 4*x)/Sqrt[3]]/Sqrt[3]

Rule 1161

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (b_)*(x_)^2 + (c_)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[(2*d)/e - b/c, 0] || (!LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 618

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{1+2x^2}{1-x^2+4x^4} dx &= \frac{1}{4} \int \frac{1}{\frac{1}{2} - \frac{\sqrt{5}x}{2} + x^2} dx + \frac{1}{4} \int \frac{1}{\frac{1}{2} + \frac{\sqrt{5}x}{2} + x^2} dx \\ &= -\left(\frac{1}{2} \text{Subst}\left(\int \frac{1}{-\frac{3}{4} - x^2} dx, x, -\frac{\sqrt{5}}{2} + 2x\right)\right) - \frac{1}{2} \text{Subst}\left(\int \frac{1}{-\frac{3}{4} - x^2} dx, x, \frac{\sqrt{5}}{2} + 2x\right) \\ &= -\frac{\tan^{-1}\left(\frac{\sqrt{5}-4x}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{\tan^{-1}\left(\frac{\sqrt{5}+4x}{\sqrt{3}}\right)}{\sqrt{3}} \end{aligned}$$

Mathematica [C] time = 0.278065, size = 101, normalized size = 2.2

$$\frac{(\sqrt{15}-5i) \tan^{-1}\left(\frac{2x}{\sqrt{\frac{1}{2}(-1-i\sqrt{15})}}\right)}{\sqrt{30}(-1-i\sqrt{15})} + \frac{(\sqrt{15}+5i) \tan^{-1}\left(\frac{2x}{\sqrt{\frac{1}{2}(-1+i\sqrt{15})}}\right)}{\sqrt{30}(-1+i\sqrt{15})}$$

Antiderivative was successfully verified.

[In] Integrate[(1 + 2*x^2)/(1 - x^2 + 4*x^4), x]

[Out] ((-5*I + Sqrt[15])*ArcTan[(2*x)/Sqrt[(-1 - I*Sqrt[15])/2]])/Sqrt[30*(-1 - I*Sqrt[15])] + ((5*I + Sqrt[15])*ArcTan[(2*x)/Sqrt[(-1 + I*Sqrt[15])/2]])/Sqrt[30*(-1 + I*Sqrt[15])]

Maple [A] time = 0.062, size = 40, normalized size = 0.9

$$\frac{\sqrt{3}}{3} \arctan\left(\frac{(4x + \sqrt{5})\sqrt{3}}{3}\right) + \frac{\sqrt{3}}{3} \arctan\left(\frac{(4x - \sqrt{5})\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*x^2+1)/(4*x^4-x^2+1), x)

[Out] 1/3*arctan(1/3*(4*x+5^(1/2))*3^(1/2))*3^(1/2)+1/3*3^(1/2)*arctan(1/3*(4*x-5^(1/2))*3^(1/2))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{2x^2+1}{4x^4-x^2+1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2+1)/(4*x^4-x^2+1), x, algorithm="maxima")

[Out] integrate((2*x^2 + 1)/(4*x^4 - x^2 + 1), x)

Fricas [A] time = 1.39883, size = 109, normalized size = 2.37

$$\frac{1}{3} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3}(4x^3 + x)\right) + \frac{1}{3} \sqrt{3} \arctan\left(\frac{2}{3} \sqrt{3}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2+1)/(4*x^4-x^2+1),x, algorithm="fricas")

[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(4*x^3 + x)) + 1/3*sqrt(3)*arctan(2/3*sqrt(3)*x)

Sympy [A] time = 0.115995, size = 42, normalized size = 0.91

$$\frac{\sqrt{3} \left(2 \operatorname{atan}\left(\frac{2\sqrt{3}x}{3}\right) + 2 \operatorname{atan}\left(\frac{4\sqrt{3}x^3}{3} + \frac{\sqrt{3}x}{3}\right) \right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x**2+1)/(4*x**4-x**2+1),x)

[Out] sqrt(3)*(2*atan(2*sqrt(3)*x/3) + 2*atan(4*sqrt(3)*x**3/3 + sqrt(3)*x/3))/6

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{2x^2 + 1}{4x^4 - x^2 + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2+1)/(4*x^4-x^2+1),x, algorithm="giac")

[Out] integrate((2*x^2 + 1)/(4*x^4 - x^2 + 1), x)

$$3.49 \quad \int \frac{1+2x^2}{1-2x^2+4x^4} dx$$

Optimal. Leaf size=44

$$\frac{\tan^{-1}(2\sqrt{2}x + \sqrt{3})}{\sqrt{2}} - \frac{\tan^{-1}(\sqrt{3} - 2\sqrt{2}x)}{\sqrt{2}}$$

[Out] -(ArcTan[Sqrt[3] - 2*Sqrt[2]*x]/Sqrt[2]) + ArcTan[Sqrt[3] + 2*Sqrt[2]*x]/Sqrt[2]

Rubi [A] time = 0.0336888, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {1161, 618, 204}

$$\frac{\tan^{-1}(2\sqrt{2}x + \sqrt{3})}{\sqrt{2}} - \frac{\tan^{-1}(\sqrt{3} - 2\sqrt{2}x)}{\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[(1 + 2*x^2)/(1 - 2*x^2 + 4*x^4), x]

[Out] -(ArcTan[Sqrt[3] - 2*Sqrt[2]*x]/Sqrt[2]) + ArcTan[Sqrt[3] + 2*Sqrt[2]*x]/Sqrt[2]

Rule 1161

Int[((d_) + (e_)*(x_)^2)/((a_) + (b_)*(x_)^2 + (c_)*(x_)^4), x_Symbol] :> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (GtQ[(2*d)/e - b/c, 0] || (!LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2], 0]))

Rule 618

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 204

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned} \int \frac{1+2x^2}{1-2x^2+4x^4} dx &= \frac{1}{4} \int \frac{1}{\frac{1}{2}-\sqrt{\frac{3}{2}}x+x^2} dx + \frac{1}{4} \int \frac{1}{\frac{1}{2}+\sqrt{\frac{3}{2}}x+x^2} dx \\ &= -\left(\frac{1}{2} \text{Subst}\left(\int \frac{1}{-\frac{1}{2}-x^2} dx, x, -\sqrt{\frac{3}{2}}+2x\right)\right) - \frac{1}{2} \text{Subst}\left(\int \frac{1}{-\frac{1}{2}-x^2} dx, x, \sqrt{\frac{3}{2}}+2x\right) \\ &= -\frac{\tan^{-1}(\sqrt{3}-2\sqrt{2}x)}{\sqrt{2}} + \frac{\tan^{-1}(\sqrt{3}+2\sqrt{2}x)}{\sqrt{2}} \end{aligned}$$

Mathematica [C] time = 0.103253, size = 99, normalized size = 2.25

$$\frac{(\sqrt{3}-3i)\tan^{-1}\left(\frac{2x}{\sqrt{-1-i\sqrt{3}}}\right)}{2\sqrt{3}(-1-i\sqrt{3})} + \frac{(\sqrt{3}+3i)\tan^{-1}\left(\frac{2x}{\sqrt{-1+i\sqrt{3}}}\right)}{2\sqrt{3}(-1+i\sqrt{3})}$$

Antiderivative was successfully verified.

[In] Integrate[(1 + 2*x^2)/(1 - 2*x^2 + 4*x^4), x]

[Out] ((-3*I + Sqrt[3])*ArcTan[(2*x)/Sqrt[-1 - I*Sqrt[3]]])/(2*Sqrt[3*(-1 - I*Sqrt[3])]) + ((3*I + Sqrt[3])*ArcTan[(2*x)/Sqrt[-1 + I*Sqrt[3]]])/(2*Sqrt[3*(-1 + I*Sqrt[3])])

Maple [A] time = 0.063, size = 40, normalized size = 0.9

$$\frac{\sqrt{2}}{2} \arctan\left(\frac{(4x-\sqrt{6})\sqrt{2}}{2}\right) + \frac{\sqrt{2}}{2} \arctan\left(\frac{(4x+\sqrt{6})\sqrt{2}}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*x^2+1)/(4*x^4-2*x^2+1), x)

[Out] 1/2*2^(1/2)*arctan(1/2*(4*x-6^(1/2))*2^(1/2))+1/2*2^(1/2)*arctan(1/2*(4*x+6^(1/2))*2^(1/2))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{2x^2+1}{4x^4-2x^2+1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2+1)/(4*x^4-2*x^2+1), x, algorithm="maxima")

[Out] integrate((2*x^2 + 1)/(4*x^4 - 2*x^2 + 1), x)

Fricas [A] time = 1.32637, size = 90, normalized size = 2.05

$$\frac{1}{2} \sqrt{2} \arctan\left(2\sqrt{2}x^3\right) + \frac{1}{2} \sqrt{2} \arctan\left(\sqrt{2}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2+1)/(4*x^4-2*x^2+1),x, algorithm="fricas")

[Out] 1/2*sqrt(2)*arctan(2*sqrt(2)*x^3) + 1/2*sqrt(2)*arctan(sqrt(2)*x)

Sympy [A] time = 0.107194, size = 29, normalized size = 0.66

$$\frac{\sqrt{2} \left(2 \operatorname{atan}\left(\sqrt{2}x\right) + 2 \operatorname{atan}\left(2\sqrt{2}x^3\right) \right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x**2+1)/(4*x**4-2*x**2+1),x)

[Out] sqrt(2)*(2*atan(sqrt(2)*x) + 2*atan(2*sqrt(2)*x**3))/4

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{2x^2 + 1}{4x^4 - 2x^2 + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2+1)/(4*x^4-2*x^2+1),x, algorithm="giac")

[Out] integrate((2*x^2 + 1)/(4*x^4 - 2*x^2 + 1), x)

$$3.50 \quad \int \frac{1+2x^2}{1-3x^2+4x^4} dx$$

Optimal. Leaf size=23

$$\tan^{-1}(4x + \sqrt{7}) - \tan^{-1}(\sqrt{7} - 4x)$$

[Out] -ArcTan[Sqrt[7] - 4*x] + ArcTan[Sqrt[7] + 4*x]

Rubi [A] time = 0.0262838, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {1161, 618, 204}

$$\tan^{-1}(4x + \sqrt{7}) - \tan^{-1}(\sqrt{7} - 4x)$$

Antiderivative was successfully verified.

[In] Int[(1 + 2*x^2)/(1 - 3*x^2 + 4*x^4), x]

[Out] -ArcTan[Sqrt[7] - 4*x] + ArcTan[Sqrt[7] + 4*x]

Rule 1161

Int[((d_) + (e_)*(x_)^2)/((a_) + (b_)*(x_)^2 + (c_)*(x_)^4), x_Symbol] :> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (GtQ[(2*d)/e - b/c, 0] || (!LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2], 0]))

Rule 618

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 204

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned} \int \frac{1+2x^2}{1-3x^2+4x^4} dx &= \frac{1}{4} \int \frac{1}{\frac{1}{2} - \frac{\sqrt{7}x}{2} + x^2} dx + \frac{1}{4} \int \frac{1}{\frac{1}{2} + \frac{\sqrt{7}x}{2} + x^2} dx \\ &= -\left(\frac{1}{2} \text{Subst}\left(\int \frac{1}{-\frac{1}{4} - x^2} dx, x, -\frac{\sqrt{7}}{2} + 2x\right)\right) - \frac{1}{2} \text{Subst}\left(\int \frac{1}{-\frac{1}{4} - x^2} dx, x, \frac{\sqrt{7}}{2} + 2x\right) \\ &= -\tan^{-1}(\sqrt{7} - 4x) + \tan^{-1}(\sqrt{7} + 4x) \end{aligned}$$

Mathematica [A] time = 0.0070199, size = 14, normalized size = 0.61

$$-\tan^{-1}\left(\frac{x}{2x^2-1}\right)$$

Antiderivative was successfully verified.

[In] Integrate[(1 + 2*x^2)/(1 - 3*x^2 + 4*x^4), x]

[Out] -ArcTan[x/(-1 + 2*x^2)]

Maple [A] time = 0.061, size = 20, normalized size = 0.9

$$\arctan(4x - \sqrt{7}) + \arctan(4x + \sqrt{7})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*x^2+1)/(4*x^4-3*x^2+1), x)

[Out] arctan(4*x-7^(1/2))+arctan(4*x+7^(1/2))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{2x^2 + 1}{4x^4 - 3x^2 + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2+1)/(4*x^4-3*x^2+1), x, algorithm="maxima")

[Out] integrate((2*x^2 + 1)/(4*x^4 - 3*x^2 + 1), x)

Fricas [A] time = 1.33888, size = 45, normalized size = 1.96

$$\arctan(4x^3 - x) + \arctan(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2+1)/(4*x^4-3*x^2+1), x, algorithm="fricas")

[Out] arctan(4*x^3 - x) + arctan(2*x)

Sympy [A] time = 0.099576, size = 12, normalized size = 0.52

$$\operatorname{atan}(2x) + \operatorname{atan}(4x^3 - x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x**2+1)/(4*x**4-3*x**2+1), x)

[Out] atan(2*x) + atan(4*x**3 - x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{2x^2 + 1}{4x^4 - 3x^2 + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2+1)/(4*x^4-3*x^2+1),x, algorithm="giac")

[Out] integrate((2*x^2 + 1)/(4*x^4 - 3*x^2 + 1), x)

$$3.51 \quad \int \frac{1+2x^2}{1-4x^2+4x^4} dx$$

Optimal. Leaf size=11

$$\frac{x}{1-2x^2}$$

[Out] x/(1 - 2*x^2)

Rubi [A] time = 0.0050256, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {28, 383}

$$\frac{x}{1-2x^2}$$

Antiderivative was successfully verified.

[In] Int[(1 + 2*x^2)/(1 - 4*x^2 + 4*x^4), x]

[Out] x/(1 - 2*x^2)

Rule 28

Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]

Rule 383

Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[c*x*(a + b*x^n)^(p + 1)/a, x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[a*d - b*c*(n*(p + 1) + 1), 0]

Rubi steps

$$\begin{aligned} \int \frac{1+2x^2}{1-4x^2+4x^4} dx &= 4 \int \frac{1+2x^2}{(-2+4x^2)^2} dx \\ &= \frac{x}{1-2x^2} \end{aligned}$$

Mathematica [A] time = 0.0057191, size = 12, normalized size = 1.09

$$-\frac{x}{2x^2-1}$$

Antiderivative was successfully verified.

[In] Integrate[(1 + 2*x^2)/(1 - 4*x^2 + 4*x^4), x]

[Out] -(x/(-1 + 2*x^2))

Maple [A] time = 0.046, size = 11, normalized size = 1.

$$-\frac{x}{2} \left(x^2 - \frac{1}{2} \right)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*x^2+1)/(4*x^4-4*x^2+1),x)

[Out] -1/2*x/(x^2-1/2)

Maxima [A] time = 0.956096, size = 16, normalized size = 1.45

$$-\frac{x}{2x^2 - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2+1)/(4*x^4-4*x^2+1),x, algorithm="maxima")

[Out] -x/(2*x^2 - 1)

Fricas [A] time = 1.31542, size = 22, normalized size = 2.

$$-\frac{x}{2x^2 - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2+1)/(4*x^4-4*x^2+1),x, algorithm="fricas")

[Out] -x/(2*x^2 - 1)

Sympy [A] time = 0.081402, size = 8, normalized size = 0.73

$$-\frac{x}{2x^2 - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x**2+1)/(4*x**4-4*x**2+1),x)

[Out] -x/(2*x**2 - 1)

Giac [A] time = 1.13147, size = 16, normalized size = 1.45

$$-\frac{x}{2x^2 - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2+1)/(4*x^4-4*x^2+1),x, algorithm="giac")

[Out] -x/(2*x^2 - 1)

$$3.52 \quad \int \frac{1+2x^2}{1-5x^2+4x^4} dx$$

Optimal. Leaf size=39

$$-\frac{1}{2} \log(1-2x) + \frac{1}{2} \log(1-x) - \frac{1}{2} \log(x+1) + \frac{1}{2} \log(2x+1)$$

[Out] -Log[1 - 2*x]/2 + Log[1 - x]/2 - Log[1 + x]/2 + Log[1 + 2*x]/2

Rubi [A] time = 0.0178852, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {1161, 616, 31}

$$-\frac{1}{2} \log(1-2x) + \frac{1}{2} \log(1-x) - \frac{1}{2} \log(x+1) + \frac{1}{2} \log(2x+1)$$

Antiderivative was successfully verified.

[In] Int[(1 + 2*x^2)/(1 - 5*x^2 + 4*x^4), x]

[Out] -Log[1 - 2*x]/2 + Log[1 - x]/2 - Log[1 + x]/2 + Log[1 + 2*x]/2

Rule 1161

Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (GtQ[(2*d)/e - b/c, 0] || (!LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2], 0]))

Rule 616

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q, Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]

Rule 31

Int[((a_) + (b_.)*(x_))^-1, x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rubi steps

$$\begin{aligned} \int \frac{1+2x^2}{1-5x^2+4x^4} dx &= \frac{1}{4} \int \frac{1}{\frac{1}{2} - \frac{3x}{2} + x^2} dx + \frac{1}{4} \int \frac{1}{\frac{1}{2} + \frac{3x}{2} + x^2} dx \\ &= \frac{1}{2} \int \frac{1}{-1+x} dx - \frac{1}{2} \int \frac{1}{-\frac{1}{2}+x} dx + \frac{1}{2} \int \frac{1}{\frac{1}{2}+x} dx - \frac{1}{2} \int \frac{1}{1+x} dx \\ &= -\frac{1}{2} \log(1-2x) + \frac{1}{2} \log(1-x) - \frac{1}{2} \log(1+x) + \frac{1}{2} \log(1+2x) \end{aligned}$$

Mathematica [A] time = 0.0058667, size = 29, normalized size = 0.74

$$\frac{1}{2} \log(-2x^2 + x + 1) - \frac{1}{2} \log(-2x^2 - x + 1)$$

Antiderivative was successfully verified.

[In] Integrate[(1 + 2*x^2)/(1 - 5*x^2 + 4*x^4), x]

[Out] -Log[1 - x - 2*x^2]/2 + Log[1 + x - 2*x^2]/2

Maple [A] time = 0.047, size = 30, normalized size = 0.8

$$-\frac{\ln(1+x)}{2} - \frac{\ln(2x-1)}{2} + \frac{\ln(-1+x)}{2} + \frac{\ln(1+2x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*x^2+1)/(4*x^4-5*x^2+1), x)

[Out] -1/2*ln(1+x)-1/2*ln(2*x-1)+1/2*ln(-1+x)+1/2*ln(1+2*x)

Maxima [A] time = 0.96804, size = 39, normalized size = 1.

$$\frac{1}{2} \log(2x + 1) - \frac{1}{2} \log(2x - 1) - \frac{1}{2} \log(x + 1) + \frac{1}{2} \log(x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2+1)/(4*x^4-5*x^2+1), x, algorithm="maxima")

[Out] 1/2*log(2*x + 1) - 1/2*log(2*x - 1) - 1/2*log(x + 1) + 1/2*log(x - 1)

Fricas [A] time = 1.31776, size = 68, normalized size = 1.74

$$-\frac{1}{2} \log(2x^2 + x - 1) + \frac{1}{2} \log(2x^2 - x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2+1)/(4*x^4-5*x^2+1), x, algorithm="fricas")

[Out] -1/2*log(2*x^2 + x - 1) + 1/2*log(2*x^2 - x - 1)

Sympy [A] time = 0.098273, size = 26, normalized size = 0.67

$$\frac{\log\left(x^2 - \frac{x}{2} - \frac{1}{2}\right)}{2} - \frac{\log\left(x^2 + \frac{x}{2} - \frac{1}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2+1)/(4*x**4-5*x**2+1),x)
```

```
[Out] log(x**2 - x/2 - 1/2)/2 - log(x**2 + x/2 - 1/2)/2
```

Giac [A] time = 1.15209, size = 45, normalized size = 1.15

$$\frac{1}{2} \log(|2x + 1|) - \frac{1}{2} \log(|2x - 1|) - \frac{1}{2} \log(|x + 1|) + \frac{1}{2} \log(|x - 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+1)/(4*x^4-5*x^2+1),x, algorithm="giac")
```

```
[Out] 1/2*log(abs(2*x + 1)) - 1/2*log(abs(2*x - 1)) - 1/2*log(abs(x + 1)) + 1/2*log(abs(x - 1))
```

$$3.53 \quad \int \frac{1+2x^2}{1-6x^2+4x^4} dx$$

Optimal. Leaf size=44

$$\frac{\tanh^{-1}(\sqrt{5}-2\sqrt{2}x)}{\sqrt{2}} - \frac{\tanh^{-1}(2\sqrt{2}x+\sqrt{5})}{\sqrt{2}}$$

[Out] ArcTanh[Sqrt[5] - 2*Sqrt[2]*x]/Sqrt[2] - ArcTanh[Sqrt[5] + 2*Sqrt[2]*x]/Sqrt[2]

Rubi [A] time = 0.0347643, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {1161, 618, 206}

$$\frac{\tanh^{-1}(\sqrt{5}-2\sqrt{2}x)}{\sqrt{2}} - \frac{\tanh^{-1}(2\sqrt{2}x+\sqrt{5})}{\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[(1 + 2*x^2)/(1 - 6*x^2 + 4*x^4), x]

[Out] ArcTanh[Sqrt[5] - 2*Sqrt[2]*x]/Sqrt[2] - ArcTanh[Sqrt[5] + 2*Sqrt[2]*x]/Sqrt[2]

Rule 1161

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (b_)*(x_)^2 + (c_)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[(2*d)/e - b/c, 0] || (!LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 618

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] :=> Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] :=> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{1+2x^2}{1-6x^2+4x^4} dx &= \frac{1}{4} \int \frac{1}{\frac{1}{2} - \sqrt{\frac{5}{2}}x + x^2} dx + \frac{1}{4} \int \frac{1}{\frac{1}{2} + \sqrt{\frac{5}{2}}x + x^2} dx \\ &= -\left(\frac{1}{2} \text{Subst}\left(\int \frac{1}{\frac{1}{2} - x^2} dx, x, -\sqrt{\frac{5}{2}} + 2x\right)\right) - \frac{1}{2} \text{Subst}\left(\int \frac{1}{\frac{1}{2} - x^2} dx, x, \sqrt{\frac{5}{2}} + 2x\right) \\ &= \frac{\tanh^{-1}(\sqrt{5} - 2\sqrt{2}x)}{\sqrt{2}} - \frac{\tanh^{-1}(\sqrt{5} + 2\sqrt{2}x)}{\sqrt{2}} \end{aligned}$$

Mathematica [A] time = 0.0134752, size = 42, normalized size = 0.95

$$\frac{\log(-2x^2 + \sqrt{2}x + 1) - \log(2x^2 + \sqrt{2}x - 1)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Integrate[(1 + 2*x^2)/(1 - 6*x^2 + 4*x^4), x]

[Out] (Log[1 + Sqrt[2]*x - 2*x^2] - Log[-1 + Sqrt[2]*x + 2*x^2])/(2*Sqrt[2])

Maple [B] time = 0.073, size = 82, normalized size = 1.9

$$-\frac{(2\sqrt{5}-10)\sqrt{5}}{10\sqrt{10}-10\sqrt{2}} \text{Arctanh}\left(8\frac{x}{2\sqrt{10}-2\sqrt{2}}\right) - \frac{2\sqrt{5}(5+\sqrt{5})}{10\sqrt{10}+10\sqrt{2}} \text{Arctanh}\left(8\frac{x}{2\sqrt{10}+2\sqrt{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*x^2+1)/(4*x^4-6*x^2+1), x)

[Out] -2/5*(5^(1/2)-5)*5^(1/2)/(2*10^(1/2)-2*2^(1/2))*arctanh(8*x/(2*10^(1/2)-2*2^(1/2)))-2/5*5^(1/2)*(5+5^(1/2))/(2*10^(1/2)+2*2^(1/2))*arctanh(8*x/(2*10^(1/2)+2*2^(1/2)))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{2x^2+1}{4x^4-6x^2+1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2+1)/(4*x^4-6*x^2+1), x, algorithm="maxima")

[Out] integrate((2*x^2 + 1)/(4*x^4 - 6*x^2 + 1), x)

Fricas [A] time = 1.38009, size = 111, normalized size = 2.52

$$\frac{1}{4} \sqrt{2} \log\left(\frac{4x^4 - 2x^2 - 2\sqrt{2}(2x^3 - x) + 1}{4x^4 - 6x^2 + 1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2+1)/(4*x^4-6*x^2+1),x, algorithm="fricas")

[Out] 1/4*sqrt(2)*log((4*x^4 - 2*x^2 - 2*sqrt(2)*(2*x^3 - x) + 1)/(4*x^4 - 6*x^2 + 1))

Sympy [A] time = 0.101562, size = 46, normalized size = 1.05

$$\frac{\sqrt{2} \log\left(x^2 - \frac{\sqrt{2}x}{2} - \frac{1}{2}\right)}{4} - \frac{\sqrt{2} \log\left(x^2 + \frac{\sqrt{2}x}{2} - \frac{1}{2}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x**2+1)/(4*x**4-6*x**2+1),x)

[Out] sqrt(2)*log(x**2 - sqrt(2)*x/2 - 1/2)/4 - sqrt(2)*log(x**2 + sqrt(2)*x/2 - 1/2)/4

Giac [B] time = 1.29373, size = 104, normalized size = 2.36

$$-\frac{1}{4} \sqrt{2} \log\left(\left|x + \frac{1}{4} \sqrt{10} + \frac{1}{4} \sqrt{2}\right|\right) + \frac{1}{4} \sqrt{2} \log\left(\left|x + \frac{1}{4} \sqrt{10} - \frac{1}{4} \sqrt{2}\right|\right) - \frac{1}{4} \sqrt{2} \log\left(\left|x - \frac{1}{4} \sqrt{10} + \frac{1}{4} \sqrt{2}\right|\right) + \frac{1}{4} \sqrt{2} \log\left(\left|x - \frac{1}{4} \sqrt{10} - \frac{1}{4} \sqrt{2}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2+1)/(4*x^4-6*x^2+1),x, algorithm="giac")

[Out] -1/4*sqrt(2)*log(abs(x + 1/4*sqrt(10) + 1/4*sqrt(2))) + 1/4*sqrt(2)*log(abs(x + 1/4*sqrt(10) - 1/4*sqrt(2))) - 1/4*sqrt(2)*log(abs(x - 1/4*sqrt(10) + 1/4*sqrt(2))) + 1/4*sqrt(2)*log(abs(x - 1/4*sqrt(10) - 1/4*sqrt(2)))

$$3.54 \quad \int \frac{1-2x^2}{1+bx^2+4x^4} dx$$

Optimal. Leaf size=66

$$\frac{\log(\sqrt{4-bx+2x^2+1})}{2\sqrt{4-b}} - \frac{\log(-\sqrt{4-bx+2x^2+1})}{2\sqrt{4-b}}$$

[Out] -Log[1 - Sqrt[4 - b]*x + 2*x^2]/(2*Sqrt[4 - b]) + Log[1 + Sqrt[4 - b]*x + 2*x^2]/(2*Sqrt[4 - b])

Rubi [A] time = 0.028782, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1164, 628}

$$\frac{\log(\sqrt{4-bx+2x^2+1})}{2\sqrt{4-b}} - \frac{\log(-\sqrt{4-bx+2x^2+1})}{2\sqrt{4-b}}$$

Antiderivative was successfully verified.

[In] Int[(1 - 2*x^2)/(1 + b*x^2 + 4*x^4), x]

[Out] -Log[1 - Sqrt[4 - b]*x + 2*x^2]/(2*Sqrt[4 - b]) + Log[1 + Sqrt[4 - b]*x + 2*x^2]/(2*Sqrt[4 - b])

Rule 1164

Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[(-2*d)/e - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && !GtQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rubi steps

$$\begin{aligned} \int \frac{1-2x^2}{1+bx^2+4x^4} dx &= -\frac{\int \frac{\frac{\sqrt{4-b}}{2}+2x}{-\frac{1}{2}-\frac{1}{2}\sqrt{4-bx-x^2}} dx}{2\sqrt{4-b}} - \frac{\int \frac{\frac{\sqrt{4-b}}{2}-2x}{-\frac{1}{2}+\frac{1}{2}\sqrt{4-bx-x^2}} dx}{2\sqrt{4-b}} \\ &= -\frac{\log(1-\sqrt{4-bx+2x^2})}{2\sqrt{4-b}} + \frac{\log(1+\sqrt{4-bx+2x^2})}{2\sqrt{4-b}} \end{aligned}$$

Mathematica [A] time = 0.0699397, size = 127, normalized size = 1.92

$$\frac{(-\sqrt{b^2-16+b+4}) \tan^{-1}\left(\frac{2\sqrt{2x}}{\sqrt{b-\sqrt{b^2-16}}}\right)}{\sqrt{b-\sqrt{b^2-16}}} - \frac{(\sqrt{b^2-16+b+4}) \tan^{-1}\left(\frac{2\sqrt{2x}}{\sqrt{\sqrt{b^2-16}+b}}\right)}{\sqrt{\sqrt{b^2-16}+b}}$$

$$\frac{\hspace{10em}}{\sqrt{2}\sqrt{b^2-16}}$$

Antiderivative was successfully verified.

[In] Integrate[(1 - 2*x^2)/(1 + b*x^2 + 4*x^4),x]

[Out] (((4 + b - Sqrt[-16 + b^2])*ArcTan[(2*Sqrt[2]*x)/Sqrt[b - Sqrt[-16 + b^2]]])/Sqrt[b - Sqrt[-16 + b^2]] - ((4 + b + Sqrt[-16 + b^2])*ArcTan[(2*Sqrt[2]*x)/Sqrt[b + Sqrt[-16 + b^2]]])/Sqrt[b + Sqrt[-16 + b^2]])/(Sqrt[2]*Sqrt[-16 + b^2])

Maple [B] time = 0.102, size = 279, normalized size = 4.2

$$-4 \frac{1}{\sqrt{(b-4)(4+b)}\sqrt{2\sqrt{(b-4)(4+b)}+2b}} \arctan\left(4 \frac{x}{\sqrt{2\sqrt{(b-4)(4+b)}+2b}}\right) - \arctan\left(4 \frac{x}{\sqrt{2\sqrt{(b-4)(4+b)}+2b}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-2*x^2+1)/(4*x^4+b*x^2+1),x)

[Out] -4/((b-4)*(4+b))^(1/2)/(2*((b-4)*(4+b))^(1/2)+2*b)^(1/2)*arctan(4*x/(2*((b-4)*(4+b))^(1/2)+2*b)^(1/2))-1/(2*((b-4)*(4+b))^(1/2)+2*b)^(1/2)*arctan(4*x/(2*((b-4)*(4+b))^(1/2)+2*b)^(1/2))-1/((b-4)*(4+b))^(1/2)/(2*((b-4)*(4+b))^(1/2)+2*b)^(1/2)*arctan(4*x/(2*((b-4)*(4+b))^(1/2)+2*b)^(1/2))*b+4/((b-4)*(4+b))^(1/2)/(-2*((b-4)*(4+b))^(1/2)+2*b)^(1/2)*arctan(4*x/(-2*((b-4)*(4+b))^(1/2)+2*b)^(1/2))-1/(-2*((b-4)*(4+b))^(1/2)+2*b)^(1/2)*arctan(4*x/(-2*((b-4)*(4+b))^(1/2)+2*b)^(1/2))+1/((b-4)*(4+b))^(1/2)/(-2*((b-4)*(4+b))^(1/2)+2*b)^(1/2)*arctan(4*x/(-2*((b-4)*(4+b))^(1/2)+2*b)^(1/2))*b

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{2x^2 - 1}{4x^4 + bx^2 + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-2*x^2+1)/(4*x^4+b*x^2+1),x, algorithm="maxima")

[Out] -integrate((2*x^2 - 1)/(4*x^4 + b*x^2 + 1), x)

Fricas [A] time = 1.30515, size = 286, normalized size = 4.33

$$\left[\frac{\sqrt{-b+4} \log\left(\frac{4x^4-(b-8)x^2+2(2x^3+x)\sqrt{-b+4}+1}{4x^4+bx^2+1}\right)}{2(b-4)}, \frac{\sqrt{b-4} \arctan\left(\frac{4x^3+(b-2)x}{\sqrt{b-4}}\right) - \sqrt{b-4} \arctan\left(\frac{2x}{\sqrt{b-4}}\right)}{b-4} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-2*x^2+1)/(4*x^4+b*x^2+1),x, algorithm="fricas")

[Out] [-1/2*sqrt(-b + 4)*log((4*x^4 - (b - 8)*x^2 + 2*(2*x^3 + x)*sqrt(-b + 4) + 1)/(4*x^4 + b*x^2 + 1))/(b - 4), (sqrt(b - 4)*arctan((4*x^3 + (b - 2)*x)/sq

$\text{rt}(b - 4) - \sqrt{b - 4} \cdot \arctan(2x/\sqrt{b - 4}) / (b - 4)$

Sympy [A] time = 0.26749, size = 94, normalized size = 1.42

$$\frac{\sqrt{-\frac{1}{b-4}} \log\left(x^2 + x\left(-\frac{b\sqrt{-\frac{1}{b-4}}}{2} + 2\sqrt{-\frac{1}{b-4}}\right) + \frac{1}{2}\right)}{2} - \frac{\sqrt{-\frac{1}{b-4}} \log\left(x^2 + x\left(\frac{b\sqrt{-\frac{1}{b-4}}}{2} - 2\sqrt{-\frac{1}{b-4}}\right) + \frac{1}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-2*x**2+1)/(4*x**4+b*x**2+1),x)

[Out] $\sqrt{-1/(b - 4)} \cdot \log(x^2 + x(-b\sqrt{-1/(b - 4)})/2 + 2\sqrt{-1/(b - 4)}) + 1/2)/2 - \sqrt{-1/(b - 4)} \cdot \log(x^2 + x(b\sqrt{-1/(b - 4)})/2 - 2\sqrt{-1/(b - 4)}) + 1/2)/2$

Giac [C] time = 1.2965, size = 3352, normalized size = 50.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-2*x^2+1)/(4*x^4+b*x^2+1),x, algorithm="giac")

[Out] $-1/4 \cdot (3 \cdot (\sqrt{2} \cdot b^2 + \sqrt{2} \cdot \sqrt{b^2 - 16}) \cdot b - 16 \cdot \sqrt{2}) \cdot \cos(5/4 \cdot \pi + 1/2 \cdot \text{real_part}(\arcsin(1/4 \cdot b)))^2 \cdot \cosh(1/2 \cdot \text{imag_part}(\arcsin(1/4 \cdot b)))^3 \cdot \sin(5/4 \cdot \pi + 1/2 \cdot \text{real_part}(\arcsin(1/4 \cdot b))) - (\sqrt{2} \cdot b^2 + \sqrt{2} \cdot \sqrt{b^2 - 16}) \cdot b - 16 \cdot \sqrt{2}) \cdot \cosh(1/2 \cdot \text{imag_part}(\arcsin(1/4 \cdot b)))^3 \cdot \sin(5/4 \cdot \pi + 1/2 \cdot \text{real_part}(\arcsin(1/4 \cdot b)))^3 - 9 \cdot (\sqrt{2} \cdot b^2 + \sqrt{2} \cdot \sqrt{b^2 - 16}) \cdot b - 16 \cdot \sqrt{2}) \cdot \cos(5/4 \cdot \pi + 1/2 \cdot \text{real_part}(\arcsin(1/4 \cdot b)))^2 \cdot \cosh(1/2 \cdot \text{imag_part}(\arcsin(1/4 \cdot b)))^2 \cdot \sin(5/4 \cdot \pi + 1/2 \cdot \text{real_part}(\arcsin(1/4 \cdot b))) \cdot \sinh(1/2 \cdot \text{imag_part}(\arcsin(1/4 \cdot b))) + 3 \cdot (\sqrt{2} \cdot b^2 + \sqrt{2} \cdot \sqrt{b^2 - 16}) \cdot b - 16 \cdot \sqrt{2}) \cdot \cosh(1/2 \cdot \text{imag_part}(\arcsin(1/4 \cdot b)))^2 \cdot \sin(5/4 \cdot \pi + 1/2 \cdot \text{real_part}(\arcsin(1/4 \cdot b)))^3 \cdot \sinh(1/2 \cdot \text{imag_part}(\arcsin(1/4 \cdot b))) + 9 \cdot (\sqrt{2} \cdot b^2 + \sqrt{2} \cdot \sqrt{b^2 - 16}) \cdot b - 16 \cdot \sqrt{2}) \cdot \cos(5/4 \cdot \pi + 1/2 \cdot \text{real_part}(\arcsin(1/4 \cdot b)))^2 \cdot \cosh(1/2 \cdot \text{imag_part}(\arcsin(1/4 \cdot b))) \cdot \sin(5/4 \cdot \pi + 1/2 \cdot \text{real_part}(\arcsin(1/4 \cdot b))) \cdot \sinh(1/2 \cdot \text{imag_part}(\arcsin(1/4 \cdot b)))^2 - 3 \cdot (\sqrt{2} \cdot b^2 + \sqrt{2} \cdot \sqrt{b^2 - 16}) \cdot b - 16 \cdot \sqrt{2}) \cdot \cosh(1/2 \cdot \text{imag_part}(\arcsin(1/4 \cdot b))) \cdot \sin(5/4 \cdot \pi + 1/2 \cdot \text{real_part}(\arcsin(1/4 \cdot b)))^3 \cdot \sinh(1/2 \cdot \text{imag_part}(\arcsin(1/4 \cdot b)))^2 - 3 \cdot (\sqrt{2} \cdot b^2 + \sqrt{2} \cdot \sqrt{b^2 - 16}) \cdot b - 16 \cdot \sqrt{2}) \cdot \cos(5/4 \cdot \pi + 1/2 \cdot \text{real_part}(\arcsin(1/4 \cdot b)))^2 \cdot \sin(5/4 \cdot \pi + 1/2 \cdot \text{real_part}(\arcsin(1/4 \cdot b))) \cdot \sinh(1/2 \cdot \text{imag_part}(\arcsin(1/4 \cdot b)))^3 + (\sqrt{2} \cdot b^2 + \sqrt{2} \cdot \sqrt{b^2 - 16}) \cdot b - 16 \cdot \sqrt{2}) \cdot \sin(5/4 \cdot \pi + 1/2 \cdot \text{real_part}(\arcsin(1/4 \cdot b)))^3 \cdot \sinh(1/2 \cdot \text{imag_part}(\arcsin(1/4 \cdot b)))^3 - (\sqrt{2} \cdot b^2 + \sqrt{2} \cdot \sqrt{b^2 - 16}) \cdot b - 16 \cdot \sqrt{2}) \cdot \cosh(1/2 \cdot \text{imag_part}(\arcsin(1/4 \cdot b))) \cdot \sin(5/4 \cdot \pi + 1/2 \cdot \text{real_part}(\arcsin(1/4 \cdot b))) + (\sqrt{2} \cdot b^2 + \sqrt{2} \cdot \sqrt{b^2 - 16}) \cdot b - 16 \cdot \sqrt{2}) \cdot \sin(5/4 \cdot \pi + 1/2 \cdot \text{real_part}(\arcsin(1/4 \cdot b))) \cdot \sinh(1/2 \cdot \text{imag_part}(\arcsin(1/4 \cdot b))) \cdot \arctan(-4 \cdot (1/4)^{(3/4)} \cdot ((1/4)^{(1/4)} \cdot \cos(5/4 \cdot \pi + 1/2 \cdot \arcsin(1/4 \cdot b)) - x) / \sin(5/4 \cdot \pi + 1/2 \cdot \arcsin(1/4 \cdot b))) / (b^2 - 16) - 1/4 \cdot (3 \cdot (\sqrt{2} \cdot b^2 + \sqrt{2} \cdot \sqrt{b^2 - 16}) \cdot b - 16 \cdot \sqrt{2}) \cdot \cos(1/4 \cdot \pi + 1/2 \cdot \text{real_part}(\arcsin(1/4 \cdot b)))^2 \cdot \cosh(1/2 \cdot \text{imag_part}(\arcsin(1/4 \cdot b)))^3 \cdot \sin(1/4 \cdot \pi + 1/2 \cdot \text{real_part}(\arcsin(1/4 \cdot b))) - (\sqrt{2} \cdot b^2 + \sqrt{2} \cdot \sqrt{b^2 - 16}) \cdot b - 16 \cdot \sqrt{2}) \cdot \cosh(1/2 \cdot \text{imag_part}(\arcsin(1/4 \cdot b)))^3 \cdot \sin(1/4 \cdot \pi + 1/2 \cdot \text{real_part}(\arcsin(1/4 \cdot b)))^3 - 9 \cdot (\sqrt{2} \cdot b^2 + \sqrt{2} \cdot \sqrt{b^2 - 16}) \cdot b - 16 \cdot \sqrt{2}) \cdot \cos(1/4 \cdot \pi + 1/2 \cdot \text{real_part}(\arcsin(1/4 \cdot b)))^2 \cdot \cosh$

$$\begin{aligned}
&) * b^2 + \sqrt{2} * \sqrt{b^2 - 16} * b - 16 * \sqrt{2} * \cos(1/4 * \pi + 1/2 * \text{real_part}(\arcsin(1/4 * b))) * \sin(1/4 * \pi + 1/2 * \text{real_part}(\arcsin(1/4 * b)))^2 * \sinh(1/2 * \text{imag_part}(\arcsin(1/4 * b)))^3 - (\sqrt{2} * b^2 + \sqrt{2} * \sqrt{b^2 - 16} * b - 16 * \sqrt{2}) * \cos(1/4 * \pi + 1/2 * \text{real_part}(\arcsin(1/4 * b))) * \cosh(1/2 * \text{imag_part}(\arcsin(1/4 * b))) + (\sqrt{2} * b^2 + \sqrt{2} * \sqrt{b^2 - 16} * b - 16 * \sqrt{2}) * \cos(1/4 * \pi + 1/2 * \text{real_part}(\arcsin(1/4 * b))) * \sinh(1/2 * \text{imag_part}(\arcsin(1/4 * b))) * \log(-2 * (1/4)^{1/4} * x * \cos(1/4 * \pi + 1/2 * \arcsin(1/4 * b)) + x^2 + 1/2) / (b^2 - 16)
\end{aligned}$$

$$3.55 \quad \int \frac{1-2x^2}{1+6x^2+4x^4} dx$$

Optimal. Leaf size=46

$$\frac{\tan^{-1}\left(\frac{2x}{\sqrt{3-\sqrt{5}}}\right)}{\sqrt{2}} - \frac{\tan^{-1}\left(\frac{2x}{\sqrt{3+\sqrt{5}}}\right)}{\sqrt{2}}$$

[Out] ArcTan[(2*x)/Sqrt[3 - Sqrt[5]]]/Sqrt[2] - ArcTan[(2*x)/Sqrt[3 + Sqrt[5]]]/Sqrt[2]

Rubi [A] time = 0.0312964, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1163, 203}

$$\frac{\tan^{-1}\left(\frac{2x}{\sqrt{3-\sqrt{5}}}\right)}{\sqrt{2}} - \frac{\tan^{-1}\left(\frac{2x}{\sqrt{3+\sqrt{5}}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[(1 - 2*x^2)/(1 + 6*x^2 + 4*x^4), x]

[Out] ArcTan[(2*x)/Sqrt[3 - Sqrt[5]]]/Sqrt[2] - ArcTan[(2*x)/Sqrt[3 + Sqrt[5]]]/Sqrt[2]

Rule 1163

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (b_)*(x_)^2 + (c_)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
- q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Eq
Q[c*d^2 - a*e^2, 0] && GtQ[b^2 - 4*a*c, 0]
```

Rule 203

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{1-2x^2}{1+6x^2+4x^4} dx &= (-1-\sqrt{5}) \int \frac{1}{3+\sqrt{5}+4x^2} dx + (-1+\sqrt{5}) \int \frac{1}{3-\sqrt{5}+4x^2} dx \\ &= \frac{\tan^{-1}\left(\frac{2x}{\sqrt{3-\sqrt{5}}}\right)}{\sqrt{2}} - \frac{\tan^{-1}\left(\frac{2x}{\sqrt{3+\sqrt{5}}}\right)}{\sqrt{2}} \end{aligned}$$

Mathematica [A] time = 0.0705627, size = 84, normalized size = 1.83

$$\frac{-(\sqrt{5}-5)\sqrt{3+\sqrt{5}}\tan^{-1}\left(\frac{2x}{\sqrt{3-\sqrt{5}}}\right) - \sqrt{3-\sqrt{5}}(5+\sqrt{5})\tan^{-1}\left(\frac{2x}{\sqrt{3+\sqrt{5}}}\right)}{4\sqrt{5}}$$

Antiderivative was successfully verified.

[In] Integrate[(1 - 2*x^2)/(1 + 6*x^2 + 4*x^4), x]

[Out] $(-((-5 + \sqrt{5})\sqrt{3 + \sqrt{5}}\operatorname{ArcTan}[(2x)/\sqrt{3 - \sqrt{5}}]) - \sqrt{3 - \sqrt{5}}(5 + \sqrt{5})\operatorname{ArcTan}[(2x)/\sqrt{3 + \sqrt{5}}]))/(4\sqrt{5})$

Maple [B] time = 0.054, size = 136, normalized size = 3.

$$-2 \frac{1}{2\sqrt{10} - 2\sqrt{2}} \arctan\left(8 \frac{x}{2\sqrt{10} - 2\sqrt{2}}\right) + 2 \frac{\sqrt{5}}{2\sqrt{10} - 2\sqrt{2}} \arctan\left(8 \frac{x}{2\sqrt{10} - 2\sqrt{2}}\right) - 2 \frac{\sqrt{5}}{2\sqrt{10} + 2\sqrt{2}} \arctan\left(8 \frac{x}{2\sqrt{10} + 2\sqrt{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-2*x^2+1)/(4*x^4+6*x^2+1), x)

[Out] $-2/(2*10^{(1/2)}-2*2^{(1/2)})*\arctan(8*x/(2*10^{(1/2)}-2*2^{(1/2)}))+2*5^{(1/2)/(2*10^{(1/2)}-2*2^{(1/2)})*\arctan(8*x/(2*10^{(1/2)}-2*2^{(1/2)}))-2*5^{(1/2)/(2*10^{(1/2)}+2*2^{(1/2)})*\arctan(8*x/(2*10^{(1/2)}+2*2^{(1/2)}))-2/(2*10^{(1/2)}+2*2^{(1/2)})*\arctan(8*x/(2*10^{(1/2)}+2*2^{(1/2)}))$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{2x^2 - 1}{4x^4 + 6x^2 + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-2*x^2+1)/(4*x^4+6*x^2+1), x, algorithm="maxima")

[Out] -integrate((2*x^2 - 1)/(4*x^4 + 6*x^2 + 1), x)

Fricas [A] time = 1.38737, size = 99, normalized size = 2.15

$$\frac{1}{2} \sqrt{2} \arctan\left(2\sqrt{2}(x^3 + x)\right) - \frac{1}{2} \sqrt{2} \arctan\left(\sqrt{2}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-2*x^2+1)/(4*x^4+6*x^2+1), x, algorithm="fricas")

[Out] $1/2*\sqrt{2}*\arctan(2*\sqrt{2}*(x^3 + x)) - 1/2*\sqrt{2}*\arctan(\sqrt{2}*x)$

Sympy [A] time = 0.113693, size = 39, normalized size = 0.85

$$\frac{\sqrt{2}(2 \operatorname{atan}(\sqrt{2}x) - 2 \operatorname{atan}(2\sqrt{2}x^3 + 2\sqrt{2}x))}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-2*x**2+1)/(4*x**4+6*x**2+1),x)

[Out] -sqrt(2)*(2*atan(sqrt(2)*x) - 2*atan(2*sqrt(2)*x**3 + 2*sqrt(2)*x))/4

Giac [A] time = 1.16846, size = 53, normalized size = 1.15

$$-\frac{1}{2}\sqrt{2}\arctan\left(\frac{4x}{\sqrt{10}+\sqrt{2}}\right)+\frac{1}{2}\sqrt{2}\arctan\left(\frac{4x}{\sqrt{10}-\sqrt{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-2*x^2+1)/(4*x^4+6*x^2+1),x, algorithm="giac")

[Out] -1/2*sqrt(2)*arctan(4*x/(sqrt(10) + sqrt(2))) + 1/2*sqrt(2)*arctan(4*x/(sqrt(10) - sqrt(2)))

$$3.56 \quad \int \frac{1-2x^2}{1+5x^2+4x^4} dx$$

Optimal. Leaf size=9

$$\tan^{-1}(2x) - \tan^{-1}(x)$$

[Out] -ArcTan[x] + ArcTan[2*x]

Rubi [A] time = 0.0092982, antiderivative size = 9, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1163, 203}

$$\tan^{-1}(2x) - \tan^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[(1 - 2*x^2)/(1 + 5*x^2 + 4*x^4), x]

[Out] -ArcTan[x] + ArcTan[2*x]

Rule 1163

Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
 > With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Eq
 Q[c*d^2 - a*e^2, 0] && GtQ[b^2 - 4*a*c, 0]

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
 [a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
 , 0] || GtQ[b, 0])

Rubi steps

$$\begin{aligned} \int \frac{1-2x^2}{1+5x^2+4x^4} dx &= 2 \int \frac{1}{1+4x^2} dx - 4 \int \frac{1}{4+4x^2} dx \\ &= -\tan^{-1}(x) + \tan^{-1}(2x) \end{aligned}$$

Mathematica [A] time = 0.0077656, size = 12, normalized size = 1.33

$$\tan^{-1}\left(\frac{x}{2x^2+1}\right)$$

Antiderivative was successfully verified.

[In] Integrate[(1 - 2*x^2)/(1 + 5*x^2 + 4*x^4), x]

[Out] ArcTan[x/(1 + 2*x^2)]

Maple [A] time = 0.048, size = 10, normalized size = 1.1

$$-\arctan(x) + \arctan(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((-2*x^2+1)/(4*x^4+5*x^2+1),x)`

[Out] `-arctan(x)+arctan(2*x)`

Maxima [A] time = 1.44267, size = 12, normalized size = 1.33

$$\arctan(2x) - \arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-2*x^2+1)/(4*x^4+5*x^2+1),x, algorithm="maxima")`

[Out] `arctan(2*x) - arctan(x)`

Fricas [A] time = 1.31286, size = 47, normalized size = 5.22

$$\arctan(4x^3 + 3x) - \arctan(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-2*x^2+1)/(4*x^4+5*x^2+1),x, algorithm="fricas")`

[Out] `arctan(4*x^3 + 3*x) - arctan(2*x)`

Sympy [A] time = 0.101973, size = 14, normalized size = 1.56

$$-\operatorname{atan}(2x) + \operatorname{atan}(4x^3 + 3x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-2*x**2+1)/(4*x**4+5*x**2+1),x)`

[Out] `-atan(2*x) + atan(4*x**3 + 3*x)`

Giac [A] time = 1.11698, size = 12, normalized size = 1.33

$$\arctan(2x) - \arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-2*x^2+1)/(4*x^4+5*x^2+1),x, algorithm="giac")`

[Out] `arctan(2*x) - arctan(x)`

$$3.57 \quad \int \frac{1-2x^2}{1+4x^2+4x^4} dx$$

Optimal. Leaf size=11

$$\frac{x}{2x^2+1}$$

[Out] x/(1 + 2*x^2)

Rubi [A] time = 0.0053507, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {28, 383}

$$\frac{x}{2x^2+1}$$

Antiderivative was successfully verified.

[In] Int[(1 - 2*x^2)/(1 + 4*x^2 + 4*x^4), x]

[Out] x/(1 + 2*x^2)

Rule 28

Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]

Rule 383

Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[c*x*(a + b*x^n)^(p + 1)/a, x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[a*d - b*c*(n*(p + 1) + 1), 0]

Rubi steps

$$\begin{aligned} \int \frac{1-2x^2}{1+4x^2+4x^4} dx &= 4 \int \frac{1-2x^2}{(2+4x^2)^2} dx \\ &= \frac{x}{1+2x^2} \end{aligned}$$

Mathematica [A] time = 0.0047324, size = 11, normalized size = 1.

$$\frac{x}{2x^2+1}$$

Antiderivative was successfully verified.

[In] Integrate[(1 - 2*x^2)/(1 + 4*x^2 + 4*x^4), x]

[Out] x/(1 + 2*x^2)

Maple [A] time = 0.046, size = 11, normalized size = 1.

$$\frac{x}{2} \left(x^2 + \frac{1}{2} \right)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-2*x^2+1)/(4*x^4+4*x^2+1),x)

[Out] 1/2*x/(x^2+1/2)

Maxima [A] time = 0.974697, size = 15, normalized size = 1.36

$$\frac{x}{2x^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-2*x^2+1)/(4*x^4+4*x^2+1),x, algorithm="maxima")

[Out] x/(2*x^2 + 1)

Fricas [A] time = 1.23776, size = 20, normalized size = 1.82

$$\frac{x}{2x^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-2*x^2+1)/(4*x^4+4*x^2+1),x, algorithm="fricas")

[Out] x/(2*x^2 + 1)

Sympy [A] time = 0.08454, size = 7, normalized size = 0.64

$$\frac{x}{2x^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-2*x**2+1)/(4*x**4+4*x**2+1),x)

[Out] x/(2*x**2 + 1)

Giac [A] time = 1.14189, size = 15, normalized size = 1.36

$$\frac{x}{2x^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-2*x^2+1)/(4*x^4+4*x^2+1),x, algorithm="giac")

[Out] x/(2*x^2 + 1)

$$3.58 \quad \int \frac{1-2x^2}{1+3x^2+4x^4} dx$$

Optimal. Leaf size=29

$$\frac{1}{2} \log(2x^2 + x + 1) - \frac{1}{2} \log(2x^2 - x + 1)$$

[Out] -Log[1 - x + 2*x^2]/2 + Log[1 + x + 2*x^2]/2

Rubi [A] time = 0.0159712, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1164, 628}

$$\frac{1}{2} \log(2x^2 + x + 1) - \frac{1}{2} \log(2x^2 - x + 1)$$

Antiderivative was successfully verified.

[In] Int[(1 - 2*x^2)/(1 + 3*x^2 + 4*x^4), x]

[Out] -Log[1 - x + 2*x^2]/2 + Log[1 + x + 2*x^2]/2

Rule 1164

Int[((d_) + (e_)*(x_)^2)/((a_) + (b_)*(x_)^2 + (c_)*(x_)^4), x_Symbol] :> With[{q = Rt[(-2*d)/e - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && !GtQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] :> Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rubi steps

$$\begin{aligned} \int \frac{1-2x^2}{1+3x^2+4x^4} dx &= -\left(\frac{1}{2} \int \frac{\frac{1}{2}+2x}{-\frac{1}{2}-\frac{x}{2}-x^2} dx\right) - \frac{1}{2} \int \frac{\frac{1}{2}-2x}{-\frac{1}{2}+\frac{x}{2}-x^2} dx \\ &= -\frac{1}{2} \log(1-x+2x^2) + \frac{1}{2} \log(1+x+2x^2) \end{aligned}$$

Mathematica [A] time = 0.0059714, size = 29, normalized size = 1.

$$\frac{1}{2} \log(2x^2 + x + 1) - \frac{1}{2} \log(2x^2 - x + 1)$$

Antiderivative was successfully verified.

[In] Integrate[(1 - 2*x^2)/(1 + 3*x^2 + 4*x^4), x]

[Out] -Log[1 - x + 2*x^2]/2 + Log[1 + x + 2*x^2]/2

Maple [A] time = 0.045, size = 26, normalized size = 0.9

$$-\frac{\ln(2x^2 - x + 1)}{2} + \frac{\ln(2x^2 + x + 1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-2*x^2+1)/(4*x^4+3*x^2+1),x)

[Out] -1/2*ln(2*x^2-x+1)+1/2*ln(2*x^2+x+1)

Maxima [A] time = 0.963789, size = 34, normalized size = 1.17

$$\frac{1}{2} \log(2x^2 + x + 1) - \frac{1}{2} \log(2x^2 - x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-2*x^2+1)/(4*x^4+3*x^2+1),x, algorithm="maxima")

[Out] 1/2*log(2*x^2 + x + 1) - 1/2*log(2*x^2 - x + 1)

Fricas [A] time = 1.29054, size = 66, normalized size = 2.28

$$\frac{1}{2} \log(2x^2 + x + 1) - \frac{1}{2} \log(2x^2 - x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-2*x^2+1)/(4*x^4+3*x^2+1),x, algorithm="fricas")

[Out] 1/2*log(2*x^2 + x + 1) - 1/2*log(2*x^2 - x + 1)

Sympy [A] time = 0.099563, size = 26, normalized size = 0.9

$$-\frac{\log\left(x^2 - \frac{x}{2} + \frac{1}{2}\right)}{2} + \frac{\log\left(x^2 + \frac{x}{2} + \frac{1}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-2*x**2+1)/(4*x**4+3*x**2+1),x)

[Out] -log(x**2 - x/2 + 1/2)/2 + log(x**2 + x/2 + 1/2)/2

Giac [A] time = 1.12616, size = 34, normalized size = 1.17

$$\frac{1}{2} \log(2x^2 + x + 1) - \frac{1}{2} \log(2x^2 - x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x^2+1)/(4*x^4+3*x^2+1),x, algorithm="giac")
```

```
[Out] 1/2*log(2*x^2 + x + 1) - 1/2*log(2*x^2 - x + 1)
```

$$3.59 \quad \int \frac{1-2x^2}{1+2x^2+4x^4} dx$$

Optimal. Leaf size=50

$$\frac{\log(2x^2 + \sqrt{2}x + 1)}{2\sqrt{2}} - \frac{\log(2x^2 - \sqrt{2}x + 1)}{2\sqrt{2}}$$

[Out] -Log[1 - Sqrt[2]*x + 2*x^2]/(2*Sqrt[2]) + Log[1 + Sqrt[2]*x + 2*x^2]/(2*Sqrt[2])

Rubi [A] time = 0.0230111, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1164, 628}

$$\frac{\log(2x^2 + \sqrt{2}x + 1)}{2\sqrt{2}} - \frac{\log(2x^2 - \sqrt{2}x + 1)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[(1 - 2*x^2)/(1 + 2*x^2 + 4*x^4), x]

[Out] -Log[1 - Sqrt[2]*x + 2*x^2]/(2*Sqrt[2]) + Log[1 + Sqrt[2]*x + 2*x^2]/(2*Sqrt[2])

Rule 1164

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(-2*d)/e - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[
*d^2 - a*e^2, 0] && !GtQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{1-2x^2}{1+2x^2+4x^4} dx &= -\frac{\int \frac{\frac{1}{\sqrt{2}}+2x}{-\frac{1}{2}-\frac{x}{\sqrt{2}}-x^2} dx}{2\sqrt{2}} - \frac{\int \frac{\frac{1}{\sqrt{2}}-2x}{-\frac{1}{2}+\frac{x}{\sqrt{2}}-x^2} dx}{2\sqrt{2}} \\ &= -\frac{\log(1 - \sqrt{2}x + 2x^2)}{2\sqrt{2}} + \frac{\log(1 + \sqrt{2}x + 2x^2)}{2\sqrt{2}} \end{aligned}$$

Mathematica [A] time = 0.0126572, size = 42, normalized size = 0.84

$$\frac{\log(2x^2 + \sqrt{2}x + 1) - \log(-2x^2 + \sqrt{2}x - 1)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Integrate[(1 - 2*x^2)/(1 + 2*x^2 + 4*x^4), x]

[Out] (-Log[-1 + Sqrt[2]*x - 2*x^2] + Log[1 + Sqrt[2]*x + 2*x^2])/(2*Sqrt[2])

Maple [A] time = 0.05, size = 39, normalized size = 0.8

$$-\frac{\ln(1 + 2x^2 - x\sqrt{2})\sqrt{2}}{4} + \frac{\ln(1 + 2x^2 + x\sqrt{2})\sqrt{2}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-2*x^2+1)/(4*x^4+2*x^2+1), x)

[Out] -1/4*ln(1+2*x^2-x*2^(1/2))*2^(1/2)+1/4*ln(1+2*x^2+x*2^(1/2))*2^(1/2)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{2x^2 - 1}{4x^4 + 2x^2 + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-2*x^2+1)/(4*x^4+2*x^2+1), x, algorithm="maxima")

[Out] -integrate((2*x^2 - 1)/(4*x^4 + 2*x^2 + 1), x)

Fricas [A] time = 1.35116, size = 111, normalized size = 2.22

$$\frac{1}{4}\sqrt{2}\log\left(\frac{4x^4 + 6x^2 + 2\sqrt{2}(2x^3 + x) + 1}{4x^4 + 2x^2 + 1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-2*x^2+1)/(4*x^4+2*x^2+1), x, algorithm="fricas")

[Out] 1/4*sqrt(2)*log((4*x^4 + 6*x^2 + 2*sqrt(2)*(2*x^3 + x) + 1)/(4*x^4 + 2*x^2 + 1))

Sympy [A] time = 0.101103, size = 46, normalized size = 0.92

$$-\frac{\sqrt{2}\log\left(x^2 - \frac{\sqrt{2}x}{2} + \frac{1}{2}\right)}{4} + \frac{\sqrt{2}\log\left(x^2 + \frac{\sqrt{2}x}{2} + \frac{1}{2}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-2*x**2+1)/(4*x**4+2*x**2+1), x)

```
[Out] -sqrt(2)*log(x**2 - sqrt(2)*x/2 + 1/2)/4 + sqrt(2)*log(x**2 + sqrt(2)*x/2 + 1/2)/4
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{2x^2 - 1}{4x^4 + 2x^2 + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x^2+1)/(4*x^4+2*x^2+1),x, algorithm="giac")
```

```
[Out] integrate(-(2*x^2 - 1)/(4*x^4 + 2*x^2 + 1), x)
```

$$3.60 \quad \int \frac{1-2x^2}{1+x^2+4x^4} dx$$

Optimal. Leaf size=50

$$\frac{\log(2x^2 + \sqrt{3}x + 1)}{2\sqrt{3}} - \frac{\log(2x^2 - \sqrt{3}x + 1)}{2\sqrt{3}}$$

[Out] $-\text{Log}[1 - \text{Sqrt}[3]*x + 2*x^2]/(2*\text{Sqrt}[3]) + \text{Log}[1 + \text{Sqrt}[3]*x + 2*x^2]/(2*\text{Sqrt}[3])$

Rubi [A] time = 0.0225422, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$, Rules used = {1164, 628}

$$\frac{\log(2x^2 + \sqrt{3}x + 1)}{2\sqrt{3}} - \frac{\log(2x^2 - \sqrt{3}x + 1)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(1 - 2*x^2)/(1 + x^2 + 4*x^4), x]$

[Out] $-\text{Log}[1 - \text{Sqrt}[3]*x + 2*x^2]/(2*\text{Sqrt}[3]) + \text{Log}[1 + \text{Sqrt}[3]*x + 2*x^2]/(2*\text{Sqrt}[3])$

Rule 1164

$\text{Int}[(d + (e_*)(x_)^2)/((a_) + (b_*)(x_)^2 + (c_*)(x_)^4), x_Symbol] :> \text{With}[\{q = \text{Rt}[(-2*d)/e - b/c, 2]\}, \text{Dist}[e/(2*c*q), \text{Int}[(q - 2*x)/\text{Simp}[d/e + q*x - x^2, x], x], x] + \text{Dist}[e/(2*c*q), \text{Int}[(q + 2*x)/\text{Simp}[d/e - q*x - x^2, x], x], x] /; \text{FreeQ}[\{a, b, c, d, e\}, x] \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& \text{EqQ}[c*d^2 - a*e^2, 0] \&\& !\text{GtQ}[b^2 - 4*a*c, 0]$

Rule 628

$\text{Int}[(d + (e_*)(x_))/((a_) + (b_*)(x_) + (c_*)(x_)^2), x_Symbol] :> \text{Simp}[(d*\text{Log}[\text{RemoveContent}[a + b*x + c*x^2, x]])/b, x] /; \text{FreeQ}[\{a, b, c, d, e\}, x] \&\& \text{EqQ}[2*c*d - b*e, 0]$

Rubi steps

$$\begin{aligned} \int \frac{1-2x^2}{1+x^2+4x^4} dx &= -\frac{\int \frac{\frac{\sqrt{3}}{2}+2x}{-\frac{1}{2}-\frac{\sqrt{3}x}{2}-x^2} dx}{2\sqrt{3}} - \frac{\int \frac{\frac{\sqrt{3}}{2}-2x}{-\frac{1}{2}+\frac{\sqrt{3}x}{2}-x^2} dx}{2\sqrt{3}} \\ &= -\frac{\log(1 - \sqrt{3}x + 2x^2)}{2\sqrt{3}} + \frac{\log(1 + \sqrt{3}x + 2x^2)}{2\sqrt{3}} \end{aligned}$$

Mathematica [A] time = 0.0131213, size = 42, normalized size = 0.84

$$\frac{\log(2x^2 + \sqrt{3}x + 1) - \log(-2x^2 + \sqrt{3}x - 1)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Integrate[(1 - 2*x^2)/(1 + x^2 + 4*x^4),x]

[Out] (-Log[-1 + Sqrt[3]*x - 2*x^2] + Log[1 + Sqrt[3]*x + 2*x^2])/(2*Sqrt[3])

Maple [A] time = 0.051, size = 39, normalized size = 0.8

$$-\frac{\ln(1 + 2x^2 - x\sqrt{3})\sqrt{3}}{6} + \frac{\ln(1 + 2x^2 + x\sqrt{3})\sqrt{3}}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-2*x^2+1)/(4*x^4+x^2+1),x)

[Out] -1/6*ln(1+2*x^2-x*3^(1/2))*3^(1/2)+1/6*ln(1+2*x^2+x*3^(1/2))*3^(1/2)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{2x^2 - 1}{4x^4 + x^2 + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-2*x^2+1)/(4*x^4+x^2+1),x, algorithm="maxima")

[Out] -integrate((2*x^2 - 1)/(4*x^4 + x^2 + 1), x)

Fricas [A] time = 1.26581, size = 108, normalized size = 2.16

$$\frac{1}{6}\sqrt{3}\log\left(\frac{4x^4 + 7x^2 + 2\sqrt{3}(2x^3 + x) + 1}{4x^4 + x^2 + 1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-2*x^2+1)/(4*x^4+x^2+1),x, algorithm="fricas")

[Out] 1/6*sqrt(3)*log((4*x^4 + 7*x^2 + 2*sqrt(3)*(2*x^3 + x) + 1)/(4*x^4 + x^2 + 1))

Sympy [A] time = 0.100796, size = 46, normalized size = 0.92

$$-\frac{\sqrt{3}\log\left(x^2 - \frac{\sqrt{3}x}{2} + \frac{1}{2}\right)}{6} + \frac{\sqrt{3}\log\left(x^2 + \frac{\sqrt{3}x}{2} + \frac{1}{2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-2*x**2+1)/(4*x**4+x**2+1),x)

```
[Out] -sqrt(3)*log(x**2 - sqrt(3)*x/2 + 1/2)/6 + sqrt(3)*log(x**2 + sqrt(3)*x/2 + 1/2)/6
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{2x^2 - 1}{4x^4 + x^2 + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x^2+1)/(4*x^4+x^2+1),x, algorithm="giac")
```

```
[Out] integrate(-(2*x^2 - 1)/(4*x^4 + x^2 + 1), x)
```

3.61 $\int \frac{1-2x^2}{1+4x^4} dx$

Optimal. Leaf size=31

$$\frac{1}{4} \log(2x^2 + 2x + 1) - \frac{1}{4} \log(2x^2 - 2x + 1)$$

[Out] -Log[1 - 2*x + 2*x^2]/4 + Log[1 + 2*x + 2*x^2]/4

Rubi [A] time = 0.0149195, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {1165, 628}

$$\frac{1}{4} \log(2x^2 + 2x + 1) - \frac{1}{4} \log(2x^2 - 2x + 1)$$

Antiderivative was successfully verified.

[In] Int[(1 - 2*x^2)/(1 + 4*x^4), x]

[Out] -Log[1 - 2*x + 2*x^2]/4 + Log[1 + 2*x + 2*x^2]/4

Rule 1165

Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rubi steps

$$\begin{aligned} \int \frac{1-2x^2}{1+4x^4} dx &= -\left(\frac{1}{4} \int \frac{1+2x}{-\frac{1}{2}-x-x^2} dx\right) - \frac{1}{4} \int \frac{1-2x}{-\frac{1}{2}+x-x^2} dx \\ &= -\frac{1}{4} \log(1-2x+2x^2) + \frac{1}{4} \log(1+2x+2x^2) \end{aligned}$$

Mathematica [A] time = 0.0046045, size = 31, normalized size = 1.

$$\frac{1}{4} \log(2x^2 + 2x + 1) - \frac{1}{4} \log(2x^2 - 2x + 1)$$

Antiderivative was successfully verified.

[In] Integrate[(1 - 2*x^2)/(1 + 4*x^4), x]

[Out] -Log[1 - 2*x + 2*x^2]/4 + Log[1 + 2*x + 2*x^2]/4

Maple [A] time = 0.046, size = 28, normalized size = 0.9

$$-\frac{\ln(2x^2 - 2x + 1)}{4} + \frac{\ln(2x^2 + 2x + 1)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-2*x^2+1)/(4*x^4+1),x)

[Out] -1/4*ln(2*x^2-2*x+1)+1/4*ln(2*x^2+2*x+1)

Maxima [A] time = 0.978012, size = 36, normalized size = 1.16

$$\frac{1}{4} \log(2x^2 + 2x + 1) - \frac{1}{4} \log(2x^2 - 2x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-2*x^2+1)/(4*x^4+1),x, algorithm="maxima")

[Out] 1/4*log(2*x^2 + 2*x + 1) - 1/4*log(2*x^2 - 2*x + 1)

Fricas [A] time = 1.39473, size = 72, normalized size = 2.32

$$\frac{1}{4} \log(2x^2 + 2x + 1) - \frac{1}{4} \log(2x^2 - 2x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-2*x^2+1)/(4*x^4+1),x, algorithm="fricas")

[Out] 1/4*log(2*x^2 + 2*x + 1) - 1/4*log(2*x^2 - 2*x + 1)

Sympy [A] time = 0.099135, size = 22, normalized size = 0.71

$$-\frac{\log\left(x^2 - x + \frac{1}{2}\right)}{4} + \frac{\log\left(x^2 + x + \frac{1}{2}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-2*x**2+1)/(4*x**4+1),x)

[Out] -log(x**2 - x + 1/2)/4 + log(x**2 + x + 1/2)/4

Giac [A] time = 1.1064, size = 46, normalized size = 1.48

$$\frac{1}{4} \log\left(x^2 + \sqrt{2} \left(\frac{1}{4}\right)^{\frac{1}{4}} x + \frac{1}{2}\right) - \frac{1}{4} \log\left(x^2 - \sqrt{2} \left(\frac{1}{4}\right)^{\frac{1}{4}} x + \frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x^2+1)/(4*x^4+1),x, algorithm="giac")
```

```
[Out] 1/4*log(x^2 + sqrt(2)*(1/4)^(1/4)*x + 1/2) - 1/4*log(x^2 - sqrt(2)*(1/4)^(1/4)*x + 1/2)
```


$$3.62 \quad \int \frac{1-2x^2}{1-x^2+4x^4} dx$$

Optimal. Leaf size=50

$$\frac{\log(2x^2 + \sqrt{5}x + 1)}{2\sqrt{5}} - \frac{\log(2x^2 - \sqrt{5}x + 1)}{2\sqrt{5}}$$

[Out] -Log[1 - Sqrt[5]*x + 2*x^2]/(2*Sqrt[5]) + Log[1 + Sqrt[5]*x + 2*x^2]/(2*Sqrt[5])

Rubi [A] time = 0.0228969, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1164, 628}

$$\frac{\log(2x^2 + \sqrt{5}x + 1)}{2\sqrt{5}} - \frac{\log(2x^2 - \sqrt{5}x + 1)}{2\sqrt{5}}$$

Antiderivative was successfully verified.

[In] Int[(1 - 2*x^2)/(1 - x^2 + 4*x^4), x]

[Out] -Log[1 - Sqrt[5]*x + 2*x^2]/(2*Sqrt[5]) + Log[1 + Sqrt[5]*x + 2*x^2]/(2*Sqrt[5])

Rule 1164

Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[(-2*d)/e - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && !GtQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[d*Log[RemoveContent[a + b*x + c*x^2, x]]/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rubi steps

$$\begin{aligned} \int \frac{1-2x^2}{1-x^2+4x^4} dx &= -\frac{\int \frac{\frac{\sqrt{5}}{2}+2x}{-\frac{1}{2}-\frac{\sqrt{5}x}{2}-x^2} dx}{2\sqrt{5}} - \frac{\int \frac{\frac{\sqrt{5}}{2}-2x}{-\frac{1}{2}+\frac{\sqrt{5}x}{2}-x^2} dx}{2\sqrt{5}} \\ &= -\frac{\log(1 - \sqrt{5}x + 2x^2)}{2\sqrt{5}} + \frac{\log(1 + \sqrt{5}x + 2x^2)}{2\sqrt{5}} \end{aligned}$$

Mathematica [A] time = 0.0137571, size = 42, normalized size = 0.84

$$\frac{\log(2x^2 + \sqrt{5}x + 1) - \log(-2x^2 + \sqrt{5}x - 1)}{2\sqrt{5}}$$

Antiderivative was successfully verified.

[In] Integrate[(1 - 2*x^2)/(1 - x^2 + 4*x^4),x]

[Out] (-Log[-1 + Sqrt[5]*x - 2*x^2] + Log[1 + Sqrt[5]*x + 2*x^2])/(2*Sqrt[5])

Maple [A] time = 0.052, size = 39, normalized size = 0.8

$$-\frac{\ln(1 + 2x^2 - x\sqrt{5})\sqrt{5}}{10} + \frac{\ln(1 + 2x^2 + x\sqrt{5})\sqrt{5}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-2*x^2+1)/(4*x^4-x^2+1),x)

[Out] -1/10*ln(1+2*x^2-x*5^(1/2))*5^(1/2)+1/10*ln(1+2*x^2+x*5^(1/2))*5^(1/2)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{2x^2 - 1}{4x^4 - x^2 + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-2*x^2+1)/(4*x^4-x^2+1),x, algorithm="maxima")

[Out] -integrate((2*x^2 - 1)/(4*x^4 - x^2 + 1), x)

Fricas [A] time = 1.42215, size = 109, normalized size = 2.18

$$\frac{1}{10}\sqrt{5}\log\left(\frac{4x^4 + 9x^2 + 2\sqrt{5}(2x^3 + x) + 1}{4x^4 - x^2 + 1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-2*x^2+1)/(4*x^4-x^2+1),x, algorithm="fricas")

[Out] 1/10*sqrt(5)*log((4*x^4 + 9*x^2 + 2*sqrt(5)*(2*x^3 + x) + 1)/(4*x^4 - x^2 + 1))

Sympy [A] time = 0.102462, size = 46, normalized size = 0.92

$$-\frac{\sqrt{5}\log\left(x^2 - \frac{\sqrt{5}x}{2} + \frac{1}{2}\right)}{10} + \frac{\sqrt{5}\log\left(x^2 + \frac{\sqrt{5}x}{2} + \frac{1}{2}\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-2*x**2+1)/(4*x**4-x**2+1),x)

[Out] $-\sqrt{5} \cdot \log(x^2 - \sqrt{5} \cdot x/2 + 1/2)/10 + \sqrt{5} \cdot \log(x^2 + \sqrt{5} \cdot x/2 + 1/2)/10$

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{2x^2 - 1}{4x^4 - x^2 + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-2*x^2+1)/(4*x^4-x^2+1),x, algorithm="giac")`

[Out] `integrate(-(2*x^2 - 1)/(4*x^4 - x^2 + 1), x)`

$$3.63 \quad \int \frac{1-2x^2}{1-2x^2+4x^4} dx$$

Optimal. Leaf size=50

$$\frac{\log(2x^2 + \sqrt{6}x + 1)}{2\sqrt{6}} - \frac{\log(2x^2 - \sqrt{6}x + 1)}{2\sqrt{6}}$$

[Out] -Log[1 - Sqrt[6]*x + 2*x^2]/(2*Sqrt[6]) + Log[1 + Sqrt[6]*x + 2*x^2]/(2*Sqrt[6])

Rubi [A] time = 0.0237402, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1164, 628}

$$\frac{\log(2x^2 + \sqrt{6}x + 1)}{2\sqrt{6}} - \frac{\log(2x^2 - \sqrt{6}x + 1)}{2\sqrt{6}}$$

Antiderivative was successfully verified.

[In] Int[(1 - 2*x^2)/(1 - 2*x^2 + 4*x^4), x]

[Out] -Log[1 - Sqrt[6]*x + 2*x^2]/(2*Sqrt[6]) + Log[1 + Sqrt[6]*x + 2*x^2]/(2*Sqrt[6])

Rule 1164

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(-2*d)/e - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[
*d^2 - a*e^2, 0] && !GtQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{1-2x^2}{1-2x^2+4x^4} dx &= -\frac{\int \frac{\sqrt{\frac{3}{2}+2x}}{-\frac{1}{2}-\sqrt{\frac{3}{2}}x-x^2} dx}{2\sqrt{6}} - \frac{\int \frac{\sqrt{\frac{3}{2}-2x}}{-\frac{1}{2}+\sqrt{\frac{3}{2}}x-x^2} dx}{2\sqrt{6}} \\ &= -\frac{\log(1 - \sqrt{6}x + 2x^2)}{2\sqrt{6}} + \frac{\log(1 + \sqrt{6}x + 2x^2)}{2\sqrt{6}} \end{aligned}$$

Mathematica [A] time = 0.0193635, size = 42, normalized size = 0.84

$$\frac{\log(2x^2 + \sqrt{6}x + 1) - \log(-2x^2 + \sqrt{6}x - 1)}{2\sqrt{6}}$$

Antiderivative was successfully verified.

[In] Integrate[(1 - 2*x^2)/(1 - 2*x^2 + 4*x^4), x]

[Out] (-Log[-1 + Sqrt[6]*x - 2*x^2] + Log[1 + Sqrt[6]*x + 2*x^2])/(2*Sqrt[6])

Maple [A] time = 0.052, size = 39, normalized size = 0.8

$$-\frac{\ln(1 + 2x^2 - x\sqrt{6})\sqrt{6}}{12} + \frac{\ln(1 + 2x^2 + x\sqrt{6})\sqrt{6}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-2*x^2+1)/(4*x^4-2*x^2+1), x)

[Out] -1/12*ln(1+2*x^2-x*6^(1/2))*6^(1/2)+1/12*ln(1+2*x^2+x*6^(1/2))*6^(1/2)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{2x^2 - 1}{4x^4 - 2x^2 + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-2*x^2+1)/(4*x^4-2*x^2+1), x, algorithm="maxima")

[Out] -integrate((2*x^2 - 1)/(4*x^4 - 2*x^2 + 1), x)

Fricas [A] time = 1.37997, size = 113, normalized size = 2.26

$$\frac{1}{12} \sqrt{6} \log\left(\frac{4x^4 + 10x^2 + 2\sqrt{6}(2x^3 + x) + 1}{4x^4 - 2x^2 + 1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-2*x^2+1)/(4*x^4-2*x^2+1), x, algorithm="fricas")

[Out] 1/12*sqrt(6)*log((4*x^4 + 10*x^2 + 2*sqrt(6)*(2*x^3 + x) + 1)/(4*x^4 - 2*x^2 + 1))

Sympy [A] time = 0.1003, size = 46, normalized size = 0.92

$$-\frac{\sqrt{6} \log\left(x^2 - \frac{\sqrt{6}x}{2} + \frac{1}{2}\right)}{12} + \frac{\sqrt{6} \log\left(x^2 + \frac{\sqrt{6}x}{2} + \frac{1}{2}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-2*x**2+1)/(4*x**4-2*x**2+1), x)

[Out] $-\sqrt{6} \cdot \log(x^2 - \sqrt{6} \cdot x/2 + 1/2)/12 + \sqrt{6} \cdot \log(x^2 + \sqrt{6} \cdot x/2 + 1/2)/12$

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{2x^2 - 1}{4x^4 - 2x^2 + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-2*x^2+1)/(4*x^4-2*x^2+1),x, algorithm="giac")`

[Out] `integrate(-(2*x^2 - 1)/(4*x^4 - 2*x^2 + 1), x)`

$$3.64 \quad \int \frac{1-2x^2}{1-3x^2+4x^4} dx$$

Optimal. Leaf size=50

$$\frac{\log(2x^2 + \sqrt{7}x + 1)}{2\sqrt{7}} - \frac{\log(2x^2 - \sqrt{7}x + 1)}{2\sqrt{7}}$$

[Out] -Log[1 - Sqrt[7]*x + 2*x^2]/(2*Sqrt[7]) + Log[1 + Sqrt[7]*x + 2*x^2]/(2*Sqrt[7])

Rubi [A] time = 0.0224819, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1164, 628}

$$\frac{\log(2x^2 + \sqrt{7}x + 1)}{2\sqrt{7}} - \frac{\log(2x^2 - \sqrt{7}x + 1)}{2\sqrt{7}}$$

Antiderivative was successfully verified.

[In] Int[(1 - 2*x^2)/(1 - 3*x^2 + 4*x^4), x]

[Out] -Log[1 - Sqrt[7]*x + 2*x^2]/(2*Sqrt[7]) + Log[1 + Sqrt[7]*x + 2*x^2]/(2*Sqrt[7])

Rule 1164

Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[(-2*d)/e - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && !GtQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rubi steps

$$\begin{aligned} \int \frac{1-2x^2}{1-3x^2+4x^4} dx &= -\frac{\int \frac{\frac{\sqrt{7}}{2}+2x}{-\frac{1}{2}-\frac{\sqrt{7}x}{2}-x^2} dx}{2\sqrt{7}} - \frac{\int \frac{\frac{\sqrt{7}}{2}-2x}{-\frac{1}{2}+\frac{\sqrt{7}x}{2}-x^2} dx}{2\sqrt{7}} \\ &= -\frac{\log(1 - \sqrt{7}x + 2x^2)}{2\sqrt{7}} + \frac{\log(1 + \sqrt{7}x + 2x^2)}{2\sqrt{7}} \end{aligned}$$

Mathematica [A] time = 0.0150176, size = 42, normalized size = 0.84

$$\frac{\log(2x^2 + \sqrt{7}x + 1) - \log(-2x^2 + \sqrt{7}x - 1)}{2\sqrt{7}}$$

Antiderivative was successfully verified.

[In] Integrate[(1 - 2*x^2)/(1 - 3*x^2 + 4*x^4),x]

[Out] (-Log[-1 + Sqrt[7]*x - 2*x^2] + Log[1 + Sqrt[7]*x + 2*x^2])/(2*Sqrt[7])

Maple [A] time = 0.051, size = 39, normalized size = 0.8

$$-\frac{\ln(1 + 2x^2 - x\sqrt{7})\sqrt{7}}{14} + \frac{\ln(1 + 2x^2 + x\sqrt{7})\sqrt{7}}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-2*x^2+1)/(4*x^4-3*x^2+1),x)

[Out] -1/14*ln(1+2*x^2-x*7^(1/2))*7^(1/2)+1/14*ln(1+2*x^2+x*7^(1/2))*7^(1/2)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{2x^2 - 1}{4x^4 - 3x^2 + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-2*x^2+1)/(4*x^4-3*x^2+1),x, algorithm="maxima")

[Out] -integrate((2*x^2 - 1)/(4*x^4 - 3*x^2 + 1), x)

Fricas [A] time = 1.35792, size = 113, normalized size = 2.26

$$\frac{1}{14} \sqrt{7} \log\left(\frac{4x^4 + 11x^2 + 2\sqrt{7}(2x^3 + x) + 1}{4x^4 - 3x^2 + 1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-2*x^2+1)/(4*x^4-3*x^2+1),x, algorithm="fricas")

[Out] 1/14*sqrt(7)*log((4*x^4 + 11*x^2 + 2*sqrt(7)*(2*x^3 + x) + 1)/(4*x^4 - 3*x^2 + 1))

Sympy [A] time = 0.103001, size = 46, normalized size = 0.92

$$-\frac{\sqrt{7} \log\left(x^2 - \frac{\sqrt{7}x}{2} + \frac{1}{2}\right)}{14} + \frac{\sqrt{7} \log\left(x^2 + \frac{\sqrt{7}x}{2} + \frac{1}{2}\right)}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-2*x**2+1)/(4*x**4-3*x**2+1),x)


```
[Out] -sqrt(7)*log(x**2 - sqrt(7)*x/2 + 1/2)/14 + sqrt(7)*log(x**2 + sqrt(7)*x/2 + 1/2)/14
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{2x^2 - 1}{4x^4 - 3x^2 + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x^2+1)/(4*x^4-3*x^2+1),x, algorithm="giac")
```

```
[Out] integrate(-(2*x^2 - 1)/(4*x^4 - 3*x^2 + 1), x)
```

$$3.65 \quad \int \frac{1-2x^2}{1-4x^2+4x^4} dx$$

Optimal. Leaf size=14

$$\frac{\tanh^{-1}(\sqrt{2}x)}{\sqrt{2}}$$

[Out] ArcTanh[Sqrt[2]*x]/Sqrt[2]

Rubi [A] time = 0.005621, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {28, 21, 206}

$$\frac{\tanh^{-1}(\sqrt{2}x)}{\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[(1 - 2*x^2)/(1 - 4*x^2 + 4*x^4),x]

[Out] ArcTanh[Sqrt[2]*x]/Sqrt[2]

Rule 28

Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]

Rule 21

Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
&& EqQ[b*c - a*d, 0] && IntegerQ[m] && (!IntegerQ[n] || SimplerQ[c + d*x,
a + b*x])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned} \int \frac{1-2x^2}{1-4x^2+4x^4} dx &= 4 \int \frac{1-2x^2}{(-2+4x^2)^2} dx \\ &= \int \frac{1}{1-2x^2} dx \\ &= \frac{\tanh^{-1}(\sqrt{2}x)}{\sqrt{2}} \end{aligned}$$

Mathematica [B] time = 0.0074046, size = 32, normalized size = 2.29

$$\frac{\log(2x + \sqrt{2}) - \log(\sqrt{2} - 2x)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Integrate[(1 - 2*x^2)/(1 - 4*x^2 + 4*x^4), x]

[Out] (-Log[Sqrt[2] - 2*x] + Log[Sqrt[2] + 2*x])/(2*Sqrt[2])

Maple [A] time = 0.04, size = 12, normalized size = 0.9

$$\frac{\operatorname{Artanh}(x\sqrt{2})\sqrt{2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-2*x^2+1)/(4*x^4-4*x^2+1), x)

[Out] 1/2*arctanh(x*2^(1/2))*2^(1/2)

Maxima [B] time = 1.49258, size = 34, normalized size = 2.43

$$-\frac{1}{4}\sqrt{2}\log\left(\frac{2x - \sqrt{2}}{2x + \sqrt{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-2*x^2+1)/(4*x^4-4*x^2+1), x, algorithm="maxima")

[Out] -1/4*sqrt(2)*log((2*x - sqrt(2))/(2*x + sqrt(2)))

Fricas [B] time = 1.2824, size = 76, normalized size = 5.43

$$\frac{1}{4}\sqrt{2}\log\left(\frac{2x^2 + 2\sqrt{2}x + 1}{2x^2 - 1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-2*x^2+1)/(4*x^4-4*x^2+1), x, algorithm="fricas")

[Out] 1/4*sqrt(2)*log((2*x^2 + 2*sqrt(2)*x + 1)/(2*x^2 - 1))

Sympy [B] time = 0.091748, size = 32, normalized size = 2.29

$$-\frac{\sqrt{2}\log\left(x - \frac{\sqrt{2}}{2}\right)}{4} + \frac{\sqrt{2}\log\left(x + \frac{\sqrt{2}}{2}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-2*x**2+1)/(4*x**4-4*x**2+1),x)

[Out] -sqrt(2)*log(x - sqrt(2)/2)/4 + sqrt(2)*log(x + sqrt(2)/2)/4

Giac [B] time = 1.13631, size = 39, normalized size = 2.79

$$\frac{1}{4} \sqrt{2} \log\left(\left|x + \frac{1}{2} \sqrt{2}\right|\right) - \frac{1}{4} \sqrt{2} \log\left(\left|x - \frac{1}{2} \sqrt{2}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-2*x^2+1)/(4*x^4-4*x^2+1),x, algorithm="giac")

[Out] 1/4*sqrt(2)*log(abs(x + 1/2*sqrt(2))) - 1/4*sqrt(2)*log(abs(x - 1/2*sqrt(2)))

$$3.66 \quad \int \frac{1-2x^2}{1-5x^2+4x^4} dx$$

Optimal. Leaf size=39

$$-\frac{1}{6} \log(1-2x) - \frac{1}{6} \log(1-x) + \frac{1}{6} \log(x+1) + \frac{1}{6} \log(2x+1)$$

[Out] -Log[1 - 2*x]/6 - Log[1 - x]/6 + Log[1 + x]/6 + Log[1 + 2*x]/6

Rubi [A] time = 0.0171199, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {1161, 616, 31}

$$-\frac{1}{6} \log(1-2x) - \frac{1}{6} \log(1-x) + \frac{1}{6} \log(x+1) + \frac{1}{6} \log(2x+1)$$

Antiderivative was successfully verified.

[In] Int[(1 - 2*x^2)/(1 - 5*x^2 + 4*x^4), x]

[Out] -Log[1 - 2*x]/6 - Log[1 - x]/6 + Log[1 + x]/6 + Log[1 + 2*x]/6

Rule 1161

Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (GtQ[(2*d)/e - b/c, 0] || (!LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2], 0]))

Rule 616

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q, Int[1/Simp[b/2 + q/2 + c*x, x], x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]

Rule 31

Int[((a_) + (b_.)*(x_)^(-1)), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rubi steps

$$\begin{aligned} \int \frac{1-2x^2}{1-5x^2+4x^4} dx &= -\left(\frac{1}{4} \int \frac{1}{-\frac{1}{2}-\frac{x}{2}+x^2} dx\right) - \frac{1}{4} \int \frac{1}{-\frac{1}{2}+\frac{x}{2}+x^2} dx \\ &= -\left(\frac{1}{6} \int \frac{1}{-1+x} dx\right) - \frac{1}{6} \int \frac{1}{-\frac{1}{2}+x} dx + \frac{1}{6} \int \frac{1}{\frac{1}{2}+x} dx + \frac{1}{6} \int \frac{1}{1+x} dx \\ &= -\frac{1}{6} \log(1-2x) - \frac{1}{6} \log(1-x) + \frac{1}{6} \log(1+x) + \frac{1}{6} \log(1+2x) \end{aligned}$$

Mathematica [A] time = 0.0056717, size = 31, normalized size = 0.79

$$\frac{1}{6} \log(2x^2 + 3x + 1) - \frac{1}{6} \log(2x^2 - 3x + 1)$$

Antiderivative was successfully verified.

[In] Integrate[(1 - 2*x^2)/(1 - 5*x^2 + 4*x^4), x]

[Out] -Log[1 - 3*x + 2*x^2]/6 + Log[1 + 3*x + 2*x^2]/6

Maple [A] time = 0.052, size = 30, normalized size = 0.8

$$\frac{\ln(1+x)}{6} - \frac{\ln(2x-1)}{6} - \frac{\ln(-1+x)}{6} + \frac{\ln(1+2x)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-2*x^2+1)/(4*x^4-5*x^2+1), x)

[Out] 1/6*ln(1+x)-1/6*ln(2*x-1)-1/6*ln(-1+x)+1/6*ln(1+2*x)

Maxima [A] time = 0.960764, size = 39, normalized size = 1.

$$\frac{1}{6} \log(2x + 1) - \frac{1}{6} \log(2x - 1) + \frac{1}{6} \log(x + 1) - \frac{1}{6} \log(x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-2*x^2+1)/(4*x^4-5*x^2+1), x, algorithm="maxima")

[Out] 1/6*log(2*x + 1) - 1/6*log(2*x - 1) + 1/6*log(x + 1) - 1/6*log(x - 1)

Fricas [A] time = 1.37242, size = 72, normalized size = 1.85

$$\frac{1}{6} \log(2x^2 + 3x + 1) - \frac{1}{6} \log(2x^2 - 3x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-2*x^2+1)/(4*x^4-5*x^2+1), x, algorithm="fricas")

[Out] 1/6*log(2*x^2 + 3*x + 1) - 1/6*log(2*x^2 - 3*x + 1)

Sympy [A] time = 0.102137, size = 29, normalized size = 0.74

$$-\frac{\log\left(x^2 - \frac{3x}{2} + \frac{1}{2}\right)}{6} + \frac{\log\left(x^2 + \frac{3x}{2} + \frac{1}{2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x**2+1)/(4*x**4-5*x**2+1),x)
```

```
[Out] -log(x**2 - 3*x/2 + 1/2)/6 + log(x**2 + 3*x/2 + 1/2)/6
```

Giac [A] time = 1.13605, size = 45, normalized size = 1.15

$$\frac{1}{6} \log(|2x + 1|) - \frac{1}{6} \log(|2x - 1|) + \frac{1}{6} \log(|x + 1|) - \frac{1}{6} \log(|x - 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x^2+1)/(4*x^4-5*x^2+1),x, algorithm="giac")
```

```
[Out] 1/6*log(abs(2*x + 1)) - 1/6*log(abs(2*x - 1)) + 1/6*log(abs(x + 1)) - 1/6*log(abs(x - 1))
```

$$3.67 \quad \int \frac{1-2x^2}{1-6x^2+4x^4} dx$$

Optimal. Leaf size=48

$$\frac{\tanh^{-1}\left(\frac{2\sqrt{2x+1}}{\sqrt{5}}\right)}{\sqrt{10}} - \frac{\tanh^{-1}\left(\frac{1-2\sqrt{2x}}{\sqrt{5}}\right)}{\sqrt{10}}$$

[Out] -(ArcTanh[(1 - 2*sqrt[2]*x)/sqrt[5]]/sqrt[10]) + ArcTanh[(1 + 2*sqrt[2]*x)/sqrt[5]]/sqrt[10]

Rubi [A] time = 0.0386751, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {1161, 618, 206}

$$\frac{\tanh^{-1}\left(\frac{2\sqrt{2x+1}}{\sqrt{5}}\right)}{\sqrt{10}} - \frac{\tanh^{-1}\left(\frac{1-2\sqrt{2x}}{\sqrt{5}}\right)}{\sqrt{10}}$$

Antiderivative was successfully verified.

[In] Int[(1 - 2*x^2)/(1 - 6*x^2 + 4*x^4),x]

[Out] -(ArcTanh[(1 - 2*sqrt[2]*x)/sqrt[5]]/sqrt[10]) + ArcTanh[(1 + 2*sqrt[2]*x)/sqrt[5]]/sqrt[10]

Rule 1161

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[(2*d)/e - b/c, 0] || (!LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[In
t[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{1-2x^2}{1-6x^2+4x^4} dx &= -\left(\frac{1}{4} \int \frac{1}{-\frac{1}{2}-\frac{x}{\sqrt{2}}+x^2} dx\right) - \frac{1}{4} \int \frac{1}{-\frac{1}{2}+\frac{x}{\sqrt{2}}+x^2} dx \\ &= \frac{1}{2} \text{Subst}\left(\int \frac{1}{\frac{5}{2}-x^2} dx, x, -\frac{1}{\sqrt{2}}+2x\right) + \frac{1}{2} \text{Subst}\left(\int \frac{1}{\frac{5}{2}-x^2} dx, x, \frac{1}{\sqrt{2}}+2x\right) \\ &= -\frac{\tanh^{-1}\left(\frac{1-2\sqrt{2}x}{\sqrt{5}}\right)}{\sqrt{10}} + \frac{\tanh^{-1}\left(\frac{1+2\sqrt{2}x}{\sqrt{5}}\right)}{\sqrt{10}} \end{aligned}$$

Mathematica [A] time = 0.0190844, size = 42, normalized size = 0.88

$$\frac{\log(2x^2 + \sqrt{10}x + 1) - \log(-2x^2 + \sqrt{10}x - 1)}{2\sqrt{10}}$$

Antiderivative was successfully verified.

[In] Integrate[(1 - 2*x^2)/(1 - 6*x^2 + 4*x^4), x]

[Out] (-Log[-1 + Sqrt[10]*x - 2*x^2] + Log[1 + Sqrt[10]*x + 2*x^2])/(2*Sqrt[10])

Maple [B] time = 0.055, size = 82, normalized size = 1.7

$$\frac{(-2 + 2\sqrt{5})\sqrt{5}}{10\sqrt{10} - 10\sqrt{2}} \text{Arctanh}\left(8 \frac{x}{2\sqrt{10} - 2\sqrt{2}}\right) + \frac{(2 + 2\sqrt{5})\sqrt{5}}{10\sqrt{10} + 10\sqrt{2}} \text{Arctanh}\left(8 \frac{x}{2\sqrt{10} + 2\sqrt{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-2*x^2+1)/(4*x^4-6*x^2+1), x)

[Out] 2/5*(5^(1/2)-1)*5^(1/2)/(2*10^(1/2)-2*2^(1/2))*arctanh(8*x/(2*10^(1/2)-2*2^(1/2)))+2/5*(5^(1/2)+1)*5^(1/2)/(2*10^(1/2)+2*2^(1/2))*arctanh(8*x/(2*10^(1/2)+2*2^(1/2)))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{2x^2 - 1}{4x^4 - 6x^2 + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-2*x^2+1)/(4*x^4-6*x^2+1), x, algorithm="maxima")

[Out] -integrate((2*x^2 - 1)/(4*x^4 - 6*x^2 + 1), x)

Fricas [A] time = 1.27381, size = 116, normalized size = 2.42

$$\frac{1}{20} \sqrt{10} \log\left(\frac{4x^4 + 14x^2 + 2\sqrt{10}(2x^3 + x) + 1}{4x^4 - 6x^2 + 1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-2*x^2+1)/(4*x^4-6*x^2+1),x, algorithm="fricas")

[Out] 1/20*sqrt(10)*log((4*x^4 + 14*x^2 + 2*sqrt(10)*(2*x^3 + x) + 1)/(4*x^4 - 6*x^2 + 1))

Sympy [A] time = 0.10442, size = 46, normalized size = 0.96

$$-\frac{\sqrt{10} \log\left(x^2 - \frac{\sqrt{10}x}{2} + \frac{1}{2}\right)}{20} + \frac{\sqrt{10} \log\left(x^2 + \frac{\sqrt{10}x}{2} + \frac{1}{2}\right)}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-2*x**2+1)/(4*x**4-6*x**2+1),x)

[Out] -sqrt(10)*log(x**2 - sqrt(10)*x/2 + 1/2)/20 + sqrt(10)*log(x**2 + sqrt(10)*x/2 + 1/2)/20

Giac [A] time = 1.18294, size = 104, normalized size = 2.17

$$\frac{1}{20} \sqrt{10} \log\left(\left|x + \frac{1}{4} \sqrt{10} + \frac{1}{4} \sqrt{2}\right|\right) + \frac{1}{20} \sqrt{10} \log\left(\left|x + \frac{1}{4} \sqrt{10} - \frac{1}{4} \sqrt{2}\right|\right) - \frac{1}{20} \sqrt{10} \log\left(\left|x - \frac{1}{4} \sqrt{10} + \frac{1}{4} \sqrt{2}\right|\right) - \frac{1}{20} \sqrt{10} \log\left(\left|x - \frac{1}{4} \sqrt{10} - \frac{1}{4} \sqrt{2}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-2*x^2+1)/(4*x^4-6*x^2+1),x, algorithm="giac")

[Out] 1/20*sqrt(10)*log(abs(x + 1/4*sqrt(10) + 1/4*sqrt(2))) + 1/20*sqrt(10)*log(abs(x + 1/4*sqrt(10) - 1/4*sqrt(2))) - 1/20*sqrt(10)*log(abs(x - 1/4*sqrt(10) + 1/4*sqrt(2))) - 1/20*sqrt(10)*log(abs(x - 1/4*sqrt(10) - 1/4*sqrt(2)))

$$3.68 \quad \int \frac{1+x^2}{1+bx^2+x^4} dx$$

Optimal. Leaf size=62

$$\frac{\tan^{-1}\left(\frac{\sqrt{2-b+2x}}{\sqrt{b+2}}\right)}{\sqrt{b+2}} - \frac{\tan^{-1}\left(\frac{\sqrt{2-b-2x}}{\sqrt{b+2}}\right)}{\sqrt{b+2}}$$

[Out] -(ArcTan[(Sqrt[2 - b] - 2*x)/Sqrt[2 + b]]/Sqrt[2 + b]) + ArcTan[(Sqrt[2 - b] + 2*x)/Sqrt[2 + b]]/Sqrt[2 + b]

Rubi [A] time = 0.0556889, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1161, 618, 204}

$$\frac{\tan^{-1}\left(\frac{\sqrt{2-b+2x}}{\sqrt{b+2}}\right)}{\sqrt{b+2}} - \frac{\tan^{-1}\left(\frac{\sqrt{2-b-2x}}{\sqrt{b+2}}\right)}{\sqrt{b+2}}$$

Antiderivative was successfully verified.

[In] Int[(1 + x^2)/(1 + b*x^2 + x^4), x]

[Out] -(ArcTan[(Sqrt[2 - b] - 2*x)/Sqrt[2 + b]]/Sqrt[2 + b]) + ArcTan[(Sqrt[2 - b] + 2*x)/Sqrt[2 + b]]/Sqrt[2 + b]

Rule 1161

Int[((d_) + (e_)*(x_)^2)/((a_) + (b_)*(x_)^2 + (c_)*(x_)^4), x_Symbol] :> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (GtQ[(2*d)/e - b/c, 0] || (!LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2], 0]))

Rule 618

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 204

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned} \int \frac{1+x^2}{1+bx^2+x^4} dx &= \frac{1}{2} \int \frac{1}{1-\sqrt{2-bx+x^2}} dx + \frac{1}{2} \int \frac{1}{1+\sqrt{2-bx+x^2}} dx \\ &= -\text{Subst} \left(\int \frac{1}{-2-b-x^2} dx, x, -\sqrt{2-b}+2x \right) - \text{Subst} \left(\int \frac{1}{-2-b-x^2} dx, x, \sqrt{2-b}+2x \right) \\ &= \frac{\tan^{-1} \left(\frac{-\sqrt{2-b}+2x}{\sqrt{2+b}} \right)}{\sqrt{2+b}} + \frac{\tan^{-1} \left(\frac{\sqrt{2-b}+2x}{\sqrt{2+b}} \right)}{\sqrt{2+b}} \end{aligned}$$

Mathematica [A] time = 0.0564094, size = 124, normalized size = 2.

$$\frac{\left(\sqrt{b^2-4-b+2}\right) \tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt{b-\sqrt{b^2-4}}}\right)}{\sqrt{b-\sqrt{b^2-4}}} + \frac{\left(\sqrt{b^2-4+b-2}\right) \tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt{\sqrt{b^2-4}+b}}\right)}{\sqrt{\sqrt{b^2-4}+b}}$$

$$\frac{\sqrt{2}\sqrt{b^2-4}}{\sqrt{2}\sqrt{b^2-4}}$$

Antiderivative was successfully verified.

[In] Integrate[(1 + x^2)/(1 + b*x^2 + x^4),x]

[Out] (((2 - b + Sqrt[-4 + b^2])*ArcTan[(Sqrt[2]*x)/Sqrt[b - Sqrt[-4 + b^2]]])/Sqrt[b - Sqrt[-4 + b^2]] + ((-2 + b + Sqrt[-4 + b^2])*ArcTan[(Sqrt[2]*x)/Sqrt[b + Sqrt[-4 + b^2]]])/Sqrt[b + Sqrt[-4 + b^2]])/(Sqrt[2]*Sqrt[-4 + b^2])

Maple [B] time = 0.138, size = 277, normalized size = 4.5

$$-2 \frac{1}{\sqrt{(-2+b)(2+b)}\sqrt{2\sqrt{(-2+b)(2+b)}+2b}} \arctan\left(2 \frac{x}{\sqrt{2\sqrt{(-2+b)(2+b)}+2b}}\right) + \arctan\left(2 \frac{x}{\sqrt{2\sqrt{(-2+b)(2+b)}+2b}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2+1)/(x^4+b*x^2+1),x)

[Out] -2/((-2+b)*(2+b))^(1/2)/(2*((-2+b)*(2+b))^(1/2)+2*b)^(1/2)*arctan(2*x/(2*((-2+b)*(2+b))^(1/2)+2*b)^(1/2))+1/(2*((-2+b)*(2+b))^(1/2)+2*b)^(1/2)*arctan(2*x/(2*((-2+b)*(2+b))^(1/2)+2*b)^(1/2))+1/((-2+b)*(2+b))^(1/2)/(2*((-2+b)*(2+b))^(1/2)+2*b)^(1/2)*arctan(2*x/(2*((-2+b)*(2+b))^(1/2)+2*b)^(1/2))*b+2/((-2+b)*(2+b))^(1/2)/(-2*((-2+b)*(2+b))^(1/2)+2*b)^(1/2)*arctan(2*x/(-2*((-2+b)*(2+b))^(1/2)+2*b)^(1/2))+1/(-2*((-2+b)*(2+b))^(1/2)+2*b)^(1/2)*arctan(2*x/(-2*((-2+b)*(2+b))^(1/2)+2*b)^(1/2))-1/((-2+b)*(2+b))^(1/2)/(-2*((-2+b)*(2+b))^(1/2)+2*b)^(1/2)*arctan(2*x/(-2*((-2+b)*(2+b))^(1/2)+2*b)^(1/2))*b

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2+1}{x^4+bx^2+1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+1)/(x^4+b*x^2+1),x, algorithm="maxima")

[Out] integrate((x^2 + 1)/(x^4 + b*x^2 + 1), x)

Fricas [A] time = 1.32819, size = 273, normalized size = 4.4

$$\left[\frac{\sqrt{-b-2} \log\left(\frac{x^4 - (b+4)x^2 - 2(x^3 - x)\sqrt{-b-2} + 1}{x^4 + bx^2 + 1}\right)}{2(b+2)}, \frac{\sqrt{b+2} \arctan\left(\frac{x^3 + (b+1)x}{\sqrt{b+2}}\right) + \sqrt{b+2} \arctan\left(\frac{x}{\sqrt{b+2}}\right)}{b+2} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+1)/(x^4+b*x^2+1),x, algorithm="fricas")

[Out] [-1/2*sqrt(-b - 2)*log((x^4 - (b + 4)*x^2 - 2*(x^3 - x)*sqrt(-b - 2) + 1)/(x^4 + b*x^2 + 1))/(b + 2), (sqrt(b + 2)*arctan((x^3 + (b + 1)*x)/sqrt(b + 2)) + sqrt(b + 2)*arctan(x/sqrt(b + 2)))/(b + 2)]

Sympy [A] time = 0.258486, size = 88, normalized size = 1.42

$$-\frac{\sqrt{-\frac{1}{b+2}} \log\left(x^2 + x\left(-b\sqrt{-\frac{1}{b+2}} - 2\sqrt{-\frac{1}{b+2}}\right) - 1\right)}{2} + \frac{\sqrt{-\frac{1}{b+2}} \log\left(x^2 + x\left(b\sqrt{-\frac{1}{b+2}} + 2\sqrt{-\frac{1}{b+2}}\right) - 1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**2+1)/(x**4+b*x**2+1),x)

[Out] -sqrt(-1/(b + 2))*log(x**2 + x*(-b*sqrt(-1/(b + 2)) - 2*sqrt(-1/(b + 2))) - 1)/2 + sqrt(-1/(b + 2))*log(x**2 + x*(b*sqrt(-1/(b + 2)) + 2*sqrt(-1/(b + 2))) - 1)/2

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+1)/(x^4+b*x^2+1),x, algorithm="giac")

[Out] Exception raised: TypeError

$$3.69 \quad \int \frac{1+x^2}{1+5x^2+x^4} dx$$

Optimal. Leaf size=49

$$\frac{\tan^{-1}\left(\sqrt{\frac{2}{5+\sqrt{21}}}x\right)}{\sqrt{7}} + \frac{\tan^{-1}\left(\sqrt{\frac{1}{2}(5+\sqrt{21})}x\right)}{\sqrt{7}}$$

[Out] ArcTan[Sqrt[2/(5 + Sqrt[21])]x]/Sqrt[7] + ArcTan[Sqrt[(5 + Sqrt[21])/2]x]/Sqrt[7]

Rubi [A] time = 0.0878152, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1163, 203}

$$\frac{\tan^{-1}\left(\sqrt{\frac{2}{5+\sqrt{21}}}x\right)}{\sqrt{7}} + \frac{\tan^{-1}\left(\sqrt{\frac{1}{2}(5+\sqrt{21})}x\right)}{\sqrt{7}}$$

Antiderivative was successfully verified.

[In] Int[(1 + x^2)/(1 + 5*x^2 + x^4), x]

[Out] ArcTan[Sqrt[2/(5 + Sqrt[21])]x]/Sqrt[7] + ArcTan[Sqrt[(5 + Sqrt[21])/2]x]/Sqrt[7]

Rule 1163

Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
 > With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && GtQ[b^2 - 4*a*c, 0]

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[Rt[b, 2]*x]/Rt[a, 2])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rubi steps

$$\begin{aligned} \int \frac{1+x^2}{1+5x^2+x^4} dx &= \frac{1}{14} (7 - \sqrt{21}) \int \frac{1}{\frac{5}{2} - \frac{\sqrt{21}}{2} + x^2} dx + \frac{1}{14} (7 + \sqrt{21}) \int \frac{1}{\frac{5}{2} + \frac{\sqrt{21}}{2} + x^2} dx \\ &= \frac{\tan^{-1}\left(\sqrt{\frac{2}{5+\sqrt{21}}}x\right)}{\sqrt{7}} + \frac{\tan^{-1}\left(\sqrt{\frac{1}{2}(5+\sqrt{21})}x\right)}{\sqrt{7}} \end{aligned}$$

Mathematica [A] time = 0.136855, size = 83, normalized size = 1.69

$$\frac{(\sqrt{21} - 3) \tan^{-1}\left(\sqrt{\frac{2}{5-\sqrt{21}}}x\right)}{\sqrt{42(5-\sqrt{21})}} + \frac{(3 + \sqrt{21}) \tan^{-1}\left(\sqrt{\frac{2}{5+\sqrt{21}}}x\right)}{\sqrt{42(5+\sqrt{21})}}$$

Antiderivative was successfully verified.

[In] Integrate[(1 + x^2)/(1 + 5*x^2 + x^4), x]

[Out] ((-3 + Sqrt[21])*ArcTan[Sqrt[2/(5 - Sqrt[21])]*x])/Sqrt[42*(5 - Sqrt[21])] + ((3 + Sqrt[21])*ArcTan[Sqrt[2/(5 + Sqrt[21])]*x])/Sqrt[42*(5 + Sqrt[21])]

Maple [B] time = 0.079, size = 136, normalized size = 2.8

$$-\frac{2\sqrt{21}}{14\sqrt{7}-14\sqrt{3}} \arctan\left(4\frac{x}{2\sqrt{7}-2\sqrt{3}}\right) + 2\frac{1}{2\sqrt{7}-2\sqrt{3}} \arctan\left(4\frac{x}{2\sqrt{7}-2\sqrt{3}}\right) + \frac{2\sqrt{21}}{14\sqrt{7}+14\sqrt{3}} \arctan\left(4\frac{x}{2\sqrt{7}+2\sqrt{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2+1)/(x^4+5*x^2+1), x)

[Out] -2/7*21^(1/2)/(2*7^(1/2)-2*3^(1/2))*arctan(4*x/(2*7^(1/2)-2*3^(1/2)))+2/(2*7^(1/2)-2*3^(1/2))*arctan(4*x/(2*7^(1/2)-2*3^(1/2)))+2/7*21^(1/2)/(2*7^(1/2)+2*3^(1/2))*arctan(4*x/(2*7^(1/2)+2*3^(1/2)))+2/(2*7^(1/2)+2*3^(1/2))*arctan(4*x/(2*7^(1/2)+2*3^(1/2)))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 + 1}{x^4 + 5x^2 + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+1)/(x^4+5*x^2+1), x, algorithm="maxima")

[Out] integrate((x^2 + 1)/(x^4 + 5*x^2 + 1), x)

Fricas [A] time = 1.31138, size = 109, normalized size = 2.22

$$\frac{1}{7}\sqrt{7} \arctan\left(\frac{1}{7}\sqrt{7}(x^3 + 6x)\right) + \frac{1}{7}\sqrt{7} \arctan\left(\frac{1}{7}\sqrt{7}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+1)/(x^4+5*x^2+1), x, algorithm="fricas")

[Out] 1/7*sqrt(7)*arctan(1/7*sqrt(7)*(x^3 + 6*x)) + 1/7*sqrt(7)*arctan(1/7*sqrt(7)*x)

Sympy [A] time = 0.108215, size = 41, normalized size = 0.84

$$\frac{\sqrt{7}\left(2 \operatorname{atan}\left(\frac{\sqrt{7}x}{7}\right) + 2 \operatorname{atan}\left(\frac{\sqrt{7}x^3}{7} + \frac{6\sqrt{7}x}{7}\right)\right)}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**2+1)/(x**4+5*x**2+1),x)

[Out] sqrt(7)*(2*atan(sqrt(7)*x/7) + 2*atan(sqrt(7)*x**3/7 + 6*sqrt(7)*x/7))/14

Giac [A] time = 1.13694, size = 35, normalized size = 0.71

$$\frac{1}{14} \sqrt{7} \left(\pi \operatorname{sgn}(x) + 2 \arctan \left(\frac{\sqrt{7}(x^2 - 1)}{7x} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+1)/(x^4+5*x^2+1),x, algorithm="giac")

[Out] 1/14*sqrt(7)*(pi*sgn(x) + 2*arctan(1/7*sqrt(7)*(x^2 - 1)/x))

$$3.70 \quad \int \frac{1+x^2}{1+4x^2+x^4} dx$$

Optimal. Leaf size=43

$$\frac{\tan^{-1}\left(\frac{x}{\sqrt{2-\sqrt{3}}}\right)}{\sqrt{6}} + \frac{\tan^{-1}\left(\frac{x}{\sqrt{2+\sqrt{3}}}\right)}{\sqrt{6}}$$

[Out] ArcTan[x/Sqrt[2 - Sqrt[3]]]/Sqrt[6] + ArcTan[x/Sqrt[2 + Sqrt[3]]]/Sqrt[6]

Rubi [A] time = 0.049938, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1163, 203}

$$\frac{\tan^{-1}\left(\frac{x}{\sqrt{2-\sqrt{3}}}\right)}{\sqrt{6}} + \frac{\tan^{-1}\left(\frac{x}{\sqrt{2+\sqrt{3}}}\right)}{\sqrt{6}}$$

Antiderivative was successfully verified.

[In] Int[(1 + x^2)/(1 + 4*x^2 + x^4), x]

[Out] ArcTan[x/Sqrt[2 - Sqrt[3]]]/Sqrt[6] + ArcTan[x/Sqrt[2 + Sqrt[3]]]/Sqrt[6]

Rule 1163

Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && GtQ[b^2 - 4*a*c, 0]

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rubi steps

$$\begin{aligned} \int \frac{1+x^2}{1+4x^2+x^4} dx &= \frac{1}{6} (3-\sqrt{3}) \int \frac{1}{2-\sqrt{3}+x^2} dx + \frac{1}{6} (3+\sqrt{3}) \int \frac{1}{2+\sqrt{3}+x^2} dx \\ &= \frac{\tan^{-1}\left(\frac{x}{\sqrt{2-\sqrt{3}}}\right)}{\sqrt{6}} + \frac{\tan^{-1}\left(\frac{x}{\sqrt{2+\sqrt{3}}}\right)}{\sqrt{6}} \end{aligned}$$

Mathematica [A] time = 0.0673384, size = 81, normalized size = 1.88

$$\frac{(\sqrt{3}-1) \tan^{-1}\left(\frac{x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{3}(2-\sqrt{3})} + \frac{(1+\sqrt{3}) \tan^{-1}\left(\frac{x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{3}(2+\sqrt{3})}$$

Antiderivative was successfully verified.

[In] Integrate[(1 + x^2)/(1 + 4*x^2 + x^4),x]

[Out] ((-1 + Sqrt[3])*ArcTan[x/Sqrt[2 - Sqrt[3]]])/(2*Sqrt[3*(2 - Sqrt[3])]) + ((1 + Sqrt[3])*ArcTan[x/Sqrt[2 + Sqrt[3]]])/(2*Sqrt[3*(2 + Sqrt[3])])

Maple [B] time = 0.065, size = 110, normalized size = 2.6

$$-\frac{\sqrt{3}}{3\sqrt{6}-3\sqrt{2}} \arctan\left(2\frac{x}{\sqrt{6}-\sqrt{2}}\right) + \frac{1}{\sqrt{6}-\sqrt{2}} \arctan\left(2\frac{x}{\sqrt{6}-\sqrt{2}}\right) + \frac{\sqrt{3}}{3\sqrt{2}+3\sqrt{6}} \arctan\left(2\frac{x}{\sqrt{2}+\sqrt{6}}\right) + \frac{1}{\sqrt{2}+\sqrt{6}} \arctan\left(2\frac{x}{\sqrt{2}+\sqrt{6}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2+1)/(x^4+4*x^2+1),x)

[Out] -1/3*3^(1/2)/(6^(1/2)-2^(1/2))*arctan(2*x/(6^(1/2)-2^(1/2)))+1/(6^(1/2)-2^(1/2))*arctan(2*x/(6^(1/2)-2^(1/2)))+1/3*3^(1/2)/(2^(1/2)+6^(1/2))*arctan(2*x/(2^(1/2)+6^(1/2)))+1/(2^(1/2)+6^(1/2))*arctan(2*x/(2^(1/2)+6^(1/2)))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 + 1}{x^4 + 4x^2 + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+1)/(x^4+4*x^2+1),x, algorithm="maxima")

[Out] integrate((x^2 + 1)/(x^4 + 4*x^2 + 1), x)

Fricas [A] time = 1.3479, size = 109, normalized size = 2.53

$$\frac{1}{6} \sqrt{6} \arctan\left(\frac{1}{6} \sqrt{6}(x^3 + 5x)\right) + \frac{1}{6} \sqrt{6} \arctan\left(\frac{1}{6} \sqrt{6}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+1)/(x^4+4*x^2+1),x, algorithm="fricas")

[Out] 1/6*sqrt(6)*arctan(1/6*sqrt(6)*(x^3 + 5*x)) + 1/6*sqrt(6)*arctan(1/6*sqrt(6)*x)

Sympy [A] time = 0.108176, size = 41, normalized size = 0.95

$$\frac{\sqrt{6} \left(2 \operatorname{atan}\left(\frac{\sqrt{6}x}{6}\right) + 2 \operatorname{atan}\left(\frac{\sqrt{6}x^3}{6} + \frac{5\sqrt{6}x}{6}\right) \right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)/(x**4+4*x**2+1),x)
```

```
[Out] sqrt(6)*(2*atan(sqrt(6)*x/6) + 2*atan(sqrt(6)*x**3/6 + 5*sqrt(6)*x/6))/12
```

Giac [A] time = 1.11523, size = 35, normalized size = 0.81

$$\frac{1}{12} \sqrt{6} \left(\pi \operatorname{sgn}(x) + 2 \arctan \left(\frac{\sqrt{6}(x^2 - 1)}{6x} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^4+4*x^2+1),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(6)*(pi*sgn(x) + 2*arctan(1/6*sqrt(6)*(x^2 - 1)/x))
```

$$3.71 \quad \int \frac{1+x^2}{1+3x^2+x^4} dx$$

Optimal. Leaf size=49

$$\frac{\tan^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}x\right)}{\sqrt{5}} + \frac{\tan^{-1}\left(\sqrt{\frac{1}{2}(3+\sqrt{5})}x\right)}{\sqrt{5}}$$

[Out] ArcTan[Sqrt[2/(3 + Sqrt[5])]*x]/Sqrt[5] + ArcTan[Sqrt[(3 + Sqrt[5])/2]*x]/Sqrt[5]

Rubi [A] time = 0.0618606, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1163, 203}

$$\frac{\tan^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}x\right)}{\sqrt{5}} + \frac{\tan^{-1}\left(\sqrt{\frac{1}{2}(3+\sqrt{5})}x\right)}{\sqrt{5}}$$

Antiderivative was successfully verified.

[In] Int[(1 + x^2)/(1 + 3*x^2 + x^4), x]

[Out] ArcTan[Sqrt[2/(3 + Sqrt[5])]*x]/Sqrt[5] + ArcTan[Sqrt[(3 + Sqrt[5])/2]*x]/Sqrt[5]

Rule 1163

Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
 > With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && GtQ[b^2 - 4*a*c, 0]

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rubi steps

$$\begin{aligned} \int \frac{1+x^2}{1+3x^2+x^4} dx &= \frac{1}{10}(5-\sqrt{5}) \int \frac{1}{\frac{3}{2}-\frac{\sqrt{5}}{2}+x^2} dx + \frac{1}{10}(5+\sqrt{5}) \int \frac{1}{\frac{3}{2}+\frac{\sqrt{5}}{2}+x^2} dx \\ &= \frac{\tan^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}x\right)}{\sqrt{5}} + \frac{\tan^{-1}\left(\sqrt{\frac{1}{2}(3+\sqrt{5})}x\right)}{\sqrt{5}} \end{aligned}$$

Mathematica [A] time = 0.0963296, size = 83, normalized size = 1.69

$$\frac{(\sqrt{5}-1) \tan^{-1}\left(\sqrt{\frac{2}{3-\sqrt{5}}}x\right)}{\sqrt{10(3-\sqrt{5})}} + \frac{(1+\sqrt{5}) \tan^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}x\right)}{\sqrt{10(3+\sqrt{5})}}$$

Antiderivative was successfully verified.

[In] Integrate[(1 + x^2)/(1 + 3*x^2 + x^4), x]

[Out] ((-1 + Sqrt[5])*ArcTan[Sqrt[2/(3 - Sqrt[5])]*x])/Sqrt[10*(3 - Sqrt[5])] + ((1 + Sqrt[5])*ArcTan[Sqrt[2/(3 + Sqrt[5])]*x])/Sqrt[10*(3 + Sqrt[5])]

Maple [B] time = 0.069, size = 104, normalized size = 2.1

$$-\frac{2\sqrt{5}}{-10+10\sqrt{5}} \arctan\left(4\frac{x}{-2+2\sqrt{5}}\right) + 2\frac{1}{-2+2\sqrt{5}} \arctan\left(4\frac{x}{-2+2\sqrt{5}}\right) + \frac{2\sqrt{5}}{10+10\sqrt{5}} \arctan\left(4\frac{x}{2+2\sqrt{5}}\right) + 2\frac{1}{2+2\sqrt{5}} \arctan\left(4\frac{x}{2+2\sqrt{5}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2+1)/(x^4+3*x^2+1), x)

[Out] -2/5*5^(1/2)/(-2+2*5^(1/2))*arctan(4*x/(-2+2*5^(1/2)))+2/(-2+2*5^(1/2))*arctan(4*x/(-2+2*5^(1/2)))+2/5*5^(1/2)/(2+2*5^(1/2))*arctan(4*x/(2+2*5^(1/2)))+2/(2+2*5^(1/2))*arctan(4*x/(2+2*5^(1/2)))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 + 1}{x^4 + 3x^2 + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+1)/(x^4+3*x^2+1), x, algorithm="maxima")

[Out] integrate((x^2 + 1)/(x^4 + 3*x^2 + 1), x)

Fricas [A] time = 1.32716, size = 109, normalized size = 2.22

$$\frac{1}{5}\sqrt{5} \arctan\left(\frac{1}{5}\sqrt{5}(x^3 + 4x)\right) + \frac{1}{5}\sqrt{5} \arctan\left(\frac{1}{5}\sqrt{5}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+1)/(x^4+3*x^2+1), x, algorithm="fricas")

[Out] 1/5*sqrt(5)*arctan(1/5*sqrt(5)*(x^3 + 4*x)) + 1/5*sqrt(5)*arctan(1/5*sqrt(5)*x)

Sympy [A] time = 0.109102, size = 41, normalized size = 0.84

$$\frac{\sqrt{5}\left(2 \operatorname{atan}\left(\frac{\sqrt{5}x}{5}\right) + 2 \operatorname{atan}\left(\frac{\sqrt{5}x^3}{5} + \frac{4\sqrt{5}x}{5}\right)\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**2+1)/(x**4+3*x**2+1),x)

[Out] sqrt(5)*(2*atan(sqrt(5)*x/5) + 2*atan(sqrt(5)*x**3/5 + 4*sqrt(5)*x/5))/10

Giac [A] time = 1.13601, size = 35, normalized size = 0.71

$$\frac{1}{10} \sqrt{5} \left(\pi \operatorname{sgn}(x) + 2 \arctan \left(\frac{\sqrt{5}(x^2 - 1)}{5x} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+1)/(x^4+3*x^2+1),x, algorithm="giac")

[Out] 1/10*sqrt(5)*(pi*sgn(x) + 2*arctan(1/5*sqrt(5)*(x^2 - 1)/x))

$$3.72 \quad \int \frac{1+x^2}{1+2x^2+x^4} dx$$

Optimal. Leaf size=2

$$\tan^{-1}(x)$$

[Out] ArcTan[x]

Rubi [A] time = 0.001515, antiderivative size = 2, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {28, 203}

$$\tan^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[(1 + x^2)/(1 + 2*x^2 + x^4), x]

[Out] ArcTan[x]

Rule 28

Int[(u_)*((a_) + (c_)*(x_)^(n2_)) + (b_)*(x_)^(n_)]^(p_), x_Symbol] :> Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]

Rule 203

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rubi steps

$$\int \frac{1+x^2}{1+2x^2+x^4} dx = \int \frac{1}{1+x^2} dx = \tan^{-1}(x)$$

Mathematica [A] time = 0.0025709, size = 2, normalized size = 1.

$$\tan^{-1}(x)$$

Antiderivative was successfully verified.

[In] Integrate[(1 + x^2)/(1 + 2*x^2 + x^4), x]

[Out] ArcTan[x]

Maple [A] time = 0.044, size = 3, normalized size = 1.5

$$\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)/(x^4+2*x^2+1),x)
```

```
[Out] arctan(x)
```

Maxima [A] time = 1.43537, size = 3, normalized size = 1.5

$$\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^4+2*x^2+1),x, algorithm="maxima")
```

```
[Out] arctan(x)
```

Fricas [A] time = 1.33545, size = 15, normalized size = 7.5

$$\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^4+2*x^2+1),x, algorithm="fricas")
```

```
[Out] arctan(x)
```

Sympy [A] time = 0.083135, size = 2, normalized size = 1.

$$\operatorname{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)/(x**4+2*x**2+1),x)
```

```
[Out] atan(x)
```

Giac [A] time = 1.10475, size = 3, normalized size = 1.5

$$\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^4+2*x^2+1),x, algorithm="giac")
```

```
[Out] arctan(x)
```


$$3.73 \quad \int \frac{1+x^2}{1+x^2+x^4} dx$$

Optimal. Leaf size=38

$$\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

[Out] -(ArcTan[(1 - 2*x)/Sqrt[3]]/Sqrt[3]) + ArcTan[(1 + 2*x)/Sqrt[3]]/Sqrt[3]

Rubi [A] time = 0.0270838, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {1161, 618, 204}

$$\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[(1 + x^2)/(1 + x^2 + x^4), x]

[Out] -(ArcTan[(1 - 2*x)/Sqrt[3]]/Sqrt[3]) + ArcTan[(1 + 2*x)/Sqrt[3]]/Sqrt[3]

Rule 1161

Int[((d_) + (e_)*(x_)^2)/((a_) + (b_)*(x_)^2 + (c_)*(x_)^4), x_Symbol] :> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (GtQ[(2*d)/e - b/c, 0] || (!LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2], 0]))

Rule 618

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 204

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned} \int \frac{1+x^2}{1+x^2+x^4} dx &= \frac{1}{2} \int \frac{1}{1-x+x^2} dx + \frac{1}{2} \int \frac{1}{1+x+x^2} dx \\ &= -\text{Subst}\left(\int \frac{1}{-3-x^2} dx, x, -1+2x\right) - \text{Subst}\left(\int \frac{1}{-3-x^2} dx, x, 1+2x\right) \\ &= \frac{\tan^{-1}\left(\frac{-1+2x}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}} \end{aligned}$$

Mathematica [C] time = 0.195295, size = 99, normalized size = 2.61

$$\frac{(\sqrt{3}-i)\tan^{-1}\left(\frac{x}{\sqrt{\frac{1}{2}(1-i\sqrt{3})}}\right)}{\sqrt{6(1-i\sqrt{3})}} + \frac{(\sqrt{3}+i)\tan^{-1}\left(\frac{x}{\sqrt{\frac{1}{2}(1+i\sqrt{3})}}\right)}{\sqrt{6(1+i\sqrt{3})}}$$

Antiderivative was successfully verified.

[In] Integrate[(1 + x^2)/(1 + x^2 + x^4),x]

[Out] ((-I + Sqrt[3])*ArcTan[x/Sqrt[(1 - I*Sqrt[3])/2]])/Sqrt[6*(1 - I*Sqrt[3])] + ((I + Sqrt[3])*ArcTan[x/Sqrt[(1 + I*Sqrt[3])/2]])/Sqrt[6*(1 + I*Sqrt[3])]

Maple [A] time = 0.045, size = 34, normalized size = 0.9

$$\frac{\sqrt{3}}{3} \arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right) + \frac{\sqrt{3}}{3} \arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2+1)/(x^4+x^2+1),x)

[Out] 1/3*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))+1/3*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)

Maxima [A] time = 1.4882, size = 45, normalized size = 1.18

$$\frac{1}{3} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3}(2x+1)\right) + \frac{1}{3} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3}(2x-1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+1)/(x^4+x^2+1),x, algorithm="maxima")

[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1))

Fricas [A] time = 1.28342, size = 109, normalized size = 2.87

$$\frac{1}{3} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3}(x^3 + 2x)\right) + \frac{1}{3} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+1)/(x^4+x^2+1),x, algorithm="fricas")

[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(x^3 + 2*x)) + 1/3*sqrt(3)*arctan(1/3*sqrt(3)*x)

Sympy [A] time = 0.105415, size = 41, normalized size = 1.08

$$\frac{\sqrt{3} \left(2 \operatorname{atan} \left(\frac{\sqrt{3}x}{3} \right) + 2 \operatorname{atan} \left(\frac{\sqrt{3}x^3}{3} + \frac{2\sqrt{3}x}{3} \right) \right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**2+1)/(x**4+x**2+1),x)

[Out] sqrt(3)*(2*atan(sqrt(3)*x/3) + 2*atan(sqrt(3)*x**3/3 + 2*sqrt(3)*x/3))/6

Giac [A] time = 1.12416, size = 35, normalized size = 0.92

$$\frac{1}{6} \sqrt{3} \left(\pi \operatorname{sgn}(x) + 2 \arctan \left(\frac{\sqrt{3}(x^2 - 1)}{3x} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+1)/(x^4+x^2+1),x, algorithm="giac")

[Out] 1/6*sqrt(3)*(pi*sgn(x) + 2*arctan(1/3*sqrt(3)*(x^2 - 1)/x))

3.74 $\int \frac{1+x^2}{1+x^4} dx$

Optimal. Leaf size=35

$$\frac{\tan^{-1}(\sqrt{2}x+1)}{\sqrt{2}} - \frac{\tan^{-1}(1-\sqrt{2}x)}{\sqrt{2}}$$

[Out] $-(\text{ArcTan}[1 - \text{Sqrt}[2]*x]/\text{Sqrt}[2]) + \text{ArcTan}[1 + \text{Sqrt}[2]*x]/\text{Sqrt}[2]$

Rubi [A] time = 0.0183083, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {1162, 617, 204}

$$\frac{\tan^{-1}(\sqrt{2}x+1)}{\sqrt{2}} - \frac{\tan^{-1}(1-\sqrt{2}x)}{\sqrt{2}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(1 + x^2)/(1 + x^4), x]$

[Out] $-(\text{ArcTan}[1 - \text{Sqrt}[2]*x]/\text{Sqrt}[2]) + \text{ArcTan}[1 + \text{Sqrt}[2]*x]/\text{Sqrt}[2]$

Rule 1162

$\text{Int}[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] \rightarrow \text{With}[\{q = \text{Rt}[(2*d)/e, 2]\}, \text{Dist}[e/(2*c), \text{Int}[1/\text{Simp}[d/e + q*x + x^2, x], x], x] + \text{Dist}[e/(2*c), \text{Int}[1/\text{Simp}[d/e - q*x + x^2, x], x], x]] /; \text{FreeQ}[\{a, c, d, e\}, x] \& \& \text{EqQ}[c*d^2 - a*e^2, 0] \& \& \text{PosQ}[d*e]$

Rule 617

$\text{Int}[(a_) + (b_)*(x_) + (c_)*(x_)^2]^{-1}, x_Symbol] \rightarrow \text{With}[\{q = 1 - 4*\text{Simplify}[(a*c)/b^2]\}, \text{Dist}[-2/b, \text{Subst}[\text{Int}[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; \text{RationalQ}[q] \& \& (\text{EqQ}[q^2, 1] \parallel \text{!RationalQ}[b^2 - 4*a*c])] /; \text{FreeQ}[\{a, b, c\}, x] \& \& \text{NeQ}[b^2 - 4*a*c, 0]$

Rule 204

$\text{Int}[(a_) + (b_)*(x_)^2]^{-1}, x_Symbol] \rightarrow -\text{Simp}[\text{ArcTan}[(\text{Rt}[-b, 2]*x)/\text{Rt}[-a, 2]]/(\text{Rt}[-a, 2]*\text{Rt}[-b, 2]), x] /; \text{FreeQ}[\{a, b\}, x] \& \& \text{PosQ}[a/b] \& \& (\text{LtQ}[a, 0] \parallel \text{LtQ}[b, 0])$

Rubi steps

$$\begin{aligned} \int \frac{1+x^2}{1+x^4} dx &= \frac{1}{2} \int \frac{1}{1-\sqrt{2}x+x^2} dx + \frac{1}{2} \int \frac{1}{1+\sqrt{2}x+x^2} dx \\ &= \frac{\text{Subst}\left(\int \frac{1}{-1-x^2} dx, x, 1-\sqrt{2}x\right)}{\sqrt{2}} - \frac{\text{Subst}\left(\int \frac{1}{-1-x^2} dx, x, 1+\sqrt{2}x\right)}{\sqrt{2}} \\ &= -\frac{\tan^{-1}(1-\sqrt{2}x)}{\sqrt{2}} + \frac{\tan^{-1}(1+\sqrt{2}x)}{\sqrt{2}} \end{aligned}$$

Mathematica [A] time = 0.0129433, size = 30, normalized size = 0.86

$$\frac{\tan^{-1}(\sqrt{2}x+1) - \tan^{-1}(1-\sqrt{2}x)}{\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Integrate[(1 + x^2)/(1 + x^4), x]

[Out] (-ArcTan[1 - Sqrt[2]*x] + ArcTan[1 + Sqrt[2]*x])/Sqrt[2]

Maple [B] time = 0.044, size = 88, normalized size = 2.5

$$\frac{\arctan(1+x\sqrt{2})\sqrt{2}}{2} + \frac{\arctan(-1+x\sqrt{2})\sqrt{2}}{2} + \frac{\sqrt{2}}{8} \ln\left(\frac{1+x^2+x\sqrt{2}}{1+x^2-x\sqrt{2}}\right) + \frac{\sqrt{2}}{8} \ln\left(\frac{1+x^2-x\sqrt{2}}{1+x^2+x\sqrt{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2+1)/(x^4+1), x)

[Out] 1/2*arctan(1+x*2^(1/2))*2^(1/2)+1/2*arctan(-1+x*2^(1/2))*2^(1/2)+1/8*2^(1/2)*ln((1+x^2+x*2^(1/2))/(1+x^2-x*2^(1/2)))+1/8*2^(1/2)*ln((1+x^2-x*2^(1/2))/(1+x^2+x*2^(1/2)))

Maxima [A] time = 1.45257, size = 53, normalized size = 1.51

$$\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(2x+\sqrt{2})\right) + \frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(2x-\sqrt{2})\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+1)/(x^4+1), x, algorithm="maxima")

[Out] 1/2*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 1/2*sqrt(2)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)))

Fricas [A] time = 1.37549, size = 107, normalized size = 3.06

$$\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(x^3+x)\right) + \frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+1)/(x^4+1), x, algorithm="fricas")

[Out] 1/2*sqrt(2)*arctan(1/2*sqrt(2)*(x^3 + x)) + 1/2*sqrt(2)*arctan(1/2*sqrt(2)*x)

Sympy [A] time = 0.101463, size = 39, normalized size = 1.11

$$\frac{\sqrt{2} \left(2 \operatorname{atan} \left(\frac{\sqrt{2}x}{2} \right) + 2 \operatorname{atan} \left(\frac{\sqrt{2}x^3}{2} + \frac{\sqrt{2}x}{2} \right) \right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**2+1)/(x**4+1),x)

[Out] sqrt(2)*(2*atan(sqrt(2)*x/2) + 2*atan(sqrt(2)*x**3/2 + sqrt(2)*x/2))/4

Giac [A] time = 1.13747, size = 53, normalized size = 1.51

$$\frac{1}{2} \sqrt{2} \arctan \left(\frac{1}{2} \sqrt{2} (2x + \sqrt{2}) \right) + \frac{1}{2} \sqrt{2} \arctan \left(\frac{1}{2} \sqrt{2} (2x - \sqrt{2}) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+1)/(x^4+1),x, algorithm="giac")

[Out] 1/2*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 1/2*sqrt(2)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)))

$$3.75 \quad \int \frac{1+x^2}{1-x^2+x^4} dx$$

Optimal. Leaf size=23

$$\tan^{-1}(2x + \sqrt{3}) - \tan^{-1}(\sqrt{3} - 2x)$$

[Out] -ArcTan[Sqrt[3] - 2*x] + ArcTan[Sqrt[3] + 2*x]

Rubi [A] time = 0.0195514, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1161, 618, 204}

$$\tan^{-1}(2x + \sqrt{3}) - \tan^{-1}(\sqrt{3} - 2x)$$

Antiderivative was successfully verified.

[In] Int[(1 + x^2)/(1 - x^2 + x^4), x]

[Out] -ArcTan[Sqrt[3] - 2*x] + ArcTan[Sqrt[3] + 2*x]

Rule 1161

Int[((d_) + (e_)*(x_)^2)/((a_) + (b_)*(x_)^2 + (c_)*(x_)^4), x_Symbol] :> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (GtQ[(2*d)/e - b/c, 0] || (!LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2], 0]))

Rule 618

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 204

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned} \int \frac{1+x^2}{1-x^2+x^4} dx &= \frac{1}{2} \int \frac{1}{1-\sqrt{3}x+x^2} dx + \frac{1}{2} \int \frac{1}{1+\sqrt{3}x+x^2} dx \\ &= -\text{Subst}\left(\int \frac{1}{-1-x^2} dx, x, -\sqrt{3}+2x\right) - \text{Subst}\left(\int \frac{1}{-1-x^2} dx, x, \sqrt{3}+2x\right) \\ &= -\tan^{-1}(\sqrt{3}-2x) + \tan^{-1}(\sqrt{3}+2x) \end{aligned}$$

Mathematica [A] time = 0.0066184, size = 12, normalized size = 0.52

$$-\tan^{-1}\left(\frac{x}{x^2-1}\right)$$

Antiderivative was successfully verified.

[In] Integrate[(1 + x^2)/(1 - x^2 + x^4),x]

[Out] -ArcTan[x/(-1 + x^2)]

Maple [A] time = 0.055, size = 20, normalized size = 0.9

$$\arctan(2x - \sqrt{3}) + \arctan(2x + \sqrt{3})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2+1)/(x^4-x^2+1),x)

[Out] arctan(2*x-3^(1/2))+arctan(2*x+3^(1/2))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 + 1}{x^4 - x^2 + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+1)/(x^4-x^2+1),x, algorithm="maxima")

[Out] integrate((x^2 + 1)/(x^4 - x^2 + 1), x)

Fricas [A] time = 1.35492, size = 34, normalized size = 1.48

$$\arctan(x^3) + \arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+1)/(x^4-x^2+1),x, algorithm="fricas")

[Out] arctan(x^3) + arctan(x)

Sympy [A] time = 0.101872, size = 7, normalized size = 0.3

$$\operatorname{atan}(x) + \operatorname{atan}(x^3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**2+1)/(x**4-x**2+1),x)

[Out] atan(x) + atan(x**3)

Giac [A] time = 1.1328, size = 41, normalized size = 1.78

$$\frac{1}{4} \pi \operatorname{sgn}(x) + \frac{1}{2} \arctan\left(\frac{x^4 - 3x^2 + 1}{2(x^3 - x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^4-x^2+1),x, algorithm="giac")
```

```
[Out] 1/4*pi*sgn(x) + 1/2*arctan(1/2*(x^4 - 3*x^2 + 1)/(x^3 - x))
```

$$3.76 \quad \int \frac{1+x^2}{1-2x^2+x^4} dx$$

Optimal. Leaf size=11

$$\frac{x}{1-x^2}$$

[Out] x/(1 - x^2)

Rubi [A] time = 0.0032829, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {28, 383}

$$\frac{x}{1-x^2}$$

Antiderivative was successfully verified.

[In] Int[(1 + x^2)/(1 - 2*x^2 + x^4),x]

[Out] x/(1 - x^2)

Rule 28

Int[(u_)*((a_) + (c_)*(x_)^(n2_)) + (b_)*(x_)^(n_)]^(p_), x_Symbol] :> Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]

Rule 383

Int[((a_) + (b_)*(x_)^(n_)]^(p_)*((c_) + (d_)*(x_)^(n_)), x_Symbol] :> Simp[(c*x*(a + b*x^n)^(p + 1))/a, x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[a*d - b*c*(n*(p + 1) + 1), 0]

Rubi steps

$$\int \frac{1+x^2}{1-2x^2+x^4} dx = \int \frac{1+x^2}{(-1+x^2)^2} dx = \frac{x}{1-x^2}$$

Mathematica [A] time = 0.0038171, size = 10, normalized size = 0.91

$$-\frac{x}{x^2-1}$$

Antiderivative was successfully verified.

[In] Integrate[(1 + x^2)/(1 - 2*x^2 + x^4),x]

[Out] -(x/(-1 + x^2))

Maple [A] time = 0.043, size = 16, normalized size = 1.5

$$-\frac{1}{-2+2x} - \frac{1}{2+2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2+1)/(x^4-2*x^2+1),x)

[Out] -1/2/(-1+x)-1/2/(1+x)

Maxima [A] time = 1.0077, size = 14, normalized size = 1.27

$$-\frac{x}{x^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+1)/(x^4-2*x^2+1),x, algorithm="maxima")

[Out] -x/(x^2 - 1)

Fricas [A] time = 1.27508, size = 19, normalized size = 1.73

$$-\frac{x}{x^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+1)/(x^4-2*x^2+1),x, algorithm="fricas")

[Out] -x/(x^2 - 1)

Sympy [A] time = 0.078523, size = 7, normalized size = 0.64

$$-\frac{x}{x^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**2+1)/(x**4-2*x**2+1),x)

[Out] -x/(x**2 - 1)

Giac [A] time = 1.14241, size = 15, normalized size = 1.36

$$-\frac{1}{x - \frac{1}{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^4-2*x^2+1),x, algorithm="giac")
```

```
[Out] -1/(x - 1/x)
```

$$3.77 \quad \int \frac{1+x^2}{1-3x^2+x^4} dx$$

Optimal. Leaf size=65

$$\frac{1}{2} \log(-2x - \sqrt{5} + 1) + \frac{1}{2} \log(-2x + \sqrt{5} + 1) - \frac{1}{2} \log(2x - \sqrt{5} + 1) - \frac{1}{2} \log(2x + \sqrt{5} + 1)$$

[Out] Log[1 - Sqrt[5] - 2*x]/2 + Log[1 + Sqrt[5] - 2*x]/2 - Log[1 - Sqrt[5] + 2*x]/2 - Log[1 + Sqrt[5] + 2*x]/2

Rubi [A] time = 0.0319407, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1161, 616, 31}

$$\frac{1}{2} \log(-2x - \sqrt{5} + 1) + \frac{1}{2} \log(-2x + \sqrt{5} + 1) - \frac{1}{2} \log(2x - \sqrt{5} + 1) - \frac{1}{2} \log(2x + \sqrt{5} + 1)$$

Antiderivative was successfully verified.

[In] Int[(1 + x^2)/(1 - 3*x^2 + x^4), x]

[Out] Log[1 - Sqrt[5] - 2*x]/2 + Log[1 + Sqrt[5] - 2*x]/2 - Log[1 - Sqrt[5] + 2*x]/2 - Log[1 + Sqrt[5] + 2*x]/2

Rule 1161

Int[((d_) + (e_)*(x_)^2)/((a_) + (b_)*(x_)^2 + (c_)*(x_)^4), x_Symbol] :> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (GtQ[(2*d)/e - b/c, 0] || (!LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2], 0]))

Rule 616

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q, Int[1/Simp[b/2 + q/2 + c*x, x], x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]

Rule 31

Int[((a_) + (b_)*(x_))^-1, x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rubi steps

$$\begin{aligned} \int \frac{1+x^2}{1-3x^2+x^4} dx &= \frac{1}{2} \int \frac{1}{1-\sqrt{5}x+x^2} dx + \frac{1}{2} \int \frac{1}{1+\sqrt{5}x+x^2} dx \\ &= \frac{1}{2} \int \frac{1}{\frac{1}{2}(-1-\sqrt{5})+x} dx - \frac{1}{2} \int \frac{1}{\frac{1}{2}(1-\sqrt{5})+x} dx + \frac{1}{2} \int \frac{1}{\frac{1}{2}(-1+\sqrt{5})+x} dx - \frac{1}{2} \int \frac{1}{\frac{1}{2}(1+\sqrt{5})+x} dx \\ &= \frac{1}{2} \log(1-\sqrt{5}-2x) + \frac{1}{2} \log(1+\sqrt{5}-2x) - \frac{1}{2} \log(1-\sqrt{5}+2x) - \frac{1}{2} \log(1+\sqrt{5}+2x) \end{aligned}$$

Mathematica [A] time = 0.0057495, size = 29, normalized size = 0.45

$$\frac{1}{2} \log(-x^2 + x + 1) - \frac{1}{2} \log(-x^2 - x + 1)$$

Antiderivative was successfully verified.

[In] Integrate[(1 + x^2)/(1 - 3*x^2 + x^4), x]

[Out] -Log[1 - x - x^2]/2 + Log[1 + x - x^2]/2

Maple [A] time = 0.044, size = 22, normalized size = 0.3

$$-\frac{\ln(x^2 + x - 1)}{2} + \frac{\ln(x^2 - x - 1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2+1)/(x^4-3*x^2+1), x)

[Out] -1/2*ln(x^2+x-1)+1/2*ln(x^2-x-1)

Maxima [A] time = 0.953934, size = 28, normalized size = 0.43

$$-\frac{1}{2} \log(x^2 + x - 1) + \frac{1}{2} \log(x^2 - x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+1)/(x^4-3*x^2+1), x, algorithm="maxima")

[Out] -1/2*log(x^2 + x - 1) + 1/2*log(x^2 - x - 1)

Fricas [A] time = 1.27954, size = 62, normalized size = 0.95

$$-\frac{1}{2} \log(x^2 + x - 1) + \frac{1}{2} \log(x^2 - x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+1)/(x^4-3*x^2+1), x, algorithm="fricas")

[Out] -1/2*log(x^2 + x - 1) + 1/2*log(x^2 - x - 1)

Sympy [A] time = 0.097323, size = 19, normalized size = 0.29

$$\frac{\log(x^2 - x - 1)}{2} - \frac{\log(x^2 + x - 1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)/(x**4-3*x**2+1),x)
```

```
[Out] log(x**2 - x - 1)/2 - log(x**2 + x - 1)/2
```

Giac [A] time = 1.13753, size = 58, normalized size = 0.89

$$-\frac{1}{4} \log\left(x + \frac{1}{x - \frac{1}{x}} - \frac{1}{x} + 2\right) + \frac{1}{4} \log\left(x + \frac{1}{x - \frac{1}{x}} - \frac{1}{x} - 2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^4-3*x^2+1),x, algorithm="giac")
```

```
[Out] -1/4*log(abs(x + 1/(x - 1/x) - 1/x + 2)) + 1/4*log(abs(x + 1/(x - 1/x) - 1/
x - 2))
```

$$3.78 \quad \int \frac{1+x^2}{1-4x^2+x^4} dx$$

Optimal. Leaf size=43

$$\frac{\tanh^{-1}(\sqrt{3}-\sqrt{2}x)}{\sqrt{2}} - \frac{\tanh^{-1}(\sqrt{2}x+\sqrt{3})}{\sqrt{2}}$$

[Out] ArcTanh[Sqrt[3] - Sqrt[2]*x]/Sqrt[2] - ArcTanh[Sqrt[3] + Sqrt[2]*x]/Sqrt[2]

Rubi [A] time = 0.0331814, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1161, 618, 206}

$$\frac{\tanh^{-1}(\sqrt{3}-\sqrt{2}x)}{\sqrt{2}} - \frac{\tanh^{-1}(\sqrt{2}x+\sqrt{3})}{\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[(1 + x^2)/(1 - 4*x^2 + x^4),x]

[Out] ArcTanh[Sqrt[3] - Sqrt[2]*x]/Sqrt[2] - ArcTanh[Sqrt[3] + Sqrt[2]*x]/Sqrt[2]

Rule 1161

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (b_)*(x_)^2 + (c_)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[(2*d)/e - b/c, 0] || (!LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 618

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] :=> Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] :=> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{1+x^2}{1-4x^2+x^4} dx &= \frac{1}{2} \int \frac{1}{1-\sqrt{6}x+x^2} dx + \frac{1}{2} \int \frac{1}{1+\sqrt{6}x+x^2} dx \\ &= -\text{Subst}\left(\int \frac{1}{2-x^2} dx, x, -\sqrt{6}+2x\right) - \text{Subst}\left(\int \frac{1}{2-x^2} dx, x, \sqrt{6}+2x\right) \\ &= \frac{\tanh^{-1}(\sqrt{3}-\sqrt{2}x)}{\sqrt{2}} - \frac{\tanh^{-1}(\sqrt{3}+\sqrt{2}x)}{\sqrt{2}} \end{aligned}$$

Mathematica [A] time = 0.0119289, size = 40, normalized size = 0.93

$$\frac{\log(-x^2 + \sqrt{2}x + 1) - \log(x^2 + \sqrt{2}x - 1)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Integrate[(1 + x^2)/(1 - 4*x^2 + x^4), x]

[Out] (Log[1 + Sqrt[2]*x - x^2] - Log[-1 + Sqrt[2]*x + x^2])/(2*Sqrt[2])

Maple [B] time = 0.067, size = 70, normalized size = 1.6

$$-\frac{(\sqrt{3} + 3)\sqrt{3}}{3\sqrt{2} + 3\sqrt{6}} \operatorname{Artanh}\left(2\frac{x}{\sqrt{2} + \sqrt{6}}\right) - \frac{(-3 + \sqrt{3})\sqrt{3}}{3\sqrt{6} - 3\sqrt{2}} \operatorname{Artanh}\left(2\frac{x}{\sqrt{6} - \sqrt{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2+1)/(x^4-4*x^2+1), x)

[Out] -1/3*(3^(1/2)+3)*3^(1/2)/(2^(1/2)+6^(1/2))*arctanh(2*x/(2^(1/2)+6^(1/2)))-1/3*(-3+3^(1/2))*3^(1/2)/(6^(1/2)-2^(1/2))*arctanh(2*x/(6^(1/2)-2^(1/2)))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 + 1}{x^4 - 4x^2 + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+1)/(x^4-4*x^2+1), x, algorithm="maxima")

[Out] integrate((x^2 + 1)/(x^4 - 4*x^2 + 1), x)

Fricas [A] time = 1.35052, size = 92, normalized size = 2.14

$$\frac{1}{4}\sqrt{2}\log\left(\frac{x^4 - 2\sqrt{2}(x^3 - x) + 1}{x^4 - 4x^2 + 1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+1)/(x^4-4*x^2+1), x, algorithm="fricas")

[Out] 1/4*sqrt(2)*log((x^4 - 2*sqrt(2)*(x^3 - x) + 1)/(x^4 - 4*x^2 + 1))

Sympy [A] time = 0.098288, size = 39, normalized size = 0.91

$$\frac{\sqrt{2}\log(x^2 - \sqrt{2}x - 1)}{4} - \frac{\sqrt{2}\log(x^2 + \sqrt{2}x - 1)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**2+1)/(x**4-4*x**2+1),x)

[Out] sqrt(2)*log(x**2 - sqrt(2)*x - 1)/4 - sqrt(2)*log(x**2 + sqrt(2)*x - 1)/4

Giac [A] time = 1.16031, size = 53, normalized size = 1.23

$$\frac{1}{4} \sqrt{2} \log \left(\frac{\left| 2x - 2\sqrt{2} - \frac{2}{x} \right|}{\left| 2x + 2\sqrt{2} - \frac{2}{x} \right|} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+1)/(x^4-4*x^2+1),x, algorithm="giac")

[Out] 1/4*sqrt(2)*log(abs(2*x - 2*sqrt(2) - 2/x)/abs(2*x + 2*sqrt(2) - 2/x))

$$3.79 \quad \int \frac{1+x^2}{1-5x^2+x^4} dx$$

Optimal. Leaf size=46

$$\frac{\tanh^{-1}\left(\frac{\sqrt{7}-2x}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{\tanh^{-1}\left(\frac{2x+\sqrt{7}}{\sqrt{3}}\right)}{\sqrt{3}}$$

[Out] ArcTanh[(Sqrt[7] - 2*x)/Sqrt[3]]/Sqrt[3] - ArcTanh[(Sqrt[7] + 2*x)/Sqrt[3]]/Sqrt[3]

Rubi [A] time = 0.0360198, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1161, 618, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{7}-2x}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{\tanh^{-1}\left(\frac{2x+\sqrt{7}}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[(1 + x^2)/(1 - 5*x^2 + x^4), x]

[Out] ArcTanh[(Sqrt[7] - 2*x)/Sqrt[3]]/Sqrt[3] - ArcTanh[(Sqrt[7] + 2*x)/Sqrt[3]]/Sqrt[3]

Rule 1161

Int[((d_) + (e_)*(x_)^2)/((a_) + (b_)*(x_)^2 + (c_)*(x_)^4), x_Symbol] :> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (GtQ[(2*d)/e - b/c, 0] || (!LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2], 0]))

Rule 618

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned} \int \frac{1+x^2}{1-5x^2+x^4} dx &= \frac{1}{2} \int \frac{1}{1-\sqrt{7}x+x^2} dx + \frac{1}{2} \int \frac{1}{1+\sqrt{7}x+x^2} dx \\ &= -\text{Subst}\left(\int \frac{1}{3-x^2} dx, x, -\sqrt{7}+2x\right) - \text{Subst}\left(\int \frac{1}{3-x^2} dx, x, \sqrt{7}+2x\right) \\ &= \frac{\tanh^{-1}\left(\frac{\sqrt{7}-2x}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{\tanh^{-1}\left(\frac{\sqrt{7}+2x}{\sqrt{3}}\right)}{\sqrt{3}} \end{aligned}$$

Mathematica [A] time = 0.0120748, size = 40, normalized size = 0.87

$$\frac{\log(-x^2 + \sqrt{3}x + 1) - \log(x^2 + \sqrt{3}x - 1)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Integrate[(1 + x^2)/(1 - 5*x^2 + x^4), x]

[Out] (Log[1 + Sqrt[3]*x - x^2] - Log[-1 + Sqrt[3]*x + x^2])/(2*Sqrt[3])

Maple [B] time = 0.073, size = 82, normalized size = 1.8

$$-\frac{(14 + 2\sqrt{21})\sqrt{21}}{42\sqrt{7} + 42\sqrt{3}} \text{Artanh}\left(4\frac{x}{2\sqrt{7} + 2\sqrt{3}}\right) - \frac{(-14 + 2\sqrt{21})\sqrt{21}}{42\sqrt{7} - 42\sqrt{3}} \text{Artanh}\left(4\frac{x}{2\sqrt{7} - 2\sqrt{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2+1)/(x^4-5*x^2+1), x)

[Out] -2/21*(7+21^(1/2))*21^(1/2)/(2*7^(1/2)+2*3^(1/2))*arctanh(4*x/(2*7^(1/2)+2*3^(1/2)))-2/21*(-7+21^(1/2))*21^(1/2)/(2*7^(1/2)-2*3^(1/2))*arctanh(4*x/(2*7^(1/2)-2*3^(1/2)))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 + 1}{x^4 - 5x^2 + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+1)/(x^4-5*x^2+1), x, algorithm="maxima")

[Out] integrate((x^2 + 1)/(x^4 - 5*x^2 + 1), x)

Fricas [A] time = 1.33009, size = 100, normalized size = 2.17

$$\frac{1}{6}\sqrt{3}\log\left(\frac{x^4 + x^2 - 2\sqrt{3}(x^3 - x) + 1}{x^4 - 5x^2 + 1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+1)/(x^4-5*x^2+1),x, algorithm="fricas")

[Out] 1/6*sqrt(3)*log((x^4 + x^2 - 2*sqrt(3)*(x^3 - x) + 1)/(x^4 - 5*x^2 + 1))

Sympy [A] time = 0.101554, size = 39, normalized size = 0.85

$$\frac{\sqrt{3} \log(x^2 - \sqrt{3}x - 1)}{6} - \frac{\sqrt{3} \log(x^2 + \sqrt{3}x - 1)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**2+1)/(x**4-5*x**2+1),x)

[Out] sqrt(3)*log(x**2 - sqrt(3)*x - 1)/6 - sqrt(3)*log(x**2 + sqrt(3)*x - 1)/6

Giac [A] time = 1.15542, size = 53, normalized size = 1.15

$$\frac{1}{6} \sqrt{3} \log \left(\frac{\left| 2x - 2\sqrt{3} - \frac{2}{x} \right|}{\left| 2x + 2\sqrt{3} - \frac{2}{x} \right|} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+1)/(x^4-5*x^2+1),x, algorithm="giac")

[Out] 1/6*sqrt(3)*log(abs(2*x - 2*sqrt(3) - 2/x)/abs(2*x + 2*sqrt(3) - 2/x))

$$3.80 \quad \int \frac{1-x^2}{1+bx^2+x^4} dx$$

Optimal. Leaf size=62

$$\frac{\log(\sqrt{2-bx+x^2+1})}{2\sqrt{2-b}} - \frac{\log(-\sqrt{2-bx+x^2+1})}{2\sqrt{2-b}}$$

[Out] -Log[1 - Sqrt[2 - b]*x + x^2]/(2*Sqrt[2 - b]) + Log[1 + Sqrt[2 - b]*x + x^2]/(2*Sqrt[2 - b])

Rubi [A] time = 0.0289177, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$, Rules used = {1164, 628}

$$\frac{\log(\sqrt{2-bx+x^2+1})}{2\sqrt{2-b}} - \frac{\log(-\sqrt{2-bx+x^2+1})}{2\sqrt{2-b}}$$

Antiderivative was successfully verified.

[In] Int[(1 - x^2)/(1 + b*x^2 + x^4), x]

[Out] -Log[1 - Sqrt[2 - b]*x + x^2]/(2*Sqrt[2 - b]) + Log[1 + Sqrt[2 - b]*x + x^2]/(2*Sqrt[2 - b])

Rule 1164

Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[(-2*d)/e - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && !GtQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rubi steps

$$\begin{aligned} \int \frac{1-x^2}{1+bx^2+x^4} dx &= -\frac{\int \frac{\sqrt{2-b+2x}}{-1-\sqrt{2-bx-x^2}} dx}{2\sqrt{2-b}} - \frac{\int \frac{\sqrt{2-b-2x}}{-1+\sqrt{2-bx-x^2}} dx}{2\sqrt{2-b}} \\ &= -\frac{\log(1-\sqrt{2-bx+x^2})}{2\sqrt{2-b}} + \frac{\log(1+\sqrt{2-bx+x^2})}{2\sqrt{2-b}} \end{aligned}$$

Mathematica [B] time = 0.0711127, size = 125, normalized size = 2.02

$$\frac{(-\sqrt{b^2-4+b+2}) \tan^{-1}\left(\frac{\sqrt{2x}}{\sqrt{b-\sqrt{b^2-4}}}\right) - (\sqrt{b^2-4+b+2}) \tan^{-1}\left(\frac{\sqrt{2x}}{\sqrt{\sqrt{b^2-4}+b}}\right)}{\sqrt{b-\sqrt{b^2-4}} - \sqrt{\sqrt{b^2-4}+b}}$$

$$\frac{\sqrt{2}\sqrt{b^2-4}}{\sqrt{b-\sqrt{b^2-4}} - \sqrt{\sqrt{b^2-4}+b}}$$

Antiderivative was successfully verified.

[In] Integrate[(1 - x^2)/(1 + b*x^2 + x^4), x]

[Out] (((2 + b - Sqrt[-4 + b^2])*ArcTan[(Sqrt[2]*x)/Sqrt[b - Sqrt[-4 + b^2]]])/Sqrt[b - Sqrt[-4 + b^2]] - ((2 + b + Sqrt[-4 + b^2])*ArcTan[(Sqrt[2]*x)/Sqrt[b + Sqrt[-4 + b^2]]])/Sqrt[b + Sqrt[-4 + b^2]])/(Sqrt[2]*Sqrt[-4 + b^2])

Maple [B] time = 0.105, size = 279, normalized size = 4.5

$$-2 \frac{1}{\sqrt{(-2+b)(2+b)}\sqrt{2\sqrt{(-2+b)(2+b)}+2b}} \arctan\left(2 \frac{x}{\sqrt{2\sqrt{(-2+b)(2+b)}+2b}}\right) - \arctan\left(2 \frac{x}{\sqrt{2\sqrt{(-2+b)(2+b)}+2b}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-x^2+1)/(x^4+b*x^2+1), x)

[Out] -2/((-2+b)*(2+b))^(1/2)/(2*((-2+b)*(2+b))^(1/2)+2*b)^(1/2)*arctan(2*x/(2*((-2+b)*(2+b))^(1/2)+2*b)^(1/2))-1/(2*((-2+b)*(2+b))^(1/2)+2*b)^(1/2)*arctan(2*x/(2*((-2+b)*(2+b))^(1/2)+2*b)^(1/2))-1/((-2+b)*(2+b))^(1/2)/(2*((-2+b)*(2+b))^(1/2)+2*b)^(1/2)*arctan(2*x/(2*((-2+b)*(2+b))^(1/2)+2*b)^(1/2))*b+2/((-2+b)*(2+b))^(1/2)/(-2*((-2+b)*(2+b))^(1/2)+2*b)^(1/2)*arctan(2*x/(-2*((-2+b)*(2+b))^(1/2)+2*b)^(1/2))-1/(-2*((-2+b)*(2+b))^(1/2)+2*b)^(1/2)*arctan(2*x/(-2*((-2+b)*(2+b))^(1/2)+2*b)^(1/2))+1/((-2+b)*(2+b))^(1/2)/(-2*((-2+b)*(2+b))^(1/2)+2*b)^(1/2)*arctan(2*x/(-2*((-2+b)*(2+b))^(1/2)+2*b)^(1/2))*b

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^2 - 1}{x^4 + bx^2 + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2+1)/(x^4+b*x^2+1), x, algorithm="maxima")

[Out] -integrate((x^2 - 1)/(x^4 + b*x^2 + 1), x)

Fricas [A] time = 1.35622, size = 273, normalized size = 4.4

$$\left[\frac{\sqrt{-b+2} \log\left(\frac{x^4-(b-4)x^2+2(x^3+x)\sqrt{-b+2}+1}{x^4+bx^2+1}\right)}{2(b-2)}, \frac{\sqrt{b-2} \arctan\left(\frac{x^3+(b-1)x}{\sqrt{b-2}}\right) - \sqrt{b-2} \arctan\left(\frac{x}{\sqrt{b-2}}\right)}{b-2} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2+1)/(x^4+b*x^2+1), x, algorithm="fricas")

[Out] [-1/2*sqrt(-b + 2)*log((x^4 - (b - 4)*x^2 + 2*(x^3 + x)*sqrt(-b + 2) + 1)/(x^4 + b*x^2 + 1))/(b - 2), (sqrt(b - 2)*arctan((x^3 + (b - 1)*x)/sqrt(b - 2

)) - sqrt(b - 2)*arctan(x/sqrt(b - 2))/(b - 2)]

Sympy [A] time = 0.251893, size = 87, normalized size = 1.4

$$\frac{\sqrt{-\frac{1}{b-2}} \log\left(x^2 + x\left(-b\sqrt{-\frac{1}{b-2}} + 2\sqrt{-\frac{1}{b-2}}\right) + 1\right)}{2} - \frac{\sqrt{-\frac{1}{b-2}} \log\left(x^2 + x\left(b\sqrt{-\frac{1}{b-2}} - 2\sqrt{-\frac{1}{b-2}}\right) + 1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x**2+1)/(x**4+b*x**2+1),x)

[Out] sqrt(-1/(b - 2))*log(x**2 + x*(-b*sqrt(-1/(b - 2)) + 2*sqrt(-1/(b - 2)))) + 1)/2 - sqrt(-1/(b - 2))*log(x**2 + x*(b*sqrt(-1/(b - 2)) - 2*sqrt(-1/(b - 2)))) + 1)/2

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2+1)/(x^4+b*x^2+1),x, algorithm="giac")

[Out] Exception raised: TypeError

$$3.81 \quad \int \frac{1-x^2}{1+5x^2+x^4} dx$$

Optimal. Leaf size=50

$$\frac{\tan^{-1}\left(\sqrt{\frac{1}{2}(5+\sqrt{21})}x\right)}{\sqrt{3}} - \frac{\tan^{-1}\left(\sqrt{\frac{2}{5+\sqrt{21}}}x\right)}{\sqrt{3}}$$

[Out] -(ArcTan[Sqrt[2/(5 + Sqrt[21])] *x]/Sqrt[3]) + ArcTan[Sqrt[(5 + Sqrt[21])/2] *x]/Sqrt[3]

Rubi [A] time = 0.0398747, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$, Rules used = {1163, 203}

$$\frac{\tan^{-1}\left(\sqrt{\frac{1}{2}(5+\sqrt{21})}x\right)}{\sqrt{3}} - \frac{\tan^{-1}\left(\sqrt{\frac{2}{5+\sqrt{21}}}x\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[(1 - x^2)/(1 + 5*x^2 + x^4), x]

[Out] -(ArcTan[Sqrt[2/(5 + Sqrt[21])] *x]/Sqrt[3]) + ArcTan[Sqrt[(5 + Sqrt[21])/2] *x]/Sqrt[3]

Rule 1163

Int[((d_) + (e_)*(x_)^2)/((a_) + (b_)*(x_)^2 + (c_)*(x_)^4), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && GtQ[b^2 - 4*a*c, 0]

Rule 203

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rubi steps

$$\begin{aligned} \int \frac{1-x^2}{1+5x^2+x^4} dx &= \frac{1}{6}(-3+\sqrt{21}) \int \frac{1}{\frac{5}{2}-\frac{\sqrt{21}}{2}+x^2} dx - \frac{1}{6}(3+\sqrt{21}) \int \frac{1}{\frac{5}{2}+\frac{\sqrt{21}}{2}+x^2} dx \\ &= -\frac{\tan^{-1}\left(\sqrt{\frac{2}{5+\sqrt{21}}}x\right)}{\sqrt{3}} + \frac{\tan^{-1}\left(\sqrt{\frac{1}{2}(5+\sqrt{21})}x\right)}{\sqrt{3}} \end{aligned}$$

Mathematica [A] time = 0.12987, size = 87, normalized size = 1.74

$$\frac{(7-\sqrt{21})\tan^{-1}\left(\sqrt{\frac{2}{5-\sqrt{21}}}x\right)}{\sqrt{42(5-\sqrt{21})}} + \frac{(-7-\sqrt{21})\tan^{-1}\left(\sqrt{\frac{2}{5+\sqrt{21}}}x\right)}{\sqrt{42(5+\sqrt{21})}}$$

Antiderivative was successfully verified.

[In] Integrate[(1 - x^2)/(1 + 5*x^2 + x^4),x]

[Out] ((7 - Sqrt[21])*ArcTan[Sqrt[2/(5 - Sqrt[21]])*x])/Sqrt[42*(5 - Sqrt[21])] + ((-7 - Sqrt[21])*ArcTan[Sqrt[2/(5 + Sqrt[21]])*x])/Sqrt[42*(5 + Sqrt[21])]

Maple [B] time = 0.058, size = 136, normalized size = 2.7

$$\frac{2\sqrt{21}}{6\sqrt{7}-6\sqrt{3}} \arctan\left(4\frac{x}{2\sqrt{7}-2\sqrt{3}}\right) - 2\frac{1}{2\sqrt{7}-2\sqrt{3}} \arctan\left(4\frac{x}{2\sqrt{7}-2\sqrt{3}}\right) - \frac{2\sqrt{21}}{6\sqrt{7}+6\sqrt{3}} \arctan\left(4\frac{x}{2\sqrt{7}+2\sqrt{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-x^2+1)/(x^4+5*x^2+1),x)

[Out] 2/3*21^(1/2)/(2*7^(1/2)-2*3^(1/2))*arctan(4*x/(2*7^(1/2)-2*3^(1/2)))-2/(2*7^(1/2)-2*3^(1/2))*arctan(4*x/(2*7^(1/2)-2*3^(1/2)))-2/3*21^(1/2)/(2*7^(1/2)+2*3^(1/2))*arctan(4*x/(2*7^(1/2)+2*3^(1/2)))-2/(2*7^(1/2)+2*3^(1/2))*arctan(4*x/(2*7^(1/2)+2*3^(1/2)))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^2 - 1}{x^4 + 5x^2 + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2+1)/(x^4+5*x^2+1),x, algorithm="maxima")

[Out] -integrate((x^2 - 1)/(x^4 + 5*x^2 + 1), x)

Fricas [A] time = 1.4109, size = 109, normalized size = 2.18

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(x^3+4x)\right) - \frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2+1)/(x^4+5*x^2+1),x, algorithm="fricas")

[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(x^3 + 4*x)) - 1/3*sqrt(3)*arctan(1/3*sqrt(3)*x)

Sympy [A] time = 0.112025, size = 42, normalized size = 0.84

$$\frac{\sqrt{3}\left(2\operatorname{atan}\left(\frac{\sqrt{3}x}{3}\right) - 2\operatorname{atan}\left(\frac{\sqrt{3}x^3}{3} + \frac{4\sqrt{3}x}{3}\right)\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x**2+1)/(x**4+5*x**2+1),x)

[Out] -sqrt(3)*(2*atan(sqrt(3)*x/3) - 2*atan(sqrt(3)*x**3/3 + 4*sqrt(3)*x/3))/6

Giac [A] time = 1.1387, size = 35, normalized size = 0.7

$$\frac{1}{6} \sqrt{3} \left(\pi \operatorname{sgn}(x) - 2 \arctan \left(\frac{\sqrt{3}(x^2 + 1)}{3x} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2+1)/(x^4+5*x^2+1),x, algorithm="giac")

[Out] 1/6*sqrt(3)*(pi*sgn(x) - 2*arctan(1/3*sqrt(3)*(x^2 + 1)/x))

$$3.82 \quad \int \frac{1-x^2}{1+4x^2+x^4} dx$$

Optimal. Leaf size=44

$$\frac{\tan^{-1}\left(\frac{x}{\sqrt{2-\sqrt{3}}}\right)}{\sqrt{2}} - \frac{\tan^{-1}\left(\frac{x}{\sqrt{2+\sqrt{3}}}\right)}{\sqrt{2}}$$

[Out] ArcTan[x/Sqrt[2 - Sqrt[3]]]/Sqrt[2] - ArcTan[x/Sqrt[2 + Sqrt[3]]]/Sqrt[2]

Rubi [A] time = 0.0288699, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$, Rules used = {1163, 203}

$$\frac{\tan^{-1}\left(\frac{x}{\sqrt{2-\sqrt{3}}}\right)}{\sqrt{2}} - \frac{\tan^{-1}\left(\frac{x}{\sqrt{2+\sqrt{3}}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[(1 - x^2)/(1 + 4*x^2 + x^4), x]

[Out] ArcTan[x/Sqrt[2 - Sqrt[3]]]/Sqrt[2] - ArcTan[x/Sqrt[2 + Sqrt[3]]]/Sqrt[2]

Rule 1163

Int[((d_) + (e_)*(x_)^2)/((a_) + (b_)*(x_)^2 + (c_)*(x_)^4), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && GtQ[b^2 - 4*a*c, 0]

Rule 203

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rubi steps

$$\begin{aligned} \int \frac{1-x^2}{1+4x^2+x^4} dx &= \frac{1}{2}(-1-\sqrt{3}) \int \frac{1}{2+\sqrt{3}+x^2} dx + \frac{1}{2}(-1+\sqrt{3}) \int \frac{1}{2-\sqrt{3}+x^2} dx \\ &= \frac{\tan^{-1}\left(\frac{x}{\sqrt{2-\sqrt{3}}}\right)}{\sqrt{2}} - \frac{\tan^{-1}\left(\frac{x}{\sqrt{2+\sqrt{3}}}\right)}{\sqrt{2}} \end{aligned}$$

Mathematica [A] time = 0.064603, size = 82, normalized size = 1.86

$$\frac{-(\sqrt{3}-3)\sqrt{2+\sqrt{3}}\tan^{-1}\left(\frac{x}{\sqrt{2-\sqrt{3}}}\right) - \sqrt{2-\sqrt{3}}(3+\sqrt{3})\tan^{-1}\left(\frac{x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Integrate[(1 - x^2)/(1 + 4*x^2 + x^4), x]

[Out] $(-((-3 + \sqrt{3})\sqrt{2 + \sqrt{3}}\text{ArcTan}[x/\sqrt{2 - \sqrt{3}}]) - \sqrt{2 - \sqrt{3}}(3 + \sqrt{3})\text{ArcTan}[x/\sqrt{2 + \sqrt{3}}]))/(2\sqrt{3})$

Maple [B] time = 0.053, size = 111, normalized size = 2.5

$$\frac{\sqrt{3}}{\sqrt{6} - \sqrt{2}} \arctan\left(2 \frac{x}{\sqrt{6} - \sqrt{2}}\right) - \frac{1}{\sqrt{6} - \sqrt{2}} \arctan\left(2 \frac{x}{\sqrt{6} - \sqrt{2}}\right) - \frac{\sqrt{3}}{\sqrt{2} + \sqrt{6}} \arctan\left(2 \frac{x}{\sqrt{2} + \sqrt{6}}\right) - \frac{1}{\sqrt{2} + \sqrt{6}} \arctan\left(2 \frac{x}{\sqrt{2} + \sqrt{6}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-x^2+1)/(x^4+4*x^2+1), x)

[Out] $3^{(1/2)}/(6^{(1/2)}-2^{(1/2)})*\arctan(2*x/(6^{(1/2)}-2^{(1/2)}))-1/(6^{(1/2)}-2^{(1/2)})*\arctan(2*x/(6^{(1/2)}-2^{(1/2)}))-3^{(1/2)}/(2^{(1/2)}+6^{(1/2)})*\arctan(2*x/(2^{(1/2)}+6^{(1/2)}))-1/(2^{(1/2)}+6^{(1/2)})*\arctan(2*x/(2^{(1/2)}+6^{(1/2)}))$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^2 - 1}{x^4 + 4x^2 + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2+1)/(x^4+4*x^2+1), x, algorithm="maxima")

[Out] -integrate((x^2 - 1)/(x^4 + 4*x^2 + 1), x)

Fricas [A] time = 1.31292, size = 109, normalized size = 2.48

$$\frac{1}{2} \sqrt{2} \arctan\left(\frac{1}{2} \sqrt{2}(x^3 + 3x)\right) - \frac{1}{2} \sqrt{2} \arctan\left(\frac{1}{2} \sqrt{2}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2+1)/(x^4+4*x^2+1), x, algorithm="fricas")

[Out] $1/2*\text{sqrt}(2)*\arctan(1/2*\text{sqrt}(2)*(x^3 + 3*x)) - 1/2*\text{sqrt}(2)*\arctan(1/2*\text{sqrt}(2)*x)$

Sympy [A] time = 0.110828, size = 42, normalized size = 0.95

$$\frac{\sqrt{2} \left(2 \operatorname{atan}\left(\frac{\sqrt{2}x}{2}\right) - 2 \operatorname{atan}\left(\frac{\sqrt{2}x^3}{2} + \frac{3\sqrt{2}x}{2}\right) \right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x**2+1)/(x**4+4*x**2+1),x)

[Out] -sqrt(2)*(2*atan(sqrt(2)*x/2) - 2*atan(sqrt(2)*x**3/2 + 3*sqrt(2)*x/2))/4

Giac [A] time = 1.14273, size = 35, normalized size = 0.8

$$\frac{1}{4} \sqrt{2} \left(\pi \operatorname{sgn}(x) - 2 \arctan \left(\frac{\sqrt{2}(x^2 + 1)}{2x} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2+1)/(x^4+4*x^2+1),x, algorithm="giac")

[Out] 1/4*sqrt(2)*(pi*sgn(x) - 2*arctan(1/2*sqrt(2)*(x^2 + 1)/x))

$$3.83 \quad \int \frac{1-x^2}{1+3x^2+x^4} dx$$

Optimal. Leaf size=39

$$\tan^{-1}\left(\sqrt{\frac{1}{2}(3+\sqrt{5})}x\right) - \tan^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}x\right)$$

[Out] -ArcTan[Sqrt[2/(3 + Sqrt[5])]*x] + ArcTan[Sqrt[(3 + Sqrt[5])/2]*x]

Rubi [A] time = 0.0327162, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$, Rules used = {1163, 203}

$$\tan^{-1}\left(\sqrt{\frac{1}{2}(3+\sqrt{5})}x\right) - \tan^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}x\right)$$

Antiderivative was successfully verified.

[In] Int[(1 - x^2)/(1 + 3*x^2 + x^4), x]

[Out] -ArcTan[Sqrt[2/(3 + Sqrt[5])]*x] + ArcTan[Sqrt[(3 + Sqrt[5])/2]*x]

Rule 1163

Int[((d_) + (e_)*(x_)^2)/((a_) + (b_)*(x_)^2 + (c_)*(x_)^4), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && GtQ[b^2 - 4*a*c, 0]

Rule 203

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rubi steps

$$\begin{aligned} \int \frac{1-x^2}{1+3x^2+x^4} dx &= \frac{1}{2}(-1-\sqrt{5}) \int \frac{1}{\frac{3}{2} + \frac{\sqrt{5}}{2} + x^2} dx + \frac{1}{2}(-1+\sqrt{5}) \int \frac{1}{\frac{3}{2} - \frac{\sqrt{5}}{2} + x^2} dx \\ &= -\tan^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}x\right) + \tan^{-1}\left(\sqrt{\frac{1}{2}(3+\sqrt{5})}x\right) \end{aligned}$$

Mathematica [A] time = 0.006976, size = 10, normalized size = 0.26

$$\tan^{-1}\left(\frac{x}{x^2+1}\right)$$

Antiderivative was successfully verified.

[In] Integrate[(1 - x^2)/(1 + 3*x^2 + x^4), x]

[Out] ArcTan[x/(1 + x^2)]

Maple [B] time = 0.054, size = 104, normalized size = 2.7

$$-2 \frac{1}{-2+2\sqrt{5}} \arctan\left(4 \frac{x}{-2+2\sqrt{5}}\right) + 2 \frac{\sqrt{5}}{-2+2\sqrt{5}} \arctan\left(4 \frac{x}{-2+2\sqrt{5}}\right) - 2 \frac{\sqrt{5}}{2+2\sqrt{5}} \arctan\left(4 \frac{x}{2+2\sqrt{5}}\right) - 2 \frac{1}{2+2\sqrt{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-x^2+1)/(x^4+3*x^2+1),x)

[Out] $-2/(-2+2*5^{(1/2)})*\arctan(4*x/(-2+2*5^{(1/2)}))+2*5^{(1/2)/(-2+2*5^{(1/2)})}*\arctan(4*x/(-2+2*5^{(1/2)}))-2*5^{(1/2)/(2+2*5^{(1/2)})}*\arctan(4*x/(2+2*5^{(1/2)}))-2/(2+2*5^{(1/2)})*\arctan(4*x/(2+2*5^{(1/2)}))$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^2 - 1}{x^4 + 3x^2 + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2+1)/(x^4+3*x^2+1),x, algorithm="maxima")

[Out] -integrate((x^2 - 1)/(x^4 + 3*x^2 + 1), x)

Fricas [A] time = 1.2397, size = 42, normalized size = 1.08

$$\arctan(x^3 + 2x) - \arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2+1)/(x^4+3*x^2+1),x, algorithm="fricas")

[Out] arctan(x^3 + 2*x) - arctan(x)

Sympy [A] time = 0.098433, size = 10, normalized size = 0.26

$$-\operatorname{atan}(x) + \operatorname{atan}(x^3 + 2x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x**2+1)/(x**4+3*x**2+1),x)

[Out] -atan(x) + atan(x**3 + 2*x)

Giac [A] time = 1.15351, size = 35, normalized size = 0.9

$$\frac{1}{4} \pi \operatorname{sgn}(x) - \frac{1}{2} \arctan\left(\frac{x^4 + x^2 + 1}{2(x^3 + x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)/(x^4+3*x^2+1),x, algorithm="giac")
```

```
[Out] 1/4*pi*sgn(x) - 1/2*arctan(1/2*(x^4 + x^2 + 1)/(x^3 + x))
```

$$3.84 \quad \int \frac{1-x^2}{1+2x^2+x^4} dx$$

Optimal. Leaf size=9

$$\frac{x}{x^2+1}$$

[Out] x/(1 + x^2)

Rubi [A] time = 0.0039017, antiderivative size = 9, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$, Rules used = {28, 383}

$$\frac{x}{x^2+1}$$

Antiderivative was successfully verified.

[In] Int[(1 - x^2)/(1 + 2*x^2 + x^4),x]

[Out] x/(1 + x^2)

Rule 28

Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]

Rule 383

Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(c*x*(a + b*x^n)^(p + 1))/a, x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[a*d - b*c*(n*(p + 1) + 1), 0]

Rubi steps

$$\int \frac{1-x^2}{1+2x^2+x^4} dx = \int \frac{1-x^2}{(1+x^2)^2} dx = \frac{x}{1+x^2}$$

Mathematica [A] time = 0.0041393, size = 9, normalized size = 1.

$$\frac{x}{x^2+1}$$

Antiderivative was successfully verified.

[In] Integrate[(1 - x^2)/(1 + 2*x^2 + x^4),x]

[Out] x/(1 + x^2)

Maple [A] time = 0.045, size = 10, normalized size = 1.1

$$\frac{x}{x^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-x^2+1)/(x^4+2*x^2+1),x)

[Out] x/(x^2+1)

Maxima [A] time = 0.980252, size = 12, normalized size = 1.33

$$\frac{x}{x^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2+1)/(x^4+2*x^2+1),x, algorithm="maxima")

[Out] x/(x^2 + 1)

Fricas [A] time = 1.33412, size = 18, normalized size = 2.

$$\frac{x}{x^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2+1)/(x^4+2*x^2+1),x, algorithm="fricas")

[Out] x/(x^2 + 1)

Sympy [A] time = 0.079362, size = 5, normalized size = 0.56

$$\frac{x}{x^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x**2+1)/(x**4+2*x**2+1),x)

[Out] x/(x**2 + 1)

Giac [A] time = 1.11334, size = 9, normalized size = 1.

$$\frac{1}{x + \frac{1}{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2+1)/(x^4+2*x^2+1),x, algorithm="giac")

[Out] 1/(x + 1/x)

$$3.85 \quad \int \frac{1-x^2}{1+x^2+x^4} dx$$

Optimal. Leaf size=25

$$\frac{1}{2} \log(x^2 + x + 1) - \frac{1}{2} \log(x^2 - x + 1)$$

[Out] -Log[1 - x + x^2]/2 + Log[1 + x + x^2]/2

Rubi [A] time = 0.0134529, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1164, 628}

$$\frac{1}{2} \log(x^2 + x + 1) - \frac{1}{2} \log(x^2 - x + 1)$$

Antiderivative was successfully verified.

[In] Int[(1 - x^2)/(1 + x^2 + x^4), x]

[Out] -Log[1 - x + x^2]/2 + Log[1 + x + x^2]/2

Rule 1164

Int[((d_) + (e_)*(x_)^2)/((a_) + (b_)*(x_)^2 + (c_)*(x_)^4), x_Symbol] :> With[{q = Rt[(-2*d)/e - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && !GtQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] :> Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rubi steps

$$\begin{aligned} \int \frac{1-x^2}{1+x^2+x^4} dx &= -\left(\frac{1}{2} \int \frac{1+2x}{-1-x-x^2} dx\right) - \frac{1}{2} \int \frac{1-2x}{-1+x-x^2} dx \\ &= -\frac{1}{2} \log(1-x+x^2) + \frac{1}{2} \log(1+x+x^2) \end{aligned}$$

Mathematica [A] time = 0.0057009, size = 25, normalized size = 1.

$$\frac{1}{2} \log(x^2 + x + 1) - \frac{1}{2} \log(x^2 - x + 1)$$

Antiderivative was successfully verified.

[In] Integrate[(1 - x^2)/(1 + x^2 + x^4), x]

[Out] -Log[1 - x + x^2]/2 + Log[1 + x + x^2]/2

Maple [A] time = 0.043, size = 22, normalized size = 0.9

$$-\frac{\ln(x^2 - x + 1)}{2} + \frac{\ln(x^2 + x + 1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-x^2+1)/(x^4+x^2+1),x)

[Out] -1/2*ln(x^2-x+1)+1/2*ln(x^2+x+1)

Maxima [A] time = 0.986035, size = 28, normalized size = 1.12

$$\frac{1}{2} \log(x^2 + x + 1) - \frac{1}{2} \log(x^2 - x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2+1)/(x^4+x^2+1),x, algorithm="maxima")

[Out] 1/2*log(x^2 + x + 1) - 1/2*log(x^2 - x + 1)

Fricas [A] time = 1.34951, size = 61, normalized size = 2.44

$$\frac{1}{2} \log(x^2 + x + 1) - \frac{1}{2} \log(x^2 - x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2+1)/(x^4+x^2+1),x, algorithm="fricas")

[Out] 1/2*log(x^2 + x + 1) - 1/2*log(x^2 - x + 1)

Sympy [A] time = 0.097416, size = 19, normalized size = 0.76

$$-\frac{\log(x^2 - x + 1)}{2} + \frac{\log(x^2 + x + 1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x**2+1)/(x**4+x**2+1),x)

[Out] -log(x**2 - x + 1)/2 + log(x**2 + x + 1)/2

Giac [A] time = 1.12756, size = 47, normalized size = 1.88

$$\frac{1}{4} \log \left(x + \frac{1}{x + \frac{1}{x}} + \frac{1}{x} + 2 \right) - \frac{1}{4} \log \left(x + \frac{1}{x + \frac{1}{x}} + \frac{1}{x} - 2 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)/(x^4+x^2+1),x, algorithm="giac")
```

```
[Out] 1/4*log(abs(x + 1/(x + 1/x) + 1/x + 2)) - 1/4*log(abs(x + 1/(x + 1/x) + 1/x - 2))
```

$$3.86 \quad \int \frac{1-x^2}{1+x^4} dx$$

Optimal. Leaf size=46

$$\frac{\log(x^2 + \sqrt{2}x + 1)}{2\sqrt{2}} - \frac{\log(x^2 - \sqrt{2}x + 1)}{2\sqrt{2}}$$

[Out] -Log[1 - Sqrt[2]*x + x^2]/(2*Sqrt[2]) + Log[1 + Sqrt[2]*x + x^2]/(2*Sqrt[2])

Rubi [A] time = 0.0194266, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {1165, 628}

$$\frac{\log(x^2 + \sqrt{2}x + 1)}{2\sqrt{2}} - \frac{\log(x^2 - \sqrt{2}x + 1)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[(1 - x^2)/(1 + x^4), x]

[Out] -Log[1 - Sqrt[2]*x + x^2]/(2*Sqrt[2]) + Log[1 + Sqrt[2]*x + x^2]/(2*Sqrt[2])

Rule 1165

Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rubi steps

$$\begin{aligned} \int \frac{1-x^2}{1+x^4} dx &= -\frac{\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2} dx}{2\sqrt{2}} - \frac{\int \frac{\sqrt{2}-2x}{-1+\sqrt{2}x-x^2} dx}{2\sqrt{2}} \\ &= -\frac{\log(1 - \sqrt{2}x + x^2)}{2\sqrt{2}} + \frac{\log(1 + \sqrt{2}x + x^2)}{2\sqrt{2}} \end{aligned}$$

Mathematica [A] time = 0.0109317, size = 40, normalized size = 0.87

$$\frac{\log(x^2 + \sqrt{2}x + 1) - \log(-x^2 + \sqrt{2}x - 1)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Integrate[(1 - x^2)/(1 + x^4),x]

[Out] (-Log[-1 + Sqrt[2]*x - x^2] + Log[1 + Sqrt[2]*x + x^2])/(2*Sqrt[2])

Maple [A] time = 0.042, size = 62, normalized size = 1.4

$$\frac{\sqrt{2}}{8} \ln\left(\frac{1+x^2+x\sqrt{2}}{1+x^2-x\sqrt{2}}\right) - \frac{\sqrt{2}}{8} \ln\left(\frac{1+x^2-x\sqrt{2}}{1+x^2+x\sqrt{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-x^2+1)/(x^4+1),x)

[Out] 1/8*2^(1/2)*ln((1+x^2+x*2^(1/2))/(1+x^2-x*2^(1/2)))-1/8*2^(1/2)*ln((1+x^2-x*2^(1/2))/(1+x^2+x*2^(1/2)))

Maxima [A] time = 1.45449, size = 46, normalized size = 1.

$$\frac{1}{4} \sqrt{2} \log(x^2 + \sqrt{2}x + 1) - \frac{1}{4} \sqrt{2} \log(x^2 - \sqrt{2}x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2+1)/(x^4+1),x, algorithm="maxima")

[Out] 1/4*sqrt(2)*log(x^2 + sqrt(2)*x + 1) - 1/4*sqrt(2)*log(x^2 - sqrt(2)*x + 1)

Fricas [A] time = 1.36964, size = 92, normalized size = 2.

$$\frac{1}{4} \sqrt{2} \log\left(\frac{x^4 + 4x^2 + 2\sqrt{2}(x^3 + x) + 1}{x^4 + 1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2+1)/(x^4+1),x, algorithm="fricas")

[Out] 1/4*sqrt(2)*log((x^4 + 4*x^2 + 2*sqrt(2)*(x^3 + x) + 1)/(x^4 + 1))

Sympy [A] time = 0.097355, size = 39, normalized size = 0.85

$$-\frac{\sqrt{2} \log(x^2 - \sqrt{2}x + 1)}{4} + \frac{\sqrt{2} \log(x^2 + \sqrt{2}x + 1)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x**2+1)/(x**4+1),x)

[Out] -sqrt(2)*log(x**2 - sqrt(2)*x + 1)/4 + sqrt(2)*log(x**2 + sqrt(2)*x + 1)/4

Giac [A] time = 1.10006, size = 46, normalized size = 1.

$$\frac{1}{4}\sqrt{2}\log\left(x^2 + \sqrt{2}x + 1\right) - \frac{1}{4}\sqrt{2}\log\left(x^2 - \sqrt{2}x + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2+1)/(x^4+1),x, algorithm="giac")

[Out] 1/4*sqrt(2)*log(x^2 + sqrt(2)*x + 1) - 1/4*sqrt(2)*log(x^2 - sqrt(2)*x + 1)

$$3.87 \quad \int \frac{1-x^2}{1-x^2+x^4} dx$$

Optimal. Leaf size=46

$$\frac{\log(x^2 + \sqrt{3}x + 1)}{2\sqrt{3}} - \frac{\log(x^2 - \sqrt{3}x + 1)}{2\sqrt{3}}$$

[Out] -Log[1 - Sqrt[3]*x + x^2]/(2*Sqrt[3]) + Log[1 + Sqrt[3]*x + x^2]/(2*Sqrt[3])

Rubi [A] time = 0.0210366, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$, Rules used = {1164, 628}

$$\frac{\log(x^2 + \sqrt{3}x + 1)}{2\sqrt{3}} - \frac{\log(x^2 - \sqrt{3}x + 1)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[(1 - x^2)/(1 - x^2 + x^4), x]

[Out] -Log[1 - Sqrt[3]*x + x^2]/(2*Sqrt[3]) + Log[1 + Sqrt[3]*x + x^2]/(2*Sqrt[3])

Rule 1164

Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[(-2*d)/e - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && !GtQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rubi steps

$$\begin{aligned} \int \frac{1-x^2}{1-x^2+x^4} dx &= -\frac{\int \frac{\sqrt{3}+2x}{-1-\sqrt{3}x-x^2} dx}{2\sqrt{3}} - \frac{\int \frac{\sqrt{3}-2x}{-1+\sqrt{3}x-x^2} dx}{2\sqrt{3}} \\ &= -\frac{\log(1-\sqrt{3}x+x^2)}{2\sqrt{3}} + \frac{\log(1+\sqrt{3}x+x^2)}{2\sqrt{3}} \end{aligned}$$

Mathematica [A] time = 0.0125238, size = 40, normalized size = 0.87

$$\frac{\log(x^2 + \sqrt{3}x + 1) - \log(-x^2 + \sqrt{3}x - 1)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Integrate[(1 - x^2)/(1 - x^2 + x^4), x]

[Out] (-Log[-1 + Sqrt[3]*x - x^2] + Log[1 + Sqrt[3]*x + x^2])/(2*Sqrt[3])

Maple [A] time = 0.049, size = 35, normalized size = 0.8

$$-\frac{\ln(1+x^2-x\sqrt{3})\sqrt{3}}{6} + \frac{\ln(1+x^2+x\sqrt{3})\sqrt{3}}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-x^2+1)/(x^4-x^2+1), x)

[Out] -1/6*ln(1+x^2-x*3^(1/2))*3^(1/2)+1/6*ln(1+x^2+x*3^(1/2))*3^(1/2)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^2 - 1}{x^4 - x^2 + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2+1)/(x^4-x^2+1), x, algorithm="maxima")

[Out] -integrate((x^2 - 1)/(x^4 - x^2 + 1), x)

Fricas [A] time = 1.32572, size = 100, normalized size = 2.17

$$\frac{1}{6} \sqrt{3} \log\left(\frac{x^4 + 5x^2 + 2\sqrt{3}(x^3 + x) + 1}{x^4 - x^2 + 1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2+1)/(x^4-x^2+1), x, algorithm="fricas")

[Out] 1/6*sqrt(3)*log((x^4 + 5*x^2 + 2*sqrt(3)*(x^3 + x) + 1)/(x^4 - x^2 + 1))

Sympy [A] time = 0.100779, size = 39, normalized size = 0.85

$$-\frac{\sqrt{3} \log(x^2 - \sqrt{3}x + 1)}{6} + \frac{\sqrt{3} \log(x^2 + \sqrt{3}x + 1)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x**2+1)/(x**4-x**2+1), x)

[Out] -sqrt(3)*log(x**2 - sqrt(3)*x + 1)/6 + sqrt(3)*log(x**2 + sqrt(3)*x + 1)/6

Giac [A] time = 1.12561, size = 53, normalized size = 1.15

$$-\frac{1}{6} \sqrt{3} \log \left(\frac{\left| 2x - 2\sqrt{3} + \frac{2}{x} \right|}{\left| 2x + 2\sqrt{3} + \frac{2}{x} \right|} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2+1)/(x^4-x^2+1),x, algorithm="giac")

[Out] -1/6*sqrt(3)*log(abs(2*x - 2*sqrt(3) + 2/x)/abs(2*x + 2*sqrt(3) + 2/x))

$$3.88 \quad \int \frac{1-x^2}{1-2x^2+x^4} dx$$

Optimal. Leaf size=2

$$\tanh^{-1}(x)$$

[Out] ArcTanh[x]

Rubi [A] time = 0.0020564, antiderivative size = 2, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.15$, Rules used = {28, 21, 207}

$$\tanh^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[(1 - x^2)/(1 - 2*x^2 + x^4), x]

[Out] ArcTanh[x]

Rule 28

Int[(u_)*((a_) + (c_)*(x_)^(n2_)) + (b_)*(x_)^(n_)]^(p_), x_Symbol] :=
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]

Rule 21

Int[(u_)*((a_) + (b_)*(v_))^(m_)*((c_) + (d_)*(v_))^(n_), x_Symbol] :=
Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
&& EqQ[b*c - a*d, 0] && IntegerQ[m] && (!IntegerQ[n] || SimplerQ[c + d*x,
a + b*x])

Rule 207

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rubi steps

$$\begin{aligned} \int \frac{1-x^2}{1-2x^2+x^4} dx &= \int \frac{1-x^2}{(-1+x^2)^2} dx \\ &= -\int \frac{1}{-1+x^2} dx \\ &= \tanh^{-1}(x) \end{aligned}$$

Mathematica [B] time = 0.0021777, size = 19, normalized size = 9.5

$$\frac{1}{2} \log(x+1) - \frac{1}{2} \log(1-x)$$

Antiderivative was successfully verified.

[In] Integrate[(1 - x^2)/(1 - 2*x^2 + x^4),x]

[Out] -Log[1 - x]/2 + Log[1 + x]/2

Maple [A] time = 0.039, size = 3, normalized size = 1.5

Artanh(x)

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-x^2+1)/(x^4-2*x^2+1),x)

[Out] arctanh(x)

Maxima [B] time = 0.969496, size = 18, normalized size = 9.

$$\frac{1}{2} \log(x + 1) - \frac{1}{2} \log(x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2+1)/(x^4-2*x^2+1),x, algorithm="maxima")

[Out] 1/2*log(x + 1) - 1/2*log(x - 1)

Fricas [B] time = 1.32515, size = 45, normalized size = 22.5

$$\frac{1}{2} \log(x + 1) - \frac{1}{2} \log(x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2+1)/(x^4-2*x^2+1),x, algorithm="fricas")

[Out] 1/2*log(x + 1) - 1/2*log(x - 1)

Sympy [B] time = 0.087152, size = 12, normalized size = 6.

$$-\frac{\log(x - 1)}{2} + \frac{\log(x + 1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x**2+1)/(x**4-2*x**2+1),x)

[Out] -log(x - 1)/2 + log(x + 1)/2

Giac [B] time = 1.14393, size = 20, normalized size = 10.

$$\frac{1}{2} \log(|x + 1|) - \frac{1}{2} \log(|x - 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)/(x^4-2*x^2+1),x, algorithm="giac")
```

```
[Out] 1/2*log(abs(x + 1)) - 1/2*log(abs(x - 1))
```

$$3.89 \quad \int \frac{1-x^2}{1-3x^2+x^4} dx$$

Optimal. Leaf size=38

$$\frac{\tanh^{-1}\left(\frac{2x+1}{\sqrt{5}}\right)}{\sqrt{5}} - \frac{\tanh^{-1}\left(\frac{1-2x}{\sqrt{5}}\right)}{\sqrt{5}}$$

[Out] -(ArcTanh[(1 - 2*x)/Sqrt[5]]/Sqrt[5]) + ArcTanh[(1 + 2*x)/Sqrt[5]]/Sqrt[5]

Rubi [A] time = 0.0294095, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.15$, Rules used = {1161, 618, 206}

$$\frac{\tanh^{-1}\left(\frac{2x+1}{\sqrt{5}}\right)}{\sqrt{5}} - \frac{\tanh^{-1}\left(\frac{1-2x}{\sqrt{5}}\right)}{\sqrt{5}}$$

Antiderivative was successfully verified.

[In] Int[(1 - x^2)/(1 - 3*x^2 + x^4), x]

[Out] -(ArcTanh[(1 - 2*x)/Sqrt[5]]/Sqrt[5]) + ArcTanh[(1 + 2*x)/Sqrt[5]]/Sqrt[5]

Rule 1161

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (b_)*(x_)^2 + (c_)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[(2*d)/e - b/c, 0] || (!LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 618

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] :=> Dist[-2, Subst[In
t[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] :=> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{1-x^2}{1-3x^2+x^4} dx &= -\left(\frac{1}{2} \int \frac{1}{-1-x+x^2} dx\right) - \frac{1}{2} \int \frac{1}{-1+x+x^2} dx \\ &= \text{Subst}\left(\int \frac{1}{5-x^2} dx, x, -1+2x\right) + \text{Subst}\left(\int \frac{1}{5-x^2} dx, x, 1+2x\right) \\ &= \frac{\tanh^{-1}\left(\frac{-1+2x}{\sqrt{5}}\right)}{\sqrt{5}} + \frac{\tanh^{-1}\left(\frac{1+2x}{\sqrt{5}}\right)}{\sqrt{5}} \end{aligned}$$

Mathematica [A] time = 0.0138923, size = 40, normalized size = 1.05

$$\frac{\log(x^2 + \sqrt{5}x + 1) - \log(-x^2 + \sqrt{5}x - 1)}{2\sqrt{5}}$$

Antiderivative was successfully verified.

[In] Integrate[(1 - x^2)/(1 - 3*x^2 + x^4), x]

[Out] (-Log[-1 + Sqrt[5]*x - x^2] + Log[1 + Sqrt[5]*x + x^2])/(2*Sqrt[5])

Maple [A] time = 0.043, size = 34, normalized size = 0.9

$$\frac{\sqrt{5}}{5} \operatorname{Arctanh}\left(\frac{(1+2x)\sqrt{5}}{5}\right) + \frac{\sqrt{5}}{5} \operatorname{Arctanh}\left(\frac{(2x-1)\sqrt{5}}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-x^2+1)/(x^4-3*x^2+1), x)

[Out] 1/5*arctanh(1/5*(1+2*x)*5^(1/2))*5^(1/2)+1/5*5^(1/2)*arctanh(1/5*(2*x-1)*5^(1/2))

Maxima [A] time = 1.44361, size = 74, normalized size = 1.95

$$-\frac{1}{10} \sqrt{5} \log\left(\frac{2x - \sqrt{5} + 1}{2x + \sqrt{5} + 1}\right) - \frac{1}{10} \sqrt{5} \log\left(\frac{2x - \sqrt{5} - 1}{2x + \sqrt{5} - 1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2+1)/(x^4-3*x^2+1), x, algorithm="maxima")

[Out] -1/10*sqrt(5)*log((2*x - sqrt(5) + 1)/(2*x + sqrt(5) + 1)) - 1/10*sqrt(5)*log((2*x - sqrt(5) - 1)/(2*x + sqrt(5) - 1))

Fricas [A] time = 1.35961, size = 104, normalized size = 2.74

$$\frac{1}{10} \sqrt{5} \log\left(\frac{x^4 + 7x^2 + 2\sqrt{5}(x^3 + x) + 1}{x^4 - 3x^2 + 1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2+1)/(x^4-3*x^2+1), x, algorithm="fricas")

[Out] 1/10*sqrt(5)*log((x^4 + 7*x^2 + 2*sqrt(5)*(x^3 + x) + 1)/(x^4 - 3*x^2 + 1))

Sympy [A] time = 0.102661, size = 39, normalized size = 1.03

$$-\frac{\sqrt{5} \log(x^2 - \sqrt{5}x + 1)}{10} + \frac{\sqrt{5} \log(x^2 + \sqrt{5}x + 1)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x**2+1)/(x**4-3*x**2+1),x)

[Out] -sqrt(5)*log(x**2 - sqrt(5)*x + 1)/10 + sqrt(5)*log(x**2 + sqrt(5)*x + 1)/10

Giac [A] time = 1.15881, size = 53, normalized size = 1.39

$$-\frac{1}{10} \sqrt{5} \log \left(\frac{\left| 2x - 2\sqrt{5} + \frac{2}{x} \right|}{\left| 2x + 2\sqrt{5} + \frac{2}{x} \right|} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2+1)/(x^4-3*x^2+1),x, algorithm="giac")

[Out] -1/10*sqrt(5)*log(abs(2*x - 2*sqrt(5) + 2/x)/abs(2*x + 2*sqrt(5) + 2/x))

$$3.90 \quad \int \frac{1-x^2}{1-4x^2+x^4} dx$$

Optimal. Leaf size=47

$$\frac{\tanh^{-1}\left(\frac{\sqrt{2x+1}}{\sqrt{3}}\right)}{\sqrt{6}} - \frac{\tanh^{-1}\left(\frac{1-\sqrt{2x}}{\sqrt{3}}\right)}{\sqrt{6}}$$

[Out] -(ArcTanh[(1 - Sqrt[2]*x)/Sqrt[3]]/Sqrt[6]) + ArcTanh[(1 + Sqrt[2]*x)/Sqrt[3]]/Sqrt[6]

Rubi [A] time = 0.0359018, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.15$, Rules used = {1161, 618, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{2x+1}}{\sqrt{3}}\right)}{\sqrt{6}} - \frac{\tanh^{-1}\left(\frac{1-\sqrt{2x}}{\sqrt{3}}\right)}{\sqrt{6}}$$

Antiderivative was successfully verified.

[In] Int[(1 - x^2)/(1 - 4*x^2 + x^4), x]

[Out] -(ArcTanh[(1 - Sqrt[2]*x)/Sqrt[3]]/Sqrt[6]) + ArcTanh[(1 + Sqrt[2]*x)/Sqrt[3]]/Sqrt[6]

Rule 1161

Int[((d_) + (e_)*(x_)^2)/((a_) + (b_)*(x_)^2 + (c_)*(x_)^4), x_Symbol] :> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (GtQ[(2*d)/e - b/c, 0] || (!LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2], 0]))

Rule 618

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned} \int \frac{1-x^2}{1-4x^2+x^4} dx &= -\left(\frac{1}{2} \int \frac{1}{-1-\sqrt{2}x+x^2} dx\right) - \frac{1}{2} \int \frac{1}{-1+\sqrt{2}x+x^2} dx \\ &= \text{Subst}\left(\int \frac{1}{6-x^2} dx, x, -\sqrt{2}+2x\right) + \text{Subst}\left(\int \frac{1}{6-x^2} dx, x, \sqrt{2}+2x\right) \\ &= \frac{\tanh^{-1}\left(\frac{-1+\sqrt{2}x}{\sqrt{3}}\right)}{\sqrt{6}} + \frac{\tanh^{-1}\left(\frac{1+\sqrt{2}x}{\sqrt{3}}\right)}{\sqrt{6}} \end{aligned}$$

Mathematica [A] time = 0.0172322, size = 40, normalized size = 0.85

$$\frac{\log(x^2 + \sqrt{6}x + 1) - \log(-x^2 + \sqrt{6}x - 1)}{2\sqrt{6}}$$

Antiderivative was successfully verified.

[In] Integrate[(1 - x^2)/(1 - 4*x^2 + x^4), x]

[Out] (-Log[-1 + Sqrt[6]*x - x^2] + Log[1 + Sqrt[6]*x + x^2])/(2*Sqrt[6])

Maple [A] time = 0.053, size = 70, normalized size = 1.5

$$\frac{(1 + \sqrt{3})\sqrt{3}}{3\sqrt{2} + 3\sqrt{6}} \text{Arctanh}\left(2 \frac{x}{\sqrt{2} + \sqrt{6}}\right) + \frac{(\sqrt{3} - 1)\sqrt{3}}{3\sqrt{6} - 3\sqrt{2}} \text{Arctanh}\left(2 \frac{x}{\sqrt{6} - \sqrt{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-x^2+1)/(x^4-4*x^2+1), x)

[Out] 1/3*(1+3^(1/2))*3^(1/2)/(2^(1/2)+6^(1/2))*arctanh(2*x/(2^(1/2)+6^(1/2)))+1/3*(3^(1/2)-1)*3^(1/2)/(6^(1/2)-2^(1/2))*arctanh(2*x/(6^(1/2)-2^(1/2)))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^2 - 1}{x^4 - 4x^2 + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2+1)/(x^4-4*x^2+1), x, algorithm="maxima")

[Out] -integrate((x^2 - 1)/(x^4 - 4*x^2 + 1), x)

Fricas [A] time = 1.34347, size = 104, normalized size = 2.21

$$\frac{1}{12} \sqrt{6} \log\left(\frac{x^4 + 8x^2 + 2\sqrt{6}(x^3 + x) + 1}{x^4 - 4x^2 + 1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2+1)/(x^4-4*x^2+1),x, algorithm="fricas")

[Out] 1/12*sqrt(6)*log((x^4 + 8*x^2 + 2*sqrt(6)*(x^3 + x) + 1)/(x^4 - 4*x^2 + 1))

Sympy [A] time = 0.103705, size = 39, normalized size = 0.83

$$-\frac{\sqrt{6} \log(x^2 - \sqrt{6}x + 1)}{12} + \frac{\sqrt{6} \log(x^2 + \sqrt{6}x + 1)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x**2+1)/(x**4-4*x**2+1),x)

[Out] -sqrt(6)*log(x**2 - sqrt(6)*x + 1)/12 + sqrt(6)*log(x**2 + sqrt(6)*x + 1)/12

Giac [A] time = 1.15901, size = 53, normalized size = 1.13

$$-\frac{1}{12} \sqrt{6} \log \left(\frac{\left| 2x - 2\sqrt{6} + \frac{2}{x} \right|}{\left| 2x + 2\sqrt{6} + \frac{2}{x} \right|} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2+1)/(x^4-4*x^2+1),x, algorithm="giac")

[Out] -1/12*sqrt(6)*log(abs(2*x - 2*sqrt(6) + 2/x)/abs(2*x + 2*sqrt(6) + 2/x))

3.91 $\int \frac{1-x^2}{1-5x^2+x^4} dx$

Optimal. Leaf size=46

$$\frac{\tanh^{-1}\left(\frac{2x+\sqrt{3}}{\sqrt{7}}\right)}{\sqrt{7}} - \frac{\tanh^{-1}\left(\frac{\sqrt{3}-2x}{\sqrt{7}}\right)}{\sqrt{7}}$$

[Out] -(ArcTanh[(Sqrt[3] - 2*x)/Sqrt[7]]/Sqrt[7]) + ArcTanh[(Sqrt[3] + 2*x)/Sqrt[7]]/Sqrt[7]

Rubi [A] time = 0.0351032, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.15$, Rules used = {1161, 618, 206}

$$\frac{\tanh^{-1}\left(\frac{2x+\sqrt{3}}{\sqrt{7}}\right)}{\sqrt{7}} - \frac{\tanh^{-1}\left(\frac{\sqrt{3}-2x}{\sqrt{7}}\right)}{\sqrt{7}}$$

Antiderivative was successfully verified.

[In] Int[(1 - x^2)/(1 - 5*x^2 + x^4), x]

[Out] -(ArcTanh[(Sqrt[3] - 2*x)/Sqrt[7]]/Sqrt[7]) + ArcTanh[(Sqrt[3] + 2*x)/Sqrt[7]]/Sqrt[7]

Rule 1161

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[(2*d)/e - b/c, 0] || ( !LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[In
t[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{1-x^2}{1-5x^2+x^4} dx &= -\left(\frac{1}{2} \int \frac{1}{-1-\sqrt{3}x+x^2} dx\right) - \frac{1}{2} \int \frac{1}{-1+\sqrt{3}x+x^2} dx \\ &= \text{Subst}\left(\int \frac{1}{7-x^2} dx, x, -\sqrt{3}+2x\right) + \text{Subst}\left(\int \frac{1}{7-x^2} dx, x, \sqrt{3}+2x\right) \\ &= -\frac{\tanh^{-1}\left(\frac{\sqrt{3}-2x}{\sqrt{7}}\right)}{\sqrt{7}} + \frac{\tanh^{-1}\left(\frac{\sqrt{3}+2x}{\sqrt{7}}\right)}{\sqrt{7}} \end{aligned}$$

Mathematica [A] time = 0.0142958, size = 40, normalized size = 0.87

$$\frac{\log(x^2 + \sqrt{7}x + 1) - \log(-x^2 + \sqrt{7}x - 1)}{2\sqrt{7}}$$

Antiderivative was successfully verified.

[In] Integrate[(1 - x^2)/(1 - 5*x^2 + x^4), x]

[Out] (-Log[-1 + Sqrt[7]*x - x^2] + Log[1 + Sqrt[7]*x + x^2])/(2*Sqrt[7])

Maple [B] time = 0.055, size = 82, normalized size = 1.8

$$\frac{(6 + 2\sqrt{21})\sqrt{21}}{42\sqrt{7} + 42\sqrt{3}} \text{Arctanh}\left(4\frac{x}{2\sqrt{7} + 2\sqrt{3}}\right) + \frac{(-6 + 2\sqrt{21})\sqrt{21}}{42\sqrt{7} - 42\sqrt{3}} \text{Arctanh}\left(4\frac{x}{2\sqrt{7} - 2\sqrt{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-x^2+1)/(x^4-5*x^2+1), x)

[Out] 2/21*(3+21^(1/2))*21^(1/2)/(2*7^(1/2)+2*3^(1/2))*arctanh(4*x/(2*7^(1/2)+2*3^(1/2)))+2/21*(-3+21^(1/2))*21^(1/2)/(2*7^(1/2)-2*3^(1/2))*arctanh(4*x/(2*7^(1/2)-2*3^(1/2)))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^2 - 1}{x^4 - 5x^2 + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2+1)/(x^4-5*x^2+1), x, algorithm="maxima")

[Out] -integrate((x^2 - 1)/(x^4 - 5*x^2 + 1), x)

Fricas [A] time = 1.39183, size = 104, normalized size = 2.26

$$\frac{1}{14}\sqrt{7}\log\left(\frac{x^4 + 9x^2 + 2\sqrt{7}(x^3 + x) + 1}{x^4 - 5x^2 + 1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2+1)/(x^4-5*x^2+1),x, algorithm="fricas")

[Out] 1/14*sqrt(7)*log((x^4 + 9*x^2 + 2*sqrt(7)*(x^3 + x) + 1)/(x^4 - 5*x^2 + 1))

Sympy [A] time = 0.101374, size = 39, normalized size = 0.85

$$-\frac{\sqrt{7}\log(x^2 - \sqrt{7}x + 1)}{14} + \frac{\sqrt{7}\log(x^2 + \sqrt{7}x + 1)}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x**2+1)/(x**4-5*x**2+1),x)

[Out] -sqrt(7)*log(x**2 - sqrt(7)*x + 1)/14 + sqrt(7)*log(x**2 + sqrt(7)*x + 1)/14

Giac [A] time = 1.14646, size = 53, normalized size = 1.15

$$-\frac{1}{14}\sqrt{7}\log\left(\frac{\left|2x - 2\sqrt{7} + \frac{2}{x}\right|}{\left|2x + 2\sqrt{7} + \frac{2}{x}\right|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2+1)/(x^4-5*x^2+1),x, algorithm="giac")

[Out] -1/14*sqrt(7)*log(abs(2*x - 2*sqrt(7) + 2/x)/abs(2*x + 2*sqrt(7) + 2/x))

$$3.92 \quad \int -\frac{1+3x^2}{1+2x^2+9x^4} dx$$

Optimal. Leaf size=43

$$\frac{\tan^{-1}\left(\frac{1-3x}{\sqrt{2}}\right)}{2\sqrt{2}} - \frac{\tan^{-1}\left(\frac{3x+1}{\sqrt{2}}\right)}{2\sqrt{2}}$$

[Out] ArcTan[(1 - 3*x)/Sqrt[2]]/(2*Sqrt[2]) - ArcTan[(1 + 3*x)/Sqrt[2]]/(2*Sqrt[2])

Rubi [A] time = 0.0348875, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.13$, Rules used = {1161, 618, 204}

$$\frac{\tan^{-1}\left(\frac{1-3x}{\sqrt{2}}\right)}{2\sqrt{2}} - \frac{\tan^{-1}\left(\frac{3x+1}{\sqrt{2}}\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[-((1 + 3*x^2)/(1 + 2*x^2 + 9*x^4)), x]

[Out] ArcTan[(1 - 3*x)/Sqrt[2]]/(2*Sqrt[2]) - ArcTan[(1 + 3*x)/Sqrt[2]]/(2*Sqrt[2])

Rule 1161

Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (GtQ[(2*d)/e - b/c, 0] || (!LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2], 0]))

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned} \int -\frac{1+3x^2}{1+2x^2+9x^4} dx &= -\left(\frac{1}{6} \int \frac{1}{\frac{1}{3}-\frac{2x}{3}+x^2} dx\right) - \frac{1}{6} \int \frac{1}{\frac{1}{3}+\frac{2x}{3}+x^2} dx \\ &= \frac{1}{3} \text{Subst}\left(\int \frac{1}{-\frac{8}{9}-x^2} dx, x, -\frac{2}{3}+2x\right) + \frac{1}{3} \text{Subst}\left(\int \frac{1}{-\frac{8}{9}-x^2} dx, x, \frac{2}{3}+2x\right) \\ &= \frac{\tan^{-1}\left(\frac{1-3x}{\sqrt{2}}\right)}{2\sqrt{2}} - \frac{\tan^{-1}\left(\frac{1+3x}{\sqrt{2}}\right)}{2\sqrt{2}} \end{aligned}$$

Mathematica [C] time = 0.100428, size = 99, normalized size = 2.3

$$-\frac{(\sqrt{2}-i)\tan^{-1}\left(\frac{3x}{\sqrt{1-2i\sqrt{2}}}\right)}{2\sqrt{2}(1-2i\sqrt{2})} - \frac{(\sqrt{2}+i)\tan^{-1}\left(\frac{3x}{\sqrt{1+2i\sqrt{2}}}\right)}{2\sqrt{2}(1+2i\sqrt{2})}$$

Antiderivative was successfully verified.

[In] Integrate[-((1 + 3*x^2)/(1 + 2*x^2 + 9*x^4)), x]

[Out] -((-I + Sqrt[2])*ArcTan[(3*x)/Sqrt[1 - (2*I)*Sqrt[2]]])/(2*Sqrt[2*(1 - (2*I)*Sqrt[2])]) - ((I + Sqrt[2])*ArcTan[(3*x)/Sqrt[1 + (2*I)*Sqrt[2]]])/(2*Sqrt[2*(1 + (2*I)*Sqrt[2])])

Maple [A] time = 0.045, size = 34, normalized size = 0.8

$$-\frac{\sqrt{2}}{4} \arctan\left(\frac{(6x-2)\sqrt{2}}{4}\right) - \frac{\sqrt{2}}{4} \arctan\left(\frac{(6x+2)\sqrt{2}}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-3*x^2-1)/(9*x^4+2*x^2+1), x)

[Out] -1/4*2^(1/2)*arctan(1/4*(6*x-2)*2^(1/2))-1/4*2^(1/2)*arctan(1/4*(6*x+2)*2^(1/2))

Maxima [A] time = 1.45843, size = 45, normalized size = 1.05

$$-\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(3x+1)\right) - \frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(3x-1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-3*x^2-1)/(9*x^4+2*x^2+1), x, algorithm="maxima")

[Out] -1/4*sqrt(2)*arctan(1/2*sqrt(2)*(3*x + 1)) - 1/4*sqrt(2)*arctan(1/2*sqrt(2)*(3*x - 1))

Fricas [A] time = 1.41658, size = 113, normalized size = 2.63

$$-\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{4}\sqrt{2}(9x^3+5x)\right)-\frac{1}{4}\sqrt{2}\arctan\left(\frac{3}{4}\sqrt{2}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-3*x^2-1)/(9*x^4+2*x^2+1),x, algorithm="fricas")

[Out] -1/4*sqrt(2)*arctan(1/4*sqrt(2)*(9*x^3 + 5*x)) - 1/4*sqrt(2)*arctan(3/4*sqrt(2)*x)

Sympy [A] time = 0.11981, size = 46, normalized size = 1.07

$$\frac{\sqrt{2}\left(2\operatorname{atan}\left(\frac{3\sqrt{2}x}{4}\right)+2\operatorname{atan}\left(\frac{9\sqrt{2}x^3}{4}+\frac{5\sqrt{2}x}{4}\right)\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-3*x**2-1)/(9*x**4+2*x**2+1),x)

[Out] -sqrt(2)*(2*atan(3*sqrt(2)*x/4) + 2*atan(9*sqrt(2)*x**3/4 + 5*sqrt(2)*x/4))/8

Giac [A] time = 1.12498, size = 45, normalized size = 1.05

$$-\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(3x+1)\right)-\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(3x-1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-3*x^2-1)/(9*x^4+2*x^2+1),x, algorithm="giac")

[Out] -1/4*sqrt(2)*arctan(1/2*sqrt(2)*(3*x + 1)) - 1/4*sqrt(2)*arctan(1/2*sqrt(2)*(3*x - 1))

$$3.93 \quad \int \frac{1+3x^2}{-1-2x^2-9x^4} dx$$

Optimal. Leaf size=43

$$\frac{\tan^{-1}\left(\frac{1-3x}{\sqrt{2}}\right)}{2\sqrt{2}} - \frac{\tan^{-1}\left(\frac{3x+1}{\sqrt{2}}\right)}{2\sqrt{2}}$$

[Out] ArcTan[(1 - 3*x)/Sqrt[2]]/(2*Sqrt[2]) - ArcTan[(1 + 3*x)/Sqrt[2]]/(2*Sqrt[2])

Rubi [A] time = 0.0323723, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {1161, 618, 204}

$$\frac{\tan^{-1}\left(\frac{1-3x}{\sqrt{2}}\right)}{2\sqrt{2}} - \frac{\tan^{-1}\left(\frac{3x+1}{\sqrt{2}}\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[(1 + 3*x^2)/(-1 - 2*x^2 - 9*x^4),x]

[Out] ArcTan[(1 - 3*x)/Sqrt[2]]/(2*Sqrt[2]) - ArcTan[(1 + 3*x)/Sqrt[2]]/(2*Sqrt[2])

Rule 1161

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[(2*d)/e - b/c, 0] || (!LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[In
t[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{1+3x^2}{-1-2x^2-9x^4} dx &= -\left(\frac{1}{6} \int \frac{1}{\frac{1}{3}-\frac{2x}{3}+x^2} dx\right) - \frac{1}{6} \int \frac{1}{\frac{1}{3}+\frac{2x}{3}+x^2} dx \\ &= \frac{1}{3} \text{Subst}\left(\int \frac{1}{-\frac{8}{9}-x^2} dx, x, -\frac{2}{3}+2x\right) + \frac{1}{3} \text{Subst}\left(\int \frac{1}{-\frac{8}{9}-x^2} dx, x, \frac{2}{3}+2x\right) \\ &= \frac{\tan^{-1}\left(\frac{1-3x}{\sqrt{2}}\right)}{2\sqrt{2}} - \frac{\tan^{-1}\left(\frac{1+3x}{\sqrt{2}}\right)}{2\sqrt{2}} \end{aligned}$$

Mathematica [C] time = 0.0329612, size = 99, normalized size = 2.3

$$-\frac{(\sqrt{2}-i)\tan^{-1}\left(\frac{3x}{\sqrt{1-2i\sqrt{2}}}\right)}{2\sqrt{2}(1-2i\sqrt{2})} - \frac{(\sqrt{2}+i)\tan^{-1}\left(\frac{3x}{\sqrt{1+2i\sqrt{2}}}\right)}{2\sqrt{2}(1+2i\sqrt{2})}$$

Antiderivative was successfully verified.

[In] Integrate[(1 + 3*x^2)/(-1 - 2*x^2 - 9*x^4), x]

[Out] -((-I + Sqrt[2])*ArcTan[(3*x)/Sqrt[1 - (2*I)*Sqrt[2]]])/(2*Sqrt[2*(1 - (2*I)*Sqrt[2])]) - ((I + Sqrt[2])*ArcTan[(3*x)/Sqrt[1 + (2*I)*Sqrt[2]]])/(2*Sqrt[2*(1 + (2*I)*Sqrt[2])])

Maple [A] time = 0.043, size = 34, normalized size = 0.8

$$-\frac{\sqrt{2}}{4} \arctan\left(\frac{(6x-2)\sqrt{2}}{4}\right) - \frac{\sqrt{2}}{4} \arctan\left(\frac{(6x+2)\sqrt{2}}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3*x^2+1)/(-9*x^4-2*x^2-1), x)

[Out] -1/4*2^(1/2)*arctan(1/4*(6*x-2)*2^(1/2))-1/4*2^(1/2)*arctan(1/4*(6*x+2)*2^(1/2))

Maxima [A] time = 1.45179, size = 45, normalized size = 1.05

$$-\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(3x+1)\right) - \frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(3x-1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x^2+1)/(-9*x^4-2*x^2-1), x, algorithm="maxima")

[Out] -1/4*sqrt(2)*arctan(1/2*sqrt(2)*(3*x + 1)) - 1/4*sqrt(2)*arctan(1/2*sqrt(2)*(3*x - 1))

Fricas [A] time = 1.45857, size = 113, normalized size = 2.63

$$-\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{4}\sqrt{2}(9x^3+5x)\right)-\frac{1}{4}\sqrt{2}\arctan\left(\frac{3}{4}\sqrt{2}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x^2+1)/(-9*x^4-2*x^2-1),x, algorithm="fricas")

[Out] -1/4*sqrt(2)*arctan(1/4*sqrt(2)*(9*x^3 + 5*x)) - 1/4*sqrt(2)*arctan(3/4*sqrt(2)*x)

Sympy [A] time = 0.123907, size = 46, normalized size = 1.07

$$\frac{\sqrt{2}\left(2\operatorname{atan}\left(\frac{3\sqrt{2}x}{4}\right)+2\operatorname{atan}\left(\frac{9\sqrt{2}x^3}{4}+\frac{5\sqrt{2}x}{4}\right)\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x**2+1)/(-9*x**4-2*x**2-1),x)

[Out] -sqrt(2)*(2*atan(3*sqrt(2)*x/4) + 2*atan(9*sqrt(2)*x**3/4 + 5*sqrt(2)*x/4))/8

Giac [A] time = 1.16648, size = 45, normalized size = 1.05

$$-\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(3x+1)\right)-\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(3x-1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x^2+1)/(-9*x^4-2*x^2-1),x, algorithm="giac")

[Out] -1/4*sqrt(2)*arctan(1/2*sqrt(2)*(3*x + 1)) - 1/4*sqrt(2)*arctan(1/2*sqrt(2)*(3*x - 1))

$$3.94 \quad \int \frac{3+2x^2}{1-2x^2+x^4} dx$$

Optimal. Leaf size=21

$$\frac{5x}{2(1-x^2)} + \frac{1}{2} \tanh^{-1}(x)$$

[Out] (5*x)/(2*(1 - x^2)) + ArcTanh[x]/2

Rubi [A] time = 0.0050243, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.15$, Rules used = {28, 385, 207}

$$\frac{5x}{2(1-x^2)} + \frac{1}{2} \tanh^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[(3 + 2*x^2)/(1 - 2*x^2 + x^4), x]

[Out] (5*x)/(2*(1 - x^2)) + ArcTanh[x]/2

Rule 28

Int[(u_)*((a_) + (c_)*(x_)^(n2_)) + (b_)*(x_)^(n_)]^(p_), x_Symbol] := Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]

Rule 385

Int[((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_)), x_Symbol] := -Simp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n + p, 0])

Rule 207

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rubi steps

$$\begin{aligned} \int \frac{3+2x^2}{1-2x^2+x^4} dx &= \int \frac{3+2x^2}{(-1+x^2)^2} dx \\ &= \frac{5x}{2(1-x^2)} - \frac{1}{2} \int \frac{1}{-1+x^2} dx \\ &= \frac{5x}{2(1-x^2)} + \frac{1}{2} \tanh^{-1}(x) \end{aligned}$$

Mathematica [A] time = 0.009797, size = 27, normalized size = 1.29

$$\frac{1}{4} \left(-\frac{10x}{x^2-1} - \log(1-x) + \log(x+1) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(3 + 2*x^2)/(1 - 2*x^2 + x^4),x]

[Out] ((-10*x)/(-1 + x^2) - Log[1 - x] + Log[1 + x])/4

Maple [A] time = 0.05, size = 28, normalized size = 1.3

$$-\frac{5}{4+4x} + \frac{\ln(1+x)}{4} - \frac{5}{-4+4x} - \frac{\ln(-1+x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*x^2+3)/(x^4-2*x^2+1),x)

[Out] -5/4/(1+x)+1/4*ln(1+x)-5/4/(-1+x)-1/4*ln(-1+x)

Maxima [A] time = 0.988093, size = 31, normalized size = 1.48

$$-\frac{5x}{2(x^2-1)} + \frac{1}{4} \log(x+1) - \frac{1}{4} \log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2+3)/(x^4-2*x^2+1),x, algorithm="maxima")

[Out] -5/2*x/(x^2 - 1) + 1/4*log(x + 1) - 1/4*log(x - 1)

Fricas [B] time = 1.29165, size = 92, normalized size = 4.38

$$\frac{(x^2-1)\log(x+1) - (x^2-1)\log(x-1) - 10x}{4(x^2-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2+3)/(x^4-2*x^2+1),x, algorithm="fricas")

[Out] 1/4*((x^2 - 1)*log(x + 1) - (x^2 - 1)*log(x - 1) - 10*x)/(x^2 - 1)

Sympy [A] time = 0.095955, size = 22, normalized size = 1.05

$$-\frac{5x}{2x^2-2} - \frac{\log(x-1)}{4} + \frac{\log(x+1)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.


```
[In] integrate((2*x**2+3)/(x**4-2*x**2+1),x)
```

```
[Out] -5*x/(2*x**2 - 2) - log(x - 1)/4 + log(x + 1)/4
```

Giac [A] time = 1.16189, size = 34, normalized size = 1.62

$$-\frac{5x}{2(x^2-1)} + \frac{1}{4} \log(|x+1|) - \frac{1}{4} \log(|x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+3)/(x^4-2*x^2+1),x, algorithm="giac")
```

```
[Out] -5/2*x/(x^2 - 1) + 1/4*log(abs(x + 1)) - 1/4*log(abs(x - 1))
```

$$3.95 \quad \int \frac{2+3x^2}{5-8x^2+3x^4} dx$$

Optimal. Leaf size=28

$$\frac{5}{2} \tanh^{-1}(x) - \frac{7}{2} \sqrt{\frac{3}{5}} \tanh^{-1}\left(\sqrt{\frac{3}{5}}x\right)$$

[Out] (5*ArcTanh[x])/2 - (7*Sqrt[3/5]*ArcTanh[Sqrt[3/5]*x])/2

Rubi [A] time = 0.0126262, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1166, 207}

$$\frac{5}{2} \tanh^{-1}(x) - \frac{7}{2} \sqrt{\frac{3}{5}} \tanh^{-1}\left(\sqrt{\frac{3}{5}}x\right)$$

Antiderivative was successfully verified.

[In] Int[(2 + 3*x^2)/(5 - 8*x^2 + 3*x^4),x]

[Out] (5*ArcTanh[x])/2 - (7*Sqrt[3/5]*ArcTanh[Sqrt[3/5]*x])/2

Rule 1166

Int[((d_) + (e_)*(x_)^2)/((a_) + (b_)*(x_)^2 + (c_)*(x_)^4), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]

Rule 207

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rubi steps

$$\begin{aligned} \int \frac{2+3x^2}{5-8x^2+3x^4} dx &= -\left(\frac{15}{2} \int \frac{1}{-3+3x^2} dx\right) + \frac{21}{2} \int \frac{1}{-5+3x^2} dx \\ &= \frac{5}{2} \tanh^{-1}(x) - \frac{7}{2} \sqrt{\frac{3}{5}} \tanh^{-1}\left(\sqrt{\frac{3}{5}}x\right) \end{aligned}$$

Mathematica [A] time = 0.0195733, size = 53, normalized size = 1.89

$$\frac{1}{20} (7\sqrt{15} \log(\sqrt{15} - 3x) - 25 \log(1 - x) + 25 \log(x + 1) - 7\sqrt{15} \log(3x + \sqrt{15}))$$

Antiderivative was successfully verified.

[In] Integrate[(2 + 3*x^2)/(5 - 8*x^2 + 3*x^4),x]

[Out] $(7\sqrt{15}\log[\sqrt{15} - 3x] - 25\log[1 - x] + 25\log[1 + x] - 7\sqrt{15}\log[\sqrt{15} + 3x])/20$

Maple [A] time = 0.045, size = 26, normalized size = 0.9

$$\frac{5 \ln(1+x)}{4} - \frac{5 \ln(-1+x)}{4} - \frac{7\sqrt{15}}{10} \operatorname{Artanh}\left(\frac{x\sqrt{15}}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((3*x^2+2)/(3*x^4-8*x^2+5),x)`

[Out] $5/4*\ln(1+x)-5/4*\ln(-1+x)-7/10*\operatorname{arctanh}(1/5*x*15^{(1/2)})*15^{(1/2)}$

Maxima [B] time = 1.4477, size = 51, normalized size = 1.82

$$\frac{7}{20} \sqrt{15} \log\left(\frac{3x - \sqrt{15}}{3x + \sqrt{15}}\right) + \frac{5}{4} \log(x+1) - \frac{5}{4} \log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((3*x^2+2)/(3*x^4-8*x^2+5),x, algorithm="maxima")`

[Out] $7/20*\sqrt{15}*\log((3*x - \sqrt{15})/(3*x + \sqrt{15})) + 5/4*\log(x + 1) - 5/4*\log(x - 1)$

Fricas [B] time = 1.3812, size = 146, normalized size = 5.21

$$\frac{7}{20} \sqrt{5}\sqrt{3} \log\left(-\frac{2\sqrt{5}\sqrt{3}x - 3x^2 - 5}{3x^2 - 5}\right) + \frac{5}{4} \log(x+1) - \frac{5}{4} \log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((3*x^2+2)/(3*x^4-8*x^2+5),x, algorithm="fricas")`

[Out] $7/20*\sqrt{5}*\sqrt{3}*\log(-(2*\sqrt{5}*\sqrt{3}*x - 3*x^2 - 5)/(3*x^2 - 5)) + 5/4*\log(x + 1) - 5/4*\log(x - 1)$

Sympy [B] time = 0.489878, size = 53, normalized size = 1.89

$$-\frac{5 \log(x-1)}{4} + \frac{5 \log(x+1)}{4} + \frac{7\sqrt{15} \log\left(x - \frac{\sqrt{15}}{3}\right)}{20} - \frac{7\sqrt{15} \log\left(x + \frac{\sqrt{15}}{3}\right)}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((3*x**2+2)/(3*x**4-8*x**2+5),x)`

[Out] $-5 \cdot \log(x - 1)/4 + 5 \cdot \log(x + 1)/4 + 7 \cdot \sqrt{15} \cdot \log(x - \sqrt{15}/3)/20 - 7 \cdot \sqrt{15} \cdot \log(x + \sqrt{15}/3)/20$

Giac [B] time = 1.10018, size = 59, normalized size = 2.11

$$\frac{7}{20} \sqrt{15} \log \left(\left| \frac{6x - 2\sqrt{15}}{6x + 2\sqrt{15}} \right| \right) + \frac{5}{4} \log(|x + 1|) - \frac{5}{4} \log(|x - 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((3*x^2+2)/(3*x^4-8*x^2+5),x, algorithm="giac")`

[Out] $7/20 \cdot \sqrt{15} \cdot \log(\text{abs}(6 \cdot x - 2 \cdot \sqrt{15})/\text{abs}(6 \cdot x + 2 \cdot \sqrt{15})) + 5/4 \cdot \log(\text{abs}(x + 1)) - 5/4 \cdot \log(\text{abs}(x - 1))$

$$3.96 \quad \int \frac{d+ex^2}{5-8x^2+3x^4} dx$$

Optimal. Leaf size=36

$$\frac{1}{2}(d+e) \tanh^{-1}(x) - \frac{(3d+5e) \tanh^{-1}\left(\sqrt{\frac{3}{5}}x\right)}{2\sqrt{15}}$$

[Out] ((d + e)*ArcTanh[x])/2 - ((3*d + 5*e)*ArcTanh[Sqrt[3/5]*x])/(2*Sqrt[15])

Rubi [A] time = 0.0399891, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1166, 207}

$$\frac{1}{2}(d+e) \tanh^{-1}(x) - \frac{(3d+5e) \tanh^{-1}\left(\sqrt{\frac{3}{5}}x\right)}{2\sqrt{15}}$$

Antiderivative was successfully verified.

[In] Int[(d + e*x^2)/(5 - 8*x^2 + 3*x^4), x]

[Out] ((d + e)*ArcTanh[x])/2 - ((3*d + 5*e)*ArcTanh[Sqrt[3/5]*x])/(2*Sqrt[15])

Rule 1166

Int[((d_) + (e_)*(x_)^2)/((a_) + (b_)*(x_)^2 + (c_)*(x_)^4), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]

Rule 207

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rubi steps

$$\begin{aligned} \int \frac{d+ex^2}{5-8x^2+3x^4} dx &= -\left(\frac{1}{2}(3(d+e)) \int \frac{1}{-3+3x^2} dx\right) + \frac{1}{2}(3d+5e) \int \frac{1}{-5+3x^2} dx \\ &= \frac{1}{2}(d+e) \tanh^{-1}(x) - \frac{(3d+5e) \tanh^{-1}\left(\sqrt{\frac{3}{5}}x\right)}{2\sqrt{15}} \end{aligned}$$

Mathematica [A] time = 0.0399562, size = 72, normalized size = 2.

$$\frac{1}{60} \left(\sqrt{15}(3d+5e) \log(\sqrt{15}-3x) - 15(d+e) \log(1-x) + 15(d+e) \log(x+1) - \sqrt{15}(3d+5e) \log(3x+\sqrt{15}) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(d + e*x^2)/(5 - 8*x^2 + 3*x^4), x]

[Out] (Sqrt[15]*(3*d + 5*e)*Log[Sqrt[15] - 3*x] - 15*(d + e)*Log[1 - x] + 15*(d + e)*Log[1 + x] - Sqrt[15]*(3*d + 5*e)*Log[Sqrt[15] + 3*x])/60

Maple [B] time = 0.049, size = 56, normalized size = 1.6

$$\frac{\ln(1+x)d}{4} + \frac{\ln(1+x)e}{4} - \frac{\ln(-1+x)d}{4} - \frac{\ln(-1+x)e}{4} - \frac{\sqrt{15}d}{10} \operatorname{Artanh}\left(\frac{x\sqrt{15}}{5}\right) - \frac{\sqrt{15}e}{6} \operatorname{Artanh}\left(\frac{x\sqrt{15}}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x^2+d)/(3*x^4-8*x^2+5), x)

[Out] 1/4*ln(1+x)*d+1/4*ln(1+x)*e-1/4*ln(-1+x)*d-1/4*ln(-1+x)*e-1/10*15^(1/2)*arc tanh(1/5*x*15^(1/2))*d-1/6*15^(1/2)*arctanh(1/5*x*15^(1/2))*e

Maxima [A] time = 1.48087, size = 69, normalized size = 1.92

$$\frac{1}{60} \sqrt{15}(3d + 5e) \log\left(\frac{3x - \sqrt{15}}{3x + \sqrt{15}}\right) + \frac{1}{4} (d + e) \log(x + 1) - \frac{1}{4} (d + e) \log(x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)/(3*x^4-8*x^2+5), x, algorithm="maxima")

[Out] 1/60*sqrt(15)*(3*d + 5*e)*log((3*x - sqrt(15))/(3*x + sqrt(15))) + 1/4*(d + e)*log(x + 1) - 1/4*(d + e)*log(x - 1)

Fricas [B] time = 1.467, size = 163, normalized size = 4.53

$$\frac{1}{60} \sqrt{15}(3d + 5e) \log\left(\frac{3x^2 - 2\sqrt{15}x + 5}{3x^2 - 5}\right) + \frac{1}{4} (d + e) \log(x + 1) - \frac{1}{4} (d + e) \log(x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)/(3*x^4-8*x^2+5), x, algorithm="fricas")

[Out] 1/60*sqrt(15)*(3*d + 5*e)*log((3*x^2 - 2*sqrt(15)*x + 5)/(3*x^2 - 5)) + 1/4*(d + e)*log(x + 1) - 1/4*(d + e)*log(x - 1)

Sympy [B] time = 1.04993, size = 474, normalized size = 13.17

$$\frac{(d + e) \log\left(x + \frac{-51d^3(d+e) - 180d^2e(d+e) - 225de^2(d+e) + 60d(d+e)^3 - 100e^3(d+e) + 75e(d+e)^3}{9d^4 + 24d^3e - 40de^3 - 25e^4}\right)}{4} - \frac{(d + e) \log\left(x + \frac{51d^3(d+e) + 180d^2e(d+e) + 225de^2(d+e) - 60d(d+e)^3 + 100e^3(d+e) - 75e(d+e)^3}{9d^4 + 24d^3e - 40de^3 - 25e^4}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x**2+d)/(3*x**4-8*x**2+5), x)

```
[Out] (d + e)*log(x + (-51*d**3*(d + e) - 180*d**2*e*(d + e) - 225*d*e**2*(d + e)
+ 60*d*(d + e)**3 - 100*e**3*(d + e) + 75*e*(d + e)**3)/(9*d**4 + 24*d**3*
e - 40*d*e**3 - 25*e**4))/4 - (d + e)*log(x + (51*d**3*(d + e) + 180*d**2*e
*(d + e) + 225*d*e**2*(d + e) - 60*d*(d + e)**3 + 100*e**3*(d + e) - 75*e*(
d + e)**3)/(9*d**4 + 24*d**3*e - 40*d*e**3 - 25*e**4))/4 + sqrt(15)*(3*d +
5*e)*log(x + (-17*sqrt(15)*d**3*(3*d + 5*e)/5 - 12*sqrt(15)*d**2*e*(3*d + 5
*e) - 15*sqrt(15)*d*e**2*(3*d + 5*e) + 4*sqrt(15)*d*(3*d + 5*e)**3/15 - 20*
sqrt(15)*e**3*(3*d + 5*e)/3 + sqrt(15)*e*(3*d + 5*e)**3/3)/(9*d**4 + 24*d**
3*e - 40*d*e**3 - 25*e**4))/60 - sqrt(15)*(3*d + 5*e)*log(x + (17*sqrt(15)*
d**3*(3*d + 5*e)/5 + 12*sqrt(15)*d**2*e*(3*d + 5*e) + 15*sqrt(15)*d*e**2*(3
*d + 5*e) - 4*sqrt(15)*d*(3*d + 5*e)**3/15 + 20*sqrt(15)*e**3*(3*d + 5*e)/3
- sqrt(15)*e*(3*d + 5*e)**3/3)/(9*d**4 + 24*d**3*e - 40*d*e**3 - 25*e**4))
/60
```

Giac [B] time = 1.10593, size = 81, normalized size = 2.25

$$\frac{1}{60} \sqrt{15}(3d + 5e) \log \left(\frac{|6x - 2\sqrt{15}|}{|6x + 2\sqrt{15}|} \right) + \frac{1}{4}(d + e) \log(|x + 1|) - \frac{1}{4}(d + e) \log(|x - 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(3*x^4-8*x^2+5),x, algorithm="giac")
```

```
[Out] 1/60*sqrt(15)*(3*d + 5*e)*log(abs(6*x - 2*sqrt(15))/abs(6*x + 2*sqrt(15)))
+ 1/4*(d + e)*log(abs(x + 1)) - 1/4*(d + e)*log(abs(x - 1))
```

$$3.97 \quad \int \frac{3+x^2}{1+3x^2+x^4} dx$$

Optimal. Leaf size=74

$$\frac{(3 + \sqrt{5})^{3/2} \tan^{-1}\left(\sqrt{\frac{1}{2}}(3 + \sqrt{5})x\right)}{2\sqrt{10}} - \frac{1}{10}\sqrt{180 - 80\sqrt{5}} \tan^{-1}\left(\sqrt{\frac{2}{3 + \sqrt{5}}}x\right)$$

[Out] -(Sqrt[180 - 80*Sqrt[5]]*ArcTan[Sqrt[2/(3 + Sqrt[5]])*x])/10 + ((3 + Sqrt[5])^(3/2)*ArcTan[Sqrt[(3 + Sqrt[5])/2]*x])/(2*Sqrt[10])

Rubi [A] time = 0.0450255, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1166, 203}

$$\frac{(3 + \sqrt{5})^{3/2} \tan^{-1}\left(\sqrt{\frac{1}{2}}(3 + \sqrt{5})x\right)}{2\sqrt{10}} - \frac{1}{10}\sqrt{180 - 80\sqrt{5}} \tan^{-1}\left(\sqrt{\frac{2}{3 + \sqrt{5}}}x\right)$$

Antiderivative was successfully verified.

[In] Int[(3 + x^2)/(1 + 3*x^2 + x^4), x]

[Out] -(Sqrt[180 - 80*Sqrt[5]]*ArcTan[Sqrt[2/(3 + Sqrt[5]])*x])/10 + ((3 + Sqrt[5])^(3/2)*ArcTan[Sqrt[(3 + Sqrt[5])/2]*x])/(2*Sqrt[10])

Rule 1166

Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[Rt[b, 2]*x]/Rt[a, 2])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rubi steps

$$\begin{aligned} \int \frac{3+x^2}{1+3x^2+x^4} dx &= \frac{1}{10}(5-3\sqrt{5}) \int \frac{1}{\frac{3}{2} + \frac{\sqrt{5}}{2} + x^2} dx + \frac{1}{10}(5+3\sqrt{5}) \int \frac{1}{\frac{3}{2} - \frac{\sqrt{5}}{2} + x^2} dx \\ &= -\frac{1}{5}\sqrt{45-20\sqrt{5}} \tan^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}x\right) + \frac{(3+\sqrt{5})^{3/2} \tan^{-1}\left(\sqrt{\frac{1}{2}}(3+\sqrt{5})x\right)}{2\sqrt{10}} \end{aligned}$$

Mathematica [A] time = 0.099855, size = 73, normalized size = 0.99

$$\frac{(3 + \sqrt{5})^{3/2} \tan^{-1}\left(\sqrt{\frac{1}{2}}(3 + \sqrt{5})x\right) - (3 - \sqrt{5})^{3/2} \tan^{-1}\left(\sqrt{\frac{2}{3 + \sqrt{5}}}x\right)}{2\sqrt{10}}$$

Antiderivative was successfully verified.

[In] Integrate[(3 + x^2)/(1 + 3*x^2 + x^4), x]

[Out] $-\left(\left(3 - \sqrt{5}\right)^{3/2} \operatorname{ArcTan}\left[\sqrt{2/(3 + \sqrt{5})}\right] x\right) + \left(3 + \sqrt{5}\right)^{3/2} \operatorname{ArcTan}\left[\sqrt{(3 + \sqrt{5})/2}\right] x\right) / (2\sqrt{10})$

Maple [B] time = 0.064, size = 104, normalized size = 1.4

$$2 \frac{1}{-2 + 2\sqrt{5}} \arctan\left(4 \frac{x}{-2 + 2\sqrt{5}}\right) + \frac{6\sqrt{5}}{-10 + 10\sqrt{5}} \arctan\left(4 \frac{x}{-2 + 2\sqrt{5}}\right) + 2 \frac{1}{2 + 2\sqrt{5}} \arctan\left(4 \frac{x}{2 + 2\sqrt{5}}\right) - \frac{6}{10 + 10\sqrt{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2+3)/(x^4+3*x^2+1), x)

[Out] $2/(-2+2*5^{(1/2)})*\arctan(4*x/(-2+2*5^{(1/2)}))+6/5*5^{(1/2)/(-2+2*5^{(1/2)})}*\arctan(4*x/(-2+2*5^{(1/2)}))+2/(2+2*5^{(1/2)})*\arctan(4*x/(2+2*5^{(1/2)}))-6/5*5^{(1/2)/(2+2*5^{(1/2)})}*\arctan(4*x/(2+2*5^{(1/2)}))$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 + 3}{x^4 + 3x^2 + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+3)/(x^4+3*x^2+1), x, algorithm="maxima")

[Out] integrate((x^2 + 3)/(x^4 + 3*x^2 + 1), x)

Fricas [B] time = 1.54818, size = 421, normalized size = 5.69

$$\frac{2}{5} \sqrt{5} \sqrt{-4\sqrt{5} + 9} \arctan\left(\frac{1}{4} \sqrt{2x^2 + \sqrt{5} + 3} (\sqrt{5}\sqrt{2} + 3\sqrt{2}) \sqrt{-4\sqrt{5} + 9} - \frac{1}{2} (\sqrt{5}x + 3x) \sqrt{-4\sqrt{5} + 9}\right) + \frac{2}{5} \sqrt{5} \sqrt{4\sqrt{5} + 9} \arctan\left(\frac{1}{4} \sqrt{2x^2 + \sqrt{5} + 3} (\sqrt{5}\sqrt{2} + 3\sqrt{2}) \sqrt{4\sqrt{5} + 9} - \frac{1}{2} (\sqrt{5}x + 3x) \sqrt{4\sqrt{5} + 9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+3)/(x^4+3*x^2+1), x, algorithm="fricas")

[Out] $2/5*\sqrt{5}*\sqrt{-4*\sqrt{5} + 9}*\arctan(1/4*\sqrt{2*x^2 + \sqrt{5} + 3}*(\sqrt{5}*\sqrt{2} + 3*\sqrt{2})*\sqrt{-4*\sqrt{5} + 9} - 1/2*(\sqrt{5}*x + 3*x)*\sqrt{-4*\sqrt{5} + 9}) + 2/5*\sqrt{5}*\sqrt{4*\sqrt{5} + 9}*\arctan(1/4*(\sqrt{2*x^2 + \sqrt{5} + 3}*(\sqrt{5}*\sqrt{2} + 3*\sqrt{2}) - 2*\sqrt{5}*x + 6*x)*\sqrt{4*\sqrt{5} + 9})$

Sympy [A] time = 0.165928, size = 46, normalized size = 0.62

$$2 \left(\frac{\sqrt{5}}{5} + \frac{1}{2}\right) \operatorname{atan}\left(\frac{2x}{-1 + \sqrt{5}}\right) - 2 \left(\frac{1}{2} - \frac{\sqrt{5}}{5}\right) \operatorname{atan}\left(\frac{2x}{1 + \sqrt{5}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**2+3)/(x**4+3*x**2+1),x)

[Out] 2*(sqrt(5)/5 + 1/2)*atan(2*x/(-1 + sqrt(5))) - 2*(1/2 - sqrt(5)/5)*atan(2*x/(1 + sqrt(5)))

Giac [A] time = 1.12001, size = 55, normalized size = 0.74

$$\frac{1}{5}(2\sqrt{5}-5)\arctan\left(\frac{2x}{\sqrt{5}+1}\right) + \frac{1}{5}(2\sqrt{5}+5)\arctan\left(\frac{2x}{\sqrt{5}-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+3)/(x^4+3*x^2+1),x, algorithm="giac")

[Out] 1/5*(2*sqrt(5) - 5)*arctan(2*x/(sqrt(5) + 1)) + 1/5*(2*sqrt(5) + 5)*arctan(2*x/(sqrt(5) - 1))

$$3.98 \quad \int \frac{a+bx^2}{1+x^2+x^4} dx$$

Optimal. Leaf size=83

$$-\frac{1}{4}(a-b)\log(x^2-x+1) + \frac{1}{4}(a-b)\log(x^2+x+1) - \frac{(a+b)\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}} + \frac{(a+b)\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{2\sqrt{3}}$$

[Out] $-\frac{(a+b)\text{ArcTan}[(1-2x)/\text{Sqrt}[3]]}{2\text{Sqrt}[3]} + \frac{(a+b)\text{ArcTan}[(1+2x)/\text{Sqrt}[3]]}{2\text{Sqrt}[3]} - \frac{(a-b)\text{Log}[1-x+x^2]}{4} + \frac{(a-b)\text{Log}[1+x+x^2]}{4}$

Rubi [A] time = 0.0548068, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {1169, 634, 618, 204, 628}

$$-\frac{1}{4}(a-b)\log(x^2-x+1) + \frac{1}{4}(a-b)\log(x^2+x+1) - \frac{(a+b)\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}} + \frac{(a+b)\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x^2)/(1 + x^2 + x^4), x]

[Out] $-\frac{(a+b)\text{ArcTan}[(1-2x)/\text{Sqrt}[3]]}{2\text{Sqrt}[3]} + \frac{(a+b)\text{ArcTan}[(1+2x)/\text{Sqrt}[3]]}{2\text{Sqrt}[3]} - \frac{(a-b)\text{Log}[1-x+x^2]}{4} + \frac{(a-b)\text{Log}[1+x+x^2]}{4}$

Rule 1169

Int[((d_) + (e_)*(x_)^2)/((a_) + (b_)*(x_)^2 + (c_)*(x_)^4), x_Symbol] :> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r + (d - e*q)*x)/(q + r*x + x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]

Rule 634

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] :> Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 618

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 204

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{a + bx^2}{1 + x^2 + x^4} dx &= \frac{1}{2} \int \frac{a - (a - b)x}{1 - x + x^2} dx + \frac{1}{2} \int \frac{a + (a - b)x}{1 + x + x^2} dx \\ &= \frac{1}{4}(a - b) \int \frac{1 + 2x}{1 + x + x^2} dx + \frac{1}{4}(-a + b) \int \frac{-1 + 2x}{1 - x + x^2} dx + \frac{1}{4}(a + b) \int \frac{1}{1 - x + x^2} dx + \frac{1}{4}(a + b) \int \frac{1}{1 + x + x^2} dx \\ &= -\frac{1}{4}(a - b) \log(1 - x + x^2) + \frac{1}{4}(a - b) \log(1 + x + x^2) + \frac{1}{2}(-a - b) \operatorname{Subst}\left(\int \frac{1}{-3 - x^2} dx, x, -1 + 2x\right) \\ &= -\frac{(a + b) \tan^{-1}\left(\frac{1 - 2x}{\sqrt{3}}\right)}{2\sqrt{3}} + \frac{(a + b) \tan^{-1}\left(\frac{1 + 2x}{\sqrt{3}}\right)}{2\sqrt{3}} - \frac{1}{4}(a - b) \log(1 - x + x^2) + \frac{1}{4}(a - b) \log(1 + x + x^2) \end{aligned}$$

Mathematica [C] time = 0.1246, size = 97, normalized size = 1.17

$$\frac{(2ia + (\sqrt{3} - i)b) \tan^{-1}\left(\frac{1}{2}(\sqrt{3} - i)x\right)}{\sqrt{6 + 6i\sqrt{3}}} + \frac{((\sqrt{3} + i)b - 2ia) \tan^{-1}\left(\frac{1}{2}(\sqrt{3} + i)x\right)}{\sqrt{6 - 6i\sqrt{3}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^2)/(1 + x^2 + x^4), x]
```

```
[Out] (((2*I)*a + (-I + Sqrt[3])*b)*ArcTan[((-I + Sqrt[3])*x)/2])/Sqrt[6 + (6*I)*
Sqrt[3]] + (((-2*I)*a + (I + Sqrt[3])*b)*ArcTan[((I + Sqrt[3])*x)/2])/Sqrt[
6 - (6*I)*Sqrt[3]]
```

Maple [A] time = 0.047, size = 114, normalized size = 1.4

$$-\frac{\ln(x^2 - x + 1)a}{4} + \frac{\ln(x^2 - x + 1)b}{4} + \frac{a\sqrt{3}}{6} \arctan\left(\frac{(2x - 1)\sqrt{3}}{3}\right) + \frac{\sqrt{3}b}{6} \arctan\left(\frac{(2x - 1)\sqrt{3}}{3}\right) + \frac{\ln(x^2 + x + 1)a}{4} - \frac{\ln(x^2 + x + 1)b}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)/(x^4+x^2+1), x)
```

```
[Out] -1/4*ln(x^2-x+1)*a+1/4*ln(x^2-x+1)*b+1/6*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))
)*a+1/6*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))*b+1/4*ln(x^2+x+1)*a-1/4*ln(x^2+
x+1)*b+1/6*3^(1/2)*arctan(1/3*(1+2*x)*3^(1/2))*a+1/6*3^(1/2)*arctan(1/3*(1+
2*x)*3^(1/2))*b
```

Maxima [A] time = 1.4756, size = 93, normalized size = 1.12

$$\frac{1}{6} \sqrt{3}(a + b) \arctan\left(\frac{1}{3} \sqrt{3}(2x + 1)\right) + \frac{1}{6} \sqrt{3}(a + b) \arctan\left(\frac{1}{3} \sqrt{3}(2x - 1)\right) + \frac{1}{4}(a - b) \log(x^2 + x + 1) - \frac{1}{4}(a - b) \log(x^2 - x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^2+a)/(x^4+x^2+1),x, algorithm="maxima")

[Out] $\frac{1}{6}\sqrt{3}(a+b)\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) + \frac{1}{6}\sqrt{3}(a+b)\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) + \frac{1}{4}(a-b)\log(x^2+x+1) - \frac{1}{4}(a-b)\log(x^2-x+1)$

Fricas [A] time = 1.39857, size = 223, normalized size = 2.69

$$\frac{1}{6}\sqrt{3}(a+b)\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) + \frac{1}{6}\sqrt{3}(a+b)\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) + \frac{1}{4}(a-b)\log(x^2+x+1) - \frac{1}{4}(a-b)\log(x^2-x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^2+a)/(x^4+x^2+1),x, algorithm="fricas")

[Out] $\frac{1}{6}\sqrt{3}(a+b)\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) + \frac{1}{6}\sqrt{3}(a+b)\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) + \frac{1}{4}(a-b)\log(x^2+x+1) - \frac{1}{4}(a-b)\log(x^2-x+1)$

Sympy [C] time = 0.885689, size = 740, normalized size = 8.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**2+a)/(x**4+x**2+1),x)

[Out] $(-a/4 + b/4 - \sqrt{3}I(a+b)/12)\log(x + (2a**3*(-a/4 + b/4 - \sqrt{3}I(a+b)/12) + 6a**2*b*(-a/4 + b/4 - \sqrt{3}I(a+b)/12) - 12a*b**2*(-a/4 + b/4 - \sqrt{3}I(a+b)/12) + 24a*(-a/4 + b/4 - \sqrt{3}I(a+b)/12)**3 + 2b**3*(-a/4 + b/4 - \sqrt{3}I(a+b)/12) - 48b*(-a/4 + b/4 - \sqrt{3}I(a+b)/12)**3)/(a**4 - a**3*b + a*b**3 - b**4) + (-a/4 + b/4 + \sqrt{3}I(a+b)/12)\log(x + (2a**3*(-a/4 + b/4 + \sqrt{3}I(a+b)/12) + 6a**2*b*(-a/4 + b/4 + \sqrt{3}I(a+b)/12) - 12a*b**2*(-a/4 + b/4 + \sqrt{3}I(a+b)/12) + 24a*(-a/4 + b/4 + \sqrt{3}I(a+b)/12)**3 + 2b**3*(-a/4 + b/4 + \sqrt{3}I(a+b)/12) - 48b*(-a/4 + b/4 + \sqrt{3}I(a+b)/12)**3)/(a**4 - a**3*b + a*b**3 - b**4) + (a/4 - b/4 - \sqrt{3}I(a+b)/12)\log(x + (2a**3*(a/4 - b/4 - \sqrt{3}I(a+b)/12) + 6a**2*b*(a/4 - b/4 - \sqrt{3}I(a+b)/12) - 12a*b**2*(a/4 - b/4 - \sqrt{3}I(a+b)/12) + 24a*(a/4 - b/4 - \sqrt{3}I(a+b)/12)**3 + 2b**3*(a/4 - b/4 - \sqrt{3}I(a+b)/12) - 48b*(a/4 - b/4 - \sqrt{3}I(a+b)/12)**3)/(a**4 - a**3*b + a*b**3 - b**4) + (a/4 - b/4 + \sqrt{3}I(a+b)/12)\log(x + (2a**3*(a/4 - b/4 + \sqrt{3}I(a+b)/12) + 6a**2*b*(a/4 - b/4 + \sqrt{3}I(a+b)/12) - 12a*b**2*(a/4 - b/4 + \sqrt{3}I(a+b)/12) + 24a*(a/4 - b/4 + \sqrt{3}I(a+b)/12)**3 + 2b**3*(a/4 - b/4 + \sqrt{3}I(a+b)/12) - 48b*(a/4 - b/4 + \sqrt{3}I(a+b)/12)**3)/(a**4 - a**3*b + a*b**3 - b**4)$

Giac [A] time = 1.12958, size = 93, normalized size = 1.12

$$\frac{1}{6}\sqrt{3}(a+b)\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) + \frac{1}{6}\sqrt{3}(a+b)\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) + \frac{1}{4}(a-b)\log(x^2+x+1) - \frac{1}{4}(a-b)\log(x^2-x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(x^4+x^2+1),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(3)*(a + b)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6*sqrt(3)*(a + b)*arc  
tan(1/3*sqrt(3)*(2*x - 1)) + 1/4*(a - b)*log(x^2 + x + 1) - 1/4*(a - b)*log  
(x^2 - x + 1)
```

$$3.99 \quad \int \frac{a+bx^2}{(1+x^2+x^4)^2} dx$$

Optimal. Leaf size=119

$$\frac{x(x^2-(a-2b))+a+b}{6(x^4+x^2+1)} - \frac{1}{8}(2a-b)\log(x^2-x+1) + \frac{1}{8}(2a-b)\log(x^2+x+1) - \frac{(4a+b)\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{12\sqrt{3}} + \frac{(4a+b)}{12\sqrt{3}}$$

[Out] (x*(a + b - (a - 2*b)*x^2))/(6*(1 + x^2 + x^4)) - ((4*a + b)*ArcTan[(1 - 2*x)/Sqrt[3]])/(12*Sqrt[3]) + ((4*a + b)*ArcTan[(1 + 2*x)/Sqrt[3]])/(12*Sqrt[3]) - ((2*a - b)*Log[1 - x + x^2])/8 + ((2*a - b)*Log[1 + x + x^2])/8

Rubi [A] time = 0.0910057, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1178, 1169, 634, 618, 204, 628}

$$\frac{x(x^2-(a-2b))+a+b}{6(x^4+x^2+1)} - \frac{1}{8}(2a-b)\log(x^2-x+1) + \frac{1}{8}(2a-b)\log(x^2+x+1) - \frac{(4a+b)\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{12\sqrt{3}} + \frac{(4a+b)}{12\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x^2)/(1 + x^2 + x^4)^2,x]

[Out] (x*(a + b - (a - 2*b)*x^2))/(6*(1 + x^2 + x^4)) - ((4*a + b)*ArcTan[(1 - 2*x)/Sqrt[3]])/(12*Sqrt[3]) + ((4*a + b)*ArcTan[(1 + 2*x)/Sqrt[3]])/(12*Sqrt[3]) - ((2*a - b)*Log[1 - x + x^2])/8 + ((2*a - b)*Log[1 + x + x^2])/8

Rule 1178

Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && IntegerQ[2*p]

Rule 1169

Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r + (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]

Rule 634

Int[((d_) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{a + bx^2}{(1 + x^2 + x^4)^2} dx &= \frac{x(a + b - (a - 2b)x^2)}{6(1 + x^2 + x^4)} + \frac{1}{6} \int \frac{5a - b + (-a + 2b)x^2}{1 + x^2 + x^4} dx \\ &= \frac{x(a + b - (a - 2b)x^2)}{6(1 + x^2 + x^4)} + \frac{1}{12} \int \frac{5a - b - (6a - 3b)x}{1 - x + x^2} dx + \frac{1}{12} \int \frac{5a - b + (6a - 3b)x}{1 + x + x^2} dx \\ &= \frac{x(a + b - (a - 2b)x^2)}{6(1 + x^2 + x^4)} + \frac{1}{8}(2a - b) \int \frac{1 + 2x}{1 + x + x^2} dx + \frac{1}{8}(-2a + b) \int \frac{-1 + 2x}{1 - x + x^2} dx + \frac{1}{24}(4a + b) \int \frac{1}{1 + x + x^2} dx \\ &= \frac{x(a + b - (a - 2b)x^2)}{6(1 + x^2 + x^4)} - \frac{1}{8}(2a - b) \log(1 - x + x^2) + \frac{1}{8}(2a - b) \log(1 + x + x^2) + \frac{1}{12}(-4a - b) \int \frac{1}{1 + x + x^2} dx \\ &= \frac{x(a + b - (a - 2b)x^2)}{6(1 + x^2 + x^4)} - \frac{(4a + b) \tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{12\sqrt{3}} + \frac{(4a + b) \tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{12\sqrt{3}} - \frac{1}{8}(2a - b) \log(1 - x + x^2) \end{aligned}$$

Mathematica [C] time = 0.243995, size = 147, normalized size = 1.24

$$\frac{x(-ax^2 + a + 2bx^2 + b)}{6(x^4 + x^2 + 1)} - \frac{((\sqrt{3} - 11i)a - 2(\sqrt{3} - 2i)b) \tan^{-1}\left(\frac{1}{2}(\sqrt{3} - i)x\right)}{6\sqrt{6 + 6i\sqrt{3}}} - \frac{((\sqrt{3} + 11i)a - 2(\sqrt{3} + 2i)b) \tan^{-1}\left(\frac{1}{2}(\sqrt{3} + i)x\right)}{6\sqrt{6 - 6i\sqrt{3}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^2)/(1 + x^2 + x^4)^2, x]
```

```
[Out] (x*(a + b - a*x^2 + 2*b*x^2))/(6*(1 + x^2 + x^4)) - (((-11*I + Sqrt[3])*a - 2*(-2*I + Sqrt[3])*b)*ArcTan[(-I + Sqrt[3])*x/2])/(6*Sqrt[6 + (6*I)*Sqrt[3]]) - (((11*I + Sqrt[3])*a - 2*(2*I + Sqrt[3])*b)*ArcTan[(I + Sqrt[3])*x/2])/(6*Sqrt[6 - (6*I)*Sqrt[3]])
```

Maple [A] time = 0.056, size = 168, normalized size = 1.4

$$-\frac{1}{4x^2 - 4x + 4} \left(\left(-\frac{2b}{3} + \frac{a}{3} \right) x + \frac{b}{3} - \frac{2a}{3} \right) - \frac{\ln(x^2 - x + 1)a}{4} + \frac{\ln(x^2 - x + 1)b}{8} + \frac{a\sqrt{3}}{9} \arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right) + \frac{\sqrt{3}b}{36} \arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^2+a)/(x^4+x^2+1)^2,x)

[Out] $-1/4 * ((-2/3 * b + 1/3 * a) * x + 1/3 * b - 2/3 * a) / (x^2 - x + 1) - 1/4 * \ln(x^2 - x + 1) * a + 1/8 * \ln(x^2 - x + 1) * b + 1/9 * 3^{1/2} * \arctan(1/3 * (2 * x - 1) * 3^{1/2}) * a + 1/36 * 3^{1/2} * \arctan(1/3 * (2 * x - 1) * 3^{1/2}) * b + 1/4 * ((2/3 * b - 1/3 * a) * x + 1/3 * b - 2/3 * a) / (x^2 + x + 1) + 1/4 * \ln(x^2 + x + 1) * a - 1/8 * \ln(x^2 + x + 1) * b + 1/9 * 3^{1/2} * \arctan(1/3 * (1 + 2 * x) * 3^{1/2}) * a + 1/36 * 3^{1/2} * \arctan(1/3 * (1 + 2 * x) * 3^{1/2}) * b$

Maxima [A] time = 1.46076, size = 142, normalized size = 1.19

$$\frac{1}{36} \sqrt{3}(4a + b) \arctan\left(\frac{1}{3} \sqrt{3}(2x + 1)\right) + \frac{1}{36} \sqrt{3}(4a + b) \arctan\left(\frac{1}{3} \sqrt{3}(2x - 1)\right) + \frac{1}{8}(2a - b) \log(x^2 + x + 1) - \frac{1}{8}(2a - b) \log(x^2 - x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^2+a)/(x^4+x^2+1)^2,x, algorithm="maxima")

[Out] $1/36 * \sqrt{3} * (4 * a + b) * \arctan(1/3 * \sqrt{3} * (2 * x + 1)) + 1/36 * \sqrt{3} * (4 * a + b) * \arctan(1/3 * \sqrt{3} * (2 * x - 1)) + 1/8 * (2 * a - b) * \log(x^2 + x + 1) - 1/8 * (2 * a - b) * \log(x^2 - x + 1) - 1/6 * ((a - 2 * b) * x^3 - (a + b) * x) / (x^4 + x^2 + 1)$

Fricas [A] time = 1.38824, size = 474, normalized size = 3.98

$$12(a - 2b)x^3 - 2\sqrt{3}((4a + b)x^4 + (4a + b)x^2 + 4a + b) \arctan\left(\frac{1}{3}\sqrt{3}(2x + 1)\right) - 2\sqrt{3}((4a + b)x^4 + (4a + b)x^2 + 4a + b) \arctan\left(\frac{1}{3}\sqrt{3}(2x - 1)\right) - 12(a + b)x - 9((2a - b)x^4 + (2a - b)x^2 + 2a - b) \log(x^2 + x + 1) + 9((2a - b)x^4 + (2a - b)x^2 + 2a - b) \log(x^2 - x + 1) / (x^4 + x^2 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^2+a)/(x^4+x^2+1)^2,x, algorithm="fricas")

[Out] $-1/72 * (12 * (a - 2 * b) * x^3 - 2 * \sqrt{3} * ((4 * a + b) * x^4 + (4 * a + b) * x^2 + 4 * a + b) * \arctan(1/3 * \sqrt{3} * (2 * x + 1)) - 2 * \sqrt{3} * ((4 * a + b) * x^4 + (4 * a + b) * x^2 + 4 * a + b) * \arctan(1/3 * \sqrt{3} * (2 * x - 1)) - 12 * (a + b) * x - 9 * ((2 * a - b) * x^4 + (2 * a - b) * x^2 + 2 * a - b) * \log(x^2 + x + 1) + 9 * ((2 * a - b) * x^4 + (2 * a - b) * x^2 + 2 * a - b) * \log(x^2 - x + 1)) / (x^4 + x^2 + 1)$

Sympy [C] time = 1.41187, size = 876, normalized size = 7.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**2+a)/(x**4+x**2+1)**2,x)

[Out] $-(x**3 * (a - 2 * b) + x * (-a - b)) / (6 * x**4 + 6 * x**2 + 6) + (-a/4 + b/8 - \sqrt{3}) * I * (4 * a + b) / 72 * \log(x + (76 * a**3 * (-a/4 + b/8 - \sqrt{3}) * I * (4 * a + b) / 72) + 948 * a**2 * b * (-a/4 + b/8 - \sqrt{3}) * I * (4 * a + b) / 72) - 816 * a * b**2 * (-a/4 + b/8 - \sqrt{3}) * I * (4 * a + b) / 72 + 12096 * a * (-a/4 + b/8 - \sqrt{3}) * I * (4 * a + b) / 72**3 + 148 * b**3 * (-a/4 + b/8 - \sqrt{3}) * I * (4 * a + b) / 72 - 8640 * b * (-a/4 + b/8 - \sqrt{3}) * I * (4 * a + b) / 72**3 / (248 * a**4 - 262 * a**3 * b + 75 * a**2 * b**2 + 11 * a * b**3)$

$$\begin{aligned}
& - 7b^4) + (-a/4 + b/8 + \sqrt{3}I(4a + b)/72) \log(x + (76a^3(-a/4 + b/8 + \sqrt{3}I(4a + b)/72) + 948a^2b(-a/4 + b/8 + \sqrt{3}I(4a + b)/72) - 816ab^2(-a/4 + b/8 + \sqrt{3}I(4a + b)/72) + 12096a(-a/4 + b/8 + \sqrt{3}I(4a + b)/72)^3 + 148b^3(-a/4 + b/8 + \sqrt{3}I(4a + b)/72) - 8640b(-a/4 + b/8 + \sqrt{3}I(4a + b)/72)^3) / (248a^4 - 262a^3b + 75a^2b^2 + 11ab^3 - 7b^4) + (a/4 - b/8 - \sqrt{3}I(4a + b)/72) \log(x + (76a^3(a/4 - b/8 - \sqrt{3}I(4a + b)/72) + 948a^2b(a/4 - b/8 - \sqrt{3}I(4a + b)/72) - 816ab^2(a/4 - b/8 - \sqrt{3}I(4a + b)/72) + 12096a(a/4 - b/8 - \sqrt{3}I(4a + b)/72)^3 + 148b^3(a/4 - b/8 - \sqrt{3}I(4a + b)/72) - 8640b(a/4 - b/8 - \sqrt{3}I(4a + b)/72)^3) / (248a^4 - 262a^3b + 75a^2b^2 + 11ab^3 - 7b^4) + (a/4 - b/8 + \sqrt{3}I(4a + b)/72) \log(x + (76a^3(a/4 - b/8 + \sqrt{3}I(4a + b)/72) + 948a^2b(a/4 - b/8 + \sqrt{3}I(4a + b)/72) - 816ab^2(a/4 - b/8 + \sqrt{3}I(4a + b)/72) + 12096a(a/4 - b/8 + \sqrt{3}I(4a + b)/72)^3 + 148b^3(a/4 - b/8 + \sqrt{3}I(4a + b)/72) - 8640b(a/4 - b/8 + \sqrt{3}I(4a + b)/72)^3) / (248a^4 - 262a^3b + 75a^2b^2 + 11ab^3 - 7b^4)
\end{aligned}$$

Giac [A] time = 1.13813, size = 147, normalized size = 1.24

$$\frac{1}{36} \sqrt{3}(4a + b) \arctan\left(\frac{1}{3} \sqrt{3}(2x + 1)\right) + \frac{1}{36} \sqrt{3}(4a + b) \arctan\left(\frac{1}{3} \sqrt{3}(2x - 1)\right) + \frac{1}{8}(2a - b) \log(x^2 + x + 1) - \frac{1}{8}(2a - b) \log(x^2 - x + 1) - \frac{1}{6}(ax^3 - 2bx^3 - ax - bx)/(x^4 + x^2 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^2+a)/(x^4+x^2+1)^2,x, algorithm="giac")

[Out] 1/36*sqrt(3)*(4*a + b)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/36*sqrt(3)*(4*a + b)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/8*(2*a - b)*log(x^2 + x + 1) - 1/8*(2*a - b)*log(x^2 - x + 1) - 1/6*(a*x^3 - 2*b*x^3 - a*x - b*x)/(x^4 + x^2 + 1)

$$3.100 \quad \int \frac{a+bx^2}{2+x^2+x^4} dx$$

Optimal. Leaf size=234

$$\frac{(a - \sqrt{2}b) \log\left(x^2 - \sqrt{2\sqrt{2}-1}x + \sqrt{2}\right)}{4\sqrt{2}(2\sqrt{2}-1)} + \frac{(a - \sqrt{2}b) \log\left(x^2 + \sqrt{2\sqrt{2}-1}x + \sqrt{2}\right)}{4\sqrt{2}(2\sqrt{2}-1)} - \frac{1}{2} \sqrt{\frac{1}{14}(2\sqrt{2}-1)} (a + \sqrt{2}b) \tan$$

```
[Out] -(Sqrt[(-1 + 2*Sqrt[2])/14]*(a + Sqrt[2]*b)*ArcTan[(Sqrt[-1 + 2*Sqrt[2]] - 2*x)/Sqrt[1 + 2*Sqrt[2]])/2 + (Sqrt[(-1 + 2*Sqrt[2])/14]*(a + Sqrt[2]*b)*ArcTan[(Sqrt[-1 + 2*Sqrt[2]] + 2*x)/Sqrt[1 + 2*Sqrt[2]])/2 - ((a - Sqrt[2]*b)*Log[Sqrt[2] - Sqrt[-1 + 2*Sqrt[2]]*x + x^2])/(4*Sqrt[2*(-1 + 2*Sqrt[2])]) + ((a - Sqrt[2]*b)*Log[Sqrt[2] + Sqrt[-1 + 2*Sqrt[2]]*x + x^2])/(4*Sqrt[2*(-1 + 2*Sqrt[2])])
```

Rubi [A] time = 0.229521, antiderivative size = 234, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {1169, 634, 618, 204, 628}

$$\frac{(a - \sqrt{2}b) \log\left(x^2 - \sqrt{2\sqrt{2}-1}x + \sqrt{2}\right)}{4\sqrt{2}(2\sqrt{2}-1)} + \frac{(a - \sqrt{2}b) \log\left(x^2 + \sqrt{2\sqrt{2}-1}x + \sqrt{2}\right)}{4\sqrt{2}(2\sqrt{2}-1)} - \frac{1}{2} \sqrt{\frac{1}{14}(2\sqrt{2}-1)} (a + \sqrt{2}b) \tan$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)/(2 + x^2 + x^4), x]
```

```
[Out] -(Sqrt[(-1 + 2*Sqrt[2])/14]*(a + Sqrt[2]*b)*ArcTan[(Sqrt[-1 + 2*Sqrt[2]] - 2*x)/Sqrt[1 + 2*Sqrt[2]])/2 + (Sqrt[(-1 + 2*Sqrt[2])/14]*(a + Sqrt[2]*b)*ArcTan[(Sqrt[-1 + 2*Sqrt[2]] + 2*x)/Sqrt[1 + 2*Sqrt[2]])/2 - ((a - Sqrt[2]*b)*Log[Sqrt[2] - Sqrt[-1 + 2*Sqrt[2]]*x + x^2])/(4*Sqrt[2*(-1 + 2*Sqrt[2])]) + ((a - Sqrt[2]*b)*Log[Sqrt[2] + Sqrt[-1 + 2*Sqrt[2]]*x + x^2])/(4*Sqrt[2*(-1 + 2*Sqrt[2])])
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r + (d - e*q)*x)/(q + r*x + x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 634

```
Int[((d_) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 628

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rubi steps

$$\int \frac{a + bx^2}{2 + x^2 + x^4} dx = \frac{\int \frac{\sqrt{-1+2\sqrt{2}a-(a-\sqrt{2}b)x}}{\sqrt{2}-\sqrt{-1+2\sqrt{2}x+x^2}} dx}{2\sqrt{2}(-1+2\sqrt{2})} + \frac{\int \frac{\sqrt{-1+2\sqrt{2}a+(a-\sqrt{2}b)x}}{\sqrt{2}+\sqrt{-1+2\sqrt{2}x+x^2}} dx}{2\sqrt{2}(-1+2\sqrt{2})}$$

$$= \frac{1}{8}(\sqrt{2}a + 2b) \int \frac{1}{\sqrt{2} - \sqrt{-1 + 2\sqrt{2}x + x^2}} dx + \frac{1}{8}(\sqrt{2}a + 2b) \int \frac{1}{\sqrt{2} + \sqrt{-1 + 2\sqrt{2}x + x^2}} dx - \frac{(a - \sqrt{2}b) \log\left(\sqrt{2} - \sqrt{-1 + 2\sqrt{2}x + x^2}\right)}{4\sqrt{2}(-1 + 2\sqrt{2})} + \frac{(a - \sqrt{2}b) \log\left(\sqrt{2} + \sqrt{-1 + 2\sqrt{2}x + x^2}\right)}{4\sqrt{2}(-1 + 2\sqrt{2})} - \frac{1}{4}(\sqrt{2}a + 2b)$$

$$= -\frac{(a + \sqrt{2}b) \tan^{-1}\left(\frac{\sqrt{-1+2\sqrt{2}-2x}}{\sqrt{1+2\sqrt{2}}}\right)}{2\sqrt{2}(1+2\sqrt{2})} + \frac{(a + \sqrt{2}b) \tan^{-1}\left(\frac{\sqrt{-1+2\sqrt{2}+2x}}{\sqrt{1+2\sqrt{2}}}\right)}{2\sqrt{2}(1+2\sqrt{2})} - \frac{(a - \sqrt{2}b) \log\left(\sqrt{2} - \sqrt{-1 + 2\sqrt{2}x + x^2}\right)}{4\sqrt{2}(-1 + 2\sqrt{2})}$$

Mathematica [C] time = 0.115918, size = 111, normalized size = 0.47

$$\frac{((\sqrt{7} + i)b - 2ia) \tan^{-1}\left(\frac{x}{\sqrt{\frac{1}{2}(1-i\sqrt{7})}}\right)}{\sqrt{14 - 14i\sqrt{7}}} + \frac{(2ia + (\sqrt{7} - i)b) \tan^{-1}\left(\frac{x}{\sqrt{\frac{1}{2}(1+i\sqrt{7})}}\right)}{\sqrt{14 + 14i\sqrt{7}}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x^2)/(2 + x^2 + x^4), x]

[Out] (((-2*I)*a + (I + Sqrt[7])*b)*ArcTan[x/Sqrt[(1 - I*Sqrt[7])/2]])/Sqrt[14 - (14*I)*Sqrt[7]] + (((2*I)*a + (-I + Sqrt[7])*b)*ArcTan[x/Sqrt[(1 + I*Sqrt[7])/2]])/Sqrt[14 + (14*I)*Sqrt[7]]

Maple [B] time = 0.08, size = 710, normalized size = 3.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^2+a)/(x^4+x^2+2), x)

```
[Out] -1/56*ln(x^2+2^(1/2)-x*(-1+2*2^(1/2))^(1/2))*2^(1/2)*(-1+2*2^(1/2))^(1/2)*a
+1/14*ln(x^2+2^(1/2)-x*(-1+2*2^(1/2))^(1/2))*2^(1/2)*(-1+2*2^(1/2))^(1/2)*b
-1/14*ln(x^2+2^(1/2)-x*(-1+2*2^(1/2))^(1/2))*(-1+2*2^(1/2))^(1/2)*a+1/28*ln
(x^2+2^(1/2)-x*(-1+2*2^(1/2))^(1/2))*(-1+2*2^(1/2))^(1/2)*b-1/28/(1+2*2^(1/
2))^(1/2)*arctan((2*x-(-1+2*2^(1/2))^(1/2))/(1+2*2^(1/2))^(1/2))*2^(1/2)*(-
1+2*2^(1/2))*a+1/7/(1+2*2^(1/2))^(1/2)*arctan((2*x-(-1+2*2^(1/2))^(1/2))/(1
+2*2^(1/2))^(1/2))*2^(1/2)*(-1+2*2^(1/2))*b-1/7/(1+2*2^(1/2))^(1/2)*arctan(
(2*x-(-1+2*2^(1/2))^(1/2))/(1+2*2^(1/2))^(1/2))*(-1+2*2^(1/2))*a+1/14/(1+2*
2^(1/2))^(1/2)*arctan((2*x-(-1+2*2^(1/2))^(1/2))/(1+2*2^(1/2))^(1/2))*(-1+2
*2^(1/2))*b+1/2/(1+2*2^(1/2))^(1/2)*arctan((2*x-(-1+2*2^(1/2))^(1/2))/(1+2*
2^(1/2))^(1/2))*2^(1/2)*a+1/56*ln(x^2+2^(1/2)+x*(-1+2*2^(1/2))^(1/2))*2^(1/
2)*(-1+2*2^(1/2))^(1/2)*a-1/14*ln(x^2+2^(1/2)+x*(-1+2*2^(1/2))^(1/2))*2^(1/
2)*(-1+2*2^(1/2))^(1/2)*b+1/14*ln(x^2+2^(1/2)+x*(-1+2*2^(1/2))^(1/2))*(-1+2
*2^(1/2))^(1/2)*a-1/28*ln(x^2+2^(1/2)+x*(-1+2*2^(1/2))^(1/2))*(-1+2*2^(1/2)
)^(1/2)*b-1/28/(1+2*2^(1/2))^(1/2)*arctan((2*x+(-1+2*2^(1/2))^(1/2))/(1+2*2
^(1/2))^(1/2))*2^(1/2)*(-1+2*2^(1/2))*a+1/7/(1+2*2^(1/2))^(1/2)*arctan((2*x
+(-1+2*2^(1/2))^(1/2))/(1+2*2^(1/2))^(1/2))*2^(1/2)*(-1+2*2^(1/2))*b-1/7/(1
+2*2^(1/2))^(1/2)*arctan((2*x+(-1+2*2^(1/2))^(1/2))/(1+2*2^(1/2))^(1/2))*(-
1+2*2^(1/2))*a+1/14/(1+2*2^(1/2))^(1/2)*arctan((2*x+(-1+2*2^(1/2))^(1/2))/(
1+2*2^(1/2))^(1/2))*(-1+2*2^(1/2))*b+1/2/(1+2*2^(1/2))^(1/2)*arctan((2*x+(
-1+2*2^(1/2))^(1/2))/(1+2*2^(1/2))^(1/2))*2^(1/2)*a
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{bx^2 + a}{x^4 + x^2 + 2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(x^4+x^2+2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)/(x^4 + x^2 + 2), x)
```

Fricas [B] time = 2.27976, size = 7710, normalized size = 32.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(x^4+x^2+2),x, algorithm="fricas")
```

```
[Out] 1/112*(28*sqrt(2)*sqrt(1/7)*(8*a^4 - 16*a^3*b + 40*a^2*b^2 - 32*a*b^3 + 32*
b^4)^(1/4)*sqrt(a^4 - 2*a^3*b + 5*a^2*b^2 - 4*a*b^3 + 4*b^4)*sqrt(a^4 - 4*a
^2*b^2 + 4*b^4)*sqrt((4*a^4 - 8*a^3*b + 20*a^2*b^2 - 16*a*b^3 + 16*b^4 - sq
rt(2)*sqrt(a^4 - 2*a^3*b + 5*a^2*b^2 - 4*a*b^3 + 4*b^4)*(a^2 - 8*a*b + 2*b^
2))/(a^4 - 4*a^2*b^2 + 4*b^4))*arctan(-1/28*(7*sqrt(1/2)*sqrt(1/7)*(8*a^4 -
16*a^3*b + 40*a^2*b^2 - 32*a*b^3 + 32*b^4)^(3/4)*(sqrt(2)*sqrt(a^4 - 2*a^3
*b + 5*a^2*b^2 - 4*a*b^3 + 4*b^4)*sqrt(a^4 - 4*a^2*b^2 + 4*b^4)*a - 2*sqrt(
a^4 - 4*a^2*b^2 + 4*b^4)*(a^2*b - a*b^2 + 2*b^3))*sqrt((4*a^4 - 8*a^3*b + 2
0*a^2*b^2 - 16*a*b^3 + 16*b^4 - sqrt(2)*sqrt(a^4 - 2*a^3*b + 5*a^2*b^2 - 4*
a*b^3 + 4*b^4)*(a^2 - 8*a*b + 2*b^2))/(a^4 - 4*a^2*b^2 + 4*b^4))*sqrt((2*(a
^4 - 2*a^3*b + 5*a^2*b^2 - 4*a*b^3 + 4*b^4)*x^2 + sqrt(1/7)*(8*a^4 - 16*a^3
*b + 40*a^2*b^2 - 32*a*b^3 + 32*b^4)^(1/4)*(sqrt(7)*sqrt(2)*sqrt(a^4 - 2*a^
3*b + 5*a^2*b^2 - 4*a*b^3 + 4*b^4))*b*x - sqrt(7)*(a^3 - a^2*b + 2*a*b^2))*x
)*sqrt((4*a^4 - 8*a^3*b + 20*a^2*b^2 - 16*a*b^3 + 16*b^4 - sqrt(2)*sqrt(a^4
```


$$\frac{t(2)\sqrt{a^4 - 2a^3b + 5a^2b^2 - 4ab^3 + 4b^4}(a^2 - 8ab + 2b^2)}{(a^4 - 4a^2b^2 + 4b^4)} + \frac{8\sqrt{2}\sqrt{a^4 - 2a^3b + 5a^2b^2 - 4ab^3 + 4b^4}(a^2 - ab + 2b^2)}{(a^4 - 2a^3b + 5a^2b^2 - 4ab^3 + 4b^4)}$$

Sympy [A] time = 0.899314, size = 122, normalized size = 0.52

$$\text{RootSum}\left(1568t^4 + t^2(-28a^2 + 224ab - 56b^2) + a^4 - 2a^3b + 5a^2b^2 - 4ab^3 + 4b^4, \left(t \mapsto t \log\left(x + \frac{112t^3a - 448t^3b + \dots}{a^4 - \dots}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**2+a)/(x**4+x**2+2),x)

[Out] RootSum(1568*_t**4 + _t**2*(-28*a**2 + 224*a*b - 56*b**2) + a**4 - 2*a**3*b + 5*a**2*b**2 - 4*a*b**3 + 4*b**4, Lambda(_t, _t*log(x + (112*_t**3*a - 448*_t**3*b + 6*_t*a**3 + 12*_t*a**2*b - 48*_t*a*b**2 + 8*_t*b**3)/(a**4 - a**3*b + 2*a*b**3 - 4*b**4))))

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{bx^2 + a}{x^4 + x^2 + 2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^2+a)/(x^4+x^2+2),x, algorithm="giac")

[Out] integrate((b*x^2 + a)/(x^4 + x^2 + 2), x)

$$3.101 \quad \int \frac{a+bx^2}{(2+x^2+x^4)^2} dx$$

Optimal. Leaf size=316

$$\frac{x(x^2(-(a-4b))+3a+2b)}{28(x^4+x^2+2)} - \frac{(\sqrt{2}(a-4b)+11a-2b)\log\left(x^2-\sqrt{2\sqrt{2}-1}x+\sqrt{2}\right)}{112\sqrt{2}(2\sqrt{2}-1)} + \frac{((11+\sqrt{2})a-2(2\sqrt{2}b+b))\log\left(x^2-\sqrt{2\sqrt{2}-1}x+\sqrt{2}\right)}{112\sqrt{2}(2\sqrt{2}-1)}$$

```
[Out] (x*(3*a + 2*b - (a - 4*b)*x^2))/(28*(2 + x^2 + x^4)) - (Sqrt[(-1 + 2*Sqrt[2])/14]*((11 - Sqrt[2])*a - (2 - 4*Sqrt[2])*b)*ArcTan[(Sqrt[-1 + 2*Sqrt[2]] - 2*x)/Sqrt[1 + 2*Sqrt[2]])/56 + (Sqrt[(-1 + 2*Sqrt[2])/14]*((11 - Sqrt[2])*a - (2 - 4*Sqrt[2])*b)*ArcTan[(Sqrt[-1 + 2*Sqrt[2]] + 2*x)/Sqrt[1 + 2*Sqrt[2]])/56 - ((11*a + Sqrt[2]*(a - 4*b) - 2*b)*Log[Sqrt[2] - Sqrt[-1 + 2*Sqrt[2]]*x + x^2])/(112*Sqrt[2]*(-1 + 2*Sqrt[2])) + (((11 + Sqrt[2])*a - 2*(b + 2*Sqrt[2]*b))*Log[Sqrt[2] + Sqrt[-1 + 2*Sqrt[2]]*x + x^2])/(112*Sqrt[2]*(-1 + 2*Sqrt[2]))
```

Rubi [A] time = 0.289108, antiderivative size = 316, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1178, 1169, 634, 618, 204, 628}

$$\frac{x(x^2(-(a-4b))+3a+2b)}{28(x^4+x^2+2)} - \frac{(\sqrt{2}(a-4b)+11a-2b)\log\left(x^2-\sqrt{2\sqrt{2}-1}x+\sqrt{2}\right)}{112\sqrt{2}(2\sqrt{2}-1)} + \frac{((11+\sqrt{2})a-2(2\sqrt{2}b+b))\log\left(x^2-\sqrt{2\sqrt{2}-1}x+\sqrt{2}\right)}{112\sqrt{2}(2\sqrt{2}-1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)/(2 + x^2 + x^4)^2, x]
```

```
[Out] (x*(3*a + 2*b - (a - 4*b)*x^2))/(28*(2 + x^2 + x^4)) - (Sqrt[(-1 + 2*Sqrt[2])/14]*((11 - Sqrt[2])*a - (2 - 4*Sqrt[2])*b)*ArcTan[(Sqrt[-1 + 2*Sqrt[2]] - 2*x)/Sqrt[1 + 2*Sqrt[2]])/56 + (Sqrt[(-1 + 2*Sqrt[2])/14]*((11 - Sqrt[2])*a - (2 - 4*Sqrt[2])*b)*ArcTan[(Sqrt[-1 + 2*Sqrt[2]] + 2*x)/Sqrt[1 + 2*Sqrt[2]])/56 - ((11*a + Sqrt[2]*(a - 4*b) - 2*b)*Log[Sqrt[2] - Sqrt[-1 + 2*Sqrt[2]]*x + x^2])/(112*Sqrt[2]*(-1 + 2*Sqrt[2])) + (((11 + Sqrt[2])*a - 2*(b + 2*Sqrt[2]*b))*Log[Sqrt[2] + Sqrt[-1 + 2*Sqrt[2]]*x + x^2])/(112*Sqrt[2]*(-1 + 2*Sqrt[2]))
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r + (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
```


$[b^2 - 4ac, 0] \ \&\& \ \text{NeQ}[c*d^2 - b*d*e + a*e^2, 0] \ \&\& \ \text{NegQ}[b^2 - 4ac]$

Rule 634

$\text{Int}[\frac{(d_.) + (e_.)x}{(a_.) + (b_.)x + (c_.)x^2}, x_Symbol] \rightarrow \text{Dist}[\frac{2cd - b^2e}{2c}, \text{Int}[\frac{1}{a + bx + cx^2}, x], x] + \text{Dist}[\frac{e}{2c}, \text{Int}[\frac{b + 2cx}{a + bx + cx^2}, x], x] /; \text{FreeQ}\{a, b, c, d, e, x\} \ \&\& \ \text{NeQ}[2cd - b^2e, 0] \ \&\& \ \text{NeQ}[b^2 - 4ac, 0] \ \&\& \ \text{!NiceSqrtQ}[b^2 - 4ac]$

Rule 618

$\text{Int}[\frac{(a_.) + (b_.)x + (c_.)x^2}{(a_.) + (b_.)x + (c_.)x^2}^{-1}, x_Symbol] \rightarrow \text{Dist}[-2, \text{Subst}[\text{Int}[\frac{1}{\text{Simp}[b^2 - 4ac - x^2, x]}, x], x, b + 2cx], x] /; \text{FreeQ}\{a, b, c, x\} \ \&\& \ \text{NeQ}[b^2 - 4ac, 0]$

Rule 204

$\text{Int}[\frac{(a_.) + (b_.)x^2}{(a_.) + (b_.)x^2}^{-1}, x_Symbol] \rightarrow -\text{Simp}[\text{ArcTan}[\frac{\text{Rt}[-b, 2]x}{\text{Rt}[-a, 2]}], x] /; \text{FreeQ}\{a, b, x\} \ \&\& \ \text{PosQ}[a/b] \ \&\& \ (\text{LtQ}[a, 0] \ || \ \text{LtQ}[b, 0])$

Rule 628

$\text{Int}[\frac{(d_.) + (e_.)x}{(a_.) + (b_.)x + (c_.)x^2}, x_Symbol] \rightarrow \text{Simp}[\frac{d \cdot \text{Log}[\text{RemoveContent}[a + bx + cx^2, x]]}{b}, x] /; \text{FreeQ}\{a, b, c, d, e, x\} \ \&\& \ \text{EqQ}[2cd - b^2e, 0]$

Rubi steps

$$\begin{aligned} \int \frac{a + bx^2}{(2 + x^2 + x^4)^2} dx &= \frac{x(3a + 2b - (a - 4b)x^2)}{28(2 + x^2 + x^4)} + \frac{1}{28} \int \frac{11a - 2b + (-a + 4b)x^2}{2 + x^2 + x^4} dx \\ &= \frac{x(3a + 2b - (a - 4b)x^2)}{28(2 + x^2 + x^4)} + \frac{\int \frac{\sqrt{-1+2\sqrt{2}(11a-2b)-(11a-2b-\sqrt{2}(-a+4b))x}}{\sqrt{2}-\sqrt{-1+2\sqrt{2}x+x^2}} dx}{56\sqrt{2}(-1+2\sqrt{2})} + \frac{\int \frac{\sqrt{-1+2\sqrt{2}(11a-2b)+(11a-2b-\sqrt{2}(-a+4b))x}}{\sqrt{2}+\sqrt{-1+2\sqrt{2}x+x^2}} dx}{56\sqrt{2}(-1+2\sqrt{2})} \\ &= \frac{x(3a + 2b - (a - 4b)x^2)}{28(2 + x^2 + x^4)} - \frac{(11a + \sqrt{2}(a - 4b) - 2b) \int \frac{-\sqrt{-1+2\sqrt{2}+2x}}{\sqrt{2}-\sqrt{-1+2\sqrt{2}x+x^2}} dx}{112\sqrt{2}(-1+2\sqrt{2})} + \frac{((11 - \sqrt{2})a - (2 - 4\sqrt{2})b) \tan^{-1}\left(\frac{\sqrt{-1+2\sqrt{2}-2x}}{\sqrt{1+2\sqrt{2}}}\right)}{56\sqrt{2}(-1+2\sqrt{2})} \\ &= \frac{x(3a + 2b - (a - 4b)x^2)}{28(2 + x^2 + x^4)} - \frac{(11a + \sqrt{2}(a - 4b) - 2b) \log\left(\sqrt{2} - \sqrt{-1 + 2\sqrt{2}x + x^2}\right)}{112\sqrt{2}(-1 + 2\sqrt{2})} + \frac{((11 + \sqrt{2})a - (2 + 4\sqrt{2})b) \tan^{-1}\left(\frac{\sqrt{-1+2\sqrt{2}+2x}}{\sqrt{1+2\sqrt{2}}}\right)}{56\sqrt{2}(-1+2\sqrt{2})} \\ &= \frac{x(3a + 2b - (a - 4b)x^2)}{28(2 + x^2 + x^4)} - \frac{((11 - \sqrt{2})a - (2 - 4\sqrt{2})b) \tan^{-1}\left(\frac{\sqrt{-1+2\sqrt{2}-2x}}{\sqrt{1+2\sqrt{2}}}\right)}{56\sqrt{2}(1 + 2\sqrt{2})} + \frac{((11 + \sqrt{2})a - (2 + 4\sqrt{2})b) \tan^{-1}\left(\frac{\sqrt{-1+2\sqrt{2}+2x}}{\sqrt{1+2\sqrt{2}}}\right)}{56\sqrt{2}(1 + 2\sqrt{2})} \end{aligned}$$

Mathematica [C] time = 0.20857, size = 165, normalized size = 0.52

$$\frac{2b(2x^3 + x) - ax(x^2 - 3)}{28(x^4 + x^2 + 2)} - \frac{((\sqrt{7} + 23i)a - 4(\sqrt{7} + 2i)b) \tan^{-1}\left(\frac{x}{\sqrt{\frac{1}{2}(1-i\sqrt{7})}}\right)}{28\sqrt{14 - 14i\sqrt{7}}} - \frac{((\sqrt{7} - 23i)a - 4(\sqrt{7} - 2i)b) \tan^{-1}\left(\frac{x}{\sqrt{\frac{1}{2}(1+i\sqrt{7})}}\right)}{28\sqrt{14 + 14i\sqrt{7}}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x^2)/(2 + x^2 + x^4)^2,x]

[Out] $(-a*x*(-3 + x^2) + 2*b*(x + 2*x^3))/(28*(2 + x^2 + x^4)) - (((23*I + \text{Sqrt}[7])*a - 4*(2*I + \text{Sqrt}[7])*b)*\text{ArcTan}[x/\text{Sqrt}[(1 - I*\text{Sqrt}[7])/2]])/(28*\text{Sqrt}[14 - (14*I)*\text{Sqrt}[7]]) - (((-23*I + \text{Sqrt}[7])*a - 4*(-2*I + \text{Sqrt}[7])*b)*\text{ArcTan}[x/\text{Sqrt}[(1 + I*\text{Sqrt}[7])/2]])/(28*\text{Sqrt}[14 + (14*I)*\text{Sqrt}[7]])$

Maple [B] time = 0.403, size = 756, normalized size = 2.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^2+a)/(x^4+x^2+2)^2,x)

[Out] $-1/784*(-(-14*a-28*2^{(1/2)}*a+112*b*2^{(1/2)}+56*b)/(1+2*2^{(1/2)})*x+1/(1+2*2^{(1/2)}))*(-1+2*2^{(1/2)})^{(1/2)}*(-70*a-42*2^{(1/2)}*a+56*b*2^{(1/2)}+28*b))/(x^2+2^{(1/2)}-x*(-1+2*2^{(1/2)})^{(1/2)})-107/1568/(1+2*2^{(1/2)})*\ln(x^2+2^{(1/2)}-x*(-1+2*2^{(1/2)})^{(1/2)})*2^{(1/2)}*(-1+2*2^{(1/2)})^{(1/2)}*a+25/784/(1+2*2^{(1/2)})*\ln(x^2+2^{(1/2)}-x*(-1+2*2^{(1/2)})^{(1/2)})*2^{(1/2)}*(-1+2*2^{(1/2)})^{(1/2)}*b-53/784/(1+2*2^{(1/2)})*\ln(x^2+2^{(1/2)}-x*(-1+2*2^{(1/2)})^{(1/2)})*(-1+2*2^{(1/2)})^{(1/2)}*a+11/196/(1+2*2^{(1/2)})*\ln(x^2+2^{(1/2)}-x*(-1+2*2^{(1/2)})^{(1/2)})*(-1+2*2^{(1/2)})^{(1/2)}*b+1/16/(1+2*2^{(1/2)})^{(3/2)}*\arctan((2*x-(-1+2*2^{(1/2)})^{(1/2)})/(1+2*2^{(1/2)})^{(1/2)})*2^{(1/2)}*a+3/8/(1+2*2^{(1/2)})^{(3/2)}*\arctan((2*x-(-1+2*2^{(1/2)})^{(1/2)})/(1+2*2^{(1/2)})^{(1/2)})*a+1/8/(1+2*2^{(1/2)})^{(3/2)}*\arctan((2*x-(-1+2*2^{(1/2)})^{(1/2)})/(1+2*2^{(1/2)})^{(1/2)})*2^{(1/2)}*b+1/784*((-14*a-28*2^{(1/2)}*a+112*b*2^{(1/2)}+56*b)/(1+2*2^{(1/2)})*x+1/(1+2*2^{(1/2)})*(-1+2*2^{(1/2)})^{(1/2)}*(-70*a-42*2^{(1/2)}*a+56*b*2^{(1/2)}+28*b))/(x^2+2^{(1/2)}+x*(-1+2*2^{(1/2)})^{(1/2)})+107/1568/(1+2*2^{(1/2)})*\ln(x^2+2^{(1/2)}+x*(-1+2*2^{(1/2)})^{(1/2)})*2^{(1/2)}*(-1+2*2^{(1/2)})^{(1/2)}*a-25/784/(1+2*2^{(1/2)})*\ln(x^2+2^{(1/2)}+x*(-1+2*2^{(1/2)})^{(1/2)})*2^{(1/2)}*(-1+2*2^{(1/2)})^{(1/2)}*b+53/784/(1+2*2^{(1/2)})*\ln(x^2+2^{(1/2)}+x*(-1+2*2^{(1/2)})^{(1/2)})*(-1+2*2^{(1/2)})^{(1/2)}*a-11/196/(1+2*2^{(1/2)})*\ln(x^2+2^{(1/2)}+x*(-1+2*2^{(1/2)})^{(1/2)})*(-1+2*2^{(1/2)})^{(1/2)}*b+1/16/(1+2*2^{(1/2)})^{(3/2)}*\arctan((2*x+(-1+2*2^{(1/2)})^{(1/2)})/(1+2*2^{(1/2)})^{(1/2)})*2^{(1/2)}*a+3/8/(1+2*2^{(1/2)})^{(3/2)}*\arctan((2*x+(-1+2*2^{(1/2)})^{(1/2)})/(1+2*2^{(1/2)})^{(1/2)})*a+1/8/(1+2*2^{(1/2)})^{(3/2)}*\arctan((2*x+(-1+2*2^{(1/2)})^{(1/2)})/(1+2*2^{(1/2)})^{(1/2)})*2^{(1/2)}*b$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{(a-4b)x^3 - (3a+2b)x}{28(x^4+x^2+2)} + \frac{1}{28} \int -\frac{(a-4b)x^2 - 11a + 2b}{x^4+x^2+2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^2+a)/(x^4+x^2+2)^2,x, algorithm="maxima")

[Out] $-1/28*((a - 4*b)*x^3 - (3*a + 2*b)*x)/(x^4 + x^2 + 2) + 1/28*\text{integrate}(-((a - 4*b)*x^2 - 11*a + 2*b)/(x^4 + x^2 + 2), x)$

$$\begin{aligned}
& 57a^2b^2 - 2332ab^3 + 484b^4)x^2 - 2^{(1/4)}\sqrt{2/7}(4489a^4 - 7102 \\
& a^3b + 5757a^2b^2 - 2332ab^3 + 484b^4)^{(1/4)}(\sqrt{7}\sqrt{2})\sqrt{4} \\
& (489a^4 - 7102a^3b + 5757a^2b^2 - 2332ab^3 + 484b^4)(a - 4b)x + s \\
& \text{qrt}(7)(737a^3 - 717a^2b + 348ab^2 - 44b^3)x)\sqrt{(35912a^4 - 5681} \\
& 6a^3b + 46056a^2b^2 - 18656ab^3 + 3872b^4 - \sqrt{2})\sqrt{4489a^4 -} \\
& 7102a^3b + 5757a^2b^2 - 2332ab^3 + 484b^4)(211a^2 - 428ab + 100* \\
& b^2))/(289a^4 - 136a^3b - 120a^2b^2 + 32ab^3 + 16b^4)) + 14\sqrt{2} \\
& *\sqrt{4489a^4 - 7102a^3b + 5757a^2b^2 - 2332ab^3 + 484b^4}(67a^2 \\
& - 53ab + 22b^2))/(4489a^4 - 7102a^3b + 5757a^2b^2 - 2332ab^3 + 48 \\
& 4b^4)) - 2^{(3/4)}\sqrt{2/7}(4489a^4 - 7102a^3b + 5757a^2b^2 - 2332a* \\
& b^3 + 484b^4)^{(3/4)}(\sqrt{2})\sqrt{4489a^4 - 7102a^3b + 5757a^2b^2 - 2} \\
& 332ab^3 + 484b^4)\sqrt{289a^4 - 136a^3b - 120a^2b^2 + 32ab^3 + 16} \\
& *b^4)(11a - 2b)x + 2\sqrt{289a^4 - 136a^3b - 120a^2b^2 + 32ab^3} \\
& + 16b^4)(67a^3 - 321a^2b + 234ab^2 - 88b^3)x)\sqrt{(35912a^4 - 56} \\
& 816a^3b + 46056a^2b^2 - 18656ab^3 + 3872b^4 - \sqrt{2})\sqrt{4489a^4} \\
& - 7102a^3b + 5757a^2b^2 - 2332ab^3 + 484b^4)(211a^2 - 428ab + 10 \\
& 0b^2))/(289a^4 - 136a^3b - 120a^2b^2 + 32ab^3 + 16b^4)) + 4\sqrt{7} \\
&)\sqrt{2}(4489a^4 - 7102a^3b + 5757a^2b^2 - 2332ab^3 + 484b^4)^{(3/} \\
& 2)\sqrt{289a^4 - 136a^3b - 120a^2b^2 + 32ab^3 + 16b^4} - 2\sqrt{7)*} \\
& (300763a^6 - 713751a^5b + 860883a^4b^2 - 617609a^3b^3 + 282678a^2b \\
& ^4 - 76956ab^5 + 10648b^6)\sqrt{289a^4 - 136a^3b - 120a^2b^2 + 32a} \\
& *b^3 + 16b^4))/(5112971a^8 - 13336819a^7b + 16286963a^6b^2 - 11087881 \\
& *a^5b^3 + 3832430a^4b^4 + 31472a^3b^5 - 641872a^2b^6 + 265232ab^7 \\
& - 42592b^8)) + 784(4489a^5 - 25058a^4b + 34165a^3b^2 - 25360a^2b^3 \\
& + 9812ab^4 - 1936b^5)x^3 - 2^{(1/4)}\sqrt{2/7}(4489a^4 - 7102a^3b + \\
& 5757a^2b^2 - 2332ab^3 + 484b^4)^{(1/4)}(\sqrt{7}\sqrt{2})((211a^2 - 428 \\
& *ab + 100b^2)x^4 + (211a^2 - 428ab + 100b^2)x^2 + 422a^2 - 856ab \\
& + 200b^2)\sqrt{4489a^4 - 7102a^3b + 5757a^2b^2 - 2332ab^3 + 484b^} \\
& 4) + 8\sqrt{7}((4489a^4 - 7102a^3b + 5757a^2b^2 - 2332ab^3 + 484b^} \\
& 4)x^4 + 8978a^4 - 14204a^3b + 11514a^2b^2 - 4664ab^3 + 968b^4 + (4 \\
& 489a^4 - 7102a^3b + 5757a^2b^2 - 2332ab^3 + 484b^4)x^2))\sqrt{(359} \\
& 12a^4 - 56816a^3b + 46056a^2b^2 - 18656ab^3 + 3872b^4 - \sqrt{2})\sqrt{2} \\
& \text{qrt}(4489a^4 - 7102a^3b + 5757a^2b^2 - 2332ab^3 + 484b^4)(211a^2 - 4 \\
& 28ab + 100b^2))/(289a^4 - 136a^3b - 120a^2b^2 + 32ab^3 + 16b^4)) \\
& *\log(32(4489a^4 - 7102a^3b + 5757a^2b^2 - 2332ab^3 + 484b^4)x^2 + \\
& 16/7*2^{(1/4)}\sqrt{2/7}(4489a^4 - 7102a^3b + 5757a^2b^2 - 2332ab^3 \\
& + 484b^4)^{(1/4)}(\sqrt{7}\sqrt{2})\sqrt{4489a^4 - 7102a^3b + 5757a^2b^2} \\
& - 2332ab^3 + 484b^4)(a - 4b)x + \sqrt{7}(737a^3 - 717a^2b + 348a \\
& *b^2 - 44b^3)x)\sqrt{(35912a^4 - 56816a^3b + 46056a^2b^2 - 18656ab} \\
& ^3 + 3872b^4 - \sqrt{2})\sqrt{4489a^4 - 7102a^3b + 5757a^2b^2 - 2332a*} \\
& b^3 + 484b^4)(211a^2 - 428ab + 100b^2))/(289a^4 - 136a^3b - 120a^} \\
& 2b^2 + 32ab^3 + 16b^4)) + 32\sqrt{2})\sqrt{4489a^4 - 7102a^3b + 5757*} \\
& a^2b^2 - 2332ab^3 + 484b^4)(67a^2 - 53ab + 22b^2)) + 2^{(1/4)}\sqrt{2/7} \\
& (2/7)(4489a^4 - 7102a^3b + 5757a^2b^2 - 2332ab^3 + 484b^4)^{(1/4)}(s \\
& \text{qrt}(7)\sqrt{2})((211a^2 - 428ab + 100b^2)x^4 + (211a^2 - 428ab + 10 \\
& 0b^2)x^2 + 422a^2 - 856ab + 200b^2)\sqrt{4489a^4 - 7102a^3b + 5757} \\
& *a^2b^2 - 2332ab^3 + 484b^4) + 8\sqrt{7}((4489a^4 - 7102a^3b + 5757} \\
& *a^2b^2 - 2332ab^3 + 484b^4)x^4 + 8978a^4 - 14204a^3b + 11514a^2b \\
& ^2 - 4664ab^3 + 968b^4 + (4489a^4 - 7102a^3b + 5757a^2b^2 - 2332a*} \\
& b^3 + 484b^4)x^2))\sqrt{(35912a^4 - 56816a^3b + 46056a^2b^2 - 18656*} \\
& ab^3 + 3872b^4 - \sqrt{2})\sqrt{4489a^4 - 7102a^3b + 5757a^2b^2 - 2332} \\
& *ab^3 + 484b^4)(211a^2 - 428ab + 100b^2))/(289a^4 - 136a^3b - 120 \\
& *a^2b^2 + 32ab^3 + 16b^4))*\log(32(4489a^4 - 7102a^3b + 5757a^2b^2} \\
& - 2332ab^3 + 484b^4)x^2 - 16/7*2^{(1/4)}\sqrt{2/7}(4489a^4 - 7102a^3*} \\
& b + 5757a^2b^2 - 2332ab^3 + 484b^4)^{(1/4)}(\sqrt{7}\sqrt{2})\sqrt{4489a} \\
& ^4 - 7102a^3b + 5757a^2b^2 - 2332ab^3 + 484b^4)(a - 4b)x + \sqrt{7} \\
&)*(737a^3 - 717a^2b + 348ab^2 - 44b^3)x)\sqrt{(35912a^4 - 56816a^3} \\
& *b + 46056a^2b^2 - 18656ab^3 + 3872b^4 - \sqrt{2})\sqrt{4489a^4 - 7102*} \\
& a^3b + 5757a^2b^2 - 2332ab^3 + 484b^4)(211a^2 - 428ab + 100b^2))
\end{aligned}$$

$$\begin{aligned} &/((289a^4 - 136a^3b - 120a^2b^2 + 32ab^3 + 16b^4)) + 32\sqrt{2}\sqrt{(4489a^4 - 7102a^3b + 5757a^2b^2 - 2332ab^3 + 484b^4)(67a^2 - 53ab + 22b^2)} \\ &- 784(13467a^5 - 12328a^4b + 3067a^3b^2 + 4518a^2b^3 - 3212ab^4 + 968b^5)x / ((4489a^4 - 7102a^3b + 5757a^2b^2 - 2332ab^3 + 484b^4)x^4 + 8978a^4 - 14204a^3b + 11514a^2b^2 - 4664ab^3 + 968b^4 + (4489a^4 - 7102a^3b + 5757a^2b^2 - 2332ab^3 + 484b^4)x^2) \end{aligned}$$

Sympy [A] time = 1.38854, size = 167, normalized size = 0.53

$$-\frac{x^3(a-4b)+x(-3a-2b)}{28x^4+28x^2+56} + \text{RootSum}\left(240945152t^4 + t^2(-1157968a^2 + 2348864ab - 548800b^2) + 4489a^4 - 7102a^3b + 5757a^2b^2 - 2332ab^3 + 484b^4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**2+a)/(x**4+x**2+2)**2,x)

[Out] $-(x^3(a-4b) + x(-3a-2b))/(28x^4 + 28x^2 + 56) + \text{RootSum}(240945152_t^{**4} + _t^{**2}(-1157968a^{**2} + 2348864a*b - 548800b^{**2}) + 4489a^{**4} - 7102a^{**3}b + 5757a^{**2}b^{**2} - 2332a*b^{**3} + 484b^{**4}, \text{Lambda}(_t, _t*\log(x + (2634240_t^{**3}a - 3161088_t^{**3}b + 11996_t*a^{**3} + 12792_t*a^{**2}b - 21936_t*a*b^{**2} + 4384_t*b^{**3})/(1139a^{**4} - 1169a^{**3}b + 318a^{**2}b^{**2} + 124a*b^{**3} - 88b^{**4})))$

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{bx^2 + a}{(x^4 + x^2 + 2)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^2+a)/(x^4+x^2+2)^2,x, algorithm="giac")

[Out] integrate((b*x^2 + a)/(x^4 + x^2 + 2)^2, x)

$$3.102 \quad \int \frac{\sqrt{2-x^2}}{1-\sqrt{2}x^2+x^4} dx$$

Optimal. Leaf size=160

$$-\frac{1}{4}\sqrt{1+\frac{1}{\sqrt{2}}}\log\left(x^2-\sqrt{2+\sqrt{2}}x+1\right)+\frac{1}{4}\sqrt{1+\frac{1}{\sqrt{2}}}\log\left(x^2+\sqrt{2+\sqrt{2}}x+1\right)-\frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{2}}-2x}{\sqrt{2-\sqrt{2}}}\right)}{2\sqrt{2+\sqrt{2}}}+\frac{\tan^{-1}\left(\frac{2x+\sqrt{2+\sqrt{2}}}{\sqrt{2-\sqrt{2}}}\right)}{2\sqrt{2+\sqrt{2}}}$$

```
[Out] -ArcTan[(Sqrt[2 + Sqrt[2]] - 2*x)/Sqrt[2 - Sqrt[2]]]/(2*Sqrt[2 + Sqrt[2]])
+ ArcTan[(Sqrt[2 + Sqrt[2]] + 2*x)/Sqrt[2 - Sqrt[2]]]/(2*Sqrt[2 + Sqrt[2]])
- (Sqrt[1 + 1/Sqrt[2]]*Log[1 - Sqrt[2 + Sqrt[2]]*x + x^2])/4 + (Sqrt[1 + 1
/Sqrt[2]]*Log[1 + Sqrt[2 + Sqrt[2]]*x + x^2])/4
```

Rubi [A] time = 0.145837, antiderivative size = 160, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {1169, 634, 618, 204, 628}

$$-\frac{1}{4}\sqrt{1+\frac{1}{\sqrt{2}}}\log\left(x^2-\sqrt{2+\sqrt{2}}x+1\right)+\frac{1}{4}\sqrt{1+\frac{1}{\sqrt{2}}}\log\left(x^2+\sqrt{2+\sqrt{2}}x+1\right)-\frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{2}}-2x}{\sqrt{2-\sqrt{2}}}\right)}{2\sqrt{2+\sqrt{2}}}+\frac{\tan^{-1}\left(\frac{2x+\sqrt{2+\sqrt{2}}}{\sqrt{2-\sqrt{2}}}\right)}{2\sqrt{2+\sqrt{2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[2] - x^2)/(1 - Sqrt[2]*x^2 + x^4), x]
```

```
[Out] -ArcTan[(Sqrt[2 + Sqrt[2]] - 2*x)/Sqrt[2 - Sqrt[2]]]/(2*Sqrt[2 + Sqrt[2]])
+ ArcTan[(Sqrt[2 + Sqrt[2]] + 2*x)/Sqrt[2 - Sqrt[2]]]/(2*Sqrt[2 + Sqrt[2]])
- (Sqrt[1 + 1/Sqrt[2]]*Log[1 - Sqrt[2 + Sqrt[2]]*x + x^2])/4 + (Sqrt[1 + 1
/Sqrt[2]]*Log[1 + Sqrt[2 + Sqrt[2]]*x + x^2])/4
```

Rule 1169

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (b_)*(x_)^2 + (c_)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
(d - e*q)*x)/(q + r*x + x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 634

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[In
t[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\sqrt{2-x^2}}{1-\sqrt{2}x^2+x^4} dx = \frac{\int \frac{\sqrt{2(2+\sqrt{2})-(1+\sqrt{2})x}}{1-\sqrt{2+\sqrt{2}x+x^2}} dx}{2\sqrt{2+\sqrt{2}}} + \frac{\int \frac{\sqrt{2(2+\sqrt{2})+(1+\sqrt{2})x}}{1+\sqrt{2+\sqrt{2}x+x^2}} dx}{2\sqrt{2+\sqrt{2}}}$$

$$= \frac{1}{4}\sqrt{3-2\sqrt{2}} \int \frac{1}{1-\sqrt{2+\sqrt{2}x+x^2}} dx + \frac{1}{4}\sqrt{3-2\sqrt{2}} \int \frac{1}{1+\sqrt{2+\sqrt{2}x+x^2}} dx + \frac{(-1-\sqrt{2})}{4\sqrt{2}}$$

$$= -\frac{1}{4}\sqrt{1+\frac{1}{\sqrt{2}}} \log\left(1-\sqrt{2+\sqrt{2}x+x^2}\right) + \frac{1}{4}\sqrt{1+\frac{1}{\sqrt{2}}} \log\left(1+\sqrt{2+\sqrt{2}x+x^2}\right) - \frac{1}{2}\sqrt{3-2\sqrt{2}}$$

$$= -\frac{1}{2}\sqrt{\frac{1}{2}(2-\sqrt{2})} \tan^{-1}\left(\frac{\sqrt{2+\sqrt{2}-2x}}{\sqrt{2-\sqrt{2}}}\right) + \frac{1}{2}\sqrt{\frac{1}{2}(2-\sqrt{2})} \tan^{-1}\left(\frac{\sqrt{2+\sqrt{2}+2x}}{\sqrt{2-\sqrt{2}}}\right) - \frac{1}{4}\sqrt{1+\frac{1}{\sqrt{2}}}$$

Mathematica [C] time = 0.0449579, size = 53, normalized size = 0.33

$$\frac{\sqrt{-1-i} \tan^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt{-1-i}}\right) + \sqrt{-1+i} \tan^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt{-1+i}}\right)}{2^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[2] - x^2)/(1 - Sqrt[2]*x^2 + x^4), x]
```

```
[Out] (Sqrt[-1 - I]*ArcTan[(2^(1/4)*x)/Sqrt[-1 - I]] + Sqrt[-1 + I]*ArcTan[(2^(1/4)*x)/Sqrt[-1 + I]])/2^(3/4)
```

Maple [A] time = 0.104, size = 199, normalized size = 1.2

$$\frac{\sqrt{2}\sqrt{2+\sqrt{2}} \ln\left(1+x^2+x\sqrt{2+\sqrt{2}}\right)}{8} + \frac{\sqrt{2}}{2\sqrt{2-\sqrt{2}}} \arctan\left(\frac{2x+\sqrt{2+\sqrt{2}}}{\sqrt{2-\sqrt{2}}}\right) - \frac{1}{2\sqrt{2-\sqrt{2}}} \arctan\left(\frac{2x+\sqrt{2+\sqrt{2}}}{\sqrt{2-\sqrt{2}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+2^(1/2))/(1+x^4-x^2*2^(1/2)), x)
```

```
[Out] 1/8*2^(1/2)*(2+2^(1/2))^(1/2)*ln(1+x^2+x*(2+2^(1/2))^(1/2))+1/2/(2-2^(1/2))^(1/2)*arctan((2*x+(2+2^(1/2))^(1/2))/(2-2^(1/2))^(1/2))*2^(1/2)-1/2/(2-2^(1/2))^(1/2)*arctan((2*x+(2+2^(1/2))^(1/2))/(2-2^(1/2))^(1/2))*2^(1/2)
```

$$\begin{aligned} & \frac{1}{2})^{\frac{1}{2}} \arctan\left(\frac{(2x + (2+2^{\frac{1}{2}})^{\frac{1}{2}})^{\frac{1}{2}}}{(2-2^{\frac{1}{2}})^{\frac{1}{2}}}\right) - \frac{1}{8} 2^{\frac{1}{2}} * \\ & (2+2^{\frac{1}{2}})^{\frac{1}{2}} \ln(1+x^2-x*(2+2^{\frac{1}{2}})^{\frac{1}{2}}) + \frac{1}{2} / (2-2^{\frac{1}{2}})^{\frac{1}{2}} \arctan \\ & \left(\frac{(2x - (2+2^{\frac{1}{2}})^{\frac{1}{2}})^{\frac{1}{2}}}{(2-2^{\frac{1}{2}})^{\frac{1}{2}}}\right) * 2^{\frac{1}{2}} - \frac{1}{2} / (2-2^{\frac{1}{2}})^{\frac{1}{2}} * a \\ & \text{rctan}\left(\frac{(2x - (2+2^{\frac{1}{2}})^{\frac{1}{2}})^{\frac{1}{2}}}{(2-2^{\frac{1}{2}})^{\frac{1}{2}}}\right) \end{aligned}$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^2 - \sqrt{2}}{x^4 - \sqrt{2}x^2 + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2+2^(1/2))/(1+x^4-x^2*2^(1/2)),x, algorithm="maxima")

[Out] -integrate((x^2 - sqrt(2))/(x^4 - sqrt(2)*x^2 + 1), x)

Fricas [C] time = 1.46472, size = 387, normalized size = 2.42

$$\frac{1}{4} \sqrt{i+1} \sqrt{2} \log\left(x + \frac{1}{2} \sqrt{2} \sqrt{i+1} \sqrt{2}\right) - \frac{1}{4} \sqrt{i+1} \sqrt{2} \log\left(x - \frac{1}{2} \sqrt{2} \sqrt{i+1} \sqrt{2}\right) + \frac{1}{4} \sqrt{-(i-1)} \sqrt{2} \log\left(x + \frac{1}{2} \sqrt{2} \sqrt{-(i-1)} \sqrt{2}\right) - \frac{1}{4} \sqrt{-(i-1)} \sqrt{2} \log\left(x - \frac{1}{2} \sqrt{2} \sqrt{-(i-1)} \sqrt{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2+2^(1/2))/(1+x^4-x^2*2^(1/2)),x, algorithm="fricas")

[Out] 1/4*sqrt((I + 1)*sqrt(2))*log(x + 1/2*sqrt(2)*sqrt((I + 1)*sqrt(2))) - 1/4*sqrt((I + 1)*sqrt(2))*log(x - 1/2*sqrt(2)*sqrt((I + 1)*sqrt(2))) + 1/4*sqrt(-(I - 1)*sqrt(2))*log(x + 1/2*sqrt(2)*sqrt(-(I - 1)*sqrt(2))) - 1/4*sqrt(-(I - 1)*sqrt(2))*log(x - 1/2*sqrt(2)*sqrt(-(I - 1)*sqrt(2)))

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: PolynomialError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x**2+2**(1/2))/(1+x**4-x**2*2**(1/2)),x)

[Out] Exception raised: PolynomialError

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x^2 - \sqrt{2}}{x^4 - \sqrt{2}x^2 + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2+2^(1/2))/(1+x^4-x^2*2^(1/2)),x, algorithm="giac")

[Out] integrate(-(x^2 - sqrt(2))/(x^4 - sqrt(2)*x^2 + 1), x)

$$3.103 \quad \int \frac{\sqrt{2+x^2}}{1+\sqrt{2x^2+x^4}} dx$$

Optimal. Leaf size=172

$$-\frac{1}{4}\sqrt{1-\frac{1}{\sqrt{2}}}\log\left(x^2-\sqrt{2-\sqrt{2}x+1}\right)+\frac{1}{4}\sqrt{1-\frac{1}{\sqrt{2}}}\log\left(x^2+\sqrt{2-\sqrt{2}x+1}\right)-\frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{2}-2x}}{\sqrt{2+\sqrt{2}}}\right)}{2\sqrt{2-\sqrt{2}}}+\frac{\tan^{-1}\left(\frac{2x+\sqrt{2-\sqrt{2}}}{\sqrt{2+\sqrt{2}}}\right)}{2\sqrt{2-\sqrt{2}}}$$

```
[Out] -ArcTan[(Sqrt[2 - Sqrt[2]] - 2*x)/Sqrt[2 + Sqrt[2]]]/(2*Sqrt[2 - Sqrt[2]])
+ ArcTan[(Sqrt[2 - Sqrt[2]] + 2*x)/Sqrt[2 + Sqrt[2]]]/(2*Sqrt[2 - Sqrt[2]])
- (Sqrt[1 - 1/Sqrt[2]]*Log[1 - Sqrt[2 - Sqrt[2]]*x + x^2])/4 + (Sqrt[1 - 1
/Sqrt[2]]*Log[1 + Sqrt[2 - Sqrt[2]]*x + x^2])/4
```

Rubi [A] time = 0.137031, antiderivative size = 172, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {1169, 634, 618, 204, 628}

$$-\frac{1}{4}\sqrt{1-\frac{1}{\sqrt{2}}}\log\left(x^2-\sqrt{2-\sqrt{2}x+1}\right)+\frac{1}{4}\sqrt{1-\frac{1}{\sqrt{2}}}\log\left(x^2+\sqrt{2-\sqrt{2}x+1}\right)-\frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{2}-2x}}{\sqrt{2+\sqrt{2}}}\right)}{2\sqrt{2-\sqrt{2}}}+\frac{\tan^{-1}\left(\frac{2x+\sqrt{2-\sqrt{2}}}{\sqrt{2+\sqrt{2}}}\right)}{2\sqrt{2-\sqrt{2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[2] + x^2)/(1 + Sqrt[2]*x^2 + x^4), x]
```

```
[Out] -ArcTan[(Sqrt[2 - Sqrt[2]] - 2*x)/Sqrt[2 + Sqrt[2]]]/(2*Sqrt[2 - Sqrt[2]])
+ ArcTan[(Sqrt[2 - Sqrt[2]] + 2*x)/Sqrt[2 + Sqrt[2]]]/(2*Sqrt[2 - Sqrt[2]])
- (Sqrt[1 - 1/Sqrt[2]]*Log[1 - Sqrt[2 - Sqrt[2]]*x + x^2])/4 + (Sqrt[1 - 1
/Sqrt[2]]*Log[1 + Sqrt[2 - Sqrt[2]]*x + x^2])/4
```

Rule 1169

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (b_)*(x_)^2 + (c_)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
(d - e*q)*x)/(q + r*x + x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 634

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[In
t[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{2+x^2}}{1+\sqrt{2}x^2+x^4} dx &= \frac{\int \frac{\sqrt{2(2-\sqrt{2})-(-1+\sqrt{2})x}}{1-\sqrt{2-\sqrt{2}x+x^2}} dx}{2\sqrt{2-\sqrt{2}}} + \frac{\int \frac{\sqrt{2(2-\sqrt{2})+(-1+\sqrt{2})x}}{1+\sqrt{2-\sqrt{2}x+x^2}} dx}{2\sqrt{2-\sqrt{2}}} \\ &= \frac{(1-\sqrt{2}) \int \frac{-\sqrt{2-\sqrt{2}+2x}}{1-\sqrt{2-\sqrt{2}x+x^2}} dx}{4\sqrt{2-\sqrt{2}}} + \frac{(-1+\sqrt{2}) \int \frac{\sqrt{2-\sqrt{2}+2x}}{1+\sqrt{2-\sqrt{2}x+x^2}} dx}{4\sqrt{2-\sqrt{2}}} + \frac{1}{4}\sqrt{3+2\sqrt{2}} \int \frac{1}{1-\sqrt{2-\sqrt{2}x+x^2}} dx \\ &= -\frac{1}{4}\sqrt{1-\frac{1}{\sqrt{2}}} \log\left(1-\sqrt{2-\sqrt{2}x+x^2}\right) + \frac{1}{4}\sqrt{1-\frac{1}{\sqrt{2}}} \log\left(1+\sqrt{2-\sqrt{2}x+x^2}\right) - \frac{1}{2}\sqrt{3+2\sqrt{2}} \operatorname{Su} \\ &= -\frac{1}{2}\sqrt{\frac{1}{2}(2+\sqrt{2})} \tan^{-1}\left(\frac{\sqrt{2-\sqrt{2}-2x}}{\sqrt{2+\sqrt{2}}}\right) + \frac{1}{2}\sqrt{\frac{1}{2}(2+\sqrt{2})} \tan^{-1}\left(\frac{\sqrt{2-\sqrt{2}+2x}}{\sqrt{2+\sqrt{2}}}\right) - \frac{1}{4}\sqrt{1-\frac{1}{\sqrt{2}}} \end{aligned}$$

Mathematica [C] time = 0.0346015, size = 53, normalized size = 0.31

$$\frac{\sqrt{1-i} \tan^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt{1-i}}\right) + \sqrt{1+i} \tan^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt{1+i}}\right)}{2^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[2] + x^2)/(1 + Sqrt[2]*x^2 + x^4), x]
```

```
[Out] (Sqrt[1 - I]*ArcTan[(2^(1/4)*x)/Sqrt[1 - I]] + Sqrt[1 + I]*ArcTan[(2^(1/4)*x)/Sqrt[1 + I]])/2^(3/4)
```

Maple [A] time = 0.151, size = 199, normalized size = 1.2

$$-\frac{\sqrt{2}\sqrt{2-\sqrt{2}} \ln\left(1+x^2-x\sqrt{2-\sqrt{2}}\right)}{8} + \frac{\sqrt{2}}{2\sqrt{2+\sqrt{2}}} \arctan\left(\frac{2x-\sqrt{2-\sqrt{2}}}{\sqrt{2+\sqrt{2}}}\right) + \frac{1}{2\sqrt{2+\sqrt{2}}} \arctan\left(\frac{2x-\sqrt{2-\sqrt{2}}}{\sqrt{2+\sqrt{2}}}\right) +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+2^(1/2))/(1+x^4+x^2*2^(1/2)), x)
```

```
[Out] -1/8*2^(1/2)*(2-2^(1/2))^(1/2)*ln(1+x^2-x*(2-2^(1/2))^(1/2))+1/2/(2+2^(1/2))^(1/2)*arctan((2*x-(2-2^(1/2))^(1/2))/(2+2^(1/2))^(1/2))*2^(1/2)+1/2/(2+2^(1/2))^(1/2)*arctan((2*x-(2-2^(1/2))^(1/2))/(2+2^(1/2))^(1/2))*2^(1/2)
```

$$\begin{aligned} & (1/2)^{(1/2)} * \arctan((2*x - (2 - 2^{(1/2)})^{(1/2)}) / (2 + 2^{(1/2)})^{(1/2)}) + 1/8 * 2^{(1/2)} * \\ & (2 - 2^{(1/2)})^{(1/2)} * \ln(1 + x^2 + x * (2 - 2^{(1/2)})^{(1/2)}) + 1/2 / (2 + 2^{(1/2)})^{(1/2)} * \arctan \\ & n((2*x + (2 - 2^{(1/2)})^{(1/2)}) / (2 + 2^{(1/2)})^{(1/2)}) * 2^{(1/2)} + 1/2 / (2 + 2^{(1/2)})^{(1/2)} * \\ & \arctan((2*x + (2 - 2^{(1/2)})^{(1/2)}) / (2 + 2^{(1/2)})^{(1/2)}) \end{aligned}$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 + \sqrt{2}}{x^4 + \sqrt{2}x^2 + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+2^(1/2))/(1+x^4+x^2*2^(1/2)),x, algorithm="maxima")

[Out] integrate((x^2 + sqrt(2))/(x^4 + sqrt(2)*x^2 + 1), x)

Fricas [C] time = 1.6733, size = 387, normalized size = 2.25

$$\frac{1}{4} \sqrt{(i-1)\sqrt{2}} \log\left(x + \frac{1}{2} \sqrt{2} \sqrt{(i-1)\sqrt{2}}\right) - \frac{1}{4} \sqrt{(i-1)\sqrt{2}} \log\left(x - \frac{1}{2} \sqrt{2} \sqrt{(i-1)\sqrt{2}}\right) + \frac{1}{4} \sqrt{-(i+1)\sqrt{2}} \log\left(x + \frac{1}{2} \sqrt{2} \sqrt{-(i+1)\sqrt{2}}\right) - \frac{1}{4} \sqrt{-(i+1)\sqrt{2}} \log\left(x - \frac{1}{2} \sqrt{2} \sqrt{-(i+1)\sqrt{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+2^(1/2))/(1+x^4+x^2*2^(1/2)),x, algorithm="fricas")

[Out] 1/4*sqrt((I - 1)*sqrt(2))*log(x + 1/2*sqrt(2)*sqrt((I - 1)*sqrt(2))) - 1/4*sqrt((I - 1)*sqrt(2))*log(x - 1/2*sqrt(2)*sqrt((I - 1)*sqrt(2))) + 1/4*sqrt(-(I + 1)*sqrt(2))*log(x + 1/2*sqrt(2)*sqrt(-(I + 1)*sqrt(2))) - 1/4*sqrt(-(I + 1)*sqrt(2))*log(x - 1/2*sqrt(2)*sqrt(-(I + 1)*sqrt(2)))

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: PolynomialError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**2+2**(1/2))/(1+x**4+x**2*2**(1/2)),x)

[Out] Exception raised: PolynomialError

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 + \sqrt{2}}{x^4 + \sqrt{2}x^2 + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+2^(1/2))/(1+x^4+x^2*2^(1/2)),x, algorithm="giac")

[Out] integrate((x^2 + sqrt(2))/(x^4 + sqrt(2)*x^2 + 1), x)

3.104 $\int \frac{\sqrt{2-x^2}}{1+bx^2+x^4} dx$

Optimal. Leaf size=160

$$-\frac{(1+\sqrt{2})\log(-\sqrt{2-bx+x^2+1})}{4\sqrt{2-b}} + \frac{(1+\sqrt{2})\log(\sqrt{2-bx+x^2+1})}{4\sqrt{2-b}} + \frac{(1-\sqrt{2})\tan^{-1}\left(\frac{\sqrt{2-b-2x}}{\sqrt{b+2}}\right)}{2\sqrt{b+2}} - \frac{(1-\sqrt{2})\tan^{-1}\left(\frac{\sqrt{2-b-2x}}{\sqrt{b+2}}\right)}{2\sqrt{b+2}}$$

[Out] ((1 - Sqrt[2])*ArcTan[(Sqrt[2 - b] - 2*x)/Sqrt[2 + b]])/(2*Sqrt[2 + b]) - ((1 - Sqrt[2])*ArcTan[(Sqrt[2 - b] + 2*x)/Sqrt[2 + b]])/(2*Sqrt[2 + b]) - ((1 + Sqrt[2])*Log[1 - Sqrt[2 - b]*x + x^2])/(4*Sqrt[2 - b]) + ((1 + Sqrt[2])*Log[1 + Sqrt[2 - b]*x + x^2])/(4*Sqrt[2 - b])

Rubi [A] time = 0.119352, antiderivative size = 160, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {1169, 634, 618, 204, 628}

$$-\frac{(1+\sqrt{2})\log(-\sqrt{2-bx+x^2+1})}{4\sqrt{2-b}} + \frac{(1+\sqrt{2})\log(\sqrt{2-bx+x^2+1})}{4\sqrt{2-b}} + \frac{(1-\sqrt{2})\tan^{-1}\left(\frac{\sqrt{2-b-2x}}{\sqrt{b+2}}\right)}{2\sqrt{b+2}} - \frac{(1-\sqrt{2})\tan^{-1}\left(\frac{\sqrt{2-b-2x}}{\sqrt{b+2}}\right)}{2\sqrt{b+2}}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[2] - x^2)/(1 + b*x^2 + x^4), x]

[Out] ((1 - Sqrt[2])*ArcTan[(Sqrt[2 - b] - 2*x)/Sqrt[2 + b]])/(2*Sqrt[2 + b]) - ((1 - Sqrt[2])*ArcTan[(Sqrt[2 - b] + 2*x)/Sqrt[2 + b]])/(2*Sqrt[2 + b]) - ((1 + Sqrt[2])*Log[1 - Sqrt[2 - b]*x + x^2])/(4*Sqrt[2 - b]) + ((1 + Sqrt[2])*Log[1 + Sqrt[2 - b]*x + x^2])/(4*Sqrt[2 - b])

Rule 1169

Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r + (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 628

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := S
 imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[2*c*d - b*e, 0]

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{2-x^2}}{1+bx^2+x^4} dx &= \frac{\int \frac{\sqrt{2}\sqrt{2-b-(1+\sqrt{2})x}}{1-\sqrt{2-bx+x^2}} dx}{2\sqrt{2-b}} + \frac{\int \frac{\sqrt{2}\sqrt{2-b+(1+\sqrt{2})x}}{1+\sqrt{2-bx+x^2}} dx}{2\sqrt{2-b}} \\ &= \frac{1}{4}(-1+\sqrt{2}) \int \frac{1}{1-\sqrt{2-bx+x^2}} dx + \frac{1}{4}(-1+\sqrt{2}) \int \frac{1}{1+\sqrt{2-bx+x^2}} dx - \frac{(1+\sqrt{2}) \int \frac{-\sqrt{2-bx+x^2}}{1-\sqrt{2-bx+x^2}} dx}{4\sqrt{2-b}} \\ &= -\frac{(1+\sqrt{2}) \log(1-\sqrt{2-bx+x^2})}{4\sqrt{2-b}} + \frac{(1+\sqrt{2}) \log(1+\sqrt{2-bx+x^2})}{4\sqrt{2-b}} + \frac{1}{2}(1-\sqrt{2}) \text{Subst}\left(\int \frac{-\sqrt{2-bx+x^2}}{1-\sqrt{2-bx+x^2}} dx, \sqrt{2-bx+x^2}\right) \\ &= \frac{(1-\sqrt{2}) \tan^{-1}\left(\frac{\sqrt{2-b-2x}}{\sqrt{2+b}}\right)}{2\sqrt{2+b}} - \frac{(1-\sqrt{2}) \tan^{-1}\left(\frac{\sqrt{2-b+2x}}{\sqrt{2+b}}\right)}{2\sqrt{2+b}} - \frac{(1+\sqrt{2}) \log(1-\sqrt{2-bx+x^2})}{4\sqrt{2-b}} + \frac{(1+\sqrt{2}) \log(1+\sqrt{2-bx+x^2})}{4\sqrt{2-b}} \end{aligned}$$

Mathematica [A] time = 0.0911318, size = 137, normalized size = 0.86

$$\frac{\frac{(-\sqrt{b^2-4+b+2\sqrt{2}}) \tan^{-1}\left(\frac{\sqrt{2x}}{\sqrt{b-\sqrt{b^2-4}}}\right)}{\sqrt{b-\sqrt{b^2-4}}} - \frac{(\sqrt{b^2-4+b+2\sqrt{2}}) \tan^{-1}\left(\frac{\sqrt{2x}}{\sqrt{\sqrt{b^2-4}+b}}\right)}{\sqrt{\sqrt{b^2-4}+b}}}{\sqrt{2}\sqrt{b^2-4}}$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[2] - x^2)/(1 + b*x^2 + x^4), x]

[Out] (((2*Sqrt[2] + b - Sqrt[-4 + b^2])*ArcTan[(Sqrt[2]*x)/Sqrt[b - Sqrt[-4 + b^2]]])/Sqrt[b - Sqrt[-4 + b^2]] - ((2*Sqrt[2] + b + Sqrt[-4 + b^2])*ArcTan[(Sqrt[2]*x)/Sqrt[b + Sqrt[-4 + b^2]]])/Sqrt[b + Sqrt[-4 + b^2]])/(Sqrt[2]*Sqrt[-4 + b^2])

Maple [B] time = 0.106, size = 285, normalized size = 1.8

$$-\arctan\left(2 \frac{x}{\sqrt{2}\sqrt{(-2+b)(2+b)}+2b}\right) \frac{1}{\sqrt{2}\sqrt{(-2+b)(2+b)}+2b} - b \arctan\left(2 \frac{x}{\sqrt{2}\sqrt{(-2+b)(2+b)}+2b}\right) \frac{1}{\sqrt{(-2+b)(2+b)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-x^2+2^(1/2))/(x^4+b*x^2+1), x)

[Out] -1/(2*((-2+b)*(2+b))^(1/2)+2*b)^(1/2)*arctan(2*x/(2*((-2+b)*(2+b))^(1/2)+2*b)^(1/2))-1/((-2+b)*(2+b))^(1/2)/(2*((-2+b)*(2+b))^(1/2)+2*b)^(1/2)*arctan(2*x/(2*((-2+b)*(2+b))^(1/2)+2*b)^(1/2))*b-2/((-2+b)*(2+b))^(1/2)/(2*((-2+b)*(2+b))^(1/2)+2*b)^(1/2)*arctan(2*x/(2*((-2+b)*(2+b))^(1/2)+2*b)^(1/2))*2^(1/2)-1/(-2*((-2+b)*(2+b))^(1/2)+2*b)^(1/2)*arctan(2*x/(-2*((-2+b)*(2+b))^(1/2)+2*b)^(1/2))+1/((-2+b)*(2+b))^(1/2)/(-2*((-2+b)*(2+b))^(1/2)+2*b)^(1/2)*arctan(2*x/(-2*((-2+b)*(2+b))^(1/2)+2*b)^(1/2))*b+2/((-2+b)*(2+b))^(1/2)/(-

$$2*((-2+b)*(2+b))^{(1/2)+2*b}^{(1/2)}*\arctan(2*x/((-2*((-2+b)*(2+b))^{(1/2)+2*b})^{(1/2)}))*2^{(1/2)}$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^2 - \sqrt{2}}{x^4 + bx^2 + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2+2^(1/2))/(x^4+b*x^2+1),x, algorithm="maxima")

[Out] -integrate((x^2 - sqrt(2))/(x^4 + b*x^2 + 1), x)

Fricas [B] time = 2.08545, size = 1226, normalized size = 7.66

$$-\frac{1}{2} \sqrt{\frac{1}{2}} \sqrt{-\frac{3b+4\sqrt{2}+\sqrt{b^2-4}}{b^2-4}} \log\left(\frac{1}{2} (2b+3\sqrt{2})x + \frac{1}{2} \sqrt{\frac{1}{2}} \left(b^2 - \frac{b^3 + \sqrt{2}b^2 - 4b - 4\sqrt{2}}{\sqrt{b^2-4}} - 4\right) \sqrt{-\frac{3b+4\sqrt{2}+\sqrt{b^2-4}}{b^2-4}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2+2^(1/2))/(x^4+b*x^2+1),x, algorithm="fricas")

[Out] -1/2*sqrt(1/2)*sqrt(-(3*b + 4*sqrt(2) + sqrt(b^2 - 4))/(b^2 - 4))*log(1/2*(2*b + 3*sqrt(2))*x + 1/2*sqrt(1/2)*(b^2 - (b^3 + sqrt(2)*b^2 - 4*b - 4*sqrt(2))/sqrt(b^2 - 4) - 4)*sqrt(-(3*b + 4*sqrt(2) + sqrt(b^2 - 4))/(b^2 - 4))) + 1/2*sqrt(1/2)*sqrt(-(3*b + 4*sqrt(2) + sqrt(b^2 - 4))/(b^2 - 4))*log(1/2*(2*b + 3*sqrt(2))*x - 1/2*sqrt(1/2)*(b^2 - (b^3 + sqrt(2)*b^2 - 4*b - 4*sqrt(2))/sqrt(b^2 - 4) - 4)*sqrt(-(3*b + 4*sqrt(2) + sqrt(b^2 - 4))/(b^2 - 4))) - 1/2*sqrt(1/2)*sqrt(-(3*b + 4*sqrt(2) - sqrt(b^2 - 4))/(b^2 - 4))*log(1/2*(2*b + 3*sqrt(2))*x + 1/2*sqrt(1/2)*(b^2 + (b^3 + sqrt(2)*b^2 - 4*b - 4*sqrt(2))/sqrt(b^2 - 4) - 4)*sqrt(-(3*b + 4*sqrt(2) - sqrt(b^2 - 4))/(b^2 - 4))) + 1/2*sqrt(1/2)*sqrt(-(3*b + 4*sqrt(2) - sqrt(b^2 - 4))/(b^2 - 4))*log(1/2*(2*b + 3*sqrt(2))*x - 1/2*sqrt(1/2)*(b^2 + (b^3 + sqrt(2)*b^2 - 4*b - 4*sqrt(2))/sqrt(b^2 - 4) - 4)*sqrt(-(3*b + 4*sqrt(2) - sqrt(b^2 - 4))/(b^2 - 4)))

Sympy [B] time = 1.80991, size = 332, normalized size = 2.08

$$-\text{RootSum}\left(t^4(16b^4 - 128b^2 + 256) + t^2(12b^3 + 16\sqrt{2}b^2 - 48b - 64\sqrt{2}) + 2b^2 + 6\sqrt{2}b + 9, \left(t \mapsto t \log\left(\frac{t^3(64b^{12} + 672b^{11} + 5760b^{10} + 12064\sqrt{2}b^9 + 17744b^8 - 27480\sqrt{2}b^7 - 154608b^6 - 141376\sqrt{2}b^5 - 69072b^4 + 672b^3 + 16\sqrt{2}b^2 - 48b - 64\sqrt{2})}{64b^{12} + 672b^{11} + 5760b^{10} + 12064\sqrt{2}b^9 + 17744b^8 - 27480\sqrt{2}b^7 - 154608b^6 - 141376\sqrt{2}b^5 - 69072b^4 + 672b^3 + 16\sqrt{2}b^2 - 48b - 64\sqrt{2}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x**2+2**(1/2))/(x**4+b*x**2+1),x)

[Out] -RootSum(_t**4*(16*b**4 - 128*b**2 + 256) + _t**2*(12*b**3 + 16*sqrt(2)*b**2 - 48*b - 64*sqrt(2)) + 2*b**2 + 6*sqrt(2)*b + 9, Lambda(_t, _t*log(_t**3*(64*b**12 + 672*sqrt(2)*b**11 + 5760*b**10 + 12064*sqrt(2)*b**9 + 17744*b**8 - 27480*sqrt(2)*b**7 - 154608*b**6 - 141376*sqrt(2)*b**5 - 69072*b**4 + 672*b**3 + 16*sqrt(2)*b**2 - 48*b - 64*sqrt(2))/(64*b**12 + 672*sqrt(2)*b**11 + 5760*b**10 + 12064*sqrt(2)*b**9 + 17744*b**8 - 27480*sqrt(2)*b**7 - 154608*b**6 - 141376*sqrt(2)*b**5 - 69072*b**4 + 672*b**3 + 16*sqrt(2)*b**2 - 48*b - 64*sqrt(2)))

$$\frac{1704\sqrt{2}b^3 + 78192b^2 - 2592\sqrt{2}b - 15552}{(8b^{10} + 88\sqrt{2}b^9 + 828b^8 + 2144\sqrt{2}b^7 + 6470b^6 + 5310\sqrt{2}b^5 + 2781b^4 - 2322\sqrt{2}b^3 - 3402b^2 + 729) + _t(16b^7 + 116\sqrt{2}b^6 + 668b^5 + 942\sqrt{2}b^4 + 1226b^3 + 144\sqrt{2}b^2 - 378b - 108\sqrt{2})} / (4b^6 + 28\sqrt{2}b^5 + 152b^4 + 192\sqrt{2}b^3 + 189b^2 - 27\sqrt{2}b - 81) + x$$

Giac [C] time = 1.25828, size = 2847, normalized size = 17.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2+2^(1/2))/(x^4+b*x^2+1),x, algorithm="giac")

[Out]
$$\begin{aligned} & -1/2*(3*(b^2 + \sqrt{b^2 - 4})b - 4)*\cos(5/4*\pi + 1/2*\text{real_part}(\arcsin(1/2*b)))^2*\cosh(1/2*\text{imag_part}(\arcsin(1/2*b)))^3*\sin(5/4*\pi + 1/2*\text{real_part}(\arcsin(1/2*b))) \\ & - (b^2 + \sqrt{b^2 - 4})b - 4)*\cosh(1/2*\text{imag_part}(\arcsin(1/2*b)))^3*\sin(5/4*\pi + 1/2*\text{real_part}(\arcsin(1/2*b)))^3 - 9*(b^2 + \sqrt{b^2 - 4})b \\ & - 4)*\cos(5/4*\pi + 1/2*\text{real_part}(\arcsin(1/2*b)))^2*\cosh(1/2*\text{imag_part}(\arcsin(1/2*b)))^2*\sin(5/4*\pi + 1/2*\text{real_part}(\arcsin(1/2*b))) \\ & *\sinh(1/2*\text{imag_part}(\arcsin(1/2*b))) + 3*(b^2 + \sqrt{b^2 - 4})b - 4)*\cosh(1/2*\text{imag_part}(\arcsin(1/2*b)))^2*\sin(5/4*\pi + 1/2*\text{real_part}(\arcsin(1/2*b)))^3 \\ & *\sinh(1/2*\text{imag_part}(\arcsin(1/2*b))) + 9*(b^2 + \sqrt{b^2 - 4})b - 4)*\cos(5/4*\pi + 1/2*\text{real_part}(\arcsin(1/2*b)))^2*\cosh(1/2*\text{imag_part}(\arcsin(1/2*b))) \\ & *\sin(5/4*\pi + 1/2*\text{real_part}(\arcsin(1/2*b)))^2*\sinh(1/2*\text{imag_part}(\arcsin(1/2*b)))^2 - 3*(b^2 + \sqrt{b^2 - 4})b - 4)*\cosh(1/2*\text{imag_part}(\arcsin(1/2*b))) \\ & *\sin(5/4*\pi + 1/2*\text{real_part}(\arcsin(1/2*b)))^3*\sinh(1/2*\text{imag_part}(\arcsin(1/2*b)))^2 - 3*(b^2 + \sqrt{b^2 - 4})b - 4)*\cos(5/4*\pi + 1/2*\text{real_part}(\arcsin(1/2*b)))^2 \\ & *\sin(5/4*\pi + 1/2*\text{real_part}(\arcsin(1/2*b)))^2*\sinh(1/2*\text{imag_part}(\arcsin(1/2*b)))^3 + (b^2 + \sqrt{b^2 - 4})b - 4)*\sin(5/4*\pi + 1/2*\text{real_part}(\arcsin(1/2*b)))^3 \\ & *\sinh(1/2*\text{imag_part}(\arcsin(1/2*b)))^3 - (\sqrt{2}*b^2 + \sqrt{2}*\sqrt{b^2 - 4})b - 4*\sqrt{2})*\cosh(1/2*\text{imag_part}(\arcsin(1/2*b)))*\sin(5/4*\pi + 1/2*\text{real_part}(\arcsin(1/2*b))) \\ & + (\sqrt{2}*b^2 + \sqrt{2}*\sqrt{b^2 - 4})b - 4*\sqrt{2})*\sin(5/4*\pi + 1/2*\text{real_part}(\arcsin(1/2*b)))*\sinh(1/2*\text{imag_part}(\arcsin(1/2*b)))*\arctan((x - \cos(5/4*\pi + 1/2*\arcsin(1/2*b)))/\sin(5/4*\pi + 1/2*\arcsin(1/2*b)))/(b^2 - 4) \\ & - 1/2*(3*(b^2 + \sqrt{b^2 - 4})b - 4)*\cos(1/4*\pi + 1/2*\text{real_part}(\arcsin(1/2*b)))^2*\cosh(1/2*\text{imag_part}(\arcsin(1/2*b)))^3*\sin(1/4*\pi + 1/2*\text{real_part}(\arcsin(1/2*b))) \\ & - (b^2 + \sqrt{b^2 - 4})b - 4)*\cosh(1/2*\text{imag_part}(\arcsin(1/2*b)))^3*\sin(1/4*\pi + 1/2*\text{real_part}(\arcsin(1/2*b)))^3 - 9*(b^2 + \sqrt{b^2 - 4})b - 4)*\cos(1/4*\pi + 1/2*\text{real_part}(\arcsin(1/2*b)))^2 \\ & *\cosh(1/2*\text{imag_part}(\arcsin(1/2*b)))^2*\sin(1/4*\pi + 1/2*\text{real_part}(\arcsin(1/2*b)))*\sinh(1/2*\text{imag_part}(\arcsin(1/2*b))) + 3*(b^2 + \sqrt{b^2 - 4})b - 4)*\cosh(1/2*\text{imag_part}(\arcsin(1/2*b)))^2 \\ & *\sin(1/4*\pi + 1/2*\text{real_part}(\arcsin(1/2*b)))^3*\sinh(1/2*\text{imag_part}(\arcsin(1/2*b))) + 9*(b^2 + \sqrt{b^2 - 4})b - 4)*\cos(1/4*\pi + 1/2*\text{real_part}(\arcsin(1/2*b)))^2 \\ & *\cosh(1/2*\text{imag_part}(\arcsin(1/2*b)))*\sin(1/4*\pi + 1/2*\text{real_part}(\arcsin(1/2*b)))^2*\sinh(1/2*\text{imag_part}(\arcsin(1/2*b)))^2 - 3*(b^2 + \sqrt{b^2 - 4})b - 4)*\cosh(1/2*\text{imag_part}(\arcsin(1/2*b))) \\ & *\sin(1/4*\pi + 1/2*\text{real_part}(\arcsin(1/2*b)))^3*\sinh(1/2*\text{imag_part}(\arcsin(1/2*b)))^2 - 3*(b^2 + \sqrt{b^2 - 4})b - 4)*\cos(1/4*\pi + 1/2*\text{real_part}(\arcsin(1/2*b)))^2 \\ & *\sin(1/4*\pi + 1/2*\text{real_part}(\arcsin(1/2*b)))*\sinh(1/2*\text{imag_part}(\arcsin(1/2*b)))^3 + (b^2 + \sqrt{b^2 - 4})b - 4)*\sin(1/4*\pi + 1/2*\text{real_part}(\arcsin(1/2*b)))^3 \\ & *\sinh(1/2*\text{imag_part}(\arcsin(1/2*b)))^3 - (\sqrt{2}*b^2 + \sqrt{2}*\sqrt{b^2 - 4})b - 4*\sqrt{2})*\cosh(1/2*\text{imag_part}(\arcsin(1/2*b)))*\sin(1/4*\pi + 1/2*\text{real_part}(\arcsin(1/2*b))) \\ & + (\sqrt{2}*b^2 + \sqrt{2}*\sqrt{b^2 - 4})b - 4*\sqrt{2})*\sin(1/4*\pi + 1/2*\text{real_part}(\arcsin(1/2*b)))*\sinh(1/2*\text{imag_part}(\arcsin(1/2*b)))*\arctan((x - \cos(1/4*\pi + 1/2*\arcsin(1/2*b)))/\sin(1/4*\pi + 1/2*\arcsin(1/2*b)))/(b^2 \end{aligned}$$

3.105 $\int \frac{\sqrt{2+x^2}}{1+bx^2+x^4} dx$

Optimal. Leaf size=160

$$\frac{(1-\sqrt{2})\log(-\sqrt{2-b}x+x^2+1)}{4\sqrt{2-b}} - \frac{(1-\sqrt{2})\log(\sqrt{2-b}x+x^2+1)}{4\sqrt{2-b}} - \frac{(1+\sqrt{2})\tan^{-1}\left(\frac{\sqrt{2-b-2x}}{\sqrt{b+2}}\right)}{2\sqrt{b+2}} + \frac{(1+\sqrt{2})\tan^{-1}\left(\frac{\sqrt{2-b+2x}}{\sqrt{b+2}}\right)}{2\sqrt{b+2}}$$

```
[Out] -((1 + Sqrt[2])*ArcTan[(Sqrt[2 - b] - 2*x)/Sqrt[2 + b]])/(2*Sqrt[2 + b]) +
((1 + Sqrt[2])*ArcTan[(Sqrt[2 - b] + 2*x)/Sqrt[2 + b]])/(2*Sqrt[2 + b]) + (
(1 - Sqrt[2])*Log[1 - Sqrt[2 - b]*x + x^2])/(4*Sqrt[2 - b]) - ((1 - Sqrt[2]
)*Log[1 + Sqrt[2 - b]*x + x^2])/(4*Sqrt[2 - b])
```

Rubi [A] time = 0.103267, antiderivative size = 160, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {1169, 634, 618, 204, 628}

$$\frac{(1-\sqrt{2})\log(-\sqrt{2-b}x+x^2+1)}{4\sqrt{2-b}} - \frac{(1-\sqrt{2})\log(\sqrt{2-b}x+x^2+1)}{4\sqrt{2-b}} - \frac{(1+\sqrt{2})\tan^{-1}\left(\frac{\sqrt{2-b-2x}}{\sqrt{b+2}}\right)}{2\sqrt{b+2}} + \frac{(1+\sqrt{2})\tan^{-1}\left(\frac{\sqrt{2-b+2x}}{\sqrt{b+2}}\right)}{2\sqrt{b+2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[2] + x^2)/(1 + b*x^2 + x^4), x]
```

```
[Out] -((1 + Sqrt[2])*ArcTan[(Sqrt[2 - b] - 2*x)/Sqrt[2 + b]])/(2*Sqrt[2 + b]) +
((1 + Sqrt[2])*ArcTan[(Sqrt[2 - b] + 2*x)/Sqrt[2 + b]])/(2*Sqrt[2 + b]) + (
(1 - Sqrt[2])*Log[1 - Sqrt[2 - b]*x + x^2])/(4*Sqrt[2 - b]) - ((1 - Sqrt[2]
)*Log[1 + Sqrt[2 - b]*x + x^2])/(4*Sqrt[2 - b])
```

Rule 1169

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (b_)*(x_)^2 + (c_)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
(d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 634

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int
[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\sqrt{2} + x^2}{1 + bx^2 + x^4} dx = \frac{\int \frac{\sqrt{2}\sqrt{2-b} - (-1+\sqrt{2})x}{1-\sqrt{2-b}x+x^2} dx}{2\sqrt{2-b}} + \frac{\int \frac{\sqrt{2}\sqrt{2-b} + (-1+\sqrt{2})x}{1+\sqrt{2-b}x+x^2} dx}{2\sqrt{2-b}}$$

$$= \frac{1}{4} (1 + \sqrt{2}) \int \frac{1}{1 - \sqrt{2-b}x + x^2} dx + \frac{1}{4} (1 + \sqrt{2}) \int \frac{1}{1 + \sqrt{2-b}x + x^2} dx + \frac{(1 - \sqrt{2}) \int \frac{-\sqrt{2-b}+2x}{1-\sqrt{2-b}x+x^2} dx}{4\sqrt{2-b}}$$

$$= \frac{(1 - \sqrt{2}) \log(1 - \sqrt{2-b}x + x^2)}{4\sqrt{2-b}} - \frac{(1 - \sqrt{2}) \log(1 + \sqrt{2-b}x + x^2)}{4\sqrt{2-b}} + \frac{1}{2} (-1 - \sqrt{2}) \text{Subst} \left(\int \frac{-2}{-2 - \sqrt{2-b}x} dx \right)$$

$$= -\frac{(1 + \sqrt{2}) \tan^{-1} \left(\frac{\sqrt{2-b}-2x}{\sqrt{2+b}} \right)}{2\sqrt{2+b}} + \frac{(1 + \sqrt{2}) \tan^{-1} \left(\frac{\sqrt{2-b}+2x}{\sqrt{2+b}} \right)}{2\sqrt{2+b}} + \frac{(1 - \sqrt{2}) \log(1 - \sqrt{2-b}x + x^2)}{4\sqrt{2-b}} - \frac{(1 - \sqrt{2}) \log(1 + \sqrt{2-b}x + x^2)}{4\sqrt{2-b}}$$

Mathematica [A] time = 0.0570214, size = 136, normalized size = 0.85

$$\frac{\left(\sqrt{b^2-4-b+2\sqrt{2}}\right) \tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt{b-\sqrt{b^2-4}}}\right)}{\sqrt{b-\sqrt{b^2-4}}} + \frac{\left(\sqrt{b^2-4+b-2\sqrt{2}}\right) \tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt{\sqrt{b^2-4}+b}}\right)}{\sqrt{\sqrt{b^2-4}+b}}$$

$$\frac{\hspace{10em}}{\sqrt{2}\sqrt{b^2-4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[2] + x^2)/(1 + b*x^2 + x^4), x]
```

```
[Out] (((2*Sqrt[2] - b + Sqrt[-4 + b^2])*ArcTan[(Sqrt[2]*x)/Sqrt[b - Sqrt[-4 + b^2]]])/Sqrt[b - Sqrt[-4 + b^2]] + ((-2*Sqrt[2] + b + Sqrt[-4 + b^2])*ArcTan[(Sqrt[2]*x)/Sqrt[b + Sqrt[-4 + b^2]]])/Sqrt[b + Sqrt[-4 + b^2]])/(Sqrt[2]*Sqrt[-4 + b^2])
```

Maple [B] time = 0.103, size = 283, normalized size = 1.8

$$\arctan\left(2 \frac{x}{\sqrt{2} \sqrt{(-2+b)(2+b)+2b}}\right) \frac{1}{\sqrt{2} \sqrt{(-2+b)(2+b)+2b}} + b \arctan\left(2 \frac{x}{\sqrt{2} \sqrt{(-2+b)(2+b)+2b}}\right) \frac{1}{\sqrt{(-2+b)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+2^(1/2))/(x^4+b*x^2+1), x)
```

```
[Out] 1/(2*((-2+b)*(2+b))^(1/2)+2*b)^(1/2)*arctan(2*x/(2*((-2+b)*(2+b))^(1/2)+2*b)^(1/2))+1/((-2+b)*(2+b))^(1/2)/(2*((-2+b)*(2+b))^(1/2)+2*b)^(1/2)*arctan(2*x/(2*((-2+b)*(2+b))^(1/2)+2*b)^(1/2))*b-2/((-2+b)*(2+b))^(1/2)/(2*((-2+b)*(2+b))^(1/2)+2*b)^(1/2)*arctan(2*x/(2*((-2+b)*(2+b))^(1/2)+2*b)^(1/2))*2^(1/2)+1/(-2*((-2+b)*(2+b))^(1/2)+2*b)^(1/2)*arctan(2*x/(-2*((-2+b)*(2+b))^(1/2)+2*b)^(1/2))-1/((-2+b)*(2+b))^(1/2)/(-2*((-2+b)*(2+b))^(1/2)+2*b)^(1/2)*arctan(2*x/(-2*((-2+b)*(2+b))^(1/2)+2*b)^(1/2))*b+2/((-2+b)*(2+b))^(1/2)/(-2
```

$*((-2+b)*(2+b))^{(1/2)+2*b}^{(1/2)}*\arctan(2*x/((-2+b)*(2+b))^{(1/2)+2*b}^{(1/2)})*2^{(1/2)}$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 + \sqrt{2}}{x^4 + bx^2 + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+2^(1/2))/(x^4+b*x^2+1),x, algorithm="maxima")

[Out] integrate((x^2 + sqrt(2))/(x^4 + b*x^2 + 1), x)

Fricas [B] time = 2.06054, size = 1224, normalized size = 7.65

$$\frac{1}{2} \sqrt{\frac{1}{2}} \sqrt{-\frac{3b-4\sqrt{2}+\sqrt{b^2-4}}{b^2-4}} \log\left(\frac{1}{2}(2b-3\sqrt{2})x + \frac{1}{2} \sqrt{\frac{1}{2}} \left(b^2 - \frac{b^3 - \sqrt{2}b^2 - 4b + 4\sqrt{2}}{\sqrt{b^2-4}} - 4\right) \sqrt{-\frac{3b-4\sqrt{2}+\sqrt{b^2-4}}{b^2-4}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+2^(1/2))/(x^4+b*x^2+1),x, algorithm="fricas")

[Out] 1/2*sqrt(1/2)*sqrt(-(3*b - 4*sqrt(2) + sqrt(b^2 - 4))/(b^2 - 4))*log(1/2*(2*b - 3*sqrt(2))*x + 1/2*sqrt(1/2)*(b^2 - (b^3 - sqrt(2)*b^2 - 4*b + 4*sqrt(2))/sqrt(b^2 - 4) - 4)*sqrt(-(3*b - 4*sqrt(2) + sqrt(b^2 - 4))/(b^2 - 4))) - 1/2*sqrt(1/2)*sqrt(-(3*b - 4*sqrt(2) + sqrt(b^2 - 4))/(b^2 - 4))*log(1/2*(2*b - 3*sqrt(2))*x - 1/2*sqrt(1/2)*(b^2 - (b^3 - sqrt(2)*b^2 - 4*b + 4*sqrt(2))/sqrt(b^2 - 4) - 4)*sqrt(-(3*b - 4*sqrt(2) + sqrt(b^2 - 4))/(b^2 - 4))) + 1/2*sqrt(1/2)*sqrt(-(3*b - 4*sqrt(2) - sqrt(b^2 - 4))/(b^2 - 4))*log(1/2*(2*b - 3*sqrt(2))*x + 1/2*sqrt(1/2)*(b^2 + (b^3 - sqrt(2)*b^2 - 4*b + 4*sqrt(2))/sqrt(b^2 - 4) - 4)*sqrt(-(3*b - 4*sqrt(2) - sqrt(b^2 - 4))/(b^2 - 4))) - 1/2*sqrt(1/2)*sqrt(-(3*b - 4*sqrt(2) - sqrt(b^2 - 4))/(b^2 - 4))*log(1/2*(2*b - 3*sqrt(2))*x - 1/2*sqrt(1/2)*(b^2 + (b^3 - sqrt(2)*b^2 - 4*b + 4*sqrt(2))/sqrt(b^2 - 4) - 4)*sqrt(-(3*b - 4*sqrt(2) - sqrt(b^2 - 4))/(b^2 - 4)))

Sympy [B] time = 1.76086, size = 330, normalized size = 2.06

$$\text{RootSum}\left(t^4(16b^4 - 128b^2 + 256) + t^2(12b^3 - 16\sqrt{2}b^2 - 48b + 64\sqrt{2}) + 2b^2 - 6\sqrt{2}b + 9, \left(t \mapsto t \log\left(\frac{t^3(64b^{12} - 672b^{11} + 5760b^{10} - 12064\sqrt{2}b^9 + 17744b^8 + 27480\sqrt{2}b^7 - 154608b^6 + 141376\sqrt{2}b^5 - 69072b^4 - 6144b^3 + 128b^2 + 256)}{t^4(16b^4 - 128b^2 + 256) + t^2(12b^3 - 16\sqrt{2}b^2 - 48b + 64\sqrt{2}) + 2b^2 - 6\sqrt{2}b + 9}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**2+2**(1/2))/(x**4+b*x**2+1),x)

[Out] RootSum(_t**4*(16*b**4 - 128*b**2 + 256) + _t**2*(12*b**3 - 16*sqrt(2)*b**2 - 48*b + 64*sqrt(2)) + 2*b**2 - 6*sqrt(2)*b + 9, Lambda(_t, _t*log(_t**3*(64*b**12 - 672*sqrt(2)*b**11 + 5760*b**10 - 12064*sqrt(2)*b**9 + 17744*b**8 + 27480*sqrt(2)*b**7 - 154608*b**6 + 141376*sqrt(2)*b**5 - 69072*b**4 - 6144*b**3 + 128*b**2 + 256)/(_t**4*(16*b**4 - 128*b**2 + 256) + _t**2*(12*b**3 - 16*sqrt(2)*b**2 - 48*b + 64*sqrt(2)) + 2*b**2 - 6*sqrt(2)*b + 9))

```
704*sqrt(2)*b**3 + 78192*b**2 + 2592*sqrt(2)*b - 15552)/(8*b**10 - 88*sqrt(
2)*b**9 + 828*b**8 - 2144*sqrt(2)*b**7 + 6470*b**6 - 5310*sqrt(2)*b**5 + 27
81*b**4 + 2322*sqrt(2)*b**3 - 3402*b**2 + 729) + _t*(16*b**7 - 116*sqrt(2)*
b**6 + 668*b**5 - 942*sqrt(2)*b**4 + 1226*b**3 - 144*sqrt(2)*b**2 - 378*b +
108*sqrt(2))/(4*b**6 - 28*sqrt(2)*b**5 + 152*b**4 - 192*sqrt(2)*b**3 + 189
*b**2 + 27*sqrt(2)*b - 81) + x)))
```

Giac [C] time = 1.25839, size = 2847, normalized size = 17.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2^(1/2))/(x^4+b*x^2+1),x, algorithm="giac")
```

```
[Out] 1/2*(3*(b^2 + sqrt(b^2 - 4)*b - 4)*cos(5/4*pi + 1/2*real_part(arcsin(1/2*b)
))^2*cosh(1/2*imag_part(arcsin(1/2*b)))^3*sin(5/4*pi + 1/2*real_part(arcsin
(1/2*b))) - (b^2 + sqrt(b^2 - 4)*b - 4)*cosh(1/2*imag_part(arcsin(1/2*b)))^
3*sin(5/4*pi + 1/2*real_part(arcsin(1/2*b)))^3 - 9*(b^2 + sqrt(b^2 - 4)*b -
4)*cos(5/4*pi + 1/2*real_part(arcsin(1/2*b)))^2*cosh(1/2*imag_part(arcsin(
1/2*b)))^2*sin(5/4*pi + 1/2*real_part(arcsin(1/2*b)))*sinh(1/2*imag_part(ar
csin(1/2*b))) + 3*(b^2 + sqrt(b^2 - 4)*b - 4)*cosh(1/2*imag_part(arcsin(1/2
*b)))^2*sin(5/4*pi + 1/2*real_part(arcsin(1/2*b)))^3*sinh(1/2*imag_part(arc
sin(1/2*b))) + 9*(b^2 + sqrt(b^2 - 4)*b - 4)*cos(5/4*pi + 1/2*real_part(arc
sin(1/2*b)))^2*cosh(1/2*imag_part(arcsin(1/2*b)))*sin(5/4*pi + 1/2*real_par
t(arcsin(1/2*b)))*sinh(1/2*imag_part(arcsin(1/2*b)))^2 - 3*(b^2 + sqrt(b^2
- 4)*b - 4)*cosh(1/2*imag_part(arcsin(1/2*b)))*sin(5/4*pi + 1/2*real_part(a
rcsin(1/2*b)))^3*sinh(1/2*imag_part(arcsin(1/2*b)))^2 - 3*(b^2 + sqrt(b^2 -
4)*b - 4)*cos(5/4*pi + 1/2*real_part(arcsin(1/2*b)))^2*sin(5/4*pi + 1/2*re
al_part(arcsin(1/2*b)))*sinh(1/2*imag_part(arcsin(1/2*b)))^3 + (b^2 + sqrt(
b^2 - 4)*b - 4)*sin(5/4*pi + 1/2*real_part(arcsin(1/2*b)))^3*sinh(1/2*imag_
part(arcsin(1/2*b)))^3 + (sqrt(2)*b^2 + sqrt(2)*sqrt(b^2 - 4)*b - 4*sqrt(2)
)*cosh(1/2*imag_part(arcsin(1/2*b)))*sin(5/4*pi + 1/2*real_part(arcsin(1/2*
b))) - (sqrt(2)*b^2 + sqrt(2)*sqrt(b^2 - 4)*b - 4*sqrt(2))*sin(5/4*pi + 1/2
*real_part(arcsin(1/2*b)))*sinh(1/2*imag_part(arcsin(1/2*b))))*arctan((x -
cos(5/4*pi + 1/2*arcsin(1/2*b)))/sin(5/4*pi + 1/2*arcsin(1/2*b)))/(b^2 - 4)
+ 1/2*(3*(b^2 + sqrt(b^2 - 4)*b - 4)*cos(1/4*pi + 1/2*real_part(arcsin(1/2
*b)))^2*cosh(1/2*imag_part(arcsin(1/2*b)))^3*sin(1/4*pi + 1/2*real_part(arc
sin(1/2*b))) - (b^2 + sqrt(b^2 - 4)*b - 4)*cosh(1/2*imag_part(arcsin(1/2*b)
))^3*sin(1/4*pi + 1/2*real_part(arcsin(1/2*b)))^3 - 9*(b^2 + sqrt(b^2 - 4)*
b - 4)*cos(1/4*pi + 1/2*real_part(arcsin(1/2*b)))^2*cosh(1/2*imag_part(arcs
in(1/2*b)))^2*sin(1/4*pi + 1/2*real_part(arcsin(1/2*b)))*sinh(1/2*imag_part
(arcsin(1/2*b))) + 3*(b^2 + sqrt(b^2 - 4)*b - 4)*cosh(1/2*imag_part(arcsin(
1/2*b)))^2*sin(1/4*pi + 1/2*real_part(arcsin(1/2*b)))^3*sinh(1/2*imag_part(
arcsin(1/2*b))) + 9*(b^2 + sqrt(b^2 - 4)*b - 4)*cos(1/4*pi + 1/2*real_part(
arcsin(1/2*b)))^2*cosh(1/2*imag_part(arcsin(1/2*b)))*sin(1/4*pi + 1/2*real_
part(arcsin(1/2*b)))*sinh(1/2*imag_part(arcsin(1/2*b)))^2 - 3*(b^2 + sqrt(b
^2 - 4)*b - 4)*cosh(1/2*imag_part(arcsin(1/2*b)))*sin(1/4*pi + 1/2*real_par
t(arcsin(1/2*b)))^3*sinh(1/2*imag_part(arcsin(1/2*b)))^2 - 3*(b^2 + sqrt(b^
2 - 4)*b - 4)*cos(1/4*pi + 1/2*real_part(arcsin(1/2*b)))^2*sin(1/4*pi + 1/2
*real_part(arcsin(1/2*b)))*sinh(1/2*imag_part(arcsin(1/2*b)))^3 + (b^2 + sq
rt(b^2 - 4)*b - 4)*sin(1/4*pi + 1/2*real_part(arcsin(1/2*b)))^3*sinh(1/2*im
ag_part(arcsin(1/2*b)))^3 + (sqrt(2)*b^2 + sqrt(2)*sqrt(b^2 - 4)*b - 4*sqrt
(2))*cosh(1/2*imag_part(arcsin(1/2*b)))*sin(1/4*pi + 1/2*real_part(arcsin(1
/2*b))) - (sqrt(2)*b^2 + sqrt(2)*sqrt(b^2 - 4)*b - 4*sqrt(2))*sin(1/4*pi +
1/2*real_part(arcsin(1/2*b)))*sinh(1/2*imag_part(arcsin(1/2*b))))*arctan((x
- cos(1/4*pi + 1/2*arcsin(1/2*b)))/sin(1/4*pi + 1/2*arcsin(1/2*b)))/(b^2 -
```


3.106 $\int \frac{2a-x^2}{a^2-ax^2+x^4} dx$

Optimal. Leaf size=114

$$-\frac{\sqrt{3} \log(-\sqrt{3}\sqrt{ax+a+x^2})}{4\sqrt{a}} + \frac{\sqrt{3} \log(\sqrt{3}\sqrt{ax+a+x^2})}{4\sqrt{a}} - \frac{\tan^{-1}\left(\sqrt{3} - \frac{2x}{\sqrt{a}}\right)}{2\sqrt{a}} + \frac{\tan^{-1}\left(\frac{2x}{\sqrt{a}} + \sqrt{3}\right)}{2\sqrt{a}}$$

[Out] -ArcTan[Sqrt[3] - (2*x)/Sqrt[a]]/(2*Sqrt[a]) + ArcTan[Sqrt[3] + (2*x)/Sqrt[a]]/(2*Sqrt[a]) - (Sqrt[3]*Log[a - Sqrt[3]*Sqrt[a]*x + x^2])/(4*Sqrt[a]) + (Sqrt[3]*Log[a + Sqrt[3]*Sqrt[a]*x + x^2])/(4*Sqrt[a])

Rubi [A] time = 0.0760081, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$, Rules used = {1169, 634, 617, 204, 628}

$$-\frac{\sqrt{3} \log(-\sqrt{3}\sqrt{ax+a+x^2})}{4\sqrt{a}} + \frac{\sqrt{3} \log(\sqrt{3}\sqrt{ax+a+x^2})}{4\sqrt{a}} - \frac{\tan^{-1}\left(\sqrt{3} - \frac{2x}{\sqrt{a}}\right)}{2\sqrt{a}} + \frac{\tan^{-1}\left(\frac{2x}{\sqrt{a}} + \sqrt{3}\right)}{2\sqrt{a}}$$

Antiderivative was successfully verified.

[In] Int[(2*a - x^2)/(a^2 - a*x^2 + x^4), x]

[Out] -ArcTan[Sqrt[3] - (2*x)/Sqrt[a]]/(2*Sqrt[a]) + ArcTan[Sqrt[3] + (2*x)/Sqrt[a]]/(2*Sqrt[a]) - (Sqrt[3]*Log[a - Sqrt[3]*Sqrt[a]*x + x^2])/(4*Sqrt[a]) + (Sqrt[3]*Log[a + Sqrt[3]*Sqrt[a]*x + x^2])/(4*Sqrt[a])

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
(d - e*q)*x)/(q + r*x + x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 634

```
Int[((d_) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := S
 imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[2*c*d - b*e, 0]

Rubi steps

$$\begin{aligned} \int \frac{2a - x^2}{a^2 - ax^2 + x^4} dx &= \frac{\int \frac{2\sqrt{3}a^{3/2} - 3ax}{a - \sqrt{3}\sqrt{ax} + x^2} dx}{2\sqrt{3}a^{3/2}} + \frac{\int \frac{2\sqrt{3}a^{3/2} + 3ax}{a + \sqrt{3}\sqrt{ax} + x^2} dx}{2\sqrt{3}a^{3/2}} \\ &= \frac{1}{4} \int \frac{1}{a - \sqrt{3}\sqrt{ax} + x^2} dx + \frac{1}{4} \int \frac{1}{a + \sqrt{3}\sqrt{ax} + x^2} dx - \frac{\sqrt{3} \int \frac{-\sqrt{3}\sqrt{a} + 2x}{a - \sqrt{3}\sqrt{ax} + x^2} dx}{4\sqrt{a}} + \frac{\sqrt{3} \int \frac{\sqrt{3}\sqrt{a} + 2x}{a + \sqrt{3}\sqrt{ax} + x^2} dx}{4\sqrt{a}} \\ &= -\frac{\sqrt{3} \log(a - \sqrt{3}\sqrt{ax} + x^2)}{4\sqrt{a}} + \frac{\sqrt{3} \log(a + \sqrt{3}\sqrt{ax} + x^2)}{4\sqrt{a}} + \frac{\text{Subst}\left(\int \frac{1}{-\frac{1}{3} - x^2} dx, x, 1 - \frac{2x}{\sqrt{3}\sqrt{a}}\right)}{2\sqrt{3}\sqrt{a}} \\ &= -\frac{\tan^{-1}\left(\sqrt{3} - \frac{2x}{\sqrt{a}}\right)}{2\sqrt{a}} + \frac{\tan^{-1}\left(\sqrt{3} + \frac{2x}{\sqrt{a}}\right)}{2\sqrt{a}} - \frac{\sqrt{3} \log(a - \sqrt{3}\sqrt{ax} + x^2)}{4\sqrt{a}} + \frac{\sqrt{3} \log(a + \sqrt{3}\sqrt{ax} + x^2)}{4\sqrt{a}} \end{aligned}$$

Mathematica [C] time = 0.175998, size = 115, normalized size = 1.01

$$\frac{\sqrt[4]{-1} \left(\sqrt{\sqrt{3} - i} (\sqrt{3} - 3i) \tanh^{-1} \left(\frac{(1+i)x}{\sqrt{\sqrt{3} + i\sqrt{a}}} \right) - \sqrt{\sqrt{3} + i} (\sqrt{3} + 3i) \tan^{-1} \left(\frac{(1+i)x}{\sqrt{\sqrt{3} - i\sqrt{a}}} \right) \right)}{2\sqrt{6}\sqrt{a}}$$

Antiderivative was successfully verified.

[In] Integrate[(2*a - x^2)/(a^2 - a*x^2 + x^4), x]

[Out] ((-1)^(1/4)*(-(Sqrt[I + Sqrt[3]]*(3*I + Sqrt[3])*ArcTan[((1 + I)*x)/(Sqrt[-I + Sqrt[3]]*Sqrt[a]])) + Sqrt[-I + Sqrt[3]]*(-3*I + Sqrt[3])*ArcTanh[((1 + I)*x)/(Sqrt[I + Sqrt[3]]*Sqrt[a]])))/(2*Sqrt[6]*Sqrt[a])

Maple [A] time = 0.067, size = 92, normalized size = 0.8

$$\frac{\sqrt{3}}{4} \ln(a + x^2 + x\sqrt{3}\sqrt{a}) \frac{1}{\sqrt{a}} + \frac{1}{2} \arctan\left(\left(2x + \sqrt{3}\sqrt{a}\right) \frac{1}{\sqrt{a}}\right) \frac{1}{\sqrt{a}} - \frac{\sqrt{3}}{4} \ln(x\sqrt{3}\sqrt{a} - x^2 - a) \frac{1}{\sqrt{a}} - \frac{1}{2} \arctan\left(\left(\sqrt{3}\sqrt{a} - 2x\right) \frac{1}{\sqrt{a}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-x^2+2*a)/(x^4-a*x^2+a^2), x)

[Out] 1/4*ln(a+x^2+x*3^(1/2)*a^(1/2))*3^(1/2)/a^(1/2)+1/2/a^(1/2)*arctan((2*x+3^(1/2)*a^(1/2))/a^(1/2))-1/4/a^(1/2)*3^(1/2)*ln(x*3^(1/2)*a^(1/2)-x^2-a)-1/2/a^(1/2)*arctan((3^(1/2)*a^(1/2)-2*x)/a^(1/2))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^2 - 2a}{x^4 - ax^2 + a^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2+2*a)/(x^4-a*x^2+a^2),x, algorithm="maxima")

[Out] -integrate((x^2 - 2*a)/(x^4 - a*x^2 + a^2), x)

Fricas [B] time = 1.81704, size = 1616, normalized size = 14.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2+2*a)/(x^4-a*x^2+a^2),x, algorithm="fricas")

[Out] $\frac{1}{24}(\sqrt{3}a\sqrt{a^{-2}} + 2\sqrt{3})\sqrt{-4a\sqrt{a^{-2}} + 8}(a^{-2})^{1/4}\log(6a^2\sqrt{a^{-2}} + 6x^2 + (\sqrt{3}a^2\sqrt{a^{-2}})x + 2\sqrt{3}ax)\sqrt{-4a\sqrt{a^{-2}} + 8}(a^{-2})^{1/4}) - \frac{1}{24}(\sqrt{3}a\sqrt{a^{-2}} + 2\sqrt{3})\sqrt{-4a\sqrt{a^{-2}} + 8}(a^{-2})^{1/4}\log(6a^2\sqrt{a^{-2}} + 6x^2 - (\sqrt{3}a^2\sqrt{a^{-2}})x + 2\sqrt{3}ax)\sqrt{-4a\sqrt{a^{-2}} + 8}(a^{-2})^{1/4}) - \frac{1}{2}\sqrt{-4a\sqrt{a^{-2}} + 8}(a^{-2})^{1/4}\arctan(\frac{1}{18}(\sqrt{6}a^2\sqrt{a^{-2}} + 2\sqrt{6}a)\sqrt{6a^2\sqrt{a^{-2}} + 6x^2 + (\sqrt{3}a^2\sqrt{a^{-2}})x + 2\sqrt{3}ax}\sqrt{-4a\sqrt{a^{-2}} + 8}(a^{-2})^{1/4})\sqrt{-4a\sqrt{a^{-2}} + 8}(a^{-2})^{3/4} - \frac{1}{3}(a^2\sqrt{a^{-2}})x + 2ax)\sqrt{-4a\sqrt{a^{-2}} + 8}(a^{-2})^{3/4} - \frac{1}{3}\sqrt{3}a\sqrt{a^{-2}} - \frac{2}{3}\sqrt{3}) - \frac{1}{2}\sqrt{-4a\sqrt{a^{-2}} + 8}(a^{-2})^{1/4}\arctan(\frac{1}{18}(\sqrt{6}a^2\sqrt{a^{-2}} + 2\sqrt{6}a)\sqrt{6a^2\sqrt{a^{-2}} + 6x^2 - (\sqrt{3}a^2\sqrt{a^{-2}})x + 2\sqrt{3}ax}\sqrt{-4a\sqrt{a^{-2}} + 8}(a^{-2})^{1/4})\sqrt{-4a\sqrt{a^{-2}} + 8}(a^{-2})^{3/4} - \frac{1}{3}(a^2\sqrt{a^{-2}})x + 2ax)\sqrt{-4a\sqrt{a^{-2}} + 8}(a^{-2})^{3/4} + \frac{1}{3}\sqrt{3}a\sqrt{a^{-2}} + \frac{2}{3}\sqrt{3})$

Sympy [A] time = 0.369861, size = 27, normalized size = 0.24

$$-\text{RootSum}(16t^4a^2 - 4t^2a + 1, (t \mapsto t \log(-2ta + x)))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x**2+2*a)/(x**4-a*x**2+a**2),x)

[Out] -RootSum(16*_t**4*a**2 - 4*_t**2*a + 1, Lambda(_t, _t*log(-2*_t*a + x)))

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x^2 - 2a}{x^4 - ax^2 + a^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2+2*a)/(x^4-a*x^2+a^2),x, algorithm="giac")

[Out] integrate(-(x^2 - 2*a)/(x^4 - a*x^2 + a^2), x)

$$3.107 \quad \int \frac{2\sqrt{a-x^2}}{a-\sqrt{ax^2+x^4}} dx$$

Optimal. Leaf size=122

$$-\frac{\sqrt{3} \log(-\sqrt{3}\sqrt[4]{ax} + \sqrt{a} + x^2)}{4\sqrt[4]{a}} + \frac{\sqrt{3} \log(\sqrt{3}\sqrt[4]{ax} + \sqrt{a} + x^2)}{4\sqrt[4]{a}} - \frac{\tan^{-1}\left(\sqrt{3} - \frac{2x}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}} + \frac{\tan^{-1}\left(\frac{2x}{\sqrt[4]{a}} + \sqrt{3}\right)}{2\sqrt[4]{a}}$$

[Out] -ArcTan[Sqrt[3] - (2*x)/a^(1/4)]/(2*a^(1/4)) + ArcTan[Sqrt[3] + (2*x)/a^(1/4)]/(2*a^(1/4)) - (Sqrt[3]*Log[Sqrt[a] - Sqrt[3]*a^(1/4)*x + x^2])/(4*a^(1/4)) + (Sqrt[3]*Log[Sqrt[a] + Sqrt[3]*a^(1/4)*x + x^2])/(4*a^(1/4))

Rubi [A] time = 0.0796739, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {1169, 634, 617, 204, 628}

$$-\frac{\sqrt{3} \log(-\sqrt{3}\sqrt[4]{ax} + \sqrt{a} + x^2)}{4\sqrt[4]{a}} + \frac{\sqrt{3} \log(\sqrt{3}\sqrt[4]{ax} + \sqrt{a} + x^2)}{4\sqrt[4]{a}} - \frac{\tan^{-1}\left(\sqrt{3} - \frac{2x}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}} + \frac{\tan^{-1}\left(\frac{2x}{\sqrt[4]{a}} + \sqrt{3}\right)}{2\sqrt[4]{a}}$$

Antiderivative was successfully verified.

[In] Int[(2*Sqrt[a] - x^2)/(a - Sqrt[a]*x^2 + x^4), x]

[Out] -ArcTan[Sqrt[3] - (2*x)/a^(1/4)]/(2*a^(1/4)) + ArcTan[Sqrt[3] + (2*x)/a^(1/4)]/(2*a^(1/4)) - (Sqrt[3]*Log[Sqrt[a] - Sqrt[3]*a^(1/4)*x + x^2])/(4*a^(1/4)) + (Sqrt[3]*Log[Sqrt[a] + Sqrt[3]*a^(1/4)*x + x^2])/(4*a^(1/4))

Rule 1169

Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r + (d - e*q)*x)/(q + r*x + x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]

Rule 634

Int[((d_) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 617

Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{2\sqrt{a} - x^2}{a - \sqrt{a}x^2 + x^4} dx &= \frac{\int \frac{2\sqrt{3}a^{3/4} - 3\sqrt{ax}}{\sqrt{a} - \sqrt{3}\sqrt[4]{ax+x^2}} dx}{2\sqrt{3}a^{3/4}} + \frac{\int \frac{2\sqrt{3}a^{3/4} + 3\sqrt{ax}}{\sqrt{a} + \sqrt{3}\sqrt[4]{ax+x^2}} dx}{2\sqrt{3}a^{3/4}} \\ &= \frac{1}{4} \int \frac{1}{\sqrt{a} - \sqrt{3}\sqrt[4]{ax+x^2}} dx + \frac{1}{4} \int \frac{1}{\sqrt{a} + \sqrt{3}\sqrt[4]{ax+x^2}} dx - \frac{\sqrt{3} \int \frac{-\sqrt{3}\sqrt[4]{a+2x}}{\sqrt{a} - \sqrt{3}\sqrt[4]{ax+x^2}} dx}{4\sqrt[4]{a}} + \frac{\sqrt{3} \int \frac{\sqrt{3}\sqrt[4]{a}}{\sqrt{a} + \sqrt{3}\sqrt[4]{ax+x^2}} dx}{4\sqrt[4]{a}} \\ &= -\frac{\sqrt{3} \log(\sqrt{a} - \sqrt{3}\sqrt[4]{ax+x^2})}{4\sqrt[4]{a}} + \frac{\sqrt{3} \log(\sqrt{a} + \sqrt{3}\sqrt[4]{ax+x^2})}{4\sqrt[4]{a}} + \frac{\text{Subst}\left(\int \frac{1}{-\frac{1}{3}-x^2} dx, x, 1 - \frac{2x}{\sqrt{3}\sqrt[4]{a}}\right)}{2\sqrt{3}\sqrt[4]{a}} \\ &= -\frac{\tan^{-1}\left(\sqrt{3} - \frac{2x}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}} + \frac{\tan^{-1}\left(\sqrt{3} + \frac{2x}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}} - \frac{\sqrt{3} \log(\sqrt{a} - \sqrt{3}\sqrt[4]{ax+x^2})}{4\sqrt[4]{a}} + \frac{\sqrt{3} \log(\sqrt{a} + \sqrt{3}\sqrt[4]{ax+x^2})}{4\sqrt[4]{a}} \end{aligned}$$

Mathematica [C] time = 0.157455, size = 115, normalized size = 0.94

$$\frac{\sqrt[4]{-1} \left(\sqrt{\sqrt{3} - i} (\sqrt{3} - 3i) \tanh^{-1} \left(\frac{(1+i)x}{\sqrt{\sqrt{3} + i} \sqrt[4]{a}} \right) - \sqrt{\sqrt{3} + i} (\sqrt{3} + 3i) \tan^{-1} \left(\frac{(1+i)x}{\sqrt{\sqrt{3} - i} \sqrt[4]{a}} \right) \right)}{2\sqrt{6}\sqrt[4]{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*Sqrt[a] - x^2)/(a - Sqrt[a]*x^2 + x^4), x]
```

```
[Out] ((-1)^(1/4)*(-(Sqrt[I + Sqrt[3]]*(3*I + Sqrt[3])*ArcTan[((1 + I)*x)/(Sqrt[-I + Sqrt[3]]*a^(1/4))]) + Sqrt[-I + Sqrt[3]]*(-3*I + Sqrt[3])*ArcTanh[((1 + I)*x)/(Sqrt[I + Sqrt[3]]*a^(1/4))]))/(2*Sqrt[6]*a^(1/4))
```

Maple [A] time = 0.073, size = 96, normalized size = 0.8

$$\frac{\sqrt{3}}{4} \ln(x^2 + \sqrt[4]{ax}\sqrt{3} + \sqrt{a}) \frac{1}{\sqrt[4]{a}} + \frac{1}{2} \arctan\left(\left(2x + \sqrt{3}\sqrt[4]{a}\right) \frac{1}{\sqrt[4]{a}}\right) \frac{1}{\sqrt[4]{a}} - \frac{\sqrt{3}}{4} \ln(\sqrt[4]{ax}\sqrt{3} - x^2 - \sqrt{a}) \frac{1}{\sqrt[4]{a}} - \frac{1}{2} \arctan\left(\left(\sqrt{3}\sqrt[4]{a}\right) \frac{1}{\sqrt[4]{a}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+2*a^(1/2))/(a+x^4-x^2*a^(1/2)), x)
```

```
[Out] 1/4*ln(x^2+a^(1/4)*x*3^(1/2)+a^(1/2))*3^(1/2)/a^(1/4)+1/2/a^(1/4)*arctan((2*x+3^(1/2)*a^(1/4))/a^(1/4))-1/4/a^(1/4)*3^(1/2)*ln(a^(1/4)*x*3^(1/2)-x^2-a^(1/2))-1/2/a^(1/4)*arctan((3^(1/2)*a^(1/4)-2*x)/a^(1/4))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^2 - 2\sqrt{a}}{x^4 - \sqrt{a}x^2 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+2*a^(1/2))/(a+x^4-x^2*a^(1/2)),x, algorithm="maxima")
```

```
[Out] -integrate((x^2 - 2*sqrt(a))/(x^4 - sqrt(a)*x^2 + a), x)
```

Fricas [B] time = 1.67147, size = 682, normalized size = 5.59

$$\frac{1}{2} \sqrt{\frac{1}{2}} \sqrt{\frac{\sqrt{3a}\sqrt{-\frac{1}{a}} + \sqrt{a}}{a}} \log\left(\sqrt{\frac{1}{2}} \sqrt{a} \sqrt{\frac{\sqrt{3a}\sqrt{-\frac{1}{a}} + \sqrt{a}}{a}} + x\right) - \frac{1}{2} \sqrt{\frac{1}{2}} \sqrt{\frac{\sqrt{3a}\sqrt{-\frac{1}{a}} + \sqrt{a}}{a}} \log\left(-\sqrt{\frac{1}{2}} \sqrt{a} \sqrt{\frac{\sqrt{3a}\sqrt{-\frac{1}{a}} + \sqrt{a}}{a}} + x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+2*a^(1/2))/(a+x^4-x^2*a^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(1/2)*sqrt((sqrt(3)*a*sqrt(-1/a) + sqrt(a))/a)*log(sqrt(1/2)*sqrt(a)
)*sqrt((sqrt(3)*a*sqrt(-1/a) + sqrt(a))/a) + x) - 1/2*sqrt(1/2)*sqrt((sqrt(
3)*a*sqrt(-1/a) + sqrt(a))/a)*log(-sqrt(1/2)*sqrt(a)*sqrt((sqrt(3)*a*sqrt(-
1/a) + sqrt(a))/a) + x) + 1/2*sqrt(1/2)*sqrt(-(sqrt(3)*a*sqrt(-1/a) - sqrt(
a))/a)*log(sqrt(1/2)*sqrt(a)*sqrt(-(sqrt(3)*a*sqrt(-1/a) - sqrt(a))/a) + x)
- 1/2*sqrt(1/2)*sqrt(-(sqrt(3)*a*sqrt(-1/a) - sqrt(a))/a)*log(-sqrt(1/2)*s
qrt(a)*sqrt(-(sqrt(3)*a*sqrt(-1/a) - sqrt(a))/a) + x)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: PolynomialError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+2*a**(1/2))/(a+x**4-x**2*a**(1/2)),x)
```

```
[Out] Exception raised: PolynomialError
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+2*a^(1/2))/(a+x^4-x^2*a^(1/2)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.108 $\int \frac{2b^{2/3}+x^2}{b^{4/3}+b^{2/3}x^2+x^4} dx$

Optimal. Leaf size=124

$$-\frac{\log(b^{2/3} - \sqrt[3]{b}x + x^2)}{4\sqrt[3]{b}} + \frac{\log(b^{2/3} + \sqrt[3]{b}x + x^2)}{4\sqrt[3]{b}} - \frac{\sqrt{3} \tan^{-1}\left(\frac{\sqrt[3]{b}-2x}{\sqrt{3}\sqrt[3]{b}}\right)}{2\sqrt[3]{b}} + \frac{\sqrt{3} \tan^{-1}\left(\frac{\sqrt[3]{b}+2x}{\sqrt{3}\sqrt[3]{b}}\right)}{2\sqrt[3]{b}}$$

[Out] $-(\text{Sqrt}[3] \cdot \text{ArcTan}[(b^{1/3} - 2x)/(\text{Sqrt}[3] \cdot b^{1/3})])/(2 \cdot b^{1/3}) + (\text{Sqrt}[3] \cdot \text{ArcTan}[(b^{1/3} + 2x)/(\text{Sqrt}[3] \cdot b^{1/3})])/(2 \cdot b^{1/3}) - \text{Log}[b^{2/3} - b^{1/3}x + x^2]/(4 \cdot b^{1/3}) + \text{Log}[b^{2/3} + b^{1/3}x + x^2]/(4 \cdot b^{1/3})$

Rubi [A] time = 0.0774642, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used = {1169, 634, 617, 204, 628}

$$-\frac{\log(b^{2/3} - \sqrt[3]{b}x + x^2)}{4\sqrt[3]{b}} + \frac{\log(b^{2/3} + \sqrt[3]{b}x + x^2)}{4\sqrt[3]{b}} - \frac{\sqrt{3} \tan^{-1}\left(\frac{\sqrt[3]{b}-2x}{\sqrt{3}\sqrt[3]{b}}\right)}{2\sqrt[3]{b}} + \frac{\sqrt{3} \tan^{-1}\left(\frac{\sqrt[3]{b}+2x}{\sqrt{3}\sqrt[3]{b}}\right)}{2\sqrt[3]{b}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(2 \cdot b^{2/3} + x^2)/(b^{4/3} + b^{2/3}x^2 + x^4), x]$

[Out] $-(\text{Sqrt}[3] \cdot \text{ArcTan}[(b^{1/3} - 2x)/(\text{Sqrt}[3] \cdot b^{1/3})])/(2 \cdot b^{1/3}) + (\text{Sqrt}[3] \cdot \text{ArcTan}[(b^{1/3} + 2x)/(\text{Sqrt}[3] \cdot b^{1/3})])/(2 \cdot b^{1/3}) - \text{Log}[b^{2/3} - b^{1/3}x + x^2]/(4 \cdot b^{1/3}) + \text{Log}[b^{2/3} + b^{1/3}x + x^2]/(4 \cdot b^{1/3})$

Rule 1169

$\text{Int}[(d + (e \cdot x^2)/(a + (b \cdot x^2 + (c \cdot x^4)), x_Symbol] :> \text{With}[\{q = \text{Rt}[a/c, 2]\}, \text{With}[\{r = \text{Rt}[2 \cdot q - b/c, 2]\}, \text{Dist}[1/(2 \cdot c \cdot q \cdot r), \text{Int}[(d \cdot r - (d - e \cdot q) \cdot x)/(q - r \cdot x + x^2), x], x] + \text{Dist}[1/(2 \cdot c \cdot q \cdot r), \text{Int}[(d \cdot r + (d - e \cdot q) \cdot x)/(q + r \cdot x + x^2), x], x]] /; \text{FreeQ}\{a, b, c, d, e\}, x] \&\& \text{NeQ}[b^2 - 4 \cdot a \cdot c, 0] \&\& \text{NeQ}[c \cdot d^2 - b \cdot d \cdot e + a \cdot e^2, 0] \&\& \text{NegQ}[b^2 - 4 \cdot a \cdot c]$

Rule 634

$\text{Int}[(d + (e \cdot x)/(a + (b \cdot x + (c \cdot x^2)), x_Symbol] :> \text{Dist}[(2 \cdot c \cdot d - b \cdot e)/(2 \cdot c), \text{Int}[1/(a + b \cdot x + c \cdot x^2), x], x] + \text{Dist}[e/(2 \cdot c), \text{Int}[(b + 2 \cdot c \cdot x)/(a + b \cdot x + c \cdot x^2), x], x] /; \text{FreeQ}\{a, b, c, d, e\}, x] \&\& \text{NeQ}[2 \cdot c \cdot d - b \cdot e, 0] \&\& \text{NeQ}[b^2 - 4 \cdot a \cdot c, 0] \&\& \text{!NiceSqrtQ}[b^2 - 4 \cdot a \cdot c]$

Rule 617

$\text{Int}[(a + (b \cdot x + (c \cdot x^2))^{-1}, x_Symbol] :> \text{With}[\{q = 1 - 4 \cdot S\text{implify}[(a \cdot c)/b^2]\}, \text{Dist}[-2/b, \text{Subst}[\text{Int}[1/(q - x^2), x], x, 1 + (2 \cdot c \cdot x)/b], x] /; \text{RationalQ}[q] \&\& (\text{EqQ}[q^2, 1] \mid \mid \text{!RationalQ}[b^2 - 4 \cdot a \cdot c])] /; \text{FreeQ}\{a, b, c\}, x] \&\& \text{NeQ}[b^2 - 4 \cdot a \cdot c, 0]$

Rule 204

$\text{Int}[(a + (b \cdot x)^{-1}, x_Symbol] :> -\text{Simp}[\text{ArcTan}[(\text{Rt}[-b, 2] \cdot x)/\text{Rt}[-a, 2]]/(\text{Rt}[-a, 2] \cdot \text{Rt}[-b, 2]), x] /; \text{FreeQ}\{a, b\}, x] \&\& \text{PosQ}[a/b] \&\& (\text{LtQ}[a, 0] \mid \mid \text{LtQ}[b, 0])$

Rule 628

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := S
 imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[2*c*d - b*e, 0]

Rubi steps

$$\begin{aligned} \int \frac{2b^{2/3} + x^2}{b^{4/3} + b^{2/3}x^2 + x^4} dx &= \int \frac{2b - b^{2/3}x}{b^{2/3} - \sqrt[3]{bx+x^2}} dx + \int \frac{2b + b^{2/3}x}{b^{2/3} + \sqrt[3]{bx+x^2}} dx \\ &= \frac{3}{4} \int \frac{1}{b^{2/3} - \sqrt[3]{bx+x^2}} dx + \frac{3}{4} \int \frac{1}{b^{2/3} + \sqrt[3]{bx+x^2}} dx - \frac{\int \frac{-\sqrt[3]{b+2x}}{b^{2/3} - \sqrt[3]{bx+x^2}} dx}{4\sqrt[3]{b}} + \frac{\int \frac{\sqrt[3]{b+2x}}{b^{2/3} + \sqrt[3]{bx+x^2}} dx}{4\sqrt[3]{b}} \\ &= -\frac{\log(b^{2/3} - \sqrt[3]{bx+x^2})}{4\sqrt[3]{b}} + \frac{\log(b^{2/3} + \sqrt[3]{bx+x^2})}{4\sqrt[3]{b}} + \frac{3 \text{Subst}\left(\int \frac{1}{-3-x^2} dx, x, 1 - \frac{2x}{\sqrt[3]{b}}\right)}{2\sqrt[3]{b}} - \frac{3 \text{Subst}\left(\int \frac{1}{-3-x^2} dx, x, 1 + \frac{2x}{\sqrt[3]{b}}\right)}{2\sqrt[3]{b}} \\ &= -\frac{\sqrt{3} \tan^{-1}\left(\frac{\sqrt[3]{b}-2x}{\sqrt{3}\sqrt[3]{b}}\right)}{2\sqrt[3]{b}} + \frac{\sqrt{3} \tan^{-1}\left(\frac{\sqrt[3]{b}+2x}{\sqrt{3}\sqrt[3]{b}}\right)}{2\sqrt[3]{b}} - \frac{\log(b^{2/3} - \sqrt[3]{bx+x^2})}{4\sqrt[3]{b}} + \frac{\log(b^{2/3} + \sqrt[3]{bx+x^2})}{4\sqrt[3]{b}} \end{aligned}$$

Mathematica [C] time = 0.134815, size = 115, normalized size = 0.93

$$\frac{\sqrt[4]{-1} \left(\sqrt{\sqrt{3}-i} (\sqrt{3}-3i) \tan^{-1}\left(\frac{(1+i)x}{\sqrt{\sqrt{3}+i}\sqrt[3]{b}}\right) - \sqrt{\sqrt{3}+i} (\sqrt{3}+3i) \tanh^{-1}\left(\frac{(1+i)x}{\sqrt{\sqrt{3}-i}\sqrt[3]{b}}\right) \right)}{2\sqrt{6}\sqrt[3]{b}}$$

Antiderivative was successfully verified.

[In] Integrate[(2*b^(2/3) + x^2)/(b^(4/3) + b^(2/3)*x^2 + x^4), x]

[Out] ((-1)^(1/4)*(Sqrt[-I + Sqrt[3]]*(-3*I + Sqrt[3])*ArcTan[((1 + I)*x)/(Sqrt[I + Sqrt[3]]*b^(1/3))]) - Sqrt[I + Sqrt[3]]*(3*I + Sqrt[3])*ArcTanh[((1 + I)*x)/(Sqrt[-I + Sqrt[3]]*b^(1/3))]))/(2*Sqrt[6]*b^(1/3))

Maple [A] time = 0.055, size = 89, normalized size = 0.7

$$-\frac{1}{4} \ln\left(b^{2/3} - \sqrt[3]{bx+x^2}\right) \frac{1}{\sqrt[3]{b}} + \frac{\sqrt{3}}{2} \arctan\left(\frac{\sqrt{3}}{3} \left(-\sqrt[3]{b} + 2x\right) \frac{1}{\sqrt[3]{b}}\right) \frac{1}{\sqrt[3]{b}} + \frac{1}{4} \ln\left(b^{2/3} + \sqrt[3]{bx+x^2}\right) \frac{1}{\sqrt[3]{b}} + \frac{\sqrt{3}}{2} \arctan\left(\frac{\sqrt{3}}{3} \left(\sqrt[3]{b} + 2x\right) \frac{1}{\sqrt[3]{b}}\right) \frac{1}{\sqrt[3]{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*b^(2/3)+x^2)/(b^(4/3)+b^(2/3)*x^2+x^4), x)

[Out] -1/4*ln(b^(2/3)-b^(1/3)*x+x^2)/b^(1/3)+1/2*3^(1/2)/b^(1/3)*arctan(1/3*(-b^(1/3)+2*x)*3^(1/2)/b^(1/3))+1/4*ln(b^(2/3)+b^(1/3)*x+x^2)/b^(1/3)+1/2*arctan(1/3*(b^(1/3)+2*x)/b^(1/3)*3^(1/2))*3^(1/2)/b^(1/3)

Maxima [C] time = 1.5106, size = 170, normalized size = 1.37

$$-\frac{i\sqrt{3} \log\left(\frac{2x-i\sqrt{3}b^{1/3}+b^{1/3}}{2x+i\sqrt{3}b^{1/3}+b^{1/3}}\right)}{4b^{1/3}} - \frac{i\sqrt{3} \log\left(\frac{2x-i\sqrt{3}b^{1/3}-b^{1/3}}{2x+i\sqrt{3}b^{1/3}-b^{1/3}}\right)}{4b^{1/3}} + \frac{\log\left(x^2 + b^{1/3}x + b^{2/3}\right)}{4b^{1/3}} - \frac{\log\left(x^2 - b^{1/3}x + b^{2/3}\right)}{4b^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*b^(2/3)+x^2)/(b^(4/3)+b^(2/3)*x^2+x^4),x, algorithm="maxima")

[Out] $-1/4*I*\sqrt{3}*\log((2*x - I*\sqrt{3}*b^{1/3} + b^{1/3})/(2*x + I*\sqrt{3}*b^{1/3} + b^{1/3}))/b^{1/3} - 1/4*I*\sqrt{3}*\log((2*x - I*\sqrt{3}*b^{1/3} - b^{1/3})/(2*x + I*\sqrt{3}*b^{1/3} - b^{1/3}))/b^{1/3} + 1/4*\log(x^2 + b^{1/3}*x + b^{2/3})/b^{1/3} - 1/4*\log(x^2 - b^{1/3}*x + b^{2/3})/b^{1/3}$

Fricas [A] time = 1.84745, size = 768, normalized size = 6.19

$$\frac{\sqrt{3}b \sqrt{-\frac{1}{b^3}} \log\left(\frac{2x^3 + \sqrt{3}\left(2b^{\frac{2}{3}}x^2 + bx - b^{\frac{4}{3}}\right) \sqrt{-\frac{1}{2} - 3b^{\frac{2}{3}}x - b}}{x^3 + b}\right) + \sqrt{3}b \sqrt{-\frac{1}{b^3}} \log\left(\frac{2x^3 + \sqrt{3}\left(2b^{\frac{2}{3}}x^2 - bx - b^{\frac{4}{3}}\right) \sqrt{-\frac{1}{2} - 3b^{\frac{2}{3}}x + b}}{x^3 - b}\right) + b^{\frac{2}{3}} \log(x^2 + b)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*b^(2/3)+x^2)/(b^(4/3)+b^(2/3)*x^2+x^4),x, algorithm="fricas")

[Out] $[1/4*(\sqrt{3}*b*\sqrt{-1/b^{2/3}})*\log((2*x^3 + \sqrt{3}*(2*b^{2/3}*x^2 + b*x - b^{4/3}))*\sqrt{-1/b^{2/3}}) - 3*b^{2/3}*x - b)/(x^3 + b)) + \sqrt{3}*b*\sqrt{-1/b^{2/3}}*\log((2*x^3 + \sqrt{3}*(2*b^{2/3}*x^2 - b*x - b^{4/3}))*\sqrt{-1/b^{2/3}}) - 3*b^{2/3}*x + b)/(x^3 - b) + b^{2/3}*\log(x^2 + b^{1/3}*x + b^{2/3})) - b^{2/3}*\log(x^2 - b^{1/3}*x + b^{2/3}))/b, 1/4*(2*\sqrt{3}*b^{2/3}*\arctan(1/3*\sqrt{3}*(2*x + b^{1/3}))/b^{1/3}) - 2*\sqrt{3}*b^{2/3}*\arctan(-1/3*\sqrt{3}*(2*x - b^{1/3}))/b^{1/3} + b^{2/3}*\log(x^2 + b^{1/3}*x + b^{2/3}) - b^{2/3}*\log(x^2 - b^{1/3}*x + b^{2/3}))/b]$

Sympy [C] time = 0.398252, size = 143, normalized size = 1.15

$$\frac{\left(-\frac{1}{4} - \frac{\sqrt{3}i}{4}\right) \log\left(2\sqrt[3]{b}\left(-\frac{1}{4} - \frac{\sqrt{3}i}{4}\right) + x\right) + \left(-\frac{1}{4} + \frac{\sqrt{3}i}{4}\right) \log\left(2\sqrt[3]{b}\left(-\frac{1}{4} + \frac{\sqrt{3}i}{4}\right) + x\right) + \left(\frac{1}{4} - \frac{\sqrt{3}i}{4}\right) \log\left(2\sqrt[3]{b}\left(\frac{1}{4} - \frac{\sqrt{3}i}{4}\right) + x\right) + \left(\frac{1}{4} + \frac{\sqrt{3}i}{4}\right) \log\left(2\sqrt[3]{b}\left(\frac{1}{4} + \frac{\sqrt{3}i}{4}\right) + x\right)}{\sqrt[3]{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*b**(2/3)+x**2)/(b**(4/3)+b**(2/3)*x**2+x**4),x)

[Out] $((-1/4 - \sqrt{3}*I/4)*\log(2*b**(1/3)*(-1/4 - \sqrt{3}*I/4) + x) + (-1/4 + \sqrt{3}*I/4)*\log(2*b**(1/3)*(-1/4 + \sqrt{3}*I/4) + x) + (1/4 - \sqrt{3}*I/4)*\log(2*b**(1/3)*(1/4 - \sqrt{3}*I/4) + x) + (1/4 + \sqrt{3}*I/4)*\log(2*b**(1/3)*(1/4 + \sqrt{3}*I/4) + x))/b**(1/3)$

Giac [A] time = 1.15824, size = 124, normalized size = 1.

$$\frac{\sqrt{3} \arctan\left(\frac{\sqrt{3}\left(2x+b^{\frac{1}{3}}\right)}{3|b|^{\frac{1}{3}}}\right)}{2|b|^{\frac{1}{3}}} + \frac{\sqrt{3} \arctan\left(\frac{\sqrt{3}\left(2x-b^{\frac{1}{3}}\right)}{3|b|^{\frac{1}{3}}}\right)}{2|b|^{\frac{1}{3}}} + \frac{\log\left(x^2 + b^{\frac{1}{3}}x + b^{\frac{2}{3}}\right)}{4b^{\frac{1}{3}}} - \frac{\log\left(x^2 - b^{\frac{1}{3}}x + b^{\frac{2}{3}}\right)}{4b^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*b^(2/3)+x^2)/(b^(4/3)+b^(2/3)*x^2+x^4),x, algorithm="giac")

[Out] 1/2*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + b^(1/3))/abs(b)^(1/3))/abs(b)^(1/3) +
 1/2*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - b^(1/3))/abs(b)^(1/3))/abs(b)^(1/3)
 + 1/4*log(x^2 + b^(1/3)*x + b^(2/3))/b^(1/3) - 1/4*log(x^2 - b^(1/3)*x + b^(2/3))/b^(1/3)

3.109 $\int \frac{A+Bx^2}{a^2-ax^2+x^4} dx$

Optimal. Leaf size=136

$$-\frac{(A-aB)\log(-\sqrt{3}\sqrt{ax+a+x^2})}{4\sqrt{3}a^{3/2}} + \frac{(A-aB)\log(\sqrt{3}\sqrt{ax+a+x^2})}{4\sqrt{3}a^{3/2}} - \frac{(aB+A)\tan^{-1}\left(\sqrt{3}-\frac{2x}{\sqrt{a}}\right)}{2a^{3/2}} + \frac{(aB+A)\tan^{-1}\left(\frac{2}{\sqrt{3}}\right)}{2a^{3/2}}$$

[Out] -((A + a*B)*ArcTan[Sqrt[3] - (2*x)/Sqrt[a]])/(2*a^(3/2)) + ((A + a*B)*ArcTan[Sqrt[3] + (2*x)/Sqrt[a]])/(2*a^(3/2)) - ((A - a*B)*Log[a - Sqrt[3]*Sqrt[a]*x + x^2])/(4*Sqrt[3]*a^(3/2)) + ((A - a*B)*Log[a + Sqrt[3]*Sqrt[a]*x + x^2])/(4*Sqrt[3]*a^(3/2))

Rubi [A] time = 0.103947, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {1169, 634, 617, 204, 628}

$$-\frac{(A-aB)\log(-\sqrt{3}\sqrt{ax+a+x^2})}{4\sqrt{3}a^{3/2}} + \frac{(A-aB)\log(\sqrt{3}\sqrt{ax+a+x^2})}{4\sqrt{3}a^{3/2}} - \frac{(aB+A)\tan^{-1}\left(\sqrt{3}-\frac{2x}{\sqrt{a}}\right)}{2a^{3/2}} + \frac{(aB+A)\tan^{-1}\left(\frac{2}{\sqrt{3}}\right)}{2a^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(A + B*x^2)/(a^2 - a*x^2 + x^4), x]

[Out] -((A + a*B)*ArcTan[Sqrt[3] - (2*x)/Sqrt[a]])/(2*a^(3/2)) + ((A + a*B)*ArcTan[Sqrt[3] + (2*x)/Sqrt[a]])/(2*a^(3/2)) - ((A - a*B)*Log[a - Sqrt[3]*Sqrt[a]*x + x^2])/(4*Sqrt[3]*a^(3/2)) + ((A - a*B)*Log[a + Sqrt[3]*Sqrt[a]*x + x^2])/(4*Sqrt[3]*a^(3/2))

Rule 1169

Int[((d_) + (e_)*(x_)^2)/((a_) + (b_)*(x_)^2 + (c_)*(x_)^4), x_Symbol] :> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r + (d - e*q)*x)/(q + r*x + x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]

Rule 634

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] :> Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 617

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 204

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[

a, 0] || LtQ[b, 0])

Rule 628

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] :> Simp[d*Log[RemoveContent[a + b*x + c*x^2, x]]/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rubi steps

$$\begin{aligned} \int \frac{A + Bx^2}{a^2 - ax^2 + x^4} dx &= \frac{\int \frac{\sqrt{3}\sqrt{a}A - (A-aB)x}{a - \sqrt{3}\sqrt{ax+x^2}} dx}{2\sqrt{3}a^{3/2}} + \frac{\int \frac{\sqrt{3}\sqrt{a}A + (A-aB)x}{a + \sqrt{3}\sqrt{ax+x^2}} dx}{2\sqrt{3}a^{3/2}} \\ &= -\frac{(A-aB) \int \frac{-\sqrt{3}\sqrt{a}+2x}{a-\sqrt{3}\sqrt{ax+x^2}} dx}{4\sqrt{3}a^{3/2}} + \frac{(A-aB) \int \frac{\sqrt{3}\sqrt{a}+2x}{a+\sqrt{3}\sqrt{ax+x^2}} dx}{4\sqrt{3}a^{3/2}} + \frac{(A+aB) \int \frac{1}{a-\sqrt{3}\sqrt{ax+x^2}} dx}{4a} + \frac{(A+aB) \int \frac{1}{a+\sqrt{3}\sqrt{ax+x^2}} dx}{4a} \\ &= -\frac{(A-aB) \log(a - \sqrt{3}\sqrt{ax+x^2})}{4\sqrt{3}a^{3/2}} + \frac{(A-aB) \log(a + \sqrt{3}\sqrt{ax+x^2})}{4\sqrt{3}a^{3/2}} + \frac{(A+aB) \operatorname{Subst}\left(\int \frac{1}{-\frac{1}{3}-x} dx\right)}{2\sqrt{3}a^{3/2}} \\ &= -\frac{(A+aB) \tan^{-1}\left(\sqrt{3} - \frac{2x}{\sqrt{a}}\right)}{2a^{3/2}} + \frac{(A+aB) \tan^{-1}\left(\sqrt{3} + \frac{2x}{\sqrt{a}}\right)}{2a^{3/2}} - \frac{(A-aB) \log(a - \sqrt{3}\sqrt{ax+x^2})}{4\sqrt{3}a^{3/2}} + \frac{(A+aB) \log(a + \sqrt{3}\sqrt{ax+x^2})}{4\sqrt{3}a^{3/2}} \end{aligned}$$

Mathematica [C] time = 0.147658, size = 130, normalized size = 0.96

$$\frac{\sqrt[4]{-1} \left(\frac{((\sqrt{3}-i)aB-2iA) \tan^{-1}\left(\frac{(1+i)x}{\sqrt{\sqrt{3}-i}\sqrt{a}}\right)}{\sqrt{\sqrt{3}-i}} - \frac{((\sqrt{3}+i)aB+2iA) \tanh^{-1}\left(\frac{(1+i)x}{\sqrt{\sqrt{3}+i}\sqrt{a}}\right)}{\sqrt{\sqrt{3}+i}} \right)}{\sqrt{6}a^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*x^2)/(a^2 - a*x^2 + x^4), x]

[Out] $((-1)^{(1/4)} * ((((-2*I)*A + (-I + \operatorname{Sqrt}[3])*a*B) * \operatorname{ArcTan}(((1 + I)*x) / (\operatorname{Sqrt}[-I + \operatorname{Sqrt}[3]] * \operatorname{Sqrt}[a])))) / \operatorname{Sqrt}[-I + \operatorname{Sqrt}[3]] - (((2*I)*A + (I + \operatorname{Sqrt}[3])*a*B) * \operatorname{ArcTan}(((1 + I)*x) / (\operatorname{Sqrt}[I + \operatorname{Sqrt}[3]] * \operatorname{Sqrt}[a])))) / \operatorname{Sqrt}[I + \operatorname{Sqrt}[3]])) / (\operatorname{Sqrt}[6] * a^{(3/2)})$

Maple [A] time = 0.061, size = 190, normalized size = 1.4

$$-\frac{B\sqrt{3}}{12} \ln(a + x^2 + x\sqrt{3}\sqrt{a}) \frac{1}{\sqrt{a}} + \frac{A\sqrt{3}}{12} \ln(a + x^2 + x\sqrt{3}\sqrt{a}) a^{-3/2} + \frac{B}{2} \arctan\left((2x + \sqrt{3}\sqrt{a}) \frac{1}{\sqrt{a}}\right) \frac{1}{\sqrt{a}} + \frac{A}{2} \arctan\left(\frac{1}{\sqrt{a}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B*x^2+A)/(x^4-a*x^2+a^2), x)

[Out] $-1/12/a^{(1/2)} * \ln(a+x^2+x*3^{(1/2)}*a^{(1/2)}) * B*3^{(1/2)} + 1/12/a^{(3/2)} * \ln(a+x^2+x*3^{(1/2)}*a^{(1/2)}) * A*3^{(1/2)} + 1/2/a^{(1/2)} * \arctan((2*x+3^{(1/2)}*a^{(1/2)})/a^{(1/2)}) * B + 1/2/a^{(3/2)} * \arctan((2*x+3^{(1/2)}*a^{(1/2)})/a^{(1/2)}) * A + 1/12/a^{(1/2)} * \ln(x*3^{(1/2)}*a^{(1/2)}-x^2-a) * B*3^{(1/2)} - 1/12/a^{(3/2)} * \ln(x*3^{(1/2)}*a^{(1/2)}-x^2-a) * A$

$*3^{(1/2)}-1/2/a^{(1/2)}*\arctan((3^{(1/2)}*a^{(1/2)}-2*x)/a^{(1/2)})*B-1/2/a^{(3/2)}*\arctan((3^{(1/2)}*a^{(1/2)}-2*x)/a^{(1/2)})*A$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Bx^2 + A}{x^4 - ax^2 + a^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^2+A)/(x^4-a*x^2+a^2),x, algorithm="maxima")

[Out] integrate((B*x^2 + A)/(x^4 - a*x^2 + a^2), x)

Fricas [B] time = 6.15726, size = 9567, normalized size = 70.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^2+A)/(x^4-a*x^2+a^2),x, algorithm="fricas")

[Out] $1/4*(4*(1/9)^{(1/4)}*a^6*\sqrt{(2*B^4*a^4 + 4*A*B^3*a^3 + 6*A^2*B^2*a^2 + 4*A^3*B*a + 2*A^4 + (B^2*a^5 + 4*A*B*a^4 + A^2*a^3)*\sqrt{(B^4*a^4 + 2*A*B^3*a^3 + 3*A^2*B^2*a^2 + 2*A^3*B*a + A^4)/a^6)}}/(B^4*a^4 - 2*A^2*B^2*a^2 + A^4))*((B^4*a^4 + 2*A*B^3*a^3 + 3*A^2*B^2*a^2 + 2*A^3*B*a + A^4)/a^6)^{(3/4)}*\sqrt{(B^4*a^4 - 2*A^2*B^2*a^2 + A^4)/a^6}*\arctan((18*\sqrt{1/3})*(1/9)^{(3/4)}*(\sqrt{1/3})*A*a^{10}*\sqrt{(B^4*a^4 + 2*A*B^3*a^3 + 3*A^2*B^2*a^2 + 2*A^3*B*a + A^4)/a^6}*\sqrt{(B^4*a^4 - 2*A^2*B^2*a^2 + A^4)/a^6} - \sqrt{1/3}*(B^3*a^{10} + A*B^2*a^9 + A^2*B*a^8)*\sqrt{(B^4*a^4 - 2*A^2*B^2*a^2 + A^4)/a^6})*\sqrt{(2*B^4*a^4 + 4*A*B^3*a^3 + 6*A^2*B^2*a^2 + 4*A^3*B*a + 2*A^4 + (B^2*a^5 + 4*A*B*a^4 + A^2*a^3)*\sqrt{(B^4*a^4 + 2*A*B^3*a^3 + 3*A^2*B^2*a^2 + 2*A^3*B*a + A^4)/a^6)}}/(B^4*a^4 - 2*A^2*B^2*a^2 + A^4))*\sqrt{((B^4*a^4 + 2*A*B^3*a^3 + 3*A^2*B^2*a^2 + 2*A^3*B*a + A^4)*x^2 + 3*\sqrt{1/3}*(1/9)^{(1/4)}*(B*a^6*x*\sqrt{(B^4*a^4 + 2*A*B^3*a^3 + 3*A^2*B^2*a^2 + 2*A^3*B*a + A^4)/a^6} - (A*B^2*a^4 + A^2*B*a^3 + A^3*a^2)*x)*\sqrt{(2*B^4*a^4 + 4*A*B^3*a^3 + 6*A^2*B^2*a^2 + 4*A^3*B*a + 2*A^4 + (B^2*a^5 + 4*A*B*a^4 + A^2*a^3)*\sqrt{(B^4*a^4 + 2*A*B^3*a^3 + 3*A^2*B^2*a^2 + 2*A^3*B*a + A^4)/a^6)}}/(B^4*a^4 - 2*A^2*B^2*a^2 + A^4))*((B^4*a^4 + 2*A*B^3*a^3 + 3*A^2*B^2*a^2 + 2*A^3*B*a + A^4)/a^6)^{(1/4)} + (B^2*a^6 + A*B*a^5 + A^2*a^4)*\sqrt{(B^4*a^4 + 2*A*B^3*a^3 + 3*A^2*B^2*a^2 + 2*A^3*B*a + A^4)/a^6}}/(B^4*a^4 + 2*A*B^3*a^3 + 3*A^2*B^2*a^2 + 2*A^3*B*a + A^4))*((B^4*a^4 + 2*A*B^3*a^3 + 3*A^2*B^2*a^2 + 2*A^3*B*a + A^4)/a^6)^{(3/4)} - 18*\sqrt{1/3}*(1/9)^{(3/4)}*(\sqrt{1/3})*A*a^{10}*x*\sqrt{(B^4*a^4 + 2*A*B^3*a^3 + 3*A^2*B^2*a^2 + 2*A^3*B*a + A^4)/a^6}*\sqrt{(B^4*a^4 - 2*A^2*B^2*a^2 + A^4)/a^6} - \sqrt{1/3}*(B^3*a^{10} + A*B^2*a^9 + A^2*B*a^8)*x*\sqrt{(B^4*a^4 - 2*A^2*B^2*a^2 + A^4)/a^6})*\sqrt{(2*B^4*a^4 + 4*A*B^3*a^3 + 6*A^2*B^2*a^2 + 4*A^3*B*a + 2*A^4 + (B^2*a^5 + 4*A*B*a^4 + A^2*a^3)*\sqrt{(B^4*a^4 + 2*A*B^3*a^3 + 3*A^2*B^2*a^2 + 2*A^3*B*a + A^4)/a^6)}}/(B^4*a^4 - 2*A^2*B^2*a^2 + A^4))*((B^4*a^4 + 2*A*B^3*a^3 + 3*A^2*B^2*a^2 + 2*A^3*B*a + A^4)/a^6)^{(3/4)} + 2*\sqrt{1/3}*(B^4*a^{10} + 2*A*B^3*a^9 + 3*A^2*B^2*a^8 + 2*A^3*B*a^7 + A^4*a^6)*\sqrt{(B^4*a^4 + 2*A*B^3*a^3 + 3*A^2*B^2*a^2 + 2*A^3*B*a + A^4)/a^6}*\sqrt{(B^4*a^4 - 2*A^2*B^2*a^2 + A^4)/a^6} + \sqrt{1/3}*(B^6*a^9 + 3*A*B^5*a^8 + 6*A^2*B^4*a^7 + 7*A^3*B^3*a^6 + 6*A^4*B^2*a^5 + 3*A^5*B*a^4 + A^6*a^3)*\sqrt{(B^4*a^4 - 2*A^2*B^2*a^2 + A^4)/a^6}}/(B^8*a^8 + 3*A*B^7*a^7 + 5*A^2*B^6*a^6 + 4*A^3*B^5*a^5 - 4*A^5*B^3*a^3 - 5*A^6*B^2*a^2 - 3*A^7*B*a - A^8)) + 4*($

$$\frac{3Ba + A^4}{a^6}^{1/4} + 2(B^2a^6 + ABa^5 + A^2a^4)\sqrt{(B^4a^4 + 2AB^3a^3 + 3A^2B^2a^2 + 2A^3Ba + A^4)/a^6}}{(B^4a^4 + 2AB^3a^3 + 3A^2B^2a^2 + 2A^3Ba + A^4)}$$

Sympy [A] time = 1.20255, size = 172, normalized size = 1.26

$$\text{RootSum}\left(144t^4a^6 + t^2(12A^2a^3 + 48ABa^4 + 12B^2a^5) + A^4 + 2A^3Ba + 3A^2B^2a^2 + 2AB^3a^3 + B^4a^4, \left(t \mapsto t \log\left(x + \frac{24t^4}{\dots}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x**2+A)/(x**4-a*x**2+a**2),x)

[Out] RootSum(144*_t**4*a**6 + _t**2*(12*A**2*a**3 + 48*A*B*a**4 + 12*B**2*a**5) + A**4 + 2*A**3*B*a + 3*A**2*B**2*a**2 + 2*A*B**3*a**3 + B**4*a**4, Lambda(_t, _t*log(x + (24*_t**3*A*a**5 + 48*_t**3*B*a**6 - 2*_t*A**3*a**2 + 6*_t*A**2*B*a**3 + 12*_t*A*B**2*a**4 + 2*_t*B**3*a**5)/(-A**4 - A**3*B*a + A*B**3*a**3 + B**4*a**4))))

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Bx^2 + A}{x^4 - ax^2 + a^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^2+A)/(x^4-a*x^2+a^2),x, algorithm="giac")

[Out] integrate((B*x^2 + A)/(x^4 - a*x^2 + a^2), x)

$$3.110 \quad \int \frac{A+Bx^2}{a-\sqrt{ax^2+x^4}} dx$$

Optimal. Leaf size=160

$$\frac{(A - \sqrt{a}B) \log(-\sqrt{3}\sqrt[4]{ax} + \sqrt{a} + x^2)}{4\sqrt{3}a^{3/4}} + \frac{(A - \sqrt{a}B) \log(\sqrt{3}\sqrt[4]{ax} + \sqrt{a} + x^2)}{4\sqrt{3}a^{3/4}} - \frac{(\sqrt{a}B + A) \tan^{-1}\left(\sqrt{3} - \frac{2x}{\sqrt[4]{a}}\right)}{2a^{3/4}} + \frac{(\sqrt{a}B + A) \tan^{-1}\left(\sqrt{3} + \frac{2x}{\sqrt[4]{a}}\right)}{2a^{3/4}}$$

[Out] $-\frac{(A + \sqrt{a}B) \operatorname{ArcTan}[\sqrt{3} - (2x)/a^{1/4}]}{(2a^{3/4})} + \frac{(A + \sqrt{a}B) \operatorname{ArcTan}[\sqrt{3} + (2x)/a^{1/4}]}{(2a^{3/4})} - \frac{(A - \sqrt{a}B) \operatorname{Log}[\sqrt{a} - \sqrt{3} \sqrt[4]{ax} + x^2]}{(4\sqrt{3} \sqrt[4]{a} a^{3/4})} + \frac{(A - \sqrt{a}B) \operatorname{Log}[\sqrt{a} + \sqrt{3} \sqrt[4]{ax} + x^2]}{(4\sqrt{3} \sqrt[4]{a} a^{3/4})}$

Rubi [A] time = 0.116336, antiderivative size = 160, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$, Rules used = {1169, 634, 617, 204, 628}

$$\frac{(A - \sqrt{a}B) \log(-\sqrt{3}\sqrt[4]{ax} + \sqrt{a} + x^2)}{4\sqrt{3}a^{3/4}} + \frac{(A - \sqrt{a}B) \log(\sqrt{3}\sqrt[4]{ax} + \sqrt{a} + x^2)}{4\sqrt{3}a^{3/4}} - \frac{(\sqrt{a}B + A) \tan^{-1}\left(\sqrt{3} - \frac{2x}{\sqrt[4]{a}}\right)}{2a^{3/4}} + \frac{(\sqrt{a}B + A) \tan^{-1}\left(\sqrt{3} + \frac{2x}{\sqrt[4]{a}}\right)}{2a^{3/4}}$$

Antiderivative was successfully verified.

[In] Int[(A + B*x^2)/(a - Sqrt[a]*x^2 + x^4), x]

[Out] $-\frac{(A + \sqrt{a}B) \operatorname{ArcTan}[\sqrt{3} - (2x)/a^{1/4}]}{(2a^{3/4})} + \frac{(A + \sqrt{a}B) \operatorname{ArcTan}[\sqrt{3} + (2x)/a^{1/4}]}{(2a^{3/4})} - \frac{(A - \sqrt{a}B) \operatorname{Log}[\sqrt{a} - \sqrt{3} \sqrt[4]{ax} + x^2]}{(4\sqrt{3} \sqrt[4]{a} a^{3/4})} + \frac{(A - \sqrt{a}B) \operatorname{Log}[\sqrt{a} + \sqrt{3} \sqrt[4]{ax} + x^2]}{(4\sqrt{3} \sqrt[4]{a} a^{3/4})}$

Rule 1169

Int[((d_) + (e_)*(x_)^2)/((a_) + (b_)*(x_)^2 + (c_)*(x_)^4), x_Symbol] :> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r + (d - e*q)*x)/(q + r*x + x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]

Rule 634

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] :> Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 617

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c]) /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 204

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[

a, 0] || LtQ[b, 0])

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{A + Bx^2}{a - \sqrt{ax^2 + x^4}} dx = \frac{\int \frac{\sqrt{3}\sqrt[4]{a}A - (A - \sqrt{a}B)x}{\sqrt{a} - \sqrt{3}\sqrt[4]{ax+x^2}} dx}{2\sqrt{3}a^{3/4}} + \frac{\int \frac{\sqrt{3}\sqrt[4]{a}A + (A - \sqrt{a}B)x}{\sqrt{a} + \sqrt{3}\sqrt[4]{ax+x^2}} dx}{2\sqrt{3}a^{3/4}}$$

$$= \frac{1}{4} \left(\frac{A}{\sqrt{a}} + B \right) \int \frac{1}{\sqrt{a} - \sqrt{3}\sqrt[4]{ax+x^2}} dx + \frac{1}{4} \left(\frac{A}{\sqrt{a}} + B \right) \int \frac{1}{\sqrt{a} + \sqrt{3}\sqrt[4]{ax+x^2}} dx - \frac{(A - \sqrt{a}B) \int \frac{1}{\sqrt{a} - \sqrt{3}\sqrt[4]{ax+x^2}} dx}{4\sqrt{3}a^{3/4}}$$

$$= -\frac{(A - \sqrt{a}B) \log(\sqrt{a} - \sqrt{3}\sqrt[4]{ax+x^2})}{4\sqrt{3}a^{3/4}} + \frac{(A - \sqrt{a}B) \log(\sqrt{a} + \sqrt{3}\sqrt[4]{ax+x^2})}{4\sqrt{3}a^{3/4}} + \frac{(A + \sqrt{a}B) \text{Subst}}{4\sqrt{3}a^{3/4}}$$

$$= -\frac{(A + \sqrt{a}B) \tan^{-1}\left(\sqrt{3} - \frac{2x}{\sqrt[4]{a}}\right)}{2a^{3/4}} + \frac{(A + \sqrt{a}B) \tan^{-1}\left(\sqrt{3} + \frac{2x}{\sqrt[4]{a}}\right)}{2a^{3/4}} - \frac{(A - \sqrt{a}B) \log(\sqrt{a} - \sqrt{3}\sqrt[4]{ax+x^2})}{4\sqrt{3}a^{3/4}}$$

Mathematica [C] time = 0.136933, size = 138, normalized size = 0.86

$$\frac{\sqrt[4]{-1} \left(\frac{((\sqrt{3}-i)\sqrt{a}B-2iA) \tan^{-1}\left(\frac{(1+i)x}{\sqrt{\sqrt{3}-i}\sqrt[4]{a}}\right)}{\sqrt{\sqrt{3}-i}} - \frac{((\sqrt{3}+i)\sqrt{a}B+2iA) \tanh^{-1}\left(\frac{(1+i)x}{\sqrt{\sqrt{3}+i}\sqrt[4]{a}}\right)}{\sqrt{\sqrt{3}+i}} \right)}{\sqrt{6}a^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(a - Sqrt[a]*x^2 + x^4), x]
```

```
[Out] ((-1)^(1/4)*((((-2*I)*A + (-I + Sqrt[3])*Sqrt[a]*B)*ArcTan[((1 + I)*x)/(Sqrt[-I + Sqrt[3]]*a^(1/4))])/Sqrt[-I + Sqrt[3]] - (((2*I)*A + (I + Sqrt[3])*Sqrt[a]*B)*ArcTanh[((1 + I)*x)/(Sqrt[I + Sqrt[3]]*a^(1/4))])/Sqrt[I + Sqrt[3]]))/Sqrt[6]*a^(3/4)
```

Maple [A] time = 0.097, size = 194, normalized size = 1.2

$$\frac{A\sqrt{3}}{12} \ln(x^2 + \sqrt[4]{ax}\sqrt{3} + \sqrt{a}) a^{-3/4} - \frac{B\sqrt{3}}{12} \ln(x^2 + \sqrt[4]{ax}\sqrt{3} + \sqrt{a}) \frac{1}{\sqrt[4]{a}} + \frac{A}{2} \arctan\left(\left(2x + \sqrt{3}\sqrt[4]{a}\right) \frac{1}{\sqrt[4]{a}}\right) a^{-3/4} + \frac{B}{2} \arctan\left(\left(2x + \sqrt{3}\sqrt[4]{a}\right) \frac{1}{\sqrt[4]{a}}\right) a^{-3/4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/(a+x^4-x^2*a^(1/2)), x)
```

```
[Out] 1/12/a^(3/4)*ln(x^2+a^(1/4)*x*3^(1/2)+a^(1/2))*A*3^(1/2)-1/12/a^(1/4)*ln(x^2+a^(1/4)*x*3^(1/2)+a^(1/2))*B*3^(1/2)+1/2/a^(3/4)*arctan((2*x+3^(1/2)*a^(1/4))/a^(1/4))*A+1/2/a^(1/4)*arctan((2*x+3^(1/2)*a^(1/4))/a^(1/4))*B-1/12/a^(3/4)
```

$$\left(\frac{3}{4}\right) \ln(x^2 - a^{1/4}) * x * 3^{1/2} + a^{1/2}) * A * 3^{1/2} + 1/12/a^{1/4} * \ln(x^2 - a^{1/4}) * x * 3^{1/2} + a^{1/2}) * B * 3^{1/2} + 1/2/a^{3/4} * \arctan((2*x - 3^{1/2}) * a^{1/4}) / a^{1/4}) * A + 1/2/a^{1/4} * \arctan((2*x - 3^{1/2}) * a^{1/4}) / a^{1/4}) * B$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Bx^2 + A}{x^4 - \sqrt{ax^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^2+A)/(a+x^4-x^2*a^(1/2)),x, algorithm="maxima")

[Out] integrate((B*x^2 + A)/(x^4 - sqrt(a)*x^2 + a), x)

Fricas [B] time = 2.4112, size = 2531, normalized size = 15.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^2+A)/(a+x^4-x^2*a^(1/2)),x, algorithm="fricas")

[Out] $\frac{1}{2} \sqrt{\frac{1}{6}} \sqrt{-(4ABa + 3\sqrt{\frac{1}{3}}a^2\sqrt{-(B^4a^2 - 2A^2B^2a + A^4)/a^3}) + (B^2a + A^2)\sqrt{a}}/a^2 \log(2(B^6a^3 - A^6)x + 3\sqrt{\frac{1}{6}}(AB^4a^3 - A^5a - \sqrt{\frac{1}{3}}(2B^3a^4 + A^2Ba^3))\sqrt{-(B^4a^2 - 2A^2B^2a + A^4)/a^3}) - (A^2B^3a^2 - A^4Ba - \sqrt{\frac{1}{3}}(AB^2a^3 - A^3a^2))\sqrt{-(B^4a^2 - 2A^2B^2a + A^4)/a^3})\sqrt{a}} \sqrt{-(4ABa + 3\sqrt{\frac{1}{3}}a^2\sqrt{-(B^4a^2 - 2A^2B^2a + A^4)/a^3}) + (B^2a + A^2)\sqrt{a}}/a^2) - \frac{1}{2} \sqrt{\frac{1}{6}} \sqrt{-(4ABa + 3\sqrt{\frac{1}{3}}a^2\sqrt{-(B^4a^2 - 2A^2B^2a + A^4)/a^3}) + (B^2a + A^2)\sqrt{a}}/a^2 \log(2(B^6a^3 - A^6)x - 3\sqrt{\frac{1}{6}}(AB^4a^3 - A^5a - \sqrt{\frac{1}{3}}(2B^3a^4 + A^2Ba^3))\sqrt{-(B^4a^2 - 2A^2B^2a + A^4)/a^3}) - (A^2B^3a^2 - A^4Ba - \sqrt{\frac{1}{3}}(AB^2a^3 - A^3a^2))\sqrt{-(B^4a^2 - 2A^2B^2a + A^4)/a^3})\sqrt{a}} \sqrt{-(4ABa + 3\sqrt{\frac{1}{3}}a^2\sqrt{-(B^4a^2 - 2A^2B^2a + A^4)/a^3}) + (B^2a + A^2)\sqrt{a}}/a^2) + \frac{1}{2} \sqrt{\frac{1}{6}} \sqrt{-(4ABa - 3\sqrt{\frac{1}{3}}a^2\sqrt{-(B^4a^2 - 2A^2B^2a + A^4)/a^3}) + (B^2a + A^2)\sqrt{a}}/a^2 \log(2(B^6a^3 - A^6)x + 3\sqrt{\frac{1}{6}}(AB^4a^3 - A^5a + \sqrt{\frac{1}{3}}(2B^3a^4 + A^2Ba^3))\sqrt{-(B^4a^2 - 2A^2B^2a + A^4)/a^3}) - (A^2B^3a^2 - A^4Ba + \sqrt{\frac{1}{3}}(AB^2a^3 - A^3a^2))\sqrt{-(B^4a^2 - 2A^2B^2a + A^4)/a^3})\sqrt{a}} \sqrt{-(4ABa - 3\sqrt{\frac{1}{3}}a^2\sqrt{-(B^4a^2 - 2A^2B^2a + A^4)/a^3}) + (B^2a + A^2)\sqrt{a}}/a^2) - \frac{1}{2} \sqrt{\frac{1}{6}} \sqrt{-(4ABa - 3\sqrt{\frac{1}{3}}a^2\sqrt{-(B^4a^2 - 2A^2B^2a + A^4)/a^3}) + (B^2a + A^2)\sqrt{a}}/a^2 \log(2(B^6a^3 - A^6)x - 3\sqrt{\frac{1}{6}}(AB^4a^3 - A^5a + \sqrt{\frac{1}{3}}(2B^3a^4 + A^2Ba^3))\sqrt{-(B^4a^2 - 2A^2B^2a + A^4)/a^3}) - (A^2B^3a^2 - A^4Ba + \sqrt{\frac{1}{3}}(AB^2a^3 - A^3a^2))\sqrt{-(B^4a^2 - 2A^2B^2a + A^4)/a^3})\sqrt{a}} \sqrt{-(4ABa - 3\sqrt{\frac{1}{3}}a^2\sqrt{-(B^4a^2 - 2A^2B^2a + A^4)/a^3}) + (B^2a + A^2)\sqrt{a}}/a^2)$

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: PolynomialError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/(a+x**4-x**2*a**(1/2)),x)
```

```
[Out] Exception raised: PolynomialError
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(a+x^4-x^2*a^(1/2)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```


$$3.111 \quad \int \frac{A+Bx^2}{a-\sqrt{ac}x^2+cx^4} dx$$

Optimal. Leaf size=414

$$\frac{\left(A - \frac{\sqrt{aB}}{\sqrt{c}}\right) \log\left(-x\sqrt{2\sqrt{a}\sqrt{c} + \sqrt{ac}} + \sqrt{a} + \sqrt{cx^2}\right)}{4\sqrt{a}\sqrt{2\sqrt{a}\sqrt{c} + \sqrt{ac}}} + \frac{\left(A - \frac{\sqrt{aB}}{\sqrt{c}}\right) \log\left(x\sqrt{2\sqrt{a}\sqrt{c} + \sqrt{ac}} + \sqrt{a} + \sqrt{cx^2}\right)}{4\sqrt{a}\sqrt{2\sqrt{a}\sqrt{c} + \sqrt{ac}}} - \frac{(\sqrt{aB} + A\sqrt{c})}{2\sqrt{a}\sqrt{2\sqrt{a}\sqrt{c} + \sqrt{ac}}}$$

```
[Out] -((Sqrt[a]*B + A*Sqrt[c])*ArcTan[(Sqrt[2*Sqrt[a]*Sqrt[c] + Sqrt[a*c]] - 2*Sqrt[c]*x)/Sqrt[2*Sqrt[a]*Sqrt[c] - Sqrt[a*c]]])/(2*Sqrt[a]*Sqrt[c]*Sqrt[2*Sqrt[a]*Sqrt[c] - Sqrt[a*c]]) + ((Sqrt[a]*B + A*Sqrt[c])*ArcTan[(Sqrt[2*Sqrt[a]*Sqrt[c] + Sqrt[a*c]] + 2*Sqrt[c]*x)/Sqrt[2*Sqrt[a]*Sqrt[c] - Sqrt[a*c]]])/(2*Sqrt[a]*Sqrt[c]*Sqrt[2*Sqrt[a]*Sqrt[c] - Sqrt[a*c]]) - ((A - (Sqrt[a]*B)/Sqrt[c])*Log[Sqrt[a] - Sqrt[2*Sqrt[a]*Sqrt[c] + Sqrt[a*c]]*x + Sqrt[c]*x^2])/(4*Sqrt[a]*Sqrt[2*Sqrt[a]*Sqrt[c] + Sqrt[a*c]]) + ((A - (Sqrt[a]*B)/Sqrt[c])*Log[Sqrt[a] + Sqrt[2*Sqrt[a]*Sqrt[c] + Sqrt[a*c]]*x + Sqrt[c]*x^2])/(4*Sqrt[a]*Sqrt[2*Sqrt[a]*Sqrt[c] + Sqrt[a*c]])
```

Rubi [A] time = 0.453327, antiderivative size = 414, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {1169, 634, 618, 204, 628}

$$\frac{\left(A - \frac{\sqrt{aB}}{\sqrt{c}}\right) \log\left(-x\sqrt{2\sqrt{a}\sqrt{c} + \sqrt{ac}} + \sqrt{a} + \sqrt{cx^2}\right)}{4\sqrt{a}\sqrt{2\sqrt{a}\sqrt{c} + \sqrt{ac}}} + \frac{\left(A - \frac{\sqrt{aB}}{\sqrt{c}}\right) \log\left(x\sqrt{2\sqrt{a}\sqrt{c} + \sqrt{ac}} + \sqrt{a} + \sqrt{cx^2}\right)}{4\sqrt{a}\sqrt{2\sqrt{a}\sqrt{c} + \sqrt{ac}}} - \frac{(\sqrt{aB} + A\sqrt{c})}{2\sqrt{a}\sqrt{2\sqrt{a}\sqrt{c} + \sqrt{ac}}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(a - Sqrt[a*c]*x^2 + c*x^4), x]
```

```
[Out] -((Sqrt[a]*B + A*Sqrt[c])*ArcTan[(Sqrt[2*Sqrt[a]*Sqrt[c] + Sqrt[a*c]] - 2*Sqrt[c]*x)/Sqrt[2*Sqrt[a]*Sqrt[c] - Sqrt[a*c]]])/(2*Sqrt[a]*Sqrt[c]*Sqrt[2*Sqrt[a]*Sqrt[c] - Sqrt[a*c]]) + ((Sqrt[a]*B + A*Sqrt[c])*ArcTan[(Sqrt[2*Sqrt[a]*Sqrt[c] + Sqrt[a*c]] + 2*Sqrt[c]*x)/Sqrt[2*Sqrt[a]*Sqrt[c] - Sqrt[a*c]]])/(2*Sqrt[a]*Sqrt[c]*Sqrt[2*Sqrt[a]*Sqrt[c] - Sqrt[a*c]]) - ((A - (Sqrt[a]*B)/Sqrt[c])*Log[Sqrt[a] - Sqrt[2*Sqrt[a]*Sqrt[c] + Sqrt[a*c]]*x + Sqrt[c]*x^2])/(4*Sqrt[a]*Sqrt[2*Sqrt[a]*Sqrt[c] + Sqrt[a*c]]) + ((A - (Sqrt[a]*B)/Sqrt[c])*Log[Sqrt[a] + Sqrt[2*Sqrt[a]*Sqrt[c] + Sqrt[a*c]]*x + Sqrt[c]*x^2])/(4*Sqrt[a]*Sqrt[2*Sqrt[a]*Sqrt[c] + Sqrt[a*c]])
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r + (d - e*q)*x)/(q + r*x + x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 634

```
Int[((d_) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
```

`[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]`

Rule 618

`Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]`

Rule 204

`Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])`

Rule 628

`Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]`

Rubi steps

$$\int \frac{A + Bx^2}{a - \sqrt{ac}x^2 + cx^4} dx = \frac{\int \frac{\frac{A\sqrt{2\sqrt{a}\sqrt{c} + \sqrt{ac}}}{\sqrt{c}} - \left(A - \frac{\sqrt{a}B}{\sqrt{c}}\right)x}{\frac{\sqrt{a}}{\sqrt{c}} - \frac{\sqrt{2\sqrt{a}\sqrt{c} + \sqrt{ac}}}{\sqrt{c}} + x^2} dx}{2\sqrt{a}\sqrt{2\sqrt{a}\sqrt{c} + \sqrt{ac}}} + \frac{\int \frac{\frac{A\sqrt{2\sqrt{a}\sqrt{c} + \sqrt{ac}}}{\sqrt{c}} + \left(A - \frac{\sqrt{a}B}{\sqrt{c}}\right)x}{\frac{\sqrt{a}}{\sqrt{c}} + \frac{\sqrt{2\sqrt{a}\sqrt{c} + \sqrt{ac}}}{\sqrt{c}} + x^2} dx}{2\sqrt{a}\sqrt{2\sqrt{a}\sqrt{c} + \sqrt{ac}}}$$

$$= \frac{\left(B + \frac{A\sqrt{c}}{\sqrt{a}}\right) \int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}} - \frac{\sqrt{2\sqrt{a}\sqrt{c} + \sqrt{ac}}}{\sqrt{c}} + x^2} dx}{4c} + \frac{\left(B + \frac{A\sqrt{c}}{\sqrt{a}}\right) \int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}} + \frac{\sqrt{2\sqrt{a}\sqrt{c} + \sqrt{ac}}}{\sqrt{c}} + x^2} dx}{4c} - \frac{\left(A - \frac{\sqrt{a}B}{\sqrt{c}}\right) \int \frac{\frac{\sqrt{2\sqrt{a}\sqrt{c}}}{\sqrt{c}}}{\frac{\sqrt{a}}{\sqrt{c}} - \frac{\sqrt{2\sqrt{a}\sqrt{c} + \sqrt{ac}}}{\sqrt{c}} + x^2} dx}{4\sqrt{a}\sqrt{2\sqrt{a}\sqrt{c} + \sqrt{ac}}}$$

$$= -\frac{\left(A - \frac{\sqrt{a}B}{\sqrt{c}}\right) \log\left(\sqrt{a} - \sqrt{2\sqrt{a}\sqrt{c} + \sqrt{ac}}x + \sqrt{cx^2}\right)}{4\sqrt{a}\sqrt{2\sqrt{a}\sqrt{c} + \sqrt{ac}}} + \frac{\left(A - \frac{\sqrt{a}B}{\sqrt{c}}\right) \log\left(\sqrt{a} + \sqrt{2\sqrt{a}\sqrt{c} + \sqrt{ac}}x + \sqrt{cx^2}\right)}{4\sqrt{a}\sqrt{2\sqrt{a}\sqrt{c} + \sqrt{ac}}}$$

$$= -\frac{\left(B + \frac{A\sqrt{c}}{\sqrt{a}}\right) \tan^{-1}\left(\frac{\sqrt{2\sqrt{a}\sqrt{c} + \sqrt{ac}} - 2\sqrt{cx}}{\sqrt{2\sqrt{a}\sqrt{c} - \sqrt{ac}}}\right)}{2\sqrt{c}\sqrt{2\sqrt{a}\sqrt{c} - \sqrt{ac}}} + \frac{\left(B + \frac{A\sqrt{c}}{\sqrt{a}}\right) \tan^{-1}\left(\frac{\sqrt{2\sqrt{a}\sqrt{c} + \sqrt{ac}} + 2\sqrt{cx}}{\sqrt{2\sqrt{a}\sqrt{c} - \sqrt{ac}}}\right)}{2\sqrt{c}\sqrt{2\sqrt{a}\sqrt{c} - \sqrt{ac}}} - \frac{\left(A - \frac{\sqrt{a}B}{\sqrt{c}}\right) \log\left(\frac{\sqrt{2\sqrt{a}\sqrt{c}}}{\sqrt{2\sqrt{a}\sqrt{c} + \sqrt{ac}}}\right)}{4\sqrt{a}\sqrt{2\sqrt{a}\sqrt{c} + \sqrt{ac}}}$$

Mathematica [C] time = 0.197111, size = 247, normalized size = 0.6

$$\frac{(\sqrt{3}\sqrt{a}B\sqrt{c} - i(B\sqrt{ac} + 2Ac)) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{-\sqrt{ac} - i\sqrt{3}\sqrt{a}\sqrt{c}}}\right)}{\sqrt{-\sqrt{ac} - i\sqrt{3}\sqrt{a}\sqrt{c}}} + \frac{(\sqrt{3}\sqrt{a}B\sqrt{c} + i(B\sqrt{ac} + 2Ac)) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{-\sqrt{ac} + i\sqrt{3}\sqrt{a}\sqrt{c}}}\right)}{\sqrt{-\sqrt{ac} + i\sqrt{3}\sqrt{a}\sqrt{c}}}$$

$$\frac{\hspace{10em}}{\sqrt{6}\sqrt{ac}}$$

Antiderivative was successfully verified.

`[In] Integrate[(A + B*x^2)/(a - Sqrt[a*c]*x^2 + c*x^4), x]`

`[Out] (((Sqrt[3]*Sqrt[a]*B*Sqrt[c] - I*(2*A*c + B*Sqrt[a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[(-I)*Sqrt[3]*Sqrt[a]*Sqrt[c] - Sqrt[a*c]]])/Sqrt[(-I)*Sqrt[3]*`

$$\sqrt{a}\sqrt{c} - \sqrt{ac}] + ((\sqrt{3}\sqrt{a}B\sqrt{c} + I(2Ac + B\sqrt{ac}))\text{ArcTan}[(\sqrt{2}\sqrt{c}x)/\sqrt{I\sqrt{3}\sqrt{a}\sqrt{c} - \sqrt{ac}}])/\sqrt{I\sqrt{3}\sqrt{a}\sqrt{c} - \sqrt{ac}})/(\sqrt{6}\sqrt{a}c)$$

Maple [A] time = 0.194, size = 404, normalized size = 1.

$$\frac{B\sqrt{3}}{12a} \ln(x\sqrt{3}\sqrt[4]{ac} - x^2\sqrt{c} - \sqrt{a})(ac)^{\frac{3}{4}}c^{-\frac{3}{2}} - \frac{A\sqrt{3}}{12c} \ln(x\sqrt{3}\sqrt[4]{ac} - x^2\sqrt{c} - \sqrt{a})(ac)^{\frac{3}{4}}a^{-\frac{3}{2}} - \frac{A}{2} \arctan\left(\sqrt{3}\sqrt[4]{ac} - 2x\sqrt{c}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B*x^2+A)/(a+c*x^4-x^2*(a*c)^(1/2)),x)

[Out] 1/12/a/c^(3/2)*ln(x*3^(1/2)*(a*c)^(1/4)-x^2*c^(1/2)-a^(1/2))*B*3^(1/2)*(a*c)^(3/4)-1/12/a^(3/2)/c*ln(x*3^(1/2)*(a*c)^(1/4)-x^2*c^(1/2)-a^(1/2))*A*3^(1/2)*(a*c)^(3/4)-1/2/a^(1/2)/(4*a^(1/2)*c^(1/2)-3*(a*c)^(1/2))^(1/2)*arctan((3^(1/2)*(a*c)^(1/4)-2*x*c^(1/2))/(4*a^(1/2)*c^(1/2)-3*(a*c)^(1/2))^(1/2))*A-1/2/c^(1/2)/(4*a^(1/2)*c^(1/2)-3*(a*c)^(1/2))^(1/2)*arctan((3^(1/2)*(a*c)^(1/4)-2*x*c^(1/2))/(4*a^(1/2)*c^(1/2)-3*(a*c)^(1/2))^(1/2))*B-1/12/a/c^(3/2)*ln(x^2*c^(1/2)+x*3^(1/2)*(a*c)^(1/4)+a^(1/2))*B*3^(1/2)*(a*c)^(3/4)+1/12/a^(3/2)/c*ln(x^2*c^(1/2)+x*3^(1/2)*(a*c)^(1/4)+a^(1/2))*A*3^(1/2)*(a*c)^(3/4)+1/2/a^(1/2)/(4*a^(1/2)*c^(1/2)-3*(a*c)^(1/2))^(1/2)*arctan((2*x*c^(1/2)+3^(1/2)*(a*c)^(1/4))/(4*a^(1/2)*c^(1/2)-3*(a*c)^(1/2))^(1/2))*A+1/2/c^(1/2)/(4*a^(1/2)*c^(1/2)-3*(a*c)^(1/2))^(1/2)*arctan((2*x*c^(1/2)+3^(1/2)*(a*c)^(1/4))/(4*a^(1/2)*c^(1/2)-3*(a*c)^(1/2))^(1/2))*B

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Bx^2 + A}{cx^4 - \sqrt{ac}x^2 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^2+A)/(a+c*x^4-x^2*(a*c)^(1/2)),x, algorithm="maxima")

[Out] integrate((B*x^2 + A)/(c*x^4 - sqrt(a*c)*x^2 + a), x)

Fricas [B] time = 3.61015, size = 3148, normalized size = 7.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^2+A)/(a+c*x^4-x^2*(a*c)^(1/2)),x, algorithm="fricas")

[Out] -1/2*sqrt(1/6)*sqrt(-(3*sqrt(1/3)*a^2*c^2*sqrt(-(B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^3*c^3)) + 4*A*B*a*c + (B^2*a + A^2*c)*sqrt(a*c)))/(a^2*c^2)*log(-2*(B^6*a^3 - A^6*c^3)*x + 3*sqrt(1/6)*(A*B^4*a^3*c - A^5*a*c^3 - sqrt(1/3))*(2*B^3*a^4*c^2 + A^2*B*a^3*c^3)*sqrt(-(B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^3*c^3)) - (A^2*B^3*a^2*c - A^4*B*a*c^2 - sqrt(1/3)*(A*B^2*a^3*c^2 - A^3*a^2*c^3)*sqrt(-(B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^3*c^3)))*sqrt(a*c))*

$$\begin{aligned} & \sqrt{-(3\sqrt{1/3})a^2c^2\sqrt{-(B^4a^2 - 2A^2B^2ac + A^4c^2)/(a^3c^3)} + 4ABac + (B^2a + A^2c)\sqrt{ac}}/(a^2c^2)) + 1/2\sqrt{1/6}\sqrt{-(3\sqrt{1/3})a^2c^2\sqrt{-(B^4a^2 - 2A^2B^2ac + A^4c^2)/(a^3c^3)} + 4ABac + (B^2a + A^2c)\sqrt{ac}}/(a^2c^2))\log(-2(B^6a^3 - A^6c^3)x - 3\sqrt{1/6}(AB^4a^3c - A^5ac^3 - \sqrt{1/3}(2B^3a^4c^2 + A^2Ba^3c^3)\sqrt{-(B^4a^2 - 2A^2B^2ac + A^4c^2)/(a^3c^3)} - (A^2B^3a^2c - A^4Bac^2 - \sqrt{1/3}(AB^2a^3c^2 - A^3a^2c^3)\sqrt{-(B^4a^2 - 2A^2B^2ac + A^4c^2)/(a^3c^3)}))\sqrt{ac})\sqrt{-(3\sqrt{1/3})a^2c^2\sqrt{-(B^4a^2 - 2A^2B^2ac + A^4c^2)/(a^3c^3)} + 4ABac + (B^2a + A^2c)\sqrt{ac}}/(a^2c^2)) - 1/2\sqrt{1/6}\sqrt{((3\sqrt{1/3})a^2c^2\sqrt{-(B^4a^2 - 2A^2B^2ac + A^4c^2)/(a^3c^3)} - 4ABac - (B^2a + A^2c)\sqrt{ac}}/(a^2c^2))\log(-2(B^6a^3 - A^6c^3)x + 3\sqrt{1/6}(AB^4a^3c - A^5ac^3 + \sqrt{1/3}(2B^3a^4c^2 + A^2Ba^3c^3)\sqrt{-(B^4a^2 - 2A^2B^2ac + A^4c^2)/(a^3c^3)} - (A^2B^3a^2c - A^4Bac^2 + \sqrt{1/3}(AB^2a^3c^2 - A^3a^2c^3)\sqrt{-(B^4a^2 - 2A^2B^2ac + A^4c^2)/(a^3c^3)}))\sqrt{ac})\sqrt{((3\sqrt{1/3})a^2c^2\sqrt{-(B^4a^2 - 2A^2B^2ac + A^4c^2)/(a^3c^3)} - 4ABac - (B^2a + A^2c)\sqrt{ac}}/(a^2c^2)) + 1/2\sqrt{1/6}\sqrt{((3\sqrt{1/3})a^2c^2\sqrt{-(B^4a^2 - 2A^2B^2ac + A^4c^2)/(a^3c^3)} - 4ABac - (B^2a + A^2c)\sqrt{ac}}/(a^2c^2))\log(-2(B^6a^3 - A^6c^3)x - 3\sqrt{1/6}(AB^4a^3c - A^5ac^3 + \sqrt{1/3}(2B^3a^4c^2 + A^2Ba^3c^3)\sqrt{-(B^4a^2 - 2A^2B^2ac + A^4c^2)/(a^3c^3)} - (A^2B^3a^2c - A^4Bac^2 + \sqrt{1/3}(AB^2a^3c^2 - A^3a^2c^3)\sqrt{-(B^4a^2 - 2A^2B^2ac + A^4c^2)/(a^3c^3)}))\sqrt{ac})\sqrt{((3\sqrt{1/3})a^2c^2\sqrt{-(B^4a^2 - 2A^2B^2ac + A^4c^2)/(a^3c^3)} - 4ABac - (B^2a + A^2c)\sqrt{ac}}/(a^2c^2))} \end{aligned}$$

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: PolynomialError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x**2+A)/(a+c*x**4-x**2*(a*c)**(1/2)),x)

[Out] Exception raised: PolynomialError

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^2+A)/(a+c*x^4-x^2*(a*c)^(1/2)),x, algorithm="giac")

[Out] Exception raised: TypeError

$$3.112 \quad \int \frac{A+Bx^2}{a-\sqrt{a}\sqrt{cx^2+cx^4}} dx$$

Optimal. Leaf size=234

$$\frac{(\sqrt{a}B + A\sqrt{c}) \tan^{-1}\left(\sqrt{3} - \frac{2\sqrt[4]{cx}}{\sqrt[4]{a}}\right)}{2a^{3/4}c^{3/4}} + \frac{(\sqrt{a}B + A\sqrt{c}) \tan^{-1}\left(\frac{2\sqrt[4]{cx}}{\sqrt[4]{a}} + \sqrt{3}\right)}{2a^{3/4}c^{3/4}} - \frac{\left(A - \frac{\sqrt{a}B}{\sqrt{c}}\right) \log\left(-\sqrt{3}\sqrt[4]{a}\sqrt[4]{cx} + \sqrt{a} + \sqrt{cx^2}\right)}{4\sqrt{3}a^{3/4}\sqrt[4]{c}}$$

[Out] -((Sqrt[a]*B + A*Sqrt[c])*ArcTan[Sqrt[3] - (2*c^(1/4)*x)/a^(1/4)])/(2*a^(3/4)*c^(3/4)) + ((Sqrt[a]*B + A*Sqrt[c])*ArcTan[Sqrt[3] + (2*c^(1/4)*x)/a^(1/4)])/(2*a^(3/4)*c^(3/4)) - ((A - (Sqrt[a]*B)/Sqrt[c])*Log[Sqrt[a] - Sqrt[3]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[3]*a^(3/4)*c^(1/4)) + ((A - (Sqrt[a]*B)/Sqrt[c])*Log[Sqrt[a] + Sqrt[3]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[3]*a^(3/4)*c^(1/4))

Rubi [A] time = 0.171647, antiderivative size = 234, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used = {1169, 634, 617, 204, 628}

$$\frac{(\sqrt{a}B + A\sqrt{c}) \tan^{-1}\left(\sqrt{3} - \frac{2\sqrt[4]{cx}}{\sqrt[4]{a}}\right)}{2a^{3/4}c^{3/4}} + \frac{(\sqrt{a}B + A\sqrt{c}) \tan^{-1}\left(\frac{2\sqrt[4]{cx}}{\sqrt[4]{a}} + \sqrt{3}\right)}{2a^{3/4}c^{3/4}} - \frac{\left(A - \frac{\sqrt{a}B}{\sqrt{c}}\right) \log\left(-\sqrt{3}\sqrt[4]{a}\sqrt[4]{cx} + \sqrt{a} + \sqrt{cx^2}\right)}{4\sqrt{3}a^{3/4}\sqrt[4]{c}}$$

Antiderivative was successfully verified.

[In] Int[(A + B*x^2)/(a - Sqrt[a]*Sqrt[c]*x^2 + c*x^4), x]

[Out] -((Sqrt[a]*B + A*Sqrt[c])*ArcTan[Sqrt[3] - (2*c^(1/4)*x)/a^(1/4)])/(2*a^(3/4)*c^(3/4)) + ((Sqrt[a]*B + A*Sqrt[c])*ArcTan[Sqrt[3] + (2*c^(1/4)*x)/a^(1/4)])/(2*a^(3/4)*c^(3/4)) - ((A - (Sqrt[a]*B)/Sqrt[c])*Log[Sqrt[a] - Sqrt[3]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[3]*a^(3/4)*c^(1/4)) + ((A - (Sqrt[a]*B)/Sqrt[c])*Log[Sqrt[a] + Sqrt[3]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[3]*a^(3/4)*c^(1/4))

Rule 1169

Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r + (d - e*q)*x)/(q + r*x + x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]

Rule 634

Int[((d_) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 617

Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[Log[RemoveContent[a + b*x + c*x^2, x]]/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{A + Bx^2}{a - \sqrt{a}\sqrt{cx^2 + cx^4}} dx = \frac{\int \frac{\frac{\sqrt{3}\sqrt[4]{a}A}{\sqrt[4]{c}} - \left(A - \frac{\sqrt{a}B}{\sqrt{c}}\right)x}{\frac{\sqrt{a}}{\sqrt{c}} - \frac{\sqrt{3}\sqrt[4]{ax}}{\sqrt[4]{c}} + x^2} dx}{2\sqrt{3}a^{3/4}\sqrt[4]{c}} + \frac{\int \frac{\frac{\sqrt{3}\sqrt[4]{a}A}{\sqrt[4]{c}} + \left(A - \frac{\sqrt{a}B}{\sqrt{c}}\right)x}{\frac{\sqrt{a}}{\sqrt{c}} + \frac{\sqrt{3}\sqrt[4]{ax}}{\sqrt[4]{c}} + x^2} dx}{2\sqrt{3}a^{3/4}\sqrt[4]{c}}$$

$$= \frac{\left(B + \frac{A\sqrt{c}}{\sqrt{a}}\right) \int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}} - \frac{\sqrt{3}\sqrt[4]{ax}}{\sqrt[4]{c}} + x^2} dx}{4c} + \frac{\left(B + \frac{A\sqrt{c}}{\sqrt{a}}\right) \int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}} + \frac{\sqrt{3}\sqrt[4]{ax}}{\sqrt[4]{c}} + x^2} dx}{4c} + \frac{(\sqrt{a}B - A\sqrt{c}) \int \frac{-\frac{\sqrt{3}\sqrt[4]{a}}{\sqrt[4]{c}} + 2x}{\frac{\sqrt{a}}{\sqrt{c}} - \frac{\sqrt{3}\sqrt[4]{ax}}{\sqrt[4]{c}} + x^2} dx}{4\sqrt{3}a^{3/4}c^{3/4}}$$

$$= \frac{(\sqrt{a}B - A\sqrt{c}) \log\left(\sqrt{a} - \sqrt{3}\sqrt[4]{a}\sqrt[4]{cx} + \sqrt{cx^2}\right)}{4\sqrt{3}a^{3/4}c^{3/4}} + \frac{\left(A - \frac{\sqrt{a}B}{\sqrt{c}}\right) \log\left(\sqrt{a} + \sqrt{3}\sqrt[4]{a}\sqrt[4]{cx} + \sqrt{cx^2}\right)}{4\sqrt{3}a^{3/4}\sqrt[4]{c}} + \dots$$

$$= -\frac{(\sqrt{a}B + A\sqrt{c}) \tan^{-1}\left(\sqrt{3} - \frac{2\sqrt[4]{cx}}{\sqrt[4]{a}}\right)}{2a^{3/4}c^{3/4}} + \frac{(\sqrt{a}B + A\sqrt{c}) \tan^{-1}\left(\sqrt{3} + \frac{2\sqrt[4]{cx}}{\sqrt[4]{a}}\right)}{2a^{3/4}c^{3/4}} + \frac{(\sqrt{a}B - A\sqrt{c}) \log}{4}$$

Mathematica [C] time = 0.184857, size = 163, normalized size = 0.7

$$\frac{\sqrt[4]{-1} \left(\frac{((\sqrt{3}-i)\sqrt{a}B-2iA\sqrt{c}) \tan^{-1}\left(\frac{(1+i)\sqrt[4]{cx}}{\sqrt{\sqrt{3}-i}\sqrt[4]{a}}\right)}{\sqrt{\sqrt{3}-i}} - \frac{((\sqrt{3}+i)\sqrt{a}B+2iA\sqrt{c}) \tanh^{-1}\left(\frac{(1+i)\sqrt[4]{cx}}{\sqrt{\sqrt{3}+i}\sqrt[4]{a}}\right)}{\sqrt{\sqrt{3}+i}} \right)}{\sqrt{6}a^{3/4}c^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(a - Sqrt[a]*Sqrt[c]*x^2 + c*x^4), x]
```

```
[Out] ((-1)^(1/4)*((( -I + Sqrt[3])*Sqrt[a]*B - (2*I)*A*Sqrt[c])*ArcTan[(((1 + I)*c^(1/4)*x)/(Sqrt[-I + Sqrt[3]]*a^(1/4)))]/Sqrt[-I + Sqrt[3]] - ((I + Sqrt[3])*Sqrt[a]*B + (2*I)*A*Sqrt[c])*ArcTanh[(((1 + I)*c^(1/4)*x)/(Sqrt[I + Sqrt[3]]*a^(1/4)))]/Sqrt[I + Sqrt[3]]))/Sqrt[6]*a^(3/4)*c^(3/4)
```

Maple [A] time = 0.092, size = 318, normalized size = 1.4

$$-\frac{A\sqrt{3}}{12} \ln\left(-\sqrt[4]{a}\sqrt[4]{cx}\sqrt{3} + \sqrt{a} + x^2\sqrt{c}\right) \frac{1}{\sqrt[4]{c}} a^{-\frac{3}{4}} + \frac{B\sqrt{3}}{12} \ln\left(-\sqrt[4]{a}\sqrt[4]{cx}\sqrt{3} + \sqrt{a} + x^2\sqrt{c}\right) c^{-\frac{3}{4}} \frac{1}{\sqrt[4]{a}} + \frac{A}{2} \arctan\left(2x\sqrt{c} - \sqrt{3}\sqrt[4]{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((B*x^2+A)/(a+c*x^4-x^2*a^{(1/2)}*c^{(1/2)}),x)$

[Out] $-1/12/c^{(1/4)}/a^{(3/4)}*\ln(-a^{(1/4)}*c^{(1/4)}*x*3^{(1/2)}+a^{(1/2)}+x^2*c^{(1/2)})*A*3^{(1/2)}+1/12/c^{(3/4)}/a^{(1/4)}*\ln(-a^{(1/4)}*c^{(1/4)}*x*3^{(1/2)}+a^{(1/2)}+x^2*c^{(1/2)})*B*3^{(1/2)}+1/2/a^{(1/2)}/(a^{(1/2)}*c^{(1/2)})^{(1/2)}*\arctan((2*x*c^{(1/2)}-3^{(1/2)}*c^{(1/4)}*a^{(1/4)})/(a^{(1/2)}*c^{(1/2)})^{(1/2)})*A+1/2/c^{(1/2)}/(a^{(1/2)}*c^{(1/2)})^{(1/2)}*\arctan((2*x*c^{(1/2)}-3^{(1/2)}*c^{(1/4)}*a^{(1/4)})/(a^{(1/2)}*c^{(1/2)})^{(1/2)})*B+1/12/c^{(1/4)}/a^{(3/4)}*\ln(a^{(1/4)}*c^{(1/4)}*x*3^{(1/2)}+a^{(1/2)}+x^2*c^{(1/2)})*A*3^{(1/2)}-1/12/c^{(3/4)}/a^{(1/4)}*\ln(a^{(1/4)}*c^{(1/4)}*x*3^{(1/2)}+a^{(1/2)}+x^2*c^{(1/2)})*B*3^{(1/2)}+1/2/a^{(1/2)}/(a^{(1/2)}*c^{(1/2)})^{(1/2)}*\arctan((2*x*c^{(1/2)}+3^{(1/2)}*c^{(1/4)}*a^{(1/4)})/(a^{(1/2)}*c^{(1/2)})^{(1/2)})*A+1/2/c^{(1/2)}/(a^{(1/2)}*c^{(1/2)})^{(1/2)}*\arctan((2*x*c^{(1/2)}+3^{(1/2)}*c^{(1/4)}*a^{(1/4)})/(a^{(1/2)}*c^{(1/2)})^{(1/2)})*B$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Bx^2 + A}{cx^4 - \sqrt{a}\sqrt{cx^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}((B*x^2+A)/(a+c*x^4-x^2*a^{(1/2)}*c^{(1/2)}),x, \text{algorithm}="maxima")$

[Out] $\text{integrate}((B*x^2 + A)/(c*x^4 - \text{sqrt}(a)*\text{sqrt}(c)*x^2 + a), x)$

Fricas [B] time = 7.48679, size = 3245, normalized size = 13.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}((B*x^2+A)/(a+c*x^4-x^2*a^{(1/2)}*c^{(1/2)}),x, \text{algorithm}="fricas")$

[Out] $-1/2*\text{sqrt}(1/6)*\text{sqrt}(-(3*\text{sqrt}(1/3)*a^2*c^2*\text{sqrt}(-(B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^3*c^3))) + 4*A*B*a*c + (B^2*a + A^2*c)*\text{sqrt}(a)*\text{sqrt}(c))/(a^2*c^2)) * \log(-2*(B^6*a^3 - A^6*c^3)*x + 3*\text{sqrt}(1/6)*(A*B^4*a^3*c - A^5*a*c^3 - (A^2*B^3*a^2*c - A^4*B*a*c^2 - \text{sqrt}(1/3)*(A*B^2*a^3*c^2 - A^3*a^2*c^3))*\text{sqrt}(-(B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^3*c^3))) * \text{sqrt}(a)*\text{sqrt}(c) - \text{sqrt}(1/3)*(2*B^3*a^4*c^2 + A^2*B*a^3*c^3)*\text{sqrt}(-(B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^3*c^3))) * \text{sqrt}(-(3*\text{sqrt}(1/3)*a^2*c^2*\text{sqrt}(-(B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^3*c^3))) + 4*A*B*a*c + (B^2*a + A^2*c)*\text{sqrt}(a)*\text{sqrt}(c))/(a^2*c^2)) + 1/2*\text{sqrt}(1/6)*\text{sqrt}(-(3*\text{sqrt}(1/3)*a^2*c^2*\text{sqrt}(-(B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^3*c^3))) + 4*A*B*a*c + (B^2*a + A^2*c)*\text{sqrt}(a)*\text{sqrt}(c))/(a^2*c^2)) * \log(-2*(B^6*a^3 - A^6*c^3)*x - 3*\text{sqrt}(1/6)*(A*B^4*a^3*c - A^5*a*c^3 - (A^2*B^3*a^2*c - A^4*B*a*c^2 - \text{sqrt}(1/3)*(A*B^2*a^3*c^2 - A^3*a^2*c^3))*\text{sqrt}(-(B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^3*c^3))) * \text{sqrt}(a)*\text{sqrt}(c) - \text{sqrt}(1/3)*(2*B^3*a^4*c^2 + A^2*B*a^3*c^3)*\text{sqrt}(-(B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^3*c^3))) * \text{sqrt}(-(3*\text{sqrt}(1/3)*a^2*c^2*\text{sqrt}(-(B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^3*c^3))) + 4*A*B*a*c + (B^2*a + A^2*c)*\text{sqrt}(a)*\text{sqrt}(c))/(a^2*c^2)) - 1/2*\text{sqrt}(1/6)*\text{sqrt}((3*\text{sqrt}(1/3)*a^2*c^2*\text{sqrt}(-(B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^3*c^3))) - 4*A*B*a*c - (B^2*a + A^2*c)*\text{sqrt}(a)*\text{sqrt}(c))/(a^2*c^2)) * \log(-2*(B^6*a^3 - A^6*c^3)*x + 3*\text{sqrt}(1/6)*(A*B^4*a^3*c - A^5*a*c^3 - (A^2*B^3*a^2*c - A^4*B*a*c^2 + \text{sqrt}(1/3)*(A*B^2*a^3*c^2 - A^3*a^2*c^3))*\text{sqrt}(-(B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^3*c^3))) * \text{sqrt}(a)*\text{sqrt}(c) + \text{sqrt}(1$

$$\begin{aligned} & /3)*(2*B^3*a^4*c^2 + A^2*B*a^3*c^3)*\sqrt{-(B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^3*c^3)}))\sqrt{(3*\sqrt{1/3}*a^2*c^2*\sqrt{-(B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^3*c^3)} - 4*A*B*a*c - (B^2*a + A^2*c)*\sqrt{a}*\sqrt{c})/(a^2*c^2))} \\ &)) + 1/2*\sqrt{1/6}*\sqrt{(3*\sqrt{1/3}*a^2*c^2*\sqrt{-(B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^3*c^3)} - 4*A*B*a*c - (B^2*a + A^2*c)*\sqrt{a}*\sqrt{c})/(a^2*c^2))} \\ & *\log(-2*(B^6*a^3 - A^6*c^3)*x - 3*\sqrt{1/6}*(A*B^4*a^3*c - A^5*a*c^3 - (A^2*B^3*a^2*c - A^4*B*a*c^2 + \sqrt{1/3}*(A*B^2*a^3*c^2 - A^3*a^2*c^3))*\sqrt{-(B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^3*c^3)}))\sqrt{a}*\sqrt{c} + \sqrt{1/3}*(2*B^3*a^4*c^2 + A^2*B*a^3*c^3)*\sqrt{-(B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^3*c^3)}))\sqrt{(3*\sqrt{1/3}*a^2*c^2*\sqrt{-(B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^3*c^3)} - 4*A*B*a*c - (B^2*a + A^2*c)*\sqrt{a}*\sqrt{c})/(a^2*c^2))} \\ &)) \end{aligned}$$

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: PolynomialError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x**2+A)/(a+c*x**4-x**2*a**(1/2)*c**(1/2)),x)

[Out] Exception raised: PolynomialError

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^2+A)/(a+c*x^4-x^2*a^(1/2)*c^(1/2)),x, algorithm="giac")

[Out] Exception raised: TypeError

$$3.113 \quad \int \frac{3-x^2}{\sqrt{3+x^2-x^4}} dx$$

Optimal. Leaf size=96

$$\sqrt{7+2\sqrt{13}}\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{2}{1+\sqrt{13}}}x\right), \frac{1}{6}(-7-\sqrt{13})\right) - \sqrt{\frac{1}{2}(\sqrt{13}-1)}E\left(\sin^{-1}\left(\sqrt{\frac{2}{1+\sqrt{13}}}x\right) \middle| \frac{1}{6}(-7-\sqrt{13})\right)$$

[Out] -(Sqrt[(-1 + Sqrt[13])/2]*EllipticE[ArcSin[Sqrt[2/(1 + Sqrt[13])]]*x], (-7 - Sqrt[13])/6) + Sqrt[7 + 2*Sqrt[13]]*EllipticF[ArcSin[Sqrt[2/(1 + Sqrt[13])]]*x], (-7 - Sqrt[13])/6]

Rubi [A] time = 0.123368, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1180, 524, 424, 419}

$$\sqrt{7+2\sqrt{13}}F\left(\sin^{-1}\left(\sqrt{\frac{2}{1+\sqrt{13}}}x\right) \middle| \frac{1}{6}(-7-\sqrt{13})\right) - \sqrt{\frac{1}{2}(\sqrt{13}-1)}E\left(\sin^{-1}\left(\sqrt{\frac{2}{1+\sqrt{13}}}x\right) \middle| \frac{1}{6}(-7-\sqrt{13})\right)$$

Antiderivative was successfully verified.

[In] Int[(3 - x^2)/Sqrt[3 + x^2 - x^4], x]

[Out] -(Sqrt[(-1 + Sqrt[13])/2]*EllipticE[ArcSin[Sqrt[2/(1 + Sqrt[13])]]*x], (-7 - Sqrt[13])/6) + Sqrt[7 + 2*Sqrt[13]]*EllipticF[ArcSin[Sqrt[2/(1 + Sqrt[13])]]*x], (-7 - Sqrt[13])/6]

Rule 1180

Int[((d_) + (e_)*(x_)^2)/Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4], x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]

Rule 524

Int[((e_) + (f_)*(x_)^(n_))/(Sqrt[(a_) + (b_)*(x_)^(n_)]*Sqrt[(c_) + (d_)*(x_)^(n_)]), x_Symbol] := Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n], x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && !(EqQ[n, 2] && ((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && (!GtQ[c, 0] || SimplerSqrtQ[-(b/a), -(d/c)]))))))

Rule 424

Int[Sqrt[(a_) + (b_)*(x_)^2]/Sqrt[(c_) + (d_)*(x_)^2], x_Symbol] := Simp[(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/Sqrt[c]*Rt[-(d/c), 2], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]

Rule 419

Int[1/(Sqrt[(a_) + (b_)*(x_)^2]*Sqrt[(c_) + (d_)*(x_)^2]), x_Symbol] := Simp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/Sqrt[a]*Sqrt[c]*Rt[-(d/c), 2], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0] && !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])

Rubi steps

$$\begin{aligned} \int \frac{3-x^2}{\sqrt{3+x^2-x^4}} dx &= 2 \int \frac{3-x^2}{\sqrt{1+\sqrt{13}-2x^2}\sqrt{-1+\sqrt{13}+2x^2}} dx \\ &= (5+\sqrt{13}) \int \frac{1}{\sqrt{1+\sqrt{13}-2x^2}\sqrt{-1+\sqrt{13}+2x^2}} dx - \int \frac{\sqrt{-1+\sqrt{13}+2x^2}}{\sqrt{1+\sqrt{13}-2x^2}} dx \\ &= -\sqrt{\frac{1}{2}(-1+\sqrt{13})} E\left(\sin^{-1}\left(\sqrt{\frac{2}{1+\sqrt{13}}}x\right)\middle|\frac{1}{6}(-7-\sqrt{13})\right) + \sqrt{7+2\sqrt{13}} F\left(\sin^{-1}\left(\sqrt{\frac{2}{1+\sqrt{13}}}x\right)\middle|\frac{1}{6}(-7+\sqrt{13})\right) \end{aligned}$$

Mathematica [C] time = 0.135745, size = 103, normalized size = 1.07

$$\frac{i\left((1+\sqrt{13})E\left(i\sinh^{-1}\left(\sqrt{\frac{2}{-1+\sqrt{13}}}x\right)\middle|\frac{1}{6}(-7+\sqrt{13})\right) - (\sqrt{13}-5)\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{\frac{2}{\sqrt{13}-1}}x\right), \frac{1}{6}(\sqrt{13}-7)\right)\right)}{\sqrt{2(1+\sqrt{13})}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(3 - x^2)/Sqrt[3 + x^2 - x^4], x]

[Out] ((-I)*((1 + Sqrt[13])*EllipticE[I*ArcSinh[Sqrt[2/(-1 + Sqrt[13])]]*x], (-7 + Sqrt[13])/6] - (-5 + Sqrt[13])*EllipticF[I*ArcSinh[Sqrt[2/(-1 + Sqrt[13])]]*x], (-7 + Sqrt[13])/6))/Sqrt[2*(1 + Sqrt[13])]

Maple [B] time = 0.31, size = 200, normalized size = 2.1

$$\frac{36\sqrt{1 - (-1/6 + 1/6\sqrt{13})x^2}\sqrt{1 - (-1/6 - 1/6\sqrt{13})x^2}\left(\text{EllipticF}\left(1/6x\sqrt{-6 + 6\sqrt{13}}, i/6\sqrt{3} + i/6\sqrt{39}\right) - \text{EllipticE}\left(1/6\sqrt{-6 + 6\sqrt{13}}\sqrt{-x^4 + x^2 + 3}(1 + \sqrt{13})\right)\right)}{\sqrt{-6 + 6\sqrt{13}}\sqrt{-x^4 + x^2 + 3}(1 + \sqrt{13})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-x^2+3)/(-x^4+x^2+3)^(1/2), x)

[Out] 36/(-6+6*13^(1/2))^(1/2)*(1-(-1/6+1/6*13^(1/2))*x^2)^(1/2)*(1-(-1/6-1/6*13^(1/2))*x^2)^(1/2)/(-x^4+x^2+3)^(1/2)/(1+13^(1/2))*(EllipticF(1/6*x*(-6+6*13^(1/2))^(1/2), 1/6*I*3^(1/2)+1/6*I*39^(1/2))-EllipticE(1/6*x*(-6+6*13^(1/2))^(1/2), 1/6*I*3^(1/2)+1/6*I*39^(1/2)))+18/(-6+6*13^(1/2))^(1/2)*(1-(-1/6+1/6*13^(1/2))*x^2)^(1/2)*(1-(-1/6-1/6*13^(1/2))*x^2)^(1/2)/(-x^4+x^2+3)^(1/2)*EllipticF(1/6*x*(-6+6*13^(1/2))^(1/2), 1/6*I*3^(1/2)+1/6*I*39^(1/2))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^2-3}{\sqrt{-x^4+x^2+3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2+3)/(-x^4+x^2+3)^(1/2),x, algorithm="maxima")

[Out] -integrate((x^2 - 3)/sqrt(-x^4 + x^2 + 3), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-x^4 + x^2 + 3}(x^2 - 3)}{x^4 - x^2 - 3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2+3)/(-x^4+x^2+3)^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(-x^4 + x^2 + 3)*(x^2 - 3)/(x^4 - x^2 - 3), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^2}{\sqrt{-x^4 + x^2 + 3}} dx - \int -\frac{3}{\sqrt{-x^4 + x^2 + 3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x**2+3)/(-x**4+x**2+3)**(1/2),x)

[Out] -Integral(x**2/sqrt(-x**4 + x**2 + 3), x) - Integral(-3/sqrt(-x**4 + x**2 + 3), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x^2 - 3}{\sqrt{-x^4 + x^2 + 3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2+3)/(-x^4+x^2+3)^(1/2),x, algorithm="giac")

[Out] integrate(-(x^2 - 3)/sqrt(-x^4 + x^2 + 3), x)

$$3.114 \quad \int \frac{3-x^2}{\sqrt{3+2x^2-x^4}} dx$$

Optimal. Leaf size=25

$$4\text{EllipticF}\left(\sin^{-1}\left(\frac{x}{\sqrt{3}}\right), -3\right) - E\left(\sin^{-1}\left(\frac{x}{\sqrt{3}}\right) \middle| -3\right)$$

[Out] -EllipticE[ArcSin[x/Sqrt[3]], -3] + 4*EllipticF[ArcSin[x/Sqrt[3]], -3]

Rubi [A] time = 0.032492, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {1180, 21, 423, 424, 419}

$$4F\left(\sin^{-1}\left(\frac{x}{\sqrt{3}}\right) \middle| -3\right) - E\left(\sin^{-1}\left(\frac{x}{\sqrt{3}}\right) \middle| -3\right)$$

Antiderivative was successfully verified.

[In] Int[(3 - x^2)/Sqrt[3 + 2*x^2 - x^4], x]

[Out] -EllipticE[ArcSin[x/Sqrt[3]], -3] + 4*EllipticF[ArcSin[x/Sqrt[3]], -3]

Rule 1180

Int[((d_) + (e_)*(x_)^2)/Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4], x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]

Rule 21

Int[(u_)*((a_) + (b_)*(v_))^(m_)*((c_) + (d_)*(v_))^(n_), x_Symbol] := Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[b*c - a*d, 0] && IntegerQ[m] && (!IntegerQ[n] || SimplerQ[c + d*x, a + b*x])

Rule 423

Int[Sqrt[(a_) + (b_)*(x_)^2]/Sqrt[(c_) + (d_)*(x_)^2], x_Symbol] := Dist[b/d, Int[Sqrt[c + d*x^2]/Sqrt[a + b*x^2], x], x] - Dist[(b*c - a*d)/d, Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c] && NegQ[b/a]

Rule 424

Int[Sqrt[(a_) + (b_)*(x_)^2]/Sqrt[(c_) + (d_)*(x_)^2], x_Symbol] := Simp[(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]

Rule 419

Int[1/(Sqrt[(a_) + (b_)*(x_)^2]*Sqrt[(c_) + (d_)*(x_)^2]), x_Symbol] := Simp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0] && !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])

Rubi steps

$$\begin{aligned}
\int \frac{3-x^2}{\sqrt{3+2x^2-x^4}} dx &= 2 \int \frac{3-x^2}{\sqrt{6-2x^2}\sqrt{2+2x^2}} dx \\
&= \int \frac{\sqrt{6-2x^2}}{\sqrt{2+2x^2}} dx \\
&= 8 \int \frac{1}{\sqrt{6-2x^2}\sqrt{2+2x^2}} dx - \int \frac{\sqrt{2+2x^2}}{\sqrt{6-2x^2}} dx \\
&= -E\left(\sin^{-1}\left(\frac{x}{\sqrt{3}}\right)\middle| -3\right) + 4F\left(\sin^{-1}\left(\frac{x}{\sqrt{3}}\right)\middle| -3\right)
\end{aligned}$$

Mathematica [C] time = 0.0532473, size = 19, normalized size = 0.76

$$-i\sqrt{3}E\left(i\sinh^{-1}(x)\middle| -\frac{1}{3}\right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[(3 - x^2)/Sqrt[3 + 2*x^2 - x^4],x]

[Out] (-I)*Sqrt[3]*EllipticE[I*ArcSinh[x], -1/3]

Maple [B] time = 0.054, size = 113, normalized size = 4.5

$$\frac{\sqrt{3}}{3}\sqrt{-3x^2+9}\sqrt{x^2+1}\left(\text{EllipticF}\left(\frac{x\sqrt{3}}{3},i\sqrt{3}\right)-\text{EllipticE}\left(\frac{x\sqrt{3}}{3},i\sqrt{3}\right)\right)\frac{1}{\sqrt{-x^4+2x^2+3}}+\sqrt{3}\sqrt{-3x^2+9}\sqrt{x^2+1}E$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-x^2+3)/(-x^4+2*x^2+3)^(1/2),x)

[Out] 1/3*3^(1/2)*(-3*x^2+9)^(1/2)*(x^2+1)^(1/2)/(-x^4+2*x^2+3)^(1/2)*(EllipticF(1/3*x*3^(1/2),I*3^(1/2))-EllipticE(1/3*x*3^(1/2),I*3^(1/2)))+3^(1/2)*(-3*x^2+9)^(1/2)*(x^2+1)^(1/2)/(-x^4+2*x^2+3)^(1/2)*EllipticF(1/3*x*3^(1/2),I*3^(1/2))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^2-3}{\sqrt{-x^4+2x^2+3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2+3)/(-x^4+2*x^2+3)^(1/2),x, algorithm="maxima")

[Out] -integrate((x^2 - 3)/sqrt(-x^4 + 2*x^2 + 3), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-x^4 + 2x^2 + 3}}{x^2 + 1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2+3)/(-x^4+2*x^2+3)^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(-x^4 + 2*x^2 + 3)/(x^2 + 1), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^2}{\sqrt{-x^4 + 2x^2 + 3}} dx - \int -\frac{3}{\sqrt{-x^4 + 2x^2 + 3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x**2+3)/(-x**4+2*x**2+3)**(1/2),x)

[Out] -Integral(x**2/sqrt(-x**4 + 2*x**2 + 3), x) - Integral(-3/sqrt(-x**4 + 2*x**2 + 3), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x^2 - 3}{\sqrt{-x^4 + 2x^2 + 3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2+3)/(-x^4+2*x^2+3)^(1/2),x, algorithm="giac")

[Out] integrate(-(x^2 - 3)/sqrt(-x^4 + 2*x^2 + 3), x)

$$3.115 \quad \int \frac{3-x^2}{\sqrt{3+3x^2-x^4}} dx$$

Optimal. Leaf size=96

$$\sqrt{9+2\sqrt{21}}\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{2}{3+\sqrt{21}}}x\right), \frac{1}{2}(-5-\sqrt{21})\right) - \sqrt{\frac{1}{2}(\sqrt{21}-3)}E\left(\sin^{-1}\left(\sqrt{\frac{2}{3+\sqrt{21}}}x\right) \middle| \frac{1}{2}(-5-\sqrt{21})\right)$$

[Out] -(Sqrt[(-3 + Sqrt[21])/2]*EllipticE[ArcSin[Sqrt[2/(3 + Sqrt[21])]]*x], (-5 - Sqrt[21])/2) + Sqrt[9 + 2*Sqrt[21]]*EllipticF[ArcSin[Sqrt[2/(3 + Sqrt[21])]]*x], (-5 - Sqrt[21])/2]

Rubi [A] time = 0.178087, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1180, 524, 424, 419}

$$\sqrt{9+2\sqrt{21}}F\left(\sin^{-1}\left(\sqrt{\frac{2}{3+\sqrt{21}}}x\right) \middle| \frac{1}{2}(-5-\sqrt{21})\right) - \sqrt{\frac{1}{2}(\sqrt{21}-3)}E\left(\sin^{-1}\left(\sqrt{\frac{2}{3+\sqrt{21}}}x\right) \middle| \frac{1}{2}(-5-\sqrt{21})\right)$$

Antiderivative was successfully verified.

[In] Int[(3 - x^2)/Sqrt[3 + 3*x^2 - x^4], x]

[Out] -(Sqrt[(-3 + Sqrt[21])/2]*EllipticE[ArcSin[Sqrt[2/(3 + Sqrt[21])]]*x], (-5 - Sqrt[21])/2) + Sqrt[9 + 2*Sqrt[21]]*EllipticF[ArcSin[Sqrt[2/(3 + Sqrt[21])]]*x], (-5 - Sqrt[21])/2]

Rule 1180

Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]

Rule 524

Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.)*(x_)^(n_)]), x_Symbol] := Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n], x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && !(EqQ[n, 2] && ((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && (!GtQ[c, 0] || SimplerSqrtQ[-(b/a), -(d/c)]))))))

Rule 424

Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] := Simp[(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]

Rule 419

Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] := Simp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0] && !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])

Rubi steps

$$\begin{aligned} \int \frac{3-x^2}{\sqrt{3+3x^2-x^4}} dx &= 2 \int \frac{3-x^2}{\sqrt{3+\sqrt{21}-2x^2}\sqrt{-3+\sqrt{21}+2x^2}} dx \\ &= (3+\sqrt{21}) \int \frac{1}{\sqrt{3+\sqrt{21}-2x^2}\sqrt{-3+\sqrt{21}+2x^2}} dx - \int \frac{\sqrt{-3+\sqrt{21}+2x^2}}{\sqrt{3+\sqrt{21}-2x^2}} dx \\ &= -\sqrt{\frac{1}{2}(-3+\sqrt{21})} E\left(\sin^{-1}\left(\sqrt{\frac{2}{3+\sqrt{21}}}x\right)\middle|\frac{1}{2}(-5-\sqrt{21})\right) + \frac{1}{2}\sqrt{36+8\sqrt{21}} F\left(\sin^{-1}\left(\sqrt{\frac{2}{3+\sqrt{21}}}x\right)\right) \end{aligned}$$

Mathematica [C] time = 0.165314, size = 103, normalized size = 1.07

$$\frac{i\left((3+\sqrt{21})E\left(i\sinh^{-1}\left(\sqrt{\frac{2}{-3+\sqrt{21}}}x\right)\middle|\frac{1}{2}(-5+\sqrt{21})\right) - (\sqrt{21}-3)\operatorname{EllipticF}\left(i\sinh^{-1}\left(\sqrt{\frac{2}{\sqrt{21}-3}}x\right), \frac{1}{2}(\sqrt{21}-5)\right)\right)}{\sqrt{2(3+\sqrt{21})}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(3 - x^2)/Sqrt[3 + 3*x^2 - x^4], x]

[Out] ((-I)*((3 + Sqrt[21])*EllipticE[I*ArcSinh[Sqrt[2/(-3 + Sqrt[21]])]*x], (-5 + Sqrt[21])/2] - (-3 + Sqrt[21])*EllipticF[I*ArcSinh[Sqrt[2/(-3 + Sqrt[21]])]*x], (-5 + Sqrt[21])/2))/Sqrt[2*(3 + Sqrt[21])]

Maple [B] time = 0.296, size = 204, normalized size = 2.1

$$\frac{36\sqrt{1 - (-1/2 + 1/6\sqrt{21})x^2}\sqrt{1 - (-1/2 - 1/6\sqrt{21})x^2}\left(\operatorname{EllipticF}\left(1/6x\sqrt{-18 + 6\sqrt{21}}, i/2\sqrt{3} + i/2\sqrt{7}\right) - \operatorname{EllipticE}\left(1/6\sqrt{-18 + 6\sqrt{21}}\sqrt{-x^4 + 3x^2 + 3}(3 + \sqrt{21})\right)\right)}{\sqrt{-18 + 6\sqrt{21}}\sqrt{-x^4 + 3x^2 + 3}(3 + \sqrt{21})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-x^2+3)/(-x^4+3*x^2+3)^(1/2), x)

[Out] 36/(-18+6*21^(1/2))^(1/2)*(1-(-1/2+1/6*21^(1/2))*x^2)^(1/2)*(1-(-1/2-1/6*21^(1/2))*x^2)^(1/2)/(-x^4+3*x^2+3)^(1/2)/(3+21^(1/2))*(EllipticF(1/6*x*(-18+6*21^(1/2))^(1/2), 1/2*I*3^(1/2)+1/2*I*7^(1/2))-EllipticE(1/6*x*(-18+6*21^(1/2))^(1/2), 1/2*I*3^(1/2)+1/2*I*7^(1/2)))+18/(-18+6*21^(1/2))^(1/2)*(1-(-1/2+1/6*21^(1/2))*x^2)^(1/2)*(1-(-1/2-1/6*21^(1/2))*x^2)^(1/2)/(-x^4+3*x^2+3)^(1/2)*EllipticF(1/6*x*(-18+6*21^(1/2))^(1/2), 1/2*I*3^(1/2)+1/2*I*7^(1/2))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^2-3}{\sqrt{-x^4+3x^2+3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2+3)/(-x^4+3*x^2+3)^(1/2),x, algorithm="maxima")

[Out] -integrate((x^2 - 3)/sqrt(-x^4 + 3*x^2 + 3), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-x^4 + 3x^2 + 3}(x^2 - 3)}{x^4 - 3x^2 - 3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2+3)/(-x^4+3*x^2+3)^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(-x^4 + 3*x^2 + 3)*(x^2 - 3)/(x^4 - 3*x^2 - 3), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^2}{\sqrt{-x^4 + 3x^2 + 3}} dx - \int -\frac{3}{\sqrt{-x^4 + 3x^2 + 3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x**2+3)/(-x**4+3*x**2+3)**(1/2),x)

[Out] -Integral(x**2/sqrt(-x**4 + 3*x**2 + 3), x) - Integral(-3/sqrt(-x**4 + 3*x**2 + 3), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x^2 - 3}{\sqrt{-x^4 + 3x^2 + 3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2+3)/(-x^4+3*x^2+3)^(1/2),x, algorithm="giac")

[Out] integrate(-(x^2 - 3)/sqrt(-x^4 + 3*x^2 + 3), x)

$$3.116 \quad \int \frac{3-x^2}{\sqrt{3-x^2-x^4}} dx$$

Optimal. Leaf size=92

$$\sqrt{5+2\sqrt{13}}\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{2}{\sqrt{13}-1}}x\right), \frac{1}{6}(\sqrt{13}-7)\right) - \sqrt{\frac{1}{2}(1+\sqrt{13})}E\left(\sin^{-1}\left(\sqrt{\frac{2}{-1+\sqrt{13}}}x\right)\middle|\frac{1}{6}(-7+\sqrt{13})\right)$$

[Out] -(Sqrt[(1 + Sqrt[13])/2]*EllipticE[ArcSin[Sqrt[2/(-1 + Sqrt[13])]]*x], (-7 + Sqrt[13])/6)) + Sqrt[5 + 2*Sqrt[13]]*EllipticF[ArcSin[Sqrt[2/(-1 + Sqrt[13])]]*x], (-7 + Sqrt[13])/6]

Rubi [A] time = 0.120274, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1180, 524, 424, 419}

$$\sqrt{5+2\sqrt{13}}F\left(\sin^{-1}\left(\sqrt{\frac{2}{-1+\sqrt{13}}}x\right)\middle|\frac{1}{6}(-7+\sqrt{13})\right) - \sqrt{\frac{1}{2}(1+\sqrt{13})}E\left(\sin^{-1}\left(\sqrt{\frac{2}{-1+\sqrt{13}}}x\right)\middle|\frac{1}{6}(-7+\sqrt{13})\right)$$

Antiderivative was successfully verified.

[In] Int[(3 - x^2)/Sqrt[3 - x^2 - x^4], x]

[Out] -(Sqrt[(1 + Sqrt[13])/2]*EllipticE[ArcSin[Sqrt[2/(-1 + Sqrt[13])]]*x], (-7 + Sqrt[13])/6)) + Sqrt[5 + 2*Sqrt[13]]*EllipticF[ArcSin[Sqrt[2/(-1 + Sqrt[13])]]*x], (-7 + Sqrt[13])/6]

Rule 1180

Int[((d_) + (e_)*(x_)^2)/Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4], x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]

Rule 524

Int[((e_) + (f_)*(x_)^(n_))/(Sqrt[(a_) + (b_)*(x_)^(n_)]*Sqrt[(c_) + (d_)*(x_)^(n_)]), x_Symbol] := Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n], x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && !(EqQ[n, 2] && ((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && (!GtQ[c, 0] || SimplerSqrtQ[-(b/a), -(d/c)]))))))

Rule 424

Int[Sqrt[(a_) + (b_)*(x_)^2]/Sqrt[(c_) + (d_)*(x_)^2], x_Symbol] := Simp[(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]

Rule 419

Int[1/(Sqrt[(a_) + (b_)*(x_)^2]*Sqrt[(c_) + (d_)*(x_)^2]), x_Symbol] := Simp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0] && !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])

Rubi steps

$$\begin{aligned} \int \frac{3-x^2}{\sqrt{3-x^2-x^4}} dx &= 2 \int \frac{3-x^2}{\sqrt{-1+\sqrt{13}-2x^2}\sqrt{1+\sqrt{13}+2x^2}} dx \\ &= (7+\sqrt{13}) \int \frac{1}{\sqrt{-1+\sqrt{13}-2x^2}\sqrt{1+\sqrt{13}+2x^2}} dx - \int \frac{\sqrt{1+\sqrt{13}+2x^2}}{\sqrt{-1+\sqrt{13}-2x^2}} dx \\ &= -\sqrt{\frac{1}{2}(1+\sqrt{13})} E\left(\sin^{-1}\left(\sqrt{\frac{2}{-1+\sqrt{13}}}x\right) \middle| \frac{1}{6}(-7+\sqrt{13})\right) + \sqrt{5+2\sqrt{13}} F\left(\sin^{-1}\left(\sqrt{\frac{2}{-1+\sqrt{13}}}x\right)\right) \end{aligned}$$

Mathematica [C] time = 0.131711, size = 107, normalized size = 1.16

$$\frac{i\left((\sqrt{13}-1)E\left(i\sinh^{-1}\left(\sqrt{\frac{2}{1+\sqrt{13}}}x\right) \middle| -\frac{7}{6}-\frac{\sqrt{13}}{6}\right) - (\sqrt{13}-7)\operatorname{EllipticF}\left(i\sinh^{-1}\left(\sqrt{\frac{2}{1+\sqrt{13}}}x\right), -\frac{7}{6}-\frac{\sqrt{13}}{6}\right)\right)}{\sqrt{2(\sqrt{13}-1)}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(3 - x^2)/Sqrt[3 - x^2 - x^4], x]

[Out] ((-I)*((-1 + Sqrt[13])*EllipticE[I*ArcSinh[Sqrt[2/(1 + Sqrt[13])]]*x], -7/6 - Sqrt[13]/6) - (-7 + Sqrt[13])*EllipticF[I*ArcSinh[Sqrt[2/(1 + Sqrt[13])]]*x], -7/6 - Sqrt[13]/6))/Sqrt[2*(-1 + Sqrt[13])]

Maple [B] time = 0.403, size = 204, normalized size = 2.2

$$\frac{36\sqrt{1-(1/6+1/6\sqrt{13})x^2}\sqrt{1-(1/6-1/6\sqrt{13})x^2}\left(\operatorname{EllipticF}\left(1/6x\sqrt{6+6\sqrt{13}}, i/6\sqrt{39}-i/6\sqrt{3}\right) - \operatorname{EllipticE}\left(1/6x\sqrt{6+6\sqrt{13}}\right)\right)}{\sqrt{6+6\sqrt{13}}\sqrt{-x^4-x^2+3}(-1+\sqrt{13})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-x^2+3)/(-x^4-x^2+3)^(1/2), x)

[Out] 36/(6+6*13^(1/2))^(1/2)*(1-(1/6+1/6*13^(1/2))*x^2)^(1/2)*(1-(1/6-1/6*13^(1/2))*x^2)^(1/2)/(-x^4-x^2+3)^(1/2)/(-1+13^(1/2))*(EllipticF(1/6*x*(6+6*13^(1/2))^(1/2), 1/6*I*39^(1/2)-1/6*I*3^(1/2))-EllipticE(1/6*x*(6+6*13^(1/2))^(1/2), 1/6*I*39^(1/2)-1/6*I*3^(1/2)))+18/(6+6*13^(1/2))^(1/2)*(1-(1/6+1/6*13^(1/2))*x^2)^(1/2)*(1-(1/6-1/6*13^(1/2))*x^2)^(1/2)/(-x^4-x^2+3)^(1/2)*EllipticF(1/6*x*(6+6*13^(1/2))^(1/2), 1/6*I*39^(1/2)-1/6*I*3^(1/2))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^2-3}{\sqrt{-x^4-x^2+3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2+3)/(-x^4-x^2+3)^(1/2),x, algorithm="maxima")

[Out] -integrate((x^2 - 3)/sqrt(-x^4 - x^2 + 3), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-x^4 - x^2 + 3}(x^2 - 3)}{x^4 + x^2 - 3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2+3)/(-x^4-x^2+3)^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(-x^4 - x^2 + 3)*(x^2 - 3)/(x^4 + x^2 - 3), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^2}{\sqrt{-x^4 - x^2 + 3}} dx - \int -\frac{3}{\sqrt{-x^4 - x^2 + 3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x**2+3)/(-x**4-x**2+3)**(1/2),x)

[Out] -Integral(x**2/sqrt(-x**4 - x**2 + 3), x) - Integral(-3/sqrt(-x**4 - x**2 + 3), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x^2 - 3}{\sqrt{-x^4 - x^2 + 3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2+3)/(-x^4-x^2+3)^(1/2),x, algorithm="giac")

[Out] integrate(-(x^2 - 3)/sqrt(-x^4 - x^2 + 3), x)

$$3.117 \quad \int \frac{3-x^2}{\sqrt{3-2x^2-x^4}} dx$$

Optimal. Leaf size=27

$$2\sqrt{3}\text{EllipticF}\left(\sin^{-1}(x), -\frac{1}{3}\right) - \sqrt{3}E\left(\sin^{-1}(x) \middle| -\frac{1}{3}\right)$$

```
[Out] -(Sqrt[3]*EllipticE[ArcSin[x], -1/3]) + 2*Sqrt[3]*EllipticF[ArcSin[x], -1/3]
```

Rubi [A] time = 0.0387074, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1180, 524, 424, 419}

$$2\sqrt{3}F\left(\sin^{-1}(x) \middle| -\frac{1}{3}\right) - \sqrt{3}E\left(\sin^{-1}(x) \middle| -\frac{1}{3}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(3 - x^2)/Sqrt[3 - 2*x^2 - x^4], x]
```

```
[Out] -(Sqrt[3]*EllipticE[ArcSin[x], -1/3]) + 2*Sqrt[3]*EllipticF[ArcSin[x], -1/3]
```

Rule 1180

```
Int[((d_) + (e_)*(x_)^2)/Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4], x_Symbol]
:> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rule 524

```
Int[((e_) + (f_)*(x_)^(n_))/(Sqrt[(a_) + (b_)*(x_)^(n_)]*Sqrt[(c_) + (d_)*(x_)^(n_)]), x_Symbol]
:> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n], x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && !(EqQ[n, 2] && ((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && (!GtQ[c, 0] || SimplerSqrtQ[-(b/a), -(d/c)]))))))
```

Rule 424

```
Int[Sqrt[(a_) + (b_)*(x_)^2]/Sqrt[(c_) + (d_)*(x_)^2], x_Symbol]
:> Simp[(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_)*(x_)^2]*Sqrt[(c_) + (d_)*(x_)^2]), x_Symbol]
:> Simp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0] && !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\begin{aligned} \int \frac{3-x^2}{\sqrt{3-2x^2-x^4}} dx &= 2 \int \frac{3-x^2}{\sqrt{2-2x^2}\sqrt{6+2x^2}} dx \\ &= 12 \int \frac{1}{\sqrt{2-2x^2}\sqrt{6+2x^2}} dx - \int \frac{\sqrt{6+2x^2}}{\sqrt{2-2x^2}} dx \\ &= -\sqrt{3}E\left(\sin^{-1}(x)\middle|-\frac{1}{3}\right) + 2\sqrt{3}F\left(\sin^{-1}(x)\middle|-\frac{1}{3}\right) \end{aligned}$$

Mathematica [C] time = 0.0641494, size = 35, normalized size = 1.3

$$-i\left(2\text{EllipticF}\left(i\sinh^{-1}\left(\frac{x}{\sqrt{3}}\right),-3\right)+E\left(i\sinh^{-1}\left(\frac{x}{\sqrt{3}}\right)\middle|-3\right)\right)$$

Antiderivative was successfully verified.

[In] Integrate[(3 - x^2)/Sqrt[3 - 2*x^2 - x^4],x]

[Out] (-I)*(EllipticE[I*ArcSinh[x/Sqrt[3]], -3] + 2*EllipticF[I*ArcSinh[x/Sqrt[3]], -3])

Maple [B] time = 0.05, size = 95, normalized size = 3.5

$$\left(\text{EllipticF}\left(x,\frac{i}{3}\sqrt{3}\right)-\text{EllipticE}\left(x,\frac{i}{3}\sqrt{3}\right)\right)\sqrt{-x^2+1}\sqrt{3x^2+9}\frac{1}{\sqrt{-x^4-2x^2+3}}+\text{EllipticF}\left(x,\frac{i}{3}\sqrt{3}\right)\sqrt{-x^2+1}\sqrt{3x^2+9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-x^2+3)/(-x^4-2*x^2+3)^(1/2),x)

[Out] (-x^2+1)^(1/2)*(3*x^2+9)^(1/2)/(-x^4-2*x^2+3)^(1/2)*(EllipticF(x,1/3*I*3^(1/2))-EllipticE(x,1/3*I*3^(1/2)))+(-x^2+1)^(1/2)*(3*x^2+9)^(1/2)/(-x^4-2*x^2+3)^(1/2)*EllipticF(x,1/3*I*3^(1/2))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^2-3}{\sqrt{-x^4-2x^2+3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2+3)/(-x^4-2*x^2+3)^(1/2),x, algorithm="maxima")

[Out] -integrate((x^2 - 3)/sqrt(-x^4 - 2*x^2 + 3), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-x^4-2x^2+3}(x^2-3)}{x^4+2x^2-3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+3)/(-x^4-2*x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-x^4 - 2*x^2 + 3)*(x^2 - 3)/(x^4 + 2*x^2 - 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^2}{\sqrt{-x^4 - 2x^2 + 3}} dx - \int -\frac{3}{\sqrt{-x^4 - 2x^2 + 3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+3)/(-x**4-2*x**2+3)**(1/2),x)
```

```
[Out] -Integral(x**2/sqrt(-x**4 - 2*x**2 + 3), x) - Integral(-3/sqrt(-x**4 - 2*x**2 + 3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x^2 - 3}{\sqrt{-x^4 - 2x^2 + 3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+3)/(-x^4-2*x^2+3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-(x^2 - 3)/sqrt(-x^4 - 2*x^2 + 3), x)
```

$$3.118 \quad \int \frac{3-x^2}{\sqrt{3-3x^2-x^4}} dx$$

Optimal. Leaf size=92

$$\sqrt{3+2\sqrt{21}}\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{2}{\sqrt{21}-3}}x\right), \frac{1}{2}(\sqrt{21}-5)\right) - \sqrt{\frac{1}{2}(3+\sqrt{21})}E\left(\sin^{-1}\left(\sqrt{\frac{2}{-3+\sqrt{21}}}x\right)\middle|\frac{1}{2}(-5+\sqrt{21})\right)$$

[Out] -(Sqrt[(3 + Sqrt[21])/2]*EllipticE[ArcSin[Sqrt[2/(-3 + Sqrt[21]])]*x], (-5 + Sqrt[21])/2)) + Sqrt[3 + 2*Sqrt[21]]*EllipticF[ArcSin[Sqrt[2/(-3 + Sqrt[21]])]*x], (-5 + Sqrt[21])/2]

Rubi [A] time = 0.165331, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1180, 524, 424, 419}

$$\sqrt{3+2\sqrt{21}}F\left(\sin^{-1}\left(\sqrt{\frac{2}{-3+\sqrt{21}}}x\right)\middle|\frac{1}{2}(-5+\sqrt{21})\right) - \sqrt{\frac{1}{2}(3+\sqrt{21})}E\left(\sin^{-1}\left(\sqrt{\frac{2}{-3+\sqrt{21}}}x\right)\middle|\frac{1}{2}(-5+\sqrt{21})\right)$$

Antiderivative was successfully verified.

[In] Int[(3 - x^2)/Sqrt[3 - 3*x^2 - x^4], x]

[Out] -(Sqrt[(3 + Sqrt[21])/2]*EllipticE[ArcSin[Sqrt[2/(-3 + Sqrt[21]])]*x], (-5 + Sqrt[21])/2)) + Sqrt[3 + 2*Sqrt[21]]*EllipticF[ArcSin[Sqrt[2/(-3 + Sqrt[21]])]*x], (-5 + Sqrt[21])/2]

Rule 1180

Int[((d_) + (e_)*(x_)^2)/Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4], x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]

Rule 524

Int[((e_) + (f_)*(x_)^(n_))/(Sqrt[(a_) + (b_)*(x_)^(n_)]*Sqrt[(c_) + (d_)*(x_)^(n_)]), x_Symbol] := Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n], x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && !(EqQ[n, 2] && ((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && (!GtQ[c, 0] || SimplerSqrtQ[-(b/a), -(d/c)]))))))

Rule 424

Int[Sqrt[(a_) + (b_)*(x_)^2]/Sqrt[(c_) + (d_)*(x_)^2], x_Symbol] := Simp[(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)]/(Sqrt[c]*Rt[-(d/c), 2])), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]

Rule 419

Int[1/(Sqrt[(a_) + (b_)*(x_)^2]*Sqrt[(c_) + (d_)*(x_)^2]), x_Symbol] := Simp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)]/(Sqrt[a]*Sqrt[c]*Rt[-(d/c), 2])), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0] && !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])

Rubi steps

$$\begin{aligned}
\int \frac{3-x^2}{\sqrt{3-3x^2-x^4}} dx &= 2 \int \frac{3-x^2}{\sqrt{-3+\sqrt{21}-2x^2}\sqrt{3+\sqrt{21}+2x^2}} dx \\
&= (9+\sqrt{21}) \int \frac{1}{\sqrt{-3+\sqrt{21}-2x^2}\sqrt{3+\sqrt{21}+2x^2}} dx - \int \frac{\sqrt{3+\sqrt{21}+2x^2}}{\sqrt{-3+\sqrt{21}-2x^2}} dx \\
&= -\sqrt{\frac{1}{2}(3+\sqrt{21})} E\left(\sin^{-1}\left(\sqrt{\frac{2}{-3+\sqrt{21}}}x\right) \middle| \frac{1}{2}(-5+\sqrt{21})\right) + \sqrt{3+2\sqrt{21}} F\left(\sin^{-1}\left(\sqrt{\frac{2}{-3+\sqrt{21}}}\right)\right)
\end{aligned}$$

Mathematica [C] time = 0.165738, size = 107, normalized size = 1.16

$$\frac{i\left((\sqrt{21}-3)E\left(i\sinh^{-1}\left(\sqrt{\frac{2}{3+\sqrt{21}}}x\right) \middle| -\frac{5}{2}-\frac{\sqrt{21}}{2}\right) - (\sqrt{21}-9)\operatorname{EllipticF}\left(i\sinh^{-1}\left(\sqrt{\frac{2}{3+\sqrt{21}}}x\right), -\frac{5}{2}-\frac{\sqrt{21}}{2}\right)\right)}{\sqrt{2(\sqrt{21}-3)}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(3 - x^2)/Sqrt[3 - 3*x^2 - x^4], x]

[Out] ((-I)*((-3 + Sqrt[21])*EllipticE[I*ArcSinh[Sqrt[2/(3 + Sqrt[21]])]*x], -5/2 - Sqrt[21]/2] - (-9 + Sqrt[21])*EllipticF[I*ArcSinh[Sqrt[2/(3 + Sqrt[21]])]*x], -5/2 - Sqrt[21]/2))/Sqrt[2*(-3 + Sqrt[21])]

Maple [B] time = 0.291, size = 204, normalized size = 2.2

$$\frac{\sqrt{1-(1/2+1/6\sqrt{21})x^2}\sqrt{1-(1/2-1/6\sqrt{21})x^2}\left(\operatorname{EllipticF}\left(1/6x\sqrt{18+6\sqrt{21}}, i/2\sqrt{7}-i/2\sqrt{3}\right) - \operatorname{EllipticE}\left(1/6x\sqrt{18+6\sqrt{21}}, i/2\sqrt{7}-i/2\sqrt{3}\right)\right)}{\sqrt{18+6\sqrt{21}}\sqrt{-x^4-3x^2+3}(-3+\sqrt{21})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-x^2+3)/(-x^4-3*x^2+3)^(1/2), x)

[Out] 36/(18+6*21^(1/2))^(1/2)*(1-(1/2+1/6*21^(1/2))*x^2)^(1/2)*(1-(1/2-1/6*21^(1/2))*x^2)^(1/2)/(-x^4-3*x^2+3)^(1/2)/(-3+21^(1/2))*(EllipticF(1/6*x*(18+6*21^(1/2))^(1/2), 1/2*I*7^(1/2)-1/2*I*3^(1/2))-EllipticE(1/6*x*(18+6*21^(1/2))^(1/2), 1/2*I*7^(1/2)-1/2*I*3^(1/2)))+18/(18+6*21^(1/2))^(1/2)*(1-(1/2+1/6*21^(1/2))*x^2)^(1/2)*(1-(1/2-1/6*21^(1/2))*x^2)^(1/2)/(-x^4-3*x^2+3)^(1/2)*EllipticF(1/6*x*(18+6*21^(1/2))^(1/2), 1/2*I*7^(1/2)-1/2*I*3^(1/2))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^2-3}{\sqrt{-x^4-3x^2+3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2+3)/(-x^4-3*x^2+3)^(1/2),x, algorithm="maxima")

[Out] -integrate((x^2 - 3)/sqrt(-x^4 - 3*x^2 + 3), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-x^4 - 3x^2 + 3}(x^2 - 3)}{x^4 + 3x^2 - 3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2+3)/(-x^4-3*x^2+3)^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(-x^4 - 3*x^2 + 3)*(x^2 - 3)/(x^4 + 3*x^2 - 3), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^2}{\sqrt{-x^4 - 3x^2 + 3}} dx - \int -\frac{3}{\sqrt{-x^4 - 3x^2 + 3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x**2+3)/(-x**4-3*x**2+3)**(1/2),x)

[Out] -Integral(x**2/sqrt(-x**4 - 3*x**2 + 3), x) - Integral(-3/sqrt(-x**4 - 3*x**2 + 3), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x^2 - 3}{\sqrt{-x^4 - 3x^2 + 3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2+3)/(-x^4-3*x^2+3)^(1/2),x, algorithm="giac")

[Out] integrate(-(x^2 - 3)/sqrt(-x^4 - 3*x^2 + 3), x)

$$3.119 \quad \int \frac{b - \sqrt{b^2 - 4ac} + 2cx^2}{\sqrt{a + bx^2 + cx^4}} dx$$

Optimal. Leaf size=296

$$\frac{\left(-\sqrt{b^2 - 4ac} + 2\sqrt{a}\sqrt{c} + b\right)\left(\sqrt{a} + \sqrt{cx^2}\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{cx^2}\right)^2}}\operatorname{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{cx}}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+bx^2+cx^4}} + \frac{2\sqrt{cx}\sqrt{a+bx^2+cx^4}}{\sqrt{a}+\sqrt{cx^2}}$$

```
[Out] (2*Sqrt[c]*x*Sqrt[a + b*x^2 + c*x^4])/(Sqrt[a] + Sqrt[c]*x^2) - (2*a^(1/4)*
c^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]
*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))
/4])/Sqrt[a + b*x^2 + c*x^4] + ((b + 2*Sqrt[a]*Sqrt[c] - Sqrt[b^2 - 4*a*c])
*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2
]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2
*a^(1/4)*c^(1/4)*Sqrt[a + b*x^2 + c*x^4])
```

Rubi [A] time = 0.118657, antiderivative size = 296, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1197, 1103, 1195}

$$\frac{\left(-\sqrt{b^2 - 4ac} + 2\sqrt{a}\sqrt{c} + b\right)\left(\sqrt{a} + \sqrt{cx^2}\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{cx^2}\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{cx}}{\sqrt[4]{a}}\right)\middle|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+bx^2+cx^4}} + \frac{2\sqrt{cx}\sqrt{a+bx^2+cx^4}}{\sqrt{a}+\sqrt{cx^2}} - \frac{2\sqrt[4]{a}}{\sqrt{a}+\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/Sqrt[a + b*x^2 + c*x^4], x]
```

```
[Out] (2*Sqrt[c]*x*Sqrt[a + b*x^2 + c*x^4])/(Sqrt[a] + Sqrt[c]*x^2) - (2*a^(1/4)*
c^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]
*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))
/4])/Sqrt[a + b*x^2 + c*x^4] + ((b + 2*Sqrt[a]*Sqrt[c] - Sqrt[b^2 - 4*a*c])
*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2
]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2
*a^(1/4)*c^(1/4)*Sqrt[a + b*x^2 + c*x^4])
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol]
:> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; NeQ[e + d*q, 0] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol]
:> With[{q = Rt[c/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]), x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol]
:> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^2*x^2))], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

$2*x^2)), x] + \text{Simp}[(d*(1 + q^2*x^2)*\text{Sqrt}[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*\text{EllipticE}[2*\text{ArcTan}[q*x], 1/2 - (b*q^2)/(4*c)])/(q*\text{Sqrt}[a + b*x^2 + c*x^4]), x] /; \text{EqQ}[e + d*q^2, 0] /; \text{FreeQ}[\{a, b, c, d, e\}, x] \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& \text{PosQ}[c/a]$

Rubi steps

$$\int \frac{b - \sqrt{b^2 - 4ac} + 2cx^2}{\sqrt{a + bx^2 + cx^4}} dx = - \left((2\sqrt{a}\sqrt{c}) \int \frac{1 - \frac{\sqrt{cx^2}}{\sqrt{a}}}{\sqrt{a + bx^2 + cx^4}} dx \right) + (b + 2\sqrt{a}\sqrt{c} - \sqrt{b^2 - 4ac}) \int \frac{1}{\sqrt{a + bx^2 + cx^4}} dx$$

$$= \frac{2\sqrt{cx}\sqrt{a + bx^2 + cx^4}}{\sqrt{a} + \sqrt{cx^2}} - \frac{2^4\sqrt{a}\sqrt{c}(\sqrt{a} + \sqrt{cx^2})\sqrt{\frac{a+bx^2+cx^4}{(\sqrt{a}+\sqrt{cx^2})^2}}E\left(2\tan^{-1}\left(\frac{\sqrt{cx}}{\sqrt{a}}\right)\right)\frac{1}{4}\left(2 - \frac{b}{\sqrt{a}\sqrt{c}}\right)}{\sqrt{a + bx^2 + cx^4}}$$

Mathematica [C] time = 0.308585, size = 187, normalized size = 0.63

$$\frac{2i\sqrt{2}a\sqrt{\frac{c}{b^2-4ac+b}}\sqrt{\frac{\sqrt{b^2-4ac+b}+2cx^2}{\sqrt{b^2-4ac+b}}}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}} + 1E\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\right)\right)\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}}{\sqrt{a + bx^2 + cx^4}}$$

Antiderivative was successfully verified.

[In] Integrate[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/Sqrt[a + b*x^2 + c*x^4], x]

[Out] ((-2*I)*Sqrt[2]*a*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])])/Sqrt[a + b*x^2 + c*x^4]

Maple [A] time = 0.075, size = 515, normalized size = 1.7

$$-ac\sqrt{2}\sqrt{4 - 2\frac{(-b + \sqrt{-4ac + b^2})x^2}{a}}\sqrt{4 + 2\frac{(b + \sqrt{-4ac + b^2})x^2}{a}}\left(\text{EllipticF}\left(\frac{x\sqrt{2}}{2}\sqrt{\frac{1}{a}(-b + \sqrt{-4ac + b^2})}, \frac{1}{2}\sqrt{-4 + 2\frac{(-b + \sqrt{-4ac + b^2})x^2}{a}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*c*x^2-(-4*a*c+b^2)^(1/2)+b)/(c*x^4+b*x^2+a)^(1/2), x)

[Out] -c*a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*x^2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2), 1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*x^2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2), 1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2)))-1/4*(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x^2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2), 1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))+1/4*b*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x^2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2), 1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))

$$\sqrt{a/c})$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{2cx^2 + b - \sqrt{b^2 - 4ac}}{\sqrt{cx^4 + bx^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*c*x^2-(-4*a*c+b^2)^(1/2)+b)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")

[Out] integrate((2*c*x^2 + b - sqrt(b^2 - 4*a*c))/sqrt(c*x^4 + b*x^2 + a), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{2cx^2 + b - \sqrt{b^2 - 4ac}}{\sqrt{cx^4 + bx^2 + a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*c*x^2-(-4*a*c+b^2)^(1/2)+b)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")

[Out] integral((2*c*x^2 + b - sqrt(b^2 - 4*a*c))/sqrt(c*x^4 + b*x^2 + a), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b + 2cx^2 - \sqrt{-4ac + b^2}}{\sqrt{a + bx^2 + cx^4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*c*x**2-(-4*a*c+b**2)**(1/2)+b)/(c*x**4+b*x**2+a)**(1/2),x)

[Out] Integral((b + 2*c*x**2 - sqrt(-4*a*c + b**2))/sqrt(a + b*x**2 + c*x**4), x)

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*c*x^2-(-4*a*c+b^2)^(1/2)+b)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")

[Out] Exception raised: TypeError

3.120 $\int (d + ex^2)^4 (a + cx^4) dx$

Optimal. Leaf size=106

$$\frac{1}{9}e^2x^9(ae^2 + 6cd^2) + \frac{4}{7}dex^7(ae^2 + cd^2) + \frac{1}{5}d^2x^5(6ae^2 + cd^2) + \frac{4}{3}ad^3ex^3 + ad^4x + \frac{4}{11}cde^3x^{11} + \frac{1}{13}ce^4x^{13}$$

[Out] a*d^4*x + (4*a*d^3*e*x^3)/3 + (d^2*(c*d^2 + 6*a*e^2)*x^5)/5 + (4*d*e*(c*d^2 + a*e^2)*x^7)/7 + (e^2*(6*c*d^2 + a*e^2)*x^9)/9 + (4*c*d*e^3*x^11)/11 + (c*e^4*x^13)/13

Rubi [A] time = 0.0826949, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {1154}

$$\frac{1}{9}e^2x^9(ae^2 + 6cd^2) + \frac{4}{7}dex^7(ae^2 + cd^2) + \frac{1}{5}d^2x^5(6ae^2 + cd^2) + \frac{4}{3}ad^3ex^3 + ad^4x + \frac{4}{11}cde^3x^{11} + \frac{1}{13}ce^4x^{13}$$

Antiderivative was successfully verified.

[In] Int[(d + e*x^2)^4*(a + c*x^4), x]

[Out] a*d^4*x + (4*a*d^3*e*x^3)/3 + (d^2*(c*d^2 + 6*a*e^2)*x^5)/5 + (4*d*e*(c*d^2 + a*e^2)*x^7)/7 + (e^2*(6*c*d^2 + a*e^2)*x^9)/9 + (4*c*d*e^3*x^11)/11 + (c*e^4*x^13)/13

Rule 1154

Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]

Rubi steps

$$\begin{aligned} \int (d + ex^2)^4 (a + cx^4) dx &= \int (ad^4 + 4ad^3ex^2 + d^2(cd^2 + 6ae^2)x^4 + 4de(cd^2 + ae^2)x^6 + e^2(6cd^2 + ae^2)x^8 + 4cde^3x^{10} \\ &\quad + ad^4x + \frac{4}{3}ad^3ex^3 + \frac{1}{5}d^2(cd^2 + 6ae^2)x^5 + \frac{4}{7}de(cd^2 + ae^2)x^7 + \frac{1}{9}e^2(6cd^2 + ae^2)x^9 + \frac{4}{11}cde^3x^{11} \\ &\quad + \frac{1}{13}ce^4x^{13}) dx \end{aligned}$$

Mathematica [A] time = 0.0206913, size = 106, normalized size = 1.

$$\frac{1}{9}e^2x^9(ae^2 + 6cd^2) + \frac{4}{7}dex^7(ae^2 + cd^2) + \frac{1}{5}d^2x^5(6ae^2 + cd^2) + \frac{4}{3}ad^3ex^3 + ad^4x + \frac{4}{11}cde^3x^{11} + \frac{1}{13}ce^4x^{13}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e*x^2)^4*(a + c*x^4), x]

[Out] a*d^4*x + (4*a*d^3*e*x^3)/3 + (d^2*(c*d^2 + 6*a*e^2)*x^5)/5 + (4*d*e*(c*d^2 + a*e^2)*x^7)/7 + (e^2*(6*c*d^2 + a*e^2)*x^9)/9 + (4*c*d*e^3*x^11)/11 + (c*e^4*x^13)/13

Maple [A] time = 0.042, size = 97, normalized size = 0.9

$$\frac{ce^4x^{13}}{13} + \frac{4cde^3x^{11}}{11} + \frac{(e^4a + 6d^2e^2c)x^9}{9} + \frac{(4ade^3 + 4cd^3e)x^7}{7} + \frac{(6ad^2e^2 + cd^4)x^5}{5} + \frac{4ad^3ex^3}{3} + ad^4x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x^2+d)^4*(c*x^4+a), x)

[Out] 1/13*c*e^4*x^13+4/11*c*d*e^3*x^11+1/9*(a*e^4+6*c*d^2*e^2)*x^9+1/7*(4*a*d*e^3+4*c*d^3*e)*x^7+1/5*(6*a*d^2*e^2+c*d^4)*x^5+4/3*a*d^3*e*x^3+a*d^4*x

Maxima [A] time = 0.995822, size = 127, normalized size = 1.2

$$\frac{1}{13}ce^4x^{13} + \frac{4}{11}cde^3x^{11} + \frac{1}{9}(6cd^2e^2 + ae^4)x^9 + \frac{4}{3}ad^3ex^3 + \frac{4}{7}(cd^3e + ade^3)x^7 + ad^4x + \frac{1}{5}(cd^4 + 6ad^2e^2)x^5$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)^4*(c*x^4+a), x, algorithm="maxima")

[Out] 1/13*c*e^4*x^13 + 4/11*c*d*e^3*x^11 + 1/9*(6*c*d^2*e^2 + a*e^4)*x^9 + 4/3*a*d^3*e*x^3 + 4/7*(c*d^3*e + a*d*e^3)*x^7 + a*d^4*x + 1/5*(c*d^4 + 6*a*d^2*e^2)*x^5

Fricas [A] time = 1.34702, size = 234, normalized size = 2.21

$$\frac{1}{13}x^{13}e^4c + \frac{4}{11}x^{11}e^3dc + \frac{2}{3}x^9e^2d^2c + \frac{1}{9}x^9e^4a + \frac{4}{7}x^7ed^3c + \frac{4}{7}x^7e^3da + \frac{1}{5}x^5d^4c + \frac{6}{5}x^5e^2d^2a + \frac{4}{3}x^3ed^3a + xd^4a$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)^4*(c*x^4+a), x, algorithm="fricas")

[Out] 1/13*x^13*e^4*c + 4/11*x^11*e^3*d*c + 2/3*x^9*e^2*d^2*c + 1/9*x^9*e^4*a + 4/7*x^7*e*d^3*c + 4/7*x^7*e^3*d*a + 1/5*x^5*d^4*c + 6/5*x^5*e^2*d^2*a + 4/3*x^3*e*d^3*a + x*d^4*a

Sympy [A] time = 0.07838, size = 110, normalized size = 1.04

$$ad^4x + \frac{4ad^3ex^3}{3} + \frac{4cde^3x^{11}}{11} + \frac{ce^4x^{13}}{13} + x^9\left(\frac{ae^4}{9} + \frac{2cd^2e^2}{3}\right) + x^7\left(\frac{4ade^3}{7} + \frac{4cd^3e}{7}\right) + x^5\left(\frac{6ad^2e^2}{5} + \frac{cd^4}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x**2+d)**4*(c*x**4+a), x)

[Out] a*d**4*x + 4*a*d**3*e*x**3/3 + 4*c*d*e**3*x**11/11 + c*e**4*x**13/13 + x**9*(a*e**4/9 + 2*c*d**2*e**2/3) + x**7*(4*a*d*e**3/7 + 4*c*d**3*e/7) + x**5*(6*a*d**2*e**2/5 + c*d**4/5)

Giac [A] time = 1.11698, size = 127, normalized size = 1.2

$$\frac{1}{13} cx^{13}e^4 + \frac{4}{11} cdx^{11}e^3 + \frac{2}{3} cd^2x^9e^2 + \frac{4}{7} cd^3x^7e + \frac{1}{9} ax^9e^4 + \frac{1}{5} cd^4x^5 + \frac{4}{7} adx^7e^3 + \frac{6}{5} ad^2x^5e^2 + \frac{4}{3} ad^3x^3e + ad^4x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)^4*(c*x^4+a),x, algorithm="giac")

[Out] 1/13*c*x^13*e^4 + 4/11*c*d*x^11*e^3 + 2/3*c*d^2*x^9*e^2 + 4/7*c*d^3*x^7*e + 1/9*a*x^9*e^4 + 1/5*c*d^4*x^5 + 4/7*a*d*x^7*e^3 + 6/5*a*d^2*x^5*e^2 + 4/3*a*d^3*x^3*e + a*d^4*x

3.121 $\int (d + ex^2)^3 (a + cx^4) dx$

Optimal. Leaf size=79

$$\frac{1}{7}ex^7 (ae^2 + 3cd^2) + \frac{1}{5}dx^5 (3ae^2 + cd^2) + ad^2ex^3 + ad^3x + \frac{1}{3}cde^2x^9 + \frac{1}{11}ce^3x^{11}$$

[Out] a*d^3*x + a*d^2*e*x^3 + (d*(c*d^2 + 3*a*e^2)*x^5)/5 + (e*(3*c*d^2 + a*e^2)*x^7)/7 + (c*d*e^2*x^9)/3 + (c*e^3*x^11)/11

Rubi [A] time = 0.0561928, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {1154}

$$\frac{1}{7}ex^7 (ae^2 + 3cd^2) + \frac{1}{5}dx^5 (3ae^2 + cd^2) + ad^2ex^3 + ad^3x + \frac{1}{3}cde^2x^9 + \frac{1}{11}ce^3x^{11}$$

Antiderivative was successfully verified.

[In] Int[(d + e*x^2)^3*(a + c*x^4),x]

[Out] a*d^3*x + a*d^2*e*x^3 + (d*(c*d^2 + 3*a*e^2)*x^5)/5 + (e*(3*c*d^2 + a*e^2)*x^7)/7 + (c*d*e^2*x^9)/3 + (c*e^3*x^11)/11

Rule 1154

Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]

Rubi steps

$$\begin{aligned} \int (d + ex^2)^3 (a + cx^4) dx &= \int (ad^3 + 3ad^2ex^2 + d(cd^2 + 3ae^2)x^4 + e(3cd^2 + ae^2)x^6 + 3cde^2x^8 + ce^3x^{10}) dx \\ &= ad^3x + ad^2ex^3 + \frac{1}{5}d(cd^2 + 3ae^2)x^5 + \frac{1}{7}e(3cd^2 + ae^2)x^7 + \frac{1}{3}cde^2x^9 + \frac{1}{11}ce^3x^{11} \end{aligned}$$

Mathematica [A] time = 0.0162709, size = 79, normalized size = 1.

$$\frac{1}{7}ex^7 (ae^2 + 3cd^2) + \frac{1}{5}dx^5 (3ae^2 + cd^2) + ad^2ex^3 + ad^3x + \frac{1}{3}cde^2x^9 + \frac{1}{11}ce^3x^{11}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e*x^2)^3*(a + c*x^4),x]

[Out] a*d^3*x + a*d^2*e*x^3 + (d*(c*d^2 + 3*a*e^2)*x^5)/5 + (e*(3*c*d^2 + a*e^2)*x^7)/7 + (c*d*e^2*x^9)/3 + (c*e^3*x^11)/11

Maple [A] time = 0.042, size = 72, normalized size = 0.9

$$\frac{ce^3x^{11}}{11} + \frac{cde^2x^9}{3} + \frac{(ae^3 + 3d^2ec)x^7}{7} + \frac{(3ade^2 + cd^3)x^5}{5} + ad^2ex^3 + ad^3x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*x^2+d)^3*(c*x^4+a),x)`

[Out] $\frac{1}{11}c^3e^3x^{11} + \frac{1}{3}cde^2x^9 + \frac{1}{7}(3cd^2e + ae^3)x^7 + ad^2ex^3 + \frac{1}{5}(cd^3 + 3ade^2)x^5 + ad^3x$

Maxima [A] time = 0.977206, size = 96, normalized size = 1.22

$$\frac{1}{11}ce^3x^{11} + \frac{1}{3}cde^2x^9 + \frac{1}{7}(3cd^2e + ae^3)x^7 + ad^2ex^3 + \frac{1}{5}(cd^3 + 3ade^2)x^5 + ad^3x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x^2+d)^3*(c*x^4+a),x, algorithm="maxima")`

[Out] $\frac{1}{11}c^3e^3x^{11} + \frac{1}{3}c^2de^2x^9 + \frac{1}{7}(3cd^2e + ae^3)x^7 + ad^2e^2x^3 + \frac{1}{5}(cd^3 + 3ade^2)x^5 + ad^3x$

Fricas [A] time = 1.41715, size = 171, normalized size = 2.16

$$\frac{1}{11}x^{11}e^3c + \frac{1}{3}x^9e^2dc + \frac{3}{7}x^7ed^2c + \frac{1}{7}x^7e^3a + \frac{1}{5}x^5d^3c + \frac{3}{5}x^5e^2da + x^3ed^2a + xd^3a$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x^2+d)^3*(c*x^4+a),x, algorithm="fricas")`

[Out] $\frac{1}{11}x^{11}e^3c + \frac{1}{3}x^9e^2d^2c + \frac{3}{7}x^7e^2d^2c + \frac{1}{7}x^7e^3a + \frac{1}{5}x^5d^3c + \frac{3}{5}x^5e^2d^2a + x^3e^2d^2a + xd^3a$

Sympy [A] time = 0.073198, size = 78, normalized size = 0.99

$$ad^3x + ad^2ex^3 + \frac{cde^2x^9}{3} + \frac{ce^3x^{11}}{11} + x^7\left(\frac{ae^3}{7} + \frac{3cd^2e}{7}\right) + x^5\left(\frac{3ade^2}{5} + \frac{cd^3}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x**2+d)**3*(c*x**4+a),x)`

[Out] $a*d**3*x + a*d**2*e*x**3 + c*d*e**2*x**9/3 + c*e**3*x**11/11 + x**7*(a*e**3/7 + 3*c*d**2*e/7) + x**5*(3*a*d*e**2/5 + c*d**3/5)$

Giac [A] time = 1.14096, size = 96, normalized size = 1.22

$$\frac{1}{11}cx^{11}e^3 + \frac{1}{3}cdx^9e^2 + \frac{3}{7}cd^2x^7e + \frac{1}{5}cd^3x^5 + \frac{1}{7}ax^7e^3 + \frac{3}{5}adx^5e^2 + ad^2x^3e + ad^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3*(c*x^4+a),x, algorithm="giac")
```

```
[Out] 1/11*c*x^11*e^3 + 1/3*c*d*x^9*e^2 + 3/7*c*d^2*x^7*e + 1/5*c*d^3*x^5 + 1/7*a*x^7*e^3 + 3/5*a*d*x^5*e^2 + a*d^2*x^3*e + a*d^3*x
```

3.122 $\int (d + ex^2)^2 (a + cx^4) dx$

Optimal. Leaf size=56

$$\frac{1}{5}x^5 (ae^2 + cd^2) + ad^2x + \frac{2}{3}adex^3 + \frac{2}{7}cdex^7 + \frac{1}{9}ce^2x^9$$

[Out] $a*d^2*x + (2*a*d*e*x^3)/3 + ((c*d^2 + a*e^2)*x^5)/5 + (2*c*d*e*x^7)/7 + (c*e^2*x^9)/9$

Rubi [A] time = 0.0321749, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {1154}

$$\frac{1}{5}x^5 (ae^2 + cd^2) + ad^2x + \frac{2}{3}adex^3 + \frac{2}{7}cdex^7 + \frac{1}{9}ce^2x^9$$

Antiderivative was successfully verified.

[In] Int[(d + e*x^2)^2*(a + c*x^4), x]

[Out] $a*d^2*x + (2*a*d*e*x^3)/3 + ((c*d^2 + a*e^2)*x^5)/5 + (2*c*d*e*x^7)/7 + (c*e^2*x^9)/9$

Rule 1154

Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]

Rubi steps

$$\begin{aligned} \int (d + ex^2)^2 (a + cx^4) dx &= \int (ad^2 + 2adex^2 + (cd^2 + ae^2)x^4 + 2cdex^6 + ce^2x^8) dx \\ &= ad^2x + \frac{2}{3}adex^3 + \frac{1}{5}(cd^2 + ae^2)x^5 + \frac{2}{7}cdex^7 + \frac{1}{9}ce^2x^9 \end{aligned}$$

Mathematica [A] time = 0.0114073, size = 56, normalized size = 1.

$$\frac{1}{5}x^5 (ae^2 + cd^2) + ad^2x + \frac{2}{3}adex^3 + \frac{2}{7}cdex^7 + \frac{1}{9}ce^2x^9$$

Antiderivative was successfully verified.

[In] Integrate[(d + e*x^2)^2*(a + c*x^4), x]

[Out] $a*d^2*x + (2*a*d*e*x^3)/3 + ((c*d^2 + a*e^2)*x^5)/5 + (2*c*d*e*x^7)/7 + (c*e^2*x^9)/9$

Maple [A] time = 0.042, size = 49, normalized size = 0.9

$$ad^2x + \frac{2adex^3}{3} + \frac{(ae^2 + cd^2)x^5}{5} + \frac{2cdex^7}{7} + \frac{ce^2x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*x^2+d)^2*(c*x^4+a),x)`

[Out] `a*d^2*x+2/3*a*d*e*x^3+1/5*(a*e^2+c*d^2)*x^5+2/7*c*d*e*x^7+1/9*c*e^2*x^9`

Maxima [A] time = 0.975884, size = 65, normalized size = 1.16

$$\frac{1}{9}ce^2x^9 + \frac{2}{7}cdex^7 + \frac{2}{3}adex^3 + \frac{1}{5}(cd^2 + ae^2)x^5 + ad^2x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x^2+d)^2*(c*x^4+a),x, algorithm="maxima")`

[Out] `1/9*c*e^2*x^9 + 2/7*c*d*e*x^7 + 2/3*a*d*e*x^3 + 1/5*(c*d^2 + a*e^2)*x^5 + a*d^2*x`

Fricas [A] time = 1.36688, size = 120, normalized size = 2.14

$$\frac{1}{9}x^9e^2c + \frac{2}{7}x^7edc + \frac{1}{5}x^5d^2c + \frac{1}{5}x^5e^2a + \frac{2}{3}x^3eda + xd^2a$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x^2+d)^2*(c*x^4+a),x, algorithm="fricas")`

[Out] `1/9*x^9*e^2*c + 2/7*x^7*e*d*c + 1/5*x^5*d^2*c + 1/5*x^5*e^2*a + 2/3*x^3*e*d*a + x*d^2*a`

Sympy [A] time = 0.065501, size = 56, normalized size = 1.

$$ad^2x + \frac{2adex^3}{3} + \frac{2cdex^7}{7} + \frac{ce^2x^9}{9} + x^5\left(\frac{ae^2}{5} + \frac{cd^2}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x**2+d)**2*(c*x**4+a),x)`

[Out] `a*d**2*x + 2*a*d*e*x**3/3 + 2*c*d*e*x**7/7 + c*e**2*x**9/9 + x**5*(a*e**2/5 + c*d**2/5)`

Giac [A] time = 1.12523, size = 68, normalized size = 1.21

$$\frac{1}{9}cx^9e^2 + \frac{2}{7}cdx^7e + \frac{1}{5}cd^2x^5 + \frac{1}{5}ax^5e^2 + \frac{2}{3}adx^3e + ad^2x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x^2+d)^2*(c*x^4+a),x, algorithm="giac")`

[Out] $\frac{1}{9}c*x^9*e^2 + \frac{2}{7}c*d*x^7*e + \frac{1}{5}c*d^2*x^5 + \frac{1}{5}a*x^5*e^2 + \frac{2}{3}a*d*x^3*e + a*d^2*x$

3.123 $\int (d + ex^2)(a + cx^4) dx$

Optimal. Leaf size=32

$$adx + \frac{1}{3}aex^3 + \frac{1}{5}cdx^5 + \frac{1}{7}cex^7$$

[Out] a*d*x + (a*e*x^3)/3 + (c*d*x^5)/5 + (c*e*x^7)/7

Rubi [A] time = 0.0137964, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {1154}

$$adx + \frac{1}{3}aex^3 + \frac{1}{5}cdx^5 + \frac{1}{7}cex^7$$

Antiderivative was successfully verified.

[In] Int[(d + e*x^2)*(a + c*x^4),x]

[Out] a*d*x + (a*e*x^3)/3 + (c*d*x^5)/5 + (c*e*x^7)/7

Rule 1154

Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]

Rubi steps

$$\begin{aligned} \int (d + ex^2)(a + cx^4) dx &= \int (ad + aex^2 + cdx^4 + cex^6) dx \\ &= adx + \frac{1}{3}aex^3 + \frac{1}{5}cdx^5 + \frac{1}{7}cex^7 \end{aligned}$$

Mathematica [A] time = 0.0016087, size = 32, normalized size = 1.

$$adx + \frac{1}{3}aex^3 + \frac{1}{5}cdx^5 + \frac{1}{7}cex^7$$

Antiderivative was successfully verified.

[In] Integrate[(d + e*x^2)*(a + c*x^4),x]

[Out] a*d*x + (a*e*x^3)/3 + (c*d*x^5)/5 + (c*e*x^7)/7

Maple [A] time = 0.041, size = 27, normalized size = 0.8

$$adx + \frac{aex^3}{3} + \frac{cdx^5}{5} + \frac{cex^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*x^2+d)*(c*x^4+a),x)`

[Out] `a*d*x+1/3*a*e*x^3+1/5*c*d*x^5+1/7*c*e*x^7`

Maxima [A] time = 0.957647, size = 35, normalized size = 1.09

$$\frac{1}{7}cex^7 + \frac{1}{5}cdx^5 + \frac{1}{3}aex^3 + adx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x^2+d)*(c*x^4+a),x, algorithm="maxima")`

[Out] `1/7*c*e*x^7 + 1/5*c*d*x^5 + 1/3*a*e*x^3 + a*d*x`

Fricas [A] time = 1.55392, size = 66, normalized size = 2.06

$$\frac{1}{7}x^7ec + \frac{1}{5}x^5dc + \frac{1}{3}x^3ea + xda$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x^2+d)*(c*x^4+a),x, algorithm="fricas")`

[Out] `1/7*x^7*e*c + 1/5*x^5*d*c + 1/3*x^3*e*a + x*d*a`

Sympy [A] time = 0.057187, size = 29, normalized size = 0.91

$$adx + \frac{aex^3}{3} + \frac{cdx^5}{5} + \frac{cex^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x**2+d)*(c*x**4+a),x)`

[Out] `a*d*x + a*e*x**3/3 + c*d*x**5/5 + c*e*x**7/7`

Giac [A] time = 1.12466, size = 38, normalized size = 1.19

$$\frac{1}{7}cx^7e + \frac{1}{5}cdx^5 + \frac{1}{3}ax^3e + adx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x^2+d)*(c*x^4+a),x, algorithm="giac")`

[Out] `1/7*c*x^7*e + 1/5*c*d*x^5 + 1/3*a*x^3*e + a*d*x`

$$3.124 \quad \int \frac{a+cx^4}{d+ex^2} dx$$

Optimal. Leaf size=55

$$\frac{(ae^2 + cd^2) \tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)}{\sqrt{de}^{5/2}} - \frac{cdx}{e^2} + \frac{cx^3}{3e}$$

[Out] $-\frac{(c*d*x)}{e^2} + \frac{(c*x^3)}{(3*e)} + \frac{((c*d^2 + a*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])}{(Sqrt[d]*e^{(5/2)})}$

Rubi [A] time = 0.0347125, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {1154, 205}

$$\frac{(ae^2 + cd^2) \tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)}{\sqrt{de}^{5/2}} - \frac{cdx}{e^2} + \frac{cx^3}{3e}$$

Antiderivative was successfully verified.

[In] Int[(a + c*x^4)/(d + e*x^2), x]

[Out] $-\frac{(c*d*x)}{e^2} + \frac{(c*x^3)}{(3*e)} + \frac{((c*d^2 + a*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])}{(Sqrt[d]*e^{(5/2)})}$

Rule 1154

Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rubi steps

$$\begin{aligned} \int \frac{a+cx^4}{d+ex^2} dx &= \int \left(-\frac{cd}{e^2} + \frac{cx^2}{e} + \frac{cd^2+ae^2}{e^2(d+ex^2)} \right) dx \\ &= -\frac{cdx}{e^2} + \frac{cx^3}{3e} + \left(a + \frac{cd^2}{e^2} \right) \int \frac{1}{d+ex^2} dx \\ &= -\frac{cdx}{e^2} + \frac{cx^3}{3e} + \frac{(cd^2+ae^2) \tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)}{\sqrt{de}^{5/2}} \end{aligned}$$

Mathematica [A] time = 0.0360523, size = 55, normalized size = 1.

$$\frac{(ae^2 + cd^2) \tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)}{\sqrt{de}^{5/2}} - \frac{cdx}{e^2} + \frac{cx^3}{3e}$$

Antiderivative was successfully verified.

[In] Integrate[(a + c*x^4)/(d + e*x^2),x]

[Out] -((c*d*x)/e^2) + (c*x^3)/(3*e) + ((c*d^2 + a*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[d]*e^(5/2))

Maple [A] time = 0.048, size = 57, normalized size = 1.

$$\frac{cx^3}{3e} - \frac{cdx}{e^2} + a \arctan\left(ex \frac{1}{\sqrt{de}}\right) \frac{1}{\sqrt{de}} + \frac{cd^2}{e^2} \arctan\left(ex \frac{1}{\sqrt{de}}\right) \frac{1}{\sqrt{de}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^4+a)/(e*x^2+d),x)

[Out] 1/3*c*x^3/e-c*d*x/e^2+1/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))*a+1/e^2/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))*c*d^2

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+a)/(e*x^2+d),x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [A] time = 1.93919, size = 293, normalized size = 5.33

$$\left[\frac{2cde^2x^3 - 6cd^2ex - 3(cd^2 + ae^2)\sqrt{-de} \log\left(\frac{ex^2 - 2\sqrt{-de}x - d}{ex^2 + d}\right)}{6de^3}, \frac{cde^2x^3 - 3cd^2ex + 3(cd^2 + ae^2)\sqrt{de} \arctan\left(\frac{\sqrt{de}x}{d}\right)}{3de^3} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+a)/(e*x^2+d),x, algorithm="fricas")

[Out] [1/6*(2*c*d*e^2*x^3 - 6*c*d^2*e*x - 3*(c*d^2 + a*e^2)*sqrt(-d*e)*log((e*x^2 - 2*sqrt(-d*e)*x - d)/(e*x^2 + d)))/(d*e^3), 1/3*(c*d*e^2*x^3 - 3*c*d^2*e*x + 3*(c*d^2 + a*e^2)*sqrt(d*e)*arctan(sqrt(d*e)*x/d))/(d*e^3)]

Sympy [B] time = 0.427377, size = 104, normalized size = 1.89

$$-\frac{cdx}{e^2} + \frac{cx^3}{3e} - \frac{\sqrt{-\frac{1}{de^5}}(ae^2 + cd^2) \log\left(-de^2\sqrt{-\frac{1}{de^5}} + x\right)}{2} + \frac{\sqrt{-\frac{1}{de^5}}(ae^2 + cd^2) \log\left(de^2\sqrt{-\frac{1}{de^5}} + x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x**4+a)/(e*x**2+d),x)

[Out] $-c*d*x/e**2 + c*x**3/(3*e) - \sqrt{-1/(d*e**5)}*(a*e**2 + c*d**2)*\log(-d*e**2*\sqrt{-1/(d*e**5)} + x)/2 + \sqrt{-1/(d*e**5)}*(a*e**2 + c*d**2)*\log(d*e**2*\sqrt{-1/(d*e**5)} + x)/2$

Giac [A] time = 1.12393, size = 59, normalized size = 1.07

$$\frac{(cd^2 + ae^2) \arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right) e^{\left(-\frac{5}{2}\right)}}{\sqrt{d}} + \frac{1}{3} (cx^3e^2 - 3cdxe) e^{(-3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+a)/(e*x^2+d),x, algorithm="giac")

[Out] $(c*d^2 + a*e^2)*\arctan(x*e^{(1/2)}/\sqrt{d})*e^{(-5/2)}/\sqrt{d} + 1/3*(c*x^3*e^2 - 3*c*d*x*e)*e^{(-3)}$

$$3.125 \quad \int \frac{a+cx^4}{(d+ex^2)^2} dx$$

Optimal. Leaf size=74

$$\frac{x\left(a + \frac{cd^2}{e^2}\right)}{2d(d+ex^2)} - \frac{(3cd^2 - ae^2)\tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)}{2d^{3/2}e^{5/2}} + \frac{cx}{e^2}$$

[Out] (c*x)/e^2 + ((a + (c*d^2)/e^2)*x)/(2*d*(d + e*x^2)) - ((3*c*d^2 - a*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(3/2)*e^(5/2))

Rubi [A] time = 0.0519109, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {1158, 388, 205}

$$\frac{x\left(a + \frac{cd^2}{e^2}\right)}{2d(d+ex^2)} - \frac{(3cd^2 - ae^2)\tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)}{2d^{3/2}e^{5/2}} + \frac{cx}{e^2}$$

Antiderivative was successfully verified.

[In] Int[(a + c*x^4)/(d + e*x^2)^2,x]

[Out] (c*x)/e^2 + ((a + (c*d^2)/e^2)*x)/(2*d*(d + e*x^2)) - ((3*c*d^2 - a*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(3/2)*e^(5/2))

Rule 1158

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] := With[{Qx = PolynomialQuotient[(a + c*x^4)^p, d + e*x^2, x], R = Coeff[PolynomialRemainder[(a + c*x^4)^p, d + e*x^2, x], x, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Simp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned} \int \frac{a + cx^4}{(d + ex^2)^2} dx &= \frac{\left(a + \frac{cd^2}{e^2}\right)x}{2d(d + ex^2)} - \frac{\int \frac{-a + \frac{cd^2}{e^2} - \frac{2cdx^2}{e}}{d + ex^2} dx}{2d} \\ &= \frac{cx}{e^2} + \frac{\left(a + \frac{cd^2}{e^2}\right)x}{2d(d + ex^2)} + \frac{\left(a - \frac{3cd^2}{e^2}\right) \int \frac{1}{d + ex^2} dx}{2d} \\ &= \frac{cx}{e^2} + \frac{\left(a + \frac{cd^2}{e^2}\right)x}{2d(d + ex^2)} + \frac{\left(a - \frac{3cd^2}{e^2}\right) \tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)}{2d^{3/2}\sqrt{e}} \end{aligned}$$

Mathematica [A] time = 0.0507658, size = 78, normalized size = 1.05

$$\frac{x(ae^2 + cd^2)}{2de^2(d + ex^2)} - \frac{(3cd^2 - ae^2) \tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)}{2d^{3/2}e^{5/2}} + \frac{cx}{e^2}$$

Antiderivative was successfully verified.

[In] Integrate[(a + c*x^4)/(d + e*x^2)^2,x]

[Out] (c*x)/e^2 + ((c*d^2 + a*e^2)*x)/(2*d*e^2*(d + e*x^2)) - ((3*c*d^2 - a*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(3/2)*e^(5/2))

Maple [A] time = 0.052, size = 82, normalized size = 1.1

$$\frac{cx}{e^2} + \frac{ax}{2d(ex^2 + d)} + \frac{cdx}{2e^2(ex^2 + d)} + \frac{a}{2d} \arctan\left(ex \frac{1}{\sqrt{de}}\right) \frac{1}{\sqrt{de}} - \frac{3cd}{2e^2} \arctan\left(ex \frac{1}{\sqrt{de}}\right) \frac{1}{\sqrt{de}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^4+a)/(e*x^2+d)^2,x)

[Out] c*x/e^2+1/2/d*x/(e*x^2+d)*a+1/2/e^2*d*x/(e*x^2+d)*c+1/2/d/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))*a-3/2/e^2*d/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))*c

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+a)/(e*x^2+d)^2,x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [A] time = 1.87591, size = 455, normalized size = 6.15

$$\left[\frac{4cd^2e^2x^3 + (3cd^3 - ade^2 + (3cd^2e - ae^3)x^2)\sqrt{-de} \log\left(\frac{ex^2 - 2\sqrt{-dex} - d}{ex^2 + d}\right) + 2(3cd^3e + ade^3)x}{4(d^2e^4x^2 + d^3e^3)}, \frac{2cd^2e^2x^3 - (3cd^3 - ade^2)}{4(d^2e^4x^2 + d^3e^3)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+a)/(e*x^2+d)^2,x, algorithm="fricas")

[Out] [1/4*(4*c*d^2*e^2*x^3 + (3*c*d^3 - a*d*e^2 + (3*c*d^2*e - a*e^3)*x^2)*sqrt(-d*e)*log((e*x^2 - 2*sqrt(-d*e)*x - d)/(e*x^2 + d)) + 2*(3*c*d^3*e + a*d*e^3)*x)/(d^2*e^4*x^2 + d^3*e^3), 1/2*(2*c*d^2*e^2*x^3 - (3*c*d^3 - a*d*e^2 + (3*c*d^2*e - a*e^3)*x^2)*sqrt(d*e)*arctan(sqrt(d*e)*x/d) + (3*c*d^3*e + a*d*e^3)*x)/(d^2*e^4*x^2 + d^3*e^3)]

Sympy [B] time = 0.639859, size = 138, normalized size = 1.86

$$\frac{cx}{e^2} + \frac{x(ae^2 + cd^2)}{2d^2e^2 + 2de^3x^2} - \frac{\sqrt{-\frac{1}{d^3e^5}}(ae^2 - 3cd^2)\log\left(-d^2e^2\sqrt{-\frac{1}{d^3e^5}} + x\right)}{4} + \frac{\sqrt{-\frac{1}{d^3e^5}}(ae^2 - 3cd^2)\log\left(d^2e^2\sqrt{-\frac{1}{d^3e^5}} + x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x**4+a)/(e*x**2+d)**2,x)

[Out] c*x/e**2 + x*(a*e**2 + c*d**2)/(2*d**2*e**2 + 2*d*e**3*x**2) - sqrt(-1/(d**3*e**5))*(a*e**2 - 3*c*d**2)*log(-d**2*e**2*sqrt(-1/(d**3*e**5)) + x)/4 + sqrt(-1/(d**3*e**5))*(a*e**2 - 3*c*d**2)*log(d**2*e**2*sqrt(-1/(d**3*e**5)) + x)/4

Giac [A] time = 1.13603, size = 84, normalized size = 1.14

$$cxe^{(-2)} - \frac{(3cd^2 - ae^2)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{5}{2}\right)}}{2d^{\frac{3}{2}}} + \frac{(cd^2x + axe^2)e^{(-2)}}{2(x^2e + d)d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+a)/(e*x^2+d)^2,x, algorithm="giac")

[Out] c*x*e^(-2) - 1/2*(3*c*d^2 - a*e^2)*arctan(x*e^(1/2)/sqrt(d))*e^(-5/2)/d^(3/2) + 1/2*(c*d^2*x + a*x*e^2)*e^(-2)/((x^2*e + d)*d)

$$3.126 \quad \int \frac{a+cx^4}{(d+ex^2)^3} dx$$

Optimal. Leaf size=93

$$\frac{x\left(\frac{3a}{d^2} - \frac{5c}{e^2}\right)}{8(d+ex^2)} + \frac{x\left(a + \frac{cd^2}{e^2}\right)}{4d(d+ex^2)^2} + \frac{3(ae^2 + cd^2)\tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)}{8d^{5/2}e^{5/2}}$$

[Out] $((a + (c*d^2)/e^2)*x)/(4*d*(d + e*x^2)^2) + (((3*a)/d^2 - (5*c)/e^2)*x)/(8*(d + e*x^2)) + (3*(c*d^2 + a*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(8*d^(5/2)*e^(5/2))$

Rubi [A] time = 0.0674083, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {1158, 385, 205}

$$\frac{x\left(\frac{3a}{d^2} - \frac{5c}{e^2}\right)}{8(d+ex^2)} + \frac{x\left(a + \frac{cd^2}{e^2}\right)}{4d(d+ex^2)^2} + \frac{3(ae^2 + cd^2)\tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)}{8d^{5/2}e^{5/2}}$$

Antiderivative was successfully verified.

[In] Int[(a + c*x^4)/(d + e*x^2)^3,x]

[Out] $((a + (c*d^2)/e^2)*x)/(4*d*(d + e*x^2)^2) + (((3*a)/d^2 - (5*c)/e^2)*x)/(8*(d + e*x^2)) + (3*(c*d^2 + a*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(8*d^(5/2)*e^(5/2))$

Rule 1158

Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] := With[{Qx = PolynomialQuotient[(a + c*x^4)^p, d + e*x^2, x], R = Coeff[PolynomialRemainder[(a + c*x^4)^p, d + e*x^2, x], x, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]

Rule 385

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := -Simp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n + p, 0])

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rubi steps

$$\begin{aligned}
\int \frac{a + cx^4}{(d + ex^2)^3} dx &= \frac{\left(a + \frac{cd^2}{e^2}\right)x}{4d(d + ex^2)^2} - \frac{\int \frac{-3a + \frac{cd^2}{e^2} - \frac{4cdx^2}{e}}{(d+ex^2)^2} dx}{4d} \\
&= \frac{\left(a + \frac{cd^2}{e^2}\right)x}{4d(d + ex^2)^2} + \frac{\left(\frac{3a}{d^2} - \frac{5c}{e^2}\right)x}{8(d + ex^2)} + \frac{1}{8} \left(3\left(\frac{a}{d^2} + \frac{c}{e^2}\right)\right) \int \frac{1}{d + ex^2} dx \\
&= \frac{\left(a + \frac{cd^2}{e^2}\right)x}{4d(d + ex^2)^2} + \frac{\left(\frac{3a}{d^2} - \frac{5c}{e^2}\right)x}{8(d + ex^2)} + \frac{3(cd^2 + ae^2) \tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)}{8d^{5/2}e^{5/2}}
\end{aligned}$$

Mathematica [A] time = 0.0628183, size = 92, normalized size = 0.99

$$\frac{ae^2x(5d + 3ex^2) - cd^2x(3d + 5ex^2)}{8d^2e^2(d + ex^2)^2} + \frac{3(ae^2 + cd^2) \tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)}{8d^{5/2}e^{5/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + c*x^4)/(d + e*x^2)^3,x]

[Out] (a*e^2*x*(5*d + 3*e*x^2) - c*d^2*x*(3*d + 5*e*x^2))/(8*d^2*e^2*(d + e*x^2)^2) + (3*(c*d^2 + a*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(8*d^(5/2)*e^(5/2))

Maple [A] time = 0.053, size = 99, normalized size = 1.1

$$\frac{1}{(ex^2 + d)^2} \left(\frac{(3ae^2 - 5cd^2)x^3}{8d^2e} + \frac{(5ae^2 - 3cd^2)x}{8de^2} \right) + \frac{3a}{8d^2} \arctan\left(ex \frac{1}{\sqrt{de}}\right) \frac{1}{\sqrt{de}} + \frac{3c}{8e^2} \arctan\left(ex \frac{1}{\sqrt{de}}\right) \frac{1}{\sqrt{de}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^4+a)/(e*x^2+d)^3,x)

[Out] (1/8*(3*a*e^2-5*c*d^2)/d^2/e*x^3+1/8*(5*a*e^2-3*c*d^2)/d/e^2*x)/(e*x^2+d)^2 +3/8/d^2/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))*a+3/8/e^2/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))*c

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+a)/(e*x^2+d)^3,x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [A] time = 1.99178, size = 635, normalized size = 6.83

$$\left[\frac{2(5cd^3e^2 - 3ade^4)x^3 + 3(cd^4 + ad^2e^2 + (cd^2e^2 + ae^4)x^4 + 2(cd^3e + ade^3)x^2)\sqrt{-de} \log\left(\frac{ex^2 - 2\sqrt{-dex-d}}{ex^2+d}\right) + 2(3cd^4e - 2d^4e^3)}{16(d^3e^5x^4 + 2d^4e^4x^2 + d^5e^3)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+a)/(e*x^2+d)^3,x, algorithm="fricas")

[Out] [-1/16*(2*(5*c*d^3*e^2 - 3*a*d*e^4)*x^3 + 3*(c*d^4 + a*d^2*e^2 + (c*d^2*e^2 + a*e^4)*x^4 + 2*(c*d^3*e + a*d*e^3)*x^2)*sqrt(-d*e)*log((e*x^2 - 2*sqrt(-d*e)*x - d)/(e*x^2 + d)) + 2*(3*c*d^4*e - 5*a*d^2*e^3)*x/(d^3*e^5*x^4 + 2*d^4*e^4*x^2 + d^5*e^3), -1/8*((5*c*d^3*e^2 - 3*a*d*e^4)*x^3 - 3*(c*d^4 + a*d^2*e^2 + (c*d^2*e^2 + a*e^4)*x^4 + 2*(c*d^3*e + a*d*e^3)*x^2)*sqrt(d*e)*arctan(sqrt(d*e)*x/d) + (3*c*d^4*e - 5*a*d^2*e^3)*x/(d^3*e^5*x^4 + 2*d^4*e^4*x^2 + d^5*e^3)]

Sympy [B] time = 0.856339, size = 219, normalized size = 2.35

$$\frac{3\sqrt{-\frac{1}{d^5e^5}}(ae^2 + cd^2) \log\left(-\frac{3d^3e^2\sqrt{-\frac{1}{d^5e^5}}(ae^2+cd^2)}{3ae^2+3cd^2} + x\right)}{16} + \frac{3\sqrt{-\frac{1}{d^5e^5}}(ae^2 + cd^2) \log\left(\frac{3d^3e^2\sqrt{-\frac{1}{d^5e^5}}(ae^2+cd^2)}{3ae^2+3cd^2} + x\right)}{16} + \frac{x^3(3ae^3 - 2d^4e^3)}{8d^4e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x**4+a)/(e*x**2+d)**3,x)

[Out] -3*sqrt(-1/(d**5*e**5))*(a*e**2 + c*d**2)*log(-3*d**3*e**2*sqrt(-1/(d**5*e**5))*(a*e**2 + c*d**2)/(3*a*e**2 + 3*c*d**2) + x)/16 + 3*sqrt(-1/(d**5*e**5))*(a*e**2 + c*d**2)*log(3*d**3*e**2*sqrt(-1/(d**5*e**5))*(a*e**2 + c*d**2)/(3*a*e**2 + 3*c*d**2) + x)/16 + (x**3*(3*a*e**3 - 5*c*d**2*e) + x*(5*a*d*e**2 - 3*c*d**3))/(8*d**4*e**2 + 16*d**3*e**3*x**2 + 8*d**2*e**4*x**4)

Giac [A] time = 1.12197, size = 104, normalized size = 1.12

$$\frac{3(cd^2 + ae^2) \arctan\left(\frac{xe^{1/2}}{\sqrt{d}}\right) e^{(-5/2)}}{8d^{5/2}} - \frac{(5cd^2x^3e + 3cd^3x - 3ax^3e^3 - 5adx^2e^2)e^{(-2)}}{8(x^2e + d)^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+a)/(e*x^2+d)^3,x, algorithm="giac")

[Out] 3/8*(c*d^2 + a*e^2)*arctan(x*e^(1/2)/sqrt(d))*e^(-5/2)/d^(5/2) - 1/8*(5*c*d^2*x^3*e + 3*c*d^3*x - 3*a*x^3*e^3 - 5*a*d*x^2*e^2)*e^(-2)/((x^2*e + d)^2*d^2)

$$3.127 \quad \int \frac{a+cx^4}{(d+ex^2)^4} dx$$

Optimal. Leaf size=123

$$\frac{x\left(\frac{5a}{d^2} + \frac{c}{e^2}\right)}{16d(d+ex^2)} + \frac{x\left(\frac{5a}{d^2} - \frac{7c}{e^2}\right)}{24(d+ex^2)^2} + \frac{x\left(a + \frac{cd^2}{e^2}\right)}{6d(d+ex^2)^3} + \frac{(5ae^2 + cd^2)\tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)}{16d^{7/2}e^{5/2}}$$

[Out] ((a + (c*d^2)/e^2)*x)/(6*d*(d + e*x^2)^3) + (((5*a)/d^2 - (7*c)/e^2)*x)/(24*(d + e*x^2)^2) + (((5*a)/d^2 + c/e^2)*x)/(16*d*(d + e*x^2)) + ((c*d^2 + 5*a*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(16*d^(7/2)*e^(5/2))

Rubi [A] time = 0.114026, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {1158, 385, 199, 205}

$$\frac{x\left(\frac{5a}{d^2} + \frac{c}{e^2}\right)}{16d(d+ex^2)} + \frac{x\left(\frac{5a}{d^2} - \frac{7c}{e^2}\right)}{24(d+ex^2)^2} + \frac{x\left(a + \frac{cd^2}{e^2}\right)}{6d(d+ex^2)^3} + \frac{(5ae^2 + cd^2)\tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)}{16d^{7/2}e^{5/2}}$$

Antiderivative was successfully verified.

[In] Int[(a + c*x^4)/(d + e*x^2)^4,x]

[Out] ((a + (c*d^2)/e^2)*x)/(6*d*(d + e*x^2)^3) + (((5*a)/d^2 - (7*c)/e^2)*x)/(24*(d + e*x^2)^2) + (((5*a)/d^2 + c/e^2)*x)/(16*d*(d + e*x^2)) + ((c*d^2 + 5*a*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(16*d^(7/2)*e^(5/2))

Rule 1158

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] := With[{Qx = PolynomialQuotient[(a + c*x^4)^p, d + e*x^2, x], R = Coeff[PolynomialRemainder[(a + c*x^4)^p, d + e*x^2, x], x, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := -Simp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n + p, 0])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := -Simp[(x*(a + b*x^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (IntegerQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denominator[p + 1/n] < Denominator[p])
```

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rubi steps

$$\begin{aligned} \int \frac{a + cx^4}{(d + ex^2)^4} dx &= \frac{\left(a + \frac{cd^2}{e^2}\right)x}{6d(d + ex^2)^3} - \frac{\int \frac{-5a + \frac{cd^2}{e^2} - \frac{6cdx^2}{e}}{(d+ex^2)^3} dx}{6d} \\ &= \frac{\left(a + \frac{cd^2}{e^2}\right)x}{6d(d + ex^2)^3} + \frac{\left(\frac{5a}{d^2} - \frac{7c}{e^2}\right)x}{24(d + ex^2)^2} + \frac{1}{8} \left(\frac{5a}{d^2} + \frac{c}{e^2}\right) \int \frac{1}{(d + ex^2)^2} dx \\ &= \frac{\left(a + \frac{cd^2}{e^2}\right)x}{6d(d + ex^2)^3} + \frac{\left(\frac{5a}{d^2} - \frac{7c}{e^2}\right)x}{24(d + ex^2)^2} + \frac{\left(\frac{5a}{d^2} + \frac{c}{e^2}\right)x}{16d(d + ex^2)} + \frac{\left(\frac{5a}{d^2} + \frac{c}{e^2}\right) \int \frac{1}{d+ex^2} dx}{16d} \\ &= \frac{\left(a + \frac{cd^2}{e^2}\right)x}{6d(d + ex^2)^3} + \frac{\left(\frac{5a}{d^2} - \frac{7c}{e^2}\right)x}{24(d + ex^2)^2} + \frac{\left(\frac{5a}{d^2} + \frac{c}{e^2}\right)x}{16d(d + ex^2)} + \frac{(cd^2 + 5ae^2) \tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)}{16d^{7/2}e^{5/2}} \end{aligned}$$

Mathematica [A] time = 0.0818858, size = 113, normalized size = 0.92

$$\frac{x \left(ae^2 (33d^2 + 40dex^2 + 15e^2x^4) + cd^2 (-3d^2 - 8dex^2 + 3e^2x^4) \right)}{48d^3e^2(d + ex^2)^3} + \frac{(5ae^2 + cd^2) \tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)}{16d^{7/2}e^{5/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + c*x^4)/(d + e*x^2)^4,x]

[Out] (x*(c*d^2*(-3*d^2 - 8*d*e*x^2 + 3*e^2*x^4) + a*e^2*(33*d^2 + 40*d*e*x^2 + 15*e^2*x^4)))/(48*d^3*e^2*(d + e*x^2)^3) + ((c*d^2 + 5*a*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(16*d^(7/2)*e^(5/2))

Maple [A] time = 0.053, size = 122, normalized size = 1.

$$\frac{1}{(ex^2 + d)^3} \left(\frac{(5ae^2 + cd^2)x^5}{16d^3} + \frac{(5ae^2 - cd^2)x^3}{6d^2e} + \frac{(11ae^2 - cd^2)x}{16de^2} \right) + \frac{5a}{16d^3} \arctan\left(ex \frac{1}{\sqrt{de}}\right) \frac{1}{\sqrt{de}} + \frac{c}{16de^2} \arctan\left(ex \frac{1}{\sqrt{de}}\right) \frac{1}{\sqrt{de}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^4+a)/(e*x^2+d)^4,x)

[Out] (1/16*(5*a*e^2+c*d^2)/d^3*x^5+1/6*(5*a*e^2-c*d^2)/d^2/e*x^3+1/16*(11*a*e^2-c*d^2)/d/e^2*x)/(e*x^2+d)^3+5/16/d^3/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))*a+1/16/d/e^2/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))*c

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+a)/(e*x^2+d)^4,x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [A] time = 1.99236, size = 875, normalized size = 7.11

$$\frac{6(cd^3e^3 + 5ade^5)x^5 - 16(cd^4e^2 - 5ad^2e^4)x^3 - 3((cd^2e^3 + 5ae^5)x^6 + cd^5 + 5ad^3e^2 + 3(cd^3e^2 + 5ade^4)x^4 + 3(cd^4e + 5ad^2e^3)x^2 + d^7e^3)}{96(d^4e^6x^6 + 3d^5e^5x^4 + 3d^6e^4x^2 + d^7e^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+a)/(e*x^2+d)^4,x, algorithm="fricas")

[Out] [1/96*(6*(c*d^3*e^3 + 5*a*d*e^5)*x^5 - 16*(c*d^4*e^2 - 5*a*d^2*e^4)*x^3 - 3*((c*d^2*e^3 + 5*a*e^5)*x^6 + c*d^5 + 5*a*d^3*e^2 + 3*(c*d^3*e^2 + 5*a*d*e^4)*x^4 + 3*(c*d^4*e + 5*a*d^2*e^3)*x^2)*sqrt(-d*e)*log((e*x^2 - 2*sqrt(-d*e)*x - d)/(e*x^2 + d)) - 6*(c*d^5*e - 11*a*d^3*e^3)*x/(d^4*e^6*x^6 + 3*d^5*e^5*x^4 + 3*d^6*e^4*x^2 + d^7*e^3), 1/48*(3*(c*d^3*e^3 + 5*a*d*e^5)*x^5 - 8*(c*d^4*e^2 - 5*a*d^2*e^4)*x^3 + 3*((c*d^2*e^3 + 5*a*e^5)*x^6 + c*d^5 + 5*a*d^3*e^2 + 3*(c*d^3*e^2 + 5*a*d*e^4)*x^4 + 3*(c*d^4*e + 5*a*d^2*e^3)*x^2)*sqrt(d*e)*arctan(sqrt(d*e)*x/d - 3*(c*d^5*e - 11*a*d^3*e^3)*x/(d^4*e^6*x^6 + 3*d^5*e^5*x^4 + 3*d^6*e^4*x^2 + d^7*e^3)]

Sympy [A] time = 1.0901, size = 204, normalized size = 1.66

$$-\frac{\sqrt{-\frac{1}{d^7e^5}}(5ae^2 + cd^2)\log\left(-d^4e^2\sqrt{-\frac{1}{d^7e^5}} + x\right)}{32} + \frac{\sqrt{-\frac{1}{d^7e^5}}(5ae^2 + cd^2)\log\left(d^4e^2\sqrt{-\frac{1}{d^7e^5}} + x\right)}{32} + \frac{x^5(15ae^4 + 3cd^2e^2) + x^3(48d^6e^2 + 144d^5e^3)}{48d^6e^2 + 144d^5e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x**4+a)/(e*x**2+d)**4,x)

[Out] -sqrt(-1/(d**7*e**5))*(5*a*e**2 + c*d**2)*log(-d**4*e**2*sqrt(-1/(d**7*e**5)) + x)/32 + sqrt(-1/(d**7*e**5))*(5*a*e**2 + c*d**2)*log(d**4*e**2*sqrt(-1/(d**7*e**5)) + x)/32 + (x**5*(15*a*e**4 + 3*c*d**2*e**2) + x**3*(40*a*d*e**3 - 8*c*d**3*e) + x*(33*a*d**2*e**2 - 3*c*d**4))/(48*d**6*e**2 + 144*d**5*e**3*x**2 + 144*d**4*e**4*x**4 + 48*d**3*e**5*x**6)

Giac [A] time = 1.13148, size = 135, normalized size = 1.1

$$\frac{(cd^2 + 5ae^2)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{5}{2}\right)}}{16d^{\frac{7}{2}}} + \frac{(3cd^2x^5e^2 - 8cd^3x^3e + 15ax^5e^4 - 3cd^4x + 40adx^3e^3 + 33ad^2xe^2)e^{(-2)}}{48(x^2e + d)^3d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+a)/(e*x^2+d)^4,x, algorithm="giac")

```
[Out] 1/16*(c*d^2 + 5*a*e^2)*arctan(x*e^(1/2)/sqrt(d))*e^(-5/2)/d^(7/2) + 1/48*(3
*c*d^2*x^5*e^2 - 8*c*d^3*x^3*e + 15*a*x^5*e^4 - 3*c*d^4*x + 40*a*d*x^3*e^3
+ 33*a*d^2*x*e^2)*e^(-2)/((x^2*e + d)^3*d^3)
```

3.128 $\int (d + ex^2)^3 (a + cx^4)^2 dx$

Optimal. Leaf size=133

$$a^2d^2ex^3 + a^2d^3x + \frac{1}{11}cex^{11}(2ae^2 + 3cd^2) + \frac{1}{9}cdx^9(6ae^2 + cd^2) + \frac{1}{7}aex^7(ae^2 + 6cd^2) + \frac{1}{5}adx^5(3ae^2 + 2cd^2) + \frac{3}{13}c^2de^2x^{13}$$

[Out] $a^2d^3x + a^2d^2e*x^3 + (a*d*(2*c*d^2 + 3*a*e^2)*x^5)/5 + (a*e*(6*c*d^2 + a*e^2)*x^7)/7 + (c*d*(c*d^2 + 6*a*e^2)*x^9)/9 + (c*e*(3*c*d^2 + 2*a*e^2)*x^{11})/11 + (3*c^2*d*e^2*x^{13})/13 + (c^2*e^3*x^{15})/15$

Rubi [A] time = 0.106869, antiderivative size = 133, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {1154}

$$a^2d^2ex^3 + a^2d^3x + \frac{1}{11}cex^{11}(2ae^2 + 3cd^2) + \frac{1}{9}cdx^9(6ae^2 + cd^2) + \frac{1}{7}aex^7(ae^2 + 6cd^2) + \frac{1}{5}adx^5(3ae^2 + 2cd^2) + \frac{3}{13}c^2de^2x^{13}$$

Antiderivative was successfully verified.

[In] Int[(d + e*x^2)^3*(a + c*x^4)^2, x]

[Out] $a^2d^3x + a^2d^2e*x^3 + (a*d*(2*c*d^2 + 3*a*e^2)*x^5)/5 + (a*e*(6*c*d^2 + a*e^2)*x^7)/7 + (c*d*(c*d^2 + 6*a*e^2)*x^9)/9 + (c*e*(3*c*d^2 + 2*a*e^2)*x^{11})/11 + (3*c^2*d*e^2*x^{13})/13 + (c^2*e^3*x^{15})/15$

Rule 1154

Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]

Rubi steps

$$\begin{aligned} \int (d + ex^2)^3 (a + cx^4)^2 dx &= \int (a^2d^3 + 3a^2d^2ex^2 + ad(2cd^2 + 3ae^2)x^4 + ae(6cd^2 + ae^2)x^6 + cd(cd^2 + 6ae^2)x^8 + ce(3cd^2 + ae^2)x^{10} + c^2de^2x^{12}) dx \\ &= a^2d^3x + a^2d^2ex^3 + \frac{1}{5}ad(2cd^2 + 3ae^2)x^5 + \frac{1}{7}ae(6cd^2 + ae^2)x^7 + \frac{1}{9}cd(cd^2 + 6ae^2)x^9 + \frac{1}{11}ce(3cd^2 + ae^2)x^{11} + \frac{3}{13}c^2de^2x^{13} \end{aligned}$$

Mathematica [A] time = 0.0216422, size = 133, normalized size = 1.

$$a^2d^2ex^3 + a^2d^3x + \frac{1}{11}cex^{11}(2ae^2 + 3cd^2) + \frac{1}{9}cdx^9(6ae^2 + cd^2) + \frac{1}{7}aex^7(ae^2 + 6cd^2) + \frac{1}{5}adx^5(3ae^2 + 2cd^2) + \frac{3}{13}c^2de^2x^{13}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e*x^2)^3*(a + c*x^4)^2, x]

[Out] $a^2d^3x + a^2d^2e*x^3 + (a*d*(2*c*d^2 + 3*a*e^2)*x^5)/5 + (a*e*(6*c*d^2 + a*e^2)*x^7)/7 + (c*d*(c*d^2 + 6*a*e^2)*x^9)/9 + (c*e*(3*c*d^2 + 2*a*e^2)*x^{11})/11 + (3*c^2*d*e^2*x^{13})/13 + (c^2*e^3*x^{15})/15$

Maple [A] time = 0.042, size = 130, normalized size = 1.

$$\frac{c^2 e^3 x^{15}}{15} + \frac{3 c^2 d e^2 x^{13}}{13} + \frac{(2 a c e^3 + 3 c^2 d^2 e) x^{11}}{11} + \frac{(6 a d e^2 c + c^2 d^3) x^9}{9} + \frac{(e^3 a^2 + 6 a c d^2 e) x^7}{7} + \frac{(3 d e^2 a^2 + 2 d^3 a c) x^5}{5} + a$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x^2+d)^3*(c*x^4+a)^2,x)

[Out] 1/15*c^2*e^3*x^15+3/13*c^2*d*e^2*x^13+1/11*(2*a*c*e^3+3*c^2*d^2*e)*x^11+1/9*(6*a*c*d*e^2+c^2*d^3)*x^9+1/7*(a^2*e^3+6*a*c*d^2*e)*x^7+1/5*(3*a^2*d*e^2+2*a*c*d^3)*x^5+a^2*d^2*e*x^3+a^2*d^3*x

Maxima [A] time = 1.00803, size = 174, normalized size = 1.31

$$\frac{1}{15} c^2 e^3 x^{15} + \frac{3}{13} c^2 d e^2 x^{13} + \frac{1}{11} (3 c^2 d^2 e + 2 a c e^3) x^{11} + \frac{1}{9} (c^2 d^3 + 6 a c d e^2) x^9 + a^2 d^2 e x^3 + \frac{1}{7} (6 a c d^2 e + a^2 e^3) x^7 + a^2 d^3 x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)^3*(c*x^4+a)^2,x, algorithm="maxima")

[Out] 1/15*c^2*e^3*x^15 + 3/13*c^2*d*e^2*x^13 + 1/11*(3*c^2*d^2*e + 2*a*c*e^3)*x^11 + 1/9*(c^2*d^3 + 6*a*c*d*e^2)*x^9 + a^2*d^2*e*x^3 + 1/7*(6*a*c*d^2*e + a^2*e^3)*x^7 + a^2*d^3*x + 1/5*(2*a*c*d^3 + 3*a^2*d*e^2)*x^5

Fricas [A] time = 1.64227, size = 304, normalized size = 2.29

$$\frac{1}{15} x^{15} e^3 c^2 + \frac{3}{13} x^{13} e^2 d c^2 + \frac{3}{11} x^{11} e d^2 c^2 + \frac{2}{11} x^{11} e^3 c a + \frac{1}{9} x^9 d^3 c^2 + \frac{2}{3} x^9 e^2 d c a + \frac{6}{7} x^7 e d^2 c a + \frac{1}{7} x^7 e^3 a^2 + \frac{2}{5} x^5 d^3 c a + \frac{3}{5} x^5 e^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)^3*(c*x^4+a)^2,x, algorithm="fricas")

[Out] 1/15*x^15*e^3*c^2 + 3/13*x^13*e^2*d*c^2 + 3/11*x^11*e*d^2*c^2 + 2/11*x^11*e^3*c*a + 1/9*x^9*d^3*c^2 + 2/3*x^9*e^2*d*c*a + 6/7*x^7*e*d^2*c*a + 1/7*x^7*e^3*a^2 + 2/5*x^5*d^3*c*a + 3/5*x^5*e^2*d*a^2 + x^3*e*d^2*a^2 + x*d^3*a^2

Sympy [A] time = 0.083892, size = 144, normalized size = 1.08

$$a^2 d^3 x + a^2 d^2 e x^3 + \frac{3 c^2 d e^2 x^{13}}{13} + \frac{c^2 e^3 x^{15}}{15} + x^{11} \left(\frac{2 a c e^3}{11} + \frac{3 c^2 d^2 e}{11} \right) + x^9 \left(\frac{2 a c d e^2}{3} + \frac{c^2 d^3}{9} \right) + x^7 \left(\frac{a^2 e^3}{7} + \frac{6 a c d^2 e}{7} \right) + x^5 \left(\frac{3 a^2 d^3}{5} + \frac{2 a^2 d^2 e}{5} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x**2+d)**3*(c*x**4+a)**2,x)

[Out] a**2*d**3*x + a**2*d**2*e*x**3 + 3*c**2*d*e**2*x**13/13 + c**2*e**3*x**15/15 + x**11*(2*a*c*e**3/11 + 3*c**2*d**2*e/11) + x**9*(2*a*c*d*e**2/3 + c**2*d**3/9) + x**7*(a**2*e**3/7 + 6*a*c*d**2*e/7) + x**5*(3*a**2*d*e**2/5 + 2*a

`*c*d**3/5)`

Giac [A] time = 1.11997, size = 173, normalized size = 1.3

$$\frac{1}{15} c^2 x^{15} e^3 + \frac{3}{13} c^2 d x^{13} e^2 + \frac{3}{11} c^2 d^2 x^{11} e + \frac{1}{9} c^2 d^3 x^9 + \frac{2}{11} a c x^{11} e^3 + \frac{2}{3} a c d x^9 e^2 + \frac{6}{7} a c d^2 x^7 e + \frac{2}{5} a c d^3 x^5 + \frac{1}{7} a^2 x^7 e^3 + \frac{3}{5} a^2 d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x^2+d)^3*(c*x^4+a)^2,x, algorithm="giac")`

[Out] `1/15*c^2*x^15*e^3 + 3/13*c^2*d*x^13*e^2 + 3/11*c^2*d^2*x^11*e + 1/9*c^2*d^3*x^9 + 2/11*a*c*x^11*e^3 + 2/3*a*c*d*x^9*e^2 + 6/7*a*c*d^2*x^7*e + 2/5*a*c*d^3*x^5 + 1/7*a^2*x^7*e^3 + 3/5*a^2*d*x^5*e^2 + a^2*d^2*x^3*e + a^2*d^3*x`

3.129 $\int (d + ex^2)^2 (a + cx^4)^2 dx$

Optimal. Leaf size=97

$$a^2d^2x + \frac{2}{3}a^2dex^3 + \frac{1}{9}cx^9(2ae^2 + cd^2) + \frac{1}{5}ax^5(ae^2 + 2cd^2) + \frac{4}{7}acdex^7 + \frac{2}{11}c^2dex^{11} + \frac{1}{13}c^2e^2x^{13}$$

[Out] $a^2d^2x + (2a^2d^2ex^3)/3 + (a(2cd^2 + ae^2)x^5)/5 + (4acdex^7)/7 + (c(c^2d^2 + 2ae^2)x^9)/9 + (2c^2dex^{11})/11 + (c^2e^2x^{13})/13$

Rubi [A] time = 0.0683933, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {1154}

$$a^2d^2x + \frac{2}{3}a^2dex^3 + \frac{1}{9}cx^9(2ae^2 + cd^2) + \frac{1}{5}ax^5(ae^2 + 2cd^2) + \frac{4}{7}acdex^7 + \frac{2}{11}c^2dex^{11} + \frac{1}{13}c^2e^2x^{13}$$

Antiderivative was successfully verified.

[In] Int[(d + e*x^2)^2*(a + c*x^4)^2,x]

[Out] $a^2d^2x + (2a^2d^2ex^3)/3 + (a(2cd^2 + ae^2)x^5)/5 + (4acdex^7)/7 + (c(c^2d^2 + 2ae^2)x^9)/9 + (2c^2dex^{11})/11 + (c^2e^2x^{13})/13$

Rule 1154

Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]

Rubi steps

$$\begin{aligned} \int (d + ex^2)^2 (a + cx^4)^2 dx &= \int (a^2d^2 + 2a^2dex^2 + a(2cd^2 + ae^2)x^4 + 4acdex^6 + c(cd^2 + 2ae^2)x^8 + 2c^2dex^{10} + c^2e^2x^{12}) dx \\ &= a^2d^2x + \frac{2}{3}a^2dex^3 + \frac{1}{5}a(2cd^2 + ae^2)x^5 + \frac{4}{7}acdex^7 + \frac{1}{9}c(cd^2 + 2ae^2)x^9 + \frac{2}{11}c^2dex^{11} + \frac{1}{13}c^2e^2x^{13} \end{aligned}$$

Mathematica [A] time = 0.0175475, size = 97, normalized size = 1.

$$a^2d^2x + \frac{2}{3}a^2dex^3 + \frac{1}{9}cx^9(2ae^2 + cd^2) + \frac{1}{5}ax^5(ae^2 + 2cd^2) + \frac{4}{7}acdex^7 + \frac{2}{11}c^2dex^{11} + \frac{1}{13}c^2e^2x^{13}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e*x^2)^2*(a + c*x^4)^2,x]

[Out] $a^2d^2x + (2a^2d^2ex^3)/3 + (a(2cd^2 + ae^2)x^5)/5 + (4acdex^7)/7 + (c(c^2d^2 + 2ae^2)x^9)/9 + (2c^2dex^{11})/11 + (c^2e^2x^{13})/13$

Maple [A] time = 0.041, size = 90, normalized size = 0.9

$$\frac{c^2 e^2 x^{13}}{13} + \frac{2c^2 dex^{11}}{11} + \frac{(2ace^2 + c^2 d^2)x^9}{9} + \frac{4acdex^7}{7} + \frac{(e^2 a^2 + 2acd^2)x^5}{5} + \frac{2a^2 dex^3}{3} + a^2 d^2 x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x^2+d)^2*(c*x^4+a)^2,x)

[Out] 1/13*c^2*e^2*x^13+2/11*c^2*d*e*x^11+1/9*(2*a*c*e^2+c^2*d^2)*x^9+4/7*a*c*d*e*x^7+1/5*(a^2*e^2+2*a*c*d^2)*x^5+2/3*a^2*d*e*x^3+a^2*d^2*x

Maxima [A] time = 1.02734, size = 120, normalized size = 1.24

$$\frac{1}{13} c^2 e^2 x^{13} + \frac{2}{11} c^2 dex^{11} + \frac{4}{7} acdex^7 + \frac{1}{9} (c^2 d^2 + 2ace^2)x^9 + \frac{2}{3} a^2 dex^3 + \frac{1}{5} (2acd^2 + a^2 e^2)x^5 + a^2 d^2 x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)^2*(c*x^4+a)^2,x, algorithm="maxima")

[Out] 1/13*c^2*e^2*x^13 + 2/11*c^2*d*e*x^11 + 4/7*a*c*d*e*x^7 + 1/9*(c^2*d^2 + 2*a*c*e^2)*x^9 + 2/3*a^2*d*e*x^3 + 1/5*(2*a*c*d^2 + a^2*e^2)*x^5 + a^2*d^2*x

Fricas [A] time = 1.48504, size = 215, normalized size = 2.22

$$\frac{1}{13} x^{13} e^2 c^2 + \frac{2}{11} x^{11} edc^2 + \frac{1}{9} x^9 d^2 c^2 + \frac{2}{9} x^9 e^2 ca + \frac{4}{7} x^7 edca + \frac{2}{5} x^5 d^2 ca + \frac{1}{5} x^5 e^2 a^2 + \frac{2}{3} x^3 eda^2 + xd^2 a^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)^2*(c*x^4+a)^2,x, algorithm="fricas")

[Out] 1/13*x^13*e^2*c^2 + 2/11*x^11*e*d*c^2 + 1/9*x^9*d^2*c^2 + 2/9*x^9*e^2*c*a + 4/7*x^7*e*d*c*a + 2/5*x^5*d^2*c*a + 1/5*x^5*e^2*a^2 + 2/3*x^3*e*d*a^2 + x*d^2*a^2

Sympy [A] time = 0.079009, size = 104, normalized size = 1.07

$$a^2 d^2 x + \frac{2a^2 dex^3}{3} + \frac{4acdex^7}{7} + \frac{2c^2 dex^{11}}{11} + \frac{c^2 e^2 x^{13}}{13} + x^9 \left(\frac{2ace^2}{9} + \frac{c^2 d^2}{9} \right) + x^5 \left(\frac{a^2 e^2}{5} + \frac{2acd^2}{5} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x**2+d)**2*(c*x**4+a)**2,x)

[Out] a**2*d**2*x + 2*a**2*d*e*x**3/3 + 4*a*c*d*e*x**7/7 + 2*c**2*d*e*x**11/11 + c**2*e**2*x**13/13 + x**9*(2*a*c*e**2/9 + c**2*d**2/9) + x**5*(a**2*e**2/5 + 2*a*c*d**2/5)

Giac [A] time = 1.16473, size = 123, normalized size = 1.27

$$\frac{1}{13} c^2 x^{13} e^2 + \frac{2}{11} c^2 d x^{11} e + \frac{1}{9} c^2 d^2 x^9 + \frac{2}{9} a c x^9 e^2 + \frac{4}{7} a c d x^7 e + \frac{2}{5} a c d^2 x^5 + \frac{1}{5} a^2 x^5 e^2 + \frac{2}{3} a^2 d x^3 e + a^2 d^2 x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)^2*(c*x^4+a)^2,x, algorithm="giac")

[Out] 1/13*c^2*x^13*e^2 + 2/11*c^2*d*x^11*e + 1/9*c^2*d^2*x^9 + 2/9*a*c*x^9*e^2 + 4/7*a*c*d*x^7*e + 2/5*a*c*d^2*x^5 + 1/5*a^2*x^5*e^2 + 2/3*a^2*d*x^3*e + a^2*d^2*x

3.130 $\int (d + ex^2)(a + cx^4)^2 dx$

Optimal. Leaf size=60

$$a^2dx + \frac{1}{3}a^2ex^3 + \frac{2}{5}acdx^5 + \frac{2}{7}acex^7 + \frac{1}{9}c^2dx^9 + \frac{1}{11}c^2ex^{11}$$

[Out] $a^2*d*x + (a^2*e*x^3)/3 + (2*a*c*d*x^5)/5 + (2*a*c*e*x^7)/7 + (c^2*d*x^9)/9 + (c^2*e*x^11)/11$

Rubi [A] time = 0.0304607, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {1154}

$$a^2dx + \frac{1}{3}a^2ex^3 + \frac{2}{5}acdx^5 + \frac{2}{7}acex^7 + \frac{1}{9}c^2dx^9 + \frac{1}{11}c^2ex^{11}$$

Antiderivative was successfully verified.

[In] Int[(d + e*x^2)*(a + c*x^4)^2,x]

[Out] $a^2*d*x + (a^2*e*x^3)/3 + (2*a*c*d*x^5)/5 + (2*a*c*e*x^7)/7 + (c^2*d*x^9)/9 + (c^2*e*x^11)/11$

Rule 1154

Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]

Rubi steps

$$\begin{aligned} \int (d + ex^2)(a + cx^4)^2 dx &= \int (a^2d + a^2ex^2 + 2acdx^4 + 2acex^6 + c^2dx^8 + c^2ex^{10}) dx \\ &= a^2dx + \frac{1}{3}a^2ex^3 + \frac{2}{5}acdx^5 + \frac{2}{7}acex^7 + \frac{1}{9}c^2dx^9 + \frac{1}{11}c^2ex^{11} \end{aligned}$$

Mathematica [A] time = 0.0023163, size = 60, normalized size = 1.

$$a^2dx + \frac{1}{3}a^2ex^3 + \frac{2}{5}acdx^5 + \frac{2}{7}acex^7 + \frac{1}{9}c^2dx^9 + \frac{1}{11}c^2ex^{11}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e*x^2)*(a + c*x^4)^2,x]

[Out] $a^2*d*x + (a^2*e*x^3)/3 + (2*a*c*d*x^5)/5 + (2*a*c*e*x^7)/7 + (c^2*d*x^9)/9 + (c^2*e*x^11)/11$

Maple [A] time = 0.041, size = 51, normalized size = 0.9

$$a^2dx + \frac{a^2ex^3}{3} + \frac{2acdx^5}{5} + \frac{2acex^7}{7} + \frac{c^2dx^9}{9} + \frac{c^2ex^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*x^2+d)*(c*x^4+a)^2,x)`

[Out] $a^2*d*x + 1/3*a^2*e*x^3 + 2/5*a*c*d*x^5 + 2/7*a*c*e*x^7 + 1/9*c^2*d*x^9 + 1/11*c^2*e*x^{11}$

Maxima [A] time = 0.93175, size = 68, normalized size = 1.13

$$\frac{1}{11}c^2ex^{11} + \frac{1}{9}c^2dx^9 + \frac{2}{7}acex^7 + \frac{2}{5}acdx^5 + \frac{1}{3}a^2ex^3 + a^2dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x^2+d)*(c*x^4+a)^2,x, algorithm="maxima")`

[Out] $1/11*c^2*e*x^{11} + 1/9*c^2*d*x^9 + 2/7*a*c*e*x^7 + 2/5*a*c*d*x^5 + 1/3*a^2*e*x^3 + a^2*d*x$

Fricas [A] time = 1.64708, size = 123, normalized size = 2.05

$$\frac{1}{11}x^{11}ec^2 + \frac{1}{9}x^9dc^2 + \frac{2}{7}x^7eca + \frac{2}{5}x^5dca + \frac{1}{3}x^3ea^2 + xda^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x^2+d)*(c*x^4+a)^2,x, algorithm="fricas")`

[Out] $1/11*x^{11}*e*c^2 + 1/9*x^9*d*c^2 + 2/7*x^7*e*c*a + 2/5*x^5*d*c*a + 1/3*x^3*e*a^2 + x*d*a^2$

Sympy [A] time = 0.067496, size = 60, normalized size = 1.

$$a^2dx + \frac{a^2ex^3}{3} + \frac{2acdx^5}{5} + \frac{2acex^7}{7} + \frac{c^2dx^9}{9} + \frac{c^2ex^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x**2+d)*(c*x**4+a)**2,x)`

[Out] $a**2*d*x + a**2*e*x**3/3 + 2*a*c*d*x**5/5 + 2*a*c*e*x**7/7 + c**2*d*x**9/9 + c**2*e*x**11/11$

Giac [A] time = 1.1765, size = 72, normalized size = 1.2

$$\frac{1}{11}c^2x^{11}e + \frac{1}{9}c^2dx^9 + \frac{2}{7}acx^7e + \frac{2}{5}acdx^5 + \frac{1}{3}a^2x^3e + a^2dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(c*x^4+a)^2,x, algorithm="giac")
```

```
[Out] 1/11*c^2*x^11*e + 1/9*c^2*d*x^9 + 2/7*a*c*x^7*e + 2/5*a*c*d*x^5 + 1/3*a^2*x^3*e + a^2*d*x
```

3.131 $\int (a + cx^4)^2 dx$

Optimal. Leaf size=25

$$a^2x + \frac{2}{5}acx^5 + \frac{c^2x^9}{9}$$

[Out] $a^2x + (2acx^5)/5 + (c^2x^9)/9$

Rubi [A] time = 0.0081181, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {194}

$$a^2x + \frac{2}{5}acx^5 + \frac{c^2x^9}{9}$$

Antiderivative was successfully verified.

[In] Int[(a + c*x^4)^2,x]

[Out] $a^2x + (2acx^5)/5 + (c^2x^9)/9$

Rule 194

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]

Rubi steps

$$\begin{aligned} \int (a + cx^4)^2 dx &= \int (a^2 + 2acx^4 + c^2x^8) dx \\ &= a^2x + \frac{2}{5}acx^5 + \frac{c^2x^9}{9} \end{aligned}$$

Mathematica [A] time = 0.0009166, size = 25, normalized size = 1.

$$a^2x + \frac{2}{5}acx^5 + \frac{c^2x^9}{9}$$

Antiderivative was successfully verified.

[In] Integrate[(a + c*x^4)^2,x]

[Out] $a^2x + (2acx^5)/5 + (c^2x^9)/9$

Maple [A] time = 0.043, size = 22, normalized size = 0.9

$$a^2x + \frac{2acx^5}{5} + \frac{c^2x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c*x^4+a)^2,x)`

[Out] $a^2x + \frac{2}{5}acx^5 + \frac{1}{9}c^2x^9$

Maxima [A] time = 1.00009, size = 28, normalized size = 1.12

$$\frac{1}{9}c^2x^9 + \frac{2}{5}acx^5 + a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x^4+a)^2,x, algorithm="maxima")`

[Out] $\frac{1}{9}c^2x^9 + \frac{2}{5}acx^5 + a^2x$

Fricas [A] time = 1.6794, size = 47, normalized size = 1.88

$$\frac{1}{9}x^9c^2 + \frac{2}{5}x^5ca + xa^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x^4+a)^2,x, algorithm="fricas")`

[Out] $\frac{1}{9}x^9c^2 + \frac{2}{5}x^5ca + xa^2$

Sympy [A] time = 0.060069, size = 22, normalized size = 0.88

$$a^2x + \frac{2acx^5}{5} + \frac{c^2x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x**4+a)**2,x)`

[Out] $a^2x + \frac{2acx^5}{5} + \frac{c^2x^9}{9}$

Giac [A] time = 1.13115, size = 28, normalized size = 1.12

$$\frac{1}{9}c^2x^9 + \frac{2}{5}acx^5 + a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x^4+a)^2,x, algorithm="giac")`

[Out] $\frac{1}{9}c^2x^9 + \frac{2}{5}acx^5 + a^2x$

$$3.132 \quad \int \frac{(a+cx^4)^2}{d+ex^2} dx$$

Optimal. Leaf size=108

$$\frac{cx^3(2ae^2+cd^2)}{3e^3} - \frac{cdx(2ae^2+cd^2)}{e^4} + \frac{(ae^2+cd^2)^2 \tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)}{\sqrt{de}^{9/2}} - \frac{c^2dx^5}{5e^2} + \frac{c^2x^7}{7e}$$

[Out] $-\left(\frac{c*d*(c*d^2 + 2*a*e^2)*x}{e^4}\right) + \left(\frac{c*(c*d^2 + 2*a*e^2)*x^3}{(3*e^3)} - \left(\frac{c^2*d*x^5}{(5*e^2)} + \frac{c^2*x^7}{(7*e)} + \left(\frac{c*d^2 + a*e^2}{e}\right)^2 * \text{ArcTan}\left[\frac{\text{Sqrt}[e]*x}{\text{Sqrt}[d]}\right]\right) / \left(\text{Sqrt}[d]*e^{(9/2)}\right)$

Rubi [A] time = 0.0767978, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {1154, 205}

$$\frac{cx^3(2ae^2+cd^2)}{3e^3} - \frac{cdx(2ae^2+cd^2)}{e^4} + \frac{(ae^2+cd^2)^2 \tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)}{\sqrt{de}^{9/2}} - \frac{c^2dx^5}{5e^2} + \frac{c^2x^7}{7e}$$

Antiderivative was successfully verified.

[In] Int[(a + c*x^4)^2/(d + e*x^2), x]

[Out] $-\left(\frac{c*d*(c*d^2 + 2*a*e^2)*x}{e^4}\right) + \left(\frac{c*(c*d^2 + 2*a*e^2)*x^3}{(3*e^3)} - \left(\frac{c^2*d*x^5}{(5*e^2)} + \frac{c^2*x^7}{(7*e)} + \left(\frac{c*d^2 + a*e^2}{e}\right)^2 * \text{ArcTan}\left[\frac{\text{Sqrt}[e]*x}{\text{Sqrt}[d]}\right]\right) / \left(\text{Sqrt}[d]*e^{(9/2)}\right)$

Rule 1154

Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rubi steps

$$\begin{aligned} \int \frac{(a+cx^4)^2}{d+ex^2} dx &= \int \left(-\frac{cd(cd^2+2ae^2)}{e^4} + \frac{c(cd^2+2ae^2)x^2}{e^3} - \frac{c^2dx^4}{e^2} + \frac{c^2x^6}{e} + \frac{c^2d^4+2acd^2e^2+a^2e^4}{e^4(d+ex^2)} \right) dx \\ &= -\frac{cd(cd^2+2ae^2)x}{e^4} + \frac{c(cd^2+2ae^2)x^3}{3e^3} - \frac{c^2dx^5}{5e^2} + \frac{c^2x^7}{7e} + \frac{(cd^2+ae^2)^2 \int \frac{1}{d+ex^2} dx}{e^4} \\ &= -\frac{cd(cd^2+2ae^2)x}{e^4} + \frac{c(cd^2+2ae^2)x^3}{3e^3} - \frac{c^2dx^5}{5e^2} + \frac{c^2x^7}{7e} + \frac{(cd^2+ae^2)^2 \tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)}{\sqrt{de}^{9/2}} \end{aligned}$$

Mathematica [A] time = 0.0798412, size = 97, normalized size = 0.9

$$\frac{cx(70ae^2(ex^2-3d) + c(35d^2ex^2 - 105d^3 - 21de^2x^4 + 15e^3x^6))}{105e^4} + \frac{(ae^2+cd^2)^2 \tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)}{\sqrt{de}^{9/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + c*x^4)^2/(d + e*x^2),x]

[Out] (c*x*(70*a*e^2*(-3*d + e*x^2) + c*(-105*d^3 + 35*d^2*e*x^2 - 21*d*e^2*x^4 + 15*e^3*x^6)))/(105*e^4) + ((c*d^2 + a*e^2)^2*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[d]*e^(9/2))

Maple [A] time = 0.047, size = 136, normalized size = 1.3

$$\frac{c^2x^7}{7e} - \frac{c^2dx^5}{5e^2} + \frac{2cx^3a}{3e} + \frac{c^2d^2x^3}{3e^3} - 2\frac{acdx}{e^2} - \frac{c^2d^3x}{e^4} + a^2 \arctan\left(\frac{ex}{\sqrt{de}}\right) \frac{1}{\sqrt{de}} + 2\frac{acd^2}{e^2\sqrt{de}} \arctan\left(\frac{ex}{\sqrt{de}}\right) + \frac{c^2d^4}{e^4} \arctan\left(\frac{ex}{\sqrt{de}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^4+a)^2/(e*x^2+d),x)

[Out] 1/7*c^2*x^7/e-1/5*c^2*d*x^5/e^2+2/3*c/e*x^3*a+1/3*c^2/e^3*d^2*x^3-2*c/e^2*a*d*x-1/e^4*c^2*d^3*x+1/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))*a^2+2/e^2/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))*a*c*d^2+1/e^4/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))*c^2*d^4

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+a)^2/(e*x^2+d),x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [A] time = 1.8622, size = 585, normalized size = 5.42

$$\left[\frac{30c^2de^4x^7 - 42c^2d^2e^3x^5 + 70(c^2d^3e^2 + 2acde^4)x^3 - 105(c^2d^4 + 2acd^2e^2 + a^2e^4)\sqrt{-de} \log\left(\frac{ex^2 - 2\sqrt{-dex} - d}{ex^2 + d}\right) - 210(c^2d^4e - 2acd^2e^2 + a^2e^4)\sqrt{-de}}{210de^5} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+a)^2/(e*x^2+d),x, algorithm="fricas")

[Out] [1/210*(30*c^2*d*e^4*x^7 - 42*c^2*d^2*e^3*x^5 + 70*(c^2*d^3*e^2 + 2*a*c*d*e^4)*x^3 - 105*(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*sqrt(-d*e)*log((e*x^2 - 2*sqrt(-d*e)*x - d)/(e*x^2 + d)) - 210*(c^2*d^4*e + 2*a*c*d^2*e^3)*x)/(d*e^5), 1/105*(15*c^2*d*e^4*x^7 - 21*c^2*d^2*e^3*x^5 + 35*(c^2*d^3*e^2 + 2*a*c*d*e^4)*x^3 + 105*(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*sqrt(d*e)*arctan(sqrt(d*e)*x/d) - 105*(c^2*d^4*e + 2*a*c*d^2*e^3)*x)/(d*e^5)]

Sympy [B] time = 0.593639, size = 235, normalized size = 2.18

$$-\frac{c^2 dx^5}{5e^2} + \frac{c^2 x^7}{7e} - \frac{\sqrt{-\frac{1}{de^9}} (ae^2 + cd^2)^2 \log\left(-\frac{de^4 \sqrt{-\frac{1}{de^9}} (ae^2 + cd^2)^2}{a^2 e^4 + 2acd^2 e^2 + c^2 d^4} + x\right)}{2} + \frac{\sqrt{-\frac{1}{de^9}} (ae^2 + cd^2)^2 \log\left(\frac{de^4 \sqrt{-\frac{1}{de^9}} (ae^2 + cd^2)^2}{a^2 e^4 + 2acd^2 e^2 + c^2 d^4} + x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x**4+a)**2/(e*x**2+d), x)

[Out] -c**2*d*x**5/(5*e**2) + c**2*x**7/(7*e) - sqrt(-1/(d*e**9))*(a*e**2 + c*d**2)**2*log(-d*e**4*sqrt(-1/(d*e**9))*(a*e**2 + c*d**2)**2/(a**2*e**4 + 2*a*c*d**2*e**2 + c**2*d**4) + x)/2 + sqrt(-1/(d*e**9))*(a*e**2 + c*d**2)**2*log(d*e**4*sqrt(-1/(d*e**9))*(a*e**2 + c*d**2)**2/(a**2*e**4 + 2*a*c*d**2*e**2 + c**2*d**4) + x)/2 + x**3*(2*a*c*e**2 + c**2*d**2)/(3*e**3) - x*(2*a*c*d*e**2 + c**2*d**3)/e**4

Giac [A] time = 1.10459, size = 142, normalized size = 1.31

$$\frac{(c^2 d^4 + 2acd^2 e^2 + a^2 e^4) \arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right) e^{\left(-\frac{9}{2}\right)}}{\sqrt{d}} + \frac{1}{105} (15c^2 x^7 e^6 - 21c^2 dx^5 e^5 + 35c^2 d^2 x^3 e^4 - 105c^2 d^3 x e^3 + 70acx^3 e^6 - 210acd^2 x^2 e^5 - 105c^2 d^3 x e^3 + 70acx^3 e^6 - 210acd^2 x^2 e^5) e^{-7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+a)^2/(e*x^2+d), x, algorithm="giac")

[Out] (c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*arctan(x*e^(1/2)/sqrt(d))*e^(-9/2)/sqrt(d) + 1/105*(15*c^2*x^7*e^6 - 21*c^2*d*x^5*e^5 + 35*c^2*d^2*x^3*e^4 - 105*c^2*d^3*x*e^3 + 70*a*c*x^3*e^6 - 210*a*c*d*x^2*e^5)*e^(-7)

$$3.133 \quad \int \frac{(a+cx^4)^2}{(d+ex^2)^2} dx$$

Optimal. Leaf size=131

$$\frac{x(ae^2 + cd^2)^2}{2de^4(d + ex^2)} + \frac{cx(2ae^2 + 3cd^2)}{e^4} - \frac{(7cd^2 - ae^2)(ae^2 + cd^2) \tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)}{2d^{3/2}e^{9/2}} - \frac{2c^2dx^3}{3e^3} + \frac{c^2x^5}{5e^2}$$

[Out] (c*(3*c*d^2 + 2*a*e^2)*x)/e^4 - (2*c^2*d*x^3)/(3*e^3) + (c^2*x^5)/(5*e^2) + ((c*d^2 + a*e^2)^2*x)/(2*d*e^4*(d + e*x^2)) - ((7*c*d^2 - a*e^2)*(c*d^2 + a*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(3/2)*e^(9/2))

Rubi [A] time = 0.187989, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {1158, 1810, 205}

$$\frac{x(ae^2 + cd^2)^2}{2de^4(d + ex^2)} + \frac{cx(2ae^2 + 3cd^2)}{e^4} - \frac{(7cd^2 - ae^2)(ae^2 + cd^2) \tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)}{2d^{3/2}e^{9/2}} - \frac{2c^2dx^3}{3e^3} + \frac{c^2x^5}{5e^2}$$

Antiderivative was successfully verified.

[In] Int[(a + c*x^4)^2/(d + e*x^2)^2,x]

[Out] (c*(3*c*d^2 + 2*a*e^2)*x)/e^4 - (2*c^2*d*x^3)/(3*e^3) + (c^2*x^5)/(5*e^2) + ((c*d^2 + a*e^2)^2*x)/(2*d*e^4*(d + e*x^2)) - ((7*c*d^2 - a*e^2)*(c*d^2 + a*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(3/2)*e^(9/2))

Rule 1158

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] := With[{Qx = PolynomialQuotient[(a + c*x^4)^p, d + e*x^2, x], R = Coeff[PolynomialRemainder[(a + c*x^4)^p, d + e*x^2, x], x, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 1810

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] := Int[ExpandIntegrand[Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a + cx^4)^2}{(d + ex^2)^2} dx &= \frac{(cd^2 + ae^2)^2 x}{2de^4 (d + ex^2)} - \frac{\int \frac{-a^2 + \frac{c^2 d^4}{e^4} + \frac{2acd^2}{e^2} - \frac{2cd(cd^2 + 2ae^2)x^2}{e^3} + \frac{2c^2 d^2 x^4}{e^2} - \frac{2c^2 dx^6}{e}}{d + ex^2} dx \\
&= \frac{(cd^2 + ae^2)^2 x}{2de^4 (d + ex^2)} - \frac{\int \left(-\frac{2cd(3cd^2 + 2ae^2)}{e^4} + \frac{4c^2 d^2 x^2}{e^3} - \frac{2c^2 dx^4}{e^2} + \frac{7c^2 d^4 + 6acd^2 e^2 - a^2 e^4}{e^4 (d + ex^2)} \right) dx}{2d} \\
&= \frac{c(3cd^2 + 2ae^2)x}{e^4} - \frac{2c^2 dx^3}{3e^3} + \frac{c^2 x^5}{5e^2} + \frac{(cd^2 + ae^2)^2 x}{2de^4 (d + ex^2)} - \frac{((7cd^2 - ae^2)(cd^2 + ae^2)) \int \frac{1}{d + ex^2} dx}{2de^4} \\
&= \frac{c(3cd^2 + 2ae^2)x}{e^4} - \frac{2c^2 dx^3}{3e^3} + \frac{c^2 x^5}{5e^2} + \frac{(cd^2 + ae^2)^2 x}{2de^4 (d + ex^2)} - \frac{(7cd^2 - ae^2)(cd^2 + ae^2) \tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)}{2d^{3/2}e^{9/2}}
\end{aligned}$$

Mathematica [A] time = 0.109365, size = 134, normalized size = 1.02

$$-\frac{(-a^2 e^4 + 6acd^2 e^2 + 7c^2 d^4) \tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)}{2d^{3/2}e^{9/2}} + \frac{x(ae^2 + cd^2)^2}{2de^4(d + ex^2)} + \frac{cx(2ae^2 + 3cd^2)}{e^4} - \frac{2c^2 dx^3}{3e^3} + \frac{c^2 x^5}{5e^2}$$

Antiderivative was successfully verified.

[In] Integrate[(a + c*x^4)^2/(d + e*x^2)^2,x]

[Out] (c*(3*c*d^2 + 2*a*e^2)*x)/e^4 - (2*c^2*d*x^3)/(3*e^3) + (c^2*x^5)/(5*e^2) + ((c*d^2 + a*e^2)^2*x)/(2*d*e^4*(d + e*x^2)) - ((7*c^2*d^4 + 6*a*c*d^2*e^2 - a^2*e^4)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(3/2)*e^(9/2))

Maple [A] time = 0.053, size = 170, normalized size = 1.3

$$\frac{c^2 x^5}{5e^2} - \frac{2c^2 dx^3}{3e^3} + 2\frac{acx}{e^2} + 3\frac{c^2 d^2 x}{e^4} + \frac{a^2 x}{2d(ex^2 + d)} + \frac{adxc}{e^2(ex^2 + d)} + \frac{d^3 xc^2}{2e^4(ex^2 + d)} + \frac{a^2}{2d} \arctan\left(ex \frac{1}{\sqrt{de}}\right) \frac{1}{\sqrt{de}} - 3\frac{ac}{e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^4+a)^2/(e*x^2+d)^2,x)

[Out] 1/5*c^2*x^5/e^2-2/3*c^2*d*x^3/e^3+2*c/e^2*a*x+3*c^2/e^4*d^2*x+1/2/d*x/(e*x^2+d)*a^2+1/e^2*d*x/(e*x^2+d)*a*c+1/2/e^4*d^3*x/(e*x^2+d)*c^2+1/2/d/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))*a^2-3/e^2*d/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))*a*c-7/2/e^4*d^3/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))*c^2

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+a)^2/(e*x^2+d)^2,x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [A] time = 1.86485, size = 807, normalized size = 6.16

$$\left[\frac{12c^2d^2e^4x^7 - 28c^2d^3e^3x^5 + 20(7c^2d^4e^2 + 6acd^2e^4)x^3 + 15(7c^2d^5 + 6acd^3e^2 - a^2de^4 + (7c^2d^4e + 6acd^2e^3 - a^2e^5)x^2)}{60(d^2e^6x^2 + d^3e^5)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+a)^2/(e*x^2+d)^2,x, algorithm="fricas")

[Out] [1/60*(12*c^2*d^2*e^4*x^7 - 28*c^2*d^3*e^3*x^5 + 20*(7*c^2*d^4*e^2 + 6*a*c*d^2*e^4)*x^3 + 15*(7*c^2*d^5 + 6*a*c*d^3*e^2 - a^2*d*e^4 + (7*c^2*d^4*e + 6*a*c*d^2*e^3 - a^2*e^5)*x^2)*sqrt(-d*e)*log((e*x^2 - 2*sqrt(-d*e)*x - d)/(e*x^2 + d)) + 30*(7*c^2*d^5*e + 6*a*c*d^3*e^3 + a^2*d*e^5)*x/(d^2*e^6*x^2 + d^3*e^5), 1/30*(6*c^2*d^2*e^4*x^7 - 14*c^2*d^3*e^3*x^5 + 10*(7*c^2*d^4*e^2 + 6*a*c*d^2*e^4)*x^3 - 15*(7*c^2*d^5 + 6*a*c*d^3*e^2 - a^2*d*e^4 + (7*c^2*d^4*e + 6*a*c*d^2*e^3 - a^2*e^5)*x^2)*sqrt(d*e)*arctan(sqrt(d*e)*x/d) + 15*(7*c^2*d^5*e + 6*a*c*d^3*e^3 + a^2*d*e^5)*x/(d^2*e^6*x^2 + d^3*e^5)]

Sympy [B] time = 1.02083, size = 314, normalized size = 2.4

$$-\frac{2c^2dx^3}{3e^3} + \frac{c^2x^5}{5e^2} + \frac{x(a^2e^4 + 2acd^2e^2 + c^2d^4)}{2d^2e^4 + 2de^5x^2} - \frac{\sqrt{-\frac{1}{d^3e^9}}(ae^2 - 7cd^2)(ae^2 + cd^2) \log\left(-\frac{d^2e^4\sqrt{-\frac{1}{d^3e^9}}(ae^2 - 7cd^2)(ae^2 + cd^2)}{a^2e^4 - 6acd^2e^2 - 7c^2d^4} + x\right)}{4} +$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x**4+a)**2/(e*x**2+d)**2,x)

[Out] -2*c**2*d*x**3/(3*e**3) + c**2*x**5/(5*e**2) + x*(a**2*e**4 + 2*a*c*d**2*e**2 + c**2*d**4)/(2*d**2*e**4 + 2*d*e**5*x**2) - sqrt(-1/(d**3*e**9))*(a*e**2 - 7*c*d**2)*(a*e**2 + c*d**2)*log(-d**2*e**4*sqrt(-1/(d**3*e**9))*(a*e**2 - 7*c*d**2)*(a*e**2 + c*d**2)/(a**2*e**4 - 6*a*c*d**2*e**2 - 7*c**2*d**4) + x)/4 + sqrt(-1/(d**3*e**9))*(a*e**2 - 7*c*d**2)*(a*e**2 + c*d**2)*log(d**2*e**4*sqrt(-1/(d**3*e**9))*(a*e**2 - 7*c*d**2)*(a*e**2 + c*d**2)/(a**2*e**4 - 6*a*c*d**2*e**2 - 7*c**2*d**4) + x)/4 + x*(2*a*c*e**2 + 3*c**2*d**2)/e**4

Giac [A] time = 1.15645, size = 173, normalized size = 1.32

$$\frac{1}{15} (3c^2x^5e^8 - 10c^2dx^3e^7 + 45c^2d^2xe^6 + 30acxe^8)e^{(-10)} - \frac{(7c^2d^4 + 6acd^2e^2 - a^2e^4) \arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right) e^{\left(-\frac{9}{2}\right)}}{2d^{\frac{3}{2}}} + \frac{(c^2d^4x + 2acd^2e^2)}{2(x^2 + d)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+a)^2/(e*x^2+d)^2,x, algorithm="giac")

```
[Out] 1/15*(3*c^2*x^5*e^8 - 10*c^2*d*x^3*e^7 + 45*c^2*d^2*x*e^6 + 30*a*c*x*e^8)*e^(-10) - 1/2*(7*c^2*d^4 + 6*a*c*d^2*e^2 - a^2*e^4)*arctan(x*e^(1/2)/sqrt(d))*e^(-9/2)/d^(3/2) + 1/2*(c^2*d^4*x + 2*a*c*d^2*x*e^2 + a^2*x*e^4)*e^(-4)/((x^2*e + d)*d)
```

$$3.134 \quad \int \frac{(a+cx^4)^2}{(d+ex^2)^3} dx$$

Optimal. Leaf size=155

$$\frac{x \left(3a^2 - \frac{10acd^2}{e^2} - \frac{13c^2d^4}{e^4} \right)}{8d^2(d+ex^2)} + \frac{(3a^2e^4 + 6acd^2e^2 + 35c^2d^4) \tan^{-1} \left(\frac{\sqrt{ex}}{\sqrt{d}} \right)}{8d^{5/2}e^{9/2}} + \frac{x(ae^2 + cd^2)^2}{4de^4(d+ex^2)^2} - \frac{3c^2dx}{e^4} + \frac{c^2x^3}{3e^3}$$

[Out] $(-3c^2dx)/e^4 + (c^2x^3)/(3e^3) + ((cd^2 + ae^2)^2x)/(4d^4e^4(d + ex^2)^2) + ((3a^2 - (13c^2d^4)/e^4 - (10acd^2)/e^2)x)/(8d^2(d + ex^2)) + ((35c^2d^4 + 6acd^2e^2 + 3a^2e^4) \text{ArcTan}[(\text{Sqrt}[e]x)/\text{Sqrt}[d]])/(8d^{5/2}e^{9/2})$

Rubi [A] time = 0.252468, antiderivative size = 155, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.21$, Rules used = {1158, 1814, 1153, 205}

$$\frac{x \left(3a^2 - \frac{10acd^2}{e^2} - \frac{13c^2d^4}{e^4} \right)}{8d^2(d+ex^2)} + \frac{(3a^2e^4 + 6acd^2e^2 + 35c^2d^4) \tan^{-1} \left(\frac{\sqrt{ex}}{\sqrt{d}} \right)}{8d^{5/2}e^{9/2}} + \frac{x(ae^2 + cd^2)^2}{4de^4(d+ex^2)^2} - \frac{3c^2dx}{e^4} + \frac{c^2x^3}{3e^3}$$

Antiderivative was successfully verified.

[In] Int[(a + c*x^4)^2/(d + e*x^2)^3,x]

[Out] $(-3c^2dx)/e^4 + (c^2x^3)/(3e^3) + ((cd^2 + ae^2)^2x)/(4d^4e^4(d + ex^2)^2) + ((3a^2 - (13c^2d^4)/e^4 - (10acd^2)/e^2)x)/(8d^2(d + ex^2)) + ((35c^2d^4 + 6acd^2e^2 + 3a^2e^4) \text{ArcTan}[(\text{Sqrt}[e]x)/\text{Sqrt}[d]])/(8d^{5/2}e^{9/2})$

Rule 1158

Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] := With[{Qx = PolynomialQuotient[(a + c*x^4)^p, d + e*x^2, x], R = Coeff[PolynomialRemainder[(a + c*x^4)^p, d + e*x^2, x], x, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x], x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]

Rule 1814

Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] := With[{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]

Rule 1153

Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rubi steps

$$\begin{aligned} \int \frac{(a + cx^4)^2}{(d + ex^2)^3} dx &= \frac{(cd^2 + ae^2)^2 x}{4de^4 (d + ex^2)^2} - \frac{\int \frac{-3a^2 + \frac{c^2 d^4}{e^4} + \frac{2acd^2}{e^2} - \frac{4cd(cd^2 + 2ae^2)x^2}{e^3} + \frac{4c^2 d^2 x^4}{e^2} - \frac{4c^2 dx^6}{e}}{(d + ex^2)^2} dx}{4d} \\ &= \frac{(cd^2 + ae^2)^2 x}{4de^4 (d + ex^2)^2} + \frac{\left(3a^2 - \frac{13c^2 d^4}{e^4} - \frac{10acd^2}{e^2}\right) x}{8d^2 (d + ex^2)} + \frac{\int \frac{3a^2 + \frac{11c^2 d^4}{e^4} + \frac{6acd^2}{e^2} - \frac{16c^2 d^3 x^2}{e^3} + \frac{8c^2 d^2 x^4}{e^2}}{d + ex^2} dx}{8d^2} \\ &= \frac{(cd^2 + ae^2)^2 x}{4de^4 (d + ex^2)^2} + \frac{\left(3a^2 - \frac{13c^2 d^4}{e^4} - \frac{10acd^2}{e^2}\right) x}{8d^2 (d + ex^2)} + \frac{\int \left(-\frac{24c^2 d^3}{e^4} + \frac{8c^2 d^2 x^2}{e^3} + \frac{35c^2 d^4 + 6acd^2 e^2 + 3a^2 e^4}{e^4 (d + ex^2)}\right) dx}{8d^2} \\ &= -\frac{3c^2 dx}{e^4} + \frac{c^2 x^3}{3e^3} + \frac{(cd^2 + ae^2)^2 x}{4de^4 (d + ex^2)^2} + \frac{\left(3a^2 - \frac{13c^2 d^4}{e^4} - \frac{10acd^2}{e^2}\right) x}{8d^2 (d + ex^2)} + \frac{(35c^2 d^4 + 6acd^2 e^2 + 3a^2 e^4) \int \frac{1}{d + ex^2}}{8d^2 e^4} \\ &= -\frac{3c^2 dx}{e^4} + \frac{c^2 x^3}{3e^3} + \frac{(cd^2 + ae^2)^2 x}{4de^4 (d + ex^2)^2} + \frac{\left(3a^2 - \frac{13c^2 d^4}{e^4} - \frac{10acd^2}{e^2}\right) x}{8d^2 (d + ex^2)} + \frac{(35c^2 d^4 + 6acd^2 e^2 + 3a^2 e^4) \tan^{-1}}{8d^{5/2} e^{9/2}} \end{aligned}$$

Mathematica [A] time = 0.111127, size = 154, normalized size = 0.99

$$\frac{x(3a^2 e^4 (5d + 3ex^2) - 6acd^2 e^2 (3d + 5ex^2) - c^2 d^2 (175d^2 ex^2 + 105d^3 + 56de^2 x^4 - 8e^3 x^6))}{24d^2 e^4 (d + ex^2)^2} + \frac{(3a^2 e^4 + 6acd^2 e^2 + 35c^2 d^4)}{8d^{5/2} e^{9/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + c*x^4)^2/(d + e*x^2)^3, x]

[Out] (x*(3*a^2*e^4*(5*d + 3*e*x^2) - 6*a*c*d^2*e^2*(3*d + 5*e*x^2) - c^2*d^2*(10*5*d^3 + 175*d^2*e*x^2 + 56*d*e^2*x^4 - 8*e^3*x^6)))/(24*d^2*e^4*(d + e*x^2)^2) + ((35*c^2*d^4 + 6*a*c*d^2*e^2 + 3*a^2*e^4)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(8*d^(5/2)*e^(9/2))

Maple [A] time = 0.053, size = 211, normalized size = 1.4

$$\frac{c^2 x^3}{3e^3} - 3 \frac{c^2 dx}{e^4} + \frac{3a^2 ex^3}{8(ex^2 + d)^2 d^2} - \frac{5ax^3 c}{4e(ex^2 + d)^2} - \frac{13d^2 x^3 c^2}{8e^3(ex^2 + d)^2} + \frac{5a^2 x}{8(ex^2 + d)^2 d} - \frac{3adxc}{4e^2(ex^2 + d)^2} - \frac{11d^3 xc^2}{8e^4(ex^2 + d)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^4+a)^2/(e*x^2+d)^3, x)

[Out] 1/3*c^2*x^3/e^3-3*c^2*d*x/e^4+3/8*e/(e*x^2+d)^2/d^2*x^3*a^2-5/4/e/(e*x^2+d)^2*x^3*a*c-13/8/e^3/(e*x^2+d)^2*d^2*x^3*c^2+5/8/(e*x^2+d)^2/d*x*a^2-3/4/e^2/(e*x^2+d)^2*d*x*a*c-11/8/e^4/(e*x^2+d)^2*d^3*x*c^2+3/8/d^2/(d*e)^(1/2)*arc

$\tan(e*x/(d*e)^{(1/2)})*a^2+3/4/e^2/(d*e)^{(1/2)}*\arctan(e*x/(d*e)^{(1/2)})*a*c+35/8/e^4*d^2/(d*e)^{(1/2)}*\arctan(e*x/(d*e)^{(1/2)})*c^2$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+a)^2/(e*x^2+d)^3,x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [A] time = 1.9364, size = 1071, normalized size = 6.91

$$\left[\frac{16c^2d^3e^4x^7 - 112c^2d^4e^3x^5 - 2(175c^2d^5e^2 + 30acd^3e^4 - 9a^2de^6)x^3 - 3(35c^2d^6 + 6acd^4e^2 + 3a^2d^2e^4 + (35c^2d^4e^2 + 6acd^2e^2 + 3a^2d^2e^2))x - 3(35c^2d^4e^2 + 6acd^2e^2 + 3a^2d^2e^2)}{48(d^3e^7x^4 + 2d^4e^6x^2 + d^5e^5)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+a)^2/(e*x^2+d)^3,x, algorithm="fricas")

[Out] [1/48*(16*c^2*d^3*e^4*x^7 - 112*c^2*d^4*e^3*x^5 - 2*(175*c^2*d^5*e^2 + 30*a*c*d^3*e^4 - 9*a^2*d*e^6)*x^3 - 3*(35*c^2*d^6 + 6*a*c*d^4*e^2 + 3*a^2*d^2*e^4 + (35*c^2*d^4*e^2 + 6*a*c*d^2*e^2 + 3*a^2*d^2*e^2))*x - 3*(35*c^2*d^4*e^2 + 6*a*c*d^2*e^2 + 3*a^2*d^2*e^2))/48*(d^3*e^7*x^4 + 2*d^4*e^6*x^2 + d^5*e^5), 1/24*(8*c^2*d^3*e^4*x^7 - 56*c^2*d^4*e^3*x^5 - (175*c^2*d^5*e^2 + 30*a*c*d^3*e^4 - 9*a^2*d*e^6)*x^3 + 3*(35*c^2*d^6 + 6*a*c*d^4*e^2 + 3*a^2*d^2*e^4 + (35*c^2*d^4*e^2 + 6*a*c*d^2*e^2 + 3*a^2*d^2*e^2))*x^2)*sqrt(-d*e)*log((e*x^2 - 2*sqrt(-d*e)*x - d)/(e*x^2 + d)) - 6*(35*c^2*d^6*e + 6*a*c*d^4*e^3 - 5*a^2*d^2*e^5)*x/(d^3*e^7*x^4 + 2*d^4*e^6*x^2 + d^5*e^5), 1/24*(8*c^2*d^3*e^4*x^7 - 56*c^2*d^4*e^3*x^5 - (175*c^2*d^5*e^2 + 30*a*c*d^3*e^4 - 9*a^2*d*e^6)*x^3 + 3*(35*c^2*d^6 + 6*a*c*d^4*e^2 + 3*a^2*d^2*e^4 + (35*c^2*d^4*e^2 + 6*a*c*d^2*e^2 + 3*a^2*d^2*e^2))*x^2)*sqrt(d*e)*arctan(sqrt(d*e)*x/d) - 3*(35*c^2*d^6*e + 6*a*c*d^4*e^3 - 5*a^2*d^2*e^5)*x/(d^3*e^7*x^4 + 2*d^4*e^6*x^2 + d^5*e^5)]

Sympy [A] time = 1.85728, size = 257, normalized size = 1.66

$$-\frac{3c^2dx}{e^4} + \frac{c^2x^3}{3e^3} - \frac{\sqrt{-\frac{1}{d^5e^9}}(3a^2e^4 + 6acd^2e^2 + 35c^2d^4)\log\left(-d^3e^4\sqrt{-\frac{1}{d^5e^9}} + x\right)}{16} + \frac{\sqrt{-\frac{1}{d^5e^9}}(3a^2e^4 + 6acd^2e^2 + 35c^2d^4)\log\left(-d^3e^4\sqrt{-\frac{1}{d^5e^9}} + x\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x**4+a)**2/(e*x**2+d)**3,x)

[Out] -3*c**2*d*x/e**4 + c**2*x**3/(3*e**3) - sqrt(-1/(d**5*e**9))*(3*a**2*e**4 + 6*a*c*d**2*e**2 + 35*c**2*d**4)*log(-d**3*e**4*sqrt(-1/(d**5*e**9)) + x)/16 + sqrt(-1/(d**5*e**9))*(3*a**2*e**4 + 6*a*c*d**2*e**2 + 35*c**2*d**4)*log(d**3*e**4*sqrt(-1/(d**5*e**9)) + x)/16 + (x**3*(3*a**2*e**5 - 10*a*c*d**2*e**3 - 13*c**2*d**4*e) + x*(5*a**2*d*e**4 - 6*a*c*d**3*e**2 - 11*c**2*d**5))/(8*d**4*e**4 + 16*d**3*e**5*x**2 + 8*d**2*e**6*x**4)

Giac [A] time = 1.19466, size = 196, normalized size = 1.26

$$\frac{1}{3} (c^2 x^3 e^6 - 9 c^2 d x e^5) e^{(-9)} + \frac{(35 c^2 d^4 + 6 a c d^2 e^2 + 3 a^2 e^4) \arctan\left(\frac{x e^{\frac{1}{2}}}{\sqrt{d}}\right) e^{\left(-\frac{9}{2}\right)}}{8 d^{\frac{5}{2}}} - \frac{(13 c^2 d^4 x^3 e + 11 c^2 d^5 x + 10 a c d^2 x^3 e^3 + \dots)}{8 (x^2 e + d)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+a)^2/(e*x^2+d)^3,x, algorithm="giac")

[Out] 1/3*(c^2*x^3*e^6 - 9*c^2*d*x*e^5)*e^(-9) + 1/8*(35*c^2*d^4 + 6*a*c*d^2*e^2 + 3*a^2*e^4)*arctan(x*e^(1/2)/sqrt(d))*e^(-9/2)/d^(5/2) - 1/8*(13*c^2*d^4*x^3*e + 11*c^2*d^5*x + 10*a*c*d^2*x^3*e^3 + 6*a*c*d^3*x*e^2 - 3*a^2*x^3*e^5 - 5*a^2*d*x*e^4)*e^(-4)/((x^2*e + d)^2*d^2)

$$3.135 \quad \int \frac{(a+cx^4)^2}{(d+ex^2)^4} dx$$

Optimal. Leaf size=184

$$\frac{x \left(5a^2 + \frac{2acd^2}{e^2} + \frac{29c^2d^4}{e^4} \right)}{16d^3(d+ex^2)} + \frac{x \left(5a^2 - \frac{14acd^2}{e^2} - \frac{19c^2d^4}{e^4} \right)}{24d^2(d+ex^2)^2} - \frac{(-5a^2e^4 - 2acd^2e^2 + 35c^2d^4) \tan^{-1} \left(\frac{\sqrt{ex}}{\sqrt{d}} \right)}{16d^{7/2}e^{9/2}} + \frac{x(ae^2 + cd^2)^2}{6de^4(d+ex^2)^3} + \frac{c^2x}{e^4}$$

[Out] (c^2*x)/e^4 + ((c*d^2 + a*e^2)^2*x)/(6*d*e^4*(d + e*x^2)^3) + ((5*a^2 - (19*c^2*d^4)/e^4 - (14*a*c*d^2)/e^2)*x)/(24*d^2*(d + e*x^2)^2) + ((5*a^2 + (29*c^2*d^4)/e^4 + (2*a*c*d^2)/e^2)*x)/(16*d^3*(d + e*x^2)) - ((35*c^2*d^4 - 2*a*c*d^2*e^2 - 5*a^2*e^4)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(16*d^(7/2)*e^(9/2))

Rubi [A] time = 0.296625, antiderivative size = 184, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {1158, 1814, 1157, 388, 205}

$$\frac{x \left(5a^2 + \frac{2acd^2}{e^2} + \frac{29c^2d^4}{e^4} \right)}{16d^3(d+ex^2)} + \frac{x \left(5a^2 - \frac{14acd^2}{e^2} - \frac{19c^2d^4}{e^4} \right)}{24d^2(d+ex^2)^2} - \frac{(-5a^2e^4 - 2acd^2e^2 + 35c^2d^4) \tan^{-1} \left(\frac{\sqrt{ex}}{\sqrt{d}} \right)}{16d^{7/2}e^{9/2}} + \frac{x(ae^2 + cd^2)^2}{6de^4(d+ex^2)^3} + \frac{c^2x}{e^4}$$

Antiderivative was successfully verified.

[In] Int[(a + c*x^4)^2/(d + e*x^2)^4, x]

[Out] (c^2*x)/e^4 + ((c*d^2 + a*e^2)^2*x)/(6*d*e^4*(d + e*x^2)^3) + ((5*a^2 - (19*c^2*d^4)/e^4 - (14*a*c*d^2)/e^2)*x)/(24*d^2*(d + e*x^2)^2) + ((5*a^2 + (29*c^2*d^4)/e^4 + (2*a*c*d^2)/e^2)*x)/(16*d^3*(d + e*x^2)) - ((35*c^2*d^4 - 2*a*c*d^2*e^2 - 5*a^2*e^4)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(16*d^(7/2)*e^(9/2))

Rule 1158

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] := With[{Qx = PolynomialQuotient[(a + c*x^4)^p, d + e*x^2, x], R = Coeff[PolynomialRemainder[(a + c*x^4)^p, d + e*x^2, x], x, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x], x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] := With[{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
```

1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]

Rule 388

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Simp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rubi steps

$$\begin{aligned} \int \frac{(a + cx^4)^2}{(d + ex^2)^4} dx &= \frac{(cd^2 + ae^2)^2 x}{6de^4 (d + ex^2)^3} - \frac{\int \frac{-5a^2 + \frac{c^2 d^4}{e^4} + \frac{2acd^2}{e^2} - \frac{6cd(cd^2 + 2ae^2)x^2}{e^3} + \frac{6c^2 d^2 x^4}{e^2} - \frac{6c^2 dx^6}{e}}{(d + ex^2)^3} dx}{6d} \\ &= \frac{(cd^2 + ae^2)^2 x}{6de^4 (d + ex^2)^3} + \frac{\left(5a^2 - \frac{19c^2 d^4}{e^4} - \frac{14acd^2}{e^2}\right) x}{24d^2 (d + ex^2)^2} + \frac{\int \frac{3\left(5a^2 + \frac{5c^2 d^4}{e^4} + \frac{2acd^2}{e^2}\right) - \frac{48c^2 d^3 x^2}{e^3} + \frac{24c^2 d^2 x^4}{e^2}}{(d + ex^2)^2} dx}{24d^2} \\ &= \frac{(cd^2 + ae^2)^2 x}{6de^4 (d + ex^2)^3} + \frac{\left(5a^2 - \frac{19c^2 d^4}{e^4} - \frac{14acd^2}{e^2}\right) x}{24d^2 (d + ex^2)^2} + \frac{\left(5a^2 + \frac{29c^2 d^4}{e^4} + \frac{2acd^2}{e^2}\right) x}{16d^3 (d + ex^2)} - \frac{\int \frac{-3\left(5a^2 - \frac{19c^2 d^4}{e^4} + \frac{2acd^2}{e^2}\right) - \frac{48c^2 d^3}{e^3}}{d + ex^2}}{48d^3} \\ &= \frac{c^2 x}{e^4} + \frac{(cd^2 + ae^2)^2 x}{6de^4 (d + ex^2)^3} + \frac{\left(5a^2 - \frac{19c^2 d^4}{e^4} - \frac{14acd^2}{e^2}\right) x}{24d^2 (d + ex^2)^2} + \frac{\left(5a^2 + \frac{29c^2 d^4}{e^4} + \frac{2acd^2}{e^2}\right) x}{16d^3 (d + ex^2)} - \frac{(35c^2 d^4 - 2acd^2 e^2)}{16d^3} \\ &= \frac{c^2 x}{e^4} + \frac{(cd^2 + ae^2)^2 x}{6de^4 (d + ex^2)^3} + \frac{\left(5a^2 - \frac{19c^2 d^4}{e^4} - \frac{14acd^2}{e^2}\right) x}{24d^2 (d + ex^2)^2} + \frac{\left(5a^2 + \frac{29c^2 d^4}{e^4} + \frac{2acd^2}{e^2}\right) x}{16d^3 (d + ex^2)} - \frac{(35c^2 d^4 - 2acd^2 e^2)}{16d^3} \end{aligned}$$

Mathematica [A] time = 0.14253, size = 174, normalized size = 0.95

$$\frac{x(a^2 e^4 (33d^2 + 40dex^2 + 15e^2 x^4) - 2acd^2 e^2 (3d^2 + 8dex^2 - 3e^2 x^4) + c^2 d^3 (280d^2 ex^2 + 105d^3 + 231de^2 x^4 + 48e^3 x^6))}{48d^3 e^4 (d + ex^2)^3}$$

Antiderivative was successfully verified.

[In] Integrate[(a + c*x^4)^2/(d + e*x^2)^4, x]

[Out] (x*(-2*a*c*d^2*e^2*(3*d^2 + 8*d*e*x^2 - 3*e^2*x^4) + a^2*e^4*(33*d^2 + 40*d*e*x^2 + 15*e^2*x^4) + c^2*d^3*(105*d^3 + 280*d^2*e*x^2 + 231*d*e^2*x^4 + 48*e^3*x^6)))/(48*d^3*e^4*(d + e*x^2)^3) - ((35*c^2*d^4 - 2*a*c*d^2*e^2 - 5*a^2*e^4)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(16*d^(7/2)*e^(9/2))

Maple [A] time = 0.054, size = 262, normalized size = 1.4

$$\frac{c^2x}{e^4} + \frac{5e^2x^5a^2}{16(ex^2+d)^3d^3} + \frac{acx^5}{8(ex^2+d)^3d} + \frac{29dx^5c^2}{16e^2(ex^2+d)^3} + \frac{5a^2ex^3}{6(ex^2+d)^3d^2} - \frac{ax^3c}{3e(ex^2+d)^3} + \frac{17d^2x^3c^2}{6e^3(ex^2+d)^3} + \frac{11}{16(ex^2+d)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^4+a)^2/(e*x^2+d)^4,x)

[Out] $c^2x/e^4 + 5/16e^2/(e*x^2+d)^3/d^3*x^5*a^2 + 1/8/(e*x^2+d)^3/d*x^5*a*c + 29/16/e^2/(e*x^2+d)^3*d*x^5*c^2 + 5/6e/(e*x^2+d)^3/d^2*x^3*a^2 - 1/3/e/(e*x^2+d)^3*x^3*a*c + 17/6/e^3/(e*x^2+d)^3*d^2*x^3*c^2 + 11/16/(e*x^2+d)^3/d*x*a^2 - 1/8/e^2/(e*x^2+d)^3*d*x*a*c + 19/16/e^4/(e*x^2+d)^3*d^3*x*c^2 + 5/16/d^3/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))*a^2 + 1/8/e^2/d/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))*a*c - 35/16/e^4*d/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))*c^2$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+a)^2/(e*x^2+d)^4,x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [A] time = 2.00367, size = 1366, normalized size = 7.42

$$\left[\frac{96c^2d^4e^4x^7 + 6(77c^2d^5e^3 + 2acd^3e^5 + 5a^2de^7)x^5 + 16(35c^2d^6e^2 - 2acd^4e^4 + 5a^2d^2e^6)x^3 + 3(35c^2d^7 - 2acd^5e^2 - 5a^2d^3e^4)}{\dots} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+a)^2/(e*x^2+d)^4,x, algorithm="fricas")

[Out] $[1/96*(96*c^2*d^4*e^4*x^7 + 6*(77*c^2*d^5*e^3 + 2*a*c*d^3*e^5 + 5*a^2*d*e^7)*x^5 + 16*(35*c^2*d^6*e^2 - 2*a*c*d^4*e^4 + 5*a^2*d^2*e^6)*x^3 + 3*(35*c^2*d^7 - 2*a*c*d^5*e^2 - 5*a^2*d^3*e^4 + (35*c^2*d^4*e^3 - 2*a*c*d^2*e^5 - 5*a^2*e^7)*x^6 + 3*(35*c^2*d^5*e^2 - 2*a*c*d^3*e^4 - 5*a^2*d*e^6)*x^4 + 3*(35*c^2*d^6*e - 2*a*c*d^4*e^3 - 5*a^2*d^2*e^5)*x^2)*sqrt(-d*e)*log((e*x^2 - 2*sqrt(-d*e)*x - d)/(e*x^2 + d)) + 6*(35*c^2*d^7*e - 2*a*c*d^5*e^3 + 11*a^2*d^3*e^5)*x)/(d^4*e^8*x^6 + 3*d^5*e^7*x^4 + 3*d^6*e^6*x^2 + d^7*e^5), 1/48*(4*8*c^2*d^4*e^4*x^7 + 3*(77*c^2*d^5*e^3 + 2*a*c*d^3*e^5 + 5*a^2*d*e^7)*x^5 + 8*(35*c^2*d^6*e^2 - 2*a*c*d^4*e^4 + 5*a^2*d^2*e^6)*x^3 - 3*(35*c^2*d^7 - 2*a*c*d^5*e^2 - 5*a^2*d^3*e^4 + (35*c^2*d^4*e^3 - 2*a*c*d^2*e^5 - 5*a^2*e^7)*x^6 + 3*(35*c^2*d^5*e^2 - 2*a*c*d^3*e^4 - 5*a^2*d*e^6)*x^4 + 3*(35*c^2*d^6*e - 2*a*c*d^4*e^3 - 5*a^2*d^2*e^5)*x^2)*sqrt(d*e)*arctan(sqrt(d*e)*x/d) + 3*(35*c^2*d^7*e - 2*a*c*d^5*e^3 + 11*a^2*d^3*e^5)*x)/(d^4*e^8*x^6 + 3*d^5*e^7*x^4 + 3*d^6*e^6*x^2 + d^7*e^5)]$

Sympy [A] time = 3.05803, size = 292, normalized size = 1.59

$$\frac{c^2 x}{e^4} - \frac{\sqrt{-\frac{1}{d^7 e^9}} (5a^2 e^4 + 2acd^2 e^2 - 35c^2 d^4) \log\left(-d^4 e^4 \sqrt{-\frac{1}{d^7 e^9}} + x\right)}{32} + \frac{\sqrt{-\frac{1}{d^7 e^9}} (5a^2 e^4 + 2acd^2 e^2 - 35c^2 d^4) \log\left(d^4 e^4 \sqrt{-\frac{1}{d^7 e^9}} + x\right)}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x**4+a)**2/(e*x**2+d)**4,x)

[Out] c**2*x/e**4 - sqrt(-1/(d**7*e**9))*(5*a**2*e**4 + 2*a*c*d**2*e**2 - 35*c**2*d**4)*log(-d**4*e**4*sqrt(-1/(d**7*e**9)) + x)/32 + sqrt(-1/(d**7*e**9))*(5*a**2*e**4 + 2*a*c*d**2*e**2 - 35*c**2*d**4)*log(d**4*e**4*sqrt(-1/(d**7*e**9)) + x)/32 + (x**5*(15*a**2*e**6 + 6*a*c*d**2*e**4 + 87*c**2*d**4*e**2) + x**3*(40*a**2*d*e**5 - 16*a*c*d**3*e**3 + 136*c**2*d**5*e) + x*(33*a**2*d**2*e**4 - 6*a*c*d**4*e**2 + 57*c**2*d**6))/ (48*d**6*e**4 + 144*d**5*e**5*x**2 + 144*d**4*e**6*x**4 + 48*d**3*e**7*x**6)

Giac [A] time = 1.15817, size = 225, normalized size = 1.22

$$c^2 x e^{(-4)} - \frac{(35 c^2 d^4 - 2 a c d^2 e^2 - 5 a^2 e^4) \arctan\left(\frac{x e^{\frac{1}{2}}}{\sqrt{d}}\right) e^{\left(-\frac{9}{2}\right)}}{16 d^{\frac{7}{2}}} + \frac{(87 c^2 d^4 x^5 e^2 + 136 c^2 d^5 x^3 e + 6 a c d^2 x^5 e^4 + 57 c^2 d^6 x - 16 a^2 d^4 x^5 e^2 + 40 a^2 d^4 x^3 e^5 + 33 a^2 d^2 x^5 e^6 - 6 a^2 c d^4 x^3 e^2 + 40 a^2 d^2 x^3 e^5 + 33 a^2 d^2 x^5 e^4) e^{(-4)}}{(x^2 e + d)^3 d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+a)^2/(e*x^2+d)^4,x, algorithm="giac")

[Out] c^2*x*e^(-4) - 1/16*(35*c^2*d^4 - 2*a*c*d^2*e^2 - 5*a^2*e^4)*arctan(x*e^(1/2)/sqrt(d))*e^(-9/2)/d^(7/2) + 1/48*(87*c^2*d^4*x^5*e^2 + 136*c^2*d^5*x^3*e + 6*a*c*d^2*x^5*e^4 + 57*c^2*d^6*x - 16*a*c*d^3*x^3*e^3 + 15*a^2*x^5*e^6 - 6*a*c*d^4*x*e^2 + 40*a^2*d*x^3*e^5 + 33*a^2*d^2*x*e^4)*e^(-4)/((x^2*e + d)^3*d^3)

$$3.136 \quad \int \frac{(a+cx^4)^2}{(d+ex^2)^5} dx$$

Optimal. Leaf size=223

$$-\frac{x(-35a^2e^4 - 6acd^2e^2 + 93c^2d^4)}{128d^4e^4(d+ex^2)} + \frac{x\left(35a^2 + \frac{6acd^2}{e^2} + \frac{163c^2d^4}{e^4}\right)}{192d^3(d+ex^2)^2} + \frac{x\left(7a^2 - \frac{18acd^2}{e^2} - \frac{25c^2d^4}{e^4}\right)}{48d^2(d+ex^2)^3} + \frac{(35a^2e^4 + 6acd^2e^2 + 35c^2d^4)}{128d^{9/2}e^{9/2}}$$

[Out] $((c*d^2 + a*e^2)^2*x)/(8*d*e^4*(d + e*x^2)^4) + ((7*a^2 - (25*c^2*d^4)/e^4 - (18*a*c*d^2)/e^2)*x)/(48*d^2*(d + e*x^2)^3) + ((35*a^2 + (163*c^2*d^4)/e^4 + (6*a*c*d^2)/e^2)*x)/(192*d^3*(d + e*x^2)^2) - ((93*c^2*d^4 - 6*a*c*d^2*e^2 - 35*a^2*e^4)*x)/(128*d^4*e^4*(d + e*x^2)) + ((35*c^2*d^4 + 6*a*c*d^2*e^2 + 35*a^2*e^4)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(128*d^(9/2)*e^(9/2))$

Rubi [A] time = 0.338634, antiderivative size = 223, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {1158, 1814, 1157, 385, 205}

$$-\frac{x(-35a^2e^4 - 6acd^2e^2 + 93c^2d^4)}{128d^4e^4(d+ex^2)} + \frac{x\left(35a^2 + \frac{6acd^2}{e^2} + \frac{163c^2d^4}{e^4}\right)}{192d^3(d+ex^2)^2} + \frac{x\left(7a^2 - \frac{18acd^2}{e^2} - \frac{25c^2d^4}{e^4}\right)}{48d^2(d+ex^2)^3} + \frac{(35a^2e^4 + 6acd^2e^2 + 35c^2d^4)}{128d^{9/2}e^{9/2}}$$

Antiderivative was successfully verified.

[In] Int[(a + c*x^4)^2/(d + e*x^2)^5, x]

[Out] $((c*d^2 + a*e^2)^2*x)/(8*d*e^4*(d + e*x^2)^4) + ((7*a^2 - (25*c^2*d^4)/e^4 - (18*a*c*d^2)/e^2)*x)/(48*d^2*(d + e*x^2)^3) + ((35*a^2 + (163*c^2*d^4)/e^4 + (6*a*c*d^2)/e^2)*x)/(192*d^3*(d + e*x^2)^2) - ((93*c^2*d^4 - 6*a*c*d^2*e^2 - 35*a^2*e^4)*x)/(128*d^4*e^4*(d + e*x^2)) + ((35*c^2*d^4 + 6*a*c*d^2*e^2 + 35*a^2*e^4)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(128*d^(9/2)*e^(9/2))$

Rule 1158

Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] := With[{Qx = PolynomialQuotient[(a + c*x^4)^p, d + e*x^2, x], R = Coeff[PolynomialRemainder[(a + c*x^4)^p, d + e*x^2, x], x, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x], x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]

Rule 1814

Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] := With[{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]

Rule 1157

Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2


```
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
p, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{(a + cx^4)^2}{(d + ex^2)^5} dx = \frac{(cd^2 + ae^2)^2 x}{8de^4 (d + ex^2)^4} - \frac{\int \frac{-7a^2 + \frac{c^2 d^4}{e^4} + \frac{2acd^2}{e^2} - \frac{8cd(cd^2 + 2ae^2)x^2}{e^3} + \frac{8c^2 d^2 x^4}{e^2} - \frac{8c^2 dx^6}{e}}{(d + ex^2)^4} dx}{8d}$$

$$= \frac{(cd^2 + ae^2)^2 x}{8de^4 (d + ex^2)^4} + \frac{\left(7a^2 - \frac{25c^2 d^4}{e^4} - \frac{18acd^2}{e^2}\right) x}{48d^2 (d + ex^2)^3} + \frac{\int \frac{35a^2 + \frac{19c^2 d^4}{e^4} + \frac{6acd^2}{e^2} - \frac{96c^2 d^3 x^2}{e^3} + \frac{48c^2 d^2 x^4}{e^2}}{(d + ex^2)^3} dx}{48d^2}$$

$$= \frac{(cd^2 + ae^2)^2 x}{8de^4 (d + ex^2)^4} + \frac{\left(7a^2 - \frac{25c^2 d^4}{e^4} - \frac{18acd^2}{e^2}\right) x}{48d^2 (d + ex^2)^3} + \frac{\left(35a^2 + \frac{163c^2 d^4}{e^4} + \frac{6acd^2}{e^2}\right) x}{192d^3 (d + ex^2)^2} - \frac{\int \frac{-3\left(35a^2 - \frac{29c^2 d^4}{e^4} + \frac{6acd^2}{e^2}\right) - 19}{(d + ex^2)^2} dx}{192d^3}$$

$$= \frac{(cd^2 + ae^2)^2 x}{8de^4 (d + ex^2)^4} + \frac{\left(7a^2 - \frac{25c^2 d^4}{e^4} - \frac{18acd^2}{e^2}\right) x}{48d^2 (d + ex^2)^3} + \frac{\left(35a^2 + \frac{163c^2 d^4}{e^4} + \frac{6acd^2}{e^2}\right) x}{192d^3 (d + ex^2)^2} - \frac{(93c^2 d^4 - 6acd^2 e^2 - 35a^2 e^4)}{128d^4 e^4 (d + ex^2)}$$

$$= \frac{(cd^2 + ae^2)^2 x}{8de^4 (d + ex^2)^4} + \frac{\left(7a^2 - \frac{25c^2 d^4}{e^4} - \frac{18acd^2}{e^2}\right) x}{48d^2 (d + ex^2)^3} + \frac{\left(35a^2 + \frac{163c^2 d^4}{e^4} + \frac{6acd^2}{e^2}\right) x}{192d^3 (d + ex^2)^2} - \frac{(93c^2 d^4 - 6acd^2 e^2 - 35a^2 e^4)}{128d^4 e^4 (d + ex^2)}$$

Mathematica [A] time = 0.190922, size = 200, normalized size = 0.9

$$\frac{\sqrt{d}\sqrt{e}(a^2 e^4 (511d^2 ex^2 + 279d^3 + 385de^2 x^4 + 105e^3 x^6) - 6acd^2 e^2 (11d^2 ex^2 + 3d^3 - 11de^2 x^4 - 3e^3 x^6) - c^2 d^4 (385d^2 ex^2 + 105d^3 + 511de^2 x^4 + 279e^3 x^6))}{(d + ex^2)^4} + 3(35a^2 e^4 - 384d^9/2e^{9/2})$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^4)^2/(d + e*x^2)^5,x]
```

```
[Out] ((Sqrt[d]*Sqrt[e]*x*(-6*a*c*d^2*e^2*(3*d^3 + 11*d^2*e*x^2 - 11*d*e^2*x^4 -
3*e^3*x^6) + a^2*e^4*(279*d^3 + 511*d^2*e*x^2 + 385*d*e^2*x^4 + 105*e^3*x^6
) - c^2*d^4*(105*d^3 + 385*d^2*e*x^2 + 511*d*e^2*x^4 + 279*e^3*x^6)))/(d +
e*x^2)^4 + 3*(35*c^2*d^4 + 6*a*c*d^2*e^2 + 35*a^2*e^4)*ArcTan[(Sqrt[e]*x)/S
```

$\text{qrt}[d]]/(384*d^{(9/2)*e^{(9/2)})}$

Maple [A] time = 0.056, size = 231, normalized size = 1.

$$\frac{1}{(ex^2 + d)^4} \left(\frac{(35a^2e^4 + 6acd^2e^2 - 93c^2d^4)x^7}{128d^4e} + \frac{(385a^2e^4 + 66acd^2e^2 - 511c^2d^4)x^5}{384d^3e^2} + \frac{(511a^2e^4 - 66acd^2e^2 - 385c^2d^4)}{384d^2e^3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c*x^4+a)^2/(e*x^2+d)^5,x)`

[Out] $(1/128*(35*a^2*e^4+6*a*c*d^2*e^2-93*c^2*d^4)/d^4/e*x^7+1/384*(385*a^2*e^4+6*6*a*c*d^2*e^2-511*c^2*d^4)/d^3/e^2*x^5+1/384*(511*a^2*e^4-66*a*c*d^2*e^2-385*c^2*d^4)/d^2/e^3*x^3+1/128*(93*a^2*e^4-6*a*c*d^2*e^2-35*c^2*d^4)/d/e^4*x)/(e*x^2+d)^4+35/128/d^4/(d*e)^{(1/2)*\arctan(e*x/(d*e)^{(1/2)})}*a^2+3/64/d^2/e^2/(d*e)^{(1/2)*\arctan(e*x/(d*e)^{(1/2)})}*a*c+35/128/e^4/(d*e)^{(1/2)*\arctan(e*x/(d*e)^{(1/2)})}*c^2$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x^4+a)^2/(e*x^2+d)^5,x, algorithm="maxima")`

[Out] Exception raised: ValueError

Fricas [A] time = 1.93905, size = 1697, normalized size = 7.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x^4+a)^2/(e*x^2+d)^5,x, algorithm="fricas")`

[Out] $[-1/768*(6*(93*c^2*d^5*e^4 - 6*a*c*d^3*e^6 - 35*a^2*d*e^8)*x^7 + 2*(511*c^2*d^6*e^3 - 66*a*c*d^4*e^5 - 385*a^2*d^2*e^7)*x^5 + 2*(385*c^2*d^7*e^2 + 66*a*c*d^5*e^4 - 511*a^2*d^3*e^6)*x^3 + 3*(35*c^2*d^8 + 6*a*c*d^6*e^2 + 35*a^2*d^4*e^4 + (35*c^2*d^4*e^4 + 6*a*c*d^2*e^6 + 35*a^2*e^8)*x^8 + 4*(35*c^2*d^5*e^3 + 6*a*c*d^3*e^5 + 35*a^2*d*e^7)*x^6 + 6*(35*c^2*d^6*e^2 + 6*a*c*d^4*e^4 + 35*a^2*d^2*e^6)*x^4 + 4*(35*c^2*d^7*e + 6*a*c*d^5*e^3 + 35*a^2*d^3*e^5)*x^2)*\sqrt{-d*e}*\log((e*x^2 - 2*\sqrt{-d*e}*x - d)/(e*x^2 + d)) + 6*(35*c^2*d^8*e + 6*a*c*d^6*e^3 - 93*a^2*d^4*e^5)*x)/(d^5*e^9*x^8 + 4*d^6*e^8*x^6 + 6*d^7*e^7*x^4 + 4*d^8*e^6*x^2 + d^9*e^5), -1/384*(3*(93*c^2*d^5*e^4 - 6*a*c*d^3*e^6 - 35*a^2*d*e^8)*x^7 + (511*c^2*d^6*e^3 - 66*a*c*d^4*e^5 - 385*a^2*d^2*e^7)*x^5 + (385*c^2*d^7*e^2 + 66*a*c*d^5*e^4 - 511*a^2*d^3*e^6)*x^3 - 3*(35*c^2*d^8 + 6*a*c*d^6*e^2 + 35*a^2*d^4*e^4 + (35*c^2*d^4*e^4 + 6*a*c*d^2*e^6 + 35*a^2*e^8)*x^8 + 4*(35*c^2*d^5*e^3 + 6*a*c*d^3*e^5 + 35*a^2*d*e^7)*x^6 + 6*(35*c^2*d^6*e^2 + 6*a*c*d^4*e^4 + 35*a^2*d^2*e^6)*x^4 + 4*(35*c^2*d^7*e + 6*a*c*d^5*e^3 + 35*a^2*d^3*e^5)*x^2)*\sqrt{d*e}*\arctan(\sqrt{d*e}*x/d)$

$$3.137 \quad \int \frac{(d+ex^2)^4}{a+cx^4} dx$$

Optimal. Leaf size=437

$$\frac{(a^2e^4 - 6acd^2e^2 - 4\sqrt{a}\sqrt{c}de(cd^2 - ae^2) + c^2d^4) \log(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{cx} + \sqrt{a} + \sqrt{cx^2})}{4\sqrt{2}a^{3/4}c^{9/4}} + \frac{(a^2e^4 - 6acd^2e^2 - 4\sqrt{a}\sqrt{c}de(cd^2 - ae^2) + c^2d^4) \log(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{cx} + \sqrt{a} + \sqrt{cx^2})}{4\sqrt{2}a^{3/4}c^{9/4}}$$

```
[Out] (e^2*(6*c*d^2 - a*e^2)*x)/c^2 + (4*d*e^3*x^3)/(3*c) + (e^4*x^5)/(5*c) - ((c^2*d^4 - 6*a*c*d^2*e^2 + a^2*e^4 + 4*Sqrt[a]*Sqrt[c]*d*e*(c*d^2 - a*e^2))*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)]/(2*Sqrt[2]*a^(3/4)*c^(9/4)) + ((c^2*d^4 - 6*a*c*d^2*e^2 + a^2*e^4 + 4*Sqrt[a]*Sqrt[c]*d*e*(c*d^2 - a*e^2))*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)]/(2*Sqrt[2]*a^(3/4)*c^(9/4)) - ((c^2*d^4 - 6*a*c*d^2*e^2 + a^2*e^4 - 4*Sqrt[a]*Sqrt[c]*d*e*(c*d^2 - a*e^2))*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*c^(9/4)) + ((c^2*d^4 - 6*a*c*d^2*e^2 + a^2*e^4 - 4*Sqrt[a]*Sqrt[c]*d*e*(c*d^2 - a*e^2))*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*c^(9/4))
```

Rubi [A] time = 0.453181, antiderivative size = 437, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {1171, 1168, 1162, 617, 204, 1165, 628}

$$\frac{(a^2e^4 - 6acd^2e^2 - 4\sqrt{a}\sqrt{c}de(cd^2 - ae^2) + c^2d^4) \log(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{cx} + \sqrt{a} + \sqrt{cx^2})}{4\sqrt{2}a^{3/4}c^{9/4}} + \frac{(a^2e^4 - 6acd^2e^2 - 4\sqrt{a}\sqrt{c}de(cd^2 - ae^2) + c^2d^4) \log(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{cx} + \sqrt{a} + \sqrt{cx^2})}{4\sqrt{2}a^{3/4}c^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^4/(a + c*x^4), x]
```

```
[Out] (e^2*(6*c*d^2 - a*e^2)*x)/c^2 + (4*d*e^3*x^3)/(3*c) + (e^4*x^5)/(5*c) - ((c^2*d^4 - 6*a*c*d^2*e^2 + a^2*e^4 + 4*Sqrt[a]*Sqrt[c]*d*e*(c*d^2 - a*e^2))*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)]/(2*Sqrt[2]*a^(3/4)*c^(9/4)) + ((c^2*d^4 - 6*a*c*d^2*e^2 + a^2*e^4 + 4*Sqrt[a]*Sqrt[c]*d*e*(c*d^2 - a*e^2))*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)]/(2*Sqrt[2]*a^(3/4)*c^(9/4)) - ((c^2*d^4 - 6*a*c*d^2*e^2 + a^2*e^4 - 4*Sqrt[a]*Sqrt[c]*d*e*(c*d^2 - a*e^2))*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*c^(9/4)) + ((c^2*d^4 - 6*a*c*d^2*e^2 + a^2*e^4 - 4*Sqrt[a]*Sqrt[c]*d*e*(c*d^2 - a*e^2))*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*c^(9/4))
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (c_.)*(x_)^4), x_Symbol] := Int[ExpandIntegrand[(d + e*x^2)^q/(a + c*x^4), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[q]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + Dist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*c)]
```

Rule 1162

Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] & EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]

Rule 617

Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 1165

Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rubi steps

$$\begin{aligned} \int \frac{(d + ex^2)^4}{a + cx^4} dx &= \int \left(\frac{e^2(6cd^2 - ae^2)}{c^2} + \frac{4de^3x^2}{c} + \frac{e^4x^4}{c} + \frac{c^2d^4 - 6acd^2e^2 + a^2e^4 + 4cde(cd^2 - ae^2)x^2}{c^2(a + cx^4)} \right) dx \\ &= \frac{e^2(6cd^2 - ae^2)x}{c^2} + \frac{4de^3x^3}{3c} + \frac{e^4x^5}{5c} + \frac{\int \frac{c^2d^4 - 6acd^2e^2 + a^2e^4 + 4cde(cd^2 - ae^2)x^2}{a + cx^4} dx}{c^2} \\ &= \frac{e^2(6cd^2 - ae^2)x}{c^2} + \frac{4de^3x^3}{3c} + \frac{e^4x^5}{5c} - \frac{\left(4cd^3e - 4ade^3 - \frac{c^2d^4 - 6acd^2e^2 + a^2e^4}{\sqrt{a}\sqrt{c}}\right) \int \frac{\sqrt{a}\sqrt{c} - cx^2}{a + cx^4} dx}{2c^2} + \frac{\left(4cd^3e - 4ade^3 - \frac{c^2d^4 - 6acd^2e^2 + a^2e^4}{\sqrt{a}\sqrt{c}}\right) \int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt{c}} + 2x}{-\frac{\sqrt{a}}{\sqrt{c}} - \frac{\sqrt{2}\sqrt[4]{ax}}{\sqrt{c}} - x^2} dx}{4\sqrt{2}\sqrt[4]{ac}^{7/4}}}{4\sqrt{2}\sqrt[4]{ac}^{7/4}} + \frac{\left(4cd^3e - 4ade^3 - \frac{c^2d^4 - 6acd^2e^2 + a^2e^4}{\sqrt{a}\sqrt{c}}\right) \log\left(\sqrt{a} - \sqrt{2}\sqrt[4]{a}\sqrt[4]{cx} + \sqrt{a + cx^4}\right)}{4\sqrt{2}\sqrt[4]{ac}^{7/4}}}{4\sqrt{2}\sqrt[4]{ac}^{7/4}} \\ &= \frac{e^2(6cd^2 - ae^2)x}{c^2} + \frac{4de^3x^3}{3c} + \frac{e^4x^5}{5c} - \frac{\left(4cd^3e - 4ade^3 + \frac{c^2d^4 - 6acd^2e^2 + a^2e^4}{\sqrt{a}\sqrt{c}}\right) \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{cx}}{\sqrt[4]{a}}\right)}{2\sqrt{2}\sqrt[4]{ac}^{7/4}} + \frac{\left(4cd^3e - 4ade^3 - \frac{c^2d^4 - 6acd^2e^2 + a^2e^4}{\sqrt{a}\sqrt{c}}\right) \log\left(\sqrt{a} - \sqrt{2}\sqrt[4]{a}\sqrt[4]{cx} + \sqrt{a + cx^4}\right)}{4\sqrt{2}\sqrt[4]{ac}^{7/4}} \end{aligned}$$

Mathematica [A] time = 0.341162, size = 444, normalized size = 1.02

$$-15\sqrt{2}\left(4a^{3/2}\sqrt{cde^3} + a^2e^4 - 4\sqrt{ac^3/2}d^3e - 6acd^2e^2 + c^2d^4\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{cx} + \sqrt{a} + \sqrt{cx^2}\right) + 15\sqrt{2}\left(4a^{3/2}\sqrt{cde^3} + a^2e^4 - \right.$$

Antiderivative was successfully verified.

[In] Integrate[(d + e*x^2)^4/(a + c*x^4),x]

[Out] $(-120a^{3/4}c^{1/4}e^2(-6cd^2 + ae^2)x + 160a^{3/4}c^{5/4}d^3e^3x^3 + 24a^{3/4}c^{5/4}e^4x^5 - 30\sqrt{2}(c^2d^4 + 4\sqrt{a}c^{3/2}d^3e - 6ac^2d^2e^2 - 4a^{3/2}\sqrt{c}d^3e + a^2e^4)\text{ArcTan}[1 - (\sqrt{2}c^{1/4}x)/a^{1/4}] + 30\sqrt{2}(c^2d^4 + 4\sqrt{a}c^{3/2}d^3e - 6ac^2d^2e^2 - 4a^{3/2}\sqrt{c}d^3e + a^2e^4)\text{ArcTan}[1 + (\sqrt{2}c^{1/4}x)/a^{1/4}] - 15\sqrt{2}(c^2d^4 - 4\sqrt{a}c^{3/2}d^3e - 6ac^2d^2e^2 + 4a^{3/2}\sqrt{c}d^3e + a^2e^4)\text{Log}[\sqrt{a} - \sqrt{2}a^{1/4}c^{1/4}x + \sqrt{c}x^2] + 15\sqrt{2}(c^2d^4 - 4\sqrt{a}c^{3/2}d^3e - 6ac^2d^2e^2 + 4a^{3/2}\sqrt{c}d^3e + a^2e^4)\text{Log}[\sqrt{a} + \sqrt{2}a^{1/4}c^{1/4}x + \sqrt{c}x^2])/(120a^{3/4}c^{9/4})$

Maple [B] time = 0.048, size = 741, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x^2+d)^4/(c*x^4+a),x)

[Out] $1/5e^4x^5/c + 4/3d^3e^3x^3/c - e^4/c^2ax + 6e^2/cd^2x + 1/4c^2(a/c)^{1/4}a^2^{1/2}\arctan(2^{1/2}/(a/c)^{1/4}x+1)e^4 - 3/2c(a/c)^{1/4}2^{1/2}\arctan(2^{1/2}/(a/c)^{1/4}x+1)d^2e^2 + 1/4(a/c)^{1/4}/a^2^{1/2}\arctan(2^{1/2}/(a/c)^{1/4}x-1)e^4 - 3/2c(a/c)^{1/4}2^{1/2}\arctan(2^{1/2}/(a/c)^{1/4}x-1)d^2e^2 + 1/4(a/c)^{1/4}/a^2^{1/2}\arctan(2^{1/2}/(a/c)^{1/4}x-1)d^4 + 1/8c^2(a/c)^{1/4}a^2^{1/2}\ln((x^2+(a/c)^{1/4}x^2^{1/2}+(a/c)^{1/2}))/((x^2-(a/c)^{1/4}x^2^{1/2}+(a/c)^{1/2}))e^4 - 3/4c(a/c)^{1/4}2^{1/2}\ln((x^2+(a/c)^{1/4}x^2^{1/2}+(a/c)^{1/2}))/((x^2-(a/c)^{1/4}x^2^{1/2}+(a/c)^{1/2}))d^2e^2 + 1/8(a/c)^{1/4}/a^2^{1/2}\ln((x^2+(a/c)^{1/4}x^2^{1/2}+(a/c)^{1/2}))/((x^2-(a/c)^{1/4}x^2^{1/2}+(a/c)^{1/2}))d^4 - 1/2c^2/(a/c)^{1/4}2^{1/2}\ln((x^2-(a/c)^{1/4}x^2^{1/2}+(a/c)^{1/2}))/((x^2+(a/c)^{1/4}x^2^{1/2}+(a/c)^{1/2}))a^2d^3e^3 + 1/2c/(a/c)^{1/4}2^{1/2}\ln((x^2-(a/c)^{1/4}x^2^{1/2}+(a/c)^{1/2}))/((x^2+(a/c)^{1/4}x^2^{1/2}+(a/c)^{1/2}))d^3e - 1/c^2/(a/c)^{1/4}2^{1/2}\arctan(2^{1/2}/(a/c)^{1/4}x+1)a^2d^3e^3 + 1/c/(a/c)^{1/4}2^{1/2}\arctan(2^{1/2}/(a/c)^{1/4}x+1)d^3e - 1/c^2/(a/c)^{1/4}2^{1/2}\arctan(2^{1/2}/(a/c)^{1/4}x-1)a^2d^3e^3 + 1/c/(a/c)^{1/4}2^{1/2}\arctan(2^{1/2}/(a/c)^{1/4}x-1)d^3e$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)^4/(c*x^4+a),x, algorithm="maxima")

$$d^{16} - 56a^7c^7d^{14}e^2 + 924a^2c^6d^{12}e^4 - 3976a^3c^5d^{10}e^6 + 6470a^4c^4d^8e^8 - 3976a^5c^3d^6e^{10} + 924a^6c^2d^4e^{12} - 56a^7c^1d^2e^{14} + a^8e^{16}) / (a^3c^9)) / (a^4) * \log((c^8d^{16} - 24a^7c^7d^{14}e^2 - 36a^2c^6d^{12}e^4 + 88a^3c^5d^{10}e^6 + 198a^4c^4d^8e^8 + 88a^5c^3d^6e^{10} - 36a^6c^2d^4e^{12} - 24a^7c^1d^2e^{14} + a^8e^{16}) * x - (a^8c^8d^{12} - 34a^2c^7d^{10}e^2 + 239a^3c^6d^8e^4 - 476a^4c^5d^6e^6 + 239a^5c^4d^4e^8 - 34a^6c^3d^2e^{10} + a^7c^2e^{12} - 4(a^3c^8d^3e - a^4c^7d^2e^3) * \sqrt{-(c^8d^{16} - 56a^7c^7d^{14}e^2 + 924a^2c^6d^{12}e^4 - 3976a^3c^5d^{10}e^6 + 6470a^4c^4d^8e^8 - 3976a^5c^3d^6e^{10} + 924a^6c^2d^4e^{12} - 56a^7c^1d^2e^{14} + a^8e^{16}) / (a^3c^9))} * \sqrt{-(8c^3d^7e - 56a^2c^2d^5e^3 + 56a^2c^2d^3e^5 - 8a^3d^2e^7 - a^4c^4 * \sqrt{-(c^8d^{16} - 56a^7c^7d^{14}e^2 + 924a^2c^6d^{12}e^4 - 3976a^3c^5d^{10}e^6 + 6470a^4c^4d^8e^8 - 3976a^5c^3d^6e^{10} + 924a^6c^2d^4e^{12} - 56a^7c^1d^2e^{14} + a^8e^{16}) / (a^3c^9))} / (a^4))) + 60(6c^2d^2e^2 - a^4e^4) * x) / c^2$$

Sympy [A] time = 4.09658, size = 500, normalized size = 1.14

$$\text{RootSum}\left(256t^4a^3c^9 + t^2(-256a^5c^5de^7 + 1792a^4c^6d^3e^5 - 1792a^3c^7d^5e^3 + 256a^2c^8d^7e) + a^8e^{16} + 8a^7cd^2e^{14} + 28a^6c^2d^4e^{12} + \dots\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**4/(c*x**4+a),x)
```

```
[Out] RootSum(256*_t**4*a**3*c**9 + _t**2*(-256*a**5*c**5*d*e**7 + 1792*a**4*c**6*d**3*e**5 - 1792*a**3*c**7*d**5*e**3 + 256*a**2*c**8*d**7*e) + a**8*e**16 + 8*a**7*c*d**2*e**14 + 28*a**6*c**2*d**4*e**12 + 56*a**5*c**3*d**6*e**10 + 70*a**4*c**4*d**8*e**8 + 56*a**3*c**5*d**10*e**6 + 28*a**2*c**6*d**12*e**4 + 8*a*c**7*d**14*e**2 + c**8*d**16, Lambda(_t, _t*log(x + (256*_t**3*a**4*c**7*d*e**3 - 256*_t**3*a**3*c**8*d**3*e + 4*_t*a**7*c**2*e**12 - 264*_t*a**6*c**3*d**2*e**10 + 1980*_t*a**5*c**4*d**4*e**8 - 3696*_t*a**4*c**5*d**6*e**6 + 1980*_t*a**3*c**6*d**8*e**4 - 264*_t*a**2*c**7*d**10*e**2 + 4*_t*a*c**8*d**12)/(a**8*e**16 - 24*a**7*c*d**2*e**14 - 36*a**6*c**2*d**4*e**12 + 88*a**5*c**3*d**6*e**10 + 198*a**4*c**4*d**8*e**8 + 88*a**3*c**5*d**10*e**6 - 36*a**2*c**6*d**12*e**4 - 24*a*c**7*d**14*e**2 + c**8*d**16)))) + 4*d*e**3*x**3/(3*c) + e**4*x**5/(5*c) - x*(a*e**4 - 6*c*d**2*e**2)/c**2
```

Giac [A] time = 1.16816, size = 672, normalized size = 1.54

$$\frac{\sqrt{2}\left((ac^3)^{\frac{1}{4}}c^3d^4 - 6(ac^3)^{\frac{1}{4}}ac^2d^2e^2 + 4(ac^3)^{\frac{3}{4}}cd^3e + (ac^3)^{\frac{1}{4}}a^2ce^4 - 4(ac^3)^{\frac{3}{4}}ade^3\right) \arctan\left(\frac{\sqrt{2}\left(2x + \sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right) + \sqrt{2}\left((ac^3)^{\frac{1}{4}}c^3d^4 - 6(ac^3)^{\frac{1}{4}}ac^2d^2e^2 + 4(ac^3)^{\frac{3}{4}}cd^3e + (ac^3)^{\frac{1}{4}}a^2ce^4 - 4(ac^3)^{\frac{3}{4}}ade^3\right)}{4ac^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^4/(c*x^4+a),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*((a*c^3)^(1/4)*c^3*d^4 - 6*(a*c^3)^(1/4)*a*c^2*d^2*e^2 + 4*(a*c^3)^(3/4)*c*d^3*e + (a*c^3)^(1/4)*a^2*c*e^4 - 4*(a*c^3)^(3/4)*a*d*e^3)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a*c^4) + 1/4*sqrt(2)*((a*c^3)^(1/4)*c^3*d^4 - 6*(a*c^3)^(1/4)*a*c^2*d^2*e^2 + 4*(a*c^3)^(3/4)*c*d^3*e + (a*c^3)^(1/4)*a^2*c*e^4 - 4*(a*c^3)^(3/4)*a*d*e^3)*arctan(1/2*sq
```


$$\begin{aligned} & \text{rt}(2) * (2 * x - \text{sqrt}(2) * (a/c)^{(1/4)}) / (a/c)^{(1/4)} / (a * c^4) + 1/8 * \text{sqrt}(2) * ((a * c^3)^{(1/4)} * c^3 * d^4 - 6 * (a * c^3)^{(1/4)} * a * c^2 * d^2 * e^2 - 4 * (a * c^3)^{(3/4)} * c * d^3 * e \\ & + (a * c^3)^{(1/4)} * a^2 * c * e^4 + 4 * (a * c^3)^{(3/4)} * a * d * e^3) * \log(x^2 + \text{sqrt}(2) * x * (a/c)^{(1/4)} + \text{sqrt}(a/c)) / (a * c^4) - 1/8 * \text{sqrt}(2) * ((a * c^3)^{(1/4)} * c^3 * d^4 - 6 * (a * c^3)^{(1/4)} * a * c^2 * d^2 * e^2 - 4 * (a * c^3)^{(3/4)} * c * d^3 * e + (a * c^3)^{(1/4)} * a^2 * c * e^4 \\ & + 4 * (a * c^3)^{(3/4)} * a * d * e^3) * \log(x^2 - \text{sqrt}(2) * x * (a/c)^{(1/4)} + \text{sqrt}(a/c)) / (a * c^4) + 1/15 * (3 * c^4 * x^5 * e^4 + 20 * c^4 * d * x^3 * e^3 + 90 * c^4 * d^2 * x * e^2 - 15 * a * c^3 * x * e^4) / c^5 \end{aligned}$$

$$3.138 \quad \int \frac{(d+ex^2)^3}{a+cx^4} dx$$

Optimal. Leaf size=370

$$-\frac{(\sqrt{cd}(cd^2 - 3ae^2) - \sqrt{ae}(3cd^2 - ae^2)) \log(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{cx} + \sqrt{a} + \sqrt{cx^2})}{4\sqrt{2}a^{3/4}c^{7/4}} + \frac{(\sqrt{cd}(cd^2 - 3ae^2) - \sqrt{ae}(3cd^2 - ae^2)) \log(\sqrt{2}\sqrt[4]{a}\sqrt[4]{cx} - \sqrt{a} + \sqrt{cx^2})}{4\sqrt{2}a^{3/4}c^{7/4}}$$

```
[Out] (3*d*e^2*x)/c + (e^3*x^3)/(3*c) - ((Sqrt[c]*d*(c*d^2 - 3*a*e^2) + Sqrt[a]*e*(3*c*d^2 - a*e^2))*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)]/(2*Sqrt[2]*a^(3/4)*c^(7/4)) + ((Sqrt[c]*d*(c*d^2 - 3*a*e^2) + Sqrt[a]*e*(3*c*d^2 - a*e^2))*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)]/(2*Sqrt[2]*a^(3/4)*c^(7/4)) - ((Sqrt[c]*d*(c*d^2 - 3*a*e^2) - Sqrt[a]*e*(3*c*d^2 - a*e^2))*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*c^(7/4)) + ((Sqrt[c]*d*(c*d^2 - 3*a*e^2) - Sqrt[a]*e*(3*c*d^2 - a*e^2))*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*c^(7/4))
```

Rubi [A] time = 0.501238, antiderivative size = 370, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {1171, 1168, 1162, 617, 204, 1165, 628}

$$-\frac{(\sqrt{cd}(cd^2 - 3ae^2) - \sqrt{ae}(3cd^2 - ae^2)) \log(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{cx} + \sqrt{a} + \sqrt{cx^2})}{4\sqrt{2}a^{3/4}c^{7/4}} + \frac{(\sqrt{cd}(cd^2 - 3ae^2) - \sqrt{ae}(3cd^2 - ae^2)) \log(\sqrt{2}\sqrt[4]{a}\sqrt[4]{cx} - \sqrt{a} + \sqrt{cx^2})}{4\sqrt{2}a^{3/4}c^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^3/(a + c*x^4), x]
```

```
[Out] (3*d*e^2*x)/c + (e^3*x^3)/(3*c) - ((Sqrt[c]*d*(c*d^2 - 3*a*e^2) + Sqrt[a]*e*(3*c*d^2 - a*e^2))*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)]/(2*Sqrt[2]*a^(3/4)*c^(7/4)) + ((Sqrt[c]*d*(c*d^2 - 3*a*e^2) + Sqrt[a]*e*(3*c*d^2 - a*e^2))*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)]/(2*Sqrt[2]*a^(3/4)*c^(7/4)) - ((Sqrt[c]*d*(c*d^2 - 3*a*e^2) - Sqrt[a]*e*(3*c*d^2 - a*e^2))*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*c^(7/4)) + ((Sqrt[c]*d*(c*d^2 - 3*a*e^2) - Sqrt[a]*e*(3*c*d^2 - a*e^2))*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*c^(7/4))
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^q/((a_) + (c_.)*(x_)^4), x_Symbol] := Int[ExpandIntegrand[(d + e*x^2)^q/(a + c*x^4), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[q]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + Dist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*c)]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
```

$/(2*c), \text{Int}[1/\text{Simp}[d/e - q*x + x^2, x], x], x] /; \text{FreeQ}\{a, c, d, e\}, x\} \& \& \text{EqQ}[c*d^2 - a*e^2, 0] \& \& \text{PosQ}[d*e]$

Rule 617

$\text{Int}[(a_ + (b_)*(x_) + (c_)*(x_)^2)^{-1}, x_Symbol] \rightarrow \text{With}\{q = 1 - 4*S \text{implify}[(a*c)/b^2]\}, \text{Dist}[-2/b, \text{Subst}[\text{Int}[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; \text{RationalQ}[q] \& \& (\text{EqQ}[q^2, 1] \parallel \text{!RationalQ}[b^2 - 4*a*c]) /; \text{FreeQ}\{a, b, c\}, x\} \& \& \text{NeQ}[b^2 - 4*a*c, 0]$

Rule 204

$\text{Int}[(a_ + (b_)*(x_)^2)^{-1}, x_Symbol] \rightarrow -\text{Simp}[\text{ArcTan}[\text{Rt}[-b, 2]*x]/\text{Rt}[-a, 2]]/\text{Rt}[-a, 2]*\text{Rt}[-b, 2], x] /; \text{FreeQ}\{a, b\}, x\} \& \& \text{PosQ}[a/b] \& \& (\text{LtQ}[a, 0] \parallel \text{LtQ}[b, 0])$

Rule 1165

$\text{Int}[(d_ + (e_)*(x_)^2)/(a_ + (c_)*(x_)^4), x_Symbol] \rightarrow \text{With}\{q = \text{Rt}[-2*d/e, 2]\}, \text{Dist}[e/(2*c*q), \text{Int}[(q - 2*x)/\text{Simp}[d/e + q*x - x^2, x], x], x] + \text{Dist}[e/(2*c*q), \text{Int}[(q + 2*x)/\text{Simp}[d/e - q*x - x^2, x], x], x] /; \text{FreeQ}\{a, c, d, e\}, x\} \& \& \text{EqQ}[c*d^2 - a*e^2, 0] \& \& \text{NegQ}[d*e]$

Rule 628

$\text{Int}[(d_ + (e_)*(x_))/(a_ + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] \rightarrow \text{Simp}[(d*\text{Log}[\text{RemoveContent}[a + b*x + c*x^2, x]])/b, x] /; \text{FreeQ}\{a, b, c, d, e\}, x\} \& \& \text{EqQ}[2*c*d - b*e, 0]$

Rubi steps

$$\begin{aligned} \int \frac{(d + ex^2)^3}{a + cx^4} dx &= \int \left(\frac{3de^2}{c} + \frac{e^3x^2}{c} + \frac{cd^3 - 3ade^2 + e(3cd^2 - ae^2)x^2}{c(a + cx^4)} \right) dx \\ &= \frac{3de^2x}{c} + \frac{e^3x^3}{3c} + \frac{\int \frac{cd^3 - 3ade^2 + e(3cd^2 - ae^2)x^2}{a + cx^4} dx}{c} \\ &= \frac{3de^2x}{c} + \frac{e^3x^3}{3c} - \frac{\left(3cd^2e - ae^3 - \frac{\sqrt{cd}(cd^2 - 3ae^2)}{\sqrt{a}}\right) \int \frac{\sqrt{a}\sqrt{c} - cx^2}{a + cx^4} dx}{2c^2} + \frac{\left(3cd^2e - ae^3 + \frac{\sqrt{cd}(cd^2 - 3ae^2)}{\sqrt{a}}\right) \int \frac{\sqrt{a}\sqrt{c} + cx^2}{a + cx^4} dx}{2c^2} \\ &= \frac{3de^2x}{c} + \frac{e^3x^3}{3c} + \frac{\left(3cd^2e - ae^3 - \frac{\sqrt{cd}(cd^2 - 3ae^2)}{\sqrt{a}}\right) \int \frac{\frac{\sqrt{2}\sqrt[4]{a} + 2x}{\sqrt[4]{c}}}{-\frac{\sqrt{a}}{\sqrt{c}} - \frac{\sqrt{2}\sqrt[4]{ax}}{\sqrt[4]{c}} - x^2} dx}{4\sqrt{2}\sqrt[4]{ac}^{7/4}} + \frac{\left(3cd^2e - ae^3 - \frac{\sqrt{cd}(cd^2 - 3ae^2)}{\sqrt{a}}\right) \int \frac{\sqrt{a}\sqrt{c} - cx^2}{a + cx^4} dx}{4\sqrt{2}\sqrt[4]{ac}^{7/4}} \\ &= \frac{3de^2x}{c} + \frac{e^3x^3}{3c} + \frac{\left(3cd^2e - ae^3 - \frac{\sqrt{cd}(cd^2 - 3ae^2)}{\sqrt{a}}\right) \log(\sqrt{a} - \sqrt{2}\sqrt[4]{a}\sqrt[4]{cx} + \sqrt{cx^2})}{4\sqrt{2}\sqrt[4]{ac}^{7/4}} - \frac{\left(3cd^2e - ae^3 - \frac{\sqrt{cd}(cd^2 - 3ae^2)}{\sqrt{a}}\right) \log(\sqrt{a} + \sqrt{2}\sqrt[4]{a}\sqrt[4]{cx} + \sqrt{cx^2})}{4\sqrt{2}\sqrt[4]{ac}^{7/4}} \\ &= \frac{3de^2x}{c} + \frac{e^3x^3}{3c} - \frac{\left(3cd^2e - ae^3 + \frac{\sqrt{cd}(cd^2 - 3ae^2)}{\sqrt{a}}\right) \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{cx}}{\sqrt[4]{a}}\right)}{2\sqrt{2}\sqrt[4]{ac}^{7/4}} + \frac{\left(3cd^2e - ae^3 + \frac{\sqrt{cd}(cd^2 - 3ae^2)}{\sqrt{a}}\right) \tan^{-1}\left(1 + \frac{\sqrt{2}\sqrt[4]{cx}}{\sqrt[4]{a}}\right)}{2\sqrt{2}\sqrt[4]{ac}^{7/4}} \end{aligned}$$

Mathematica [A] time = 0.280898, size = 360, normalized size = 0.97

$$-3\sqrt{2} \left(a^{3/2}e^3 - 3\sqrt{acd^2}e - 3a\sqrt{cde^2} + c^{3/2}d^3 \right) \log \left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{cx} + \sqrt{a} + \sqrt{cx^2} \right) + 3\sqrt{2} \left(a^{3/2}e^3 - 3\sqrt{acd^2}e - 3a\sqrt{cde^2} + c^{3/2}d^3 \right) \log \left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{cx} + \sqrt{a} + \sqrt{cx^2} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(d + e*x^2)^3/(a + c*x^4),x]

[Out] $(72*a^{3/4}*c^{3/4}*d*e^2*x + 8*a^{3/4}*c^{3/4}*e^3*x^3 + 6*\sqrt{2}*(-(c^{3/2}*d^3) - 3*\sqrt{a}*c*d^2*e + 3*a*\sqrt{c}*d*e^2 + a^{3/2}*e^3)*\text{ArcTan}[1 - (\sqrt{2}*c^{1/4}*x)/a^{1/4}] + 6*\sqrt{2}*(c^{3/2}*d^3 + 3*\sqrt{a}*c*d^2*e - 3*a*\sqrt{c}*d*e^2 - a^{3/2}*e^3)*\text{ArcTan}[1 + (\sqrt{2}*c^{1/4}*x)/a^{1/4}] - 3*\sqrt{2}*(c^{3/2}*d^3 - 3*\sqrt{a}*c*d^2*e - 3*a*\sqrt{c}*d*e^2 + a^{3/2}*e^3)*\text{Log}[\sqrt{a} - \sqrt{2}*a^{1/4}*c^{1/4}*x + \sqrt{c}*x^2] + 3*\sqrt{2}*(c^{3/2}*d^3 - 3*\sqrt{a}*c*d^2*e - 3*a*\sqrt{c}*d*e^2 + a^{3/2}*e^3)*\text{Log}[\sqrt{a} + \sqrt{2}*a^{1/4}*c^{1/4}*x + \sqrt{c}*x^2])/(24*a^{3/4}*c^{7/4})$

Maple [A] time = 0.049, size = 572, normalized size = 1.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x^2+d)^3/(c*x^4+a),x)

[Out] $1/3*e^3*x^3/c+3*d*e^2*x/c-3/4/c*(a/c)^{1/4}*2^{1/2}*\arctan(2^{1/2}/(a/c)^{1/4}*x-1)*d*e^2+1/4*(a/c)^{1/4}/a*2^{1/2}*\arctan(2^{1/2}/(a/c)^{1/4}*x-1)*d^3-3/8/c*(a/c)^{1/4}*2^{1/2}*\ln((x^2+(a/c)^{1/4}*x*2^{1/2}+(a/c)^{1/2}))/((x^2-(a/c)^{1/4}*x*2^{1/2}+(a/c)^{1/2}))*d*e^2+1/8*(a/c)^{1/4}/a*2^{1/2}*\ln((x^2+(a/c)^{1/4}*x*2^{1/2}+(a/c)^{1/2}))/((x^2-(a/c)^{1/4}*x*2^{1/2}+(a/c)^{1/2}))*d^3-3/4/c*(a/c)^{1/4}*2^{1/2}*\arctan(2^{1/2}/(a/c)^{1/4}*x+1)*d*e^2+1/4*(a/c)^{1/4}/a*2^{1/2}*\arctan(2^{1/2}/(a/c)^{1/4}*x+1)*d^3-1/8/c^2/(a/c)^{1/4}*2^{1/2}*\ln((x^2-(a/c)^{1/4}*x*2^{1/2}+(a/c)^{1/2}))/((x^2+(a/c)^{1/4}*x*2^{1/2}+(a/c)^{1/2}))*a*e^3+3/8/c/(a/c)^{1/4}*2^{1/2}*\ln((x^2-(a/c)^{1/4}*x*2^{1/2}+(a/c)^{1/2}))/((x^2+(a/c)^{1/4}*x*2^{1/2}+(a/c)^{1/2}))*d^2*e-1/4/c^2/(a/c)^{1/4}*2^{1/2}*\arctan(2^{1/2}/(a/c)^{1/4}*x-1)*a*e^3+3/4/c/(a/c)^{1/4}*2^{1/2}*\arctan(2^{1/2}/(a/c)^{1/4}*x-1)*d^2*e-1/4/c^2/(a/c)^{1/4}*2^{1/2}*\arctan(2^{1/2}/(a/c)^{1/4}*x+1)*a*e^3+3/4/c/(a/c)^{1/4}*2^{1/2}*\arctan(2^{1/2}/(a/c)^{1/4}*x+1)*d^2*e$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)^3/(c*x^4+a),x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [B] time = 8.33023, size = 4462, normalized size = 12.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)^3/(c*x^4+a),x, algorithm="fricas")

[Out] $\frac{1}{12} \cdot (4e^3x^3 + 36d^2e^2x - 3c\sqrt{-(6c^2d^5e - 20a^2cd^3e^3 + 6a^2d^5e + a^3c^3\sqrt{-(c^6d^{12} - 30a^2c^5d^{10}e^2 + 255a^2c^4d^8e^4 - 452a^3c^3d^6e^6 + 255a^4c^2d^4e^8 - 30a^5cd^2e^{10} + a^6e^{12})})/(a^3c^7)))/(a^3c^3)) \cdot \log(-(c^6d^{12} - 12a^2c^5d^{10}e^2 - 27a^2c^4d^8e^4 + 27a^4c^2d^4e^8 + 12a^5cd^2e^{10} - a^6e^{12}))x + (a^6d^9 - 18a^2c^5d^7e^2 + 60a^3c^4d^5e^4 - 46a^4c^3d^3e^6 + 3a^5c^2d^2e^8 + (3a^3c^6d^2e - a^4c^5e^3)\sqrt{-(c^6d^{12} - 30a^2c^5d^{10}e^2 + 255a^2c^4d^8e^4 - 452a^3c^3d^6e^6 + 255a^4c^2d^4e^8 - 30a^5cd^2e^{10} + a^6e^{12})})/(a^3c^7)))/\sqrt{-(6c^2d^5e - 20a^2cd^3e^3 + 6a^2d^5e + a^3c^3\sqrt{-(c^6d^{12} - 30a^2c^5d^{10}e^2 + 255a^2c^4d^8e^4 - 452a^3c^3d^6e^6 + 255a^4c^2d^4e^8 - 30a^5cd^2e^{10} + a^6e^{12})})/(a^3c^7)))/(a^3c^3)) + 3c\sqrt{-(6c^2d^5e - 20a^2cd^3e^3 + 6a^2d^5e + a^3c^3\sqrt{-(c^6d^{12} - 30a^2c^5d^{10}e^2 + 255a^2c^4d^8e^4 - 452a^3c^3d^6e^6 + 255a^4c^2d^4e^8 - 30a^5cd^2e^{10} + a^6e^{12})})/(a^3c^7)))/(a^3c^3)) \cdot \log(-(c^6d^{12} - 12a^2c^5d^{10}e^2 - 27a^2c^4d^8e^4 + 27a^4c^2d^4e^8 + 12a^5cd^2e^{10} - a^6e^{12}))x - (a^6d^9 - 18a^2c^5d^7e^2 + 60a^3c^4d^5e^4 - 46a^4c^3d^3e^6 + 3a^5c^2d^2e^8 + (3a^3c^6d^2e - a^4c^5e^3)\sqrt{-(c^6d^{12} - 30a^2c^5d^{10}e^2 + 255a^2c^4d^8e^4 - 452a^3c^3d^6e^6 + 255a^4c^2d^4e^8 - 30a^5cd^2e^{10} + a^6e^{12})})/(a^3c^7)))/\sqrt{-(6c^2d^5e - 20a^2cd^3e^3 + 6a^2d^5e + a^3c^3\sqrt{-(c^6d^{12} - 30a^2c^5d^{10}e^2 + 255a^2c^4d^8e^4 - 452a^3c^3d^6e^6 + 255a^4c^2d^4e^8 - 30a^5cd^2e^{10} + a^6e^{12})})/(a^3c^7)))/(a^3c^3)) - 3c\sqrt{-(6c^2d^5e - 20a^2cd^3e^3 + 6a^2d^5e - a^3c^3\sqrt{-(c^6d^{12} - 30a^2c^5d^{10}e^2 + 255a^2c^4d^8e^4 - 452a^3c^3d^6e^6 + 255a^4c^2d^4e^8 - 30a^5cd^2e^{10} + a^6e^{12})})/(a^3c^7)))/(a^3c^3)) \cdot \log(-(c^6d^{12} - 12a^2c^5d^{10}e^2 - 27a^2c^4d^8e^4 + 27a^4c^2d^4e^8 + 12a^5cd^2e^{10} - a^6e^{12}))x + (a^6d^9 - 18a^2c^5d^7e^2 + 60a^3c^4d^5e^4 - 46a^4c^3d^3e^6 + 3a^5c^2d^2e^8 - (3a^3c^6d^2e - a^4c^5e^3)\sqrt{-(c^6d^{12} - 30a^2c^5d^{10}e^2 + 255a^2c^4d^8e^4 - 452a^3c^3d^6e^6 + 255a^4c^2d^4e^8 - 30a^5cd^2e^{10} + a^6e^{12})})/(a^3c^7)))/\sqrt{-(6c^2d^5e - 20a^2cd^3e^3 + 6a^2d^5e - a^3c^3\sqrt{-(c^6d^{12} - 30a^2c^5d^{10}e^2 + 255a^2c^4d^8e^4 - 452a^3c^3d^6e^6 + 255a^4c^2d^4e^8 - 30a^5cd^2e^{10} + a^6e^{12})})/(a^3c^7)))/(a^3c^3)) + 3c\sqrt{-(6c^2d^5e - 20a^2cd^3e^3 + 6a^2d^5e - a^3c^3\sqrt{-(c^6d^{12} - 30a^2c^5d^{10}e^2 + 255a^2c^4d^8e^4 - 452a^3c^3d^6e^6 + 255a^4c^2d^4e^8 - 30a^5cd^2e^{10} + a^6e^{12})})/(a^3c^7)))/(a^3c^3)) \cdot \log(-(c^6d^{12} - 12a^2c^5d^{10}e^2 - 27a^2c^4d^8e^4 + 27a^4c^2d^4e^8 + 12a^5cd^2e^{10} - a^6e^{12}))x - (a^6d^9 - 18a^2c^5d^7e^2 + 60a^3c^4d^5e^4 - 46a^4c^3d^3e^6 + 3a^5c^2d^2e^8 - (3a^3c^6d^2e - a^4c^5e^3)\sqrt{-(c^6d^{12} - 30a^2c^5d^{10}e^2 + 255a^2c^4d^8e^4 - 452a^3c^3d^6e^6 + 255a^4c^2d^4e^8 - 30a^5cd^2e^{10} + a^6e^{12})})/(a^3c^7)))/\sqrt{-(6c^2d^5e - 20a^2cd^3e^3 + 6a^2d^5e - a^3c^3\sqrt{-(c^6d^{12} - 30a^2c^5d^{10}e^2 + 255a^2c^4d^8e^4 - 452a^3c^3d^6e^6 + 255a^4c^2d^4e^8 - 30a^5cd^2e^{10} + a^6e^{12})})/(a^3c^7)))/(a^3c^3)) + 3c\sqrt{-(6c^2d^5e - 20a^2cd^3e^3 + 6a^2d^5e - a^3c^3\sqrt{-(c^6d^{12} - 30a^2c^5d^{10}e^2 + 255a^2c^4d^8e^4 - 452a^3c^3d^6e^6 + 255a^4c^2d^4e^8 - 30a^5cd^2e^{10} + a^6e^{12})})/(a^3c^7)))/(a^3c^3)) \cdot \log(-(c^6d^{12} - 12a^2c^5d^{10}e^2 - 27a^2c^4d^8e^4 + 27a^4c^2d^4e^8 + 12a^5cd^2e^{10} - a^6e^{12}))x - (a^6d^9 - 18a^2c^5d^7e^2 + 60a^3c^4d^5e^4 - 46a^4c^3d^3e^6 + 3a^5c^2d^2e^8 - (3a^3c^6d^2e - a^4c^5e^3)\sqrt{-(c^6d^{12} - 30a^2c^5d^{10}e^2 + 255a^2c^4d^8e^4 - 452a^3c^3d^6e^6 + 255a^4c^2d^4e^8 - 30a^5cd^2e^{10} + a^6e^{12})})/(a^3c^7)))/\sqrt{-(6c^2d^5e - 20a^2cd^3e^3 + 6a^2d^5e - a^3c^3\sqrt{-(c^6d^{12} - 30a^2c^5d^{10}e^2 + 255a^2c^4d^8e^4 - 452a^3c^3d^6e^6 + 255a^4c^2d^4e^8 - 30a^5cd^2e^{10} + a^6e^{12})})/(a^3c^7)))/(a^3c^3)))/c$

Sympy [A] time = 2.36805, size = 350, normalized size = 0.95

$$\text{RootSum}\left(256t^4a^3c^7 + t^2(192a^4c^4de^5 - 640a^3c^5d^3e^3 + 192a^2c^6d^5e) + a^6e^{12} + 6a^5cd^2e^{10} + 15a^4c^2d^4e^8 + 20a^3c^3d^6e^6 + \dots\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x**2+d)**3/(c*x**4+a),x)

```
[Out] RootSum(256*_t**4*a**3*c**7 + _t**2*(192*a**4*c**4*d*e**5 - 640*a**3*c**5*d
**3*e**3 + 192*a**2*c**6*d**5*e) + a**6*e**12 + 6*a**5*c*d**2*e**10 + 15*a
*4*c**2*d**4*e**8 + 20*a**3*c**3*d**6*e**6 + 15*a**2*c**4*d**8*e**4 + 6*a*c
**5*d**10*e**2 + c**6*d**12, Lambda(_t, _t*log(x + (-64*_t**3*a**4*c**5*e**
3 + 192*_t**3*a**3*c**6*d**2*e - 36*_t*a**5*c**2*d*e**8 + 336*_t*a**4*c**3*
d**3*e**6 - 504*_t*a**3*c**4*d**5*e**4 + 144*_t*a**2*c**5*d**7*e**2 - 4*_t*
a*c**6*d**9)/(a**6*e**12 - 12*a**5*c*d**2*e**10 - 27*a**4*c**2*d**4*e**8 +
27*a**2*c**4*d**8*e**4 + 12*a*c**5*d**10*e**2 - c**6*d**12)))) + 3*d*e**2*x
/c + e**3*x**3/(3*c)
```

Giac [A] time = 1.17081, size = 547, normalized size = 1.48

$$\frac{c^2x^3e^3 + 9c^2dxe^2}{3c^3} + \frac{\sqrt{2}\left((ac^3)^{\frac{1}{4}}c^3d^3 - 3(ac^3)^{\frac{1}{4}}ac^2de^2 + 3(ac^3)^{\frac{3}{4}}cd^2e - (ac^3)^{\frac{3}{4}}ae^3\right)\arctan\left(\frac{\sqrt{2}\left(2x + \sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{4ac^4} + \frac{\sqrt{2}\left((ac^3)^{\frac{1}{4}}c^3d^3 - 3(ac^3)^{\frac{1}{4}}ac^2de^2 + 3(ac^3)^{\frac{3}{4}}cd^2e - (ac^3)^{\frac{3}{4}}ae^3\right)\arctan\left(\frac{\sqrt{2}\left(2x - \sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{4ac^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3/(c*x^4+a),x, algorithm="giac")
```

```
[Out] 1/3*(c^2*x^3*e^3 + 9*c^2*d*x*e^2)/c^3 + 1/4*sqrt(2)*((a*c^3)^(1/4)*c^3*d^3
- 3*(a*c^3)^(1/4)*a*c^2*d*e^2 + 3*(a*c^3)^(3/4)*c*d^2*e - (a*c^3)^(3/4)*a*e
^3)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a*c^4) + 1
/4*sqrt(2)*((a*c^3)^(1/4)*c^3*d^3 - 3*(a*c^3)^(1/4)*a*c^2*d*e^2 + 3*(a*c^3)
^(3/4)*c*d^2*e - (a*c^3)^(3/4)*a*e^3)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/
c)^(1/4))/(a/c)^(1/4))/(a*c^4) + 1/8*sqrt(2)*((a*c^3)^(1/4)*c^3*d^3 - 3*(a*
c^3)^(1/4)*a*c^2*d*e^2 - 3*(a*c^3)^(3/4)*c*d^2*e + (a*c^3)^(3/4)*a*e^3)*log
(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a*c^4) - 1/8*sqrt(2)*((a*c^3)^(1
/4)*c^3*d^3 - 3*(a*c^3)^(1/4)*a*c^2*d*e^2 - 3*(a*c^3)^(3/4)*c*d^2*e + (a*c^
3)^(3/4)*a*e^3)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a*c^4)
```

$$3.139 \quad \int \frac{(d+ex^2)^2}{a+cx^4} dx$$

Optimal. Leaf size=297

$$\frac{(-2\sqrt{a}\sqrt{cde} - ae^2 + cd^2) \log(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{cx} + \sqrt{a} + \sqrt{cx^2})}{4\sqrt{2}a^{3/4}c^{5/4}} + \frac{(-2\sqrt{a}\sqrt{cde} - ae^2 + cd^2) \log(\sqrt{2}\sqrt[4]{a}\sqrt[4]{cx} + \sqrt{a} + \sqrt{cx^2})}{4\sqrt{2}a^{3/4}c^{5/4}}$$

```
[Out] (e^2*x)/c - ((c*d^2 + 2*Sqrt[a]*Sqrt[c]*d*e - a*e^2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*c^(5/4)) + ((c*d^2 + 2*Sqrt[a]*Sqrt[c]*d*e - a*e^2)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*c^(5/4)) - ((c*d^2 - 2*Sqrt[a]*Sqrt[c]*d*e - a*e^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*c^(5/4)) + ((c*d^2 - 2*Sqrt[a]*Sqrt[c]*d*e - a*e^2)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*c^(5/4))
```

Rubi [A] time = 0.292803, antiderivative size = 297, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {1171, 1168, 1162, 617, 204, 1165, 628}

$$\frac{(-2\sqrt{a}\sqrt{cde} - ae^2 + cd^2) \log(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{cx} + \sqrt{a} + \sqrt{cx^2})}{4\sqrt{2}a^{3/4}c^{5/4}} + \frac{(-2\sqrt{a}\sqrt{cde} - ae^2 + cd^2) \log(\sqrt{2}\sqrt[4]{a}\sqrt[4]{cx} + \sqrt{a} + \sqrt{cx^2})}{4\sqrt{2}a^{3/4}c^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^2/(a + c*x^4), x]
```

```
[Out] (e^2*x)/c - ((c*d^2 + 2*Sqrt[a]*Sqrt[c]*d*e - a*e^2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*c^(5/4)) + ((c*d^2 + 2*Sqrt[a]*Sqrt[c]*d*e - a*e^2)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*c^(5/4)) - ((c*d^2 - 2*Sqrt[a]*Sqrt[c]*d*e - a*e^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*c^(5/4)) + ((c*d^2 - 2*Sqrt[a]*Sqrt[c]*d*e - a*e^2)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*c^(5/4))
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := Int[ExpandIntegrand[(d + e*x^2)^q/(a + c*x^4), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[q]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + Dist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*c)]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x] /; FreeQ[{a, c, d, e}, x] &
```

& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]

Rule 617

Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 1165

Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rubi steps

$$\begin{aligned} \int \frac{(d + ex^2)^2}{a + cx^4} dx &= \int \left(\frac{e^2}{c} + \frac{cd^2 - ae^2 + 2cdex^2}{c(a + cx^4)} \right) dx \\ &= \frac{e^2 x}{c} + \frac{\int \frac{cd^2 - ae^2 + 2cdex^2}{a + cx^4} dx}{c} \\ &= \frac{e^2 x}{c} + \frac{(cd^2 - 2\sqrt{a}\sqrt{cde} - ae^2) \int \frac{\sqrt{a}\sqrt{c} - cx^2}{a + cx^4} dx}{2\sqrt{ac}^{3/2}} + \frac{(cd^2 + 2\sqrt{a}\sqrt{cde} - ae^2) \int \frac{\sqrt{a}\sqrt{c} + cx^2}{a + cx^4} dx}{2\sqrt{ac}^{3/2}} \\ &= \frac{e^2 x}{c} - \frac{(cd^2 - 2\sqrt{a}\sqrt{cde} - ae^2) \int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt{c}} + 2x}{-\frac{\sqrt{a}}{\sqrt{c}} - \frac{\sqrt{2}\sqrt[4]{ax}}{\sqrt{c}} - x^2} dx}{4\sqrt{2}a^{3/4}c^{5/4}} - \frac{(cd^2 - 2\sqrt{a}\sqrt{cde} - ae^2) \int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt{c}} - 2x}{-\frac{\sqrt{a}}{\sqrt{c}} + \frac{\sqrt{2}\sqrt[4]{ax}}{\sqrt{c}} - x^2} dx}{4\sqrt{2}a^{3/4}c^{5/4}} + \frac{(cd^2 + 2\sqrt{a}\sqrt{cde} - ae^2) \int \frac{\sqrt{a}\sqrt{c} - cx^2}{a + cx^4} dx}{2\sqrt{ac}^{3/2}} \\ &= \frac{e^2 x}{c} - \frac{(cd^2 - 2\sqrt{a}\sqrt{cde} - ae^2) \log(\sqrt{a} - \sqrt{2}\sqrt[4]{a}\sqrt[4]{cx} + \sqrt{cx^2})}{4\sqrt{2}a^{3/4}c^{5/4}} + \frac{(cd^2 - 2\sqrt{a}\sqrt{cde} - ae^2) \log(\sqrt{a} + \sqrt{2}\sqrt[4]{a}\sqrt[4]{cx} + \sqrt{cx^2})}{4\sqrt{2}a^{3/4}c^{5/4}} \\ &= \frac{e^2 x}{c} - \frac{(cd^2 + 2\sqrt{a}\sqrt{cde} - ae^2) \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{cx}}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}c^{5/4}} + \frac{(cd^2 + 2\sqrt{a}\sqrt{cde} - ae^2) \tan^{-1}\left(1 + \frac{\sqrt{2}\sqrt[4]{cx}}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}c^{5/4}} - \frac{(cd^2 + 2\sqrt{a}\sqrt{cde} - ae^2) \int \frac{\sqrt{a}\sqrt{c} + cx^2}{a + cx^4} dx}{2\sqrt{ac}^{3/2}} \end{aligned}$$

Mathematica [A] time = 0.26163, size = 269, normalized size = 0.91

$$\frac{8a^{3/4}\sqrt[4]{ce^2x} + \sqrt{2}(2\sqrt{a}\sqrt{cde} + ae^2 - cd^2) \log(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{cx} + \sqrt{a} + \sqrt{cx^2}) + \sqrt{2}(-2\sqrt{a}\sqrt{cde} - ae^2 + cd^2) \log(\sqrt{2}\sqrt[4]{a}\sqrt[4]{cx} + \sqrt{a} + \sqrt{cx^2})}{8a^{3/4}c^{5/4}}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e*x^2)^2/(a + c*x^4), x]

[Out] $(8*a^{3/4}*c^{1/4}*e^2*x - 2*\sqrt{2}*(c*d^2 + 2*\sqrt{a}*\sqrt{c}*d*e - a*e^2)*\text{ArcTan}[1 - (\sqrt{2}*c^{1/4}*x)/a^{1/4}] + 2*\sqrt{2}*(c*d^2 + 2*\sqrt{a}*\sqrt{c}*d*e - a*e^2)*\text{ArcTan}[1 + (\sqrt{2}*c^{1/4}*x)/a^{1/4}] + \sqrt{2}*(-(c*d^2) + 2*\sqrt{a}*\sqrt{c}*d*e + a*e^2)*\text{Log}[\sqrt{a} - \sqrt{2}*a^{1/4}*c^{1/4}*x + \sqrt{c}*x^2] + \sqrt{2}*(c*d^2 - 2*\sqrt{a}*\sqrt{c}*d*e - a*e^2)*\text{Log}[\sqrt{a} + \sqrt{2}*a^{1/4}*c^{1/4}*x + \sqrt{c}*x^2])/(8*a^{3/4}*c^{5/4})$

Maple [A] time = 0.046, size = 412, normalized size = 1.4

$$\frac{e^2x}{c} - \frac{\sqrt{2}e^2}{4c} \sqrt[4]{\frac{a}{c}} \arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{c}}} - 1\right) + \frac{\sqrt{2}d^2}{4a} \sqrt[4]{\frac{a}{c}} \arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{c}}} - 1\right) - \frac{\sqrt{2}e^2}{8c} \sqrt[4]{\frac{a}{c}} \ln\left(\left(x^2 + \sqrt[4]{\frac{a}{c}}x\sqrt{2} + \sqrt{\frac{a}{c}}\right)\left(x^2\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x^2+d)^2/(c*x^4+a), x)

[Out] $e^2*x/c - 1/4/c*(a/c)^{1/4}*2^{1/2}*\arctan(2^{1/2}/(a/c)^{1/4}*x-1)*e^{2+1/4}*(a/c)^{1/4}/a*2^{1/2}*\arctan(2^{1/2}/(a/c)^{1/4}*x-1)*d^{2-1/8}/c*(a/c)^{1/4}*2^{1/2}*ln((x^2+(a/c)^{1/4}*x*2^{1/2}+(a/c)^{1/2}))/((x^2-(a/c)^{1/4}*x*2^{1/2}+(a/c)^{1/2}))*e^{2+1/8}*(a/c)^{1/4}/a*2^{1/2}*ln((x^2+(a/c)^{1/4}*x*2^{1/2}+(a/c)^{1/2}))/((x^2-(a/c)^{1/4}*x*2^{1/2}+(a/c)^{1/2}))*d^{2-1/4}/c*(a/c)^{1/4}*2^{1/2}*\arctan(2^{1/2}/(a/c)^{1/4}*x+1)*e^{2+1/4}*(a/c)^{1/4}/a*2^{1/2}*\arctan(2^{1/2}/(a/c)^{1/4}*x+1)*d^{2+1/4}/c*d*e/(a/c)^{1/4}*2^{1/2}*ln((x^2-(a/c)^{1/4}*x*2^{1/2}+(a/c)^{1/2}))/((x^2+(a/c)^{1/4}*x*2^{1/2}+(a/c)^{1/2}))+1/2/c*d*e/(a/c)^{1/4}*2^{1/2}*\arctan(2^{1/2}/(a/c)^{1/4}*x+1)+1/2/c*d*e/(a/c)^{1/4}*2^{1/2}*\arctan(2^{1/2}/(a/c)^{1/4}*x-1)$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)^2/(c*x^4+a), x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [B] time = 3.21992, size = 3004, normalized size = 10.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)^2/(c*x^4+a), x, algorithm="fricas")

[Out] $1/4*(4*e^2*x + c*\sqrt{-(4*c*d^3*e - 4*a*d*e^3 + a*c^2*\sqrt{-(c^4*d^8 - 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12*a^3*c*d^2*e^6 + a^4*e^8)})/(a^3*c^5)))/(a*c^2)*\log((c^4*d^8 - 4*a*c^3*d^6*e^2 - 10*a^2*c^2*d^4*e^4 - 4*a^3*c*d^2$

```

*e^6 + a^4*e^8)*x + (a*c^4*d^6 - 7*a^2*c^3*d^4*e^2 + 7*a^3*c^2*d^2*e^4 - a^
4*c*e^6 + 2*a^3*c^4*d*e*sqrt(-(c^4*d^8 - 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*
e^4 - 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^3*c^5)))*sqrt(-(4*c*d^3*e - 4*a*d*e^3
+ a*c^2*sqrt(-(c^4*d^8 - 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12*a^3*c*d
^2*e^6 + a^4*e^8)/(a^3*c^5)))/(a*c^2)) - c*sqrt(-(4*c*d^3*e - 4*a*d*e^3 +
a*c^2*sqrt(-(c^4*d^8 - 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12*a^3*c*d^2
*e^6 + a^4*e^8)/(a^3*c^5)))/(a*c^2))*log((c^4*d^8 - 4*a*c^3*d^6*e^2 - 10*a^
2*c^2*d^4*e^4 - 4*a^3*c*d^2*e^6 + a^4*e^8)*x - (a*c^4*d^6 - 7*a^2*c^3*d^4*e
^2 + 7*a^3*c^2*d^2*e^4 - a^4*c*e^6 + 2*a^3*c^4*d*e*sqrt(-(c^4*d^8 - 12*a*c^
3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^3*c^5)))*sq
rt(-(4*c*d^3*e - 4*a*d*e^3 + a*c^2*sqrt(-(c^4*d^8 - 12*a*c^3*d^6*e^2 + 38*a
^2*c^2*d^4*e^4 - 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^3*c^5)))/(a*c^2))) + c*sqrt
(-(4*c*d^3*e - 4*a*d*e^3 - a*c^2*sqrt(-(c^4*d^8 - 12*a*c^3*d^6*e^2 + 38*a^2
*c^2*d^4*e^4 - 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^3*c^5)))/(a*c^2))*log((c^4*d^
8 - 4*a*c^3*d^6*e^2 - 10*a^2*c^2*d^4*e^4 - 4*a^3*c*d^2*e^6 + a^4*e^8)*x + (
a*c^4*d^6 - 7*a^2*c^3*d^4*e^2 + 7*a^3*c^2*d^2*e^4 - a^4*c*e^6 - 2*a^3*c^4*d
*e*sqrt(-(c^4*d^8 - 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12*a^3*c*d^2*e^
6 + a^4*e^8)/(a^3*c^5)))*sqrt(-(4*c*d^3*e - 4*a*d*e^3 - a*c^2*sqrt(-(c^4*d^
8 - 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^
3*c^5)))/(a*c^2)) - c*sqrt(-(4*c*d^3*e - 4*a*d*e^3 - a*c^2*sqrt(-(c^4*d^8
- 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^3*
c^5)))/(a*c^2))*log((c^4*d^8 - 4*a*c^3*d^6*e^2 - 10*a^2*c^2*d^4*e^4 - 4*a^3
*c*d^2*e^6 + a^4*e^8)*x - (a*c^4*d^6 - 7*a^2*c^3*d^4*e^2 + 7*a^3*c^2*d^2*e^
4 - a^4*c*e^6 - 2*a^3*c^4*d*e*sqrt(-(c^4*d^8 - 12*a*c^3*d^6*e^2 + 38*a^2*c^
2*d^4*e^4 - 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^3*c^5)))*sqrt(-(4*c*d^3*e - 4*a*
d*e^3 - a*c^2*sqrt(-(c^4*d^8 - 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12*a
^3*c*d^2*e^6 + a^4*e^8)/(a^3*c^5)))/(a*c^2))))/c

```

Sympy [A] time = 1.50119, size = 238, normalized size = 0.8

RootSum(256t^4a^3c^5 + t^2(-128a^3c^3de^3 + 128a^2c^4d^3e) + a^4e^8 + 4a^3cd^2e^6 + 6a^2c^2d^4e^4 + 4ac^3d^6e^2 + c^4d^8, (t ↦ t log(x +

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x**2+d)**2/(c*x**4+a),x)

[Out] RootSum(256*_t**4*a**3*c**5 + _t**2*(-128*a**3*c**3*d*e**3 + 128*a**2*c**4*d**3*e) + a**4*e**8 + 4*a**3*c*d**2*e**6 + 6*a**2*c**2*d**4*e**4 + 4*a*c**3*d**6*e**2 + c**4*d**8, Lambda(_t, _t*log(x + (-128*_t**3*a**3*c**4*d*e - 4*_t*a**4*c*e**6 + 60*_t*a**3*c**2*d**2*e**4 - 60*_t*a**2*c**3*d**4*e**2 + 4*_t*a*c**4*d**6)/(a**4*e**8 - 4*a**3*c*d**2*e**6 - 10*a**2*c**2*d**4*e**4 - 4*a*c**3*d**6*e**2 + c**4*d**8)))) + e**2*x/c

Giac [A] time = 1.14799, size = 454, normalized size = 1.53

$$\frac{xe^2}{c} + \frac{\sqrt{2}\left((ac^3)^{\frac{1}{4}}c^2d^2 - (ac^3)^{\frac{1}{4}}ace^2 + 2(ac^3)^{\frac{3}{4}}de\right)\arctan\left(\frac{\sqrt{2}\left(2x - \sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{4ac^3} - \frac{\sqrt{2}\left((ac^3)^{\frac{1}{4}}c^2d^2 - (ac^3)^{\frac{1}{4}}ace^2 - 2(ac^3)^{\frac{3}{4}}d\right)}{8ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)^2/(c*x^4+a),x, algorithm="giac")

```
[Out] x*e^2/c + 1/4*sqrt(2)*((a*c^3)^(1/4)*c^2*d^2 - (a*c^3)^(1/4)*a*c*e^2 + 2*(a
*c^3)^(3/4)*d*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4)
)/(a*c^3) - 1/8*sqrt(2)*((a*c^3)^(1/4)*c^2*d^2 - (a*c^3)^(1/4)*a*c*e^2 - 2*
(a*c^3)^(3/4)*d*e)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a*c^3) + 1
/4*sqrt(2)*((a*c^3)^(1/4)*a*c^4*d^2 - (a*c^3)^(1/4)*a^2*c^3*e^2 + 2*(a*c^3)
^(3/4)*a*c^2*d*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4)
))/(a^2*c^5) + 1/8*sqrt(2)*((a*c^3)^(1/4)*a*c^4*d^2 - (a*c^3)^(1/4)*a^2*c^3
*e^2 - 2*(a*c^3)^(3/4)*a*c^2*d*e)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/
c))/(a^2*c^5)
```

3.140 $\int \frac{d+ex^2}{a+cx^4} dx$

Optimal. Leaf size=247

$$\frac{(\sqrt{cd} - \sqrt{ae}) \log(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{cx} + \sqrt{a} + \sqrt{cx^2})}{4\sqrt{2}a^{3/4}c^{3/4}} + \frac{(\sqrt{cd} - \sqrt{ae}) \log(\sqrt{2}\sqrt[4]{a}\sqrt[4]{cx} + \sqrt{a} + \sqrt{cx^2})}{4\sqrt{2}a^{3/4}c^{3/4}} - \frac{(\sqrt{ae} + \sqrt{cd}) \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{cx} + \sqrt{a} + \sqrt{cx^2}}{2\sqrt{2}a^{3/4}c^{3/4}}\right)}{2\sqrt{2}a^{3/4}c^{3/4}}$$

```
[Out] -((Sqrt[c]*d + Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*c^(3/4)) + ((Sqrt[c]*d + Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*c^(3/4)) - ((Sqrt[c]*d - Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*c^(3/4)) + ((Sqrt[c]*d - Sqrt[a]*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*c^(3/4))
```

Rubi [A] time = 0.152343, antiderivative size = 247, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {1168, 1162, 617, 204, 1165, 628}

$$\frac{(\sqrt{cd} - \sqrt{ae}) \log(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{cx} + \sqrt{a} + \sqrt{cx^2})}{4\sqrt{2}a^{3/4}c^{3/4}} + \frac{(\sqrt{cd} - \sqrt{ae}) \log(\sqrt{2}\sqrt[4]{a}\sqrt[4]{cx} + \sqrt{a} + \sqrt{cx^2})}{4\sqrt{2}a^{3/4}c^{3/4}} - \frac{(\sqrt{ae} + \sqrt{cd}) \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{cx} + \sqrt{a} + \sqrt{cx^2}}{2\sqrt{2}a^{3/4}c^{3/4}}\right)}{2\sqrt{2}a^{3/4}c^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)/(a + c*x^4), x]
```

```
[Out] -((Sqrt[c]*d + Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*c^(3/4)) + ((Sqrt[c]*d + Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*c^(3/4)) - ((Sqrt[c]*d - Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*c^(3/4)) + ((Sqrt[c]*d - Sqrt[a]*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*c^(3/4))
```

Rule 1168

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + Dist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*c)]
```

Rule 1162

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 1165

Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rubi steps

$$\begin{aligned} \int \frac{d + ex^2}{a + cx^4} dx &= \frac{\left(\frac{\sqrt{cd}}{\sqrt{a}} - e\right) \int \frac{\sqrt{a}\sqrt{c}-cx^2}{a+cx^4} dx}{2c} + \frac{\left(\frac{\sqrt{cd}}{\sqrt{a}} + e\right) \int \frac{\sqrt{a}\sqrt{c}+cx^2}{a+cx^4} dx}{2c} \\ &= \frac{\left(\frac{\sqrt{cd}}{\sqrt{a}} + e\right) \int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}} - \frac{\sqrt{2}\sqrt[4]{ax}}{\sqrt[4]{c}} + x^2} dx}{4c} + \frac{\left(\frac{\sqrt{cd}}{\sqrt{a}} + e\right) \int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}} + \frac{\sqrt{2}\sqrt[4]{ax}}{\sqrt[4]{c}} + x^2} dx}{4c} - \frac{(\sqrt{cd} - \sqrt{ae}) \int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}} + 2x}{-\frac{\sqrt{a}}{\sqrt{c}} - \frac{\sqrt{2}\sqrt[4]{ax}}{\sqrt[4]{c}} - x^2} dx}{4\sqrt{2}a^{3/4}c^{3/4}}}{4\sqrt{2}a^{3/4}c^{3/4}} - \frac{(\sqrt{cd} + \sqrt{ae}) \int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}} + 2x}{-\frac{\sqrt{a}}{\sqrt{c}} - \frac{\sqrt{2}\sqrt[4]{ax}}{\sqrt[4]{c}} - x^2} dx}{4\sqrt{2}a^{3/4}c^{3/4}}}{4\sqrt{2}a^{3/4}c^{3/4}} \\ &= -\frac{(\sqrt{cd} - \sqrt{ae}) \log(\sqrt{a} - \sqrt{2}\sqrt[4]{a}\sqrt[4]{cx} + \sqrt{cx^2})}{4\sqrt{2}a^{3/4}c^{3/4}} + \frac{(\sqrt{cd} - \sqrt{ae}) \log(\sqrt{a} + \sqrt{2}\sqrt[4]{a}\sqrt[4]{cx} + \sqrt{cx^2})}{4\sqrt{2}a^{3/4}c^{3/4}} + \frac{(\sqrt{cd} + \sqrt{ae}) \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{cx}}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}c^{3/4}} + \frac{(\sqrt{cd} + \sqrt{ae}) \tan^{-1}\left(1 + \frac{\sqrt{2}\sqrt[4]{cx}}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}c^{3/4}} - \frac{(\sqrt{cd} - \sqrt{ae}) \log(\sqrt{a} - \sqrt{2}\sqrt[4]{a}\sqrt[4]{cx} + \sqrt{cx^2})}{4\sqrt{2}a^{3/4}c^{3/4}} \end{aligned}$$

Mathematica [A] time = 0.0579915, size = 183, normalized size = 0.74

$$\frac{-(\sqrt{cd} - \sqrt{ae}) \left(\log(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{cx} + \sqrt{a} + \sqrt{cx^2}) - \log(\sqrt{2}\sqrt[4]{a}\sqrt[4]{cx} + \sqrt{a} + \sqrt{cx^2}) \right) - 2(\sqrt{ae} + \sqrt{cd}) \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{cx}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{3/4}c^{3/4}}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e*x^2)/(a + c*x^4), x]

[Out] (-2*(Sqrt[c]*d + Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + 2*(Sqrt[c]*d + Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)] - (Sqrt[c]*d - Sqrt[a]*e)*(Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2] - Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*c^(3/4))

Maple [A] time = 0.046, size = 260, normalized size = 1.1

$$\frac{d\sqrt{2}}{8a} \frac{\sqrt[4]{a}}{\sqrt{c}} \ln\left(\left(x^2 + \sqrt[4]{\frac{a}{c}}x\sqrt{2} + \sqrt{\frac{a}{c}}\right)\left(x^2 - \sqrt[4]{\frac{a}{c}}x\sqrt{2} + \sqrt{\frac{a}{c}}\right)^{-1}\right) + \frac{d\sqrt{2}}{4a} \frac{\sqrt[4]{a}}{\sqrt{c}} \arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{c}}} + 1\right) + \frac{d\sqrt{2}}{4a} \frac{\sqrt[4]{a}}{\sqrt{c}} \arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{c}}} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*x^2+d)/(c*x^4+a),x)`

[Out] $\frac{1}{8}d*(a/c)^{(1/4)}/a*2^{(1/2)}*\ln((x^2+(a/c)^{(1/4)}*x*2^{(1/2)}+(a/c)^{(1/2)})/(x^2-(a/c)^{(1/4)}*x*2^{(1/2)}+(a/c)^{(1/2)}))+1/4*d*(a/c)^{(1/4)}/a*2^{(1/2)}*\arctan(2^{(1/2)}/(a/c)^{(1/4)}*x+1)+1/4*d*(a/c)^{(1/4)}/a*2^{(1/2)}*\arctan(2^{(1/2)}/(a/c)^{(1/4)}*x-1)+1/8*e/c/(a/c)^{(1/4)}*2^{(1/2)}*\ln((x^2-(a/c)^{(1/4)}*x*2^{(1/2)}+(a/c)^{(1/2)})/(x^2+(a/c)^{(1/4)}*x*2^{(1/2)}+(a/c)^{(1/2)}))+1/4*e/c/(a/c)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(a/c)^{(1/4)}*x+1)+1/4*e/c/(a/c)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(a/c)^{(1/4)}*x-1)$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x^2+d)/(c*x^4+a),x, algorithm="maxima")`

[Out] Exception raised: ValueError

Fricas [B] time = 1.97057, size = 1544, normalized size = 6.25

$$-\frac{1}{4}\sqrt{-\frac{ac\sqrt{-\frac{c^2d^4-2acd^2e^2+a^2e^4}{a^3c^3}}+2de}{ac}}\log\left(-\left(c^2d^4-a^2e^4\right)x+\left(a^3c^2e\sqrt{-\frac{c^2d^4-2acd^2e^2+a^2e^4}{a^3c^3}}+ac^2d^3-a^2cde^2\right)\sqrt{-\frac{ac\sqrt{-\frac{c^2d^4-2acd^2e^2+a^2e^4}{a^3c^3}}+2de}{ac}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x^2+d)/(c*x^4+a),x, algorithm="fricas")`

[Out] $-1/4*\sqrt{-\left(a*c*\sqrt{-\left(c^2*d^4-2*a*c*d^2*e^2+a^2*e^4\right)/\left(a^3*c^3\right)}+2*d*e\right)/\left(a*c\right)}*\log\left(-\left(c^2*d^4-a^2*e^4\right)*x+\left(a^3*c^2*e*\sqrt{-\left(c^2*d^4-2*a*c*d^2*e^2+a^2*e^4\right)/\left(a^3*c^3\right)}+a*c^2*d^3-a^2*c*d*e^2\right)*\sqrt{-\left(a*c*\sqrt{-\left(c^2*d^4-2*a*c*d^2*e^2+a^2*e^4\right)/\left(a^3*c^3\right)}+2*d*e\right)/\left(a*c\right)}\right)+1/4*\sqrt{-\left(a*c*\sqrt{-\left(c^2*d^4-2*a*c*d^2*e^2+a^2*e^4\right)/\left(a^3*c^3\right)}+2*d*e\right)/\left(a*c\right)}*\log\left(-\left(c^2*d^4-a^2*e^4\right)*x-\left(a^3*c^2*e*\sqrt{-\left(c^2*d^4-2*a*c*d^2*e^2+a^2*e^4\right)/\left(a^3*c^3\right)}+a*c^2*d^3-a^2*c*d*e^2\right)*\sqrt{-\left(a*c*\sqrt{-\left(c^2*d^4-2*a*c*d^2*e^2+a^2*e^4\right)/\left(a^3*c^3\right)}+2*d*e\right)/\left(a*c\right)}\right)+1/4*\sqrt{\left(a*c*\sqrt{-\left(c^2*d^4-2*a*c*d^2*e^2+a^2*e^4\right)/\left(a^3*c^3\right)}-2*d*e\right)/\left(a*c\right)}*\log\left(-\left(c^2*d^4-a^2*e^4\right)*x+\left(a^3*c^2*e*\sqrt{-\left(c^2*d^4-2*a*c*d^2*e^2+a^2*e^4\right)/\left(a^3*c^3\right)}-a*c^2*d^3+a^2*c*d*e^2\right)*\sqrt{\left(a*c*\sqrt{-\left(c^2*d^4-2*a*c*d^2*e^2+a^2*e^4\right)/\left(a^3*c^3\right)}-2*d*e\right)/\left(a*c\right)}\right)-1/4*\sqrt{\left(a*c*\sqrt{-\left(c^2*d^4-2*a*c*d^2*e^2+a^2*e^4\right)/\left(a^3*c^3\right)}-2*d*e\right)/\left(a*c\right)}*\log\left(-\left(c^2*d^4-a^2*e^4\right)*x-\left(a^3*c^2*e*\sqrt{-\left(c^2*d^4-2*a*c*d^2*e^2+a^2*e^4\right)/\left(a^3*c^3\right)}-a*c^2*d^3+a^2*c*d*e^2\right)*\sqrt{\left(a*c*\sqrt{-\left(c^2*d^4-2*a*c*d^2*e^2+a^2*e^4\right)/\left(a^3*c^3\right)}-2*d*e\right)/\left(a*c\right)}\right)$

Sympy [A] time = 0.641222, size = 109, normalized size = 0.44

$$\text{RootSum}\left(256t^4a^3c^3+64t^2a^2c^2de+a^2e^4+2acd^2e^2+c^2d^4,\left(t\mapsto t\log\left(x+\frac{64t^3a^3c^2e+12ta^2cde^2-4tac^2d^3}{a^2e^4-c^2d^4}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x**2+d)/(c*x**4+a),x)

[Out] RootSum(256*_t**4*a**3*c**3 + 64*_t**2*a**2*c**2*d*e + a**2*e**4 + 2*a*c*d*
*2*e**2 + c**2*d**4, Lambda(_t, _t*log(x + (64*_t**3*a**3*c**2*e + 12*_t*a*
*2*c*d*e**2 - 4*_t*a*c**2*d**3)/(a**2*e**4 - c**2*d**4))))

Giac [A] time = 1.1514, size = 331, normalized size = 1.34

$$\frac{\sqrt{2}\left((ac^3)^{\frac{1}{4}}c^2d + (ac^3)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{4ac^3} + \frac{\sqrt{2}\left((ac^3)^{\frac{1}{4}}c^2d + (ac^3)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{4ac^3} + \frac{\sqrt{2}\left((ac^3)^{\frac{1}{4}}c^2d + (ac^3)^{\frac{3}{4}}e\right)\log\left(x^2 + \sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{c}}\right)}{4ac^3} - \frac{\sqrt{2}\left((ac^3)^{\frac{1}{4}}c^2d + (ac^3)^{\frac{3}{4}}e\right)\log\left(x^2 - \sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{c}}\right)}{4ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)/(c*x^4+a),x, algorithm="giac")

[Out] 1/4*sqrt(2)*((a*c^3)^(1/4)*c^2*d + (a*c^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x
+ sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a*c^3) + 1/4*sqrt(2)*((a*c^3)^(1/4)*c
^2*d + (a*c^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c
)^(1/4))/(a*c^3) + 1/8*sqrt(2)*((a*c^3)^(1/4)*c^2*d - (a*c^3)^(3/4)*e)*log(
x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a*c^3) - 1/8*sqrt(2)*((a*c^3)^(1/
4)*c^2*d - (a*c^3)^(3/4)*e)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a
*c^3)

3.141 $\int \frac{1}{a+cx^4} dx$

Optimal. Leaf size=185

$$-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{cx} + \sqrt{a} + \sqrt{cx^2}\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{c}} + \frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{cx} + \sqrt{a} + \sqrt{cx^2}\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{c}} - \frac{\tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{cx}}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{c}} + \frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{cx}}{\sqrt[4]{a}} + 1\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{c}}$$

[Out] -ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)]/(2*Sqrt[2]*a^(3/4)*c^(1/4)) + ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)]/(2*Sqrt[2]*a^(3/4)*c^(1/4)) - Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2]/(4*Sqrt[2]*a^(3/4)*c^(1/4)) + Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2]/(4*Sqrt[2]*a^(3/4)*c^(1/4))

Rubi [A] time = 0.110959, antiderivative size = 185, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {211, 1165, 628, 1162, 617, 204}

$$-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{cx} + \sqrt{a} + \sqrt{cx^2}\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{c}} + \frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{cx} + \sqrt{a} + \sqrt{cx^2}\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{c}} - \frac{\tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{cx}}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{c}} + \frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{cx}}{\sqrt[4]{a}} + 1\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{c}}$$

Antiderivative was successfully verified.

[In] Int[(a + c*x^4)^(-1), x]

[Out] -ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)]/(2*Sqrt[2]*a^(3/4)*c^(1/4)) + ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)]/(2*Sqrt[2]*a^(3/4)*c^(1/4)) - Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2]/(4*Sqrt[2]*a^(3/4)*c^(1/4)) + Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2]/(4*Sqrt[2]*a^(3/4)*c^(1/4))

Rule 211

Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] := With[{r = Numerator[Rt[a/b, 2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4), x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] && AtomQ[SplitProduct[SumBaseQ, b]]))

Rule 1165

Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 1162

Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e

/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]

Rule 617

Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c]) /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-
a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a,
0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned} \int \frac{1}{a+cx^4} dx &= \frac{\int \frac{\sqrt{a}-\sqrt{cx^2}}{a+cx^4} dx}{2\sqrt{a}} + \frac{\int \frac{\sqrt{a}+\sqrt{cx^2}}{a+cx^4} dx}{2\sqrt{a}} \\ &= \frac{\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}} - \frac{\sqrt{2}\sqrt[4]{ax}}{\sqrt{c}} + x^2} dx}{4\sqrt{a}\sqrt{c}} + \frac{\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}} + \frac{\sqrt{2}\sqrt[4]{ax}}{\sqrt{c}} + x^2} dx}{4\sqrt{a}\sqrt{c}} - \frac{\int \frac{\frac{\sqrt{2}\sqrt[4]{a}+2x}{\sqrt[4]{c}}}{-\frac{\sqrt{a}}{\sqrt{c}} - \frac{\sqrt{2}\sqrt[4]{ax}}{\sqrt{c}} - x^2} dx}{4\sqrt{2}a^{3/4}\sqrt[4]{c}} - \frac{\int \frac{\frac{\sqrt{2}\sqrt[4]{a}-2x}{\sqrt[4]{c}}}{-\frac{\sqrt{a}}{\sqrt{c}} + \frac{\sqrt{2}\sqrt[4]{ax}}{\sqrt{c}} - x^2} dx}{4\sqrt{2}a^{3/4}\sqrt[4]{c}} \\ &= -\frac{\log(\sqrt{a} - \sqrt{2}\sqrt[4]{a}\sqrt[4]{cx} + \sqrt{cx^2})}{4\sqrt{2}a^{3/4}\sqrt[4]{c}} + \frac{\log(\sqrt{a} + \sqrt{2}\sqrt[4]{a}\sqrt[4]{cx} + \sqrt{cx^2})}{4\sqrt{2}a^{3/4}\sqrt[4]{c}} + \frac{\text{Subst}\left(\int \frac{1}{-1-x^2} dx, x, 1 - \frac{\sqrt{2}\sqrt[4]{cx}}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{c}} \\ &= -\frac{\tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{cx}}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{c}} + \frac{\tan^{-1}\left(1 + \frac{\sqrt{2}\sqrt[4]{cx}}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{c}} - \frac{\log(\sqrt{a} - \sqrt{2}\sqrt[4]{a}\sqrt[4]{cx} + \sqrt{cx^2})}{4\sqrt{2}a^{3/4}\sqrt[4]{c}} + \frac{\log(\sqrt{a} + \sqrt{2}\sqrt[4]{a}\sqrt[4]{cx} + \sqrt{cx^2})}{4\sqrt{2}a^{3/4}\sqrt[4]{c}} \end{aligned}$$

Mathematica [A] time = 0.0188434, size = 134, normalized size = 0.72

$$\frac{-\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{cx} + \sqrt{a} + \sqrt{cx^2}\right) + \log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{cx} + \sqrt{a} + \sqrt{cx^2}\right) - 2\tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{cx}}{\sqrt[4]{a}}\right) + 2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{cx}}{\sqrt[4]{a}} + 1\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{c}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + c*x^4)^(-1), x]

[Out] (-2*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + 2*ArcTan[1 + (Sqrt[2]*c^(1/4)*
*x)/a^(1/4)] - Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2] + Log
[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*c^(
1/4))

Maple [A] time = 0.043, size = 128, normalized size = 0.7

$$\frac{\sqrt{2}}{8a}\sqrt[4]{\frac{a}{c}}\ln\left(\left(x^2 + \sqrt[4]{\frac{a}{c}}x\sqrt{2} + \sqrt{\frac{a}{c}}\right)\left(x^2 - \sqrt[4]{\frac{a}{c}}x\sqrt{2} + \sqrt{\frac{a}{c}}\right)^{-1}\right) + \frac{\sqrt{2}}{4a}\sqrt[4]{\frac{a}{c}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{c}}} + 1\right) + \frac{\sqrt{2}}{4a}\sqrt[4]{\frac{a}{c}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{c}}} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(c*x^4+a),x)

[Out] $\frac{1}{8} \left(\frac{a}{c}\right)^{\frac{1}{4}} / a^{2^{\frac{1}{2}}} \ln\left(\frac{x^2 + \left(\frac{a}{c}\right)^{\frac{1}{4}} x^{2^{\frac{1}{2}}} + \left(\frac{a}{c}\right)^{\frac{1}{2}}}{x^2 - \left(\frac{a}{c}\right)^{\frac{1}{4}} x^{2^{\frac{1}{2}}} + \left(\frac{a}{c}\right)^{\frac{1}{2}}}\right) + \frac{1}{4} \left(\frac{a}{c}\right)^{\frac{1}{4}} / a^{2^{\frac{1}{2}}} \arctan\left(\frac{2^{\frac{1}{2}}}{\left(\frac{a}{c}\right)^{\frac{1}{4}} x + 1}\right) + \frac{1}{4} \left(\frac{a}{c}\right)^{\frac{1}{4}} / a^{2^{\frac{1}{2}}} \arctan\left(\frac{2^{\frac{1}{2}}}{\left(\frac{a}{c}\right)^{\frac{1}{4}} x - 1}\right)$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x^4+a),x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [A] time = 1.99861, size = 306, normalized size = 1.65

$$\left(-\frac{1}{a^3c}\right)^{\frac{1}{4}} \arctan\left(-a^2cx\left(-\frac{1}{a^3c}\right)^{\frac{3}{4}} + \sqrt{a^2\sqrt{-\frac{1}{a^3c}} + x^2a^2c}\left(-\frac{1}{a^3c}\right)^{\frac{3}{4}}\right) + \frac{1}{4}\left(-\frac{1}{a^3c}\right)^{\frac{1}{4}} \log\left(a\left(-\frac{1}{a^3c}\right)^{\frac{1}{4}} + x\right) - \frac{1}{4}\left(-\frac{1}{a^3c}\right)^{\frac{1}{4}} \log\left(\dots\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x^4+a),x, algorithm="fricas")

[Out] $\left(-\frac{1}{a^3c}\right)^{\frac{1}{4}} \arctan\left(-a^2cx\left(-\frac{1}{a^3c}\right)^{\frac{3}{4}} + \sqrt{a^2\sqrt{-\frac{1}{a^3c}} + x^2a^2c}\left(-\frac{1}{a^3c}\right)^{\frac{3}{4}}\right) + \frac{1}{4}\left(-\frac{1}{a^3c}\right)^{\frac{1}{4}} \log\left(a\left(-\frac{1}{a^3c}\right)^{\frac{1}{4}} + x\right) - \frac{1}{4}\left(-\frac{1}{a^3c}\right)^{\frac{1}{4}} \log\left(-a\left(-\frac{1}{a^3c}\right)^{\frac{1}{4}} + x\right)$

Sympy [A] time = 0.157485, size = 20, normalized size = 0.11

$$\text{RootSum}\left(256t^4a^3c + 1, (t \mapsto t \log(4ta + x))\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x**4+a),x)

[Out] RootSum(256*_t**4*a**3*c + 1, Lambda(_t, _t*log(4*_t*a + x)))

Giac [A] time = 1.12837, size = 242, normalized size = 1.31

$$\frac{\sqrt{2}\left(ac^3\right)^{\frac{1}{4}} \arctan\left(\frac{\sqrt{2}\left(2x + \sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{4ac} + \frac{\sqrt{2}\left(ac^3\right)^{\frac{1}{4}} \arctan\left(\frac{\sqrt{2}\left(2x - \sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{4ac} + \frac{\sqrt{2}\left(ac^3\right)^{\frac{1}{4}} \log\left(x^2 + \sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{c}}\right)}{8ac} - \frac{\sqrt{2}\left(ac^3\right)^{\frac{1}{4}} \log\left(x^2 - \sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{c}}\right)}{8ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+a),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*(a*c^3)^(1/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a*c) + 1/4*sqrt(2)*(a*c^3)^(1/4)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a*c) + 1/8*sqrt(2)*(a*c^3)^(1/4)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a*c) - 1/8*sqrt(2)*(a*c^3)^(1/4)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a*c)
```

3.142 $\int \frac{1}{(d+ex^2)(a+cx^4)} dx$

Optimal. Leaf size=336

$$-\frac{\sqrt[4]{c}(\sqrt{ae} + \sqrt{cd}) \log(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{cx} + \sqrt{a} + \sqrt{cx^2})}{4\sqrt{2}a^{3/4}(ae^2 + cd^2)} + \frac{\sqrt[4]{c}(\sqrt{ae} + \sqrt{cd}) \log(\sqrt{2}\sqrt[4]{a}\sqrt[4]{cx} + \sqrt{a} + \sqrt{cx^2})}{4\sqrt{2}a^{3/4}(ae^2 + cd^2)} - \frac{\sqrt[4]{c}(\sqrt{cd} - \sqrt{ae})}{2\sqrt{2}a^{3/4}}$$

```
[Out] (e^(3/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[d]*(c*d^2 + a*e^2)) - (c^(1/4)*
(Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]
*a^(3/4)*(c*d^2 + a*e^2)) + (c^(1/4)*(Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 + (Sq
rt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2)) - (c^(1/4)*(
Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^
2])/(4*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2)) + (c^(1/4)*(Sqrt[c]*d + Sqrt[a]*e)*
Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*
(c*d^2 + a*e^2))
```

Rubi [A] time = 0.269766, antiderivative size = 336, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {1171, 205, 1168, 1162, 617, 204, 1165, 628}

$$-\frac{\sqrt[4]{c}(\sqrt{ae} + \sqrt{cd}) \log(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{cx} + \sqrt{a} + \sqrt{cx^2})}{4\sqrt{2}a^{3/4}(ae^2 + cd^2)} + \frac{\sqrt[4]{c}(\sqrt{ae} + \sqrt{cd}) \log(\sqrt{2}\sqrt[4]{a}\sqrt[4]{cx} + \sqrt{a} + \sqrt{cx^2})}{4\sqrt{2}a^{3/4}(ae^2 + cd^2)} - \frac{\sqrt[4]{c}(\sqrt{cd} - \sqrt{ae})}{2\sqrt{2}a^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x^2)*(a + c*x^4)), x]
```

```
[Out] (e^(3/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[d]*(c*d^2 + a*e^2)) - (c^(1/4)*
(Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]
*a^(3/4)*(c*d^2 + a*e^2)) + (c^(1/4)*(Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 + (Sq
rt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2)) - (c^(1/4)*(
Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^
2])/(4*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2)) + (c^(1/4)*(Sqrt[c]*d + Sqrt[a]*e)*
Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*
(c*d^2 + a*e^2))
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (c_.)*(x_)^4), x_Symbol] := Int[Expa
ndIntegrand[(d + e*x^2)^q/(a + c*x^4), x], x] /; FreeQ[{a, c, d, e}, x] &&
NeQ[c*d^2 + a*e^2, 0] && IntegerQ[q]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
```

c)]

Rule 1162

Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]

Rule 617

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 204

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 1165

Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]

Rule 628

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rubi steps

$$\begin{aligned}
 \int \frac{1}{(d+ex^2)(a+cx^4)} dx &= \int \left(\frac{e^2}{(cd^2+ae^2)(d+ex^2)} + \frac{c(d-ex^2)}{(cd^2+ae^2)(a+cx^4)} \right) dx \\
 &= \frac{c \int \frac{d-ex^2}{a+cx^4} dx}{cd^2+ae^2} + \frac{e^2 \int \frac{1}{d+ex^2} dx}{cd^2+ae^2} \\
 &= \frac{e^{3/2} \tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)}{\sqrt{d}(cd^2+ae^2)} + \frac{\left(\frac{\sqrt{cd}}{\sqrt{a}}-e\right) \int \frac{\sqrt{a}\sqrt{c+cx^2}}{a+cx^4} dx}{2(cd^2+ae^2)} + \frac{\left(\frac{\sqrt{cd}}{\sqrt{a}}+e\right) \int \frac{\sqrt{a}\sqrt{c-cx^2}}{a+cx^4} dx}{2(cd^2+ae^2)} \\
 &= \frac{e^{3/2} \tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)}{\sqrt{d}(cd^2+ae^2)} + \frac{\left(\frac{\sqrt{cd}}{\sqrt{a}}-e\right) \int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}-\sqrt{2}\sqrt[4]{ax}+x^2} dx}{4(cd^2+ae^2)} + \frac{\left(\frac{\sqrt{cd}}{\sqrt{a}}+e\right) \int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}+\sqrt{2}\sqrt[4]{ax}+x^2} dx}{4(cd^2+ae^2)} - \frac{\left(\sqrt[4]{c}(\sqrt{cd}+\sqrt{ae})\log(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{cx}+\sqrt{cx^2})\right)}{4\sqrt{2}a^{3/4}(cd^2+ae^2)} + \frac{\sqrt[4]{c}(\sqrt{cd}+\sqrt{ae})\log(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{cx}+\sqrt{cx^2})}{4\sqrt{2}a^{3/4}(cd^2+ae^2)} \\
 &= \frac{e^{3/2} \tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)}{\sqrt{d}(cd^2+ae^2)} - \frac{\sqrt[4]{c}(\sqrt{cd}+\sqrt{ae})\log(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{cx}+\sqrt{cx^2})}{4\sqrt{2}a^{3/4}(cd^2+ae^2)} + \frac{\sqrt[4]{c}(\sqrt{cd}+\sqrt{ae})\log(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{cx}+\sqrt{cx^2})}{4\sqrt{2}a^{3/4}(cd^2+ae^2)} \\
 &= \frac{e^{3/2} \tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)}{\sqrt{d}(cd^2+ae^2)} - \frac{\sqrt[4]{c}(\sqrt{cd}-\sqrt{ae})\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{cx}}{\sqrt{a}}\right)}{2\sqrt{2}a^{3/4}(cd^2+ae^2)} + \frac{\sqrt[4]{c}(\sqrt{cd}-\sqrt{ae})\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{cx}}{\sqrt{a}}\right)}{2\sqrt{2}a^{3/4}(cd^2+ae^2)}
 \end{aligned}$$

Mathematica [A] time = 0.155439, size = 234, normalized size = 0.7

$$\frac{8a^{3/4}e^{3/2} \tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right) + \sqrt{2}\sqrt[4]{c}\sqrt{d}\left(-(\sqrt{ae} + \sqrt{cd})\left(\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{cx} + \sqrt{a} + \sqrt{cx^2}\right) - \log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{cx} + \sqrt{a} + \sqrt{cx^2}\right)\right) + (2\right)}{8a^{3/4}\sqrt{d}(ae^2 + cd^2)}$$

Antiderivative was successfully verified.

[In] Integrate[1/((d + e*x^2)*(a + c*x^4)),x]

[Out] (8*a^(3/4)*e^(3/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]] + Sqrt[2]*c^(1/4)*Sqrt[d]*((-2*Sqrt[c]*d + 2*Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + 2*(Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)] - (Sqrt[c]*d + Sqrt[a]*e)*(Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2] - Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2]))/(8*a^(3/4)*Sqrt[d]*(c*d^2 + a*e^2))

Maple [A] time = 0.054, size = 363, normalized size = 1.1

$$\frac{cd\sqrt{2}}{(8ae^2 + 8cd^2)a}\sqrt[4]{\frac{a}{c}}\ln\left(\left(x^2 + \sqrt[4]{\frac{a}{c}}x\sqrt{2} + \sqrt{\frac{a}{c}}\right)\left(x^2 - \sqrt[4]{\frac{a}{c}}x\sqrt{2} + \sqrt{\frac{a}{c}}\right)^{-1}\right) + \frac{cd\sqrt{2}}{(4ae^2 + 4cd^2)a}\sqrt[4]{\frac{a}{c}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{c}}} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(e*x^2+d)/(c*x^4+a),x)

[Out] 1/8/(a*e^2+c*d^2)*c*d*(a/c)^(1/4)/a*2^(1/2)*ln((x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))+1/4/(a*e^2+c*d^2)*c*d*(a/c)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)+1/4/(a*e^2+c*d^2)*c*d*(a/c)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)-1/8/(a*e^2+c*d^2)*e/(a/c)^(1/4)*2^(1/2)*ln((x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))-1/4/(a*e^2+c*d^2)*e/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)-1/4/(a*e^2+c*d^2)*e/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)+e^2/(a*e^2+c*d^2)/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x^2+d)/(c*x^4+a),x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [B] time = 6.76989, size = 7792, normalized size = 23.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

$$\begin{aligned} & (a^6 c^2 d^2 e^6 + a^7 e^8) / (a^2 c^2 d^4 + 2 a^2 c^2 d^2 e^2 + a^3 e^4) - (c d^2 + a e^2) \sqrt{(2 c^2 d e - (a^2 c^2 d^4 + 2 a^2 c^2 d^2 e^2 + a^3 e^4) \sqrt{(c^3 d^4 - 2 a^2 c^2 d^2 e^2 + a^2 c^2 e^4) / (a^3 c^4 d^8 + 4 a^4 c^3 d^6 e^2 + 6 a^5 c^2 d^4 e^4 + 4 a^6 c^2 d^2 e^6 + a^7 e^8)})} / (a^2 c^2 d^4 + 2 a^2 c^2 d^2 e^2 + a^3 e^4) \\ & \log(- (c^2 d^2 - a c e^2) x + (a^2 c^2 d^3 - a^2 c^2 d e^2 - (a^3 c^2 d^4 e + 2 a^4 c^2 d^2 e^3 + a^5 e^5) \sqrt{(c^3 d^4 - 2 a^2 c^2 d^2 e^2 + a^2 c^2 e^4) / (a^3 c^4 d^8 + 4 a^4 c^3 d^6 e^2 + 6 a^5 c^2 d^4 e^4 + 4 a^6 c^2 d^2 e^6 + a^7 e^8)})} \sqrt{(2 c^2 d e - (a^2 c^2 d^4 + 2 a^2 c^2 d^2 e^2 + a^3 e^4) \sqrt{(c^3 d^4 - 2 a^2 c^2 d^2 e^2 + a^2 c^2 e^4) / (a^3 c^4 d^8 + 4 a^4 c^3 d^6 e^2 + 6 a^5 c^2 d^4 e^4 + 4 a^6 c^2 d^2 e^6 + a^7 e^8)})} \sqrt{(2 c^2 d e - (a^2 c^2 d^4 + 2 a^2 c^2 d^2 e^2 + a^3 e^4) \sqrt{(c^3 d^4 - 2 a^2 c^2 d^2 e^2 + a^2 c^2 e^4) / (a^3 c^4 d^8 + 4 a^4 c^3 d^6 e^2 + 6 a^5 c^2 d^4 e^4 + 4 a^6 c^2 d^2 e^6 + a^7 e^8)})} / (a^2 c^2 d^4 + 2 a^2 c^2 d^2 e^2 + a^3 e^4) \\ & + (c d^2 + a e^2) \sqrt{(2 c^2 d e - (a^2 c^2 d^4 + 2 a^2 c^2 d^2 e^2 + a^3 e^4) \sqrt{(c^3 d^4 - 2 a^2 c^2 d^2 e^2 + a^2 c^2 e^4) / (a^3 c^4 d^8 + 4 a^4 c^3 d^6 e^2 + 6 a^5 c^2 d^4 e^4 + 4 a^6 c^2 d^2 e^6 + a^7 e^8)})} \sqrt{(2 c^2 d e - (a^2 c^2 d^4 + 2 a^2 c^2 d^2 e^2 + a^3 e^4) \sqrt{(c^3 d^4 - 2 a^2 c^2 d^2 e^2 + a^2 c^2 e^4) / (a^3 c^4 d^8 + 4 a^4 c^3 d^6 e^2 + 6 a^5 c^2 d^4 e^4 + 4 a^6 c^2 d^2 e^6 + a^7 e^8)})} \sqrt{(2 c^2 d e - (a^2 c^2 d^4 + 2 a^2 c^2 d^2 e^2 + a^3 e^4) \sqrt{(c^3 d^4 - 2 a^2 c^2 d^2 e^2 + a^2 c^2 e^4) / (a^3 c^4 d^8 + 4 a^4 c^3 d^6 e^2 + 6 a^5 c^2 d^4 e^4 + 4 a^6 c^2 d^2 e^6 + a^7 e^8)})} / (a^2 c^2 d^4 + 2 a^2 c^2 d^2 e^2 + a^3 e^4) \log(- (c^2 d^2 - a c e^2) x - (a^2 c^2 d^3 - a^2 c^2 d e^2 - (a^3 c^2 d^4 e + 2 a^4 c^2 d^2 e^3 + a^5 e^5) \sqrt{(c^3 d^4 - 2 a^2 c^2 d^2 e^2 + a^2 c^2 e^4) / (a^3 c^4 d^8 + 4 a^4 c^3 d^6 e^2 + 6 a^5 c^2 d^4 e^4 + 4 a^6 c^2 d^2 e^6 + a^7 e^8)})} \sqrt{(2 c^2 d e - (a^2 c^2 d^4 + 2 a^2 c^2 d^2 e^2 + a^3 e^4) \sqrt{(c^3 d^4 - 2 a^2 c^2 d^2 e^2 + a^2 c^2 e^4) / (a^3 c^4 d^8 + 4 a^4 c^3 d^6 e^2 + 6 a^5 c^2 d^4 e^4 + 4 a^6 c^2 d^2 e^6 + a^7 e^8)})} \sqrt{(2 c^2 d e - (a^2 c^2 d^4 + 2 a^2 c^2 d^2 e^2 + a^3 e^4) \sqrt{(c^3 d^4 - 2 a^2 c^2 d^2 e^2 + a^2 c^2 e^4) / (a^3 c^4 d^8 + 4 a^4 c^3 d^6 e^2 + 6 a^5 c^2 d^4 e^4 + 4 a^6 c^2 d^2 e^6 + a^7 e^8)})} / (a^2 c^2 d^4 + 2 a^2 c^2 d^2 e^2 + a^3 e^4)) / (c d^2 + a e^2) \end{aligned}$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x**2+d)/(c*x**4+a),x)

[Out] Timed out

Giac [A] time = 1.13057, size = 458, normalized size = 1.36

$$\frac{\left((ac^3)^{\frac{1}{4}} c^2 d - (ac^3)^{\frac{3}{4}} e \right) \arctan\left(\frac{\sqrt{2} \left(2x + \sqrt{2} \left(\frac{a}{c} \right)^{\frac{1}{4}} \right)}{2 \left(\frac{a}{c} \right)^{\frac{1}{4}}} \right)}{2 \left(\sqrt{2} ac^3 d^2 + \sqrt{2} a^2 c^2 e^2 \right)} + \frac{\left((ac^3)^{\frac{1}{4}} c^2 d - (ac^3)^{\frac{3}{4}} e \right) \arctan\left(\frac{\sqrt{2} \left(2x - \sqrt{2} \left(\frac{a}{c} \right)^{\frac{1}{4}} \right)}{2 \left(\frac{a}{c} \right)^{\frac{1}{4}}} \right)}{2 \left(\sqrt{2} ac^3 d^2 + \sqrt{2} a^2 c^2 e^2 \right)} + \frac{\left((ac^3)^{\frac{1}{4}} c^2 d + (ac^3)^{\frac{3}{4}} e \right)}{4 \left(\sqrt{2} ac^3 d^2 + \sqrt{2} a^2 c^2 e^2 \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x^2+d)/(c*x^4+a),x, algorithm="giac")

[Out] $\frac{1}{2} \left((a^2 c^3)^{\frac{1}{4}} c^2 d - (a^2 c^3)^{\frac{3}{4}} e \right) \arctan\left(\frac{1}{2} \sqrt{2} \left(2x + \sqrt{2} \left(\frac{a}{c} \right)^{\frac{1}{4}} \right) / \left(\frac{a}{c} \right)^{\frac{1}{4}} \right) / \left(\sqrt{2} a^2 c^3 d^2 + \sqrt{2} a^2 c^2 e^2 \right) + \frac{1}{2} \left((a^2 c^3)^{\frac{1}{4}} c^2 d - (a^2 c^3)^{\frac{3}{4}} e \right) \arctan\left(\frac{1}{2} \sqrt{2} \left(2x - \sqrt{2} \left(\frac{a}{c} \right)^{\frac{1}{4}} \right) / \left(\frac{a}{c} \right)^{\frac{1}{4}} \right) / \left(\sqrt{2} a^2 c^3 d^2 + \sqrt{2} a^2 c^2 e^2 \right) + \frac{1}{4} \left((a^2 c^3)^{\frac{1}{4}} c^2 d + (a^2 c^3)^{\frac{3}{4}} e \right) \log\left(x^2 + \sqrt{2} x \left(\frac{a}{c} \right)^{\frac{1}{4}} + \sqrt{2} \left(\frac{a}{c} \right)^{\frac{1}{4}} \right) / \left(\sqrt{2} a^2 c^3 d^2 + \sqrt{2} a^2 c^2 e^2 \right) - \frac{1}{4} \left((a^2 c^3)^{\frac{1}{4}} c^2 d + (a^2 c^3)^{\frac{3}{4}} e \right) \log\left(x^2 - \sqrt{2} x \left(\frac{a}{c} \right)^{\frac{1}{4}} + \sqrt{2} \left(\frac{a}{c} \right)^{\frac{1}{4}} \right) / \left(\sqrt{2} a^2 c^3 d^2 + \sqrt{2} a^2 c^2 e^2 \right) + \arctan\left(x e^{\frac{1}{2}} / \sqrt{d} \right) e^{\frac{3}{2}} / \left((c d^2 + a e^2) \sqrt{d} \right)$

$$3.143 \quad \int \frac{1}{(d+ex^2)^2(a+cx^4)} dx$$

Optimal. Leaf size=453

$$\frac{c^{3/4} (2\sqrt{a}\sqrt{cde} - ae^2 + cd^2) \log(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{cx} + \sqrt{a} + \sqrt{cx^2})}{4\sqrt{2}a^{3/4} (ae^2 + cd^2)^2} + \frac{c^{3/4} (2\sqrt{a}\sqrt{cde} - ae^2 + cd^2) \log(\sqrt{2}\sqrt[4]{a}\sqrt[4]{cx} + \sqrt{a} + \sqrt{cx^2})}{4\sqrt{2}a^{3/4} (ae^2 + cd^2)^2}$$

```
[Out] (e^2*x)/(2*d*(c*d^2 + a*e^2)*(d + e*x^2)) + (2*c*Sqrt[d]*e^(3/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(c*d^2 + a*e^2)^2 + (e^(3/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(3/2)*(c*d^2 + a*e^2)) - (c^(3/4)*(c*d^2 - 2*Sqrt[a]*Sqrt[c]*d*e - a*e^2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2)^2) + (c^(3/4)*(c*d^2 - 2*Sqrt[a]*Sqrt[c]*d*e - a*e^2)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2)^2) - (c^(3/4)*(c*d^2 + 2*Sqrt[a]*Sqrt[c]*d*e - a*e^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2)^2) + (c^(3/4)*(c*d^2 + 2*Sqrt[a]*Sqrt[c]*d*e - a*e^2)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2)^2)
```

Rubi [A] time = 0.384018, antiderivative size = 453, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {1171, 199, 205, 1168, 1162, 617, 204, 1165, 628}

$$\frac{c^{3/4} (2\sqrt{a}\sqrt{cde} - ae^2 + cd^2) \log(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{cx} + \sqrt{a} + \sqrt{cx^2})}{4\sqrt{2}a^{3/4} (ae^2 + cd^2)^2} + \frac{c^{3/4} (2\sqrt{a}\sqrt{cde} - ae^2 + cd^2) \log(\sqrt{2}\sqrt[4]{a}\sqrt[4]{cx} + \sqrt{a} + \sqrt{cx^2})}{4\sqrt{2}a^{3/4} (ae^2 + cd^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x^2)^2*(a + c*x^4)), x]
```

```
[Out] (e^2*x)/(2*d*(c*d^2 + a*e^2)*(d + e*x^2)) + (2*c*Sqrt[d]*e^(3/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(c*d^2 + a*e^2)^2 + (e^(3/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(3/2)*(c*d^2 + a*e^2)) - (c^(3/4)*(c*d^2 - 2*Sqrt[a]*Sqrt[c]*d*e - a*e^2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2)^2) + (c^(3/4)*(c*d^2 - 2*Sqrt[a]*Sqrt[c]*d*e - a*e^2)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2)^2) - (c^(3/4)*(c*d^2 + 2*Sqrt[a]*Sqrt[c]*d*e - a*e^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2)^2) + (c^(3/4)*(c*d^2 + 2*Sqrt[a]*Sqrt[c]*d*e - a*e^2)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2)^2)
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (c_.)*(x_)^4), x_Symbol] := Int[ExpandIntegrand[(d + e*x^2)^q/(a + c*x^4), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[q]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := -Simp[(x*(a + b*x^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (IntegerQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
```

ator[p + 1/n] < Denominator[p])

Rule 205

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 1168

Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + Dist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*c)]

Rule 1162

Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]

Rule 617

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 204

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 1165

Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]

Rule 628

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rubi steps

$$\begin{aligned}
\int \frac{1}{(d+ex^2)^2(a+cx^4)} dx &= \int \left(\frac{e^2}{(cd^2+ae^2)(d+ex^2)^2} + \frac{2cde^2}{(cd^2+ae^2)^2(d+ex^2)} + \frac{c(cd^2-ae^2-2cdex^2)}{(cd^2+ae^2)^2(a+cx^4)} \right) dx \\
&= \frac{c \int \frac{cd^2-ae^2-2cdex^2}{a+cx^4} dx}{(cd^2+ae^2)^2} + \frac{(2cde^2) \int \frac{1}{d+ex^2} dx}{(cd^2+ae^2)^2} + \frac{e^2 \int \frac{1}{(d+ex^2)^2} dx}{cd^2+ae^2} \\
&= \frac{e^2 x}{2d(cd^2+ae^2)(d+ex^2)} + \frac{2c\sqrt{d}e^{3/2} \tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)}{(cd^2+ae^2)^2} + \frac{(\sqrt{c}(cd^2-2\sqrt{a}\sqrt{cde}-ae^2)) \int \frac{\sqrt{a}\sqrt{c}}{a+cx^4} dx}{2\sqrt{a}(cd^2+ae^2)^2} \\
&= \frac{e^2 x}{2d(cd^2+ae^2)(d+ex^2)} + \frac{2c\sqrt{d}e^{3/2} \tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)}{(cd^2+ae^2)^2} + \frac{e^{3/2} \tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)}{2d^{3/2}(cd^2+ae^2)} + \frac{(\sqrt{c}(cd^2-2\sqrt{a}\sqrt{cde}-ae^2)) \int \frac{\sqrt{a}\sqrt{c}}{a+cx^4} dx}{4\sqrt{a}(cd^2+ae^2)^2} \\
&= \frac{e^2 x}{2d(cd^2+ae^2)(d+ex^2)} + \frac{2c\sqrt{d}e^{3/2} \tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)}{(cd^2+ae^2)^2} + \frac{e^{3/2} \tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)}{2d^{3/2}(cd^2+ae^2)} - \frac{c^{3/4}(cd^2+2\sqrt{a}\sqrt{cde}-ae^2) \int \frac{\sqrt{a}\sqrt{c}}{a+cx^4} dx}{2\sqrt{2}(cd^2+ae^2)^2} \\
&= \frac{e^2 x}{2d(cd^2+ae^2)(d+ex^2)} + \frac{2c\sqrt{d}e^{3/2} \tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)}{(cd^2+ae^2)^2} + \frac{e^{3/2} \tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)}{2d^{3/2}(cd^2+ae^2)} - \frac{c^{3/4}(cd^2-2\sqrt{a}\sqrt{cde}-ae^2) \int \frac{\sqrt{a}\sqrt{c}}{a+cx^4} dx}{2\sqrt{2}(cd^2+ae^2)^2}
\end{aligned}$$

Mathematica [A] time = 0.493526, size = 362, normalized size = 0.8

$$\frac{\sqrt{2}c^{3/4}(-2\sqrt{a}\sqrt{cde+ae^2}-cd^2) \log(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{cx+\sqrt{a}+\sqrt{cx^2}})}{a^{3/4}} + \frac{\sqrt{2}c^{3/4}(2\sqrt{a}\sqrt{cde-ae^2}+cd^2) \log(\sqrt{2}\sqrt[4]{a}\sqrt[4]{cx+\sqrt{a}+\sqrt{cx^2}})}{a^{3/4}} + \frac{2\sqrt{2}c^{3/4}(2\sqrt{a}\sqrt{cde+ae^2}-cd^2) \tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)}{a^{3/4}}}{8(ae^2+cd^2)^2}$$

Antiderivative was successfully verified.

[In] Integrate[1/((d + e*x^2)^2*(a + c*x^4)), x]

[Out] $((4e^2(c^2d^2 + ae^2)x)/(d(d + ex^2)) + (4e^{3/2}(5c^2d^2 + ae^2) \operatorname{ArcTan}[\sqrt{e}x/\sqrt{d}])/d^{3/2} + (2\sqrt{2}c^{3/4}(-c^2d^2 + 2\sqrt{a}\sqrt{cde+ae^2} + ae^2) \operatorname{ArcTan}[1 - (\sqrt{2}c^{1/4}x/a^{1/4})]/a^{3/4} - (2\sqrt{2}c^{3/4}(-c^2d^2 + 2\sqrt{a}\sqrt{cde+ae^2} + ae^2) \operatorname{ArcTan}[1 + (\sqrt{2}c^{1/4}x/a^{1/4})]/a^{3/4} + (\sqrt{2}c^{3/4}(-c^2d^2 - 2\sqrt{a}\sqrt{cde-ae^2} + ae^2) \operatorname{Log}[\sqrt{a} - \sqrt{2}a^{1/4}c^{1/4}x + \sqrt{c}x^2])/a^{3/4} + (\sqrt{2}c^{3/4}(c^2d^2 + 2\sqrt{a}\sqrt{cde-ae^2} + ae^2) \operatorname{Log}[\sqrt{a} + \sqrt{2}a^{1/4}c^{1/4}x + \sqrt{c}x^2])/a^{3/4})/(8(c^2d^2 + ae^2)^2)$

Maple [A] time = 0.056, size = 650, normalized size = 1.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(e*x^2+d)^2/(c*x^4+a), x)

[Out] $-1/4c/(ae^2+c^2d^2)^2(a/c)^{1/4}2^{1/2} \operatorname{arctan}(2^{1/2}/(a/c)^{1/4}x-1)e^{-2} + 1/4c^2/(ae^2+c^2d^2)^2(a/c)^{1/4}/a2^{1/2} \operatorname{arctan}(2^{1/2}/(a/c)^{1/4}$

$$\begin{aligned} &) * x - 1) * d^{-2-1/8} * c / (a * e^2 + c * d^2)^2 * (a/c)^{1/4} * 2^{1/2} * \ln((x^2 + (a/c)^{1/4} * x * \\ & 2^{1/2} + (a/c)^{1/2}) / (x^2 - (a/c)^{1/4} * x * 2^{1/2} + (a/c)^{1/2})) * e^{2+1/8} * c^2 / (\\ & a * e^2 + c * d^2)^2 * (a/c)^{1/4} / a * 2^{1/2} * \ln((x^2 + (a/c)^{1/4} * x * 2^{1/2} + (a/c)^{1/2}) / \\ & (x^2 - (a/c)^{1/4} * x * 2^{1/2} + (a/c)^{1/2})) * d^{-2-1/4} * c / (a * e^2 + c * d^2)^2 * (a/c)^{1/4} * 2^{1/2} * \\ & \arctan(2^{1/2} / (a/c)^{1/4} * x + 1) * e^{2+1/4} * c^2 / (a * e^2 + c * d^2)^2 * (a/c)^{1/4} / a * 2^{1/2} * \\ & \arctan(2^{1/2} / (a/c)^{1/4} * x + 1) * d^{-2-1/4} * c / (a * e^2 + c * d^2)^2 * d * e / (a/c)^{1/4} * 2^{1/2} * \\ & \ln((x^2 - (a/c)^{1/4} * x * 2^{1/2} + (a/c)^{1/2}) / (x^2 + (a/c)^{1/4} * x * 2^{1/2} + (a/c)^{1/2})) - 1/2 * c / (a * e^2 + c * d^2)^2 * d * e / (a/c)^{1/4} * 2^{1/2} * \\ & \arctan(2^{1/2} / (a/c)^{1/4} * x + 1) - 1/2 * c / (a * e^2 + c * d^2)^2 * d * e / (a/c)^{1/4} * 2^{1/2} * \\ & \arctan(2^{1/2} / (a/c)^{1/4} * x - 1) + 1/2 * e^4 / (a * e^2 + c * d^2)^2 * d * x / (e * x^2 + d) * a + 1/2 * e^2 / (a * e^2 + c * d^2)^2 * d * x / (e * x^2 + d) * c + 1/2 * e^4 / (a * e^2 + c * d^2)^2 * \\ & d / (d * e)^{1/2} * \arctan(e * x / (d * e)^{1/2}) * a + 5/2 * e^2 / (a * e^2 + c * d^2)^2 * d / (d * e)^{1/2} * \arctan(e * x / (d * e)^{1/2}) * c \end{aligned}$$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x^2+d)^2/(c*x^4+a),x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [B] time = 121.687, size = 16741, normalized size = 36.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x^2+d)^2/(c*x^4+a),x, algorithm="fricas")

[Out]
$$\begin{aligned} & [1/4 * ((c^2 * d^6 + 2 * a * c * d^4 * e^2 + a^2 * d^2 * e^4 + (c^2 * d^5 * e + 2 * a * c * d^3 * e^3 + \\ & a^2 * d * e^5) * x^2) * \sqrt{(4 * c^3 * d^3 * e - 4 * a * c^2 * d * e^3 + (a * c^4 * d^8 + 4 * a^2 * c^3 * d^6 * e^2 + 6 * a^3 * c^2 * d^4 * e^4 + 4 * a^4 * c * d^2 * e^6 + a^5 * e^8) * \sqrt{-(c^7 * d^8 - \\ & 12 * a * c^6 * d^6 * e^2 + 38 * a^2 * c^5 * d^4 * e^4 - 12 * a^3 * c^4 * d^2 * e^6 + a^4 * c^3 * e^8) / (a^3 * c^8 * d^16 + 8 * a^4 * c^7 * d^14 * e^2 + 28 * a^5 * c^6 * d^12 * e^4 + 56 * a^6 * c^5 * d^10 * e^6 + 70 * a^7 * c^4 * d^8 * e^8 + 56 * a^8 * c^3 * d^6 * e^10 + 28 * a^9 * c^2 * d^4 * e^12 + 8 * a^10 * c * d^2 * e^14 + a^11 * e^16))} / (a * c^4 * d^8 + 4 * a^2 * c^3 * d^6 * e^2 + 6 * a^3 * c^2 * d^4 * e^4 + 4 * a^4 * c * d^2 * e^6 + a^5 * e^8)) * \log((c^4 * d^4 - 6 * a * c^3 * d^2 * e^2 + a^2 * c^2 * e^4) * x + (a * c^4 * d^6 - 7 * a^2 * c^3 * d^4 * e^2 + 7 * a^3 * c^2 * d^2 * e^4 - a^4 * c * e^6 + 2 * (a^3 * c^4 * d^9 * e + 4 * a^4 * c^3 * d^7 * e^3 + 6 * a^5 * c^2 * d^5 * e^5 + 4 * a^6 * c * d^3 * e^7 + a^7 * d * e^9) * \sqrt{-(c^7 * d^8 - 12 * a * c^6 * d^6 * e^2 + 38 * a^2 * c^5 * d^4 * e^4 - 12 * a^3 * c^4 * d^2 * e^6 + a^4 * c^3 * e^8) / (a^3 * c^8 * d^16 + 8 * a^4 * c^7 * d^14 * e^2 + 28 * a^5 * c^6 * d^12 * e^4 + 56 * a^6 * c^5 * d^10 * e^6 + 70 * a^7 * c^4 * d^8 * e^8 + 56 * a^8 * c^3 * d^6 * e^10 + 28 * a^9 * c^2 * d^4 * e^12 + 8 * a^10 * c * d^2 * e^14 + a^11 * e^16))} * \sqrt{(4 * c^3 * d^3 * e - 4 * a * c^2 * d * e^3 + (a * c^4 * d^8 + 4 * a^2 * c^3 * d^6 * e^2 + 6 * a^3 * c^2 * d^4 * e^4 + 4 * a^4 * c * d^2 * e^6 + a^5 * e^8) * \sqrt{-(c^7 * d^8 - 12 * a * c^6 * d^6 * e^2 + 38 * a^2 * c^5 * d^4 * e^4 - 12 * a^3 * c^4 * d^2 * e^6 + a^4 * c^3 * e^8) / (a^3 * c^8 * d^16 + 8 * a^4 * c^7 * d^14 * e^2 + 28 * a^5 * c^6 * d^12 * e^4 + 56 * a^6 * c^5 * d^10 * e^6 + 70 * a^7 * c^4 * d^8 * e^8 + 56 * a^8 * c^3 * d^6 * e^10 + 28 * a^9 * c^2 * d^4 * e^12 + 8 * a^10 * c * d^2 * e^14 + a^11 * e^16))} / (a * c^4 * d^8 + 4 * a^2 * c^3 * d^6 * e^2 + 6 * a^3 * c^2 * d^4 * e^4 + 4 * a^4 * c * d^2 * e^6 + a^5 * e^8)) - (c^2 * d^6 + 2 * a * c * d^4 * e^2 + a^2 * d^2 * e^4 + (c^2 * d^5 * e + 2 * a * c * d^3 * e^3 + a^2 * d * e^5) * x^2) * \sqrt{(4 * c^3 * d^3 * e - 4 * a * c^2 * d * e^3 + (a * c^4 * d^8 + 4 * a^2 * c^3 * d^6 * e^2 + 6 * a^3 * c^2 * d^4 * e^4 + 4 * a^4 * c * d^2 * e^6 + a^5 * e^8) * \sqrt{-(c^7 * d^8 - 12 * a * c^6 * d^6 * e^2 + 38 * a^2 * c^5 * d^4 * e^4 - 12 * a^3 * c^4 * d^2 * e^6 + a^4 * c^3 * e^8) / (a^3 * c^8 * d^16 + 8 * a^4 * c^7 * d^14 * e^2 + 28 * a^5 * c^6 * d^12 * e^4 + 56 * a^6 * c^5 * d^10 * e^6 + 70 * a^7 * c^4 * d^8 * e^8 + 56 * a^8 * c^3 * d^6 * e^10 + 28 * a^9 * c^2 * d^4 * e^12 + 8 * a^10 * c * d^2 * e^14 + a^11 * e^16))} / (a * c^4 * d^8 + 4 * a^2 * c^3 * d^6 * e^2 + 6 * a^3 * c^2 * d^4 * e^4 + 4 * a^4 * c * d^2 * e^6 + a^5 * e^8)) \end{aligned}$$

$$\begin{aligned}
& *c^6*d^6*e^2 + 38*a^2*c^5*d^4*e^4 - 12*a^3*c^4*d^2*e^6 + a^4*c^3*e^8)/(a^3*c^8*d^16 + 8*a^4*c^7*d^14*e^2 + 28*a^5*c^6*d^12*e^4 + 56*a^6*c^5*d^10*e^6 + \\
& 70*a^7*c^4*d^8*e^8 + 56*a^8*c^3*d^6*e^10 + 28*a^9*c^2*d^4*e^12 + 8*a^10*c*d^2*e^14 + a^11*e^16)))/(a*c^4*d^8 + 4*a^2*c^3*d^6*e^2 + 6*a^3*c^2*d^4*e^4 + \\
& 4*a^4*c*d^2*e^6 + a^5*e^8))*\log((c^4*d^4 - 6*a*c^3*d^2*e^2 + a^2*c^2*e^4) \\
& *x - (a*c^4*d^6 - 7*a^2*c^3*d^4*e^2 + 7*a^3*c^2*d^2*e^4 - a^4*c*e^6 + 2*(a^3*c^4*d^9*e + 4*a^4*c^3*d^7*e^3 + 6*a^5*c^2*d^5*e^5 + 4*a^6*c*d^3*e^7 + a^7 \\
& *d*e^9)*\sqrt{-(c^7*d^8 - 12*a*c^6*d^6*e^2 + 38*a^2*c^5*d^4*e^4 - 12*a^3*c^4*d^2*e^6 + a^4*c^3*e^8)/(a^3*c^8*d^16 + 8*a^4*c^7*d^14*e^2 + 28*a^5*c^6*d^12 \\
& *e^4 + 56*a^6*c^5*d^10*e^6 + 70*a^7*c^4*d^8*e^8 + 56*a^8*c^3*d^6*e^10 + 28*a^9*c^2*d^4*e^12 + 8*a^10*c*d^2*e^14 + a^11*e^16)))*\sqrt{((4*c^3*d^3*e - 4* \\
& a*c^2*d*e^3 + (a*c^4*d^8 + 4*a^2*c^3*d^6*e^2 + 6*a^3*c^2*d^4*e^4 + 4*a^4*c*d^2*e^6 + a^5*e^8)*\sqrt{-(c^7*d^8 - 12*a*c^6*d^6*e^2 + 38*a^2*c^5*d^4*e^4 - \\
& 12*a^3*c^4*d^2*e^6 + a^4*c^3*e^8)/(a^3*c^8*d^16 + 8*a^4*c^7*d^14*e^2 + 28*a^5*c^6*d^12*e^4 + 56*a^6*c^5*d^10*e^6 + 70*a^7*c^4*d^8*e^8 + 56*a^8*c^3*d^6 \\
& *e^10 + 28*a^9*c^2*d^4*e^12 + 8*a^10*c*d^2*e^14 + a^11*e^16)))/(a*c^4*d^8 + 4*a^2*c^3*d^6*e^2 + 6*a^3*c^2*d^4*e^4 + 4*a^4*c*d^2*e^6 + a^5*e^8))) + (c \\
& ^2*d^6 + 2*a*c*d^4*e^2 + a^2*d^2*e^4 + (c^2*d^5*e + 2*a*c*d^3*e^3 + a^2*d*e^5) \\
& ^5)*x^2)*\sqrt{((4*c^3*d^3*e - 4*a*c^2*d*e^3 - (a*c^4*d^8 + 4*a^2*c^3*d^6*e^2 + 6*a^3*c^2*d^4*e^4 + 4*a^4*c*d^2*e^6 + a^5*e^8)*\sqrt{-(c^7*d^8 - 12*a*c^6 \\
& *d^6*e^2 + 38*a^2*c^5*d^4*e^4 - 12*a^3*c^4*d^2*e^6 + a^4*c^3*e^8)/(a^3*c^8*d^16 + 8*a^4*c^7*d^14*e^2 + 28*a^5*c^6*d^12*e^4 + 56*a^6*c^5*d^10*e^6 + 70*a^7*c^4*d^8 \\
& *e^8 + 56*a^8*c^3*d^6*e^10 + 28*a^9*c^2*d^4*e^12 + 8*a^10*c*d^2*e^14 + a^11*e^16)))/(a*c^4*d^8 + 4*a^2*c^3*d^6*e^2 + 6*a^3*c^2*d^4*e^4 + 4*a^4*c*d^2*e^6 + a^5 \\
& *e^8))*\log((c^4*d^4 - 6*a*c^3*d^2*e^2 + a^2*c^2*e^4)*x + \\
& (a*c^4*d^6 - 7*a^2*c^3*d^4*e^2 + 7*a^3*c^2*d^2*e^4 - a^4*c*e^6 - 2*(a^3*c^4*d^9*e + 4*a^4*c^3*d^7*e^3 + 6*a^5*c^2*d^5*e^5 + 4*a^6*c*d^3*e^7 + a^7*d*e^9) \\
& *\sqrt{-(c^7*d^8 - 12*a*c^6*d^6*e^2 + 38*a^2*c^5*d^4*e^4 - 12*a^3*c^4*d^2*e^6 + a^4*c^3*e^8)/(a^3*c^8*d^16 + 8*a^4*c^7*d^14*e^2 + 28*a^5*c^6*d^12*e^4 + 56*a^6*c^5 \\
& *d^10*e^6 + 70*a^7*c^4*d^8*e^8 + 56*a^8*c^3*d^6*e^10 + 28*a^9*c^2*d^4*e^12 + 8*a^10*c*d^2*e^14 + a^11*e^16)))*\sqrt{((4*c^3*d^3*e - 4*a*c^2*d*e^3 - (a*c^4*d^8 + 4*a^2 \\
& *c^3*d^6*e^2 + 6*a^3*c^2*d^4*e^4 + 4*a^4*c*d^2*e^6 + a^5*e^8)*\sqrt{-(c^7*d^8 - 12*a*c^6*d^6 \\
& *e^2 + 38*a^2*c^5*d^4*e^4 - 12*a^3*c^4*d^2*e^6 + a^4*c^3*e^8)/(a^3*c^8*d^16 + 8*a^4*c^7*d^14*e^2 + 28*a^5*c^6*d^12*e^4 + 56*a^6*c^5*d^10*e^6 + 70*a^7*c^4*d^8 \\
& *e^8 + 56*a^8*c^3*d^6*e^10 + 28*a^9*c^2*d^4*e^12 + 8*a^10*c*d^2*e^14 + a^11*e^16)))/(a*c^4*d^8 + 4*a^2*c^3*d^6*e^2 + 6*a^3*c^2*d^4*e^4 + 4*a^4*c*d^2*e^6 + a^5 \\
& *e^8))) - (c^2*d^6 + 2*a*c*d^4*e^2 + a^2*d^2*e^4 + (c^2*d^5*e + 2*a*c*d^3*e^3 + a^2*d*e^5) \\
& ^5)*x^2)*\sqrt{((4*c^3*d^3*e - 4*a*c^2*d*e^3 - (a*c^4*d^8 + 4*a^2*c^3*d^6*e^2 + 6 \\
& *a^3*c^2*d^4*e^4 + 4*a^4*c*d^2*e^6 + a^5*e^8)*\sqrt{-(c^7*d^8 - 12*a*c^6*d^6 \\
& *e^2 + 38*a^2*c^5*d^4*e^4 - 12*a^3*c^4*d^2*e^6 + a^4*c^3*e^8)/(a^3*c^8*d^16 + 8*a^4*c^7*d^14*e^2 + 28*a^5*c^6*d^12*e^4 + 56*a^6*c^5*d^10*e^6 + 70*a^7*c^4*d^8 \\
& *e^8 + 56*a^8*c^3*d^6*e^10 + 28*a^9*c^2*d^4*e^12 + 8*a^10*c*d^2*e^14 + a^11*e^16)))/(a*c^4*d^8 + 4*a^2*c^3*d^6*e^2 + 6*a^3*c^2*d^4*e^4 + 4*a^4*c*d^2*e^6 + a^5 \\
& *e^8))) + (5*c*d^3*e + a*d*e^3 + (5*c*d^2*e^2 + a*e^4)*x^2)*\sqrt{(-e/d)*\log((e*x^2 + 2*d*x*\sqrt{(-e/d) - d)/(e*x^2 + d)} + 2*(c*d^2*e^2 + a*e^4)*x)/(c^2*d^6 + 2*a*c*d^4*e^2
\end{aligned}$$

$$d^{12}e^4 + 56a^6c^5d^{10}e^6 + 70a^7c^4d^8e^8 + 56a^8c^3d^6e^{10} + 28a^9c^2d^4e^{12} + 8a^{10}cd^2e^{14} + a^{11}e^{16}) / (a^4c^4d^8 + 4a^2c^3d^6e^2 + 6a^3c^2d^4e^4 + 4a^4cd^2e^6 + a^5e^8) - (c^2d^6 + 2a^2cd^4e^2 + a^2d^2e^4 + (c^2d^5e + 2a^2cd^3e^3 + a^2d^2e^5)x^2) \sqrt{(4c^3d^3e - 4a^2cd^2e^3 - (a^4c^4d^8 + 4a^2c^3d^6e^2 + 6a^3c^2d^4e^4 + 4a^4cd^2e^6 + a^5e^8)) \sqrt{-(c^7d^8 - 12a^6c^6d^6e^2 + 38a^2c^5d^4e^4 - 12a^3c^4d^2e^6 + a^4c^3e^8) / (a^3c^8d^{16} + 8a^4c^7d^{14}e^2 + 28a^5c^6d^{12}e^4 + 56a^6c^5d^{10}e^6 + 70a^7c^4d^8e^8 + 56a^8c^3d^6e^{10} + 28a^9c^2d^4e^{12} + 8a^{10}cd^2e^{14} + a^{11}e^{16})} / (a^4c^4d^8 + 4a^2c^3d^6e^2 + 6a^3c^2d^4e^4 + 4a^4cd^2e^6 + a^5e^8)) \log((c^4d^4 - 6a^3c^3d^2e^2 + a^2c^2e^4)x - (a^4c^4d^6 - 7a^2c^3d^4e^2 + 7a^3c^2d^2e^4 - a^4c^2e^6 - 2(a^3c^4d^9e + 4a^4c^3d^7e^3 + 6a^5c^2d^5e^5 + 4a^6cd^3e^7 + a^7d^2e^9)) \sqrt{-(c^7d^8 - 12a^6c^6d^6e^2 + 38a^2c^5d^4e^4 - 12a^3c^4d^2e^6 + a^4c^3e^8) / (a^3c^8d^{16} + 8a^4c^7d^{14}e^2 + 28a^5c^6d^{12}e^4 + 56a^6c^5d^{10}e^6 + 70a^7c^4d^8e^8 + 56a^8c^3d^6e^{10} + 28a^9c^2d^4e^{12} + 8a^{10}cd^2e^{14} + a^{11}e^{16})} \sqrt{(4c^3d^3e - 4a^2cd^2e^3 - (a^4c^4d^8 + 4a^2c^3d^6e^2 + 6a^3c^2d^4e^4 + 4a^4cd^2e^6 + a^5e^8)) \sqrt{-(c^7d^8 - 12a^6c^6d^6e^2 + 38a^2c^5d^4e^4 - 12a^3c^4d^2e^6 + a^4c^3e^8) / (a^3c^8d^{16} + 8a^4c^7d^{14}e^2 + 28a^5c^6d^{12}e^4 + 56a^6c^5d^{10}e^6 + 70a^7c^4d^8e^8 + 56a^8c^3d^6e^{10} + 28a^9c^2d^4e^{12} + 8a^{10}cd^2e^{14} + a^{11}e^{16})} / (a^4c^4d^8 + 4a^2c^3d^6e^2 + 6a^3c^2d^4e^4 + 4a^4cd^2e^6 + a^5e^8)) + 2(c^2d^2e^2 + a^2e^4)x / (c^2d^6 + 2a^2cd^4e^2 + a^2d^2e^4 + (c^2d^5e + 2a^2cd^3e^3 + a^2d^2e^5)x^2)]$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x**2+d)**2/(c*x**4+a), x)

[Out] Timed out

Giac [A] time = 1.25946, size = 698, normalized size = 1.54

$$\frac{(5cd^2e^2 + ae^4) \arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right) e^{\left(-\frac{1}{2}\right)} \left(\left(ac^3\right)^{\frac{1}{4}} c^2d^2 - \left(ac^3\right)^{\frac{1}{4}} ace^2 - 2\left(ac^3\right)^{\frac{3}{4}} de\right) \arctan\left(\frac{\sqrt{2}\left(2x + \sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{2\left(c^2d^5 + 2acd^3e^2 + a^2de^4\right)\sqrt{d}} + \frac{\left(\left(ac^3\right)^{\frac{1}{4}} c^2d^2 - \left(ac^3\right)^{\frac{1}{4}} ace^2 - 2\left(ac^3\right)^{\frac{3}{4}} de\right) \arctan\left(\frac{\sqrt{2}\left(2x + \sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}ac^3d^4 + 2\sqrt{2}a^2c^2d^2e^2 + \sqrt{2}a^3ce^4\right)} + \frac{\left(\left(ac^3\right)^{\frac{1}{4}} c^2d^2\right)}{2\left(\sqrt{2}ac^3d^4 + 2\sqrt{2}a^2c^2d^2e^2 + \sqrt{2}a^3ce^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x^2+d)^2/(c*x^4+a), x, algorithm="giac")

[Out] 1/2*(5*c*d^2*e^2 + a*e^4)*arctan(x*e^(1/2)/sqrt(d))*e^(-1/2)/((c^2*d^5 + 2*a*c*d^3*e^2 + a^2*d*e^4)*sqrt(d)) + 1/2*((a*c^3)^(1/4)*c^2*d^2 - (a*c^3)^(1/4)*a*c*e^2 - 2*(a*c^3)^(3/4)*d*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a*c^3*d^4 + 2*sqrt(2)*a^2*c^2*d^2*e^2 + sqrt(2)*a^3*c*e^4) + 1/2*((a*c^3)^(1/4)*c^2*d^2 - (a*c^3)^(1/4)*a*c*e^2 - 2*(a*c^3)^(3/4)*d*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(

$$\begin{aligned} & \sqrt{2} * a * c^3 * d^4 + 2 * \sqrt{2} * a^2 * c^2 * d^2 * e^2 + \sqrt{2} * a^3 * c * e^4 + 1/4 * ((\\ & a * c^3)^{1/4} * c^2 * d^2 - (a * c^3)^{1/4} * a * c * e^2 + 2 * (a * c^3)^{3/4} * d * e) * \log(x^2 \\ & + \sqrt{2} * x * (a/c)^{1/4} + \sqrt{a/c}) / (\sqrt{2} * a * c^3 * d^4 + 2 * \sqrt{2} * a^2 * c^2 * d^2 * e^2 + \sqrt{2} * a^3 * c * e^4) \\ & - 1/4 * ((a * c^3)^{1/4} * c^2 * d^2 - (a * c^3)^{1/4} * a * c * e^2 + 2 * (a * c^3)^{3/4} * d * e) * \log(x^2 - \sqrt{2} * x * (a/c)^{1/4} + \sqrt{a/c}) \\ &) / (\sqrt{2} * a * c^3 * d^4 + 2 * \sqrt{2} * a^2 * c^2 * d^2 * e^2 + \sqrt{2} * a^3 * c * e^4) + 1/2 * x * e^2 / ((c * d^3 + a * d * e^2) * (x^2 * e + d)) \end{aligned}$$

$$3.144 \quad \int \frac{(d+ex^2)^3}{(a+cx^4)^2} dx$$

Optimal. Leaf size=363

$$\frac{3(\sqrt{cd} - \sqrt{ae})(ae^2 + cd^2) \log(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{cx} + \sqrt{a} + \sqrt{cx^2})}{16\sqrt{2}a^{7/4}c^{7/4}} + \frac{3(\sqrt{cd} - \sqrt{ae})(ae^2 + cd^2) \log(\sqrt{2}\sqrt[4]{a}\sqrt[4]{cx} + \sqrt{a} + \sqrt{cx^2})}{16\sqrt{2}a^{7/4}c^{7/4}}$$

```
[Out] -((e^3*x^3)/(c*(a + c*x^4))) + (x*(d*(c*d^2 - 3*a*e^2) + 3*e*(c*d^2 + a*e^2)
)*x^2)/(4*a*c*(a + c*x^4)) - (3*(Sqrt[c]*d + Sqrt[a]*e)*(c*d^2 + a*e^2)*Ar
cTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)]/(8*Sqrt[2]*a^(7/4)*c^(7/4)) + (3*(Sq
rt[c]*d + Sqrt[a]*e)*(c*d^2 + a*e^2)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)
]/(8*Sqrt[2]*a^(7/4)*c^(7/4)) - (3*(Sqrt[c]*d - Sqrt[a]*e)*(c*d^2 + a*e^2)
*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)
)*c^(7/4)) + (3*(Sqrt[c]*d - Sqrt[a]*e)*(c*d^2 + a*e^2)*Log[Sqrt[a] + Sqrt[
2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*c^(7/4))
```

Rubi [A] time = 0.410194, antiderivative size = 363, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {1207, 1858, 1168, 1162, 617, 204, 1165, 628}

$$\frac{3(\sqrt{cd} - \sqrt{ae})(ae^2 + cd^2) \log(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{cx} + \sqrt{a} + \sqrt{cx^2})}{16\sqrt{2}a^{7/4}c^{7/4}} + \frac{3(\sqrt{cd} - \sqrt{ae})(ae^2 + cd^2) \log(\sqrt{2}\sqrt[4]{a}\sqrt[4]{cx} + \sqrt{a} + \sqrt{cx^2})}{16\sqrt{2}a^{7/4}c^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^3/(a + c*x^4)^2, x]
```

```
[Out] -((e^3*x^3)/(c*(a + c*x^4))) + (x*(d*(c*d^2 - 3*a*e^2) + 3*e*(c*d^2 + a*e^2)
)*x^2)/(4*a*c*(a + c*x^4)) - (3*(Sqrt[c]*d + Sqrt[a]*e)*(c*d^2 + a*e^2)*Ar
cTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)]/(8*Sqrt[2]*a^(7/4)*c^(7/4)) + (3*(Sq
rt[c]*d + Sqrt[a]*e)*(c*d^2 + a*e^2)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)
]/(8*Sqrt[2]*a^(7/4)*c^(7/4)) - (3*(Sqrt[c]*d - Sqrt[a]*e)*(c*d^2 + a*e^2)
*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)
)*c^(7/4)) + (3*(Sqrt[c]*d - Sqrt[a]*e)*(c*d^2 + a*e^2)*Log[Sqrt[a] + Sqrt[
2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*c^(7/4))
```

Rule 1207

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] := Sim
p[(e^q*x^(2*q - 3)*(a + c*x^4)^(p + 1))/(c*(4*p + 2*q + 1)), x] + Dist[1/(c
*(4*p + 2*q + 1)), Int[(a + c*x^4)^p*ExpandToSum[c*(4*p + 2*q + 1)*(d + e*x
^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x]
, x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[q, 1]
```

Rule 1858

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, D
ist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*Expan
dToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x] - Simp[(x*R*(a +
b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x] /; GeQ[q, n]]
```

/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]

Rule 1168

Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + Dist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*c)]

Rule 1162

Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]

Rule 617

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 204

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 1165

Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]

Rule 628

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rubi steps

$$\begin{aligned}
\int \frac{(d+ex^2)^3}{(a+cx^4)^2} dx &= -\frac{e^3 x^3}{c(a+cx^4)} - \frac{\int \frac{-cd^3-3e(cd^2+ae^2)x^2-3cde^2x^4}{(a+cx^4)^2} dx}{c} \\
&= -\frac{e^3 x^3}{c(a+cx^4)} + \frac{x(d(cd^2-3ae^2)+3e(cd^2+ae^2)x^2)}{4ac(a+cx^4)} + \frac{\int \frac{3cd(cd^2+ae^2)+3ce(cd^2+ae^2)x^2}{a+cx^4} dx}{4ac^2} \\
&= -\frac{e^3 x^3}{c(a+cx^4)} + \frac{x(d(cd^2-3ae^2)+3e(cd^2+ae^2)x^2)}{4ac(a+cx^4)} + \frac{(3(\sqrt{cd}-\sqrt{ae})(cd^2+ae^2)) \int \frac{\sqrt{a}\sqrt{c-cx^2}}{a+cx^4} dx}{8a^{3/2}c^2} \\
&= -\frac{e^3 x^3}{c(a+cx^4)} + \frac{x(d(cd^2-3ae^2)+3e(cd^2+ae^2)x^2)}{4ac(a+cx^4)} - \frac{(3(\sqrt{cd}-\sqrt{ae})(cd^2+ae^2)) \int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt{c}}+2x}{\sqrt{c}-\frac{\sqrt{2}\sqrt[4]{ax}}{\sqrt{c}}} dx}{16\sqrt{2}a^{7/4}c^{7/4}} \\
&= -\frac{e^3 x^3}{c(a+cx^4)} + \frac{x(d(cd^2-3ae^2)+3e(cd^2+ae^2)x^2)}{4ac(a+cx^4)} - \frac{3(\sqrt{cd}-\sqrt{ae})(cd^2+ae^2) \log(\sqrt{a}-\sqrt{2}\sqrt[4]{ax})}{16\sqrt{2}a^{7/4}c^{7/4}} \\
&= -\frac{e^3 x^3}{c(a+cx^4)} + \frac{x(d(cd^2-3ae^2)+3e(cd^2+ae^2)x^2)}{4ac(a+cx^4)} - \frac{3(\sqrt{cd}+\sqrt{ae})(cd^2+ae^2) \tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{ax}}{\sqrt{a}}\right)}{8\sqrt{2}a^{7/4}c^{7/4}}
\end{aligned}$$

Mathematica [A] time = 0.269247, size = 371, normalized size = 1.02

$$-\frac{8a^{3/4}c^{3/4}(ae^2x(3d+ex^2)-cd^2x(d+3ex^2))}{a+cx^4} + 3\sqrt{2}(a^{3/2}e^3 + \sqrt{acd^2e} - a\sqrt{cde^2} - c^{3/2}d^3) \log(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{cx} + \sqrt{a} + \sqrt{cx^2}) + 3\sqrt{2}(-$$

Antiderivative was successfully verified.

[In] Integrate[(d + e*x^2)^3/(a + c*x^4)^2,x]

[Out] $((-8a^{(3/4)}c^{(3/4)}(ae^2x(3d+ex^2)-cd^2x(d+3ex^2)))/(a+c*x^4) - 6*\text{Sqrt}[2]*(c^{(3/2)}d^3 + \text{Sqrt}[a]*c*d^2*e + a*\text{Sqrt}[c]*d*e^2 + a^{(3/2)}e^3)*\text{ArcTan}[1 - (\text{Sqrt}[2]*c^{(1/4)}*x)/a^{(1/4)}] + 6*\text{Sqrt}[2]*(c^{(3/2)}d^3 + \text{Sqrt}[a]*c*d^2*e + a*\text{Sqrt}[c]*d*e^2 + a^{(3/2)}e^3)*\text{ArcTan}[1 + (\text{Sqrt}[2]*c^{(1/4)}*x)/a^{(1/4)}] + 3*\text{Sqrt}[2]*(-c^{(3/2)}d^3) + \text{Sqrt}[a]*c*d^2*e - a*\text{Sqrt}[c]*d*e^2 + a^{(3/2)}e^3)*\text{Log}[\text{Sqrt}[a] - \text{Sqrt}[2]*a^{(1/4)}*c^{(1/4)}*x + \text{Sqrt}[c]*x^2] + 3*\text{Sqrt}[2]*(c^{(3/2)}d^3 - \text{Sqrt}[a]*c*d^2*e + a*\text{Sqrt}[c]*d*e^2 - a^{(3/2)}e^3)*\text{Log}[\text{Sqrt}[a] + \text{Sqrt}[2]*a^{(1/4)}*c^{(1/4)}*x + \text{Sqrt}[c]*x^2])/(32*a^{(7/4)}*c^{(7/4)})$

Maple [B] time = 0.058, size = 624, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x^2+d)^3/(c*x^4+a)^2,x)

[Out] $(-1/4*e*(a*e^2-3*c*d^2)/a/c*x^3-1/4*d*(3*a*e^2-c*d^2)/a/c*x)/(c*x^4+a)+3/32/a/c*d*(a/c)^{(1/4)}*2^{(1/2)}*\ln((x^2+(a/c)^{(1/4)}*x*2^{(1/2)}+(a/c)^{(1/2)})/(x^2-(a/c)^{(1/4)}*x*2^{(1/2)}+(a/c)^{(1/2)}))e^2+3/32/a^2*d^3*(a/c)^{(1/4)}*2^{(1/2)}*\ln((x^2+(a/c)^{(1/4)}*x*2^{(1/2)}+(a/c)^{(1/2)})/(x^2-(a/c)^{(1/4)}*x*2^{(1/2)}+(a/c)^{(1/2)}))$

$$\begin{aligned} & 1/2)) + 3/16/a/c*d*(a/c)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(a/c)^{(1/4)}*x+1)*e^{2+3} \\ & /16/a^2*d^3*(a/c)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(a/c)^{(1/4)}*x+1) + 3/16/a/c*d* \\ & (a/c)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(a/c)^{(1/4)}*x-1)*e^{2+3}/16/a^2*d^3*(a/c)^{(1/4)} \\ & *2^{(1/2)}*\arctan(2^{(1/2)}/(a/c)^{(1/4)}*x-1) + 3/32/c^2*e^3/(a/c)^{(1/4)}*2^{(1/2)} \\ & * \ln((x^2-(a/c)^{(1/4)}*x*2^{(1/2)}+(a/c)^{(1/2)})/(x^2+(a/c)^{(1/4)}*x*2^{(1/2)}+(a/c)^{(1/2)})) \\ & + 3/32/a/c*e/(a/c)^{(1/4)}*2^{(1/2)}*\ln((x^2-(a/c)^{(1/4)}*x*2^{(1/2)}+(a/c)^{(1/2)})/(x^2+(a/c)^{(1/4)}*x*2^{(1/2)}+(a/c)^{(1/2)})) \\ & *d^2 + 3/16/c^2*e^3/(a/c)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(a/c)^{(1/4)}*x+1) + 3/16/a/c*e/(a/c)^{(1/4)}*2^{(1/2)} \\ & *\arctan(2^{(1/2)}/(a/c)^{(1/4)}*x+1)*d^2 + 3/16/c^2*e^3/(a/c)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(a/c)^{(1/4)}*x-1) \\ & + 3/16/a/c*e/(a/c)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(a/c)^{(1/4)}*x-1)*d^2 \end{aligned}$$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)^3/(c*x^4+a)^2,x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [B] time = 2.41217, size = 4196, normalized size = 11.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)^3/(c*x^4+a)^2,x, algorithm="fricas")

[Out]
$$\begin{aligned} & 1/16*(4*(3*c*d^2*e - a*e^3)*x^3 - 3*(a*c^2*x^4 + a^2*c)*\sqrt{-(2*c^2*d^5*e + 4*a*c*d^3*e^3 + 2*a^2*d*e^5 + a^3*c^3*\sqrt{-(c^6*d^12 + 2*a*c^5*d^10*e^2 - a^2*c^4*d^8*e^4 - 4*a^3*c^3*d^6*e^6 - a^4*c^2*d^4*e^8 + 2*a^5*c*d^2*e^10 + a^6*e^12)/(a^7*c^7))})/(a^3*c^3))*\log(-27*(c^5*d^10 + 3*a*c^4*d^8*e^2 + 2*a^2*c^3*d^6*e^4 - 2*a^3*c^2*d^4*e^6 - 3*a^4*c*d^2*e^8 - a^5*e^10)*x + 27*(a^2*c^5*d^7 + a^3*c^4*d^5*e^2 - a^4*c^3*d^3*e^4 - a^5*c^2*d*e^6 + a^6*c^5*e*\sqrt{-(c^6*d^12 + 2*a*c^5*d^10*e^2 - a^2*c^4*d^8*e^4 - 4*a^3*c^3*d^6*e^6 - a^4*c^2*d^4*e^8 + 2*a^5*c*d^2*e^10 + a^6*e^12)/(a^7*c^7))})*\sqrt{-(2*c^2*d^5*e + 4*a*c*d^3*e^3 + 2*a^2*d*e^5 + a^3*c^3*\sqrt{-(c^6*d^12 + 2*a*c^5*d^10*e^2 - a^2*c^4*d^8*e^4 - 4*a^3*c^3*d^6*e^6 - a^4*c^2*d^4*e^8 + 2*a^5*c*d^2*e^10 + a^6*e^12)/(a^7*c^7))})/(a^3*c^3)) + 3*(a*c^2*x^4 + a^2*c)*\sqrt{-(2*c^2*d^5*e + 4*a*c*d^3*e^3 + 2*a^2*d*e^5 + a^3*c^3*\sqrt{-(c^6*d^12 + 2*a*c^5*d^10*e^2 - a^2*c^4*d^8*e^4 - 4*a^3*c^3*d^6*e^6 - a^4*c^2*d^4*e^8 + 2*a^5*c*d^2*e^10 + a^6*e^12)/(a^7*c^7))})/(a^3*c^3))*\log(-27*(c^5*d^10 + 3*a*c^4*d^8*e^2 + 2*a^2*c^3*d^6*e^4 - 2*a^3*c^2*d^4*e^6 - 3*a^4*c*d^2*e^8 - a^5*e^10)*x - 27*(a^2*c^5*d^7 + a^3*c^4*d^5*e^2 - a^4*c^3*d^3*e^4 - a^5*c^2*d*e^6 + a^6*c^5*e*\sqrt{-(c^6*d^12 + 2*a*c^5*d^10*e^2 - a^2*c^4*d^8*e^4 - 4*a^3*c^3*d^6*e^6 - a^4*c^2*d^4*e^8 + 2*a^5*c*d^2*e^10 + a^6*e^12)/(a^7*c^7))})*\sqrt{-(2*c^2*d^5*e + 4*a*c*d^3*e^3 + 2*a^2*d*e^5 + a^3*c^3*\sqrt{-(c^6*d^12 + 2*a*c^5*d^10*e^2 - a^2*c^4*d^8*e^4 - 4*a^3*c^3*d^6*e^6 - a^4*c^2*d^4*e^8 + 2*a^5*c*d^2*e^10 + a^6*e^12)/(a^7*c^7))})/(a^3*c^3)) - 3*(a*c^2*x^4 + a^2*c)*\sqrt{-(2*c^2*d^5*e + 4*a*c*d^3*e^3 + 2*a^2*d*e^5 - a^3*c^3*\sqrt{-(c^6*d^12 + 2*a*c^5*d^10*e^2 - a^2*c^4*d^8*e^4 - 4*a^3*c^3*d^6*e^6 - a^4*c^2*d^4*e^8 + 2*a^5*c*d^2*e^10 + a^6*e^12)/(a^7*c^7))})/(a^3*c^3))*\log(-27*(c^5*d^10 + 3*a*c^4*d^8*e^2 + 2*a^2*c^3*d^6*e^4 - 2*a^3*c^2*d^4*e^6 - 3*a^4*c*d^2*e^8 - a^5*e^10)*x \end{aligned}$$

$$\begin{aligned} & ^{10}x + 27*(a^2*c^5*d^7 + a^3*c^4*d^5*e^2 - a^4*c^3*d^3*e^4 - a^5*c^2*d*e^6 \\ & - a^6*c^5*e*\sqrt{-(c^6*d^{12} + 2*a*c^5*d^{10}*e^2 - a^2*c^4*d^8*e^4 - 4*a^3*c^3*d^6*e^6 - a^4*c^2*d^4*e^8 + 2*a^5*c*d^2*e^{10} + a^6*e^{12})/(a^7*c^7)})) * \sqrt{-(2*c^2*d^5*e + 4*a*c*d^3*e^3 + 2*a^2*d*e^5 - a^3*c^3*\sqrt{-(c^6*d^{12} + 2*a*c^5*d^{10}*e^2 - a^2*c^4*d^8*e^4 - 4*a^3*c^3*d^6*e^6 - a^4*c^2*d^4*e^8 + 2*a^5*c*d^2*e^{10} + a^6*e^{12})/(a^7*c^7)})/(a^3*c^3))} + 3*(a*c^2*x^4 + a^2*c) * \sqrt{-(2*c^2*d^5*e + 4*a*c*d^3*e^3 + 2*a^2*d*e^5 - a^3*c^3*\sqrt{-(c^6*d^{12} + 2*a*c^5*d^{10}*e^2 - a^2*c^4*d^8*e^4 - 4*a^3*c^3*d^6*e^6 - a^4*c^2*d^4*e^8 + 2*a^5*c*d^2*e^{10} + a^6*e^{12})/(a^7*c^7)})/(a^3*c^3))} * \log(-27*(c^5*d^{10} + 3*a*c^4*d^8*e^2 + 2*a^2*c^3*d^6*e^4 - 2*a^3*c^2*d^4*e^6 - 3*a^4*c*d^2*e^8 - a^5*e^{10})*x - 27*(a^2*c^5*d^7 + a^3*c^4*d^5*e^2 - a^4*c^3*d^3*e^4 - a^5*c^2*d*e^6 - a^6*c^5*e*\sqrt{-(c^6*d^{12} + 2*a*c^5*d^{10}*e^2 - a^2*c^4*d^8*e^4 - 4*a^3*c^3*d^6*e^6 - a^4*c^2*d^4*e^8 + 2*a^5*c*d^2*e^{10} + a^6*e^{12})/(a^7*c^7)})) * \sqrt{-(2*c^2*d^5*e + 4*a*c*d^3*e^3 + 2*a^2*d*e^5 - a^3*c^3*\sqrt{-(c^6*d^{12} + 2*a*c^5*d^{10}*e^2 - a^2*c^4*d^8*e^4 - 4*a^3*c^3*d^6*e^6 - a^4*c^2*d^4*e^8 + 2*a^5*c*d^2*e^{10} + a^6*e^{12})/(a^7*c^7)})/(a^3*c^3))} + 4*(c*d^3 - 3*a*d*e^2)*x)/(a*c^2*x^4 + a^2*c) \end{aligned}$$

Sympy [A] time = 3.67759, size = 352, normalized size = 0.97

$$\text{RootSum}\left(65536t^4a^7c^7 + t^2(9216a^6c^4de^5 + 18432a^5c^5d^3e^3 + 9216a^4c^6d^5e) + 81a^6e^{12} + 486a^5cd^2e^{10} + 1215a^4c^2d^4e^8\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x**2+d)**3/(c*x**4+a)**2,x)

[Out] RootSum(65536*_t**4*a**7*c**7 + _t**2*(9216*a**6*c**4*d*e**5 + 18432*a**5*c**5*d**3*e**3 + 9216*a**4*c**6*d**5*e) + 81*a**6*e**12 + 486*a**5*c*d**2*e**10 + 1215*a**4*c**2*d**4*e**8 + 1620*a**3*c**3*d**6*e**6 + 1215*a**2*c**4*d**8*e**4 + 486*a*c**5*d**10*e**2 + 81*c**6*d**12, Lambda(_t, _t*log(x + (4096*_t**3*a**6*c**5*e + 432*_t*a**5*c**2*d*e**6 + 720*_t*a**4*c**3*d**3*e**4 + 144*_t*a**3*c**4*d**5*e**2 - 144*_t*a**2*c**5*d**7)/(27*a**5*e**10 + 81*a**4*c*d**2*e**8 + 54*a**3*c**2*d**4*e**6 - 54*a**2*c**3*d**6*e**4 - 81*a*c**4*d**8*e**2 - 27*c**5*d**10)))) - (x**3*(a*e**3 - 3*c*d**2*e) + x*(3*a*d**2 - c*d**3))/(4*a**2*c + 4*a*c**2*x**4)

Giac [A] time = 1.18259, size = 574, normalized size = 1.58

$$\frac{3cd^2x^3e + cd^3x - ax^3e^3 - 3adxe^2}{4(cx^4 + a)ac} + \frac{3\sqrt{2}\left((ac^3)^{\frac{1}{4}}c^3d^3 + (ac^3)^{\frac{1}{4}}ac^2de^2 + (ac^3)^{\frac{3}{4}}cd^2e + (ac^3)^{\frac{3}{4}}ae^3\right)\arctan\left(\frac{\sqrt{2}(2x + \sqrt{2})}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{16a^2c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)^3/(c*x^4+a)^2,x, algorithm="giac")

[Out] 1/4*(3*c*d^2*x^3*e + c*d^3*x - a*x^3*e^3 - 3*a*d*x*e^2)/((c*x^4 + a)*a*c) + 3/16*sqrt(2)*((a*c^3)^(1/4)*c^3*d^3 + (a*c^3)^(1/4)*a*c^2*d*e^2 + (a*c^3)^(3/4)*c*d^2*e + (a*c^3)^(3/4)*a*e^3)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a^2*c^4) + 3/16*sqrt(2)*((a*c^3)^(1/4)*c^3*d^3 + (a*c^3)^(1/4)*a*c^2*d*e^2 + (a*c^3)^(3/4)*c*d^2*e + (a*c^3)^(3/4)*a*e^3)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a^2*c^4)

$$\begin{aligned} & n(1/2*\sqrt{2}*(2*x - \sqrt{2}*(a/c)^{(1/4)})/(a/c)^{(1/4)})/(a^2*c^4) + 3/32*\sqrt{2} \\ & *((a*c^3)^{(1/4)}*c^3*d^3 + (a*c^3)^{(1/4)}*a*c^2*d*e^2 - (a*c^3)^{(3/4)}*c*d \\ & ^2*e - (a*c^3)^{(3/4)}*a*e^3)*\log(x^2 + \sqrt{2}*x*(a/c)^{(1/4)} + \sqrt{a/c})/(a \\ & ^2*c^4) - 3/32*\sqrt{2}*((a*c^3)^{(1/4)}*c^3*d^3 + (a*c^3)^{(1/4)}*a*c^2*d*e^2 - \\ & (a*c^3)^{(3/4)}*c*d^2*e - (a*c^3)^{(3/4)}*a*e^3)*\log(x^2 - \sqrt{2}*x*(a/c)^{(1/4)} \\ & + \sqrt{a/c})/(a^2*c^4) \end{aligned}$$

$$3.145 \quad \int \frac{(d+ex^2)^2}{(a+cx^4)^2} dx$$

Optimal. Leaf size=349

$$\frac{(-2\sqrt{a}\sqrt{cde} + ae^2 + 3cd^2) \log(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{cx} + \sqrt{a} + \sqrt{cx^2})}{16\sqrt{2}a^{7/4}c^{5/4}} + \frac{(-2\sqrt{a}\sqrt{cde} + ae^2 + 3cd^2) \log(\sqrt{2}\sqrt[4]{a}\sqrt[4]{cx} + \sqrt{a} + \sqrt{cx^2})}{16\sqrt{2}a^{7/4}c^{5/4}}$$

```
[Out] -(e^2*x)/(3*c*(a + c*x^4)) + (x*(3*c*d^2 + a*e^2 + 6*c*d*e*x^2))/(12*a*c*(a
+ c*x^4)) - ((3*c*d^2 + 2*Sqrt[a]*Sqrt[c]*d*e + a*e^2)*ArcTan[1 - (Sqrt[2]
*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*c^(5/4)) + ((3*c*d^2 + 2*Sqrt[a]*S
qrt[c]*d*e + a*e^2)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(
7/4)*c^(5/4)) - ((3*c*d^2 - 2*Sqrt[a]*Sqrt[c]*d*e + a*e^2)*Log[Sqrt[a] - Sq
rt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*c^(5/4)) + ((3*
c*d^2 - 2*Sqrt[a]*Sqrt[c]*d*e + a*e^2)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4
)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*c^(5/4))
```

Rubi [A] time = 0.312522, antiderivative size = 349, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {1207, 1179, 1168, 1162, 617, 204, 1165, 628}

$$\frac{(-2\sqrt{a}\sqrt{cde} + ae^2 + 3cd^2) \log(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{cx} + \sqrt{a} + \sqrt{cx^2})}{16\sqrt{2}a^{7/4}c^{5/4}} + \frac{(-2\sqrt{a}\sqrt{cde} + ae^2 + 3cd^2) \log(\sqrt{2}\sqrt[4]{a}\sqrt[4]{cx} + \sqrt{a} + \sqrt{cx^2})}{16\sqrt{2}a^{7/4}c^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^2/(a + c*x^4)^2, x]
```

```
[Out] -(e^2*x)/(3*c*(a + c*x^4)) + (x*(3*c*d^2 + a*e^2 + 6*c*d*e*x^2))/(12*a*c*(a
+ c*x^4)) - ((3*c*d^2 + 2*Sqrt[a]*Sqrt[c]*d*e + a*e^2)*ArcTan[1 - (Sqrt[2]
*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*c^(5/4)) + ((3*c*d^2 + 2*Sqrt[a]*S
qrt[c]*d*e + a*e^2)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(
7/4)*c^(5/4)) - ((3*c*d^2 - 2*Sqrt[a]*Sqrt[c]*d*e + a*e^2)*Log[Sqrt[a] - Sq
rt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*c^(5/4)) + ((3*
c*d^2 - 2*Sqrt[a]*Sqrt[c]*d*e + a*e^2)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4
)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*c^(5/4))
```

Rule 1207

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] := Sim
p[(e^q*x^(2*q - 3)*(a + c*x^4)^(p + 1))/(c*(4*p + 2*q + 1)), x] + Dist[1/(c
*(4*p + 2*q + 1)), Int[(a + c*x^4)^p*ExpandToSum[c*(4*p + 2*q + 1)*(d + e*x
^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x]
, x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[q, 1]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)^(p_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] := -Simp[(x
*(d + e*x^2)*(a + c*x^4)^(p + 1))/(4*a*(p + 1)), x] + Dist[1/(4*a*(p + 1)),
Int[Simp[d*(4*p + 5) + e*(4*p + 7)*x^2, x]*(a + c*x^4)^(p + 1), x], x] /;
FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && IntegerQ[2
*p]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(d+ex^2)^2}{(a+cx^4)^2} dx &= -\frac{e^2x}{3c(a+cx^4)} - \frac{\int \frac{-3cd^2-ae^2-6cdex^2}{(a+cx^4)^2} dx}{3c} \\
&= -\frac{e^2x}{3c(a+cx^4)} + \frac{x(3cd^2+ae^2+6cdex^2)}{12ac(a+cx^4)} + \frac{\int \frac{3(3cd^2+ae^2)+6cdex^2}{a+cx^4} dx}{12ac} \\
&= -\frac{e^2x}{3c(a+cx^4)} + \frac{x(3cd^2+ae^2+6cdex^2)}{12ac(a+cx^4)} + \frac{(3cd^2-2\sqrt{a}\sqrt{cde}+ae^2) \int \frac{\sqrt{a}\sqrt{c}-cx^2}{a+cx^4} dx}{8a^{3/2}c^{3/2}} + \frac{(3cd^2+2\sqrt{a}\sqrt{cde}+ae^2) \int \frac{\sqrt{2}\sqrt[4]{a}+2x}{\sqrt{c}-\frac{\sqrt{2}\sqrt[4]{ax}}{\sqrt{c}}-x^2} dx}{16\sqrt{2}a^{7/4}c^{5/4}} \\
&= -\frac{e^2x}{3c(a+cx^4)} + \frac{x(3cd^2+ae^2+6cdex^2)}{12ac(a+cx^4)} - \frac{(3cd^2-2\sqrt{a}\sqrt{cde}+ae^2) \log(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{cx}+\sqrt{cx^2})}{16\sqrt{2}a^{7/4}c^{5/4}} - \frac{(3cd^2+2\sqrt{a}\sqrt{cde}+ae^2) \tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{cx}}{\sqrt{a}}\right)}{8\sqrt{2}a^{7/4}c^{5/4}} + \frac{(3cd^2+2\sqrt{a}\sqrt{cde}+ae^2) \log(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{cx}+\sqrt{cx^2})}{16\sqrt{2}a^{7/4}c^{5/4}}
\end{aligned}$$

Mathematica [A] time = 0.171625, size = 295, normalized size = 0.85

$$-\frac{8a^{3/4}\sqrt[4]{c}(ae^2x-cdx(d+2ex^2))}{a+cx^4} - \sqrt{2}(-2\sqrt{a}\sqrt{cde}+ae^2+3cd^2) \log(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{cx}+\sqrt{a}+\sqrt{cx^2}) + \sqrt{2}(-2\sqrt{a}\sqrt{cde}+ae^2+3cd^2) \log(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{cx}+\sqrt{cx^2})$$

Antiderivative was successfully verified.

[In] Integrate[(d + e*x^2)^2/(a + c*x^4)^2,x]

[Out] $((-8*a^{(3/4)}*c^{(1/4)}*(a*e^2*x - c*d*x*(d + 2*e*x^2)))/(a + c*x^4) - 2*\text{Sqrt}[2]*(3*c*d^2 + 2*\text{Sqrt}[a]*\text{Sqrt}[c]*d*e + a*e^2)*\text{ArcTan}[1 - (\text{Sqrt}[2]*c^{(1/4)}*x)/a^{(1/4)}] + 2*\text{Sqrt}[2]*(3*c*d^2 + 2*\text{Sqrt}[a]*\text{Sqrt}[c]*d*e + a*e^2)*\text{ArcTan}[1 + (\text{Sqrt}[2]*c^{(1/4)}*x)/a^{(1/4)}] - \text{Sqrt}[2]*(3*c*d^2 - 2*\text{Sqrt}[a]*\text{Sqrt}[c]*d*e + a*e^2)*\text{Log}[\text{Sqrt}[a] - \text{Sqrt}[2]*a^{(1/4)}*c^{(1/4)}*x + \text{Sqrt}[c]*x^2] + \text{Sqrt}[2]*(3*c*d^2 - 2*\text{Sqrt}[a]*\text{Sqrt}[c]*d*e + a*e^2)*\text{Log}[\text{Sqrt}[a] + \text{Sqrt}[2]*a^{(1/4)}*c^{(1/4)}*x + \text{Sqrt}[c]*x^2])/(32*a^{(7/4)}*c^{(5/4)})$

Maple [A] time = 0.052, size = 464, normalized size = 1.3

$$\frac{1}{cx^4+a} \left(\frac{dex^3}{2a} - \frac{(ae^2-cd^2)x}{4ac} \right) + \frac{\sqrt{2}e^2}{16ac} \sqrt[4]{\frac{a}{c}} \arctan \left(x\sqrt{2} \frac{1}{\sqrt[4]{\frac{a}{c}}} - 1 \right) + \frac{3\sqrt{2}d^2}{16a^2} \sqrt[4]{\frac{a}{c}} \arctan \left(x\sqrt{2} \frac{1}{\sqrt[4]{\frac{a}{c}}} - 1 \right) + \frac{\sqrt{2}e^2}{32ac} \sqrt[4]{\frac{a}{c}} \ln \left(\frac{x^2+a}{c} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x^2+d)^2/(c*x^4+a)^2,x)

[Out] $(1/2*d*e/a*x^3-1/4*(a*e^2-c*d^2)/a/c*x)/(c*x^4+a)+1/16/a/c*(a/c)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(a/c)^{(1/4)}*x-1)*e^2+3/16/a^2*(a/c)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(a/c)^{(1/4)}*x-1)*d^2+1/32/a/c*(a/c)^{(1/4)}*2^{(1/2)}*\ln((x^2+a)/c)$

$$\begin{aligned} & /4)*x*2^{(1/2)}+(a/c)^{(1/2)})/(x^2-(a/c)^{(1/4)}*x*2^{(1/2)}+(a/c)^{(1/2)})))*e^{2+3/3} \\ & 2/a^2*(a/c)^{(1/4)}*2^{(1/2)}*\ln((x^2+(a/c)^{(1/4)}*x*2^{(1/2)}+(a/c)^{(1/2)})/(x^2-(a/c)^{(1/4)}*x*2^{(1/2)}+(a/c)^{(1/2)})))*d^{2+1/16}/a/c*(a/c)^{(1/4)}*2^{(1/2)}*\arctan(\\ & 2^{(1/2)}/(a/c)^{(1/4)}*x+1)*e^{2+3/16}/a^2*(a/c)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(a/c)^{(1/4)}*x+1)*d^{2+1/16}/a/c*d*e/(a/c)^{(1/4)}*2^{(1/2)}*\ln((x^2-(a/c)^{(1/4)}*x*2^{(1/2)}+(a/c)^{(1/2)})/(x^2+(a/c)^{(1/4)}*x*2^{(1/2)}+(a/c)^{(1/2)})))+1/8/a/c*d*e/(a/c)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(a/c)^{(1/4)}*x+1)+1/8/a/c*d*e/(a/c)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(a/c)^{(1/4)}*x-1) \end{aligned}$$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2/(c*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.92838, size = 3308, normalized size = 9.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2/(c*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] 1/16*(8*c*d*e*x^3 + (a*c^2*x^4 + a^2*c)*sqrt(-(a^3*c^2*sqrt(-(81*c^4*d^8 + 36*a*c^3*d^6*e^2 + 22*a^2*c^2*d^4*e^4 + 4*a^3*c*d^2*e^6 + a^4*e^8))/(a^7*c^5)) + 12*c*d^3*e + 4*a*d*e^3)/(a^3*c^2))*log((81*c^4*d^8 + 108*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 + 12*a^3*c*d^2*e^6 + a^4*e^8)*x + (2*a^6*c^4*d*e*sqrt(-(81*c^4*d^8 + 36*a*c^3*d^6*e^2 + 22*a^2*c^2*d^4*e^4 + 4*a^3*c*d^2*e^6 + a^4*e^8))/(a^7*c^5)) + 27*a^2*c^4*d^6 + 15*a^3*c^3*d^4*e^2 + 5*a^4*c^2*d^2*e^4 + a^5*c*e^6)*sqrt(-(a^3*c^2*sqrt(-(81*c^4*d^8 + 36*a*c^3*d^6*e^2 + 22*a^2*c^2*d^4*e^4 + 4*a^3*c*d^2*e^6 + a^4*e^8))/(a^7*c^5)) + 12*c*d^3*e + 4*a*d*e^3)/(a^3*c^2))) - (a*c^2*x^4 + a^2*c)*sqrt(-(a^3*c^2*sqrt(-(81*c^4*d^8 + 36*a*c^3*d^6*e^2 + 22*a^2*c^2*d^4*e^4 + 4*a^3*c*d^2*e^6 + a^4*e^8))/(a^7*c^5)) + 12*c*d^3*e + 4*a*d*e^3)/(a^3*c^2))*log((81*c^4*d^8 + 108*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 + 12*a^3*c*d^2*e^6 + a^4*e^8)*x - (2*a^6*c^4*d*e*sqrt(-(81*c^4*d^8 + 36*a*c^3*d^6*e^2 + 22*a^2*c^2*d^4*e^4 + 4*a^3*c*d^2*e^6 + a^4*e^8))/(a^7*c^5)) + 27*a^2*c^4*d^6 + 15*a^3*c^3*d^4*e^2 + 5*a^4*c^2*d^2*e^4 + a^5*c*e^6)*sqrt(-(a^3*c^2*sqrt(-(81*c^4*d^8 + 36*a*c^3*d^6*e^2 + 22*a^2*c^2*d^4*e^4 + 4*a^3*c*d^2*e^6 + a^4*e^8))/(a^7*c^5)) + 12*c*d^3*e + 4*a*d*e^3)/(a^3*c^2))) - (a*c^2*x^4 + a^2*c)*sqrt((a^3*c^2*sqrt(-(81*c^4*d^8 + 36*a*c^3*d^6*e^2 + 22*a^2*c^2*d^4*e^4 + 4*a^3*c*d^2*e^6 + a^4*e^8))/(a^7*c^5)) - 12*c*d^3*e - 4*a*d*e^3)/(a^3*c^2))*log((81*c^4*d^8 + 108*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 + 12*a^3*c*d^2*e^6 + a^4*e^8)*x + (2*a^6*c^4*d*e*sqrt(-(81*c^4*d^8 + 36*a*c^3*d^6*e^2 + 22*a^2*c^2*d^4*e^4 + 4*a^3*c*d^2*e^6 + a^4*e^8))/(a^7*c^5)) - 27*a^2*c^4*d^6 - 15*a^3*c^3*d^4*e^2 - 5*a^4*c^2*d^2*e^4 - a^5*c*e^6)*sqrt((a^3*c^2*sqrt(-(81*c^4*d^8 + 36*a*c^3*d^6*e^2 + 22*a^2*c^2*d^4*e^4 + 4*a^3*c*d^2*e^6 + a^4*e^8))/(a^7*c^5)) - 12*c*d^3*e - 4*a*d*e^3)/(a^3*c^2))) + (a*c^2*x^4 + a^2*c)*sqrt((a^3*c^2*sqrt(-(81*c^4*d^8 + 36*a*c^3*d^6*e^2 + 22*a^2*c^2*d^4*e^4 + 4*a^3*c*d^2*e^6 + a^4*e^8))/(a^7*c^5)) - 12*c*d^3*e - 4*a*d*e^3)/(a^3*c^2))*log((81*c^4*d^8 + 108*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 + 12*a^3*c*d^2*e^6 + a^4*e^8)*x - (2*a^6*c^4*d*e*sqrt(-(81*c^4
```

$$\begin{aligned} & *d^8 + 36*a*c^3*d^6*e^2 + 22*a^2*c^2*d^4*e^4 + 4*a^3*c*d^2*e^6 + a^4*e^8)/(\\ & a^7*c^5)) - 27*a^2*c^4*d^6 - 15*a^3*c^3*d^4*e^2 - 5*a^4*c^2*d^2*e^4 - a^5*c \\ & *e^6)*\sqrt{(a^3*c^2*\sqrt{-(81*c^4*d^8 + 36*a*c^3*d^6*e^2 + 22*a^2*c^2*d^4*e \\ & ^4 + 4*a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5)) - 12*c*d^3*e - 4*a*d*e^3)/(a^3*c \\ & ^2))) + 4*(c*d^2 - a*e^2)*x)/(a*c^2*x^4 + a^2*c) \end{aligned}$$

Sympy [A] time = 2.12239, size = 275, normalized size = 0.79

$$\text{RootSum}\left(65536t^4a^7c^5 + t^2(2048a^5c^3de^3 + 6144a^4c^4d^3e) + a^4e^8 + 20a^3cd^2e^6 + 118a^2c^2d^4e^4 + 180ac^3d^6e^2 + 81c^4d^8, \dots\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x**2+d)**2/(c*x**4+a)**2,x)

[Out] RootSum(65536*_t**4*a**7*c**5 + _t**2*(2048*a**5*c**3*d**e**3 + 6144*a**4*c**4*d**3*e) + a**4*e**8 + 20*a**3*c*d**2*e**6 + 118*a**2*c**2*d**4*e**4 + 180*a*c**3*d**6*e**2 + 81*c**4*d**8, Lambda(_t, _t*log(x + (-8192*_t**3*a**6*c**4*d*e + 16*_t*a**5*c*e**6 - 48*_t*a**4*c**2*d**2*e**4 - 144*_t*a**3*c**3*d**4*e**2 + 432*_t*a**2*c**4*d**6)/(a**4*e**8 + 12*a**3*c*d**2*e**6 + 38*a**2*c**2*d**4*e**4 + 108*a*c**3*d**6*e**2 + 81*c**4*d**8)))) + (2*c*d*e*x**3 + x*(-a*e**2 + c*d**2))/(4*a**2*c + 4*a*c**2*x**4)

Giac [A] time = 1.13706, size = 497, normalized size = 1.42

$$\frac{2cdx^3e + cd^2x - axe^2}{4(cx^4 + a)ac} + \frac{\sqrt{2}\left(3(ac^3)^{\frac{1}{4}}c^2d^2 + (ac^3)^{\frac{1}{4}}ace^2 + 2(ac^3)^{\frac{3}{4}}de\right)\arctan\left(\frac{\sqrt{2}\left(2x - \sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{16a^2c^3} - \frac{\sqrt{2}\left(3(ac^3)^{\frac{1}{4}}c^2d^2\right)}{16a^2c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)^2/(c*x^4+a)^2,x, algorithm="giac")

[Out] 1/4*(2*c*d*x^3*e + c*d^2*x - a*x*e^2)/((c*x^4 + a)*a*c) + 1/16*sqrt(2)*(3*(a*c^3)^(1/4)*c^2*d^2 + (a*c^3)^(1/4)*a*c*e^2 + 2*(a*c^3)^(3/4)*d*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a^2*c^3) - 1/32*sqrt(2)*(3*(a*c^3)^(1/4)*c^2*d^2 + (a*c^3)^(1/4)*a*c*e^2 - 2*(a*c^3)^(3/4)*d*e)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a^2*c^3) + 1/16*sqrt(2)*(3*(a*c^3)^(1/4)*a*c^4*d^2 + (a*c^3)^(1/4)*a^2*c^3*e^2 + 2*(a*c^3)^(3/4)*a*c^2*d*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a^3*c^5) + 1/32*sqrt(2)*(3*(a*c^3)^(1/4)*a*c^4*d^2 + (a*c^3)^(1/4)*a^2*c^3*e^2 - 2*(a*c^3)^(3/4)*a*c^2*d*e)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a^3*c^5)

$$3.146 \quad \int \frac{d+ex^2}{(a+cx^4)^2} dx$$

Optimal. Leaf size=275

$$\frac{(3\sqrt{cd} - \sqrt{ae}) \log(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{cx} + \sqrt{a} + \sqrt{cx^2})}{16\sqrt{2}a^{7/4}c^{3/4}} + \frac{(3\sqrt{cd} - \sqrt{ae}) \log(\sqrt{2}\sqrt[4]{a}\sqrt[4]{cx} + \sqrt{a} + \sqrt{cx^2})}{16\sqrt{2}a^{7/4}c^{3/4}} - \frac{(\sqrt{ae} + 3\sqrt{cd}) \tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{cx} + \sqrt{a} + \sqrt{cx^2}}{8\sqrt{2}a^{7/4}c^{3/4}}\right)}{8\sqrt{2}a^{7/4}c^{3/4}}$$

[Out] (x*(d + e*x^2))/(4*a*(a + c*x^4)) - ((3*Sqrt[c]*d + Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*c^(3/4)) + ((3*Sqrt[c]*d + Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*c^(3/4)) - ((3*Sqrt[c]*d - Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*c^(3/4)) + ((3*Sqrt[c]*d - Sqrt[a]*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*c^(3/4))

Rubi [A] time = 0.203391, antiderivative size = 275, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {1179, 1168, 1162, 617, 204, 1165, 628}

$$\frac{(3\sqrt{cd} - \sqrt{ae}) \log(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{cx} + \sqrt{a} + \sqrt{cx^2})}{16\sqrt{2}a^{7/4}c^{3/4}} + \frac{(3\sqrt{cd} - \sqrt{ae}) \log(\sqrt{2}\sqrt[4]{a}\sqrt[4]{cx} + \sqrt{a} + \sqrt{cx^2})}{16\sqrt{2}a^{7/4}c^{3/4}} - \frac{(\sqrt{ae} + 3\sqrt{cd}) \tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{cx} + \sqrt{a} + \sqrt{cx^2}}{8\sqrt{2}a^{7/4}c^{3/4}}\right)}{8\sqrt{2}a^{7/4}c^{3/4}}$$

Antiderivative was successfully verified.

[In] Int[(d + e*x^2)/(a + c*x^4)^2, x]

[Out] (x*(d + e*x^2))/(4*a*(a + c*x^4)) - ((3*Sqrt[c]*d + Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*c^(3/4)) + ((3*Sqrt[c]*d + Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*c^(3/4)) - ((3*Sqrt[c]*d - Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*c^(3/4)) + ((3*Sqrt[c]*d - Sqrt[a]*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*c^(3/4))

Rule 1179

Int[((d_) + (e_)*(x_)^2)*((a_) + (c_)*(x_)^4)^(p_), x_Symbol] := -Simp[(x*(d + e*x^2)*(a + c*x^4)^(p + 1))/(4*a*(p + 1)), x] + Dist[1/(4*a*(p + 1)), Int[Simp[d*(4*p + 5) + e*(4*p + 7)*x^2, x]*(a + c*x^4)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && IntegerQ[2*p]

Rule 1168

Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + Dist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*c)]

Rule 1162

Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e

/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x] /; FreeQ[{a, c, d, e}, x] & EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]

Rule 617

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])]; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 204

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 1165

Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]

Rule 628

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rubi steps

$$\begin{aligned} \int \frac{d + ex^2}{(a + cx^4)^2} dx &= \frac{x(d + ex^2)}{4a(a + cx^4)} - \frac{\int \frac{-3d - ex^2}{a + cx^4} dx}{4a} \\ &= \frac{x(d + ex^2)}{4a(a + cx^4)} + \frac{\left(\frac{3\sqrt{cd}}{\sqrt{a}} - e\right) \int \frac{\sqrt{a}\sqrt{c} - cx^2}{a + cx^4} dx}{8ac} + \frac{\left(\frac{3\sqrt{cd}}{\sqrt{a}} + e\right) \int \frac{\sqrt{a}\sqrt{c} + cx^2}{a + cx^4} dx}{8ac} \\ &= \frac{x(d + ex^2)}{4a(a + cx^4)} + \frac{\left(\frac{3\sqrt{cd}}{\sqrt{a}} + e\right) \int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}} - \frac{\sqrt{2}\sqrt[4]{ax}}{\sqrt[4]{c}} + x^2} dx}{16ac} + \frac{\left(\frac{3\sqrt{cd}}{\sqrt{a}} + e\right) \int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}} + \frac{\sqrt{2}\sqrt[4]{ax}}{\sqrt[4]{c}} + x^2} dx}{16ac} - \frac{(3\sqrt{cd} - \sqrt{ae}) \int \frac{1}{\sqrt{a} - \sqrt{2}\sqrt[4]{a}\sqrt[4]{cx} + \sqrt{cx^2}} dx}{16\sqrt{2}a^{7/4}c^{3/4}} \\ &= \frac{x(d + ex^2)}{4a(a + cx^4)} - \frac{(3\sqrt{cd} - \sqrt{ae}) \log(\sqrt{a} - \sqrt{2}\sqrt[4]{a}\sqrt[4]{cx} + \sqrt{cx^2})}{16\sqrt{2}a^{7/4}c^{3/4}} + \frac{(3\sqrt{cd} - \sqrt{ae}) \log(\sqrt{a} + \sqrt{2}\sqrt[4]{a}\sqrt[4]{cx} + \sqrt{cx^2})}{16\sqrt{2}a^{7/4}c^{3/4}} \\ &= \frac{x(d + ex^2)}{4a(a + cx^4)} - \frac{(3\sqrt{cd} + \sqrt{ae}) \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{cx}}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}c^{3/4}} + \frac{(3\sqrt{cd} + \sqrt{ae}) \tan^{-1}\left(1 + \frac{\sqrt{2}\sqrt[4]{cx}}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}c^{3/4}} - \frac{(3\sqrt{cd} - \sqrt{ae}) \log(\sqrt{a} - \sqrt{2}\sqrt[4]{a}\sqrt[4]{cx} + \sqrt{cx^2})}{16\sqrt{2}a^{7/4}c^{3/4}} + \frac{(3\sqrt{cd} - \sqrt{ae}) \log(\sqrt{a} + \sqrt{2}\sqrt[4]{a}\sqrt[4]{cx} + \sqrt{cx^2})}{16\sqrt{2}a^{7/4}c^{3/4}} \end{aligned}$$

Mathematica [A] time = 0.287757, size = 267, normalized size = 0.97

$$\frac{\sqrt{2}(a^{3/4}e - 3\sqrt[4]{a}\sqrt{cd}) \log(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{cx} + \sqrt{a} + \sqrt{cx^2})}{c^{3/4}} + \frac{\sqrt{2}(3\sqrt[4]{a}\sqrt{cd} - a^{3/4}e) \log(\sqrt{2}\sqrt[4]{a}\sqrt[4]{cx} + \sqrt{a} + \sqrt{cx^2})}{c^{3/4}} - \frac{2\sqrt{2}\sqrt[4]{a}(\sqrt{ae} + 3\sqrt{cd}) \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{cx}}{\sqrt[4]{a}}\right)}{c^{3/4}} + \frac{2\sqrt{2}\sqrt[4]{a}(\sqrt{ae} - 3\sqrt{cd}) \tan^{-1}\left(1 + \frac{\sqrt{2}\sqrt[4]{cx}}{\sqrt[4]{a}}\right)}{c^{3/4}} + \frac{x(d + ex^2)}{4a(a + cx^4)}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e*x^2)/(a + c*x^4)^2,x]

[Out]
$$\begin{aligned} & ((8*a*x*(d + e*x^2))/(a + c*x^4) - (2*\sqrt{2}*a^{(1/4)}*(3*\sqrt{c}*d + \sqrt{a} \\ &]*e)*\text{ArcTan}[1 - (\sqrt{2}*c^{(1/4)}*x)/a^{(1/4)}])/c^{(3/4)} + (2*\sqrt{2}*a^{(1/4)} \\ & (3*\sqrt{c}*d + \sqrt{a}*e)*\text{ArcTan}[1 + (\sqrt{2}*c^{(1/4)}*x)/a^{(1/4)}])/c^{(3/4)} \\ & + (\sqrt{2}*(-3*a^{(1/4)}*\sqrt{c}*d + a^{(3/4)}*e)*\text{Log}[\sqrt{a} - \sqrt{2}*a^{(1/4)} \\ & *c^{(1/4)}*x + \sqrt{c}*x^2])/c^{(3/4)} + (\sqrt{2}*(3*a^{(1/4)}*\sqrt{c}*d - a^{(3/4)} \\ &)*e)*\text{Log}[\sqrt{a} + \sqrt{2}*a^{(1/4)}*c^{(1/4)}*x + \sqrt{c}*x^2])/c^{(3/4)})/(32*a \\ & ^2) \end{aligned}$$

Maple [A] time = 0.05, size = 303, normalized size = 1.1

$$\frac{dx}{4a(cx^4+a)} + \frac{3d\sqrt{2}}{32a^2} \sqrt[4]{\frac{a}{c}} \ln \left(\left(x^2 + \sqrt[4]{\frac{a}{c}}x\sqrt{2} + \sqrt{\frac{a}{c}} \right) \left(x^2 - \sqrt[4]{\frac{a}{c}}x\sqrt{2} + \sqrt{\frac{a}{c}} \right)^{-1} \right) + \frac{3d\sqrt{2}}{16a^2} \sqrt[4]{\frac{a}{c}} \arctan \left(x\sqrt{2} \frac{1}{\sqrt[4]{\frac{a}{c}}} + 1 \right) + \frac{3e}{16a^2} \sqrt[4]{\frac{a}{c}} \arctan \left(x\sqrt{2} \frac{1}{\sqrt[4]{\frac{a}{c}}} - 1 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x^2+d)/(c*x^4+a)^2,x)

[Out]
$$\begin{aligned} & 1/4*d*x/a/(c*x^4+a)+3/32*d/a^2*(a/c)^{(1/4)}*2^{(1/2)}*\ln((x^2+(a/c)^{(1/4)}*x*2^{(1/2)} \\ & +(a/c)^{(1/2)})/(x^2-(a/c)^{(1/4)}*x*2^{(1/2)}+(a/c)^{(1/2)}))+3/16*d/a^2*(a/c)^{(1/4)}*2^{(1/2)} \\ & *\arctan(2^{(1/2)}/(a/c)^{(1/4)}*x+1)+3/16*d/a^2*(a/c)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(a/c)^{(1/4)}*x-1) \\ & +1/4*e*x^3/a/(c*x^4+a)+1/32*e/a/c/(a/c)^{(1/4)}*2^{(1/2)}*\ln((x^2-(a/c)^{(1/4)}*x*2^{(1/2)}+(a/c)^{(1/2)})/(x^2+(a/c)^{(1/4)}*x \\ & *2^{(1/2)}+(a/c)^{(1/2)}))+1/16*e/a/c/(a/c)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(a/c)^{(1/4)}*x+1) \\ & +1/16*e/a/c/(a/c)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(a/c)^{(1/4)}*x-1) \end{aligned}$$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)/(c*x^4+a)^2,x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [B] time = 1.78671, size = 1825, normalized size = 6.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)/(c*x^4+a)^2,x, algorithm="fricas")

[Out]
$$\begin{aligned} & 1/16*(4*e*x^3 - (a*c*x^4 + a^2)*\text{sqrt}(-(a^3*c*\text{sqrt}(-(81*c^2*d^4 - 18*a*c*d^2 \\ & *e^2 + a^2*e^4)/(a^7*c^3)) + 6*d*e)/(a^3*c)))*\text{log}(-(81*c^2*d^4 - a^2*e^4)*x \\ & + (a^6*c^2*e*\text{sqrt}(-(81*c^2*d^4 - 18*a*c*d^2*e^2 + a^2*e^4)/(a^7*c^3)) + 27* \\ & a^2*c^2*d^3 - 3*a^3*c*d*e^2)*\text{sqrt}(-(a^3*c*\text{sqrt}(-(81*c^2*d^4 - 18*a*c*d^2*e^2 \\ & + a^2*e^4)/(a^7*c^3)) + 6*d*e)/(a^3*c))) + (a*c*x^4 + a^2)*\text{sqrt}(-(a^3*c*s \end{aligned}$$

```

qrt(-(81*c^2*d^4 - 18*a*c*d^2*e^2 + a^2*e^4)/(a^7*c^3)) + 6*d*e)/(a^3*c))*1
og(-(81*c^2*d^4 - a^2*e^4)*x - (a^6*c^2*e*sqrt(-(81*c^2*d^4 - 18*a*c*d^2*e^
2 + a^2*e^4)/(a^7*c^3)) + 27*a^2*c^2*d^3 - 3*a^3*c*d*e^2)*sqrt(-(a^3*c*sqrt
(-(81*c^2*d^4 - 18*a*c*d^2*e^2 + a^2*e^4)/(a^7*c^3)) + 6*d*e)/(a^3*c))) + (
a*c*x^4 + a^2)*sqrt((a^3*c*sqrt(-(81*c^2*d^4 - 18*a*c*d^2*e^2 + a^2*e^4)/(a
^7*c^3)) - 6*d*e)/(a^3*c))*log(-(81*c^2*d^4 - a^2*e^4)*x + (a^6*c^2*e*sqrt(
-(81*c^2*d^4 - 18*a*c*d^2*e^2 + a^2*e^4)/(a^7*c^3)) - 27*a^2*c^2*d^3 + 3*a^
3*c*d*e^2)*sqrt((a^3*c*sqrt(-(81*c^2*d^4 - 18*a*c*d^2*e^2 + a^2*e^4)/(a^7*c
^3)) - 6*d*e)/(a^3*c))) - (a*c*x^4 + a^2)*sqrt((a^3*c*sqrt(-(81*c^2*d^4 - 1
8*a*c*d^2*e^2 + a^2*e^4)/(a^7*c^3)) - 6*d*e)/(a^3*c))*log(-(81*c^2*d^4 - a^
2*e^4)*x - (a^6*c^2*e*sqrt(-(81*c^2*d^4 - 18*a*c*d^2*e^2 + a^2*e^4)/(a^7*c^
3)) - 27*a^2*c^2*d^3 + 3*a^3*c*d*e^2)*sqrt((a^3*c*sqrt(-(81*c^2*d^4 - 18*a*
c*d^2*e^2 + a^2*e^4)/(a^7*c^3)) - 6*d*e)/(a^3*c))) + 4*d*x)/(a*c*x^4 + a^2)

```

Sympy [A] time = 1.16455, size = 136, normalized size = 0.49

$$\text{RootSum}\left(65536t^4a^7c^3 + 3072t^2a^4c^2de + a^2e^4 + 18acd^2e^2 + 81c^2d^4, \left(t \mapsto t \log\left(x + \frac{4096t^3a^6c^2e + 144ta^3cde^2 - 432}{a^2e^4 - 81c^2d^4}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x**2+d)/(c*x**4+a)**2,x)

[Out] RootSum(65536*_t**4*a**7*c**3 + 3072*_t**2*a**4*c**2*d*e + a**2*e**4 + 18*a*c*d**2*e**2 + 81*c**2*d**4, Lambda(_t, _t*log(x + (4096*_t**3*a**6*c**2*e + 144*_t*a**3*c*d*e**2 - 432*_t*a**2*c**2*d**3)/(a**2*e**4 - 81*c**2*d**4))) + (d*x + e*x**3)/(4*a**2 + 4*a*c*x**4)

Giac [A] time = 1.12612, size = 369, normalized size = 1.34

$$\frac{x^3e + dx}{4(cx^4 + a)a} + \frac{\sqrt{2}\left(3(ac^3)^{\frac{1}{4}}c^2d + (ac^3)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2x + \sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{16a^2c^3} + \frac{\sqrt{2}\left(3(ac^3)^{\frac{1}{4}}c^2d + (ac^3)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2x - \sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{16a^2c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)/(c*x^4+a)^2,x, algorithm="giac")

[Out] 1/4*(x^3*e + d*x)/((c*x^4 + a)*a) + 1/16*sqrt(2)*(3*(a*c^3)^(1/4)*c^2*d + (a*c^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a^2*c^3) + 1/16*sqrt(2)*(3*(a*c^3)^(1/4)*c^2*d + (a*c^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a^2*c^3) + 1/32*sqrt(2)*(3*(a*c^3)^(1/4)*c^2*d - (a*c^3)^(3/4)*e)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a^2*c^3) - 1/32*sqrt(2)*(3*(a*c^3)^(1/4)*c^2*d - (a*c^3)^(3/4)*e)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a^2*c^3)

$$3.147 \quad \int \frac{1}{(a+cx^4)^2} dx$$

Optimal. Leaf size=202

$$-\frac{3 \log(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{cx} + \sqrt{a} + \sqrt{cx^2})}{16\sqrt{2}a^{7/4}\sqrt[4]{c}} + \frac{3 \log(\sqrt{2}\sqrt[4]{a}\sqrt[4]{cx} + \sqrt{a} + \sqrt{cx^2})}{16\sqrt{2}a^{7/4}\sqrt[4]{c}} - \frac{3 \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{cx}}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}\sqrt[4]{c}} + \frac{3 \tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{cx}}{\sqrt[4]{a}} + 1\right)}{8\sqrt{2}a^{7/4}\sqrt[4]{c}} + \frac{1}{4}$$

[Out] x/(4*a*(a + c*x^4)) - (3*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*c^(1/4)) + (3*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*c^(1/4)) - (3*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*c^(1/4)) + (3*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*c^(1/4))

Rubi [A] time = 0.132941, antiderivative size = 202, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.778$, Rules used = {199, 211, 1165, 628, 1162, 617, 204}

$$-\frac{3 \log(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{cx} + \sqrt{a} + \sqrt{cx^2})}{16\sqrt{2}a^{7/4}\sqrt[4]{c}} + \frac{3 \log(\sqrt{2}\sqrt[4]{a}\sqrt[4]{cx} + \sqrt{a} + \sqrt{cx^2})}{16\sqrt{2}a^{7/4}\sqrt[4]{c}} - \frac{3 \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{cx}}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}\sqrt[4]{c}} + \frac{3 \tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{cx}}{\sqrt[4]{a}} + 1\right)}{8\sqrt{2}a^{7/4}\sqrt[4]{c}} + \frac{1}{4}$$

Antiderivative was successfully verified.

[In] Int[(a + c*x^4)^(-2), x]

[Out] x/(4*a*(a + c*x^4)) - (3*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*c^(1/4)) + (3*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*c^(1/4)) - (3*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*c^(1/4)) + (3*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*c^(1/4))

Rule 199

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := -Simp[(x*(a + b*x^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (IntegerQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denominator[p + 1/n] < Denominator[p])

Rule 211

Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] := With[{r = Numerator[Rt[a/b, 2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4), x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] && AtomQ[SplitProduct[SumBaseQ, b]]))

Rule 1165

Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]

Rule 628

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := S
 imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[2*c*d - b*e, 0]

Rule 1162

Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[
 (2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
 /(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x] /; FreeQ[{a, c, d, e}, x] &
 & EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]

Rule 617

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
 imply[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c))] /; Free
 Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 204

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
 -a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
 a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned} \int \frac{1}{(a+cx^4)^2} dx &= \frac{x}{4a(a+cx^4)} + \frac{3 \int \frac{1}{a+cx^4} dx}{4a} \\ &= \frac{x}{4a(a+cx^4)} + \frac{3 \int \frac{\sqrt{a}-\sqrt{cx^2}}{a+cx^4} dx}{8a^{3/2}} + \frac{3 \int \frac{\sqrt{a}+\sqrt{cx^2}}{a+cx^4} dx}{8a^{3/2}} \\ &= \frac{x}{4a(a+cx^4)} + \frac{3 \int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}} - \frac{\sqrt{2}\sqrt[4]{ax}}{\sqrt[4]{c}} + x^2} dx}{16a^{3/2}\sqrt{c}} + \frac{3 \int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}} + \frac{\sqrt{2}\sqrt[4]{ax}}{\sqrt[4]{c}} + x^2} dx}{16a^{3/2}\sqrt{c}} - \frac{3 \int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}} + 2x}{-\frac{\sqrt{a}}{\sqrt{c}} - \frac{\sqrt{2}\sqrt[4]{ax}}{\sqrt[4]{c}} - x^2} dx}{16\sqrt{2}a^{7/4}\sqrt[4]{c}} - \frac{3 \int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}} - 2x}{-\frac{\sqrt{a}}{\sqrt{c}} + \frac{\sqrt{2}\sqrt[4]{ax}}{\sqrt[4]{c}}} dx}{16\sqrt{2}a^{7/4}\sqrt[4]{c}} \\ &= \frac{x}{4a(a+cx^4)} - \frac{3 \log(\sqrt{a} - \sqrt{2}\sqrt[4]{a}\sqrt[4]{cx} + \sqrt{cx^2})}{16\sqrt{2}a^{7/4}\sqrt[4]{c}} + \frac{3 \log(\sqrt{a} + \sqrt{2}\sqrt[4]{a}\sqrt[4]{cx} + \sqrt{cx^2})}{16\sqrt{2}a^{7/4}\sqrt[4]{c}} + \frac{3 \operatorname{Subst}\left(\int \frac{1}{u^2} du\right)}{8\sqrt{2}a^{7/4}\sqrt[4]{c}} \\ &= \frac{x}{4a(a+cx^4)} - \frac{3 \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{cx}}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}\sqrt[4]{c}} + \frac{3 \tan^{-1}\left(1 + \frac{\sqrt{2}\sqrt[4]{cx}}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}\sqrt[4]{c}} - \frac{3 \log(\sqrt{a} - \sqrt{2}\sqrt[4]{a}\sqrt[4]{cx} + \sqrt{cx^2})}{16\sqrt{2}a^{7/4}\sqrt[4]{c}} + \frac{3 \log(\sqrt{a} + \sqrt{2}\sqrt[4]{a}\sqrt[4]{cx} + \sqrt{cx^2})}{16\sqrt{2}a^{7/4}\sqrt[4]{c}} \end{aligned}$$

Mathematica [A] time = 0.112389, size = 183, normalized size = 0.91

$$\frac{8a^{3/4}x}{a+cx^4} - \frac{3\sqrt{2}\log(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{cx} + \sqrt{a} + \sqrt{cx^2})}{\sqrt[4]{c}} + \frac{3\sqrt{2}\log(\sqrt{2}\sqrt[4]{a}\sqrt[4]{cx} + \sqrt{a} + \sqrt{cx^2})}{\sqrt[4]{c}} - \frac{6\sqrt{2}\tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{cx}}{\sqrt[4]{a}}\right)}{\sqrt[4]{c}} + \frac{6\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{cx}}{\sqrt[4]{a}} + 1\right)}{\sqrt[4]{c}}$$

$$32a^{7/4}$$

Antiderivative was successfully verified.

[In] Integrate[(a + c*x^4)^(-2), x]

[Out] $((8*a^{(3/4)*x}/(a + c*x^4) - (6*\text{Sqrt}[2]*\text{ArcTan}[1 - (\text{Sqrt}[2]*c^{(1/4)*x})/a^{(1/4)}])/c^{(1/4)} + (6*\text{Sqrt}[2]*\text{ArcTan}[1 + (\text{Sqrt}[2]*c^{(1/4)*x})/a^{(1/4)}])/c^{(1/4)} - (3*\text{Sqrt}[2]*\text{Log}[\text{Sqrt}[a] - \text{Sqrt}[2]*a^{(1/4)*c^{(1/4)*x} + \text{Sqrt}[c]*x^2])/c^{(1/4)} + (3*\text{Sqrt}[2]*\text{Log}[\text{Sqrt}[a] + \text{Sqrt}[2]*a^{(1/4)*c^{(1/4)*x} + \text{Sqrt}[c]*x^2])/c^{(1/4)})/(32*a^{(7/4)})$

Maple [A] time = 0.049, size = 143, normalized size = 0.7

$$\frac{x}{4a(cx^4 + a)} + \frac{3\sqrt{2}}{32a^2} \sqrt{\frac{a}{c}} \ln \left(\left(x^2 + \sqrt{\frac{a}{c}} x \sqrt{2} + \sqrt{\frac{a}{c}} \right) \left(x^2 - \sqrt{\frac{a}{c}} x \sqrt{2} + \sqrt{\frac{a}{c}} \right)^{-1} \right) + \frac{3\sqrt{2}}{16a^2} \sqrt{\frac{a}{c}} \arctan \left(x \sqrt{2} \frac{1}{\sqrt{\frac{a}{c}}} + 1 \right) + \frac{3\sqrt{2}}{16a^2} \sqrt{\frac{a}{c}} \arctan \left(x \sqrt{2} \frac{1}{\sqrt{\frac{a}{c}}} - 1 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(c*x^4+a)^2,x)`

[Out] $1/4*x/a/(c*x^4+a) + 3/32/a^2*(a/c)^{(1/4)}*2^{(1/2)}*\ln((x^2+(a/c)^{(1/4)}*x*2^{(1/2)}+(a/c)^{(1/2)})/(x^2-(a/c)^{(1/4)}*x*2^{(1/2)}+(a/c)^{(1/2)})) + 3/16/a^2*(a/c)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(a/c)^{(1/4)}*x+1) + 3/16/a^2*(a/c)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(a/c)^{(1/4)}*x-1)$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(c*x^4+a)^2,x, algorithm="maxima")`

[Out] Exception raised: ValueError

Fricas [A] time = 1.69677, size = 414, normalized size = 2.05

$$\frac{12(acx^4 + a^2) \left(-\frac{1}{a^7c} \right)^{\frac{1}{4}} \arctan \left(-a^5cx \left(-\frac{1}{a^7c} \right)^{\frac{3}{4}} + \sqrt{a^4 \sqrt{-\frac{1}{a^7c}} + x^2 a^5c} \left(-\frac{1}{a^7c} \right)^{\frac{3}{4}} \right) + 3(acx^4 + a^2) \left(-\frac{1}{a^7c} \right)^{\frac{1}{4}} \log \left(a^2 \left(-\frac{1}{a^7c} \right)^{\frac{1}{4}} + \sqrt{a^4 \sqrt{-\frac{1}{a^7c}} + x^2 a^5c} \left(-\frac{1}{a^7c} \right)^{\frac{3}{4}} \right)}{16(acx^4 + a^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(c*x^4+a)^2,x, algorithm="fricas")`

[Out] $1/16*(12*(a*c*x^4 + a^2)*(-1/(a^7*c))^{(1/4)}*\arctan(-a^5*c*x*(-1/(a^7*c))^{(3/4)} + \text{sqrt}(a^4*\text{sqrt}(-1/(a^7*c)) + x^2)*a^5*c*(-1/(a^7*c))^{(3/4)}) + 3*(a*c*x^4 + a^2)*(-1/(a^7*c))^{(1/4)}*\log(a^2*(-1/(a^7*c))^{(1/4)} + \text{sqrt}(a^4*\text{sqrt}(-1/(a^7*c)) + x^2)*a^5*c*(-1/(a^7*c))^{(3/4)}) - 3*(a*c*x^4 + a^2)*(-1/(a^7*c))^{(1/4)}*\log(-a^2*(-1/(a^7*c))^{(1/4)} + \text{sqrt}(a^4*\text{sqrt}(-1/(a^7*c)) + x^2)*a^5*c*(-1/(a^7*c))^{(3/4)}) + 4*x)/(a*c*x^4 + a^2)$

Sympy [A] time = 0.502584, size = 39, normalized size = 0.19

$$\frac{x}{4a^2 + 4acx^4} + \text{RootSum}\left(65536t^4a^7c + 81, \left(t \mapsto t \log\left(\frac{16ta^2}{3} + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x**4+a)**2,x)

[Out] x/(4*a**2 + 4*a*c*x**4) + RootSum(65536*_t**4*a**7*c + 81, Lambda(_t, _t*log(16*_t*a**2/3 + x)))

Giac [A] time = 1.11033, size = 262, normalized size = 1.3

$$\frac{x}{4(cx^4 + a)a} + \frac{3\sqrt{2}(ac^3)^{\frac{1}{4}} \arctan\left(\frac{\sqrt{2}\left(2x + \sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{16a^2c} + \frac{3\sqrt{2}(ac^3)^{\frac{1}{4}} \arctan\left(\frac{\sqrt{2}\left(2x - \sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{16a^2c} + \frac{3\sqrt{2}(ac^3)^{\frac{1}{4}} \log\left(x^2 + \sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}} + \left(\frac{a}{c}\right)^{\frac{1}{2}}\right)}{32a^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x^4+a)^2,x, algorithm="giac")

[Out] 1/4*x/((c*x^4 + a)*a) + 3/16*sqrt(2)*(a*c^3)^(1/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a^2*c) + 3/16*sqrt(2)*(a*c^3)^(1/4)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a^2*c) + 3/32*sqrt(2)*(a*c^3)^(1/4)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a^2*c) - 3/32*sqrt(2)*(a*c^3)^(1/4)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a^2*c)

$$3.148 \quad \int \frac{1}{(d+ex^2)(a+cx^4)^2} dx$$

Optimal. Leaf size=689

$$\frac{\sqrt[4]{ce^2}(\sqrt{ae} + \sqrt{cd}) \log(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{cx} + \sqrt{a} + \sqrt{cx^2})}{4\sqrt{2}a^{3/4}(ae^2 + cd^2)^2} + \frac{\sqrt[4]{ce^2}(\sqrt{ae} + \sqrt{cd}) \log(\sqrt{2}\sqrt[4]{a}\sqrt[4]{cx} + \sqrt{a} + \sqrt{cx^2})}{4\sqrt{2}a^{3/4}(ae^2 + cd^2)^2} - \frac{\sqrt[4]{c}(\sqrt{ae} + 3\sqrt{cd})}{4\sqrt{2}a^{3/4}(ae^2 + cd^2)^2}$$

[Out] (c*x*(d - e*x^2))/(4*a*(c*d^2 + a*e^2)*(a + c*x^4)) + (e^(7/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[d]*(c*d^2 + a*e^2)^2) - (c^(1/4)*e^2*(Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2)^2) - (c^(1/4)*(3*Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*(c*d^2 + a*e^2)) + (c^(1/4)*e^2*(Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2)^2) + (c^(1/4)*(3*Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*(c*d^2 + a*e^2)) - (c^(1/4)*e^2*(Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2)^2) - (c^(1/4)*(3*Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*(c*d^2 + a*e^2)) + (c^(1/4)*e^2*(Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2)^2) + (c^(1/4)*(3*Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*(c*d^2 + a*e^2))

Rubi [A] time = 0.622971, antiderivative size = 689, normalized size of antiderivative = 1., number of steps used = 22, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {1239, 205, 1179, 1168, 1162, 617, 204, 1165, 628}

$$\frac{\sqrt[4]{ce^2}(\sqrt{ae} + \sqrt{cd}) \log(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{cx} + \sqrt{a} + \sqrt{cx^2})}{4\sqrt{2}a^{3/4}(ae^2 + cd^2)^2} + \frac{\sqrt[4]{ce^2}(\sqrt{ae} + \sqrt{cd}) \log(\sqrt{2}\sqrt[4]{a}\sqrt[4]{cx} + \sqrt{a} + \sqrt{cx^2})}{4\sqrt{2}a^{3/4}(ae^2 + cd^2)^2} - \frac{\sqrt[4]{c}(\sqrt{ae} + 3\sqrt{cd})}{4\sqrt{2}a^{3/4}(ae^2 + cd^2)^2}$$

Antiderivative was successfully verified.

[In] Int[1/((d + e*x^2)*(a + c*x^4)^2), x]

[Out] (c*x*(d - e*x^2))/(4*a*(c*d^2 + a*e^2)*(a + c*x^4)) + (e^(7/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[d]*(c*d^2 + a*e^2)^2) - (c^(1/4)*e^2*(Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2)^2) - (c^(1/4)*(3*Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*(c*d^2 + a*e^2)) + (c^(1/4)*e^2*(Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2)^2) + (c^(1/4)*(3*Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*(c*d^2 + a*e^2)) - (c^(1/4)*e^2*(Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2)^2) - (c^(1/4)*(3*Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*(c*d^2 + a*e^2)) + (c^(1/4)*e^2*(Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2)^2) + (c^(1/4)*(3*Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*(c*d^2 + a*e^2))

Rule 1239

Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] := Int[ExpandIntegrand[(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a, c, d, e,

p, q, x && ((IntegerQ[p] && IntegerQ[q]) || IGtQ[p, 0])

Rule 205

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 1179

Int[((d_) + (e_)*(x_)^2)*((a_) + (c_)*(x_)^4)^(p_), x_Symbol] := -Simp[(x*(d + e*x^2)*(a + c*x^4)^(p + 1))/(4*a*(p + 1)), x] + Dist[1/(4*a*(p + 1)), Int[Simp[d*(4*p + 5) + e*(4*p + 7)*x^2, x]*(a + c*x^4)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && IntegerQ[2*p]

Rule 1168

Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + Dist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*c)]

Rule 1162

Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]

Rule 617

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c))] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 204

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 1165

Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]

Rule 628

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rubi steps

$$\begin{aligned}
\int \frac{1}{(d+ex^2)(a+cx^4)^2} dx &= \int \left(\frac{e^4}{(cd^2+ae^2)^2(d+ex^2)} + \frac{c(d-ex^2)}{(cd^2+ae^2)(a+cx^4)^2} - \frac{ce^2(-d+ex^2)}{(cd^2+ae^2)^2(a+cx^4)} \right) dx \\
&= -\frac{(ce^2) \int \frac{-d+ex^2}{a+cx^4} dx}{(cd^2+ae^2)^2} + \frac{e^4 \int \frac{1}{d+ex^2} dx}{(cd^2+ae^2)^2} + \frac{c \int \frac{d-ex^2}{(a+cx^4)^2} dx}{cd^2+ae^2} \\
&= \frac{cx(d-ex^2)}{4a(cd^2+ae^2)(a+cx^4)} + \frac{e^{7/2} \tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)}{\sqrt{d}(cd^2+ae^2)^2} + \frac{\left(\left(\frac{\sqrt{cd}}{\sqrt{a}}-e\right)e^2\right) \int \frac{\sqrt{a}\sqrt{c}+cx^2}{a+cx^4} dx}{2(cd^2+ae^2)^2} + \frac{\left(e^2\left(\frac{\sqrt{cd}}{\sqrt{a}}+e\right)\right)}{2(cd^2+ae^2)^2} \\
&= \frac{cx(d-ex^2)}{4a(cd^2+ae^2)(a+cx^4)} + \frac{e^{7/2} \tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)}{\sqrt{d}(cd^2+ae^2)^2} + \frac{\left(\left(\frac{\sqrt{cd}}{\sqrt{a}}-e\right)e^2\right) \int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{ax}}{\sqrt[4]{c}}+x^2} dx}{4(cd^2+ae^2)^2} + \frac{\left(\left(\frac{\sqrt{cd}}{\sqrt{a}}+e\right)e^2\right)}{4(cd^2+ae^2)^2} \\
&= \frac{cx(d-ex^2)}{4a(cd^2+ae^2)(a+cx^4)} + \frac{e^{7/2} \tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)}{\sqrt{d}(cd^2+ae^2)^2} - \frac{\sqrt[4]{ce^2}(\sqrt{cd}+\sqrt{ae}) \log(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{cx}+\sqrt{c})}{4\sqrt{2}a^{3/4}(cd^2+ae^2)^2} + \frac{\sqrt[4]{ce^2}(\sqrt{cd}-\sqrt{ae}) \tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{cx}}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}(cd^2+ae^2)^2} + \frac{\sqrt[4]{ce^2}}{\sqrt[4]{c}} \\
&= \frac{cx(d-ex^2)}{4a(cd^2+ae^2)(a+cx^4)} + \frac{e^{7/2} \tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)}{\sqrt{d}(cd^2+ae^2)^2} - \frac{\sqrt[4]{ce^2}(\sqrt{cd}-\sqrt{ae}) \tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{cx}}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}(cd^2+ae^2)^2} - \frac{\sqrt[4]{ce^2}}{\sqrt[4]{c}}
\end{aligned}$$

Mathematica [A] time = 0.314477, size = 429, normalized size = 0.62

$$-\frac{\sqrt{2}\sqrt[4]{c}(5a^{3/2}e^3+\sqrt{acd^2e+7a\sqrt{c}de^2+3c^{3/2}d^3})\log(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{cx}+\sqrt{a}+\sqrt{cx^2})}{a^{7/4}} + \frac{\sqrt{2}\sqrt[4]{c}(5a^{3/2}e^3+\sqrt{acd^2e+7a\sqrt{c}de^2+3c^{3/2}d^3})\log(\sqrt{2}\sqrt[4]{a}\sqrt[4]{cx}+\sqrt{a}+\sqrt{cx^2})}{a^{7/4}} + \frac{2\sqrt{2}\sqrt[4]{ce^2}}{\sqrt[4]{c}}$$

32

Antiderivative was successfully verified.

[In] Integrate[1/((d + e*x^2)*(a + c*x^4)^2), x]

[Out] ((8*c*(c*d^2 + a*e^2)*x*(d - e*x^2))/(a*(a + c*x^4)) + (32*e^(7/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[d] + (2*Sqrt[2]*c^(1/4)*(-3*c^(3/2)*d^3 + Sqrt[a]*c*d^2*e - 7*a*Sqrt[c]*d*e^2 + 5*a^(3/2)*e^3)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)]/a^(7/4) - (2*Sqrt[2]*c^(1/4)*(-3*c^(3/2)*d^3 + Sqrt[a]*c*d^2*e - 7*a*Sqrt[c]*d*e^2 + 5*a^(3/2)*e^3)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)]/a^(7/4) - (Sqrt[2]*c^(1/4)*(3*c^(3/2)*d^3 + Sqrt[a]*c*d^2*e + 7*a*Sqrt[c]*d*e^2 + 5*a^(3/2)*e^3)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2]/a^(7/4) + (Sqrt[2]*c^(1/4)*(3*c^(3/2)*d^3 + Sqrt[a]*c*d^2*e + 7*a*Sqrt[c]*d*e^2 + 5*a^(3/2)*e^3)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2]/a^(7/4))/(32*(c*d^2 + a*e^2)^2)

Maple [A] time = 0.061, size = 873, normalized size = 1.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(e*x^2+d)/(c*x^4+a)^2,x)`

[Out]
$$-1/4*c/(a*e^2+c*d^2)^2/(c*x^4+a)*e^3*x^3-1/4*c^2/(a*e^2+c*d^2)^2/(c*x^4+a)*e/a*x^3*d^2+1/4*c/(a*e^2+c*d^2)^2/(c*x^4+a)*d*x*e^2+1/4*c^2/(a*e^2+c*d^2)^2/(c*x^4+a)*d^3/a*x+7/16*c/(a*e^2+c*d^2)^2/a*(a/c)^{1/4}*2^{1/2}*arctan(2^{1/2}/(a/c)^{1/4}*x-1)*d*e^2+3/16*c^2/(a*e^2+c*d^2)^2/a^2*(a/c)^{1/4}*2^{1/2}*arctan(2^{1/2}/(a/c)^{1/4}*x-1)*d^3+7/32*c/(a*e^2+c*d^2)^2/a*(a/c)^{1/4}*2^{1/2}*ln((x^2+(a/c)^{1/4}*x*2^{1/2}+(a/c)^{1/2})/(x^2-(a/c)^{1/4}*x*2^{1/2}+(a/c)^{1/2}))+d*e^2+3/32*c^2/(a*e^2+c*d^2)^2/a^2*(a/c)^{1/4}*2^{1/2}*ln((x^2+(a/c)^{1/4}*x*2^{1/2}+(a/c)^{1/2})/(x^2-(a/c)^{1/4}*x*2^{1/2}+(a/c)^{1/2}))*d^3+7/16*c/(a*e^2+c*d^2)^2/a*(a/c)^{1/4}*2^{1/2}*arctan(2^{1/2}/(a/c)^{1/4}*x+1)*d*e^2+3/16*c^2/(a*e^2+c*d^2)^2/a^2*(a/c)^{1/4}*2^{1/2}*arctan(2^{1/2}/(a/c)^{1/4}*x+1)*d^3-5/32/(a*e^2+c*d^2)^2/(a/c)^{1/4}*2^{1/2}*ln((x^2-(a/c)^{1/4}*x*2^{1/2}+(a/c)^{1/2})/(x^2+(a/c)^{1/4}*x*2^{1/2}+(a/c)^{1/2}))*e^3-1/32*c/(a*e^2+c*d^2)^2/a/(a/c)^{1/4}*2^{1/2}*ln((x^2-(a/c)^{1/4}*x*2^{1/2}+(a/c)^{1/2})/(x^2+(a/c)^{1/4}*x*2^{1/2}+(a/c)^{1/2}))*d^2*e-5/16/(a*e^2+c*d^2)^2/(a/c)^{1/4}*2^{1/2}*arctan(2^{1/2}/(a/c)^{1/4}*x-1)*e^3-1/16*c/(a*e^2+c*d^2)^2/a/(a/c)^{1/4}*2^{1/2}*arctan(2^{1/2}/(a/c)^{1/4}*x-1)*d^2*e-5/16/(a*e^2+c*d^2)^2/(a/c)^{1/4}*2^{1/2}*arctan(2^{1/2}/(a/c)^{1/4}*x+1)*e^3-1/16*c/(a*e^2+c*d^2)^2/a/(a/c)^{1/4}*2^{1/2}*arctan(2^{1/2}/(a/c)^{1/4}*x+1)*d^2*e+e^4/(a*e^2+c*d^2)^2/(d*e)^{1/2}*arctan(e*x/(d*e)^{1/2})$$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="maxima")`

[Out] Exception raised: ValueError

Fricas [B] time = 114.867, size = 20650, normalized size = 29.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="fricas")`

[Out]
$$[-1/16*(4*(c^2*d^2*e + a*c*e^3)*x^3 + (a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4 + (a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)*x^4)*sqrt((6*c^3*d^5*e + 44*a*c^2*d^3*e^3 + 70*a^2*c*d*e^5 + (a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)*sqrt(-(81*c^7*d^12 + 738*a*c^6*d^10*e^2 + 2383*a^2*c^5*d^8*e^4 + 2748*a^3*c^4*d^6*e^6 - 529*a^4*c^3*d^4*e^8 - 1950*a^5*c^2*d^2*e^10 + 625*a^6*c*e^12)/(a^7*c^8*d^16 + 8*a^8*c^7*d^14*e^2 + 28*a^9*c^6*d^12*e^4 + 56*a^10*c^5*d^10*e^6 + 70*a^11*c^4*d^8*e^8 + 56*a^12*c^3*d^6*e^10 + 28*a^13*c^2*d^4*e^12 + 8*a^14*c*d^2*e^14 + a^15*e^16)))/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)*log(-(81*c^5*d^8 + 594*a*c^4*d^6*e^2 + 1376*a^2*c^3*d^4*e^4 + 750*a^3*c^2*d^2*e^6 - 625*a^4*c*e^8)*x + (27*a^2*c^5*d^9 + 186*a^3*c^4*d^7*e^2 + 404*a^4*c^3*d^5*e^4 + 198*a^5*c^2*d^3*e^6 - 175*a^6*c*d*e^8 + (a^6*c^5*d^$$

$$\begin{aligned}
& 10e + 9a^7c^4d^8e^3 + 26a^8c^3d^6e^5 + 34a^9c^2d^4e^7 + 21a^{10}c^1d^2e^9 + 5a^{11}e^{11})\sqrt{-(81c^7d^{12} + 738a^6c^6d^{10}e^2 + 2383a^2c^5d^8e^4 + 2748a^3c^4d^6e^6 - 529a^4c^3d^4e^8 - 1950a^5c^2d^2e^{10} + 625a^6c^1e^{12})/(a^7c^8d^{16} + 8a^8c^7d^{14}e^2 + 28a^9c^6d^{12}e^4 + 56a^{10}c^5d^{10}e^6 + 70a^{11}c^4d^8e^8 + 56a^{12}c^3d^6e^{10} + 28a^{13}c^2d^4e^{12} + 8a^{14}c^1d^2e^{14} + a^{15}e^{16}))}\sqrt{(6c^3d^5e + 44a^2c^2d^3e^3 + 70a^2c^1d^1e^5 + (a^3c^4d^8 + 4a^4c^3d^6e^2 + 6a^5c^2d^4e^4 + 4a^6c^1d^2e^6 + a^7e^8))\sqrt{-(81c^7d^{12} + 738a^6c^6d^{10}e^2 + 2383a^2c^5d^8e^4 + 2748a^3c^4d^6e^6 - 529a^4c^3d^4e^8 - 1950a^5c^2d^2e^{10} + 625a^6c^1e^{12})/(a^7c^8d^{16} + 8a^8c^7d^{14}e^2 + 28a^9c^6d^{12}e^4 + 56a^{10}c^5d^{10}e^6 + 70a^{11}c^4d^8e^8 + 56a^{12}c^3d^6e^{10} + 28a^{13}c^2d^4e^{12} + 8a^{14}c^1d^2e^{14} + a^{15}e^{16})))/(a^3c^4d^8 + 4a^4c^3d^6e^2 + 6a^5c^2d^4e^4 + 4a^6c^1d^2e^6 + a^7e^8))} - (a^2c^2d^4 + 2a^3c^1d^2e^2 + a^4e^4 + (a^3c^1d^4 + 2a^2c^2d^2e^2 + a^3c^1e^4)*x^4)\sqrt{(6c^3d^5e + 44a^2c^2d^3e^3 + 70a^2c^1d^1e^5 + (a^3c^4d^8 + 4a^4c^3d^6e^2 + 6a^5c^2d^4e^4 + 4a^6c^1d^2e^6 + a^7e^8))\sqrt{-(81c^7d^{12} + 738a^6c^6d^{10}e^2 + 2383a^2c^5d^8e^4 + 2748a^3c^4d^6e^6 - 529a^4c^3d^4e^8 - 1950a^5c^2d^2e^{10} + 625a^6c^1e^{12})/(a^7c^8d^{16} + 8a^8c^7d^{14}e^2 + 28a^9c^6d^{12}e^4 + 56a^{10}c^5d^{10}e^6 + 70a^{11}c^4d^8e^8 + 56a^{12}c^3d^6e^{10} + 28a^{13}c^2d^4e^{12} + 8a^{14}c^1d^2e^{14} + a^{15}e^{16})))/(a^3c^4d^8 + 4a^4c^3d^6e^2 + 6a^5c^2d^4e^4 + 4a^6c^1d^2e^6 + a^7e^8))*\log(-(81c^5d^8 + 594a^4c^4d^6e^2 + 1376a^2c^3d^4e^4 + 750a^3c^2d^2e^6 - 625a^4c^1e^8)*x - (27a^2c^5d^9 + 186a^3c^4d^7e^2 + 404a^4c^3d^5e^4 + 198a^5c^2d^3e^6 - 175a^6c^1d^1e^8 + (a^6c^5d^{10}e + 9a^7c^4d^8e^3 + 26a^8c^3d^6e^5 + 34a^9c^2d^4e^7 + 21a^{10}c^1d^2e^9 + 5a^{11}e^{11})\sqrt{-(81c^7d^{12} + 738a^6c^6d^{10}e^2 + 2383a^2c^5d^8e^4 + 2748a^3c^4d^6e^6 - 529a^4c^3d^4e^8 - 1950a^5c^2d^2e^{10} + 625a^6c^1e^{12})/(a^7c^8d^{16} + 8a^8c^7d^{14}e^2 + 28a^9c^6d^{12}e^4 + 56a^{10}c^5d^{10}e^6 + 70a^{11}c^4d^8e^8 + 56a^{12}c^3d^6e^{10} + 28a^{13}c^2d^4e^{12} + 8a^{14}c^1d^2e^{14} + a^{15}e^{16}))}\sqrt{(6c^3d^5e + 44a^2c^2d^3e^3 + 70a^2c^1d^1e^5 + (a^3c^4d^8 + 4a^4c^3d^6e^2 + 6a^5c^2d^4e^4 + 4a^6c^1d^2e^6 + a^7e^8))\sqrt{-(81c^7d^{12} + 738a^6c^6d^{10}e^2 + 2383a^2c^5d^8e^4 + 2748a^3c^4d^6e^6 - 529a^4c^3d^4e^8 - 1950a^5c^2d^2e^{10} + 625a^6c^1e^{12})/(a^7c^8d^{16} + 8a^8c^7d^{14}e^2 + 28a^9c^6d^{12}e^4 + 56a^{10}c^5d^{10}e^6 + 70a^{11}c^4d^8e^8 + 56a^{12}c^3d^6e^{10} + 28a^{13}c^2d^4e^{12} + 8a^{14}c^1d^2e^{14} + a^{15}e^{16})))/(a^3c^4d^8 + 4a^4c^3d^6e^2 + 6a^5c^2d^4e^4 + 4a^6c^1d^2e^6 + a^7e^8))} + (a^2c^2d^4 + 2a^3c^1d^2e^2 + a^4e^4 + (a^3c^1d^4 + 2a^2c^2d^2e^2 + a^3c^1e^4)*x^4)\sqrt{(6c^3d^5e + 44a^2c^2d^3e^3 + 70a^2c^1d^1e^5 - (a^3c^4d^8 + 4a^4c^3d^6e^2 + 6a^5c^2d^4e^4 + 4a^6c^1d^2e^6 + a^7e^8))\sqrt{-(81c^7d^{12} + 738a^6c^6d^{10}e^2 + 2383a^2c^5d^8e^4 + 2748a^3c^4d^6e^6 - 529a^4c^3d^4e^8 - 1950a^5c^2d^2e^{10} + 625a^6c^1e^{12})/(a^7c^8d^{16} + 8a^8c^7d^{14}e^2 + 28a^9c^6d^{12}e^4 + 56a^{10}c^5d^{10}e^6 + 70a^{11}c^4d^8e^8 + 56a^{12}c^3d^6e^{10} + 28a^{13}c^2d^4e^{12} + 8a^{14}c^1d^2e^{14} + a^{15}e^{16})))/(a^3c^4d^8 + 4a^4c^3d^6e^2 + 6a^5c^2d^4e^4 + 4a^6c^1d^2e^6 + a^7e^8))*\log(-(81c^5d^8 + 594a^4c^4d^6e^2 + 1376a^2c^3d^4e^4 + 750a^3c^2d^2e^6 - 625a^4c^1e^8)*x + (27a^2c^5d^9 + 186a^3c^4d^7e^2 + 404a^4c^3d^5e^4 + 198a^5c^2d^3e^6 - 175a^6c^1d^1e^8 - (a^6c^5d^{10}e + 9a^7c^4d^8e^3 + 26a^8c^3d^6e^5 + 34a^9c^2d^4e^7 + 21a^{10}c^1d^2e^9 + 5a^{11}e^{11})\sqrt{-(81c^7d^{12} + 738a^6c^6d^{10}e^2 + 2383a^2c^5d^8e^4 + 2748a^3c^4d^6e^6 - 529a^4c^3d^4e^8 - 1950a^5c^2d^2e^{10} + 625a^6c^1e^{12})/(a^7c^8d^{16} + 8a^8c^7d^{14}e^2 + 28a^9c^6d^{12}e^4 + 56a^{10}c^5d^{10}e^6 + 70a^{11}c^4d^8e^8 + 56a^{12}c^3d^6e^{10} + 28a^{13}c^2d^4e^{12} + 8a^{14}c^1d^2e^{14} + a^{15}e^{16}))}\sqrt{(6c^3d^5e + 44a^2c^2d^3e^3 + 70a^2c^1d^1e^5 - (a^3c^4d^8 + 4a^4c^3d^6e^2 + 6a^5c^2d^4e^4 + 4a^6c^1d^2e^6 + a^7e^8))\sqrt{-(81c^7d^{12} + 738a^6c^6d^{10}e^2 + 2383a^2c^5d^8e^4 + 2748a^3c^4d^6e^6 - 529a^4c^3d^4e^8 - 1950a^5c^2d^2e^{10} + 625a^6c^1e^{12})/(a^7c^8d^{16} + 8a^8c^7d^{14}e^2 + 28a^9c^6d^{12}e^4 + 56a^{10}c^5d^{10}e^6 + 70a^{11}c^4d^8e^8 + 56a^{12}c^3d^6e^{10} + 28a^{13}c^2d^4e^{12} + 8a^{14}c^1d^2e^{14} + a^{15}e^{16})))/(a^3c^4d^8 + 4a^4c^3d^6e^2 + 6a^5c^2d^4e^4 + 4a^6c^1d^2e^6 + a^7e^8))}
\end{aligned}$$

$$\begin{aligned}
& e^{12}) / (a^7 c^8 d^{16} + 8 a^8 c^7 d^{14} e^2 + 28 a^9 c^6 d^{12} e^4 + 56 a^{10} c^5 d^{10} e^6 + 70 a^{11} c^4 d^8 e^8 + 56 a^{12} c^3 d^6 e^{10} + 28 a^{13} c^2 d^4 e^{12} + 8 a^{14} c d^2 e^{14} + a^{15} e^{16})) / (a^3 c^4 d^8 + 4 a^4 c^3 d^6 e^2 + 6 a^5 c^2 d^4 e^4 + 4 a^6 c d^2 e^6 + a^7 e^8)) - (a^2 c^2 d^4 + 2 a^3 c d^2 e^2 + a^4 e^4 + (a c^3 d^4 + 2 a^2 c^2 d^2 e^2 + a^3 c e^4) x^4) \sqrt{((6 c^3 d^5 e + 44 a c^2 d^3 e^3 + 70 a^2 c d e^5 - (a^3 c^4 d^8 + 4 a^4 c^3 d^6 e^2 + 6 a^5 c^2 d^4 e^4 + 4 a^6 c d^2 e^6 + a^7 e^8) \sqrt{-81 c^7 d^{12} + 738 a c^6 d^{10} e^2 + 2383 a^2 c^5 d^8 e^4 + 2748 a^3 c^4 d^6 e^6 - 529 a^4 c^3 d^4 e^8 - 1950 a^5 c^2 d^2 e^{10} + 625 a^6 c e^{12})}) / (a^7 c^8 d^{16} + 8 a^8 c^7 d^{14} e^2 + 28 a^9 c^6 d^{12} e^4 + 56 a^{10} c^5 d^{10} e^6 + 70 a^{11} c^4 d^8 e^8 + 56 a^{12} c^3 d^6 e^{10} + 28 a^{13} c^2 d^4 e^{12} + 8 a^{14} c d^2 e^{14} + a^{15} e^{16}))} / (a^3 c^4 d^8 + 4 a^4 c^3 d^6 e^2 + 6 a^5 c^2 d^4 e^4 + 4 a^6 c d^2 e^6 + a^7 e^8)) \log(-81 c^5 d^8 + 594 a c^4 d^6 e^2 + 1376 a^2 c^3 d^4 e^4 + 750 a^3 c^2 d^2 e^6 - 625 a^4 c e^8) x - (27 a^2 c^5 d^9 + 186 a^3 c^4 d^7 e^2 + 404 a^4 c^3 d^5 e^4 + 198 a^5 c^2 d^3 e^6 - 175 a^6 c d e^8 - (a^6 c^5 d^{10} e + 9 a^7 c^4 d^8 e^3 + 26 a^8 c^3 d^6 e^5 + 34 a^9 c^2 d^4 e^7 + 21 a^{10} c d^2 e^9 + 5 a^{11} e^{11}) \sqrt{-81 c^7 d^{12} + 738 a c^6 d^{10} e^2 + 2383 a^2 c^5 d^8 e^4 + 2748 a^3 c^4 d^6 e^6 - 529 a^4 c^3 d^4 e^8 - 1950 a^5 c^2 d^2 e^{10} + 625 a^6 c e^{12})}) / (a^7 c^8 d^{16} + 8 a^8 c^7 d^{14} e^2 + 28 a^9 c^6 d^{12} e^4 + 56 a^{10} c^5 d^{10} e^6 + 70 a^{11} c^4 d^8 e^8 + 56 a^{12} c^3 d^6 e^{10} + 28 a^{13} c^2 d^4 e^{12} + 8 a^{14} c d^2 e^{14} + a^{15} e^{16})) \sqrt{((6 c^3 d^5 e + 44 a c^2 d^3 e^3 + 70 a^2 c d e^5 - (a^3 c^4 d^8 + 4 a^4 c^3 d^6 e^2 + 6 a^5 c^2 d^4 e^4 + 4 a^6 c d^2 e^6 + a^7 e^8) \sqrt{-81 c^7 d^{12} + 738 a c^6 d^{10} e^2 + 2383 a^2 c^5 d^8 e^4 + 2748 a^3 c^4 d^6 e^6 - 529 a^4 c^3 d^4 e^8 - 1950 a^5 c^2 d^2 e^{10} + 625 a^6 c e^{12})}) / (a^7 c^8 d^{16} + 8 a^8 c^7 d^{14} e^2 + 28 a^9 c^6 d^{12} e^4 + 56 a^{10} c^5 d^{10} e^6 + 70 a^{11} c^4 d^8 e^8 + 56 a^{12} c^3 d^6 e^{10} + 28 a^{13} c^2 d^4 e^{12} + 8 a^{14} c d^2 e^{14} + a^{15} e^{16}))} \sqrt{((6 c^3 d^5 e + 44 a c^2 d^3 e^3 + 70 a^2 c d e^5 - (a^3 c^4 d^8 + 4 a^4 c^3 d^6 e^2 + 6 a^5 c^2 d^4 e^4 + 4 a^6 c d^2 e^6 + a^7 e^8) \sqrt{-81 c^7 d^{12} + 738 a c^6 d^{10} e^2 + 2383 a^2 c^5 d^8 e^4 + 2748 a^3 c^4 d^6 e^6 - 529 a^4 c^3 d^4 e^8 - 1950 a^5 c^2 d^2 e^{10} + 625 a^6 c e^{12})}) / (a^7 c^8 d^{16} + 8 a^8 c^7 d^{14} e^2 + 28 a^9 c^6 d^{12} e^4 + 56 a^{10} c^5 d^{10} e^6 + 70 a^{11} c^4 d^8 e^8 + 56 a^{12} c^3 d^6 e^{10} + 28 a^{13} c^2 d^4 e^{12} + 8 a^{14} c d^2 e^{14} + a^{15} e^{16}))} - 8 (a c e^3 x^4 + a^2 e^3) \sqrt{-e/d} \log((e x^2 + 2 d x \sqrt{-e/d} - d) / (e x^2 + d)) - 4 (c^2 d^3 + a c d e^2) x / (a^2 c^2 d^4 + 2 a^3 c d^2 e^2 + a^4 e^4 + (a c^3 d^4 + 2 a^2 c^2 d^2 e^2 + a^3 c e^4) x^4), -1/16 (4 (c^2 d^2 e + a c e^3) x^3 - 16 (a c e^3 x^4 + a^2 e^3)) \sqrt{e/d} \arctan(x \sqrt{e/d}) + (a^2 c^2 d^4 + 2 a^3 c d^2 e^2 + a^4 e^4 + (a c^3 d^4 + 2 a^2 c^2 d^2 e^2 + a^3 c e^4) x^4) \sqrt{((6 c^3 d^5 e + 44 a c^2 d^3 e^3 + 70 a^2 c d e^5 + (a^3 c^4 d^8 + 4 a^4 c^3 d^6 e^2 + 6 a^5 c^2 d^4 e^4 + 4 a^6 c d^2 e^6 + a^7 e^8) \sqrt{-81 c^7 d^{12} + 738 a c^6 d^{10} e^2 + 2383 a^2 c^5 d^8 e^4 + 2748 a^3 c^4 d^6 e^6 - 529 a^4 c^3 d^4 e^8 - 1950 a^5 c^2 d^2 e^{10} + 625 a^6 c e^{12})}) / (a^7 c^8 d^{16} + 8 a^8 c^7 d^{14} e^2 + 28 a^9 c^6 d^{12} e^4 + 56 a^{10} c^5 d^{10} e^6 + 70 a^{11} c^4 d^8 e^8 + 56 a^{12} c^3 d^6 e^{10} + 28 a^{13} c^2 d^4 e^{12} + 8 a^{14} c d^2 e^{14} + a^{15} e^{16}))} / (a^3 c^4 d^8 + 4 a^4 c^3 d^6 e^2 + 6 a^5 c^2 d^4 e^4 + 4 a^6 c d^2 e^6 + a^7 e^8)) \log(-81 c^5 d^8 + 594 a c^4 d^6 e^2 + 1376 a^2 c^3 d^4 e^4 + 750 a^3 c^2 d^2 e^6 - 625 a^4 c e^8) x + (27 a^2 c^5 d^9 + 186 a^3 c^4 d^7 e^2 + 404 a^4 c^3 d^5 e^4 + 198 a^5 c^2 d^3 e^6 - 175 a^6 c d e^8 + (a^6 c^5 d^{10} e + 9 a^7 c^4 d^8 e^3 + 26 a^8 c^3 d^6 e^5 + 34 a^9 c^2 d^4 e^7 + 21 a^{10} c d^2 e^9 + 5 a^{11} e^{11}) \sqrt{-81 c^7 d^{12} + 738 a c^6 d^{10} e^2 + 2383 a^2 c^5 d^8 e^4 + 2748 a^3 c^4 d^6 e^6 - 529 a^4 c^3 d^4 e^8 - 1950 a^5 c^2 d^2 e^{10} + 625 a^6 c e^{12})}) / (a^7 c^8 d^{16} + 8 a^8 c^7 d^{14} e^2 + 28 a^9 c^6 d^{12} e^4 + 56 a^{10} c^5 d^{10} e^6 + 70 a^{11} c^4 d^8 e^8 + 56 a^{12} c^3 d^6 e^{10} + 28 a^{13} c^2 d^4 e^{12} + 8 a^{14} c d^2 e^{14} + a^{15} e^{16})) \sqrt{((6 c^3 d^5 e + 44 a c^2 d^3 e^3 + 70 a^2 c d e^5 + (a^3 c^4 d^8 + 4 a^4 c^3 d^6 e^2 + 6 a^5 c^2 d^4 e^4 + 4 a^6 c d^2 e^6 + a^7 e^8) \sqrt{-81 c^7 d^{12} + 738 a c^6 d^{10} e^2 + 2383 a^2 c^5 d^8 e^4 + 2748 a^3 c^4 d^6 e^6 - 529 a^4 c^3 d^4 e^8 - 1950 a^5 c^2 d^2 e^{10} + 625 a^6 c e^{12})}) / (a^7 c^8 d^{16} + 8 a^8 c^7 d^{14} e^2 + 28 a^9 c^6 d^{12} e^4 + 56 a^{10} c^5 d^{10} e^6 + 70 a^{11} c^4 d^8 e^8 + 56 a^{12} c^3 d^6 e^{10} + 28 a^{13} c^2 d^4 e^{12} + 8 a^{14} c d^2 e^{14} + a^{15} e^{16}))} / (a^3 c^4 d^8 + 4 a^4 c^3 d^6 e^2 + 6 a^5 c^2 d^4 e^4 + 4 a^6 c d^2 e^6 + a^7 e^8)) - (a^2 c^2 d^4 + 2 a^3 c d^2 e^2 + a^4 e^4 + (a c^3 d^4 + 2 a^2 c^2 d^2 e^2 + a^3 c e^4) x^4) \sqrt{((6 c^3 d^5 e + 44 a c^2 d^3 e^3 + 70 a^2 c d e^5 + 70 a^2 c d e^5
\end{aligned}$$

$$\begin{aligned}
& *d^5 + (a^3c^4d^8 + 4a^4c^3d^6e^2 + 6a^5c^2d^4e^4 + 4a^6cd^2e^6 + a^7e^8)\sqrt{-(81c^7d^{12} + 738aac^6d^{10}e^2 + 2383a^2c^5d^8e^4 + 2748a^3c^4d^6e^6 - 529a^4c^3d^4e^8 - 1950a^5c^2d^2e^{10} + 625a^6c^0e^{12})/(a^7c^8d^{16} + 8a^8c^7d^{14}e^2 + 28a^9c^6d^{12}e^4 + 56a^{10}c^5d^{10}e^6 + 70a^{11}c^4d^8e^8 + 56a^{12}c^3d^6e^{10} + 28a^{13}c^2d^4e^{12} + 8a^{14}cd^2e^{14} + a^{15}e^{16})))/(a^3c^4d^8 + 4a^4c^3d^6e^2 + 6a^5c^2d^4e^4 + 4a^6cd^2e^6 + a^7e^8)) * \log(-(81c^5d^8 + 594aac^4d^6e^2 + 1376a^2c^3d^4e^4 + 750a^3c^2d^2e^6 - 625a^4c^0e^8) * x - (27a^2c^5d^9 + 186a^3c^4d^7e^2 + 404a^4c^3d^5e^4 + 198a^5c^2d^3e^6 - 175a^6cd^0e^8 + (a^6c^5d^{10}e + 9a^7c^4d^8e^3 + 26a^8c^3d^6e^5 + 34a^9c^2d^4e^7 + 21a^{10}cd^2e^9 + 5a^{11}e^{11})) * \sqrt{-(81c^7d^{12} + 738aac^6d^{10}e^2 + 2383a^2c^5d^8e^4 + 2748a^3c^4d^6e^6 - 529a^4c^3d^4e^8 - 1950a^5c^2d^2e^{10} + 625a^6c^0e^{12})/(a^7c^8d^{16} + 8a^8c^7d^{14}e^2 + 28a^9c^6d^{12}e^4 + 56a^{10}c^5d^{10}e^6 + 70a^{11}c^4d^8e^8 + 56a^{12}c^3d^6e^{10} + 28a^{13}c^2d^4e^{12} + 8a^{14}cd^2e^{14} + a^{15}e^{16}))} * \sqrt{((6c^3d^5e + 44aac^2d^3e^3 + 70a^2cd^0e^5 + (a^3c^4d^8 + 4a^4c^3d^6e^2 + 6a^5c^2d^4e^4 + 4a^6cd^2e^6 + a^7e^8)) * \sqrt{-(81c^7d^{12} + 738aac^6d^{10}e^2 + 2383a^2c^5d^8e^4 + 2748a^3c^4d^6e^6 - 529a^4c^3d^4e^8 - 1950a^5c^2d^2e^{10} + 625a^6c^0e^{12})/(a^7c^8d^{16} + 8a^8c^7d^{14}e^2 + 28a^9c^6d^{12}e^4 + 56a^{10}c^5d^{10}e^6 + 70a^{11}c^4d^8e^8 + 56a^{12}c^3d^6e^{10} + 28a^{13}c^2d^4e^{12} + 8a^{14}cd^2e^{14} + a^{15}e^{16}))} + (a^2c^2d^4 + 2a^3cd^2e^2 + a^4e^4 + (ac^3d^4 + 2a^2c^2d^2e^2 + a^3ce^4) * x^4) * \sqrt{((6c^3d^5e + 44aac^2d^3e^3 + 70a^2cd^0e^5 - (a^3c^4d^8 + 4a^4c^3d^6e^2 + 6a^5c^2d^4e^4 + 4a^6cd^2e^6 + a^7e^8)) * \sqrt{-(81c^7d^{12} + 738aac^6d^{10}e^2 + 2383a^2c^5d^8e^4 + 2748a^3c^4d^6e^6 - 529a^4c^3d^4e^8 - 1950a^5c^2d^2e^{10} + 625a^6c^0e^{12})/(a^7c^8d^{16} + 8a^8c^7d^{14}e^2 + 28a^9c^6d^{12}e^4 + 56a^{10}c^5d^{10}e^6 + 70a^{11}c^4d^8e^8 + 56a^{12}c^3d^6e^{10} + 28a^{13}c^2d^4e^{12} + 8a^{14}cd^2e^{14} + a^{15}e^{16}))} / (a^3c^4d^8 + 4a^4c^3d^6e^2 + 6a^5c^2d^4e^4 + 4a^6cd^2e^6 + a^7e^8)) * \log(-(81c^5d^8 + 594aac^4d^6e^2 + 1376a^2c^3d^4e^4 + 750a^3c^2d^2e^6 - 625a^4c^0e^8) * x + (27a^2c^5d^9 + 186a^3c^4d^7e^2 + 404a^4c^3d^5e^4 + 198a^5c^2d^3e^6 - 175a^6cd^0e^8 - (a^6c^5d^{10}e + 9a^7c^4d^8e^3 + 26a^8c^3d^6e^5 + 34a^9c^2d^4e^7 + 21a^{10}cd^2e^9 + 5a^{11}e^{11})) * \sqrt{-(81c^7d^{12} + 738aac^6d^{10}e^2 + 2383a^2c^5d^8e^4 + 2748a^3c^4d^6e^6 - 529a^4c^3d^4e^8 - 1950a^5c^2d^2e^{10} + 625a^6c^0e^{12})/(a^7c^8d^{16} + 8a^8c^7d^{14}e^2 + 28a^9c^6d^{12}e^4 + 56a^{10}c^5d^{10}e^6 + 70a^{11}c^4d^8e^8 + 56a^{12}c^3d^6e^{10} + 28a^{13}c^2d^4e^{12} + 8a^{14}cd^2e^{14} + a^{15}e^{16}))} * \sqrt{((6c^3d^5e + 44aac^2d^3e^3 + 70a^2cd^0e^5 - (a^3c^4d^8 + 4a^4c^3d^6e^2 + 6a^5c^2d^4e^4 + 4a^6cd^2e^6 + a^7e^8)) * \sqrt{-(81c^7d^{12} + 738aac^6d^{10}e^2 + 2383a^2c^5d^8e^4 + 2748a^3c^4d^6e^6 - 529a^4c^3d^4e^8 - 1950a^5c^2d^2e^{10} + 625a^6c^0e^{12})/(a^7c^8d^{16} + 8a^8c^7d^{14}e^2 + 28a^9c^6d^{12}e^4 + 56a^{10}c^5d^{10}e^6 + 70a^{11}c^4d^8e^8 + 56a^{12}c^3d^6e^{10} + 28a^{13}c^2d^4e^{12} + 8a^{14}cd^2e^{14} + a^{15}e^{16}))} / (a^3c^4d^8 + 4a^4c^3d^6e^2 + 6a^5c^2d^4e^4 + 4a^6cd^2e^6 + a^7e^8)) - (a^2c^2d^4 + 2a^3cd^2e^2 + a^4e^4 + (ac^3d^4 + 2a^2c^2d^2e^2 + a^3ce^4) * x^4) * \sqrt{((6c^3d^5e + 44aac^2d^3e^3 + 70a^2cd^0e^5 - (a^3c^4d^8 + 4a^4c^3d^6e^2 + 6a^5c^2d^4e^4 + 4a^6cd^2e^6 + a^7e^8)) * \sqrt{-(81c^7d^{12} + 738aac^6d^{10}e^2 + 2383a^2c^5d^8e^4 + 2748a^3c^4d^6e^6 - 529a^4c^3d^4e^8 - 1950a^5c^2d^2e^{10} + 625a^6c^0e^{12})/(a^7c^8d^{16} + 8a^8c^7d^{14}e^2 + 28a^9c^6d^{12}e^4 + 56a^{10}c^5d^{10}e^6 + 70a^{11}c^4d^8e^8 + 56a^{12}c^3d^6e^{10} + 28a^{13}c^2d^4e^{12} + 8a^{14}cd^2e^{14} + a^{15}e^{16}))} / (a^3c^4d^8 + 4a^4c^3d^6e^2 + 6a^5c^2d^4e^4 + 4a^6cd^2e^6 + a^7e^8)) * \log(-(81c^5d^8 + 594aac^4d^6e^2 + 1376a^2c^3d^4e^4 + 750a^3c^2d^2e^6 - 625a^4c^0e^8) * x - (27a^2c^5d^9 + 186a^3c^4d^7e^2 + 404a^4c^3d^5e^4 + 198a^5c^2d^3e^6 - 175a^6cd^0e^8 - (a^6c^5d^{10}e + 9a^7c^4d^8e^3 + 26a^8c^3d^6e^5 + 34a^9c^2d^4e^7 + 21a^{10}cd^2e^9 + 5a^{11}e^{11})) * \sqrt{-(81c^7d^{12} + 738aac^6d^{10}e^2 + 2383a^2c^5d^8e^4 + 2748a^3c^4d^6e^6 - 529a^4c^3d^4e^8 - 1950a^5c^2d^2e^{10} + 625a^6c^0e^{12})/(a^7c^8d^{16} + 8a^8c^7d^{14}e^2 + 28a^9c^6d^{12}e^4 + 56a^{10}c^5d^{10}e^6 + 70a^{11}c^4d^8e^8 + 56a^{12}c^3d^6e^{10} + 28a^{13}c^2d^4e^{12} + 8a^{14}cd^2e^{14} + a^{15}e^{16}))} / (a^3c^4d^8 + 4a^4c^3d^6e^2 + 6a^5c^2d^4e^4 + 4a^6cd^2e^6 + a^7e^8)) * \log(-(81c^5d^8 + 594aac^4d^6e^2 + 1376a^2c^3d^4e^4 + 750a^3c^2d^2e^6 - 625a^4c^0e^8) * x - (27a^2c^5d^9 + 186a^3c^4d^7e^2 + 404a^4c^3d^5e^4 + 198a^5c^2d^3e^6 - 175a^6cd^0e^8 - (a^6c^5d^{10}e + 9a^7c^4d^8e^3 + 26a^8c^3d^6e^5 + 34a^9c^2d^4e^7 + 21a^{10}cd^2e^9 + 5a^{11}e^{11})) * \sqrt{-(81c^7d^{12} + 738aac^6d^{10}e^2 + 2383a^2c^5d^8e^4 + 2748a^3c^4d^6e^6 - 529a^4c^3d^4e^8 - 1950a^5c^2d^2e^{10} + 625a^6c^0e^{12})/(a^7c^8d^{16} + 8a^8c^7d^{14}e^2 + 28a^9c^6d^{12}e^4 + 56a^{10}c^5d^{10}e^6 + 70a^{11}c^4d^8e^8 + 56a^{12}c^3d^6e^{10} + 28a^{13}c^2d^4e^{12} + 8a^{14}cd^2e^{14} + a^{15}e^{16}))} / (a^3c^4d^8 + 4a^4c^3d^6e^2 + 6a^5c^2d^4e^4 + 4a^6cd^2e^6 + a^7e^8))
\end{aligned}$$

$$c^5d^{10}e + 9a^7c^4d^8e^3 + 26a^8c^3d^6e^5 + 34a^9c^2d^4e^7 + 21a^{10}c*d^2e^9 + 5a^{11}e^{11})\sqrt{-(81c^7d^{12} + 738a*c^6d^{10}e^2 + 2383a^2c^5d^8e^4 + 2748a^3c^4d^6e^6 - 529a^4c^3d^4e^8 - 1950a^5c^2d^2e^{10} + 625a^6c*e^{12})/(a^7c^8d^{16} + 8a^8c^7d^{14}e^2 + 28a^9c^6d^{12}e^4 + 56a^{10}c^5d^{10}e^6 + 70a^{11}c^4d^8e^8 + 56a^{12}c^3d^6e^{10} + 28a^{13}c^2d^4e^{12} + 8a^{14}c*d^2e^{14} + a^{15}e^{16}))}\sqrt{((6c^3d^5e + 44a*c^2d^3e^3 + 70a^2*c*d*e^5 - (a^3c^4d^8 + 4a^4c^3d^6e^2 + 6a^5c^2d^4e^4 + 4a^6c*d^2e^6 + a^7e^8))\sqrt{-(81c^7d^{12} + 738a*c^6d^{10}e^2 + 2383a^2c^5d^8e^4 + 2748a^3c^4d^6e^6 - 529a^4c^3d^4e^8 - 1950a^5c^2d^2e^{10} + 625a^6c*e^{12})/(a^7c^8d^{16} + 8a^8c^7d^{14}e^2 + 28a^9c^6d^{12}e^4 + 56a^{10}c^5d^{10}e^6 + 70a^{11}c^4d^8e^8 + 56a^{12}c^3d^6e^{10} + 28a^{13}c^2d^4e^{12} + 8a^{14}c*d^2e^{14} + a^{15}e^{16})))/(a^3c^4d^8 + 4a^4c^3d^6e^2 + 6a^5c^2d^4e^4 + 4a^6c*d^2e^6 + a^7e^8))} - 4*(c^2d^3 + a*c*d*e^2)*x)/(a^2c^2d^4 + 2a^3c*d^2e^2 + a^4e^4 + (a*c^3d^4 + 2a^2c^2d^2e^2 + a^3c*e^4)*x^4)]$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x**2+d)/(c*x**4+a)**2,x)

[Out] Timed out

Giac [A] time = 1.17323, size = 814, normalized size = 1.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="giac")

[Out] $\frac{1}{8}*(3*(a*c^3)^{(1/4)}*c^3*d^3 + 7*(a*c^3)^{(1/4)}*a*c^2*d*e^2 - (a*c^3)^{(3/4)}*c*d^2*e - 5*(a*c^3)^{(3/4)}*a*e^3)*\arctan(1/2*\sqrt{2}*(2*x + \sqrt{2})*(a/c)^{(1/4)})/(a/c)^{(1/4)})/(\sqrt{2}*a^2*c^4*d^4 + 2*\sqrt{2}*a^3*c^3*d^2*e^2 + \sqrt{2}*a^4*c^2*e^4) + \frac{1}{8}*(3*(a*c^3)^{(1/4)}*c^3*d^3 + 7*(a*c^3)^{(1/4)}*a*c^2*d*e^2 - (a*c^3)^{(3/4)}*c*d^2*e - 5*(a*c^3)^{(3/4)}*a*e^3)*\arctan(1/2*\sqrt{2}*(2*x - \sqrt{2})*(a/c)^{(1/4)})/(a/c)^{(1/4)})/(\sqrt{2}*a^2*c^4*d^4 + 2*\sqrt{2}*a^3*c^3*d^2*e^2 + \sqrt{2}*a^4*c^2*e^4) + \frac{1}{16}*(3*(a*c^3)^{(1/4)}*c^3*d^3 + 7*(a*c^3)^{(1/4)}*a*c^2*d*e^2 + (a*c^3)^{(3/4)}*c*d^2*e + 5*(a*c^3)^{(3/4)}*a*e^3)*\log(x^2 + \sqrt{2}*x*(a/c)^{(1/4)} + \sqrt{a/c})/(\sqrt{2}*a^2*c^4*d^4 + 2*\sqrt{2}*a^3*c^3*d^2*e^2 + \sqrt{2}*a^4*c^2*e^4) - \frac{1}{16}*(3*(a*c^3)^{(1/4)}*c^3*d^3 + 7*(a*c^3)^{(1/4)}*a*c^2*d*e^2 + (a*c^3)^{(3/4)}*c*d^2*e + 5*(a*c^3)^{(3/4)}*a*e^3)*\log(x^2 - \sqrt{2}*x*(a/c)^{(1/4)} + \sqrt{a/c})/(\sqrt{2}*a^2*c^4*d^4 + 2*\sqrt{2}*a^3*c^3*d^2*e^2 + \sqrt{2}*a^4*c^2*e^4) + \arctan(x*e^{(1/2)}/\sqrt{d})*e^{(7/2)}/((c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*\sqrt{d}) - 1/4*(c*x^3*e - c*d*x)/((c*x^4 + a)*(a*c*d^2 + a^2*e^2))$

$$3.149 \quad \int \frac{1}{(d+ex^2)^2(a+cx^4)^2} dx$$

Optimal. Leaf size=864

$$\frac{xe^4}{2d(cd^2 + ae^2)^2(ex^2 + d)} + \frac{\tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)e^{7/2}}{2d^{3/2}(cd^2 + ae^2)^2} + \frac{4c\sqrt{d}\tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)e^{7/2}}{(cd^2 + ae^2)^3} - \frac{c^{3/4}(3cd^2 - 4\sqrt{a}\sqrt{ced} - ae^2)\tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{cx}}{\sqrt{a}}\right)}{2\sqrt{2}a^{3/4}(cd^2 + ae^2)^3}$$

```
[Out] (e^4*x)/(2*d*(c*d^2 + a*e^2)^2*(d + e*x^2)) + (c*x*(c*d^2 - a*e^2 - 2*c*d*e*x^2))/(4*a*(c*d^2 + a*e^2)^2*(a + c*x^4)) + (4*c*Sqrt[d]*e^(7/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(c*d^2 + a*e^2)^3 + (e^(7/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(3/2)*(c*d^2 + a*e^2)^2) - (c^(3/4)*e^2*(3*c*d^2 - 4*Sqrt[a]*Sqrt[c]*d*e - a*e^2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)]/(2*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2)^3) - (c^(3/4)*(3*c*d^2 - 2*Sqrt[a]*Sqrt[c]*d*e - 3*a*e^2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)]/(8*Sqrt[2]*a^(7/4)*(c*d^2 + a*e^2)^2) + (c^(3/4)*e^2*(3*c*d^2 - 4*Sqrt[a]*Sqrt[c]*d*e - a*e^2)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)]/(2*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2)^3) + (c^(3/4)*(3*c*d^2 - 2*Sqrt[a]*Sqrt[c]*d*e - 3*a*e^2)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)]/(8*Sqrt[2]*a^(7/4)*(c*d^2 + a*e^2)^2) - (c^(3/4)*e^2*(3*c*d^2 + 4*Sqrt[a]*Sqrt[c]*d*e - a*e^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2)^3) - (c^(3/4)*(3*c*d^2 + 2*Sqrt[a]*Sqrt[c]*d*e - 3*a*e^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*(c*d^2 + a*e^2)^2) + (c^(3/4)*e^2*(3*c*d^2 + 4*Sqrt[a]*Sqrt[c]*d*e - a*e^2)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2)^3) + (c^(3/4)*(3*c*d^2 + 2*Sqrt[a]*Sqrt[c]*d*e - 3*a*e^2)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*(c*d^2 + a*e^2)^2)
```

Rubi [A] time = 0.906108, antiderivative size = 864, normalized size of antiderivative = 1., number of steps used = 24, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.526$, Rules used = {1239, 199, 205, 1179, 1168, 1162, 617, 204, 1165, 628}

$$\frac{xe^4}{2d(cd^2 + ae^2)^2(ex^2 + d)} + \frac{\tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)e^{7/2}}{2d^{3/2}(cd^2 + ae^2)^2} + \frac{4c\sqrt{d}\tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)e^{7/2}}{(cd^2 + ae^2)^3} - \frac{c^{3/4}(3cd^2 - 4\sqrt{a}\sqrt{ced} - ae^2)\tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{cx}}{\sqrt{a}}\right)}{2\sqrt{2}a^{3/4}(cd^2 + ae^2)^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x^2)^2*(a + c*x^4)^2), x]
```

```
[Out] (e^4*x)/(2*d*(c*d^2 + a*e^2)^2*(d + e*x^2)) + (c*x*(c*d^2 - a*e^2 - 2*c*d*e*x^2))/(4*a*(c*d^2 + a*e^2)^2*(a + c*x^4)) + (4*c*Sqrt[d]*e^(7/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(c*d^2 + a*e^2)^3 + (e^(7/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(3/2)*(c*d^2 + a*e^2)^2) - (c^(3/4)*e^2*(3*c*d^2 - 4*Sqrt[a]*Sqrt[c]*d*e - a*e^2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)]/(2*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2)^3) - (c^(3/4)*(3*c*d^2 - 2*Sqrt[a]*Sqrt[c]*d*e - 3*a*e^2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)]/(8*Sqrt[2]*a^(7/4)*(c*d^2 + a*e^2)^2) + (c^(3/4)*e^2*(3*c*d^2 - 4*Sqrt[a]*Sqrt[c]*d*e - a*e^2)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)]/(2*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2)^3) + (c^(3/4)*(3*c*d^2 - 2*Sqrt[a]*Sqrt[c]*d*e - 3*a*e^2)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)]/(8*Sqrt[2]*a^(7/4)*(c*d^2 + a*e^2)^2) - (c^(3/4)*e^2*(3*c*d^2 + 4*Sqrt[a]*Sqrt[c]*d*e - a*e^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2)^3) - (c^(3/4)*(3*c*d^2 + 2*Sqrt[a]*Sqrt[c]*d*e - 3*a*e^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*(c*d^2 + a*e^2)^2) + (c^(3/4)*e^2*(3*c*d^2 +
```

$$\frac{4\sqrt{a}\sqrt{c}d\sqrt{e - a e^2} \operatorname{Log}[\sqrt{a} + \sqrt{2} a^{1/4} c^{1/4} x + \sqrt{c} x^2]}{(4\sqrt{2} a^{3/4} (c d^2 + a e^2)^3) + (c^{3/4} (3 c d^2 + 2 \sqrt{a} \sqrt{c} d \sqrt{e - 3 a e^2}) \operatorname{Log}[\sqrt{a} + \sqrt{2} a^{1/4} c^{1/4} x + \sqrt{c} x^2])} / (16 \sqrt{2} a^{7/4} (c d^2 + a e^2)^2)$$

Rule 1239

$$\operatorname{Int}[(d + (e \cdot x)^2)^q (a + (c \cdot x)^4)^p, x] \rightarrow \operatorname{Int}[\operatorname{ExpandIntegrand}[(d + e x^2)^q (a + c x^4)^p, x], x] /; \operatorname{FreeQ}\{a, c, d, e, p, q\}, x \ \&\& \ ((\operatorname{IntegerQ}[p] \ \&\& \ \operatorname{IntegerQ}[q]) \ || \ \operatorname{IGtQ}[p, 0])$$

Rule 199

$$\operatorname{Int}[(a + (b \cdot x)^n)^p, x] \rightarrow -\operatorname{Simp}[(x(a + b x^n)^{p+1}) / (a n (p+1)), x] + \operatorname{Dist}[(n(p+1) + 1) / (a n (p+1)), \operatorname{Int}[(a + b x^n)^{p+1}, x], x] /; \operatorname{FreeQ}\{a, b\}, x \ \&\& \ \operatorname{IGtQ}[n, 0] \ \&\& \ \operatorname{LtQ}[p, -1] \ \&\& \ (\operatorname{IntegerQ}[2p] \ || \ (n == 2 \ \&\& \ \operatorname{IntegerQ}[4p]) \ || \ (n == 2 \ \&\& \ \operatorname{IntegerQ}[3p]) \ || \ \operatorname{Denominator}[p + 1/n] < \operatorname{Denominator}[p])$$

Rule 205

$$\operatorname{Int}[(a + (b \cdot x)^2)^{-1}, x] \rightarrow \operatorname{Simp}[(\operatorname{Rt}[a/b, 2] \operatorname{ArcTan}[x/\operatorname{Rt}[a/b, 2]]) / a, x] /; \operatorname{FreeQ}\{a, b\}, x \ \&\& \ \operatorname{PosQ}[a/b]$$

Rule 1179

$$\operatorname{Int}[(d + (e \cdot x)^2) (a + (c \cdot x)^4)^p, x] \rightarrow -\operatorname{Simp}[(x(d + e x^2) (a + c x^4)^{p+1}) / (4 a (p+1)), x] + \operatorname{Dist}[1 / (4 a (p+1)), \operatorname{Int}[\operatorname{Simp}[d(4p+5) + e(4p+7) x^2, x] (a + c x^4)^{p+1}, x], x] /; \operatorname{FreeQ}\{a, c, d, e\}, x \ \&\& \ \operatorname{NeQ}[c d^2 + a e^2, 0] \ \&\& \ \operatorname{LtQ}[p, -1] \ \&\& \ \operatorname{IntegerQ}[2p]$$

Rule 1168

$$\operatorname{Int}[(d + (e \cdot x)^2) / ((a + (c \cdot x)^4)), x] \rightarrow \operatorname{With}[\{q = \operatorname{Rt}[a c, 2]\}, \operatorname{Dist}[(d q + a e) / (2 a c), \operatorname{Int}[(q + c x^2) / (a + c x^4), x], x] + \operatorname{Dist}[(d q - a e) / (2 a c), \operatorname{Int}[(q - c x^2) / (a + c x^4), x], x] /; \operatorname{FreeQ}\{a, c, d, e\}, x \ \&\& \ \operatorname{NeQ}[c d^2 + a e^2, 0] \ \&\& \ \operatorname{NeQ}[c d^2 - a e^2, 0] \ \&\& \ \operatorname{NegQ}[-(a c)]$$

Rule 1162

$$\operatorname{Int}[(d + (e \cdot x)^2) / ((a + (c \cdot x)^4)), x] \rightarrow \operatorname{With}[\{q = \operatorname{Rt}[(2d)/e, 2]\}, \operatorname{Dist}[e / (2c), \operatorname{Int}[1 / \operatorname{Simp}[d/e + q x + x^2, x], x], x] + \operatorname{Dist}[e / (2c), \operatorname{Int}[1 / \operatorname{Simp}[d/e - q x + x^2, x], x], x] /; \operatorname{FreeQ}\{a, c, d, e\}, x \ \&\& \ \operatorname{EqQ}[c d^2 - a e^2, 0] \ \&\& \ \operatorname{PosQ}[d e]$$

Rule 617

$$\operatorname{Int}[(a + (b \cdot x) + (c \cdot x)^2)^{-1}, x] \rightarrow \operatorname{With}[\{q = 1 - 4 \operatorname{Simplify}[(a c) / b^2]\}, \operatorname{Dist}[-2/b, \operatorname{Subst}[\operatorname{Int}[1 / (q - x^2), x], x, 1 + (2 c x) / b], x] /; \operatorname{RationalQ}[q] \ \&\& \ (\operatorname{EqQ}[q^2, 1] \ || \ \operatorname{!RationalQ}[b^2 - 4 a c]) /; \operatorname{FreeQ}\{a, b, c\}, x \ \&\& \ \operatorname{NeQ}[b^2 - 4 a c, 0]$$

Rule 204

$$\operatorname{Int}[(a + (b \cdot x)^2)^{-1}, x] \rightarrow -\operatorname{Simp}[\operatorname{ArcTan}[(\operatorname{Rt}[-b, 2] x) / \operatorname{Rt}[-a, 2]] / (\operatorname{Rt}[-a, 2] \operatorname{Rt}[-b, 2]), x] /; \operatorname{FreeQ}\{a, b\}, x \ \&\& \ \operatorname{PosQ}[a/b] \ \&\& \ (\operatorname{LtQ}[$$

a, 0] || LtQ[b, 0])

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{(d+ex^2)^2(a+cx^4)^2} dx &= \int \left(\frac{e^4}{(cd^2+ae^2)^2(d+ex^2)^2} + \frac{4cde^4}{(cd^2+ae^2)^3(d+ex^2)} + \frac{c(cd^2-ae^2-2cdex^2)}{(cd^2+ae^2)^2(a+cx^4)^2} - \frac{ce^2(-3cd^2+ae^2)}{(cd^2+ae^2)^3} \right) dx \\ &= -\frac{(ce^2) \int \frac{-3cd^2+ae^2+4cdex^2}{a+cx^4} dx}{(cd^2+ae^2)^3} + \frac{(4cde^4) \int \frac{1}{d+ex^2} dx}{(cd^2+ae^2)^3} + \frac{c \int \frac{cd^2-ae^2-2cdex^2}{(a+cx^4)^2} dx}{(cd^2+ae^2)^2} + \frac{e^4 \int \frac{1}{(d+ex^2)^2} dx}{(cd^2+ae^2)^2} \\ &= \frac{e^4 x}{2d(cd^2+ae^2)^2(d+ex^2)} + \frac{cx(cd^2-ae^2-2cdex^2)}{4a(cd^2+ae^2)^2(a+cx^4)} + \frac{4c\sqrt{d}e^{7/2} \tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)}{(cd^2+ae^2)^3} + \frac{(\sqrt{ce^2}(3cd^2+ae^2)) \tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)}{2d^{3/2}(cd^2+ae^2)^2} \\ &= \frac{e^4 x}{2d(cd^2+ae^2)^2(d+ex^2)} + \frac{cx(cd^2-ae^2-2cdex^2)}{4a(cd^2+ae^2)^2(a+cx^4)} + \frac{4c\sqrt{d}e^{7/2} \tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)}{(cd^2+ae^2)^3} + \frac{e^{7/2} \tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)}{2d^{3/2}(cd^2+ae^2)^2} \\ &= \frac{e^4 x}{2d(cd^2+ae^2)^2(d+ex^2)} + \frac{cx(cd^2-ae^2-2cdex^2)}{4a(cd^2+ae^2)^2(a+cx^4)} + \frac{4c\sqrt{d}e^{7/2} \tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)}{(cd^2+ae^2)^3} + \frac{e^{7/2} \tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)}{2d^{3/2}(cd^2+ae^2)^2} \\ &= \frac{e^4 x}{2d(cd^2+ae^2)^2(d+ex^2)} + \frac{cx(cd^2-ae^2-2cdex^2)}{4a(cd^2+ae^2)^2(a+cx^4)} + \frac{4c\sqrt{d}e^{7/2} \tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)}{(cd^2+ae^2)^3} + \frac{e^{7/2} \tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)}{2d^{3/2}(cd^2+ae^2)^2} \\ &= \frac{e^4 x}{2d(cd^2+ae^2)^2(d+ex^2)} + \frac{cx(cd^2-ae^2-2cdex^2)}{4a(cd^2+ae^2)^2(a+cx^4)} + \frac{4c\sqrt{d}e^{7/2} \tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)}{(cd^2+ae^2)^3} + \frac{e^{7/2} \tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)}{2d^{3/2}(cd^2+ae^2)^2} \end{aligned}$$

Mathematica [A] time = 0.623756, size = 540, normalized size = 0.62

$$-\frac{\sqrt{2}c^{3/4}(18a^{3/2}\sqrt{cde^3-7a^2e^4+2\sqrt{ac^3}d^3e+12acd^2e^2+3c^2d^4)} \log(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{cx+\sqrt{a}+\sqrt{cx^2}})}{a^{7/4}} + \frac{\sqrt{2}c^{3/4}(18a^{3/2}\sqrt{cde^3-7a^2e^4+2\sqrt{ac^3}d^3e+12acd^2e^2+3c^2d^4)} \log(\sqrt{2}\sqrt[4]{a}\sqrt[4]{cx+\sqrt{a}+\sqrt{cx^2}})}{a^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x^2)^2*(a + c*x^4)^2), x]
```

```
[Out] ((16*e^4*(c*d^2 + a*e^2)*x)/(d*(d + e*x^2)) + (8*c*(c*d^2 + a*e^2)*x*(-(a*e^2 + c*d*(d - 2*e*x^2)))/(a*(a + c*x^4)) + (16*e^(7/2)*(9*c*d^2 + a*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/d^(3/2) + (2*Sqrt[2]*c^(3/4)*(-3*c^2*d^4 + 2*Sqrt[a]*c^(3/2)*d^3*e - 12*a*c*d^2*e^2 + 18*a^(3/2)*Sqrt[c]*d*e^3 + 7*a^2*e^4)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)]/a^(7/4) - (2*Sqrt[2]*c^(3/4)*(-3*c^2*d^4 + 2*Sqrt[a]*c^(3/2)*d^3*e - 12*a*c*d^2*e^2 + 18*a^(3/2)*Sqrt[c]*d*e^3 + 7*a^2*e^4)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)]/a^(7/4) - (Sqrt[2]*c^(3/4)*(3*c^2*d^4 + 2*Sqrt[a]*c^(3/2)*d^3*e + 12*a*c*d^2*e^2 + 18*a^(3/2)*Sqrt[c]*d*e^3 - 7*a^2*e^4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/a^(7/4) + (Sqrt[2]*c^(3/4)*(3*c^2*d^4 + 2*Sqrt[a]*c^(3/2)*d^3*e + 12*a*c*d^2*e^2 + 18*a^(3/2)*Sqrt[c]*d*e^3 - 7*a^2*e^4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/a^(7/4))/(32*(c*d^2 + a*e^2)^3)
```

Maple [A] time = 0.066, size = 1169, normalized size = 1.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d)^2/(c*x^4+a)^2,x)
```

```
[Out] -1/2*c^2/(a*e^2+c*d^2)^3/(c*x^4+a)*d*e^3*x^3-1/2*c^3/(a*e^2+c*d^2)^3/(c*x^4+a)*d^3*e/a*x^3-1/4*c/(a*e^2+c*d^2)^3/(c*x^4+a)*x*a*e^4+1/4*c^3/(a*e^2+c*d^2)^3/(c*x^4+a)*x/a*d^4-7/16*c/(a*e^2+c*d^2)^3*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)*e^4+3/4*c^2/(a*e^2+c*d^2)^3/a*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)*d^2*e^2+3/16*c^3/(a*e^2+c*d^2)^3/a^2*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)*e^4+3/4*c^2/(a*e^2+c*d^2)^3/a*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)*d^2*e^2+3/16*c^3/(a*e^2+c*d^2)^3/a^2*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)*d^4-7/32*c/(a*e^2+c*d^2)^3*(a/c)^(1/4)*2^(1/2)*ln((x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))*e^4+3/8*c^2/(a*e^2+c*d^2)^3/a*(a/c)^(1/4)*2^(1/2)*ln((x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))*d^2*e^2+3/32*c^3/(a*e^2+c*d^2)^3/a^2*(a/c)^(1/4)*2^(1/2)*ln((x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))*d^4-9/16*c/(a*e^2+c*d^2)^3/(a/c)^(1/4)*2^(1/2)*ln((x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))*d*e^3-1/16*c^2/(a*e^2+c*d^2)^3/a/(a/c)^(1/4)*2^(1/2)*ln((x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))*d^3*e-9/8*c/(a*e^2+c*d^2)^3/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)*d*e^3-1/8*c^2/(a*e^2+c*d^2)^3/a/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)*d^3*e-9/8*c/(a*e^2+c*d^2)^3/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)*d*e^3-1/8*c^2/(a*e^2+c*d^2)^3/a/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)*d^3*e+1/2*e^6/(a*e^2+c*d^2)^3/d*x/(e*x^2+d)*a+1/2*e^4/(a*e^2+c*d^2)^3*d*x/(e*x^2+d)*c+1/2*e^6/(a*e^2+c*d^2)^3/d/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^2/(c*x^4+a)^2,x, algorithm="maxima")
```

[Out] Exception raised: ValueError

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x^2+d)^2/(c*x^4+a)^2,x, algorithm="fricas")

[Out] Timed out

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x**2+d)**2/(c*x**4+a)**2,x)

[Out] Timed out

Giac [A] time = 1.15424, size = 1146, normalized size = 1.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x^2+d)^2/(c*x^4+a)^2,x, algorithm="giac")

[Out]
$$\frac{1}{2} \cdot (9cd^2e^4 + a^6) \arctan\left(\frac{x\sqrt{d}}{\sqrt{d}}\right) e^{-1/2} / \left((c^3d^7 + 3a^2cd^5e^2 + 3a^2cd^3e^4 + a^3d^2e^6) \sqrt{d} \right) + \frac{1}{8} \cdot (3(a^3c^3)^{1/4} c^3d^4 + 12(a^3c^3)^{1/4} a^2cd^2e^2 - 2(a^3c^3)^{3/4} cd^3e - 7(a^3c^3)^{1/4} a^2ce^4 - 18(a^3c^3)^{3/4} ad^3e^3) \arctan\left(\frac{1}{2} \sqrt{2} (2x + \sqrt{2}) \left(\frac{a}{c}\right)^{1/4}\right) / \left(\sqrt{2} a^2c^4d^6 + 3\sqrt{2} a^3c^3d^4e^2 + 3\sqrt{2} a^4c^2d^2e^4 + \sqrt{2} a^5ce^6 \right) + \frac{1}{8} \cdot (3(a^3c^3)^{1/4} c^3d^4 + 12(a^3c^3)^{1/4} a^2cd^2e^2 - 2(a^3c^3)^{3/4} cd^3e - 7(a^3c^3)^{1/4} a^2ce^4 - 18(a^3c^3)^{3/4} ad^3e^3) \arctan\left(\frac{1}{2} \sqrt{2} (2x - \sqrt{2}) \left(\frac{a}{c}\right)^{1/4}\right) / \left(\sqrt{2} a^2c^4d^6 + 3\sqrt{2} a^3c^3d^4e^2 + 3\sqrt{2} a^4c^2d^2e^4 + \sqrt{2} a^5ce^6 \right) + \frac{1}{16} \cdot (3(a^3c^3)^{1/4} c^3d^4 + 12(a^3c^3)^{1/4} a^2cd^2e^2 + 2(a^3c^3)^{3/4} cd^3e - 7(a^3c^3)^{1/4} a^2ce^4 + 18(a^3c^3)^{3/4} ad^3e^3) \log(x^2 + \sqrt{2} x \left(\frac{a}{c}\right)^{1/4} + \sqrt{a/c}) / \left(\sqrt{2} a^2c^4d^6 + 3\sqrt{2} a^3c^3d^4e^2 + 3\sqrt{2} a^4c^2d^2e^4 + \sqrt{2} a^5ce^6 \right) - \frac{1}{16} \cdot (3(a^3c^3)^{1/4} c^3d^4 + 12(a^3c^3)^{1/4} a^2cd^2e^2 + 2(a^3c^3)^{3/4} cd^3e - 7(a^3c^3)^{1/4} a^2ce^4 + 18(a^3c^3)^{3/4} ad^3e^3) \log(x^2 - \sqrt{2} x \left(\frac{a}{c}\right)^{1/4} + \sqrt{a/c}) / \left(\sqrt{2} a^2c^4d^6 + 3\sqrt{2} a^3c^3d^4e^2 + 3\sqrt{2} a^4c^2d^2e^4 + \sqrt{2} a^5ce^6 \right) - \frac{1}{4} \cdot (2c^2d^2x^5e^2 + c^2d^3x^3e - 2ac^2x^5e^4 - c^2d^4x + ac^2d^2x^3e^3 + ac^2d^2x^2e^2 - 2a^2x^2e^4) / \left((a^2cd^5 + 2a^2cd^3e^2 + a^3d^2e^4) (cx^6e + cd^2x^4 + ax^2e + ad) \right)$$

$$3.150 \quad \int \frac{(d+ex^2)^4}{\sqrt{a+cx^4}} dx$$

Optimal. Leaf size=388

$$\frac{(\sqrt{a} + \sqrt{cx^2}) \sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{cx^2})^2}} (-252a^{3/2}\sqrt{cde^3} + 25a^2e^4 + 420\sqrt{ac}^{3/2}d^3e - 210acd^2e^2 + 105c^2d^4) \text{EllipticF}\left(2 \tan^{-1}\left(\frac{\sqrt{cx}}{\sqrt{a}}\right)\right)}{210\sqrt[4]{ac}^{9/4}\sqrt{a+cx^4}}$$

[Out] (e^2*(42*c*d^2 - 5*a*e^2)*x*Sqrt[a + c*x^4])/(21*c^2) + (4*d*e^3*x^3*Sqrt[a + c*x^4])/(5*c) + (e^4*x^5*Sqrt[a + c*x^4])/(7*c) + (4*d*e*(5*c*d^2 - 3*a*e^2)*x*Sqrt[a + c*x^4])/(5*c^(3/2)*(Sqrt[a] + Sqrt[c]*x^2)) - (4*a^(1/4)*d*e*(5*c*d^2 - 3*a*e^2)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(5*c^(7/4)*Sqrt[a + c*x^4]) + ((105*c^2*d^4 + 420*Sqrt[a]*c^(3/2)*d^3*e - 210*a*c*d^2*e^2 - 252*a^(3/2)*Sqrt[c]*d*e^3 + 25*a^2*e^4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(210*a^(1/4)*c^(9/4)*Sqrt[a + c*x^4])

Rubi [A] time = 0.415187, antiderivative size = 386, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {1207, 1888, 1198, 220, 1196}

$$\frac{(\sqrt{a} + \sqrt{cx^2}) \sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{cx^2})^2}} (5(5a^2e^4 - 42acd^2e^2 + 21c^2d^4) + 84\sqrt{a}\sqrt{cde}(5cd^2 - 3ae^2)) F\left(2 \tan^{-1}\left(\frac{\sqrt{cx}}{\sqrt{a}}\right) \middle| \frac{1}{2}\right)}{210\sqrt[4]{ac}^{9/4}\sqrt{a+cx^4}} + \frac{e^2x\sqrt{a}}{\dots}$$

Antiderivative was successfully verified.

[In] Int[(d + e*x^2)^4/Sqrt[a + c*x^4], x]

[Out] (e^2*(42*c*d^2 - 5*a*e^2)*x*Sqrt[a + c*x^4])/(21*c^2) + (4*d*e^3*x^3*Sqrt[a + c*x^4])/(5*c) + (e^4*x^5*Sqrt[a + c*x^4])/(7*c) + (4*d*e*(5*c*d^2 - 3*a*e^2)*x*Sqrt[a + c*x^4])/(5*c^(3/2)*(Sqrt[a] + Sqrt[c]*x^2)) - (4*a^(1/4)*d*e*(5*c*d^2 - 3*a*e^2)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(5*c^(7/4)*Sqrt[a + c*x^4]) + ((84*Sqrt[a]*Sqrt[c]*d*e*(5*c*d^2 - 3*a*e^2) + 5*(21*c^2*d^4 - 42*a*c*d^2*e^2 + 5*a^2*e^4))*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(210*a^(1/4)*c^(9/4)*Sqrt[a + c*x^4])

Rule 1207

Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] := Simp[(e^q*x^(2*q - 3)*(a + c*x^4)^(p + 1))/(c*(4*p + 2*q + 1)), x] + Dist[1/(c*(4*p + 2*q + 1)), Int[(a + c*x^4)^p*ExpandToSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[q, 1]

Rule 1888

Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{q = Expon[Pq, x]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(q + n*p + 1)), Int[ExpandToSum[b*(q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*(q - n + 1)*x^(q - n), x]*(a + b*x^n)^p, x], x] + Simp[(Pqq*x^(q - n + 1)*(a + b*x^n)^(p + 1))/(b*(q + n*p + 1

)), x]] /; NeQ[q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[p + (q + 1)/(2*n)]) /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]

Rule 1198

Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] := With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0] /; FreeQ[{a, c, d, e}, x] && PosQ[c/a]

Rule 220

Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] := With[{q = Rt[b/a, 4]}, Simp[(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x], 1/2]]/(2*q*Sqrt[a + b*x^4]), x] /; FreeQ[{a, b}, x] && PosQ[b/a]

Rule 1196

Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] := With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x], 1/2]]/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0] /; FreeQ[{a, c, d, e}, x] && PosQ[c/a]

Rubi steps

$$\begin{aligned} \int \frac{(d + ex^2)^4}{\sqrt{a + cx^4}} dx &= \frac{e^4 x^5 \sqrt{a + cx^4}}{7c} + \frac{\int \frac{7cd^4 + 28cd^3 ex^2 + e^2(42cd^2 - 5ae^2)x^4 + 28cde^3 x^6}{\sqrt{a + cx^4}} dx}{7c} \\ &= \frac{4de^3 x^3 \sqrt{a + cx^4}}{5c} + \frac{e^4 x^5 \sqrt{a + cx^4}}{7c} + \frac{\int \frac{35c^2 d^4 + 28cde(5cd^2 - 3ae^2)x^2 + 5ce^2(42cd^2 - 5ae^2)x^4}{\sqrt{a + cx^4}} dx}{35c^2} \\ &= \frac{e^2(42cd^2 - 5ae^2)x\sqrt{a + cx^4}}{21c^2} + \frac{4de^3 x^3 \sqrt{a + cx^4}}{5c} + \frac{e^4 x^5 \sqrt{a + cx^4}}{7c} + \frac{\int \frac{5c(21c^2 d^4 - 42acd^2 e^2 + 5a^2 e^4) + 84c^2 de(5cd^2 - 3ae^2)}{\sqrt{a + cx^4}} dx}{105c^3} \\ &= \frac{e^2(42cd^2 - 5ae^2)x\sqrt{a + cx^4}}{21c^2} + \frac{4de^3 x^3 \sqrt{a + cx^4}}{5c} + \frac{e^4 x^5 \sqrt{a + cx^4}}{7c} - \frac{(4\sqrt{ade}(5cd^2 - 3ae^2)) \int \frac{1 - \frac{\sqrt{cx^2}}{\sqrt{a + cx^4}}}{\sqrt{a + cx^4}} dx}{5c^{3/2}} \\ &= \frac{e^2(42cd^2 - 5ae^2)x\sqrt{a + cx^4}}{21c^2} + \frac{4de^3 x^3 \sqrt{a + cx^4}}{5c} + \frac{e^4 x^5 \sqrt{a + cx^4}}{7c} + \frac{4de(5cd^2 - 3ae^2)x\sqrt{a + cx^4}}{5c^{3/2}(\sqrt{a} + \sqrt{cx^2})} - \frac{4\sqrt{ade}(5cd^2 - 3ae^2)}{5c^{3/2}} \end{aligned}$$

Mathematica [C] time = 0.216292, size = 203, normalized size = 0.52

$$\frac{5x\sqrt{\frac{cx^4}{a}} + 1(5a^2e^4 - 42acd^2e^2 + 21c^2d^4) {}_2F_1\left(\frac{1}{4}, \frac{1}{2}; \frac{5}{4}; -\frac{cx^4}{a}\right) + ex\left(-25a^2e^3 + 28cdx^2\sqrt{\frac{cx^4}{a}} + 1(5cd^2 - 3ae^2) {}_2F_1\left(\frac{1}{2}, \frac{3}{4}; \frac{7}{4}; -\frac{cx^4}{a}\right)\right)}{105c^2\sqrt{a + cx^4}}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e*x^2)^4/Sqrt[a + c*x^4], x]

[Out] (5*(21*c^2*d^4 - 42*a*c*d^2*e^2 + 5*a^2*e^4)*x*Sqrt[1 + (c*x^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -(c*x^4)/a] + e*x*(-25*a^2*e^3 + 2*a*c*e*(105*d^2 + 42*d*e*x^2 - 5*e^2*x^4) + 3*c^2*e*x^4*(70*d^2 + 28*d*e*x^2 + 5*e^2*x^4)

+ 28*c*d*(5*c*d^2 - 3*a*e^2)*x^2*sqrt[1 + (c*x^4)/a]*Hypergeometric2F1[1/2, 3/4, 7/4, -((c*x^4)/a)]/(105*c^2*sqrt[a + c*x^4])

Maple [C] time = 0.192, size = 506, normalized size = 1.3

$$e^4 \left(\frac{x^5}{7c} \sqrt{cx^4 + a} - \frac{5ax}{21c^2} \sqrt{cx^4 + a} + \frac{5a^2}{21c^2} \sqrt{1 - ix^2\sqrt{c}\frac{1}{\sqrt{a}}} \sqrt{1 + ix^2\sqrt{c}\frac{1}{\sqrt{a}}} \operatorname{EllipticF} \left(x \sqrt{i\sqrt{c}\frac{1}{\sqrt{a}}}, i \right) \frac{1}{\sqrt{i\sqrt{c}\frac{1}{\sqrt{a}}}} \frac{1}{\sqrt{cx^4 + a}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x^2+d)^4/(c*x^4+a)^(1/2), x)

[Out] e^4*(1/7/c*x^5*(c*x^4+a)^(1/2)-5/21*a/c^2*x*(c*x^4+a)^(1/2)+5/21*a^2/c^2/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2), I))+4*d*e^3*(1/5/c*x^3*(c*x^4+a)^(1/2)-3/5*I*a^(3/2)/c^(3/2)/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*(EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2), I)-EllipticE(x*(I/a^(1/2)*c^(1/2))^(1/2), I)))+6*d^2*e^2*(1/3/c*x*(c*x^4+a)^(1/2)-1/3*a/c/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2), I))+4*I*d^3*e*a^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)/c^(1/2)*(EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2), I)-EllipticE(x*(I/a^(1/2)*c^(1/2))^(1/2), I))+d^4/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2), I)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^2 + d)^4}{\sqrt{cx^4 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)^4/(c*x^4+a)^(1/2), x, algorithm="maxima")

[Out] integrate((e*x^2 + d)^4/sqrt(c*x^4 + a), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral} \left(\frac{e^4 x^8 + 4 d e^3 x^6 + 6 d^2 e^2 x^4 + 4 d^3 e x^2 + d^4}{\sqrt{cx^4 + a}}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)^4/(c*x^4+a)^(1/2), x, algorithm="fricas")

[Out] integral((e^4*x^8 + 4*d*e^3*x^6 + 6*d^2*e^2*x^4 + 4*d^3*e*x^2 + d^4)/sqrt(c*x^4 + a), x)

Sympy [C] time = 4.70302, size = 214, normalized size = 0.55

$$\frac{d^4 x \Gamma\left(\frac{1}{4}\right) {}_2F_1\left(\frac{1}{4}, \frac{1}{2} \middle| \frac{cx^4 e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{5}{4}\right)} + \frac{d^3 e x^3 \Gamma\left(\frac{3}{4}\right) {}_2F_1\left(\frac{1}{2}, \frac{3}{4} \middle| \frac{cx^4 e^{i\pi}}{a}\right)}{\sqrt{a}\Gamma\left(\frac{7}{4}\right)} + \frac{3d^2 e^2 x^5 \Gamma\left(\frac{5}{4}\right) {}_2F_1\left(\frac{1}{2}, \frac{5}{4} \middle| \frac{cx^4 e^{i\pi}}{a}\right)}{2\sqrt{a}\Gamma\left(\frac{9}{4}\right)} + \frac{d e^3 x^7 \Gamma\left(\frac{7}{4}\right) {}_2F_1\left(\frac{1}{2}, \frac{7}{4} \middle| \frac{cx^4 e^{i\pi}}{a}\right)}{\sqrt{a}\Gamma\left(\frac{11}{4}\right)} + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x**2+d)**4/(c*x**4+a)**(1/2),x)

[Out] d**4*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), c*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(5/4)) + d**3*e*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), c*x**4*exp_polar(I*pi)/a)/(sqrt(a)*gamma(7/4)) + 3*d**2*e**2*x**5*gamma(5/4)*hyper((1/2, 5/4), (9/4,), c*x**4*exp_polar(I*pi)/a)/(2*sqrt(a)*gamma(9/4)) + d*e**3*x**7*gamma(7/4)*hyper((1/2, 7/4), (11/4,), c*x**4*exp_polar(I*pi)/a)/(sqrt(a)*gamma(11/4)) + e**4*x**9*gamma(9/4)*hyper((1/2, 9/4), (13/4,), c*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(13/4))

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^2 + d)^4}{\sqrt{cx^4 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)^4/(c*x^4+a)^(1/2),x, algorithm="giac")

[Out] integrate((e*x^2 + d)^4/sqrt(c*x^4 + a), x)

$$3.151 \quad \int \frac{(d+ex^2)^3}{\sqrt{a+cx^4}} dx$$

Optimal. Leaf size=326

$$\frac{\sqrt[4]{a}(\sqrt{a} + \sqrt{cx^2}) \sqrt{\frac{a+cx^4}{(\sqrt{a} + \sqrt{cx^2})^2}} \left(\frac{5\sqrt{cd}(cd^2 - ae^2)}{\sqrt{a}} - 3ae^3 + 15cd^2e \right) \text{EllipticF}\left(2 \tan^{-1}\left(\frac{\sqrt[4]{cx}}{\sqrt[4]{a}}\right), \frac{1}{2}\right)}{10c^{7/4}\sqrt{a+cx^4}} + \frac{3ex\sqrt{a+cx^4}(5cd^2 - ae^2)}{5c^{3/2}(\sqrt{a} + \sqrt{cx^2})}$$

```
[Out] (d*e^2*x*Sqrt[a + c*x^4])/c + (e^3*x^3*Sqrt[a + c*x^4])/(5*c) + (3*e*(5*c*d^2 - a*e^2)*x*Sqrt[a + c*x^4])/(5*c^(3/2)*(Sqrt[a] + Sqrt[c]*x^2)) - (3*a^(1/4)*e*(5*c*d^2 - a*e^2)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(5*c^(7/4)*Sqrt[a + c*x^4]) + (a^(1/4)*(15*c*d^2*e - 3*a*e^3 + (5*Sqrt[c]*d*(c*d^2 - a*e^2))/Sqrt[a])*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(10*c^(7/4)*Sqrt[a + c*x^4])
```

Rubi [A] time = 0.286931, antiderivative size = 326, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {1207, 1888, 1198, 220, 1196}

$$\frac{3ex\sqrt{a+cx^4}(5cd^2 - ae^2)}{5c^{3/2}(\sqrt{a} + \sqrt{cx^2})} + \frac{\sqrt[4]{a}(\sqrt{a} + \sqrt{cx^2}) \sqrt{\frac{a+cx^4}{(\sqrt{a} + \sqrt{cx^2})^2}} \left(\frac{5\sqrt{cd}(cd^2 - ae^2)}{\sqrt{a}} - 3ae^3 + 15cd^2e \right) F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{cx}}{\sqrt[4]{a}}\right), \frac{1}{2}\right)}{10c^{7/4}\sqrt{a+cx^4}} - \frac{3\sqrt[4]{a}}{10c^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^3/Sqrt[a + c*x^4], x]
```

```
[Out] (d*e^2*x*Sqrt[a + c*x^4])/c + (e^3*x^3*Sqrt[a + c*x^4])/(5*c) + (3*e*(5*c*d^2 - a*e^2)*x*Sqrt[a + c*x^4])/(5*c^(3/2)*(Sqrt[a] + Sqrt[c]*x^2)) - (3*a^(1/4)*e*(5*c*d^2 - a*e^2)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(5*c^(7/4)*Sqrt[a + c*x^4]) + (a^(1/4)*(15*c*d^2*e - 3*a*e^3 + (5*Sqrt[c]*d*(c*d^2 - a*e^2))/Sqrt[a])*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(10*c^(7/4)*Sqrt[a + c*x^4])
```

Rule 1207

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] := Simp[(e^q*x^(2*q - 3)*(a + c*x^4)^(p + 1))/(c*(4*p + 2*q + 1)), x] + Dist[1/(c*(4*p + 2*q + 1)), Int[(a + c*x^4)^p*ExpandToSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - c*(4*p + 2*q + 1)*e^q*x^(2*q)], x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[q, 1]
```

Rule 1888

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{q = Expon[Pq, x]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(q + n*p + 1)), Int[ExpandToSum[b*(q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*(q - n + 1)*x^(q - n), x]*(a + b*x^n)^p, x], x] + Simp[(Pqq*x^(q - n + 1)*(a + b*x^n)^(p + 1))/(b*(q + n*p + 1)), x]] /; NeQ[q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[
```

$p + (q + 1)/(2*n)]]) /; \text{FreeQ}\{a, b, p\}, x\} \&\& \text{PolyQ}\{Pq, x\} \&\& \text{IGtQ}\{n, 0\}$

Rule 1198

$\text{Int}[(d + e*x^2)/\text{Sqrt}[a + c*x^4], x_Symbol] := \text{With}\{q = \text{Rt}[c/a, 2]\}, \text{Dist}[(e + d*q)/q, \text{Int}[1/\text{Sqrt}[a + c*x^4], x], x] - \text{Dist}[e/q, \text{Int}[(1 - q*x^2)/\text{Sqrt}[a + c*x^4], x], x] /; \text{NeQ}[e + d*q, 0] /; \text{FreeQ}\{a, c, d, e\}, x\} \&\& \text{PosQ}[c/a]$

Rule 220

$\text{Int}[1/\text{Sqrt}[(a + b*x^4)], x_Symbol] := \text{With}\{q = \text{Rt}[b/a, 4]\}, \text{Simp}[(1 + q^2*x^2)*\text{Sqrt}[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*\text{EllipticF}[2*\text{ArcTan}[q*x, 1/2]]/(2*q*\text{Sqrt}[a + b*x^4]), x] /; \text{FreeQ}\{a, b\}, x\} \&\& \text{PosQ}[b/a]$

Rule 1196

$\text{Int}[(d + e*x^2)/\text{Sqrt}[(a + c*x^4)], x_Symbol] := \text{With}\{q = \text{Rt}[c/a, 4]\}, -\text{Simp}[(d*x*\text{Sqrt}[a + c*x^4])/(a*(1 + q^2*x^2)), x] + \text{Simp}[(d*(1 + q^2*x^2)*\text{Sqrt}[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*\text{EllipticE}[2*\text{ArcTan}[q*x, 1/2]])/(q*\text{Sqrt}[a + c*x^4]), x] /; \text{EqQ}[e + d*q^2, 0] /; \text{FreeQ}\{a, c, d, e\}, x\} \&\& \text{PosQ}[c/a]$

Rubi steps

$$\begin{aligned} \int \frac{(d + ex^2)^3}{\sqrt{a + cx^4}} dx &= \frac{e^3 x^3 \sqrt{a + cx^4}}{5c} + \frac{\int \frac{5cd^3 + 3e(5cd^2 - ae^2)x^2 + 15cde^2 x^4}{\sqrt{a + cx^4}} dx}{5c} \\ &= \frac{de^2 x \sqrt{a + cx^4}}{c} + \frac{e^3 x^3 \sqrt{a + cx^4}}{5c} + \frac{\int \frac{15cd(cd^2 - ae^2) + 9ce(5cd^2 - ae^2)x^2}{\sqrt{a + cx^4}} dx}{15c^2} \\ &= \frac{de^2 x \sqrt{a + cx^4}}{c} + \frac{e^3 x^3 \sqrt{a + cx^4}}{5c} - \frac{(3\sqrt{ae}(5cd^2 - ae^2)) \int \frac{1 - \sqrt{cx^2}}{\sqrt{a + cx^4}} dx}{5c^{3/2}} + \frac{(5\sqrt{cd}(cd^2 - ae^2) + 3\sqrt{ae}(5cd^2 - ae^2))}{5c^{3/2}} \\ &= \frac{de^2 x \sqrt{a + cx^4}}{c} + \frac{e^3 x^3 \sqrt{a + cx^4}}{5c} + \frac{3e(5cd^2 - ae^2)x\sqrt{a + cx^4}}{5c^{3/2}(\sqrt{a} + \sqrt{cx^2})} - \frac{3^4 \sqrt{ae}(5cd^2 - ae^2)(\sqrt{a} + \sqrt{cx^2}) \sqrt{\frac{a}{\sqrt{a} + \sqrt{cx^2}}}}{5c^{7/4} \sqrt{a + cx^4}} \end{aligned}$$

Mathematica [C] time = 0.144001, size = 140, normalized size = 0.43

$$\frac{5dx\sqrt{\frac{cx^4}{a} + 1}(cd^2 - ae^2) {}_2F_1\left(\frac{1}{4}, \frac{1}{2}; \frac{5}{4}; -\frac{cx^4}{a}\right) + ex\left(x^2\sqrt{\frac{cx^4}{a} + 1}(5cd^2 - ae^2) {}_2F_1\left(\frac{1}{2}, \frac{3}{4}; \frac{7}{4}; -\frac{cx^4}{a}\right) + e(a + cx^4)(5d + ex^2)\right)}{5c\sqrt{a + cx^4}}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e*x^2)^3/Sqrt[a + c*x^4],x]

[Out] (5*d*(c*d^2 - a*e^2)*x*Sqrt[1 + (c*x^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -((c*x^4)/a)] + e*x*(e*(5*d + e*x^2)*(a + c*x^4) + (5*c*d^2 - a*e^2)*x^2*Sqrt[1 + (c*x^4)/a]*Hypergeometric2F1[1/2, 3/4, 7/4, -((c*x^4)/a)])/(5*c*Sqrt[a + c*x^4])

Maple [C] time = 0.052, size = 388, normalized size = 1.2

$$e^3 \left(\frac{x^3}{5c} \sqrt{cx^4 + a} - \frac{3i}{5} a^{\frac{3}{2}} \sqrt{1 - ix^2 \sqrt{c} \frac{1}{\sqrt{a}}} \sqrt{1 + ix^2 \sqrt{c} \frac{1}{\sqrt{a}}} \left(\text{EllipticF} \left(x \sqrt{i \sqrt{c} \frac{1}{\sqrt{a}}}, i \right) - \text{EllipticE} \left(x \sqrt{i \sqrt{c} \frac{1}{\sqrt{a}}}, i \right) \right) c^{-\frac{3}{2}} \sqrt{\frac{1}{a}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x^2+d)^3/(c*x^4+a)^(1/2), x)

[Out] $e^3 \frac{(1/5/c*x^3*(c*x^4+a)^{(1/2)} - 3/5*I*a^{(3/2)}/c^{(3/2)}/(I/a^{(1/2)}*c^{(1/2)})^{(1/2)} * (1-I/a^{(1/2)}*c^{(1/2)}*x^2)^{(1/2)} * (1+I/a^{(1/2)}*c^{(1/2)}*x^2)^{(1/2)} / (c*x^4+a)^{(1/2)} * (\text{EllipticF}(x*(I/a^{(1/2)}*c^{(1/2)})^{(1/2)}, I) - \text{EllipticE}(x*(I/a^{(1/2)}*c^{(1/2)})^{(1/2)}, I)) + 3*d*e^2*(1/3/c*x*(c*x^4+a)^{(1/2)} - 1/3*a/c/(I/a^{(1/2)}*c^{(1/2)})^{(1/2)} * (1-I/a^{(1/2)}*c^{(1/2)}*x^2)^{(1/2)} * (1+I/a^{(1/2)}*c^{(1/2)}*x^2)^{(1/2)} / (c*x^4+a)^{(1/2)} * \text{EllipticF}(x*(I/a^{(1/2)}*c^{(1/2)})^{(1/2)}, I) + 3*I*d^2*e*a^{(1/2)}/(I/a^{(1/2)}*c^{(1/2)})^{(1/2)} * (1-I/a^{(1/2)}*c^{(1/2)}*x^2)^{(1/2)} * (1+I/a^{(1/2)}*c^{(1/2)}*x^2)^{(1/2)} / (c*x^4+a)^{(1/2)} / c^{(1/2)} * (\text{EllipticF}(x*(I/a^{(1/2)}*c^{(1/2)})^{(1/2)}, I) - \text{EllipticE}(x*(I/a^{(1/2)}*c^{(1/2)})^{(1/2)}, I)) + d^3/(I/a^{(1/2)}*c^{(1/2)})^{(1/2)} * (1-I/a^{(1/2)}*c^{(1/2)}*x^2)^{(1/2)} * (1+I/a^{(1/2)}*c^{(1/2)}*x^2)^{(1/2)} / (c*x^4+a)^{(1/2)} * \text{EllipticF}(x*(I/a^{(1/2)}*c^{(1/2)})^{(1/2)}, I)$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^2 + d)^3}{\sqrt{cx^4 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)^3/(c*x^4+a)^(1/2), x, algorithm="maxima")

[Out] integrate((e*x^2 + d)^3/sqrt(c*x^4 + a), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral} \left(\frac{e^3 x^6 + 3 d e^2 x^4 + 3 d^2 e x^2 + d^3}{\sqrt{cx^4 + a}}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)^3/(c*x^4+a)^(1/2), x, algorithm="fricas")

[Out] integral((e^3*x^6 + 3*d*e^2*x^4 + 3*d^2*e*x^2 + d^3)/sqrt(c*x^4 + a), x)

Sympy [C] time = 3.58309, size = 173, normalized size = 0.53

$$\frac{d^3 x \Gamma\left(\frac{1}{4}\right) {}_2F_1\left(\frac{1}{4}, \frac{1}{2} \left| \frac{cx^4 e^{i\pi}}{a} \right.\right)}{4\sqrt{a} \Gamma\left(\frac{5}{4}\right)} + \frac{3d^2 e x^3 \Gamma\left(\frac{3}{4}\right) {}_2F_1\left(\frac{3}{4}, \frac{3}{4} \left| \frac{cx^4 e^{i\pi}}{a} \right.\right)}{4\sqrt{a} \Gamma\left(\frac{7}{4}\right)} + \frac{3d e^2 x^5 \Gamma\left(\frac{5}{4}\right) {}_2F_1\left(\frac{5}{4}, \frac{5}{4} \left| \frac{cx^4 e^{i\pi}}{a} \right.\right)}{4\sqrt{a} \Gamma\left(\frac{9}{4}\right)} + \frac{e^3 x^7 \Gamma\left(\frac{7}{4}\right) {}_2F_1\left(\frac{7}{4}, \frac{7}{4} \left| \frac{cx^4 e^{i\pi}}{a} \right.\right)}{4\sqrt{a} \Gamma\left(\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x**2+d)**3/(c*x**4+a)**(1/2),x)

[Out] d**3*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), c*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(5/4)) + 3*d**2*e*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), c*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(7/4)) + 3*d*e**2*x**5*gamma(5/4)*hyper((1/2, 5/4), (9/4,), c*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(9/4)) + e**3*x**7*gamma(7/4)*hyper((1/2, 7/4), (11/4,), c*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(11/4))

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^2 + d)^3}{\sqrt{cx^4 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)^3/(c*x^4+a)^(1/2),x, algorithm="giac")

[Out] integrate((e*x^2 + d)^3/sqrt(c*x^4 + a), x)

$$3.152 \quad \int \frac{(d+ex^2)^2}{\sqrt{a+cx^4}} dx$$

Optimal. Leaf size=264

$$\frac{(\sqrt{a} + \sqrt{cx^2}) \sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{cx^2})^2}} (6\sqrt{a}\sqrt{cde} - ae^2 + 3cd^2) \text{EllipticF}\left(2 \tan^{-1}\left(\frac{\sqrt[4]{cx}}{\sqrt[4]{a}}\right), \frac{1}{2}\right) - 2\sqrt[4]{ade}(\sqrt{a} + \sqrt{cx^2}) \sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{cx^2})^2}}}{6\sqrt[4]{ac^5}\sqrt{a+cx^4}} - \frac{2\sqrt[4]{ade}(\sqrt{a} + \sqrt{cx^2}) \sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{cx^2})^2}}}{c^{3/4}\sqrt{a+cx^4}}$$

[Out] (e^2*x*Sqrt[a + c*x^4])/(3*c) + (2*d*e*x*Sqrt[a + c*x^4])/(Sqrt[c]*(Sqrt[a] + Sqrt[c]*x^2)) - (2*a^(1/4)*d*e*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(c^(3/4)*Sqrt[a + c*x^4]) + ((3*c*d^2 + 6*Sqrt[a]*Sqrt[c]*d*e - a*e^2)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(6*a^(1/4)*c^(5/4)*Sqrt[a + c*x^4])

Rubi [A] time = 0.129349, antiderivative size = 264, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.19$, Rules used = {1207, 1198, 220, 1196}

$$\frac{(\sqrt{a} + \sqrt{cx^2}) \sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{cx^2})^2}} (6\sqrt{a}\sqrt{cde} - ae^2 + 3cd^2) F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{cx}}{\sqrt[4]{a}}\right) \middle| \frac{1}{2}\right) - 2\sqrt[4]{ade}(\sqrt{a} + \sqrt{cx^2}) \sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{cx^2})^2}} E\left(2 \tan^{-1}\left(\frac{\sqrt[4]{cx}}{\sqrt[4]{a}}\right) \middle| \frac{1}{2}\right)}{6\sqrt[4]{ac^5}\sqrt{a+cx^4}} - \frac{2\sqrt[4]{ade}(\sqrt{a} + \sqrt{cx^2}) \sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{cx^2})^2}}}{c^{3/4}\sqrt{a+cx^4}}$$

Antiderivative was successfully verified.

[In] Int[(d + e*x^2)^2/Sqrt[a + c*x^4], x]

[Out] (e^2*x*Sqrt[a + c*x^4])/(3*c) + (2*d*e*x*Sqrt[a + c*x^4])/(Sqrt[c]*(Sqrt[a] + Sqrt[c]*x^2)) - (2*a^(1/4)*d*e*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(c^(3/4)*Sqrt[a + c*x^4]) + ((3*c*d^2 + 6*Sqrt[a]*Sqrt[c]*d*e - a*e^2)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(6*a^(1/4)*c^(5/4)*Sqrt[a + c*x^4])

Rule 1207

Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] := Simp[(e^q*x^(2*q - 3)*(a + c*x^4)^(p + 1))/(c*(4*p + 2*q + 1)), x] + Dist[1/(c*(4*p + 2*q + 1)), Int[(a + c*x^4)^p*ExpandToSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[q, 1]

Rule 1198

Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] := With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0] /; FreeQ[{a, c, d, e}, x] && PosQ[c/a]

Rule 220

Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] := With[{q = Rt[b/a, 4]}, Simp[(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x

, 1/2]]/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]

Rule 1196

Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] := With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x], 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e}, x] && PosQ[c/a]

Rubi steps

$$\begin{aligned} \int \frac{(d + ex^2)^2}{\sqrt{a + cx^4}} dx &= \frac{e^2 x \sqrt{a + cx^4}}{3c} + \frac{\int \frac{3cd^2 - ae^2 + 6cdex^2}{\sqrt{a + cx^4}} dx}{3c} \\ &= \frac{e^2 x \sqrt{a + cx^4}}{3c} - \frac{(2\sqrt{ade}) \int \frac{1 - \frac{\sqrt{cx^2}}{\sqrt{a}}}{\sqrt{a + cx^4}} dx}{\sqrt{c}} + \frac{(3cd^2 + 6\sqrt{a}\sqrt{cde} - ae^2) \int \frac{1}{\sqrt{a + cx^4}} dx}{3c} \\ &= \frac{e^2 x \sqrt{a + cx^4}}{3c} + \frac{2dex\sqrt{a + cx^4}}{\sqrt{c}(\sqrt{a} + \sqrt{cx^2})} - \frac{2^4 \sqrt{ade} (\sqrt{a} + \sqrt{cx^2}) \sqrt{\frac{a + cx^4}{(\sqrt{a} + \sqrt{cx^2})^2}} E\left(2 \tan^{-1}\left(\frac{\sqrt{cx^2}}{\sqrt{a}}\right) \middle| \frac{1}{2}\right)}{c^{3/4} \sqrt{a + cx^4}} + \dots \end{aligned}$$

Mathematica [C] time = 0.0952858, size = 120, normalized size = 0.45

$$\frac{x\sqrt{\frac{cx^4}{a} + 1} (3cd^2 - ae^2) {}_2F_1\left(\frac{1}{4}, \frac{1}{2}; \frac{5}{4}; -\frac{cx^4}{a}\right) + ex \left(2cdx^2 \sqrt{\frac{cx^4}{a}} + 1 {}_2F_1\left(\frac{1}{2}, \frac{3}{4}; \frac{7}{4}; -\frac{cx^4}{a}\right) + e(a + cx^4)\right)}{3c\sqrt{a + cx^4}}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e*x^2)^2/Sqrt[a + c*x^4],x]

[Out] ((3*c*d^2 - a*e^2)*x*Sqrt[1 + (c*x^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -((c*x^4)/a)] + e*x*(e*(a + c*x^4) + 2*c*d*x^2*Sqrt[1 + (c*x^4)/a]*Hypergeometric2F1[1/2, 3/4, 7/4, -((c*x^4)/a)]))/(3*c*Sqrt[a + c*x^4])

Maple [C] time = 0.051, size = 266, normalized size = 1.

$$e^2 \left(\frac{x}{3c} \sqrt{cx^4 + a} - \frac{a}{3c} \sqrt{1 - ix^2 \sqrt{c} \frac{1}{\sqrt{a}}} \sqrt{1 + ix^2 \sqrt{c} \frac{1}{\sqrt{a}}} \text{EllipticF}\left(x \sqrt{i\sqrt{c} \frac{1}{\sqrt{a}}}, i\right) \frac{1}{\sqrt{i\sqrt{c} \frac{1}{\sqrt{a}}}} \frac{1}{\sqrt{cx^4 + a}} \right) + 2ide\sqrt{a} \sqrt{1 - ix^2 \sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x^2+d)^2/(c*x^4+a)^(1/2),x)

[Out] e^2*(1/3/c*x*(c*x^4+a)^(1/2)-1/3*a/c/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)+2*I*d*e*a^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)/c^(1/2)*(EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*c^(1/2))^(1/2),I))

$$\frac{1}{2} * c^{(1/2)} \left(\frac{1}{2}, I \right) + d^2 / \left(I/a^{(1/2)} * c^{(1/2)} \right)^{(1/2)} * \left(1 - I/a^{(1/2)} * c^{(1/2)} * x^2 \right)^{(1/2)} * \left(1 + I/a^{(1/2)} * c^{(1/2)} * x^2 \right)^{(1/2)} / \left(c * x^4 + a \right)^{(1/2)} * \text{EllipticF} \left(x * \left(I/a^{(1/2)} * c^{(1/2)} \right)^{(1/2)}, I \right)$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^2 + d)^2}{\sqrt{cx^4 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)^2/(c*x^4+a)^(1/2),x, algorithm="maxima")

[Out] integrate((e*x^2 + d)^2/sqrt(c*x^4 + a), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral} \left(\frac{e^2 x^4 + 2 d e x^2 + d^2}{\sqrt{c x^4 + a}}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)^2/(c*x^4+a)^(1/2),x, algorithm="fricas")

[Out] integral((e^2*x^4 + 2*d*e*x^2 + d^2)/sqrt(c*x^4 + a), x)

Sympy [C] time = 2.72874, size = 124, normalized size = 0.47

$$\frac{d^2 x \Gamma\left(\frac{1}{4}\right) {}_2F_1\left(\frac{1}{4}, \frac{1}{2} \middle| \frac{c x^4 e^{i\pi}}{a}\right)}{4 \sqrt{a} \Gamma\left(\frac{5}{4}\right)} + \frac{d e x^3 \Gamma\left(\frac{3}{4}\right) {}_2F_1\left(\frac{3}{4}, \frac{3}{4} \middle| \frac{c x^4 e^{i\pi}}{a}\right)}{2 \sqrt{a} \Gamma\left(\frac{7}{4}\right)} + \frac{e^2 x^5 \Gamma\left(\frac{5}{4}\right) {}_2F_1\left(\frac{5}{4}, \frac{5}{4} \middle| \frac{c x^4 e^{i\pi}}{a}\right)}{4 \sqrt{a} \Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x**2+d)**2/(c*x**4+a)**(1/2),x)

[Out] d**2*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), c*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(5/4)) + d*e*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), c*x**4*exp_polar(I*pi)/a)/(2*sqrt(a)*gamma(7/4)) + e**2*x**5*gamma(5/4)*hyper((1/2, 5/4), (9/4,), c*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(9/4))

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^2 + d)^2}{\sqrt{cx^4 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2/(c*x^4+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^2/sqrt(c*x^4 + a), x)
```

3.153 $\int \frac{d+ex^2}{\sqrt{a+cx^4}} dx$

Optimal. Leaf size=226

$$\frac{\sqrt[4]{a}(\sqrt{a} + \sqrt{cx^2}) \sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{cx^2})^2}} \left(\frac{\sqrt{cd}}{\sqrt{a}} + e\right) \text{EllipticF}\left(2 \tan^{-1}\left(\frac{\sqrt[4]{cx}}{\sqrt[4]{a}}\right), \frac{1}{2}\right)}{2c^{3/4}\sqrt{a+cx^4}} - \frac{\sqrt[4]{ae}(\sqrt{a} + \sqrt{cx^2}) \sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{cx^2})^2}} E\left(2 \tan^{-1}\left(\frac{\sqrt[4]{cx}}{\sqrt[4]{a}}\right)\right)}{c^{3/4}\sqrt{a+cx^4}}$$

[Out] (e*x*Sqrt[a + c*x^4])/(Sqrt[c]*(Sqrt[a] + Sqrt[c]*x^2)) - (a^(1/4)*e*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(c^(3/4)*Sqrt[a + c*x^4]) + (a^(1/4)*((Sqrt[c]*d)/Sqrt[a] + e)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*c^(3/4)*Sqrt[a + c*x^4])

Rubi [A] time = 0.0694483, antiderivative size = 226, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {1198, 220, 1196}

$$\frac{\sqrt[4]{a}(\sqrt{a} + \sqrt{cx^2}) \sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{cx^2})^2}} \left(\frac{\sqrt{cd}}{\sqrt{a}} + e\right) F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{cx}}{\sqrt[4]{a}}\right) \middle| \frac{1}{2}\right)}{2c^{3/4}\sqrt{a+cx^4}} - \frac{\sqrt[4]{ae}(\sqrt{a} + \sqrt{cx^2}) \sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{cx^2})^2}} E\left(2 \tan^{-1}\left(\frac{\sqrt[4]{cx}}{\sqrt[4]{a}}\right) \middle| \frac{1}{2}\right)}{c^{3/4}\sqrt{a+cx^4}} +$$

Antiderivative was successfully verified.

[In] Int[(d + e*x^2)/Sqrt[a + c*x^4], x]

[Out] (e*x*Sqrt[a + c*x^4])/(Sqrt[c]*(Sqrt[a] + Sqrt[c]*x^2)) - (a^(1/4)*e*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(c^(3/4)*Sqrt[a + c*x^4]) + (a^(1/4)*((Sqrt[c]*d)/Sqrt[a] + e)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*c^(3/4)*Sqrt[a + c*x^4])

Rule 1198

Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] := With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0] /; FreeQ[{a, c, d, e}, x] && PosQ[c/a]

Rule 220

Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] := With[{q = Rt[b/a, 4]}, Simp[(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2]*EllipticF[2*ArcTan[q*x], 1/2])/(2*q*Sqrt[a + b*x^4]), x] /; FreeQ[{a, b}, x] && PosQ[b/a]

Rule 1196

Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] := With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2]*EllipticE[2*ArcTan[q*x], 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0] /; FreeQ[{a, c, d, e}, x] && PosQ[c/a]

Rubi steps

$$\int \frac{d + ex^2}{\sqrt{a + cx^4}} dx = -\frac{(\sqrt{ae}) \int \frac{1 - \frac{\sqrt{cx^2}}{\sqrt{a}}}{\sqrt{a+cx^4}} dx}{\sqrt{c}} + \left(d + \frac{\sqrt{ae}}{\sqrt{c}}\right) \int \frac{1}{\sqrt{a + cx^4}} dx$$

$$= \frac{ex\sqrt{a + cx^4}}{\sqrt{c}(\sqrt{a} + \sqrt{cx^2})} - \frac{\sqrt[4]{ae}(\sqrt{a} + \sqrt{cx^2}) \sqrt{\frac{a+cx^4}{(\sqrt{a} + \sqrt{cx^2})^2}} E\left(2 \tan^{-1}\left(\frac{\sqrt[4]{cx}}{\sqrt[4]{a}}\right) \middle| \frac{1}{2}\right)}{c^{3/4}\sqrt{a + cx^4}} + \frac{(\sqrt{cd} + \sqrt{ae})(\sqrt{a} + \sqrt{cx^2})}{2\sqrt[4]{ac^3}}$$

Mathematica [C] time = 0.0301316, size = 77, normalized size = 0.34

$$\frac{\sqrt{\frac{cx^4}{a} + 1} \left(3dx {}_2F_1\left(\frac{1}{4}, \frac{1}{2}; \frac{5}{4}; -\frac{cx^4}{a}\right) + ex^3 {}_2F_1\left(\frac{1}{2}, \frac{3}{4}; \frac{7}{4}; -\frac{cx^4}{a}\right) \right)}{3\sqrt{a + cx^4}}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e*x^2)/Sqrt[a + c*x^4],x]

[Out] (Sqrt[1 + (c*x^4)/a]*(3*d*x*Hypergeometric2F1[1/4, 1/2, 5/4, -((c*x^4)/a)] + e*x^3*Hypergeometric2F1[1/2, 3/4, 7/4, -((c*x^4)/a)]))/(3*Sqrt[a + c*x^4])

Maple [C] time = 0.046, size = 169, normalized size = 0.8

$$ie\sqrt{a}\sqrt{1 - ix^2\sqrt{c}\frac{1}{\sqrt{a}}}\sqrt{1 + ix^2\sqrt{c}\frac{1}{\sqrt{a}}}\left(\text{EllipticF}\left(x\sqrt{i\sqrt{c}\frac{1}{\sqrt{a}}}, i\right) - \text{EllipticE}\left(x\sqrt{i\sqrt{c}\frac{1}{\sqrt{a}}}, i\right)\right) \frac{1}{\sqrt{i\sqrt{c}\frac{1}{\sqrt{a}}}} \frac{1}{\sqrt{cx^4 + a}} \frac{1}{\sqrt{c}} + d\sqrt{\dots}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x^2+d)/(c*x^4+a)^(1/2),x)

[Out] I*e*a^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)/c^(1/2)*(EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*c^(1/2))^(1/2),I))+d/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex^2 + d}{\sqrt{cx^4 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)/(c*x^4+a)^(1/2),x, algorithm="maxima")

[Out] integrate((e*x^2 + d)/sqrt(c*x^4 + a), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{ex^2 + d}{\sqrt{cx^4 + a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)/(c*x^4+a)^(1/2),x, algorithm="fricas")

[Out] integral((e*x^2 + d)/sqrt(c*x^4 + a), x)

Sympy [C] time = 1.65146, size = 78, normalized size = 0.35

$$\frac{dx\Gamma\left(\frac{1}{4}\right) {}_2F_1\left(\frac{1}{4}, \frac{1}{2} \middle| \frac{cx^4 e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{5}{4}\right)} + \frac{ex^3\Gamma\left(\frac{3}{4}\right) {}_2F_1\left(\frac{1}{2}, \frac{3}{4} \middle| \frac{cx^4 e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x**2+d)/(c*x**4+a)**(1/2),x)

[Out] d*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), c*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(5/4)) + e*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), c*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(7/4))

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex^2 + d}{\sqrt{cx^4 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)/(c*x^4+a)^(1/2),x, algorithm="giac")

[Out] integrate((e*x^2 + d)/sqrt(c*x^4 + a), x)

$$3.154 \quad \int \frac{1}{(d+ex^2)\sqrt{a+cx^4}} dx$$

Optimal. Leaf size=334

$$\frac{\sqrt[4]{c}(\sqrt{a} + \sqrt{cx^2}) \sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{cx^2})^2}} \text{EllipticF}\left(2 \tan^{-1}\left(\frac{\sqrt[4]{cx}}{\sqrt[4]{a}}\right), \frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt{a+cx^4}(\sqrt{cd} - \sqrt{ae})} - \frac{a^{3/4}(\sqrt{a} + \sqrt{cx^2}) \sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{cx^2})^2}} \left(\frac{\sqrt{cd}}{\sqrt{a}} + e\right)^2 \Pi\left(-\frac{(\sqrt{cd}-\sqrt{ae})^2}{4\sqrt{a}\sqrt{cde}}; 2\right)}{4\sqrt[4]{cd}\sqrt{a+cx^4}(cd^2 - ae^2)}$$

[Out] (Sqrt[e]*ArcTan[(Sqrt[c*d^2 + a*e^2]*x)/(Sqrt[d]*Sqrt[e]*Sqrt[a + c*x^4])]) / (2*Sqrt[d]*Sqrt[c*d^2 + a*e^2]) + (c^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2]) / (2*a^(1/4)*(Sqrt[c]*d - Sqrt[a]*e)*Sqrt[a + c*x^4]) - (a^(3/4)*((Sqrt[c]*d)/Sqrt[a] + e)^2*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticPi[-(Sqrt[c]*d - Sqrt[a]*e)^2/(4*Sqrt[a]*Sqrt[c]*d*e), 2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2]) / (4*c^(1/4)*d*(c*d^2 - a*e^2)*Sqrt[a + c*x^4])

Rubi [A] time = 0.265683, antiderivative size = 334, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1217, 220, 1707}

$$\frac{a^{3/4}(\sqrt{a} + \sqrt{cx^2}) \sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{cx^2})^2}} \left(\frac{\sqrt{cd}}{\sqrt{a}} + e\right)^2 \Pi\left(-\frac{(\sqrt{cd}-\sqrt{ae})^2}{4\sqrt{a}\sqrt{cde}}; 2 \tan^{-1}\left(\frac{\sqrt[4]{cx}}{\sqrt[4]{a}}\right) \middle| \frac{1}{2}\right)}{4\sqrt[4]{cd}\sqrt{a+cx^4}(cd^2 - ae^2)} + \frac{\sqrt{e} \tan^{-1}\left(\frac{x\sqrt{ae^2+cd^2}}{\sqrt{d}\sqrt{e}\sqrt{a+cx^4}}\right)}{2\sqrt{d}\sqrt{ae^2 + cd^2}} + \frac{\sqrt[4]{c}(\sqrt{a} + \sqrt{cx^2})}{2\sqrt[4]{a}}$$

Antiderivative was successfully verified.

[In] Int[1/((d + e*x^2)*Sqrt[a + c*x^4]),x]

[Out] (Sqrt[e]*ArcTan[(Sqrt[c*d^2 + a*e^2]*x)/(Sqrt[d]*Sqrt[e]*Sqrt[a + c*x^4])]) / (2*Sqrt[d]*Sqrt[c*d^2 + a*e^2]) + (c^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2]) / (2*a^(1/4)*(Sqrt[c]*d - Sqrt[a]*e)*Sqrt[a + c*x^4]) - (a^(3/4)*((Sqrt[c]*d)/Sqrt[a] + e)^2*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticPi[-(Sqrt[c]*d - Sqrt[a]*e)^2/(4*Sqrt[a]*Sqrt[c]*d*e), 2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2]) / (4*c^(1/4)*d*(c*d^2 - a*e^2)*Sqrt[a + c*x^4])

Rule 1217

Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4], x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]

Rule 220

Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2]]*EllipticF[2*ArcTan[q*x], 1/2]) / (2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]

Rule 1707


```
Int[((A_) + (B_)*(x_)^2)/(((d_) + (e_)*(x_)^2)*Sqrt[(a_) + (c_)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*ArcTan[(Rt[(c*d)/e
+ (a*e)/d, 2]*x)/Sqrt[a + c*x^4]])/(2*d*e*Rt[(c*d)/e + (a*e)/d, 2]), x] +
Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a + c*x^4))/(a*(A + B*x^2)^2)]*Ell
ipticPi[Cancel[-((B*d - A*e)^2/(4*d*e*A*B))], 2*ArcTan[q*x], 1/2])/(4*d*e*A
*q*Sqrt[a + c*x^4]), x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{1}{(d+ex^2)\sqrt{a+cx^4}} dx = \frac{\sqrt{c} \int \frac{1}{\sqrt{a+cx^4}} dx}{\sqrt{cd} - \sqrt{ae}} - \frac{(\sqrt{ae}) \int \frac{1 + \frac{\sqrt{cx^2}}{\sqrt{a}}}{(d+ex^2)\sqrt{a+cx^4}} dx}{\sqrt{cd} - \sqrt{ae}}$$

$$= \frac{\sqrt{e} \tan^{-1}\left(\frac{\sqrt{cd^2+ae^2}x}{\sqrt{d}\sqrt{e}\sqrt{a+cx^4}}\right)}{2\sqrt{d}\sqrt{cd^2+ae^2}} + \frac{\sqrt[4]{c}(\sqrt{a} + \sqrt{cx^2}) \sqrt{\frac{a+cx^4}{(\sqrt{a} + \sqrt{cx^2})^2}} F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right) \middle| \frac{1}{2}\right)}{2\sqrt[4]{a}(\sqrt{cd} - \sqrt{ae})\sqrt{a+cx^4}} - \frac{\sqrt[4]{a}\left(\frac{\sqrt{cd}}{\sqrt{a}} + \dots\right)}{2\sqrt[4]{a}(\sqrt{cd} - \sqrt{ae})\sqrt{a+cx^4}}$$

Mathematica [C] time = 0.15367, size = 95, normalized size = 0.28

$$\frac{i\sqrt{\frac{cx^4}{a}} + 1\Pi\left(-\frac{i\sqrt{ae}}{\sqrt{cd}}; i \sinh^{-1}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}x\right)\right) - 1}{d\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\sqrt{a+cx^4}}$$

Antiderivative was successfully verified.

[In] Integrate[1/((d + e*x^2)*Sqrt[a + c*x^4]),x]

[Out] ((-I)*Sqrt[1 + (c*x^4)/a]*EllipticPi[(-I)*Sqrt[a]*e)/(Sqrt[c]*d), I*ArcSin h[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x], -1)]/(Sqrt[(I*Sqrt[c])/Sqrt[a]]*d*Sqrt[a + c*x^4])

Maple [C] time = 0.191, size = 107, normalized size = 0.3

$$\frac{1}{d} \sqrt{1 - ix^2 \sqrt{c} \frac{1}{\sqrt{a}}} \sqrt{1 + ix^2 \sqrt{c} \frac{1}{\sqrt{a}}} \text{EllipticPi} \left(x \sqrt{i\sqrt{c} \frac{1}{\sqrt{a}}}, \frac{ie}{d} \sqrt{a} \frac{1}{\sqrt{c}}, \sqrt{-i\sqrt{c} \frac{1}{\sqrt{a}}} \frac{1}{\sqrt{i\sqrt{c} \frac{1}{\sqrt{a}}}} \right) \frac{1}{\sqrt{i\sqrt{c} \frac{1}{\sqrt{a}}}} \frac{1}{\sqrt{cx^4 + a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(e*x^2+d)/(c*x^4+a)^(1/2),x)

[Out] 1/d/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticPi(x*(I/a^(1/2)*c^(1/2))^(1/2), I*a^(1/2)/c^(1/2)*e/d, (-I/a^(1/2)*c^(1/2))^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^4 + a}(ex^2 + d)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x^2+d)/(c*x^4+a)^(1/2),x, algorithm="maxima")

[Out] integrate(1/(sqrt(c*x^4 + a)*(e*x^2 + d)), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^4 + a}}{cex^6 + cdx^4 + aex^2 + ad}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x^2+d)/(c*x^4+a)^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(c*x^4 + a)/(c*e*x^6 + c*d*x^4 + a*e*x^2 + a*d), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a + cx^4}(d + ex^2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x**2+d)/(c*x**4+a)**(1/2),x)

[Out] Integral(1/(sqrt(a + c*x**4)*(d + e*x**2)), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^4 + a}(ex^2 + d)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x^2+d)/(c*x^4+a)^(1/2),x, algorithm="giac")

[Out] integrate(1/(sqrt(c*x^4 + a)*(e*x^2 + d)), x)

$$3.155 \quad \int \frac{1}{(d+ex^2)^2 \sqrt{a+cx^4}} dx$$

Optimal. Leaf size=581

$$\frac{\sqrt[4]{c}(\sqrt{a} + \sqrt{cx^2}) \sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{cx^2})^2}} \text{EllipticF}\left(2 \tan^{-1}\left(\frac{\sqrt[4]{cx}}{\sqrt[4]{a}}\right), \frac{1}{2}\right)}{2\sqrt[4]{ad}\sqrt{a+cx^4}(\sqrt{cd} - \sqrt{ae})} + \frac{e^2 x \sqrt{a+cx^4}}{2d(d+ex^2)(ae^2+cd^2)} - \frac{\sqrt{cex}\sqrt{a+cx^4}}{2d(\sqrt{a} + \sqrt{cx^2})(ae^2+cd^2)}$$

[Out] $-(\text{Sqrt}[c]*e*x*\text{Sqrt}[a + c*x^4])/(2*d*(c*d^2 + a*e^2)*(\text{Sqrt}[a] + \text{Sqrt}[c]*x^2)) + (e^2*x*\text{Sqrt}[a + c*x^4])/(2*d*(c*d^2 + a*e^2)*(d + e*x^2)) + (\text{Sqrt}[e]*(3*c*d^2 + a*e^2)*\text{ArcTan}[(\text{Sqrt}[c*d^2 + a*e^2]*x)/(\text{Sqrt}[d]*\text{Sqrt}[e]*\text{Sqrt}[a + c*x^4])])/(4*d^(3/2)*(c*d^2 + a*e^2)^(3/2)) + (a^(1/4)*c^(1/4)*e*(\text{Sqrt}[a] + \text{Sqrt}[c]*x^2)*\text{Sqrt}[(a + c*x^4)/(\text{Sqrt}[a] + \text{Sqrt}[c]*x^2)^2]*\text{EllipticE}[2*\text{ArcTan}[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*d*(c*d^2 + a*e^2)*\text{Sqrt}[a + c*x^4]) + (c^(1/4)*(\text{Sqrt}[a] + \text{Sqrt}[c]*x^2)*\text{Sqrt}[(a + c*x^4)/(\text{Sqrt}[a] + \text{Sqrt}[c]*x^2)^2]*\text{EllipticF}[2*\text{ArcTan}[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(1/4)*d*(\text{Sqrt}[c]*d - \text{Sqrt}[a]*e)*\text{Sqrt}[a + c*x^4]) - ((\text{Sqrt}[c]*d + \text{Sqrt}[a]*e)*(3*c*d^2 + a*e^2)*(\text{Sqrt}[a] + \text{Sqrt}[c]*x^2)*\text{Sqrt}[(a + c*x^4)/(\text{Sqrt}[a] + \text{Sqrt}[c]*x^2)^2]*\text{EllipticPi}[-(\text{Sqrt}[c]*d - \text{Sqrt}[a]*e)^2/(4*\text{Sqrt}[a]*\text{Sqrt}[c]*d*e), 2*\text{ArcTan}[(c^(1/4)*x)/a^(1/4)], 1/2])/(8*a^(1/4)*c^(1/4)*d^2*(\text{Sqrt}[c]*d - \text{Sqrt}[a]*e)*(c*d^2 + a*e^2)*\text{Sqrt}[a + c*x^4])$

Rubi [A] time = 0.762326, antiderivative size = 581, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {1224, 1715, 1196, 1709, 220, 1707}

$$\frac{e^2 x \sqrt{a+cx^4}}{2d(d+ex^2)(ae^2+cd^2)} - \frac{\sqrt{cex}\sqrt{a+cx^4}}{2d(\sqrt{a} + \sqrt{cx^2})(ae^2+cd^2)} + \frac{\sqrt{e}(ae^2+3cd^2) \tan^{-1}\left(\frac{x\sqrt{ae^2+cd^2}}{\sqrt{d}\sqrt{e}\sqrt{a+cx^4}}\right)}{4d^{3/2}(ae^2+cd^2)^{3/2}} + \frac{\sqrt[4]{a}\sqrt[4]{ce}(\sqrt{a} + \sqrt{cx^2})}{2d\sqrt{a}}$$

Antiderivative was successfully verified.

[In] Int[1/((d + e*x^2)^2*Sqrt[a + c*x^4]), x]

[Out] $-(\text{Sqrt}[c]*e*x*\text{Sqrt}[a + c*x^4])/(2*d*(c*d^2 + a*e^2)*(\text{Sqrt}[a] + \text{Sqrt}[c]*x^2)) + (e^2*x*\text{Sqrt}[a + c*x^4])/(2*d*(c*d^2 + a*e^2)*(d + e*x^2)) + (\text{Sqrt}[e]*(3*c*d^2 + a*e^2)*\text{ArcTan}[(\text{Sqrt}[c*d^2 + a*e^2]*x)/(\text{Sqrt}[d]*\text{Sqrt}[e]*\text{Sqrt}[a + c*x^4])])/(4*d^(3/2)*(c*d^2 + a*e^2)^(3/2)) + (a^(1/4)*c^(1/4)*e*(\text{Sqrt}[a] + \text{Sqrt}[c]*x^2)*\text{Sqrt}[(a + c*x^4)/(\text{Sqrt}[a] + \text{Sqrt}[c]*x^2)^2]*\text{EllipticE}[2*\text{ArcTan}[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*d*(c*d^2 + a*e^2)*\text{Sqrt}[a + c*x^4]) + (c^(1/4)*(\text{Sqrt}[a] + \text{Sqrt}[c]*x^2)*\text{Sqrt}[(a + c*x^4)/(\text{Sqrt}[a] + \text{Sqrt}[c]*x^2)^2]*\text{EllipticF}[2*\text{ArcTan}[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(1/4)*d*(\text{Sqrt}[c]*d - \text{Sqrt}[a]*e)*\text{Sqrt}[a + c*x^4]) - ((\text{Sqrt}[c]*d + \text{Sqrt}[a]*e)*(3*c*d^2 + a*e^2)*(\text{Sqrt}[a] + \text{Sqrt}[c]*x^2)*\text{Sqrt}[(a + c*x^4)/(\text{Sqrt}[a] + \text{Sqrt}[c]*x^2)^2]*\text{EllipticPi}[-(\text{Sqrt}[c]*d - \text{Sqrt}[a]*e)^2/(4*\text{Sqrt}[a]*\text{Sqrt}[c]*d*e), 2*\text{ArcTan}[(c^(1/4)*x)/a^(1/4)], 1/2])/(8*a^(1/4)*c^(1/4)*d^2*(\text{Sqrt}[c]*d - \text{Sqrt}[a]*e)*(c*d^2 + a*e^2)*\text{Sqrt}[a + c*x^4])$

Rule 1224

Int[((d_) + (e_.)*(x_)^2)^(q_)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] := -Simp[(e^2*x*(d + e*x^2)^(q + 1)*Sqrt[a + c*x^4])/(2*d*(q + 1)*(c*d^2 + a*e^2)), x] + Dist[1/(2*d*(q + 1)*(c*d^2 + a*e^2)), Int[((d + e*x^2)^(q + 1)*Simp[

$a^2e^{2q+3} + 2cd^2(q+1) - 2ecd(q+1)x^2 + ce^{2q+5}x^4, x) / \sqrt{a+cx^4}, x, x /; \text{FreeQ}\{a, c, d, e, x\} \ \&\& \ \text{ILtQ}[q, -1]$

Rule 1715

$\text{Int}[(P4x_)/(((d_) + (e_)*(x_)^2)*\sqrt{(a_) + (c_)*(x_)^4}), x_Symbol] \rightarrow$
 $\text{With}\{q = \text{Rt}[c/a, 2], A = \text{Coeff}[P4x, x, 0], B = \text{Coeff}[P4x, x, 2], C = \text{Coeff}[P4x, x, 4]\}, -\text{Dist}[C/(e*q), \text{Int}[(1 - q*x^2)/\sqrt{a+cx^4}, x], x] + \text{Dist}[1/(c*e), \text{Int}[(A*c*e + a*C*d*q + (B*c*e - C*(c*d - a*e*q))*x^2]/((d + e*x^2)*\sqrt{a+cx^4}), x], x] /; \text{FreeQ}\{a, c, d, e, x\} \ \&\& \ \text{PolyQ}[P4x, x^2, 2] \ \&\& \ \text{NeQ}[c*d^2 + a*e^2, 0] \ \&\& \ \text{NeQ}[c*d^2 - a*e^2, 0] \ \&\& \ \text{PosQ}[c/a]$

Rule 1196

$\text{Int}(((d_) + (e_)*(x_)^2)/\sqrt{(a_) + (c_)*(x_)^4}), x_Symbol] \rightarrow \text{With}\{q = \text{Rt}[c/a, 4]\}, -\text{Simp}[(d*x*\sqrt{a+cx^4})/(a*(1+q^2*x^2)), x] + \text{Simp}[(d*(1+q^2*x^2)*\sqrt{(a+cx^4)})/(a*(1+q^2*x^2)^2)*\text{EllipticE}[2*\text{ArcTan}[q*x], 1/2)]/(q*\sqrt{a+cx^4}), x] /; \text{EqQ}[e + d*q^2, 0] /; \text{FreeQ}\{a, c, d, e, x\} \ \&\& \ \text{PosQ}[c/a]$

Rule 1709

$\text{Int}(((A_) + (B_)*(x_)^2)/(((d_) + (e_)*(x_)^2)*\sqrt{(a_) + (c_)*(x_)^4}), x_Symbol] \rightarrow \text{With}\{q = \text{Rt}[c/a, 2]\}, \text{Dist}[(A*(c*d + a*e*q) - a*B*(e + d*q))/(c*d^2 - a*e^2), \text{Int}[1/\sqrt{a+cx^4}, x], x] + \text{Dist}[(a*(B*d - A*e)*(e + d*q))/(c*d^2 - a*e^2), \text{Int}[(1 + q*x^2)/((d + e*x^2)*\sqrt{a+cx^4}), x], x] /; \text{FreeQ}\{a, c, d, e, A, B, x\} \ \&\& \ \text{NeQ}[c*d^2 + a*e^2, 0] \ \&\& \ \text{NeQ}[c*d^2 - a*e^2, 0] \ \&\& \ \text{PosQ}[c/a] \ \&\& \ \text{NeQ}[c*A^2 - a*B^2, 0]$

Rule 220

$\text{Int}[1/\sqrt{(a_) + (b_)*(x_)^4}), x_Symbol] \rightarrow \text{With}\{q = \text{Rt}[b/a, 4]\}, \text{Simp}[(1 + q^2*x^2)*\sqrt{(a + b*x^4)})/(a*(1 + q^2*x^2)^2)*\text{EllipticF}[2*\text{ArcTan}[q*x], 1/2)]/(2*q*\sqrt{a + b*x^4}), x] /; \text{FreeQ}\{a, b, x\} \ \&\& \ \text{PosQ}[b/a]$

Rule 1707

$\text{Int}(((A_) + (B_)*(x_)^2)/(((d_) + (e_)*(x_)^2)*\sqrt{(a_) + (c_)*(x_)^4}), x_Symbol] \rightarrow \text{With}\{q = \text{Rt}[B/A, 2]\}, -\text{Simp}[(B*d - A*e)*\text{ArcTan}[(\text{Rt}[(c*d)/e + (a*e)/d, 2]*x)/\sqrt{a+cx^4}]]/(2*d*e*\text{Rt}[(c*d)/e + (a*e)/d, 2]), x] + \text{Simp}[(B*d + A*e)*(A + B*x^2)*\sqrt{(A^2*(a + cx^4))}/(a*(A + B*x^2)^2)*\text{EllipticPi}[\text{Cancel}[-(B*d - A*e)^2/(4*d*e*A*B)], 2*\text{ArcTan}[q*x], 1/2)]/(4*d*e*A*q*\sqrt{a + cx^4}), x] /; \text{FreeQ}\{a, c, d, e, A, B, x\} \ \&\& \ \text{NeQ}[c*d^2 + a*e^2, 0] \ \&\& \ \text{NeQ}[c*d^2 - a*e^2, 0] \ \&\& \ \text{PosQ}[c/a] \ \&\& \ \text{EqQ}[c*A^2 - a*B^2, 0]$

Rubi steps

$$\begin{aligned}
\int \frac{1}{(d+ex^2)^2 \sqrt{a+cx^4}} dx &= \frac{e^2 x \sqrt{a+cx^4}}{2d(cd^2+ae^2)(d+ex^2)} - \frac{\int \frac{-2cd^2-ae^2+2cdex^2+ce^2x^4}{(d+ex^2)\sqrt{a+cx^4}} dx}{2d(cd^2+ae^2)} \\
&= \frac{e^2 x \sqrt{a+cx^4}}{2d(cd^2+ae^2)(d+ex^2)} - \frac{\int \frac{\sqrt{ac}^{3/2}de^2+ce(-2cd^2-ae^2)+(2c^2de^2-ce^2(cd-\sqrt{a}\sqrt{ce}))x^2}{(d+ex^2)\sqrt{a+cx^4}} dx}{2cde(cd^2+ae^2)} + \frac{(\sqrt{a}\sqrt{ce}) \int}{2d(cd^2+ae^2)} \\
&= -\frac{\sqrt{cex}\sqrt{a+cx^4}}{2d(cd^2+ae^2)(\sqrt{a}+\sqrt{cx^2})} + \frac{e^2 x \sqrt{a+cx^4}}{2d(cd^2+ae^2)(d+ex^2)} + \frac{\sqrt[4]{a}\sqrt[4]{ce}(\sqrt{a}+\sqrt{cx^2})\sqrt{\frac{a+c}{(\sqrt{a}+\sqrt{cx^2})^2}}}{2d(cd^2+ae^2)} \\
&= -\frac{\sqrt{cex}\sqrt{a+cx^4}}{2d(cd^2+ae^2)(\sqrt{a}+\sqrt{cx^2})} + \frac{e^2 x \sqrt{a+cx^4}}{2d(cd^2+ae^2)(d+ex^2)} + \frac{\sqrt{e}(3cd^2+ae^2)\tan^{-1}\left(\frac{\sqrt{cd^2}}{\sqrt{a}\sqrt{e}}\right)}{4d^{3/2}(cd^2+ae^2)^{3/2}}
\end{aligned}$$

Mathematica [C] time = 0.77866, size = 522, normalized size = 0.9

$$\sqrt{cd}\sqrt{\frac{cx^4}{a}+1}(d+ex^2)(\sqrt{ae}+i\sqrt{cd})\text{EllipticF}\left(i\sinh^{-1}\left(x\sqrt{\frac{i\sqrt{c}}{a}}\right),-1\right)-3icd^2ex^2\sqrt{\frac{cx^4}{a}+1}\Pi\left(-\frac{i\sqrt{ae}}{\sqrt{cd}};i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{c}}{a}}x\right)\right)$$

Antiderivative was successfully verified.

[In] Integrate[1/((d + e*x^2)^2*Sqrt[a + c*x^4]),x]

[Out] (a*Sqrt[(I*Sqrt[c])/Sqrt[a]]*d*e^2*x + Sqrt[(I*Sqrt[c])/Sqrt[a]]*c*d*e^2*x^5 - Sqrt[a]*Sqrt[c]*d*e*(d + e*x^2)*Sqrt[1 + (c*x^4)/a]*EllipticE[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x], -1] + Sqrt[c]*d*(I*Sqrt[c]*d + Sqrt[a]*e)*(d + e*x^2)*Sqrt[1 + (c*x^4)/a]*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x], -1] - (3*I)*c*d^3*Sqrt[1 + (c*x^4)/a]*EllipticPi[(-I)*Sqrt[a]*e]/(Sqrt[c]*d), I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x], -1] - I*a*d*e^2*Sqrt[1 + (c*x^4)/a]*EllipticPi[(-I)*Sqrt[a]*e]/(Sqrt[c]*d), I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x], -1] - (3*I)*c*d^2*e*x^2*Sqrt[1 + (c*x^4)/a]*EllipticPi[(-I)*Sqrt[a]*e]/(Sqrt[c]*d), I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x], -1] - I*a*e^3*x^2*Sqrt[1 + (c*x^4)/a]*EllipticPi[(-I)*Sqrt[a]*e]/(Sqrt[c]*d), I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x], -1)]/(2*Sqrt[(I*Sqrt[c])/Sqrt[a]]*d^2*(c*d^2 + a*e^2)*(d + e*x^2)*Sqrt[a + c*x^4])

Maple [C] time = 0.197, size = 556, normalized size = 1.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(e*x^2+d)^2/(c*x^4+a)^(1/2),x)

[Out] 1/2*e^2*x*(c*x^4+a)^(1/2)/d/(a*e^2+c*d^2)/(e*x^2+d)-1/2/(a*e^2+c*d^2)*c/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)-1/2*I*c^(1/2)*e/(a*e^2+c*d^2)/d*a^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)

$$\begin{aligned} & (1/2)*x^2)^{(1/2)}*(1+I/a^{(1/2)}*c^{(1/2)}*x^2)^{(1/2)}/(c*x^4+a)^{(1/2)}*EllipticF(\\ & x*(I/a^{(1/2)}*c^{(1/2)})^{(1/2)}, I)+1/2*I*c^{(1/2)}*e/(a*e^2+c*d^2)/d*a^{(1/2)}/(I/a \\ & ^{(1/2)}*c^{(1/2)})^{(1/2)}*(1-I/a^{(1/2)}*c^{(1/2)}*x^2)^{(1/2)}*(1+I/a^{(1/2)}*c^{(1/2)}* \\ & x^2)^{(1/2)}/(c*x^4+a)^{(1/2)}*EllipticE(x*(I/a^{(1/2)}*c^{(1/2)})^{(1/2)}, I)+1/2/(a* \\ & e^2+c*d^2)/d^2*e^2/(I/a^{(1/2)}*c^{(1/2)})^{(1/2)}*(1-I/a^{(1/2)}*c^{(1/2)}*x^2)^{(1/2)} \\ &)*(1+I/a^{(1/2)}*c^{(1/2)}*x^2)^{(1/2)}/(c*x^4+a)^{(1/2)}*EllipticPi(x*(I/a^{(1/2)}*c \\ & ^{(1/2)})^{(1/2)}, I*a^{(1/2)}/c^{(1/2)}*e/d, (-I/a^{(1/2)}*c^{(1/2)})^{(1/2)}/(I/a^{(1/2)}*c \\ & ^{(1/2)})^{(1/2)})*a+3/2/(a*e^2+c*d^2)/(I/a^{(1/2)}*c^{(1/2)})^{(1/2)}*(1-I/a^{(1/2)}*c \\ & ^{(1/2)}*x^2)^{(1/2)}*(1+I/a^{(1/2)}*c^{(1/2)}*x^2)^{(1/2)}/(c*x^4+a)^{(1/2)}*EllipticP \\ & i(x*(I/a^{(1/2)}*c^{(1/2)})^{(1/2)}, I*a^{(1/2)}/c^{(1/2)}*e/d, (-I/a^{(1/2)}*c^{(1/2)})^{(1 \\ & /2)}/(I/a^{(1/2)}*c^{(1/2)})^{(1/2)})*c \end{aligned}$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^4 + a}(ex^2 + d)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x^2+d)^2/(c*x^4+a)^(1/2),x, algorithm="maxima")

[Out] integrate(1/(sqrt(c*x^4 + a)*(e*x^2 + d)^2), x)

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x^2+d)^2/(c*x^4+a)^(1/2),x, algorithm="fricas")

[Out] Timed out

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a + cx^4}(d + ex^2)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x**2+d)**2/(c*x**4+a)**(1/2),x)

[Out] Integral(1/(sqrt(a + c*x**4)*(d + e*x**2)**2), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^4 + a}(ex^2 + d)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^2/(c*x^4+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(c*x^4 + a)*(e*x^2 + d)^2), x)
```

$$3.156 \quad \int \frac{1}{(d+ex^2)^3 \sqrt{a+cx^4}} dx$$

Optimal. Leaf size=729

$$\frac{\sqrt[4]{c}(\sqrt{a} + \sqrt{cx^2}) \sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{cx^2})^2}} (-\sqrt{a}\sqrt{cde} + 3ae^2 + 4cd^2) \text{EllipticF}\left(2 \tan^{-1}\left(\frac{\sqrt[4]{cx}}{\sqrt{a}}\right), \frac{1}{2}\right) + 3\sqrt{e}(a^2e^4 + 2acd^2e^2 + 5c^2d^4) \tan^{-1}\left(\frac{x\sqrt{ae^2+cd^2}}{\sqrt{a}\sqrt{e}\sqrt{a+cx^4}}\right)}{8\sqrt[4]{ad^2}\sqrt{a+cx^4}(\sqrt{cd} - \sqrt{ae})(ae^2 + cd^2) + 16d^{5/2}(ae^2 + cd^2)}$$

[Out] (-3*Sqrt[c]*e*(3*c*d^2 + a*e^2)*x*Sqrt[a + c*x^4])/(8*d^2*(c*d^2 + a*e^2)^2*(Sqrt[a] + Sqrt[c]*x^2)) + (e^2*x*Sqrt[a + c*x^4])/(4*d*(c*d^2 + a*e^2)*(d + e*x^2)^2) + (3*e^2*(3*c*d^2 + a*e^2)*x*Sqrt[a + c*x^4])/(8*d^2*(c*d^2 + a*e^2)^2*(d + e*x^2)) + (3*Sqrt[e]*(5*c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*ArcTan[(Sqrt[c*d^2 + a*e^2]*x)/(Sqrt[d]*Sqrt[e]*Sqrt[a + c*x^4])])/(16*d^(5/2)*(c*d^2 + a*e^2)^(5/2)) + (3*a^(1/4)*c^(1/4)*e*(3*c*d^2 + a*e^2)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(8*d^2*(c*d^2 + a*e^2)^2*Sqrt[a + c*x^4]) + (c^(1/4)*(4*c*d^2 - Sqrt[a]*Sqrt[c]*d*e + 3*a*e^2)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(8*a^(1/4)*d^2*(Sqrt[c]*d - Sqrt[a]*e)*(c*d^2 + a*e^2)*Sqrt[a + c*x^4]) - (3*(Sqrt[c]*d + Sqrt[a]*e)*(5*c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticPi[-(Sqrt[c]*d - Sqrt[a]*e)^2/(4*Sqrt[a]*Sqrt[c]*d*e), 2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(32*a^(1/4)*c^(1/4)*d^3*(Sqrt[c]*d - Sqrt[a]*e)*(c*d^2 + a*e^2)^2*Sqrt[a + c*x^4])

Rubi [A] time = 1.25044, antiderivative size = 729, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1224, 1697, 1715, 1196, 1709, 220, 1707}

$$\frac{3\sqrt{e}(a^2e^4 + 2acd^2e^2 + 5c^2d^4) \tan^{-1}\left(\frac{x\sqrt{ae^2+cd^2}}{\sqrt{a}\sqrt{e}\sqrt{a+cx^4}}\right)}{16d^{5/2}(ae^2 + cd^2)^{5/2}} - \frac{3(\sqrt{a} + \sqrt{cx^2}) \sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{cx^2})^2}} (\sqrt{ae} + \sqrt{cd})(a^2e^4 + 2acd^2e^2 + 5c^2d^4) \text{EllipticF}\left(2 \tan^{-1}\left(\frac{\sqrt[4]{cx}}{\sqrt{a}}\right), \frac{1}{2}\right) + 3\sqrt{e}(a^2e^4 + 2acd^2e^2 + 5c^2d^4) \tan^{-1}\left(\frac{x\sqrt{ae^2+cd^2}}{\sqrt{a}\sqrt{e}\sqrt{a+cx^4}}\right)}{32\sqrt[4]{a}\sqrt[4]{cd^3}\sqrt{a+cx^4}(\sqrt{cd} - \sqrt{ae})(ae^2 + cd^2)}$$

Antiderivative was successfully verified.

[In] Int[1/((d + e*x^2)^3*Sqrt[a + c*x^4]),x]

[Out] (-3*Sqrt[c]*e*(3*c*d^2 + a*e^2)*x*Sqrt[a + c*x^4])/(8*d^2*(c*d^2 + a*e^2)^2*(Sqrt[a] + Sqrt[c]*x^2)) + (e^2*x*Sqrt[a + c*x^4])/(4*d*(c*d^2 + a*e^2)*(d + e*x^2)^2) + (3*e^2*(3*c*d^2 + a*e^2)*x*Sqrt[a + c*x^4])/(8*d^2*(c*d^2 + a*e^2)^2*(d + e*x^2)) + (3*Sqrt[e]*(5*c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*ArcTan[(Sqrt[c*d^2 + a*e^2]*x)/(Sqrt[d]*Sqrt[e]*Sqrt[a + c*x^4])])/(16*d^(5/2)*(c*d^2 + a*e^2)^(5/2)) + (3*a^(1/4)*c^(1/4)*e*(3*c*d^2 + a*e^2)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(8*d^2*(c*d^2 + a*e^2)^2*Sqrt[a + c*x^4]) + (c^(1/4)*(4*c*d^2 - Sqrt[a]*Sqrt[c]*d*e + 3*a*e^2)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(8*a^(1/4)*d^2*(Sqrt[c]*d - Sqrt[a]*e)*(c*d^2 + a*e^2)*Sqrt[a + c*x^4]) - (3*(Sqrt[c]*d + Sqrt[a]*e)*(5*c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticPi[-(Sqrt[c]*d - Sqrt[a]*e)^2/(4*Sqrt[a]*Sqrt[c]*d*e), 2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(32*a^(1/4)*c^(1/4)*d^3*(Sqrt[c]*d - Sqrt[a]*e)*(c*d^2 + a*e^2)^2*Sqrt[a + c*x^4])

Rule 1224

Int[((d_) + (e_)*(x_)^2)^(q_)/Sqrt[(a_) + (c_)*(x_)^4], x_Symbol] := -Simp[(e^2*x*(d + e*x^2)^(q + 1)*Sqrt[a + c*x^4])/(2*d*(q + 1)*(c*d^2 + a*e^2)), x] + Dist[1/(2*d*(q + 1)*(c*d^2 + a*e^2)), Int[((d + e*x^2)^(q + 1)*Simp[a*e^2*(2*q + 3) + 2*c*d^2*(q + 1) - 2*e*c*d*(q + 1)*x^2 + c*e^2*(2*q + 5)*x^4, x])/Sqrt[a + c*x^4], x], x] /; FreeQ[{a, c, d, e}, x] && ILtQ[q, -1]

Rule 1697

Int(((P4x_)*((d_) + (e_)*(x_)^2)^(q_))/Sqrt[(a_) + (c_)*(x_)^4], x_Symbol] := With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[P4x, x, 4]}, -Simp[((C*d^2 - B*d*e + A*e^2)*x*(d + e*x^2)^(q + 1)*Sqrt[a + c*x^4])/(2*d*(q + 1)*(c*d^2 + a*e^2)), x] + Dist[1/(2*d*(q + 1)*(c*d^2 + a*e^2)), Int[((d + e*x^2)^(q + 1)*Simp[a*d*(C*d - B*e) + A*(a*e^2*(2*q + 3) + 2*c*d^2*(q + 1)) + 2*d*(B*c*d - A*c*e + a*C*e)*(q + 1)*x^2 + c*(C*d^2 - B*d*e + A*e^2)*(2*q + 5)*x^4, x])/Sqrt[a + c*x^4], x], x] /; FreeQ[{a, c, d, e}, x] && PolyQ[P4x, x^2] && LeQ[Expon[P4x, x], 4] && NeQ[c*d^2 + a*e^2, 0] && ILtQ[q, -1]

Rule 1715

Int[(P4x_)/(((d_) + (e_)*(x_)^2)*Sqrt[(a_) + (c_)*(x_)^4]), x_Symbol] := With[{q = Rt[c/a, 2], A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[P4x, x, 4]}, -Dist[C/(e*q), Int[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] + Dist[1/(c*e), Int[(A*c*e + a*C*d*q + (B*c*e - C*(c*d - a*e*q))*x^2]/((d + e*x^2)*Sqrt[a + c*x^4]), x], x] /; FreeQ[{a, c, d, e}, x] && PolyQ[P4x, x^2, 2] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]

Rule 1196

Int[((d_) + (e_)*(x_)^2)/Sqrt[(a_) + (c_)*(x_)^4], x_Symbol] := With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x], 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0] /; FreeQ[{a, c, d, e}, x] && PosQ[c/a]

Rule 1709

Int(((A_) + (B_)*(x_)^2)/(((d_) + (e_)*(x_)^2)*Sqrt[(a_) + (c_)*(x_)^4]), x_Symbol] := With[{q = Rt[c/a, 2]}, Dist[(A*(c*d + a*e*q) - a*B*(e + d*q))/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4], x], x] + Dist[(a*(B*d - A*e)*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + c*x^4]), x], x] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && NeQ[c*A^2 - a*B^2, 0]

Rule 220

Int[1/Sqrt[(a_) + (b_)*(x_)^4], x_Symbol] := With[{q = Rt[b/a, 4]}, Simp[(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x], 1/2])/(2*q*Sqrt[a + b*x^4]), x] /; FreeQ[{a, b}, x] && PosQ[b/a]

Rule 1707

Int(((A_) + (B_)*(x_)^2)/(((d_) + (e_)*(x_)^2)*Sqrt[(a_) + (c_)*(x_)^4]), x_Symbol] := With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*ArcTan[(Rt[(c*d)/e + (a*e)/d, 2]*x)/Sqrt[a + c*x^4])]/(2*d*e*Rt[(c*d)/e + (a*e)/d, 2]), x] + Simp[(B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a + c*x^4)/(a*(A + B*x^2)^2)]*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)], 2*ArcTan[q*x], 1/2])/(4*d*e*A*q*Sqrt[a + c*x^4]), x] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e

$\sqrt{a+cx^4}$, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]

Rubi steps

$$\begin{aligned} \int \frac{1}{(d+ex^2)^3 \sqrt{a+cx^4}} dx &= \frac{e^2 x \sqrt{a+cx^4}}{4d(cd^2+ae^2)(d+ex^2)^2} - \frac{\int \frac{-4cd^2-3ae^2+4cdex^2-ce^2x^4}{(d+ex^2)^2 \sqrt{a+cx^4}} dx}{4d(cd^2+ae^2)} \\ &= \frac{e^2 x \sqrt{a+cx^4}}{4d(cd^2+ae^2)(d+ex^2)^2} + \frac{3e^2(3cd^2+ae^2)x\sqrt{a+cx^4}}{8d^2(cd^2+ae^2)^2(d+ex^2)} + \frac{\int \frac{8c^2d^4+5acd^2e^2+3a^2e^4-4cde(4cd^2+ae^2)x^2}{(d+ex^2)\sqrt{a+cx^4}}}{8d^2(cd^2+ae^2)^2} \\ &= \frac{e^2 x \sqrt{a+cx^4}}{4d(cd^2+ae^2)(d+ex^2)^2} + \frac{3e^2(3cd^2+ae^2)x\sqrt{a+cx^4}}{8d^2(cd^2+ae^2)^2(d+ex^2)} + \frac{\int \frac{-3\sqrt{a}c^{3/2}de^2(3cd^2+ae^2)+ce(8c^2d^4+5acd^2e^2+3a^2e^4-4cde(4cd^2+ae^2)x^2)}{(d+ex^2)\sqrt{a+cx^4}}}{8d^2(cd^2+ae^2)^2} \\ &= -\frac{3\sqrt{ce}(3cd^2+ae^2)x\sqrt{a+cx^4}}{8d^2(cd^2+ae^2)^2(\sqrt{a}+\sqrt{cx^2})} + \frac{e^2 x \sqrt{a+cx^4}}{4d(cd^2+ae^2)(d+ex^2)^2} + \frac{3e^2(3cd^2+ae^2)x\sqrt{a+cx^4}}{8d^2(cd^2+ae^2)^2(d+ex^2)} \\ &= -\frac{3\sqrt{ce}(3cd^2+ae^2)x\sqrt{a+cx^4}}{8d^2(cd^2+ae^2)^2(\sqrt{a}+\sqrt{cx^2})} + \frac{e^2 x \sqrt{a+cx^4}}{4d(cd^2+ae^2)(d+ex^2)^2} + \frac{3e^2(3cd^2+ae^2)x\sqrt{a+cx^4}}{8d^2(cd^2+ae^2)^2(d+ex^2)} \end{aligned}$$

Mathematica [C] time = 1.12101, size = 332, normalized size = 0.46

$$\frac{de^2x(a+cx^4)(ae^2(5d+3ex^2)+cd^2(11d+9ex^2))}{(d+ex^2)^2} + \frac{\sqrt{\frac{cx^4}{a}+1} \left(i \left(\sqrt{cd}(-3ia^{3/2}e^3-9i\sqrt{acd^2e+a}\sqrt{cde^2+7c^{3/2}d^3}) \text{EllipticF} \left(i \sinh^{-1} \left(x \sqrt{\frac{i\sqrt{c}}{a}} \right), -1 \right) - 3(a^2e^4+2acd^2e^2+5c^2d^3) \right) \right)}{\sqrt{\frac{i\sqrt{c}}{a}}}$$

$$8d^3\sqrt{a+cx^4}(ae^2+cd^2)^2$$

Antiderivative was successfully verified.

[In] Integrate[1/((d + e*x^2)^3*Sqrt[a + c*x^4]),x]

[Out] ((d*e^2*x*(a + c*x^4)*(a*e^2*(5*d + 3*e*x^2) + c*d^2*(11*d + 9*e*x^2)))/(d + e*x^2)^2 + (Sqrt[1 + (c*x^4)/a]*(-3*Sqrt[a]*Sqrt[c]*d*e*(3*c*d^2 + a*e^2)*EllipticE[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x], -1] + I*(Sqrt[c]*d*(7*c^(3/2)*d^3 - (9*I)*Sqrt[a]*c*d^2*e + a*Sqrt[c]*d*e^2 - (3*I)*a^(3/2)*e^3)*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x], -1] - 3*(5*c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*EllipticPi[(-I)*Sqrt[a]*e/(Sqrt[c]*d), I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x], -1]))/Sqrt[(I*Sqrt[c])/Sqrt[a]])/(8*d^3*(c*d^2 + a*e^2)^2*Sqrt[a + c*x^4])

Maple [C] time = 0.195, size = 1018, normalized size = 1.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(e*x^2+d)^3/(c*x^4+a)^(1/2),x)

```
[Out] 1/4*e^2*x*(c*x^4+a)^(1/2)/d/(a*e^2+c*d^2)/(e*x^2+d)^2+3/8*e^2*(a*e^2+3*c*d^2)*x*(c*x^4+a)^(1/2)/d^2/(a*e^2+c*d^2)^2/(e*x^2+d)-1/8*c/d/(a*e^2+c*d^2)^2/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)*a*e^2-7/8*c^2*d/(a*e^2+c*d^2)^2/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)+3/8*I*c^(1/2)*e^3/(a*e^2+c*d^2)^2/d^2*a^(3/2)/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticE(x*(I/a^(1/2)*c^(1/2))^(1/2),I)-3/8*I*c^(1/2)*e^3/(a*e^2+c*d^2)^2/d^2*a^(3/2)/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)+9/8*I*c^(3/2)*e/(a*e^2+c*d^2)^2*a^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticE(x*(I/a^(1/2)*c^(1/2))^(1/2),I)-9/8*I*c^(3/2)*e/(a*e^2+c*d^2)^2*a^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)+3/8/(a*e^2+c*d^2)^2/d^3*e^4/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticPi(x*(I/a^(1/2)*c^(1/2))^(1/2),I*a^(1/2)/c^(1/2))*e/d,(-I/a^(1/2)*c^(1/2))^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2))*a^2+3/4/(a*e^2+c*d^2)^2/d*e^2/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticPi(x*(I/a^(1/2)*c^(1/2))^(1/2),I*a^(1/2)/c^(1/2))*e/d,(-I/a^(1/2)*c^(1/2))^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2))*a*c+15/8/(a*e^2+c*d^2)^2*d/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticPi(x*(I/a^(1/2)*c^(1/2))^(1/2),I*a^(1/2)/c^(1/2))*e/d,(-I/a^(1/2)*c^(1/2))^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2))*c^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^4 + a}(ex^2 + d)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^3/(c*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^4 + a)*(e*x^2 + d)^3), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^3/(c*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a + cx^4}(d + ex^2)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x**2+d)**3/(c*x**4+a)**(1/2),x)

[Out] Integral(1/(sqrt(a + c*x**4)*(d + e*x**2)**3), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^4 + a}(ex^2 + d)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x^2+d)^3/(c*x^4+a)^(1/2),x, algorithm="giac")

[Out] integrate(1/(sqrt(c*x^4 + a)*(e*x^2 + d)^3), x)

$$3.157 \quad \int \frac{(d+ex^2)^3}{\sqrt{a-cx^4}} dx$$

Optimal. Leaf size=213

$$\frac{a^{3/4} \sqrt{1 - \frac{cx^4}{a}} \left(\frac{5\sqrt{cd}(ae^2+cd^2)}{\sqrt{a}} - 3e(ae^2 + 5cd^2) \right) \text{EllipticF} \left(\sin^{-1} \left(\frac{\sqrt[4]{cx}}{\sqrt[4]{a}} \right), -1 \right)}{5c^{7/4} \sqrt{a - cx^4}} + \frac{3a^{3/4} e \sqrt{1 - \frac{cx^4}{a}} (ae^2 + 5cd^2) E \left(\sin^{-1} \left(\frac{\sqrt[4]{cx}}{\sqrt[4]{a}} \right) \right)}{5c^{7/4} \sqrt{a - cx^4}}$$

[Out] -((d*e^2*x*Sqrt[a - c*x^4])/c) - (e^3*x^3*Sqrt[a - c*x^4])/(5*c) + (3*a^(3/4)*e*(5*c*d^2 + a*e^2)*Sqrt[1 - (c*x^4)/a]*EllipticE[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(5*c^(7/4)*Sqrt[a - c*x^4]) + (a^(3/4)*((5*Sqrt[c]*d*(c*d^2 + a*e^2))/Sqrt[a] - 3*e*(5*c*d^2 + a*e^2))*Sqrt[1 - (c*x^4)/a]*EllipticF[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(5*c^(7/4)*Sqrt[a - c*x^4])

Rubi [A] time = 0.281801, antiderivative size = 213, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {1207, 1888, 1201, 224, 221, 1200, 1199, 424}

$$\frac{a^{3/4} \sqrt{1 - \frac{cx^4}{a}} \left(\frac{5\sqrt{cd}(ae^2+cd^2)}{\sqrt{a}} - 3e(ae^2 + 5cd^2) \right) F \left(\sin^{-1} \left(\frac{\sqrt[4]{cx}}{\sqrt[4]{a}} \right) \middle| -1 \right)}{5c^{7/4} \sqrt{a - cx^4}} + \frac{3a^{3/4} e \sqrt{1 - \frac{cx^4}{a}} (ae^2 + 5cd^2) E \left(\sin^{-1} \left(\frac{\sqrt[4]{cx}}{\sqrt[4]{a}} \right) \middle| -1 \right)}{5c^{7/4} \sqrt{a - cx^4}}$$

Antiderivative was successfully verified.

[In] Int[(d + e*x^2)^3/Sqrt[a - c*x^4], x]

[Out] -((d*e^2*x*Sqrt[a - c*x^4])/c) - (e^3*x^3*Sqrt[a - c*x^4])/(5*c) + (3*a^(3/4)*e*(5*c*d^2 + a*e^2)*Sqrt[1 - (c*x^4)/a]*EllipticE[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(5*c^(7/4)*Sqrt[a - c*x^4]) + (a^(3/4)*((5*Sqrt[c]*d*(c*d^2 + a*e^2))/Sqrt[a] - 3*e*(5*c*d^2 + a*e^2))*Sqrt[1 - (c*x^4)/a]*EllipticF[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(5*c^(7/4)*Sqrt[a - c*x^4])

Rule 1207

Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(e^q*x^(2*q - 3)*(a + c*x^4)^(p + 1))/(c*(4*p + 2*q + 1)), x] + Dist[1/(c*(4*p + 2*q + 1)), Int[(a + c*x^4)^p*ExpandToSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - c*(4*p + 2*q + 1)*e^q*x^(2*q)], x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[q, 1]

Rule 1888

Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{q = Expon[Pq, x]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(q + n*p + 1)), Int[ExpandToSum[b*(q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*(q - n + 1)*x^(q - n), x]*(a + b*x^n)^p, x], x] + Simp[(Pqq*x^(q - n + 1)*(a + b*x^n)^(p + 1))/(b*(q + n*p + 1)), x]] /; NeQ[q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[p + (q + 1)/(2*n)]) /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]

Rule 1201

Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-(c/a), 2]}, Dist[(d*q - e)/q, Int[1/Sqrt[a + c*x^4], x], x] + Dist[e/q, Int[(1 + q*x^2)/Sqrt[a + c*x^4], x], x]] /; FreeQ[{a, c, d, e}, x] && Neg

$Q[c/a] \ \&\& \ \text{NeQ}[c*d^2 + a*e^2, 0]$

Rule 224

$\text{Int}[1/\text{Sqrt}[(a_) + (b_)*(x_)^4], x_Symbol] \text{ :> } \text{Dist}[\text{Sqrt}[1 + (b*x^4)/a]/\text{Sqrt}[a + b*x^4], \text{Int}[1/\text{Sqrt}[1 + (b*x^4)/a], x], x] \text{ /; } \text{FreeQ}\{a, b\}, x] \ \&\& \ \text{NegQ}[b/a] \ \&\& \ \text{!GtQ}[a, 0]$

Rule 221

$\text{Int}[1/\text{Sqrt}[(a_) + (b_)*(x_)^4], x_Symbol] \text{ :> } \text{Simp}[\text{EllipticF}[\text{ArcSin}[\text{Rt}[-b, 4]*x]/\text{Rt}[a, 4]], -1]/(\text{Rt}[a, 4]*\text{Rt}[-b, 4]), x] \text{ /; } \text{FreeQ}\{a, b\}, x] \ \&\& \ \text{NegQ}[b/a] \ \&\& \ \text{GtQ}[a, 0]$

Rule 1200

$\text{Int}[(d_) + (e_)*(x_)^2]/\text{Sqrt}[(a_) + (c_)*(x_)^4], x_Symbol] \text{ :> } \text{Dist}[\text{Sqrt}[1 + (c*x^4)/a]/\text{Sqrt}[a + c*x^4], \text{Int}[(d + e*x^2)/\text{Sqrt}[1 + (c*x^4)/a], x], x] \text{ /; } \text{FreeQ}\{a, c, d, e\}, x] \ \&\& \ \text{NegQ}[c/a] \ \&\& \ \text{EqQ}[c*d^2 + a*e^2, 0] \ \&\& \ \text{!GtQ}[a, 0]$

Rule 1199

$\text{Int}[(d_) + (e_)*(x_)^2]/\text{Sqrt}[(a_) + (c_)*(x_)^4], x_Symbol] \text{ :> } \text{Dist}[d/\text{Sqrt}[a], \text{Int}[\text{Sqrt}[1 + (e*x^2)/d]/\text{Sqrt}[1 - (e*x^2)/d], x], x] \text{ /; } \text{FreeQ}\{a, c, d, e\}, x] \ \&\& \ \text{NegQ}[c/a] \ \&\& \ \text{EqQ}[c*d^2 + a*e^2, 0] \ \&\& \ \text{GtQ}[a, 0]$

Rule 424

$\text{Int}[\text{Sqrt}[(a_) + (b_)*(x_)^2]/\text{Sqrt}[(c_) + (d_)*(x_)^2], x_Symbol] \text{ :> } \text{Simp}[(\text{Sqrt}[a]*\text{EllipticE}[\text{ArcSin}[\text{Rt}[-(d/c), 2]*x], (b*c)/(a*d)])/(\text{Sqrt}[c]*\text{Rt}[-(d/c), 2]), x] \text{ /; } \text{FreeQ}\{a, b, c, d\}, x] \ \&\& \ \text{NegQ}[d/c] \ \&\& \ \text{GtQ}[c, 0] \ \&\& \ \text{GtQ}[a, 0]$

Rubi steps

$$\begin{aligned} \int \frac{(d + ex^2)^3}{\sqrt{a - cx^4}} dx &= -\frac{e^3 x^3 \sqrt{a - cx^4}}{5c} - \frac{\int \frac{-5cd^3 - 3e(5cd^2 + ae^2)x^2 - 15cde^2 x^4}{\sqrt{a - cx^4}} dx}{5c} \\ &= -\frac{de^2 x \sqrt{a - cx^4}}{c} - \frac{e^3 x^3 \sqrt{a - cx^4}}{5c} + \frac{\int \frac{15cd(cd^2 + ae^2) + 9ce(5cd^2 + ae^2)x^2}{\sqrt{a - cx^4}} dx}{15c^2} \\ &= -\frac{de^2 x \sqrt{a - cx^4}}{c} - \frac{e^3 x^3 \sqrt{a - cx^4}}{5c} + \frac{(3\sqrt{ae}(5cd^2 + ae^2)) \int \frac{1 + \frac{\sqrt{cx^2}}{\sqrt{a}}}{\sqrt{a - cx^4}} dx}{5c^{3/2}} + \frac{(5\sqrt{cd}(cd^2 + ae^2) - 3\sqrt{ae}(5cd^2 + ae^2))}{5c^{3/2}} \\ &= -\frac{de^2 x \sqrt{a - cx^4}}{c} - \frac{e^3 x^3 \sqrt{a - cx^4}}{5c} + \frac{(3\sqrt{ae}(5cd^2 + ae^2) \sqrt{1 - \frac{cx^4}{a}}) \int \frac{1 + \frac{\sqrt{cx^2}}{\sqrt{a}}}{\sqrt{1 - \frac{cx^4}{a}}} dx}{5c^{3/2} \sqrt{a - cx^4}} + \frac{((5\sqrt{cd}(cd^2 + ae^2) - 3\sqrt{ae}(5cd^2 + ae^2)) \sqrt{1 - \frac{cx^4}{a}}) F\left(\sin^{-1}\left(\frac{\sqrt[4]{cx}}{\sqrt[4]{a}}\right)\right)}{5c^{7/4} \sqrt{a - cx^4}} \\ &= -\frac{de^2 x \sqrt{a - cx^4}}{c} - \frac{e^3 x^3 \sqrt{a - cx^4}}{5c} + \frac{3a^{3/4} e (5cd^2 + ae^2) \sqrt{1 - \frac{cx^4}{a}} E\left(\sin^{-1}\left(\frac{\sqrt[4]{cx}}{\sqrt[4]{a}}\right)\right) - 1}{5c^{7/4} \sqrt{a - cx^4}} + \frac{\sqrt[4]{a} (5\sqrt{cd}(cd^2 + ae^2) - 3\sqrt{ae}(5cd^2 + ae^2))}{5c^{7/4} \sqrt{a - cx^4}} \end{aligned}$$

Mathematica [C] time = 0.167146, size = 141, normalized size = 0.66

$$\frac{5dx\sqrt{1-\frac{cx^4}{a}}(ae^2+cd^2) {}_2F_1\left(\frac{1}{4}, \frac{1}{2}; \frac{5}{4}; \frac{cx^4}{a}\right) + ex\left(x^2\sqrt{1-\frac{cx^4}{a}}(ae^2+5cd^2) {}_2F_1\left(\frac{1}{2}, \frac{3}{4}; \frac{7}{4}; \frac{cx^4}{a}\right) + e(cx^4-a)(5d+ex^2)\right)}{5c\sqrt{a-cx^4}}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e*x^2)^3/Sqrt[a - c*x^4], x]

[Out] (5*d*(c*d^2 + a*e^2)*x*Sqrt[1 - (c*x^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, (c*x^4)/a] + e*x*(e*(5*d + e*x^2)*(-a + c*x^4) + (5*c*d^2 + a*e^2)*x^2*Sqrt[1 - (c*x^4)/a]*Hypergeometric2F1[1/2, 3/4, 7/4, (c*x^4)/a]))/(5*c*Sqrt[a - c*x^4])

Maple [B] time = 0.291, size = 360, normalized size = 1.7

$$e^3 \left(-\frac{x^3}{5c} \sqrt{-cx^4 + a} - \frac{3}{5} a^{\frac{3}{2}} \sqrt{1 - x^2 \sqrt{c} \frac{1}{\sqrt{a}}} \sqrt{1 + x^2 \sqrt{c} \frac{1}{\sqrt{a}}} \left(\text{EllipticF} \left(x \sqrt{\sqrt{c} \frac{1}{\sqrt{a}}}, i \right) - \text{EllipticE} \left(x \sqrt{\sqrt{c} \frac{1}{\sqrt{a}}}, i \right) \right) c^{-\frac{3}{2}} \frac{1}{\sqrt{a}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x^2+d)^3/(-c*x^4+a)^(1/2), x)

[Out] e^3*(-1/5/c*x^3*(-c*x^4+a)^(1/2)-3/5*a^(3/2)/c^(3/2)/(1/a^(1/2)*c^(1/2))^(1/2)*(1-1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+1/a^(1/2)*c^(1/2)*x^2)^(1/2)/(-c*x^4+a)^(1/2)*(EllipticF(x*(1/a^(1/2)*c^(1/2))^(1/2), I)-EllipticE(x*(1/a^(1/2)*c^(1/2))^(1/2), I))+3*d*e^2*(-1/3/c*x*(-c*x^4+a)^(1/2)+1/3*a/c/(1/a^(1/2)*c^(1/2))^(1/2)*(1-1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+1/a^(1/2)*c^(1/2)*x^2)^(1/2)/(-c*x^4+a)^(1/2)*EllipticF(x*(1/a^(1/2)*c^(1/2))^(1/2), I)-3*d^2*e*a^(1/2)/(1/a^(1/2)*c^(1/2))^(1/2)*(1-1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+1/a^(1/2)*c^(1/2)*x^2)^(1/2)/(-c*x^4+a)^(1/2)/c^(1/2)*(EllipticF(x*(1/a^(1/2)*c^(1/2))^(1/2), I)-EllipticE(x*(1/a^(1/2)*c^(1/2))^(1/2), I))+d^3/(1/a^(1/2)*c^(1/2))^(1/2)*(1-1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+1/a^(1/2)*c^(1/2)*x^2)^(1/2)/(-c*x^4+a)^(1/2)*EllipticF(x*(1/a^(1/2)*c^(1/2))^(1/2), I)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^2 + d)^3}{\sqrt{-cx^4 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)^3/(-c*x^4+a)^(1/2), x, algorithm="maxima")

[Out] integrate((e*x^2 + d)^3/sqrt(-c*x^4 + a), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral} \left(-\frac{(e^3x^6 + 3de^2x^4 + 3d^2ex^2 + d^3)\sqrt{-cx^4 + a}}{cx^4 - a}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)^3/(-c*x^4+a)^(1/2),x, algorithm="fricas")

[Out] integral(-(e^3*x^6 + 3*d*e^2*x^4 + 3*d^2*e*x^2 + d^3)*sqrt(-c*x^4 + a)/(c*x^4 - a), x)

Sympy [A] time = 3.68116, size = 180, normalized size = 0.85

$$\frac{d^3 x \Gamma\left(\frac{1}{4}\right) {}_2F_1\left(\frac{1}{4}, \frac{1}{2} \middle| \frac{cx^4 e^{2i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{5}{4}\right)} + \frac{3d^2 e x^3 \Gamma\left(\frac{3}{4}\right) {}_2F_1\left(\frac{3}{4}, \frac{3}{4} \middle| \frac{cx^4 e^{2i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{7}{4}\right)} + \frac{3de^2 x^5 \Gamma\left(\frac{5}{4}\right) {}_2F_1\left(\frac{5}{4}, \frac{5}{4} \middle| \frac{cx^4 e^{2i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{9}{4}\right)} + \frac{e^3 x^7 \Gamma\left(\frac{7}{4}\right) {}_2F_1\left(\frac{7}{4}, \frac{7}{4} \middle| \frac{cx^4 e^{2i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x**2+d)**3/(-c*x**4+a)**(1/2),x)

[Out] d**3*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), c*x**4*exp_polar(2*I*pi)/a)/(4*sqrt(a)*gamma(5/4)) + 3*d**2*e*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), c*x**4*exp_polar(2*I*pi)/a)/(4*sqrt(a)*gamma(7/4)) + 3*d*e**2*x**5*gamma(5/4)*hyper((1/2, 5/4), (9/4,), c*x**4*exp_polar(2*I*pi)/a)/(4*sqrt(a)*gamma(9/4)) + e**3*x**7*gamma(7/4)*hyper((1/2, 7/4), (11/4,), c*x**4*exp_polar(2*I*pi)/a)/(4*sqrt(a)*gamma(11/4))

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^2 + d)^3}{\sqrt{-cx^4 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)^3/(-c*x^4+a)^(1/2),x, algorithm="giac")

[Out] integrate((e*x^2 + d)^3/sqrt(-c*x^4 + a), x)

$$3.158 \quad \int \frac{(d+ex^2)^2}{\sqrt{a-cx^4}} dx$$

Optimal. Leaf size=162

$$\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}(-6\sqrt{a}\sqrt{cde}+ae^2+3cd^2)\operatorname{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt[4]{cx}}{\sqrt[4]{a}}\right),-1\right)}{3c^{5/4}\sqrt{a-cx^4}} + \frac{2a^{3/4}de\sqrt{1-\frac{cx^4}{a}}E\left(\sin^{-1}\left(\frac{\sqrt[4]{cx}}{\sqrt[4]{a}}\right)\right)-1}{c^{3/4}\sqrt{a-cx^4}} - \frac{e^2x\sqrt{a-cx^4}}{3c}$$

[Out] $-(e^2x\sqrt{a-cx^4})/(3c) + (2a^{3/4}d\sqrt{1-(cx^4)/a})\operatorname{EllipticE}[\operatorname{ArcSin}[(c^{1/4}x)/a^{1/4}], -1]/(c^{3/4}\sqrt{a-cx^4}) + (a^{1/4})(3cd^2 - 6\sqrt{a}\sqrt{c}d\sqrt{e} + ae^2)\sqrt{1-(cx^4)/a}\operatorname{EllipticF}[\operatorname{ArcSin}[(c^{1/4}x)/a^{1/4}], -1]/(3c^{5/4}\sqrt{a-cx^4})$

Rubi [A] time = 0.144698, antiderivative size = 162, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {1207, 1201, 224, 221, 1200, 1199, 424}

$$\frac{2a^{3/4}de\sqrt{1-\frac{cx^4}{a}}E\left(\sin^{-1}\left(\frac{\sqrt[4]{cx}}{\sqrt[4]{a}}\right)\right)-1}{c^{3/4}\sqrt{a-cx^4}} + \frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}(-6\sqrt{a}\sqrt{cde}+ae^2+3cd^2)F\left(\sin^{-1}\left(\frac{\sqrt[4]{cx}}{\sqrt[4]{a}}\right)\right)-1}{3c^{5/4}\sqrt{a-cx^4}} - \frac{e^2x\sqrt{a-cx^4}}{3c}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(d+ex^2)^2/\sqrt{a-cx^4}, x]$

[Out] $-(e^2x\sqrt{a-cx^4})/(3c) + (2a^{3/4}d\sqrt{1-(cx^4)/a})\operatorname{EllipticE}[\operatorname{ArcSin}[(c^{1/4}x)/a^{1/4}], -1]/(c^{3/4}\sqrt{a-cx^4}) + (a^{1/4})(3cd^2 - 6\sqrt{a}\sqrt{c}d\sqrt{e} + ae^2)\sqrt{1-(cx^4)/a}\operatorname{EllipticF}[\operatorname{ArcSin}[(c^{1/4}x)/a^{1/4}], -1]/(3c^{5/4}\sqrt{a-cx^4})$

Rule 1207

$\operatorname{Int}[(d + (e \cdot x)^2)^q \cdot ((a + (c \cdot x)^4)^p), x_Symbol] \rightarrow \operatorname{Simp}[(e^q x^{2q-3} (a + cx^4)^{p+1}) / (c(4p+2q+1)), x] + \operatorname{Dist}[1 / (c(4p+2q+1)), \operatorname{Int}[(a + cx^4)^p \operatorname{ExpandToSum}[c(4p+2q+1)(d + ex^2)^q - a(2q-3)e^q x^{2q-4} - c(4p+2q+1)e^q x^{2q}], x], x] /;$ $\operatorname{FreeQ}\{a, c, d, e, p\}, x \ \&\& \operatorname{NeQ}[c \cdot d^2 + a \cdot e^2, 0] \ \&\& \operatorname{IGtQ}[q, 1]$

Rule 1201

$\operatorname{Int}[(d + (e \cdot x)^2) / \sqrt{(a + (c \cdot x)^4)}, x_Symbol] \rightarrow \operatorname{With}\{q = \operatorname{Rt}[-(c/a), 2]\}, \operatorname{Dist}[(d \cdot q - e) / q, \operatorname{Int}[1 / \sqrt{a + cx^4}], x], x] + \operatorname{Dist}[e / q, \operatorname{Int}[(1 + qx^2) / \sqrt{a + cx^4}], x], x] /;$ $\operatorname{FreeQ}\{a, c, d, e\}, x \ \&\& \operatorname{NegQ}[c/a] \ \&\& \operatorname{NeQ}[c \cdot d^2 + a \cdot e^2, 0]$

Rule 224

$\operatorname{Int}[1 / \sqrt{(a + (b \cdot x)^4)}, x_Symbol] \rightarrow \operatorname{Dist}[\sqrt{1 + (bx^4)/a} / \sqrt{a + bx^4}, \operatorname{Int}[1 / \sqrt{1 + (bx^4)/a}], x], x] /;$ $\operatorname{FreeQ}\{a, b\}, x \ \&\& \operatorname{NegQ}[b/a] \ \&\& \operatorname{!GtQ}[a, 0]$

Rule 221

$\operatorname{Int}[1 / \sqrt{(a + (b \cdot x)^4)}, x_Symbol] \rightarrow \operatorname{Simp}[\operatorname{EllipticF}[\operatorname{ArcSin}[(\operatorname{Rt}[-b, 4] \cdot x) / \operatorname{Rt}[a, 4]], -1] / (\operatorname{Rt}[a, 4] \cdot \operatorname{Rt}[-b, 4]), x] /;$ $\operatorname{FreeQ}\{a, b\}, x \ \&\& \operatorname{NegQ}[$

b/a] && GtQ[a, 0]

Rule 1200

Int[((d_) + (e_)*(x_)^2)/Sqrt[(a_) + (c_)*(x_)^4], x_Symbol] := Dist[Sqrt[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + (c*x^4)/a], x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && !GtQ[a, 0]

Rule 1199

Int[((d_) + (e_)*(x_)^2)/Sqrt[(a_) + (c_)*(x_)^4], x_Symbol] := Dist[d/Sqrt[a], Int[Sqrt[1 + (e*x^2)/d]/Sqrt[1 - (e*x^2)/d], x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]

Rule 424

Int[Sqrt[(a_) + (b_)*(x_)^2]/Sqrt[(c_) + (d_)*(x_)^2], x_Symbol] := Simp[(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]

Rubi steps

$$\begin{aligned} \int \frac{(d + ex^2)^2}{\sqrt{a - cx^4}} dx &= -\frac{e^2 x \sqrt{a - cx^4}}{3c} - \frac{\int \frac{-3cd^2 - ae^2 - 6cdex^2}{\sqrt{a - cx^4}} dx}{3c} \\ &= -\frac{e^2 x \sqrt{a - cx^4}}{3c} + \frac{(2\sqrt{ade}) \int \frac{1 + \frac{\sqrt{cx^2}}{\sqrt{a}}}{\sqrt{a - cx^4}} dx}{\sqrt{c}} - \frac{(-3cd^2 + 6\sqrt{a}\sqrt{cde} - ae^2) \int \frac{1}{\sqrt{a - cx^4}} dx}{3c} \\ &= -\frac{e^2 x \sqrt{a - cx^4}}{3c} + \frac{\left(2\sqrt{ade}\sqrt{1 - \frac{cx^4}{a}}\right) \int \frac{1 + \frac{\sqrt{cx^2}}{\sqrt{a}}}{\sqrt{1 - \frac{cx^4}{a}}} dx}{\sqrt{c}\sqrt{a - cx^4}} - \frac{\left((-3cd^2 + 6\sqrt{a}\sqrt{cde} - ae^2)\sqrt{1 - \frac{cx^4}{a}}\right) \int \frac{1}{\sqrt{1 - \frac{cx^4}{a}}} dx}{3c\sqrt{a - cx^4}} \\ &= -\frac{e^2 x \sqrt{a - cx^4}}{3c} + \frac{\sqrt[4]{a} (3cd^2 - 6\sqrt{a}\sqrt{cde} + ae^2) \sqrt{1 - \frac{cx^4}{a}} F\left(\sin^{-1}\left(\frac{\sqrt[4]{cx}}{\sqrt[4]{a}}\right) \middle| -1\right)}{3c^{5/4}\sqrt{a - cx^4}} + \frac{\left(2\sqrt{ade}\sqrt{1 - \frac{cx^4}{a}}\right) \int \frac{1}{\sqrt{1 - \frac{cx^4}{a}}} dx}{\sqrt{c}\sqrt{a - cx^4}} \\ &= -\frac{e^2 x \sqrt{a - cx^4}}{3c} + \frac{2a^{3/4}de\sqrt{1 - \frac{cx^4}{a}} E\left(\sin^{-1}\left(\frac{\sqrt[4]{cx}}{\sqrt[4]{a}}\right) \middle| -1\right)}{c^{3/4}\sqrt{a - cx^4}} + \frac{\sqrt[4]{a} (3cd^2 - 6\sqrt{a}\sqrt{cde} + ae^2) \sqrt{1 - \frac{cx^4}{a}} F\left(\sin^{-1}\left(\frac{\sqrt[4]{cx}}{\sqrt[4]{a}}\right) \middle| -1\right)}{3c^{5/4}\sqrt{a - cx^4}} \end{aligned}$$

Mathematica [C] time = 0.104588, size = 121, normalized size = 0.75

$$\frac{x\sqrt{1 - \frac{cx^4}{a}} \left(ae^2 + 3cd^2 \right) {}_2F_1\left(\frac{1}{4}, \frac{1}{2}; \frac{5}{4}; \frac{cx^4}{a}\right) + ex \left(2cdx^2 \sqrt{1 - \frac{cx^4}{a}} {}_2F_1\left(\frac{1}{2}, \frac{3}{4}; \frac{7}{4}; \frac{cx^4}{a}\right) - ae + cex^4 \right)}{3c\sqrt{a - cx^4}}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e*x^2)^2/Sqrt[a - c*x^4], x]

[Out] ((3*c*d^2 + a*e^2)*x*Sqrt[1 - (c*x^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, (c*x^4)/a] + e*x*(-(a*e) + c*e*x^4 + 2*c*d*x^2*Sqrt[1 - (c*x^4)/a]*Hypergeom

etric2F1[1/2, 3/4, 7/4, (c*x^4)/a]))/(3*c*Sqrt[a - c*x^4])

Maple [A] time = 0.055, size = 246, normalized size = 1.5

$$e^2 \left(-\frac{x}{3c} \sqrt{-cx^4 + a} + \frac{a}{3c} \sqrt{1 - x^2 \sqrt{c} \frac{1}{\sqrt{a}}} \sqrt{1 + x^2 \sqrt{c} \frac{1}{\sqrt{a}}} \operatorname{EllipticF} \left(x \sqrt{\sqrt{c} \frac{1}{\sqrt{a}}}, i \right) \frac{1}{\sqrt{\sqrt{c} \frac{1}{\sqrt{a}}}} \frac{1}{\sqrt{-cx^4 + a}} \right) - 2 \frac{de\sqrt{a}}{\sqrt{-cx^4 + a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x^2+d)^2/(-c*x^4+a)^(1/2),x)

[Out] $e^2 * (-1/3/c*x*(-c*x^4+a)^{(1/2)} + 1/3*a/c/(1/a^{(1/2)}*c^{(1/2)})^{(1/2)}*(1-1/a^{(1/2)}*c^{(1/2)}*x^2)^{(1/2)}*(1+1/a^{(1/2)}*c^{(1/2)}*x^2)^{(1/2)}/(-c*x^4+a)^{(1/2)}*\operatorname{EllipticF}(x*(1/a^{(1/2)}*c^{(1/2)})^{(1/2)},I)) - 2*d*e*a^{(1/2)}/(1/a^{(1/2)}*c^{(1/2)})^{(1/2)}*(1-1/a^{(1/2)}*c^{(1/2)}*x^2)^{(1/2)}*(1+1/a^{(1/2)}*c^{(1/2)}*x^2)^{(1/2)}/(-c*x^4+a)^{(1/2)}/c^{(1/2)}*(\operatorname{EllipticF}(x*(1/a^{(1/2)}*c^{(1/2)})^{(1/2)},I) - \operatorname{EllipticE}(x*(1/a^{(1/2)}*c^{(1/2)})^{(1/2)},I)) + d^2/(1/a^{(1/2)}*c^{(1/2)})^{(1/2)}*(1-1/a^{(1/2)}*c^{(1/2)}*x^2)^{(1/2)}*(1+1/a^{(1/2)}*c^{(1/2)}*x^2)^{(1/2)}/(-c*x^4+a)^{(1/2)}*\operatorname{EllipticF}(x*(1/a^{(1/2)}*c^{(1/2)})^{(1/2)},I)$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^2 + d)^2}{\sqrt{-cx^4 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)^2/(-c*x^4+a)^(1/2),x, algorithm="maxima")

[Out] integrate((e*x^2 + d)^2/sqrt(-c*x^4 + a), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral} \left(-\frac{(e^2x^4 + 2dex^2 + d^2)\sqrt{-cx^4 + a}}{cx^4 - a}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)^2/(-c*x^4+a)^(1/2),x, algorithm="fricas")

[Out] integral(-(e^2*x^4 + 2*d*e*x^2 + d^2)*sqrt(-c*x^4 + a)/(c*x^4 - a), x)

Sympy [A] time = 2.81924, size = 129, normalized size = 0.8

$$\frac{d^2 x \Gamma\left(\frac{1}{4}\right) {}_2F_1\left(\frac{1}{4}, \frac{1}{2} \middle| \frac{cx^4 e^{2i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{5}{4}\right)} + \frac{dex^3 \Gamma\left(\frac{3}{4}\right) {}_2F_1\left(\frac{3}{4}, \frac{3}{4} \middle| \frac{cx^4 e^{2i\pi}}{a}\right)}{2\sqrt{a}\Gamma\left(\frac{7}{4}\right)} + \frac{e^2 x^5 \Gamma\left(\frac{5}{4}\right) {}_2F_1\left(\frac{5}{4}, \frac{5}{4} \middle| \frac{cx^4 e^{2i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x**2+d)**2/(-c*x**4+a)**(1/2),x)

[Out] d**2*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), c*x**4*exp_polar(2*I*pi)/a)/(4*sqrt(a)*gamma(5/4)) + d*e*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), c*x**4*exp_polar(2*I*pi)/a)/(2*sqrt(a)*gamma(7/4)) + e**2*x**5*gamma(5/4)*hyper((1/2, 5/4), (9/4,), c*x**4*exp_polar(2*I*pi)/a)/(4*sqrt(a)*gamma(9/4))

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^2 + d)^2}{\sqrt{-cx^4 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)^2/(-c*x^4+a)^(1/2),x, algorithm="giac")

[Out] integrate((e*x^2 + d)^2/sqrt(-c*x^4 + a), x)

$$3.159 \quad \int \frac{d+ex^2}{\sqrt{a-cx^4}} dx$$

Optimal. Leaf size=124

$$\frac{a^{3/4} \sqrt{1 - \frac{cx^4}{a}} \left(\frac{\sqrt{cd}}{\sqrt{a}} - e \right) \text{EllipticF} \left(\sin^{-1} \left(\frac{\sqrt[4]{cx}}{\sqrt[4]{a}} \right), -1 \right)}{c^{3/4} \sqrt{a - cx^4}} + \frac{a^{3/4} e \sqrt{1 - \frac{cx^4}{a}} E \left(\sin^{-1} \left(\frac{\sqrt[4]{cx}}{\sqrt[4]{a}} \right) \middle| -1 \right)}{c^{3/4} \sqrt{a - cx^4}}$$

[Out] (a^(3/4)*e*Sqrt[1 - (c*x^4)/a]*EllipticE[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(c^(3/4)*Sqrt[a - c*x^4]) + (a^(3/4)*((Sqrt[c]*d)/Sqrt[a] - e)*Sqrt[1 - (c*x^4)/a]*EllipticF[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(c^(3/4)*Sqrt[a - c*x^4])

Rubi [A] time = 0.0888635, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.3$, Rules used = {1201, 224, 221, 1200, 1199, 424}

$$\frac{a^{3/4} \sqrt{1 - \frac{cx^4}{a}} \left(\frac{\sqrt{cd}}{\sqrt{a}} - e \right) F \left(\sin^{-1} \left(\frac{\sqrt[4]{cx}}{\sqrt[4]{a}} \right) \middle| -1 \right)}{c^{3/4} \sqrt{a - cx^4}} + \frac{a^{3/4} e \sqrt{1 - \frac{cx^4}{a}} E \left(\sin^{-1} \left(\frac{\sqrt[4]{cx}}{\sqrt[4]{a}} \right) \middle| -1 \right)}{c^{3/4} \sqrt{a - cx^4}}$$

Antiderivative was successfully verified.

[In] Int[(d + e*x^2)/Sqrt[a - c*x^4], x]

[Out] (a^(3/4)*e*Sqrt[1 - (c*x^4)/a]*EllipticE[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(c^(3/4)*Sqrt[a - c*x^4]) + (a^(3/4)*((Sqrt[c]*d)/Sqrt[a] - e)*Sqrt[1 - (c*x^4)/a]*EllipticF[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(c^(3/4)*Sqrt[a - c*x^4])

Rule 1201

Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] := With[{q = Rt[-(c/a), 2]}, Dist[(d*q - e)/q, Int[1/Sqrt[a + c*x^4], x], x] + Dist[e/q, Int[(1 + q*x^2)/Sqrt[a + c*x^4], x], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && NeQ[c*d^2 + a*e^2, 0]

Rule 224

Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] := Dist[Sqrt[1 + (b*x^4)/a]/Sqrt[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[b/a] && !GtQ[a, 0]

Rule 221

Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] := Simp[EllipticF[ArcSin[(Rt[-b, 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[b/a] && GtQ[a, 0]

Rule 1200

Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] := Dist[Sqrt[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + (c*x^4)/a], x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && !GtQ[a, 0]

Rule 1199

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] := Dist[d/Sq
rt[a], Int[Sqrt[1 + (e*x^2)/d]/Sqrt[1 - (e*x^2)/d], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] := Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{d + ex^2}{\sqrt{a - cx^4}} dx &= \frac{(\sqrt{ae}) \int \frac{1 + \frac{\sqrt{cx^2}}{\sqrt{a}}}{\sqrt{a - cx^4}} dx}{\sqrt{c}} + \left(d - \frac{\sqrt{ae}}{\sqrt{c}}\right) \int \frac{1}{\sqrt{a - cx^4}} dx \\ &= \frac{\left(\sqrt{ae}\sqrt{1 - \frac{cx^4}{a}}\right) \int \frac{1 + \frac{\sqrt{cx^2}}{\sqrt{a}}}{\sqrt{1 - \frac{cx^4}{a}}} dx}{\sqrt{c}\sqrt{a - cx^4}} + \frac{\left(\left(d - \frac{\sqrt{ae}}{\sqrt{c}}\right)\sqrt{1 - \frac{cx^4}{a}}\right) \int \frac{1}{\sqrt{1 - \frac{cx^4}{a}}} dx}{\sqrt{a - cx^4}} \\ &= \frac{\sqrt[4]{a}\left(d - \frac{\sqrt{ae}}{\sqrt{c}}\right)\sqrt{1 - \frac{cx^4}{a}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{cx}}{\sqrt[4]{a}}\right)\right) - 1}{\sqrt[4]{c}\sqrt{a - cx^4}} + \frac{\left(\sqrt{ae}\sqrt{1 - \frac{cx^4}{a}}\right) \int \frac{\sqrt{1 + \frac{\sqrt{cx^2}}{\sqrt{a}}}}{\sqrt{1 - \frac{\sqrt{cx^2}}{\sqrt{a}}}} dx}{\sqrt{c}\sqrt{a - cx^4}} \\ &= \frac{a^{3/4}e\sqrt{1 - \frac{cx^4}{a}}E\left(\sin^{-1}\left(\frac{\sqrt[4]{cx}}{\sqrt[4]{a}}\right)\right) - 1}{c^{3/4}\sqrt{a - cx^4}} + \frac{\sqrt[4]{a}\left(d - \frac{\sqrt{ae}}{\sqrt{c}}\right)\sqrt{1 - \frac{cx^4}{a}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{cx}}{\sqrt[4]{a}}\right)\right) - 1}{\sqrt[4]{c}\sqrt{a - cx^4}} \end{aligned}$$

Mathematica [C] time = 0.0312186, size = 77, normalized size = 0.62

$$\frac{\sqrt{1 - \frac{cx^4}{a}} \left(3dx {}_2F_1\left(\frac{1}{4}, \frac{1}{2}; \frac{5}{4}; \frac{cx^4}{a}\right) + ex^3 {}_2F_1\left(\frac{1}{2}, \frac{3}{4}; \frac{7}{4}; \frac{cx^4}{a}\right)\right)}{3\sqrt{a - cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)/Sqrt[a - c*x^4],x]
```

```
[Out] (Sqrt[1 - (c*x^4)/a]*(3*d*x*Hypergeometric2F1[1/4, 1/2, 5/4, (c*x^4)/a] + e
*x^3*Hypergeometric2F1[1/2, 3/4, 7/4, (c*x^4)/a]))/(3*Sqrt[a - c*x^4])
```

Maple [A] time = 0.05, size = 154, normalized size = 1.2

$$-e\sqrt{a}\sqrt{1 - x^2\sqrt{c}\frac{1}{\sqrt{a}}}\sqrt{1 + x^2\sqrt{c}\frac{1}{\sqrt{a}}}\left(\text{EllipticF}\left(x\sqrt{\sqrt{c}\frac{1}{\sqrt{a}}}, i\right) - \text{EllipticE}\left(x\sqrt{\sqrt{c}\frac{1}{\sqrt{a}}}, i\right)\right) \frac{1}{\sqrt{\sqrt{c}\frac{1}{\sqrt{a}}}} \frac{1}{\sqrt{-cx^4 + a}} \frac{1}{\sqrt{c}} + d\sqrt{1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)/(-c*x^4+a)^(1/2),x)
```

```
[Out] -e*a^(1/2)/(1/a^(1/2)*c^(1/2))^(1/2)*(1-1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+1/a^(1/2)*c^(1/2)*x^2)^(1/2)/(-c*x^4+a)^(1/2)/c^(1/2)*(EllipticF(x*(1/a^(1/2)*c^(1/2))^(1/2),I)-EllipticE(x*(1/a^(1/2)*c^(1/2))^(1/2),I))+d/(1/a^(1/2)*c^(1/2))^(1/2)*(1-1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+1/a^(1/2)*c^(1/2)*x^2)^(1/2)/(-c*x^4+a)^(1/2)*EllipticF(x*(1/a^(1/2)*c^(1/2))^(1/2),I)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex^2 + d}{\sqrt{-cx^4 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(-c*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)/sqrt(-c*x^4 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-cx^4 + a}(ex^2 + d)}{cx^4 - a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(-c*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-c*x^4 + a)*(e*x^2 + d)/(c*x^4 - a), x)
```

Sympy [A] time = 1.70771, size = 82, normalized size = 0.66

$$\frac{dx\Gamma\left(\frac{1}{4}\right) {}_2F_1\left(\frac{1}{4}, \frac{1}{2} \middle| \frac{cx^4 e^{2i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{5}{4}\right)} + \frac{ex^3\Gamma\left(\frac{3}{4}\right) {}_2F_1\left(\frac{1}{2}, \frac{3}{4} \middle| \frac{cx^4 e^{2i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)/(-c*x**4+a)**(1/2),x)
```

```
[Out] d*x*gamma(1/4)*hyper((1/4, 1/2), (5/4, ), c*x**4*exp_polar(2*I*pi)/a)/(4*sqrt(a)*gamma(5/4)) + e*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4, ), c*x**4*exp_polar(2*I*pi)/a)/(4*sqrt(a)*gamma(7/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex^2 + d}{\sqrt{-cx^4 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(-c*x^4+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)/sqrt(-c*x^4 + a), x)
```


$$3.160 \quad \int \frac{1}{(d+ex^2)\sqrt{a-cx^4}} dx$$

Optimal. Leaf size=72

$$\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\Pi\left(-\frac{\sqrt{ae}}{\sqrt{cd}}; \sin^{-1}\left(\frac{\sqrt[4]{cx}}{\sqrt[4]{a}}\right)\right)-1}{\sqrt[4]{cd}\sqrt{a-cx^4}}$$

[Out] (a^(1/4)*Sqrt[1 - (c*x^4)/a]*EllipticPi[-((Sqrt[a]*e)/(Sqrt[c]*d)), ArcSin[(c^(1/4)*x)/a^(1/4)], -1)]/(c^(1/4)*d*Sqrt[a - c*x^4])

Rubi [A] time = 0.0409227, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1219, 1218}

$$\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\Pi\left(-\frac{\sqrt{ae}}{\sqrt{cd}}; \sin^{-1}\left(\frac{\sqrt[4]{cx}}{\sqrt[4]{a}}\right)\right)-1}{\sqrt[4]{cd}\sqrt{a-cx^4}}$$

Antiderivative was successfully verified.

[In] Int[1/((d + e*x^2)*Sqrt[a - c*x^4]), x]

[Out] (a^(1/4)*Sqrt[1 - (c*x^4)/a]*EllipticPi[-((Sqrt[a]*e)/(Sqrt[c]*d)), ArcSin[(c^(1/4)*x)/a^(1/4)], -1)]/(c^(1/4)*d*Sqrt[a - c*x^4])

Rule 1219

Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] := Dist[Sqrt[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + (c*x^4)/a]), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && !GtQ[a, 0]

Rule 1218

Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] := With[{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1)]/(d*Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]

Rubi steps

$$\begin{aligned} \int \frac{1}{(d+ex^2)\sqrt{a-cx^4}} dx &= \frac{\sqrt{1-\frac{cx^4}{a}} \int \frac{1}{(d+ex^2)\sqrt{1-\frac{cx^4}{a}}} dx}{\sqrt{a-cx^4}} \\ &= \frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\Pi\left(-\frac{\sqrt{ae}}{\sqrt{cd}}; \sin^{-1}\left(\frac{\sqrt[4]{cx}}{\sqrt[4]{a}}\right)\right)-1}{\sqrt[4]{cd}\sqrt{a-cx^4}} \end{aligned}$$

Mathematica [C] time = 0.152069, size = 91, normalized size = 1.26

$$\frac{i\sqrt{1-\frac{cx^4}{a}}\Pi\left(-\frac{\sqrt{ae}}{\sqrt{cd}}; i\sinh^{-1}\left(\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}x\right)\right)-1}{d\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\sqrt{a-cx^4}}$$

Antiderivative was successfully verified.

[In] Integrate[1/((d + e*x^2)*Sqrt[a - c*x^4]),x]

[Out] ((-I)*Sqrt[1 - (c*x^4)/a]*EllipticPi[-((Sqrt[a]*e)/(Sqrt[c]*d)), I*ArcSinh[Sqrt[-(Sqrt[c]/Sqrt[a])]*x], -1])/(Sqrt[-(Sqrt[c]/Sqrt[a])]*d*Sqrt[a - c*x^4])

Maple [A] time = 0.193, size = 97, normalized size = 1.4

$$\frac{1}{d} \sqrt{1 - x^2 \sqrt{c} \frac{1}{\sqrt{a}}} \sqrt{1 + x^2 \sqrt{c} \frac{1}{\sqrt{a}}} \text{EllipticPi} \left(x \sqrt{\sqrt{c} \frac{1}{\sqrt{a}}}, -\frac{e}{d} \sqrt{a} \frac{1}{\sqrt{c}}, \sqrt{-\sqrt{c} \frac{1}{\sqrt{a}}} \frac{1}{\sqrt{\sqrt{c} \frac{1}{\sqrt{a}}}} \right) \frac{1}{\sqrt{\sqrt{c} \frac{1}{\sqrt{a}}}} \frac{1}{\sqrt{-cx^4 + a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(e*x^2+d)/(-c*x^4+a)^(1/2),x)

[Out] 1/d/(1/a^(1/2)*c^(1/2))^(1/2)*(1-1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+1/a^(1/2)*c^(1/2)*x^2)^(1/2)/(-c*x^4+a)^(1/2)*EllipticPi(x*(1/a^(1/2)*c^(1/2))^(1/2), -e*a^(1/2)/d/c^(1/2), (-1/a^(1/2)*c^(1/2))^(1/2)/(1/a^(1/2)*c^(1/2))^(1/2))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-cx^4 + a}(ex^2 + d)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x^2+d)/(-c*x^4+a)^(1/2),x, algorithm="maxima")

[Out] integrate(1/(sqrt(-c*x^4 + a)*(e*x^2 + d)), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral} \left(-\frac{\sqrt{-cx^4 + a}}{cex^6 + cdx^4 - aex^2 - ad}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x^2+d)/(-c*x^4+a)^(1/2),x, algorithm="fricas")

[Out] integral(-sqrt(-c*x^4 + a)/(c*e*x^6 + c*d*x^4 - a*e*x^2 - a*d), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a - cx^4}(d + ex^2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x**2+d)/(-c*x**4+a)**(1/2),x)

[Out] Integral(1/(sqrt(a - c*x**4)*(d + e*x**2)), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-cx^4 + a}(ex^2 + d)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x^2+d)/(-c*x^4+a)^(1/2),x, algorithm="giac")

[Out] integrate(1/(sqrt(-c*x^4 + a)*(e*x^2 + d)), x)

$$3.161 \quad \int \frac{1}{(d+ex^2)^2 \sqrt{a-cx^4}} dx$$

Optimal. Leaf size=299

$$\frac{\sqrt[4]{a}\sqrt[4]{c}\sqrt{1-\frac{cx^4}{a}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt[4]{cx}}{\sqrt[4]{a}}\right), -1\right)}{2d\sqrt{a-cx^4}(\sqrt{ae}+\sqrt{cd})} - \frac{a^{3/4}\sqrt[4]{ce}\sqrt{1-\frac{cx^4}{a}}E\left(\sin^{-1}\left(\frac{\sqrt[4]{cx}}{\sqrt[4]{a}}\right)\right)-1}{2d\sqrt{a-cx^4}(cd^2-ae^2)} - \frac{e^2x\sqrt{a-cx^4}}{2d(d+ex^2)(cd^2-ae^2)} + \frac{\sqrt[4]{a}\sqrt[4]{c}\sqrt{1-\frac{cx^4}{a}}\text{EllipticE}\left(\sin^{-1}\left(\frac{\sqrt[4]{cx}}{\sqrt[4]{a}}\right)\right)}{2d\sqrt{a-cx^4}(cd^2-ae^2)}$$

[Out] $-(e^2*x*\text{Sqrt}[a - c*x^4])/(2*d*(c*d^2 - a*e^2)*(d + e*x^2)) - (a^{(3/4)}*c^{(1/4)}*e*\text{Sqrt}[1 - (c*x^4)/a]*\text{EllipticE}[\text{ArcSin}[(c^{(1/4)}*x)/a^{(1/4)}], -1])/(2*d*(c*d^2 - a*e^2)*\text{Sqrt}[a - c*x^4]) - (a^{(1/4)}*c^{(1/4)}*\text{Sqrt}[1 - (c*x^4)/a]*\text{EllipticF}[\text{ArcSin}[(c^{(1/4)}*x)/a^{(1/4)}], -1])/(2*d*(\text{Sqrt}[c]*d + \text{Sqrt}[a]*e)*\text{Sqrt}[a - c*x^4]) + (a^{(1/4)}*(3*c*d^2 - a*e^2)*\text{Sqrt}[1 - (c*x^4)/a]*\text{EllipticPi}[-((\text{Sqrt}[a]*e)/(\text{Sqrt}[c]*d)), \text{ArcSin}[(c^{(1/4)}*x)/a^{(1/4)}], -1])/(2*c^{(1/4)}*d^2*(c*d^2 - a*e^2)*\text{Sqrt}[a - c*x^4])$

Rubi [A] time = 0.356134, antiderivative size = 299, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 10, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.454$, Rules used = {1224, 1717, 1201, 224, 221, 1200, 1199, 424, 1219, 1218}

$$\frac{a^{3/4}\sqrt[4]{ce}\sqrt{1-\frac{cx^4}{a}}E\left(\sin^{-1}\left(\frac{\sqrt[4]{cx}}{\sqrt[4]{a}}\right)\right)-1}{2d\sqrt{a-cx^4}(cd^2-ae^2)} - \frac{e^2x\sqrt{a-cx^4}}{2d(d+ex^2)(cd^2-ae^2)} + \frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}(3cd^2-ae^2)\Pi\left(-\frac{\sqrt{ae}}{\sqrt{cd}}; \sin^{-1}\left(\frac{\sqrt[4]{cx}}{\sqrt[4]{a}}\right)\right)-1}{2\sqrt[4]{cd^2}\sqrt{a-cx^4}(cd^2-ae^2)}$$

Antiderivative was successfully verified.

[In] Int[1/((d + e*x^2)^2*Sqrt[a - c*x^4]),x]

[Out] $-(e^2*x*\text{Sqrt}[a - c*x^4])/(2*d*(c*d^2 - a*e^2)*(d + e*x^2)) - (a^{(3/4)}*c^{(1/4)}*e*\text{Sqrt}[1 - (c*x^4)/a]*\text{EllipticE}[\text{ArcSin}[(c^{(1/4)}*x)/a^{(1/4)}], -1])/(2*d*(c*d^2 - a*e^2)*\text{Sqrt}[a - c*x^4]) - (a^{(1/4)}*c^{(1/4)}*\text{Sqrt}[1 - (c*x^4)/a]*\text{EllipticF}[\text{ArcSin}[(c^{(1/4)}*x)/a^{(1/4)}], -1])/(2*d*(\text{Sqrt}[c]*d + \text{Sqrt}[a]*e)*\text{Sqrt}[a - c*x^4]) + (a^{(1/4)}*(3*c*d^2 - a*e^2)*\text{Sqrt}[1 - (c*x^4)/a]*\text{EllipticPi}[-((\text{Sqrt}[a]*e)/(\text{Sqrt}[c]*d)), \text{ArcSin}[(c^{(1/4)}*x)/a^{(1/4)}], -1])/(2*c^{(1/4)}*d^2*(c*d^2 - a*e^2)*\text{Sqrt}[a - c*x^4])$

Rule 1224

Int[((d_) + (e_.)*(x_)^2)^(q_)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> -Simp[(e^2*x*(d + e*x^2)^(q + 1)*Sqrt[a + c*x^4])/(2*d*(q + 1)*(c*d^2 + a*e^2)), x] + Dist[1/(2*d*(q + 1)*(c*d^2 + a*e^2)), Int[((d + e*x^2)^(q + 1)*Simp[a*e^2*(2*q + 3) + 2*c*d^2*(q + 1) - 2*e*c*d*(q + 1)*x^2 + c*e^2*(2*q + 5)*x^4, x])/Sqrt[a + c*x^4], x], x] /; FreeQ[{a, c, d, e}, x] && ILtQ[q, -1]

Rule 1717

Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[P4x, x, 4]}, -Dist[(e^2)^(-1), Int[(C*d - B*e - C*e*x^2)/Sqrt[a + c*x^4], x], x] + Dist[(C*d^2 - B*d*e + A*e^2)/e^2, Int[1/((d + e*x^2)*Sqrt[a + c*x^4]), x], x] /; FreeQ[{a, c, d, e}, x] && PolyQ[P4x, x^2, 2] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0]

Rule 1201

```
Int[((d_) + (e_)*(x_)^2)/Sqrt[(a_) + (c_)*(x_)^4], x_Symbol] := With[{q =
  Rt[-(c/a), 2]}, Dist[(d*q - e)/q, Int[1/Sqrt[a + c*x^4], x], x] + Dist[e/q
, Int[(1 + q*x^2)/Sqrt[a + c*x^4], x], x]] /; FreeQ[{a, c, d, e}, x] && Neg
Q[c/a] && NeQ[c*d^2 + a*e^2, 0]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_)*(x_)^4], x_Symbol] := Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && !GtQ[a, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_)*(x_)^4], x_Symbol] := Simp[EllipticF[ArcSin[(Rt[-b,
  4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 1200

```
Int[((d_) + (e_)*(x_)^2)/Sqrt[(a_) + (c_)*(x_)^4], x_Symbol] := Dist[Sqrt
[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + (c*x^4)/a], x], x
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && !GtQ[
a, 0]
```

Rule 1199

```
Int[((d_) + (e_)*(x_)^2)/Sqrt[(a_) + (c_)*(x_)^4], x_Symbol] := Dist[d/Sq
rt[a], Int[Sqrt[1 + (e*x^2)/d]/Sqrt[1 - (e*x^2)/d], x], x] /; FreeQ[{a, c,
  d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_)*(x_)^2]/Sqrt[(c_) + (d_)*(x_)^2], x_Symbol] := Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 1219

```
Int[1/(((d_) + (e_)*(x_)^2)*Sqrt[(a_) + (c_)*(x_)^4]), x_Symbol] := Dist[
Sqrt[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + (c*x^4)/a
]), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && !GtQ[a, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_)*(x_)^2)*Sqrt[(a_) + (c_)*(x_)^4]), x_Symbol] := With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(d+ex^2)^2 \sqrt{a-cx^4}} dx &= -\frac{e^2 x \sqrt{a-cx^4}}{2d(cd^2-ae^2)(d+ex^2)} + \frac{\int \frac{2cd^2-ae^2-2cdex^2-ce^2x^4}{(d+ex^2)\sqrt{a-cx^4}} dx}{2d(cd^2-ae^2)} \\
&= -\frac{e^2 x \sqrt{a-cx^4}}{2d(cd^2-ae^2)(d+ex^2)} - \frac{\int \frac{cde^2+ce^3x^2}{\sqrt{a-cx^4}} dx}{2de^2(cd^2-ae^2)} + \frac{(3cd^2-ae^2) \int \frac{1}{(d+ex^2)\sqrt{a-cx^4}} dx}{2d(cd^2-ae^2)} \\
&= -\frac{e^2 x \sqrt{a-cx^4}}{2d(cd^2-ae^2)(d+ex^2)} - \frac{\sqrt{c} \int \frac{1}{\sqrt{a-cx^4}} dx}{2d(\sqrt{cd}+\sqrt{ae})} - \frac{(\sqrt{a}\sqrt{ce}) \int \frac{1+\frac{\sqrt{cx^2}}{\sqrt{a}}}{\sqrt{a-cx^4}} dx}{2d(cd^2-ae^2)} + \frac{\left((3cd^2-ae^2)\sqrt{1-\frac{cx^4}{a}}\right)}{2d(cd^2-ae^2)} \\
&= -\frac{e^2 x \sqrt{a-cx^4}}{2d(cd^2-ae^2)(d+ex^2)} + \frac{\sqrt[4]{a}(3cd^2-ae^2)\sqrt{1-\frac{cx^4}{a}}\Pi\left(-\frac{\sqrt{ae}}{\sqrt{cd}}; \sin^{-1}\left(\frac{\sqrt[4]{cx}}{\sqrt[4]{a}}\right)\right)-1}{2\sqrt[4]{cd^2}(cd^2-ae^2)\sqrt{a-cx^4}} - \frac{\left(\sqrt{c}\sqrt{1-\frac{cx^4}{a}}\right)}{2d(\sqrt{cd}+\sqrt{ae})} \\
&= -\frac{e^2 x \sqrt{a-cx^4}}{2d(cd^2-ae^2)(d+ex^2)} - \frac{\sqrt[4]{a}\sqrt[4]{c}\sqrt{1-\frac{cx^4}{a}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{cx}}{\sqrt[4]{a}}\right)\right)-1}{2d(\sqrt{cd}+\sqrt{ae})\sqrt{a-cx^4}} + \frac{\sqrt[4]{a}(3cd^2-ae^2)\sqrt{1-\frac{cx^4}{a}}}{2\sqrt[4]{cd^2}(cd^2-ae^2)} \\
&= -\frac{e^2 x \sqrt{a-cx^4}}{2d(cd^2-ae^2)(d+ex^2)} - \frac{a^{3/4}\sqrt[4]{c}e\sqrt{1-\frac{cx^4}{a}}E\left(\sin^{-1}\left(\frac{\sqrt[4]{cx}}{\sqrt[4]{a}}\right)\right)-1}{2d(cd^2-ae^2)\sqrt{a-cx^4}} - \frac{\sqrt[4]{a}\sqrt[4]{c}\sqrt{1-\frac{cx^4}{a}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{cx}}{\sqrt[4]{a}}\right)\right)}{2d(\sqrt{cd}+\sqrt{ae})}
\end{aligned}$$

Mathematica [C] time = 0.974078, size = 508, normalized size = 1.7

$$-i\sqrt{cd}\sqrt{1-\frac{cx^4}{a}}(d+ex^2)(\sqrt{ae}-\sqrt{cd})\text{EllipticF}\left(i\sinh^{-1}\left(x\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\right),-1\right)-3icd^2ex^2\sqrt{1-\frac{cx^4}{a}}\Pi\left(-\frac{\sqrt{ae}}{\sqrt{cd}};i\sinh^{-1}\left(\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\right)\right)$$

Antiderivative was successfully verified.

[In] Integrate[1/((d + e*x^2)^2*Sqrt[a - c*x^4]),x]

[Out] $(-(a\sqrt{-\left(\frac{\sqrt{c}}{\sqrt{a}}\right)})*d*e^2*x) + \sqrt{-\left(\frac{\sqrt{c}}{\sqrt{a}}\right)}*c*d*e^2*x^5 + I*\sqrt{a}*\sqrt{c}*d*e*(d + e*x^2)*\sqrt{1 - \left(\frac{c*x^4}{a}\right)}*\text{EllipticE}\left[I*\text{ArcSinh}\left[\sqrt{-\left(\frac{\sqrt{c}}{\sqrt{a}}\right)}*x\right], -1\right] - I*\sqrt{c}*d*(-\left(\frac{\sqrt{c}}{\sqrt{a}}\right)*d + \sqrt{a}*e)*(d + e*x^2)*\sqrt{1 - \left(\frac{c*x^4}{a}\right)}*\text{EllipticF}\left[I*\text{ArcSinh}\left[\sqrt{-\left(\frac{\sqrt{c}}{\sqrt{a}}\right)}*x\right], -1\right] - (3*I)*c*d^3*\sqrt{1 - \left(\frac{c*x^4}{a}\right)}*\text{EllipticPi}\left[-\left(\frac{\sqrt{a}*e}{\sqrt{c}*d}\right), I*\text{ArcSinh}\left[\sqrt{-\left(\frac{\sqrt{c}}{\sqrt{a}}\right)}*x\right], -1\right] + I*a*d*e^2*\sqrt{1 - \left(\frac{c*x^4}{a}\right)}*\text{EllipticPi}\left[-\left(\frac{\sqrt{a}*e}{\sqrt{c}*d}\right), I*\text{ArcSinh}\left[\sqrt{-\left(\frac{\sqrt{c}}{\sqrt{a}}\right)}*x\right], -1\right] - (3*I)*c*d^2*e*x^2*\sqrt{1 - \left(\frac{c*x^4}{a}\right)}*\text{EllipticPi}\left[-\left(\frac{\sqrt{a}*e}{\sqrt{c}*d}\right), I*\text{ArcSinh}\left[\sqrt{-\left(\frac{\sqrt{c}}{\sqrt{a}}\right)}*x\right], -1\right] + I*a*e^3*x^2*\sqrt{1 - \left(\frac{c*x^4}{a}\right)}*\text{EllipticPi}\left[-\left(\frac{\sqrt{a}*e}{\sqrt{c}*d}\right), I*\text{ArcSinh}\left[\sqrt{-\left(\frac{\sqrt{c}}{\sqrt{a}}\right)}*x\right], -1\right]\right)/(2*\sqrt{-\left(\frac{\sqrt{c}}{\sqrt{a}}\right)})*d^2*(c*d^2 - a*e^2)*(d + e*x^2)*\sqrt{a - c*x^4})$

Maple [B] time = 0.368, size = 523, normalized size = 1.8

$$\frac{e^2 x}{(2ae^2 - 2cd^2)d(ex^2 + d)}\sqrt{-cx^4 + a} + \frac{c}{2ae^2 - 2cd^2}\sqrt{1 - x^2\sqrt{c}\frac{1}{\sqrt{a}}}\sqrt{1 + x^2\sqrt{c}\frac{1}{\sqrt{a}}}\text{EllipticF}\left(x\sqrt{\sqrt{c}\frac{1}{\sqrt{a}}}, i\right)\frac{1}{\sqrt{\sqrt{c}\frac{1}{\sqrt{a}}}\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(e*x^2+d)^2/(-c*x^4+a)^(1/2),x)`

[Out] $\frac{1}{2}e^2/(ae^2-cd^2)/d*x*(-cx^4+a)^{1/2}/(e^2x^2+d)+\frac{1}{2}c/(ae^2-cd^2)/(1/a^{1/2}*c^{1/2})^{1/2}*(1-1/a^{1/2}*c^{1/2}*x^2)^{1/2}*(1+1/a^{1/2}*c^{1/2}*x^2)^{1/2}/(-cx^4+a)^{1/2}*EllipticF(x*(1/a^{1/2}*c^{1/2})^{1/2},I)-\frac{1}{2}c^{1/2}*e/(ae^2-cd^2)/d*a^{1/2}/(1/a^{1/2}*c^{1/2})^{1/2}*(1-1/a^{1/2}*c^{1/2}*x^2)^{1/2}*(1+1/a^{1/2}*c^{1/2}*x^2)^{1/2}/(-cx^4+a)^{1/2}*EllipticF(x*(1/a^{1/2}*c^{1/2})^{1/2},I)+\frac{1}{2}c^{1/2}*e/(ae^2-cd^2)/d*a^{1/2}/(1/a^{1/2}*c^{1/2})^{1/2}*(1-1/a^{1/2}*c^{1/2}*x^2)^{1/2}*(1+1/a^{1/2}*c^{1/2}*x^2)^{1/2}/(-cx^4+a)^{1/2}*EllipticE(x*(1/a^{1/2}*c^{1/2})^{1/2},I)+\frac{1}{2}/(ae^2-cd^2)/d^2*e^2/(1/a^{1/2}*c^{1/2})^{1/2}*(1-1/a^{1/2}*c^{1/2}*x^2)^{1/2}*(1+1/a^{1/2}*c^{1/2}*x^2)^{1/2}/(-cx^4+a)^{1/2}*EllipticPi(x*(1/a^{1/2}*c^{1/2})^{1/2},-e*a^{1/2}/d/c^{1/2},(-1/a^{1/2}*c^{1/2})^{1/2}/(1/a^{1/2}*c^{1/2})^{1/2})*a-3/2/(ae^2-cd^2)/(1/a^{1/2}*c^{1/2})^{1/2}*(1-1/a^{1/2}*c^{1/2}*x^2)^{1/2}*(1+1/a^{1/2}*c^{1/2}*x^2)^{1/2}/(-cx^4+a)^{1/2}*EllipticPi(x*(1/a^{1/2}*c^{1/2})^{1/2},-e*a^{1/2}/d/c^{1/2},(-1/a^{1/2}*c^{1/2})^{1/2}/(1/a^{1/2}*c^{1/2})^{1/2})*c$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-cx^4 + a}(ex^2 + d)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(e*x^2+d)^2/(-c*x^4+a)^(1/2),x, algorithm="maxima")`

[Out] `integrate(1/(sqrt(-c*x^4 + a)*(e*x^2 + d)^2), x)`

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(e*x^2+d)^2/(-c*x^4+a)^(1/2),x, algorithm="fricas")`

[Out] Timed out

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a - cx^4} (d + ex^2)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(e*x**2+d)**2/(-c*x**4+a)**(1/2),x)`

[Out] Integral(1/(sqrt(a - c*x**4)*(d + e*x**2)**2), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-cx^4 + a}(ex^2 + d)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x^2+d)^2/(-c*x^4+a)^(1/2),x, algorithm="giac")

[Out] integrate(1/(sqrt(-c*x^4 + a)*(e*x^2 + d)^2), x)

$$3.162 \quad \int \frac{1}{(d+ex^2)^3 \sqrt{a-cx^4}} dx$$

Optimal. Leaf size=425

$$\frac{\sqrt[4]{a}\sqrt[4]{c}\sqrt{1-\frac{cx^4}{a}}(-2\sqrt{a}\sqrt{cde}-3ae^2+7cd^2)\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt[4]{cx}}{\sqrt[4]{a}}\right),-1\right)}{8d^2\sqrt{a-cx^4}(\sqrt{ae}+\sqrt{cd})(cd^2-ae^2)} + \frac{3\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}(a^2e^4-2acd^2e^2+5c^2d^4)\Pi}{8\sqrt[4]{cd^3}\sqrt{a-cx^4}(cd^2-ae^2)}$$

```
[Out] -(e^2*x*Sqrt[a - c*x^4])/(4*d*(c*d^2 - a*e^2)*(d + e*x^2)^2) - (3*e^2*(3*c*d^2 - a*e^2)*x*Sqrt[a - c*x^4])/(8*d^2*(c*d^2 - a*e^2)^2*(d + e*x^2)) - (3*a^(3/4)*c^(1/4)*e*(3*c*d^2 - a*e^2)*Sqrt[1 - (c*x^4)/a]*EllipticE[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(8*d^2*(c*d^2 - a*e^2)^2*Sqrt[a - c*x^4]) - (a^(1/4)*c^(1/4)*(7*c*d^2 - 2*Sqrt[a]*Sqrt[c]*d*e - 3*a*e^2)*Sqrt[1 - (c*x^4)/a]*EllipticF[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(8*d^2*(Sqrt[c]*d + Sqrt[a]*e)*(c*d^2 - a*e^2)*Sqrt[a - c*x^4]) + (3*a^(1/4)*(5*c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)*Sqrt[1 - (c*x^4)/a]*EllipticPi[-((Sqrt[a]*e)/(Sqrt[c]*d)), ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(8*c^(1/4)*d^3*(c*d^2 - a*e^2)^2*Sqrt[a - c*x^4])
```

Rubi [A] time = 0.750702, antiderivative size = 425, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 11, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$, Rules used = {1224, 1697, 1717, 1201, 224, 221, 1200, 1199, 424, 1219, 1218}

$$\frac{3\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}(a^2e^4-2acd^2e^2+5c^2d^4)\Pi\left(-\frac{\sqrt{ae}}{\sqrt{cd}}; \sin^{-1}\left(\frac{\sqrt[4]{cx}}{\sqrt[4]{a}}\right)\right)-1}{8\sqrt[4]{cd^3}\sqrt{a-cx^4}(cd^2-ae^2)^2} - \frac{3a^{3/4}\sqrt[4]{ce}\sqrt{1-\frac{cx^4}{a}}(3cd^2-ae^2)E\left(\sin^{-1}\left(\frac{\sqrt[4]{cx}}{\sqrt[4]{a}}\right)\right)}{8d^2\sqrt{a-cx^4}(cd^2-ae^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x^2)^3*Sqrt[a - c*x^4]), x]
```

```
[Out] -(e^2*x*Sqrt[a - c*x^4])/(4*d*(c*d^2 - a*e^2)*(d + e*x^2)^2) - (3*e^2*(3*c*d^2 - a*e^2)*x*Sqrt[a - c*x^4])/(8*d^2*(c*d^2 - a*e^2)^2*(d + e*x^2)) - (3*a^(3/4)*c^(1/4)*e*(3*c*d^2 - a*e^2)*Sqrt[1 - (c*x^4)/a]*EllipticE[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(8*d^2*(c*d^2 - a*e^2)^2*Sqrt[a - c*x^4]) - (a^(1/4)*c^(1/4)*(7*c*d^2 - 2*Sqrt[a]*Sqrt[c]*d*e - 3*a*e^2)*Sqrt[1 - (c*x^4)/a]*EllipticF[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(8*d^2*(Sqrt[c]*d + Sqrt[a]*e)*(c*d^2 - a*e^2)*Sqrt[a - c*x^4]) + (3*a^(1/4)*(5*c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)*Sqrt[1 - (c*x^4)/a]*EllipticPi[-((Sqrt[a]*e)/(Sqrt[c]*d)), ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(8*c^(1/4)*d^3*(c*d^2 - a*e^2)^2*Sqrt[a - c*x^4])
```

Rule 1224

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] := -Simp[(e^2*x*(d + e*x^2)^(q + 1)*Sqrt[a + c*x^4])/(2*d*(q + 1)*(c*d^2 + a*e^2)), x] + Dist[1/(2*d*(q + 1)*(c*d^2 + a*e^2)), Int[((d + e*x^2)^(q + 1)*Simp[a*e^2*(2*q + 3) + 2*c*d^2*(q + 1) - 2*e*c*d*(q + 1)*x^2 + c*e^2*(2*q + 5)*x^4, x])/Sqrt[a + c*x^4], x], x] /; FreeQ[{a, c, d, e}, x] && ILtQ[q, -1]
```

Rule 1697

```
Int[((P4x_)*((d_) + (e_.)*(x_)^2)^(q_))/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] := With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[P4x, x, 4]}
```

```

}, -Simp[((C*d^2 - B*d*e + A*e^2)*x*(d + e*x^2)^(q + 1)*Sqrt[a + c*x^4])/(2
*d*(q + 1)*(c*d^2 + a*e^2)), x] + Dist[1/(2*d*(q + 1)*(c*d^2 + a*e^2)), Int
[((d + e*x^2)^(q + 1)*Simp[a*d*(C*d - B*e) + A*(a*e^2*(2*q + 3) + 2*c*d^2*(
q + 1)) + 2*d*(B*c*d - A*c*e + a*C*e)*(q + 1)*x^2 + c*(C*d^2 - B*d*e + A*e^
2)*(2*q + 5)*x^4, x])/Sqrt[a + c*x^4], x], x] /; FreeQ[{a, c, d, e}, x] &&
PolyQ[P4x, x^2] && LeQ[Expon[P4x, x], 4] && NeQ[c*d^2 + a*e^2, 0] && ILtQ[
q, -1]

```

Rule 1717

```

Int[(P4x_)/(((d_) + (e_)*(x_)^2)*Sqrt[(a_) + (c_)*(x_)^4]), x_Symbol] :=
With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[P4x, x, 4]}, -D
ist[(e^2)^(-1), Int[(C*d - B*e - C*e*x^2)/Sqrt[a + c*x^4], x], x] + Dist[(C
*d^2 - B*d*e + A*e^2)/e^2, Int[1/((d + e*x^2)*Sqrt[a + c*x^4]), x], x] /;
FreeQ[{a, c, d, e}, x] && PolyQ[P4x, x^2, 2] && NeQ[c*d^2 + a*e^2, 0] && Ne
Q[c*d^2 - a*e^2, 0]

```

Rule 1201

```

Int[((d_) + (e_)*(x_)^2)/Sqrt[(a_) + (c_)*(x_)^4], x_Symbol] := With[{q =
Rt[-(c/a), 2]}, Dist[(d*q - e)/q, Int[1/Sqrt[a + c*x^4], x], x] + Dist[e/q
, Int[(1 + q*x^2)/Sqrt[a + c*x^4], x], x]] /; FreeQ[{a, c, d, e}, x] && Neg
Q[c/a] && NeQ[c*d^2 + a*e^2, 0]

```

Rule 224

```

Int[1/Sqrt[(a_) + (b_)*(x_)^4], x_Symbol] := Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && !GtQ[a, 0]

```

Rule 221

```

Int[1/Sqrt[(a_) + (b_)*(x_)^4], x_Symbol] := Simp[EllipticF[ArcSin[(Rt[-b,
4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]

```

Rule 1200

```

Int[((d_) + (e_)*(x_)^2)/Sqrt[(a_) + (c_)*(x_)^4], x_Symbol] := Dist[Sqrt
[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + (c*x^4)/a], x], x
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && !GtQ[
a, 0]

```

Rule 1199

```

Int[((d_) + (e_)*(x_)^2)/Sqrt[(a_) + (c_)*(x_)^4], x_Symbol] := Dist[d/Sq
rt[a], Int[Sqrt[1 + (e*x^2)/d]/Sqrt[1 - (e*x^2)/d], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]

```

Rule 424

```

Int[Sqrt[(a_) + (b_)*(x_)^2]/Sqrt[(c_) + (d_)*(x_)^2], x_Symbol] := Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]

```

Rule 1219

```

Int[1/(((d_) + (e_)*(x_)^2)*Sqrt[(a_) + (c_)*(x_)^4]), x_Symbol] := Dist[
Sqrt[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + (c*x^4)/a]

```

), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && !GtQ[a, 0]

Rule 1218

Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] := With[{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]

Rubi steps

$$\begin{aligned} \int \frac{1}{(d+ex^2)^3 \sqrt{a-cx^4}} dx &= -\frac{e^2 x \sqrt{a-cx^4}}{4d(cd^2-ae^2)(d+ex^2)^2} + \frac{\int \frac{4cd^2-3ae^2-4cdex^2+ce^2x^4}{(d+ex^2)^2 \sqrt{a-cx^4}} dx}{4d(cd^2-ae^2)} \\ &= -\frac{e^2 x \sqrt{a-cx^4}}{4d(cd^2-ae^2)(d+ex^2)^2} - \frac{3e^2(3cd^2-ae^2)x\sqrt{a-cx^4}}{8d^2(cd^2-ae^2)^2(d+ex^2)} + \frac{\int \frac{8c^2d^4-5acd^2e^2+3a^2e^4-4cde(4cd^2-ae^2)}{(d+ex^2)\sqrt{a-cx^4}} dx}{8d^2(cd^2-ae^2)^2} \\ &= -\frac{e^2 x \sqrt{a-cx^4}}{4d(cd^2-ae^2)(d+ex^2)^2} - \frac{3e^2(3cd^2-ae^2)x\sqrt{a-cx^4}}{8d^2(cd^2-ae^2)^2(d+ex^2)} - \frac{\int \frac{-3cde^2(3cd^2-ae^2)+4cde^2(4cd^2-ae^2)}{\sqrt{a-cx^4}} dx}{8d^2e^2(cd^2-ae^2)} \\ &= -\frac{e^2 x \sqrt{a-cx^4}}{4d(cd^2-ae^2)(d+ex^2)^2} - \frac{3e^2(3cd^2-ae^2)x\sqrt{a-cx^4}}{8d^2(cd^2-ae^2)^2(d+ex^2)} - \frac{(\sqrt{c}(\sqrt{cd}-\sqrt{ae}))(7cd^2-2ae^2)}{8d^2(cd^2-ae^2)^2} \\ &= -\frac{e^2 x \sqrt{a-cx^4}}{4d(cd^2-ae^2)(d+ex^2)^2} - \frac{3e^2(3cd^2-ae^2)x\sqrt{a-cx^4}}{8d^2(cd^2-ae^2)^2(d+ex^2)} + \frac{3\sqrt[4]{a}(5c^2d^4-2acd^2e^2+a^2e^4)}{8\sqrt[4]{cd^3}(cd^2-ae^2)^2} \\ &= -\frac{e^2 x \sqrt{a-cx^4}}{4d(cd^2-ae^2)(d+ex^2)^2} - \frac{3e^2(3cd^2-ae^2)x\sqrt{a-cx^4}}{8d^2(cd^2-ae^2)^2(d+ex^2)} - \frac{\sqrt[4]{a}\sqrt[4]{c}(\sqrt{cd}-\sqrt{ae})(7cd^2-2ae^2)}{8d^2(cd^2-ae^2)^2} \\ &= -\frac{e^2 x \sqrt{a-cx^4}}{4d(cd^2-ae^2)(d+ex^2)^2} - \frac{3e^2(3cd^2-ae^2)x\sqrt{a-cx^4}}{8d^2(cd^2-ae^2)^2(d+ex^2)} - \frac{3a^{3/4}\sqrt[4]{ce}(3cd^2-ae^2)\sqrt{1-\frac{c}{a}}}{8d^2(cd^2-ae^2)^2} \end{aligned}$$

Mathematica [C] time = 1.27414, size = 321, normalized size = 0.76

$$\frac{de^2x(a-cx^4)(ae^2(5d+3ex^2)-cd^2(11d+9ex^2))}{(d+ex^2)^2} - \frac{i\sqrt{1-\frac{cx^4}{a}}\left((-3a^{3/2}\sqrt{cde^3+9\sqrt{ac}^{3/2}d^3e+acd^2e^2-7c^2d^4})\text{EllipticF}\left(i\sinh^{-1}\left(x\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\right), -1\right)+3(a^2e^4-2acd^2e^2+5c^2d^4)\right)}{\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}}$$

$$8d^3\sqrt{a-cx^4}(cd^2-ae^2)^2$$

Antiderivative was successfully verified.

[In] Integrate[1/((d + e*x^2)^3*Sqrt[a - c*x^4]), x]

[Out] ((d*e^2*x*(a - c*x^4)*(a*e^2*(5*d + 3*e*x^2) - c*d^2*(11*d + 9*e*x^2)))/(d + e*x^2)^2 - (I*Sqrt[1 - (c*x^4)/a]*(3*Sqrt[a]*Sqrt[c]*d*e*(-3*c*d^2 + a*e^2)*EllipticE[I*ArcSinh[Sqrt[-(Sqrt[c]/Sqrt[a])]*x], -1] + (-7*c^2*d^4 + 9*Sqrt[a]*c^(3/2)*d^3*e + a*c*d^2*e^2 - 3*a^(3/2)*Sqrt[c]*d*e^3)*EllipticF[I*a

```
rcSinh[Sqrt[-(Sqrt[c]/Sqrt[a])*x], -1] + 3*(5*c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)*EllipticPi[-((Sqrt[a]*e)/(Sqrt[c]*d)), I*ArcSinh[Sqrt[-(Sqrt[c]/Sqrt[a])*x], -1])/Sqrt[-(Sqrt[c]/Sqrt[a])]/(8*d^3*(c*d^2 - a*e^2)^2*Sqrt[a - c*x^4])
```

Maple [B] time = 0.286, size = 961, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d)^3/(-c*x^4+a)^(1/2),x)
```

```
[Out] 1/4*e^2/(a*e^2-c*d^2)/d*x*(-c*x^4+a)^(1/2)/(e*x^2+d)^2+3/8*e^2*(a*e^2-3*c*d^2)/(a*e^2-c*d^2)^2/d^2*x*(-c*x^4+a)^(1/2)/(e*x^2+d)+1/8*c/d/(a*e^2-c*d^2)^2/(1/a^(1/2)*c^(1/2))^(1/2)*(1-1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+1/a^(1/2)*c^(1/2)*x^2)^(1/2)/(-c*x^4+a)^(1/2)*EllipticF(x*(1/a^(1/2)*c^(1/2))^(1/2),I)*a*e^2-7/8*c^2*d/(a*e^2-c*d^2)^2/(1/a^(1/2)*c^(1/2))^(1/2)*(1-1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+1/a^(1/2)*c^(1/2)*x^2)^(1/2)/(-c*x^4+a)^(1/2)*EllipticF(x*(1/a^(1/2)*c^(1/2))^(1/2),I)-3/8*c^(1/2)*e^3/(a*e^2-c*d^2)^2/d^2*a^(3/2)/(1/a^(1/2)*c^(1/2))^(1/2)*(1-1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+1/a^(1/2)*c^(1/2)*x^2)^(1/2)/(-c*x^4+a)^(1/2)*EllipticE(x*(1/a^(1/2)*c^(1/2))^(1/2),I)+9/8*c^(3/2)*e/(a*e^2-c*d^2)^2*a^(1/2)/(1/a^(1/2)*c^(1/2))^(1/2)*(1-1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+1/a^(1/2)*c^(1/2)*x^2)^(1/2)/(-c*x^4+a)^(1/2)*EllipticF(x*(1/a^(1/2)*c^(1/2))^(1/2),I)+3/8*c^(1/2)*e^3/(a*e^2-c*d^2)^2/d^2*a^(3/2)/(1/a^(1/2)*c^(1/2))^(1/2)*(1-1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+1/a^(1/2)*c^(1/2)*x^2)^(1/2)/(-c*x^4+a)^(1/2)*EllipticE(x*(1/a^(1/2)*c^(1/2))^(1/2),I)-9/8*c^(3/2)*e/(a*e^2-c*d^2)^2*a^(1/2)/(1/a^(1/2)*c^(1/2))^(1/2)*(1-1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+1/a^(1/2)*c^(1/2)*x^2)^(1/2)/(-c*x^4+a)^(1/2)*EllipticE(x*(1/a^(1/2)*c^(1/2))^(1/2),I)+3/8/(a*e^2-c*d^2)^2/d^3*e^4/(1/a^(1/2)*c^(1/2))^(1/2)*(1-1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+1/a^(1/2)*c^(1/2)*x^2)^(1/2)/(-c*x^4+a)^(1/2)*EllipticPi(x*(1/a^(1/2)*c^(1/2))^(1/2),-e*a^(1/2)/d/c^(1/2),(-1/a^(1/2)*c^(1/2))^(1/2)/(1/a^(1/2)*c^(1/2))^(1/2))*a^2-3/4/(a*e^2-c*d^2)^2/d*e^2/(1/a^(1/2)*c^(1/2))^(1/2)*(1-1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+1/a^(1/2)*c^(1/2)*x^2)^(1/2)/(-c*x^4+a)^(1/2)*EllipticPi(x*(1/a^(1/2)*c^(1/2))^(1/2),-e*a^(1/2)/d/c^(1/2),(-1/a^(1/2)*c^(1/2))^(1/2)/(1/a^(1/2)*c^(1/2))^(1/2))*a*c+15/8/(a*e^2-c*d^2)^2*d/(1/a^(1/2)*c^(1/2))^(1/2)*(1-1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+1/a^(1/2)*c^(1/2)*x^2)^(1/2)/(-c*x^4+a)^(1/2)*EllipticPi(x*(1/a^(1/2)*c^(1/2))^(1/2),-e*a^(1/2)/d/c^(1/2),(-1/a^(1/2)*c^(1/2))^(1/2)/(1/a^(1/2)*c^(1/2))^(1/2))*c^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-cx^4 + a}(ex^2 + d)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^3/(-c*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-c*x^4 + a)*(e*x^2 + d)^3), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x^2+d)^3/(-c*x^4+a)^(1/2),x, algorithm="fricas")

[Out] Timed out

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a - cx^4} (d + ex^2)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x**2+d)**3/(-c*x**4+a)**(1/2),x)

[Out] Integral(1/(sqrt(a - c*x**4)*(d + e*x**2)**3), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-cx^4 + a} (ex^2 + d)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x^2+d)^3/(-c*x^4+a)^(1/2),x, algorithm="giac")

[Out] integrate(1/(sqrt(-c*x^4 + a)*(e*x^2 + d)^3), x)

$$3.163 \quad \int \frac{1}{(d+ex^2)^4 \sqrt{a-cx^4}} dx$$

Optimal. Leaf size=563

$$\frac{\sqrt[4]{a}\sqrt[4]{c}\sqrt{1-\frac{cx^4}{a}}(10a^{3/2}\sqrt{cde^3+15a^2e^4-30\sqrt{a}c^{3/2}d^3e-32acd^2e^2+57c^2d^4)}\operatorname{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt[4]{cx}}{\sqrt[4]{a}}\right),-1\right)}{48d^3\sqrt{a-cx^4}(\sqrt{cd}-\sqrt{ae})^2(\sqrt{ae}+\sqrt{cd})^3} - \frac{e^2x\sqrt{a-cx^4}}{16d^3}$$

[Out] $-(e^2*x*\operatorname{Sqrt}[a-c*x^4])/(6*d*(c*d^2-a*e^2)*(d+e*x^2)^3)-(5*e^2*(3*c*d^2-a*e^2)*x*\operatorname{Sqrt}[a-c*x^4])/(24*d^2*(c*d^2-a*e^2)^2*(d+e*x^2)^2)-(e^2*(29*c^2*d^4-14*a*c*d^2*e^2+5*a^2*e^4)*x*\operatorname{Sqrt}[a-c*x^4])/(16*d^3*(c*d^2-a*e^2)^3*(d+e*x^2))-(a^{3/4}*c^{1/4}*e*(29*c^2*d^4-14*a*c*d^2*e^2+5*a^2*e^4)*\operatorname{Sqrt}[1-(c*x^4)/a]*\operatorname{EllipticE}[\operatorname{ArcSin}[(c^{1/4}*x)/a^{1/4}],-1])/(16*d^3*(c*d^2-a*e^2)^3*\operatorname{Sqrt}[a-c*x^4])-(a^{1/4}*c^{1/4}*(57*c^2*d^4-30*\operatorname{Sqrt}[a]*c^{3/2}*d^3*e-32*a*c*d^2*e^2+10*a^{3/2}*\operatorname{Sqrt}[c]*d*e^3+15*a^2*e^4)*\operatorname{Sqrt}[1-(c*x^4)/a]*\operatorname{EllipticF}[\operatorname{ArcSin}[(c^{1/4}*x)/a^{1/4}],-1])/(48*d^3*(\operatorname{Sqrt}[c]*d-\operatorname{Sqrt}[a]*e)^2*(\operatorname{Sqrt}[c]*d+\operatorname{Sqrt}[a]*e)^3*\operatorname{Sqrt}[a-c*x^4])+(a^{1/4}*(35*c^3*d^6-7*a*c^2*d^4*e^2+17*a^2*c*d^2*e^4-5*a^3*e^6)*\operatorname{Sqrt}[1-(c*x^4)/a]*\operatorname{EllipticPi}[-((\operatorname{Sqrt}[a]*e)/(\operatorname{Sqrt}[c]*d)),\operatorname{ArcSin}[(c^{1/4}*x)/a^{1/4}],-1])/(16*c^{1/4}*d^4*(c*d^2-a*e^2)^3*\operatorname{Sqrt}[a-c*x^4])$

Rubi [A] time = 1.20551, antiderivative size = 563, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 11, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$, Rules used = {1224, 1697, 1717, 1201, 224, 221, 1200, 1199, 424, 1219, 1218}

$$\frac{e^2x\sqrt{a-cx^4}(5a^2e^4-14acd^2e^2+29c^2d^4)}{16d^3(d+ex^2)(cd^2-ae^2)^3} - \frac{\sqrt[4]{a}\sqrt[4]{c}\sqrt{1-\frac{cx^4}{a}}(10a^{3/2}\sqrt{cde^3+15a^2e^4-30\sqrt{a}c^{3/2}d^3e-32acd^2e^2+57c^2d^4)}{48d^3\sqrt{a-cx^4}(\sqrt{cd}-\sqrt{ae})^2(\sqrt{ae}+\sqrt{cd})^3}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[1/((d+e*x^2)^4*\operatorname{Sqrt}[a-c*x^4]),x]$

[Out] $-(e^2*x*\operatorname{Sqrt}[a-c*x^4])/(6*d*(c*d^2-a*e^2)*(d+e*x^2)^3)-(5*e^2*(3*c*d^2-a*e^2)*x*\operatorname{Sqrt}[a-c*x^4])/(24*d^2*(c*d^2-a*e^2)^2*(d+e*x^2)^2)-(e^2*(29*c^2*d^4-14*a*c*d^2*e^2+5*a^2*e^4)*x*\operatorname{Sqrt}[a-c*x^4])/(16*d^3*(c*d^2-a*e^2)^3*(d+e*x^2))-(a^{3/4}*c^{1/4}*e*(29*c^2*d^4-14*a*c*d^2*e^2+5*a^2*e^4)*\operatorname{Sqrt}[1-(c*x^4)/a]*\operatorname{EllipticE}[\operatorname{ArcSin}[(c^{1/4}*x)/a^{1/4}],-1])/(16*d^3*(c*d^2-a*e^2)^3*\operatorname{Sqrt}[a-c*x^4])-(a^{1/4}*c^{1/4}*(57*c^2*d^4-30*\operatorname{Sqrt}[a]*c^{3/2}*d^3*e-32*a*c*d^2*e^2+10*a^{3/2}*\operatorname{Sqrt}[c]*d*e^3+15*a^2*e^4)*\operatorname{Sqrt}[1-(c*x^4)/a]*\operatorname{EllipticF}[\operatorname{ArcSin}[(c^{1/4}*x)/a^{1/4}],-1])/(48*d^3*(\operatorname{Sqrt}[c]*d-\operatorname{Sqrt}[a]*e)^2*(\operatorname{Sqrt}[c]*d+\operatorname{Sqrt}[a]*e)^3*\operatorname{Sqrt}[a-c*x^4])+(a^{1/4}*(35*c^3*d^6-7*a*c^2*d^4*e^2+17*a^2*c*d^2*e^4-5*a^3*e^6)*\operatorname{Sqrt}[1-(c*x^4)/a]*\operatorname{EllipticPi}[-((\operatorname{Sqrt}[a]*e)/(\operatorname{Sqrt}[c]*d)),\operatorname{ArcSin}[(c^{1/4}*x)/a^{1/4}],-1])/(16*c^{1/4}*d^4*(c*d^2-a*e^2)^3*\operatorname{Sqrt}[a-c*x^4])$

Rule 1224

$\operatorname{Int}[(d_+ + (e_+)*(x_+)^2)^{(q_+)}/\operatorname{Sqrt}[(a_+ + (c_+)*(x_+)^4], x_Symbol] :> -\operatorname{Simp}[(e^2*x*(d+e*x^2)^{(q+1)}*\operatorname{Sqrt}[a+c*x^4])/(2*d*(q+1)*(c*d^2+a*e^2)), x] + \operatorname{Dist}[1/(2*d*(q+1)*(c*d^2+a*e^2)), \operatorname{Int}[(d+e*x^2)^{(q+1)}*\operatorname{Simp}[a*e^2*(2*q+3)+2*c*d^2*(q+1)-2*e*c*d*(q+1)*x^2+c*e^2*(2*q+5)*x^4, x]]/\operatorname{Sqrt}[a+c*x^4], x] /; \operatorname{FreeQ}\{a, c, d, e\}, x \&\& \operatorname{ILtQ}[q, -1]$

Rule 1697

```
Int[((P4x_)*((d_) + (e_)*(x_)^2)^(q_))/Sqrt[(a_) + (c_)*(x_)^4], x_Symbol]
:> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[P4x, x, 4]
}, -Simp[((C*d^2 - B*d*e + A*e^2)*x*(d + e*x^2)^(q + 1)*Sqrt[a + c*x^4])/(2
*d*(q + 1)*(c*d^2 + a*e^2)), x] + Dist[1/(2*d*(q + 1)*(c*d^2 + a*e^2)), Int
[((d + e*x^2)^(q + 1)*Simp[a*d*(C*d - B*e) + A*(a*e^2*(2*q + 3) + 2*c*d^2*(
q + 1)) + 2*d*(B*c*d - A*c*e + a*C*e)*(q + 1)*x^2 + c*(C*d^2 - B*d*e + A*e^
2)*(2*q + 5)*x^4, x])/Sqrt[a + c*x^4], x], x]] /; FreeQ[{a, c, d, e}, x] &&
PolyQ[P4x, x^2] && LeQ[Expon[P4x, x], 4] && NeQ[c*d^2 + a*e^2, 0] && ILtQ[
q, -1]
```

Rule 1717

```
Int[(P4x_)/(((d_) + (e_)*(x_)^2)*Sqrt[(a_) + (c_)*(x_)^4]), x_Symbol] :>
With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[P4x, x, 4]}, -D
ist[(e^2)^(-1), Int[(C*d - B*e - C*e*x^2)/Sqrt[a + c*x^4], x], x] + Dist[(C
*d^2 - B*d*e + A*e^2)/e^2, Int[1/((d + e*x^2)*Sqrt[a + c*x^4]), x], x]] /;
FreeQ[{a, c, d, e}, x] && PolyQ[P4x, x^2, 2] && NeQ[c*d^2 + a*e^2, 0] && Ne
Q[c*d^2 - a*e^2, 0]
```

Rule 1201

```
Int[((d_) + (e_)*(x_)^2)/Sqrt[(a_) + (c_)*(x_)^4], x_Symbol] :> With[{q =
Rt[-(c/a), 2]}, Dist[(d*q - e)/q, Int[1/Sqrt[a + c*x^4], x], x] + Dist[e/q
, Int[(1 + q*x^2)/Sqrt[a + c*x^4], x], x]] /; FreeQ[{a, c, d, e}, x] && Neg
Q[c/a] && NeQ[c*d^2 + a*e^2, 0]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && !GtQ[a, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 1200

```
Int[((d_) + (e_)*(x_)^2)/Sqrt[(a_) + (c_)*(x_)^4], x_Symbol] :> Dist[Sqrt
[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + (c*x^4)/a], x], x
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && !GtQ[
a, 0]
```

Rule 1199

```
Int[((d_) + (e_)*(x_)^2)/Sqrt[(a_) + (c_)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + (e*x^2)/d]/Sqrt[1 - (e*x^2)/d], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_)*(x_)^2]/Sqrt[(c_) + (d_)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 1219

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] := Dist[
Sqrt[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + (c*x^4)/a]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && !GtQ[a, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] := With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{(d+ex^2)^4 \sqrt{a-cx^4}} dx &= -\frac{e^2 x \sqrt{a-cx^4}}{6d(cd^2-ae^2)(d+ex^2)^3} + \frac{\int \frac{6cd^2-5ae^2-6cdex^2+3ce^2x^4}{(d+ex^2)^3 \sqrt{a-cx^4}} dx}{6d(cd^2-ae^2)} \\ &= -\frac{e^2 x \sqrt{a-cx^4}}{6d(cd^2-ae^2)(d+ex^2)^3} - \frac{5e^2(3cd^2-ae^2)x\sqrt{a-cx^4}}{24d^2(cd^2-ae^2)^2(d+ex^2)^2} + \frac{\int \frac{24c^2d^4-29acd^2e^2+15a^2e^4-8cde(6cd^2-ae^2)}{(d+ex^2)^2 \sqrt{a-cx^4}} dx}{24d^2(cd^2-ae^2)} \\ &= -\frac{e^2 x \sqrt{a-cx^4}}{6d(cd^2-ae^2)(d+ex^2)^3} - \frac{5e^2(3cd^2-ae^2)x\sqrt{a-cx^4}}{24d^2(cd^2-ae^2)^2(d+ex^2)^2} - \frac{e^2(29c^2d^4-14acd^2e^2+5a^2e^4)}{16d^3(cd^2-ae^2)^3(d+ex^2)} \\ &= -\frac{e^2 x \sqrt{a-cx^4}}{6d(cd^2-ae^2)(d+ex^2)^3} - \frac{5e^2(3cd^2-ae^2)x\sqrt{a-cx^4}}{24d^2(cd^2-ae^2)^2(d+ex^2)^2} - \frac{e^2(29c^2d^4-14acd^2e^2+5a^2e^4)}{16d^3(cd^2-ae^2)^3(d+ex^2)} \\ &= -\frac{e^2 x \sqrt{a-cx^4}}{6d(cd^2-ae^2)(d+ex^2)^3} - \frac{5e^2(3cd^2-ae^2)x\sqrt{a-cx^4}}{24d^2(cd^2-ae^2)^2(d+ex^2)^2} - \frac{e^2(29c^2d^4-14acd^2e^2+5a^2e^4)}{16d^3(cd^2-ae^2)^3(d+ex^2)} \\ &= -\frac{e^2 x \sqrt{a-cx^4}}{6d(cd^2-ae^2)(d+ex^2)^3} - \frac{5e^2(3cd^2-ae^2)x\sqrt{a-cx^4}}{24d^2(cd^2-ae^2)^2(d+ex^2)^2} - \frac{e^2(29c^2d^4-14acd^2e^2+5a^2e^4)}{16d^3(cd^2-ae^2)^3(d+ex^2)} \\ &= -\frac{e^2 x \sqrt{a-cx^4}}{6d(cd^2-ae^2)(d+ex^2)^3} - \frac{5e^2(3cd^2-ae^2)x\sqrt{a-cx^4}}{24d^2(cd^2-ae^2)^2(d+ex^2)^2} - \frac{e^2(29c^2d^4-14acd^2e^2+5a^2e^4)}{16d^3(cd^2-ae^2)^3(d+ex^2)} \\ &= -\frac{e^2 x \sqrt{a-cx^4}}{6d(cd^2-ae^2)(d+ex^2)^3} - \frac{5e^2(3cd^2-ae^2)x\sqrt{a-cx^4}}{24d^2(cd^2-ae^2)^2(d+ex^2)^2} - \frac{e^2(29c^2d^4-14acd^2e^2+5a^2e^4)}{16d^3(cd^2-ae^2)^3(d+ex^2)} \\ &= -\frac{e^2 x \sqrt{a-cx^4}}{6d(cd^2-ae^2)(d+ex^2)^3} - \frac{5e^2(3cd^2-ae^2)x\sqrt{a-cx^4}}{24d^2(cd^2-ae^2)^2(d+ex^2)^2} - \frac{e^2(29c^2d^4-14acd^2e^2+5a^2e^4)}{16d^3(cd^2-ae^2)^3(d+ex^2)} \end{aligned}$$

Mathematica [C] time = 1.99735, size = 458, normalized size = 0.81

$$\frac{de^2x(a-cx^4)\left(3(d+ex^2)^2(5a^2e^4-14acd^2e^2+29c^2d^4)+10d(d+ex^2)(cd^2-ae^2)(3cd^2-ae^2)+8(cd^3-ade^2)^2\right)}{(d+ex^2)^3(cd^2-ae^2)^3} - \frac{i\sqrt{1-\frac{cx^4}{a}}\left(\sqrt{cd}\left(42a^{3/2}cd^2e^3+5a^2\sqrt{cde^4}-15a^{5/2}e^5-2ac^3\right)\right)}{(d+ex^2)^3(cd^2-ae^2)^3}$$

Antiderivative was successfully verified.

[In] Integrate[1/((d + e*x^2)^4*Sqrt[a - c*x^4]),x]

[Out]
$$\frac{-((d^2 e^2 x^2 (a - c x^4) (8 (c d^3 - a d e^2)^2 + 10 d (c d^2 - a e^2) (3 c d^2 - a e^2) (d + e x^2) + 3 (29 c^2 d^4 - 14 a c d^2 e^2 + 5 a^2 e^4) (d + e x^2)^2)) / ((c d^2 - a e^2)^3 (d + e x^2)^3) - (I \sqrt{1 - (c x^4)/a}) (3 \sqrt{a} \sqrt{c} d e (29 c^2 d^4 - 14 a c d^2 e^2 + 5 a^2 e^4) \text{EllipticE}[I \text{ArcSinh}[\sqrt{-(\sqrt{c}/\sqrt{a})}] x], -1) + \sqrt{c} d (57 c^{5/2} d^5 - 87 \sqrt{a} c^2 d^4 e - 2 a c^{3/2} d^3 e^2 + 42 a^{3/2} c d^2 e^3 + 5 a^2 \sqrt{c} d e^4 - 15 a^{5/2} e^5) \text{EllipticF}[I \text{ArcSinh}[\sqrt{-(\sqrt{c}/\sqrt{a})}] x], -1) + 3 (-35 c^3 d^6 + 7 a c^2 d^4 e^2 - 17 a^2 c d^2 e^4 + 5 a^3 e^6) \text{EllipticPi}[-((\sqrt{a} e)/(\sqrt{c} d)), I \text{ArcSinh}[\sqrt{-(\sqrt{c}/\sqrt{a})}] x], -1)}{(\sqrt{-(\sqrt{c}/\sqrt{a})}) (-c d^2 + a e^2)^3 (48 d^4 \sqrt{a - c x^4})}$$

Maple [B] time = 0.195, size = 1420, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(e*x^2+d)^4/(-c*x^4+a)^(1/2),x)

[Out]
$$\frac{1}{6} e^2 (a e^2 - c d^2) / d x (-c x^4 + a)^{1/2} / (e x^2 + d)^3 + 5/24 e^2 (a e^2 - 3 c d^2) / (a e^2 - c d^2)^2 / d^2 x (-c x^4 + a)^{1/2} / (e x^2 + d)^2 + 1/16 e^2 (5 a^2 e^4 - 14 a c d^2 e^2 + 29 c^2 d^4) / (a e^2 - c d^2)^3 / d^3 x (-c x^4 + a)^{1/2} / (e x^2 + d) - 7/8 c^{3/2} e^3 / (a e^2 - c d^2)^3 / d a^{3/2} / (1/a^{1/2} c^{1/2})^{1/2} * (1 - 1/a^{1/2} c^{1/2} x^2)^{1/2} * (1 + 1/a^{1/2} c^{1/2} x^2)^{1/2} / (-c x^4 + a)^{1/2} * \text{EllipticE}(x * (1/a^{1/2} c^{1/2})^{1/2}, I) + 29/16 c^{5/2} e / (a e^2 - c d^2)^3 d a^{1/2} / (1/a^{1/2} c^{1/2})^{1/2} * (1 - 1/a^{1/2} c^{1/2} x^2)^{1/2} * (1 + 1/a^{1/2} c^{1/2} x^2)^{1/2} / (-c x^4 + a)^{1/2} * \text{EllipticE}(x * (1/a^{1/2} c^{1/2})^{1/2}, I) + 5/48 c/d^2 / (a e^2 - c d^2)^3 / (1/a^{1/2} c^{1/2})^{1/2} * (1 - 1/a^{1/2} c^{1/2} x^2)^{1/2} * (1 + 1/a^{1/2} c^{1/2} x^2)^{1/2} / (-c x^4 + a)^{1/2} * \text{EllipticF}(x * (1/a^{1/2} c^{1/2})^{1/2}, I) * a^2 e^4 - 1/24 c^2 / (a e^2 - c d^2)^3 / (1/a^{1/2} c^{1/2})^{1/2} * (1 - 1/a^{1/2} c^{1/2} x^2)^{1/2} * (1 + 1/a^{1/2} c^{1/2} x^2)^{1/2} / (-c x^4 + a)^{1/2} * \text{EllipticF}(x * (1/a^{1/2} c^{1/2})^{1/2}, I) * a e^2 + 5/16 / (a e^2 - c d^2)^3 / d^4 e^6 / (1/a^{1/2} c^{1/2})^{1/2} * (1 - 1/a^{1/2} c^{1/2} x^2)^{1/2} * (1 + 1/a^{1/2} c^{1/2} x^2)^{1/2} / (-c x^4 + a)^{1/2} * \text{EllipticPi}(x * (1/a^{1/2} c^{1/2})^{1/2}, -e a^{1/2} / d c^{1/2}, (-1/a^{1/2} c^{1/2})^{1/2} / (1/a^{1/2} c^{1/2})^{1/2}) * a^3 + 7/16 / (a e^2 - c d^2)^3 e^2 / (1/a^{1/2} c^{1/2})^{1/2} * (1 - 1/a^{1/2} c^{1/2} x^2)^{1/2} * (1 + 1/a^{1/2} c^{1/2} x^2)^{1/2} / (-c x^4 + a)^{1/2} * \text{EllipticPi}(x * (1/a^{1/2} c^{1/2})^{1/2}, -e a^{1/2} / d c^{1/2}, (-1/a^{1/2} c^{1/2})^{1/2} / (1/a^{1/2} c^{1/2})^{1/2}) * a c^2 - 35/16 / (a e^2 - c d^2)^3 d^2 / (1/a^{1/2} c^{1/2})^{1/2} * (1 - 1/a^{1/2} c^{1/2} x^2)^{1/2} * (1 + 1/a^{1/2} c^{1/2} x^2)^{1/2} / (-c x^4 + a)^{1/2} * \text{EllipticPi}(x * (1/a^{1/2} c^{1/2})^{1/2}, -e a^{1/2} / d c^{1/2}, (-1/a^{1/2} c^{1/2})^{1/2} / (1/a^{1/2} c^{1/2})^{1/2}) * c^3 + 19/16 c^3 d^2 / (a e^2 - c d^2)^3 / (1/a^{1/2} c^{1/2})^{1/2} * (1 - 1/a^{1/2} c^{1/2} x^2)^{1/2} * (1 + 1/a^{1/2} c^{1/2} x^2)^{1/2} / (-c x^4 + a)^{1/2} * \text{EllipticF}(x * (1/a^{1/2} c^{1/2})^{1/2}, I) - 5/16 c^{1/2} e^5 / (a e^2 - c d^2)^3 / d^3 a^{5/2} / (1/a^{1/2} c^{1/2})^{1/2} * (1 - 1/a^{1/2} c^{1/2} x^2)^{1/2} * (1 + 1/a^{1/2} c^{1/2} x^2)^{1/2} / (-c x^4 + a)^{1/2} * \text{EllipticF}(x * (1/a^{1/2} c^{1/2})^{1/2}, I) + 7/8 c^{3/2} e^3 / (a e^2 - c d^2)^3 / d a^{3/2} / (1/a^{1/2} c^{1/2})^{1/2} * (1 - 1/a^{1/2} c^{1/2} x^2)^{1/2} * (1 + 1/a^{1/2} c^{1/2} x^2)^{1/2} / (-c x^4 + a)^{1/2} * \text{EllipticF}(x * (1/a^{1/2} c^{1/2})^{1/2}, I) - 29/16 c^{5/2} e / (a e^2 - c d^2)^3 d a^{1/2} / (1/a^{1/2} c^{1/2})^{1/2} * (1 - 1/a^{1/2} c^{1/2} x^2)^{1/2} * (1 + 1/a^{1/2} c^{1/2} x^2)^{1/2} / (-c x^4 + a)^{1/2} * \text{EllipticF}(x * (1/a^{1/2} c^{1/2})^{1/2}, I) + 5/16 c^{1/2} e^5 / (a e^2 - c d^2)^3 / d^3 a^{5/2} / (1/a^{1/2} c^{1/2})^{1/2} * (1 - 1/a^{1/2} c^{1/2} x^2)^{1/2} * (1 + 1/a^{1/2} c^{1/2} x^2)^{1/2} / (-c x^4 + a)^{1/2}$$

$/2) * \text{EllipticE}(x * (1/a^{1/2} * c^{1/2})^{1/2}, I) - 17/16 / (a * e^2 - c * d^2)^{3/2} * e^4 / (1/a^{1/2} * c^{1/2})^{1/2} * (1 - 1/a^{1/2} * c^{1/2} * x^2)^{1/2} * (1 + 1/a^{1/2} * c^{1/2} * x^2)^{1/2} / (-c * x^4 + a)^{1/2} * \text{EllipticPi}(x * (1/a^{1/2} * c^{1/2})^{1/2}, -e * a^{1/2} / d / c^{1/2}, (-1/a^{1/2} * c^{1/2})^{1/2} / (1/a^{1/2} * c^{1/2})^{1/2}) * a^2 * c$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-cx^4 + a}(ex^2 + d)^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x^2+d)^4/(-c*x^4+a)^(1/2),x, algorithm="maxima")

[Out] integrate(1/(sqrt(-c*x^4 + a)*(e*x^2 + d)^4), x)

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x^2+d)^4/(-c*x^4+a)^(1/2),x, algorithm="fricas")

[Out] Timed out

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a - cx^4}(d + ex^2)^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x**2+d)**4/(-c*x**4+a)**(1/2),x)

[Out] Integral(1/(sqrt(a - c*x**4)*(d + e*x**2)**4), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-cx^4 + a}(ex^2 + d)^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x^2+d)^4/(-c*x^4+a)^(1/2),x, algorithm="giac")

[Out] integrate(1/(sqrt(-c*x^4 + a)*(e*x^2 + d)^4), x)

$$3.164 \quad \int \frac{d+ex^2}{\sqrt{-a+cx^4}} dx$$

Optimal. Leaf size=126

$$\frac{a^{3/4} \sqrt{1 - \frac{cx^4}{a}} \left(\frac{\sqrt{cd}}{\sqrt{a}} - e \right) \text{EllipticF} \left(\sin^{-1} \left(\frac{\sqrt[4]{cx}}{\sqrt[4]{a}} \right), -1 \right)}{c^{3/4} \sqrt{cx^4 - a}} + \frac{a^{3/4} e \sqrt{1 - \frac{cx^4}{a}} E \left(\sin^{-1} \left(\frac{\sqrt[4]{cx}}{\sqrt[4]{a}} \right) \middle| -1 \right)}{c^{3/4} \sqrt{cx^4 - a}}$$

[Out] (a^(3/4)*e*Sqrt[1 - (c*x^4)/a]*EllipticE[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(c^(3/4)*Sqrt[-a + c*x^4]) + (a^(3/4)*((Sqrt[c]*d)/Sqrt[a] - e)*Sqrt[1 - (c*x^4)/a]*EllipticF[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(c^(3/4)*Sqrt[-a + c*x^4])

Rubi [A] time = 0.0828779, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {1201, 224, 221, 1200, 1199, 424}

$$\frac{a^{3/4} \sqrt{1 - \frac{cx^4}{a}} \left(\frac{\sqrt{cd}}{\sqrt{a}} - e \right) F \left(\sin^{-1} \left(\frac{\sqrt[4]{cx}}{\sqrt[4]{a}} \right) \middle| -1 \right)}{c^{3/4} \sqrt{cx^4 - a}} + \frac{a^{3/4} e \sqrt{1 - \frac{cx^4}{a}} E \left(\sin^{-1} \left(\frac{\sqrt[4]{cx}}{\sqrt[4]{a}} \right) \middle| -1 \right)}{c^{3/4} \sqrt{cx^4 - a}}$$

Antiderivative was successfully verified.

[In] Int[(d + e*x^2)/Sqrt[-a + c*x^4], x]

[Out] (a^(3/4)*e*Sqrt[1 - (c*x^4)/a]*EllipticE[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(c^(3/4)*Sqrt[-a + c*x^4]) + (a^(3/4)*((Sqrt[c]*d)/Sqrt[a] - e)*Sqrt[1 - (c*x^4)/a]*EllipticF[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(c^(3/4)*Sqrt[-a + c*x^4])

Rule 1201

Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-(c/a), 2]}, Dist[(d*q - e)/q, Int[1/Sqrt[a + c*x^4], x], x] + Dist[e/q, Int[(1 + q*x^2)/Sqrt[a + c*x^4], x], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && NeQ[c*d^2 + a*e^2, 0]

Rule 224

Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[b/a] && !GtQ[a, 0]

Rule 221

Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b, 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[b/a] && GtQ[a, 0]

Rule 1200

Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + (c*x^4)/a], x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && !GtQ[a, 0]

Rule 1199

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] := Dist[d/Sq
rt[a], Int[Sqrt[1 + (e*x^2)/d]/Sqrt[1 - (e*x^2)/d], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] := Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{d + ex^2}{\sqrt{-a + cx^4}} dx &= \frac{(\sqrt{ae}) \int \frac{1 + \frac{\sqrt{cx^2}}{\sqrt{a}}}{\sqrt{-a + cx^4}} dx}{\sqrt{c}} + \left(d - \frac{\sqrt{ae}}{\sqrt{c}}\right) \int \frac{1}{\sqrt{-a + cx^4}} dx \\ &= \frac{\left(\sqrt{ae}\sqrt{1 - \frac{cx^4}{a}}\right) \int \frac{1 + \frac{\sqrt{cx^2}}{\sqrt{a}}}{\sqrt{1 - \frac{cx^4}{a}}} dx}{\sqrt{c}\sqrt{-a + cx^4}} + \frac{\left(\left(d - \frac{\sqrt{ae}}{\sqrt{c}}\right)\sqrt{1 - \frac{cx^4}{a}}\right) \int \frac{1}{\sqrt{1 - \frac{cx^4}{a}}} dx}{\sqrt{-a + cx^4}} \\ &= \frac{\sqrt[4]{a}\left(d - \frac{\sqrt{ae}}{\sqrt{c}}\right)\sqrt{1 - \frac{cx^4}{a}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{cx}}{\sqrt[4]{a}}\right)\right) - 1}{\sqrt[4]{c}\sqrt{-a + cx^4}} + \frac{\left(\sqrt{ae}\sqrt{1 - \frac{cx^4}{a}}\right) \int \frac{\sqrt{1 + \frac{\sqrt{cx^2}}{\sqrt{a}}}}{\sqrt{1 - \frac{cx^4}{a}}} dx}{\sqrt{c}\sqrt{-a + cx^4}} \\ &= \frac{a^{3/4}e\sqrt{1 - \frac{cx^4}{a}}E\left(\sin^{-1}\left(\frac{\sqrt[4]{cx}}{\sqrt[4]{a}}\right)\right) - 1}{c^{3/4}\sqrt{-a + cx^4}} + \frac{\sqrt[4]{a}\left(d - \frac{\sqrt{ae}}{\sqrt{c}}\right)\sqrt{1 - \frac{cx^4}{a}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{cx}}{\sqrt[4]{a}}\right)\right) - 1}{\sqrt[4]{c}\sqrt{-a + cx^4}} \end{aligned}$$

Mathematica [C] time = 0.0345568, size = 78, normalized size = 0.62

$$\frac{\sqrt{1 - \frac{cx^4}{a}} \left(3dx {}_2F_1\left(\frac{1}{4}, \frac{1}{2}; \frac{5}{4}; \frac{cx^4}{a}\right) + ex^3 {}_2F_1\left(\frac{1}{2}, \frac{3}{4}; \frac{7}{4}; \frac{cx^4}{a}\right)\right)}{3\sqrt{cx^4 - a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)/Sqrt[-a + c*x^4],x]
```

```
[Out] (Sqrt[1 - (c*x^4)/a]*(3*d*x*Hypergeometric2F1[1/4, 1/2, 5/4, (c*x^4)/a] + e
*x^3*Hypergeometric2F1[1/2, 3/4, 7/4, (c*x^4)/a]))/(3*Sqrt[-a + c*x^4])
```

Maple [A] time = 0.181, size = 160, normalized size = 1.3

$$e\sqrt{a}\sqrt{1 + x^2\sqrt{c}\frac{1}{\sqrt{a}}}\sqrt{1 - x^2\sqrt{c}\frac{1}{\sqrt{a}}}\left(\text{EllipticF}\left(x\sqrt{-\sqrt{c}\frac{1}{\sqrt{a}}}, i\right) - \text{EllipticE}\left(x\sqrt{-\sqrt{c}\frac{1}{\sqrt{a}}}, i\right)\right) \frac{1}{\sqrt{-\sqrt{c}\frac{1}{\sqrt{a}}}} \frac{1}{\sqrt{cx^4 - a}} \frac{1}{\sqrt{c}} + d\sqrt{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)/(c*x^4-a)^(1/2),x)
```

```
[Out] e*a^(1/2)/(-1/a^(1/2)*c^(1/2))^(1/2)*(1+1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1-1/a
^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4-a)^(1/2)/c^(1/2)*(EllipticF(x*(-1/a^(1/2)*
c^(1/2))^(1/2),I)-EllipticE(x*(-1/a^(1/2)*c^(1/2))^(1/2),I))+d/(-1/a^(1/2)*
c^(1/2))^(1/2)*(1+1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1-1/a^(1/2)*c^(1/2)*x^2)^(1
/2)/(c*x^4-a)^(1/2)*EllipticF(x*(-1/a^(1/2)*c^(1/2))^(1/2),I)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex^2 + d}{\sqrt{cx^4 - a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(c*x^4-a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)/sqrt(c*x^4 - a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{ex^2 + d}{\sqrt{cx^4 - a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(c*x^4-a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((e*x^2 + d)/sqrt(c*x^4 - a), x)
```

Sympy [A] time = 1.66242, size = 73, normalized size = 0.58

$$\frac{id x \Gamma\left(\frac{1}{4}\right) {}_2F_1\left(\frac{1}{4}, \frac{1}{2} \middle| \frac{cx^4}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{5}{4}\right)} - \frac{ie x^3 \Gamma\left(\frac{3}{4}\right) {}_2F_1\left(\frac{1}{2}, \frac{3}{4} \middle| \frac{cx^4}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)/(c*x**4-a)**(1/2),x)
```

```
[Out] -I*d*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), c*x**4/a)/(4*sqrt(a)*gamma(5/4)
) - I*e*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), c*x**4/a)/(4*sqrt(a)*gamm
a(7/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex^2 + d}{\sqrt{cx^4 - a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(c*x^4-a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)/sqrt(c*x^4 - a), x)
```

$$3.165 \quad \int \frac{1}{(d+ex^2)\sqrt{-a+cx^4}} dx$$

Optimal. Leaf size=73

$$\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\Pi\left(-\frac{\sqrt{ae}}{\sqrt{cd}}; \sin^{-1}\left(\frac{\sqrt[4]{cx}}{\sqrt[4]{a}}\right)\right)-1}{\sqrt[4]{cd}\sqrt{cx^4-a}}$$

[Out] (a^(1/4)*Sqrt[1 - (c*x^4)/a]*EllipticPi[-((Sqrt[a]*e)/(Sqrt[c]*d)), ArcSin[(c^(1/4)*x)/a^(1/4)], -1)]/(c^(1/4)*d*Sqrt[-a + c*x^4])

Rubi [A] time = 0.041897, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {1219, 1218}

$$\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\Pi\left(-\frac{\sqrt{ae}}{\sqrt{cd}}; \sin^{-1}\left(\frac{\sqrt[4]{cx}}{\sqrt[4]{a}}\right)\right)-1}{\sqrt[4]{cd}\sqrt{cx^4-a}}$$

Antiderivative was successfully verified.

[In] Int[1/((d + e*x^2)*Sqrt[-a + c*x^4]),x]

[Out] (a^(1/4)*Sqrt[1 - (c*x^4)/a]*EllipticPi[-((Sqrt[a]*e)/(Sqrt[c]*d)), ArcSin[(c^(1/4)*x)/a^(1/4)], -1)]/(c^(1/4)*d*Sqrt[-a + c*x^4])

Rule 1219

Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] := Dist[Sqrt[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + (c*x^4)/a]), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && !GtQ[a, 0]

Rule 1218

Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] := With[{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1)]/(d*Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]

Rubi steps

$$\begin{aligned} \int \frac{1}{(d+ex^2)\sqrt{-a+cx^4}} dx &= \frac{\sqrt{1-\frac{cx^4}{a}} \int \frac{1}{(d+ex^2)\sqrt{1-\frac{cx^4}{a}}} dx}{\sqrt{-a+cx^4}} \\ &= \frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\Pi\left(-\frac{\sqrt{ae}}{\sqrt{cd}}; \sin^{-1}\left(\frac{\sqrt[4]{cx}}{\sqrt[4]{a}}\right)\right)-1}{\sqrt[4]{cd}\sqrt{-a+cx^4}} \end{aligned}$$

Mathematica [C] time = 0.149835, size = 92, normalized size = 1.26

$$\frac{i\sqrt{1-\frac{cx^4}{a}}\Pi\left(-\frac{\sqrt{ae}}{\sqrt{cd}}; i\sinh^{-1}\left(\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}x\right)\right)-1}{d\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\sqrt{cx^4-a}}$$

Antiderivative was successfully verified.

[In] Integrate[1/((d + e*x^2)*Sqrt[-a + c*x^4]),x]

[Out] ((-I)*Sqrt[1 - (c*x^4)/a]*EllipticPi[-((Sqrt[a]*e)/(Sqrt[c]*d)), I*ArcSinh[Sqrt[-(Sqrt[c]/Sqrt[a])]*x], -1])/(Sqrt[-(Sqrt[c]/Sqrt[a])]*d*Sqrt[-a + c*x^4])

Maple [A] time = 0.18, size = 99, normalized size = 1.4

$$\frac{1}{d} \sqrt{1 + x^2 \sqrt{c} \frac{1}{\sqrt{a}}} \sqrt{1 - x^2 \sqrt{c} \frac{1}{\sqrt{a}}} \text{EllipticPi} \left(x \sqrt{-\sqrt{c} \frac{1}{\sqrt{a}}}, \frac{e}{d} \sqrt{a} \frac{1}{\sqrt{c}}, \sqrt{\sqrt{c} \frac{1}{\sqrt{a}}} \frac{1}{\sqrt{-\sqrt{c} \frac{1}{\sqrt{a}}}} \right) \frac{1}{\sqrt{-\sqrt{c} \frac{1}{\sqrt{a}}}} \frac{1}{\sqrt{cx^4 - a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(e*x^2+d)/(c*x^4-a)^(1/2),x)

[Out] 1/d/(-1/a^(1/2)*c^(1/2))^(1/2)*(1+1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1-1/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4-a)^(1/2)*EllipticPi(x*(-1/a^(1/2)*c^(1/2))^(1/2), e*a^(1/2)/d/c^(1/2), (1/a^(1/2)*c^(1/2))^(1/2)/(-1/a^(1/2)*c^(1/2))^(1/2))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^4 - a}(ex^2 + d)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x^2+d)/(c*x^4-a)^(1/2),x, algorithm="maxima")

[Out] integrate(1/(sqrt(c*x^4 - a)*(e*x^2 + d)), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral} \left(\frac{\sqrt{cx^4 - a}}{cex^6 + cdx^4 - aex^2 - ad}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x^2+d)/(c*x^4-a)^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(c*x^4 - a)/(c*e*x^6 + c*d*x^4 - a*e*x^2 - a*d), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-a + cx^4}(d + ex^2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x**2+d)/(c*x**4-a)**(1/2),x)

[Out] Integral(1/(sqrt(-a + c*x**4)*(d + e*x**2)), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^4 - a}(ex^2 + d)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x^2+d)/(c*x^4-a)^(1/2),x, algorithm="giac")

[Out] integrate(1/(sqrt(c*x^4 - a)*(e*x^2 + d)), x)

$$3.166 \quad \int \frac{\sqrt{a} + \sqrt{cx^2}}{\sqrt{-a + cx^4}} dx$$

Optimal. Leaf size=54

$$\frac{a^{3/4} \sqrt{1 - \frac{cx^4}{a}} E\left(\sin^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right) \middle| -1\right)}{\sqrt[4]{c} \sqrt{cx^4 - a}}$$

[Out] (a^(3/4)*Sqrt[1 - (c*x^4)/a]*EllipticE[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(c^(1/4)*Sqrt[-a + c*x^4])

Rubi [A] time = 0.0492816, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {1200, 1199, 424}

$$\frac{a^{3/4} \sqrt{1 - \frac{cx^4}{a}} E\left(\sin^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right) \middle| -1\right)}{\sqrt[4]{c} \sqrt{cx^4 - a}}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[a] + Sqrt[c]*x^2)/Sqrt[-a + c*x^4], x]

[Out] (a^(3/4)*Sqrt[1 - (c*x^4)/a]*EllipticE[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(c^(1/4)*Sqrt[-a + c*x^4])

Rule 1200

Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] := Dist[Sqrt[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + (c*x^4)/a], x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && !GtQ[a, 0]

Rule 1199

Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] := Dist[d/Sqrt[a], Int[Sqrt[1 + (e*x^2)/d]/Sqrt[1 - (e*x^2)/d], x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]

Rule 424

Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] := Simp[(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]

Rubi steps

$$\int \frac{\sqrt{a} + \sqrt{cx^2}}{\sqrt{-a + cx^4}} dx = \frac{\sqrt{1 - \frac{cx^4}{a}} \int \frac{\sqrt{a + \sqrt{cx^2}}}{\sqrt{1 - \frac{cx^4}{a}}} dx}{\sqrt{-a + cx^4}}$$

$$= \frac{\left(\sqrt{a} \sqrt{1 - \frac{cx^4}{a}}\right) \int \frac{\sqrt{1 + \frac{\sqrt{cx^2}}{\sqrt{a}}}}{\sqrt{1 - \frac{\sqrt{cx^2}}{\sqrt{a}}}} dx}{\sqrt{-a + cx^4}}$$

$$= \frac{a^{3/4} \sqrt{1 - \frac{cx^4}{a}} \left(\sin^{-1} \left(\frac{\sqrt[4]{cx}}{\sqrt[4]{a}} \right) - 1 \right)}{\sqrt[4]{c} \sqrt{-a + cx^4}}$$

Mathematica [C] time = 0.0369814, size = 86, normalized size = 1.59

$$\frac{\sqrt{1 - \frac{cx^4}{a}} \left(\sqrt{cx^3} {}_2F_1 \left(\frac{1}{2}, \frac{3}{4}; \frac{7}{4}; \frac{cx^4}{a} \right) + 3\sqrt{ax} {}_2F_1 \left(\frac{1}{4}, \frac{1}{2}; \frac{5}{4}; \frac{cx^4}{a} \right) \right)}{3\sqrt{cx^4 - a}}$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[a] + Sqrt[c]*x^2)/Sqrt[-a + c*x^4],x]

[Out] (Sqrt[1 - (c*x^4)/a]*(3*Sqrt[a]*x*Hypergeometric2F1[1/4, 1/2, 5/4, (c*x^4)/a] + Sqrt[c]*x^3*Hypergeometric2F1[1/2, 3/4, 7/4, (c*x^4)/a]))/(3*Sqrt[-a + c*x^4])

Maple [B] time = 0.072, size = 158, normalized size = 2.9

$$\sqrt{a} \sqrt{1 + x^2 \sqrt{c} \frac{1}{\sqrt{a}}} \sqrt{1 - x^2 \sqrt{c} \frac{1}{\sqrt{a}}} \left(\text{EllipticF} \left(x \sqrt{-\sqrt{c} \frac{1}{\sqrt{a}}}, i \right) - \text{EllipticE} \left(x \sqrt{-\sqrt{c} \frac{1}{\sqrt{a}}}, i \right) \right) \frac{1}{\sqrt{-\sqrt{c} \frac{1}{\sqrt{a}}}} \frac{1}{\sqrt{cx^4 - a}} + \sqrt{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a^(1/2)+x^2*c^(1/2))/(c*x^4-a)^(1/2),x)

[Out] a^(1/2)/(-1/a^(1/2)*c^(1/2))^(1/2)*(1+1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1-1/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4-a)^(1/2)*(EllipticF(x*(-1/a^(1/2)*c^(1/2))^(1/2),I)-EllipticE(x*(-1/a^(1/2)*c^(1/2))^(1/2),I))+a^(1/2)/(-1/a^(1/2)*c^(1/2))^(1/2)*(1+1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1-1/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4-a)^(1/2)*EllipticF(x*(-1/a^(1/2)*c^(1/2))^(1/2),I)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^2 + \sqrt{a}}}{\sqrt{cx^4 - a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a^(1/2)+x^2*c^(1/2))/(c*x^4-a)^(1/2),x, algorithm="maxima")

[Out] integrate((sqrt(c)*x^2 + sqrt(a))/sqrt(c*x^4 - a), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^2 + \sqrt{a}}}{\sqrt{cx^4 - a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a^(1/2)+x^2*c^(1/2))/(c*x^4-a)^(1/2),x, algorithm="fricas")

[Out] integral((sqrt(c)*x^2 + sqrt(a))/sqrt(c*x^4 - a), x)

Sympy [A] time = 1.79627, size = 70, normalized size = 1.3

$$\frac{ix\Gamma\left(\frac{1}{4}\right) {}_2F_1\left(\frac{1}{4}, \frac{1}{2} \middle| \frac{cx^4}{a}\right)}{4\Gamma\left(\frac{5}{4}\right)} - \frac{i\sqrt{c}x^3\Gamma\left(\frac{3}{4}\right) {}_2F_1\left(\frac{3}{4}, \frac{3}{4} \middle| \frac{cx^4}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a**(1/2)+x**2*c**(1/2))/(c*x**4-a)**(1/2),x)

[Out] -I*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), c*x**4/a)/(4*gamma(5/4)) - I*sqrt(c)*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), c*x**4/a)/(4*sqrt(a)*gamma(7/4))

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a^(1/2)+x^2*c^(1/2))/(c*x^4-a)^(1/2),x, algorithm="giac")

[Out] Exception raised: TypeError

$$3.167 \quad \int \frac{1 + \sqrt{\frac{c}{a}}x^2}{\sqrt{-a + cx^4}} dx$$

Optimal. Leaf size=52

$$\frac{\sqrt{1 - \frac{cx^4}{a}} E\left(\sin^{-1}\left(\sqrt[4]{\frac{c}{a}}x\right) \middle| -1\right)}{\sqrt[4]{\frac{c}{a}}\sqrt{cx^4 - a}}$$

[Out] (Sqrt[1 - (c*x^4)/a]*EllipticE[ArcSin[(c/a)^(1/4)*x], -1])/((c/a)^(1/4)*Sqrt[-a + c*x^4])

Rubi [A] time = 0.0463449, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {1200, 1199, 424}

$$\frac{\sqrt{1 - \frac{cx^4}{a}} E\left(\sin^{-1}\left(\sqrt[4]{\frac{c}{a}}x\right) \middle| -1\right)}{\sqrt[4]{\frac{c}{a}}\sqrt{cx^4 - a}}$$

Antiderivative was successfully verified.

[In] Int[(1 + Sqrt[c/a]*x^2)/Sqrt[-a + c*x^4], x]

[Out] (Sqrt[1 - (c*x^4)/a]*EllipticE[ArcSin[(c/a)^(1/4)*x], -1])/((c/a)^(1/4)*Sqrt[-a + c*x^4])

Rule 1200

Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + (c*x^4)/a], x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && !GtQ[a, 0]

Rule 1199

Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sqrt[a], Int[Sqrt[1 + (e*x^2)/d]/Sqrt[1 - (e*x^2)/d], x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]

Rule 424

Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]

Rubi steps

$$\begin{aligned}
\int \frac{1 + \sqrt{\frac{c}{a}}x^2}{\sqrt{-a + cx^4}} dx &= \frac{\sqrt{1 - \frac{cx^4}{a}} \int \frac{1 + \sqrt{\frac{c}{a}}x^2}{\sqrt{1 - \frac{cx^4}{a}}} dx}{\sqrt{-a + cx^4}} \\
&= \frac{\sqrt{1 - \frac{cx^4}{a}} \int \frac{\sqrt{1 + \sqrt{\frac{c}{a}}x^2}}{\sqrt{1 - \sqrt{\frac{c}{a}}x^2}} dx}{\sqrt{-a + cx^4}} \\
&= \frac{\sqrt{1 - \frac{cx^4}{a}} E\left(\sin^{-1}\left(\sqrt{\frac{c}{a}}x\right) \middle| -1\right)}{\sqrt{\frac{c}{a}}\sqrt{-a + cx^4}}
\end{aligned}$$

Mathematica [C] time = 0.0305537, size = 85, normalized size = 1.63

$$\frac{\sqrt{1 - \frac{cx^4}{a}} \left(x^3 \sqrt{\frac{c}{a}} {}_2F_1\left(\frac{1}{2}, \frac{3}{4}; \frac{7}{4}; \frac{cx^4}{a}\right) + 3x {}_2F_1\left(\frac{1}{4}, \frac{1}{2}; \frac{5}{4}; \frac{cx^4}{a}\right) \right)}{3\sqrt{cx^4 - a}}$$

Antiderivative was successfully verified.

[In] Integrate[(1 + Sqrt[c/a]*x^2)/Sqrt[-a + c*x^4],x]

[Out] (Sqrt[1 - (c*x^4)/a]*(3*x*Hypergeometric2F1[1/4, 1/2, 5/4, (c*x^4)/a] + Sqrt[c/a]*x^3*Hypergeometric2F1[1/2, 3/4, 7/4, (c*x^4)/a]))/(3*Sqrt[-a + c*x^4])

Maple [B] time = 0.071, size = 165, normalized size = 3.2

$$\sqrt{1 + x^2\sqrt{\frac{c}{a}}}\sqrt{\frac{1}{\sqrt{a}}}\sqrt{1 - x^2\sqrt{\frac{c}{a}}}\sqrt{\frac{1}{\sqrt{a}}}\text{EllipticF}\left(x\sqrt{-\sqrt{\frac{c}{a}}}\sqrt{\frac{1}{\sqrt{a}}}, i\right) \frac{1}{\sqrt{-\sqrt{\frac{c}{a}}}\sqrt{\frac{1}{\sqrt{a}}}} \frac{1}{\sqrt{cx^4 - a}} + \sqrt{\frac{c}{a}}\sqrt{a}\sqrt{1 + x^2\sqrt{\frac{c}{a}}}\sqrt{\frac{1}{\sqrt{a}}}\sqrt{1 - x^2\sqrt{\frac{c}{a}}}\sqrt{\frac{1}{\sqrt{a}}}\left(\text{EllipticE}\left(x\sqrt{-\sqrt{\frac{c}{a}}}\sqrt{\frac{1}{\sqrt{a}}}, i\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1+x^2*(c/a)^(1/2))/(c*x^4-a)^(1/2),x)

[Out] 1/(-1/a^(1/2)*c^(1/2))^(1/2)*(1+1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1-1/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4-a)^(1/2)*EllipticF(x*(-1/a^(1/2)*c^(1/2))^(1/2),I) + (c/a)^(1/2)*a^(1/2)/(-1/a^(1/2)*c^(1/2))^(1/2)*(1+1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1-1/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4-a)^(1/2)/c^(1/2)*(EllipticF(x*(-1/a^(1/2)*c^(1/2))^(1/2),I)-EllipticE(x*(-1/a^(1/2)*c^(1/2))^(1/2),I))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2\sqrt{\frac{c}{a}} + 1}{\sqrt{cx^4 - a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+x^2*(c/a)^(1/2))/(c*x^4-a)^(1/2),x, algorithm="maxima")

[Out] integrate((x^2*sqrt(c/a) + 1)/sqrt(c*x^4 - a), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^2\sqrt{\frac{c}{a}}+1}{\sqrt{cx^4-a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+x^2*(c/a)^(1/2))/(c*x^4-a)^(1/2),x, algorithm="fricas")

[Out] integral((x^2*sqrt(c/a) + 1)/sqrt(c*x^4 - a), x)

Sympy [B] time = 1.75264, size = 76, normalized size = 1.46

$$\frac{ix^3\sqrt{\frac{c}{a}}\Gamma\left(\frac{3}{4}\right) {}_2F_1\left(\frac{1}{2}, \frac{3}{4} \middle| \frac{cx^4}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{7}{4}\right)} - \frac{ix\Gamma\left(\frac{1}{4}\right) {}_2F_1\left(\frac{1}{4}, \frac{1}{2} \middle| \frac{cx^4}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+x**2*(c/a)**(1/2))/(c*x**4-a)**(1/2),x)

[Out] -I*x**3*sqrt(c/a)*gamma(3/4)*hyper((1/2, 3/4), (7/4,), c*x**4/a)/(4*sqrt(a)*gamma(7/4)) - I*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), c*x**4/a)/(4*sqrt(a)*gamma(5/4))

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2\sqrt{\frac{c}{a}}+1}{\sqrt{cx^4-a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+x^2*(c/a)^(1/2))/(c*x^4-a)^(1/2),x, algorithm="giac")

[Out] integrate((x^2*sqrt(c/a) + 1)/sqrt(c*x^4 - a), x)

3.168 $\int \frac{d+ex^2}{\sqrt{-a-cx^4}} dx$

Optimal. Leaf size=236

$$\frac{\sqrt[4]{a}(\sqrt{a} + \sqrt{cx^2}) \sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{cx^2})^2}} \left(\frac{\sqrt{cd}}{\sqrt{a}} + e\right) \text{EllipticF}\left(2 \tan^{-1}\left(\frac{\sqrt[4]{cx}}{\sqrt[4]{a}}\right), \frac{1}{2}\right)}{2c^{3/4}\sqrt{-a-cx^4}} - \frac{\sqrt[4]{ae}(\sqrt{a} + \sqrt{cx^2}) \sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{cx^2})^2}} E\left(2 \tan^{-1}\left(\frac{\sqrt[4]{cx}}{\sqrt[4]{a}}\right)\right)}{c^{3/4}\sqrt{-a-cx^4}}$$

[Out] $-\left(\frac{e*x*\text{Sqrt}[-a - c*x^4]}{\text{Sqrt}[c]*(\text{Sqrt}[a] + \text{Sqrt}[c]*x^2)}\right) - (a^{1/4})*e*(\text{Sqrt}[a] + \text{Sqrt}[c]*x^2)*\text{Sqrt}[(a + c*x^4)/(\text{Sqrt}[a] + \text{Sqrt}[c]*x^2)^2]*\text{EllipticE}[2*\text{ArcTan}[(c^{1/4})*x]/a^{1/4}], 1/2)/\left(c^{3/4}*\text{Sqrt}[-a - c*x^4]\right) + (a^{1/4})*((\text{Sqrt}[c]*d)/\text{Sqrt}[a] + e)*(\text{Sqrt}[a] + \text{Sqrt}[c]*x^2)*\text{Sqrt}[(a + c*x^4)/(\text{Sqrt}[a] + \text{Sqrt}[c]*x^2)^2]*\text{EllipticF}[2*\text{ArcTan}[(c^{1/4})*x]/a^{1/4}], 1/2)/(2*c^{3/4}*\text{Sqrt}[-a - c*x^4])$

Rubi [A] time = 0.0692306, antiderivative size = 236, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {1198, 220, 1196}

$$\frac{\sqrt[4]{a}(\sqrt{a} + \sqrt{cx^2}) \sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{cx^2})^2}} \left(\frac{\sqrt{cd}}{\sqrt{a}} + e\right) F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{cx}}{\sqrt[4]{a}}\right) \middle| \frac{1}{2}\right)}{2c^{3/4}\sqrt{-a-cx^4}} - \frac{\sqrt[4]{ae}(\sqrt{a} + \sqrt{cx^2}) \sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{cx^2})^2}} E\left(2 \tan^{-1}\left(\frac{\sqrt[4]{cx}}{\sqrt[4]{a}}\right) \middle| \frac{1}{2}\right)}{c^{3/4}\sqrt{-a-cx^4}} - \frac{ex}{\sqrt{c}}$$

Antiderivative was successfully verified.

[In] Int[(d + e*x^2)/Sqrt[-a - c*x^4], x]

[Out] $-\left(\frac{e*x*\text{Sqrt}[-a - c*x^4]}{\text{Sqrt}[c]*(\text{Sqrt}[a] + \text{Sqrt}[c]*x^2)}\right) - (a^{1/4})*e*(\text{Sqrt}[a] + \text{Sqrt}[c]*x^2)*\text{Sqrt}[(a + c*x^4)/(\text{Sqrt}[a] + \text{Sqrt}[c]*x^2)^2]*\text{EllipticE}[2*\text{ArcTan}[(c^{1/4})*x]/a^{1/4}], 1/2)/\left(c^{3/4}*\text{Sqrt}[-a - c*x^4]\right) + (a^{1/4})*((\text{Sqrt}[c]*d)/\text{Sqrt}[a] + e)*(\text{Sqrt}[a] + \text{Sqrt}[c]*x^2)*\text{Sqrt}[(a + c*x^4)/(\text{Sqrt}[a] + \text{Sqrt}[c]*x^2)^2]*\text{EllipticF}[2*\text{ArcTan}[(c^{1/4})*x]/a^{1/4}], 1/2)/(2*c^{3/4}*\text{Sqrt}[-a - c*x^4])$

Rule 1198

Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] := With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0] /; FreeQ[{a, c, d, e}, x] && PosQ[c/a]

Rule 220

Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] := With[{q = Rt[b/a, 4]}, Simp[(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x], 1/2]]/(2*q*Sqrt[a + b*x^4]), x] /; FreeQ[{a, b}, x] && PosQ[b/a]

Rule 1196

Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] := With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x], 1/2]]/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0] /; FreeQ[{a, c, d, e}, x] && PosQ[c/a]

Rubi steps

$$\int \frac{d + ex^2}{\sqrt{-a - cx^4}} dx = -\frac{(\sqrt{ae}) \int \frac{1 - \sqrt{cx^2}}{\sqrt{-a - cx^4}} dx}{\sqrt{c}} + \left(d + \frac{\sqrt{ae}}{\sqrt{c}}\right) \int \frac{1}{\sqrt{-a - cx^4}} dx$$

$$= -\frac{ex\sqrt{-a - cx^4}}{\sqrt{c}(\sqrt{a} + \sqrt{cx^2})} - \frac{\sqrt[4]{ae}(\sqrt{a} + \sqrt{cx^2}) \sqrt{\frac{a+cx^4}{(\sqrt{a} + \sqrt{cx^2})^2}} E\left(2 \tan^{-1}\left(\frac{\sqrt[4]{cx}}{\sqrt{a}}\right) \middle| \frac{1}{2}\right)}{c^{3/4}\sqrt{-a - cx^4}} + \frac{(\sqrt{cd} + \sqrt{ae})(\sqrt{a} + \sqrt{cx^2})}{2\sqrt{c}}$$

Mathematica [C] time = 0.037163, size = 80, normalized size = 0.34

$$\frac{\sqrt{\frac{cx^4}{a} + 1} \left(3dx {}_2F_1\left(\frac{1}{4}, \frac{1}{2}; \frac{5}{4}; -\frac{cx^4}{a}\right) + ex^3 {}_2F_1\left(\frac{1}{2}, \frac{3}{4}; \frac{7}{4}; -\frac{cx^4}{a}\right)\right)}{3\sqrt{-a - cx^4}}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e*x^2)/Sqrt[-a - c*x^4], x]

[Out] (Sqrt[1 + (c*x^4)/a]*(3*d*x*Hypergeometric2F1[1/4, 1/2, 5/4, -((c*x^4)/a)] + e*x^3*Hypergeometric2F1[1/2, 3/4, 7/4, -((c*x^4)/a)]))/(3*Sqrt[-a - c*x^4])

Maple [C] time = 0.175, size = 175, normalized size = 0.7

$$-ie\sqrt{a}\sqrt{1 + ix^2\sqrt{c}\frac{1}{\sqrt{a}}}\sqrt{1 - ix^2\sqrt{c}\frac{1}{\sqrt{a}}}\left(\text{EllipticF}\left(x\sqrt{-i\sqrt{c}\frac{1}{\sqrt{a}}}, i\right) - \text{EllipticE}\left(x\sqrt{-i\sqrt{c}\frac{1}{\sqrt{a}}}, i\right)\right) \frac{1}{\sqrt{-i\sqrt{c}\frac{1}{\sqrt{a}}}} \frac{1}{\sqrt{-cx^4 - a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x^2+d)/(-c*x^4-a)^(1/2), x)

[Out] -I*e*a^(1/2)/(-I/a^(1/2)*c^(1/2))^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(-c*x^4-a)^(1/2)/c^(1/2)*(EllipticF(x*(-I/a^(1/2)*c^(1/2))^(1/2), I)-EllipticE(x*(-I/a^(1/2)*c^(1/2))^(1/2), I))+d/(-I/a^(1/2)*c^(1/2))^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(-c*x^4-a)^(1/2)*EllipticF(x*(-I/a^(1/2)*c^(1/2))^(1/2), I)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex^2 + d}{\sqrt{-cx^4 - a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)/(-c*x^4-a)^(1/2), x, algorithm="maxima")

[Out] integrate((e*x^2 + d)/sqrt(-c*x^4 - a), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-cx^4 - a}(ex^2 + d)}{cx^4 + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)/(-c*x^4-a)^(1/2),x, algorithm="fricas")

[Out] integral(-sqrt(-c*x^4 - a)*(e*x^2 + d)/(c*x^4 + a), x)

Sympy [C] time = 1.63576, size = 83, normalized size = 0.35

$$\frac{id x \Gamma\left(\frac{1}{4}\right) {}_2F_1\left(\frac{1}{4}, \frac{1}{2} \middle| \frac{cx^4 e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{5}{4}\right)} - \frac{ie x^3 \Gamma\left(\frac{3}{4}\right) {}_2F_1\left(\frac{1}{2}, \frac{3}{4} \middle| \frac{cx^4 e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x**2+d)/(-c*x**4-a)**(1/2),x)

[Out] -I*d*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), c*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(5/4)) - I*e*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), c*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(7/4))

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex^2 + d}{\sqrt{-cx^4 - a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)/(-c*x^4-a)^(1/2),x, algorithm="giac")

[Out] integrate((e*x^2 + d)/sqrt(-c*x^4 - a), x)

$$3.169 \quad \int \frac{1}{(d+ex^2)\sqrt{-a-cx^4}} dx$$

Optimal. Leaf size=347

$$\frac{\sqrt[4]{c}(\sqrt{a} + \sqrt{cx^2}) \sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{cx^2})^2}} \text{EllipticF}\left(2 \tan^{-1}\left(\frac{\sqrt[4]{cx}}{\sqrt{a}}\right), \frac{1}{2}\right) - a^{3/4}(\sqrt{a} + \sqrt{cx^2}) \sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{cx^2})^2}} \left(\frac{\sqrt{cd}}{\sqrt{a}} + e\right)^2 \Pi\left(-\frac{(\sqrt{cd}-\sqrt{ae})^2}{4\sqrt{a}\sqrt{cde}}\right)}{2\sqrt[4]{a}\sqrt{-a-cx^4}(\sqrt{cd}-\sqrt{ae}) - 4\sqrt[4]{cd}\sqrt{-a-cx^4}(cd^2-ae^2)}$$

```
[Out] (Sqrt[e]*ArcTan[(Sqrt[-(c*d^2) - a*e^2]*x)/(Sqrt[d]*Sqrt[e]*Sqrt[-a - c*x^4
]])/(2*Sqrt[d]*Sqrt[-(c*d^2) - a*e^2]) + (c^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*
Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/
a^(1/4)], 1/2])/(2*a^(1/4)*(Sqrt[c]*d - Sqrt[a]*e)*Sqrt[-a - c*x^4]) - (a^(
3/4)*((Sqrt[c]*d)/Sqrt[a] + e)^2*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(
Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticPi[-(Sqrt[c]*d - Sqrt[a]*e)^2/(4*Sqrt[a]*
Sqrt[c]*d*e), 2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(4*c^(1/4)*d*(c*d^2 - a*
e^2)*Sqrt[-a - c*x^4])
```

Rubi [A] time = 0.295035, antiderivative size = 347, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1217, 220, 1707}

$$\frac{a^{3/4}(\sqrt{a} + \sqrt{cx^2}) \sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{cx^2})^2}} \left(\frac{\sqrt{cd}}{\sqrt{a}} + e\right)^2 \Pi\left(-\frac{(\sqrt{cd}-\sqrt{ae})^2}{4\sqrt{a}\sqrt{cde}}; 2 \tan^{-1}\left(\frac{\sqrt[4]{cx}}{\sqrt{a}}\right) \middle| \frac{1}{2}\right) + \frac{\sqrt{e} \tan^{-1}\left(\frac{x\sqrt{-ae^2-cd^2}}{\sqrt{d}\sqrt{e}\sqrt{-a-cx^4}}\right)}{2\sqrt{d}\sqrt{-ae^2-cd^2}} + \frac{\sqrt[4]{c}(\sqrt{a} + \sqrt{cx^2})}{2\sqrt[4]{a}\sqrt{-a-cx^4}(\sqrt{cd}-\sqrt{ae})} + \frac{a^{3/4}(\sqrt{a} + \sqrt{cx^2}) \sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{cx^2})^2}} \left(\frac{\sqrt{cd}}{\sqrt{a}} + e\right)^2 \Pi\left(-\frac{(\sqrt{cd}-\sqrt{ae})^2}{4\sqrt{a}\sqrt{cde}}; 2 \tan^{-1}\left(\frac{\sqrt[4]{cx}}{\sqrt{a}}\right) \middle| \frac{1}{2}\right)}{4\sqrt[4]{cd}\sqrt{-a-cx^4}(cd^2-ae^2)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x^2)*Sqrt[-a - c*x^4]), x]
```

```
[Out] (Sqrt[e]*ArcTan[(Sqrt[-(c*d^2) - a*e^2]*x)/(Sqrt[d]*Sqrt[e]*Sqrt[-a - c*x^4
]])/(2*Sqrt[d]*Sqrt[-(c*d^2) - a*e^2]) + (c^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*
Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/
a^(1/4)], 1/2])/(2*a^(1/4)*(Sqrt[c]*d - Sqrt[a]*e)*Sqrt[-a - c*x^4]) - (a^(
3/4)*((Sqrt[c]*d)/Sqrt[a] + e)^2*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(
Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticPi[-(Sqrt[c]*d - Sqrt[a]*e)^2/(4*Sqrt[a]*
Sqrt[c]*d*e), 2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(4*c^(1/4)*d*(c*d^2 - a*
e^2)*Sqrt[-a - c*x^4])
```

Rule 1217

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] := With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] := With[{q = Rt[b/a, 4]}, Simp[
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1707

```
Int[((A_) + (B_)*(x_)^2)/(((d_) + (e_)*(x_)^2)*Sqrt[(a_) + (c_)*(x_)^4])
, x_Symbol] := With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*ArcTan[(Rt[(c*d)/
+ (a*e)/d, 2]*x)/Sqrt[a + c*x^4]])/(2*d*e*Rt[(c*d)/e + (a*e)/d, 2]), x] +
Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a + c*x^4))/(a*(A + B*x^2)^2)]*Ell
ipticPi[Cancel[-((B*d - A*e)^2/(4*d*e*A*B))], 2*ArcTan[q*x], 1/2])/(4*d*e*A
*q*Sqrt[a + c*x^4]), x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{1}{(d + ex^2)\sqrt{-a - cx^4}} dx = \frac{\sqrt{c} \int \frac{1}{\sqrt{-a - cx^4}} dx}{\sqrt{cd} - \sqrt{ae}} - \frac{(\sqrt{ae}) \int \frac{1 + \frac{\sqrt{cx^2}}{\sqrt{a}}}{(d+ex^2)\sqrt{-a-cx^4}} dx}{\sqrt{cd} - \sqrt{ae}}$$

$$= \frac{\sqrt{e} \tan^{-1}\left(\frac{\sqrt{-cd^2 - ae^2}x}{\sqrt{d}\sqrt{e}\sqrt{-a - cx^4}}\right)}{2\sqrt{d}\sqrt{-cd^2 - ae^2}} + \frac{\sqrt[4]{c}(\sqrt{a} + \sqrt{cx^2})\sqrt{\frac{a+cx^4}{(\sqrt{a} + \sqrt{cx^2})^2}} F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right) \middle| \frac{1}{2}\right)}{2\sqrt[4]{a}(\sqrt{cd} - \sqrt{ae})\sqrt{-a - cx^4}} - \frac{\sqrt[4]{a}\left(\frac{\sqrt{cd}}{\sqrt{a}} + \dots\right)}{2\sqrt[4]{a}(\sqrt{cd} - \sqrt{ae})\sqrt{-a - cx^4}}$$

Mathematica [C] time = 0.147194, size = 98, normalized size = 0.28

$$\frac{i\sqrt{\frac{cx^4}{a}} + 1\Pi\left(-\frac{i\sqrt{ae}}{\sqrt{cd}}; i \sinh^{-1}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}x\right) \middle| -1\right)}{d\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\sqrt{-a - cx^4}}$$

Antiderivative was successfully verified.

[In] Integrate[1/((d + e*x^2)*Sqrt[-a - c*x^4]),x]

[Out] ((-I)*Sqrt[1 + (c*x^4)/a]*EllipticPi[(-I)*Sqrt[a]*e]/(Sqrt[c]*d), I*ArcSin h[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x], -1]/(Sqrt[(I*Sqrt[c])/Sqrt[a]]*d*Sqrt[-a - c*x^4])

Maple [C] time = 0.178, size = 110, normalized size = 0.3

$$\frac{1}{d}\sqrt{1 + ix^2\sqrt{c}\frac{1}{\sqrt{a}}}\sqrt{1 - ix^2\sqrt{c}\frac{1}{\sqrt{a}}}\text{EllipticPi}\left(x\sqrt{-i\sqrt{c}\frac{1}{\sqrt{a}}}, \frac{-ie}{d}\sqrt{a}\frac{1}{\sqrt{c}}, \sqrt{i\sqrt{c}\frac{1}{\sqrt{a}}}\frac{1}{\sqrt{-i\sqrt{c}\frac{1}{\sqrt{a}}}}\right)\frac{1}{\sqrt{-i\sqrt{c}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{-cx^4 - a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(e*x^2+d)/(-c*x^4-a)^(1/2),x)

[Out] 1/d/(-I/a^(1/2)*c^(1/2))^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(-c*x^4-a)^(1/2)*EllipticPi(x*(-I/a^(1/2)*c^(1/2))^(1/2), -I*a^(1/2)/c^(1/2)*e/d, (I/a^(1/2)*c^(1/2))^(1/2)/(-I/a^(1/2)*c^(1/2))^(1/2))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-cx^4 - a}(ex^2 + d)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x^2+d)/(-c*x^4-a)^(1/2),x, algorithm="maxima")

[Out] integrate(1/(sqrt(-c*x^4 - a)*(e*x^2 + d)), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-cx^4 - a}}{cex^6 + cdx^4 + aex^2 + ad}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x^2+d)/(-c*x^4-a)^(1/2),x, algorithm="fricas")

[Out] integral(-sqrt(-c*x^4 - a)/(c*e*x^6 + c*d*x^4 + a*e*x^2 + a*d), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-a - cx^4}(d + ex^2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x**2+d)/(-c*x**4-a)**(1/2),x)

[Out] Integral(1/(sqrt(-a - c*x**4)*(d + e*x**2)), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-cx^4 - a}(ex^2 + d)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x^2+d)/(-c*x^4-a)^(1/2),x, algorithm="giac")

[Out] integrate(1/(sqrt(-c*x^4 - a)*(e*x^2 + d)), x)

$$3.170 \quad \int \frac{1}{(a+bx^2)\sqrt{4-5x^4}} dx$$

Optimal. Leaf size=40

$$\frac{\Pi\left(-\frac{2b}{\sqrt{5a}}; \sin^{-1}\left(\frac{\sqrt[4]{5x}}{\sqrt{2}}\right) \middle| -1\right)}{\sqrt{2}\sqrt[4]{5a}}$$

[Out] EllipticPi[(-2*b)/(Sqrt[5]*a), ArcSin[(5^(1/4)*x)/Sqrt[2]], -1]/(Sqrt[2]*5^(1/4)*a)

Rubi [A] time = 0.0630028, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {1213, 537}

$$\frac{\Pi\left(-\frac{2b}{\sqrt{5a}}; \sin^{-1}\left(\frac{\sqrt[4]{5x}}{\sqrt{2}}\right) \middle| -1\right)}{\sqrt{2}\sqrt[4]{5a}}$$

Antiderivative was successfully verified.

[In] Int[1/((a + b*x^2)*Sqrt[4 - 5*x^4]),x]

[Out] EllipticPi[(-2*b)/(Sqrt[5]*a), ArcSin[(5^(1/4)*x)/Sqrt[2]], -1]/(Sqrt[2]*5^(1/4)*a)

Rule 1213

Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[-(a*c), 2]}, Dist[Sqrt[-c], Int[1/((d + e*x^2)*Sqrt[q + c*x^2]*Sqrt[q - c*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && GtQ[a, 0] && LtQ[c, 0]

Rule 537

Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d, e, f}, x] && !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] && !(GtQ[f/e, 0] && SimplifySqrtQ[-(f/e), -(d/c)])

Rubi steps

$$\begin{aligned} \int \frac{1}{(a+bx^2)\sqrt{4-5x^4}} dx &= \sqrt{5} \int \frac{1}{\sqrt{2\sqrt{5}-5x^2}\sqrt{2\sqrt{5}+5x^2}(a+bx^2)} dx \\ &= \frac{\Pi\left(-\frac{2b}{\sqrt{5a}}; \sin^{-1}\left(\frac{\sqrt[4]{5x}}{\sqrt{2}}\right) \middle| -1\right)}{\sqrt{2}\sqrt[4]{5a}} \end{aligned}$$

Mathematica [A] time = 0.123363, size = 43, normalized size = 1.08

$$\frac{\Pi\left(-\frac{2b}{\sqrt{5a}}; -\sin^{-1}\left(\frac{\sqrt[4]{5x}}{\sqrt{2}}\right) \middle| -1\right)}{\sqrt{2}\sqrt[4]{5a}}$$

Antiderivative was successfully verified.

[In] Integrate[1/((a + b*x^2)*Sqrt[4 - 5*x^4]),x]

[Out] -(EllipticPi[(-2*b)/(Sqrt[5]*a), -ArcSin[(5^(1/4)*x)/Sqrt[2]], -1]/(Sqrt[2]*5^(1/4)*a))

Maple [B] time = 0.191, size = 79, normalized size = 2.

$$\frac{\sqrt{25}^{\frac{3}{4}}}{5a} \sqrt{1 - \frac{x^2\sqrt{5}}{2}} \sqrt{1 + \frac{x^2\sqrt{5}}{2}} \text{EllipticPi} \left(\frac{\sqrt[4]{5}x\sqrt{2}}{2}, -\frac{2\sqrt{5}b}{5a}, \frac{\sqrt{-\frac{\sqrt{5}}{2}}\sqrt{25}^{\frac{3}{4}}}{5} \right) \frac{1}{\sqrt{-5x^4+4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b*x^2+a)/(-5*x^4+4)^(1/2),x)

[Out] 1/5/a*2^(1/2)*5^(3/4)*(1-1/2*x^2*5^(1/2))^(1/2)*(1+1/2*x^2*5^(1/2))^(1/2)/(-5*x^4+4)^(1/2)*EllipticPi(1/2*5^(1/4)*x*2^(1/2),-2/5*b/a*5^(1/2),1/5*(-1/2*5^(1/2))^(1/2)*2^(1/2)*5^(3/4))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-5x^4+4}(bx^2+a)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x^2+a)/(-5*x^4+4)^(1/2),x, algorithm="maxima")

[Out] integrate(1/(sqrt(-5*x^4 + 4)*(b*x^2 + a)), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral} \left(-\frac{\sqrt{-5x^4+4}}{5bx^6+5ax^4-4bx^2-4a}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x^2+a)/(-5*x^4+4)^(1/2),x, algorithm="fricas")

[Out] integral(-sqrt(-5*x^4 + 4)/(5*b*x^6 + 5*a*x^4 - 4*b*x^2 - 4*a), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{4-5x^4}(a+bx^2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x**2+a)/(-5*x**4+4)**(1/2),x)

[Out] Integral(1/(sqrt(4 - 5*x**4)*(a + b*x**2)), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-5x^4 + 4}(bx^2 + a)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x^2+a)/(-5*x^4+4)^(1/2),x, algorithm="giac")

[Out] integrate(1/(sqrt(-5*x^4 + 4)*(b*x^2 + a)), x)

$$3.171 \quad \int \frac{1}{(a+bx^2)\sqrt{4+5x^4}} dx$$

Optimal. Leaf size=310

$$\frac{\sqrt[4]{5}(\sqrt{5x^2+2})\sqrt{\frac{5x^4+4}{(\sqrt{5x^2+2})^2}}(\sqrt{5a+2b})\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{5}x}{\sqrt{2}}\right), \frac{1}{2}\right)}{2\sqrt{2}\sqrt{5x^4+4}(5a^2-4b^2)} + \frac{\sqrt{b}\tan^{-1}\left(\frac{x\sqrt{5a^2+4b^2}}{\sqrt{a}\sqrt{b}\sqrt{5x^4+4}}\right)}{2\sqrt{a}\sqrt{5a^2+4b^2}} - \frac{(\sqrt{5x^2+2})\sqrt{\frac{5x^4+4}{(\sqrt{5x^2+2})^2}}}{4\sqrt{2}\sqrt[4]{5}}$$

[Out] (Sqrt[b]*ArcTan[(Sqrt[5*a^2 + 4*b^2]*x)/(Sqrt[a]*Sqrt[b]*Sqrt[4 + 5*x^4])]) / (2*Sqrt[a]*Sqrt[5*a^2 + 4*b^2]) + (5^(1/4)*(Sqrt[5]*a + 2*b)*(2 + Sqrt[5]*x^2)*Sqrt[(4 + 5*x^4)/(2 + Sqrt[5]*x^2)^2]*EllipticF[2*ArcTan[(5^(1/4)*x)/Sqrt[2]], 1/2]) / (2*Sqrt[2]*(5*a^2 - 4*b^2)*Sqrt[4 + 5*x^4]) - ((Sqrt[5]*a + 2*b)^2*(2 + Sqrt[5]*x^2)*Sqrt[(4 + 5*x^4)/(2 + Sqrt[5]*x^2)^2]*EllipticPi[-(Sqrt[5]*a - 2*b)^2/(8*Sqrt[5]*a*b), 2*ArcTan[(5^(1/4)*x)/Sqrt[2]], 1/2]) / (4*Sqrt[2]*5^(1/4)*a*(5*a^2 - 4*b^2)*Sqrt[4 + 5*x^4])

Rubi [A] time = 0.272687, antiderivative size = 310, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1217, 220, 1707}

$$\frac{\sqrt{b}\tan^{-1}\left(\frac{x\sqrt{5a^2+4b^2}}{\sqrt{a}\sqrt{b}\sqrt{5x^4+4}}\right)}{2\sqrt{a}\sqrt{5a^2+4b^2}} + \frac{\sqrt[4]{5}(\sqrt{5x^2+2})\sqrt{\frac{5x^4+4}{(\sqrt{5x^2+2})^2}}(\sqrt{5a+2b})F\left(2\tan^{-1}\left(\frac{\sqrt[4]{5}x}{\sqrt{2}}\right), \frac{1}{2}\right)}{2\sqrt{2}\sqrt{5x^4+4}(5a^2-4b^2)} - \frac{(\sqrt{5x^2+2})\sqrt{\frac{5x^4+4}{(\sqrt{5x^2+2})^2}}}{4\sqrt{2}\sqrt[4]{5}}$$

Antiderivative was successfully verified.

[In] Int[1/((a + b*x^2)*Sqrt[4 + 5*x^4]), x]

[Out] (Sqrt[b]*ArcTan[(Sqrt[5*a^2 + 4*b^2]*x)/(Sqrt[a]*Sqrt[b]*Sqrt[4 + 5*x^4])]) / (2*Sqrt[a]*Sqrt[5*a^2 + 4*b^2]) + (5^(1/4)*(Sqrt[5]*a + 2*b)*(2 + Sqrt[5]*x^2)*Sqrt[(4 + 5*x^4)/(2 + Sqrt[5]*x^2)^2]*EllipticF[2*ArcTan[(5^(1/4)*x)/Sqrt[2]], 1/2]) / (2*Sqrt[2]*(5*a^2 - 4*b^2)*Sqrt[4 + 5*x^4]) - ((Sqrt[5]*a + 2*b)^2*(2 + Sqrt[5]*x^2)*Sqrt[(4 + 5*x^4)/(2 + Sqrt[5]*x^2)^2]*EllipticPi[-(Sqrt[5]*a - 2*b)^2/(8*Sqrt[5]*a*b), 2*ArcTan[(5^(1/4)*x)/Sqrt[2]], 1/2]) / (4*Sqrt[2]*5^(1/4)*a*(5*a^2 - 4*b^2)*Sqrt[4 + 5*x^4])

Rule 1217

Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] := With[{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4], x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]

Rule 220

Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] := With[{q = Rt[b/a, 4]}, Simp[(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x], 1/2]) / (2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]

Rule 1707

Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] := With[{q = Rt[B/A, 2]}, -Simp[(B*d - A*e)*ArcTan[(Rt[(c*d)/e

+ (a*e)/d, 2]*x)/Sqrt[a + c*x^4]]/(2*d*e*Rt[(c*d)/e + (a*e)/d, 2]), x] +
 Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a + c*x^4))/(a*(A + B*x^2)^2)]*EllipticPi[Cancel[-((B*d - A*e)^2/(4*d*e*A*B))], 2]*ArcTan[q*x], 1/2])/(4*d*e*A
 *q*Sqrt[a + c*x^4]), x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
 ^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]

Rubi steps

$$\int \frac{1}{(a + bx^2)\sqrt{4 + 5x^4}} dx = -\frac{(2b(\sqrt{5}a + 2b)) \int \frac{1 + \frac{\sqrt{5}x^2}{2}}{(a+bx^2)\sqrt{4+5x^4}} dx}{5a^2 - 4b^2} + \frac{(5a + 2\sqrt{5}b) \int \frac{1}{\sqrt{4+5x^4}} dx}{5a^2 - 4b^2}$$

$$= \frac{\sqrt{b} \tan^{-1}\left(\frac{\sqrt{5a^2+4b^2}x}{\sqrt{a}\sqrt{b}\sqrt{4+5x^4}}\right)}{2\sqrt{a}\sqrt{5a^2+4b^2}} + \frac{\sqrt[4]{5}(\sqrt{5}a + 2b)(2 + \sqrt{5}x^2) \sqrt{\frac{4+5x^4}{(2+\sqrt{5}x^2)^2}} F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{5}x}{\sqrt{2}}\right) \middle| \frac{1}{2}\right)}{2\sqrt{2}(5a^2 - 4b^2)\sqrt{4 + 5x^4}} - \dots$$

Mathematica [C] time = 0.103303, size = 50, normalized size = 0.16

$$\frac{\left(\frac{1}{2} + \frac{i}{2}\right) \Pi\left(-\frac{2ib}{\sqrt{5a}}; i \sinh^{-1}\left(\left(\frac{1}{2} + \frac{i}{2}\right) \sqrt[4]{5x}\right) \middle| -1\right)}{\sqrt[4]{5a}}$$

Antiderivative was successfully verified.

[In] Integrate[1/((a + b*x^2)*Sqrt[4 + 5*x^4]),x]

[Out] ((-1/2 - I/2)*EllipticPi[(-2*I)*b]/(Sqrt[5]*a), I*ArcSinh[(1/2 + I/2)*5^(1/4)*x], -1)]/(5^(1/4)*a)

Maple [C] time = 0.382, size = 86, normalized size = 0.3

$$\frac{1}{a\sqrt{\frac{i}{2}\sqrt{5}}}\sqrt{1 - \frac{i}{2}x^2\sqrt{5}}\sqrt{1 + \frac{i}{2}x^2\sqrt{5}}\text{EllipticPi}\left(\sqrt{\frac{i}{2}\sqrt{5}x}, \frac{\frac{2i}{5}\sqrt{5}b}{a}, \sqrt{\frac{-i}{2}\sqrt{5}}\right)\frac{1}{\sqrt{5x^4 + 4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b*x^2+a)/(5*x^4+4)^(1/2),x)

[Out] 1/a/(1/2*I*5^(1/2))^(1/2)*(1-1/2*I*x^2*5^(1/2))^(1/2)*(1+1/2*I*x^2*5^(1/2))^(1/2)/(5*x^4+4)^(1/2)*EllipticPi((1/2*I*5^(1/2))^(1/2)*x,2/5*I*5^(1/2)*b/a,(-1/2*I*5^(1/2))^(1/2)/(1/2*I*5^(1/2))^(1/2))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{5x^4 + 4}(bx^2 + a)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x^2+a)/(5*x^4+4)^(1/2),x, algorithm="maxima")

[Out] integrate(1/(sqrt(5*x^4 + 4)*(b*x^2 + a)), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{5x^4 + 4}}{5bx^6 + 5ax^4 + 4bx^2 + 4a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x^2+a)/(5*x^4+4)^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(5*x^4 + 4)/(5*b*x^6 + 5*a*x^4 + 4*b*x^2 + 4*a), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a + bx^2)\sqrt{5x^4 + 4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x**2+a)/(5*x**4+4)**(1/2),x)

[Out] Integral(1/((a + b*x**2)*sqrt(5*x**4 + 4)), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{5x^4 + 4}(bx^2 + a)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x^2+a)/(5*x^4+4)^(1/2),x, algorithm="giac")

[Out] integrate(1/(sqrt(5*x^4 + 4)*(b*x^2 + a)), x)

$$3.172 \quad \int \frac{1}{(a+bx^2)\sqrt{4-dx^4}} dx$$

Optimal. Leaf size=40

$$\frac{\Pi\left(-\frac{2b}{a\sqrt{d}}; \sin^{-1}\left(\frac{\sqrt[4]{dx}}{\sqrt{2}}\right) \middle| -1\right)}{\sqrt{2}a\sqrt[4]{d}}$$

[Out] EllipticPi[(-2*b)/(a*Sqrt[d]), ArcSin[(d^(1/4)*x)/Sqrt[2]], -1]/(Sqrt[2]*a*d^(1/4))

Rubi [A] time = 0.0179991, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {1218}

$$\frac{\Pi\left(-\frac{2b}{a\sqrt{d}}; \sin^{-1}\left(\frac{\sqrt[4]{dx}}{\sqrt{2}}\right) \middle| -1\right)}{\sqrt{2}a\sqrt[4]{d}}$$

Antiderivative was successfully verified.

[In] Int[1/((a + b*x^2)*Sqrt[4 - d*x^4]),x]

[Out] EllipticPi[(-2*b)/(a*Sqrt[d]), ArcSin[(d^(1/4)*x)/Sqrt[2]], -1]/(Sqrt[2]*a*d^(1/4))

Rule 1218

Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]

Rubi steps

$$\int \frac{1}{(a+bx^2)\sqrt{4-dx^4}} dx = \frac{\Pi\left(-\frac{2b}{a\sqrt{d}}; \sin^{-1}\left(\frac{\sqrt[4]{dx}}{\sqrt{2}}\right) \middle| -1\right)}{\sqrt{2}a\sqrt[4]{d}}$$

Mathematica [C] time = 0.125321, size = 59, normalized size = 1.48

$$\frac{i\Pi\left(-\frac{2b}{a\sqrt{d}}; i \sinh^{-1}\left(\frac{\sqrt{-\sqrt{dx}}}{\sqrt{2}}\right) \middle| -1\right)}{\sqrt{2}a\sqrt{-\sqrt{d}}}$$

Antiderivative was successfully verified.

[In] Integrate[1/((a + b*x^2)*Sqrt[4 - d*x^4]),x]

[Out] ((-I)*EllipticPi[(-2*b)/(a*Sqrt[d]), I*ArcSinh[(Sqrt[-Sqrt[d]]*x)/Sqrt[2]], -1])/(Sqrt[2]*a*Sqrt[-Sqrt[d]])

Maple [B] time = 0.095, size = 78, normalized size = 2.

$$\frac{\sqrt{2}}{a} \sqrt{1 - \frac{x^2}{2} \sqrt{d}} \sqrt{1 + \frac{x^2}{2} \sqrt{d}} \operatorname{EllipticPi} \left(\frac{x\sqrt{2}}{2} \sqrt[4]{d}, -2 \frac{b}{a\sqrt{d}}, \sqrt{2} \sqrt{-\frac{1}{2} \sqrt{d} \frac{1}{\sqrt[4]{d}}} \right) \frac{1}{\sqrt[4]{d}} \frac{1}{\sqrt{-dx^4 + 4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b*x^2+a)/(-d*x^4+4)^(1/2), x)

[Out] 1/a*2^(1/2)/d^(1/4)*(1-1/2*x^2*d^(1/2))^(1/2)*(1+1/2*x^2*d^(1/2))^(1/2)/(-d*x^4+4)^(1/2)*EllipticPi(1/2*d^(1/4)*x*2^(1/2), -2*b/a/d^(1/2), (-1/2*d^(1/2))^(1/2)*2^(1/2)/d^(1/4))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-dx^4 + 4}(bx^2 + a)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x^2+a)/(-d*x^4+4)^(1/2), x, algorithm="maxima")

[Out] integrate(1/(sqrt(-d*x^4 + 4)*(b*x^2 + a)), x)

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x^2+a)/(-d*x^4+4)^(1/2), x, algorithm="fricas")

[Out] Timed out

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a + bx^2) \sqrt{-dx^4 + 4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x**2+a)/(-d*x**4+4)**(1/2), x)

[Out] Integral(1/((a + b*x**2)*sqrt(-d*x**4 + 4)), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-dx^4 + 4}(bx^2 + a)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(-d*x^4+4)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-d*x^4 + 4)*(b*x^2 + a)), x)
```

$$3.173 \quad \int \frac{1}{(a+bx^2)\sqrt{4+dx^4}} dx$$

Optimal. Leaf size=300

$$\frac{\sqrt[4]{d}(\sqrt{dx^2+2})\sqrt{\frac{dx^4+4}{(\sqrt{dx^2+2})^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt{2}}\right), \frac{1}{2}\right)}{2\sqrt{2}\sqrt{dx^4+4}(2b-a\sqrt{d})} + \frac{\sqrt{b}\tan^{-1}\left(\frac{x\sqrt{a^2d+4b^2}}{\sqrt{a}\sqrt{b}\sqrt{dx^4+4}}\right)}{2\sqrt{a}\sqrt{a^2d+4b^2}} + \frac{(\sqrt{dx^2+2})\sqrt{\frac{dx^4+4}{(\sqrt{dx^2+2})^2}}(a\sqrt{d}+2b)\Pi\left(-\frac{d\sqrt{dx^2+2}}{2\sqrt{a}\sqrt{a^2d+4b^2}}\right)}{4\sqrt{2}a\sqrt[4]{d}\sqrt{dx^4+4}}$$

[Out] (Sqrt[b]*ArcTan[(Sqrt[4*b^2 + a^2*d]*x)/(Sqrt[a]*Sqrt[b]*Sqrt[4 + d*x^4])]) / (2*Sqrt[a]*Sqrt[4*b^2 + a^2*d]) - (d^(1/4)*(2 + Sqrt[d]*x^2)*Sqrt[(4 + d*x^4)/(2 + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/Sqrt[2]], 1/2]) / (2*Sqrt[2]*(2*b - a*Sqrt[d])*Sqrt[4 + d*x^4]) + ((2*b + a*Sqrt[d])*(2 + Sqrt[d]*x^2)*Sqrt[(4 + d*x^4)/(2 + Sqrt[d]*x^2)^2]*EllipticPi[-(2*b - a*Sqrt[d])^2/(8*a*b*Sqrt[d]), 2*ArcTan[(d^(1/4)*x)/Sqrt[2]], 1/2]) / (4*Sqrt[2]*a*(2*b - a*Sqrt[d])*d^(1/4)*Sqrt[4 + d*x^4])

Rubi [A] time = 0.223249, antiderivative size = 300, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1217, 220, 1707}

$$\frac{\sqrt{b}\tan^{-1}\left(\frac{x\sqrt{a^2d+4b^2}}{\sqrt{a}\sqrt{b}\sqrt{dx^4+4}}\right)}{2\sqrt{a}\sqrt{a^2d+4b^2}} - \frac{\sqrt[4]{d}(\sqrt{dx^2+2})\sqrt{\frac{dx^4+4}{(\sqrt{dx^2+2})^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt{2}}\right), \frac{1}{2}\right)}{2\sqrt{2}\sqrt{dx^4+4}(2b-a\sqrt{d})} + \frac{(\sqrt{dx^2+2})\sqrt{\frac{dx^4+4}{(\sqrt{dx^2+2})^2}}(a\sqrt{d}+2b)\Pi\left(-\frac{d\sqrt{dx^2+2}}{2\sqrt{a}\sqrt{a^2d+4b^2}}\right)}{4\sqrt{2}a\sqrt[4]{d}\sqrt{dx^4+4}}$$

Antiderivative was successfully verified.

[In] Int[1/((a + b*x^2)*Sqrt[4 + d*x^4]), x]

[Out] (Sqrt[b]*ArcTan[(Sqrt[4*b^2 + a^2*d]*x)/(Sqrt[a]*Sqrt[b]*Sqrt[4 + d*x^4])]) / (2*Sqrt[a]*Sqrt[4*b^2 + a^2*d]) - (d^(1/4)*(2 + Sqrt[d]*x^2)*Sqrt[(4 + d*x^4)/(2 + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/Sqrt[2]], 1/2]) / (2*Sqrt[2]*(2*b - a*Sqrt[d])*Sqrt[4 + d*x^4]) + ((2*b + a*Sqrt[d])*(2 + Sqrt[d]*x^2)*Sqrt[(4 + d*x^4)/(2 + Sqrt[d]*x^2)^2]*EllipticPi[-(2*b - a*Sqrt[d])^2/(8*a*b*Sqrt[d]), 2*ArcTan[(d^(1/4)*x)/Sqrt[2]], 1/2]) / (4*Sqrt[2]*a*(2*b - a*Sqrt[d])*d^(1/4)*Sqrt[4 + d*x^4])

Rule 1217

Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] := With[{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4], x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]

Rule 220

Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] := With[{q = Rt[b/a, 4]}, Simp[(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2]*EllipticF[2*ArcTan[q*x], 1/2]) / (2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]

Rule 1707

Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] := With[{q = Rt[B/A, 2]}, -Simp[(B*d - A*e)*ArcTan[(Rt[(c*d)/e

+ (a*e)/d, 2]*x)/Sqrt[a + c*x^4]]/(2*d*e*Rt[(c*d)/e + (a*e)/d, 2]), x] +
 Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a + c*x^4))/(a*(A + B*x^2)^2)]*EllipticPi[Cancel[-((B*d - A*e)^2/(4*d*e*A*B))], 2]*ArcTan[q*x], 1/2])/(4*d*e*A*q*Sqrt[a + c*x^4]), x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]

Rubi steps

$$\int \frac{1}{(a + bx^2)\sqrt{4 + dx^4}} dx = \frac{(2b) \int \frac{1 + \frac{\sqrt{dx^2}}{2}}{(a+bx^2)\sqrt{4+dx^4}} dx}{2b - a\sqrt{d}} - \frac{\sqrt{d} \int \frac{1}{\sqrt{4+dx^4}} dx}{2b - a\sqrt{d}}$$

$$= \frac{\sqrt{b} \tan^{-1}\left(\frac{\sqrt{4b^2+a^2} dx}{\sqrt{a}\sqrt{b}\sqrt{4+dx^4}}\right)}{2\sqrt{a}\sqrt{4b^2+a^2}d} - \frac{\sqrt[4]{d}(2 + \sqrt{dx^2}) \sqrt{\frac{4+dx^4}{(2+\sqrt{dx^2})^2}} F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{dx}}{\sqrt{2}}\right) \middle| \frac{1}{2}\right)}{2\sqrt{2}(2b - a\sqrt{d})\sqrt{4 + dx^4}} + \frac{(2b + a\sqrt{d})(2 - \sqrt{dx^2})}{2\sqrt{2}(2b - a\sqrt{d})\sqrt{4 + dx^4}}$$

Mathematica [C] time = 0.111654, size = 65, normalized size = 0.22

$$\frac{i\pi \left(-\frac{2ib}{a\sqrt{d}}; i \sinh^{-1}\left(\frac{\sqrt{i\sqrt{d}x}}{\sqrt{2}}\right) \middle| -1 \right)}{\sqrt{2}a\sqrt{i\sqrt{d}}}$$

Antiderivative was successfully verified.

[In] Integrate[1/((a + b*x^2)*Sqrt[4 + d*x^4]),x]

[Out] ((-I)*EllipticPi[((-2*I)*b)/(a*Sqrt[d]), I*ArcSinh[(Sqrt[I*Sqrt[d]]*x)/Sqrt[2]], -1])/(Sqrt[2]*a*Sqrt[I*Sqrt[d]])

Maple [C] time = 0.023, size = 86, normalized size = 0.3

$$\frac{1}{a} \sqrt{1 - \frac{i}{2}\sqrt{dx^2}} \sqrt{1 + \frac{i}{2}\sqrt{dx^2}} \text{EllipticPi}\left(\sqrt{\frac{i}{2}\sqrt{dx}}, \frac{2ib}{a} \frac{1}{\sqrt{d}}, \sqrt{-\frac{i}{2}\sqrt{d}} \frac{1}{\sqrt{\frac{i}{2}\sqrt{d}}}\right) \frac{1}{\sqrt{\frac{i}{2}\sqrt{d}}} \frac{1}{\sqrt{dx^4 + 4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b*x^2+a)/(d*x^4+4)^(1/2),x)

[Out] 1/a/(1/2*I*d^(1/2))^(1/2)*(1-1/2*I*d^(1/2)*x^2)^(1/2)*(1+1/2*I*d^(1/2)*x^2)^(1/2)/(d*x^4+4)^(1/2)*EllipticPi((1/2*I*d^(1/2))^(1/2)*x,2*I/d^(1/2)*b/a,(-1/2*I*d^(1/2))^(1/2)/(1/2*I*d^(1/2))^(1/2))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{dx^4 + 4}(bx^2 + a)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x^2+a)/(d*x^4+4)^(1/2),x, algorithm="maxima")

[Out] integrate(1/(sqrt(d*x^4 + 4)*(b*x^2 + a)), x)

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x^2+a)/(d*x^4+4)^(1/2),x, algorithm="fricas")

[Out] Timed out

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a + bx^2)\sqrt{dx^4 + 4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x**2+a)/(d*x**4+4)**(1/2),x)

[Out] Integral(1/((a + b*x**2)*sqrt(d*x**4 + 4)), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{dx^4 + 4}(bx^2 + a)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x^2+a)/(d*x^4+4)^(1/2),x, algorithm="giac")

[Out] integrate(1/(sqrt(d*x^4 + 4)*(b*x^2 + a)), x)

$$3.174 \quad \int \frac{\sqrt{a+bx^2}}{\sqrt{1-x^4}} dx$$

Optimal. Leaf size=112

$$\frac{a\sqrt{1-x^2}\sqrt{\frac{a(x^2+1)}{a+bx^2}}\Pi\left(\frac{b}{a+b};\sin^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{bx^2+a}}\right)\middle|-\frac{a-b}{a+b}\right)}{\sqrt{x^2+1}\sqrt{a+b}\sqrt{\frac{a(1-x^2)}{a+bx^2}}}$$

[Out] (a*Sqrt[1 - x^2]*Sqrt[(a*(1 + x^2))/(a + b*x^2)]*EllipticPi[b/(a + b), ArcSin[(Sqrt[a + b]*x)/Sqrt[a + b*x^2]], -((a - b)/(a + b))])/(Sqrt[a + b]*Sqrt[1 + x^2]*Sqrt[(a*(1 - x^2))/(a + b*x^2)])

Rubi [F] time = 0.0094724, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{1-x^4}} dx$$

Verification is Not applicable to the result.

[In] Int[Sqrt[a + b*x^2]/Sqrt[1 - x^4], x]

[Out] Defer[Int][Sqrt[a + b*x^2]/Sqrt[1 - x^4], x]

Rubi steps

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{1-x^4}} dx = \int \frac{\sqrt{a+bx^2}}{\sqrt{1-x^4}} dx$$

Mathematica [F] time = 0.0598551, size = 0, normalized size = 0.

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{1-x^4}} dx$$

Verification is Not applicable to the result.

[In] Integrate[Sqrt[a + b*x^2]/Sqrt[1 - x^4], x]

[Out] Integrate[Sqrt[a + b*x^2]/Sqrt[1 - x^4], x]

Maple [F] time = 0.144, size = 0, normalized size = 0.

$$\int \sqrt{bx^2 + a} \frac{1}{\sqrt{-x^4 + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^2+a)^(1/2)/(-x^4+1)^(1/2),x)`

[Out] `int((b*x^2+a)^(1/2)/(-x^4+1)^(1/2),x)`

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx^2 + a}}{\sqrt{-x^4 + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2+a)^(1/2)/(-x^4+1)^(1/2),x, algorithm="maxima")`

[Out] `integrate(sqrt(b*x^2 + a)/sqrt(-x^4 + 1), x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-x^4 + 1}\sqrt{bx^2 + a}}{x^4 - 1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2+a)^(1/2)/(-x^4+1)^(1/2),x, algorithm="fricas")`

[Out] `integral(-sqrt(-x^4 + 1)*sqrt(b*x^2 + a)/(x^4 - 1), x)`

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a + bx^2}}{\sqrt{-(x-1)(x+1)(x^2+1)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**2+a)**(1/2)/(-x**4+1)**(1/2),x)`

[Out] `Integral(sqrt(a + b*x**2)/sqrt(-(x - 1)*(x + 1)*(x**2 + 1)), x)`

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx^2 + a}}{\sqrt{-x^4 + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2+a)^(1/2)/(-x^4+1)^(1/2),x, algorithm="giac")`

[Out] `integrate(sqrt(b*x^2 + a)/sqrt(-x^4 + 1), x)`

$$3.175 \quad \int (c + ex^2)^q (a + bx^4)^p dx$$

Optimal. Leaf size=21

$$\text{Unintegrable}\left(\left(a + bx^4\right)^p \left(c + ex^2\right)^q, x\right)$$

[Out] Defer[Int][(c + e*x^2)^q*(a + b*x^4)^p, x]

Rubi [A] time = 0.0080599, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int (c + ex^2)^q (a + bx^4)^p dx$$

Verification is Not applicable to the result.

[In] Int[(c + e*x^2)^q*(a + b*x^4)^p,x]

[Out] Defer[Int][(c + e*x^2)^q*(a + b*x^4)^p, x]

Rubi steps

$$\int (c + ex^2)^q (a + bx^4)^p dx = \int (c + ex^2)^q (a + bx^4)^p dx$$

Mathematica [A] time = 0.0762001, size = 0, normalized size = 0.

$$\int (c + ex^2)^q (a + bx^4)^p dx$$

Verification is Not applicable to the result.

[In] Integrate[(c + e*x^2)^q*(a + b*x^4)^p,x]

[Out] Integrate[(c + e*x^2)^q*(a + b*x^4)^p, x]

Maple [A] time = 0.102, size = 0, normalized size = 0.

$$\int (ex^2 + c)^q (bx^4 + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x^2+c)^q*(b*x^4+a)^p,x)

[Out] int((e*x^2+c)^q*(b*x^4+a)^p,x)

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int (bx^4 + a)^p (ex^2 + c)^q dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+c)^q*(b*x^4+a)^p,x, algorithm="maxima")

[Out] integrate((b*x^4 + a)^p*(e*x^2 + c)^q, x)

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(bx^4 + a\right)^p \left(ex^2 + c\right)^q, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+c)^q*(b*x^4+a)^p,x, algorithm="fricas")

[Out] integral((b*x^4 + a)^p*(e*x^2 + c)^q, x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x**2+c)**q*(b*x**4+a)**p,x)

[Out] Timed out

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int (bx^4 + a)^p (ex^2 + c)^q dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+c)^q*(b*x^4+a)^p,x, algorithm="giac")

[Out] integrate((b*x^4 + a)^p*(e*x^2 + c)^q, x)

3.176 $\int (c + ex^2)^3 (a + bx^4)^p dx$

Optimal. Leaf size=204

$$\frac{ex^3 (a + bx^4)^p \left(\frac{bx^4}{a} + 1\right)^{-p} (ae^2 - bc^2(4p + 7)) {}_2F_1\left(\frac{3}{4}, -p; \frac{7}{4}; -\frac{bx^4}{a}\right)}{b(4p + 7)} + c^3 x (a + bx^4)^p \left(\frac{bx^4}{a} + 1\right)^{-p} {}_2F_1\left(\frac{1}{4}, -p; \frac{5}{4}; -\frac{bx^4}{a}\right) +$$

[Out] (e³x³(a + b*x⁴)^(1 + p))/(b*(7 + 4*p)) + (c³*x*(a + b*x⁴)^p*Hypergeometric2F1[1/4, -p, 5/4, -((b*x⁴)/a)]/(1 + (b*x⁴)/a)^p - (e*(a*e² - b*c²*2*(7 + 4*p))*x³(a + b*x⁴)^p*Hypergeometric2F1[3/4, -p, 7/4, -((b*x⁴)/a)])/(b*(7 + 4*p)*(1 + (b*x⁴)/a)^p + (3*c*e²*x⁵(a + b*x⁴)^p*Hypergeometric2F1[5/4, -p, 9/4, -((b*x⁴)/a)]/(5*(1 + (b*x⁴)/a)^p)

Rubi [A] time = 0.230033, antiderivative size = 196, normalized size of antiderivative = 0.96, number of steps used = 9, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {1207, 1893, 246, 245, 365, 364}

$$ex^3 (a + bx^4)^p \left(\frac{bx^4}{a} + 1\right)^{-p} \left(c^2 - \frac{ae^2}{4bp + 7b}\right) {}_2F_1\left(\frac{3}{4}, -p; \frac{7}{4}; -\frac{bx^4}{a}\right) + c^3 x (a + bx^4)^p \left(\frac{bx^4}{a} + 1\right)^{-p} {}_2F_1\left(\frac{1}{4}, -p; \frac{5}{4}; -\frac{bx^4}{a}\right) + \frac{3}{5}$$

Antiderivative was successfully verified.

[In] Int[(c + e*x²)³(a + b*x⁴)^p, x]

[Out] (e³x³(a + b*x⁴)^(1 + p))/(b*(7 + 4*p)) + (c³*x*(a + b*x⁴)^p*Hypergeometric2F1[1/4, -p, 5/4, -((b*x⁴)/a)]/(1 + (b*x⁴)/a)^p + (e*(c² - (a*e²)/(7*b + 4*b*p))*x³(a + b*x⁴)^p*Hypergeometric2F1[3/4, -p, 7/4, -((b*x⁴)/a)])/(1 + (b*x⁴)/a)^p + (3*c*e²*x⁵(a + b*x⁴)^p*Hypergeometric2F1[5/4, -p, 9/4, -((b*x⁴)/a)]/(5*(1 + (b*x⁴)/a)^p)

Rule 1207

Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] := Simp[(e^q*x^(2*q - 3)(a + c*x⁴)^(p + 1))/(c*(4*p + 2*q + 1)), x] + Dist[1/(c*(4*p + 2*q + 1)), Int[(a + c*x⁴)^p*ExpandToSum[c*(4*p + 2*q + 1)*(d + e*x²)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d² + a*e², 0] && IGtQ[q, 1]

Rule 1893

Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Int[ExpandIntegrand[Pq*(a + b*xⁿ)^p, x], x] /; FreeQ[{a, b, n, p}, x] && (PolyQ[Pq, x] || PolyQ[Pq, xⁿ])

Rule 246

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[(a^{IntPart[p]}(a + b*xⁿ)^{FracPart[p]})/(1 + (b*xⁿ)/a)^{FracPart[p]}, Int[(1 + (b*xⁿ)/a)^p, x], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])

Rule 245

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[a^p*x*Hypergeometric2F1[-p, 1/n, 1/n + 1, -((b*xⁿ)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p

, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] || GtQ[a, 0])

Rule 365

Int[((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && !(ILtQ[p, 0] || GtQ[a, 0])

Rule 364

Int[((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Simp[(a^p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])

Rubi steps

$$\begin{aligned} \int (c + ex^2)^3 (a + bx^4)^p dx &= \frac{e^3 x^3 (a + bx^4)^{1+p}}{b(7 + 4p)} + \frac{\int (a + bx^4)^p (bc^3(7 + 4p) - 3e(ae^2 - bc^2(7 + 4p))x^2 + 3bce^2(7 + 4p)) dx}{b(7 + 4p)} \\ &= \frac{e^3 x^3 (a + bx^4)^{1+p}}{b(7 + 4p)} + \frac{\int (bc^3(7 + 4p)(a + bx^4)^p + 3e(-ae^2 + bc^2(7 + 4p))x^2 (a + bx^4)^p + 3bce^2(7 + 4p)(a + bx^4)^p) dx}{b(7 + 4p)} \\ &= \frac{e^3 x^3 (a + bx^4)^{1+p}}{b(7 + 4p)} + c^3 \int (a + bx^4)^p dx + (3ce^2) \int x^4 (a + bx^4)^p dx + \left(3e \left(c^2 - \frac{ae^2}{7b + 4bp}\right) \int (a + bx^4)^p dx\right) \\ &= \frac{e^3 x^3 (a + bx^4)^{1+p}}{b(7 + 4p)} + \left(c^3 (a + bx^4)^p \left(1 + \frac{bx^4}{a}\right)^{-p}\right) \int \left(1 + \frac{bx^4}{a}\right)^p dx + \left(3ce^2 (a + bx^4)^p \left(1 + \frac{bx^4}{a}\right)^{-p}\right) \int \left(1 + \frac{bx^4}{a}\right)^p dx \\ &= \frac{e^3 x^3 (a + bx^4)^{1+p}}{b(7 + 4p)} + c^3 x (a + bx^4)^p \left(1 + \frac{bx^4}{a}\right)^{-p} {}_2F_1\left(\frac{1}{4}, -p; \frac{5}{4}; -\frac{bx^4}{a}\right) + e \left(c^2 - \frac{ae^2}{7b + 4bp}\right) \int (a + bx^4)^p dx \end{aligned}$$

Mathematica [A] time = 0.0666942, size = 136, normalized size = 0.67

$$\frac{1}{35} x (a + bx^4)^p \left(\frac{bx^4}{a} + 1\right)^{-p} \left(ex^2 \left(35c^2 {}_2F_1\left(\frac{3}{4}, -p; \frac{7}{4}; -\frac{bx^4}{a}\right) + ex^2 \left(21c {}_2F_1\left(\frac{5}{4}, -p; \frac{9}{4}; -\frac{bx^4}{a}\right) + 5ex^2 {}_2F_1\left(\frac{7}{4}, -p; \frac{11}{4}; -\frac{bx^4}{a}\right) \right) \right) \right) / (35(1 + (bx^4)/a)^p)$$

Antiderivative was successfully verified.

[In] Integrate[(c + e*x^2)^3*(a + b*x^4)^p,x]

[Out] (x*(a + b*x^4)^p*(35*c^3*Hypergeometric2F1[1/4, -p, 5/4, -((b*x^4)/a)] + e*x^2*(35*c^2*Hypergeometric2F1[3/4, -p, 7/4, -((b*x^4)/a)] + e*x^2*(21*c*Hypergeometric2F1[5/4, -p, 9/4, -((b*x^4)/a)] + 5*e*x^2*Hypergeometric2F1[7/4, -p, 11/4, -((b*x^4)/a)])))/(35*(1 + (b*x^4)/a)^p)

Maple [F] time = 0.051, size = 0, normalized size = 0.

$$\int (ex^2 + c)^3 (bx^4 + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*x^2+c)^3*(b*x^4+a)^p,x)`

[Out] `int((e*x^2+c)^3*(b*x^4+a)^p,x)`

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (ex^2 + c)^3 (bx^4 + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x^2+c)^3*(b*x^4+a)^p,x, algorithm="maxima")`

[Out] `integrate((e*x^2 + c)^3*(b*x^4 + a)^p, x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(e^3x^6 + 3ce^2x^4 + 3c^2ex^2 + c^3\right)(bx^4 + a)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x^2+c)^3*(b*x^4+a)^p,x, algorithm="fricas")`

[Out] `integral((e^3*x^6 + 3*c*e^2*x^4 + 3*c^2*e*x^2 + c^3)*(b*x^4 + a)^p, x)`

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x**2+c)**3*(b*x**4+a)**p,x)`

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (ex^2 + c)^3 (bx^4 + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x^2+c)^3*(b*x^4+a)^p,x, algorithm="giac")`

[Out] `integrate((e*x^2 + c)^3*(b*x^4 + a)^p, x)`

3.177 $\int (c + ex^2)^2 (a + bx^4)^p dx$

Optimal. Leaf size=150

$$\frac{x(a + bx^4)^p \left(\frac{bx^4}{a} + 1\right)^{-p} (ae^2 - bc^2(4p + 5)) {}_2F_1\left(\frac{1}{4}, -p; \frac{5}{4}; -\frac{bx^4}{a}\right)}{b(4p + 5)} + \frac{2}{3} cex^3 (a + bx^4)^p \left(\frac{bx^4}{a} + 1\right)^{-p} {}_2F_1\left(\frac{3}{4}, -p; \frac{7}{4}; -\frac{bx^4}{a}\right)$$

[Out] $(e^{2*x*(a + b*x^4)^{(1 + p)}})/(b*(5 + 4*p)) - ((a*e^2 - b*c^2*(5 + 4*p))*x*(a + b*x^4)^p*Hypergeometric2F1[1/4, -p, 5/4, -(b*x^4)/a])/(b*(5 + 4*p)*(1 + (b*x^4)/a)^p) + (2*c*e*x^3*(a + b*x^4)^p*Hypergeometric2F1[3/4, -p, 7/4, -(b*x^4)/a])/(3*(1 + (b*x^4)/a)^p)$

Rubi [A] time = 0.131657, antiderivative size = 142, normalized size of antiderivative = 0.95, number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {1207, 1204, 246, 245, 365, 364}

$$x(a + bx^4)^p \left(\frac{bx^4}{a} + 1\right)^{-p} \left(c^2 - \frac{ae^2}{4bp + 5b}\right) {}_2F_1\left(\frac{1}{4}, -p; \frac{5}{4}; -\frac{bx^4}{a}\right) + \frac{2}{3} cex^3 (a + bx^4)^p \left(\frac{bx^4}{a} + 1\right)^{-p} {}_2F_1\left(\frac{3}{4}, -p; \frac{7}{4}; -\frac{bx^4}{a}\right)$$

Antiderivative was successfully verified.

[In] Int[(c + e*x^2)^2*(a + b*x^4)^p,x]

[Out] $(e^{2*x*(a + b*x^4)^{(1 + p)}})/(b*(5 + 4*p)) + ((c^2 - (a*e^2)/(5*b + 4*b*p))*x*(a + b*x^4)^p*Hypergeometric2F1[1/4, -p, 5/4, -(b*x^4)/a])/(1 + (b*x^4)/a)^p + (2*c*e*x^3*(a + b*x^4)^p*Hypergeometric2F1[3/4, -p, 7/4, -(b*x^4)/a])/(3*(1 + (b*x^4)/a)^p)$

Rule 1207

Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] := Simp[(e^q*x^(2*q - 3)*(a + c*x^4)^(p + 1))/(c*(4*p + 2*q + 1)), x] + Dist[1/(c*(4*p + 2*q + 1)), Int[(a + c*x^4)^p*ExpandToSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[q, 1]

Rule 1204

Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] := Int[ExpandIntegrand[(d + e*x^2)*(a + c*x^4)^p, x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0]

Rule 246

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])

Rule 245

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[a^p*x*Hypergeometric2F1[-p, 1/n, 1/n + 1, -(b*x^n)/a], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||

GtQ[a, 0])

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
&& !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/((c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned} \int (c + ex^2)^2 (a + bx^4)^p dx &= \frac{e^2 x (a + bx^4)^{1+p}}{b(5 + 4p)} + \frac{\int (-ae^2 + bc^2(5 + 4p) + 2bce(5 + 4p)x^2) (a + bx^4)^p dx}{b(5 + 4p)} \\ &= \frac{e^2 x (a + bx^4)^{1+p}}{b(5 + 4p)} + \frac{\int \left(-ae^2 \left(1 - \frac{bc^2(5+4p)}{ae^2}\right) (a + bx^4)^p + 2bce(5 + 4p)x^2 (a + bx^4)^p\right) dx}{b(5 + 4p)} \\ &= \frac{e^2 x (a + bx^4)^{1+p}}{b(5 + 4p)} + (2ce) \int x^2 (a + bx^4)^p dx - \left(-c^2 + \frac{ae^2}{5b + 4bp}\right) \int (a + bx^4)^p dx \\ &= \frac{e^2 x (a + bx^4)^{1+p}}{b(5 + 4p)} + \left(2ce (a + bx^4)^p \left(1 + \frac{bx^4}{a}\right)^{-p}\right) \int x^2 \left(1 + \frac{bx^4}{a}\right)^p dx - \left(\left(-c^2 + \frac{ae^2}{5b + 4bp}\right)\right) \int (a + bx^4)^p dx \\ &= \frac{e^2 x (a + bx^4)^{1+p}}{b(5 + 4p)} + \left(c^2 - \frac{ae^2}{5b + 4bp}\right) x (a + bx^4)^p \left(1 + \frac{bx^4}{a}\right)^{-p} {}_2F_1\left(\frac{1}{4}, -p; \frac{5}{4}; -\frac{bx^4}{a}\right) + \frac{2}{3} cex^3 \end{aligned}$$

Mathematica [A] time = 0.0412566, size = 106, normalized size = 0.71

$$\frac{1}{15} x (a + bx^4)^p \left(\frac{bx^4}{a} + 1\right)^{-p} \left(15c^2 {}_2F_1\left(\frac{1}{4}, -p; \frac{5}{4}; -\frac{bx^4}{a}\right) + ex^2 \left(10c {}_2F_1\left(\frac{3}{4}, -p; \frac{7}{4}; -\frac{bx^4}{a}\right) + 3ex^2 {}_2F_1\left(\frac{5}{4}, -p; \frac{9}{4}; -\frac{bx^4}{a}\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + e*x^2)^2*(a + b*x^4)^p,x]
```

```
[Out] (x*(a + b*x^4)^p*(15*c^2*Hypergeometric2F1[1/4, -p, 5/4, -((b*x^4)/a)] + e*
x^2*(10*c*Hypergeometric2F1[3/4, -p, 7/4, -((b*x^4)/a)] + 3*e*x^2*Hypergeom
etric2F1[5/4, -p, 9/4, -((b*x^4)/a)]))/(15*(1 + (b*x^4)/a)^p)
```

Maple [F] time = 0.04, size = 0, normalized size = 0.

$$\int (ex^2 + c)^2 (bx^4 + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+c)^2*(b*x^4+a)^p,x)
```

[Out] $\text{int}((e*x^2+c)^2*(b*x^4+a)^p, x)$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (ex^2 + c)^2 (bx^4 + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}((e*x^2+c)^2*(b*x^4+a)^p, x, \text{algorithm}="maxima")$

[Out] $\text{integrate}((e*x^2 + c)^2*(b*x^4 + a)^p, x)$

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(e^2x^4 + 2cex^2 + c^2\right)(bx^4 + a)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}((e*x^2+c)^2*(b*x^4+a)^p, x, \text{algorithm}="fricas")$

[Out] $\text{integral}((e^2*x^4 + 2*c*e*x^2 + c^2)*(b*x^4 + a)^p, x)$

Sympy [C] time = 118.77, size = 119, normalized size = 0.79

$$\frac{a^p c^2 x \Gamma\left(\frac{1}{4}\right) {}_2F_1\left(\frac{1}{4}, -p \left| \frac{bx^4 e^{i\pi}}{a} \right.\right)}{4\Gamma\left(\frac{5}{4}\right)} + \frac{a^p c e x^3 \Gamma\left(\frac{3}{4}\right) {}_2F_1\left(\frac{3}{4}, -p \left| \frac{bx^4 e^{i\pi}}{a} \right.\right)}{2\Gamma\left(\frac{7}{4}\right)} + \frac{a^p e^2 x^5 \Gamma\left(\frac{5}{4}\right) {}_2F_1\left(\frac{5}{4}, -p \left| \frac{bx^4 e^{i\pi}}{a} \right.\right)}{4\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}((e*x**2+c)**2*(b*x**4+a)**p, x)$

[Out] $a**p*c**2*x*\text{gamma}(1/4)*\text{hyper}((1/4, -p), (5/4,), b*x**4*\text{exp_polar}(I*\text{pi})/a)/(4*\text{gamma}(5/4)) + a**p*c*e*x**3*\text{gamma}(3/4)*\text{hyper}((3/4, -p), (7/4,), b*x**4*\text{exp_polar}(I*\text{pi})/a)/(2*\text{gamma}(7/4)) + a**p*e**2*x**5*\text{gamma}(5/4)*\text{hyper}((5/4, -p), (9/4,), b*x**4*\text{exp_polar}(I*\text{pi})/a)/(4*\text{gamma}(9/4))$

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (ex^2 + c)^2 (bx^4 + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}((e*x^2+c)^2*(b*x^4+a)^p, x, \text{algorithm}="giac")$

[Out] $\text{integrate}((e*x^2 + c)^2*(b*x^4 + a)^p, x)$

3.178 $\int (c + ex^2) (a + bx^4)^p dx$

Optimal. Leaf size=96

$$cx(a + bx^4)^p \left(\frac{bx^4}{a} + 1\right)^{-p} {}_2F_1\left(\frac{1}{4}, -p; \frac{5}{4}; -\frac{bx^4}{a}\right) + \frac{1}{3}ex^3(a + bx^4)^p \left(\frac{bx^4}{a} + 1\right)^{-p} {}_2F_1\left(\frac{3}{4}, -p; \frac{7}{4}; -\frac{bx^4}{a}\right)$$

[Out] (c*x*(a + b*x^4)^p*Hypergeometric2F1[1/4, -p, 5/4, -((b*x^4)/a)]/(1 + (b*x^4)/a)^p + (e*x^3*(a + b*x^4)^p*Hypergeometric2F1[3/4, -p, 7/4, -((b*x^4)/a)])/(3*(1 + (b*x^4)/a)^p)

Rubi [A] time = 0.0507824, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {1204, 246, 245, 365, 364}

$$cx(a + bx^4)^p \left(\frac{bx^4}{a} + 1\right)^{-p} {}_2F_1\left(\frac{1}{4}, -p; \frac{5}{4}; -\frac{bx^4}{a}\right) + \frac{1}{3}ex^3(a + bx^4)^p \left(\frac{bx^4}{a} + 1\right)^{-p} {}_2F_1\left(\frac{3}{4}, -p; \frac{7}{4}; -\frac{bx^4}{a}\right)$$

Antiderivative was successfully verified.

[In] Int[(c + e*x^2)*(a + b*x^4)^p,x]

[Out] (c*x*(a + b*x^4)^p*Hypergeometric2F1[1/4, -p, 5/4, -((b*x^4)/a)]/(1 + (b*x^4)/a)^p + (e*x^3*(a + b*x^4)^p*Hypergeometric2F1[3/4, -p, 7/4, -((b*x^4)/a)])/(3*(1 + (b*x^4)/a)^p)

Rule 1204

Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] := Int[ExpandIntegrand[(d + e*x^2)*(a + c*x^4)^p, x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0]

Rule 246

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])

Rule 245

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[a^p*x*Hypergeometric2F1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] || GtQ[a, 0])

Rule 365

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && !(ILtQ[p, 0] || GtQ[a, 0])

Rule 364

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(a^p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a)

)]/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])

Rubi steps

$$\begin{aligned} \int (c + ex^2)(a + bx^4)^p dx &= \int \left(c(a + bx^4)^p + ex^2(a + bx^4)^p \right) dx \\ &= c \int (a + bx^4)^p dx + e \int x^2 (a + bx^4)^p dx \\ &= \left(c(a + bx^4)^p \left(1 + \frac{bx^4}{a} \right)^{-p} \right) \int \left(1 + \frac{bx^4}{a} \right)^p dx + \left(e(a + bx^4)^p \left(1 + \frac{bx^4}{a} \right)^{-p} \right) \int x^2 \left(1 + \frac{bx^4}{a} \right)^p dx \\ &= cx(a + bx^4)^p \left(1 + \frac{bx^4}{a} \right)^{-p} {}_2F_1 \left(\frac{1}{4}, -p; \frac{5}{4}; -\frac{bx^4}{a} \right) + \frac{1}{3} ex^3 (a + bx^4)^p \left(1 + \frac{bx^4}{a} \right)^{-p} {}_2F_1 \left(\frac{3}{4}, -p; \frac{7}{4}; -\frac{bx^4}{a} \right) \end{aligned}$$

Mathematica [A] time = 0.0227502, size = 75, normalized size = 0.78

$$\frac{1}{3} x (a + bx^4)^p \left(\frac{bx^4}{a} + 1 \right)^{-p} \left(3c {}_2F_1 \left(\frac{1}{4}, -p; \frac{5}{4}; -\frac{bx^4}{a} \right) + ex^2 {}_2F_1 \left(\frac{3}{4}, -p; \frac{7}{4}; -\frac{bx^4}{a} \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(c + e*x^2)*(a + b*x^4)^p,x]

[Out] (x*(a + b*x^4)^p*(3*c*Hypergeometric2F1[1/4, -p, 5/4, -(b*x^4)/a]) + e*x^2*Hypergeometric2F1[3/4, -p, 7/4, -(b*x^4)/a])/(3*(1 + (b*x^4)/a)^p)

Maple [F] time = 0.028, size = 0, normalized size = 0.

$$\int (ex^2 + c)(bx^4 + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x^2+c)*(b*x^4+a)^p,x)

[Out] int((e*x^2+c)*(b*x^4+a)^p,x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (ex^2 + c)(bx^4 + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+c)*(b*x^4+a)^p,x, algorithm="maxima")

[Out] integrate((e*x^2 + c)*(b*x^4 + a)^p, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((ex^2 + c)(bx^4 + a)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+c)*(b*x^4+a)^p,x, algorithm="fricas")

[Out] integral((e*x^2 + c)*(b*x^4 + a)^p, x)

Sympy [C] time = 56.8967, size = 75, normalized size = 0.78

$$\frac{a^p c x \Gamma\left(\frac{1}{4}\right) {}_2F_1\left(\frac{1}{4}, -p \middle| \frac{bx^4 e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{5}{4}\right)} + \frac{a^p e x^3 \Gamma\left(\frac{3}{4}\right) {}_2F_1\left(\frac{3}{4}, -p \middle| \frac{bx^4 e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x**2+c)*(b*x**4+a)**p,x)

[Out] a**p*c*x*gamma(1/4)*hyper((1/4, -p), (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(5/4)) + a**p*e*x**3*gamma(3/4)*hyper((3/4, -p), (7/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(7/4))

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (ex^2 + c)(bx^4 + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+c)*(b*x^4+a)^p,x, algorithm="giac")

[Out] integrate((e*x^2 + c)*(b*x^4 + a)^p, x)

3.179 $\int (a + bx^4)^p dx$

Optimal. Leaf size=44

$$x(a + bx^4)^p \left(\frac{bx^4}{a} + 1\right)^{-p} {}_2F_1\left(\frac{1}{4}, -p; \frac{5}{4}; -\frac{bx^4}{a}\right)$$

[Out] $(x*(a + b*x^4)^p*Hypergeometric2F1[1/4, -p, 5/4, -(b*x^4)/a])/(1 + (b*x^4)/a)^p$

Rubi [A] time = 0.0097065, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {246, 245}

$$x(a + bx^4)^p \left(\frac{bx^4}{a} + 1\right)^{-p} {}_2F_1\left(\frac{1}{4}, -p; \frac{5}{4}; -\frac{bx^4}{a}\right)$$

Antiderivative was successfully verified.

[In] Int[(a + b*x^4)^p,x]

[Out] $(x*(a + b*x^4)^p*Hypergeometric2F1[1/4, -p, 5/4, -(b*x^4)/a])/(1 + (b*x^4)/a)^p$

Rule 246

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])

Rule 245

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[a^p*x*Hypergeometric2F1[-p, 1/n, 1/n + 1, -(b*x^n)/a], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] || GtQ[a, 0])

Rubi steps

$$\begin{aligned} \int (a + bx^4)^p dx &= \left((a + bx^4)^p \left(1 + \frac{bx^4}{a}\right)^{-p} \right) \int \left(1 + \frac{bx^4}{a}\right)^p dx \\ &= x(a + bx^4)^p \left(1 + \frac{bx^4}{a}\right)^{-p} {}_2F_1\left(\frac{1}{4}, -p; \frac{5}{4}; -\frac{bx^4}{a}\right) \end{aligned}$$

Mathematica [A] time = 0.0032691, size = 44, normalized size = 1.

$$x(a + bx^4)^p \left(\frac{bx^4}{a} + 1\right)^{-p} {}_2F_1\left(\frac{1}{4}, -p; \frac{5}{4}; -\frac{bx^4}{a}\right)$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x^4)^p,x]

[Out] (x*(a + b*x^4)^p*Hypergeometric2F1[1/4, -p, 5/4, -((b*x^4)/a)])/(1 + (b*x^4)/a)^p

Maple [F] time = 0.021, size = 0, normalized size = 0.

$$\int (bx^4 + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^4+a)^p,x)

[Out] int((b*x^4+a)^p,x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (bx^4 + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^4+a)^p,x, algorithm="maxima")

[Out] integrate((b*x^4 + a)^p, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(bx^4 + a\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^4+a)^p,x, algorithm="fricas")

[Out] integral((b*x^4 + a)^p, x)

Sympy [C] time = 12.4092, size = 34, normalized size = 0.77

$$\frac{a^p x \Gamma\left(\frac{1}{4}\right) {}_2F_1\left(\frac{1}{4}, -p \middle| \frac{bx^4 e^{i\pi}}{a}\right)}{4 \Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**4+a)**p,x)


```
[Out] a**p*x*gamma(1/4)*hyper((1/4, -p), (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(5/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (bx^4 + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^p,x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + a)^p, x)
```

$$3.180 \quad \int \frac{(a+bx^4)^p}{c+ex^2} dx$$

Optimal. Leaf size=123

$$\frac{x(a+bx^4)^p \left(\frac{bx^4}{a} + 1\right)^{-p} F_1\left(\frac{1}{4}; -p, 1; \frac{5}{4}; -\frac{bx^4}{a}, \frac{e^2x^4}{c^2}\right)}{c} - \frac{ex^3(a+bx^4)^p \left(\frac{bx^4}{a} + 1\right)^{-p} F_1\left(\frac{3}{4}; -p, 1; \frac{7}{4}; -\frac{bx^4}{a}, \frac{e^2x^4}{c^2}\right)}{3c^2}$$

[Out] (x*(a + b*x^4)^p*AppellF1[1/4, -p, 1, 5/4, -((b*x^4)/a), (e^2*x^4)/c^2])/(c*(1 + (b*x^4)/a)^p) - (e*x^3*(a + b*x^4)^p*AppellF1[3/4, -p, 1, 7/4, -((b*x^4)/a), (e^2*x^4)/c^2])/(3*c^2*(1 + (b*x^4)/a)^p)

Rubi [A] time = 0.126569, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {1240, 430, 429, 511, 510}

$$\frac{x(a+bx^4)^p \left(\frac{bx^4}{a} + 1\right)^{-p} F_1\left(\frac{1}{4}; -p, 1; \frac{5}{4}; -\frac{bx^4}{a}, \frac{e^2x^4}{c^2}\right)}{c} - \frac{ex^3(a+bx^4)^p \left(\frac{bx^4}{a} + 1\right)^{-p} F_1\left(\frac{3}{4}; -p, 1; \frac{7}{4}; -\frac{bx^4}{a}, \frac{e^2x^4}{c^2}\right)}{3c^2}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x^4)^p/(c + e*x^2), x]

[Out] (x*(a + b*x^4)^p*AppellF1[1/4, -p, 1, 5/4, -((b*x^4)/a), (e^2*x^4)/c^2])/(c*(1 + (b*x^4)/a)^p) - (e*x^3*(a + b*x^4)^p*AppellF1[3/4, -p, 1, 7/4, -((b*x^4)/a), (e^2*x^4)/c^2])/(3*c^2*(1 + (b*x^4)/a)^p)

Rule 1240

Int[((d_) + (e_)*(x_)^2)^(q_)*((a_) + (c_)*(x_)^4)^(p_), x_Symbol] := Int[ExpandIntegrand[(a + c*x^4)^p, (d/(d^2 - e^2*x^4) - (e*x^2)/(d^2 - e^2*x^4))^(-q), x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] && ! IntegerQ[p] && ILtQ[q, 0]

Rule 430

Int[((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_), x_Symbol] := Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] && !(IntegerQ[p] || GtQ[a, 0])

Rule 429

Int[((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_), x_Symbol] := Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])

Rule 511

Int[((e_)*(x_)^(m_))*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_), x_Symbol] := Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n - 1] && !(IntegerQ[p] || GtQ[a, 0])

Rule 510

Int[((e_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)]/(e*(m + 1)), x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])

Rubi steps

$$\begin{aligned} \int \frac{(a + bx^4)^p}{c + ex^2} dx &= \int \left(\frac{c(a + bx^4)^p}{c^2 - e^2x^4} + \frac{ex^2(a + bx^4)^p}{-c^2 + e^2x^4} \right) dx \\ &= c \int \frac{(a + bx^4)^p}{c^2 - e^2x^4} dx + e \int \frac{x^2(a + bx^4)^p}{-c^2 + e^2x^4} dx \\ &= \left(c(a + bx^4)^p \left(1 + \frac{bx^4}{a} \right)^{-p} \right) \int \frac{\left(1 + \frac{bx^4}{a} \right)^p}{c^2 - e^2x^4} dx + \left(e(a + bx^4)^p \left(1 + \frac{bx^4}{a} \right)^{-p} \right) \int \frac{x^2 \left(1 + \frac{bx^4}{a} \right)^p}{-c^2 + e^2x^4} dx \\ &= \frac{x(a + bx^4)^p \left(1 + \frac{bx^4}{a} \right)^{-p} F_1\left(\frac{1}{4}; -p, 1; \frac{5}{4}; -\frac{bx^4}{a}, \frac{e^2x^4}{c^2}\right)}{c} - \frac{ex^3(a + bx^4)^p \left(1 + \frac{bx^4}{a} \right)^{-p} F_1\left(\frac{3}{4}; -p, 1; \frac{7}{4}; -\frac{bx^4}{a}\right)}{3c^2} \end{aligned}$$

Mathematica [F] time = 0.127536, size = 0, normalized size = 0.

$$\int \frac{(a + bx^4)^p}{c + ex^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[(a + b*x^4)^p/(c + e*x^2), x]

[Out] Integrate[(a + b*x^4)^p/(c + e*x^2), x]

Maple [F] time = 0.042, size = 0, normalized size = 0.

$$\int \frac{(bx^4 + a)^p}{ex^2 + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^4+a)^p/(e*x^2+c), x)

[Out] int((b*x^4+a)^p/(e*x^2+c), x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^4 + a)^p}{ex^2 + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^4+a)^p/(e*x^2+c),x, algorithm="maxima")

[Out] integrate((b*x^4 + a)^p/(e*x^2 + c), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx^4 + a)^p}{ex^2 + c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^4+a)^p/(e*x^2+c),x, algorithm="fricas")

[Out] integral((b*x^4 + a)^p/(e*x^2 + c), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**4+a)**p/(e*x**2+c),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^4 + a)^p}{ex^2 + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^4+a)^p/(e*x^2+c),x, algorithm="giac")

[Out] integrate((b*x^4 + a)^p/(e*x^2 + c), x)

$$3.181 \quad \int \frac{(a+bx^4)^p}{(c+ex^2)^2} dx$$

Optimal. Leaf size=189

$$\frac{x(a+bx^4)^p \left(\frac{bx^4}{a} + 1\right)^{-p} F_1\left(\frac{1}{4}; -p, 2; \frac{5}{4}; -\frac{bx^4}{a}, \frac{e^2x^4}{c^2}\right)}{c^2} - \frac{2ex^3(a+bx^4)^p \left(\frac{bx^4}{a} + 1\right)^{-p} F_1\left(\frac{3}{4}; -p, 2; \frac{7}{4}; -\frac{bx^4}{a}, \frac{e^2x^4}{c^2}\right)}{3c^3} + \frac{e^2x^5(a+bx^4)^p \left(\frac{bx^4}{a} + 1\right)^{-p} F_1\left(\frac{5}{4}; -p, 2; \frac{9}{4}; -\frac{bx^4}{a}, \frac{e^2x^4}{c^2}\right)}{5c^4}$$

[Out] (x*(a + b*x^4)^p*AppellF1[1/4, -p, 2, 5/4, -((b*x^4)/a), (e^2*x^4)/c^2])/(c^2*(1 + (b*x^4)/a)^p) - (2*e*x^3*(a + b*x^4)^p*AppellF1[3/4, -p, 2, 7/4, -((b*x^4)/a), (e^2*x^4)/c^2])/(3*c^3*(1 + (b*x^4)/a)^p) + (e^2*x^5*(a + b*x^4)^p*AppellF1[5/4, -p, 2, 9/4, -((b*x^4)/a), (e^2*x^4)/c^2])/(5*c^4*(1 + (b*x^4)/a)^p)

Rubi [A] time = 0.194056, antiderivative size = 189, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {1240, 430, 429, 511, 510}

$$\frac{x(a+bx^4)^p \left(\frac{bx^4}{a} + 1\right)^{-p} F_1\left(\frac{1}{4}; -p, 2; \frac{5}{4}; -\frac{bx^4}{a}, \frac{e^2x^4}{c^2}\right)}{c^2} - \frac{2ex^3(a+bx^4)^p \left(\frac{bx^4}{a} + 1\right)^{-p} F_1\left(\frac{3}{4}; -p, 2; \frac{7}{4}; -\frac{bx^4}{a}, \frac{e^2x^4}{c^2}\right)}{3c^3} + \frac{e^2x^5(a+bx^4)^p \left(\frac{bx^4}{a} + 1\right)^{-p} F_1\left(\frac{5}{4}; -p, 2; \frac{9}{4}; -\frac{bx^4}{a}, \frac{e^2x^4}{c^2}\right)}{5c^4}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x^4)^p/(c + e*x^2)^2,x]

[Out] (x*(a + b*x^4)^p*AppellF1[1/4, -p, 2, 5/4, -((b*x^4)/a), (e^2*x^4)/c^2])/(c^2*(1 + (b*x^4)/a)^p) - (2*e*x^3*(a + b*x^4)^p*AppellF1[3/4, -p, 2, 7/4, -((b*x^4)/a), (e^2*x^4)/c^2])/(3*c^3*(1 + (b*x^4)/a)^p) + (e^2*x^5*(a + b*x^4)^p*AppellF1[5/4, -p, 2, 9/4, -((b*x^4)/a), (e^2*x^4)/c^2])/(5*c^4*(1 + (b*x^4)/a)^p)

Rule 1240

Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] := Int[ExpandIntegrand[(a + c*x^4)^p, (d/(d^2 - e^2*x^4) - (e*x^2)/(d^2 - e^2*x^4))^(-q), x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] && ! IntegerQ[p] && ILtQ[q, 0]

Rule 430

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] && !(IntegerQ[p] || GtQ[a, 0])

Rule 429

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n - 1] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)]/(e*(m + 1)), x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{(a + bx^4)^p}{(c + ex^2)^2} dx &= \int \left(\frac{c^2 (a + bx^4)^p}{(c^2 - e^2 x^4)^2} - \frac{2cex^2 (a + bx^4)^p}{(c^2 - e^2 x^4)^2} + \frac{e^2 x^4 (a + bx^4)^p}{(-c^2 + e^2 x^4)^2} \right) dx \\ &= c^2 \int \frac{(a + bx^4)^p}{(c^2 - e^2 x^4)^2} dx - (2ce) \int \frac{x^2 (a + bx^4)^p}{(c^2 - e^2 x^4)^2} dx + e^2 \int \frac{x^4 (a + bx^4)^p}{(-c^2 + e^2 x^4)^2} dx \\ &= \left(c^2 (a + bx^4)^p \left(1 + \frac{bx^4}{a} \right)^{-p} \right) \int \frac{\left(1 + \frac{bx^4}{a} \right)^p}{(c^2 - e^2 x^4)^2} dx - \left(2ce (a + bx^4)^p \left(1 + \frac{bx^4}{a} \right)^{-p} \right) \int \frac{x^2 \left(1 + \frac{bx^4}{a} \right)^p}{(c^2 - e^2 x^4)^2} dx + \left(e^2 (a + bx^4)^p \left(1 + \frac{bx^4}{a} \right)^{-p} \right) \int \frac{x^4 \left(1 + \frac{bx^4}{a} \right)^p}{(-c^2 + e^2 x^4)^2} dx \\ &= \frac{x (a + bx^4)^p \left(1 + \frac{bx^4}{a} \right)^{-p} F_1 \left(\frac{1}{4}; -p, 2; \frac{5}{4}; -\frac{bx^4}{a}, \frac{e^2 x^4}{c^2} \right)}{c^2} - \frac{2ex^3 (a + bx^4)^p \left(1 + \frac{bx^4}{a} \right)^{-p} F_1 \left(\frac{3}{4}; -p, 2; \frac{7}{4}; -\frac{bx^4}{a}, \frac{e^2 x^4}{c^2} \right)}{3c^3} \end{aligned}$$

Mathematica [F] time = 0.240951, size = 0, normalized size = 0.

$$\int \frac{(a + bx^4)^p}{(c + ex^2)^2} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*x^4)^p/(c + e*x^2)^2, x]
```

```
[Out] Integrate[(a + b*x^4)^p/(c + e*x^2)^2, x]
```

Maple [F] time = 0.053, size = 0, normalized size = 0.

$$\int \frac{(bx^4 + a)^p}{(ex^2 + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)^p/(e*x^2+c)^2, x)
```

```
[Out] int((b*x^4+a)^p/(e*x^2+c)^2, x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^4 + a)^p}{(ex^2 + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^4+a)^p/(e*x^2+c)^2,x, algorithm="maxima")

[Out] integrate((b*x^4 + a)^p/(e*x^2 + c)^2, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx^4 + a)^p}{e^2x^4 + 2cex^2 + c^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^4+a)^p/(e*x^2+c)^2,x, algorithm="fricas")

[Out] integral((b*x^4 + a)^p/(e^2*x^4 + 2*c*e*x^2 + c^2), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**4+a)**p/(e*x**2+c)**2,x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^4 + a)^p}{(ex^2 + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^4+a)^p/(e*x^2+c)^2,x, algorithm="giac")

[Out] integrate((b*x^4 + a)^p/(e*x^2 + c)^2, x)

3.182 $\int (1 - x^2)^3 (1 + bx^4)^p dx$

Optimal. Leaf size=108

$$x {}_2F_1\left(\frac{1}{4}, -p; \frac{5}{4}; -bx^4\right) + \frac{x^3(1 - b(4p + 7)) {}_2F_1\left(\frac{3}{4}, -p; \frac{7}{4}; -bx^4\right)}{b(4p + 7)} + \frac{3}{5}x^5 {}_2F_1\left(\frac{5}{4}, -p; \frac{9}{4}; -bx^4\right) - \frac{x^3(bx^4 + 1)^{p+1}}{b(4p + 7)}$$

[Out] $-(x^3(1 + b*x^4)^{(1 + p)})/(b*(7 + 4*p)) + x*Hypergeometric2F1[1/4, -p, 5/4, -(b*x^4)] + ((1 - b*(7 + 4*p))*x^3*Hypergeometric2F1[3/4, -p, 7/4, -(b*x^4)])/(b*(7 + 4*p)) + (3*x^5*Hypergeometric2F1[5/4, -p, 9/4, -(b*x^4)])/5$

Rubi [A] time = 0.11461, antiderivative size = 103, normalized size of antiderivative = 0.95, number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.21$, Rules used = {1207, 1893, 245, 364}

$$x {}_2F_1\left(\frac{1}{4}, -p; \frac{5}{4}; -bx^4\right) - x^3\left(1 - \frac{1}{4bp + 7b}\right) {}_2F_1\left(\frac{3}{4}, -p; \frac{7}{4}; -bx^4\right) + \frac{3}{5}x^5 {}_2F_1\left(\frac{5}{4}, -p; \frac{9}{4}; -bx^4\right) - \frac{x^3(bx^4 + 1)^{p+1}}{b(4p + 7)}$$

Antiderivative was successfully verified.

[In] Int[(1 - x^2)^3*(1 + b*x^4)^p,x]

[Out] $-(x^3(1 + b*x^4)^{(1 + p)})/(b*(7 + 4*p)) + x*Hypergeometric2F1[1/4, -p, 5/4, -(b*x^4)] - (1 - (7*b + 4*b*p)^{-1})*x^3*Hypergeometric2F1[3/4, -p, 7/4, -(b*x^4)] + (3*x^5*Hypergeometric2F1[5/4, -p, 9/4, -(b*x^4)])/5$

Rule 1207

Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] := Simp[(e^q*x^(2*q - 3)*(a + c*x^4)^(p + 1))/(c*(4*p + 2*q + 1)), x] + Dist[1/(c*(4*p + 2*q + 1)), Int[(a + c*x^4)^p*ExpandToSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[q, 1]

Rule 1893

Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Int[ExpandIntegrand[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n, p}, x] && (PolyQ[Pq, x] || PolyQ[Pq, x^n])

Rule 245

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[a^p*x*Hypergeometric2F1[-p, 1/n, 1/n + 1, -(b*x^n)/a], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] || GtQ[a, 0])

Rule 364

Int[((c_.)*(x_)^(m_.))*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(a^p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -(b*x^n)/a])/((c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])

Rubi steps

$$\begin{aligned}
\int (1-x^2)^3 (1+bx^4)^p dx &= -\frac{x^3(1+bx^4)^{1+p}}{b(7+4p)} + \frac{\int (1+bx^4)^p (b(7+4p) + 3(1-b(7+4p))x^2 + 3b(7+4p)x^4) dx}{b(7+4p)} \\
&= -\frac{x^3(1+bx^4)^{1+p}}{b(7+4p)} + \frac{\int (b(7+4p)(1+bx^4)^p + 3(1-b(7+4p))x^2(1+bx^4)^p + 3b(7+4p)x^4(1+bx^4)^p) dx}{b(7+4p)} \\
&= -\frac{x^3(1+bx^4)^{1+p}}{b(7+4p)} + 3 \int x^4(1+bx^4)^p dx - \left(3\left(1 - \frac{1}{7b+4bp}\right)\right) \int x^2(1+bx^4)^p dx + \int (1-x^2)^3 (1+bx^4)^p dx \\
&= -\frac{x^3(1+bx^4)^{1+p}}{b(7+4p)} + x {}_2F_1\left(\frac{1}{4}, -p; \frac{5}{4}; -bx^4\right) - \left(1 - \frac{1}{7b+4bp}\right) x^3 {}_2F_1\left(\frac{3}{4}, -p; \frac{7}{4}; -bx^4\right) + \frac{3}{5} x^5 {}_2F_1\left(\frac{5}{4}, -p; \frac{9}{4}; -bx^4\right) - x^7 {}_2F_1\left(\frac{7}{4}, -p; \frac{11}{4}; -bx^4\right)
\end{aligned}$$

Mathematica [A] time = 0.0172261, size = 86, normalized size = 0.8

$$-\frac{1}{7}x^7 {}_2F_1\left(\frac{7}{4}, -p; \frac{11}{4}; -bx^4\right) + \frac{3}{5}x^5 {}_2F_1\left(\frac{5}{4}, -p; \frac{9}{4}; -bx^4\right) - x^3 {}_2F_1\left(\frac{3}{4}, -p; \frac{7}{4}; -bx^4\right) + x {}_2F_1\left(\frac{1}{4}, -p; \frac{5}{4}; -bx^4\right)$$

Antiderivative was successfully verified.

[In] Integrate[(1 - x^2)^3*(1 + b*x^4)^p, x]

[Out] x*Hypergeometric2F1[1/4, -p, 5/4, -(b*x^4)] - x^3*Hypergeometric2F1[3/4, -p, 7/4, -(b*x^4)] + (3*x^5*Hypergeometric2F1[5/4, -p, 9/4, -(b*x^4)])/5 - (x^7*Hypergeometric2F1[7/4, -p, 11/4, -(b*x^4)])/7

Maple [A] time = 0.105, size = 75, normalized size = 0.7

$$-\frac{x^7}{7} {}_2F_1\left(\frac{7}{4}, -p; \frac{11}{4}; -bx^4\right) + \frac{3x^5}{5} {}_2F_1\left(\frac{5}{4}, -p; \frac{9}{4}; -bx^4\right) - x^3 {}_2F_1\left(\frac{3}{4}, -p; \frac{7}{4}; -bx^4\right) + x {}_2F_1\left(\frac{1}{4}, -p; \frac{5}{4}; -bx^4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-x^2+1)^3*(b*x^4+1)^p, x)

[Out] -1/7*x^7*hypergeom([7/4, -p], [11/4], -b*x^4)+3/5*x^5*hypergeom([5/4, -p], [9/4], -b*x^4)-x^3*hypergeom([3/4, -p], [7/4], -b*x^4)+x*hypergeom([1/4, -p], [5/4], -b*x^4)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int (x^2-1)^3 (bx^4+1)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2+1)^3*(b*x^4+1)^p, x, algorithm="maxima")

[Out] -integrate((x^2 - 1)^3*(b*x^4 + 1)^p, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\left(x^6 - 3x^4 + 3x^2 - 1\right)\left(bx^4 + 1\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2+1)^3*(b*x^4+1)^p,x, algorithm="fricas")

[Out] integral(-(x^6 - 3*x^4 + 3*x^2 - 1)*(b*x^4 + 1)^p, x)

Sympy [C] time = 176.712, size = 129, normalized size = 1.19

$$\frac{x^7 \Gamma\left(\frac{7}{4}\right) {}_2F_1\left(\frac{7}{4}, -p \middle| \frac{11}{4}, bx^4 e^{i\pi}\right)}{4\Gamma\left(\frac{11}{4}\right)} + \frac{3x^5 \Gamma\left(\frac{5}{4}\right) {}_2F_1\left(\frac{5}{4}, -p \middle| \frac{9}{4}, bx^4 e^{i\pi}\right)}{4\Gamma\left(\frac{9}{4}\right)} - \frac{3x^3 \Gamma\left(\frac{3}{4}\right) {}_2F_1\left(\frac{3}{4}, -p \middle| \frac{7}{4}, bx^4 e^{i\pi}\right)}{4\Gamma\left(\frac{7}{4}\right)} + \frac{x \Gamma\left(\frac{1}{4}\right) {}_2F_1\left(\frac{1}{4}, -p \middle| \frac{5}{4}, bx^4 e^{i\pi}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x**2+1)**3*(b*x**4+1)**p,x)

[Out] -x**7*gamma(7/4)*hyper((7/4, -p), (11/4,), b*x**4*exp_polar(I*pi))/(4*gamma(11/4)) + 3*x**5*gamma(5/4)*hyper((5/4, -p), (9/4,), b*x**4*exp_polar(I*pi))/(4*gamma(9/4)) - 3*x**3*gamma(3/4)*hyper((3/4, -p), (7/4,), b*x**4*exp_polar(I*pi))/(4*gamma(7/4)) + x*gamma(1/4)*hyper((1/4, -p), (5/4,), b*x**4*exp_polar(I*pi))/(4*gamma(5/4))

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -(x^2 - 1)^3 (bx^4 + 1)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2+1)^3*(b*x^4+1)^p,x, algorithm="giac")

[Out] integrate(-(x^2 - 1)^3*(b*x^4 + 1)^p, x)

3.183 $\int (1 - x^2)^2 (1 + bx^4)^p dx$

Optimal. Leaf size=86

$$-\frac{x(1 - b(4p + 5)) {}_2F_1\left(\frac{1}{4}, -p; \frac{5}{4}; -bx^4\right)}{b(4p + 5)} - \frac{2}{3}x^3 {}_2F_1\left(\frac{3}{4}, -p; \frac{7}{4}; -bx^4\right) + \frac{x(bx^4 + 1)^{p+1}}{b(4p + 5)}$$

[Out] (x*(1 + b*x^4)^(1 + p))/(b*(5 + 4*p)) - ((1 - b*(5 + 4*p))*x*Hypergeometric2F1[1/4, -p, 5/4, -(b*x^4)])/(b*(5 + 4*p)) - (2*x^3*Hypergeometric2F1[3/4, -p, 7/4, -(b*x^4)])/3

Rubi [A] time = 0.0683756, antiderivative size = 79, normalized size of antiderivative = 0.92, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.21$, Rules used = {1207, 1204, 245, 364}

$$x\left(1 - \frac{1}{4bp + 5b}\right) {}_2F_1\left(\frac{1}{4}, -p; \frac{5}{4}; -bx^4\right) - \frac{2}{3}x^3 {}_2F_1\left(\frac{3}{4}, -p; \frac{7}{4}; -bx^4\right) + \frac{x(bx^4 + 1)^{p+1}}{b(4p + 5)}$$

Antiderivative was successfully verified.

[In] Int[(1 - x^2)^2*(1 + b*x^4)^p,x]

[Out] (x*(1 + b*x^4)^(1 + p))/(b*(5 + 4*p)) + (1 - (5*b + 4*b*p)^(-1))*x*Hypergeometric2F1[1/4, -p, 5/4, -(b*x^4)] - (2*x^3*Hypergeometric2F1[3/4, -p, 7/4, -(b*x^4)])/3

Rule 1207

Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] := Simp[e^q*x^(2*q - 3)*(a + c*x^4)^(p + 1)]/(c*(4*p + 2*q + 1)), x] + Dist[1/(c*(4*p + 2*q + 1)), Int[(a + c*x^4)^p*ExpandToSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[q, 1]

Rule 1204

Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] := Int[ExpandIntegrand[(d + e*x^2)*(a + c*x^4)^p, x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0]

Rule 245

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[a^p*x*Hypergeometric2F1[-p, 1/n, 1/n + 1, -(b*x^n)/a], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] || GtQ[a, 0])

Rule 364

Int[((c_.)*(x_)^(m_.))*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(a^p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -(b*x^n)/a])/((c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])

Rubi steps

$$\begin{aligned}
\int (1-x^2)^2 (1+bx^4)^p dx &= \frac{x(1+bx^4)^{1+p}}{b(5+4p)} + \frac{\int (-1+b(5+4p)-2b(5+4p)x^2)(1+bx^4)^p dx}{b(5+4p)} \\
&= \frac{x(1+bx^4)^{1+p}}{b(5+4p)} + \frac{\int ((-1+b(5+4p))(1+bx^4)^p - 2b(5+4p)x^2(1+bx^4)^p) dx}{b(5+4p)} \\
&= \frac{x(1+bx^4)^{1+p}}{b(5+4p)} - 2 \int x^2 (1+bx^4)^p dx + \left(1 - \frac{1}{5b+4bp}\right) \int (1+bx^4)^p dx \\
&= \frac{x(1+bx^4)^{1+p}}{b(5+4p)} + \left(1 - \frac{1}{5b+4bp}\right) x {}_2F_1\left(\frac{1}{4}, -p; \frac{5}{4}; -bx^4\right) - \frac{2}{3} x^3 {}_2F_1\left(\frac{3}{4}, -p; \frac{7}{4}; -bx^4\right)
\end{aligned}$$

Mathematica [A] time = 0.0107821, size = 65, normalized size = 0.76

$$\frac{1}{5} x^5 {}_2F_1\left(\frac{5}{4}, -p; \frac{9}{4}; -bx^4\right) - \frac{2}{3} x^3 {}_2F_1\left(\frac{3}{4}, -p; \frac{7}{4}; -bx^4\right) + x {}_2F_1\left(\frac{1}{4}, -p; \frac{5}{4}; -bx^4\right)$$

Antiderivative was successfully verified.

[In] Integrate[(1 - x^2)^2*(1 + b*x^4)^p,x]

[Out] x*Hypergeometric2F1[1/4, -p, 5/4, -(b*x^4)] - (2*x^3*Hypergeometric2F1[3/4, -p, 7/4, -(b*x^4)])/3 + (x^5*Hypergeometric2F1[5/4, -p, 9/4, -(b*x^4)])/5

Maple [A] time = 0.045, size = 56, normalized size = 0.7

$$\frac{x^5}{5} {}_2F_1\left(\frac{5}{4}, -p; \frac{9}{4}; -bx^4\right) - \frac{2x^3}{3} {}_2F_1\left(\frac{3}{4}, -p; \frac{7}{4}; -bx^4\right) + x {}_2F_1\left(\frac{1}{4}, -p; \frac{5}{4}; -bx^4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-x^2+1)^2*(b*x^4+1)^p,x)

[Out] 1/5*x^5*hypergeom([5/4,-p],[9/4],-b*x^4)-2/3*x^3*hypergeom([3/4,-p],[7/4],-b*x^4)+x*hypergeom([1/4,-p],[5/4],-b*x^4)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (x^2 - 1)^2 (bx^4 + 1)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2+1)^2*(b*x^4+1)^p,x, algorithm="maxima")

[Out] integrate((x^2 - 1)^2*(b*x^4 + 1)^p, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(x^4 - 2x^2 + 1\right)\left(bx^4 + 1\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2+1)^2*(b*x^4+1)^p,x, algorithm="fricas")

[Out] integral((x^4 - 2*x^2 + 1)*(b*x^4 + 1)^p, x)

Sympy [C] time = 99.1881, size = 94, normalized size = 1.09

$$\frac{x^5 \Gamma\left(\frac{5}{4}\right) {}_2F_1\left(\frac{5}{4}, -p \middle| \frac{9}{4}; bx^4 e^{i\pi}\right)}{4 \Gamma\left(\frac{9}{4}\right)} - \frac{x^3 \Gamma\left(\frac{3}{4}\right) {}_2F_1\left(\frac{3}{4}, -p \middle| \frac{7}{4}; bx^4 e^{i\pi}\right)}{2 \Gamma\left(\frac{7}{4}\right)} + \frac{x \Gamma\left(\frac{1}{4}\right) {}_2F_1\left(\frac{1}{4}, -p \middle| \frac{5}{4}; bx^4 e^{i\pi}\right)}{4 \Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x**2+1)**2*(b*x**4+1)**p,x)

[Out] x**5*gamma(5/4)*hyper((5/4, -p), (9/4,), b*x**4*exp_polar(I*pi))/(4*gamma(9/4)) - x**3*gamma(3/4)*hyper((3/4, -p), (7/4,), b*x**4*exp_polar(I*pi))/(2*gamma(7/4)) + x*gamma(1/4)*hyper((1/4, -p), (5/4,), b*x**4*exp_polar(I*pi))/(4*gamma(5/4))

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (x^2 - 1)^2 (bx^4 + 1)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2+1)^2*(b*x^4+1)^p,x, algorithm="giac")

[Out] integrate((x^2 - 1)^2*(b*x^4 + 1)^p, x)

3.184 $\int (1 - x^2)(1 + bx^4)^p dx$

Optimal. Leaf size=42

$$x {}_2F_1\left(\frac{1}{4}, -p; \frac{5}{4}; -bx^4\right) - \frac{1}{3}x^3 {}_2F_1\left(\frac{3}{4}, -p; \frac{7}{4}; -bx^4\right)$$

[Out] x*Hypergeometric2F1[1/4, -p, 5/4, -(b*x^4)] - (x^3*Hypergeometric2F1[3/4, -p, 7/4, -(b*x^4)])/3

Rubi [A] time = 0.0205814, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {1204, 245, 364}

$$x {}_2F_1\left(\frac{1}{4}, -p; \frac{5}{4}; -bx^4\right) - \frac{1}{3}x^3 {}_2F_1\left(\frac{3}{4}, -p; \frac{7}{4}; -bx^4\right)$$

Antiderivative was successfully verified.

[In] Int[(1 - x^2)*(1 + b*x^4)^p, x]

[Out] x*Hypergeometric2F1[1/4, -p, 5/4, -(b*x^4)] - (x^3*Hypergeometric2F1[3/4, -p, 7/4, -(b*x^4)])/3

Rule 1204

Int[((d_) + (e_)*(x_)^2)*((a_) + (c_)*(x_)^4)^(p_), x_Symbol] := Int[ExpandIntegrand[(d + e*x^2)*(a + c*x^4)^p, x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0]

Rule 245

Int[((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Simp[a^p*x*Hypergeometric2F1[-p, 1/n, 1/n + 1, -(b*x^n)/a], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] || GtQ[a, 0])

Rule 364

Int[((c_)*(x_)^(m_))*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Simp[(a^p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -(b*x^n)/a])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])

Rubi steps

$$\begin{aligned} \int (1 - x^2)(1 + bx^4)^p dx &= \int \left((1 + bx^4)^p - x^2(1 + bx^4)^p \right) dx \\ &= \int (1 + bx^4)^p dx - \int x^2(1 + bx^4)^p dx \\ &= x {}_2F_1\left(\frac{1}{4}, -p; \frac{5}{4}; -bx^4\right) - \frac{1}{3}x^3 {}_2F_1\left(\frac{3}{4}, -p; \frac{7}{4}; -bx^4\right) \end{aligned}$$

Mathematica [A] time = 0.006765, size = 42, normalized size = 1.

$$x {}_2F_1\left(\frac{1}{4}, -p; \frac{5}{4}; -bx^4\right) - \frac{1}{3}x^3 {}_2F_1\left(\frac{3}{4}, -p; \frac{7}{4}; -bx^4\right)$$

Antiderivative was successfully verified.

[In] Integrate[(1 - x^2)*(1 + b*x^4)^p, x]

[Out] x*Hypergeometric2F1[1/4, -p, 5/4, -(b*x^4)] - (x^3*Hypergeometric2F1[3/4, -p, 7/4, -(b*x^4)])/3

Maple [A] time = 0.027, size = 37, normalized size = 0.9

$$x {}_2F_1\left(\frac{1}{4}, -p; \frac{5}{4}; -bx^4\right) - \frac{x^3}{3} {}_2F_1\left(\frac{3}{4}, -p; \frac{7}{4}; -bx^4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-x^2+1)*(b*x^4+1)^p, x)

[Out] x*hypergeom([1/4, -p], [5/4], -b*x^4) - 1/3*x^3*hypergeom([3/4, -p], [7/4], -b*x^4)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$- \int (x^2 - 1)(bx^4 + 1)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2+1)*(b*x^4+1)^p, x, algorithm="maxima")

[Out] -integrate((x^2 - 1)*(b*x^4 + 1)^p, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(- (x^2 - 1)(bx^4 + 1)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2+1)*(b*x^4+1)^p, x, algorithm="fricas")

[Out] integral(-(x^2 - 1)*(b*x^4 + 1)^p, x)

Sympy [C] time = 45.9657, size = 61, normalized size = 1.45

$$-\frac{x^3 \Gamma\left(\frac{3}{4}\right) {}_2F_1\left(\frac{3}{4}, -p; \frac{7}{4}; bx^4 e^{i\pi}\right)}{4\Gamma\left(\frac{7}{4}\right)} + \frac{x \Gamma\left(\frac{1}{4}\right) {}_2F_1\left(\frac{1}{4}, -p; \frac{5}{4}; bx^4 e^{i\pi}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x**2+1)*(b*x**4+1)**p,x)

[Out] -x**3*gamma(3/4)*hyper((3/4, -p), (7/4,), b*x**4*exp_polar(I*pi))/(4*gamma(7/4)) + x*gamma(1/4)*hyper((1/4, -p), (5/4,), b*x**4*exp_polar(I*pi))/(4*gamma(5/4))

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -(x^2 - 1)(bx^4 + 1)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2+1)*(b*x^4+1)^p,x, algorithm="giac")

[Out] integrate(-(x^2 - 1)*(b*x^4 + 1)^p, x)

3.185 $\int (1 + bx^4)^p dx$

Optimal. Leaf size=18

$$x {}_2F_1\left(\frac{1}{4}, -p; \frac{5}{4}; -bx^4\right)$$

[Out] x*Hypergeometric2F1[1/4, -p, 5/4, -(b*x^4)]

Rubi [A] time = 0.0042675, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {245}

$$x {}_2F_1\left(\frac{1}{4}, -p; \frac{5}{4}; -bx^4\right)$$

Antiderivative was successfully verified.

[In] Int[(1 + b*x^4)^p, x]

[Out] x*Hypergeometric2F1[1/4, -p, 5/4, -(b*x^4)]

Rule 245

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] || GtQ[a, 0])

Rubi steps

$$\int (1 + bx^4)^p dx = x {}_2F_1\left(\frac{1}{4}, -p; \frac{5}{4}; -bx^4\right)$$

Mathematica [A] time = 0.0016625, size = 18, normalized size = 1.

$$x {}_2F_1\left(\frac{1}{4}, -p; \frac{5}{4}; -bx^4\right)$$

Antiderivative was successfully verified.

[In] Integrate[(1 + b*x^4)^p, x]

[Out] x*Hypergeometric2F1[1/4, -p, 5/4, -(b*x^4)]

Maple [A] time = 0.021, size = 17, normalized size = 0.9

$$x {}_2F_1\left(\frac{1}{4}, -p; \frac{5}{4}; -bx^4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^4+1)^p,x)`

[Out] `x*hypergeom([1/4,-p],[5/4],-b*x^4)`

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (bx^4 + 1)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^4+1)^p,x, algorithm="maxima")`

[Out] `integrate((b*x^4 + 1)^p, x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(bx^4 + 1\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^4+1)^p,x, algorithm="fricas")`

[Out] `integral((b*x^4 + 1)^p, x)`

Sympy [C] time = 9.93296, size = 29, normalized size = 1.61

$$\frac{x\Gamma\left(\frac{1}{4}\right) {}_2F_1\left(\frac{1}{4}, -p \middle| \frac{5}{4} \right) bx^4 e^{i\pi}}{4\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**4+1)**p,x)`

[Out] `x*gamma(1/4)*hyper((1/4, -p), (5/4,), b*x**4*exp_polar(I*pi))/(4*gamma(5/4))`

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (bx^4 + 1)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^4+1)^p,x, algorithm="giac")`

[Out] `integrate((b*x^4 + 1)^p, x)`

$$3.186 \quad \int \frac{(1+bx^4)^p}{1-x^2} dx$$

Optimal. Leaf size=50

$$\frac{1}{3}x^3F_1\left(\frac{3}{4};1,-p;\frac{7}{4};x^4,-bx^4\right) + xF_1\left(\frac{1}{4};1,-p;\frac{5}{4};x^4,-bx^4\right)$$

[Out] x*AppellF1[1/4, 1, -p, 5/4, x^4, -(b*x^4)] + (x^3*AppellF1[3/4, 1, -p, 7/4, x^4, -(b*x^4)])/3

Rubi [A] time = 0.0539169, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {1240, 429, 510}

$$\frac{1}{3}x^3F_1\left(\frac{3}{4};1,-p;\frac{7}{4};x^4,-bx^4\right) + xF_1\left(\frac{1}{4};1,-p;\frac{5}{4};x^4,-bx^4\right)$$

Antiderivative was successfully verified.

[In] Int[(1 + b*x^4)^p/(1 - x^2),x]

[Out] x*AppellF1[1/4, 1, -p, 5/4, x^4, -(b*x^4)] + (x^3*AppellF1[3/4, 1, -p, 7/4, x^4, -(b*x^4)])/3

Rule 1240

Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] := Int[ExpandIntegrand[(a + c*x^4)^p, (d/(d^2 - e^2*x^4) - (e*x^2)/(d^2 - e^2*x^4))^(-q), x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[p] && ILtQ[q, 0]

Rule 429

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -(b*x^n)/a, -(d*x^n)/c], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])

Rule 510

Int[((e_.)*(x_)^(m_.))*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := Simp[(a^p*c^q*(e*x)^(m+1)*AppellF1[(m+1)/n, -p, -q, 1 + (m+1)/n, -(b*x^n)/a, -(d*x^n)/c])/(e*(m+1)), x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])

Rubi steps

$$\begin{aligned} \int \frac{(1+bx^4)^p}{1-x^2} dx &= \int \left(\frac{(1+bx^4)^p}{1-x^4} - \frac{x^2(1+bx^4)^p}{-1+x^4} \right) dx \\ &= \int \frac{(1+bx^4)^p}{1-x^4} dx - \int \frac{x^2(1+bx^4)^p}{-1+x^4} dx \\ &= xF_1\left(\frac{1}{4}; 1, -p; \frac{5}{4}; x^4, -bx^4\right) + \frac{1}{3}x^3F_1\left(\frac{3}{4}; 1, -p; \frac{7}{4}; x^4, -bx^4\right) \end{aligned}$$

Mathematica [F] time = 0.0804455, size = 0, normalized size = 0.

$$\int \frac{(1+bx^4)^p}{1-x^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[(1 + b*x^4)^p/(1 - x^2), x]

[Out] Integrate[(1 + b*x^4)^p/(1 - x^2), x]

Maple [F] time = 0.043, size = 0, normalized size = 0.

$$\int \frac{(bx^4+1)^p}{-x^2+1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^4+1)^p/(-x^2+1), x)

[Out] int((b*x^4+1)^p/(-x^2+1), x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$- \int \frac{(bx^4+1)^p}{x^2-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^4+1)^p/(-x^2+1), x, algorithm="maxima")

[Out] -integrate((b*x^4 + 1)^p/(x^2 - 1), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{(bx^4+1)^p}{x^2-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+1)^p/(-x^2+1),x, algorithm="fricas")
```

```
[Out] integral(-(b*x^4 + 1)^p/(x^2 - 1), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+1)**p/(-x**2+1),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{(bx^4 + 1)^p}{x^2 - 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+1)^p/(-x^2+1),x, algorithm="giac")
```

```
[Out] integrate(-(b*x^4 + 1)^p/(x^2 - 1), x)
```

$$3.187 \quad \int \frac{(1+bx^4)^p}{(1-x^2)^2} dx$$

Optimal. Leaf size=77

$$\frac{1}{5}x^5F_1\left(\frac{5}{4}; 2, -p; \frac{9}{4}; x^4, -bx^4\right) + \frac{2}{3}x^3F_1\left(\frac{3}{4}; 2, -p; \frac{7}{4}; x^4, -bx^4\right) + xF_1\left(\frac{1}{4}; 2, -p; \frac{5}{4}; x^4, -bx^4\right)$$

[Out] x*AppellF1[1/4, 2, -p, 5/4, x^4, -(b*x^4)] + (2*x^3*AppellF1[3/4, 2, -p, 7/4, x^4, -(b*x^4)])/3 + (x^5*AppellF1[5/4, 2, -p, 9/4, x^4, -(b*x^4)])/5

Rubi [A] time = 0.0817376, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {1240, 429, 510}

$$\frac{1}{5}x^5F_1\left(\frac{5}{4}; 2, -p; \frac{9}{4}; x^4, -bx^4\right) + \frac{2}{3}x^3F_1\left(\frac{3}{4}; 2, -p; \frac{7}{4}; x^4, -bx^4\right) + xF_1\left(\frac{1}{4}; 2, -p; \frac{5}{4}; x^4, -bx^4\right)$$

Antiderivative was successfully verified.

[In] Int[(1 + b*x^4)^p/(1 - x^2)^2,x]

[Out] x*AppellF1[1/4, 2, -p, 5/4, x^4, -(b*x^4)] + (2*x^3*AppellF1[3/4, 2, -p, 7/4, x^4, -(b*x^4)])/3 + (x^5*AppellF1[5/4, 2, -p, 9/4, x^4, -(b*x^4)])/5

Rule 1240

Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + c*x^4)^p, (d/(d^2 - e^2*x^4) - (e*x^2)/(d^2 - e^2*x^4))^(-q), x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] && ! IntegerQ[p] && ILtQ[q, 0]

Rule 429

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] :> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])

Rule 510

Int[((e_.)*(x_)^(m_.))*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m+1)*AppellF1[(m+1)/n, -p, -q, 1 + (m+1)/n, -((b*x^n)/a), -((d*x^n)/c)]/(e*(m+1)), x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])

Rubi steps

$$\begin{aligned}
\int \frac{(1+bx^4)^p}{(1-x^2)^2} dx &= \int \left(\frac{(1+bx^4)^p}{(-1+x^4)^2} + \frac{2x^2(1+bx^4)^p}{(-1+x^4)^2} + \frac{x^4(1+bx^4)^p}{(-1+x^4)^2} \right) dx \\
&= 2 \int \frac{x^2(1+bx^4)^p}{(-1+x^4)^2} dx + \int \frac{(1+bx^4)^p}{(-1+x^4)^2} dx + \int \frac{x^4(1+bx^4)^p}{(-1+x^4)^2} dx \\
&= xF_1\left(\frac{1}{4}; 2, -p; \frac{5}{4}; x^4, -bx^4\right) + \frac{2}{3}x^3F_1\left(\frac{3}{4}; 2, -p; \frac{7}{4}; x^4, -bx^4\right) + \frac{1}{5}x^5F_1\left(\frac{5}{4}; 2, -p; \frac{9}{4}; x^4, -bx^4\right)
\end{aligned}$$

Mathematica [F] time = 0.103737, size = 0, normalized size = 0.

$$\int \frac{(1+bx^4)^p}{(1-x^2)^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[(1 + b*x^4)^p/(1 - x^2)^2, x]

[Out] Integrate[(1 + b*x^4)^p/(1 - x^2)^2, x]

Maple [F] time = 0.056, size = 0, normalized size = 0.

$$\int \frac{(bx^4+1)^p}{(-x^2+1)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^4+1)^p/(-x^2+1)^2, x)

[Out] int((b*x^4+1)^p/(-x^2+1)^2, x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^4+1)^p}{(x^2-1)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^4+1)^p/(-x^2+1)^2, x, algorithm="maxima")

[Out] integrate((b*x^4 + 1)^p/(x^2 - 1)^2, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx^4+1)^p}{x^4-2x^2+1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^4+1)^p/(-x^2+1)^2,x, algorithm="fricas")

[Out] integral((b*x^4 + 1)^p/(x^4 - 2*x^2 + 1), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**4+1)**p/(-x**2+1)**2,x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^4 + 1)^p}{(x^2 - 1)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^4+1)^p/(-x^2+1)^2,x, algorithm="giac")

[Out] integrate((b*x^4 + 1)^p/(x^2 - 1)^2, x)

$$3.188 \quad \int \frac{(1+bx^4)^p}{(1-x^2)^3} dx$$

Optimal. Leaf size=101

$$\frac{1}{7}x^7F_1\left(\frac{7}{4};3,-p;\frac{11}{4};x^4,-bx^4\right) + \frac{3}{5}x^5F_1\left(\frac{5}{4};3,-p;\frac{9}{4};x^4,-bx^4\right) + x^3F_1\left(\frac{3}{4};3,-p;\frac{7}{4};x^4,-bx^4\right) + xF_1\left(\frac{1}{4};3,-p;\frac{5}{4};x^4,-bx^4\right)$$

[Out] x*AppellF1[1/4, 3, -p, 5/4, x^4, -(b*x^4)] + x^3*AppellF1[3/4, 3, -p, 7/4, x^4, -(b*x^4)] + (3*x^5*AppellF1[5/4, 3, -p, 9/4, x^4, -(b*x^4)])/5 + (x^7*AppellF1[7/4, 3, -p, 11/4, x^4, -(b*x^4)])/7

Rubi [A] time = 0.114012, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {1240, 429, 510}

$$\frac{1}{7}x^7F_1\left(\frac{7}{4};3,-p;\frac{11}{4};x^4,-bx^4\right) + \frac{3}{5}x^5F_1\left(\frac{5}{4};3,-p;\frac{9}{4};x^4,-bx^4\right) + x^3F_1\left(\frac{3}{4};3,-p;\frac{7}{4};x^4,-bx^4\right) + xF_1\left(\frac{1}{4};3,-p;\frac{5}{4};x^4,-bx^4\right)$$

Antiderivative was successfully verified.

[In] Int[(1 + b*x^4)^p/(1 - x^2)^3,x]

[Out] x*AppellF1[1/4, 3, -p, 5/4, x^4, -(b*x^4)] + x^3*AppellF1[3/4, 3, -p, 7/4, x^4, -(b*x^4)] + (3*x^5*AppellF1[5/4, 3, -p, 9/4, x^4, -(b*x^4)])/5 + (x^7*AppellF1[7/4, 3, -p, 11/4, x^4, -(b*x^4)])/7

Rule 1240

Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] := Int[ExpandIntegrand[(a + c*x^4)^p, (d/(d^2 - e^2*x^4) - (e*x^2)/(d^2 - e^2*x^4))^(-q), x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[p] && ILtQ[q, 0]

Rule 429

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -(b*x^n)/a, -(d*x^n)/c], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])

Rule 510

Int[((e_.)*(x_)^(m_.))*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := Simp[(a^p*c^q*(e*x)^(m+1)*AppellF1[(m+1)/n, -p, -q, 1 + (m+1)/n, -(b*x^n)/a, -(d*x^n)/c])/(e*(m+1)), x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n-1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])

Rubi steps

$$\begin{aligned} \int \frac{(1+bx^4)^p}{(1-x^2)^3} dx &= \int \left(-\frac{(1+bx^4)^p}{(-1+x^4)^3} - \frac{3x^2(1+bx^4)^p}{(-1+x^4)^3} - \frac{3x^4(1+bx^4)^p}{(-1+x^4)^3} - \frac{x^6(1+bx^4)^p}{(-1+x^4)^3} \right) dx \\ &= -\left(3 \int \frac{x^2(1+bx^4)^p}{(-1+x^4)^3} dx \right) - 3 \int \frac{x^4(1+bx^4)^p}{(-1+x^4)^3} dx - \int \frac{(1+bx^4)^p}{(-1+x^4)^3} dx - \int \frac{x^6(1+bx^4)^p}{(-1+x^4)^3} dx \\ &= xF_1\left(\frac{1}{4}; 3, -p; \frac{5}{4}; x^4, -bx^4\right) + x^3F_1\left(\frac{3}{4}; 3, -p; \frac{7}{4}; x^4, -bx^4\right) + \frac{3}{5}x^5F_1\left(\frac{5}{4}; 3, -p; \frac{9}{4}; x^4, -bx^4\right) + \frac{1}{7}x^7F_1\left(\frac{7}{4}; \end{aligned}$$

Mathematica [F] time = 0.148215, size = 0, normalized size = 0.

$$\int \frac{(1+bx^4)^p}{(1-x^2)^3} dx$$

Verification is Not applicable to the result.

[In] Integrate[(1 + b*x^4)^p/(1 - x^2)^3, x]

[Out] Integrate[(1 + b*x^4)^p/(1 - x^2)^3, x]

Maple [F] time = 0.072, size = 0, normalized size = 0.

$$\int \frac{(bx^4+1)^p}{(-x^2+1)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^4+1)^p/(-x^2+1)^3, x)

[Out] int((b*x^4+1)^p/(-x^2+1)^3, x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$- \int \frac{(bx^4+1)^p}{(x^2-1)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^4+1)^p/(-x^2+1)^3, x, algorithm="maxima")

[Out] -integrate((b*x^4 + 1)^p/(x^2 - 1)^3, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{(bx^4+1)^p}{x^6-3x^4+3x^2-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+1)^p/(-x^2+1)^3,x, algorithm="fricas")
```

```
[Out] integral(-(b*x^4 + 1)^p/(x^6 - 3*x^4 + 3*x^2 - 1), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+1)**p/(-x**2+1)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{(bx^4 + 1)^p}{(x^2 - 1)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+1)^p/(-x^2+1)^3,x, algorithm="giac")
```

```
[Out] integrate(-(b*x^4 + 1)^p/(x^2 - 1)^3, x)
```

$$3.189 \quad \int \frac{(d+ex^2)^4}{d^2-e^2x^4} dx$$

Optimal. Leaf size=51

$$\frac{8d^{5/2} \tanh^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)}{\sqrt{e}} - 7d^2x - \frac{4}{3}dex^3 - \frac{1}{5}e^2x^5$$

[Out] $-7*d^2*x - (4*d*e*x^3)/3 - (e^2*x^5)/5 + (8*d^{(5/2)}*ArcTanh[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[e]$

Rubi [A] time = 0.0410136, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1150, 390, 208}

$$\frac{8d^{5/2} \tanh^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)}{\sqrt{e}} - 7d^2x - \frac{4}{3}dex^3 - \frac{1}{5}e^2x^5$$

Antiderivative was successfully verified.

[In] Int[(d + e*x^2)^4/(d^2 - e^2*x^4),x]

[Out] $-7*d^2*x - (4*d*e*x^3)/3 - (e^2*x^5)/5 + (8*d^{(5/2)}*ArcTanh[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[e]$

Rule 1150

Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[(d + e*x^2)^(p + q)*(a/d + (c*x^2)/e)^p, x] /; FreeQ[{a, c, d, e, q}, x] && EqQ[c*d^2 + a*e^2, 0] && IntegerQ[p]

Rule 390

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] :> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rubi steps

$$\begin{aligned}
\int \frac{(d+ex^2)^4}{d^2-e^2x^4} dx &= \int \frac{(d+ex^2)^3}{d-ex^2} dx \\
&= \int \left(-7d^2 - 4dex^2 - e^2x^4 + \frac{8d^3}{d-ex^2} \right) dx \\
&= -7d^2x - \frac{4}{3}dex^3 - \frac{e^2x^5}{5} + (8d^3) \int \frac{1}{d-ex^2} dx \\
&= -7d^2x - \frac{4}{3}dex^3 - \frac{e^2x^5}{5} + \frac{8d^{5/2} \tanh^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)}{\sqrt{e}}
\end{aligned}$$

Mathematica [A] time = 0.023367, size = 51, normalized size = 1.

$$\frac{8d^{5/2} \tanh^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)}{\sqrt{e}} - 7d^2x - \frac{4}{3}dex^3 - \frac{1}{5}e^2x^5$$

Antiderivative was successfully verified.

[In] Integrate[(d + e*x^2)^4/(d^2 - e^2*x^4), x]

[Out] -7*d^2*x - (4*d*e*x^3)/3 - (e^2*x^5)/5 + (8*d^(5/2)*ArcTanh[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[e]

Maple [A] time = 0.006, size = 42, normalized size = 0.8

$$-\frac{e^2x^5}{5} - \frac{4dex^3}{3} - 7d^2x + 8 \frac{d^3}{\sqrt{de}} \operatorname{Artanh}\left(\frac{ex}{\sqrt{de}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x^2+d)^4/(-e^2*x^4+d^2), x)

[Out] -1/5*e^2*x^5-4/3*d*e*x^3-7*d^2*x+8*d^3/(d*e)^(1/2)*arctanh(x*e/(d*e)^(1/2))

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)^4/(-e^2*x^4+d^2), x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [A] time = 1.84779, size = 255, normalized size = 5.

$$\left[-\frac{1}{5}e^2x^5 - \frac{4}{3}dex^3 + 4d^2\sqrt{\frac{d}{e}} \log\left(\frac{ex^2 + 2ex\sqrt{\frac{d}{e}} + d}{ex^2 - d}\right) - 7d^2x, -\frac{1}{5}e^2x^5 - \frac{4}{3}dex^3 - 8d^2\sqrt{-\frac{d}{e}} \arctan\left(\frac{ex\sqrt{-\frac{d}{e}}}{d}\right) - 7d^2x \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)^4/(-e^2*x^4+d^2),x, algorithm="fricas")

[Out] [-1/5*e^2*x^5 - 4/3*d*e*x^3 + 4*d^2*sqrt(d/e)*log((e*x^2 + 2*e*x*sqrt(d/e) + d)/(e*x^2 - d)) - 7*d^2*x, -1/5*e^2*x^5 - 4/3*d*e*x^3 - 8*d^2*sqrt(-d/e)*arctan(e*x*sqrt(-d/e)/d) - 7*d^2*x]

Sympy [A] time = 0.475067, size = 75, normalized size = 1.47

$$-7d^2x - \frac{4dex^3}{3} - \frac{e^2x^5}{5} - 4\sqrt{\frac{d^5}{e}} \log\left(x - \frac{\sqrt{\frac{d^5}{e}}}{d^2}\right) + 4\sqrt{\frac{d^5}{e}} \log\left(x + \frac{\sqrt{\frac{d^5}{e}}}{d^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x**2+d)**4/(-e**2*x**4+d**2),x)

[Out] -7*d**2*x - 4*d*e*x**3/3 - e**2*x**5/5 - 4*sqrt(d**5/e)*log(x - sqrt(d**5/e)/d**2) + 4*sqrt(d**5/e)*log(x + sqrt(d**5/e)/d**2)

Giac [B] time = 1.18287, size = 194, normalized size = 3.8

$$4\left(\left(d^2\right)^{\frac{1}{4}}d^2e^{\frac{11}{2}} - \left(d^2\right)^{\frac{1}{4}}d|d|e^{\frac{11}{2}}\right) \arctan\left(\frac{xe^{\frac{1}{2}}}{\left(d^2\right)^{\frac{1}{4}}}\right) e^{(-6)} + 2\left(\left(d^2\right)^{\frac{1}{4}}d^2e^{\frac{15}{2}} + \left(d^2\right)^{\frac{3}{4}}de^{\frac{15}{2}}\right) e^{(-8)} \log\left(\left|\left(d^2\right)^{\frac{1}{4}}e^{\left(-\frac{1}{2}\right)} + x\right|\right) - 2\left(\left(d^2\right)^{\frac{1}{4}}d^2e^{\frac{11}{2}} - \left(d^2\right)^{\frac{1}{4}}d|d|e^{\frac{11}{2}}\right) \arctan\left(\frac{xe^{\frac{1}{2}}}{\left(d^2\right)^{\frac{1}{4}}}\right) e^{(-6)} + 2\left(\left(d^2\right)^{\frac{1}{4}}d^2e^{\frac{15}{2}} + \left(d^2\right)^{\frac{3}{4}}de^{\frac{15}{2}}\right) e^{(-8)} \log\left(\left|\left(d^2\right)^{\frac{1}{4}}e^{\left(-\frac{1}{2}\right)} + x\right|\right) - 2\left(\left(d^2\right)^{\frac{1}{4}}d^2e^{\frac{11}{2}} + \left(d^2\right)^{\frac{1}{4}}d*abs(d)*e^{\frac{11}{2}}\right) e^{(-6)} \log(abs\left(-\left(d^2\right)^{\frac{1}{4}}e^{\left(-\frac{1}{2}\right)} + x\right)) - 1/15*(3*x^5*e^12 + 20*d*x^3*e^11 + 105*d^2*x*e^10)*e^(-10)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)^4/(-e^2*x^4+d^2),x, algorithm="giac")

[Out] 4*((d^2)^(1/4)*d^2*e^(11/2) - (d^2)^(1/4)*d*abs(d)*e^(11/2))*arctan(x*e^(1/2)/(d^2)^(1/4))*e^(-6) + 2*((d^2)^(1/4)*d^2*e^(15/2) + (d^2)^(3/4)*d*e^(15/2))*e^(-8)*log(abs((d^2)^(1/4)*e^(-1/2) + x)) - 2*((d^2)^(1/4)*d^2*e^(11/2) + (d^2)^(1/4)*d*abs(d)*e^(11/2))*e^(-6)*log(abs(- (d^2)^(1/4)*e^(-1/2) + x)) - 1/15*(3*x^5*e^12 + 20*d*x^3*e^11 + 105*d^2*x*e^10)*e^(-10)

$$3.190 \quad \int \frac{(d+ex^2)^3}{d^2-e^2x^4} dx$$

Optimal. Leaf size=38

$$\frac{4d^{3/2} \tanh^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)}{\sqrt{e}} - 3dx - \frac{ex^3}{3}$$

[Out] $-3*d*x - (e*x^3)/3 + (4*d^{(3/2)}*ArcTanh[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[e]$

Rubi [A] time = 0.0337036, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1150, 390, 208}

$$\frac{4d^{3/2} \tanh^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)}{\sqrt{e}} - 3dx - \frac{ex^3}{3}$$

Antiderivative was successfully verified.

[In] Int[(d + e*x^2)^3/(d^2 - e^2*x^4), x]

[Out] $-3*d*x - (e*x^3)/3 + (4*d^{(3/2)}*ArcTanh[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[e]$

Rule 1150

Int[((d_) + (e_)*(x_)^2)^(q_)*((a_) + (c_)*(x_)^4)^(p_), x_Symbol] :> Int[(d + e*x^2)^(p + q)*(a/d + (c*x^2)/e)^p, x] /; FreeQ[{a, c, d, e, q}, x] && EqQ[c*d^2 + a*e^2, 0] && IntegerQ[p]

Rule 390

Int[((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_), x_Symbol] :> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]

Rule 208

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rubi steps

$$\begin{aligned} \int \frac{(d+ex^2)^3}{d^2-e^2x^4} dx &= \int \frac{(d+ex^2)^2}{d-ex^2} dx \\ &= \int \left(-3d-ex^2 + \frac{4d^2}{d-ex^2}\right) dx \\ &= -3dx - \frac{ex^3}{3} + (4d^2) \int \frac{1}{d-ex^2} dx \\ &= -3dx - \frac{ex^3}{3} + \frac{4d^{3/2} \tanh^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)}{\sqrt{e}} \end{aligned}$$

Mathematica [A] time = 0.0171963, size = 38, normalized size = 1.

$$\frac{4d^{3/2} \tanh^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)}{\sqrt{e}} - 3dx - \frac{ex^3}{3}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e*x^2)^3/(d^2 - e^2*x^4), x]

[Out] -3*d*x - (e*x^3)/3 + (4*d^(3/2)*ArcTanh[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[e]

Maple [A] time = 0.002, size = 31, normalized size = 0.8

$$-\frac{ex^3}{3} - 3dx + 4 \frac{d^2}{\sqrt{de}} \operatorname{Artanh}\left(\frac{ex}{\sqrt{de}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x^2+d)^3/(-e^2*x^4+d^2), x)

[Out] -1/3*e*x^3-3*d*x+4*d^2/(d*e)^(1/2)*arctanh(x*e/(d*e)^(1/2))

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)^3/(-e^2*x^4+d^2), x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [A] time = 1.8496, size = 201, normalized size = 5.29

$$\left[-\frac{1}{3}ex^3 + 2d\sqrt{\frac{d}{e}} \log\left(\frac{ex^2 + 2ex\sqrt{\frac{d}{e}} + d}{ex^2 - d}\right) - 3dx, -\frac{1}{3}ex^3 - 4d\sqrt{-\frac{d}{e}} \arctan\left(\frac{ex\sqrt{-\frac{d}{e}}}{d}\right) - 3dx \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)^3/(-e^2*x^4+d^2), x, algorithm="fricas")

[Out] [-1/3*e*x^3 + 2*d*sqrt(d/e)*log((e*x^2 + 2*e*x*sqrt(d/e) + d)/(e*x^2 - d)) - 3*d*x, -1/3*e*x^3 - 4*d*sqrt(-d/e)*arctan(e*x*sqrt(-d/e)/d) - 3*d*x]

Sympy [A] time = 0.404648, size = 58, normalized size = 1.53

$$-3dx - \frac{ex^3}{3} - 2\sqrt{\frac{d^3}{e}} \log\left(x - \frac{\sqrt{\frac{d^3}{e}}}{d}\right) + 2\sqrt{\frac{d^3}{e}} \log\left(x + \frac{\sqrt{\frac{d^3}{e}}}{d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x**2+d)**3/(-e**2*x**4+d**2),x)

[Out] -3*d*x - e*x**3/3 - 2*sqrt(d**3/e)*log(x - sqrt(d**3/e)/d) + 2*sqrt(d**3/e)*log(x + sqrt(d**3/e)/d)

Giac [B] time = 1.1757, size = 166, normalized size = 4.37

$$2 \left((d^2)^{\frac{1}{4}} d e^{\frac{11}{2}} - (d^2)^{\frac{1}{4}} |d| e^{\frac{11}{2}} \right) \arctan \left(\frac{x e^{\frac{1}{2}}}{(d^2)^{\frac{1}{4}}} \right) e^{(-6)} + \left((d^2)^{\frac{1}{4}} d e^{\frac{15}{2}} + (d^2)^{\frac{3}{4}} e^{\frac{15}{2}} \right) e^{(-8)} \log \left(\left((d^2)^{\frac{1}{4}} e^{\left(-\frac{1}{2}\right)} + x \right) \right) - \left((d^2)^{\frac{1}{4}} d e^{\frac{11}{2}} + \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)^3/(-e^2*x^4+d^2),x, algorithm="giac")

[Out] 2*((d^2)^(1/4)*d*e^(11/2) - (d^2)^(1/4)*abs(d)*e^(11/2))*arctan(x*e^(1/2)/((d^2)^(1/4))*e^(-6) + ((d^2)^(1/4)*d*e^(15/2) + (d^2)^(3/4)*e^(15/2))*e^(-8)*log(abs((d^2)^(1/4)*e^(-1/2) + x)) - ((d^2)^(1/4)*d*e^(11/2) + (d^2)^(1/4)*abs(d)*e^(11/2))*e^(-6)*log(abs(-(d^2)^(1/4)*e^(-1/2) + x)) - 1/3*(x^3*e^7 + 9*d*x*e^6)*e^(-6)

$$3.191 \quad \int \frac{(d+ex^2)^2}{d^2-e^2x^4} dx$$

Optimal. Leaf size=29

$$\frac{2\sqrt{d} \tanh^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)}{\sqrt{e}} - x$$

[Out] $-x + (2*\text{Sqrt}[d]*\text{ArcTanh}[(\text{Sqrt}[e]*x)/\text{Sqrt}[d]])/\text{Sqrt}[e]$

Rubi [A] time = 0.023093, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1150, 388, 208}

$$\frac{2\sqrt{d} \tanh^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)}{\sqrt{e}} - x$$

Antiderivative was successfully verified.

[In] $\text{Int}[(d + e*x^2)^2/(d^2 - e^2*x^4), x]$

[Out] $-x + (2*\text{Sqrt}[d]*\text{ArcTanh}[(\text{Sqrt}[e]*x)/\text{Sqrt}[d]])/\text{Sqrt}[e]$

Rule 1150

$\text{Int}[(d + e*x^2)^2/(d^2 - e^2*x^4), x] \text{ :> Int}[(d + e*x^2)^2/(d^2 - e^2*x^4), x] \text{ ; FreeQ}\{a, c, d, e, q\}, x \text{ \&\& EqQ}[c*d^2 + a*e^2, 0] \text{ \&\& IntegerQ}[p]$

Rule 388

$\text{Int}[(a + b*x^n)^p/(c + d*x^n), x] \text{ :> Simp}[(d*x*(a + b*x^n)^{p+1}/(b*(n*(p+1)+1)), x] - \text{Dist}[(a*d - b*c*(n*(p+1)+1))/(b*(n*(p+1)+1)), \text{Int}[(a + b*x^n)^p, x], x] \text{ ; FreeQ}\{a, b, c, d, n\}, x \text{ \&\& NeQ}[b*c - a*d, 0] \text{ \&\& NeQ}[n*(p+1)+1, 0]$

Rule 208

$\text{Int}[(a + b*x^2)^{-1}, x] \text{ :> Simp}[(\text{Rt}[-(a/b), 2]*\text{ArcTanh}[x/\text{Rt}[-(a/b), 2]])/a, x] \text{ ; FreeQ}\{a, b\}, x \text{ \&\& NegQ}[a/b]$

Rubi steps

$$\begin{aligned} \int \frac{(d+ex^2)^2}{d^2-e^2x^4} dx &= \int \frac{d+ex^2}{d-ex^2} dx \\ &= -x + (2d) \int \frac{1}{d-ex^2} dx \\ &= -x + \frac{2\sqrt{d} \tanh^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)}{\sqrt{e}} \end{aligned}$$

Mathematica [A] time = 0.0089009, size = 29, normalized size = 1.

$$\frac{2\sqrt{d} \tanh^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)}{\sqrt{e}} - x$$

Antiderivative was successfully verified.

[In] Integrate[(d + e*x^2)^2/(d^2 - e^2*x^4), x]

[Out] -x + (2*Sqrt[d]*ArcTanh[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[e]

Maple [A] time = 0.004, size = 22, normalized size = 0.8

$$-x + 2 \frac{d}{\sqrt{de}} \operatorname{Artanh}\left(\frac{ex}{\sqrt{de}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x^2+d)^2/(-e^2*x^4+d^2), x)

[Out] -x+2*d/(d*e)^(1/2)*arctanh(x*e/(d*e)^(1/2))

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)^2/(-e^2*x^4+d^2), x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [A] time = 1.87925, size = 149, normalized size = 5.14

$$\left[\sqrt{\frac{d}{e}} \log\left(\frac{ex^2 + 2ex\sqrt{\frac{d}{e}} + d}{ex^2 - d}\right) - x, -2\sqrt{\frac{d}{e}} \arctan\left(\frac{ex\sqrt{\frac{d}{e}}}{d}\right) - x \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)^2/(-e^2*x^4+d^2), x, algorithm="fricas")

[Out] [sqrt(d/e)*log((e*x^2 + 2*e*x*sqrt(d/e) + d)/(e*x^2 - d)) - x, -2*sqrt(-d/e)*arctan(e*x*sqrt(-d/e)/d) - x]

Sympy [A] time = 0.359195, size = 34, normalized size = 1.17

$$-x - \sqrt{\frac{d}{e}} \log\left(x - \sqrt{\frac{d}{e}}\right) + \sqrt{\frac{d}{e}} \log\left(x + \sqrt{\frac{d}{e}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x**2+d)**2/(-e**2*x**4+d**2),x)

[Out] -x - sqrt(d/e)*log(x - sqrt(d/e)) + sqrt(d/e)*log(x + sqrt(d/e))

Giac [B] time = 1.15496, size = 159, normalized size = 5.48

$$\frac{\left((d^2)^{\frac{1}{4}} d e^{\frac{7}{2}} - (d^2)^{\frac{1}{4}} |d| e^{\frac{7}{2}} \right) \arctan\left(\frac{x e^{\frac{1}{2}}}{(d^2)^{\frac{1}{4}}} \right) e^{(-4)} + \frac{\left((d^2)^{\frac{1}{4}} d e^{\frac{11}{2}} + (d^2)^{\frac{3}{4}} e^{\frac{11}{2}} \right) e^{(-6)} \log\left(\left| (d^2)^{\frac{1}{4}} e^{\left(-\frac{1}{2}\right)} + x \right| \right)}{2d} - \frac{\left((d^2)^{\frac{1}{4}} d e^{\frac{7}{2}} + (d^2)^{\frac{1}{4}} |d| e^{\frac{7}{2}} \right) \arctan\left(\frac{x e^{\frac{1}{2}}}{(d^2)^{\frac{1}{4}}} \right) e^{(-4)}}{d}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)^2/(-e^2*x^4+d^2),x, algorithm="giac")

[Out] ((d^2)^(1/4)*d*e^(7/2) - (d^2)^(1/4)*abs(d)*e^(7/2))*arctan(x*e^(1/2)/(d^2)^(1/4))*e^(-4)/d + 1/2*((d^2)^(1/4)*d*e^(11/2) + (d^2)^(3/4)*e^(11/2))*e^(-6)*log(abs((d^2)^(1/4)*e^(-1/2) + x))/d - 1/2*((d^2)^(1/4)*d*e^(7/2) + (d^2)^(1/4)*abs(d)*e^(7/2))*e^(-4)*log(abs(-(d^2)^(1/4)*e^(-1/2) + x))/d - x

$$3.192 \quad \int \frac{d+ex^2}{d^2-e^2x^4} dx$$

Optimal. Leaf size=24

$$\frac{\tanh^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)}{\sqrt{d}\sqrt{e}}$$

[Out] ArcTanh[(Sqrt[e]*x)/Sqrt[d]]/(Sqrt[d]*Sqrt[e])

Rubi [A] time = 0.0122413, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1150, 208}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)}{\sqrt{d}\sqrt{e}}$$

Antiderivative was successfully verified.

[In] Int[(d + e*x^2)/(d^2 - e^2*x^4), x]

[Out] ArcTanh[(Sqrt[e]*x)/Sqrt[d]]/(Sqrt[d]*Sqrt[e])

Rule 1150

Int[((d_) + (e_)*(x_)^2)^(q_)*((a_) + (c_)*(x_)^4)^(p_), x_Symbol] :> Int[(d + e*x^2)^(p + q)*(a/d + (c*x^2)/e)^p, x] /; FreeQ[{a, c, d, e, q}, x] && EqQ[c*d^2 + a*e^2, 0] && IntegerQ[p]

Rule 208

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rubi steps

$$\begin{aligned} \int \frac{d+ex^2}{d^2-e^2x^4} dx &= \int \frac{1}{d-ex^2} dx \\ &= \frac{\tanh^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)}{\sqrt{d}\sqrt{e}} \end{aligned}$$

Mathematica [A] time = 0.0043265, size = 24, normalized size = 1.

$$\frac{\tanh^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)}{\sqrt{d}\sqrt{e}}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e*x^2)/(d^2 - e^2*x^4), x]

[Out] ArcTanh[(Sqrt[e]*x)/Sqrt[d]]/(Sqrt[d]*Sqrt[e])

Maple [A] time = 0.003, size = 16, normalized size = 0.7

$$\operatorname{Arctanh}\left(ex\frac{1}{\sqrt{de}}\right)\frac{1}{\sqrt{de}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x^2+d)/(-e^2*x^4+d^2),x)

[Out] 1/(d*e)^(1/2)*arctanh(x*e/(d*e)^(1/2))

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)/(-e^2*x^4+d^2),x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [A] time = 1.90441, size = 151, normalized size = 6.29

$$\left[\frac{\sqrt{de} \log\left(\frac{ex^2+2\sqrt{dex}+d}{ex^2-d}\right)}{2de}, -\frac{\sqrt{-de} \arctan\left(\frac{\sqrt{-dex}}{d}\right)}{de} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)/(-e^2*x^4+d^2),x, algorithm="fricas")

[Out] [1/2*sqrt(d*e)*log((e*x^2 + 2*sqrt(d*e)*x + d)/(e*x^2 - d))/(d*e), -sqrt(-d*e)*arctan(sqrt(-d*e)*x/d)/(d*e)]

Sympy [B] time = 0.186596, size = 46, normalized size = 1.92

$$-\frac{\sqrt{\frac{1}{de}} \log\left(-d\sqrt{\frac{1}{de}} + x\right)}{2} + \frac{\sqrt{\frac{1}{de}} \log\left(d\sqrt{\frac{1}{de}} + x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x**2+d)/(-e**2*x**4+d**2),x)

[Out] -sqrt(1/(d*e))*log(-d*sqrt(1/(d*e)) + x)/2 + sqrt(1/(d*e))*log(d*sqrt(1/(d*e)) + x)/2

Giac [B] time = 1.16564, size = 157, normalized size = 6.54

$$\frac{\left((d^2)^{\frac{1}{4}}de^{\frac{7}{2}} - (d^2)^{\frac{1}{4}}|d|e^{\frac{7}{2}}\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{(d^2)^{\frac{1}{4}}}\right)e^{(-4)}}{2d^2} + \frac{\left((d^2)^{\frac{1}{4}}de^{\frac{11}{2}} + (d^2)^{\frac{3}{4}}e^{\frac{11}{2}}\right)e^{(-6)}\log\left(\left|(d^2)^{\frac{1}{4}}e^{\left(-\frac{1}{2}\right)} + x\right|\right)}{4d^2} - \frac{\left((d^2)^{\frac{1}{4}}de^{\frac{7}{2}} + (d^2)^{\frac{3}{4}}e^{\frac{7}{2}}\right)e^{(-4)}\log\left(\left|(d^2)^{\frac{1}{4}}e^{\left(-\frac{1}{2}\right)} + x\right|\right)}{4d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)/(-e^2*x^4+d^2),x, algorithm="giac")

[Out] 1/2*((d^2)^(1/4)*d*e^(7/2) - (d^2)^(1/4)*abs(d)*e^(7/2))*arctan(x*e^(1/2)/(d^2)^(1/4))*e^(-4)/d^2 + 1/4*((d^2)^(1/4)*d*e^(11/2) + (d^2)^(3/4)*e^(11/2))*e^(-6)*log(abs((d^2)^(1/4)*e^(-1/2) + x))/d^2 - 1/4*((d^2)^(1/4)*d*e^(7/2) + (d^2)^(1/4)*abs(d)*e^(7/2))*e^(-4)*log(abs(-(d^2)^(1/4)*e^(-1/2) + x))/d^2

$$3.193 \quad \int \frac{1}{(d+ex^2)(d^2-e^2x^4)} dx$$

Optimal. Leaf size=72

$$\frac{x}{4d^2(d+ex^2)} + \frac{\tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)}{2d^{5/2}\sqrt{e}} + \frac{\tanh^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)}{4d^{5/2}\sqrt{e}}$$

[Out] x/(4*d^2*(d + e*x^2)) + ArcTan[(Sqrt[e]*x)/Sqrt[d]]/(2*d^(5/2)*Sqrt[e]) + ArcTanh[(Sqrt[e]*x)/Sqrt[d]]/(4*d^(5/2)*Sqrt[e])

Rubi [A] time = 0.0565035, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {1150, 414, 522, 208, 205}

$$\frac{x}{4d^2(d+ex^2)} + \frac{\tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)}{2d^{5/2}\sqrt{e}} + \frac{\tanh^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)}{4d^{5/2}\sqrt{e}}$$

Antiderivative was successfully verified.

[In] Int[1/((d + e*x^2)*(d^2 - e^2*x^4)),x]

[Out] x/(4*d^2*(d + e*x^2)) + ArcTan[(Sqrt[e]*x)/Sqrt[d]]/(2*d^(5/2)*Sqrt[e]) + ArcTanh[(Sqrt[e]*x)/Sqrt[d]]/(4*d^(5/2)*Sqrt[e])

Rule 1150

Int[((d_) + (e_)*(x_)^2)^(q_)*((a_) + (c_)*(x_)^4)^(p_), x_Symbol] :> Int[(d + e*x^2)^(p + q)*(a/d + (c*x^2)/e)^p, x] /; FreeQ[{a, c, d, e, q}, x] && EqQ[c*d^2 + a*e^2, 0] && IntegerQ[p]

Rule 414

Int[((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_), x_Symbol] :> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c - a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && !(IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, n, p, q, x]

Rule 522

Int[((e_) + (f_)*(x_)^(n_))/((a_) + (b_)*(x_)^(n_))*((c_) + (d_)*(x_)^(n_)), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x]

Rule 208

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 205


```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{(d+ex^2)(d^2-e^2x^4)} dx &= \int \frac{1}{(d-ex^2)(d+ex^2)^2} dx \\ &= \frac{x}{4d^2(d+ex^2)} - \frac{\int \frac{-3de+e^2x^2}{(d-ex^2)(d+ex^2)} dx}{4d^2e} \\ &= \frac{x}{4d^2(d+ex^2)} + \frac{\int \frac{1}{d-ex^2} dx}{4d^2} + \frac{\int \frac{1}{d+ex^2} dx}{2d^2} \\ &= \frac{x}{4d^2(d+ex^2)} + \frac{\tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)}{2d^{5/2}\sqrt{e}} + \frac{\tanh^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)}{4d^{5/2}\sqrt{e}} \end{aligned}$$

Mathematica [A] time = 0.0344483, size = 65, normalized size = 0.9

$$\frac{\frac{\sqrt{dx}}{d+ex^2} + \frac{2 \tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)}{\sqrt{e}} + \frac{\tanh^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)}{\sqrt{e}}}{4d^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x^2)*(d^2 - e^2*x^4)), x]
```

```
[Out] ((Sqrt[d]*x)/(d + e*x^2) + (2*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[e] + ArcTan
h[(Sqrt[e]*x)/Sqrt[d]]/Sqrt[e])/(4*d^(5/2))
```

Maple [A] time = 0.016, size = 55, normalized size = 0.8

$$\frac{x}{4d^2(ex^2+d)} + \frac{1}{2d^2} \arctan\left(ex \frac{1}{\sqrt{de}}\right) \frac{1}{\sqrt{de}} + \frac{1}{4d^2} \operatorname{Artanh}\left(ex \frac{1}{\sqrt{de}}\right) \frac{1}{\sqrt{de}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d)/(-e^2*x^4+d^2), x)
```

```
[Out] 1/4*x/d^2/(e*x^2+d)+1/2/d^2/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))+1/4/d^2/(d*
e)^(1/2)*arctanh(x*e/(d*e)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(-e^2*x^4+d^2), x, algorithm="maxima")
```

[Out] Exception raised: ValueError

Fricas [A] time = 1.85425, size = 421, normalized size = 5.85

$$\left[\frac{2dex + 4(ex^2 + d)\sqrt{de} \arctan\left(\frac{\sqrt{dex}}{d}\right) + (ex^2 + d)\sqrt{de} \log\left(\frac{ex^2 + 2\sqrt{dex} + d}{ex^2 - d}\right)}{8(d^3e^2x^2 + d^4e)}, \frac{dex - (ex^2 + d)\sqrt{-de} \arctan\left(\frac{\sqrt{-dex}}{d}\right) - (ex^2 + d)\sqrt{-de} \log\left(\frac{ex^2 - 2\sqrt{-dex} + d}{ex^2 + d}\right)}{4(d^3e^2x^2 + d^4e)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x^2+d)/(-e^2*x^4+d^2),x, algorithm="fricas")

[Out] [1/8*(2*d*e*x + 4*(e*x^2 + d)*sqrt(d*e)*arctan(sqrt(d*e)*x/d) + (e*x^2 + d)*sqrt(d*e)*log((e*x^2 + 2*sqrt(d*e)*x + d)/(e*x^2 - d)))/(d^3*e^2*x^2 + d^4*e), 1/4*(d*e*x - (e*x^2 + d)*sqrt(-d*e)*arctan(sqrt(-d*e)*x/d) - (e*x^2 + d)*sqrt(-d*e)*log((e*x^2 - 2*sqrt(-d*e)*x - d)/(e*x^2 + d)))/(d^3*e^2*x^2 + d^4*e)]

Sympy [B] time = 0.653694, size = 226, normalized size = 3.14

$$\frac{x}{4d^3 + 4d^2ex^2} - \frac{\sqrt{\frac{1}{d^5e}} \log\left(-\frac{d^8e\left(\frac{1}{d^5e}\right)^{\frac{3}{2}}}{10} - \frac{9d^3\sqrt{\frac{1}{d^5e}}}{10} + x\right)}{8} + \frac{\sqrt{\frac{1}{d^5e}} \log\left(\frac{d^8e\left(\frac{1}{d^5e}\right)^{\frac{3}{2}}}{10} + \frac{9d^3\sqrt{\frac{1}{d^5e}}}{10} + x\right)}{8} - \frac{\sqrt{-\frac{1}{d^5e}} \log\left(-\frac{4d^8e\left(-\frac{1}{d^5e}\right)^{\frac{3}{2}}}{5} - \frac{9d^3\sqrt{-\frac{1}{d^5e}}}{5} + x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x**2+d)/(-e**2*x**4+d**2),x)

[Out] x/(4*d**3 + 4*d**2*e*x**2) - sqrt(1/(d**5*e))*log(-d**8*e*(1/(d**5*e))**(3/2)/10 - 9*d**3*sqrt(1/(d**5*e))/10 + x)/8 + sqrt(1/(d**5*e))*log(d**8*e*(1/(d**5*e))**(3/2)/10 + 9*d**3*sqrt(1/(d**5*e))/10 + x)/8 - sqrt(-1/(d**5*e))*log(-4*d**8*e*(-1/(d**5*e))**(3/2)/5 - 9*d**3*sqrt(-1/(d**5*e))/5 + x)/4 + sqrt(-1/(d**5*e))*log(4*d**8*e*(-1/(d**5*e))**(3/2)/5 + 9*d**3*sqrt(-1/(d**5*e))/5 + x)/4

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x^2+d)/(-e^2*x^4+d^2),x, algorithm="giac")

[Out] Exception raised: NotImplementedError

$$3.194 \quad \int \frac{1}{(d+ex^2)^2(d^2-e^2x^4)} dx$$

Optimal. Leaf size=89

$$\frac{5x}{16d^3(d+ex^2)} + \frac{x}{8d^2(d+ex^2)^2} + \frac{7 \tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)}{16d^{7/2}\sqrt{e}} + \frac{\tanh^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)}{8d^{7/2}\sqrt{e}}$$

[Out] x/(8*d^2*(d + e*x^2)^2) + (5*x)/(16*d^3*(d + e*x^2)) + (7*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(16*d^(7/2)*Sqrt[e]) + ArcTanh[(Sqrt[e]*x)/Sqrt[d]]/(8*d^(7/2)*Sqrt[e])

Rubi [A] time = 0.0827575, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$, Rules used = {1150, 414, 527, 522, 208, 205}

$$\frac{5x}{16d^3(d+ex^2)} + \frac{x}{8d^2(d+ex^2)^2} + \frac{7 \tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)}{16d^{7/2}\sqrt{e}} + \frac{\tanh^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)}{8d^{7/2}\sqrt{e}}$$

Antiderivative was successfully verified.

[In] Int[1/((d + e*x^2)^2*(d^2 - e^2*x^4)), x]

[Out] x/(8*d^2*(d + e*x^2)^2) + (5*x)/(16*d^3*(d + e*x^2)) + (7*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(16*d^(7/2)*Sqrt[e]) + ArcTanh[(Sqrt[e]*x)/Sqrt[d]]/(8*d^(7/2)*Sqrt[e])

Rule 1150

Int[((d_) + (e_)*(x_)^2)^(q_)*((a_) + (c_)*(x_)^4)^(p_), x_Symbol] :> Int[(d + e*x^2)^(p + q)*(a/d + (c*x^2)/e)^p, x] /; FreeQ[{a, c, d, e, q}, x] && EqQ[c*d^2 + a*e^2, 0] && IntegerQ[p]

Rule 414

Int[((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_), x_Symbol] :> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c - a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && !(IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, n, p, q, x]

Rule 527

Int[((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_)*((e_) + (f_)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x] /; FreeQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] := Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{(d+ex^2)^2(d^2-e^2x^4)} dx &= \int \frac{1}{(d-ex^2)(d+ex^2)^3} dx \\ &= \frac{x}{8d^2(d+ex^2)^2} - \frac{\int \frac{-7de+3e^2x^2}{(d-ex^2)(d+ex^2)^2} dx}{8d^2e} \\ &= \frac{x}{8d^2(d+ex^2)^2} + \frac{5x}{16d^3(d+ex^2)} + \frac{\int \frac{18d^2e^2-10de^3x^2}{(d-ex^2)(d+ex^2)} dx}{32d^4e^2} \\ &= \frac{x}{8d^2(d+ex^2)^2} + \frac{5x}{16d^3(d+ex^2)} + \frac{\int \frac{1}{d-ex^2} dx}{8d^3} + \frac{7 \int \frac{1}{d+ex^2} dx}{16d^3} \\ &= \frac{x}{8d^2(d+ex^2)^2} + \frac{5x}{16d^3(d+ex^2)} + \frac{7 \tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)}{16d^{7/2}\sqrt{e}} + \frac{\tanh^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)}{8d^{7/2}\sqrt{e}} \end{aligned}$$

Mathematica [A] time = 0.0584231, size = 76, normalized size = 0.85

$$\frac{\frac{\sqrt{dx}(7d+5ex^2)}{(d+ex^2)^2} + \frac{7 \tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)}{\sqrt{e}} + \frac{2 \tanh^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)}{\sqrt{e}}}{16d^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x^2)^2*(d^2 - e^2*x^4)),x]
```

```
[Out] ((Sqrt[d]*x*(7*d + 5*e*x^2))/(d + e*x^2)^2 + (7*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[e] + (2*ArcTanh[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[e])/(16*d^(7/2))
```

Maple [A] time = 0.01, size = 73, normalized size = 0.8

$$\frac{5ex^3}{16d^3(ex^2+d)^2} + \frac{7x}{16d^2(ex^2+d)^2} + \frac{7}{16d^3} \arctan\left(ex \frac{1}{\sqrt{de}}\right) \frac{1}{\sqrt{de}} + \frac{1}{8d^3} \operatorname{Artanh}\left(ex \frac{1}{\sqrt{de}}\right) \frac{1}{\sqrt{de}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(e*x^2+d)^2/(-e^2*x^4+d^2),x)`

[Out] $5/16/d^3/(e*x^2+d)^2*x^3*e+7/16*x/d^2/(e*x^2+d)^2+7/16/d^3/(d*e)^{(1/2)}*\arctan(x*e/(d*e)^{(1/2)})+1/8/d^3/(d*e)^{(1/2)}*\operatorname{arctanh}(x*e/(d*e)^{(1/2)})$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(e*x^2+d)^2/(-e^2*x^4+d^2),x, algorithm="maxima")`

[Out] Exception raised: ValueError

Fricas [B] time = 1.82766, size = 608, normalized size = 6.83

$$\left[\frac{5de^2x^3 + 7d^2ex + 7(e^2x^4 + 2dex^2 + d^2)\sqrt{de} \arctan\left(\frac{\sqrt{dex}}{d}\right) + (e^2x^4 + 2dex^2 + d^2)\sqrt{de} \log\left(\frac{ex^2 + 2\sqrt{dex} + d}{ex^2 - d}\right)}{16(d^4e^3x^4 + 2d^5e^2x^2 + d^6e)}, \frac{10de^2x^3 + \dots}{\dots} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(e*x^2+d)^2/(-e^2*x^4+d^2),x, algorithm="fricas")`

[Out] $[1/16*(5*d*e^2*x^3 + 7*d^2*e*x + 7*(e^2*x^4 + 2*d*e*x^2 + d^2)*\sqrt{d*e}*\arctan(\sqrt{d*e}*x/d) + (e^2*x^4 + 2*d*e*x^2 + d^2)*\sqrt{d*e}*\log((e*x^2 + 2*\sqrt{d*e}*x + d)/(e*x^2 - d)))/(d^4*e^3*x^4 + 2*d^5*e^2*x^2 + d^6*e), 1/32*(10*d*e^2*x^3 + 14*d^2*e*x - 4*(e^2*x^4 + 2*d*e*x^2 + d^2)*\sqrt{-d*e}*\arctan(\sqrt{-d*e}*x/d) - 7*(e^2*x^4 + 2*d*e*x^2 + d^2)*\sqrt{-d*e}*\log((e*x^2 - 2*\sqrt{-d*e}*x - d)/(e*x^2 + d)))/(d^4*e^3*x^4 + 2*d^5*e^2*x^2 + d^6*e)]$

Sympy [B] time = 0.90657, size = 255, normalized size = 2.87

$$\frac{\sqrt{\frac{1}{d^7e}} \log\left(-\frac{20d^{11}e\left(\frac{1}{d^7e}\right)^{\frac{3}{2}}}{371} - \frac{351d^4\sqrt{\frac{1}{d^7e}}}{371} + x\right)}{16} + \frac{\sqrt{\frac{1}{d^7e}} \log\left(\frac{20d^{11}e\left(\frac{1}{d^7e}\right)^{\frac{3}{2}}}{371} + \frac{351d^4\sqrt{\frac{1}{d^7e}}}{371} + x\right)}{16} - \frac{7\sqrt{-\frac{1}{d^7e}} \log\left(-\frac{245d^{11}e\left(-\frac{1}{d^7e}\right)^{\frac{3}{2}}}{106} - \dots\right)}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(e*x**2+d)**2/(-e**2*x**4+d**2),x)`

[Out] $-\sqrt{1/(d**7*e)}*\log(-20*d**11*e*(1/(d**7*e))**(3/2)/371 - 351*d**4*\sqrt{1/(d**7*e)}/371 + x)/16 + \sqrt{1/(d**7*e)}*\log(20*d**11*e*(1/(d**7*e))**(3/2)/371 + 351*d**4*\sqrt{1/(d**7*e)}/371 + x)/16 - 7*\sqrt{-1/(d**7*e)}*\log(-245*d**11*e*(-1/(d**7*e))**(3/2)/106 - 351*d**4*\sqrt{-1/(d**7*e)}/106 + x)/32 + 7*\sqrt{-1/(d**7*e)}*\log(245*d**11*e*(-1/(d**7*e))**(3/2)/106 + 351*d**4*\sqrt{-1/(d**7*e)}/106 + x)/32 + (7*d*x + 5*e*x**3)/(16*d**5 + 32*d**4*e*x**2 + 16*d**3*e**2*x**4)$

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(e*x^2+d)^2/(-e^2*x^4+d^2),x, algorithm="giac")`

[Out] Exception raised: NotImplementedError

$$3.195 \quad \int \frac{(d+ex^2)^{3/2}}{d^2-e^2x^4} dx$$

Optimal. Leaf size=62

$$\frac{\sqrt{2} \tanh^{-1}\left(\frac{\sqrt{2}\sqrt{ex}}{\sqrt{d+ex^2}}\right)}{\sqrt{e}} - \frac{\tanh^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d+ex^2}}\right)}{\sqrt{e}}$$

[Out] -(ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]/Sqrt[e]) + (Sqrt[2]*ArcTanh[(Sqrt[2]*Sqrt[e]*x)/Sqrt[d + e*x^2]])/Sqrt[e]

Rubi [A] time = 0.0442698, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {1150, 402, 217, 206, 377, 208}

$$\frac{\sqrt{2} \tanh^{-1}\left(\frac{\sqrt{2}\sqrt{ex}}{\sqrt{d+ex^2}}\right)}{\sqrt{e}} - \frac{\tanh^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d+ex^2}}\right)}{\sqrt{e}}$$

Antiderivative was successfully verified.

[In] Int[(d + e*x^2)^(3/2)/(d^2 - e^2*x^4), x]

[Out] -(ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]/Sqrt[e]) + (Sqrt[2]*ArcTanh[(Sqrt[2]*Sqrt[e]*x)/Sqrt[d + e*x^2]])/Sqrt[e]

Rule 1150

Int[((d_) + (e_)*(x_)^2)^(q_)*((a_) + (c_)*(x_)^4)^(p_), x_Symbol] :> Int[(d + e*x^2)^(p + q)*(a/d + (c*x^2)/e)^p, x] /; FreeQ[{a, c, d, e, q}, x] && EqQ[c*d^2 + a*e^2, 0] && IntegerQ[p]

Rule 402

Int[((a_) + (b_)*(x_)^2)^(p_)/((c_) + (d_)*(x_)^2), x_Symbol] :> Dist[b/d, Int[(a + b*x^2)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^2)^(p - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4])

Rule 217

Int[1/Sqrt[(a_) + (b_)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 377

Int[((a_) + (b_)*(x_)^(n_))^(p_)/((c_) + (d_)*(x_)^(n_)), x_Symbol] :> Subst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]

Rule 208

$\text{Int}[(a_ + (b_ \cdot)(x_)^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(\text{Rt}[-(a/b), 2] \cdot \text{ArcTanh}[x/\text{Rt}[-(a/b), 2]])/a, x] /; \text{FreeQ}\{a, b\}, x] \ \&\& \ \text{NegQ}[a/b]$

Rubi steps

$$\begin{aligned} \int \frac{(d + ex^2)^{3/2}}{d^2 - e^2x^4} dx &= \int \frac{\sqrt{d + ex^2}}{d - ex^2} dx \\ &= (2d) \int \frac{1}{(d - ex^2)\sqrt{d + ex^2}} dx - \int \frac{1}{\sqrt{d + ex^2}} dx \\ &= (2d) \text{Subst}\left(\int \frac{1}{d - 2dex^2} dx, x, \frac{x}{\sqrt{d + ex^2}}\right) - \text{Subst}\left(\int \frac{1}{1 - ex^2} dx, x, \frac{x}{\sqrt{d + ex^2}}\right) \\ &= -\frac{\tanh^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d+ex^2}}\right)}{\sqrt{e}} + \frac{\sqrt{2} \tanh^{-1}\left(\frac{\sqrt{2}\sqrt{ex}}{\sqrt{d+ex^2}}\right)}{\sqrt{e}} \end{aligned}$$

Mathematica [A] time = 0.0283889, size = 61, normalized size = 0.98

$$\frac{\sqrt{2} \tanh^{-1}\left(\frac{\sqrt{2}\sqrt{ex}}{\sqrt{d+ex^2}}\right) - \log\left(\sqrt{e}\sqrt{d + ex^2} + ex\right)}{\sqrt{e}}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e*x^2)^(3/2)/(d^2 - e^2*x^4), x]

[Out] (Sqrt[2]*ArcTanh[(Sqrt[2]*Sqrt[e]*x)/Sqrt[d + e*x^2]] - Log[e*x + Sqrt[e]*Sqrt[d + e*x^2]])/Sqrt[e]

Maple [B] time = 0.054, size = 1442, normalized size = 23.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x^2+d)^(3/2)/(-e^2*x^4+d^2), x)

[Out]
$$\begin{aligned} & -1/6*e/((-d*e)^{(1/2)}+(d*e)^{(1/2)})/((-d*e)^{(1/2)}+(d*e)^{(1/2)})/(-d*e)^{(1/2)}* \\ & ((x+(-d*e)^{(1/2)}/e)^2*e^{-2}*(-d*e)^{(1/2)}*(x+(-d*e)^{(1/2)}/e))^{(3/2)}+1/4*e/((-d \\ & *e)^{(1/2)}+(d*e)^{(1/2)})/((-d*e)^{(1/2)}+(d*e)^{(1/2)})*((x+(-d*e)^{(1/2)}/e)^2*e^{-2} \\ & *(-d*e)^{(1/2)}*(x+(-d*e)^{(1/2)}/e))^{(1/2)}*x+1/4*e^{(1/2)}/((-d*e)^{(1/2)}+(d*e)^{(1/2)})/ \\ & (-d*e)^{(1/2)}+(d*e)^{(1/2)})*d*\ln((x+(-d*e)^{(1/2)}/e)*e-(-d*e)^{(1/2)}) \\ & /e^{(1/2)}+((x+(-d*e)^{(1/2)}/e)^2*e^{-2}*(-d*e)^{(1/2)}*(x+(-d*e)^{(1/2)}/e))^{(1/2)}+ \\ & 1/6*e/((-d*e)^{(1/2)}+(d*e)^{(1/2)})/((-d*e)^{(1/2)}+(d*e)^{(1/2)})/(-d*e)^{(1/2)}* \\ & (x-(-d*e)^{(1/2)}/e)^2*e+2*(-d*e)^{(1/2)}*(x-(-d*e)^{(1/2)}/e))^{(3/2)}+1/4*e/((-d* \\ & e)^{(1/2)}+(d*e)^{(1/2)})/((-d*e)^{(1/2)}+(d*e)^{(1/2)})*((x-(-d*e)^{(1/2)}/e)^2*e+2 \\ & *(-d*e)^{(1/2)}*(x-(-d*e)^{(1/2)}/e))^{(1/2)}*x+1/4*e^{(1/2)}/((-d*e)^{(1/2)}+(d*e)^{(1/2)})/ \\ & (-d*e)^{(1/2)}+(d*e)^{(1/2)})*d*\ln(((x-(-d*e)^{(1/2)}/e)*e+(-d*e)^{(1/2)})/ \\ & e^{(1/2)}+((x-(-d*e)^{(1/2)}/e)^2*e+2*(-d*e)^{(1/2)}*(x-(-d*e)^{(1/2)}/e))^{(1/2)}+1 \\ & /6*e/(d*e)^{(1/2)}/((-d*e)^{(1/2)}+(d*e)^{(1/2)})/((-d*e)^{(1/2)}+(d*e)^{(1/2)})*((x \\ & +(d*e)^{(1/2)}/e)^2*e^{-2}*(d*e)^{(1/2)}*(x+(d*e)^{(1/2)}/e)+2*d)^{(3/2)}-1/4*e/((-d*e \\ &)^{(1/2)}+(d*e)^{(1/2)})/((-d*e)^{(1/2)}+(d*e)^{(1/2)})*((x+(d*e)^{(1/2)}/e)^2*e^{-2}* \end{aligned}$$

$$d^{1/2} * (x + (d * e)^{1/2} / e + 2 * d)^{1/2} * x^{-5/4} * e^{1/2} / ((-d * e)^{1/2} + (d * e)^{1/2}) / (-(-d * e)^{1/2} + (d * e)^{1/2}) * d * \ln(((x + (d * e)^{1/2} / e) * e - (d * e)^{1/2}) / e^{1/2} + ((x + (d * e)^{1/2} / e)^2 * e^{-2} * (d * e)^{1/2} * (x + (d * e)^{1/2} / e) + 2 * d)^{1/2}) + e / (d * e)^{1/2} / ((-d * e)^{1/2} + (d * e)^{1/2}) / (-(-d * e)^{1/2} + (d * e)^{1/2}) * d * ((x + (d * e)^{1/2} / e)^2 * e^{-2} * (d * e)^{1/2} * (x + (d * e)^{1/2} / e) + 2 * d)^{1/2} - e / (d * e)^{1/2} / ((-d * e)^{1/2} + (d * e)^{1/2}) / (-(-d * e)^{1/2} + (d * e)^{1/2}) * d^{3/2} * 2^{1/2} * \ln((4 * d - 2 * (d * e)^{1/2} * (x + (d * e)^{1/2} / e) + 2 * 2^{1/2} * d^{1/2} * ((x + (d * e)^{1/2} / e)^2 * e^{-2} * (d * e)^{1/2} * (x + (d * e)^{1/2} / e) + 2 * d)^{1/2}) / (x + (d * e)^{1/2} / e) - 1/6 * e / (d * e)^{1/2} / ((-d * e)^{1/2} + (d * e)^{1/2}) / (-(-d * e)^{1/2} + (d * e)^{1/2}) * ((x - (d * e)^{1/2} / e)^2 * e + 2 * (d * e)^{1/2} * (x - (d * e)^{1/2} / e) + 2 * d)^{3/2} - 1/4 * e / ((-d * e)^{1/2} + (d * e)^{1/2}) / (-(-d * e)^{1/2} + (d * e)^{1/2}) * ((x - (d * e)^{1/2} / e)^2 * e + 2 * (d * e)^{1/2} * (x - (d * e)^{1/2} / e) + 2 * d)^{1/2} * x^{-5/4} * e^{1/2} / ((-d * e)^{1/2} + (d * e)^{1/2}) / (-(-d * e)^{1/2} + (d * e)^{1/2}) * d * \ln(((x - (d * e)^{1/2} / e) * e + (d * e)^{1/2}) / e^{1/2} + ((x - (d * e)^{1/2} / e)^2 * e + 2 * (d * e)^{1/2} * (x - (d * e)^{1/2} / e) + 2 * d)^{1/2}) - e / (d * e)^{1/2} / ((-d * e)^{1/2} + (d * e)^{1/2}) / (-(-d * e)^{1/2} + (d * e)^{1/2}) * d * ((x - (d * e)^{1/2} / e)^2 * e + 2 * (d * e)^{1/2} * (x - (d * e)^{1/2} / e) + 2 * d)^{1/2} + e / (d * e)^{1/2} / ((-d * e)^{1/2} + (d * e)^{1/2}) / (-(-d * e)^{1/2} + (d * e)^{1/2}) * d^{3/2} * 2^{1/2} * \ln((4 * d + 2 * (d * e)^{1/2} * (x - (d * e)^{1/2} / e) + 2 * 2^{1/2} * d^{1/2} * ((x - (d * e)^{1/2} / e)^2 * e + 2 * (d * e)^{1/2} * (x - (d * e)^{1/2} / e) + 2 * d)^{1/2}) / (x - (d * e)^{1/2} / e))$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$- \int \frac{(ex^2 + d)^{3/2}}{e^2 x^4 - d^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)^(3/2)/(-e^2*x^4+d^2),x, algorithm="maxima")

[Out] -integrate((e*x^2 + d)^(3/2)/(e^2*x^4 - d^2), x)

Fricas [A] time = 2.05638, size = 491, normalized size = 7.92

$$\left[\frac{\sqrt{2} \sqrt{e} \log \left(\frac{17 e^2 x^4 + 14 d e x^2 + d^2 + \frac{4 \sqrt{2} (3 e^2 x^3 + d e x) \sqrt{e x^2 + d}}{\sqrt{e}}}{e^2 x^4 - 2 d e x^2 + d^2} \right) + 2 \sqrt{e} \log \left(-2 e x^2 + 2 \sqrt{e x^2 + d} \sqrt{e x - d} \right)}{4 e}, \sqrt{2} e \sqrt{-\frac{1}{e}} \arctan \left(\frac{\sqrt{2} (3 e x^2 + d)}{4} \right) \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)^(3/2)/(-e^2*x^4+d^2),x, algorithm="fricas")

[Out] [1/4*(sqrt(2)*sqrt(e)*log((17*e^2*x^4 + 14*d*e*x^2 + d^2 + 4*sqrt(2)*(3*e^2*x^3 + d*e*x)*sqrt(e*x^2 + d)/sqrt(e))/(e^2*x^4 - 2*d*e*x^2 + d^2)) + 2*sqrt(e)*log(-2*e*x^2 + 2*sqrt(e*x^2 + d)*sqrt(e)*x - d))/e, -1/2*(sqrt(2)*e*sqrt(-1/e)*arctan(1/4*sqrt(2)*(3*e*x^2 + d)*sqrt(e*x^2 + d)*sqrt(-1/e)/(e*x^3 + d*x)) - 2*sqrt(-e)*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)))/e]

Sympy [F] time = 0., size = 0, normalized size = 0.

$$- \int \frac{\sqrt{d + ex^2}}{-d + ex^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x**2+d)**(3/2)/(-e**2*x**4+d**2),x)

[Out] -Integral(sqrt(d + e*x**2)/(-d + e*x**2), x)

Giac [A] time = 1.17519, size = 32, normalized size = 0.52

$$\frac{1}{2} e^{\left(-\frac{1}{2}\right)} \log \left(\left(x e^{\frac{1}{2}} - \sqrt{x^2 e + d} \right)^2 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)^(3/2)/(-e^2*x^4+d^2),x, algorithm="giac")

[Out] 1/2*e^(-1/2)*log((x*e^(1/2) - sqrt(x^2*e + d))^2)

$$3.196 \quad \int \frac{\sqrt{d+ex^2}}{d^2-e^2x^4} dx$$

Optimal. Leaf size=38

$$\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{ex}}{\sqrt{d+ex^2}}\right)}{\sqrt{2d}\sqrt{e}}$$

[Out] ArcTanh[(Sqrt[2]*Sqrt[e]*x)/Sqrt[d + e*x^2]]/(Sqrt[2]*d*Sqrt[e])

Rubi [A] time = 0.0256809, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1150, 377, 208}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{ex}}{\sqrt{d+ex^2}}\right)}{\sqrt{2d}\sqrt{e}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[d + e*x^2]/(d^2 - e^2*x^4), x]

[Out] ArcTanh[(Sqrt[2]*Sqrt[e]*x)/Sqrt[d + e*x^2]]/(Sqrt[2]*d*Sqrt[e])

Rule 1150

Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[(d + e*x^2)^(p + q)*(a/d + (c*x^2)/e)^p, x] /; FreeQ[{a, c, d, e, q}, x] && EqQ[c*d^2 + a*e^2, 0] && IntegerQ[p]

Rule 377

Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Subst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{d+ex^2}}{d^2-e^2x^4} dx &= \int \frac{1}{(d-ex^2)\sqrt{d+ex^2}} dx \\ &= \text{Subst}\left(\int \frac{1}{d-2dex^2} dx, x, \frac{x}{\sqrt{d+ex^2}}\right) \\ &= \frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{ex}}{\sqrt{d+ex^2}}\right)}{\sqrt{2d}\sqrt{e}} \end{aligned}$$

Mathematica [A] time = 0.156767, size = 38, normalized size = 1.

$$\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{ex}}{\sqrt{d+ex^2}}\right)}{\sqrt{2d}\sqrt{e}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[d + e*x^2]/(d^2 - e^2*x^4), x]

[Out] ArcTanh[(Sqrt[2]*Sqrt[e]*x)/Sqrt[d + e*x^2]]/(Sqrt[2]*d*Sqrt[e])

Maple [B] time = 0.027, size = 986, normalized size = 26.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x^2+d)^(1/2)/(-e^2*x^4+d^2), x)

[Out]
$$\frac{1}{2}e^{1/2} \left(\frac{(-d*e)^{1/2} + (d*e)^{1/2}}{(-d*e)^{1/2} - (d*e)^{1/2}} \right) \frac{1}{(-d*e)^{1/2}} \left(\frac{x + (-d*e)^{1/2}/e}{e} \right)^{1/2} - \frac{1}{2}e^{1/2} \frac{1}{(-d*e)^{1/2} + (d*e)^{1/2}} \frac{1}{(-d*e)^{1/2} - (d*e)^{1/2}} \ln \left(\frac{(x + (-d*e)^{1/2}/e)^{1/2}}{e} \right) e - (-d*e)^{1/2} / e^{1/2} + \left(\frac{x + (-d*e)^{1/2}/e}{e} \right)^{1/2} - \frac{1}{2}e^{1/2} \frac{1}{(-d*e)^{1/2} + (d*e)^{1/2}} \frac{1}{(-d*e)^{1/2} - (d*e)^{1/2}} \frac{1}{(-d*e)^{1/2}} \left(\frac{x - (-d*e)^{1/2}/e}{e} \right)^{1/2} e + 2 \frac{(-d*e)^{1/2}}{e} \left(\frac{x - (-d*e)^{1/2}/e}{e} \right)^{1/2} - \frac{1}{2}e^{1/2} \frac{1}{(d*e)^{1/2}} \frac{1}{(-d*e)^{1/2} + (d*e)^{1/2}} \frac{1}{(-d*e)^{1/2} - (d*e)^{1/2}} \left(\frac{x + (d*e)^{1/2}/e}{e} \right)^{1/2} e + 2 \frac{(d*e)^{1/2}}{e} \left(\frac{x + (d*e)^{1/2}/e}{e} \right)^{1/2} + \frac{1}{2}e^{1/2} \frac{1}{(-d*e)^{1/2} + (d*e)^{1/2}} \frac{1}{(-d*e)^{1/2} - (d*e)^{1/2}} \ln \left(\frac{(x + (d*e)^{1/2}/e)^{1/2}}{e} \right) e - (d*e)^{1/2} / e^{1/2} + \left(\frac{x + (d*e)^{1/2}/e}{e} \right)^{1/2} e + 2 \frac{(d*e)^{1/2}}{e} \left(\frac{x + (d*e)^{1/2}/e}{e} \right)^{1/2} + \frac{1}{2}e^{1/2} \frac{1}{(d*e)^{1/2}} \frac{1}{(-d*e)^{1/2} + (d*e)^{1/2}} \frac{1}{(-d*e)^{1/2} - (d*e)^{1/2}} \ln \left(\frac{(x - (d*e)^{1/2}/e)^{1/2}}{e} \right) e + (d*e)^{1/2} / e^{1/2} + \left(\frac{x - (d*e)^{1/2}/e}{e} \right)^{1/2} e + 2 \frac{(d*e)^{1/2}}{e} \left(\frac{x - (d*e)^{1/2}/e}{e} \right)^{1/2} - \frac{1}{2}e^{1/2} \frac{1}{(d*e)^{1/2}} \frac{1}{(-d*e)^{1/2} + (d*e)^{1/2}} \frac{1}{(-d*e)^{1/2} - (d*e)^{1/2}} d^{1/2} 2^{1/2} \ln \left(\frac{(4*d - 2*(d*e)^{1/2} * (x + (d*e)^{1/2}/e) + 2*2^{1/2} * d^{1/2} * ((x + (d*e)^{1/2}/e)^{1/2} e - 2*(d*e)^{1/2} * (x + (d*e)^{1/2}/e) + 2*d)^{1/2}}{(x + (d*e)^{1/2}/e) + 1/2 * e / (d*e)^{1/2}} \right) \frac{1}{(-d*e)^{1/2} + (d*e)^{1/2}} \frac{1}{(-d*e)^{1/2} - (d*e)^{1/2}} \left(\frac{x - (d*e)^{1/2}/e}{e} \right)^{1/2} e + 2 \frac{(d*e)^{1/2}}{e} \left(\frac{x - (d*e)^{1/2}/e}{e} \right)^{1/2} + \frac{1}{2}e^{1/2} \frac{1}{(-d*e)^{1/2} + (d*e)^{1/2}} \frac{1}{(-d*e)^{1/2} - (d*e)^{1/2}} \ln \left(\frac{(x - (d*e)^{1/2}/e)^{1/2}}{e} \right) e + (d*e)^{1/2} / e^{1/2} + \left(\frac{x - (d*e)^{1/2}/e}{e} \right)^{1/2} e + 2 \frac{(d*e)^{1/2}}{e} \left(\frac{x - (d*e)^{1/2}/e}{e} \right)^{1/2} \right)$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\sqrt{ex^2+d}}{e^2x^4-d^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)^(1/2)/(-e^2*x^4+d^2), x, algorithm="maxima")

[Out] -integrate(sqrt(e*x^2 + d)/(e^2*x^4 - d^2), x)

Fricas [A] time = 2.15559, size = 340, normalized size = 8.95

$$\left[\frac{\sqrt{2} \log\left(\frac{17e^2x^4 + 14dex^2 + 4\sqrt{2}(3ex^3 + dx)\sqrt{ex^2 + d}\sqrt{e + d^2}}{e^2x^4 - 2dex^2 + d^2}\right)}{8d\sqrt{e}}, -\frac{\sqrt{2}\sqrt{-e} \arctan\left(\frac{\sqrt{2}(3ex^2 + d)\sqrt{ex^2 + d}\sqrt{-e}}{4(e^2x^3 + dex)}\right)}{4de} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)^(1/2)/(-e^2*x^4+d^2),x, algorithm="fricas")

[Out] [1/8*sqrt(2)*log((17*e^2*x^4 + 14*d*e*x^2 + 4*sqrt(2)*(3*e*x^3 + d*x)*sqrt(e*x^2 + d)*sqrt(e) + d^2)/(e^2*x^4 - 2*d*e*x^2 + d^2))/(d*sqrt(e)), -1/4*sqrt(2)*sqrt(-e)*arctan(1/4*sqrt(2)*(3*e*x^2 + d)*sqrt(e*x^2 + d)*sqrt(-e)/(e^2*x^3 + d*e*x))/(d*e)]

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{-d\sqrt{d + ex^2} + ex^2\sqrt{d + ex^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x**2+d)**(1/2)/(-e**2*x**4+d**2),x)

[Out] -Integral(1/(-d*sqrt(d + e*x**2) + e*x**2*sqrt(d + e*x**2)), x)

Giac [B] time = 1.45393, size = 177, normalized size = 4.66

$$\frac{\left(\sqrt{2}i \arctan\left(\frac{e^{\frac{1}{2}}}{\sqrt{-\frac{de + \sqrt{d^2}e}{d}}}\right) e^{\frac{1}{2}} - \sqrt{2}i \arctan\left(\frac{e^{\frac{1}{2}}}{\sqrt{-\frac{de - \sqrt{d^2}e}{d}}}\right) e^{\frac{1}{2}}\right) e^{(-1)\operatorname{sgn}(x)}}{4|d|} + \frac{\sqrt{2}i \arctan\left(\frac{\sqrt{\frac{d}{x^2} + e}}{\sqrt{-\frac{des\operatorname{gn}(x) + \sqrt{d^2}e}{d\operatorname{sgn}(x)}}}\right) e^{\left(-\frac{1}{2}\right)}}{2|d|\operatorname{sgn}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)^(1/2)/(-e^2*x^4+d^2),x, algorithm="giac")

[Out] -1/4*(sqrt(2)*i*arctan(e^(1/2)/sqrt(-(d*e + sqrt(d^2)*e)/d))*e^(1/2) - sqrt(2)*i*arctan(e^(1/2)/sqrt(-(d*e - sqrt(d^2)*e)/d))*e^(1/2))*e^(-1)*sgn(x)/abs(d) + 1/2*sqrt(2)*i*arctan(sqrt(d/x^2 + e)/sqrt(-(d*e*sgn(x) + sqrt(d^2)*e)/(d*sgn(x))))*e^(-1/2)/(abs(d)*abs(sgn(x)))

$$3.197 \quad \int \frac{1}{\sqrt{d+ex^2}(d^2-e^2x^4)} dx$$

Optimal. Leaf size=61

$$\frac{x}{2d^2\sqrt{d+ex^2}} + \frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{ex}}{\sqrt{d+ex^2}}\right)}{2\sqrt{2}d^2\sqrt{e}}$$

[Out] x/(2*d^2*Sqrt[d + e*x^2]) + ArcTanh[(Sqrt[2]*Sqrt[e]*x)/Sqrt[d + e*x^2]]/(2*Sqrt[2]*d^2*Sqrt[e])

Rubi [A] time = 0.0390935, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1150, 382, 377, 208}

$$\frac{x}{2d^2\sqrt{d+ex^2}} + \frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{ex}}{\sqrt{d+ex^2}}\right)}{2\sqrt{2}d^2\sqrt{e}}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[d + e*x^2]*(d^2 - e^2*x^4)),x]

[Out] x/(2*d^2*Sqrt[d + e*x^2]) + ArcTanh[(Sqrt[2]*Sqrt[e]*x)/Sqrt[d + e*x^2]]/(2*Sqrt[2]*d^2*Sqrt[e])

Rule 1150

Int[((d_) + (e_)*(x_)^2)^(q_)*((a_) + (c_)*(x_)^4)^(p_), x_Symbol] := Int[(d + e*x^2)^(p + q)*(a/d + (c*x^2)/e)^p, x] /; FreeQ[{a, c, d, e, q}, x] && EqQ[c*d^2 + a*e^2, 0] && IntegerQ[p]

Rule 382

Int[((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_), x_Symbol] := -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c - a*d)), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] || !LtQ[q, -1]) && NeQ[p, -1]

Rule 377

Int[((a_) + (b_)*(x_)^(n_))^(p_)/((c_) + (d_)*(x_)^(n_)), x_Symbol] := Subst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]

Rule 208

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{d+ex^2}(d^2-e^2x^4)} dx &= \int \frac{1}{(d-ex^2)(d+ex^2)^{3/2}} dx \\
&= \frac{x}{2d^2\sqrt{d+ex^2}} + \frac{\int \frac{1}{(d-ex^2)\sqrt{d+ex^2}} dx}{2d} \\
&= \frac{x}{2d^2\sqrt{d+ex^2}} + \frac{\text{Subst}\left(\int \frac{1}{d-2dex^2} dx, x, \frac{x}{\sqrt{d+ex^2}}\right)}{2d} \\
&= \frac{x}{2d^2\sqrt{d+ex^2}} + \frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{ex}}{\sqrt{d+ex^2}}\right)}{2\sqrt{2}d^2\sqrt{e}}
\end{aligned}$$

Mathematica [A] time = 0.127252, size = 108, normalized size = 1.77

$$\frac{\frac{4x}{\sqrt{d+ex^2}} - \frac{\sqrt{2} \tanh^{-1}\left(\frac{\sqrt{d}-\sqrt{ex}}{\sqrt{2}\sqrt{d+ex^2}}\right)}{\sqrt{e}} + \frac{\sqrt{2} \tanh^{-1}\left(\frac{\sqrt{d}+\sqrt{ex}}{\sqrt{2}\sqrt{d+ex^2}}\right)}{\sqrt{e}}}{8d^2}$$

Antiderivative was successfully verified.

[In] Integrate[1/(Sqrt[d + e*x^2]*(d^2 - e^2*x^4)), x]

[Out] ((4*x)/Sqrt[d + e*x^2] - (Sqrt[2]*ArcTanh[(Sqrt[d] - Sqrt[e]*x)/(Sqrt[2]*Sqrt[d + e*x^2])])/Sqrt[e] + (Sqrt[2]*ArcTanh[(Sqrt[d] + Sqrt[e]*x)/(Sqrt[2]*Sqrt[d + e*x^2])])/Sqrt[e])/(8*d^2)

Maple [B] time = 0.023, size = 441, normalized size = 7.2

$$-\frac{1}{2d} \sqrt{\left(x + \frac{1}{e} \sqrt{-de}\right)^2 e - 2 \sqrt{-de} \left(x + \frac{\sqrt{-de}}{e}\right)} \left(\sqrt{-de} + \sqrt{de}\right)^{-1} \left(\sqrt{-de} - \sqrt{de}\right)^{-1} \left(x + \frac{1}{e} \sqrt{-de}\right)^{-1} - \frac{1}{2d} \sqrt{\left(x - \frac{1}{e} \sqrt{-de}\right)^2 e - 2 \sqrt{-de} \left(x - \frac{\sqrt{-de}}{e}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(e*x^2+d)^(1/2)/(-e^2*x^4+d^2), x)

[Out] -1/2/((-d*e)^(1/2)+(d*e)^(1/2))/((-d*e)^(1/2)-(d*e)^(1/2))/d/(x+(-d*e)^(1/2)/e)*((x+(-d*e)^(1/2)/e)^2*e-2*(-d*e)^(1/2)*(x+(-d*e)^(1/2)/e))^(1/2)-1/2/((-d*e)^(1/2)+(d*e)^(1/2))/((-d*e)^(1/2)-(d*e)^(1/2))/d/(x-(-d*e)^(1/2)/e)*((x-(-d*e)^(1/2)/e)^2*e+2*(-d*e)^(1/2)*(x-(-d*e)^(1/2)/e))^(1/2)+1/4*e/(d*e)^(1/2)/((-d*e)^(1/2)+(d*e)^(1/2))/((-d*e)^(1/2)-(d*e)^(1/2))*2^(1/2)/d^(1/2)*ln((4*d-2*(d*e)^(1/2)*(x+(d*e)^(1/2)/e)+2*2^(1/2)*d^(1/2))*((x+(d*e)^(1/2)/e)^2*e-2*(d*e)^(1/2)*(x+(d*e)^(1/2)/e)+2*d)^(1/2)/(x+(d*e)^(1/2)/e))-1/4*e/(d*e)^(1/2)/((-d*e)^(1/2)+(d*e)^(1/2))/((-d*e)^(1/2)-(d*e)^(1/2))*2^(1/2)/d^(1/2)*ln((4*d+2*(d*e)^(1/2)*(x-(d*e)^(1/2)/e)+2*2^(1/2)*d^(1/2))*((x-(d*e)^(1/2)/e)^2*e+2*(d*e)^(1/2)*(x-(d*e)^(1/2)/e)+2*d)^(1/2)/(x-(d*e)^(1/2)/e))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{(e^2x^4 - d^2)\sqrt{ex^2 + d}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x^2+d)^(1/2)/(-e^2*x^4+d^2),x, algorithm="maxima")

[Out] -integrate(1/((e^2*x^4 - d^2)*sqrt(e*x^2 + d)), x)

Fricas [B] time = 2.01701, size = 490, normalized size = 8.03

$$\left[\frac{\sqrt{2}(ex^2 + d)\sqrt{e} \log\left(\frac{17e^2x^4 + 14dex^2 + 4\sqrt{2}(3ex^3 + dx)\sqrt{ex^2 + d}\sqrt{e + d^2}}{e^2x^4 - 2dex^2 + d^2}\right) + 8\sqrt{ex^2 + dex} \sqrt{2}(ex^2 + d)\sqrt{-e} \arctan\left(\frac{\sqrt{2}(3ex^2 + d)\sqrt{ex^2 + d}}{4(e^2x^3 + dex)}\right)}{16(d^2e^2x^2 + d^3e)}, -\frac{\sqrt{2}(ex^2 + d)\sqrt{-e} \arctan\left(\frac{\sqrt{2}(3ex^2 + d)\sqrt{ex^2 + d}}{4(e^2x^3 + dex)}\right)}{8(d^2e^2x^2 + d^3e)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x^2+d)^(1/2)/(-e^2*x^4+d^2),x, algorithm="fricas")

[Out] [1/16*(sqrt(2)*(e*x^2 + d)*sqrt(e)*log((17*e^2*x^4 + 14*d*e*x^2 + 4*sqrt(2)*(3*e*x^3 + d*x)*sqrt(e*x^2 + d)*sqrt(e) + d^2)/(e^2*x^4 - 2*d*e*x^2 + d^2)) + 8*sqrt(e*x^2 + d)*e*x)/(d^2*e^2*x^2 + d^3*e), -1/8*(sqrt(2)*(e*x^2 + d)*sqrt(-e)*arctan(1/4*sqrt(2)*(3*e*x^2 + d)*sqrt(e*x^2 + d)*sqrt(-e)/(e^2*x^3 + d*e*x)) - 4*sqrt(e*x^2 + d)*e*x)/(d^2*e^2*x^2 + d^3*e)]

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{-d^2\sqrt{d + ex^2} + e^2x^4\sqrt{d + ex^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x**2+d)**(1/2)/(-e**2*x**4+d**2),x)

[Out] -Integral(1/(-d**2*sqrt(d + e*x**2) + e**2*x**4*sqrt(d + e*x**2)), x)

Giac [A] time = 1.22753, size = 1, normalized size = 0.02

+∞

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x^2+d)^(1/2)/(-e^2*x^4+d^2),x, algorithm="giac")

[Out] +Infinity

$$3.198 \quad \int \frac{1}{(d+ex^2)^{3/2}(d^2-e^2x^4)} dx$$

Optimal. Leaf size=80

$$\frac{7x}{12d^3\sqrt{d+ex^2}} + \frac{x}{6d^2(d+ex^2)^{3/2}} + \frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{ex}}{\sqrt{d+ex^2}}\right)}{4\sqrt{2}d^3\sqrt{e}}$$

[Out] x/(6*d^2*(d + e*x^2)^(3/2)) + (7*x)/(12*d^3*Sqrt[d + e*x^2]) + ArcTanh[(Sqrt[2]*Sqrt[e]*x)/Sqrt[d + e*x^2]]/(4*Sqrt[2]*d^3*Sqrt[e])

Rubi [A] time = 0.0688886, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {1150, 414, 527, 12, 377, 208}

$$\frac{7x}{12d^3\sqrt{d+ex^2}} + \frac{x}{6d^2(d+ex^2)^{3/2}} + \frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{ex}}{\sqrt{d+ex^2}}\right)}{4\sqrt{2}d^3\sqrt{e}}$$

Antiderivative was successfully verified.

[In] Int[1/((d + e*x^2)^(3/2)*(d^2 - e^2*x^4)),x]

[Out] x/(6*d^2*(d + e*x^2)^(3/2)) + (7*x)/(12*d^3*Sqrt[d + e*x^2]) + ArcTanh[(Sqrt[2]*Sqrt[e]*x)/Sqrt[d + e*x^2]]/(4*Sqrt[2]*d^3*Sqrt[e])

Rule 1150

Int[((d_) + (e_)*(x_)^2)^(q_)*((a_) + (c_)*(x_)^4)^(p_), x_Symbol] := Int[(d + e*x^2)^(p + q)*(a/d + (c*x^2)/e)^p, x] /; FreeQ[{a, c, d, e, q}, x] && EqQ[c*d^2 + a*e^2, 0] && IntegerQ[p]

Rule 414

Int[((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_), x_Symbol] := -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c - a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && !(IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, n, p, q, x]

Rule 527

Int[((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_)*((e_) + (f_)*(x_)^(n_)), x_Symbol] := -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x] /; FreeQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 377

Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Subst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rubi steps

$$\begin{aligned} \int \frac{1}{(d+ex^2)^{3/2}(d^2-e^2x^4)} dx &= \int \frac{1}{(d-ex^2)(d+ex^2)^{5/2}} dx \\ &= \frac{x}{6d^2(d+ex^2)^{3/2}} - \frac{\int \frac{-5de+2e^2x^2}{(d-ex^2)(d+ex^2)^{3/2}} dx}{6d^2e} \\ &= \frac{x}{6d^2(d+ex^2)^{3/2}} + \frac{7x}{12d^3\sqrt{d+ex^2}} + \frac{\int \frac{3d^2e^2}{(d-ex^2)\sqrt{d+ex^2}} dx}{12d^4e^2} \\ &= \frac{x}{6d^2(d+ex^2)^{3/2}} + \frac{7x}{12d^3\sqrt{d+ex^2}} + \frac{\int \frac{1}{(d-ex^2)\sqrt{d+ex^2}} dx}{4d^2} \\ &= \frac{x}{6d^2(d+ex^2)^{3/2}} + \frac{7x}{12d^3\sqrt{d+ex^2}} + \frac{\text{Subst}\left(\int \frac{1}{d-2dex^2} dx, x, \frac{x}{\sqrt{d+ex^2}}\right)}{4d^2} \\ &= \frac{x}{6d^2(d+ex^2)^{3/2}} + \frac{7x}{12d^3\sqrt{d+ex^2}} + \frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{ex}}{\sqrt{d+ex^2}}\right)}{4\sqrt{2}d^3\sqrt{e}} \end{aligned}$$

Mathematica [C] time = 1.68192, size = 345, normalized size = 4.31

$$\frac{384e^4x^8(d+ex^2)^2 \text{HypergeometricPFQ}\left(\{2,2,2\}, \left\{1, \frac{9}{2}\right\}, -\frac{2ex^2}{d-ex^2}\right)}{ex^2-d} + \frac{384e^4x^8(4d^2+7dex^2+3e^2x^4) {}_2F_1\left(2, \frac{9}{2}; -\frac{2ex^2}{d-ex^2}\right)}{ex^2-d} + \frac{35\sqrt{2}\sqrt{\frac{ex^2}{ex^2-d}}(-5d^2ex^2-15d^3+12de^2x^4+8e^3x^6)}{ex^2-d}$$

$$2520d^5e^3x^5\sqrt{d+ex^2}\left(1-\frac{e^2x^4}{d^2}\right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[1/((d + e*x^2)^(3/2)*(d^2 - e^2*x^4)), x]

[Out] ((35*Sqrt[2]*Sqrt[(e*x^2)/(-d + e*x^2)]*(-15*d^3 - 5*d^2*e*x^2 + 12*d*e^2*x^4 + 8*e^3*x^6)*(Sqrt[2]*Sqrt[(e*x^2)/(-d + e*x^2)]*Sqrt[(d + e*x^2)/(d - e*x^2)]*(-3*d^2 - 2*d*e*x^2 + 5*e^2*x^4) + 3*(d + e*x^2)^2*ArcSin[Sqrt[2]*Sqrt[(e*x^2)/(-d + e*x^2)]])/Sqrt[(d + e*x^2)/(d - e*x^2)] + (384*e^4*x^8*(4*d^2 + 7*d*e*x^2 + 3*e^2*x^4)*Hypergeometric2F1[2, 2, 9/2, (-2*e*x^2)/(d - e*x^2)]/(-d + e*x^2) + (384*e^4*x^8*(d + e*x^2)^2*HypergeometricPFQ[{2, 2, 2}, {1, 9/2}, (-2*e*x^2)/(d - e*x^2)]/(-d + e*x^2)))/(2520*d^5*e^3*x^5*Sqrt[d + e*x^2]*(1 - (e^2*x^4)/d^2))

Maple [B] time = 0.022, size = 911, normalized size = 11.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\int (1/(e*x^2+d)^{3/2}/(-e^2*x^4+d^2), x)$

[Out]
$$-1/6/((-d*e)^{(1/2)}+(d*e)^{(1/2)})/((-d*e)^{(1/2)}-(d*e)^{(1/2)})/d/(x+(-d*e)^{(1/2)}/e)/((x+(-d*e)^{(1/2)}/e)^{2*e-2*(-d*e)^{(1/2)}*(x+(-d*e)^{(1/2)}/e)}^{(1/2)}-1/3*e/((-d*e)^{(1/2)}+(d*e)^{(1/2)})/((-d*e)^{(1/2)}-(d*e)^{(1/2)})/d^2/(x+(-d*e)^{(1/2)}/e)^{2*e-2*(-d*e)^{(1/2)}*(x+(-d*e)^{(1/2)}/e)}^{(1/2)}*x-1/6/((-d*e)^{(1/2)}+(d*e)^{(1/2)})/((-d*e)^{(1/2)}-(d*e)^{(1/2)})/d/(x-(-d*e)^{(1/2)}/e)/((x-(-d*e)^{(1/2)}/e)^{2*e+2*(-d*e)^{(1/2)}*(x-(-d*e)^{(1/2)}/e)}^{(1/2)}-1/3*e/((-d*e)^{(1/2)}+(d*e)^{(1/2)})/((-d*e)^{(1/2)}-(d*e)^{(1/2)})/d^2/(x-(-d*e)^{(1/2)}/e)^{2*e+2*(-d*e)^{(1/2)}*(x-(-d*e)^{(1/2)}/e)}^{(1/2)}*x-1/4*e/(d*e)^{(1/2)}/((-d*e)^{(1/2)}+(d*e)^{(1/2)})/((-d*e)^{(1/2)}-(d*e)^{(1/2)})/d/((x+(d*e)^{(1/2)}/e)^{2*e-2*(d*e)^{(1/2)}*(x+(d*e)^{(1/2)}/e)+2*d)^{(1/2)}-1/4*e/((-d*e)^{(1/2)}+(d*e)^{(1/2)})/((-d*e)^{(1/2)}-(d*e)^{(1/2)})/d^2/((x+(d*e)^{(1/2)}/e)^{2*e-2*(d*e)^{(1/2)}*(x+(d*e)^{(1/2)}/e)+2*d)^{(1/2)}*x+1/8*e/(d*e)^{(1/2)}/((-d*e)^{(1/2)}+(d*e)^{(1/2)})/((-d*e)^{(1/2)}-(d*e)^{(1/2)})/d^{(3/2)}*2^{(1/2)}*\ln((4*d-2*(d*e)^{(1/2)}*(x+(d*e)^{(1/2)}/e)+2*2^{(1/2)}*d^{(1/2)}*((x+(d*e)^{(1/2)}/e)^{2*e-2*(d*e)^{(1/2)}*(x+(d*e)^{(1/2)}/e)+2*d)^{(1/2)})/(x+(d*e)^{(1/2)}/e))+1/4*e/(d*e)^{(1/2)}/((-d*e)^{(1/2)}+(d*e)^{(1/2)})/((-d*e)^{(1/2)}-(d*e)^{(1/2)})/d/((x-(d*e)^{(1/2)}/e)^{2*e+2*(d*e)^{(1/2)}*(x-(d*e)^{(1/2)}/e)+2*d)^{(1/2)}-1/4*e/((-d*e)^{(1/2)}+(d*e)^{(1/2)})/((-d*e)^{(1/2)}-(d*e)^{(1/2)})/d^2/((x-(d*e)^{(1/2)}/e)^{2*e+2*(d*e)^{(1/2)}*(x-(d*e)^{(1/2)}/e)+2*d)^{(1/2)}*x-1/8*e/(d*e)^{(1/2)}/((-d*e)^{(1/2)}+(d*e)^{(1/2)})/((-d*e)^{(1/2)}-(d*e)^{(1/2)})/d^{(3/2)}*2^{(1/2)}*\ln((4*d+2*(d*e)^{(1/2)}*(x-(d*e)^{(1/2)}/e)+2*2^{(1/2)}*d^{(1/2)}*((x-(d*e)^{(1/2)}/e)^{2*e+2*(d*e)^{(1/2)}*(x-(d*e)^{(1/2)}/e)+2*d)^{(1/2)})/(x-(d*e)^{(1/2)}/e))$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{(e^2x^4 - d^2)(ex^2 + d)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(1/(e*x^2+d)^{3/2}/(-e^2*x^4+d^2), x, \text{algorithm}="maxima")$

[Out] $-\text{integrate}(1/((e^2*x^4 - d^2)*(e*x^2 + d)^{3/2}), x)$

Fricas [B] time = 2.10236, size = 632, normalized size = 7.9

$$\left[\frac{3\sqrt{2}(e^2x^4 + 2dex^2 + d^2)\sqrt{e} \log\left(\frac{17e^2x^4 + 14dex^2 + 4\sqrt{2}(3ex^3 + dx)\sqrt{ex^2 + d}\sqrt{e + d^2}}{e^2x^4 - 2dex^2 + d^2}\right) + 8(7e^2x^3 + 9dex)\sqrt{ex^2 + d} - 3\sqrt{2}(e^2x^4 + 2dex^2 + d^2)\sqrt{e}}{96(d^3e^3x^4 + 2d^4e^2x^2 + d^5e)}, \dots \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(1/(e*x^2+d)^{3/2}/(-e^2*x^4+d^2), x, \text{algorithm}="fricas")$

[Out] $[1/96*(3*\sqrt{2})*(e^2*x^4 + 2*d*e*x^2 + d^2)*\sqrt{e}*\log((17*e^2*x^4 + 14*d*e*x^2 + 4*\sqrt{2})*(3*e*x^3 + d*x)*\sqrt{e*x^2 + d}*\sqrt{e} + d^2)/(e^2*x^4 - 2*d*e*x^2 + d^2) + 8*(7*e^2*x^3 + 9*d*e*x)*\sqrt{e*x^2 + d} - 3*\sqrt{2}*(e^2*x^4 + 2*d*e*x^2 + d^2)*\sqrt{e}]/96*(d^3*e^3*x^4 + 2*d^4*e^2*x^2 + d^5*e)$

- 2*d*e*x^2 + d^2)) + 8*(7*e^2*x^3 + 9*d*e*x)*sqrt(e*x^2 + d))/(d^3*e^3*x^4 + 2*d^4*e^2*x^2 + d^5*e), -1/48*(3*sqrt(2)*(e^2*x^4 + 2*d*e*x^2 + d^2)*sqrt(-e)*arctan(1/4*sqrt(2)*(3*e*x^2 + d)*sqrt(e*x^2 + d)*sqrt(-e)/(e^2*x^3 + d*e*x)) - 4*(7*e^2*x^3 + 9*d*e*x)*sqrt(e*x^2 + d))/(d^3*e^3*x^4 + 2*d^4*e^2*x^2 + d^5*e)]

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{-d^3\sqrt{d+ex^2} - d^2ex^2\sqrt{d+ex^2} + de^2x^4\sqrt{d+ex^2} + e^3x^6\sqrt{d+ex^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x**2+d)**(3/2)/(-e**2*x**4+d**2),x)

[Out] -Integral(1/(-d**3*sqrt(d + e*x**2) - d**2*e*x**2*sqrt(d + e*x**2) + d*e**2*x**4*sqrt(d + e*x**2) + e**3*x**6*sqrt(d + e*x**2)), x)

Giac [F] time = 0., size = 0, normalized size = 0.

[undef, undef, undef, -1]

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x^2+d)^(3/2)/(-e^2*x^4+d^2),x, algorithm="giac")

[Out] [undef, undef, undef, -1]

$$3.199 \quad \int \frac{(a+bx^2)^{5/2}}{\sqrt{a^2-b^2x^4}} dx$$

Optimal. Leaf size=153

$$-\frac{9ax(a-bx^2)\sqrt{a+bx^2}}{8\sqrt{a^2-b^2x^4}} - \frac{x(a-bx^2)(a+bx^2)^{3/2}}{4\sqrt{a^2-b^2x^4}} + \frac{19a^2\sqrt{a-bx^2}\sqrt{a+bx^2}\tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a-bx^2}}\right)}{8\sqrt{b}\sqrt{a^2-b^2x^4}}$$

[Out] (-9*a*x*(a - b*x^2)*Sqrt[a + b*x^2])/(8*Sqrt[a^2 - b^2*x^4]) - (x*(a - b*x^2)*(a + b*x^2)^(3/2))/(4*Sqrt[a^2 - b^2*x^4]) + (19*a^2*Sqrt[a - b*x^2]*Sqrt[a + b*x^2]*ArcTan[(Sqrt[b]*x)/Sqrt[a - b*x^2]])/(8*Sqrt[b]*Sqrt[a^2 - b^2*x^4])

Rubi [A] time = 0.0547431, antiderivative size = 153, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {1152, 416, 388, 217, 203}

$$-\frac{9ax(a-bx^2)\sqrt{a+bx^2}}{8\sqrt{a^2-b^2x^4}} - \frac{x(a-bx^2)(a+bx^2)^{3/2}}{4\sqrt{a^2-b^2x^4}} + \frac{19a^2\sqrt{a-bx^2}\sqrt{a+bx^2}\tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a-bx^2}}\right)}{8\sqrt{b}\sqrt{a^2-b^2x^4}}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x^2)^(5/2)/Sqrt[a^2 - b^2*x^4], x]

[Out] (-9*a*x*(a - b*x^2)*Sqrt[a + b*x^2])/(8*Sqrt[a^2 - b^2*x^4]) - (x*(a - b*x^2)*(a + b*x^2)^(3/2))/(4*Sqrt[a^2 - b^2*x^4]) + (19*a^2*Sqrt[a - b*x^2]*Sqrt[a + b*x^2]*ArcTan[(Sqrt[b]*x)/Sqrt[a - b*x^2]])/(8*Sqrt[b]*Sqrt[a^2 - b^2*x^4])

Rule 1152

Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist[(a + c*x^4)^FracPart[p]/((d + e*x^2)^FracPart[p]*(a/d + (c*x^2)/e)^FracPart[p]), Int[(d + e*x^2)^(p + q)*(a/d + (c*x^2)/e)^p, x], x] /; FreeQ[{a, c, d, e, p, q}, x] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p]

Rule 416

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] :> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)), x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q - 1) + 1))*x^n, x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] && !IGtQ[p, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]

Rule 388

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]

Rule 217

Int[1/Sqrt[(a_) + (b_)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 203

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rubi steps

$$\begin{aligned} \int \frac{(a + bx^2)^{5/2}}{\sqrt{a^2 - b^2x^4}} dx &= \frac{\left(\sqrt{a - bx^2}\sqrt{a + bx^2}\right) \int \frac{(a+bx^2)^2}{\sqrt{a-bx^2}} dx}{\sqrt{a^2 - b^2x^4}} \\ &= -\frac{x(a - bx^2)(a + bx^2)^{3/2}}{4\sqrt{a^2 - b^2x^4}} - \frac{\left(\sqrt{a - bx^2}\sqrt{a + bx^2}\right) \int \frac{-5a^2b - 9ab^2x^2}{\sqrt{a-bx^2}} dx}{4b\sqrt{a^2 - b^2x^4}} \\ &= -\frac{9ax(a - bx^2)\sqrt{a + bx^2}}{8\sqrt{a^2 - b^2x^4}} - \frac{x(a - bx^2)(a + bx^2)^{3/2}}{4\sqrt{a^2 - b^2x^4}} + \frac{\left(19a^2\sqrt{a - bx^2}\sqrt{a + bx^2}\right) \int \frac{1}{\sqrt{a-bx^2}} dx}{8\sqrt{a^2 - b^2x^4}} \\ &= -\frac{9ax(a - bx^2)\sqrt{a + bx^2}}{8\sqrt{a^2 - b^2x^4}} - \frac{x(a - bx^2)(a + bx^2)^{3/2}}{4\sqrt{a^2 - b^2x^4}} + \frac{\left(19a^2\sqrt{a - bx^2}\sqrt{a + bx^2}\right) \text{Subst}\left(\int \frac{1}{1+bx^2} dx, x, \frac{\sqrt{bx}}{\sqrt{a-bx^2}}\right)}{8\sqrt{a^2 - b^2x^4}} \\ &= -\frac{9ax(a - bx^2)\sqrt{a + bx^2}}{8\sqrt{a^2 - b^2x^4}} - \frac{x(a - bx^2)(a + bx^2)^{3/2}}{4\sqrt{a^2 - b^2x^4}} + \frac{19a^2\sqrt{a - bx^2}\sqrt{a + bx^2} \tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a-bx^2}}\right)}{8\sqrt{b}\sqrt{a^2 - b^2x^4}} \end{aligned}$$

Mathematica [C] time = 0.164907, size = 98, normalized size = 0.64

$$-\frac{(11ax + 2bx^3)\sqrt{a^2 - b^2x^4}}{8\sqrt{a + bx^2}} + \frac{19ia^2 \log\left(\frac{2\sqrt{a^2 - b^2x^4}}{\sqrt{a+bx^2}} - 2i\sqrt{bx}\right)}{8\sqrt{b}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x^2)^(5/2)/Sqrt[a^2 - b^2*x^4], x]

[Out] -((11*a*x + 2*b*x^3)*Sqrt[a^2 - b^2*x^4])/(8*Sqrt[a + b*x^2]) + (((19*I)/8)*a^2*Log[(-2*I)*Sqrt[b]*x + (2*Sqrt[a^2 - b^2*x^4])/Sqrt[a + b*x^2]])/Sqrt[b]

Maple [A] time = 0.089, size = 132, normalized size = 0.9

$$-\frac{1}{8}\sqrt{-b^2x^4 + a^2} \left(2x^3b^{3/2}\sqrt{-bx^2 + a} + 11\sqrt{b}\sqrt{-bx^2 + a}ax + 13 \arctan\left(\frac{x\sqrt{b}}{\sqrt{-bx^2 + a}}\right)a^2 - 32 \arctan\left(x\sqrt{b}\frac{1}{\sqrt{\frac{(-bx+\sqrt{ab})(bx-a)}{b}}}\right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^2+a)^(5/2)/(-b^2*x^4+a^2)^(1/2), x)

[Out] -1/8*(-b^2*x^4+a^2)^(1/2)*(2*x^3*b^(3/2)*(-b*x^2+a)^(1/2)+11*b^(1/2)*(-b*x^2+a)^(1/2)*x*a+13*arctan(x*b^(1/2)/(-b*x^2+a)^(1/2))*a^2-32*arctan(b^(1/2)*

$x/((-b*x+(a*b)^{(1/2)})/b*(b*x+(a*b)^{(1/2)}))^{(1/2)}*a^2/(b*x^2+a)^{(1/2)/(-b*x^2+a)^{(1/2)}/b^{(1/2)}$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)^{\frac{5}{2}}}{\sqrt{-b^2x^4 + a^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^2+a)^(5/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="maxima")

[Out] integrate((b*x^2 + a)^(5/2)/sqrt(-b^2*x^4 + a^2), x)

Fricas [A] time = 2.08634, size = 549, normalized size = 3.59

$$\left[\frac{19(a^2bx^2 + a^3)\sqrt{-b} \log\left(-\frac{2b^2x^4+abx^2-2\sqrt{-b^2x^4+a^2}\sqrt{bx^2+a}\sqrt{-bx-a^2}}{bx^2+a}\right) + 2\sqrt{-b^2x^4+a^2}(2b^2x^3+11abx)\sqrt{bx^2+a} - 19(a^2b^2x^4+a^2)\sqrt{-b}}{16(b^2x^2+ab)}, - \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^2+a)^(5/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="fricas")

[Out] [-1/16*(19*(a^2*b*x^2 + a^3)*sqrt(-b)*log(-(2*b^2*x^4 + a*b*x^2 - 2*sqrt(-b^2*x^4 + a^2)*sqrt(b*x^2 + a)*sqrt(-b)*x - a^2)/(b*x^2 + a)) + 2*sqrt(-b^2*x^4 + a^2)*(2*b^2*x^3 + 11*a*b*x)*sqrt(b*x^2 + a))/(b^2*x^2 + a*b), -1/8*(19*(a^2*b*x^2 + a^3)*sqrt(b)*arctan(sqrt(-b^2*x^4 + a^2)*sqrt(b*x^2 + a)*sqrt(b)/(b^2*x^3 + a*b*x)) + sqrt(-b^2*x^4 + a^2)*(2*b^2*x^3 + 11*a*b*x)*sqrt(b*x^2 + a))/(b^2*x^2 + a*b)]

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + bx^2)^{\frac{5}{2}}}{\sqrt{-(-a + bx^2)(a + bx^2)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**2+a)**(5/2)/(-b**2*x**4+a**2)**(1/2),x)

[Out] Integral((a + b*x**2)**(5/2)/sqrt(-(-a + b*x**2)*(a + b*x**2)), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)^{\frac{5}{2}}}{\sqrt{-b^2x^4 + a^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(5/2)/sqrt(-b^2*x^4 + a^2), x)
```


$$3.200 \quad \int \frac{(a+bx^2)^{3/2}}{\sqrt{a^2-b^2x^4}} dx$$

Optimal. Leaf size=110

$$\frac{3a\sqrt{a-bx^2}\sqrt{a+bx^2}\tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a-bx^2}}\right)}{2\sqrt{b}\sqrt{a^2-b^2x^4}} - \frac{x(a-bx^2)\sqrt{a+bx^2}}{2\sqrt{a^2-b^2x^4}}$$

[Out] $-(x*(a - b*x^2)*\text{Sqrt}[a + b*x^2])/(2*\text{Sqrt}[a^2 - b^2*x^4]) + (3*a*\text{Sqrt}[a - b*x^2]*\text{Sqrt}[a + b*x^2]*\text{ArcTan}[(\text{Sqrt}[b]*x)/\text{Sqrt}[a - b*x^2]])/(2*\text{Sqrt}[b]*\text{Sqrt}[a^2 - b^2*x^4])$

Rubi [A] time = 0.0346587, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1152, 388, 217, 203}

$$\frac{3a\sqrt{a-bx^2}\sqrt{a+bx^2}\tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a-bx^2}}\right)}{2\sqrt{b}\sqrt{a^2-b^2x^4}} - \frac{x(a-bx^2)\sqrt{a+bx^2}}{2\sqrt{a^2-b^2x^4}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a + b*x^2)^{(3/2)}/\text{Sqrt}[a^2 - b^2*x^4], x]$

[Out] $-(x*(a - b*x^2)*\text{Sqrt}[a + b*x^2])/(2*\text{Sqrt}[a^2 - b^2*x^4]) + (3*a*\text{Sqrt}[a - b*x^2]*\text{Sqrt}[a + b*x^2]*\text{ArcTan}[(\text{Sqrt}[b]*x)/\text{Sqrt}[a - b*x^2]])/(2*\text{Sqrt}[b]*\text{Sqrt}[a^2 - b^2*x^4])$

Rule 1152

$\text{Int}[(d + e*x^2)^q*(a + c*x^4)^p, x_Symbol] \rightarrow \text{Dist}[(a + c*x^4)^{\text{FracPart}[p]}/((d + e*x^2)^{\text{FracPart}[p]}*(a/d + (c*x^2)/e)^{\text{FracPart}[p]})]$, $\text{Int}[(d + e*x^2)^{(p+q)}*(a/d + (c*x^2)/e)^p, x]$, x /; $\text{FreeQ}\{a, c, d, e, p, q\}, x$ && $\text{EqQ}[c*d^2 + a*e^2, 0]$ && $! \text{IntegerQ}[p]$

Rule 388

$\text{Int}[(a + b*x^n)^p*(c + d*x^n), x_Symbol] \rightarrow \text{Simp}[(d*x*(a + b*x^n)^{(p+1)})/(b*(n*(p+1) + 1)), x] - \text{Dist}[(a*d - b*c*(n*(p+1) + 1))/(b*(n*(p+1) + 1)), \text{Int}[(a + b*x^n)^p, x], x]$ /; $\text{FreeQ}\{a, b, c, d, n\}, x$ && $\text{NeQ}[b*c - a*d, 0]$ && $\text{NeQ}[n*(p+1) + 1, 0]$

Rule 217

$\text{Int}[1/\text{Sqrt}[(a + b*x^2)], x_Symbol] \rightarrow \text{Subst}[\text{Int}[1/(1 - b*x^2), x], x, x/\text{Sqrt}[a + b*x^2]]$ /; $\text{FreeQ}\{a, b\}, x$ && $! \text{GtQ}[a, 0]$

Rule 203

$\text{Int}[(a + b*x^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(1*\text{ArcTan}[(\text{Rt}[b, 2]*x)/\text{Rt}[a, 2]])/(\text{Rt}[a, 2]*\text{Rt}[b, 2]), x]$ /; $\text{FreeQ}\{a, b\}, x$ && $\text{PosQ}[a/b]$ && $(\text{GtQ}[a, 0] \parallel \text{GtQ}[b, 0])$

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx^2)^{3/2}}{\sqrt{a^2-b^2x^4}} dx &= \frac{(\sqrt{a-bx^2}\sqrt{a+bx^2}) \int \frac{a+bx^2}{\sqrt{a-bx^2}} dx}{\sqrt{a^2-b^2x^4}} \\
&= -\frac{x(a-bx^2)\sqrt{a+bx^2}}{2\sqrt{a^2-b^2x^4}} + \frac{(3a\sqrt{a-bx^2}\sqrt{a+bx^2}) \int \frac{1}{\sqrt{a-bx^2}} dx}{2\sqrt{a^2-b^2x^4}} \\
&= -\frac{x(a-bx^2)\sqrt{a+bx^2}}{2\sqrt{a^2-b^2x^4}} + \frac{(3a\sqrt{a-bx^2}\sqrt{a+bx^2}) \text{Subst}\left(\int \frac{1}{1+bx^2} dx, x, \frac{x}{\sqrt{a-bx^2}}\right)}{2\sqrt{a^2-b^2x^4}} \\
&= -\frac{x(a-bx^2)\sqrt{a+bx^2}}{2\sqrt{a^2-b^2x^4}} + \frac{3a\sqrt{a-bx^2}\sqrt{a+bx^2} \tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a-bx^2}}\right)}{2\sqrt{b}\sqrt{a^2-b^2x^4}}
\end{aligned}$$

Mathematica [C] time = 0.0743558, size = 86, normalized size = 0.78

$$-\frac{x\sqrt{a^2-b^2x^4}}{2\sqrt{a+bx^2}} + \frac{3ia \log\left(\frac{2\sqrt{a^2-b^2x^4}}{\sqrt{a+bx^2}} - 2i\sqrt{bx}\right)}{2\sqrt{b}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x^2)^(3/2)/Sqrt[a^2 - b^2*x^4], x]

[Out] -(x*Sqrt[a^2 - b^2*x^4])/(2*Sqrt[a + b*x^2]) + (((3*I)/2)*a*Log[(-2*I)*Sqrt[b]*x + (2*Sqrt[a^2 - b^2*x^4])/Sqrt[a + b*x^2]])/Sqrt[b]

Maple [A] time = 0.02, size = 107, normalized size = 1.

$$-\frac{1}{2}\sqrt{-b^2x^4+a^2}\left(x\sqrt{-bx^2+a}\sqrt{b}+a\arctan\left(x\sqrt{b}\frac{1}{\sqrt{-bx^2+a}}\right)-4\arctan\left(x\sqrt{b}\frac{1}{\sqrt{\frac{(-bx+\sqrt{ab})(bx+\sqrt{ab})}{b}}}\right)a\right)\frac{1}{\sqrt{bx^2+a}}\frac{1}{\sqrt{-bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^2+a)^(3/2)/(-b^2*x^4+a^2)^(1/2), x)

[Out] -1/2/(b*x^2+a)^(1/2)*(-b^2*x^4+a^2)^(1/2)*(x*(-b*x^2+a)^(1/2)*b^(1/2)+a*arctan(x*b^(1/2)/(-b*x^2+a)^(1/2))-4*arctan(b^(1/2)*x/((-b*x+(a*b)^(1/2))/b*(b*x+(a*b)^(1/2))))^(1/2)*a)/(-b*x^2+a)^(1/2)/b^(1/2)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2+a)^{3/2}}{\sqrt{-b^2x^4+a^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^2+a)^(3/2)/(-b^2*x^4+a^2)^(1/2), x, algorithm="maxima")

[Out] integrate((b*x^2 + a)^(3/2)/sqrt(-b^2*x^4 + a^2), x)

Fricas [A] time = 2.10198, size = 489, normalized size = 4.45

$$\left[\frac{2\sqrt{-b^2x^4 + a^2}\sqrt{bx^2 + abx} + 3(abx^2 + a^2)\sqrt{-b}\log\left(-\frac{2b^2x^4 + abx^2 - 2\sqrt{-b^2x^4 + a^2}\sqrt{bx^2 + a}\sqrt{-bx - a^2}}{bx^2 + a}\right)}{4(b^2x^2 + ab)}, \frac{\sqrt{-b^2x^4 + a^2}\sqrt{bx^2 + ab}}{4(b^2x^2 + ab)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^2+a)^(3/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="fricas")

[Out] [-1/4*(2*sqrt(-b^2*x^4 + a^2)*sqrt(b*x^2 + a)*b*x + 3*(a*b*x^2 + a^2)*sqrt(-b)*log(-(2*b^2*x^4 + a*b*x^2 - 2*sqrt(-b^2*x^4 + a^2)*sqrt(b*x^2 + a)*sqrt(-b)*x - a^2)/(b*x^2 + a)))/(b^2*x^2 + a*b), -1/2*(sqrt(-b^2*x^4 + a^2)*sqrt(b*x^2 + a)*b*x + 3*(a*b*x^2 + a^2)*sqrt(b)*arctan(sqrt(-b^2*x^4 + a^2)*sqrt(b*x^2 + a)*sqrt(b)/(b^2*x^3 + a*b*x)))/(b^2*x^2 + a*b)]

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + bx^2)^{\frac{3}{2}}}{\sqrt{-(-a + bx^2)(a + bx^2)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**2+a)**(3/2)/(-b**2*x**4+a**2)**(1/2),x)

[Out] Integral((a + b*x**2)**(3/2)/sqrt(-(-a + b*x**2)*(a + b*x**2)), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)^{\frac{3}{2}}}{\sqrt{-b^2x^4 + a^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^2+a)^(3/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="giac")

[Out] integrate((b*x^2 + a)^(3/2)/sqrt(-b^2*x^4 + a^2), x)

$$3.201 \quad \int \frac{\sqrt{a+bx^2}}{\sqrt{a^2-b^2x^4}} dx$$

Optimal. Leaf size=65

$$\frac{\sqrt{a-bx^2}\sqrt{a+bx^2}\tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a-bx^2}}\right)}{\sqrt{b}\sqrt{a^2-b^2x^4}}$$

[Out] (Sqrt[a - b*x^2]*Sqrt[a + b*x^2]*ArcTan[(Sqrt[b]*x)/Sqrt[a - b*x^2]])/(Sqrt[b]*Sqrt[a^2 - b^2*x^4])

Rubi [A] time = 0.0220998, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {1152, 217, 203}

$$\frac{\sqrt{a-bx^2}\sqrt{a+bx^2}\tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a-bx^2}}\right)}{\sqrt{b}\sqrt{a^2-b^2x^4}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + b*x^2]/Sqrt[a^2 - b^2*x^4], x]

[Out] (Sqrt[a - b*x^2]*Sqrt[a + b*x^2]*ArcTan[(Sqrt[b]*x)/Sqrt[a - b*x^2]])/(Sqrt[b]*Sqrt[a^2 - b^2*x^4])

Rule 1152

Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist[(a + c*x^4)^FracPart[p]/((d + e*x^2)^FracPart[p]*(a/d + (c*x^2)/e)^FracPart[p]), Int[(d + e*x^2)^(p + q)*(a/d + (c*x^2)/e)^p, x], x] /; FreeQ[{a, c, d, e, p, q}, x] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p]

Rule 217

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{a+bx^2}}{\sqrt{a^2-b^2x^4}} dx &= \frac{(\sqrt{a-bx^2}\sqrt{a+bx^2}) \int \frac{1}{\sqrt{a-bx^2}} dx}{\sqrt{a^2-b^2x^4}} \\ &= \frac{(\sqrt{a-bx^2}\sqrt{a+bx^2}) \text{Subst}\left(\int \frac{1}{1+bx^2} dx, x, \frac{x}{\sqrt{a-bx^2}}\right)}{\sqrt{a^2-b^2x^4}} \\ &= \frac{\sqrt{a-bx^2}\sqrt{a+bx^2}\tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a-bx^2}}\right)}{\sqrt{b}\sqrt{a^2-b^2x^4}} \end{aligned}$$

Mathematica [C] time = 0.0428736, size = 50, normalized size = 0.77

$$\frac{i \log\left(\frac{2\sqrt{a^2-b^2x^4}}{\sqrt{a+bx^2}} - 2i\sqrt{bx}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a + b*x^2]/Sqrt[a^2 - b^2*x^4], x]

[Out] (I*Log[(-2*I)*Sqrt[b]*x + (2*Sqrt[a^2 - b^2*x^4])/Sqrt[a + b*x^2]])/Sqrt[b]

Maple [A] time = 0.02, size = 69, normalized size = 1.1

$$\sqrt{-b^2x^4 + a^2} \arctan\left(x\sqrt{b} \frac{1}{\sqrt{\frac{1}{b}(-bx + \sqrt{ab})(bx + \sqrt{ab})}}\right) \frac{1}{\sqrt{bx^2 + a}} \frac{1}{\sqrt{-bx^2 + a}} \frac{1}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^2+a)^(1/2)/(-b^2*x^4+a^2)^(1/2), x)

[Out] 1/(b*x^2+a)^(1/2)*(-b^2*x^4+a^2)^(1/2)*arctan(b^(1/2)*x/((-b*x+(a*b)^(1/2))/b*(b*x+(a*b)^(1/2))))^(1/2)/(-b*x^2+a)^(1/2)/b^(1/2)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx^2 + a}}{\sqrt{-b^2x^4 + a^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^2+a)^(1/2)/(-b^2*x^4+a^2)^(1/2), x, algorithm="maxima")

[Out] integrate(sqrt(b*x^2 + a)/sqrt(-b^2*x^4 + a^2), x)

Fricas [A] time = 1.98855, size = 270, normalized size = 4.15

$$\left[\frac{\sqrt{-b} \log\left(-\frac{2b^2x^4+abx^2-2\sqrt{-b^2x^4+a^2}\sqrt{bx^2+a}\sqrt{-bx-a^2}}{bx^2+a}\right)}{2b}, -\frac{\arctan\left(\frac{\sqrt{-b^2x^4+a^2}\sqrt{bx^2+a}\sqrt{b}}{b^2x^3+abx}\right)}{\sqrt{b}} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^2+a)^(1/2)/(-b^2*x^4+a^2)^(1/2), x, algorithm="fricas")

[Out] [-1/2*sqrt(-b)*log(-(2*b^2*x^4 + a*b*x^2 - 2*sqrt(-b^2*x^4 + a^2)*sqrt(b*x^2 + a)*sqrt(-b)*x - a^2)/(b*x^2 + a))/b, -arctan(sqrt(-b^2*x^4 + a^2)*sqrt(b*x^2 + a)*sqrt(b)/(b^2*x^3 + a*b*x))/sqrt(b)]

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a + bx^2}}{\sqrt{-(-a + bx^2)(a + bx^2)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**2+a)**(1/2)/(-b**2*x**4+a**2)**(1/2), x)

[Out] Integral(sqrt(a + b*x**2)/sqrt(-(-a + b*x**2)*(a + b*x**2)), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx^2 + a}}{\sqrt{-b^2x^4 + a^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^2+a)^(1/2)/(-b^2*x^4+a^2)^(1/2), x, algorithm="giac")

[Out] integrate(sqrt(b*x^2 + a)/sqrt(-b^2*x^4 + a^2), x)

$$3.202 \quad \int \frac{1}{\sqrt{a+bx^2}\sqrt{a^2-b^2x^4}} dx$$

Optimal. Leaf size=78

$$\frac{\sqrt{a-bx^2}\sqrt{a+bx^2} \tan^{-1}\left(\frac{\sqrt{2}\sqrt{bx}}{\sqrt{a-bx^2}}\right)}{\sqrt{2a}\sqrt{b}\sqrt{a^2-b^2x^4}}$$

[Out] (Sqrt[a - b*x^2]*Sqrt[a + b*x^2]*ArcTan[(Sqrt[2]*Sqrt[b]*x)/Sqrt[a - b*x^2]])/(Sqrt[2]*a*Sqrt[b]*Sqrt[a^2 - b^2*x^4])

Rubi [A] time = 0.035809, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {1152, 377, 205}

$$\frac{\sqrt{a-bx^2}\sqrt{a+bx^2} \tan^{-1}\left(\frac{\sqrt{2}\sqrt{bx}}{\sqrt{a-bx^2}}\right)}{\sqrt{2a}\sqrt{b}\sqrt{a^2-b^2x^4}}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[a + b*x^2]*Sqrt[a^2 - b^2*x^4]), x]

[Out] (Sqrt[a - b*x^2]*Sqrt[a + b*x^2]*ArcTan[(Sqrt[2]*Sqrt[b]*x)/Sqrt[a - b*x^2]])/(Sqrt[2]*a*Sqrt[b]*Sqrt[a^2 - b^2*x^4])

Rule 1152

Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist[(a + c*x^4)^FracPart[p]/((d + e*x^2)^FracPart[p]*(a/d + (c*x^2)/e)^FracPart[p]), Int[(d + e*x^2)^(p + q)*(a/d + (c*x^2)/e)^p, x], x] /; FreeQ[{a, c, d, e, p, q}, x] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p]

Rule 377

Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Subst[Int[1/(c - (b*c - a*d)*x^n), x], x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{a+bx^2}\sqrt{a^2-b^2x^4}} dx &= \frac{\left(\sqrt{a-bx^2}\sqrt{a+bx^2}\right) \int \frac{1}{\sqrt{a-bx^2}(a+bx^2)} dx}{\sqrt{a^2-b^2x^4}} \\ &= \frac{\left(\sqrt{a-bx^2}\sqrt{a+bx^2}\right) \text{Subst}\left(\int \frac{1}{a+2abx^2} dx, x, \frac{x}{\sqrt{a-bx^2}}\right)}{\sqrt{a^2-b^2x^4}} \\ &= \frac{\sqrt{a-bx^2}\sqrt{a+bx^2} \tan^{-1}\left(\frac{\sqrt{2}\sqrt{bx}}{\sqrt{a-bx^2}}\right)}{\sqrt{2a}\sqrt{b}\sqrt{a^2-b^2x^4}} \end{aligned}$$

Mathematica [A] time = 0.0542164, size = 78, normalized size = 1.

$$\frac{\sqrt{a^2 - b^2x^4} \tan^{-1}\left(\frac{\sqrt{2}\sqrt{bx}}{\sqrt{a-bx^2}}\right)}{\sqrt{2a}\sqrt{b}\sqrt{a - bx^2}\sqrt{a + bx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(Sqrt[a + b*x^2]*Sqrt[a^2 - b^2*x^4]),x]

[Out] (Sqrt[a^2 - b^2*x^4]*ArcTan[(Sqrt[2]*Sqrt[b]*x)/Sqrt[a - b*x^2]])/(Sqrt[2]*a*Sqrt[b]*Sqrt[a - b*x^2]*Sqrt[a + b*x^2])

Maple [B] time = 0.043, size = 249, normalized size = 3.2

$$-\frac{1}{2}\sqrt{-b^2x^4 + a^2}\sqrt{b}\left(\sqrt{a}\sqrt{2}\ln\left(2\frac{b\left(\sqrt{2}\sqrt{a}\sqrt{-bx^2 + a} - \sqrt{-abx + a}\right)}{bx - \sqrt{-ab}}\right)\sqrt{b} - \sqrt{a}\sqrt{2}\ln\left(2\frac{b\left(\sqrt{2}\sqrt{a}\sqrt{-bx^2 + a} + \sqrt{-abx + a}\right)}{bx + \sqrt{-ab}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b*x^2+a)^(1/2)/(-b^2*x^4+a^2)^(1/2),x)

[Out] -1/2*(-b^2*x^4+a^2)^(1/2)*b^(1/2)*(a^(1/2)*2^(1/2)*ln(2*b*(2^(1/2)*a^(1/2)*(-b*x^2+a)^(1/2)-(-a*b)^(1/2)*x+a)/(b*x-(-a*b)^(1/2)))*b^(1/2)-a^(1/2)*2^(1/2)*ln(2*b*(2^(1/2)*a^(1/2)*(-b*x^2+a)^(1/2)+(-a*b)^(1/2)*x+a)/(b*x+(-a*b)^(1/2)))*b^(1/2)+2*(-a*b)^(1/2)*arctan(x*b^(1/2)/(-b*x^2+a)^(1/2))-2*arctan(b^(1/2)*x/((-b*x+(a*b)^(1/2))/b*(b*x+(a*b)^(1/2)))^(1/2))*(-a*b)^(1/2)/(b*x^2+a)^(1/2)/(-b*x^2+a)^(1/2)/((-a*b)^(1/2)+(a*b)^(1/2))/(-(-a*b)^(1/2)+(a*b)^(1/2))/(-a*b)^(1/2)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-b^2x^4 + a^2}\sqrt{bx^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x^2+a)^(1/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="maxima")

[Out] integrate(1/(sqrt(-b^2*x^4 + a^2)*sqrt(b*x^2 + a)), x)

Fricas [A] time = 2.16544, size = 359, normalized size = 4.6

$$\left[\frac{\sqrt{2}\sqrt{-b}\log\left(-\frac{3b^2x^4+2abx^2-2\sqrt{2}\sqrt{-b^2x^4+a^2}\sqrt{bx^2+a}\sqrt{-bx-a^2}}{b^2x^4+2abx^2+a^2}\right)}{4ab}, -\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-b^2x^4+a^2}\sqrt{bx^2+a}\sqrt{b}}{2(b^2x^3+abx)}\right)}{2a\sqrt{b}} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x^2+a)^(1/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="fricas")

[Out] [-1/4*sqrt(2)*sqrt(-b)*log(-(3*b^2*x^4 + 2*a*b*x^2 - 2*sqrt(2)*sqrt(-b^2*x^4 + a^2))*sqrt(b*x^2 + a)*sqrt(-b)*x - a^2)/(b^2*x^4 + 2*a*b*x^2 + a^2))/(a*b), -1/2*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-b^2*x^4 + a^2)*sqrt(b*x^2 + a)*sqrt(b)/(b^2*x^3 + a*b*x))/(a*sqrt(b))]

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-(-a + bx^2)(a + bx^2)}\sqrt{a + bx^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x**2+a)**(1/2)/(-b**2*x**4+a**2)**(1/2),x)

[Out] Integral(1/(sqrt(-(-a + b*x**2)*(a + b*x**2))*sqrt(a + b*x**2)), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-b^2x^4 + a^2}\sqrt{bx^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x^2+a)^(1/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="giac")

[Out] integrate(1/(sqrt(-b^2*x^4 + a^2)*sqrt(b*x^2 + a)), x)

$$3.203 \quad \int \frac{1}{(a+bx^2)^{3/2} \sqrt{a^2-b^2x^4}} dx$$

Optimal. Leaf size=125

$$\frac{x(a-bx^2)}{4a^2\sqrt{a+bx^2}\sqrt{a^2-b^2x^4}} + \frac{3\sqrt{a+bx^2}\sqrt{a-bx^2} \tan^{-1}\left(\frac{\sqrt{2}\sqrt{bx}}{\sqrt{a-bx^2}}\right)}{4\sqrt{2}a^2\sqrt{b}\sqrt{a^2-b^2x^4}}$$

[Out] (x*(a - b*x^2))/(4*a^2*Sqrt[a + b*x^2]*Sqrt[a^2 - b^2*x^4]) + (3*Sqrt[a - b*x^2]*Sqrt[a + b*x^2]*ArcTan[(Sqrt[2]*Sqrt[b]*x)/Sqrt[a - b*x^2]])/(4*Sqrt[2]*a^2*Sqrt[b]*Sqrt[a^2 - b^2*x^4])

Rubi [A] time = 0.0511505, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1152, 382, 377, 205}

$$\frac{x(a-bx^2)}{4a^2\sqrt{a+bx^2}\sqrt{a^2-b^2x^4}} + \frac{3\sqrt{a+bx^2}\sqrt{a-bx^2} \tan^{-1}\left(\frac{\sqrt{2}\sqrt{bx}}{\sqrt{a-bx^2}}\right)}{4\sqrt{2}a^2\sqrt{b}\sqrt{a^2-b^2x^4}}$$

Antiderivative was successfully verified.

[In] Int[1/((a + b*x^2)^(3/2)*Sqrt[a^2 - b^2*x^4]), x]

[Out] (x*(a - b*x^2))/(4*a^2*Sqrt[a + b*x^2]*Sqrt[a^2 - b^2*x^4]) + (3*Sqrt[a - b*x^2]*Sqrt[a + b*x^2]*ArcTan[(Sqrt[2]*Sqrt[b]*x)/Sqrt[a - b*x^2]])/(4*Sqrt[2]*a^2*Sqrt[b]*Sqrt[a^2 - b^2*x^4])

Rule 1152

Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist[(a + c*x^4)^FracPart[p]/((d + e*x^2)^FracPart[p]*(a/d + (c*x^2)/e)^FracPart[p]), Int[(d + e*x^2)^(p + q)*(a/d + (c*x^2)/e)^p, x], x] /; FreeQ[{a, c, d, e, p, q}, x] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p]

Rule 382

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] :> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c - a*d)), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] || !LtQ[q, -1]) && NeQ[p, -1]

Rule 377

Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Subst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a+bx^2)^{3/2} \sqrt{a^2-b^2x^4}} dx &= \frac{\left(\sqrt{a-bx^2}\sqrt{a+bx^2}\right) \int \frac{1}{\sqrt{a-bx^2}(a+bx^2)^2} dx}{\sqrt{a^2-b^2x^4}} \\
&= \frac{x(a-bx^2)}{4a^2\sqrt{a+bx^2}\sqrt{a^2-b^2x^4}} + \frac{\left(3\sqrt{a-bx^2}\sqrt{a+bx^2}\right) \int \frac{1}{\sqrt{a-bx^2}(a+bx^2)} dx}{4a\sqrt{a^2-b^2x^4}} \\
&= \frac{x(a-bx^2)}{4a^2\sqrt{a+bx^2}\sqrt{a^2-b^2x^4}} + \frac{\left(3\sqrt{a-bx^2}\sqrt{a+bx^2}\right) \text{Subst}\left(\int \frac{1}{a+2abx^2} dx, x, \frac{x}{\sqrt{a-bx^2}}\right)}{4a\sqrt{a^2-b^2x^4}} \\
&= \frac{x(a-bx^2)}{4a^2\sqrt{a+bx^2}\sqrt{a^2-b^2x^4}} + \frac{3\sqrt{a-bx^2}\sqrt{a+bx^2} \tan^{-1}\left(\frac{\sqrt{2}\sqrt{bx}}{\sqrt{a-bx^2}}\right)}{4\sqrt{2}a^2\sqrt{b}\sqrt{a^2-b^2x^4}}
\end{aligned}$$

Mathematica [A] time = 0.0842704, size = 111, normalized size = 0.89

$$\frac{\sqrt{a^2-b^2x^4} \left(2\sqrt{bx}\sqrt{a-bx^2} + 3\sqrt{2}(a+bx^2) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{bx}}{\sqrt{a-bx^2}}\right)\right)}{8a^2\sqrt{b}\sqrt{a-bx^2}(a+bx^2)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/((a + b*x^2)^(3/2)*Sqrt[a^2 - b^2*x^4]), x]

[Out] (Sqrt[a^2 - b^2*x^4]*(2*Sqrt[b]*x*Sqrt[a - b*x^2] + 3*Sqrt[2]*(a + b*x^2)*ArcTan[(Sqrt[2]*Sqrt[b]*x)/Sqrt[a - b*x^2]])/(8*a^2*Sqrt[b]*Sqrt[a - b*x^2]*(a + b*x^2)^(3/2))

Maple [B] time = 0.063, size = 488, normalized size = 3.9

$$-\frac{1}{4}\sqrt{-b^2x^4+a^2}b^{\frac{5}{2}}\left(3\ln\left(2\frac{b\left(\sqrt{2}\sqrt{a}\sqrt{-bx^2+a}-\sqrt{-abx+a}\right)}{bx-\sqrt{-ab}}\right)\sqrt{2}x^2b^{\frac{3}{2}}\sqrt{a}-3\ln\left(2\frac{b\left(\sqrt{2}\sqrt{a}\sqrt{-bx^2+a}+\sqrt{-abx+a}\right)}{bx+\sqrt{-ab}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b*x^2+a)^(3/2)/(-b^2*x^4+a^2)^(1/2), x)

[Out] -1/4*(-b^2*x^4+a^2)^(1/2)*b^(5/2)*(3*ln(2*b*(2^(1/2)*a^(1/2)*(-b*x^2+a)^(1/2)-(-a*b)^(1/2)*x+a)/(b*x-(-a*b)^(1/2)))*2^(1/2)*x^2*b^(3/2)*a^(1/2)-3*ln(2*b*(2^(1/2)*a^(1/2)*(-b*x^2+a)^(1/2)+(-a*b)^(1/2)*x+a)/(b*x+(-a*b)^(1/2)))*2^(1/2)*x^2*b^(3/2)*a^(1/2)+3*ln(2*b*(2^(1/2)*a^(1/2)*(-b*x^2+a)^(1/2)-(-a*b)^(1/2)*x+a)/(b*x-(-a*b)^(1/2)))*2^(1/2)*a^(3/2)*b^(1/2)-3*ln(2*b*(2^(1/2)*a^(1/2)*(-b*x^2+a)^(1/2)+(-a*b)^(1/2)*x+a)/(b*x+(-a*b)^(1/2)))*2^(1/2)*a^(3/2)*b^(1/2)+4*arctan(x*b^(1/2)/(-b*x^2+a)^(1/2))*x^2*b*(-a*b)^(1/2)-4*arctan(b^(1/2)*x/((-b*x+(a*b)^(1/2))/b*(b*x+(a*b)^(1/2))))^(1/2)*x^2*b*(-a*b)^(1/2)-4*b^(1/2)*(-a*b)^(1/2)*(-b*x^2+a)^(1/2)*x+4*arctan(x*b^(1/2)/(-b*x^2+a)^(1/2))*a*(-a*b)^(1/2)-4*arctan(b^(1/2)*x/((-b*x+(a*b)^(1/2))/b*(b*x+(a*b)^(1/2))))^(1/2)*a*(-a*b)^(1/2))/(b*x^2+a)^(1/2)/(-b*x^2+a)^(1/2)/((-a*b)^(1/2)+(a*b)^(1/2))^2/(-(-a*b)^(1/2)+(a*b)^(1/2))^2/(-a*b)^(1/2)/(b*x+(-a*b)^(1/2))

$1/2)) / (b*x - (-a*b)^{(1/2)})$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-b^2x^4 + a^2}(bx^2 + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x^2+a)^(3/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="maxima")

[Out] integrate(1/(sqrt(-b^2*x^4 + a^2)*(b*x^2 + a)^(3/2)), x)

Fricas [A] time = 2.23803, size = 655, normalized size = 5.24

$$\left[\frac{4\sqrt{-b^2x^4 + a^2}\sqrt{bx^2 + a}bx - 3\sqrt{2}(b^2x^4 + 2abx^2 + a^2)\sqrt{-b}\log\left(\frac{-3b^2x^4 + 2abx^2 - 2\sqrt{2}\sqrt{-b^2x^4 + a^2}\sqrt{bx^2 + a}\sqrt{-bx - a^2}}{b^2x^4 + 2abx^2 + a^2}\right)}{16(a^2b^3x^4 + 2a^3b^2x^2 + a^4b)}, \frac{2\sqrt{-b^2x^4 + a^2}}{16(a^2b^3x^4 + 2a^3b^2x^2 + a^4b)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x^2+a)^(3/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="fricas")

[Out] [1/16*(4*sqrt(-b^2*x^4 + a^2)*sqrt(b*x^2 + a)*b*x - 3*sqrt(2)*(b^2*x^4 + 2*a*b*x^2 + a^2)*sqrt(-b)*log(-(3*b^2*x^4 + 2*a*b*x^2 - 2*sqrt(2)*sqrt(-b^2*x^4 + a^2)*sqrt(b*x^2 + a)*sqrt(-b)*x - a^2)/(b^2*x^4 + 2*a*b*x^2 + a^2)))/(a^2*b^3*x^4 + 2*a^3*b^2*x^2 + a^4*b), 1/8*(2*sqrt(-b^2*x^4 + a^2)*sqrt(b*x^2 + a)*b*x - 3*sqrt(2)*(b^2*x^4 + 2*a*b*x^2 + a^2)*sqrt(b)*arctan(1/2*sqrt(2)*sqrt(-b^2*x^4 + a^2)*sqrt(b*x^2 + a)*sqrt(b)/(b^2*x^3 + a*b*x)))/(a^2*b^3*x^4 + 2*a^3*b^2*x^2 + a^4*b)]

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-(-a + bx^2)(a + bx^2)}(a + bx^2)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x**2+a)**(3/2)/(-b**2*x**4+a**2)**(1/2),x)

[Out] Integral(1/(sqrt(-(-a + b*x**2)*(a + b*x**2))*(a + b*x**2)**(3/2)), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-b^2x^4 + a^2}(bx^2 + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(3/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-b^2*x^4 + a^2)*(b*x^2 + a)^(3/2)), x)
```

$$3.204 \quad \int \frac{1}{(a+bx^2)^{5/2} \sqrt{a^2-b^2x^4}} dx$$

Optimal. Leaf size=168

$$\frac{9x(a-bx^2)}{32a^3\sqrt{a+bx^2}\sqrt{a^2-b^2x^4}} + \frac{x(a-bx^2)}{8a^2(a+bx^2)^{3/2}\sqrt{a^2-b^2x^4}} + \frac{19\sqrt{a+bx^2}\sqrt{a-bx^2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{bx}}{\sqrt{a-bx^2}}\right)}{32\sqrt{2}a^3\sqrt{b}\sqrt{a^2-b^2x^4}}$$

[Out] (x*(a - b*x^2))/(8*a^2*(a + b*x^2)^(3/2)*Sqrt[a^2 - b^2*x^4]) + (9*x*(a - b*x^2))/(32*a^3*Sqrt[a + b*x^2]*Sqrt[a^2 - b^2*x^4]) + (19*Sqrt[a - b*x^2]*Sqrt[a + b*x^2]*ArcTan[(Sqrt[2]*Sqrt[b]*x)/Sqrt[a - b*x^2]])/(32*Sqrt[2]*a^3*Sqrt[b]*Sqrt[a^2 - b^2*x^4])

Rubi [A] time = 0.0862913, antiderivative size = 168, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {1152, 414, 527, 12, 377, 205}

$$\frac{9x(a-bx^2)}{32a^3\sqrt{a+bx^2}\sqrt{a^2-b^2x^4}} + \frac{x(a-bx^2)}{8a^2(a+bx^2)^{3/2}\sqrt{a^2-b^2x^4}} + \frac{19\sqrt{a+bx^2}\sqrt{a-bx^2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{bx}}{\sqrt{a-bx^2}}\right)}{32\sqrt{2}a^3\sqrt{b}\sqrt{a^2-b^2x^4}}$$

Antiderivative was successfully verified.

[In] Int[1/((a + b*x^2)^(5/2)*Sqrt[a^2 - b^2*x^4]),x]

[Out] (x*(a - b*x^2))/(8*a^2*(a + b*x^2)^(3/2)*Sqrt[a^2 - b^2*x^4]) + (9*x*(a - b*x^2))/(32*a^3*Sqrt[a + b*x^2]*Sqrt[a^2 - b^2*x^4]) + (19*Sqrt[a - b*x^2]*Sqrt[a + b*x^2]*ArcTan[(Sqrt[2]*Sqrt[b]*x)/Sqrt[a - b*x^2]])/(32*Sqrt[2]*a^3*Sqrt[b]*Sqrt[a^2 - b^2*x^4])

Rule 1152

Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist[(a + c*x^4)^FracPart[p]/((d + e*x^2)^FracPart[p]*(a/d + (c*x^2)/e)^FracPart[p]), Int[(d + e*x^2)^(p + q)*(a/d + (c*x^2)/e)^p, x], x] /; FreeQ[{a, c, d, e, p, q}, x] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p]

Rule 414

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] :> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c - a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && !(IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, n, p, q, x]

Rule 527

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x] /; FreeQ

[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 377

Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Subst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rubi steps

$$\begin{aligned} \int \frac{1}{(a+bx^2)^{5/2} \sqrt{a^2-b^2x^4}} dx &= \frac{\left(\sqrt{a-bx^2}\sqrt{a+bx^2}\right) \int \frac{1}{\sqrt{a-bx^2}(a+bx^2)^3} dx}{\sqrt{a^2-b^2x^4}} \\ &= \frac{x(a-bx^2)}{8a^2(a+bx^2)^{3/2} \sqrt{a^2-b^2x^4}} - \frac{\left(\sqrt{a-bx^2}\sqrt{a+bx^2}\right) \int \frac{-7ab+2b^2x^2}{\sqrt{a-bx^2}(a+bx^2)^2} dx}{8a^2b\sqrt{a^2-b^2x^4}} \\ &= \frac{x(a-bx^2)}{8a^2(a+bx^2)^{3/2} \sqrt{a^2-b^2x^4}} + \frac{9x(a-bx^2)}{32a^3\sqrt{a+bx^2}\sqrt{a^2-b^2x^4}} + \frac{\left(\sqrt{a-bx^2}\sqrt{a+bx^2}\right) \int \frac{1}{\sqrt{a-bx^2}} dx}{32a^4b^2\sqrt{a^2-b^2x^4}} \\ &= \frac{x(a-bx^2)}{8a^2(a+bx^2)^{3/2} \sqrt{a^2-b^2x^4}} + \frac{9x(a-bx^2)}{32a^3\sqrt{a+bx^2}\sqrt{a^2-b^2x^4}} + \frac{\left(19\sqrt{a-bx^2}\sqrt{a+bx^2}\right) \int \frac{1}{\sqrt{a-bx^2}} dx}{32a^2\sqrt{a^2-b^2x^4}} \\ &= \frac{x(a-bx^2)}{8a^2(a+bx^2)^{3/2} \sqrt{a^2-b^2x^4}} + \frac{9x(a-bx^2)}{32a^3\sqrt{a+bx^2}\sqrt{a^2-b^2x^4}} + \frac{\left(19\sqrt{a-bx^2}\sqrt{a+bx^2}\right) \text{Subst}\left[\int \frac{1}{\sqrt{a-bx^2}} dx, x, \frac{x}{\sqrt{a-bx^2}}\right]}{32a^2\sqrt{a^2-b^2x^4}} \\ &= \frac{x(a-bx^2)}{8a^2(a+bx^2)^{3/2} \sqrt{a^2-b^2x^4}} + \frac{9x(a-bx^2)}{32a^3\sqrt{a+bx^2}\sqrt{a^2-b^2x^4}} + \frac{19\sqrt{a-bx^2}\sqrt{a+bx^2} \tan^{-1}\left(\frac{\sqrt{2}\sqrt{bx^2}}{\sqrt{a-bx^2}}\right)}{32\sqrt{2}a^3\sqrt{b}\sqrt{a^2-b^2x^4}} \end{aligned}$$

Mathematica [A] time = 0.108498, size = 123, normalized size = 0.73

$$\frac{\sqrt{a^2-b^2x^4} \left(2\sqrt{bx^2}\sqrt{a-bx^2} (13a+9bx^2) + 19\sqrt{2} (a+bx^2)^2 \tan^{-1}\left(\frac{\sqrt{2}\sqrt{bx^2}}{\sqrt{a-bx^2}}\right) \right)}{64a^3\sqrt{b}\sqrt{a-bx^2} (a+bx^2)^{5/2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/((a + b*x^2)^(5/2)*Sqrt[a^2 - b^2*x^4]), x]

[Out] (Sqrt[a^2 - b^2*x^4]*(2*Sqrt[b]*x*Sqrt[a - b*x^2]*(13*a + 9*b*x^2) + 19*Sqrt[2]*(a + b*x^2)^2*ArcTan[(Sqrt[2]*Sqrt[b]*x)/Sqrt[a - b*x^2]]))/(64*a^3*Sqrt[b]*Sqrt[a - b*x^2]*(a + b*x^2)^(5/2))

Maple [B] time = 0.054, size = 711, normalized size = 4.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(1/(b*x^2+a)^{(5/2)}/(-b^2*x^4+a^2)^{(1/2)},x)$

[Out]
$$-1/16*(-b^2*x^4+a^2)^{(1/2)}*b^{(9/2)}*(19*\ln(2*b*(2^{(1/2)}*a^{(1/2)}*(-b*x^2+a)^{(1/2)}-(-a*b)^{(1/2)}*x+a)/(b*x-(-a*b)^{(1/2)})))*2^{(1/2)}*x^4*b^{(5/2)}*a^{(1/2)}-19*\ln(2*b*(2^{(1/2)}*a^{(1/2)}*(-b*x^2+a)^{(1/2)}+(-a*b)^{(1/2)}*x+a)/(b*x+(-a*b)^{(1/2)})))*2^{(1/2)}*x^4*b^{(5/2)}*a^{(1/2)}+38*\ln(2*b*(2^{(1/2)}*a^{(1/2)}*(-b*x^2+a)^{(1/2)}-(-a*b)^{(1/2)}*x+a)/(b*x-(-a*b)^{(1/2)})))*2^{(1/2)}*x^2*a^{(3/2)}*b^{(3/2)}-38*\ln(2*b*(2^{(1/2)}*a^{(1/2)}*(-b*x^2+a)^{(1/2)}+(-a*b)^{(1/2)}*x+a)/(b*x+(-a*b)^{(1/2)})))*2^{(1/2)}*x^2*a^{(3/2)}*b^{(3/2)}-16*\arctan(b^{(1/2)}*x/((-b*x+(a*b)^{(1/2)})/b*(b*x+(a*b)^{(1/2)})))^{(1/2)}*x^4*b^2*(-a*b)^{(1/2)}+16*\arctan(x*b^{(1/2)}/(-b*x^2+a)^{(1/2)})*x^4*b^2*(-a*b)^{(1/2)}-36*b^{(3/2)}*(-a*b)^{(1/2)}*(-b*x^2+a)^{(1/2)}*x^3+19*\ln(2*b*(2^{(1/2)}*a^{(1/2)}*(-b*x^2+a)^{(1/2)}-(-a*b)^{(1/2)}*x+a)/(b*x-(-a*b)^{(1/2)})))*2^{(1/2)}*a^{(5/2)}*b^{(1/2)}-19*\ln(2*b*(2^{(1/2)}*a^{(1/2)}*(-b*x^2+a)^{(1/2)}+(-a*b)^{(1/2)}*x+a)/(b*x+(-a*b)^{(1/2)})))*2^{(1/2)}*a^{(5/2)}*b^{(1/2)}-32*\arctan(b^{(1/2)}*x/((-b*x+(a*b)^{(1/2)})/b*(b*x+(a*b)^{(1/2)})))^{(1/2)}*x^2*a*b*(-a*b)^{(1/2)}+32*\arctan(x*b^{(1/2)}/(-b*x^2+a)^{(1/2)})*x^2*a*b*(-a*b)^{(1/2)}-52*b^{(1/2)}*a*(-a*b)^{(1/2)}*(-b*x^2+a)^{(1/2)}*x-16*\arctan(b^{(1/2)}*x/((-b*x+(a*b)^{(1/2)})/b*(b*x+(a*b)^{(1/2)})))^{(1/2)}*a^2*(-a*b)^{(1/2)}+16*\arctan(x*b^{(1/2)}/(-b*x^2+a)^{(1/2)})*a^2*(-a*b)^{(1/2)}/(b*x^2+a)^{(1/2)}/(-b*x^2+a)^{(1/2)}/(-a*b)^{(1/2)}/((-a*b)^{(1/2)}+(a*b)^{(1/2)})^3/((-a*b)^{(1/2)}+(a*b)^{(1/2)})^3/(b*x+(-a*b)^{(1/2)})^2/(b*x-(-a*b)^{(1/2)})^2$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-b^2x^4 + a^2}(bx^2 + a)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(1/(b*x^2+a)^{(5/2)}/(-b^2*x^4+a^2)^{(1/2)},x, \text{algorithm}="maxima")$

[Out] $\text{integrate}(1/(\text{sqrt}(-b^2*x^4 + a^2)*(b*x^2 + a)^{(5/2)}), x)$

Fricas [A] time = 1.87349, size = 801, normalized size = 4.77

$$\left[\frac{19\sqrt{2}(b^3x^6 + 3ab^2x^4 + 3a^2bx^2 + a^3)\sqrt{-b} \log\left(-\frac{3b^2x^4 + 2abx^2 - 2\sqrt{2}\sqrt{-b^2x^4 + a^2}\sqrt{bx^2 + a}\sqrt{-bx - a^2}}{b^2x^4 + 2abx^2 + a^2}\right) - 4\sqrt{-b^2x^4 + a^2}(9b^2x^3 + 13abx + 4a^2)}{128(a^3b^4x^6 + 3a^4b^3x^4 + 3a^5b^2x^2 + a^6b)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(1/(b*x^2+a)^{(5/2)}/(-b^2*x^4+a^2)^{(1/2)},x, \text{algorithm}="fricas")$

[Out] $[-1/128*(19*\text{sqrt}(2)*(b^3*x^6 + 3*a*b^2*x^4 + 3*a^2*b*x^2 + a^3)*\text{sqrt}(-b)*\log(-(3*b^2*x^4 + 2*a*b*x^2 - 2*\text{sqrt}(2)*\text{sqrt}(-b^2*x^4 + a^2)*\text{sqrt}(b*x^2 + a)*$


```
sqrt(-b)*x - a^2)/(b^2*x^4 + 2*a*b*x^2 + a^2)) - 4*sqrt(-b^2*x^4 + a^2)*(9*
b^2*x^3 + 13*a*b*x)*sqrt(b*x^2 + a))/(a^3*b^4*x^6 + 3*a^4*b^3*x^4 + 3*a^5*b
^2*x^2 + a^6*b), -1/64*(19*sqrt(2)*(b^3*x^6 + 3*a*b^2*x^4 + 3*a^2*b*x^2 + a
^3)*sqrt(b)*arctan(1/2*sqrt(2)*sqrt(-b^2*x^4 + a^2)*sqrt(b*x^2 + a)*sqrt(b)
/(b^2*x^3 + a*b*x)) - 2*sqrt(-b^2*x^4 + a^2)*(9*b^2*x^3 + 13*a*b*x)*sqrt(b*
x^2 + a))/(a^3*b^4*x^6 + 3*a^4*b^3*x^4 + 3*a^5*b^2*x^2 + a^6*b)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-(-a + bx^2)(a + bx^2)}(a + bx^2)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**(5/2)/(-b**2*x**4+a**2)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-(-a + b*x**2)*(a + b*x**2))*(a + b*x**2)**(5/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-b^2x^4 + a^2}(bx^2 + a)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(5/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-b^2*x^4 + a^2)*(b*x^2 + a)^(5/2)), x)
```

$$3.205 \quad \int \frac{(a-bx^2)^{5/2}}{\sqrt{a^2-b^2x^4}} dx$$

Optimal. Leaf size=152

$$-\frac{9ax\sqrt{a-bx^2}(a+bx^2)}{8\sqrt{a^2-b^2x^4}} - \frac{x(a-bx^2)^{3/2}(a+bx^2)}{4\sqrt{a^2-b^2x^4}} + \frac{19a^2\sqrt{a-bx^2}\sqrt{a+bx^2}\tanh^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a+bx^2}}\right)}{8\sqrt{b}\sqrt{a^2-b^2x^4}}$$

[Out] $(-9*a*x*\text{Sqrt}[a - b*x^2]*(a + b*x^2))/(8*\text{Sqrt}[a^2 - b^2*x^4]) - (x*(a - b*x^2)^{(3/2)}*(a + b*x^2))/(4*\text{Sqrt}[a^2 - b^2*x^4]) + (19*a^2*\text{Sqrt}[a - b*x^2]*\text{Sqrt}[a + b*x^2]*\text{ArcTanh}[(\text{Sqrt}[b]*x)/\text{Sqrt}[a + b*x^2]])/(8*\text{Sqrt}[b]*\text{Sqrt}[a^2 - b^2*x^4])$

Rubi [A] time = 0.0545413, antiderivative size = 152, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {1152, 416, 388, 217, 206}

$$-\frac{9ax\sqrt{a-bx^2}(a+bx^2)}{8\sqrt{a^2-b^2x^4}} - \frac{x(a-bx^2)^{3/2}(a+bx^2)}{4\sqrt{a^2-b^2x^4}} + \frac{19a^2\sqrt{a-bx^2}\sqrt{a+bx^2}\tanh^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a+bx^2}}\right)}{8\sqrt{b}\sqrt{a^2-b^2x^4}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a - b*x^2)^{(5/2)}/\text{Sqrt}[a^2 - b^2*x^4], x]$

[Out] $(-9*a*x*\text{Sqrt}[a - b*x^2]*(a + b*x^2))/(8*\text{Sqrt}[a^2 - b^2*x^4]) - (x*(a - b*x^2)^{(3/2)}*(a + b*x^2))/(4*\text{Sqrt}[a^2 - b^2*x^4]) + (19*a^2*\text{Sqrt}[a - b*x^2]*\text{Sqrt}[a + b*x^2]*\text{ArcTanh}[(\text{Sqrt}[b]*x)/\text{Sqrt}[a + b*x^2]])/(8*\text{Sqrt}[b]*\text{Sqrt}[a^2 - b^2*x^4])$

Rule 1152

$\text{Int}[(d + e*x^2)^{(q_1)}*(a + c*x^2)^{(p_1)}, x_Symbol] \rightarrow \text{Dist}[(a + c*x^2)^{\text{FracPart}[p_1]}/((d + e*x^2)^{\text{FracPart}[p_1]}*(a/d + (c*x^2)/e)^{\text{FracPart}[p_1]})], \text{Int}[(d + e*x^2)^{(p + q)}*(a/d + (c*x^2)/e)^p, x], x] /;$ $\text{FreeQ}\{a, c, d, e, p, q\}, x] \ \&\& \ \text{EqQ}[c*d^2 + a*e^2, 0] \ \&\& \ !\text{IntegerQ}[p]$

Rule 416

$\text{Int}[(a + b*x^n)^{(p)}*((c + d*x^n)^{(q)}), x_Symbol] \rightarrow \text{Simp}[(d*x*(a + b*x^n)^{(p + 1)}*(c + d*x^n)^{(q - 1)})/(b*(n*(p + q) + 1)), x] + \text{Dist}[1/(b*(n*(p + q) + 1)), \text{Int}[(a + b*x^n)^p*(c + d*x^n)^{(q - 2)}*\text{Simp}[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q - 1) + 1))*x^n, x], x] /;$ $\text{FreeQ}\{a, b, c, d, n, p\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{GtQ}[q, 1] \ \&\& \ \text{NeQ}[n*(p + q) + 1, 0] \ \&\& \ !\text{IGtQ}[p, 1] \ \&\& \ \text{IntBinomialQ}[a, b, c, d, n, p, q, x]$

Rule 388

$\text{Int}[(a + b*x^n)^{(p)}*((c + d*x^n)^{(q)}), x_Symbol] \rightarrow \text{Simp}[(d*x*(a + b*x^n)^{(p + 1)})/(b*(n*(p + 1) + 1)), x] - \text{Dist}[(a*d - b*c*(n*(p + 1) + 1))/(b*(n*(p + 1) + 1)), \text{Int}[(a + b*x^n)^p, x], x] /;$ $\text{FreeQ}\{a, b, c, d, n\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{NeQ}[n*(p + 1) + 1, 0]$

Rule 217

`Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]`

Rule 206

`Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])`

Rubi steps

$$\begin{aligned} \int \frac{(a - bx^2)^{5/2}}{\sqrt{a^2 - b^2x^4}} dx &= \frac{\left(\sqrt{a - bx^2}\sqrt{a + bx^2}\right) \int \frac{(a - bx^2)^2}{\sqrt{a + bx^2}} dx}{\sqrt{a^2 - b^2x^4}} \\ &= -\frac{x(a - bx^2)^{3/2}(a + bx^2)}{4\sqrt{a^2 - b^2x^4}} + \frac{\left(\sqrt{a - bx^2}\sqrt{a + bx^2}\right) \int \frac{5a^2b - 9ab^2x^2}{\sqrt{a + bx^2}} dx}{4b\sqrt{a^2 - b^2x^4}} \\ &= -\frac{9ax\sqrt{a - bx^2}(a + bx^2)}{8\sqrt{a^2 - b^2x^4}} - \frac{x(a - bx^2)^{3/2}(a + bx^2)}{4\sqrt{a^2 - b^2x^4}} + \frac{\left(19a^2\sqrt{a - bx^2}\sqrt{a + bx^2}\right) \int \frac{1}{\sqrt{a + bx^2}} dx}{8\sqrt{a^2 - b^2x^4}} \\ &= -\frac{9ax\sqrt{a - bx^2}(a + bx^2)}{8\sqrt{a^2 - b^2x^4}} - \frac{x(a - bx^2)^{3/2}(a + bx^2)}{4\sqrt{a^2 - b^2x^4}} + \frac{\left(19a^2\sqrt{a - bx^2}\sqrt{a + bx^2}\right) \text{Subst}\left(\int \frac{1}{1 - bx^2} dx\right)}{8\sqrt{a^2 - b^2x^4}} \\ &= -\frac{9ax\sqrt{a - bx^2}(a + bx^2)}{8\sqrt{a^2 - b^2x^4}} - \frac{x(a - bx^2)^{3/2}(a + bx^2)}{4\sqrt{a^2 - b^2x^4}} + \frac{19a^2\sqrt{a - bx^2}\sqrt{a + bx^2} \tanh^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a + bx^2}}\right)}{8\sqrt{b}\sqrt{a^2 - b^2x^4}} \end{aligned}$$

Mathematica [A] time = 0.200751, size = 123, normalized size = 0.81

$$\frac{1}{8} \left(\frac{x(2bx^2 - 11a)\sqrt{a^2 - b^2x^4}}{\sqrt{a - bx^2}} + \frac{19a^2 \log\left(\sqrt{b}\sqrt{a - bx^2}\sqrt{a^2 - b^2x^4} + abx - b^2x^3\right)}{\sqrt{b}} - \frac{19a^2 \log(bx^2 - a)}{\sqrt{b}} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(a - b*x^2)^(5/2)/Sqrt[a^2 - b^2*x^4], x]

[Out] ((x*(-11*a + 2*b*x^2)*Sqrt[a^2 - b^2*x^4])/Sqrt[a - b*x^2] - (19*a^2*Log[-a + b*x^2])/Sqrt[b] + (19*a^2*Log[a*b*x - b^2*x^3 + Sqrt[b]*Sqrt[a - b*x^2]*Sqrt[a^2 - b^2*x^4])/Sqrt[b])/8

Maple [A] time = 0.017, size = 105, normalized size = 0.7

$$-\frac{1}{8bx^2 - 8a} \sqrt{-bx^2 + a} \sqrt{-b^2x^4 + a^2} \left(2x^3b^{3/2}\sqrt{bx^2 + a} - 11xa\sqrt{bx^2 + a}\sqrt{b} + 19 \ln\left(x\sqrt{b} + \sqrt{bx^2 + a}\right)a^2 \right) \frac{1}{\sqrt{bx^2 + a}} \frac{1}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-b*x^2+a)^(5/2)/(-b^2*x^4+a^2)^(1/2), x)

[Out] -1/8*(-b*x^2+a)^(1/2)*(-b^2*x^4+a^2)^(1/2)*(2*x^3*b^(3/2)*(b*x^2+a)^(1/2)-11*x*a*(b*x^2+a)^(1/2)*b^(1/2)+19*ln(x*b^(1/2)+(b*x^2+a)^(1/2))*a^2)/(b*x^2-

$a/(b*x^2+a)^{(1/2)}/b^{(1/2)}$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(-bx^2 + a)^{\frac{5}{2}}}{\sqrt{-b^2x^4 + a^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b*x^2+a)^(5/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="maxima")

[Out] integrate((-b*x^2 + a)^(5/2)/sqrt(-b^2*x^4 + a^2), x)

Fricas [A] time = 1.99549, size = 551, normalized size = 3.62

$$\left[\frac{19(a^2bx^2 - a^3)\sqrt{b} \log\left(\frac{2b^2x^4 - abx^2 - 2\sqrt{-b^2x^4 + a^2}\sqrt{-bx^2 + a}\sqrt{bx - a^2}}{bx^2 - a}\right) - 2\sqrt{-b^2x^4 + a^2}(2b^2x^3 - 11abx)\sqrt{-bx^2 + a} - 19(a^2bx^2 - a^3)}{16(b^2x^2 - ab)}, \dots \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b*x^2+a)^(5/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="fricas")

[Out] [1/16*(19*(a^2*b*x^2 - a^3)*sqrt(b)*log((2*b^2*x^4 - a*b*x^2 - 2*sqrt(-b^2*x^4 + a^2)*sqrt(-b*x^2 + a)*sqrt(b)*x - a^2)/(b*x^2 - a)) - 2*sqrt(-b^2*x^4 + a^2)*(2*b^2*x^3 - 11*a*b*x)*sqrt(-b*x^2 + a))/(b^2*x^2 - a*b), 1/8*(19*(a^2*b*x^2 - a^3)*sqrt(-b)*arctan(sqrt(-b^2*x^4 + a^2)*sqrt(-b*x^2 + a)*sqrt(-b)/(b^2*x^3 - a*b*x)) - sqrt(-b^2*x^4 + a^2)*(2*b^2*x^3 - 11*a*b*x)*sqrt(-b*x^2 + a))/(b^2*x^2 - a*b)]

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a - bx^2)^{\frac{5}{2}}}{\sqrt{-(-a + bx^2)(a + bx^2)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b*x**2+a)**(5/2)/(-b**2*x**4+a**2)**(1/2),x)

[Out] Integral((a - b*x**2)**(5/2)/sqrt(-(-a + b*x**2)*(a + b*x**2)), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(-bx^2 + a)^{\frac{5}{2}}}{\sqrt{-b^2x^4 + a^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(5/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((-b*x^2 + a)^(5/2)/sqrt(-b^2*x^4 + a^2), x)
```

$$3.206 \quad \int \frac{(a-bx^2)^{3/2}}{\sqrt{a^2-b^2x^4}} dx$$

Optimal. Leaf size=109

$$\frac{3a\sqrt{a-bx^2}\sqrt{a+bx^2}\tanh^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}\sqrt{a^2-b^2x^4}} - \frac{x\sqrt{a-bx^2}(a+bx^2)}{2\sqrt{a^2-b^2x^4}}$$

[Out] $-(x\sqrt{a-bx^2}(a+bx^2))/(2\sqrt{a^2-b^2x^4}) + (3a\sqrt{a-bx^2}\sqrt{a+bx^2}\text{ArcTanh}[(\sqrt{bx})/\sqrt{a+bx^2}])/(2\sqrt{b}\sqrt{a^2-b^2x^4})$

Rubi [A] time = 0.0352851, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {1152, 388, 217, 206}

$$\frac{3a\sqrt{a-bx^2}\sqrt{a+bx^2}\tanh^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}\sqrt{a^2-b^2x^4}} - \frac{x\sqrt{a-bx^2}(a+bx^2)}{2\sqrt{a^2-b^2x^4}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a-bx^2)^{(3/2)}/\sqrt{a^2-b^2x^4}, x]$

[Out] $-(x\sqrt{a-bx^2}(a+bx^2))/(2\sqrt{a^2-b^2x^4}) + (3a\sqrt{a-bx^2}\sqrt{a+bx^2}\text{ArcTanh}[(\sqrt{bx})/\sqrt{a+bx^2}])/(2\sqrt{b}\sqrt{a^2-b^2x^4})$

Rule 1152

$\text{Int}[(d + e \cdot x^2)^q \cdot (a + c \cdot x^4)^p, x_Symbol] \rightarrow \text{Dist}[(a + c \cdot x^4)^{\text{FracPart}[p]} / ((d + e \cdot x^2)^{\text{FracPart}[p]} \cdot (a/d + (c \cdot x^2)/e)^{\text{FracPart}[p]}), \text{Int}[(d + e \cdot x^2)^{p+q} \cdot (a/d + (c \cdot x^2)/e)^p, x], x] /;$ FreeQ[{a, c, d, e, p, q}, x] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p]

Rule 388

$\text{Int}[(a + b \cdot x^n)^p \cdot (c + d \cdot x^n), x_Symbol] \rightarrow \text{Simp}[(d \cdot x \cdot (a + b \cdot x^n)^{p+1}) / (b \cdot (n \cdot (p+1) + 1)), x] - \text{Dist}[(a \cdot d - b \cdot c \cdot (n \cdot (p+1) + 1)) / (b \cdot (n \cdot (p+1) + 1)), \text{Int}[(a + b \cdot x^n)^p, x], x] /;$ FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p+1) + 1, 0]

Rule 217

$\text{Int}[1/\sqrt{(a + b \cdot x^2)}, x_Symbol] \rightarrow \text{Subst}[\text{Int}[1/(1 - b \cdot x^2), x], x, x/\sqrt{a + b \cdot x^2}] /;$ FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 206

$\text{Int}[(a + b \cdot x^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(1 \cdot \text{ArcTanh}[(\text{Rt}[-b, 2] \cdot x) / \text{Rt}[a, 2]]) / (\text{Rt}[a, 2] \cdot \text{Rt}[-b, 2]), x] /;$ FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned}
\int \frac{(a - bx^2)^{3/2}}{\sqrt{a^2 - b^2x^4}} dx &= \frac{(\sqrt{a - bx^2}\sqrt{a + bx^2}) \int \frac{a - bx^2}{\sqrt{a + bx^2}} dx}{\sqrt{a^2 - b^2x^4}} \\
&= -\frac{x\sqrt{a - bx^2}(a + bx^2)}{2\sqrt{a^2 - b^2x^4}} + \frac{(3a\sqrt{a - bx^2}\sqrt{a + bx^2}) \int \frac{1}{\sqrt{a + bx^2}} dx}{2\sqrt{a^2 - b^2x^4}} \\
&= -\frac{x\sqrt{a - bx^2}(a + bx^2)}{2\sqrt{a^2 - b^2x^4}} + \frac{(3a\sqrt{a - bx^2}\sqrt{a + bx^2}) \text{Subst}\left(\int \frac{1}{1 - bx^2} dx, x, \frac{x}{\sqrt{a + bx^2}}\right)}{2\sqrt{a^2 - b^2x^4}} \\
&= -\frac{x\sqrt{a - bx^2}(a + bx^2)}{2\sqrt{a^2 - b^2x^4}} + \frac{3a\sqrt{a - bx^2}\sqrt{a + bx^2} \tanh^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a + bx^2}}\right)}{2\sqrt{b}\sqrt{a^2 - b^2x^4}}
\end{aligned}$$

Mathematica [A] time = 0.115382, size = 110, normalized size = 1.01

$$\frac{1}{2} \left(-\frac{x\sqrt{a^2 - b^2x^4}}{\sqrt{a - bx^2}} + \frac{3a \log\left(\sqrt{b}\sqrt{a - bx^2}\sqrt{a^2 - b^2x^4} + abx - b^2x^3\right)}{\sqrt{b}} - \frac{3a \log(bx^2 - a)}{\sqrt{b}} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(a - b*x^2)^(3/2)/Sqrt[a^2 - b^2*x^4], x]

[Out] (-((x*Sqrt[a^2 - b^2*x^4])/Sqrt[a - b*x^2]) - (3*a*Log[-a + b*x^2])/Sqrt[b] + (3*a*Log[a*b*x - b^2*x^3 + Sqrt[b]*Sqrt[a - b*x^2]*Sqrt[a^2 - b^2*x^4]])/Sqrt[b])/2

Maple [A] time = 0.016, size = 85, normalized size = 0.8

$$-\frac{1}{2bx^2 - 2a} \sqrt{-bx^2 + a} \sqrt{-b^2x^4 + a^2} \left(-x\sqrt{bx^2 + a}\sqrt{b} + 3 \ln\left(x\sqrt{b} + \sqrt{bx^2 + a}\right) a \right) \frac{1}{\sqrt{bx^2 + a}} \frac{1}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-b*x^2+a)^(3/2)/(-b^2*x^4+a^2)^(1/2), x)

[Out] -1/2*(-b*x^2+a)^(1/2)*(-b^2*x^4+a^2)^(1/2)*(-x*(b*x^2+a)^(1/2)*b^(1/2)+3*ln(x*b^(1/2)+(b*x^2+a)^(1/2))*a)/(b*x^2-a)/(b*x^2+a)^(1/2)/b^(1/2)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(-bx^2 + a)^{3/2}}{\sqrt{-b^2x^4 + a^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b*x^2+a)^(3/2)/(-b^2*x^4+a^2)^(1/2), x, algorithm="maxima")

[Out] integrate((-b*x^2 + a)^(3/2)/sqrt(-b^2*x^4 + a^2), x)

Fricas [A] time = 1.97421, size = 490, normalized size = 4.5

$$\left[\frac{2\sqrt{-b^2x^4 + a^2}\sqrt{-bx^2 + abx} + 3(abx^2 - a^2)\sqrt{b} \log\left(\frac{2b^2x^4 - abx^2 - 2\sqrt{-b^2x^4 + a^2}\sqrt{-bx^2 + a}\sqrt{bx - a^2}}{bx^2 - a}\right)}{4(b^2x^2 - ab)}, \frac{\sqrt{-b^2x^4 + a^2}\sqrt{-bx^2 + abx} + 3(abx^2 - a^2)\sqrt{b}}{4(b^2x^2 - ab)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b*x^2+a)^(3/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="fricas")

[Out] [1/4*(2*sqrt(-b^2*x^4 + a^2)*sqrt(-b*x^2 + a)*b*x + 3*(a*b*x^2 - a^2)*sqrt(b)*log((2*b^2*x^4 - a*b*x^2 - 2*sqrt(-b^2*x^4 + a^2)*sqrt(-b*x^2 + a)*sqrt(b)*x - a^2)/(b*x^2 - a)))/(b^2*x^2 - a*b), 1/2*(sqrt(-b^2*x^4 + a^2)*sqrt(-b*x^2 + a)*b*x + 3*(a*b*x^2 - a^2)*sqrt(-b)*arctan(sqrt(-b^2*x^4 + a^2)*sqrt(-b*x^2 + a)*sqrt(-b)/(b^2*x^3 - a*b*x)))/(b^2*x^2 - a*b)]

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a - bx^2)^{\frac{3}{2}}}{\sqrt{-(-a + bx^2)(a + bx^2)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b*x**2+a)**(3/2)/(-b**2*x**4+a**2)**(1/2),x)

[Out] Integral((a - b*x**2)**(3/2)/sqrt(-(-a + b*x**2)*(a + b*x**2)), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(-bx^2 + a)^{\frac{3}{2}}}{\sqrt{-b^2x^4 + a^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b*x^2+a)^(3/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="giac")

[Out] integrate((-b*x^2 + a)^(3/2)/sqrt(-b^2*x^4 + a^2), x)

$$3.207 \quad \int \frac{\sqrt{a-bx^2}}{\sqrt{a^2-b^2x^4}} dx$$

Optimal. Leaf size=64

$$\frac{\sqrt{a-bx^2}\sqrt{a+bx^2}\tanh^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a+bx^2}}\right)}{\sqrt{b}\sqrt{a^2-b^2x^4}}$$

[Out] (Sqrt[a - b*x^2]*Sqrt[a + b*x^2]*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(Sqrt[b]*Sqrt[a^2 - b^2*x^4])

Rubi [A] time = 0.0254866, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {1152, 217, 206}

$$\frac{\sqrt{a-bx^2}\sqrt{a+bx^2}\tanh^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a+bx^2}}\right)}{\sqrt{b}\sqrt{a^2-b^2x^4}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a - b*x^2]/Sqrt[a^2 - b^2*x^4], x]

[Out] (Sqrt[a - b*x^2]*Sqrt[a + b*x^2]*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(Sqrt[b]*Sqrt[a^2 - b^2*x^4])

Rule 1152

Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist[(a + c*x^4)^FracPart[p]/((d + e*x^2)^FracPart[p]*(a/d + (c*x^2)/e)^FracPart[p]), Int[(d + e*x^2)^(p + q)*(a/d + (c*x^2)/e)^p, x], x] /; FreeQ[{a, c, d, e, p, q}, x] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p]

Rule 217

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{a-bx^2}}{\sqrt{a^2-b^2x^4}} dx &= \frac{(\sqrt{a-bx^2}\sqrt{a+bx^2}) \int \frac{1}{\sqrt{a+bx^2}} dx}{\sqrt{a^2-b^2x^4}} \\ &= \frac{(\sqrt{a-bx^2}\sqrt{a+bx^2}) \text{Subst}\left(\int \frac{1}{1-bx^2} dx, x, \frac{x}{\sqrt{a+bx^2}}\right)}{\sqrt{a^2-b^2x^4}} \\ &= \frac{\sqrt{a-bx^2}\sqrt{a+bx^2}\tanh^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a+bx^2}}\right)}{\sqrt{b}\sqrt{a^2-b^2x^4}} \end{aligned}$$

Mathematica [A] time = 0.0411674, size = 67, normalized size = 1.05

$$\frac{\log\left(\sqrt{b}\sqrt{a-bx^2}\sqrt{a^2-b^2x^4}+abx-b^2x^3\right)-\log\left(bx^2-a\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a - b*x^2]/Sqrt[a^2 - b^2*x^4], x]

[Out] (-Log[-a + b*x^2] + Log[a*b*x - b^2*x^3 + Sqrt[b]*Sqrt[a - b*x^2]*Sqrt[a^2 - b^2*x^4]])/Sqrt[b]

Maple [A] time = 0.014, size = 54, normalized size = 0.8

$$\sqrt{-b^2x^4+a^2}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)\frac{1}{\sqrt{-bx^2+a}}\frac{1}{\sqrt{bx^2+a}}\frac{1}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-b*x^2+a)^(1/2)/(-b^2*x^4+a^2)^(1/2), x)

[Out] 1/(-b*x^2+a)^(1/2)/(b*x^2+a)^(1/2)/b^(1/2)*(-b^2*x^4+a^2)^(1/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-bx^2+a}}{\sqrt{-b^2x^4+a^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b*x^2+a)^(1/2)/(-b^2*x^4+a^2)^(1/2), x, algorithm="maxima")

[Out] integrate(sqrt(-b*x^2 + a)/sqrt(-b^2*x^4 + a^2), x)

Fricas [A] time = 1.94114, size = 269, normalized size = 4.2

$$\left[\frac{\log\left(\frac{2b^2x^4-abx^2-2\sqrt{-b^2x^4+a^2}\sqrt{-bx^2+a}\sqrt{bx-a^2}}{bx^2-a}\right)}{2\sqrt{b}}, \frac{\sqrt{-b}\arctan\left(\frac{\sqrt{-b^2x^4+a^2}\sqrt{-bx^2+a}\sqrt{-b}}{b^2x^3-abx}\right)}{b} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b*x^2+a)^(1/2)/(-b^2*x^4+a^2)^(1/2), x, algorithm="fricas")

[Out] [1/2*log((2*b^2*x^4 - a*b*x^2 - 2*sqrt(-b^2*x^4 + a^2)*sqrt(-b*x^2 + a)*sqrt(b)*x - a^2)/(b*x^2 - a))/sqrt(b), sqrt(-b)*arctan(sqrt(-b^2*x^4 + a^2)*sqrt(-b*x^2 + a)*sqrt(-b)/(b^2*x^3 - a*b*x))/b]

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a - bx^2}}{\sqrt{-(-a + bx^2)(a + bx^2)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b*x**2+a)**(1/2)/(-b**2*x**4+a**2)**(1/2), x)

[Out] Integral(sqrt(a - b*x**2)/sqrt(-(-a + b*x**2)*(a + b*x**2)), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-bx^2 + a}}{\sqrt{-b^2x^4 + a^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b*x^2+a)^(1/2)/(-b^2*x^4+a^2)^(1/2), x, algorithm="giac")

[Out] integrate(sqrt(-b*x^2 + a)/sqrt(-b^2*x^4 + a^2), x)

$$3.208 \quad \int \frac{1}{\sqrt{a-bx^2}\sqrt{a^2-b^2x^4}} dx$$

Optimal. Leaf size=77

$$\frac{\sqrt{a-bx^2}\sqrt{a+bx^2} \tanh^{-1}\left(\frac{\sqrt{2}\sqrt{bx}}{\sqrt{a+bx^2}}\right)}{\sqrt{2a}\sqrt{b}\sqrt{a^2-b^2x^4}}$$

[Out] (Sqrt[a - b*x^2]*Sqrt[a + b*x^2]*ArcTanh[(Sqrt[2]*Sqrt[b]*x)/Sqrt[a + b*x^2]])/(Sqrt[2]*a*Sqrt[b]*Sqrt[a^2 - b^2*x^4])

Rubi [A] time = 0.0358898, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {1152, 377, 208}

$$\frac{\sqrt{a-bx^2}\sqrt{a+bx^2} \tanh^{-1}\left(\frac{\sqrt{2}\sqrt{bx}}{\sqrt{a+bx^2}}\right)}{\sqrt{2a}\sqrt{b}\sqrt{a^2-b^2x^4}}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[a - b*x^2]*Sqrt[a^2 - b^2*x^4]), x]

[Out] (Sqrt[a - b*x^2]*Sqrt[a + b*x^2]*ArcTanh[(Sqrt[2]*Sqrt[b]*x)/Sqrt[a + b*x^2]])/(Sqrt[2]*a*Sqrt[b]*Sqrt[a^2 - b^2*x^4])

Rule 1152

Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist[(a + c*x^4)^FracPart[p]/((d + e*x^2)^FracPart[p]*(a/d + (c*x^2)/e)^FracPart[p]), Int[(d + e*x^2)^(p + q)*(a/d + (c*x^2)/e)^p, x], x] /; FreeQ[{a, c, d, e, p, q}, x] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p]

Rule 377

Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Subst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{a-bx^2}\sqrt{a^2-b^2x^4}} dx &= \frac{\left(\sqrt{a-bx^2}\sqrt{a+bx^2}\right) \int \frac{1}{(a-bx^2)\sqrt{a+bx^2}} dx}{\sqrt{a^2-b^2x^4}} \\ &= \frac{\left(\sqrt{a-bx^2}\sqrt{a+bx^2}\right) \text{Subst}\left(\int \frac{1}{a-2abx^2} dx, x, \frac{x}{\sqrt{a+bx^2}}\right)}{\sqrt{a^2-b^2x^4}} \\ &= \frac{\sqrt{a-bx^2}\sqrt{a+bx^2} \tanh^{-1}\left(\frac{\sqrt{2}\sqrt{bx}}{\sqrt{a+bx^2}}\right)}{\sqrt{2a}\sqrt{b}\sqrt{a^2-b^2x^4}} \end{aligned}$$

Mathematica [A] time = 0.0509518, size = 77, normalized size = 1.

$$\frac{\sqrt{a^2 - b^2x^4} \tanh^{-1}\left(\frac{\sqrt{2}\sqrt{bx}}{\sqrt{a+bx^2}}\right)}{\sqrt{2a}\sqrt{b}\sqrt{a - bx^2}\sqrt{a + bx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(Sqrt[a - b*x^2]*Sqrt[a^2 - b^2*x^4]), x]

[Out] (Sqrt[a^2 - b^2*x^4]*ArcTanh[(Sqrt[2]*Sqrt[b]*x)/Sqrt[a + b*x^2]])/(Sqrt[2]*a*Sqrt[b]*Sqrt[a - b*x^2]*Sqrt[a + b*x^2])

Maple [B] time = 0.057, size = 267, normalized size = 3.5

$$\frac{1}{2bx^2 - 2a} \sqrt{-bx^2 + a} \sqrt{-b^2x^4 + a^2} \sqrt{b} \left(\sqrt{a} \sqrt{2} \ln \left(2 \frac{b \left(\sqrt{2} \sqrt{a} \sqrt{bx^2 + a} + \sqrt{abx + a} \right)}{bx - \sqrt{ab}} \right) \right) \sqrt{b} - \sqrt{a} \sqrt{2} \ln \left(2 \frac{b \left(\sqrt{2} \sqrt{a} \sqrt{bx^2 - a} + \sqrt{abx - a} \right)}{bx + \sqrt{ab}} \right) \sqrt{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(-b*x^2+a)^(1/2)/(-b^2*x^4+a^2)^(1/2), x)

[Out] 1/2*(-b*x^2+a)^(1/2)*(-b^2*x^4+a^2)^(1/2)*b^(1/2)*(a^(1/2)*2^(1/2)*ln(2*b*(2^(1/2)*a^(1/2)*(b*x^2+a)^(1/2)+(a*b)^(1/2)*x+a)/(b*x-(a*b)^(1/2)))*b^(1/2)-a^(1/2)*2^(1/2)*ln(2*b*(2^(1/2)*a^(1/2)*(b*x^2+a)^(1/2)-(a*b)^(1/2)*x+a)/(b*x+(a*b)^(1/2)))*b^(1/2)+2*ln((b^(1/2)*(-b*x+(-a*b)^(1/2))/b*(-b*x+(-a*b)^(1/2)))^(1/2)+b*x)/b^(1/2))*(a*b)^(1/2)-2*(a*b)^(1/2)*ln((b^(1/2)*(b*x^2+a)^(1/2)+b*x)/b^(1/2)))/(b*x^2-a)/(b*x^2+a)^(1/2)/((-a*b)^(1/2)+(a*b)^(1/2))/((-a*b)^(1/2)-(a*b)^(1/2))/(a*b)^(1/2)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-b^2x^4 + a^2}\sqrt{-bx^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-b*x^2+a)^(1/2)/(-b^2*x^4+a^2)^(1/2), x, algorithm="maxima")

[Out] integrate(1/(sqrt(-b^2*x^4 + a^2)*sqrt(-b*x^2 + a)), x)

Fricas [A] time = 1.99997, size = 359, normalized size = 4.66

$$\left[\frac{\sqrt{2} \log\left(-\frac{3b^2x^4 - 2abx^2 - 2\sqrt{2}\sqrt{-b^2x^4 + a^2}\sqrt{-bx^2 + a}\sqrt{bx - a^2}}{b^2x^4 - 2abx^2 + a^2}\right)}{4a\sqrt{b}}, \frac{\sqrt{2}\sqrt{-b} \arctan\left(\frac{\sqrt{2}\sqrt{-b^2x^4 + a^2}\sqrt{-bx^2 + a}\sqrt{-b}}{2(b^2x^3 - abx)}\right)}{2ab} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-b*x^2+a)^(1/2)/(-b^2*x^4+a^2)^(1/2), x, algorithm="fricas")

[Out] $\left[\frac{1}{4}\sqrt{2}\log\left(-\frac{3b^2x^4 - 2abx^2 - 2\sqrt{2}\sqrt{-b^2x^4 + a^2}\sqrt{-bx^2 + a}\sqrt{bx - a^2}}{b^2x^4 - 2abx^2 + a^2}\right) / (a\sqrt{b}), \frac{1}{2}\sqrt{2}\sqrt{-b}\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{-b^2x^4 + a^2}\sqrt{-bx^2 + a}\sqrt{-b}\right) / (b^2x^3 - abx) \right] / (ab)$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-(-a + bx^2)(a + bx^2)}\sqrt{a - bx^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-b*x**2+a)**(1/2)/(-b**2*x**4+a**2)**(1/2), x)

[Out] Integral(1/(sqrt(-(-a + b*x**2)*(a + b*x**2))*sqrt(a - b*x**2)), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-b^2x^4 + a^2}\sqrt{-bx^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-b*x^2+a)^(1/2)/(-b^2*x^4+a^2)^(1/2), x, algorithm="giac")

[Out] integrate(1/(sqrt(-b^2*x^4 + a^2)*sqrt(-b*x^2 + a)), x)

$$3.209 \quad \int \frac{1}{(a-bx^2)^{3/2} \sqrt{a^2-b^2x^4}} dx$$

Optimal. Leaf size=124

$$\frac{x(a+bx^2)}{4a^2\sqrt{a-bx^2}\sqrt{a^2-b^2x^4}} + \frac{3\sqrt{a-bx^2}\sqrt{a+bx^2} \tanh^{-1}\left(\frac{\sqrt{2}\sqrt{bx}}{\sqrt{a+bx^2}}\right)}{4\sqrt{2}a^2\sqrt{b}\sqrt{a^2-b^2x^4}}$$

[Out] (x*(a + b*x^2))/(4*a^2*Sqrt[a - b*x^2]*Sqrt[a^2 - b^2*x^4]) + (3*Sqrt[a - b*x^2]*Sqrt[a + b*x^2]*ArcTanh[(Sqrt[2]*Sqrt[b]*x)/Sqrt[a + b*x^2]])/(4*Sqrt[2]*a^2*Sqrt[b]*Sqrt[a^2 - b^2*x^4])

Rubi [A] time = 0.0538451, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {1152, 382, 377, 208}

$$\frac{x(a+bx^2)}{4a^2\sqrt{a-bx^2}\sqrt{a^2-b^2x^4}} + \frac{3\sqrt{a-bx^2}\sqrt{a+bx^2} \tanh^{-1}\left(\frac{\sqrt{2}\sqrt{bx}}{\sqrt{a+bx^2}}\right)}{4\sqrt{2}a^2\sqrt{b}\sqrt{a^2-b^2x^4}}$$

Antiderivative was successfully verified.

[In] Int[1/((a - b*x^2)^(3/2)*Sqrt[a^2 - b^2*x^4]),x]

[Out] (x*(a + b*x^2))/(4*a^2*Sqrt[a - b*x^2]*Sqrt[a^2 - b^2*x^4]) + (3*Sqrt[a - b*x^2]*Sqrt[a + b*x^2]*ArcTanh[(Sqrt[2]*Sqrt[b]*x)/Sqrt[a + b*x^2]])/(4*Sqrt[2]*a^2*Sqrt[b]*Sqrt[a^2 - b^2*x^4])

Rule 1152

Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist[(a + c*x^4)^FracPart[p]/((d + e*x^2)^FracPart[p]*(a/d + (c*x^2)/e)^FracPart[p]), Int[(d + e*x^2)^(p + q)*(a/d + (c*x^2)/e)^p, x], x] /; FreeQ[{a, c, d, e, p, q}, x] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p]

Rule 382

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] :> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c - a*d)), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)], Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] || !LtQ[q, -1]) && NeQ[p, -1]

Rule 377

Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Subst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rubi steps

$$\begin{aligned} \int \frac{1}{(a-bx^2)^{3/2} \sqrt{a^2-b^2x^4}} dx &= \frac{\left(\sqrt{a-bx^2}\sqrt{a+bx^2}\right) \int \frac{1}{(a-bx^2)^2 \sqrt{a+bx^2}} dx}{\sqrt{a^2-b^2x^4}} \\ &= \frac{x(a+bx^2)}{4a^2\sqrt{a-bx^2}\sqrt{a^2-b^2x^4}} + \frac{\left(3\sqrt{a-bx^2}\sqrt{a+bx^2}\right) \int \frac{1}{(a-bx^2)\sqrt{a+bx^2}} dx}{4a\sqrt{a^2-b^2x^4}} \\ &= \frac{x(a+bx^2)}{4a^2\sqrt{a-bx^2}\sqrt{a^2-b^2x^4}} + \frac{\left(3\sqrt{a-bx^2}\sqrt{a+bx^2}\right) \text{Subst}\left(\int \frac{1}{a-2abx^2} dx, x, \frac{x}{\sqrt{a+bx^2}}\right)}{4a\sqrt{a^2-b^2x^4}} \\ &= \frac{x(a+bx^2)}{4a^2\sqrt{a-bx^2}\sqrt{a^2-b^2x^4}} + \frac{3\sqrt{a-bx^2}\sqrt{a+bx^2} \tanh^{-1}\left(\frac{\sqrt{2}\sqrt{bx}}{\sqrt{a+bx^2}}\right)}{4\sqrt{2}a^2\sqrt{b}\sqrt{a^2-b^2x^4}} \end{aligned}$$

Mathematica [A] time = 0.083406, size = 110, normalized size = 0.89

$$\frac{\sqrt{a^2-b^2x^4} \left(2\sqrt{bx}\sqrt{a+bx^2} + 3\sqrt{2}(a-bx^2) \tanh^{-1}\left(\frac{\sqrt{2}\sqrt{bx}}{\sqrt{a+bx^2}}\right)\right)}{8a^2\sqrt{b}(a-bx^2)^{3/2}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/((a - b*x^2)^(3/2)*Sqrt[a^2 - b^2*x^4]), x]

[Out] (Sqrt[a^2 - b^2*x^4]*(2*Sqrt[b]*x*Sqrt[a + b*x^2] + 3*Sqrt[2]*(a - b*x^2)*ArcTanh[(Sqrt[2]*Sqrt[b]*x)/Sqrt[a + b*x^2]]))/(8*a^2*Sqrt[b]*(a - b*x^2)^(3/2)*Sqrt[a + b*x^2])

Maple [B] time = 0.041, size = 510, normalized size = 4.1

$$-\frac{1}{4bx^2-4a}\sqrt{-bx^2+a}\sqrt{-b^2x^4+a^2b^2} \left(3 \ln \left(2 \frac{b\left(\sqrt{2}\sqrt{a}\sqrt{bx^2+a} + \sqrt{ab}x + a\right)}{bx - \sqrt{ab}}\right)\right) \sqrt{2}x^2b^{3/2}\sqrt{a} - 3 \ln \left(2 \frac{b\left(\sqrt{2}\sqrt{a}\sqrt{bx^2+a} + \sqrt{ab}x + a\right)}{bx + \sqrt{ab}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((-b*x^2+a)^(3/2)/(-b^2*x^4+a^2)^(1/2)), x)

[Out] -1/4*(-b*x^2+a)^(1/2)*(-b^2*x^4+a^2)^(1/2)*b^(5/2)*(3*ln(2*b*(2^(1/2)*a^(1/2)*(b*x^2+a)^(1/2)+(a*b)^(1/2)*x+a)/(b*x-(a*b)^(1/2)))*2^(1/2)*x^2*b^(3/2)*a^(1/2)-3*ln(2*b*(2^(1/2)*a^(1/2)*(b*x^2+a)^(1/2)-(a*b)^(1/2)*x+a)/(b*x+(a*b)^(1/2)))*2^(1/2)*x^2*b^(3/2)*a^(1/2)-3*ln(2*b*(2^(1/2)*a^(1/2)*(b*x^2+a)^(1/2)+(a*b)^(1/2)*x+a)/(b*x-(a*b)^(1/2)))*2^(1/2)*a^(3/2)*b^(1/2)+3*ln(2*b*(2^(1/2)*a^(1/2)*(b*x^2+a)^(1/2)-(a*b)^(1/2)*x+a)/(b*x+(a*b)^(1/2)))*2^(1/2)*a^(3/2)*b^(1/2)+4*ln((b^(1/2)*(-b*x+(-a*b)^(1/2))/b*(-b*x+(-a*b)^(1/2)))^(1/2)+b*x)/b^(1/2))*x^2*b*(a*b)^(1/2)-4*ln((b^(1/2)*(b*x^2+a)^(1/2)+b*x)/b^(1/2))*x^2*b*(a*b)^(1/2)-4*b^(1/2)*(a*b)^(1/2)*(b*x^2+a)^(1/2)*x-4*ln((b^(1/2)*(-b*x+(-a*b)^(1/2))/b*(-b*x+(-a*b)^(1/2)))^(1/2)+b*x)/b^(1/2))*a*(a*b)^(1/2)+4*ln((b^(1/2)*(b*x^2+a)^(1/2)+b*x)/b^(1/2))*a*(a*b)^(1/2)/(b*x^2-a)/(b*x^2+a)^(1/2)/((-a*b)^(1/2)+(a*b)^(1/2))^2/((-a*b)^(1/2)-(a*b)^(1/2))^2/(b*x+(a*b)^(1/2))/(b*x-(a*b)^(1/2))/(a*b)^(1/2)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-b^2x^4 + a^2}(-bx^2 + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-b*x^2+a)^(3/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="maxima")

[Out] integrate(1/(sqrt(-b^2*x^4 + a^2)*(-b*x^2 + a)^(3/2)), x)

Fricas [A] time = 2.35277, size = 660, normalized size = 5.32

$$\left[\frac{4\sqrt{-b^2x^4 + a^2}\sqrt{-bx^2 + a}bx + 3\sqrt{2}(b^2x^4 - 2abx^2 + a^2)\sqrt{b}\log\left(-\frac{3b^2x^4 - 2abx^2 - 2\sqrt{2}\sqrt{-b^2x^4 + a^2}\sqrt{-bx^2 + a}\sqrt{bx - a^2}}{b^2x^4 - 2abx^2 + a^2}\right)}{16(a^2b^3x^4 - 2a^3b^2x^2 + a^4b)}, \frac{2\sqrt{-b^2x^4 + a^2}}{16(a^2b^3x^4 - 2a^3b^2x^2 + a^4b)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-b*x^2+a)^(3/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="fricas")

[Out] [1/16*(4*sqrt(-b^2*x^4 + a^2)*sqrt(-b*x^2 + a)*b*x + 3*sqrt(2)*(b^2*x^4 - 2*a*b*x^2 + a^2)*sqrt(b)*log(-(3*b^2*x^4 - 2*a*b*x^2 - 2*sqrt(2)*sqrt(-b^2*x^4 + a^2)*sqrt(-b*x^2 + a)*sqrt(b)*x - a^2)/(b^2*x^4 - 2*a*b*x^2 + a^2)))/(a^2*b^3*x^4 - 2*a^3*b^2*x^2 + a^4*b), 1/8*(2*sqrt(-b^2*x^4 + a^2)*sqrt(-b*x^2 + a)*b*x + 3*sqrt(2)*(b^2*x^4 - 2*a*b*x^2 + a^2)*sqrt(-b)*arctan(1/2*sqrt(2)*sqrt(-b^2*x^4 + a^2)*sqrt(-b*x^2 + a)*sqrt(-b)/(b^2*x^3 - a*b*x)))/(a^2*b^3*x^4 - 2*a^3*b^2*x^2 + a^4*b)]

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-(-a + bx^2)(a + bx^2)}(a - bx^2)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-b*x**2+a)**(3/2)/(-b**2*x**4+a**2)**(1/2),x)

[Out] Integral(1/(sqrt(-(-a + b*x**2)*(a + b*x**2))*(a - b*x**2)**(3/2)), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-b^2x^4 + a^2}(-bx^2 + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^(3/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-b^2*x^4 + a^2)*(-b*x^2 + a)^(3/2)), x)
```

$$3.210 \quad \int \frac{1}{(a-bx^2)^{5/2} \sqrt{a^2-b^2x^4}} dx$$

Optimal. Leaf size=167

$$\frac{9x(a+bx^2)}{32a^3\sqrt{a-bx^2}\sqrt{a^2-b^2x^4}} + \frac{x(a+bx^2)}{8a^2(a-bx^2)^{3/2}\sqrt{a^2-b^2x^4}} + \frac{19\sqrt{a-bx^2}\sqrt{a+bx^2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{bx}}{\sqrt{a+bx^2}}\right)}{32\sqrt{2}a^3\sqrt{b}\sqrt{a^2-b^2x^4}}$$

[Out] (x*(a + b*x^2))/(8*a^2*(a - b*x^2)^(3/2)*Sqrt[a^2 - b^2*x^4]) + (9*x*(a + b*x^2))/(32*a^3*Sqrt[a - b*x^2]*Sqrt[a^2 - b^2*x^4]) + (19*Sqrt[a - b*x^2]*Sqrt[a + b*x^2]*ArcTanh[(Sqrt[2]*Sqrt[b]*x)/Sqrt[a + b*x^2]])/(32*Sqrt[2]*a^3*Sqrt[b]*Sqrt[a^2 - b^2*x^4])

Rubi [A] time = 0.0873148, antiderivative size = 167, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {1152, 414, 527, 12, 377, 208}

$$\frac{9x(a+bx^2)}{32a^3\sqrt{a-bx^2}\sqrt{a^2-b^2x^4}} + \frac{x(a+bx^2)}{8a^2(a-bx^2)^{3/2}\sqrt{a^2-b^2x^4}} + \frac{19\sqrt{a-bx^2}\sqrt{a+bx^2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{bx}}{\sqrt{a+bx^2}}\right)}{32\sqrt{2}a^3\sqrt{b}\sqrt{a^2-b^2x^4}}$$

Antiderivative was successfully verified.

[In] Int[1/((a - b*x^2)^(5/2)*Sqrt[a^2 - b^2*x^4]),x]

[Out] (x*(a + b*x^2))/(8*a^2*(a - b*x^2)^(3/2)*Sqrt[a^2 - b^2*x^4]) + (9*x*(a + b*x^2))/(32*a^3*Sqrt[a - b*x^2]*Sqrt[a^2 - b^2*x^4]) + (19*Sqrt[a - b*x^2]*Sqrt[a + b*x^2]*ArcTanh[(Sqrt[2]*Sqrt[b]*x)/Sqrt[a + b*x^2]])/(32*Sqrt[2]*a^3*Sqrt[b]*Sqrt[a^2 - b^2*x^4])

Rule 1152

Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (c_.)*(x_)^(4))^(p_), x_Symbol] :> Dist[(a + c*x^4)^FracPart[p]/((d + e*x^2)^FracPart[p]*(a/d + (c*x^2)/e)^FracPart[p]), Int[(d + e*x^2)^(p + q)*(a/d + (c*x^2)/e)^p, x], x] /; FreeQ[{a, c, d, e, p, q}, x] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p]

Rule 414

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] :> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c - a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && !(!IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, n, p, q, x]

Rule 527

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ

[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 377

Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Subst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rubi steps

$$\begin{aligned} \int \frac{1}{(a-bx^2)^{5/2} \sqrt{a^2-b^2x^4}} dx &= \frac{\left(\sqrt{a-bx^2}\sqrt{a+bx^2}\right) \int \frac{1}{(a-bx^2)^3 \sqrt{a+bx^2}} dx}{\sqrt{a^2-b^2x^4}} \\ &= \frac{x(a+bx^2)}{8a^2(a-bx^2)^{3/2} \sqrt{a^2-b^2x^4}} + \frac{\left(\sqrt{a-bx^2}\sqrt{a+bx^2}\right) \int \frac{7ab+2b^2x^2}{(a-bx^2)^2 \sqrt{a+bx^2}} dx}{8a^2b\sqrt{a^2-b^2x^4}} \\ &= \frac{x(a+bx^2)}{8a^2(a-bx^2)^{3/2} \sqrt{a^2-b^2x^4}} + \frac{9x(a+bx^2)}{32a^3\sqrt{a-bx^2}\sqrt{a^2-b^2x^4}} + \frac{\left(\sqrt{a-bx^2}\sqrt{a+bx^2}\right) \int \frac{19a}{(a-bx^2)} dx}{32a^4b^2\sqrt{a^2-b^2x^4}} \\ &= \frac{x(a+bx^2)}{8a^2(a-bx^2)^{3/2} \sqrt{a^2-b^2x^4}} + \frac{9x(a+bx^2)}{32a^3\sqrt{a-bx^2}\sqrt{a^2-b^2x^4}} + \frac{\left(19\sqrt{a-bx^2}\sqrt{a+bx^2}\right) \int \frac{19a}{(a-bx^2)} dx}{32a^2\sqrt{a^2-b^2x^4}} \\ &= \frac{x(a+bx^2)}{8a^2(a-bx^2)^{3/2} \sqrt{a^2-b^2x^4}} + \frac{9x(a+bx^2)}{32a^3\sqrt{a-bx^2}\sqrt{a^2-b^2x^4}} + \frac{\left(19\sqrt{a-bx^2}\sqrt{a+bx^2}\right) \text{Subst}}{32a^2\sqrt{a^2-b^2x^4}} \\ &= \frac{x(a+bx^2)}{8a^2(a-bx^2)^{3/2} \sqrt{a^2-b^2x^4}} + \frac{9x(a+bx^2)}{32a^3\sqrt{a-bx^2}\sqrt{a^2-b^2x^4}} + \frac{19\sqrt{a-bx^2}\sqrt{a+bx^2} \tanh^{-1}\left(\frac{\sqrt{2}\sqrt{bx}}{\sqrt{a+bx^2}}\right)}{32\sqrt{2}a^3\sqrt{b}\sqrt{a^2-b^2x^4}} \end{aligned}$$

Mathematica [A] time = 0.10748, size = 122, normalized size = 0.73

$$\frac{\sqrt{a^2-b^2x^4} \left(2\sqrt{bx} (13a-9bx^2) \sqrt{a+bx^2} + 19\sqrt{2} (a-bx^2)^2 \tanh^{-1} \left(\frac{\sqrt{2}\sqrt{bx}}{\sqrt{a+bx^2}} \right) \right)}{64a^3\sqrt{b} (a-bx^2)^{5/2} \sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/((a - b*x^2)^(5/2)*Sqrt[a^2 - b^2*x^4]), x]

[Out] (Sqrt[a^2 - b^2*x^4]*(2*Sqrt[b]*x*(13*a - 9*b*x^2)*Sqrt[a + b*x^2] + 19*Sqrt[2]*(a - b*x^2)^2*ArcTanh[(Sqrt[2]*Sqrt[b]*x)/Sqrt[a + b*x^2]])/(64*a^3*Sqrt[b]*(a - b*x^2)^(5/2)*Sqrt[a + b*x^2])

Maple [B] time = 0.05, size = 739, normalized size = 4.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\int (1/(-b*x^2+a)^{(5/2)} / (-b^2*x^4+a^2)^{(1/2)}, x)$

[Out] $\frac{1}{16}(-b*x^2+a)^{(1/2)}*(-b^2*x^4+a^2)^{(1/2)}*b^{(9/2)}*(-19*\ln(2*b*(2^{(1/2)}*a^{(1/2)}*(b*x^2+a)^{(1/2)}-(a*b)^{(1/2)}*x+a)/(b*x+(a*b)^{(1/2)}))^{(1/2)}*x^4*b^{(5/2)}*a^{(1/2)}+19*\ln(2*b*(2^{(1/2)}*a^{(1/2)}*(b*x^2+a)^{(1/2)}+(a*b)^{(1/2)}*x+a)/(b*x-(a*b)^{(1/2)}))^{(1/2)}*x^4*b^{(5/2)}*a^{(1/2)}+16*\ln((b^{(1/2)}*(-(b*x+(-a*b)^{(1/2)})))/b*(-b*x+(-a*b)^{(1/2)}))^{(1/2)}+b*x)/b^{(1/2)}*x^4*b^2*(a*b)^{(1/2)}+38*\ln(2*b*(2^{(1/2)}*a^{(1/2)}*(b*x^2+a)^{(1/2)}-(a*b)^{(1/2)}*x+a)/(b*x+(a*b)^{(1/2)}))^{(1/2)}*x^2*a^{(3/2)}*b^{(3/2)}-38*\ln(2*b*(2^{(1/2)}*a^{(1/2)}*(b*x^2+a)^{(1/2)}+(a*b)^{(1/2)}*x+a)/(b*x-(a*b)^{(1/2)}))^{(1/2)}*x^2*a^{(3/2)}*b^{(3/2)}-16*\ln((b^{(1/2)}*(b*x^2+a)^{(1/2)}+b*x)/b^{(1/2)}*x^4*b^2*(a*b)^{(1/2)}-36*b^{(3/2)}*(a*b)^{(1/2)}*(b*x^2+a)^{(1/2)}*x^3-32*\ln((b^{(1/2)}*(-(b*x+(-a*b)^{(1/2)})))/b*(-b*x+(-a*b)^{(1/2)}))^{(1/2)}+b*x)/b^{(1/2)}*x^2*a*b*(a*b)^{(1/2)}-19*\ln(2*b*(2^{(1/2)}*a^{(1/2)}*(b*x^2+a)^{(1/2)}-(a*b)^{(1/2)}*x+a)/(b*x+(a*b)^{(1/2)}))^{(1/2)}*a^{(5/2)}*b^{(1/2)}+19*\ln(2*b*(2^{(1/2)}*a^{(1/2)}*(b*x^2+a)^{(1/2)}+(a*b)^{(1/2)}*x+a)/(b*x-(a*b)^{(1/2)}))^{(1/2)}*a^{(5/2)}*b^{(1/2)}+32*\ln((b^{(1/2)}*(b*x^2+a)^{(1/2)}+b*x)/b^{(1/2)}*x^2*a*b*(a*b)^{(1/2)}+52*b^{(1/2)}*a*(a*b)^{(1/2)}*(b*x^2+a)^{(1/2)}*x+16*\ln((b^{(1/2)}*(-(b*x+(-a*b)^{(1/2)})))/b*(-b*x+(-a*b)^{(1/2)}))^{(1/2)}+b*x)/b^{(1/2)}*a^2*(a*b)^{(1/2)}-16*\ln((b^{(1/2)}*(b*x^2+a)^{(1/2)}+b*x)/b^{(1/2)}*a^2*(a*b)^{(1/2)})/(b*x^2-a)/(b*x^2+a)^{(1/2)}/((-a*b)^{(1/2)}+(a*b)^{(1/2)})^3/((-a*b)^{(1/2)}-(a*b)^{(1/2)})^3/(b*x-(a*b)^{(1/2)})^2/(a*b)^{(1/2)}/(b*x+(a*b)^{(1/2)})^2$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-b^2x^4 + a^2}(-bx^2 + a)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(1/(-b*x^2+a)^{(5/2)} / (-b^2*x^4+a^2)^{(1/2)}, x, \text{algorithm}="maxima")$

[Out] $\text{integrate}(1/(\text{sqrt}(-b^2*x^4 + a^2)*(-b*x^2 + a)^{(5/2)}), x)$

Fricas [A] time = 2.39237, size = 803, normalized size = 4.81

$$\frac{19\sqrt{2}(b^3x^6 - 3ab^2x^4 + 3a^2bx^2 - a^3)\sqrt{b}\log\left(\frac{-3b^2x^4 - 2abx^2 - 2\sqrt{2}\sqrt{-b^2x^4+a^2}\sqrt{-bx^2+a}\sqrt{bx-a^2}}{b^2x^4 - 2abx^2 + a^2}\right) + 4\sqrt{-b^2x^4 + a^2}(9b^2x^3 - 13abx + a^2)}{128(a^3b^4x^6 - 3a^4b^3x^4 + 3a^5b^2x^2 - a^6b)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(1/(-b*x^2+a)^{(5/2)} / (-b^2*x^4+a^2)^{(1/2)}, x, \text{algorithm}="fricas")$

[Out] $[1/128*(19*\text{sqrt}(2))*(b^3*x^6 - 3*a*b^2*x^4 + 3*a^2*b*x^2 - a^3)*\text{sqrt}(b)*\log(-3*b^2*x^4 - 2*a*b*x^2 - 2*\text{sqrt}(2)*\text{sqrt}(-b^2*x^4 + a^2)*\text{sqrt}(-b*x^2 + a)*\text{sqrt}(b) + 4*\text{sqrt}(-b^2*x^4 + a^2)*(9*b^2*x^3 - 13*abx + a^2))$

```

qrt(b)*x - a^2)/(b^2*x^4 - 2*a*b*x^2 + a^2)) + 4*sqrt(-b^2*x^4 + a^2)*(9*b^
2*x^3 - 13*a*b*x)*sqrt(-b*x^2 + a))/(a^3*b^4*x^6 - 3*a^4*b^3*x^4 + 3*a^5*b^
2*x^2 - a^6*b), 1/64*(19*sqrt(2)*(b^3*x^6 - 3*a*b^2*x^4 + 3*a^2*b*x^2 - a^3
)*sqrt(-b)*arctan(1/2*sqrt(2)*sqrt(-b^2*x^4 + a^2)*sqrt(-b*x^2 + a)*sqrt(-b
)/(b^2*x^3 - a*b*x)) + 2*sqrt(-b^2*x^4 + a^2)*(9*b^2*x^3 - 13*a*b*x)*sqrt(-
b*x^2 + a))/(a^3*b^4*x^6 - 3*a^4*b^3*x^4 + 3*a^5*b^2*x^2 - a^6*b)]

```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-(-a + bx^2)(a + bx^2)}(a - bx^2)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x**2+a)**(5/2)/(-b**2*x**4+a**2)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-(-a + b*x**2)*(a + b*x**2))*(a - b*x**2)**(5/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-b^2x^4 + a^2}(-bx^2 + a)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^(5/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-b^2*x^4 + a^2)*(-b*x^2 + a)^(5/2)), x)
```

$$3.211 \quad \int \frac{\sqrt{-1+x^2}}{\sqrt{-1+x^4}} dx$$

Optimal. Leaf size=30

$$\frac{\sqrt{x^2-1}\sqrt{x^2+1}\sinh^{-1}(x)}{\sqrt{x^4-1}}$$

[Out] (Sqrt[-1 + x^2]*Sqrt[1 + x^2]*ArcSinh[x])/Sqrt[-1 + x^4]

Rubi [A] time = 0.0100729, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {1152, 215}

$$\frac{\sqrt{x^2-1}\sqrt{x^2+1}\sinh^{-1}(x)}{\sqrt{x^4-1}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[-1 + x^2]/Sqrt[-1 + x^4], x]

[Out] (Sqrt[-1 + x^2]*Sqrt[1 + x^2]*ArcSinh[x])/Sqrt[-1 + x^4]

Rule 1152

Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] := Dist[(a + c*x^4)^FracPart[p]/((d + e*x^2)^FracPart[p]*(a/d + (c*x^2)/e)^FracPart[p]), Int[(d + e*x^2)^(p + q)*(a/d + (c*x^2)/e)^p, x], x] /; FreeQ[{a, c, d, e, p, q}, x] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p]

Rule 215

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSinh[(Rt[b, 2]*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{-1+x^2}}{\sqrt{-1+x^4}} dx &= \frac{\left(\sqrt{-1+x^2}\sqrt{1+x^2}\right) \int \frac{1}{\sqrt{1+x^2}} dx}{\sqrt{-1+x^4}} \\ &= \frac{\sqrt{-1+x^2}\sqrt{1+x^2}\sinh^{-1}(x)}{\sqrt{-1+x^4}} \end{aligned}$$

Mathematica [A] time = 0.0226663, size = 38, normalized size = 1.27

$$\log\left(x^3 + \sqrt{x^2-1}\sqrt{x^4-1} - x\right) - \log(1-x^2)$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[-1 + x^2]/Sqrt[-1 + x^4], x]

[Out] $-\text{Log}[1 - x^2] + \text{Log}[-x + x^3 + \text{Sqrt}[-1 + x^2]*\text{Sqrt}[-1 + x^4]]$

Maple [A] time = 0.01, size = 25, normalized size = 0.8

$$\text{Arcsinh}(x) \sqrt{x^4 - 1} \frac{1}{\sqrt{x^2 - 1}} \frac{1}{\sqrt{x^2 + 1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((x^2-1)^{(1/2)}/(x^4-1)^{(1/2)},x)$

[Out] $1/(x^2-1)^{(1/2)}*(x^4-1)^{(1/2)}/(x^2+1)^{(1/2)}*\text{arcsinh}(x)$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2 - 1}}{\sqrt{x^4 - 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}((x^2-1)^{(1/2)}/(x^4-1)^{(1/2)},x, \text{algorithm}=\text{"maxima"})$

[Out] $\text{integrate}(\text{sqrt}(x^2 - 1)/\text{sqrt}(x^4 - 1), x)$

Fricas [B] time = 2.21912, size = 165, normalized size = 5.5

$$\frac{1}{2} \log\left(\frac{x^3 + \sqrt{x^4 - 1}\sqrt{x^2 - 1} - x}{x^3 - x}\right) - \frac{1}{2} \log\left(-\frac{x^3 - \sqrt{x^4 - 1}\sqrt{x^2 - 1} - x}{x^3 - x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}((x^2-1)^{(1/2)}/(x^4-1)^{(1/2)},x, \text{algorithm}=\text{"fricas"})$

[Out] $1/2*\log((x^3 + \text{sqrt}(x^4 - 1)*\text{sqrt}(x^2 - 1) - x)/(x^3 - x)) - 1/2*\log(-(x^3 - \text{sqrt}(x^4 - 1)*\text{sqrt}(x^2 - 1) - x)/(x^3 - x))$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{(x-1)(x+1)}}{\sqrt{(x-1)(x+1)(x^2+1)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}((x**2-1)**(1/2)/(x**4-1)**(1/2),x)$

[Out] $\text{Integral}(\text{sqrt}((x - 1)*(x + 1))/\text{sqrt}((x - 1)*(x + 1)*(x**2 + 1)), x)$

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2 - 1}}{\sqrt{x^4 - 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)^(1/2)/(x^4-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^2 - 1)/sqrt(x^4 - 1), x)
```

$$3.212 \quad \int \frac{\sqrt{1+x^2}}{\sqrt{-1+x^4}} dx$$

Optimal. Leaf size=24

$$-\frac{\sqrt{x^4-1} \sin^{-1}(x)}{\sqrt{1-x^4}}$$

[Out] -((Sqrt[-1 + x^4]*ArcSin[x])/Sqrt[1 - x^4])

Rubi [A] time = 0.0105591, antiderivative size = 40, normalized size of antiderivative = 1.67, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {1152, 217, 206}

$$\frac{\sqrt{x^2-1}\sqrt{x^2+1} \tanh^{-1}\left(\frac{x}{\sqrt{x^2-1}}\right)}{\sqrt{x^4-1}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[1 + x^2]/Sqrt[-1 + x^4], x]

[Out] (Sqrt[-1 + x^2]*Sqrt[1 + x^2]*ArcTanh[x/Sqrt[-1 + x^2]])/Sqrt[-1 + x^4]

Rule 1152

Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist[(a + c*x^4)^FracPart[p]/((d + e*x^2)^FracPart[p]*(a/d + (c*x^2)/e)^FracPart[p]), Int[(d + e*x^2)^(p + q)*(a/d + (c*x^2)/e)^p, x], x] /; FreeQ[{a, c, d, e, p, q}, x] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p]

Rule 217

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{1+x^2}}{\sqrt{-1+x^4}} dx &= \frac{\left(\sqrt{-1+x^2}\sqrt{1+x^2}\right) \int \frac{1}{\sqrt{-1+x^2}} dx}{\sqrt{-1+x^4}} \\ &= \frac{\left(\sqrt{-1+x^2}\sqrt{1+x^2}\right) \text{Subst}\left(\int \frac{1}{1-x^2} dx, x, \frac{x}{\sqrt{-1+x^2}}\right)}{\sqrt{-1+x^4}} \\ &= \frac{\sqrt{-1+x^2}\sqrt{1+x^2} \tanh^{-1}\left(\frac{x}{\sqrt{-1+x^2}}\right)}{\sqrt{-1+x^4}} \end{aligned}$$

Mathematica [A] time = 0.0201424, size = 34, normalized size = 1.42

$$\log\left(x^3 + \sqrt{x^2 + 1}\sqrt{x^4 - 1} + x\right) - \log(x^2 + 1)$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[1 + x^2]/Sqrt[-1 + x^4], x]

[Out] -Log[1 + x^2] + Log[x + x^3 + Sqrt[1 + x^2]*Sqrt[-1 + x^4]]

Maple [A] time = 0.01, size = 33, normalized size = 1.4

$$\sqrt{x^4 - 1} \ln\left(x + \sqrt{x^2 - 1}\right) \frac{1}{\sqrt{x^2 + 1}} \frac{1}{\sqrt{x^2 - 1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2+1)^(1/2)/(x^4-1)^(1/2), x)

[Out] 1/(x^2+1)^(1/2)*(x^4-1)^(1/2)/(x^2-1)^(1/2)*ln(x+(x^2-1)^(1/2))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2 + 1}}{\sqrt{x^4 - 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+1)^(1/2)/(x^4-1)^(1/2), x, algorithm="maxima")

[Out] integrate(sqrt(x^2 + 1)/sqrt(x^4 - 1), x)

Fricas [B] time = 2.08377, size = 165, normalized size = 6.88

$$\frac{1}{2} \log\left(\frac{x^3 + \sqrt{x^4 - 1}\sqrt{x^2 + 1} + x}{x^3 + x}\right) - \frac{1}{2} \log\left(-\frac{x^3 - \sqrt{x^4 - 1}\sqrt{x^2 + 1} + x}{x^3 + x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+1)^(1/2)/(x^4-1)^(1/2), x, algorithm="fricas")

[Out] 1/2*log((x^3 + sqrt(x^4 - 1)*sqrt(x^2 + 1) + x)/(x^3 + x)) - 1/2*log(-(x^3 - sqrt(x^4 - 1)*sqrt(x^2 + 1) + x)/(x^3 + x))

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2 + 1}}{\sqrt{(x - 1)(x + 1)(x^2 + 1)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**2+1)**(1/2)/(x**4-1)**(1/2),x)

[Out] Integral(sqrt(x**2 + 1)/sqrt((x - 1)*(x + 1)*(x**2 + 1)), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2 + 1}}{\sqrt{x^4 - 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+1)^(1/2)/(x^4-1)^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(x^2 + 1)/sqrt(x^4 - 1), x)

$$3.213 \quad \int \frac{-\sqrt{-1+x^2} + \sqrt{1+x^2}}{\sqrt{-1+x^4}} dx$$

Optimal. Leaf size=73

$$\frac{\sqrt{x^2-1}\sqrt{x^4-1}\sinh^{-1}(x)}{(1-x^2)\sqrt{x^2+1}} - \frac{\sqrt{x^4-1}\sin^{-1}(x)}{\sqrt{1-x^2}\sqrt{x^2+1}}$$

[Out] -((Sqrt[-1 + x^4]*ArcSin[x])/(Sqrt[1 - x^2]*Sqrt[1 + x^2])) + (Sqrt[-1 + x^2]*Sqrt[-1 + x^4]*ArcSinh[x])/((1 - x^2)*Sqrt[1 + x^2])

Rubi [A] time = 0.117558, antiderivative size = 72, normalized size of antiderivative = 0.99, number of steps used = 7, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {6742, 1152, 215, 217, 206}

$$\frac{\sqrt{x^2-1}\sqrt{x^2+1}\tanh^{-1}\left(\frac{x}{\sqrt{x^2-1}}\right)}{\sqrt{x^4-1}} - \frac{\sqrt{x^2-1}\sqrt{x^2+1}\sinh^{-1}(x)}{\sqrt{x^4-1}}$$

Antiderivative was successfully verified.

[In] Int[(-Sqrt[-1 + x^2] + Sqrt[1 + x^2])/Sqrt[-1 + x^4], x]

[Out] -((Sqrt[-1 + x^2]*Sqrt[1 + x^2]*ArcSinh[x])/Sqrt[-1 + x^4]) + (Sqrt[-1 + x^2]*Sqrt[1 + x^2]*ArcTanh[x/Sqrt[-1 + x^2]])/Sqrt[-1 + x^4]

Rule 6742

Int[u_, x_Symbol] :=> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]

Rule 1152

Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :=> Dist[(a + c*x^4)^FracPart[p]/((d + e*x^2)^FracPart[p]*(a/d + (c*x^2)/e)^FracPart[p]), Int[(d + e*x^2)^(p + q)*(a/d + (c*x^2)/e)^p, x], x] /; FreeQ[{a, c, d, e, p, q}, x] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p]

Rule 215

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :=> Simp[ArcSinh[(Rt[b, 2]*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

Rule 217

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :=> Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :=> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned}
\int \frac{-\sqrt{-1+x^2} + \sqrt{1+x^2}}{\sqrt{-1+x^4}} dx &= \int \left(-\frac{\sqrt{-1+x^2}}{\sqrt{-1+x^4}} + \frac{\sqrt{1+x^2}}{\sqrt{-1+x^4}} \right) dx \\
&= -\int \frac{\sqrt{-1+x^2}}{\sqrt{-1+x^4}} dx + \int \frac{\sqrt{1+x^2}}{\sqrt{-1+x^4}} dx \\
&= \frac{(\sqrt{-1+x^2}\sqrt{1+x^2}) \int \frac{1}{\sqrt{-1+x^2}} dx}{\sqrt{-1+x^4}} - \frac{(\sqrt{-1+x^2}\sqrt{1+x^2}) \int \frac{1}{\sqrt{1+x^2}} dx}{\sqrt{-1+x^4}} \\
&= -\frac{\sqrt{-1+x^2}\sqrt{1+x^2} \sinh^{-1}(x)}{\sqrt{-1+x^4}} + \frac{(\sqrt{-1+x^2}\sqrt{1+x^2}) \operatorname{Subst}\left(\int \frac{1}{1-x^2} dx, x, \frac{x}{\sqrt{-1+x^2}}\right)}{\sqrt{-1+x^4}} \\
&= -\frac{\sqrt{-1+x^2}\sqrt{1+x^2} \sinh^{-1}(x)}{\sqrt{-1+x^4}} + \frac{\sqrt{-1+x^2}\sqrt{1+x^2} \tanh^{-1}\left(\frac{x}{\sqrt{-1+x^2}}\right)}{\sqrt{-1+x^4}}
\end{aligned}$$

Mathematica [A] time = 0.0573332, size = 71, normalized size = 0.97

$$\log(1-x^2) - \log(x^2+1) - \log(x^3 + \sqrt{x^2-1}\sqrt{x^4-1} - x) + \log(x^3 + \sqrt{x^2+1}\sqrt{x^4-1} + x)$$

Antiderivative was successfully verified.

[In] Integrate[(-Sqrt[-1 + x^2] + Sqrt[1 + x^2])/Sqrt[-1 + x^4], x]

[Out] Log[1 - x^2] - Log[1 + x^2] - Log[-x + x^3 + Sqrt[-1 + x^2]*Sqrt[-1 + x^4]] + Log[x + x^3 + Sqrt[1 + x^2]*Sqrt[-1 + x^4]]

Maple [A] time = 0.004, size = 59, normalized size = 0.8

$$-\operatorname{Arcsinh}(x) \sqrt{x^4-1} \frac{1}{\sqrt{x^2-1}} \frac{1}{\sqrt{x^2+1}} + \sqrt{x^4-1} \ln\left(x + \sqrt{x^2-1}\right) \frac{1}{\sqrt{x^2-1}} \frac{1}{\sqrt{x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((- (x^2-1)^(1/2) + (x^2+1)^(1/2)) / (x^4-1)^(1/2), x)

[Out] -1/(x^2-1)^(1/2) * (x^4-1)^(1/2) / (x^2+1)^(1/2) * arcsinh(x) + 1/(x^2+1)^(1/2) * (x^4-1)^(1/2) / (x^2-1)^(1/2) * ln(x + (x^2-1)^(1/2))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2+1} - \sqrt{x^2-1}}{\sqrt{x^4-1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((- (x^2-1)^(1/2) + (x^2+1)^(1/2)) / (x^4-1)^(1/2), x, algorithm="maxima")

[Out] integrate((sqrt(x^2 + 1) - sqrt(x^2 - 1)) / sqrt(x^4 - 1), x)

Fricas [B] time = 1.9218, size = 331, normalized size = 4.53

$$\frac{1}{2} \log\left(\frac{x^3 + \sqrt{x^4 - 1}\sqrt{x^2 + 1} + x}{x^3 + x}\right) - \frac{1}{2} \log\left(-\frac{x^3 - \sqrt{x^4 - 1}\sqrt{x^2 + 1} + x}{x^3 + x}\right) - \frac{1}{2} \log\left(\frac{x^3 + \sqrt{x^4 - 1}\sqrt{x^2 - 1} - x}{x^3 - x}\right) + \frac{1}{2} \log\left(-\frac{x^3 - \sqrt{x^4 - 1}\sqrt{x^2 - 1} - x}{x^3 - x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2-1)^(1/2)+(x^2+1)^(1/2))/(x^4-1)^(1/2),x, algorithm="fricas")

[Out] 1/2*log((x^3 + sqrt(x^4 - 1)*sqrt(x^2 + 1) + x)/(x^3 + x)) - 1/2*log(-(x^3 - sqrt(x^4 - 1)*sqrt(x^2 + 1) + x)/(x^3 + x)) - 1/2*log((x^3 + sqrt(x^4 - 1)*sqrt(x^2 - 1) - x)/(x^3 - x)) + 1/2*log(-(x^3 - sqrt(x^4 - 1)*sqrt(x^2 - 1) - x)/(x^3 - x))

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{-\sqrt{x^2 - 1} + \sqrt{x^2 + 1}}{\sqrt{(x - 1)(x + 1)(x^2 + 1)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x**2-1)**(1/2)+(x**2+1)**(1/2))/(x**4-1)**(1/2),x)

[Out] Integral((-sqrt(x**2 - 1) + sqrt(x**2 + 1))/sqrt((x - 1)*(x + 1)*(x**2 + 1)), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2 + 1} - \sqrt{x^2 - 1}}{\sqrt{x^4 - 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2-1)^(1/2)+(x^2+1)^(1/2))/(x^4-1)^(1/2),x, algorithm="giac")

[Out] integrate((sqrt(x^2 + 1) - sqrt(x^2 - 1))/sqrt(x^4 - 1), x)

$$3.214 \quad \int \frac{(d+ex^2)^4}{-cd^2+bde+be^2x^2+ce^2x^4} dx$$

Optimal. Leaf size=121

$$\frac{x(b^2e^2 - 5bcde + 7c^2d^2)}{c^3} + \frac{ex^3(4cd - be)}{3c^2} - \frac{(2cd - be)^3 \tanh^{-1}\left(\frac{\sqrt{c}\sqrt{ex}}{\sqrt{cd-be}}\right)}{c^{7/2}\sqrt{e}\sqrt{cd - be}} + \frac{e^2x^5}{5c}$$

[Out] $((7*c^2*d^2 - 5*b*c*d*e + b^2*e^2)*x)/c^3 + (e*(4*c*d - b*e)*x^3)/(3*c^2) + (e^2*x^5)/(5*c) - ((2*c*d - b*e)^3*ArcTanh[(Sqrt[c]*Sqrt[e]*x)/Sqrt[c*d - b*e]])/(c^{(7/2)*Sqrt[e]*Sqrt[c*d - b*e]})$

Rubi [A] time = 0.159732, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1149, 390, 208}

$$\frac{x(b^2e^2 - 5bcde + 7c^2d^2)}{c^3} + \frac{ex^3(4cd - be)}{3c^2} - \frac{(2cd - be)^3 \tanh^{-1}\left(\frac{\sqrt{c}\sqrt{ex}}{\sqrt{cd-be}}\right)}{c^{7/2}\sqrt{e}\sqrt{cd - be}} + \frac{e^2x^5}{5c}$$

Antiderivative was successfully verified.

[In] Int[(d + e*x^2)^4/(-(c*d^2) + b*d*e + b*e^2*x^2 + c*e^2*x^4), x]

[Out] $((7*c^2*d^2 - 5*b*c*d*e + b^2*e^2)*x)/c^3 + (e*(4*c*d - b*e)*x^3)/(3*c^2) + (e^2*x^5)/(5*c) - ((2*c*d - b*e)^3*ArcTanh[(Sqrt[c]*Sqrt[e]*x)/Sqrt[c*d - b*e]])/(c^{(7/2)*Sqrt[e]*Sqrt[c*d - b*e]})$

Rule 1149

Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[(d + e*x^2)^(p + q)*(a/d + (c*x^2)/e)^p, x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]

Rule 390

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] :> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rubi steps

$$\begin{aligned}
\int \frac{(d+ex^2)^4}{-cd^2+bde+be^2x^2+ce^2x^4} dx &= \int \frac{(d+ex^2)^3}{\frac{-cd^2+bde}{d}+cex^2} dx \\
&= \int \left(\frac{7c^2d^2-5bcde+b^2e^2}{c^3} + \frac{e(4cd-be)x^2}{c^2} + \frac{e^2x^4}{c} + \frac{8c^3d^3-12bc^2d^2e+6b^2cde^2}{c^3(-cd+be+cex^2)} \right) dx \\
&= \frac{(7c^2d^2-5bcde+b^2e^2)x}{c^3} + \frac{e(4cd-be)x^3}{3c^2} + \frac{e^2x^5}{5c} + \frac{(2cd-be)^3 \int \frac{1}{-cd+be+cex^2} dx}{c^3} \\
&= \frac{(7c^2d^2-5bcde+b^2e^2)x}{c^3} + \frac{e(4cd-be)x^3}{3c^2} + \frac{e^2x^5}{5c} - \frac{(2cd-be)^3 \tanh^{-1}\left(\frac{\sqrt{c}\sqrt{ex}}{\sqrt{cd-be}}\right)}{c^{7/2}\sqrt{e}\sqrt{cd-be}}
\end{aligned}$$

Mathematica [A] time = 0.0766953, size = 121, normalized size = 1.

$$-\frac{x(-b^2e^2+5bcde-7c^2d^2)}{c^3} - \frac{ex^3(be-4cd)}{3c^2} - \frac{(be-2cd)^3 \tan^{-1}\left(\frac{\sqrt{c}\sqrt{ex}}{\sqrt{be-cd}}\right)}{c^{7/2}\sqrt{e}\sqrt{be-cd}} + \frac{e^2x^5}{5c}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e*x^2)^4/(-(c*d^2) + b*d*e + b*e^2*x^2 + c*e^2*x^4), x]

[Out] -(((-7*c^2*d^2 + 5*b*c*d*e - b^2*e^2)*x)/c^3) - (e*(-4*c*d + b*e)*x^3)/(3*c^2) + (e^2*x^5)/(5*c) - ((-2*c*d + b*e)^3*ArcTan[(Sqrt[c]*Sqrt[e]*x)/Sqrt[-(c*d) + b*e]])/(c^(7/2)*Sqrt[e]*Sqrt[-(c*d) + b*e])

Maple [B] time = 0.01, size = 226, normalized size = 1.9

$$\frac{e^2x^5}{5c} - \frac{bx^3e^2}{3c^2} + \frac{4dex^3}{3c} + \frac{b^2e^2x}{c^3} - 5\frac{bdex}{c^2} + 7\frac{d^2x}{c} - \frac{b^3e^3}{c^3} \arctan\left(cex\frac{1}{\sqrt{(be-cd)ce}}\right) \frac{1}{\sqrt{(be-cd)ce}} + 6\frac{b^2de^2}{c^2\sqrt{(be-cd)ce}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x^2+d)^4/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2), x)

[Out] 1/5*e^2*x^5/c-1/3/c^2*x^3*b*e^2+4/3/c*x^3*d*e+1/c^3*b^2*e^2*x-5/c^2*b*d*e*x+7/c*d^2*x-1/c^3/((b*e-c*d)*c*e)^(1/2)*arctan(c*e*x/((b*e-c*d)*c*e)^(1/2))*b^3*e^3+6/c^2/((b*e-c*d)*c*e)^(1/2)*arctan(c*e*x/((b*e-c*d)*c*e)^(1/2))*b^2*d*e^2-12/c/((b*e-c*d)*c*e)^(1/2)*arctan(c*e*x/((b*e-c*d)*c*e)^(1/2))*b*d^2*e+8/((b*e-c*d)*c*e)^(1/2)*arctan(c*e*x/((b*e-c*d)*c*e)^(1/2))*d^3

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)^4/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2), x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [B] time = 1.94085, size = 903, normalized size = 7.46

$$\frac{6(c^4de^3 - bc^3e^4)x^5 + 10(4c^4d^2e^2 - 5bc^3de^3 + b^2c^2e^4)x^3 - 15(8c^3d^3 - 12bc^2d^2e + 6b^2cde^2 - b^3e^3)\sqrt{c^2de - bce^2} \log\left(\frac{cx}{c^2de - bce^2}\right)}{30(c^5de - bc^4e^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)^4/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm="fricas")

[Out] [1/30*(6*(c^4*d*e^3 - b*c^3*e^4)*x^5 + 10*(4*c^4*d^2*e^2 - 5*b*c^3*d*e^3 + b^2*c^2*e^4)*x^3 - 15*(8*c^3*d^3 - 12*b*c^2*d^2*e + 6*b^2*c*d*e^2 - b^3*e^3)*sqrt(c^2*d*e - b*c*e^2)*log((c*e*x^2 + c*d - b*e + 2*sqrt(c^2*d*e - b*c*e^2)*x)/(c*e*x^2 - c*d + b*e)) + 30*(7*c^4*d^3*e - 12*b*c^3*d^2*e^2 + 6*b^2*c^2*d*e^3 - b^3*c*e^4)*x)/(c^5*d*e - b*c^4*e^2), 1/15*(3*(c^4*d*e^3 - b*c^3*e^4)*x^5 + 5*(4*c^4*d^2*e^2 - 5*b*c^3*d*e^3 + b^2*c^2*e^4)*x^3 - 15*(8*c^3*d^3 - 12*b*c^2*d^2*e + 6*b^2*c*d*e^2 - b^3*e^3)*sqrt(-c^2*d*e + b*c*e^2)*arctan(-sqrt(-c^2*d*e + b*c*e^2)*x/(c*d - b*e)) + 15*(7*c^4*d^3*e - 12*b*c^3*d^2*e^2 + 6*b^2*c^2*d*e^3 - b^3*c*e^4)*x)/(c^5*d*e - b*c^4*e^2)]

Sympy [B] time = 0.954936, size = 343, normalized size = 2.83

$$\frac{\sqrt{-\frac{1}{c^7e^{be-cd}}}(be-2cd)^3 \log\left(x + \frac{-bc^3e\sqrt{-\frac{1}{c^7e^{be-cd}}}(be-2cd)^3 + c^4d\sqrt{-\frac{1}{c^7e^{be-cd}}}(be-2cd)^3}}{b^3e^3 - 6b^2cde^2 + 12bc^2d^2e - 8c^3d^3}\right)}{2} - \frac{\sqrt{-\frac{1}{c^7e^{be-cd}}}(be-2cd)^3 \log\left(x + \frac{bc^3e\sqrt{-\frac{1}{c^7e^{be-cd}}}(be-2cd)^3}}{b^3e^3 - 6b^2cde^2 + 12bc^2d^2e - 8c^3d^3}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x**2+d)**4/(c*e**2*x**4+b*e**2*x**2+b*d*e-c*d**2),x)

[Out] sqrt(-1/(c**7*e*(b*e - c*d)))*(b*e - 2*c*d)**3*log(x + (-b*c**3*e*sqrt(-1/(c**7*e*(b*e - c*d)))*(b*e - 2*c*d)**3 + c**4*d*sqrt(-1/(c**7*e*(b*e - c*d)))*(b*e - 2*c*d)**3)/(b**3*e**3 - 6*b**2*c*d*e**2 + 12*b*c**2*d**2*e - 8*c**3*d**3))/2 - sqrt(-1/(c**7*e*(b*e - c*d)))*(b*e - 2*c*d)**3*log(x + (b*c**3*e*sqrt(-1/(c**7*e*(b*e - c*d)))*(b*e - 2*c*d)**3 - c**4*d*sqrt(-1/(c**7*e*(b*e - c*d)))*(b*e - 2*c*d)**3)/(b**3*e**3 - 6*b**2*c*d*e**2 + 12*b*c**2*d**2*e - 8*c**3*d**3))/2 + e**2*x**5/(5*c) - x**3*(b*e**2 - 4*c*d*e)/(3*c**2) + x*(b**2*e**2 - 5*b*c*d*e + 7*c**2*d**2)/c**3

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)^4/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm="giac")

[Out] Timed out

$$3.215 \quad \int \frac{(d+ex^2)^3}{-cd^2+bde+be^2x^2+ce^2x^4} dx$$

Optimal. Leaf size=86

$$\frac{x(3cd-be)}{c^2} - \frac{(2cd-be)^2 \tanh^{-1}\left(\frac{\sqrt{c}\sqrt{ex}}{\sqrt{cd-be}}\right)}{c^{5/2}\sqrt{e}\sqrt{cd-be}} + \frac{ex^3}{3c}$$

[Out] $((3*c*d - b*e)*x)/c^2 + (e*x^3)/(3*c) - ((2*c*d - b*e)^2*ArcTanh[(Sqrt[c]*Sqrt[e]*x)/Sqrt[c*d - b*e]])/(c^{5/2}*Sqrt[e]*Sqrt[c*d - b*e])$

Rubi [A] time = 0.107268, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1149, 390, 208}

$$\frac{x(3cd-be)}{c^2} - \frac{(2cd-be)^2 \tanh^{-1}\left(\frac{\sqrt{c}\sqrt{ex}}{\sqrt{cd-be}}\right)}{c^{5/2}\sqrt{e}\sqrt{cd-be}} + \frac{ex^3}{3c}$$

Antiderivative was successfully verified.

[In] Int[(d + e*x^2)^3/(-(c*d^2) + b*d*e + b*e^2*x^2 + c*e^2*x^4), x]

[Out] $((3*c*d - b*e)*x)/c^2 + (e*x^3)/(3*c) - ((2*c*d - b*e)^2*ArcTanh[(Sqrt[c]*Sqrt[e]*x)/Sqrt[c*d - b*e]])/(c^{5/2}*Sqrt[e]*Sqrt[c*d - b*e])$

Rule 1149

Int[((d_) + (e_)*(x_)^2)^(q_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] := Int[(d + e*x^2)^(p+q)*(a/d + (c*x^2)/e)^p, x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]

Rule 390

Int[((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_), x_Symbol] := Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]

Rule 208

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rubi steps

$$\begin{aligned}
\int \frac{(d + ex^2)^3}{-cd^2 + bde + be^2x^2 + ce^2x^4} dx &= \int \frac{(d + ex^2)^2}{\frac{-cd^2 + bde}{d} + cex^2} dx \\
&= \int \left(\frac{3cd - be}{c^2} + \frac{ex^2}{c} + \frac{4c^2d^2 - 4bcde + b^2e^2}{c^2(-cd + be + cex^2)} \right) dx \\
&= \frac{(3cd - be)x}{c^2} + \frac{ex^3}{3c} + \frac{(2cd - be)^2}{c^2} \int \frac{1}{-cd + be + cex^2} dx \\
&= \frac{(3cd - be)x}{c^2} + \frac{ex^3}{3c} - \frac{(2cd - be)^2 \tanh^{-1} \left(\frac{\sqrt{c}\sqrt{ex}}{\sqrt{cd - be}} \right)}{c^{5/2}\sqrt{e}\sqrt{cd - be}}
\end{aligned}$$

Mathematica [A] time = 0.0464509, size = 84, normalized size = 0.98

$$-\frac{x(be - 3cd)}{c^2} + \frac{(be - 2cd)^2 \tan^{-1} \left(\frac{\sqrt{c}\sqrt{ex}}{\sqrt{be - cd}} \right)}{c^{5/2}\sqrt{e}\sqrt{be - cd}} + \frac{ex^3}{3c}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e*x^2)^3/(-(c*d^2) + b*d*e + b*e^2*x^2 + c*e^2*x^4),x]

[Out] -(((-3*c*d + b*e)*x)/c^2) + (e*x^3)/(3*c) + ((-2*c*d + b*e)^2*ArcTan[(Sqrt[c]*Sqrt[e]*x)/Sqrt[-(c*d) + b*e]])/(c^(5/2)*Sqrt[e]*Sqrt[-(c*d) + b*e])

Maple [A] time = 0.004, size = 142, normalized size = 1.7

$$\frac{ex^3}{3c} - \frac{bex}{c^2} + 3\frac{dx}{c} + \frac{b^2e^2}{c^2} \arctan\left(\frac{cex}{\sqrt{(be - cd)ce}}\right) \frac{1}{\sqrt{(be - cd)ce}} - 4\frac{bde}{c\sqrt{(be - cd)ce}} \arctan\left(\frac{cex}{\sqrt{(be - cd)ce}}\right) + 4\frac{d^2}{\sqrt{(be - cd)ce}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x^2+d)^3/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x)

[Out] 1/3*e*x^3/c-1/c^2*b*e*x+3/c*d*x+1/c^2/((b*e-c*d)*c*e)^(1/2)*arctan(c*e*x/((b*e-c*d)*c*e)^(1/2))*b^2*e^2-4/c/((b*e-c*d)*c*e)^(1/2)*arctan(c*e*x/((b*e-c*d)*c*e)^(1/2))*b*d*e+4/((b*e-c*d)*c*e)^(1/2)*arctan(c*e*x/((b*e-c*d)*c*e)^(1/2))*d^2

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)^3/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [A] time = 1.9537, size = 632, normalized size = 7.35

$$\frac{2(c^3de^2 - bc^2e^3)x^3 + 3(4c^2d^2 - 4bcde + b^2e^2)\sqrt{c^2de - bce^2} \log\left(\frac{cex^2 + cd - be - 2\sqrt{c^2de - bce^2}x}{cex^2 - cd + be}\right) + 6(3c^3d^2e - 4bc^2de^2 + b^2ce^3)}{6(c^4de - bc^3e^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)^3/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm="fricas")

[Out] [1/6*(2*(c^3*d*e^2 - b*c^2*e^3)*x^3 + 3*(4*c^2*d^2 - 4*b*c*d*e + b^2*e^2)*sqrt(c^2*d*e - b*c*e^2)*log((c*e*x^2 + c*d - b*e - 2*sqrt(c^2*d*e - b*c*e^2)*x)/(c*e*x^2 - c*d + b*e)) + 6*(3*c^3*d^2*e - 4*b*c^2*d*e^2 + b^2*c*e^3)*x)/(c^4*d*e - b*c^3*e^2), 1/3*((c^3*d*e^2 - b*c^2*e^3)*x^3 - 3*(4*c^2*d^2 - 4*b*c*d*e + b^2*e^2)*sqrt(-c^2*d*e + b*c*e^2)*arctan(-sqrt(-c^2*d*e + b*c*e^2)*x/(c*d - b*e)) + 3*(3*c^3*d^2*e - 4*b*c^2*d*e^2 + b^2*c*e^3)*x)/(c^4*d*e - b*c^3*e^2)]

Sympy [B] time = 0.739891, size = 275, normalized size = 3.2

$$\frac{\sqrt{-\frac{1}{c^5e(be-cd)}}(be-2cd)^2 \log\left(x + \frac{-bc^2e\sqrt{-\frac{1}{c^5e(be-cd)}}(be-2cd)^2 + c^3d\sqrt{-\frac{1}{c^5e(be-cd)}}(be-2cd)^2}{b^2e^2 - 4bcde + 4c^2d^2}\right)}{2} + \frac{\sqrt{-\frac{1}{c^5e(be-cd)}}(be-2cd)^2 \log\left(x + \frac{bc^2e}{b^2e^2 - 4bcde + 4c^2d^2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x**2+d)**3/(c*e**2*x**4+b*e**2*x**2+b*d*e-c*d**2),x)

[Out] -sqrt(-1/(c**5*e*(b*e - c*d)))*(b*e - 2*c*d)**2*log(x + (-b*c**2*e*sqrt(-1/(c**5*e*(b*e - c*d)))*(b*e - 2*c*d)**2 + c**3*d*sqrt(-1/(c**5*e*(b*e - c*d)))*(b*e - 2*c*d)**2)/(b**2*e**2 - 4*b*c*d*e + 4*c**2*d**2))/2 + sqrt(-1/(c**5*e*(b*e - c*d)))*(b*e - 2*c*d)**2*log(x + (b*c**2*e*sqrt(-1/(c**5*e*(b*e - c*d)))*(b*e - 2*c*d)**2 - c**3*d*sqrt(-1/(c**5*e*(b*e - c*d)))*(b*e - 2*c*d)**2)/(b**2*e**2 - 4*b*c*d*e + 4*c**2*d**2))/2 + e*x**3/(3*c) - x*(b*e - 3*c*d)/c**2

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)^3/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm="giac")

[Out] Timed out

$$3.216 \quad \int \frac{(d+ex^2)^2}{-cd^2+bde+be^2x^2+ce^2x^4} dx$$

Optimal. Leaf size=64

$$\frac{x}{c} - \frac{(2cd - be) \tanh^{-1}\left(\frac{\sqrt{c}\sqrt{ex}}{\sqrt{cd-be}}\right)}{c^{3/2}\sqrt{e}\sqrt{cd-be}}$$

[Out] x/c - ((2*c*d - b*e)*ArcTanh[(Sqrt[c]*Sqrt[e]*x)/Sqrt[c*d - b*e]]/(c^(3/2)*Sqrt[e]*Sqrt[c*d - b*e])

Rubi [A] time = 0.078468, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1149, 388, 208}

$$\frac{x}{c} - \frac{(2cd - be) \tanh^{-1}\left(\frac{\sqrt{c}\sqrt{ex}}{\sqrt{cd-be}}\right)}{c^{3/2}\sqrt{e}\sqrt{cd-be}}$$

Antiderivative was successfully verified.

[In] Int[(d + e*x^2)^2/(-c*d^2) + b*d*e + b*e^2*x^2 + c*e^2*x^4), x]

[Out] x/c - ((2*c*d - b*e)*ArcTanh[(Sqrt[c]*Sqrt[e]*x)/Sqrt[c*d - b*e]]/(c^(3/2)*Sqrt[e]*Sqrt[c*d - b*e])

Rule 1149

Int[((d_) + (e_)*(x_)^2)^(q_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] :> Int[(d + e*x^2)^(p + q)*(a/d + (c*x^2)/e)^p, x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]

Rule 388

Int[((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_)), x_Symbol] :> Simp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]

Rule 208

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rubi steps

$$\int \frac{(d + ex^2)^2}{-cd^2 + bde + be^2x^2 + ce^2x^4} dx = \int \frac{d + ex^2}{\frac{-cd^2 + bde}{d} + cex^2} dx$$

$$= \frac{x}{c} - \frac{\left(-cde + \frac{e(-cd^2 + bde)}{d}\right) \int \frac{1}{\frac{-cd^2 + bde}{d} + cex^2} dx}{ce}$$

$$= \frac{x}{c} - \frac{(2cd - be) \tanh^{-1}\left(\frac{\sqrt{c}\sqrt{ex}}{\sqrt{cd - be}}\right)}{c^{3/2}\sqrt{e}\sqrt{cd - be}}$$

Mathematica [A] time = 0.0551712, size = 63, normalized size = 0.98

$$\frac{x}{c} - \frac{(be - 2cd) \tan^{-1}\left(\frac{\sqrt{c}\sqrt{ex}}{\sqrt{be - cd}}\right)}{c^{3/2}\sqrt{e}\sqrt{be - cd}}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e*x^2)^2/(-(c*d^2) + b*d*e + b*e^2*x^2 + c*e^2*x^4), x]

[Out] x/c - ((-2*c*d + b*e)*ArcTan[(Sqrt[c]*Sqrt[e]*x)/Sqrt[-(c*d) + b*e]]/(c^(3/2)*Sqrt[e]*Sqrt[-(c*d) + b*e])

Maple [A] time = 0.003, size = 79, normalized size = 1.2

$$\frac{x}{c} - \frac{be}{c} \arctan\left(\frac{cex}{\sqrt{(be - cd)ce}}\right) - \frac{1}{\sqrt{(be - cd)ce}} + 2 \frac{d}{\sqrt{(be - cd)ce}} \arctan\left(\frac{cex}{\sqrt{(be - cd)ce}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x^2+d)^2/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2), x)

[Out] x/c - 1/c/((b*e - c*d)*c*e)^(1/2)*arctan(c*e*x/((b*e - c*d)*c*e)^(1/2))*b*e + 2/((b*e - c*d)*c*e)^(1/2)*arctan(c*e*x/((b*e - c*d)*c*e)^(1/2))*d

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)^2/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2), x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [A] time = 2.00839, size = 424, normalized size = 6.62

$$\left[\frac{\sqrt{c^2de - bce^2}(2cd - be) \log\left(\frac{cex^2 + cd - be + 2\sqrt{c^2de - bce^2}x}{cex^2 - cd + be}\right) - 2(c^2de - bce^2)x}{2(c^3de - bc^2e^2)}, -\frac{\sqrt{-c^2de + bce^2}(2cd - be) \arctan\left(-\frac{\sqrt{-c^2de + bce^2}}{cd - be}\right)}{c^3de - bc^2e^2} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)^2/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm="fricas")

[Out] [-1/2*(sqrt(c^2*d*e - b*c*e^2)*(2*c*d - b*e)*log((c*e*x^2 + c*d - b*e + 2*sqrt(c^2*d*e - b*c*e^2)*x)/(c*e*x^2 - c*d + b*e)) - 2*(c^2*d*e - b*c*e^2)*x)/(c^3*d*e - b*c^2*e^2), -(sqrt(-c^2*d*e + b*c*e^2)*(2*c*d - b*e)*arctan(-sqrt(-c^2*d*e + b*c*e^2)*x/(c*d - b*e)) - (c^2*d*e - b*c*e^2)*x)/(c^3*d*e - b*c^2*e^2)]

Sympy [B] time = 0.568459, size = 212, normalized size = 3.31

$$\frac{\sqrt{-\frac{1}{c^3e^{be-cd}}}(be - 2cd) \log\left(x + \frac{-bce\sqrt{-\frac{1}{c^3e^{be-cd}}}(be-2cd)+c^2d\sqrt{-\frac{1}{c^3e^{be-cd}}}(be-2cd)}}{be-2cd}\right)}{2} - \frac{\sqrt{-\frac{1}{c^3e^{be-cd}}}(be - 2cd) \log\left(x + \frac{bce\sqrt{-\frac{1}{c^3e^{be-cd}}}(be-2cd)}}{be-2cd}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x**2+d)**2/(c*e**2*x**4+b*e**2*x**2+b*d*e-c*d**2),x)

[Out] sqrt(-1/(c**3*e*(b*e - c*d)))*(b*e - 2*c*d)*log(x + (-b*c*e*sqrt(-1/(c**3*e*(b*e - c*d)))*(b*e - c*d)))/(b*e - 2*c*d) + c**2*d*sqrt(-1/(c**3*e*(b*e - c*d)))*(b*e - 2*c*d)/(b*e - 2*c*d)/2 - sqrt(-1/(c**3*e*(b*e - c*d)))*(b*e - 2*c*d)*log(x + (b*c*e*sqrt(-1/(c**3*e*(b*e - c*d)))*(b*e - 2*c*d) - c**2*d*sqrt(-1/(c**3*e*(b*e - c*d)))*(b*e - 2*c*d))/(b*e - 2*c*d))/2 + x/c

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)^2/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm="giac")

[Out] Timed out

$$3.217 \quad \int \frac{d+ex^2}{-cd^2+bde+be^2x^2+ce^2x^4} dx$$

Optimal. Leaf size=49

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{ex}}{\sqrt{cd-be}}\right)}{\sqrt{c}\sqrt{e}\sqrt{cd-be}}$$

[Out] -(ArcTanh[(Sqrt[c]*Sqrt[e]*x)/Sqrt[c*d - b*e]]/(Sqrt[c]*Sqrt[e]*Sqrt[c*d - b*e]))

Rubi [A] time = 0.0288674, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.054$, Rules used = {1149, 208}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{ex}}{\sqrt{cd-be}}\right)}{\sqrt{c}\sqrt{e}\sqrt{cd-be}}$$

Antiderivative was successfully verified.

[In] Int[(d + e*x^2)/(-(c*d^2) + b*d*e + b*e^2*x^2 + c*e^2*x^4), x]

[Out] -(ArcTanh[(Sqrt[c]*Sqrt[e]*x)/Sqrt[c*d - b*e]]/(Sqrt[c]*Sqrt[e]*Sqrt[c*d - b*e]))

Rule 1149

Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[(d + e*x^2)^(p + q)*(a/d + (c*x^2)/e)^p, x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rubi steps

$$\begin{aligned} \int \frac{d+ex^2}{-cd^2+bde+be^2x^2+ce^2x^4} dx &= \int \frac{1}{\frac{-cd^2+bde}{d}+ce^2x^2} dx \\ &= -\frac{\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{ex}}{\sqrt{cd-be}}\right)}{\sqrt{c}\sqrt{e}\sqrt{cd-be}} \end{aligned}$$

Mathematica [A] time = 0.0125817, size = 48, normalized size = 0.98

$$\frac{\tan^{-1}\left(\frac{\sqrt{c}\sqrt{ex}}{\sqrt{be-cd}}\right)}{\sqrt{c}\sqrt{e}\sqrt{be-cd}}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e*x^2)/(-(c*d^2) + b*d*e + b*e^2*x^2 + c*e^2*x^4),x]

[Out] ArcTan[(Sqrt[c]*Sqrt[e]*x)/Sqrt[-(c*d) + b*e]]/(Sqrt[c]*Sqrt[e]*Sqrt[-(c*d) + b*e])

Maple [A] time = 0.003, size = 33, normalized size = 0.7

$$\arctan\left(cex\frac{1}{\sqrt{(be-cd)ce}}\right)\frac{1}{\sqrt{(be-cd)ce}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x^2+d)/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x)

[Out] 1/((b*e-c*d)*c*e)^(1/2)*arctan(c*e*x/((b*e-c*d)*c*e)^(1/2))

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [A] time = 1.60306, size = 279, normalized size = 5.69

$$\left[\frac{\log\left(\frac{cex^2+cd-be-2\sqrt{c^2de-bce^2}x}{cex^2-cd+be}\right)}{2\sqrt{c^2de-bce^2}}, -\frac{\sqrt{-c^2de+bce^2}\arctan\left(-\frac{\sqrt{-c^2de+bce^2}x}{cd-be}\right)}{c^2de-bce^2} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm="fricas")

[Out] [1/2*log((c*e*x^2 + c*d - b*e - 2*sqrt(c^2*d*e - b*c*e^2)*x)/(c*e*x^2 - c*d + b*e))/sqrt(c^2*d*e - b*c*e^2), -sqrt(-c^2*d*e + b*c*e^2)*arctan(-sqrt(-c^2*d*e + b*c*e^2)*x/(c*d - b*e))/(c^2*d*e - b*c*e^2)]

Sympy [B] time = 0.218389, size = 124, normalized size = 2.53

$$-\frac{\sqrt{-\frac{1}{ce(be-cd)}}\log\left(-be\sqrt{-\frac{1}{ce(be-cd)}}+cd\sqrt{-\frac{1}{ce(be-cd)}}+x\right)}{2}+\frac{\sqrt{-\frac{1}{ce(be-cd)}}\log\left(be\sqrt{-\frac{1}{ce(be-cd)}}-cd\sqrt{-\frac{1}{ce(be-cd)}}+x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)/(c*e**2*x**4+b*e**2*x**2+b*d*e-c*d**2),x)
```

```
[Out] -sqrt(-1/(c*e*(b*e - c*d)))*log(-b*e*sqrt(-1/(c*e*(b*e - c*d))) + c*d*sqrt(-1/(c*e*(b*e - c*d))) + x)/2 + sqrt(-1/(c*e*(b*e - c*d)))*log(b*e*sqrt(-1/(c*e*(b*e - c*d))) - c*d*sqrt(-1/(c*e*(b*e - c*d))) + x)/2
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm="giac")
```

```
[Out] Timed out
```

$$3.218 \quad \int \frac{1}{(d+ex^2)(-cd^2+bde+be^2x^2+ce^2x^4)} dx$$

Optimal. Leaf size=136

$$\frac{c^{3/2} \tanh^{-1}\left(\frac{\sqrt{c}\sqrt{ex}}{\sqrt{cd-be}}\right)}{\sqrt{e}\sqrt{cd-be}(2cd-be)^2} - \frac{(4cd-be) \tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)}{2d^{3/2}\sqrt{e}(2cd-be)^2} - \frac{x}{2d(d+ex^2)(2cd-be)}$$

[Out] $-x/(2*d*(2*c*d - b*e)*(d + e*x^2)) - ((4*c*d - b*e)*\text{ArcTan}[(\text{Sqrt}[e]*x)/\text{Sqrt}[d]])/(2*d^{(3/2)}*\text{Sqrt}[e]*(2*c*d - b*e)^2) - (c^{(3/2)}*\text{ArcTanh}[(\text{Sqrt}[c]*\text{Sqrt}[e]*x)/\text{Sqrt}[c*d - b*e]])/(\text{Sqrt}[e]*\text{Sqrt}[c*d - b*e]*(2*c*d - b*e)^2)$

Rubi [A] time = 0.178477, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}} = 0.128$, Rules used = {1149, 414, 522, 205, 208}

$$\frac{c^{3/2} \tanh^{-1}\left(\frac{\sqrt{c}\sqrt{ex}}{\sqrt{cd-be}}\right)}{\sqrt{e}\sqrt{cd-be}(2cd-be)^2} - \frac{(4cd-be) \tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)}{2d^{3/2}\sqrt{e}(2cd-be)^2} - \frac{x}{2d(d+ex^2)(2cd-be)}$$

Antiderivative was successfully verified.

[In] $\text{Int}[1/((d + e*x^2)*(-c*d^2) + b*d*e + b*e^2*x^2 + c*e^2*x^4), x]$

[Out] $-x/(2*d*(2*c*d - b*e)*(d + e*x^2)) - ((4*c*d - b*e)*\text{ArcTan}[(\text{Sqrt}[e]*x)/\text{Sqrt}[d]])/(2*d^{(3/2)}*\text{Sqrt}[e]*(2*c*d - b*e)^2) - (c^{(3/2)}*\text{ArcTanh}[(\text{Sqrt}[c]*\text{Sqrt}[e]*x)/\text{Sqrt}[c*d - b*e]])/(\text{Sqrt}[e]*\text{Sqrt}[c*d - b*e]*(2*c*d - b*e)^2)$

Rule 1149

$\text{Int}(((d_) + (e_)*(x_)^2)^(q_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] \rightarrow \text{Int}[(d + e*x^2)^(p+q)*(a/d + (c*x^2)/e)^p, x] /; \text{FreeQ}\{a, b, c, d, e, q\}, x] \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& \text{EqQ}[c*d^2 - b*d*e + a*e^2, 0] \&\& \text{IntegerQ}[p]$

Rule 414

$\text{Int}(((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_), x_Symbol] \rightarrow -\text{Simp}[(b*x*(a + b*x^n)^(p+1)*(c + d*x^n)^(q+1))/(a*n*(p+1)*(b*c - a*d)), x] + \text{Dist}[1/(a*n*(p+1)*(b*c - a*d)), \text{Int}[(a + b*x^n)^(p+1)*(c + d*x^n)^q*\text{Simp}[b*c + n*(p+1)*(b*c - a*d) + d*b*(n*(p+q+2)+1)*x^n, x], x] /; \text{FreeQ}\{a, b, c, d, n, q\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{LtQ}[p, -1] \&\& !(!\text{IntegerQ}[p] \&\& \text{IntegerQ}[q] \&\& \text{LtQ}[q, -1]) \&\& \text{IntBinomialQ}[a, b, c, d, n, p, q, x]$

Rule 522

$\text{Int}(((e_) + (f_)*(x_)^(n_))/(((a_) + (b_)*(x_)^(n_))*((c_) + (d_)*(x_)^(n_))), x_Symbol] \rightarrow \text{Dist}[(b*e - a*f)/(b*c - a*d), \text{Int}[1/(a + b*x^n), x], x] - \text{Dist}[(d*e - c*f)/(b*c - a*d), \text{Int}[1/(c + d*x^n), x], x] /; \text{FreeQ}\{a, b, c, d, e, f, n\}, x]$

Rule 205

$\text{Int}(((a_) + (b_)*(x_)^2)^(-1), x_Symbol] \rightarrow \text{Simp}[(\text{Rt}[a/b, 2]*\text{ArcTan}[x/\text{Rt}[a/b, 2]])/a, x] /; \text{FreeQ}\{a, b\}, x] \&\& \text{PosQ}[a/b]$

Rule 208

$\text{Int}[(a_ + (b_ \cdot)(x_)^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(\text{Rt}[-(a/b), 2] \cdot \text{ArcTanh}[x/\text{Rt}[-(a/b), 2]])/a, x] /; \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{NegQ}[a/b]$

Rubi steps

$$\begin{aligned} \int \frac{1}{(d+ex^2)(-cd^2+bde+be^2x^2+ce^2x^4)} dx &= \int \frac{1}{(d+ex^2)^2 \left(\frac{-cd^2+bde}{d} + cex^2 \right)} dx \\ &= -\frac{x}{2d(2cd-be)(d+ex^2)} + \frac{\int \frac{e(3cd-be)-ce^2x^2}{(d+ex^2) \left(\frac{-cd^2+bde}{d} + cex^2 \right)} dx}{2de(2cd-be)} \\ &= -\frac{x}{2d(2cd-be)(d+ex^2)} + \frac{c^2 \int \frac{1}{\frac{-cd^2+bde}{d} + cex^2} dx}{(2cd-be)^2} - \frac{(4cd-be) \int \frac{1}{d+ex^2} dx}{2d(2cd-be)^2} \\ &= -\frac{x}{2d(2cd-be)(d+ex^2)} - \frac{(4cd-be) \tan^{-1} \left(\frac{\sqrt{ex}}{\sqrt{d}} \right)}{2d^{3/2} \sqrt{e}(2cd-be)^2} - \frac{c^{3/2} \tanh^{-1} \left(\frac{\sqrt{c}}{\sqrt{ca}} \right)}{\sqrt{e} \sqrt{cd-be}(2cd-be)} \end{aligned}$$

Mathematica [A] time = 0.20775, size = 133, normalized size = 0.98

$$\frac{c^{3/2} \tan^{-1} \left(\frac{\sqrt{c} \sqrt{ex}}{\sqrt{be-cd}} \right)}{\sqrt{e}(be-2cd)^2 \sqrt{be-cd}} + \frac{(be-4cd) \tan^{-1} \left(\frac{\sqrt{ex}}{\sqrt{d}} \right)}{2d^{3/2} \sqrt{e}(2cd-be)^2} - \frac{x}{2d(d+ex^2)(2cd-be)}$$

Antiderivative was successfully verified.

[In] Integrate[1/((d + e*x^2)*(-(c*d^2) + b*d*e + b*e^2*x^2 + c*e^2*x^4)),x]

[Out] -x/(2*d*(2*c*d - b*e)*(d + e*x^2)) + ((-4*c*d + b*e)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(3/2)*Sqrt[e]*(2*c*d - b*e)^2) + (c^(3/2)*ArcTan[(Sqrt[c]*Sqrt[e]*x)/Sqrt[-(c*d) + b*e]])/(Sqrt[e]*(-2*c*d + b*e)^2*Sqrt[-(c*d) + b*e])

Maple [A] time = 0.02, size = 155, normalized size = 1.1

$$\frac{bx e}{2 (be - 2cd)^2 d (ex^2 + d)} - \frac{cx}{(be - 2cd)^2 (ex^2 + d)} + \frac{be}{2 (be - 2cd)^2 d} \arctan \left(ex \frac{1}{\sqrt{de}} \right) \frac{1}{\sqrt{de}} - 2 \frac{c}{(be - 2cd)^2 \sqrt{de}} \arctan \left(\frac{c}{\sqrt{de}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(e*x^2+d)/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x)

[Out] 1/2/(b*e-2*c*d)^2/d*x/(e*x^2+d)*b*e-1/(b*e-2*c*d)^2*x/(e*x^2+d)*c+1/2/(b*e-2*c*d)^2/d/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))*b*e-2/(b*e-2*c*d)^2/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))*c+c^2/(b*e-2*c*d)^2/((b*e-c*d)*c*e)^(1/2)*arctan(c*e*x/((b*e-c*d)*c*e)^(1/2))

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.54581, size = 1829, normalized size = 13.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*(c*d^2*e^2*x^2 + c*d^3*e)*sqrt(c/(c*d*e - b*e^2))*log((c*e*x^2 - 2*(c*d*e - b*e^2)*x*sqrt(c/(c*d*e - b*e^2)) + c*d - b*e)/(c*e*x^2 - c*d + b*e)) + (4*c*d^2 - b*d*e + (4*c*d*e - b*e^2)*x^2)*sqrt(-d*e)*log((e*x^2 - 2*sqrt(-d*e)*x - d)/(e*x^2 + d)) - 2*(2*c*d^2*e - b*d*e^2)*x/(4*c^2*d^5*e - 4*b*c*d^4*e^2 + b^2*d^3*e^3 + (4*c^2*d^4*e^2 - 4*b*c*d^3*e^3 + b^2*d^2*e^4)*x^2), -1/2*((4*c*d^2 - b*d*e + (4*c*d*e - b*e^2)*x^2)*sqrt(d*e)*arctan(sqrt(d*e)*x/d) - (c*d^2*e^2*x^2 + c*d^3*e)*sqrt(c/(c*d*e - b*e^2))*log((c*e*x^2 - 2*(c*d*e - b*e^2)*x*sqrt(c/(c*d*e - b*e^2)) + c*d - b*e)/(c*e*x^2 - c*d + b*e)) + (2*c*d^2*e - b*d*e^2)*x/(4*c^2*d^5*e - 4*b*c*d^4*e^2 + b^2*d^3*e^3 + (4*c^2*d^4*e^2 - 4*b*c*d^3*e^3 + b^2*d^2*e^4)*x^2), 1/4*(4*(c*d^2*e^2*x^2 + c*d^3*e)*sqrt(-c/(c*d*e - b*e^2))*arctan(e*x*sqrt(-c/(c*d*e - b*e^2))) + (4*c*d^2 - b*d*e + (4*c*d*e - b*e^2)*x^2)*sqrt(-d*e)*log((e*x^2 - 2*sqrt(-d*e)*x - d)/(e*x^2 + d)) - 2*(2*c*d^2*e - b*d*e^2)*x/(4*c^2*d^5*e - 4*b*c*d^4*e^2 + b^2*d^3*e^3 + (4*c^2*d^4*e^2 - 4*b*c*d^3*e^3 + b^2*d^2*e^4)*x^2), 1/2*(2*(c*d^2*e^2*x^2 + c*d^3*e)*sqrt(-c/(c*d*e - b*e^2))*arctan(e*x*sqrt(-c/(c*d*e - b*e^2))) - (4*c*d^2 - b*d*e + (4*c*d*e - b*e^2)*x^2)*sqrt(d*e)*arctan(sqrt(d*e)*x/d) - (2*c*d^2*e - b*d*e^2)*x/(4*c^2*d^5*e - 4*b*c*d^4*e^2 + b^2*d^3*e^3 + (4*c^2*d^4*e^2 - 4*b*c*d^3*e^3 + b^2*d^2*e^4)*x^2)]
```

Sympy [B] time = 22.4644, size = 2664, normalized size = 19.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)/(c*e**2*x**4+b*e**2*x**2+b*d*e-c*d**2),x)
```

```
[Out] x/(2*b*d**2*e - 4*c*d**3 + x**2*(2*b*d*e**2 - 4*c*d**2*e)) - sqrt(-1/(d**3*e))*(b*e - 4*c*d)*log(x + (-b**7*d**3*e**8*(-1/(d**3*e))**3/2)*(b*e - 4*c*d)**3/(2*(b*e - 2*c*d)**6) + 7*b**6*c*d**4*e**7*(-1/(d**3*e))**3/2)*(b*e - 4*c*d)**3/(b*e - 2*c*d)**6 - 79*b**5*c**2*d**5*e**6*(-1/(d**3*e))**3/2)*(b*e - 4*c*d)**3/(2*(b*e - 2*c*d)**6) - b**5*e**5*sqrt(-1/(d**3*e))*(b*e - 4*c*d)/(2*(b*e - 2*c*d)**2) + 117*b**4*c**3*d**6*e**5*(-1/(d**3*e))**3/2)*(b*e - 4*c*d)**3/(b*e - 2*c*d)**6 + 7*b**4*c*d*e**4*sqrt(-1/(d**3*e))*(b*e - 4*c*d)/(b*e - 2*c*d)**2 - 196*b**3*c**4*d**7*e**4*(-1/(d**3*e))**3/2)*(b*e - 4*c*d)**3/(b*e - 2*c*d)**6 - 73*b**3*c**2*d**2*e**3*sqrt(-1/(d**3*e))*(b*e - 4*c*d)/(2*(b*e - 2*c*d)**2) + 184*b**2*c**5*d**8*e**3*(-1/(d**3*e))**3/2)*(b*e - 4*c*d)**3/(b*e - 2*c*d)**6 + 86*b**2*c**3*d**3*e**2*sqrt(-1/(d
```

```

**3*e))*(b*e - 4*c*d)/(b*e - 2*c*d)**2 - 88*b*c**6*d**9*e**2*(-1/(d**3*e))
*(3/2)*(b*e - 4*c*d)**3/(b*e - 2*c*d)**6 - 88*b*c**4*d**4*e*sqrt(-1/(d**3*
e))*(b*e - 4*c*d)/(b*e - 2*c*d)**2 + 16*c**7*d**10*e*(-1/(d**3*e))**3/2*(b
*e - 4*c*d)**3/(b*e - 2*c*d)**6 + 36*c**5*d**5*sqrt(-1/(d**3*e))*(b*e - 4*c
*d)/(b*e - 2*c*d)**2)/(b**2*c**2*e**2 - 9*b*c**3*d*e + 20*c**4*d**2))/(4*(b
*e - 2*c*d)**2) + sqrt(-1/(d**3*e))*(b*e - 4*c*d)*log(x + (b**7*d**3*e**8*(
-1/(d**3*e))**3/2*(b*e - 4*c*d)**3/(2*(b*e - 2*c*d)**6) - 7*b**6*c*d**4*e
**7*(-1/(d**3*e))**3/2*(b*e - 4*c*d)**3/(b*e - 2*c*d)**6 + 79*b**5*c**2*d
**5*e**6*(-1/(d**3*e))**3/2*(b*e - 4*c*d)**3/(2*(b*e - 2*c*d)**6) + b**5*
e**5*sqrt(-1/(d**3*e))*(b*e - 4*c*d)/(2*(b*e - 2*c*d)**2) - 117*b**4*c**3*d
**6*e**5*(-1/(d**3*e))**3/2*(b*e - 4*c*d)**3/(b*e - 2*c*d)**6 - 7*b**4*c*
d**4*sqrt(-1/(d**3*e))*(b*e - 4*c*d)/(b*e - 2*c*d)**2 + 196*b**3*c**4*d**
7*e**4*(-1/(d**3*e))**3/2*(b*e - 4*c*d)**3/(b*e - 2*c*d)**6 + 73*b**3*c**
2*d**2*e**3*sqrt(-1/(d**3*e))*(b*e - 4*c*d)/(2*(b*e - 2*c*d)**2) - 184*b**2
*c**5*d**8*e**3*(-1/(d**3*e))**3/2*(b*e - 4*c*d)**3/(b*e - 2*c*d)**6 - 86
*b**2*c**3*d**3*e**2*sqrt(-1/(d**3*e))*(b*e - 4*c*d)/(b*e - 2*c*d)**2 + 88*
b*c**6*d**9*e**2*(-1/(d**3*e))**3/2*(b*e - 4*c*d)**3/(b*e - 2*c*d)**6 + 8
8*b*c**4*d**4*e*sqrt(-1/(d**3*e))*(b*e - 4*c*d)/(b*e - 2*c*d)**2 - 16*c**7*
d**10*e*(-1/(d**3*e))**3/2*(b*e - 4*c*d)**3/(b*e - 2*c*d)**6 - 36*c**5*d*
**5*sqrt(-1/(d**3*e))*(b*e - 4*c*d)/(b*e - 2*c*d)**2)/(b**2*c**2*e**2 - 9*b*
c**3*d*e + 20*c**4*d**2))/(4*(b*e - 2*c*d)**2) - sqrt(-c**3/(e*(b*e - c*d))
)*log(x + (-4*b**7*d**3*e**8*(-c**3/(e*(b*e - c*d)))**3/2)/(b*e - 2*c*d)**
6 + 56*b**6*c*d**4*e**7*(-c**3/(e*(b*e - c*d)))**3/2)/(b*e - 2*c*d)**6 - 3
16*b**5*c**2*d**5*e**6*(-c**3/(e*(b*e - c*d)))**3/2)/(b*e - 2*c*d)**6 - b*
**5*e**5*sqrt(-c**3/(e*(b*e - c*d)))/(b*e - 2*c*d)**2 + 936*b**4*c**3*d**6*e
**5*(-c**3/(e*(b*e - c*d)))**3/2)/(b*e - 2*c*d)**6 + 14*b**4*c*d**4*sqrt
(-c**3/(e*(b*e - c*d)))/(b*e - 2*c*d)**2 - 1568*b**3*c**4*d**7*e**4*(-c**3/
(e*(b*e - c*d)))**3/2)/(b*e - 2*c*d)**6 - 73*b**3*c**2*d**2*e**3*sqrt(-c**
3/(e*(b*e - c*d)))/(b*e - 2*c*d)**2 + 1472*b**2*c**5*d**8*e**3*(-c**3/(e*(b
*e - c*d)))**3/2)/(b*e - 2*c*d)**6 + 172*b**2*c**3*d**3*e**2*sqrt(-c**3/(e
*(b*e - c*d)))/(b*e - 2*c*d)**2 - 704*b*c**6*d**9*e**2*(-c**3/(e*(b*e - c*d
)))**3/2)/(b*e - 2*c*d)**6 - 176*b*c**4*d**4*e*sqrt(-c**3/(e*(b*e - c*d))
)/(b*e - 2*c*d)**2 + 128*c**7*d**10*e*(-c**3/(e*(b*e - c*d)))**3/2)/(b*e -
2*c*d)**6 + 72*c**5*d**5*sqrt(-c**3/(e*(b*e - c*d)))/(b*e - 2*c*d)**2)/(b**
2*c**2*e**2 - 9*b*c**3*d*e + 20*c**4*d**2))/(2*(b*e - 2*c*d)**2) + sqrt(-c*
**3/(e*(b*e - c*d)))*log(x + (4*b**7*d**3*e**8*(-c**3/(e*(b*e - c*d)))**3/2
)/(b*e - 2*c*d)**6 - 56*b**6*c*d**4*e**7*(-c**3/(e*(b*e - c*d)))**3/2)/(b*
e - 2*c*d)**6 + 316*b**5*c**2*d**5*e**6*(-c**3/(e*(b*e - c*d)))**3/2)/(b*e
- 2*c*d)**6 + b**5*e**5*sqrt(-c**3/(e*(b*e - c*d)))/(b*e - 2*c*d)**2 - 936
*b**4*c**3*d**6*e**5*(-c**3/(e*(b*e - c*d)))**3/2)/(b*e - 2*c*d)**6 - 14*b
**4*c*d**4*sqrt(-c**3/(e*(b*e - c*d)))/(b*e - 2*c*d)**2 + 1568*b**3*c**4*
d**7*e**4*(-c**3/(e*(b*e - c*d)))**3/2)/(b*e - 2*c*d)**6 + 73*b**3*c**2*d*
**2*e**3*sqrt(-c**3/(e*(b*e - c*d)))/(b*e - 2*c*d)**2 - 1472*b**2*c**5*d**8*
e**3*(-c**3/(e*(b*e - c*d)))**3/2)/(b*e - 2*c*d)**6 - 172*b**2*c**3*d**3*e
**2*sqrt(-c**3/(e*(b*e - c*d)))/(b*e - 2*c*d)**2 + 704*b*c**6*d**9*e**2*(-c
**3/(e*(b*e - c*d)))**3/2)/(b*e - 2*c*d)**6 + 176*b*c**4*d**4*e*sqrt(-c**3
/(e*(b*e - c*d)))/(b*e - 2*c*d)**2 - 128*c**7*d**10*e*(-c**3/(e*(b*e - c*d)
))**3/2)/(b*e - 2*c*d)**6 - 72*c**5*d**5*sqrt(-c**3/(e*(b*e - c*d)))/(b*e
- 2*c*d)**2)/(b**2*c**2*e**2 - 9*b*c**3*d*e + 20*c**4*d**2))/(2*(b*e - 2*c*
d)**2)

```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x^2+d)/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm="giac"

)

[Out] Timed out

$$3.219 \quad \int \frac{1}{(d+ex^2)^2(-cd^2+bde+be^2x^2+ce^2x^4)} dx$$

Optimal. Leaf size=187

$$\frac{(3b^2e^2 - 16bcde + 28c^2d^2) \tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)}{8d^{5/2}\sqrt{e}(2cd - be)^3} - \frac{c^{5/2} \tanh^{-1}\left(\frac{\sqrt{c}\sqrt{ex}}{\sqrt{cd-be}}\right)}{\sqrt{e}\sqrt{cd-be}(2cd - be)^3} - \frac{x(10cd - 3be)}{8d^2(d + ex^2)(2cd - be)^2} - \frac{x}{4d(d + ex^2)^2(2cd - be)}$$

[Out] $-x/(4*d*(2*c*d - b*e)*(d + e*x^2)^2) - ((10*c*d - 3*b*e)*x)/(8*d^2*(2*c*d - b*e)^2*(d + e*x^2)) - ((28*c^2*d^2 - 16*b*c*d*e + 3*b^2*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(8*d^(5/2)*Sqrt[e]*(2*c*d - b*e)^3) - (c^(5/2)*ArcTanh[(Sqrt[c]*Sqrt[e]*x)/Sqrt[c*d - b*e]])/(Sqrt[e]*Sqrt[c*d - b*e]*(2*c*d - b*e)^3)$

Rubi [A] time = 0.277802, antiderivative size = 187, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1149, 414, 527, 522, 205, 208}

$$\frac{(3b^2e^2 - 16bcde + 28c^2d^2) \tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)}{8d^{5/2}\sqrt{e}(2cd - be)^3} - \frac{c^{5/2} \tanh^{-1}\left(\frac{\sqrt{c}\sqrt{ex}}{\sqrt{cd-be}}\right)}{\sqrt{e}\sqrt{cd-be}(2cd - be)^3} - \frac{x(10cd - 3be)}{8d^2(d + ex^2)(2cd - be)^2} - \frac{x}{4d(d + ex^2)^2(2cd - be)}$$

Antiderivative was successfully verified.

[In] $\text{Int}[1/((d + e*x^2)^2*(-(c*d^2) + b*d*e + b*e^2*x^2 + c*e^2*x^4)),x]$

[Out] $-x/(4*d*(2*c*d - b*e)*(d + e*x^2)^2) - ((10*c*d - 3*b*e)*x)/(8*d^2*(2*c*d - b*e)^2*(d + e*x^2)) - ((28*c^2*d^2 - 16*b*c*d*e + 3*b^2*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(8*d^(5/2)*Sqrt[e]*(2*c*d - b*e)^3) - (c^(5/2)*ArcTanh[(Sqrt[c]*Sqrt[e]*x)/Sqrt[c*d - b*e]])/(Sqrt[e]*Sqrt[c*d - b*e]*(2*c*d - b*e)^3)$

Rule 1149

$\text{Int}(((d_) + (e_)*(x_)^2)^(q_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] := \text{Int}[(d + e*x^2)^(p + q)*(a/d + (c*x^2)/e)^p, x] /;$ FreeQ[{a, b, c, d, e, q}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]

Rule 414

$\text{Int}(((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_), x_Symbol] := -\text{Simp}[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c - a*d)), x] + \text{Dist}[1/(a*n*(p + 1)*(b*c - a*d)), \text{Int}[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*\text{Simp}[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x], x], x] /;$ FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && !(IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, n, p, q, x]

Rule 527

$\text{Int}(((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_)*((e_) + (f_)*(x_)^(n_)), x_Symbol] := -\text{Simp}[(b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1)), x] + \text{Dist}[1/(a*n*(b*c - a*d)*(p + 1)), \text{Int}[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*\text{Simp}[c*(b*e - a*f) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /;$ FreeQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]

Rule 522

```
Int[((e_) + (f_)*(x_)^(n_))/(((a_) + (b_)*(x_)^(n_))*((c_) + (d_)*(x_)^(n_))), x_Symbol] := Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x] - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x]
```

Rule 205

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{(d+ex^2)^2(-cd^2+bde+be^2x^2+ce^2x^4)} dx &= \int \frac{1}{(d+ex^2)^3 \left(\frac{-cd^2+bde}{d} + cex^2 \right)} dx \\ &= -\frac{x}{4d(2cd-be)(d+ex^2)^2} + \frac{\int \frac{e(7cd-3be)-3ce^2x^2}{(d+ex^2)^2 \left(\frac{-cd^2+bde}{d} + cex^2 \right)} dx}{4de(2cd-be)} \\ &= -\frac{x}{4d(2cd-be)(d+ex^2)^2} - \frac{(10cd-3be)x}{8d^2(2cd-be)^2(d+ex^2)} + \frac{\int \frac{e^2(18c^2d^2-13bcde-3ce^2x^2)}{(d+ex^2)^3} dx}{8d^2e^2} \\ &= -\frac{x}{4d(2cd-be)(d+ex^2)^2} - \frac{(10cd-3be)x}{8d^2(2cd-be)^2(d+ex^2)} + \frac{c^3 \int \frac{1}{\frac{-cd^2+bde}{d} + cex^2}}{(2cd-be)^3} \\ &= -\frac{x}{4d(2cd-be)(d+ex^2)^2} - \frac{(10cd-3be)x}{8d^2(2cd-be)^2(d+ex^2)} - \frac{(28c^2d^2-16bcd)}{8d^{5/2}\sqrt{e}} \end{aligned}$$

Mathematica [A] time = 0.424915, size = 177, normalized size = 0.95

$$\frac{1}{8} \left(\frac{(3b^2e^2 - 16bcde + 28c^2d^2) \tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)}{d^{5/2}\sqrt{e}(2cd-be)^3} - \frac{\frac{8c^{5/2} \tan^{-1}\left(\frac{\sqrt{c}\sqrt{ex}}{\sqrt{be-cd}}\right)}{\sqrt{e}\sqrt{be-cd}} + \frac{x(be-2cd)(2cd(7d+5ex^2)-be(5d+3ex^2))}{d^2(d+ex^2)^2}}{(be-2cd)^3} \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x^2)^2*(-(c*d^2) + b*d*e + b*e^2*x^2 + c*e^2*x^4)),x]
```

```
[Out] (-(((28*c^2*d^2 - 16*b*c*d*e + 3*b^2*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(d^(5/2)*Sqrt[e]*(2*c*d - b*e)^3)) - (((-2*c*d + b*e)*x*(-(b*e*(5*d + 3*e*x^2)) + 2*c*d*(7*d + 5*e*x^2)))/(d^2*(d + e*x^2)^2) + (8*c^(5/2)*ArcTan[(Sqrt[c]*Sqrt[e]*x)/Sqrt[-(c*d) + b*e]]/(Sqrt[e]*Sqrt[-(c*d) + b*e]))/(-2*c*d + b*e)^3)/8
```

Maple [A] time = 0.013, size = 319, normalized size = 1.7

$$\frac{3e^3x^3b^2}{8(b e - 2cd)^3(ex^2 + d)^2d^2} - 2\frac{e^2x^3bc}{(be - 2cd)^3(ex^2 + d)^2d} + \frac{5ex^3c^2}{2(be - 2cd)^3(ex^2 + d)^2} + \frac{5xb^2e^2}{8(be - 2cd)^3(ex^2 + d)^2d} - 3\frac{c^2}{(be - 2cd)^3(ex^2 + d)^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(e*x^2+d)^2/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2), x)

[Out] 3/8/(b*e-2*c*d)^3/(e*x^2+d)^2*e^3/d^2*x^3*b^2-2/(b*e-2*c*d)^3/(e*x^2+d)^2*e^2/d*x^3*b*c+5/2/(b*e-2*c*d)^3/(e*x^2+d)^2*e*x^3*c^2+5/8/(b*e-2*c*d)^3/(e*x^2+d)^2/d*x*b^2*e^2-3/(b*e-2*c*d)^3/(e*x^2+d)^2*x*b*c*e+7/2/(b*e-2*c*d)^3/(e*x^2+d)^2*d*x*c^2+3/8/(b*e-2*c*d)^3/d^2/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))*b^2*e^2-2/(b*e-2*c*d)^3/d/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))*b*c*e+7/2/(b*e-2*c*d)^3/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))*c^2-c^3/(b*e-2*c*d)^3/((b*e-c*d)*c*e)^(1/2)*arctan(c*e*x/((b*e-c*d)*c*e)^(1/2))

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x^2+d)^2/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2), x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [B] time = 7.54463, size = 3611, normalized size = 19.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x^2+d)^2/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2), x, algorithm="fricas")

[Out] [-1/16*(2*(20*c^2*d^3*e^2 - 16*b*c*d^2*e^3 + 3*b^2*d*e^4)*x^3 + 8*(c^2*d^3*e^3*x^4 + 2*c^2*d^4*e^2*x^2 + c^2*d^5*e)*sqrt(c/(c*d*e - b*e^2))*log((c*e*x^2 + 2*(c*d*e - b*e^2)*x*sqrt(c/(c*d*e - b*e^2)) + c*d - b*e)/(c*e*x^2 - c*d + b*e)) - (28*c^2*d^4 - 16*b*c*d^3*e + 3*b^2*d^2*e^2 + (28*c^2*d^2*e^2 - 16*b*c*d*e^3 + 3*b^2*e^4)*x^4 + 2*(28*c^2*d^3*e - 16*b*c*d^2*e^2 + 3*b^2*d*e^3)*x^2)*sqrt(-d*e)*log((e*x^2 - 2*sqrt(-d*e)*x - d)/(e*x^2 + d)) + 2*(28*c^2*d^4*e - 24*b*c*d^3*e^2 + 5*b^2*d^2*e^3)*x)/(8*c^3*d^8*e - 12*b*c^2*d^7*e^2 + 6*b^2*c*d^6*e^3 - b^3*d^5*e^4 + (8*c^3*d^6*e^3 - 12*b*c^2*d^5*e^4 + 6*b^2*c*d^4*e^5 - b^3*d^3*e^6)*x^4 + 2*(8*c^3*d^7*e^2 - 12*b*c^2*d^6*e^3 + 6*b^2*c*d^5*e^4 - b^3*d^4*e^5)*x^2), -1/8*((20*c^2*d^3*e^2 - 16*b*c*d^2*e^3 + 3*b^2*d*e^4)*x^3 + (28*c^2*d^4 - 16*b*c*d^3*e + 3*b^2*d^2*e^2 + (28*c^2*d^2*e^2 - 16*b*c*d*e^3 + 3*b^2*e^4)*x^4 + 2*(28*c^2*d^3*e - 16*b*c*d^2*e^2 + 3*b^2*d*e^3)*x^2)*sqrt(d*e)*arctan(sqrt(d*e)*x/d) + 4*(c^2*d^3*e^3*x^4 + 2*c^2*d^4*e^2*x^2 + c^2*d^5*e)*sqrt(c/(c*d*e - b*e^2))*log((c*e*x^2 + 2*(c*d*e - b*e^2)*x*sqrt(c/(c*d*e - b*e^2)) + c*d - b*e)/(c*e*x^2 - c*d + b*e)) +

$$\begin{aligned} & (28*c^2*d^4*e - 24*b*c*d^3*e^2 + 5*b^2*d^2*e^3)*x)/(8*c^3*d^8*e - 12*b*c^2*d^7*e^2 + 6*b^2*c*d^6*e^3 - b^3*d^5*e^4 + (8*c^3*d^6*e^3 - 12*b*c^2*d^5*e^4 + 6*b^2*c*d^4*e^5 - b^3*d^3*e^6)*x^4 + 2*(8*c^3*d^7*e^2 - 12*b*c^2*d^6*e^3 + 6*b^2*c*d^5*e^4 - b^3*d^4*e^5)*x^2), -1/16*(2*(20*c^2*d^3*e^2 - 16*b*c*d^2*e^3 + 3*b^2*d*e^4)*x^3 - 16*(c^2*d^3*e^3*x^4 + 2*c^2*d^4*e^2*x^2 + c^2*d^5*e)*sqrt(-c/(c*d*e - b*e^2))*arctan(e*x*sqrt(-c/(c*d*e - b*e^2)))) - (28*c^2*d^4 - 16*b*c*d^3*e + 3*b^2*d^2*e^2 + (28*c^2*d^2*e^2 - 16*b*c*d*e^3 + 3*b^2*e^4)*x^4 + 2*(28*c^2*d^3*e - 16*b*c*d^2*e^2 + 3*b^2*d*e^3)*x^2)*sqrt(-d*e)*log((e*x^2 - 2*sqrt(-d*e)*x - d)/(e*x^2 + d)) + 2*(28*c^2*d^4*e - 24*b*c*d^3*e^2 + 5*b^2*d^2*e^3)*x)/(8*c^3*d^8*e - 12*b*c^2*d^7*e^2 + 6*b^2*c*d^6*e^3 - b^3*d^5*e^4 + (8*c^3*d^6*e^3 - 12*b*c^2*d^5*e^4 + 6*b^2*c*d^4*e^5 - b^3*d^3*e^6)*x^4 + 2*(8*c^3*d^7*e^2 - 12*b*c^2*d^6*e^3 + 6*b^2*c*d^5*e^4 - b^3*d^4*e^5)*x^2), -1/8*((20*c^2*d^3*e^2 - 16*b*c*d^2*e^3 + 3*b^2*d*e^4)*x^3 - 8*(c^2*d^3*e^3*x^4 + 2*c^2*d^4*e^2*x^2 + c^2*d^5*e)*sqrt(-c/(c*d*e - b*e^2))*arctan(e*x*sqrt(-c/(c*d*e - b*e^2)))) + (28*c^2*d^4 - 16*b*c*d^3*e + 3*b^2*d^2*e^2 + (28*c^2*d^2*e^2 - 16*b*c*d*e^3 + 3*b^2*e^4)*x^4 + 2*(28*c^2*d^3*e - 16*b*c*d^2*e^2 + 3*b^2*d*e^3)*x^2)*sqrt(d*e)*arctan(sqrt(d*e)*x/d) + (28*c^2*d^4*e - 24*b*c*d^3*e^2 + 5*b^2*d^2*e^3)*x)/(8*c^3*d^8*e - 12*b*c^2*d^7*e^2 + 6*b^2*c*d^6*e^3 - b^3*d^5*e^4 + (8*c^3*d^6*e^3 - 12*b*c^2*d^5*e^4 + 6*b^2*c*d^4*e^5 - b^3*d^3*e^6)*x^4 + 2*(8*c^3*d^7*e^2 - 12*b*c^2*d^6*e^3 + 6*b^2*c*d^5*e^4 - b^3*d^4*e^5)*x^2)] \end{aligned}$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x**2+d)**2/(c*e**2*x**4+b*e**2*x**2+b*d*e-c*d**2), x)

[Out] Timed out

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x^2+d)^2/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2), x, algorithm="giac")

[Out] Timed out

$$3.220 \quad \int \frac{(d+ex^2)^{5/2}}{-cd^2+bde+be^2x^2+ce^2x^4} dx$$

Optimal. Leaf size=139

$$-\frac{(2cd-be)^{3/2} \tanh^{-1}\left(\frac{\sqrt{ex}\sqrt{2cd-be}}{\sqrt{d+ex^2}\sqrt{cd-be}}\right)}{c^2\sqrt{e}\sqrt{cd-be}} + \frac{(5cd-2be) \tanh^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d+ex^2}}\right)}{2c^2\sqrt{e}} + \frac{x\sqrt{d+ex^2}}{2c}$$

[Out] (x*sqrt[d + e*x^2])/(2*c) + ((5*c*d - 2*b*e)*ArcTanh[(sqrt[e]*x)/sqrt[d + e*x^2]])/(2*c^2*sqrt[e]) - ((2*c*d - b*e)^(3/2)*ArcTanh[(sqrt[e]*sqrt[2*c*d - b*e]*x)/(sqrt[c*d - b*e]*sqrt[d + e*x^2])])/(c^2*sqrt[e]*sqrt[c*d - b*e])

Rubi [A] time = 0.276271, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {1149, 416, 523, 217, 206, 377, 208}

$$-\frac{(2cd-be)^{3/2} \tanh^{-1}\left(\frac{\sqrt{ex}\sqrt{2cd-be}}{\sqrt{d+ex^2}\sqrt{cd-be}}\right)}{c^2\sqrt{e}\sqrt{cd-be}} + \frac{(5cd-2be) \tanh^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d+ex^2}}\right)}{2c^2\sqrt{e}} + \frac{x\sqrt{d+ex^2}}{2c}$$

Antiderivative was successfully verified.

[In] Int[(d + e*x^2)^(5/2)/(-(c*d^2) + b*d*e + b*e^2*x^2 + c*e^2*x^4),x]

[Out] (x*sqrt[d + e*x^2])/(2*c) + ((5*c*d - 2*b*e)*ArcTanh[(sqrt[e]*x)/sqrt[d + e*x^2]])/(2*c^2*sqrt[e]) - ((2*c*d - b*e)^(3/2)*ArcTanh[(sqrt[e]*sqrt[2*c*d - b*e]*x)/(sqrt[c*d - b*e]*sqrt[d + e*x^2])])/(c^2*sqrt[e]*sqrt[c*d - b*e])

Rule 1149

Int[((d_) + (e_)*(x_)^2)^(q_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] := Int[(d + e*x^2)^(p + q)*(a/d + (c*x^2)/e)^p, x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]

Rule 416

Int[((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_), x_Symbol] := Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)), x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q - 1) + 1))*x^n, x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] && !IGtQ[p, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]

Rule 523

Int[((e_) + (f_)*(x_)^(n_))/(((a_) + (b_)*(x_)^(n_))*sqrt[(c_) + (d_)*(x_)^(n_)]), x_Symbol] := Dist[f/b, Int[1/sqrt[c + d*x^n], x], x] + Dist[(b*e - a*f)/b, Int[1/((a + b*x^n)*sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x]

Rule 217

Int[1/sqrt[(a_) + (b_)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 377

Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Subst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rubi steps

$$\begin{aligned} \int \frac{(d+ex^2)^{5/2}}{-cd^2+bde+be^2x^2+ce^2x^4} dx &= \int \frac{(d+ex^2)^{3/2}}{\frac{-cd^2+bde}{d}+ce^2x^2} dx \\ &= \frac{x\sqrt{d+ex^2}}{2c} + \frac{\int \frac{de(3cd-be)+e^2(5cd-2be)x^2}{\sqrt{d+ex^2}\left(\frac{-cd^2+bde}{d}+ce^2x^2\right)} dx}{2ce} \\ &= \frac{x\sqrt{d+ex^2}}{2c} + \frac{(5cd-2be) \int \frac{1}{\sqrt{d+ex^2}} dx}{2c^2} + \frac{(2cd-be)^2 \int \frac{1}{\sqrt{d+ex^2}\left(\frac{-cd^2+bde}{d}+ce^2x^2\right)} dx}{c^2} \\ &= \frac{x\sqrt{d+ex^2}}{2c} + \frac{(5cd-2be) \operatorname{Subst}\left(\int \frac{1}{1-ex^2} dx, x, \frac{x}{\sqrt{d+ex^2}}\right)}{2c^2} + \frac{(2cd-be)^2 \operatorname{Subst}\left(\int \frac{1}{\frac{-cd^2+bde}{d}+ce^2x^2} dx, x, \frac{x}{\sqrt{d+ex^2}}\right)}{c^2} \\ &= \frac{x\sqrt{d+ex^2}}{2c} + \frac{(5cd-2be) \tanh^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d+ex^2}}\right)}{2c^2\sqrt{e}} - \frac{(2cd-be)^{3/2} \tanh^{-1}\left(\frac{\sqrt{e}\sqrt{2cd-bex}}{\sqrt{cd-be}\sqrt{d+ex^2}}\right)}{c^2\sqrt{e}\sqrt{cd-be}} \end{aligned}$$

Mathematica [A] time = 0.264286, size = 134, normalized size = 0.96

$$\frac{(2be-5cd) \log\left(\sqrt{e}\sqrt{d+ex^2}+ex\right)}{\sqrt{e}} - \frac{2(be-2cd)^{3/2} \tanh^{-1}\left(\frac{\sqrt{ex}\sqrt{be-2cd}}{\sqrt{d+ex^2}\sqrt{be-cd}}\right)}{\sqrt{e}\sqrt{be-cd}} - cx\sqrt{d+ex^2} - \frac{2c^2}{2c^2}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e*x^2)^(5/2)/(-(c*d^2) + b*d*e + b*e^2*x^2 + c*e^2*x^4), x]

[Out] -(c*x*Sqrt[d + e*x^2]) - (2*(-2*c*d + b*e)^(3/2)*ArcTanh[(Sqrt[e]*Sqrt[-2*c*d + b*e]*x)/(Sqrt[-(c*d) + b*e]*Sqrt[d + e*x^2])])/(Sqrt[e]*Sqrt[-(c*d) + b*e]) + ((-5*c*d + 2*b*e)*Log[e*x + Sqrt[e]*Sqrt[d + e*x^2]])/Sqrt[e])/(2*c^2)

Maple [B] time = 0.05, size = 7043, normalized size = 50.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^(5/2)/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^2 + d)^{\frac{5}{2}}}{ce^2x^4 + be^2x^2 - cd^2 + bde} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(5/2)/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)^(5/2)/(c*e^2*x^4 + b*e^2*x^2 - c*d^2 + b*d*e), x)
```

Fricas [A] time = 4.7133, size = 2303, normalized size = 16.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(5/2)/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*sqrt(e*x^2 + d)*c*e*x - (5*c*d - 2*b*e)*sqrt(e)*log(-2*e*x^2 + 2*sqrt(e*x^2 + d)*sqrt(e)*x - d) - (2*c*d*e - b*e^2)*sqrt((2*c*d - b*e)/(c*d*e - b*e^2))*log((c^2*d^4 - 2*b*c*d^3*e + b^2*d^2*e^2 + (17*c^2*d^2*e^2 - 24*b*c*d*e^3 + 8*b^2*e^4)*x^4 + 2*(7*c^2*d^3*e - 11*b*c*d^2*e^2 + 4*b^2*d*e^3)*x^2 + 4*((3*c^2*d^2*e^2 - 5*b*c*d*e^3 + 2*b^2*e^4)*x^3 + (c^2*d^3*e - 2*b*c*d^2*e^2 + b^2*d*e^3)*x)*sqrt(e*x^2 + d)*sqrt((2*c*d - b*e)/(c*d*e - b*e^2)))/(c^2*e^2*x^4 + c^2*d^2 - 2*b*c*d*e + b^2*e^2 - 2*(c^2*d*e - b*c*e^2)*x^2)))/(c^2*e), 1/4*(2*sqrt(e*x^2 + d)*c*e*x - 2*(5*c*d - 2*b*e)*sqrt(-e)*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)) - (2*c*d*e - b*e^2)*sqrt((2*c*d - b*e)/(c*d*e - b*e^2))*log((c^2*d^4 - 2*b*c*d^3*e + b^2*d^2*e^2 + (17*c^2*d^2*e^2 - 24*b*c*d*e^3 + 8*b^2*e^4)*x^4 + 2*(7*c^2*d^3*e - 11*b*c*d^2*e^2 + 4*b^2*d*e^3)*x^2 + 4*((3*c^2*d^2*e^2 - 5*b*c*d*e^3 + 2*b^2*e^4)*x^3 + (c^2*d^3*e - 2*b*c*d^2*e^2 + b^2*d*e^3)*x)*sqrt(e*x^2 + d)*sqrt((2*c*d - b*e)/(c*d*e - b*e^2)))/(c^2*e^2*x^4 + c^2*d^2 - 2*b*c*d*e + b^2*e^2 - 2*(c^2*d*e - b*c*e^2)*x^2)))/(c^2*e), 1/4*(2*sqrt(e*x^2 + d)*c*e*x + 2*(2*c*d*e - b*e^2)*sqrt(-(2*c*d - b*e)/(c*d*e - b*e^2))*arctan(1/2*(c*d^2 - b*d*e + (3*c*d*e - 2*b*e^2)*x^2)*sqrt(e*x^2 + d)*sqrt(-(2*c*d - b*e)/(c*d*e - b*e^2)))/((2*c*d*e - b*e^2)*x^3 + (2*c*d^2 - b*d*e)*x)) - (5*c*d - 2*b*e)*sqrt(e)*log(-2*e*x^2 + 2*sqrt(e*x^2 + d)*sqrt(e)*x - d))/(c^2*e), 1/2*(sqrt(e*x^2 + d)*c*e*x - (5*c*d - 2*b*e)*sqrt(-e)*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)) + (2*c*d*e - b*e^2)*sqrt(-(2*c*d - b*e)/(c*d*e - b*e^2))*arctan(1/2*(c*d^2 - b*d*e + (3*c*d*e - 2*b*e^2)*x^2)*sqrt(e*x^2 + d)*sqrt(-(2*c*d - b*e)/(c*d*e - b*e^2)))/((2*c*d*e - b*e^2)*x^3 + (2*c*d^2 - b*d*e)*x)))/(c^2*e)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(d + ex^2)^{\frac{3}{2}}}{be - cd + cex^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x**2+d)**(5/2)/(c*e**2*x**4+b*e**2*x**2+b*d*e-c*d**2),x)

[Out] Integral((d + e*x**2)**(3/2)/(b*e - c*d + c*e*x**2), x)

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)^(5/2)/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm="giac")

[Out] Timed out

$$3.221 \quad \int \frac{(d+ex^2)^{3/2}}{-cd^2+bde+be^2x^2+ce^2x^4} dx$$

Optimal. Leaf size=108

$$\frac{\tanh^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d+ex^2}}\right)}{c\sqrt{e}} - \frac{\sqrt{2cd-be} \tanh^{-1}\left(\frac{\sqrt{ex}\sqrt{2cd-be}}{\sqrt{d+ex^2}\sqrt{cd-be}}\right)}{c\sqrt{e}\sqrt{cd-be}}$$

[Out] ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]/(c*Sqrt[e]) - (Sqrt[2*c*d - b*e]*ArcTanh[(Sqrt[e]*Sqrt[2*c*d - b*e]*x)/(Sqrt[c*d - b*e]*Sqrt[d + e*x^2])])/(c*Sqrt[e]*Sqrt[c*d - b*e])

Rubi [A] time = 0.126344, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}} = 0.146$, Rules used = {1149, 402, 217, 206, 377, 208}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d+ex^2}}\right)}{c\sqrt{e}} - \frac{\sqrt{2cd-be} \tanh^{-1}\left(\frac{\sqrt{ex}\sqrt{2cd-be}}{\sqrt{d+ex^2}\sqrt{cd-be}}\right)}{c\sqrt{e}\sqrt{cd-be}}$$

Antiderivative was successfully verified.

[In] Int[(d + e*x^2)^(3/2)/(-(c*d^2) + b*d*e + b*e^2*x^2 + c*e^2*x^4), x]

[Out] ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]/(c*Sqrt[e]) - (Sqrt[2*c*d - b*e]*ArcTanh[(Sqrt[e]*Sqrt[2*c*d - b*e]*x)/(Sqrt[c*d - b*e]*Sqrt[d + e*x^2])])/(c*Sqrt[e]*Sqrt[c*d - b*e])

Rule 1149

Int[((d_) + (e_)*(x_)^2)^(q_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] := Int[(d + e*x^2)^(p + q)*(a/d + (c*x^2)/e)^p, x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]

Rule 402

Int[((a_) + (b_)*(x_)^2)^(p_)/((c_) + (d_)*(x_)^2), x_Symbol] := Dist[b/d, Int[(a + b*x^2)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^2)^(p - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4])

Rule 217

Int[1/Sqrt[(a_) + (b_)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 377

Int[((a_) + (b_)*(x_)^(n_))^(p_)/((c_) + (d_)*(x_)^(n_)), x_Symbol] := Subst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]

Rule 208

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rubi steps

$$\begin{aligned} \int \frac{(d+ex^2)^{3/2}}{-cd^2+bde+be^2x^2+ce^2x^4} dx &= \int \frac{\sqrt{d+ex^2}}{\frac{-cd^2+bde}{d}+ce^2x^2} dx \\ &= \frac{\int \frac{1}{\sqrt{d+ex^2}} dx}{c} - \frac{\left(-cde + \frac{e(-cd^2+bde)}{d}\right) \int \frac{1}{\sqrt{d+ex^2} \left(\frac{-cd^2+bde}{d}+ce^2x^2\right)} dx}{ce} \\ &= \frac{\text{Subst}\left(\int \frac{1}{1-ex^2} dx, x, \frac{x}{\sqrt{d+ex^2}}\right)}{c} - \frac{\left(-cde + \frac{e(-cd^2+bde)}{d}\right) \text{Subst}\left(\int \frac{1}{\frac{-cd^2+bde}{d} - \left(-cde + \frac{e(-cd^2+bde)}{d}\right)} dx\right)}{ce} \\ &= \frac{\tanh^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d+ex^2}}\right)}{c\sqrt{e}} - \frac{\sqrt{2cd-be} \tanh^{-1}\left(\frac{\sqrt{e}\sqrt{2cd-bex}}{\sqrt{cd-be}\sqrt{d+ex^2}}\right)}{c\sqrt{e}\sqrt{cd-be}} \end{aligned}$$

Mathematica [A] time = 0.0882227, size = 103, normalized size = 0.95

$$\frac{\frac{\sqrt{be-2cd} \tanh^{-1}\left(\frac{\sqrt{ex}\sqrt{be-2cd}}{\sqrt{d+ex^2}\sqrt{be-cd}}\right)}{\sqrt{be-cd}} - \log\left(\sqrt{e}\sqrt{d+ex^2}+ex\right)}{c\sqrt{e}}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e*x^2)^(3/2)/(-(c*d^2) + b*d*e + b*e^2*x^2 + c*e^2*x^4), x]

[Out] -(((Sqrt[-2*c*d + b*e]*ArcTanh[(Sqrt[e]*Sqrt[-2*c*d + b*e]*x)/(Sqrt[-(c*d + b*e)*Sqrt[d + e*x^2]])]/Sqrt[-(c*d + b*e)] - Log[e*x + Sqrt[e]*Sqrt[d + e*x^2]])/(c*Sqrt[e]))

Maple [B] time = 0.02, size = 4308, normalized size = 39.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x^2+d)^(3/2)/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2), x)

[Out] 1/6*c^2*e/((-d*e)^(1/2)*c+(-(b*e-c*d)*c*e)^(1/2))/((-d*e)^(1/2)*c+(-(b*e-c*d)*c*e)^(1/2))/(-(b*e-c*d)*c*e)^(1/2)*((x-(-(b*e-c*d)*c*e)^(1/2)/c/e)^2*e+2*(-(b*e-c*d)*c*e)^(1/2)/c*(x-(-(b*e-c*d)*c*e)^(1/2)/c/e)-(b*e-2*c*d)/c)^(3/2)+1/4*c*e/((-d*e)^(1/2)*c+(-(b*e-c*d)*c*e)^(1/2))/((-d*e)^(1/2)*c+(-(b*e-c*d)*c*e)^(1/2))*((x-(-(b*e-c*d)*c*e)^(1/2)/c/e)^2*e+2*(-(b*e-c*d)*c*e)^(1/2)/c*(x-(-(b*e-c*d)*c*e)^(1/2)/c/e)-(b*e-2*c*d)/c)^(3/2)+1/4*c*e/((-d*e)^(1/2)*c+(-(b*e-c*d)*c*e)^(1/2))/((-d*e)^(1/2)*c+(-(b*e-c*d)*c*e)^(1/2))*((x-(-(b*e-c*d)*c*e)^(1/2)/c/e)^2*e+2*(-(b*e-c*d)*c*e)^(1/2)/c*(x-(-(b*e-c*d)*c*e)^(1/2)/c/e)-(b*e-2*c*d)/c)^(3/2)

$$\begin{aligned}
& /2)/c*(x-(-(b*e-c*d)*c*e)^{(1/2)}/c/e)-(b*e-2*c*d)/c)^{(1/2)}*x+5/4*c*e^{(1/2)}/ \\
& ((-d*e)^{(1/2)}*c+(-(b*e-c*d)*c*e)^{(1/2)})/((-d*e)^{(1/2)}*c+(-(b*e-c*d)*c*e)^{(1/2)}) \\
&)*\ln(((b*e-c*d)*c*e)^{(1/2)}/c+(x-(-(b*e-c*d)*c*e)^{(1/2)}/c/e)*e)/e^{(1/2)} \\
& +((x-(-(b*e-c*d)*c*e)^{(1/2)}/c/e)^{2*e+2}*(-(b*e-c*d)*c*e)^{(1/2)}/c*(x-(-(b*e-c \\
& *d)*c*e)^{(1/2)}/c/e)-(b*e-2*c*d)/c)^{(1/2)}*d-1/2*c*e^2/((-d*e)^{(1/2)}*c+(-(b* \\
& e-c*d)*c*e)^{(1/2)})/((-d*e)^{(1/2)}*c+(-(b*e-c*d)*c*e)^{(1/2)})/(-(b*e-c*d)*c*e \\
&)^{(1/2)}*((x-(-(b*e-c*d)*c*e)^{(1/2)}/c/e)^{2*e+2}*(-(b*e-c*d)*c*e)^{(1/2)}/c*(x-(\\
& -(b*e-c*d)*c*e)^{(1/2)}/c/e)-(b*e-2*c*d)/c)^{(1/2)}*b+c^2*e/((-d*e)^{(1/2)}*c+(-(\\
& b*e-c*d)*c*e)^{(1/2)})/((-d*e)^{(1/2)}*c+(-(b*e-c*d)*c*e)^{(1/2)})/(-(b*e-c*d)*c \\
& *e)^{(1/2)}*((x-(-(b*e-c*d)*c*e)^{(1/2)}/c/e)^{2*e+2}*(-(b*e-c*d)*c*e)^{(1/2)}/c*(x \\
& -(-(b*e-c*d)*c*e)^{(1/2)}/c/e)-(b*e-2*c*d)/c)^{(1/2)}*d-1/2*e^{(3/2)}/((-d*e)^{(1/2) \\
&)*c+(-(b*e-c*d)*c*e)^{(1/2)})/((-d*e)^{(1/2)}*c+(-(b*e-c*d)*c*e)^{(1/2)})*\ln(((\\
& -(b*e-c*d)*c*e)^{(1/2)}/c+(x-(-(b*e-c*d)*c*e)^{(1/2)}/c/e)*e)/e^{(1/2)}+((x-(-(b* \\
& e-c*d)*c*e)^{(1/2)}/c/e)^{2*e+2}*(-(b*e-c*d)*c*e)^{(1/2)}/c*(x-(-(b*e-c*d)*c*e)^{(\\
& 1/2)}/c/e)-(b*e-2*c*d)/c)^{(1/2)}*b-1/2*e^3/((-d*e)^{(1/2)}*c+(-(b*e-c*d)*c*e)^{(\\
& 1/2)})/((-d*e)^{(1/2)}*c+(-(b*e-c*d)*c*e)^{(1/2)})/(-(b*e-c*d)*c*e)^{(1/2)}/(-(b \\
& *e-2*c*d)/c)^{(1/2)}*\ln((-2*(b*e-2*c*d)/c+2*(-(b*e-c*d)*c*e)^{(1/2)}/c*(x-(-(b* \\
& e-c*d)*c*e)^{(1/2)}/c/e)+2*(-(b*e-2*c*d)/c)^{(1/2)}*((x-(-(b*e-c*d)*c*e)^{(1/2)}/ \\
& c/e)^{2*e+2}*(-(b*e-c*d)*c*e)^{(1/2)}/c*(x-(-(b*e-c*d)*c*e)^{(1/2)}/c/e)-(b*e-2*c \\
& *d)/c)^{(1/2)})/(x-(-(b*e-c*d)*c*e)^{(1/2)}/c/e))*b^2+2*c*e^2/((-d*e)^{(1/2)}*c+(\\
& -(b*e-c*d)*c*e)^{(1/2)})/((-d*e)^{(1/2)}*c+(-(b*e-c*d)*c*e)^{(1/2)})/(-(b*e-c*d) \\
& *c*e)^{(1/2)}/(-(b*e-2*c*d)/c)^{(1/2)}*\ln((-2*(b*e-2*c*d)/c+2*(-(b*e-c*d)*c*e)^{(\\
& 1/2)}/c*(x-(-(b*e-c*d)*c*e)^{(1/2)}/c/e)+2*(-(b*e-2*c*d)/c)^{(1/2)}*((x-(-(b*e- \\
& c*d)*c*e)^{(1/2)}/c/e)^{2*e+2}*(-(b*e-c*d)*c*e)^{(1/2)}/c*(x-(-(b*e-c*d)*c*e)^{(1/ \\
& 2)}/c/e)-(b*e-2*c*d)/c)^{(1/2)})/(x-(-(b*e-c*d)*c*e)^{(1/2)}/c/e))*b*d-2*c^2*e/(\\
& (-d*e)^{(1/2)}*c+(-(b*e-c*d)*c*e)^{(1/2)})/((-d*e)^{(1/2)}*c+(-(b*e-c*d)*c*e)^{(1 \\
& /2)})/(-(b*e-c*d)*c*e)^{(1/2)}/(-(b*e-2*c*d)/c)^{(1/2)}*\ln((-2*(b*e-2*c*d)/c+2*(\\
& -(b*e-c*d)*c*e)^{(1/2)}/c*(x-(-(b*e-c*d)*c*e)^{(1/2)}/c/e)+2*(-(b*e-2*c*d)/c)^{(\\
& 1/2)}*((x-(-(b*e-c*d)*c*e)^{(1/2)}/c/e)^{2*e+2}*(-(b*e-c*d)*c*e)^{(1/2)}/c*(x-(-(b \\
& *e-c*d)*c*e)^{(1/2)}/c/e)-(b*e-2*c*d)/c)^{(1/2)})/(x-(-(b*e-c*d)*c*e)^{(1/2)}/c/e \\
&))*d^2+1/6*c*e/(-d*e)^{(1/2)}/((-d*e)^{(1/2)}*c+(-(b*e-c*d)*c*e)^{(1/2)})/((-d*e \\
&)^{(1/2)}*c+(-(b*e-c*d)*c*e)^{(1/2)})*((x+(-d*e)^{(1/2)}/e)^{2*e-2*(-d*e)^{(1/2)}*(x \\
& +(-d*e)^{(1/2)}/e))^{(3/2)}-1/4*c*e/((-d*e)^{(1/2)}*c+(-(b*e-c*d)*c*e)^{(1/2)})/((- \\
& -d*e)^{(1/2)}*c+(-(b*e-c*d)*c*e)^{(1/2)})*((x+(-d*e)^{(1/2)}/e)^{2*e-2*(-d*e)^{(1/2) \\
&)*(x+(-d*e)^{(1/2)}/e))^{(1/2)}*x-1/4*c*e^{(1/2)}/((-d*e)^{(1/2)}*c+(-(b*e-c*d)*c*e \\
&)^{(1/2)})/((-d*e)^{(1/2)}*c+(-(b*e-c*d)*c*e)^{(1/2)})*d*\ln(((x+(-d*e)^{(1/2)}/e)* \\
& e-(-d*e)^{(1/2)}/e)^{(1/2)}+((x+(-d*e)^{(1/2)}/e)^{2*e-2*(-d*e)^{(1/2)}*(x+(-d*e)^{(1 \\
& /2)}/e))^{(1/2)}-1/6*c*e/(-d*e)^{(1/2)}/((-d*e)^{(1/2)}*c+(-(b*e-c*d)*c*e)^{(1/2)}) \\
&)/((-d*e)^{(1/2)}*c+(-(b*e-c*d)*c*e)^{(1/2)})*((x-(-d*e)^{(1/2)}/e)^{2*e+2*(-d*e)^{(\\
& 1/2)}*(x-(-d*e)^{(1/2)}/e))^{(3/2)}-1/4*c*e/((-d*e)^{(1/2)}*c+(-(b*e-c*d)*c*e)^{(1 \\
& /2)})/((-d*e)^{(1/2)}*c+(-(b*e-c*d)*c*e)^{(1/2)})*((x-(-d*e)^{(1/2)}/e)^{2*e+2*(-d \\
& *e)^{(1/2)}*(x-(-d*e)^{(1/2)}/e))^{(1/2)}*x-1/4*c*e^{(1/2)}/((-d*e)^{(1/2)}*c+(-(b*e- \\
& c*d)*c*e)^{(1/2)})/((-d*e)^{(1/2)}*c+(-(b*e-c*d)*c*e)^{(1/2)})*d*\ln(((x-(-d*e)^{(\\
& 1/2)}/e)*e+(-d*e)^{(1/2)}/e)^{(1/2)}+((x-(-d*e)^{(1/2)}/e)^{2*e+2*(-d*e)^{(1/2)}*(x- \\
& -d*e)^{(1/2)}/e))^{(1/2)}-1/6*c^2*e/((-d*e)^{(1/2)}*c+(-(b*e-c*d)*c*e)^{(1/2)})/((- \\
& -d*e)^{(1/2)}*c+(-(b*e-c*d)*c*e)^{(1/2)})/(-(b*e-c*d)*c*e)^{(1/2)}*((x+(-(b*e-c* \\
& d)*c*e)^{(1/2)}/c/e)^{2*e-2*(-(b*e-c*d)*c*e)^{(1/2)}/c*(x+(-(b*e-c*d)*c*e)^{(1/2) \\
& /c/e)-(b*e-2*c*d)/c)^{(3/2)}+1/4*c*e/((-d*e)^{(1/2)}*c+(-(b*e-c*d)*c*e)^{(1/2)})/ \\
& (-d*e)^{(1/2)}*c+(-(b*e-c*d)*c*e)^{(1/2)})*((x+(-(b*e-c*d)*c*e)^{(1/2)}/c/e)^{2* \\
& e-2*(-(b*e-c*d)*c*e)^{(1/2)}/c*(x+(-(b*e-c*d)*c*e)^{(1/2)}/c/e)-(b*e-2*c*d)/c)^{(\\
& 1/2)}*x+5/4*c*e^{(1/2)}/((-d*e)^{(1/2)}*c+(-(b*e-c*d)*c*e)^{(1/2)})/((-d*e)^{(1/2) \\
&)*c+(-(b*e-c*d)*c*e)^{(1/2)})*\ln((-d*e)^{(1/2)}/c+(x+(-(b*e-c*d)*c* \\
& e)^{(1/2)}/c/e)*e)/e^{(1/2)}+((x+(-(b*e-c*d)*c*e)^{(1/2)}/c/e)^{2*e-2*(-(b*e-c*d)* \\
& c*e)^{(1/2)}/c*(x+(-(b*e-c*d)*c*e)^{(1/2)}/c/e)-(b*e-2*c*d)/c)^{(1/2)}*d+1/2*c*e \\
& ^2/((-d*e)^{(1/2)}*c+(-(b*e-c*d)*c*e)^{(1/2)})/((-d*e)^{(1/2)}*c+(-(b*e-c*d)*c*e \\
&)^{(1/2)})/(-(b*e-c*d)*c*e)^{(1/2)}*((x+(-(b*e-c*d)*c*e)^{(1/2)}/c/e)^{2*e-2*(-(b* \\
& e-c*d)*c*e)^{(1/2)}/c*(x+(-(b*e-c*d)*c*e)^{(1/2)}/c/e)-(b*e-2*c*d)/c)^{(1/2)}*b-c \\
& ^2*e/((-d*e)^{(1/2)}*c+(-(b*e-c*d)*c*e)^{(1/2)})/((-d*e)^{(1/2)}*c+(-(b*e-c*d)*c \\
& *e)^{(1/2)})/(-(b*e-c*d)*c*e)^{(1/2)}*((x+(-(b*e-c*d)*c*e)^{(1/2)}/c/e)^{2*e-2*(-(
\end{aligned}$$

$$b^2 e^{-c^2 d} c^2 e^{(1/2)} / c^2 (x + (-b^2 e^{-c^2 d} c^2 e^{(1/2)}) / c^2 e - (b^2 e^{-2 c^2 d}) / c^2)^{(1/2)} d^{-1/2} e^{(3/2)} / ((-d^2 e)^{(1/2)} c^2 + (-b^2 e^{-c^2 d} c^2 e^{(1/2)}) / (-(-d^2 e)^{(1/2)} c^2 + (-b^2 e^{-c^2 d} c^2 e^{(1/2)}) * \ln((-(-b^2 e^{-c^2 d} c^2 e^{(1/2)}) / c^2 + (x + (-b^2 e^{-c^2 d} c^2 e^{(1/2)}) / c^2 e) * e) / e^{(1/2)} + ((x + (-b^2 e^{-c^2 d} c^2 e^{(1/2)}) / c^2 e)^2 e^{-2} (-b^2 e^{-c^2 d} c^2 e^{(1/2)}) / c^2 (x + (-b^2 e^{-c^2 d} c^2 e^{(1/2)}) / c^2 e - (b^2 e^{-2 c^2 d}) / c^2)^{(1/2)} * b^{1/2} e^{3/2} / ((-d^2 e)^{(1/2)} c^2 + (-b^2 e^{-c^2 d} c^2 e^{(1/2)}) / (-(-d^2 e)^{(1/2)} c^2 + (-b^2 e^{-c^2 d} c^2 e^{(1/2)}) / (-b^2 e^{-c^2 d} c^2 e^{(1/2)}) / (-b^2 e^{-2 c^2 d}) / c^2)^{(1/2)} * \ln((-2 * (b^2 e^{-2 c^2 d}) / c^2 - 2 * (-b^2 e^{-c^2 d} c^2 e^{(1/2)}) / c^2 (x + (-b^2 e^{-c^2 d} c^2 e^{(1/2)}) / c^2 e) + 2 * (-b^2 e^{-2 c^2 d}) / c^2) * ((x + (-b^2 e^{-c^2 d} c^2 e^{(1/2)}) / c^2 e)^2 e^{-2} (-b^2 e^{-c^2 d} c^2 e^{(1/2)}) / c^2 (x + (-b^2 e^{-c^2 d} c^2 e^{(1/2)}) / c^2 e - (b^2 e^{-2 c^2 d}) / c^2)^{(1/2)} / (x + (-b^2 e^{-c^2 d} c^2 e^{(1/2)}) / c^2 e) * b^{2-2 c^2 e^2} / ((-d^2 e)^{(1/2)} c^2 + (-b^2 e^{-c^2 d} c^2 e^{(1/2)}) / (-(-d^2 e)^{(1/2)} c^2 + (-b^2 e^{-c^2 d} c^2 e^{(1/2)}) * e^{(1/2)}) / (-b^2 e^{-c^2 d} c^2 e^{(1/2)}) / (-b^2 e^{-2 c^2 d}) / c^2)^{(1/2)} * \ln((-2 * (b^2 e^{-2 c^2 d}) / c^2 - 2 * (-b^2 e^{-c^2 d} c^2 e^{(1/2)}) / c^2 (x + (-b^2 e^{-c^2 d} c^2 e^{(1/2)}) / c^2 e) + 2 * (-b^2 e^{-2 c^2 d}) / c^2) * ((x + (-b^2 e^{-c^2 d} c^2 e^{(1/2)}) / c^2 e)^2 e^{-2} (-b^2 e^{-c^2 d} c^2 e^{(1/2)}) / c^2 (x + (-b^2 e^{-c^2 d} c^2 e^{(1/2)}) / c^2 e - (b^2 e^{-2 c^2 d}) / c^2)^{(1/2)} / (x + (-b^2 e^{-c^2 d} c^2 e^{(1/2)}) / c^2 e) * b^2 d + 2 * c^2 e^2 / ((-d^2 e)^{(1/2)} c^2 + (-b^2 e^{-c^2 d} c^2 e^{(1/2)}) / (-(-d^2 e)^{(1/2)} c^2 + (-b^2 e^{-c^2 d} c^2 e^{(1/2)}) / (-b^2 e^{-2 c^2 d}) / c^2)^{(1/2)} * \ln((-2 * (b^2 e^{-2 c^2 d}) / c^2 - 2 * (-b^2 e^{-c^2 d} c^2 e^{(1/2)}) / c^2 (x + (-b^2 e^{-c^2 d} c^2 e^{(1/2)}) / c^2 e) + 2 * (-b^2 e^{-2 c^2 d}) / c^2) * ((x + (-b^2 e^{-c^2 d} c^2 e^{(1/2)}) / c^2 e)^2 e^{-2} (-b^2 e^{-c^2 d} c^2 e^{(1/2)}) / c^2 (x + (-b^2 e^{-c^2 d} c^2 e^{(1/2)}) / c^2 e - (b^2 e^{-2 c^2 d}) / c^2)^{(1/2)} / (x + (-b^2 e^{-c^2 d} c^2 e^{(1/2)}) / c^2 e) * d^2$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^2 + d)^{\frac{3}{2}}}{ce^2x^4 + be^2x^2 - cd^2 + bde} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)^(3/2)/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm="maxima")

[Out] integrate((e*x^2 + d)^(3/2)/(c*e^2*x^4 + b*e^2*x^2 - c*d^2 + b*d*e), x)

Fricas [A] time = 2.55881, size = 1985, normalized size = 18.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)^(3/2)/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm="fricas")

[Out] [1/4*(e*sqrt((2*c*d - b*e)/(c*d*e - b*e^2))*log((c^2*d^4 - 2*b*c*d^3*e + b^2*d^2*e^2 + (17*c^2*d^2*e^2 - 24*b*c*d*e^3 + 8*b^2*e^4)*x^4 + 2*(7*c^2*d^3*e - 11*b*c*d^2*e^2 + 4*b^2*d*e^3)*x^2 - 4*((3*c^2*d^2*e^2 - 5*b*c*d*e^3 + 2*b^2*e^4)*x^3 + (c^2*d^3*e - 2*b*c*d^2*e^2 + b^2*d*e^3)*x)*sqrt(e*x^2 + d)*sqrt((2*c*d - b*e)/(c*d*e - b*e^2)))/(c^2*e^2*x^4 + c^2*d^2 - 2*b*c*d*e + b^2*e^2 - 2*(c^2*d*e - b*c*e^2)*x^2) + 2*sqrt(e)*log(-2*e*x^2 - 2*sqrt(e*x^2 + d)*sqrt(e)*x - d)/(c*e), 1/4*(e*sqrt((2*c*d - b*e)/(c*d*e - b*e^2))*log((c^2*d^4 - 2*b*c*d^3*e + b^2*d^2*e^2 + (17*c^2*d^2*e^2 - 24*b*c*d*e^3 + 8*b^2*e^4)*x^4 + 2*(7*c^2*d^3*e - 11*b*c*d^2*e^2 + 4*b^2*d*e^3)*x^2 - 4*((3*c^2*d^2*e^2 - 5*b*c*d*e^3 + 2*b^2*e^4)*x^3 + (c^2*d^3*e - 2*b*c*d^2*e^2 + b^2*d*e^3)*x)*sqrt(e*x^2 + d)*sqrt((2*c*d - b*e)/(c*d*e - b*e^2)))/(c^2*e^2

$$x^4 + c^2d^2 - 2b^2cde + b^2e^2 - 2(c^2de - b^2e^2)x^2) - 4\sqrt{-e} \arctan(\sqrt{-e}x/\sqrt{ex^2 + d})/(ce), 1/2(e\sqrt{-(2cd - b^2e)/(cde - b^2e^2)}) \arctan(1/2(c^2d^2 - b^2de + (3cde - 2b^2e^2)x^2)\sqrt{ex^2 + d}\sqrt{-(2cd - b^2e)/(cde - b^2e^2)})/((2cde - b^2e^2)x^3 + (2c^2d^2 - b^2de)x)) + \sqrt{e} \log(-2ex^2 - 2\sqrt{ex^2 + d}\sqrt{e}x - d)/(ce), 1/2(e\sqrt{-(2cd - b^2e)/(cde - b^2e^2)}) \arctan(1/2(c^2d^2 - b^2de + (3cde - 2b^2e^2)x^2)\sqrt{ex^2 + d}\sqrt{-(2cd - b^2e)/(cde - b^2e^2)})/((2cde - b^2e^2)x^3 + (2c^2d^2 - b^2de)x)) - 2\sqrt{-e} \arctan(\sqrt{-e}x/\sqrt{ex^2 + d})/(ce)]$$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{d + ex^2}}{be - cd + cex^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x**2+d)**(3/2)/(c*e**2*x**4+b*e**2*x**2+b*d*e-c*d**2),x)

[Out] Integral(sqrt(d + e*x**2)/(b*e - c*d + c*e*x**2), x)

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)^(3/2)/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm="giac")

[Out] Timed out

$$3.222 \quad \int \frac{\sqrt{d+ex^2}}{-cd^2+bde+be^2x^2+ce^2x^4} dx$$

Optimal. Leaf size=76

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{ex}\sqrt{2cd-be}}{\sqrt{d+ex^2}\sqrt{cd-be}}\right)}{\sqrt{e}\sqrt{cd-be}\sqrt{2cd-be}}$$

[Out] $-(\text{ArcTanh}[(\text{Sqrt}[e]*\text{Sqrt}[2*c*d - b*e]*x)/(\text{Sqrt}[c*d - b*e]*\text{Sqrt}[d + e*x^2])]) / (\text{Sqrt}[e]*\text{Sqrt}[c*d - b*e]*\text{Sqrt}[2*c*d - b*e])$

Rubi [A] time = 0.0704533, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}} = 0.073$, Rules used = {1149, 377, 208}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{ex}\sqrt{2cd-be}}{\sqrt{d+ex^2}\sqrt{cd-be}}\right)}{\sqrt{e}\sqrt{cd-be}\sqrt{2cd-be}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Sqrt}[d + e*x^2]/(-(c*d^2) + b*d*e + b*e^2*x^2 + c*e^2*x^4), x]$

[Out] $-(\text{ArcTanh}[(\text{Sqrt}[e]*\text{Sqrt}[2*c*d - b*e]*x)/(\text{Sqrt}[c*d - b*e]*\text{Sqrt}[d + e*x^2])]) / (\text{Sqrt}[e]*\text{Sqrt}[c*d - b*e]*\text{Sqrt}[2*c*d - b*e])$

Rule 1149

$\text{Int}[(d + e*x^2)^q * (a + b*x^2 + c*x^4)^p, x_Symbol] \rightarrow \text{Int}[(d + e*x^2)^{p+q} * (a/d + (c*x^2)/e)^p, x] /;$ FreeQ[{a, b, c, d, e, q}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]

Rule 377

$\text{Int}[(a + b*x^n)^p / (c + d*x^n), x_Symbol] \rightarrow \text{Subst}[\text{Int}[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^{1/n}] /;$ FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]

Rule 208

$\text{Int}[(a + b*x^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(\text{Rt}[-(a/b), 2]*\text{ArcTanh}[x/\text{Rt}[-(a/b), 2]])/a, x] /;$ FreeQ[{a, b}, x] && NegQ[a/b]

Rubi steps

$$\int \frac{\sqrt{d+ex^2}}{-cd^2+bde+be^2x^2+ce^2x^4} dx = \int \frac{1}{\sqrt{d+ex^2} \left(\frac{-cd^2+bde}{d} + cex^2 \right)} dx$$

$$= \text{Subst} \left(\int \frac{1}{\frac{-cd^2+bde}{d} - \left(-cde + \frac{e(-cd^2+bde)}{d} \right) x^2} dx, x, \frac{x}{\sqrt{d+ex^2}} \right)$$

$$= -\frac{\tanh^{-1} \left(\frac{\sqrt{e}\sqrt{2cd-be}}{\sqrt{cd-be}\sqrt{d+ex^2}} \right)}{\sqrt{e}\sqrt{cd-be}\sqrt{2cd-be}}$$

Mathematica [A] time = 0.067776, size = 76, normalized size = 1.

$$\frac{\tanh^{-1} \left(\frac{\sqrt{ex}\sqrt{2cd-be}}{\sqrt{d+ex^2}\sqrt{cd-be}} \right)}{\sqrt{e}\sqrt{cd-be}\sqrt{2cd-be}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[d + e*x^2]/(-(c*d^2) + b*d*e + b*e^2*x^2 + c*e^2*x^4), x]

[Out] -(ArcTanh[(Sqrt[e]*Sqrt[2*c*d - b*e]*x)/(Sqrt[c*d - b*e]*Sqrt[d + e*x^2])]/(Sqrt[e]*Sqrt[c*d - b*e]*Sqrt[2*c*d - b*e]))

Maple [B] time = 0.02, size = 2252, normalized size = 29.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x^2+d)^(1/2)/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2), x)

[Out] $\frac{1}{2}c^2e/((-d*e)^{(1/2)}*c+(-b*e-c*d)*c*e)^{(1/2)}/(-(-d*e)^{(1/2)}*c+(-b*e-c*d)*c*e)^{(1/2)}/(-b*e-c*d)*c*e)^{(1/2)}*((x-(-b*e-c*d)*c*e)^{(1/2)}/c/e)^{2e+2*(-b*e-c*d)*c*e)^{(1/2)}/c*(x-(-b*e-c*d)*c*e)^{(1/2)}/c/e-(b*e-2*c*d)/c)^{(1/2)+1/2*c*e^{(1/2)}/((-d*e)^{(1/2)}*c+(-b*e-c*d)*c*e)^{(1/2)}/(-(-d*e)^{(1/2)}*c+(-b*e-c*d)*c*e)^{(1/2)})*\ln(((b*e-c*d)*c*e)^{(1/2)}/c+(x-(-b*e-c*d)*c*e)^{(1/2)}/c/e)*e/e^{(1/2)}+((x-(-b*e-c*d)*c*e)^{(1/2)}/c/e)^{2e+2*(-b*e-c*d)*c*e)^{(1/2)}/c*(x-(-b*e-c*d)*c*e)^{(1/2)}/c/e-(b*e-2*c*d)/c)^{(1/2)}+1/2*c*e^2/((-d*e)^{(1/2)}*c+(-b*e-c*d)*c*e)^{(1/2)}/(-(-d*e)^{(1/2)}*c+(-b*e-c*d)*c*e)^{(1/2)}/(-b*e-c*d)*c*e)^{(1/2)}/(-b*e-2*c*d)/c)^{(1/2)}*\ln((-2*(b*e-2*c*d)/c+2*(-b*e-c*d)*c*e)^{(1/2)}/c*(x-(-b*e-c*d)*c*e)^{(1/2)}/c/e)+2*(-b*e-2*c*d)/c)^{(1/2)}*((x-(-b*e-c*d)*c*e)^{(1/2)}/c/e)^{2e+2*(-b*e-c*d)*c*e)^{(1/2)}/c*(x-(-b*e-c*d)*c*e)^{(1/2)}/c/e-(b*e-2*c*d)/c)^{(1/2)}/(x-(-b*e-c*d)*c*e)^{(1/2)}/c/e)*b-c^2e/((-d*e)^{(1/2)}*c+(-b*e-c*d)*c*e)^{(1/2)}/(-(-d*e)^{(1/2)}*c+(-b*e-c*d)*c*e)^{(1/2)}/(-b*e-2*c*d)/c)^{(1/2)}*\ln((-2*(b*e-2*c*d)/c+2*(-b*e-c*d)*c*e)^{(1/2)}/c*(x-(-b*e-c*d)*c*e)^{(1/2)}/c/e)+2*(-b*e-2*c*d)/c)^{(1/2)}*((x-(-b*e-c*d)*c*e)^{(1/2)}/c/e)^{2e+2*(-b*e-c*d)*c*e)^{(1/2)}/c*(x-(-b*e-c*d)*c*e)^{(1/2)}/c/e-(b*e-2*c*d)/c)^{(1/2)}/(x-(-b*e-c*d)*c*e)^{(1/2)}/c/e)*d+1/2*c*e/(-d*e)^{(1/2)}/((-d*e)^{(1/2)}*c+(-b*e-c*d)*c*e)^{(1/2)}/(-(-d*e)^{(1/2)}*c+(-b*e-c*d)*c*e)^{(1/2)}*((x+(-d*e)^{(1/2)}/e)^{2e-2*(-d*e)^{(1/2)}*(x+(-d*e)^{(1/2)}/e)}^{(1/2)}-1/2*c*e^{(1/2)}/((-d*e)^{(1/2)}*c+(-b*e-c*d)*c*e)^{(1/2)}/(-(-d*e)^{(1/2)}*c+(-b*e-c*d)*c*e)^{(1/2)})*\ln(((x+(-d*e)^{(1/2)}/e)*e$

$$\begin{aligned} & -(-d*e)^{(1/2)}/e^{(1/2)}+((x+(-d*e)^{(1/2)}/e)^{2*e-2*(-d*e)^{(1/2)}*(x+(-d*e)^{(1/2)}/e)} \\ & ^{(1/2)}-1/2*c*e/(-d*e)^{(1/2)}/((-d*e)^{(1/2)*c+(-b*e-c*d)*c*e)^{(1/2)})/ \\ & ((-d*e)^{(1/2)*c+(-b*e-c*d)*c*e)^{(1/2)}*((x-(-d*e)^{(1/2)}/e)^{2*e+2*(-d*e)^{(1/2)}/e} \\ & *(x-(-d*e)^{(1/2)}/e))^{(1/2)}-1/2*c*e^{(1/2)}/((-d*e)^{(1/2)*c+(-b*e-c*d)*c*e} \\ & ^{(1/2)})/((-d*e)^{(1/2)*c+(-b*e-c*d)*c*e)^{(1/2)}*\ln(((x-(-d*e)^{(1/2)}/e)*e \\ & +(-d*e)^{(1/2)}/e)^{(1/2)}+((x-(-d*e)^{(1/2)}/e)^{2*e+2*(-d*e)^{(1/2)}/e}*(x-(-d*e)^{(1/2)}/e))^{(1/2)} \\ & -1/2*c^2*e/((-d*e)^{(1/2)*c+(-b*e-c*d)*c*e)^{(1/2)})/((-d*e)^{(1/2)*c+(-b*e-c*d)*c*e} \\ & ^{(1/2)}*((x+(-b*e-c*d)*c*e)^{(1/2)}/c/e)^{2*e-2*(-b*e-c*d)*c*e)^{(1/2)}/c*(x+(-b*e-c*d)*c*e)^{(1/2)}/c/e- \\ & (b*e-2*c*d)/c)^{(1/2)}+1/2*c*e^{(1/2)}/((-d*e)^{(1/2)*c+(-b*e-c*d)*c*e)^{(1/2)})/((-d*e)^{(1/2)*c+(-b*e-c*d)*c*e} \\ & ^{(1/2)}*\ln((-(-b*e-c*d)*c*e)^{(1/2)}/c+(x+(-b*e-c*d)*c*e)^{(1/2)}/c/e)*e/e)^{(1/2)}+((x+(-b*e-c*d)*c*e)^{(1/2)}/c/e)^{2*e-2*(-b*e-c*d)*c*e} \\ & ^{(1/2)}/c*(x+(-b*e-c*d)*c*e)^{(1/2)}/c/e-(b*e-2*c*d)/c)^{(1/2)} \\ & -1/2*c*e^2/((-d*e)^{(1/2)*c+(-b*e-c*d)*c*e)^{(1/2)})/((-d*e)^{(1/2)*c+(-b*e-c*d)*c*e} \\ & ^{(1/2)})/((-b*e-c*d)*c*e)^{(1/2)}/((-b*e-2*c*d)/c)^{(1/2)}*\ln((-2*(b*e-2*c*d)/c-2*(-b*e-c*d)*c*e)^{(1/2)}/c*(x+(-b*e-c*d)*c*e)^{(1/2)}/c/e+2*(-b*e-2*c*d)/c)^{(1/2)}*((x+(-b*e-c*d)*c*e)^{(1/2)}/c/e)^{2*e-2*(-b*e-c*d)*c*e} \\ & ^{(1/2)}/c*(x+(-b*e-c*d)*c*e)^{(1/2)}/c/e-(b*e-2*c*d)/c)^{(1/2)})/(x+(-b*e-c*d)*c*e)^{(1/2)}/c/e)*b+c^2*e/((-d*e)^{(1/2)*c+(-b*e-c*d)*c*e} \\ & ^{(1/2)})/((-d*e)^{(1/2)*c+(-b*e-c*d)*c*e} \\ & ^{(1/2)})/((-b*e-c*d)*c*e)^{(1/2)}/((-b*e-2*c*d)/c)^{(1/2)}* \\ & \ln((-2*(b*e-2*c*d)/c-2*(-b*e-c*d)*c*e)^{(1/2)}/c*(x+(-b*e-c*d)*c*e)^{(1/2)}/c/e+2*(-b*e-2*c*d)/c)^{(1/2)}*((x+(-b*e-c*d)*c*e)^{(1/2)}/c/e)^{2*e-2*(-b*e-c*d)*c*e} \\ & ^{(1/2)}/c*(x+(-b*e-c*d)*c*e)^{(1/2)}/c/e-(b*e-2*c*d)/c)^{(1/2)})/(x+(-b*e-c*d)*c*e)^{(1/2)}/c/e)*d \end{aligned}$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ex^2 + d}}{ce^2x^4 + be^2x^2 - cd^2 + bde} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)^(1/2)/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm="maxima")

[Out] integrate(sqrt(e*x^2 + d)/(c*e^2*x^4 + b*e^2*x^2 - c*d^2 + b*d*e), x)

Fricas [B] time = 2.31571, size = 907, normalized size = 11.93

$$\left[\log \left(\frac{c^2d^4 - 2bcd^3e + b^2d^2e^2 + (17c^2d^2e^2 - 24bcde^3 + 8b^2e^4)x^4 + 2(7c^2d^3e - 11bcd^2e^2 + 4b^2de^3)x^2 - 4\sqrt{2c^2d^2e - 3bcde^2 + b^2e^3}((3cde - 2be^2)x^3 + (cd^2 - bde)x)\sqrt{ex^2 + d}}{c^2e^2x^4 + c^2d^2 - 2bcde + b^2e^2 - 2(c^2de - bce^2)x^2} \right) \right. \\ \left. \frac{4\sqrt{2c^2d^2e - 3bcde^2 + b^2e^3}}{4\sqrt{2c^2d^2e - 3bcde^2 + b^2e^3}} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)^(1/2)/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm="fricas")

[Out] [1/4*log((c^2*d^4 - 2*b*c*d^3*e + b^2*d^2*e^2 + (17*c^2*d^2*e^2 - 24*b*c*d*e^3 + 8*b^2*e^4)*x^4 + 2*(7*c^2*d^3*e - 11*b*c*d^2*e^2 + 4*b^2*d*e^3)*x^2 - 4*sqrt(2*c^2*d^2*e - 3*b*c*d*e^2 + b^2*e^3)*((3*c*d*e - 2*b*e^2)*x^3 + (c*d^2 - b*d*e)*x)*sqrt(e*x^2 + d))/(c^2*e^2*x^4 + c^2*d^2 - 2*b*c*d*e + b^2*e^2 - 2*(c^2*d*e - b*c*e^2)*x^2))/sqrt(2*c^2*d^2*e - 3*b*c*d*e^2 + b^2*e^3), -1/2*sqrt(-2*c^2*d^2*e + 3*b*c*d*e^2 - b^2*e^3)*arctan(-1/2*sqrt(-2*c^2*d^2

$$\frac{2*e + 3*b*c*d*e^2 - b^2*e^3)*(c*d^2 - b*d*e + (3*c*d*e - 2*b*e^2)*x^2)*\sqrt{(e*x^2 + d)/((2*c^2*d^2*e^2 - 3*b*c*d*e^3 + b^2*e^4)*x^3 + (2*c^2*d^3*e - 3*b*c*d^2*e^2 + b^2*d*e^3)*x)}}{(2*c^2*d^2*e - 3*b*c*d*e^2 + b^2*e^3)}$$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{d + ex^2} (be - cd + cex^2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x**2+d)**(1/2)/(c*e**2*x**4+b*e**2*x**2+b*d*e-c*d**2),x)

[Out] Integral(1/(sqrt(d + e*x**2)*(b*e - c*d + c*e*x**2)), x)

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)^(1/2)/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm="giac")

[Out] Timed out

$$3.223 \quad \int \frac{1}{\sqrt{d+ex^2}(-cd^2+bde+be^2x^2+ce^2x^4)} dx$$

Optimal. Leaf size=106

$$-\frac{x}{d\sqrt{d+ex^2}(2cd-be)} - \frac{c \tanh^{-1}\left(\frac{\sqrt{ex}\sqrt{2cd-be}}{\sqrt{d+ex^2}\sqrt{cd-be}}\right)}{\sqrt{e}\sqrt{cd-be}(2cd-be)^{3/2}}$$

[Out] $-(x/(d*(2*c*d - b*e)*Sqrt[d + e*x^2])) - (c*ArcTanh[(Sqrt[e]*Sqrt[2*c*d - b*e]*x)/(Sqrt[c*d - b*e]*Sqrt[d + e*x^2])])/(Sqrt[e]*Sqrt[c*d - b*e]*(2*c*d - b*e)^{(3/2)})$

Rubi [A] time = 0.11729, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}} = 0.098$, Rules used = {1149, 382, 377, 208}

$$-\frac{x}{d\sqrt{d+ex^2}(2cd-be)} - \frac{c \tanh^{-1}\left(\frac{\sqrt{ex}\sqrt{2cd-be}}{\sqrt{d+ex^2}\sqrt{cd-be}}\right)}{\sqrt{e}\sqrt{cd-be}(2cd-be)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[d + e*x^2]*(-(c*d^2) + b*d*e + b*e^2*x^2 + c*e^2*x^4)),x]

[Out] $-(x/(d*(2*c*d - b*e)*Sqrt[d + e*x^2])) - (c*ArcTanh[(Sqrt[e]*Sqrt[2*c*d - b*e]*x)/(Sqrt[c*d - b*e]*Sqrt[d + e*x^2])])/(Sqrt[e]*Sqrt[c*d - b*e]*(2*c*d - b*e)^{(3/2)})$

Rule 1149

Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[(d + e*x^2)^(p + q)*(a/d + (c*x^2)/e)^p, x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]

Rule 382

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] :> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c - a*d)), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] || !LtQ[q, -1]) && NeQ[p, -1]

Rule 377

Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Subst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{d+ex^2}(-cd^2+bde+be^2x^2+ce^2x^4)} dx &= \int \frac{1}{(d+ex^2)^{3/2} \left(\frac{-cd^2+bde}{d} + cex^2 \right)} dx \\
&= -\frac{x}{d(2cd-be)\sqrt{d+ex^2}} + \frac{c \int \frac{1}{\sqrt{d+ex^2} \left(\frac{-cd^2+bde}{d} + cex^2 \right)} dx}{2cd-be} \\
&= -\frac{x}{d(2cd-be)\sqrt{d+ex^2}} + \frac{c \operatorname{Subst} \left(\int \frac{1}{\frac{-cd^2+bde}{d} - \left(-cde + \frac{e(-cd^2+bde)}{d} \right) x^2} dx, x, \sqrt{d+ex^2} \right)}{2cd-be} \\
&= -\frac{x}{d(2cd-be)\sqrt{d+ex^2}} - \frac{c \tanh^{-1} \left(\frac{\sqrt{e}\sqrt{2cd-bex}}{\sqrt{cd-be}\sqrt{d+ex^2}} \right)}{\sqrt{e}\sqrt{cd-be}(2cd-be)^{3/2}}
\end{aligned}$$

Mathematica [C] time = 0.800298, size = 418, normalized size = 3.94

$$x \left(-\frac{2cex^2 \left(\frac{ex^2(be-2cd)}{(d+ex^2)(be-cd)} \right)^{5/2} {}_2F_1 \left(2, \frac{5}{2}; \frac{7}{2}; \frac{e(be-2cd)x^2}{(be-cd)(ex^2+d)} \right)}{cd-be} + 2 \left(\frac{ex^2(be-2cd)}{(d+ex^2)(be-cd)} \right)^{5/2} {}_2F_1 \left(2, \frac{5}{2}; \frac{7}{2}; \frac{e(be-2cd)x^2}{(be-cd)(ex^2+d)} \right) + \frac{10cex^2 \sqrt{\frac{ex^2(be-2cd)}{(d+ex^2)(be-cd)}}}{cd-be} - 15 \sqrt{\frac{ex^2(be-2cd)}{(d+ex^2)(be-cd)}} \right) - \frac{5(d+ex^2)^{3/2}(cd-be) \left(\frac{ex^2(be-2cd)}{(d+ex^2)(be-cd)} \right)^{3/2}}{cd-be}$$

Warning: Unable to verify antiderivative.

[In] Integrate[1/(Sqrt[d + e*x^2]*(-(c*d^2) + b*d*e + b*e^2*x^2 + c*e^2*x^4)), x]

[Out] -(x*(-15*Sqrt[(e*(-2*c*d + b*e))*x^2]/((-c*d) + b*e)*(d + e*x^2)) + (10*c*e*x^2*Sqrt[(e*(-2*c*d + b*e))*x^2]/((-c*d) + b*e)*(d + e*x^2)))/(c*d - b*e) + 15*ArcTanh[Sqrt[(e*(-2*c*d + b*e))*x^2]/((-c*d) + b*e)*(d + e*x^2)]] - (10*c*e*x^2*ArcTanh[Sqrt[(e*(-2*c*d + b*e))*x^2]/((-c*d) + b*e)*(d + e*x^2)]])/(c*d - b*e) + 2*((e*(-2*c*d + b*e))*x^2)/((-c*d) + b*e)*(d + e*x^2))^(5/2)*Hypergeometric2F1[2, 5/2, 7/2, (e*(-2*c*d + b*e))*x^2]/((-c*d) + b*e)*(d + e*x^2))] - (2*c*e*x^2*((e*(-2*c*d + b*e))*x^2)/((-c*d) + b*e)*(d + e*x^2))^(5/2)*Hypergeometric2F1[2, 5/2, 7/2, (e*(-2*c*d + b*e))*x^2]/((-c*d) + b*e)*(d + e*x^2)]]/(c*d - b*e))/(5*(c*d - b*e)*((e*(-2*c*d + b*e))*x^2)/((-c*d) + b*e)*(d + e*x^2))^(3/2)*(d + e*x^2)^(3/2))

Maple [B] time = 0.018, size = 771, normalized size = 7.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(e*x^2+d)^(1/2)/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2), x)

[Out] -1/2*c^2*e/((-d*e)^(1/2)*c+(-(b*e-c*d)*c*e)^(1/2))/((-d*e)^(1/2)*c+(-(b*e-c*d)*c*e)^(1/2))/(-(b*e-c*d)*c*e)^(1/2)/(-(b*e-2*c*d)/c)^(1/2)*ln((-2*(b*e-2*c*d)/c+2*(-(b*e-c*d)*c*e)^(1/2)/c*(x-(-(b*e-c*d)*c*e)^(1/2)/c/e)+2*(-(b*e-2*c*d)/c)^(1/2)*((x-(-(b*e-c*d)*c*e)^(1/2)/c/e)^2*e+2*(-(b*e-c*d)*c*e)^(1/2)/c))

$$\frac{2}{c} \frac{(x - (-b^2e - c^2d) * c * e)^{1/2} / c / e - (b^2e - 2 * c * d) / c)^{1/2}}{(x - (-b^2e - c^2d) * c * e)^{1/2} / c / e) - 1/2 * c / d / ((-d * e)^{1/2} * c + (-b^2e - c^2d) * c * e)^{1/2}} / ((-d * e)^{1/2} * c + (-b^2e - c^2d) * c * e)^{1/2} / (x + (-d * e)^{1/2} / e) * ((x + (-d * e)^{1/2} / e)^{2 * e - 2 * (-d * e)^{1/2}} * (x + (-d * e)^{1/2} / e))^{1/2} - 1/2 * c / d / ((-d * e)^{1/2} * c + (-b^2e - c^2d) * c * e)^{1/2}} / ((-d * e)^{1/2} * c + (-b^2e - c^2d) * c * e)^{1/2} / (x - (-d * e)^{1/2} / e) * ((x - (-d * e)^{1/2} / e)^{2 * e + 2 * (-d * e)^{1/2}} * (x - (-d * e)^{1/2} / e))^{1/2} + 1/2 * c^2 * e / ((-d * e)^{1/2} * c + (-b^2e - c^2d) * c * e)^{1/2}} / ((-d * e)^{1/2} * c + (-b^2e - c^2d) * c * e)^{1/2} / (-b^2e - c^2d) / c)^{1/2} * \ln((-2 * (b^2e - 2 * c * d) / c - 2 * (-b^2e - c^2d) * c * e)^{1/2} / c * (x + (-b^2e - c^2d) * c * e)^{1/2} / c / e) + 2 * (-b^2e - c^2d) / c)^{1/2} * ((x + (-b^2e - c^2d) * c * e)^{1/2} / c / e)^{2 * e - 2 * (-b^2e - c^2d) * c * e)^{1/2} / c * (x + (-b^2e - c^2d) * c * e)^{1/2} / c / e - (b^2e - 2 * c * d) / c)^{1/2}} / (x + (-b^2e - c^2d) * c * e)^{1/2} / c / e)$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(ce^2x^4 + be^2x^2 - cd^2 + bde)\sqrt{ex^2 + d}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x^2+d)^(1/2)/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm="maxima")

[Out] integrate(1/((c*e^2*x^4 + b*e^2*x^2 - c*d^2 + b*d*e)*sqrt(e*x^2 + d)), x)

Fricas [B] time = 2.78176, size = 1440, normalized size = 13.58

$$\frac{4(2c^2d^2e - 3bcde^2 + b^2e^3)\sqrt{ex^2 + d} + \sqrt{2c^2d^2e - 3bcde^2 + b^2e^3}(cdex^2 + cd^2) \log\left(\frac{c^2d^4 - 2bcd^3e + b^2d^2e^2 + (17c^2d^2e^2 - 24bcde^3 + b^2e^4)x^2 + 4(4c^3d^5e - 8bc^2d^4e^2 + 5b^2cd^3e^3 - b^3d^2e^4 + (4c^3d^4e^2 - 8bc^2d^3e^3 - b^3d^2e^4 + 4c^3d^4e^2 - 8bc^2d^3e^3 + 5b^2cd^2e^4 - b^3d^2e^5)x^2)}{4(4c^3d^5e - 8bc^2d^4e^2 + 5b^2cd^3e^3 - b^3d^2e^4 + (4c^3d^4e^2 - 8bc^2d^3e^3 - b^3d^2e^4 + 4c^3d^4e^2 - 8bc^2d^3e^3 + 5b^2cd^2e^4 - b^3d^2e^5)x^2)}\right)}{4(4c^3d^5e - 8bc^2d^4e^2 + 5b^2cd^3e^3 - b^3d^2e^4 + (4c^3d^4e^2 - 8bc^2d^3e^3 - b^3d^2e^4 + 4c^3d^4e^2 - 8bc^2d^3e^3 + 5b^2cd^2e^4 - b^3d^2e^5)x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x^2+d)^(1/2)/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm="fricas")

[Out] [-1/4*(4*(2*c^2*d^2*e - 3*b*c*d*e^2 + b^2*e^3)*sqrt(e*x^2 + d)*x + sqrt(2*c^2*d^2*e - 3*b*c*d*e^2 + b^2*e^3)*(c*d*e*x^2 + c*d^2)*log((c^2*d^4 - 2*b*c*d^3*e + b^2*d^2*e^2 + (17*c^2*d^2*e^2 - 24*b*c*d^3e + 8*b^2*e^4)*x^4 + 2*(7*c^2*d^3e - 11*b*c*d^2e^2 + 4*b^2*d^3e^3)*x^2 + 4*sqrt(2*c^2*d^2*e - 3*b*c*d*e^2 + b^2*e^3)*((3*c*d*e - 2*b*e^2)*x^3 + (c*d^2 - b*d*e)*x)*sqrt(e*x^2 + d))/(c^2*e^2*x^4 + c^2*d^2 - 2*b*c*d*e + b^2*e^2 - 2*(c^2*d*e - b*c*e^2)*x^2)))/(4*c^3*d^5*e - 8*b*c^2*d^4*e^2 + 5*b^2*c*d^3*e^3 - b^3*d^2*e^4 + (4*c^3*d^4*e^2 - 8*b*c^2*d^3e^3 + 5*b^2*c*d^2e^4 - b^3*d^2e^5)*x^2), -1/2*(2*(2*c^2*d^2*e - 3*b*c*d*e^2 + b^2*e^3)*sqrt(e*x^2 + d)*x + sqrt(-2*c^2*d^2*e + 3*b*c*d*e^2 - b^2*e^3)*(c*d*e*x^2 + c*d^2)*arctan(-1/2*sqrt(-2*c^2*d^2*e + 3*b*c*d*e^2 - b^2*e^3)*(c*d^2 - b*d*e + (3*c*d*e - 2*b*e^2)*x^2)*sqrt(e*x^2 + d)/((2*c^2*d^2*e^2 - 3*b*c*d^3e + b^2*e^4)*x^3 + (2*c^2*d^3e - 3*b*c*d^2e^2 + b^2*d^3e^3)*x)))/(4*c^3*d^5*e - 8*b*c^2*d^4e^2 + 5*b^2*c*d^3e^3 - b^3*d^2e^4 + (4*c^3*d^4e^2 - 8*b*c^2*d^3e^3 + 5*b^2*c*d^2e^4 - b^3*d^2e^5)*x^2)]

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(d + ex^2)^{\frac{3}{2}} (be - cd + cex^2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x**2+d)**(1/2)/(c*e**2*x**4+b*e**2*x**2+b*d*e-c*d**2),x)

[Out] Integral(1/((d + e*x**2)**(3/2)*(b*e - c*d + c*e*x**2)), x)

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x^2+d)^(1/2)/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm="giac")

[Out] Timed out

$$3.224 \quad \int \frac{1}{(d+ex^2)^{3/2}(-cd^2+bde+be^2x^2+ce^2x^4)} dx$$

Optimal. Leaf size=149

$$-\frac{c^2 \tanh^{-1}\left(\frac{\sqrt{ex}\sqrt{2cd-be}}{\sqrt{d+ex^2}\sqrt{cd-be}}\right)}{\sqrt{e}\sqrt{cd-be}(2cd-be)^{5/2}} - \frac{x(7cd-2be)}{3d^2\sqrt{d+ex^2}(2cd-be)^2} - \frac{x}{3d(d+ex^2)^{3/2}(2cd-be)}$$

[Out] $-x/(3*d*(2*c*d - b*e)*(d + e*x^2)^{(3/2)}) - ((7*c*d - 2*b*e)*x)/(3*d^2*(2*c*d - b*e)^2*\text{Sqrt}[d + e*x^2]) - (c^2*\text{ArcTanh}[(\text{Sqrt}[e]*\text{Sqrt}[2*c*d - b*e]*x)/(\text{Sqrt}[c*d - b*e]*\text{Sqrt}[d + e*x^2])]) / (\text{Sqrt}[e]*\text{Sqrt}[c*d - b*e]*(2*c*d - b*e)^{(5/2)})$

Rubi [A] time = 0.268589, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}} = 0.146$, Rules used = {1149, 414, 527, 12, 377, 208}

$$-\frac{c^2 \tanh^{-1}\left(\frac{\sqrt{ex}\sqrt{2cd-be}}{\sqrt{d+ex^2}\sqrt{cd-be}}\right)}{\sqrt{e}\sqrt{cd-be}(2cd-be)^{5/2}} - \frac{x(7cd-2be)}{3d^2\sqrt{d+ex^2}(2cd-be)^2} - \frac{x}{3d(d+ex^2)^{3/2}(2cd-be)}$$

Antiderivative was successfully verified.

[In] $\text{Int}[1/((d + e*x^2)^{(3/2)}*(-(c*d^2) + b*d*e + b*e^2*x^2 + c*e^2*x^4)), x]$

[Out] $-x/(3*d*(2*c*d - b*e)*(d + e*x^2)^{(3/2)}) - ((7*c*d - 2*b*e)*x)/(3*d^2*(2*c*d - b*e)^2*\text{Sqrt}[d + e*x^2]) - (c^2*\text{ArcTanh}[(\text{Sqrt}[e]*\text{Sqrt}[2*c*d - b*e]*x)/(\text{Sqrt}[c*d - b*e]*\text{Sqrt}[d + e*x^2])]) / (\text{Sqrt}[e]*\text{Sqrt}[c*d - b*e]*(2*c*d - b*e)^{(5/2)})$

Rule 1149

$\text{Int}[(d + e*x^2)^{(p+q)}*(a/d + (c*x^2)/e)^p, x] /;$ FreeQ[{a, b, c, d, e, q}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]

Rule 414

$\text{Int}[(a + b*x^n)^{(p+1)}*(c + d*x^n)^{(q+1)}/(a*n*(p+1)*(b*c - a*d)), x] + \text{Dist}[1/(a*n*(p+1)*(b*c - a*d)), \text{Int}[(a + b*x^n)^{(p+1)}*(c + d*x^n)^q*\text{Simp}[b*c + n*(p+1)*(b*c - a*d) + d*b*(n*(p+q+2) + 1)*x^n, x], x] /;$ FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && !(IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, n, p, q, x]

Rule 527

$\text{Int}[(a + b*x^n)^{(p+1)}*(c + d*x^n)^q*\text{Simp}[c*(b*e - a*f) + e*n*(b*c - a*d)*(p+1) + d*(b*e - a*f)*(n*(p+q+2) + 1)*x^n, x], x] /;$ FreeQ

[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 377

Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Subst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rubi steps

$$\begin{aligned}
 \int \frac{1}{(d+ex^2)^{3/2}(-cd^2+bde+be^2x^2+ce^2x^4)} dx &= \int \frac{1}{(d+ex^2)^{5/2} \left(\frac{-cd^2+bde}{d} + cex^2 \right)} dx \\
 &= -\frac{x}{3d(2cd-be)(d+ex^2)^{3/2}} + \frac{\int \frac{e(5cd-2be)-2ce^2x^2}{(d+ex^2)^{3/2} \left(\frac{-cd^2+bde}{d} + cex^2 \right)} dx}{3de(2cd-be)} \\
 &= -\frac{x}{3d(2cd-be)(d+ex^2)^{3/2}} - \frac{(7cd-2be)x}{3d^2(2cd-be)^2\sqrt{d+ex^2}} + \frac{\int \frac{3c^2}{\sqrt{d+ex^2}} dx}{3d^2e^2} \\
 &= -\frac{x}{3d(2cd-be)(d+ex^2)^{3/2}} - \frac{(7cd-2be)x}{3d^2(2cd-be)^2\sqrt{d+ex^2}} + \frac{c^2 \int \frac{1}{\sqrt{d+ex^2}} dx}{(2cd-be)^2} \\
 &= -\frac{x}{3d(2cd-be)(d+ex^2)^{3/2}} - \frac{(7cd-2be)x}{3d^2(2cd-be)^2\sqrt{d+ex^2}} + \frac{c^2 \operatorname{Subst}\left(\int \frac{1}{\sqrt{u}} du\right)}{(2cd-be)^2} \\
 &= -\frac{x}{3d(2cd-be)(d+ex^2)^{3/2}} - \frac{(7cd-2be)x}{3d^2(2cd-be)^2\sqrt{d+ex^2}} - \frac{c^2 \tanh^{-1}\left(\frac{x}{\sqrt{e}\sqrt{cd-be}}\right)}{\sqrt{e}\sqrt{cd-be}}
 \end{aligned}$$

Mathematica [C] time = 3.49204, size = 1058, normalized size = 7.1

$$x \left(-\frac{56c^2e^2 \left(\frac{e(be-2cd)x^2}{(be-cd)(ex^2+d)} \right)^{3/2}}{(cd-be)^2} + \frac{168c^2e^2 \tanh^{-1}\left(\sqrt{\frac{e(be-2cd)x^2}{(be-cd)(ex^2+d)}} \right)}{(cd-be)^2} + \frac{36c^2e^2 \left(\frac{e(be-2cd)x^2}{(be-cd)(ex^2+d)} \right)^{7/2}}{(cd-be)^2} {}_2F_1\left(2, \frac{7}{2}; \frac{9}{2}; \frac{e(be-2cd)x^2}{(be-cd)(ex^2+d)} \right) + \frac{12c^2e^2 \left(\frac{e(be-2cd)x^2}{(be-cd)(ex^2+d)} \right)}{(cd-be)^2} \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[1/((d + e*x^2)^(3/2)*(-(c*d^2) + b*d*e + b*e^2*x^2 + c*e^2*x^4)), x]

[Out] $-(x*(-315*\sqrt{(e*(-2*c*d + b*e)*x^2)/((-c*d) + b*e)*(d + e*x^2)}) + (420*c*e*x^2*\sqrt{(e*(-2*c*d + b*e)*x^2)/((-c*d) + b*e)*(d + e*x^2)})/(c*d - b*e) - (168*c^2*e^2*x^4*\sqrt{(e*(-2*c*d + b*e)*x^2)/((-c*d) + b*e)*(d + e*x^2)})/(c*d - b*e)^2 - 105*((e*(-2*c*d + b*e)*x^2)/((-c*d) + b*e)*(d + e*x^2))^{3/2} + (140*c*e*x^2*((e*(-2*c*d + b*e)*x^2)/((-c*d) + b*e)*(d + e*x^2))^{3/2})/(c*d - b*e) - (56*c^2*e^2*x^4*((e*(-2*c*d + b*e)*x^2)/((-c*d) + b*e)*(d + e*x^2))^{3/2})/(c*d - b*e)^2 + 315*\text{ArcTanh}[\sqrt{(e*(-2*c*d + b*e)*x^2)/((-c*d) + b*e)*(d + e*x^2)}] - (420*c*e*x^2*\text{ArcTanh}[\sqrt{(e*(-2*c*d + b*e)*x^2)/((-c*d) + b*e)*(d + e*x^2)}])/(c*d - b*e) + (168*c^2*e^2*x^4*\text{ArcTanh}[\sqrt{(e*(-2*c*d + b*e)*x^2)/((-c*d) + b*e)*(d + e*x^2)}])/(c*d - b*e)^2 + 48*((e*(-2*c*d + b*e)*x^2)/((-c*d) + b*e)*(d + e*x^2))^{7/2}*\text{Hypergeometric2F1}[2, 7/2, 9/2, (e*(-2*c*d + b*e)*x^2)/((-c*d) + b*e)*(d + e*x^2)] - (84*c*e*x^2*((e*(-2*c*d + b*e)*x^2)/((-c*d) + b*e)*(d + e*x^2))^{7/2}*\text{Hypergeometric2F1}[2, 7/2, 9/2, (e*(-2*c*d + b*e)*x^2)/((-c*d) + b*e)*(d + e*x^2)])/(c*d - b*e) + (36*c^2*e^2*x^4*((e*(-2*c*d + b*e)*x^2)/((-c*d) + b*e)*(d + e*x^2))^{7/2}*\text{Hypergeometric2F1}[2, 7/2, 9/2, (e*(-2*c*d + b*e)*x^2)/((-c*d) + b*e)*(d + e*x^2)])/(c*d - b*e)^2 + 12*((e*(-2*c*d + b*e)*x^2)/((-c*d) + b*e)*(d + e*x^2))^{7/2}*\text{HypergeometricPFQ}[\{2, 2, 7/2\}, \{1, 9/2\}, (e*(-2*c*d + b*e)*x^2)/((-c*d) + b*e)*(d + e*x^2)] - (24*c*e*x^2*((e*(-2*c*d + b*e)*x^2)/((-c*d) + b*e)*(d + e*x^2))^{7/2}*\text{HypergeometricPFQ}[\{2, 2, 7/2\}, \{1, 9/2\}, (e*(-2*c*d + b*e)*x^2)/((-c*d) + b*e)*(d + e*x^2)])/(c*d - b*e) + (12*c^2*e^2*x^4*((e*(-2*c*d + b*e)*x^2)/((-c*d) + b*e)*(d + e*x^2))^{7/2}*\text{HypergeometricPFQ}[\{2, 2, 7/2\}, \{1, 9/2\}, (e*(-2*c*d + b*e)*x^2)/((-c*d) + b*e)*(d + e*x^2)])/(c*d - b*e)^2)/(63*(c*d - b*e)*((e*(-2*c*d + b*e)*x^2)/((-c*d) + b*e)*(d + e*x^2))^{5/2}*(d + e*x^2)^{5/2})$

Maple [B] time = 0.02, size = 1637, normalized size = 11.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(e*x^2+d)^(3/2)/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2), x)

[Out] $-1/2*c^3*e/((-d*e)^{1/2}*c+(-(b*e-c*d)*c*e)^{1/2})/(-(-d*e)^{1/2}*c+(-(b*e-c*d)*c*e)^{1/2})/(-(b*e-c*d)*c*e)^{1/2}/(b*e-2*c*d)/((x-(-(b*e-c*d)*c*e)^{1/2}/c/e)^2*e+2*(-(b*e-c*d)*c*e)^{1/2}/c*(x-(-(b*e-c*d)*c*e)^{1/2}/c/e)-(b*e-2*c*d)/c)^{1/2}+1/2*c^2*e/((-d*e)^{1/2}*c+(-(b*e-c*d)*c*e)^{1/2})/(-(-d*e)^{1/2}*c+(-(b*e-c*d)*c*e)^{1/2})/((b*e-2*c*d)/d)/((x-(-(b*e-c*d)*c*e)^{1/2}/c/e)^2*e+2*(-(b*e-c*d)*c*e)^{1/2}/c*(x-(-(b*e-c*d)*c*e)^{1/2}/c/e)-(b*e-2*c*d)/c)^{1/2}*x+1/2*c^3*e/((-d*e)^{1/2}*c+(-(b*e-c*d)*c*e)^{1/2})/(-(-d*e)^{1/2}*c+(-(b*e-c*d)*c*e)^{1/2})/((b*e-2*c*d)/c)^{1/2}*ln((-2*(b*e-2*c*d)/c+2*(-(b*e-c*d)*c*e)^{1/2}/c*(x-(-(b*e-c*d)*c*e)^{1/2}/c/e)+2*(-(b*e-2*c*d)/c)^{1/2}*((x-(-(b*e-c*d)*c*e)^{1/2}/c/e)^2*e+2*(-(b*e-c*d)*c*e)^{1/2}/c*(x-(-(b*e-c*d)*c*e)^{1/2}/c/e)-(b*e-2*c*d)/c)^{1/2})/(x-(-(b*e-c*d)*c*e)^{1/2}/c/e)-1/6*c/d/((-d*e)^{1/2}*c+(-(b*e-c*d)*c*e)^{1/2})/(-(-d*e)^{1/2}*c+(-(b*e-c*d)*c*e)^{1/2})/(x+(-d*e)^{1/2}/e)/((x+(-d*e)^{1/2}/e)^2*e-2*(-d*e)^{1/2}*(x+(-d*e)^{1/2}/e))^{1/2}-1/3*c*e/d^2/((-d*e)^{1/2}*c+(-(b*e-c*d)*c*e)^{1/2})/(-(-d*e)^{1/2}*c+(-(b*e-c*d)*c*e)^{1/2})/((x+(-d*e)^{1/2}/e)^2*e-2*(-d*e)^{1/2}*(x+(-d*e)^{1/2}/e))^{1/2}*x-1/6*c/d/((-d*e)^{1/2}*c+(-(b*e-c*d)*c*e)^{1/2})/(-(-d*e)^{1/2}*c+(-(b*e-c*d)*c*e)^{1/2})/((x-(-d*e)^{1/2}/e)/((x-(-d*e)^{1/2}/e)^2*e+2*(-d*e)^{1/2}*(x-(-d*e)^{1/2}/e))^{1/2}-1/3*c*e/d^2/((-d*e)^{1/2}*c+(-(b*e-c*d)*c*e)^{1/2})/(-$

$$\frac{(-d*e)^{(1/2)*c+(-(b*e-c*d)*c*e)^{(1/2)}}/((x-(-d*e)^{(1/2)}/e)^{2*e+2*(-d*e)^{(1/2)}}*(x-(-d*e)^{(1/2)}/e))^{(1/2)*x+1/2*c^3*e/((-d*e)^{(1/2)*c+(-(b*e-c*d)*c*e)^{(1/2)}}/(-(-d*e)^{(1/2)*c+(-(b*e-c*d)*c*e)^{(1/2)}}/(-b*e-c*d)/c*e)^{(1/2)}/(b*e-2*c*d)/((x+(-b*e-c*d)*c*e)^{(1/2)}/c/e)^{2*e-2*(-b*e-c*d)*c*e)^{(1/2)}/c*(x+(-b*e-c*d)*c*e)^{(1/2)}/c/e)-(b*e-2*c*d)/c)^{(1/2)+1/2*c^2*e/((-d*e)^{(1/2)*c+(-(b*e-c*d)*c*e)^{(1/2)}}/(-(-d*e)^{(1/2)*c+(-(b*e-c*d)*c*e)^{(1/2)}}/(-b*e-c*d)/d)/((x+(-b*e-c*d)*c*e)^{(1/2)}/c/e)^{2*e-2*(-b*e-c*d)*c*e)^{(1/2)}/c*(x+(-b*e-c*d)*c*e)^{(1/2)}/c/e)-(b*e-2*c*d)/c)^{(1/2)*x-1/2*c^3*e/((-d*e)^{(1/2)*c+(-(b*e-c*d)*c*e)^{(1/2)}}/(-(-d*e)^{(1/2)*c+(-(b*e-c*d)*c*e)^{(1/2)}}/(-b*e-c*d)*c*e)^{(1/2)}/(b*e-2*c*d)/(-b*e-2*c*d)/c)^{(1/2)*\ln((-2*(b*e-2*c*d)/c-2*(-b*e-c*d)*c*e)^{(1/2)}/c*(x+(-b*e-c*d)*c*e)^{(1/2)}/c/e)+2*(-b*e-2*c*d)/c)^{(1/2)*((x+(-b*e-c*d)*c*e)^{(1/2)}/c/e)^{2*e-2*(-b*e-c*d)*c*e)^{(1/2)}/c*(x+(-b*e-c*d)*c*e)^{(1/2)}/c/e)-(b*e-2*c*d)/c)^{(1/2)}/(x+(-b*e-c*d)*c*e)^{(1/2)}/c/e))$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(ce^2x^4 + be^2x^2 - cd^2 + bde)(ex^2 + d)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x^2+d)^(3/2)/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm="maxima")

[Out] integrate(1/((c*e^2*x^4 + b*e^2*x^2 - c*d^2 + b*d*e)*(e*x^2 + d)^(3/2)), x)

Fricas [B] time = 5.15639, size = 2169, normalized size = 14.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x^2+d)^(3/2)/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm="fricas")

[Out] [1/12*(3*(c^2*d^2*e^2*x^4 + 2*c^2*d^3*e*x^2 + c^2*d^4)*sqrt(2*c^2*d^2*e - 3*b*c*d*e^2 + b^2*e^3)*log((c^2*d^4 - 2*b*c*d^3*e + b^2*d^2*e^2 + (17*c^2*d^2*e^2 - 24*b*c*d*e^3 + 8*b^2*e^4)*x^4 + 2*(7*c^2*d^3*e - 11*b*c*d^2*e^2 + 4*b^2*d*e^3)*x^2 - 4*sqrt(2*c^2*d^2*e - 3*b*c*d*e^2 + b^2*e^3))*((3*c*d*e - 2*b*e^2)*x^3 + (c*d^2 - b*d*e)*x)*sqrt(e*x^2 + d))/(c^2*e^2*x^4 + c^2*d^2 - 2*b*c*d*e + b^2*e^2 - 2*(c^2*d*e - b*c*e^2)*x^2) - 4*((14*c^3*d^3*e^2 - 25*b*c^2*d^2*e^3 + 13*b^2*c*d*e^4 - 2*b^3*e^5)*x^3 + 3*(6*c^3*d^4*e - 11*b*c^2*d^3*e^2 + 6*b^2*c*d^2*e^3 - b^3*d*e^4)*x)*sqrt(e*x^2 + d))/(8*c^4*d^8*e - 20*b*c^3*d^7*e^2 + 18*b^2*c^2*d^6*e^3 - 7*b^3*c*d^5*e^4 + b^4*d^4*e^5 + (8*c^4*d^6*e^3 - 20*b*c^3*d^5*e^4 + 18*b^2*c^2*d^4*e^5 - 7*b^3*c*d^3*e^6 + b^4*d^2*e^7)*x^4 + 2*(8*c^4*d^7*e^2 - 20*b*c^3*d^6*e^3 + 18*b^2*c^2*d^5*e^4 - 7*b^3*c*d^4*e^5 + b^4*d^3*e^6)*x^2), -1/6*(3*(c^2*d^2*e^2*x^4 + 2*c^2*d^3*e*x^2 + c^2*d^4)*sqrt(-2*c^2*d^2*e + 3*b*c*d*e^2 - b^2*e^3)*arctan(-1/2*sqrt(-2*c^2*d^2*e + 3*b*c*d*e^2 - b^2*e^3)*(c*d^2 - b*d*e + (3*c*d*e - 2*b*e^2)*x^2)*sqrt(e*x^2 + d)/((2*c^2*d^2*e^2 - 3*b*c*d*e^3 + b^2*e^4)*x^3 + (2*c^2*d^3*e - 3*b*c*d^2*e^2 + b^2*d*e^3)*x)) + 2*((14*c^3*d^3*e^2 - 25*b*c^2*d^2*e^3 + 13*b^2*c*d*e^4 - 2*b^3*e^5)*x^3 + 3*(6*c^3*d^4*e - 11*b*c^2*d^3*e^2 + 6*b^2*c*d^2*e^3 - b^3*d*e^4)*x)*sqrt(e*x^2 + d))/(8*c^4*d^8*e - 20*b*c^3*d^7*e^2 + 18*b^2*c^2*d^6*e^3 - 7*b^3*c*d^5*e^4 + b^4*d^4*e^5 + (8*c^4*d^6*e^3 - 20*b*c^3*d^5*e^4 + 18*b^2*c^2*d^4*e^5 - 7*b^3*c*d^3*e^6 + b^4*d^2*e^7)*x^4 + 2*(8*c^4*d^7*e^2 - 20*b*c^3*d^6*e^3 + 18*b^2*c^2*d^5*e^4 - 7*b^3*c*d^4*e^5 + b^4*d^3*e^6)*x^2)

$$e^3 - 20*b*c^3*d^5*e^4 + 18*b^2*c^2*d^4*e^5 - 7*b^3*c*d^3*e^6 + b^4*d^2*e^7) * x^4 + 2*(8*c^4*d^7*e^2 - 20*b*c^3*d^6*e^3 + 18*b^2*c^2*d^5*e^4 - 7*b^3*c*d^4*e^5 + b^4*d^3*e^6) * x^2]$$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(d + ex^2)^{\frac{5}{2}} (be - cd + cex^2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x**2+d)**(3/2)/(c*e**2*x**4+b*e**2*x**2+b*d*e-c*d**2),x)

[Out] Integral(1/((d + e*x**2)**(5/2)*(b*e - c*d + c*e*x**2)), x)

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x^2+d)^(3/2)/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm="giac")

[Out] Timed out

3.225 $\int (1 + x^2)^3 \sqrt{1 + x^2 + x^4} dx$

Optimal. Leaf size=183

$$\frac{7(x^2 + 1) \sqrt{\frac{x^4 + x^2 + 1}{(x^2 + 1)^2}} \text{EllipticF}\left(2 \tan^{-1}(x), \frac{1}{4}\right)}{15\sqrt{x^4 + x^2 + 1}} + \frac{1}{9}(x^4 + x^2 + 1)^{3/2} x^3 + \frac{1}{3}(x^4 + x^2 + 1)^{3/2} x + \frac{2}{45}(6x^2 + 7) \sqrt{x^4 + x^2 + 1}$$

[Out] (26*x*Sqrt[1 + x^2 + x^4])/(45*(1 + x^2)) + (2*x*(7 + 6*x^2)*Sqrt[1 + x^2 + x^4])/45 + (x*(1 + x^2 + x^4)^(3/2))/3 + (x^3*(1 + x^2 + x^4)^(3/2))/9 - (26*(1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*EllipticE[2*ArcTan[x], 1/4])/(45*Sqrt[1 + x^2 + x^4]) + (7*(1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*EllipticF[2*ArcTan[x], 1/4])/(15*Sqrt[1 + x^2 + x^4])

Rubi [A] time = 0.0882292, antiderivative size = 183, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.3$, Rules used = {1206, 1679, 1176, 1197, 1103, 1195}

$$\frac{1}{9}(x^4 + x^2 + 1)^{3/2} x^3 + \frac{1}{3}(x^4 + x^2 + 1)^{3/2} x + \frac{2}{45}(6x^2 + 7) \sqrt{x^4 + x^2 + 1} x + \frac{26\sqrt{x^4 + x^2 + 1}x}{45(x^2 + 1)} + \frac{7(x^2 + 1) \sqrt{\frac{x^4 + x^2 + 1}{(x^2 + 1)^2}}}{15\sqrt{x^4 + x^2 + 1}}$$

Antiderivative was successfully verified.

[In] Int[(1 + x^2)^3*Sqrt[1 + x^2 + x^4], x]

[Out] (26*x*Sqrt[1 + x^2 + x^4])/(45*(1 + x^2)) + (2*x*(7 + 6*x^2)*Sqrt[1 + x^2 + x^4])/45 + (x*(1 + x^2 + x^4)^(3/2))/3 + (x^3*(1 + x^2 + x^4)^(3/2))/9 - (26*(1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*EllipticE[2*ArcTan[x], 1/4])/(45*Sqrt[1 + x^2 + x^4]) + (7*(1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*EllipticF[2*ArcTan[x], 1/4])/(15*Sqrt[1 + x^2 + x^4])

Rule 1206

Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := Simp[(e^q*x^(2*q - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(4*p + 2*q + 1)), x] + Dist[1/(c*(4*p + 2*q + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*q + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1]

Rule 1679

Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := With[{q = Expon[Pq, x^2], e = Coeff[Pq, x^2, Expon[Pq, x^2]]}, Simp[(e*x^(2*q - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(2*q + 4*p + 1)), x] + Dist[1/(c*(2*q + 4*p + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*q + 4*p + 1)*Pq - a*e*(2*q - 3)*x^(2*q - 4) - b*e*(2*q + 2*p - 1)*x^(2*q - 2) - c*e*(2*q + 4*p + 1)*x^(2*q), x], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && !LtQ[p, -1]

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:= Simp[(x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*(a + b*x^2 + c*x^4)^p)/(c*(4*p + 1)*(4*p + 3)), x] + Dist[(2*p)/(c*(4*p + 1)*(4*p + 3)),
Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) -
b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol]
:= With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; NeQ[e + d*q, 0] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] := With[{q = Rt[c/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]), x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol]
:= With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c*x^4]), x] /; EqQ[e + d*q^2, 0] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\begin{aligned} \int (1+x^2)^3 \sqrt{1+x^2+x^4} dx &= \frac{1}{9}x^3(1+x^2+x^4)^{3/2} + \frac{1}{9} \int \sqrt{1+x^2+x^4} (9+24x^2+21x^4) dx \\ &= \frac{1}{3}x(1+x^2+x^4)^{3/2} + \frac{1}{9}x^3(1+x^2+x^4)^{3/2} + \frac{1}{63} \int (42+84x^2) \sqrt{1+x^2+x^4} dx \\ &= \frac{2}{45}x(7+6x^2) \sqrt{1+x^2+x^4} + \frac{1}{3}x(1+x^2+x^4)^{3/2} + \frac{1}{9}x^3(1+x^2+x^4)^{3/2} + \frac{1}{945} \int \frac{336+336x^2}{\sqrt{1+x^2+x^4}} dx \\ &= \frac{2}{45}x(7+6x^2) \sqrt{1+x^2+x^4} + \frac{1}{3}x(1+x^2+x^4)^{3/2} + \frac{1}{9}x^3(1+x^2+x^4)^{3/2} - \frac{26}{45} \int \frac{1-x^2}{\sqrt{1+x^2+x^4}} dx \\ &= \frac{26x\sqrt{1+x^2+x^4}}{45(1+x^2)} + \frac{2}{45}x(7+6x^2) \sqrt{1+x^2+x^4} + \frac{1}{3}x(1+x^2+x^4)^{3/2} + \frac{1}{9}x^3(1+x^2+x^4)^{3/2} \end{aligned}$$

Mathematica [C] time = 0.313023, size = 169, normalized size = 0.92

$$\frac{2(-1)^{5/6} (4\sqrt{3} + 9i) \sqrt{\sqrt[3]{-1}x^2 + 1} \sqrt{1 - (-1)^{2/3}x^2} \text{EllipticF}\left(i \sinh^{-1}\left((-1)^{5/6}x\right), (-1)^{2/3}\right) + x(5x^{10} + 25x^8 + 57x^6 + 81x^4 + 45\sqrt{x^4 + x^2 + 1})}{45\sqrt{x^4 + x^2 + 1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^2)^3*Sqrt[1 + x^2 + x^4],x]
```

```
[Out] (x*(29 + 61*x^2 + 81*x^4 + 57*x^6 + 25*x^8 + 5*x^10) + 26*(-1)^(1/3)*Sqrt[1
+ (-1)^(1/3)*x^2]*Sqrt[1 - (-1)^(2/3)*x^2]*EllipticE[I*ArcSinh[(-1)^(5/6)*
x], (-1)^(2/3)] + 2*(-1)^(5/6)*(9*I + 4*Sqrt[3])*Sqrt[1 + (-1)^(1/3)*x^2]*S
qrt[1 - (-1)^(2/3)*x^2]*EllipticF[I*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)])/(45
*Sqrt[1 + x^2 + x^4])
```

Maple [C] time = 0.174, size = 263, normalized size = 1.4

$$\frac{x^7}{9}\sqrt{x^4+x^2+1} + \frac{4x^5}{9}\sqrt{x^4+x^2+1} + \frac{32x^3}{45}\sqrt{x^4+x^2+1} + \frac{29x}{45}\sqrt{x^4+x^2+1} + \frac{32}{45\sqrt{-2+2i\sqrt{3}}}\sqrt{1-\left(-\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)^3*(x^4+x^2+1)^(1/2), x)
```

```
[Out] 1/9*x^7*(x^4+x^2+1)^(1/2)+4/9*x^5*(x^4+x^2+1)^(1/2)+32/45*x^3*(x^4+x^2+1)^(
1/2)+29/45*x*(x^4+x^2+1)^(1/2)+32/45/(-2+2*I*3^(1/2))^(1/2)*(1-(-1/2+1/2*I*
3^(1/2))*x^2)^(1/2)*(1-(-1/2-1/2*I*3^(1/2))*x^2)^(1/2)/(x^4+x^2+1)^(1/2)*El
lipticF(1/2*x*(-2+2*I*3^(1/2))^(1/2), 1/2*(-2+2*I*3^(1/2))^(1/2))-104/45/(-2
+2*I*3^(1/2))^(1/2)*(1-(-1/2+1/2*I*3^(1/2))*x^2)^(1/2)*(1-(-1/2-1/2*I*3^(1/
2))*x^2)^(1/2)/(x^4+x^2+1)^(1/2)/(I*3^(1/2)+1)*(EllipticF(1/2*x*(-2+2*I*3^(
1/2))^(1/2), 1/2*(-2+2*I*3^(1/2))^(1/2))-EllipticE(1/2*x*(-2+2*I*3^(1/2))^(1
/2), 1/2*(-2+2*I*3^(1/2))^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{x^4+x^2+1}(x^2+1)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^3*(x^4+x^2+1)^(1/2), x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^4 + x^2 + 1)*(x^2 + 1)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(x^6+3x^4+3x^2+1\right)\sqrt{x^4+x^2+1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^3*(x^4+x^2+1)^(1/2), x, algorithm="fricas")
```

```
[Out] integral((x^6 + 3*x^4 + 3*x^2 + 1)*sqrt(x^4 + x^2 + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{(x^2-x+1)(x^2+x+1)}(x^2+1)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**2+1)**3*(x**4+x**2+1)**(1/2),x)

[Out] Integral(sqrt((x**2 - x + 1)*(x**2 + x + 1))*(x**2 + 1)**3, x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{x^4 + x^2 + 1}(x^2 + 1)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+1)^3*(x^4+x^2+1)^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(x^4 + x^2 + 1)*(x^2 + 1)^3, x)

3.226 $\int (1 + x^2)^2 \sqrt{1 + x^2 + x^4} dx$

Optimal. Leaf size=164

$$\frac{4(x^2 + 1) \sqrt{\frac{x^4 + x^2 + 1}{(x^2 + 1)^2}} \text{EllipticF}\left(2 \tan^{-1}(x), \frac{1}{4}\right)}{7\sqrt{x^4 + x^2 + 1}} + \frac{1}{7}x(x^4 + x^2 + 1)^{3/2} + \frac{2}{21}x(3x^2 + 4)\sqrt{x^4 + x^2 + 1} + \frac{2x\sqrt{x^4 + x^2 + 1}}{3(x^2 + 1)}$$

```
[Out] (2*x*Sqrt[1 + x^2 + x^4])/(3*(1 + x^2)) + (2*x*(4 + 3*x^2)*Sqrt[1 + x^2 + x^4])/21 + (x*(1 + x^2 + x^4)^(3/2))/7 - (2*(1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*EllipticE[2*ArcTan[x], 1/4])/(3*Sqrt[1 + x^2 + x^4]) + (4*(1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*EllipticF[2*ArcTan[x], 1/4])/(7*Sqrt[1 + x^2 + x^4])
```

Rubi [A] time = 0.0605862, antiderivative size = 164, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$, Rules used = {1206, 1176, 1197, 1103, 1195}

$$\frac{1}{7}x(x^4 + x^2 + 1)^{3/2} + \frac{2}{21}x(3x^2 + 4)\sqrt{x^4 + x^2 + 1} + \frac{2x\sqrt{x^4 + x^2 + 1}}{3(x^2 + 1)} + \frac{4(x^2 + 1) \sqrt{\frac{x^4 + x^2 + 1}{(x^2 + 1)^2}} F\left(2 \tan^{-1}(x) \middle| \frac{1}{4}\right)}{7\sqrt{x^4 + x^2 + 1}} - \frac{2(x^2 + 1)}{3(x^2 + 1)}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^2)^2*Sqrt[1 + x^2 + x^4], x]
```

```
[Out] (2*x*Sqrt[1 + x^2 + x^4])/(3*(1 + x^2)) + (2*x*(4 + 3*x^2)*Sqrt[1 + x^2 + x^4])/21 + (x*(1 + x^2 + x^4)^(3/2))/7 - (2*(1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*EllipticE[2*ArcTan[x], 1/4])/(3*Sqrt[1 + x^2 + x^4]) + (4*(1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*EllipticF[2*ArcTan[x], 1/4])/(7*Sqrt[1 + x^2 + x^4])
```

Rule 1206

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := Simp[(e^q*x^(2*q - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(4*p + 2*q + 1)), x] + Dist[1/(c*(4*p + 2*q + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*q + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := Simp[(x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*(a + b*x^2 + c*x^4)^p)/(c*(4*p + 1)*(4*p + 3)), x] + Dist[(2*p)/(c*(4*p + 1)*(4*p + 3)), Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) - b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol]
:> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; NeQ[e + d*q, 0] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]), x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol]
:> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c*x^4]), x] /; EqQ[e + d*q^2, 0] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\begin{aligned} \int (1+x^2)^2 \sqrt{1+x^2+x^4} dx &= \frac{1}{7}x(1+x^2+x^4)^{3/2} + \frac{1}{7} \int (6+10x^2) \sqrt{1+x^2+x^4} dx \\ &= \frac{2}{21}x(4+3x^2) \sqrt{1+x^2+x^4} + \frac{1}{7}x(1+x^2+x^4)^{3/2} + \frac{1}{105} \int \frac{50+70x^2}{\sqrt{1+x^2+x^4}} dx \\ &= \frac{2}{21}x(4+3x^2) \sqrt{1+x^2+x^4} + \frac{1}{7}x(1+x^2+x^4)^{3/2} - \frac{2}{3} \int \frac{1-x^2}{\sqrt{1+x^2+x^4}} dx + \frac{8}{7} \int \frac{1}{\sqrt{1+x^2}} dx \\ &= \frac{2x\sqrt{1+x^2+x^4}}{3(1+x^2)} + \frac{2}{21}x(4+3x^2) \sqrt{1+x^2+x^4} + \frac{1}{7}x(1+x^2+x^4)^{3/2} - \frac{2(1+x^2) \sqrt{\frac{1+x^2}{1+x^2}}}{3\sqrt{1+x^2}} \end{aligned}$$

Mathematica [C] time = 0.154316, size = 162, normalized size = 0.99

$$\frac{2\sqrt[3]{-1}(5\sqrt[3]{-1}-7)\sqrt{\sqrt[3]{-1}x^2+1}\sqrt{1-(-1)^{2/3}x^2}\text{EllipticF}\left(i\sinh^{-1}\left((-1)^{5/6}x\right),(-1)^{2/3}\right)+x(3x^8+12x^6+23x^4+20x^2+11)}{21\sqrt{x^4+x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^2)^2*Sqrt[1 + x^2 + x^4], x]
```

```
[Out] (x*(11 + 20*x^2 + 23*x^4 + 12*x^6 + 3*x^8) + 14*(-1)^(1/3)*Sqrt[1 + (-1)^(1/3)*x^2]*Sqrt[1 - (-1)^(2/3)*x^2]*EllipticE[I*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)] + 2*(-1)^(1/3)*(-7 + 5*(-1)^(1/3))*Sqrt[1 + (-1)^(1/3)*x^2]*Sqrt[1 - (-1)^(2/3)*x^2]*EllipticF[I*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)]/(21*Sqrt[1 + x^2 + x^4])
```

Maple [C] time = 0.006, size = 248, normalized size = 1.5

$$\frac{x^5\sqrt{x^4+x^2+1} + \frac{3x^3}{7}\sqrt{x^4+x^2+1} + \frac{11x}{21}\sqrt{x^4+x^2+1} + \frac{20}{21\sqrt{-2+2i\sqrt{3}}}\sqrt{1-\left(-\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)x^2}\sqrt{1-\left(-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)x^2}}{21\sqrt{x^4+x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^2+1)^2*(x^4+x^2+1)^(1/2),x)`

[Out] $1/7*x^5*(x^4+x^2+1)^{(1/2)}+3/7*x^3*(x^4+x^2+1)^{(1/2)}+11/21*x*(x^4+x^2+1)^{(1/2)}+20/21/(-2+2*I*3^{(1/2)})^{(1/2)}*(1-(-1/2+1/2*I*3^{(1/2)})*x^2)^{(1/2)}*(1-(-1/2-1/2*I*3^{(1/2)})*x^2)^{(1/2)}/(x^4+x^2+1)^{(1/2)}*EllipticF(1/2*x*(-2+2*I*3^{(1/2)})^{(1/2)},1/2*(-2+2*I*3^{(1/2)})^{(1/2)})-8/3/(-2+2*I*3^{(1/2)})^{(1/2)}*(1-(-1/2+1/2*I*3^{(1/2)})*x^2)^{(1/2)}*(1-(-1/2-1/2*I*3^{(1/2)})*x^2)^{(1/2)}/(x^4+x^2+1)^{(1/2)}/(I*3^{(1/2)}+1)*(EllipticF(1/2*x*(-2+2*I*3^{(1/2)})^{(1/2)},1/2*(-2+2*I*3^{(1/2)})^{(1/2)})^{(1/2)})-EllipticE(1/2*x*(-2+2*I*3^{(1/2)})^{(1/2)},1/2*(-2+2*I*3^{(1/2)})^{(1/2)})$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{x^4 + x^2 + 1} (x^2 + 1)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^2+1)^2*(x^4+x^2+1)^(1/2),x, algorithm="maxima")`

[Out] `integrate(sqrt(x^4 + x^2 + 1)*(x^2 + 1)^2, x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(x^4 + 2x^2 + 1\right)\sqrt{x^4 + x^2 + 1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^2+1)^2*(x^4+x^2+1)^(1/2),x, algorithm="fricas")`

[Out] `integral((x^4 + 2*x^2 + 1)*sqrt(x^4 + x^2 + 1), x)`

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{(x^2 - x + 1)(x^2 + x + 1)} (x^2 + 1)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x**2+1)**2*(x**4+x**2+1)**(1/2),x)`

[Out] `Integral(sqrt((x**2 - x + 1)*(x**2 + x + 1))*(x**2 + 1)**2, x)`

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{x^4 + x^2 + 1} (x^2 + 1)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^2*(x^4+x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^4 + x^2 + 1)*(x^2 + 1)^2, x)
```

3.227 $\int (1 + x^2) \sqrt{1 + x^2 + x^4} dx$

Optimal. Leaf size=145

$$\frac{3(x^2 + 1) \sqrt{\frac{x^4 + x^2 + 1}{(x^2 + 1)^2}} \operatorname{EllipticF}\left(2 \tan^{-1}(x), \frac{1}{4}\right)}{5\sqrt{x^4 + x^2 + 1}} + \frac{1}{5}(x^2 + 2) \sqrt{x^4 + x^2 + 1}x + \frac{3\sqrt{x^4 + x^2 + 1}x}{5(x^2 + 1)} - \frac{3(x^2 + 1) \sqrt{\frac{x^4 + x^2 + 1}{(x^2 + 1)^2}} E\left(2 \tan^{-1}(x)\right)}{5\sqrt{x^4 + x^2 + 1}}$$

```
[Out] (3*x*Sqrt[1 + x^2 + x^4])/(5*(1 + x^2)) + (x*(2 + x^2)*Sqrt[1 + x^2 + x^4])
/5 - (3*(1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*EllipticE[2*ArcTan[x],
1/4])/(5*Sqrt[1 + x^2 + x^4]) + (3*(1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)
^2]*EllipticF[2*ArcTan[x], 1/4])/(5*Sqrt[1 + x^2 + x^4])
```

Rubi [A] time = 0.0425715, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {1176, 1197, 1103, 1195}

$$\frac{1}{5}(x^2 + 2) \sqrt{x^4 + x^2 + 1}x + \frac{3\sqrt{x^4 + x^2 + 1}x}{5(x^2 + 1)} + \frac{3(x^2 + 1) \sqrt{\frac{x^4 + x^2 + 1}{(x^2 + 1)^2}} F\left(2 \tan^{-1}(x) \middle| \frac{1}{4}\right)}{5\sqrt{x^4 + x^2 + 1}} - \frac{3(x^2 + 1) \sqrt{\frac{x^4 + x^2 + 1}{(x^2 + 1)^2}} E\left(2 \tan^{-1}(x)\right)}{5\sqrt{x^4 + x^2 + 1}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^2)*Sqrt[1 + x^2 + x^4], x]
```

```
[Out] (3*x*Sqrt[1 + x^2 + x^4])/(5*(1 + x^2)) + (x*(2 + x^2)*Sqrt[1 + x^2 + x^4])
/5 - (3*(1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*EllipticE[2*ArcTan[x],
1/4])/(5*Sqrt[1 + x^2 + x^4]) + (3*(1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)
^2]*EllipticF[2*ArcTan[x], 1/4])/(5*Sqrt[1 + x^2 + x^4])
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[(x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*(a + b*x^2 + c
*x^4)^p)/(c*(4*p + 1)*(4*p + 3)), x] + Dist[(2*p)/(c*(4*p + 1)*(4*p + 3)),
Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) -
b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol]
:> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4],
x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; NeQ[e + d*q, 0] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)]/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

Int[((d_) + (e_)*(x_)^2)/Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4], x_Symbol] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c*x^4]), x] /; EqQ[e + d*q^2, 0] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]

Rubi steps

$$\begin{aligned} \int (1+x^2)\sqrt{1+x^2+x^4} dx &= \frac{1}{5}x(2+x^2)\sqrt{1+x^2+x^4} + \frac{1}{15} \int \frac{9+9x^2}{\sqrt{1+x^2+x^4}} dx \\ &= \frac{1}{5}x(2+x^2)\sqrt{1+x^2+x^4} - \frac{3}{5} \int \frac{1-x^2}{\sqrt{1+x^2+x^4}} dx + \frac{6}{5} \int \frac{1}{\sqrt{1+x^2+x^4}} dx \\ &= \frac{3x\sqrt{1+x^2+x^4}}{5(1+x^2)} + \frac{1}{5}x(2+x^2)\sqrt{1+x^2+x^4} - \frac{3(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}}E\left(2\tan^{-1}(x)\middle|\frac{1}{4}\right)}{5\sqrt{1+x^2+x^4}} + \dots \end{aligned}$$

Mathematica [C] time = 0.183043, size = 168, normalized size = 1.16

$$\frac{\frac{3}{2}\sqrt{2+(1-i\sqrt{3})x^2}\sqrt{2+(1+i\sqrt{3})x^2}\text{EllipticF}\left(\sin^{-1}\left(\frac{1}{2}(x+i\sqrt{3}x)\right), \frac{1}{2}i(\sqrt{3}+i)\right) + x^7 + 3x^5 + 3x^3 + 3\sqrt{-1}\sqrt{\sqrt{3}-1x^2}}{5\sqrt{x^4+x^2+1}}}$$

Antiderivative was successfully verified.

[In] Integrate[(1 + x^2)*Sqrt[1 + x^2 + x^4], x]

[Out] (2*x + 3*x^3 + 3*x^5 + x^7 + 3*(-1)^(1/3)*Sqrt[1 + (-1)^(1/3)*x^2]*Sqrt[1 - (-1)^(2/3)*x^2]*EllipticE[I*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)] + (3*Sqrt[2 + (1 - I*Sqrt[3])*x^2]*Sqrt[2 + (1 + I*Sqrt[3])*x^2]*EllipticF[ArcSin[(x + I*Sqrt[3]*x)/2], (I/2)*(I + Sqrt[3])])/2)/(5*Sqrt[1 + x^2 + x^4])

Maple [C] time = 0.006, size = 233, normalized size = 1.6

$$\frac{x^3\sqrt{x^4+x^2+1} + \frac{2x}{5}\sqrt{x^4+x^2+1} + \frac{6}{5\sqrt{-2+2i\sqrt{3}}}\sqrt{1-\left(-\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)x^2}\sqrt{1-\left(-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)x^2}\text{EllipticF}\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2}\right)}{5\sqrt{x^4+x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2+1)*(x^4+x^2+1)^(1/2), x)

[Out] 1/5*x^3*(x^4+x^2+1)^(1/2)+2/5*x*(x^4+x^2+1)^(1/2)+6/5/(-2+2*I*3^(1/2))^(1/2)*(1-(-1/2+1/2*I*3^(1/2))*x^2)^(1/2)*(1-(-1/2-1/2*I*3^(1/2))*x^2)^(1/2)/(x^4+x^2+1)^(1/2)*EllipticF(1/2*x*(-2+2*I*3^(1/2))^(1/2), 1/2*(-2+2*I*3^(1/2))^(1/2))-12/5/(-2+2*I*3^(1/2))^(1/2)*(1-(-1/2+1/2*I*3^(1/2))*x^2)^(1/2)*(1-(-1/2-1/2*I*3^(1/2))*x^2)^(1/2)/(x^4+x^2+1)^(1/2)/(I*3^(1/2)+1)*(EllipticF(1/2*x*(-2+2*I*3^(1/2))^(1/2), 1/2*(-2+2*I*3^(1/2))^(1/2))-EllipticE(1/2*x*(-2+2*I*3^(1/2))^(1/2), 1/2*(-2+2*I*3^(1/2))^(1/2)))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{x^4 + x^2 + 1}(x^2 + 1) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+1)*(x^4+x^2+1)^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(x^4 + x^2 + 1)*(x^2 + 1), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{x^4 + x^2 + 1}(x^2 + 1), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+1)*(x^4+x^2+1)^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(x^4 + x^2 + 1)*(x^2 + 1), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{(x^2 - x + 1)(x^2 + x + 1)}(x^2 + 1) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**2+1)*(x**4+x**2+1)**(1/2),x)

[Out] Integral(sqrt((x**2 - x + 1)*(x**2 + x + 1))*(x**2 + 1), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{x^4 + x^2 + 1}(x^2 + 1) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+1)*(x^4+x^2+1)^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(x^4 + x^2 + 1)*(x^2 + 1), x)

$$3.228 \quad \int \frac{\sqrt{1+x^2+x^4}}{1+x^2} dx$$

Optimal. Leaf size=137

$$\frac{3(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}\text{EllipticF}\left(2\tan^{-1}(x), \frac{1}{4}\right)}{4\sqrt{x^4+x^2+1}} + \frac{\sqrt{x^4+x^2+1}x}{x^2+1} + \frac{1}{2}\tan^{-1}\left(\frac{x}{\sqrt{x^4+x^2+1}}\right) - \frac{(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}E\left(2\tan^{-1}(x)\right)}{\sqrt{x^4+x^2+1}}$$

[Out] (x*Sqrt[1 + x^2 + x^4])/(1 + x^2) + ArcTan[x/Sqrt[1 + x^2 + x^4]]/2 - ((1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*EllipticE[2*ArcTan[x], 1/4])/Sqrt[1 + x^2 + x^4] + (3*(1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*EllipticF[2*ArcTan[x], 1/4])/(4*Sqrt[1 + x^2 + x^4])

Rubi [A] time = 0.0861591, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.35$, Rules used = {1208, 1139, 1103, 1195, 1210, 1698, 203}

$$\frac{\sqrt{x^4+x^2+1}x}{x^2+1} + \frac{1}{2}\tan^{-1}\left(\frac{x}{\sqrt{x^4+x^2+1}}\right) + \frac{3(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}F\left(2\tan^{-1}(x)\middle|\frac{1}{4}\right)}{4\sqrt{x^4+x^2+1}} - \frac{(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}E\left(2\tan^{-1}(x)\middle|\frac{1}{4}\right)}{\sqrt{x^4+x^2+1}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[1 + x^2 + x^4]/(1 + x^2), x]

[Out] (x*Sqrt[1 + x^2 + x^4])/(1 + x^2) + ArcTan[x/Sqrt[1 + x^2 + x^4]]/2 - ((1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*EllipticE[2*ArcTan[x], 1/4])/Sqrt[1 + x^2 + x^4] + (3*(1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*EllipticF[2*ArcTan[x], 1/4])/(4*Sqrt[1 + x^2 + x^4])

Rule 1208

Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_)/((d_) + (e_.)*(x_)^2), x_Symbol] := -Dist[(e^2)^(-1), Int[(c*d - b*e - c*e*x^2)*(a + b*x^2 + c*x^4)^(p - 1), x], x] + Dist[(c*d^2 - b*d*e + a*e^2)/e^2, Int[(a + b*x^2 + c*x^4)^(p - 1)/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p + 1/2, 0]

Rule 1139

Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] := With[{q = Rt[c/a, 2]}, Dist[1/q, Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]

Rule 1103

Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] := With[{q = Rt[c/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]

Rule 1195

```
Int[((d_) + (e_)*(x_)^2)/Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4], x_Symbol]
:= With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^2*x^2)), x] +
Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c*x^4]), x] /;
EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1210

```
Int[1/(((d_) + (e_)*(x_)^2)*Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4]), x_Symbol]
:= Dist[1/(2*d), Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] + Dist[1/(2*d), Int[(d - e*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x] /;
FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[c*d^2 - a*e^2, 0]
```

Rule 1698

```
Int[((A_) + (B_)*(x_)^2)/(((d_) + (e_)*(x_)^2)*Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4]), x_Symbol]
:= Dist[A, Subst[Int[1/(d - (b*d - 2*a*e)*x^2), x], x, x/Sqrt[a + b*x^2 + c*x^4]], x] /; FreeQ[{a, b, c, d, e, A, B}, x] &&
NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[c*d^2 - a*e^2, 0] && EqQ[B*d + A*e, 0]
```

Rule 203

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /;
FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{1+x^2+x^4}}{1+x^2} dx &= \int \frac{x^2}{\sqrt{1+x^2+x^4}} dx + \int \frac{1}{(1+x^2)\sqrt{1+x^2+x^4}} dx \\ &= \frac{1}{2} \int \frac{1}{\sqrt{1+x^2+x^4}} dx + \frac{1}{2} \int \frac{1-x^2}{(1+x^2)\sqrt{1+x^2+x^4}} dx + \int \frac{1}{\sqrt{1+x^2+x^4}} dx - \int \frac{1-x^2}{\sqrt{1+x^2+x^4}} dx \\ &= \frac{x\sqrt{1+x^2+x^4}}{1+x^2} - \frac{(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}} E\left(2 \tan^{-1}(x) \middle| \frac{1}{4}\right)}{\sqrt{1+x^2+x^4}} + \frac{3(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}} F\left(2 \tan^{-1}(x) \middle| \frac{1}{4}\right)}{4\sqrt{1+x^2+x^4}} + \dots \\ &= \frac{x\sqrt{1+x^2+x^4}}{1+x^2} + \frac{1}{2} \tan^{-1}\left(\frac{x}{\sqrt{1+x^2+x^4}}\right) - \frac{(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}} E\left(2 \tan^{-1}(x) \middle| \frac{1}{4}\right)}{\sqrt{1+x^2+x^4}} + \frac{3(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}} F\left(2 \tan^{-1}(x) \middle| \frac{1}{4}\right)}{4\sqrt{1+x^2+x^4}} + \dots \end{aligned}$$

Mathematica [C] time = 0.0951932, size = 118, normalized size = 0.86

$$\frac{\sqrt[3]{-1}\sqrt{\sqrt[3]{-1}x^2 + 1}\sqrt{1 - (-1)^{2/3}x^2} \left(\text{EllipticF}\left(i \sinh^{-1}\left((-1)^{5/6}x\right), (-1)^{2/3}\right) - E\left(i \sinh^{-1}\left((-1)^{5/6}x\right) \middle| (-1)^{2/3}\right) + \sqrt[3]{-1}\Pi\left(\dots\right) \right)}{\sqrt{x^4 + x^2 + 1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + x^2 + x^4]/(1 + x^2), x]
```

```
[Out] -(((-1)^(1/3)*Sqrt[1 + (-1)^(1/3)*x^2]*Sqrt[1 - (-1)^(2/3)*x^2]*(-EllipticE[I*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)] + EllipticF[I*ArcSinh[(-1)^(5/6)*x],
```

$(-1)^{(2/3)} + (-1)^{(1/3)} * \text{EllipticPi} [(-1)^{(1/3)}, (-I) * \text{ArcSinh} [(-1)^{(5/6)} * x], (-1)^{(2/3)}] / \text{Sqrt} [1 + x^2 + x^4]$

Maple [C] time = 0.052, size = 293, normalized size = 2.1

$$-4 \frac{\sqrt{1 + 1/2 x^2 - i/2 x^2 \sqrt{3}} \sqrt{1 + 1/2 x^2 + i/2 x^2 \sqrt{3}} \text{EllipticF} \left(1/2 x \sqrt{-2 + 2 i \sqrt{3}}, 1/2 \sqrt{-2 + 2 i \sqrt{3}} \right)}{\sqrt{-2 + 2 i \sqrt{3}} \sqrt{x^4 + x^2 + 1} (i \sqrt{3} + 1)} + 4 \frac{\sqrt{1 + 1/2 x^2 - i/2 x^2 \sqrt{3}}}{\sqrt{-2 + 2 i \sqrt{3}} \sqrt{x^4 + x^2 + 1} (i \sqrt{3} + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^4+x^2+1)^(1/2)/(x^2+1),x)

[Out] $-4/(-2+2*I*3^{(1/2)})^{(1/2)}*(1+1/2*x^2-1/2*I*x^2*3^{(1/2)})^{(1/2)}*(1+1/2*x^2+1/2*I*x^2*3^{(1/2)})^{(1/2)}/(x^4+x^2+1)^{(1/2)}/(I*3^{(1/2)}+1)*\text{EllipticF}(1/2*x*(-2+2*I*3^{(1/2)})^{(1/2)},1/2*(-2+2*I*3^{(1/2)})^{(1/2)})+4/(-2+2*I*3^{(1/2)})^{(1/2)}*(1+1/2*x^2-1/2*I*x^2*3^{(1/2)})^{(1/2)}*(1+1/2*x^2+1/2*I*x^2*3^{(1/2)})^{(1/2)}/(x^4+x^2+1)^{(1/2)}/(I*3^{(1/2)}+1)*\text{EllipticE}(1/2*x*(-2+2*I*3^{(1/2)})^{(1/2)},1/2*(-2+2*I*3^{(1/2)})^{(1/2)})+1/(-1/2+1/2*I*3^{(1/2)})^{(1/2)}*(1+1/2*x^2-1/2*I*x^2*3^{(1/2)})^{(1/2)}*(1+1/2*x^2+1/2*I*x^2*3^{(1/2)})^{(1/2)}/(x^4+x^2+1)^{(1/2)}*\text{EllipticPi}((-1/2+1/2*I*3^{(1/2)})^{(1/2)}*x,-1/(-1/2+1/2*I*3^{(1/2)}),(-1/2-1/2*I*3^{(1/2)})^{(1/2)}/(-1/2+1/2*I*3^{(1/2)})^{(1/2)})$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^4 + x^2 + 1}}{x^2 + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4+x^2+1)^(1/2)/(x^2+1),x, algorithm="maxima")

[Out] integrate(sqrt(x^4 + x^2 + 1)/(x^2 + 1), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral} \left(\frac{\sqrt{x^4 + x^2 + 1}}{x^2 + 1}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4+x^2+1)^(1/2)/(x^2+1),x, algorithm="fricas")

[Out] integral(sqrt(x^4 + x^2 + 1)/(x^2 + 1), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{(x^2 - x + 1)(x^2 + x + 1)}}{x^2 + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**4+x**2+1)**(1/2)/(x**2+1),x)

[Out] Integral(sqrt((x**2 - x + 1)*(x**2 + x + 1))/(x**2 + 1), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^4 + x^2 + 1}}{x^2 + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4+x^2+1)^(1/2)/(x^2+1),x, algorithm="giac")

[Out] integrate(sqrt(x^4 + x^2 + 1)/(x^2 + 1), x)

$$3.229 \quad \int \frac{\sqrt{1+x^2+x^4}}{(1+x^2)^2} dx$$

Optimal. Leaf size=49

$$\frac{(x^2 + 1) \sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}} E\left(2 \tan^{-1}(x) \middle| \frac{1}{4}\right)}{2\sqrt{x^4 + x^2 + 1}}$$

[Out] ((1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*EllipticE[2*ArcTan[x], 1/4])/(2*Sqrt[1 + x^2 + x^4])

Rubi [A] time = 0.0106672, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.05$, Rules used = {1225}

$$\frac{(x^2 + 1) \sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}} E\left(2 \tan^{-1}(x) \middle| \frac{1}{4}\right)}{2\sqrt{x^4 + x^2 + 1}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[1 + x^2 + x^4]/(1 + x^2)^2,x]

[Out] ((1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*EllipticE[2*ArcTan[x], 1/4])/(2*Sqrt[1 + x^2 + x^4])

Rule 1225

Int[Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]/((d_) + (e_.)*(x_)^2)^2, x_Symbol] :> With[{q = Rt[e/d, 2]}, Simp[(c*(d + e*x^2)*Sqrt[(e^2*(a + b*x^2 + c*x^4))/(c*(d + e*x^2)^2)]*EllipticE[2*ArcTan[q*x], (2*c*d - b*e)/(4*c*d)]]/(2*d*e^2*q*Sqrt[a + b*x^2 + c*x^4]), x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[c*d^2 - a*e^2, 0] && PosQ[e/d]

Rubi steps

$$\int \frac{\sqrt{1+x^2+x^4}}{(1+x^2)^2} dx = \frac{(1+x^2) \sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}} E\left(2 \tan^{-1}(x) \middle| \frac{1}{4}\right)}{2\sqrt{1+x^2+x^4}}$$

Mathematica [C] time = 0.352647, size = 164, normalized size = 3.35

$$\frac{(-1)^{2/3} \sqrt[3]{\sqrt{-1}x^2 + 1} \sqrt{1 - (-1)^{2/3}x^2} \text{EllipticF}\left(i \sinh^{-1}\left((-1)^{5/6}x\right), (-1)^{2/3}\right) + \sqrt[3]{-1} \sqrt[3]{\sqrt{-1}x^2 + 1} \sqrt{1 - (-1)^{2/3}x^2} \left(\text{EllipticF}\left(i \sinh^{-1}\left((-1)^{5/6}x\right), (-1)^{2/3}\right)\right)}{2\sqrt{x^4 + x^2 + 1}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[1 + x^2 + x^4]/(1 + x^2)^2,x]

```
[Out] ((x + x^3 + x^5)/(1 + x^2) + (-1)^(2/3)*Sqrt[1 + (-1)^(1/3)*x^2]*Sqrt[1 - (-1)^(2/3)*x^2]*EllipticF[I*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)] + (-1)^(1/3)*Sqrt[1 + (-1)^(1/3)*x^2]*Sqrt[1 - (-1)^(2/3)*x^2]*(-EllipticE[I*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)] + EllipticF[I*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)])) / (2*Sqrt[1 + x^2 + x^4])
```

Maple [C] time = 0.017, size = 224, normalized size = 4.6

$$\frac{x}{2x^2+2}\sqrt{x^4+x^2+1} + \frac{1}{\sqrt{-2+2i\sqrt{3}}}\sqrt{1-\left(-\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)x^2}\sqrt{1-\left(-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)x^2}\text{EllipticF}\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2}, \frac{\sqrt{-2+2i\sqrt{3}}}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+x^2+1)^(1/2)/(x^2+1)^2,x)
```

```
[Out] 1/2*x*(x^4+x^2+1)^(1/2)/(x^2+1)+1/(-2+2*I*3^(1/2))^(1/2)*(1-(-1/2+1/2*I*3^(1/2))*x^2)^(1/2)*(1-(-1/2-1/2*I*3^(1/2))*x^2)^(1/2)/(x^4+x^2+1)^(1/2)*EllipticF(1/2*x*(-2+2*I*3^(1/2))^(1/2),1/2*(-2+2*I*3^(1/2))^(1/2))+2/(-2+2*I*3^(1/2))^(1/2)*(1-(-1/2+1/2*I*3^(1/2))*x^2)^(1/2)*(1-(-1/2-1/2*I*3^(1/2))*x^2)^(1/2)/(x^4+x^2+1)^(1/2)/(I*3^(1/2)+1)*(EllipticF(1/2*x*(-2+2*I*3^(1/2))^(1/2),1/2*(-2+2*I*3^(1/2))^(1/2))-EllipticE(1/2*x*(-2+2*I*3^(1/2))^(1/2),1/2*(-2+2*I*3^(1/2))^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^4 + x^2 + 1}}{(x^2 + 1)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+x^2+1)^(1/2)/(x^2+1)^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^4 + x^2 + 1)/(x^2 + 1)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{x^4 + x^2 + 1}}{x^4 + 2x^2 + 1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+x^2+1)^(1/2)/(x^2+1)^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^4 + x^2 + 1)/(x^4 + 2*x^2 + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{(x^2 - x + 1)(x^2 + x + 1)}}{(x^2 + 1)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**4+x**2+1)**(1/2)/(x**2+1)**2,x)

[Out] Integral(sqrt((x**2 - x + 1)*(x**2 + x + 1))/(x**2 + 1)**2, x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^4 + x^2 + 1}}{(x^2 + 1)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4+x^2+1)^(1/2)/(x^2+1)^2,x, algorithm="giac")

[Out] integrate(sqrt(x^4 + x^2 + 1)/(x^2 + 1)^2, x)

$$3.230 \quad \int \frac{\sqrt{1+x^2+x^4}}{(1+x^2)^3} dx$$

Optimal. Leaf size=93

$$\frac{\sqrt{x^4+x^2+1}x}{4(x^2+1)^2} + \frac{1}{4} \tan^{-1}\left(\frac{x}{\sqrt{x^4+x^2+1}}\right) + \frac{(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}E\left(2\tan^{-1}(x)\middle|\frac{1}{4}\right)}{4\sqrt{x^4+x^2+1}}$$

[Out] (x*sqrt[1 + x^2 + x^4])/(4*(1 + x^2)^2) + ArcTan[x/sqrt[1 + x^2 + x^4]]/4 + ((1 + x^2)*sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*EllipticE[2*ArcTan[x], 1/4])/(4*sqrt[1 + x^2 + x^4])

Rubi [A] time = 0.508155, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 23, number of rules used = 13, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.65$, Rules used = {1228, 1223, 1696, 1593, 1712, 1195, 1700, 1103, 1698, 203, 12, 1317, 1210}

$$\frac{\sqrt{x^4+x^2+1}x}{4(x^2+1)^2} + \frac{1}{4} \tan^{-1}\left(\frac{x}{\sqrt{x^4+x^2+1}}\right) + \frac{(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}E\left(2\tan^{-1}(x)\middle|\frac{1}{4}\right)}{4\sqrt{x^4+x^2+1}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[1 + x^2 + x^4]/(1 + x^2)^3,x]

[Out] (x*sqrt[1 + x^2 + x^4])/(4*(1 + x^2)^2) + ArcTan[x/sqrt[1 + x^2 + x^4]]/4 + ((1 + x^2)*sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*EllipticE[2*ArcTan[x], 1/4])/(4*sqrt[1 + x^2 + x^4])

Rule 1228

Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{aa, bb, cc}, Int[ExpandIntegrand[1/Sqrt[aa + bb*x^2 + c*x^4], (d + e*x^2)^q*(aa + bb*x^2 + cc*x^4)^(p + 1/2), x] /. {aa -> a, bb -> b, cc -> c}, x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, 0] && IntegerQ[p + 1/2]

Rule 1223

Int[((d_) + (e_.)*(x_)^2)^(q_)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> -Simp[(e^2*x*(d + e*x^2)^(q + 1)*sqrt[a + b*x^2 + c*x^4])/(2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2)), Int[((d + e*x^2)^(q + 1)*Simp[a*e^2*(2*q + 3) + 2*d*(c*d - b*e)*(q + 1) - 2*e*(c*d*(q + 1) - b*e*(q + 2))*x^2 + c*e^2*(2*q + 5)*x^4, x])/sqrt[a + b*x^2 + c*x^4], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && ILtQ[q, -1]

Rule 1696

Int[((P4x_)*((d_) + (e_.)*(x_)^2)^(q_))/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[P4x, x, 4]}, -Simp[((C*d^2 - B*d*e + A*e^2)*x*(d + e*x^2)^(q + 1)*sqrt[a + b*x^2 + c*x^4])/(2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(2*

```
d*(q + 1)*(c*d^2 - b*d*e + a*e^2), Int[((d + e*x^2)^(q + 1)*Simp[a*d*(C*d
- B*e) + A*(a*e^2*(2*q + 3) + 2*d*(c*d - b*e)*(q + 1)) - 2*((B*d - A*e)*(b*
e*(q + 2) - c*d*(q + 1)) - C*d*(b*d + a*e*(q + 1)))*x^2 + c*(C*d^2 - B*d*e
+ A*e^2)*(2*q + 5)*x^4, x])/Sqrt[a + b*x^2 + c*x^4], x], x] /; FreeQ[{a, b
, c, d, e}, x] && PolyQ[P4x, x^2] && LeQ[Expon[P4x, x], 4] && NeQ[b^2 - 4*a
*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, -1]
```

Rule 1593

```
Int[(u_)*((a_)*(x_)^(p_) + (b_)*(x_)^(q_))^(n_), x_Symbol] := Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1712

```
Int[(P4x_)/(((d_) + (e_)*(x_)^2)*Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4])
, x_Symbol] := With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[
P4x, x, 4]}, -Dist[C/e^2, Int[(d - e*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] +
Dist[1/e^2, Int[(C*d^2 + A*e^2 + B*e^2*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 +
c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[P4x, x^2, 2] && NeQ[
b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[c*d^2 - a*e^2, 0]
```

Rule 1195

```
Int[((d_) + (e_)*(x_)^2)/Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4], x_Symbol]
:= With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rule 1700

```
Int[((A_) + (B_)*(x_)^2)/(((d_) + (e_)*(x_)^2)*Sqrt[(a_) + (b_)*(x_)^2 +
(c_)*(x_)^4]), x_Symbol] := Dist[(B*d + A*e)/(2*d*e), Int[1/Sqrt[a + b*x^
2 + c*x^4], x], x] - Dist[(B*d - A*e)/(2*d*e), Int[(d - e*x^2)/((d + e*x^2)
*Sqrt[a + b*x^2 + c*x^4]), x], x] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[c*d^2 - a*e^2, 0]
&& NeQ[B*d + A*e, 0]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4], x_Symbol] := With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1698

```
Int[((A_) + (B_)*(x_)^2)/(((d_) + (e_)*(x_)^2)*Sqrt[(a_) + (b_)*(x_)^2 +
(c_)*(x_)^4]), x_Symbol] := Dist[A, Subst[Int[1/(d - (b*d - 2*a*e)*x^2),
x], x, x/Sqrt[a + b*x^2 + c*x^4]], x] /; FreeQ[{a, b, c, d, e, A, B}, x] &&
NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[c*d^2 - a*e^2,
0] && EqQ[B*d + A*e, 0]
```

Rule 203

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[Rt[b, 2]*x]/Rt
[a, 2])]/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

, 0] || GtQ[b, 0])

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 1317

Int[(x_)^2/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_Symbol] := Dist[d/(2*d*e), Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[d/(2*d*e), Int[(d - e*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && PosQ[c/a] && EqQ[c*d^2 - a*e^2, 0]

Rule 1210

Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_Symbol] := Dist[1/(2*d), Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] + Dist[1/(2*d), Int[(d - e*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[c*d^2 - a*e^2, 0]

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{1+x^2+x^4}}{(1+x^2)^3} dx &= \int \left(\frac{1}{(1+x^2)^3 \sqrt{1+x^2+x^4}} - \frac{1}{(1+x^2)^2 \sqrt{1+x^2+x^4}} + \frac{1}{(1+x^2) \sqrt{1+x^2+x^4}} \right) dx \\
&= \int \frac{1}{(1+x^2)^3 \sqrt{1+x^2+x^4}} dx - \int \frac{1}{(1+x^2)^2 \sqrt{1+x^2+x^4}} dx + \int \frac{1}{(1+x^2) \sqrt{1+x^2+x^4}} dx \\
&= \frac{x\sqrt{1+x^2+x^4}}{4(1+x^2)^2} - \frac{x\sqrt{1+x^2+x^4}}{2(1+x^2)} - \frac{1}{4} \int \frac{-3+2x^2-x^4}{(1+x^2)^2 \sqrt{1+x^2+x^4}} dx + \frac{1}{2} \int \frac{1}{\sqrt{1+x^2+x^4}} dx + \frac{1}{2} \int \frac{1}{\sqrt{1+x^2+x^4}} dx \\
&= \frac{x\sqrt{1+x^2+x^4}}{4(1+x^2)^2} + \frac{x\sqrt{1+x^2+x^4}}{4(1+x^2)} + \frac{(1+x^2) \sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}} F\left(2 \tan^{-1}(x) \middle| \frac{1}{4}\right)}{4\sqrt{1+x^2+x^4}} + \frac{1}{8} \int \frac{-10x^2-6x^4}{(1+x^2) \sqrt{1+x^2+x^4}} dx \\
&= \frac{x\sqrt{1+x^2+x^4}}{4(1+x^2)^2} + \frac{3x\sqrt{1+x^2+x^4}}{4(1+x^2)} + \frac{1}{2} \tan^{-1}\left(\frac{x}{\sqrt{1+x^2+x^4}}\right) - \frac{(1+x^2) \sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}} E\left(2 \tan^{-1}(x) \middle| \frac{1}{4}\right)}{2\sqrt{1+x^2+x^4}} \\
&= \frac{x\sqrt{1+x^2+x^4}}{4(1+x^2)^2} + \frac{3x\sqrt{1+x^2+x^4}}{4(1+x^2)} + \frac{1}{2} \tan^{-1}\left(\frac{x}{\sqrt{1+x^2+x^4}}\right) - \frac{(1+x^2) \sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}} E\left(2 \tan^{-1}(x) \middle| \frac{1}{4}\right)}{2\sqrt{1+x^2+x^4}} \\
&= \frac{x\sqrt{1+x^2+x^4}}{4(1+x^2)^2} + \frac{1}{2} \tan^{-1}\left(\frac{x}{\sqrt{1+x^2+x^4}}\right) + \frac{(1+x^2) \sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}} E\left(2 \tan^{-1}(x) \middle| \frac{1}{4}\right)}{4\sqrt{1+x^2+x^4}} + \frac{(1+x^2) \sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}} E\left(2 \tan^{-1}(x) \middle| \frac{1}{4}\right)}{2\sqrt{1+x^2+x^4}} \\
&= \frac{x\sqrt{1+x^2+x^4}}{4(1+x^2)^2} + \frac{(1+x^2) \sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}} E\left(2 \tan^{-1}(x) \middle| \frac{1}{4}\right)}{4\sqrt{1+x^2+x^4}} + \frac{1}{4} \text{Subst}\left(\int \frac{1}{1+x^2} dx, x, \frac{x}{\sqrt{1+x^2+x^4}}\right) \\
&= \frac{x\sqrt{1+x^2+x^4}}{4(1+x^2)^2} + \frac{1}{4} \tan^{-1}\left(\frac{x}{\sqrt{1+x^2+x^4}}\right) + \frac{(1+x^2) \sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}} E\left(2 \tan^{-1}(x) \middle| \frac{1}{4}\right)}{4\sqrt{1+x^2+x^4}}
\end{aligned}$$

Mathematica [C] time = 0.301769, size = 176, normalized size = 1.89

$$\frac{\sqrt[3]{-1} \sqrt{\sqrt[3]{-1} x^2 + 1} \sqrt{1 - (-1)^{2/3} x^2} \left(\text{EllipticF}\left(i \sinh^{-1}\left((-1)^{5/6} x\right), (-1)^{2/3}\right) - E\left(i \sinh^{-1}\left((-1)^{5/6} x\right) \middle| (-1)^{2/3}\right) \right) + \frac{x(x^2+2)(x^4+x^2)}{(x^2+1)^2}}{4\sqrt{x^4+x^2+1}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[1 + x^2 + x^4]/(1 + x^2)^3, x]

[Out] ((x*(2 + x^2)*(1 + x^2 + x^4))/(1 + x^2)^2 + (-1)^(1/3)*Sqrt[1 + (-1)^(1/3)*x^2]*Sqrt[1 - (-1)^(2/3)*x^2]*(-EllipticE[I*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)] + EllipticF[I*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)]) - 2*(-1)^(2/3)*Sqrt[1 + (-1)^(1/3)*x^2]*Sqrt[1 - (-1)^(2/3)*x^2]*EllipticPi[(-1)^(1/3), (-I)*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)]/(4*Sqrt[1 + x^2 + x^4])

Maple [C] time = 0.022, size = 333, normalized size = 3.6

$$\frac{x}{4(x^2+1)^2}\sqrt{x^4+x^2+1} + \frac{x}{4x^2+4}\sqrt{x^4+x^2+1} + \frac{1}{\sqrt{-2+2i\sqrt{3}}(i\sqrt{3}+1)}\sqrt{1+\frac{x^2}{2}-\frac{i}{2}x^2\sqrt{3}}\sqrt{1+\frac{x^2}{2}+\frac{i}{2}x^2\sqrt{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^4+x^2+1)^(1/2)/(x^2+1)^3,x)

[Out] 1/4*x*(x^4+x^2+1)^(1/2)/(x^2+1)^2+1/4*x*(x^4+x^2+1)^(1/2)/(x^2+1)+1/(-2+2*I*3^(1/2))^(1/2)*(1+1/2*x^2-1/2*I*x^2*3^(1/2))^(1/2)*(1+1/2*x^2+1/2*I*x^2*3^(1/2))^(1/2)/(x^4+x^2+1)^(1/2)/(I*3^(1/2)+1)*EllipticF(1/2*x*(-2+2*I*3^(1/2))^(1/2),1/2*(-2+2*I*3^(1/2))^(1/2))-1/(-2+2*I*3^(1/2))^(1/2)*(1+1/2*x^2-1/2*I*x^2*3^(1/2))^(1/2)*(1+1/2*x^2+1/2*I*x^2*3^(1/2))^(1/2)/(x^4+x^2+1)^(1/2)/(I*3^(1/2)+1)*EllipticE(1/2*x*(-2+2*I*3^(1/2))^(1/2),1/2*(-2+2*I*3^(1/2))^(1/2))+1/2/(-1/2+1/2*I*3^(1/2))^(1/2)*(1+1/2*x^2-1/2*I*x^2*3^(1/2))^(1/2)*(1+1/2*x^2+1/2*I*x^2*3^(1/2))^(1/2)/(x^4+x^2+1)^(1/2)*EllipticPi((-1/2+1/2*I*3^(1/2))^(1/2)*x,-1/(-1/2+1/2*I*3^(1/2)),(-1/2-1/2*I*3^(1/2))^(1/2)/(-1/2+1/2*I*3^(1/2))^(1/2))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^4+x^2+1}}{(x^2+1)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4+x^2+1)^(1/2)/(x^2+1)^3,x, algorithm="maxima")

[Out] integrate(sqrt(x^4 + x^2 + 1)/(x^2 + 1)^3, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{x^4+x^2+1}}{x^6+3x^4+3x^2+1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4+x^2+1)^(1/2)/(x^2+1)^3,x, algorithm="fricas")

[Out] integral(sqrt(x^4 + x^2 + 1)/(x^6 + 3*x^4 + 3*x^2 + 1), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{(x^2-x+1)(x^2+x+1)}}{(x^2+1)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**4+x**2+1)**(1/2)/(x**2+1)**3,x)

[Out] Integral(sqrt((x**2 - x + 1)*(x**2 + x + 1))/(x**2 + 1)**3, x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^4 + x^2 + 1}}{(x^2 + 1)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4+x^2+1)^(1/2)/(x^2+1)^3,x, algorithm="giac")

[Out] integrate(sqrt(x^4 + x^2 + 1)/(x^2 + 1)^3, x)

$$3.231 \quad \int \frac{\sqrt{1+x^2+x^4}}{(1+x^2)^4} dx$$

Optimal. Leaf size=166

$$\frac{(x^2+1) \sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}} \text{EllipticF}\left(2 \tan^{-1}(x), \frac{1}{4}\right)}{8\sqrt{x^4+x^2+1}} + \frac{\sqrt{x^4+x^2+1}x}{6(x^2+1)^2} + \frac{\sqrt{x^4+x^2+1}x}{6(x^2+1)^3} + \frac{1}{4} \tan^{-1}\left(\frac{x}{\sqrt{x^4+x^2+1}}\right) + \frac{(x^2+1)}{3\sqrt{x^4+x^2+1}}$$

[Out] (x*Sqrt[1 + x^2 + x^4])/(6*(1 + x^2)^3) + (x*Sqrt[1 + x^2 + x^4])/(6*(1 + x^2)^2) + ArcTan[x/Sqrt[1 + x^2 + x^4]]/4 + ((1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*EllipticE[2*ArcTan[x], 1/4])/(3*Sqrt[1 + x^2 + x^4]) - ((1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*EllipticF[2*ArcTan[x], 1/4])/(8*Sqrt[1 + x^2 + x^4])

Rubi [A] time = 0.615431, antiderivative size = 166, normalized size of antiderivative = 1., number of steps used = 26, number of rules used = 14, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.7$, Rules used = {1228, 1223, 1696, 1586, 1197, 1103, 1195, 1593, 1712, 1700, 1698, 203, 12, 1317}

$$\frac{\sqrt{x^4+x^2+1}x}{6(x^2+1)^2} + \frac{\sqrt{x^4+x^2+1}x}{6(x^2+1)^3} + \frac{1}{4} \tan^{-1}\left(\frac{x}{\sqrt{x^4+x^2+1}}\right) - \frac{(x^2+1) \sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}} F\left(2 \tan^{-1}(x) \middle| \frac{1}{4}\right)}{8\sqrt{x^4+x^2+1}} + \frac{(x^2+1) \sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}}{3\sqrt{x^4+x^2+1}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[1 + x^2 + x^4]/(1 + x^2)^4, x]

[Out] (x*Sqrt[1 + x^2 + x^4])/(6*(1 + x^2)^3) + (x*Sqrt[1 + x^2 + x^4])/(6*(1 + x^2)^2) + ArcTan[x/Sqrt[1 + x^2 + x^4]]/4 + ((1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*EllipticE[2*ArcTan[x], 1/4])/(3*Sqrt[1 + x^2 + x^4]) - ((1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*EllipticF[2*ArcTan[x], 1/4])/(8*Sqrt[1 + x^2 + x^4])

Rule 1228

Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{aa, bb, cc}, Int[ExpandIntegrand[1/Sqrt[aa + bb*x^2 + c*x^4], (d + e*x^2)^q*(aa + bb*x^2 + cc*x^4)^(p + 1/2), x] /. {aa -> a, bb -> b, cc -> c}, x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, 0] && IntegerQ[p + 1/2]

Rule 1223

Int[((d_) + (e_.)*(x_)^2)^(q_)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> -Simp[(e^2*x*(d + e*x^2)^(q + 1)*Sqrt[a + b*x^2 + c*x^4])/(2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2)), Int[((d + e*x^2)^(q + 1)*Simp[a*e^2*(2*q + 3) + 2*d*(c*d - b*e)*(q + 1) - 2*e*(c*d*(q + 1) - b*e*(q + 2))*x^2 + c*e^2*(2*q + 5)*x^4, x])/Sqrt[a + b*x^2 + c*x^4], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && ILtQ[q, -1]

Rule 1696

```
Int[((P4x_)*((d_) + (e_)*(x_)^2)^(q_))/Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4], x_Symbol] := With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[P4x, x, 4]}, -Simp[((C*d^2 - B*d*e + A*e^2)*x*(d + e*x^2)^(q + 1)*Sqrt[a + b*x^2 + c*x^4])/(2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2)), Int[((d + e*x^2)^(q + 1)*Simp[a*d*(C*d - B*e) + A*(a*e^2*(2*q + 3) + 2*d*(c*d - b*e)*(q + 1)) - 2*((B*d - A*e)*(b*e*(q + 2) - c*d*(q + 1)) - C*d*(b*d + a*e*(q + 1)))*x^2 + c*(C*d^2 - B*d*e + A*e^2)*(2*q + 5)*x^4, x])/Sqrt[a + b*x^2 + c*x^4], x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[P4x, x^2] && LeQ[Expon[P4x, x], 4] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, -1]
```

Rule 1586

```
Int[(u_)*(Px_)^(p_)*(Qx_)^(q_), x_Symbol] := Int[u*PolynomialQuotient[Px, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] && EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 1197

```
Int[((d_) + (e_)*(x_)^2)/Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4], x_Symbol] := With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; NeQ[e + d*q, 0] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4], x_Symbol] := With[{q = Rt[c/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_)*(x_)^2)/Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4], x_Symbol] := With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c*x^4]), x] /; EqQ[e + d*q^2, 0] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1593

```
Int[(u_)*((a_)*(x_)^(p_) + (b_)*(x_)^(q_))^(n_), x_Symbol] := Int[u*x^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] && PosQ[q - p]
```

Rule 1712

```
Int[(P4x_)/(((d_) + (e_)*(x_)^2)*Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4]), x_Symbol] := With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[P4x, x, 4]}, -Dist[C/e^2, Int[(d - e*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] + Dist[1/e^2, Int[(C*d^2 + A*e^2 + B*e^2*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[P4x, x^2, 2] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[c*d^2 - a*e^2, 0]
```

Rule 1700

```
Int[((A_) + (B_)*(x_)^2)/(((d_) + (e_)*(x_)^2)*Sqrt[(a_) + (b_)*(x_)^2 +
```

```
(c_.)*(x_)^4]), x_Symbol] := Dist[(B*d + A*e)/(2*d*e), Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[(B*d - A*e)/(2*d*e), Int[(d - e*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[c*d^2 - a*e^2, 0] && NeQ[B*d + A*e, 0]
```

Rule 1698

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_Symbol] := Dist[A, Subst[Int[1/(d - (b*d - 2*a*e)*x^2), x], x, x/Sqrt[a + b*x^2 + c*x^4]], x] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[c*d^2 - a*e^2, 0] && EqQ[B*d + A*e, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1317

```
Int[(x_)^2/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_Symbol] := Dist[d/(2*d*e), Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[d/(2*d*e), Int[(d - e*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && PosQ[c/a] && EqQ[c*d^2 - a*e^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{1+x^2+x^4}}{(1+x^2)^4} dx &= \int \left(\frac{1}{(1+x^2)^4 \sqrt{1+x^2+x^4}} - \frac{1}{(1+x^2)^3 \sqrt{1+x^2+x^4}} + \frac{1}{(1+x^2)^2 \sqrt{1+x^2+x^4}} \right) dx \\
&= \int \frac{1}{(1+x^2)^4 \sqrt{1+x^2+x^4}} dx - \int \frac{1}{(1+x^2)^3 \sqrt{1+x^2+x^4}} dx + \int \frac{1}{(1+x^2)^2 \sqrt{1+x^2+x^4}} dx \\
&= \frac{x\sqrt{1+x^2+x^4}}{6(1+x^2)^3} - \frac{x\sqrt{1+x^2+x^4}}{4(1+x^2)^2} + \frac{x\sqrt{1+x^2+x^4}}{2(1+x^2)} - \frac{1}{6} \int \frac{-5+2x^2-3x^4}{(1+x^2)^3 \sqrt{1+x^2+x^4}} dx + \frac{1}{4} \int \frac{-3}{(1+x^2)^2 \sqrt{1+x^2+x^4}} dx \\
&= \frac{x\sqrt{1+x^2+x^4}}{6(1+x^2)^3} + \frac{x\sqrt{1+x^2+x^4}}{6(1+x^2)^2} - \frac{x\sqrt{1+x^2+x^4}}{4(1+x^2)} + \frac{1}{24} \int \frac{10-8x^2+10x^4}{(1+x^2)^2 \sqrt{1+x^2+x^4}} dx - \frac{1}{8} \int \frac{-3}{(1+x^2) \sqrt{1+x^2+x^4}} dx \\
&= \frac{x\sqrt{1+x^2+x^4}}{6(1+x^2)^3} + \frac{x\sqrt{1+x^2+x^4}}{6(1+x^2)^2} - \frac{x\sqrt{1+x^2+x^4}}{6(1+x^2)} + \frac{(1+x^2) \sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}} E\left(2 \tan^{-1}(x) \middle| \frac{1}{4}\right)}{2\sqrt{1+x^2+x^4}} - \frac{1}{48} \int \frac{-3}{(1+x^2) \sqrt{1+x^2+x^4}} dx \\
&= \frac{x\sqrt{1+x^2+x^4}}{6(1+x^2)^3} + \frac{x\sqrt{1+x^2+x^4}}{6(1+x^2)^2} - \frac{x\sqrt{1+x^2+x^4}}{6(1+x^2)} + \frac{(1+x^2) \sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}} E\left(2 \tan^{-1}(x) \middle| \frac{1}{4}\right)}{2\sqrt{1+x^2+x^4}} - \frac{1}{48} \int \frac{-3}{(1+x^2) \sqrt{1+x^2+x^4}} dx \\
&= \frac{x\sqrt{1+x^2+x^4}}{6(1+x^2)^3} + \frac{x\sqrt{1+x^2+x^4}}{6(1+x^2)^2} + \frac{7x\sqrt{1+x^2+x^4}}{12(1+x^2)} - \frac{(1+x^2) \sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}} E\left(2 \tan^{-1}(x) \middle| \frac{1}{4}\right)}{4\sqrt{1+x^2+x^4}} - \frac{1}{48} \int \frac{-3}{(1+x^2) \sqrt{1+x^2+x^4}} dx \\
&= \frac{x\sqrt{1+x^2+x^4}}{6(1+x^2)^3} + \frac{x\sqrt{1+x^2+x^4}}{6(1+x^2)^2} + \frac{1}{2} \tan^{-1}\left(\frac{x}{\sqrt{1+x^2+x^4}}\right) + \frac{(1+x^2) \sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}} E\left(2 \tan^{-1}(x) \middle| \frac{1}{4}\right)}{3\sqrt{1+x^2+x^4}} - \frac{1}{48} \int \frac{-3}{(1+x^2) \sqrt{1+x^2+x^4}} dx \\
&= \frac{x\sqrt{1+x^2+x^4}}{6(1+x^2)^3} + \frac{x\sqrt{1+x^2+x^4}}{6(1+x^2)^2} + \frac{1}{4} \tan^{-1}\left(\frac{x}{\sqrt{1+x^2+x^4}}\right) + \frac{(1+x^2) \sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}} E\left(2 \tan^{-1}(x) \middle| \frac{1}{4}\right)}{3\sqrt{1+x^2+x^4}} - \frac{1}{48} \int \frac{-3}{(1+x^2) \sqrt{1+x^2+x^4}} dx
\end{aligned}$$

Mathematica [C] time = 0.419021, size = 240, normalized size = 1.45

$$-(-1)^{2/3} \sqrt{\sqrt[3]{-1}x^2 + 1} \sqrt{1 - (-1)^{2/3}x^2} \text{EllipticF}\left(i \sinh^{-1}\left((-1)^{5/6}x\right), (-1)^{2/3}\right) - 2\sqrt[3]{-1} \sqrt{\sqrt[3]{-1}x^2 + 1} \sqrt{1 - (-1)^{2/3}x^2} \left(E\left(i \sinh^{-1}\left((-1)^{5/6}x\right), (-1)^{2/3}\right)\right)$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[1 + x^2 + x^4]/(1 + x^2)^4, x]

[Out] ((x*(1 + x^2 + x^4)*(4 + 5*x^2 + 2*x^4))/(1 + x^2)^3 - 2*(-1)^(1/3)*Sqrt[1 + (-1)^(1/3)*x^2]*Sqrt[1 - (-1)^(2/3)*x^2]*(EllipticE[I*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)] - EllipticF[I*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)]) - (-1)^(2/3)*Sqrt[1 + (-1)^(1/3)*x^2]*Sqrt[1 - (-1)^(2/3)*x^2]*EllipticF[I*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)] - 3*(-1)^(2/3)*Sqrt[1 + (-1)^(1/3)*x^2]*Sqrt[1 - (-1)^(2/3)*x^2]*EllipticPi[(-1)^(1/3), (-I)*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)])/(6*Sqrt[1 + x^2 + x^4])

Maple [C] time = 0.024, size = 438, normalized size = 2.6

$$\frac{x}{6(x^2+1)^3}\sqrt{x^4+x^2+1} + \frac{x}{6(x^2+1)^2}\sqrt{x^4+x^2+1} + \frac{x}{3x^2+3}\sqrt{x^4+x^2+1} - \frac{1}{3\sqrt{-2+2i\sqrt{3}}}\sqrt{1+\frac{x^2}{2}-\frac{i}{2}x^2\sqrt{3}}\sqrt{1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^4+x^2+1)^(1/2)/(x^2+1)^4,x)

[Out] 1/6*x*(x^4+x^2+1)^(1/2)/(x^2+1)^3+1/6*x*(x^4+x^2+1)^(1/2)/(x^2+1)^2+1/3*x*(x^4+x^2+1)^(1/2)/(x^2+1)-1/3/(-2+2*I*3^(1/2))^(1/2)*(1+1/2*x^2-1/2*I*x^2*3^(1/2))^(1/2)*(1+1/2*x^2+1/2*I*x^2*3^(1/2))^(1/2)/(x^4+x^2+1)^(1/2)*EllipticF(1/2*x*(-2+2*I*3^(1/2))^(1/2),1/2*(-2+2*I*3^(1/2))^(1/2))+4/3/(-2+2*I*3^(1/2))^(1/2)*(1+1/2*x^2-1/2*I*x^2*3^(1/2))^(1/2)*(1+1/2*x^2+1/2*I*x^2*3^(1/2))^(1/2)/(x^4+x^2+1)^(1/2)/(I*3^(1/2)+1)*EllipticF(1/2*x*(-2+2*I*3^(1/2))^(1/2),1/2*(-2+2*I*3^(1/2))^(1/2))-4/3/(-2+2*I*3^(1/2))^(1/2)*(1+1/2*x^2-1/2*I*x^2*3^(1/2))^(1/2)*(1+1/2*x^2+1/2*I*x^2*3^(1/2))^(1/2)/(x^4+x^2+1)^(1/2)/(I*3^(1/2)+1)*EllipticE(1/2*x*(-2+2*I*3^(1/2))^(1/2),1/2*(-2+2*I*3^(1/2))^(1/2))+1/2/(-1/2+1/2*I*3^(1/2))^(1/2)*(1+1/2*x^2-1/2*I*x^2*3^(1/2))^(1/2)*(1+1/2*x^2+1/2*I*x^2*3^(1/2))^(1/2)/(x^4+x^2+1)^(1/2)*EllipticPi((-1/2+1/2*I*3^(1/2))^(1/2)*x,-1/(-1/2+1/2*I*3^(1/2)),(-1/2-1/2*I*3^(1/2))^(1/2)/(-1/2+1/2*I*3^(1/2))^(1/2))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^4+x^2+1}}{(x^2+1)^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4+x^2+1)^(1/2)/(x^2+1)^4,x, algorithm="maxima")

[Out] integrate(sqrt(x^4 + x^2 + 1)/(x^2 + 1)^4, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{x^4+x^2+1}}{x^8+4x^6+6x^4+4x^2+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4+x^2+1)^(1/2)/(x^2+1)^4,x, algorithm="fricas")

[Out] integral(sqrt(x^4 + x^2 + 1)/(x^8 + 4*x^6 + 6*x^4 + 4*x^2 + 1), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{(x^2-x+1)(x^2+x+1)}}{(x^2+1)^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**4+x**2+1)**(1/2)/(x**2+1)**4,x)

[Out] Integral(sqrt((x**2 - x + 1)*(x**2 + x + 1))/(x**2 + 1)**4, x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^4 + x^2 + 1}}{(x^2 + 1)^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4+x^2+1)^(1/2)/(x^2+1)^4,x, algorithm="giac")

[Out] integrate(sqrt(x^4 + x^2 + 1)/(x^2 + 1)^4, x)

$$3.232 \quad \int \frac{(1+x^2)^3}{\sqrt{1+x^2+x^4}} dx$$

Optimal. Leaf size=159

$$\frac{3(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}\text{EllipticF}\left(2\tan^{-1}(x), \frac{1}{4}\right)}{5\sqrt{x^4+x^2+1}} + \frac{1}{5}\sqrt{x^4+x^2+1}x^3 + \frac{14\sqrt{x^4+x^2+1}x}{15(x^2+1)} + \frac{11}{15}\sqrt{x^4+x^2+1}x - \frac{14(x^2+1)}{15\sqrt{x^4+x^2+1}}$$

[Out] (11*x*Sqrt[1 + x^2 + x^4])/15 + (x^3*Sqrt[1 + x^2 + x^4])/5 + (14*x*Sqrt[1 + x^2 + x^4])/(15*(1 + x^2)) - (14*(1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*EllipticE[2*ArcTan[x], 1/4])/(15*Sqrt[1 + x^2 + x^4]) + (3*(1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*EllipticF[2*ArcTan[x], 1/4])/(5*Sqrt[1 + x^2 + x^4])

Rubi [A] time = 0.0714685, antiderivative size = 159, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$, Rules used = {1206, 1679, 1197, 1103, 1195}

$$\frac{1}{5}\sqrt{x^4+x^2+1}x^3 + \frac{14\sqrt{x^4+x^2+1}x}{15(x^2+1)} + \frac{11}{15}\sqrt{x^4+x^2+1}x + \frac{3(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}F\left(2\tan^{-1}(x)\middle|\frac{1}{4}\right)}{5\sqrt{x^4+x^2+1}} - \frac{14(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}}{15\sqrt{x^4+x^2+1}}$$

Antiderivative was successfully verified.

[In] Int[(1 + x^2)^3/Sqrt[1 + x^2 + x^4], x]

[Out] (11*x*Sqrt[1 + x^2 + x^4])/15 + (x^3*Sqrt[1 + x^2 + x^4])/5 + (14*x*Sqrt[1 + x^2 + x^4])/(15*(1 + x^2)) - (14*(1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*EllipticE[2*ArcTan[x], 1/4])/(15*Sqrt[1 + x^2 + x^4]) + (3*(1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*EllipticF[2*ArcTan[x], 1/4])/(5*Sqrt[1 + x^2 + x^4])

Rule 1206

Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(e^q*x^(2*q - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(4*p + 2*q + 1)), x] + Dist[1/(c*(4*p + 2*q + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*q + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x] /;

FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1]

Rule 1679

Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{q = Expon[Pq, x^2], e = Coeff[Pq, x^2, Expon[Pq, x^2]]}, Simp[(e*x^(2*q - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(2*q + 4*p + 1)), x] + Dist[1/(c*(2*q + 4*p + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*q + 4*p + 1)*Pq - a*e*(2*q - 3)*x^(2*q - 4) - b*e*(2*q + 2*p - 1)*x^(2*q - 2) - c*e*(2*q + 4*p + 1)*x^(2*q), x], x] /;

FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && !LtQ[p, -1]

Rule 1197

```
Int[((d_) + (e_)*(x_)^2)/Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4], x_Symbol]
:= With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4], x], x]
- Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; NeQ[e + d*q, 0] /;
FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4], x_Symbol] := With[{q = Rt[c/a, 4]},
Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]), x] /;
FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_)*(x_)^2)/Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4], x_Symbol]
:= With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^2*x^2)), x]
+ Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c*x^4]), x] /;
EqQ[e + d*q^2, 0] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\begin{aligned} \int \frac{(1+x^2)^3}{\sqrt{1+x^2+x^4}} dx &= \frac{1}{5}x^3\sqrt{1+x^2+x^4} + \frac{1}{5} \int \frac{5+12x^2+11x^4}{\sqrt{1+x^2+x^4}} dx \\ &= \frac{11}{15}x\sqrt{1+x^2+x^4} + \frac{1}{5}x^3\sqrt{1+x^2+x^4} + \frac{1}{15} \int \frac{4+14x^2}{\sqrt{1+x^2+x^4}} dx \\ &= \frac{11}{15}x\sqrt{1+x^2+x^4} + \frac{1}{5}x^3\sqrt{1+x^2+x^4} - \frac{14}{15} \int \frac{1-x^2}{\sqrt{1+x^2+x^4}} dx + \frac{6}{5} \int \frac{1}{\sqrt{1+x^2+x^4}} dx \\ &= \frac{11}{15}x\sqrt{1+x^2+x^4} + \frac{1}{5}x^3\sqrt{1+x^2+x^4} + \frac{14x\sqrt{1+x^2+x^4}}{15(1+x^2)} - \frac{14(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}}E\left(2\tan^{-1}(x)\middle|\frac{1}{4}\right)}{15\sqrt{1+x^2+x^4}} \end{aligned}$$

Mathematica [C] time = 0.173071, size = 157, normalized size = 0.99

$$\frac{2\sqrt[3]{-1}(2\sqrt[3]{-1}-7)\sqrt{\sqrt[3]{-1}x^2+1}\sqrt{1-(-1)^{2/3}x^2}\text{EllipticF}\left(i\sinh^{-1}\left((-1)^{5/6}x\right),(-1)^{2/3}\right)+x(3x^6+14x^4+14x^2+11)+14\sqrt[3]{-1}}{15\sqrt{x^4+x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^2)^3/Sqrt[1 + x^2 + x^4], x]
```

```
[Out] (x*(11 + 14*x^2 + 14*x^4 + 3*x^6) + 14*(-1)^(1/3)*Sqrt[1 + (-1)^(1/3)*x^2]*
Sqrt[1 - (-1)^(2/3)*x^2]*EllipticE[I*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)] + 2
*(-1)^(1/3)*(-7 + 2*(-1)^(1/3))*Sqrt[1 + (-1)^(1/3)*x^2]*Sqrt[1 - (-1)^(2/3)
)*x^2]*EllipticF[I*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)]/(15*Sqrt[1 + x^2 + x^4])
```

Maple [C] time = 0.027, size = 233, normalized size = 1.5

$$\frac{x^3}{5}\sqrt{x^4+x^2+1} + \frac{11x}{15}\sqrt{x^4+x^2+1} + \frac{8}{15\sqrt{-2+2i\sqrt{3}}}\sqrt{1-\left(-\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)x^2}\sqrt{1-\left(-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)x^2}\text{EllipticF}\left(\frac{x\sqrt{-2}}{\dots}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2+1)^3/(x^4+x^2+1)^(1/2), x)

[Out] 1/5*x^3*(x^4+x^2+1)^(1/2)+11/15*x*(x^4+x^2+1)^(1/2)+8/15/(-2+2*I*3^(1/2))^(1/2)*(1-(-1/2+1/2*I*3^(1/2))*x^2)^(1/2)*(1-(-1/2-1/2*I*3^(1/2))*x^2)^(1/2)/(x^4+x^2+1)^(1/2)*EllipticF(1/2*x*(-2+2*I*3^(1/2))^(1/2),1/2*(-2+2*I*3^(1/2))^(1/2))-56/15/(-2+2*I*3^(1/2))^(1/2)*(1-(-1/2+1/2*I*3^(1/2))*x^2)^(1/2)*(1-(-1/2-1/2*I*3^(1/2))*x^2)^(1/2)/(x^4+x^2+1)^(1/2)/(I*3^(1/2)+1)*(EllipticF(1/2*x*(-2+2*I*3^(1/2))^(1/2),1/2*(-2+2*I*3^(1/2))^(1/2))-EllipticE(1/2*x*(-2+2*I*3^(1/2))^(1/2),1/2*(-2+2*I*3^(1/2))^(1/2)))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(x^2+1)^3}{\sqrt{x^4+x^2+1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+1)^3/(x^4+x^2+1)^(1/2), x, algorithm="maxima")

[Out] integrate((x^2 + 1)^3/sqrt(x^4 + x^2 + 1), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^6+3x^4+3x^2+1}{\sqrt{x^4+x^2+1}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+1)^3/(x^4+x^2+1)^(1/2), x, algorithm="fricas")

[Out] integral((x^6 + 3*x^4 + 3*x^2 + 1)/sqrt(x^4 + x^2 + 1), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(x^2+1)^3}{\sqrt{(x^2-x+1)(x^2+x+1)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**2+1)**3/(x**4+x**2+1)**(1/2), x)

```
[Out] Integral((x**2 + 1)**3/sqrt((x**2 - x + 1)*(x**2 + x + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(x^2 + 1)^3}{\sqrt{x^4 + x^2 + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^3/(x^4+x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x^2 + 1)^3/sqrt(x^4 + x^2 + 1), x)
```

$$3.233 \quad \int \frac{(1+x^2)^2}{\sqrt{1+x^2+x^4}} dx$$

Optimal. Leaf size=137

$$\frac{(x^2+1) \sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}} \text{EllipticF}\left(2 \tan^{-1}(x), \frac{1}{4}\right)}{\sqrt{x^4+x^2+1}} + \frac{4\sqrt{x^4+x^2+1}x}{3(x^2+1)} + \frac{1}{3}\sqrt{x^4+x^2+1}x - \frac{4(x^2+1) \sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}} E\left(2 \tan^{-1}(x)\right)}{3\sqrt{x^4+x^2+1}}$$

[Out] (x*Sqrt[1 + x^2 + x^4])/3 + (4*x*Sqrt[1 + x^2 + x^4])/(3*(1 + x^2)) - (4*(1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*EllipticE[2*ArcTan[x], 1/4])/(3*Sqrt[1 + x^2 + x^4]) + ((1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*EllipticF[2*ArcTan[x], 1/4])/Sqrt[1 + x^2 + x^4]

Rubi [A] time = 0.0449664, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$, Rules used = {1206, 1197, 1103, 1195}

$$\frac{4\sqrt{x^4+x^2+1}x}{3(x^2+1)} + \frac{1}{3}\sqrt{x^4+x^2+1}x + \frac{(x^2+1) \sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}} F\left(2 \tan^{-1}(x) \middle| \frac{1}{4}\right)}{\sqrt{x^4+x^2+1}} - \frac{4(x^2+1) \sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}} E\left(2 \tan^{-1}(x) \middle| \frac{1}{4}\right)}{3\sqrt{x^4+x^2+1}}$$

Antiderivative was successfully verified.

[In] Int[(1 + x^2)^2/Sqrt[1 + x^2 + x^4], x]

[Out] (x*Sqrt[1 + x^2 + x^4])/3 + (4*x*Sqrt[1 + x^2 + x^4])/(3*(1 + x^2)) - (4*(1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*EllipticE[2*ArcTan[x], 1/4])/(3*Sqrt[1 + x^2 + x^4]) + ((1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*EllipticF[2*ArcTan[x], 1/4])/Sqrt[1 + x^2 + x^4]

Rule 1206

Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(e^q*x^(2*q - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(4*p + 2*q + 1)), x] + Dist[1/(c*(4*p + 2*q + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*p + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1]

Rule 1197

Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; NeQ[e + d*q, 0] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]

Rule 1103

Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4])

), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]

Rule 1195

Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]

Rubi steps

$$\int \frac{(1+x^2)^2}{\sqrt{1+x^2+x^4}} dx = \frac{1}{3}x\sqrt{1+x^2+x^4} + \frac{1}{3} \int \frac{2+4x^2}{\sqrt{1+x^2+x^4}} dx$$

$$= \frac{1}{3}x\sqrt{1+x^2+x^4} - \frac{4}{3} \int \frac{1-x^2}{\sqrt{1+x^2+x^4}} dx + 2 \int \frac{1}{\sqrt{1+x^2+x^4}} dx$$

$$= \frac{1}{3}x\sqrt{1+x^2+x^4} + \frac{4x\sqrt{1+x^2+x^4}}{3(1+x^2)} - \frac{4(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}}E\left(2\tan^{-1}(x)\middle|\frac{1}{4}\right)}{3\sqrt{1+x^2+x^4}} + \frac{(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}}F\left(\frac{1}{4}\right)}{\sqrt{1+x^2+x^4}}$$

Mathematica [C] time = 0.136815, size = 143, normalized size = 1.04

$$\frac{2\sqrt[3]{-1}(\sqrt[3]{-1}-2)\sqrt{\sqrt[3]{-1}x^2+1}\sqrt{1-(-1)^{2/3}x^2}\text{EllipticF}\left(i\sinh^{-1}\left((-1)^{5/6}x\right),(-1)^{2/3}\right)+x^5+x^3+4\sqrt[3]{-1}\sqrt{\sqrt[3]{-1}x^2+1}\sqrt{1-(-1)^{2/3}x^2}}{3\sqrt{x^4+x^2+1}}$$

Antiderivative was successfully verified.

[In] Integrate[(1 + x^2)^2/Sqrt[1 + x^2 + x^4], x]

[Out] (x + x^3 + x^5 + 4*(-1)^(1/3)*Sqrt[1 + (-1)^(1/3)*x^2]*Sqrt[1 - (-1)^(2/3)*x^2]*EllipticE[I*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)] + 2*(-1)^(1/3)*(-2 + (-1)^(1/3))*Sqrt[1 + (-1)^(1/3)*x^2]*Sqrt[1 - (-1)^(2/3)*x^2]*EllipticF[I*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)]/(3*Sqrt[1 + x^2 + x^4])

Maple [C] time = 0.007, size = 218, normalized size = 1.6

$$\frac{x}{3}\sqrt{x^4+x^2+1} + \frac{4}{3\sqrt{-2+2i\sqrt{3}}}\sqrt{1-\left(-\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)x^2}\sqrt{1-\left(-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)x^2}\text{EllipticF}\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2}, \frac{\sqrt{-2+2i\sqrt{3}}}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2+1)^2/(x^4+x^2+1)^(1/2), x)

[Out] 1/3*x*(x^4+x^2+1)^(1/2)+4/3/(-2+2*I*3^(1/2))^(1/2)*(1-(-1/2+1/2*I*3^(1/2))*x^2)^(1/2)*(1-(-1/2-1/2*I*3^(1/2))*x^2)^(1/2)/(x^4+x^2+1)^(1/2)*EllipticF(1/2*x*(-2+2*I*3^(1/2))^(1/2), 1/2*(-2+2*I*3^(1/2))^(1/2))-16/3/(-2+2*I*3^(1/2))^(1/2)*(1-(-1/2+1/2*I*3^(1/2))*x^2)^(1/2)*(1-(-1/2-1/2*I*3^(1/2))*x^2)^(1/2)/(x^4+x^2+1)^(1/2)/(I*3^(1/2)+1)*(EllipticF(1/2*x*(-2+2*I*3^(1/2))^(1/2), 1/2*(-2+2*I*3^(1/2))^(1/2))

,1/2*(-2+2*I*3^(1/2))^(1/2))-EllipticE(1/2*x*(-2+2*I*3^(1/2))^(1/2),1/2*(-2+2*I*3^(1/2))^(1/2)))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(x^2 + 1)^2}{\sqrt{x^4 + x^2 + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+1)^2/(x^4+x^2+1)^(1/2),x, algorithm="maxima")

[Out] integrate((x^2 + 1)^2/sqrt(x^4 + x^2 + 1), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^4 + 2x^2 + 1}{\sqrt{x^4 + x^2 + 1}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+1)^2/(x^4+x^2+1)^(1/2),x, algorithm="fricas")

[Out] integral((x^4 + 2*x^2 + 1)/sqrt(x^4 + x^2 + 1), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(x^2 + 1)^2}{\sqrt{(x^2 - x + 1)(x^2 + x + 1)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**2+1)**2/(x**4+x**2+1)**(1/2),x)

[Out] Integral((x**2 + 1)**2/sqrt((x**2 - x + 1)*(x**2 + x + 1)), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(x^2 + 1)^2}{\sqrt{x^4 + x^2 + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+1)^2/(x^4+x^2+1)^(1/2),x, algorithm="giac")

[Out] integrate((x^2 + 1)^2/sqrt(x^4 + x^2 + 1), x)

$$3.234 \quad \int \frac{1+x^2}{\sqrt{1+x^2+x^4}} dx$$

Optimal. Leaf size=115

$$\frac{(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}\text{EllipticF}\left(2\tan^{-1}(x), \frac{1}{4}\right)}{\sqrt{x^4+x^2+1}} + \frac{\sqrt{x^4+x^2+1}x}{x^2+1} - \frac{(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}E\left(2\tan^{-1}(x)\middle|\frac{1}{4}\right)}{\sqrt{x^4+x^2+1}}$$

[Out] (x*Sqrt[1 + x^2 + x^4])/(1 + x^2) - ((1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*EllipticE[2*ArcTan[x], 1/4])/Sqrt[1 + x^2 + x^4] + ((1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*EllipticF[2*ArcTan[x], 1/4])/Sqrt[1 + x^2 + x^4]

Rubi [A] time = 0.0221631, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1197, 1103, 1195}

$$\frac{\sqrt{x^4+x^2+1}x}{x^2+1} + \frac{(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}F\left(2\tan^{-1}(x)\middle|\frac{1}{4}\right)}{\sqrt{x^4+x^2+1}} - \frac{(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}E\left(2\tan^{-1}(x)\middle|\frac{1}{4}\right)}{\sqrt{x^4+x^2+1}}$$

Antiderivative was successfully verified.

[In] Int[(1 + x^2)/Sqrt[1 + x^2 + x^4], x]

[Out] (x*Sqrt[1 + x^2 + x^4])/(1 + x^2) - ((1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*EllipticE[2*ArcTan[x], 1/4])/Sqrt[1 + x^2 + x^4] + ((1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*EllipticF[2*ArcTan[x], 1/4])/Sqrt[1 + x^2 + x^4]

Rule 1197

Int[((d_) + (e_)*(x_)^2)/Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4], x_Symbol] := With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; NeQ[e + d*q, 0] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]

Rule 1103

Int[1/Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4], x_Symbol] := With[{q = Rt[c/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]), x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]

Rule 1195

Int[((d_) + (e_)*(x_)^2)/Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4], x_Symbol] := With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c*x^4]), x] /; EqQ[e + d*q^2, 0] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]

Rubi steps

$$\int \frac{1+x^2}{\sqrt{1+x^2+x^4}} dx = 2 \int \frac{1}{\sqrt{1+x^2+x^4}} dx - \int \frac{1-x^2}{\sqrt{1+x^2+x^4}} dx$$

$$= \frac{x\sqrt{1+x^2+x^4}}{1+x^2} - \frac{(1+x^2) \sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}} E\left(2 \tan^{-1}(x) \middle| \frac{1}{4}\right)}{\sqrt{1+x^2+x^4}} + \frac{(1+x^2) \sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}} F\left(2 \tan^{-1}(x) \middle| \frac{1}{4}\right)}{\sqrt{1+x^2+x^4}}$$

Mathematica [C] time = 0.0701471, size = 94, normalized size = 0.82

$$\frac{\sqrt[3]{-1} \sqrt{\sqrt[3]{-1} x^2 + 1} \sqrt{1 - (-1)^{2/3} x^2} \left((\sqrt[3]{-1} - 1) \operatorname{EllipticF}\left(i \sinh^{-1}\left((-1)^{5/6} x\right), (-1)^{2/3}\right) + E\left(i \sinh^{-1}\left((-1)^{5/6} x\right) \middle| (-1)^{2/3}\right) \right)}{\sqrt{x^4 + x^2 + 1}}$$

Antiderivative was successfully verified.

[In] Integrate[(1 + x^2)/Sqrt[1 + x^2 + x^4], x]

[Out] ((-1)^(1/3)*Sqrt[1 + (-1)^(1/3)*x^2]*Sqrt[1 - (-1)^(2/3)*x^2]*(EllipticE[I*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)] + (-1 + (-1)^(1/3))*EllipticF[I*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)])/Sqrt[1 + x^2 + x^4]

Maple [C] time = 0.005, size = 205, normalized size = 1.8

$$-4 \frac{\sqrt{1 - (-1/2 + i/2\sqrt{3})x^2} \sqrt{1 - (-1/2 - i/2\sqrt{3})x^2} \left(\operatorname{EllipticF}\left(1/2 x \sqrt{-2 + 2i\sqrt{3}}, 1/2 \sqrt{-2 + 2i\sqrt{3}}\right) - \operatorname{EllipticE}\left(1/2 \sqrt{-2 + 2i\sqrt{3}}\right) \right)}{\sqrt{-2 + 2i\sqrt{3}} \sqrt{x^4 + x^2 + 1} (i\sqrt{3} + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2+1)/(x^4+x^2+1)^(1/2), x)

[Out] -4/(-2+2*I*3^(1/2))^(1/2)*(1-(-1/2+1/2*I*3^(1/2))*x^2)^(1/2)*(1-(-1/2-1/2*I*3^(1/2))*x^2)^(1/2)/(x^4+x^2+1)^(1/2)/(I*3^(1/2)+1)*(EllipticF(1/2*x*(-2+2*I*3^(1/2))^(1/2), 1/2*(-2+2*I*3^(1/2))^(1/2))-EllipticE(1/2*x*(-2+2*I*3^(1/2))^(1/2), 1/2*(-2+2*I*3^(1/2))^(1/2)))+2/(-2+2*I*3^(1/2))^(1/2)*(1-(-1/2+1/2*I*3^(1/2))*x^2)^(1/2)*(1-(-1/2-1/2*I*3^(1/2))*x^2)^(1/2)/(x^4+x^2+1)^(1/2)*EllipticF(1/2*x*(-2+2*I*3^(1/2))^(1/2), 1/2*(-2+2*I*3^(1/2))^(1/2))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 + 1}{\sqrt{x^4 + x^2 + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+1)/(x^4+x^2+1)^(1/2), x, algorithm="maxima")

[Out] integrate((x^2 + 1)/sqrt(x^4 + x^2 + 1), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^2 + 1}{\sqrt{x^4 + x^2 + 1}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+1)/(x^4+x^2+1)^(1/2),x, algorithm="fricas")

[Out] integral((x^2 + 1)/sqrt(x^4 + x^2 + 1), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 + 1}{\sqrt{(x^2 - x + 1)(x^2 + x + 1)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**2+1)/(x**4+x**2+1)**(1/2),x)

[Out] Integral((x**2 + 1)/sqrt((x**2 - x + 1)*(x**2 + x + 1)), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 + 1}{\sqrt{x^4 + x^2 + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+1)/(x^4+x^2+1)^(1/2),x, algorithm="giac")

[Out] integrate((x^2 + 1)/sqrt(x^4 + x^2 + 1), x)

$$3.235 \quad \int \frac{1}{(1+x^2)\sqrt{1+x^2+x^4}} dx$$

Optimal. Leaf size=69

$$\frac{(x^2 + 1) \sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}} \text{EllipticF}\left(2 \tan^{-1}(x), \frac{1}{4}\right)}{4\sqrt{x^4 + x^2 + 1}} + \frac{1}{2} \tan^{-1}\left(\frac{x}{\sqrt{x^4 + x^2 + 1}}\right)$$

[Out] ArcTan[x/Sqrt[1 + x^2 + x^4]]/2 + ((1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*EllipticF[2*ArcTan[x], 1/4])/(4*Sqrt[1 + x^2 + x^4])

Rubi [A] time = 0.058729, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$, Rules used = {1210, 1103, 1698, 203}

$$\frac{1}{2} \tan^{-1}\left(\frac{x}{\sqrt{x^4 + x^2 + 1}}\right) + \frac{(x^2 + 1) \sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}} F\left(2 \tan^{-1}(x) \middle| \frac{1}{4}\right)}{4\sqrt{x^4 + x^2 + 1}}$$

Antiderivative was successfully verified.

[In] Int[1/((1 + x^2)*Sqrt[1 + x^2 + x^4]), x]

[Out] ArcTan[x/Sqrt[1 + x^2 + x^4]]/2 + ((1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*EllipticF[2*ArcTan[x], 1/4])/(4*Sqrt[1 + x^2 + x^4])

Rule 1210

Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_Symbol] :> Dist[1/(2*d), Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] + Dist[1/(2*d), Int[(d - e*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[c*d^2 - a*e^2, 0]

Rule 1103

Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)]/(2*q*Sqrt[a + b*x^2 + c*x^4]), x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]

Rule 1698

Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_Symbol] :> Dist[A, Subst[Int[1/(d - (b*d - 2*a*e)*x^2), x], x, x/Sqrt[a + b*x^2 + c*x^4]], x] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[c*d^2 - a*e^2, 0] && EqQ[B*d + A*e, 0]

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rubi steps

$$\begin{aligned} \int \frac{1}{(1+x^2)\sqrt{1+x^2+x^4}} dx &= \frac{1}{2} \int \frac{1}{\sqrt{1+x^2+x^4}} dx + \frac{1}{2} \int \frac{1-x^2}{(1+x^2)\sqrt{1+x^2+x^4}} dx \\ &= \frac{(1+x^2) \sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}} F\left(2 \tan^{-1}(x) \middle| \frac{1}{4}\right)}{4\sqrt{1+x^2+x^4}} + \frac{1}{2} \text{Subst}\left(\int \frac{1}{1+x^2} dx, x, \frac{x}{\sqrt{1+x^2+x^4}}\right) \\ &= \frac{1}{2} \tan^{-1}\left(\frac{x}{\sqrt{1+x^2+x^4}}\right) + \frac{(1+x^2) \sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}} F\left(2 \tan^{-1}(x) \middle| \frac{1}{4}\right)}{4\sqrt{1+x^2+x^4}} \end{aligned}$$

Mathematica [C] time = 0.0686553, size = 73, normalized size = 1.06

$$\frac{(-1)^{2/3} \sqrt{\sqrt[3]{-1}x^2 + 1} \sqrt{1 - (-1)^{2/3}x^2} \Pi\left(\sqrt[3]{-1}; -i \sinh^{-1}\left((-1)^{5/6}x\right) \middle| (-1)^{2/3}\right)}{\sqrt{x^4 + x^2 + 1}}$$

Antiderivative was successfully verified.

[In] Integrate[1/((1 + x^2)*Sqrt[1 + x^2 + x^4]),x]

[Out] -(((-1)^(2/3)*Sqrt[1 + (-1)^(1/3)*x^2]*Sqrt[1 - (-1)^(2/3)*x^2]*EllipticPi[(-1)^(1/3), (-I)*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)])/Sqrt[1 + x^2 + x^4])

Maple [C] time = 0.016, size = 104, normalized size = 1.5

$$\frac{1}{\sqrt{-\frac{1}{2} + \frac{i}{2}\sqrt{3}}} \sqrt{1 + \frac{x^2}{2} - \frac{i}{2}x^2\sqrt{3}} \sqrt{1 + \frac{x^2}{2} + \frac{i}{2}x^2\sqrt{3}} \text{EllipticPi}\left(\sqrt{-\frac{1}{2} + \frac{i}{2}\sqrt{3}}x, -\left(-\frac{1}{2} + \frac{i}{2}\sqrt{3}\right)^{-1}, \frac{\sqrt{-\frac{1}{2} - \frac{i}{2}\sqrt{3}}}{\sqrt{-\frac{1}{2} + \frac{i}{2}\sqrt{3}}}\right) \frac{1}{\sqrt{x^4 + x^2 + 1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^2+1)/(x^4+x^2+1)^(1/2),x)

[Out] 1/((-1/2+1/2*I*3^(1/2))^(1/2)*(1+1/2*x^2-1/2*I*x^2*3^(1/2))^(1/2)*(1+1/2*x^2+1/2*I*x^2*3^(1/2))^(1/2)/(x^4+x^2+1)^(1/2)*EllipticPi((-1/2+1/2*I*3^(1/2))^(1/2)*x,-1/((-1/2+1/2*I*3^(1/2))),(1/2-1/2*I*3^(1/2))^(1/2)/(-1/2+1/2*I*3^(1/2))^(1/2))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^4 + x^2 + 1}(x^2 + 1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^2+1)/(x^4+x^2+1)^(1/2),x, algorithm="maxima")

[Out] integrate(1/(sqrt(x^4 + x^2 + 1)*(x^2 + 1)), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{x^4 + x^2 + 1}}{x^6 + 2x^4 + 2x^2 + 1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^2+1)/(x^4+x^2+1)^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(x^4 + x^2 + 1)/(x^6 + 2*x^4 + 2*x^2 + 1), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{(x^2 - x + 1)(x^2 + x + 1)(x^2 + 1)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x**2+1)/(x**4+x**2+1)**(1/2),x)

[Out] Integral(1/(sqrt((x**2 - x + 1)*(x**2 + x + 1))*(x**2 + 1)), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^4 + x^2 + 1}(x^2 + 1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^2+1)/(x^4+x^2+1)^(1/2),x, algorithm="giac")

[Out] integrate(1/(sqrt(x^4 + x^2 + 1)*(x^2 + 1)), x)

$$3.236 \quad \int \frac{1}{(1+x^2)^2 \sqrt{1+x^2+x^4}} dx$$

Optimal. Leaf size=118

$$-\frac{(x^2+1) \sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}} \text{EllipticF}\left(2 \tan^{-1}(x), \frac{1}{4}\right)}{4\sqrt{x^4+x^2+1}} + \frac{1}{2} \tan^{-1}\left(\frac{x}{\sqrt{x^4+x^2+1}}\right) + \frac{(x^2+1) \sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}} E\left(2 \tan^{-1}(x) \middle| \frac{1}{4}\right)}{2\sqrt{x^4+x^2+1}}$$

[Out] ArcTan[x/Sqrt[1 + x^2 + x^4]]/2 + ((1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*EllipticE[2*ArcTan[x], 1/4])/(2*Sqrt[1 + x^2 + x^4]) - ((1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*EllipticF[2*ArcTan[x], 1/4])/(4*Sqrt[1 + x^2 + x^4])

Rubi [A] time = 0.13507, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.4$, Rules used = {1223, 1712, 1195, 12, 1317, 1103, 1698, 203}

$$\frac{1}{2} \tan^{-1}\left(\frac{x}{\sqrt{x^4+x^2+1}}\right) - \frac{(x^2+1) \sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}} F\left(2 \tan^{-1}(x) \middle| \frac{1}{4}\right)}{4\sqrt{x^4+x^2+1}} + \frac{(x^2+1) \sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}} E\left(2 \tan^{-1}(x) \middle| \frac{1}{4}\right)}{2\sqrt{x^4+x^2+1}}$$

Antiderivative was successfully verified.

[In] Int[1/((1 + x^2)^2*Sqrt[1 + x^2 + x^4]),x]

[Out] ArcTan[x/Sqrt[1 + x^2 + x^4]]/2 + ((1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*EllipticE[2*ArcTan[x], 1/4])/(2*Sqrt[1 + x^2 + x^4]) - ((1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*EllipticF[2*ArcTan[x], 1/4])/(4*Sqrt[1 + x^2 + x^4])

Rule 1223

Int[((d_) + (e_.)*(x_)^2)^(q_)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] := -Simp[(e^2*x*(d + e*x^2)^(q + 1)*Sqrt[a + b*x^2 + c*x^4])/(2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2)), Int[((d + e*x^2)^(q + 1)*Simp[a*e^2*(2*q + 3) + 2*d*(c*d - b*e)*(q + 1) - 2*e*(c*d*(q + 1) - b*e*(q + 2))*x^2 + c*e^2*(2*q + 5)*x^4, x])/Sqrt[a + b*x^2 + c*x^4], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && ILtQ[q, -1]

Rule 1712

Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_Symbol] := With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[P4x, x, 4]}, -Dist[C/e^2, Int[(d - e*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] + Dist[1/e^2, Int[(C*d^2 + A*e^2 + B*e^2*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[P4x, x^2, 2] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[c*d^2 - a*e^2, 0]

Rule 1195

Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] := With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2))], x]

2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)]/(q*Sqrt[a + b*x^2 + c*x^4]), x] /; EqQ[e + d*q^2, 0] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 1317

Int[(x_)^2/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_Symbol] := Dist[d/(2*d*e), Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[d/(2*d*e), Int[(d - e*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && PosQ[c/a] && EqQ[c*d^2 - a*e^2, 0]

Rule 1103

Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] := With[{q = Rt[c/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]), x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]

Rule 1698

Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_Symbol] := Dist[A, Subst[Int[1/(d - (b*d - 2*a*e)*x^2), x], x, x/Sqrt[a + b*x^2 + c*x^4]], x] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[c*d^2 - a*e^2, 0] && EqQ[B*d + A*e, 0]

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rubi steps

$$\begin{aligned}
\int \frac{1}{(1+x^2)^2 \sqrt{1+x^2+x^4}} dx &= \frac{x\sqrt{1+x^2+x^4}}{2(1+x^2)} - \frac{1}{2} \int \frac{-1+2x^2+x^4}{(1+x^2)\sqrt{1+x^2+x^4}} dx \\
&= \frac{x\sqrt{1+x^2+x^4}}{2(1+x^2)} + \frac{1}{2} \int \frac{1-x^2}{\sqrt{1+x^2+x^4}} dx - \frac{1}{2} \int \frac{2x^2}{(1+x^2)\sqrt{1+x^2+x^4}} dx \\
&= \frac{(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}} E\left(2 \tan^{-1}(x) \middle| \frac{1}{4}\right)}{2\sqrt{1+x^2+x^4}} - \int \frac{x^2}{(1+x^2)\sqrt{1+x^2+x^4}} dx \\
&= \frac{(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}} E\left(2 \tan^{-1}(x) \middle| \frac{1}{4}\right)}{2\sqrt{1+x^2+x^4}} - \frac{1}{2} \int \frac{1}{\sqrt{1+x^2+x^4}} dx + \frac{1}{2} \int \frac{1-x^2}{(1+x^2)\sqrt{1+x^2+x^4}} dx \\
&= \frac{(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}} E\left(2 \tan^{-1}(x) \middle| \frac{1}{4}\right)}{2\sqrt{1+x^2+x^4}} - \frac{(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}} F\left(2 \tan^{-1}(x) \middle| \frac{1}{4}\right)}{4\sqrt{1+x^2+x^4}} + \frac{1}{2} \text{Subst} \left(\int \frac{1-x^2}{(1+x^2)\sqrt{1+x^2+x^4}} dx \right) \\
&= \frac{1}{2} \tan^{-1} \left(\frac{x}{\sqrt{1+x^2+x^4}} \right) + \frac{(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}} E\left(2 \tan^{-1}(x) \middle| \frac{1}{4}\right)}{2\sqrt{1+x^2+x^4}} - \frac{(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}} F\left(2 \tan^{-1}(x) \middle| \frac{1}{4}\right)}{4\sqrt{1+x^2+x^4}}
\end{aligned}$$

Mathematica [C] time = 0.407151, size = 226, normalized size = 1.92

$$-(-1)^{2/3} \sqrt{\sqrt[3]{-1}x^2 + 1} \sqrt{1 - (-1)^{2/3}x^2} \text{EllipticF}\left(i \sinh^{-1}\left((-1)^{5/6}x\right), (-1)^{2/3}\right) + \sqrt[3]{-1} \sqrt{\sqrt[3]{-1}x^2 + 1} \sqrt{1 - (-1)^{2/3}x^2} \left(\text{EllipticF}\left(i \sinh^{-1}\left((-1)^{5/6}x\right), (-1)^{2/3}\right) + \sqrt[3]{-1} \sqrt{\sqrt[3]{-1}x^2 + 1} \sqrt{1 - (-1)^{2/3}x^2} \text{EllipticE}\left(i \sinh^{-1}\left((-1)^{5/6}x\right), (-1)^{2/3}\right)\right)$$

Antiderivative was successfully verified.

[In] Integrate[1/((1 + x^2)^2*Sqrt[1 + x^2 + x^4]),x]

[Out] ((x + x^3 + x^5)/(1 + x^2) - (-1)^(2/3)*Sqrt[1 + (-1)^(1/3)*x^2]*Sqrt[1 - (-1)^(2/3)*x^2]*EllipticF[I*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)] + (-1)^(1/3)*Sqrt[1 + (-1)^(1/3)*x^2]*Sqrt[1 - (-1)^(2/3)*x^2]*(-EllipticE[I*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)] + EllipticF[I*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)]) - 2*(-1)^(2/3)*Sqrt[1 + (-1)^(1/3)*x^2]*Sqrt[1 - (-1)^(2/3)*x^2]*EllipticPi[(-1)^(1/3), (-I)*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)]/(2*Sqrt[1 + x^2 + x^4])

Maple [C] time = 0.019, size = 397, normalized size = 3.4

$$\frac{x}{2x^2+2} \sqrt{x^4+x^2+1} - \frac{1}{\sqrt{-2+2i\sqrt{3}}} \sqrt{1+\frac{x^2}{2}-\frac{i}{2}x^2\sqrt{3}} \sqrt{1+\frac{x^2}{2}+\frac{i}{2}x^2\sqrt{3}} \text{EllipticF}\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2}, \frac{\sqrt{-2+2i\sqrt{3}}}{2}\right) \sqrt{-2+2i\sqrt{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^2+1)^2/(x^4+x^2+1)^(1/2),x)

[Out] 1/2*x*(x^4+x^2+1)^(1/2)/(x^2+1)-1/(-2+2*I*3^(1/2))^(1/2)*(1+1/2*x^2-1/2*I*x^2*3^(1/2))^(1/2)*(1+1/2*x^2+1/2*I*x^2*3^(1/2))^(1/2)/(x^4+x^2+1)^(1/2)*E11


```

ipticF(1/2*x*(-2+2*I*3^(1/2))^(1/2),1/2*(-2+2*I*3^(1/2))^(1/2))+2/(-2+2*I*3
^(1/2))^(1/2)*(1+1/2*x^2-1/2*I*x^2*3^(1/2))^(1/2)*(1+1/2*x^2+1/2*I*x^2*3^(
1/2))^(1/2)/(x^4+x^2+1)^(1/2)/(I*3^(1/2)+1)*EllipticF(1/2*x*(-2+2*I*3^(1/2))
^(1/2),1/2*(-2+2*I*3^(1/2))^(1/2))-2/(-2+2*I*3^(1/2))^(1/2)*(1+1/2*x^2-1/2*
I*x^2*3^(1/2))^(1/2)*(1+1/2*x^2+1/2*I*x^2*3^(1/2))^(1/2)/(x^4+x^2+1)^(1/2)/
(I*3^(1/2)+1)*EllipticE(1/2*x*(-2+2*I*3^(1/2))^(1/2),1/2*(-2+2*I*3^(1/2))^(
1/2))+1/(-1/2+1/2*I*3^(1/2))^(1/2)*(1+1/2*x^2-1/2*I*x^2*3^(1/2))^(1/2)*(1+1
/2*x^2+1/2*I*x^2*3^(1/2))^(1/2)/(x^4+x^2+1)^(1/2)*EllipticPi((-1/2+1/2*I*3^
(1/2))^(1/2)*x,-1/(-1/2+1/2*I*3^(1/2)),(-1/2-1/2*I*3^(1/2))^(1/2)/(-1/2+1/2
*I*3^(1/2))^(1/2))

```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^4 + x^2 + 1}(x^2 + 1)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^2/(x^4+x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(x^4 + x^2 + 1)*(x^2 + 1)^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{x^4 + x^2 + 1}}{x^8 + 3x^6 + 4x^4 + 3x^2 + 1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^2/(x^4+x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^4 + x^2 + 1)/(x^8 + 3*x^6 + 4*x^4 + 3*x^2 + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{(x^2 - x + 1)(x^2 + x + 1)}(x^2 + 1)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2+1)**2/(x**4+x**2+1)**(1/2),x)
```

```
[Out] Integral(1/(sqrt((x**2 - x + 1)*(x**2 + x + 1))*(x**2 + 1)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^4 + x^2 + 1}(x^2 + 1)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^2/(x^4+x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(x^4 + x^2 + 1)*(x^2 + 1)^2), x)
```

$$3.237 \quad \int \frac{1}{(1+x^2)^3 \sqrt{1+x^2+x^4}} dx$$

Optimal. Leaf size=142

$$\frac{(x^2+1) \sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}} \operatorname{EllipticF}\left(2 \tan^{-1}(x), \frac{1}{4}\right)}{2\sqrt{x^4+x^2+1}} + \frac{\sqrt{x^4+x^2+1}x}{4(x^2+1)^2} + \frac{1}{4} \tan^{-1}\left(\frac{x}{\sqrt{x^4+x^2+1}}\right) + \frac{3(x^2+1) \sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}} E\left(2 \tan^{-1}(x)\right)}{4\sqrt{x^4+x^2+1}}$$

```
[Out] (x*Sqrt[1 + x^2 + x^4])/(4*(1 + x^2)^2) + ArcTan[x/Sqrt[1 + x^2 + x^4]]/4 +
(3*(1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*EllipticE[2*ArcTan[x], 1/4]
)/(4*Sqrt[1 + x^2 + x^4]) - ((1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*El
lipticF[2*ArcTan[x], 1/4])/(2*Sqrt[1 + x^2 + x^4])
```

Rubi [A] time = 0.277109, antiderivative size = 142, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.45$, Rules used = {1223, 1696, 1593, 1712, 1195, 1700, 1103, 1698, 203}

$$\frac{\sqrt{x^4+x^2+1}x}{4(x^2+1)^2} + \frac{1}{4} \tan^{-1}\left(\frac{x}{\sqrt{x^4+x^2+1}}\right) - \frac{(x^2+1) \sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}} F\left(2 \tan^{-1}(x)\right)}{2\sqrt{x^4+x^2+1}} + \frac{3(x^2+1) \sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}} E\left(2 \tan^{-1}(x)\right)}{4\sqrt{x^4+x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 + x^2)^3*Sqrt[1 + x^2 + x^4]), x]
```

```
[Out] (x*Sqrt[1 + x^2 + x^4])/(4*(1 + x^2)^2) + ArcTan[x/Sqrt[1 + x^2 + x^4]]/4 +
(3*(1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*EllipticE[2*ArcTan[x], 1/4]
)/(4*Sqrt[1 + x^2 + x^4]) - ((1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*El
lipticF[2*ArcTan[x], 1/4])/(2*Sqrt[1 + x^2 + x^4])
```

Rule 1223

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_
Symbol] := -Simp[(e^2*x*(d + e*x^2)^(q + 1)*Sqrt[a + b*x^2 + c*x^4])/(2*d*(
q + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(2*d*(q + 1)*(c*d^2 - b*d*e +
a*e^2)), Int[((d + e*x^2)^(q + 1)*Simp[a*e^2*(2*q + 3) + 2*d*(c*d - b*e)*(q
+ 1) - 2*e*(c*d*(q + 1) - b*e*(q + 2))*x^2 + c*e^2*(2*q + 5)*x^4, x])/Sqrt
[a + b*x^2 + c*x^4], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c
, 0] && ILtQ[q, -1]
```

Rule 1696

```
Int[((P4x_)*((d_) + (e_.)*(x_)^2)^(q_))/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x
_)^4], x_Symbol] := With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = C
oeff[P4x, x, 4]}, -Simp[((C*d^2 - B*d*e + A*e^2)*x*(d + e*x^2)^(q + 1)*Sqrt
[a + b*x^2 + c*x^4])/(2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(2*
d*(q + 1)*(c*d^2 - b*d*e + a*e^2)), Int[((d + e*x^2)^(q + 1)*Simp[a*d*(C*d
- B*e) + A*(a*e^2*(2*q + 3) + 2*d*(c*d - b*e)*(q + 1)) - 2*((B*d - A*e)*(b
e*(q + 2) - c*d*(q + 1)) - C*d*(b*d + a*e*(q + 1)))*x^2 + c*(C*d^2 - B*d*e
+ A*e^2)*(2*q + 5)*x^4, x])/Sqrt[a + b*x^2 + c*x^4], x], x] /; FreeQ[{a, b
, c, d, e}, x] && PolyQ[P4x, x^2] && LeQ[Expon[P4x, x], 4] && NeQ[b^2 - 4*a
*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, -1]
```

Rule 1593

$\text{Int}[(u_)*((a_)*(x_)^{(p_)} + (b_)*(x_)^{(q_}))^{(n_)}, x_Symbol] \rightarrow \text{Int}[u*x^{(n*p)}*(a + b*x^{(q-p)})^n, x] /; \text{FreeQ}\{a, b, p, q, x\} \&\& \text{IntegerQ}[n] \&\& \text{PosQ}[q - p]$

Rule 1712

$\text{Int}[(P4x_)/(((d_)+(e_)*(x_)^2)*\text{Sqrt}[(a_)+(b_)*(x_)^2+(c_)*(x_)^4]), x_Symbol] \rightarrow \text{With}\{A = \text{Coeff}[P4x, x, 0], B = \text{Coeff}[P4x, x, 2], C = \text{Coeff}[P4x, x, 4]\}, -\text{Dist}[C/e^2, \text{Int}[(d - e*x^2)/\text{Sqrt}[a + b*x^2 + c*x^4], x], x] + \text{Dist}[1/e^2, \text{Int}[(C*d^2 + A*e^2 + B*e^2*x^2)/((d + e*x^2)*\text{Sqrt}[a + b*x^2 + c*x^4]), x], x] /; \text{FreeQ}\{a, b, c, d, e, x\} \&\& \text{PolyQ}[P4x, x^2, 2] \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& \text{NeQ}[c*d^2 - b*d*e + a*e^2, 0] \&\& \text{EqQ}[c*d^2 - a*e^2, 0]$

Rule 1195

$\text{Int}[(d_)+(e_)*(x_)^2)/\text{Sqrt}[(a_)+(b_)*(x_)^2+(c_)*(x_)^4], x_Symbol] \rightarrow \text{With}\{q = \text{Rt}[c/a, 4]\}, -\text{Simp}[(d*x*\text{Sqrt}[a + b*x^2 + c*x^4])/(a*(1 + q^2*x^2)), x] + \text{Simp}[(d*(1 + q^2*x^2)*\text{Sqrt}[a + b*x^2 + c*x^4])/(a*(1 + q^2*x^2)^2)]*\text{EllipticE}[2*\text{ArcTan}[q*x], 1/2 - (b*q^2)/(4*c)]/(q*\text{Sqrt}[a + b*x^2 + c*x^4]), x] /; \text{EqQ}[e + d*q^2, 0] /; \text{FreeQ}\{a, b, c, d, e, x\} \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& \text{PosQ}[c/a]$

Rule 1700

$\text{Int}[(A_)+(B_)*(x_)^2)/(((d_)+(e_)*(x_)^2)*\text{Sqrt}[(a_)+(b_)*(x_)^2+(c_)*(x_)^4]), x_Symbol] \rightarrow \text{Dist}[(B*d + A*e)/(2*d*e), \text{Int}[1/\text{Sqrt}[a + b*x^2 + c*x^4], x], x] - \text{Dist}[(B*d - A*e)/(2*d*e), \text{Int}[(d - e*x^2)/((d + e*x^2)*\text{Sqrt}[a + b*x^2 + c*x^4]), x], x] /; \text{FreeQ}\{a, b, c, d, e, A, B, x\} \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& \text{NeQ}[c*d^2 - b*d*e + a*e^2, 0] \&\& \text{EqQ}[c*d^2 - a*e^2, 0] \&\& \text{NeQ}[B*d + A*e, 0]$

Rule 1103

$\text{Int}[1/\text{Sqrt}[(a_)+(b_)*(x_)^2+(c_)*(x_)^4], x_Symbol] \rightarrow \text{With}\{q = \text{Rt}[c/a, 4]\}, \text{Simp}[(1 + q^2*x^2)*\text{Sqrt}[a + b*x^2 + c*x^4])/(a*(1 + q^2*x^2)^2)]*\text{EllipticF}[2*\text{ArcTan}[q*x], 1/2 - (b*q^2)/(4*c)]/(2*q*\text{Sqrt}[a + b*x^2 + c*x^4]), x] /; \text{FreeQ}\{a, b, c, x\} \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& \text{PosQ}[c/a]$

Rule 1698

$\text{Int}[(A_)+(B_)*(x_)^2)/(((d_)+(e_)*(x_)^2)*\text{Sqrt}[(a_)+(b_)*(x_)^2+(c_)*(x_)^4]), x_Symbol] \rightarrow \text{Dist}[A, \text{Subst}[\text{Int}[1/(d - (b*d - 2*a*e)*x^2), x], x, x/\text{Sqrt}[a + b*x^2 + c*x^4]], x] /; \text{FreeQ}\{a, b, c, d, e, A, B, x\} \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& \text{NeQ}[c*d^2 - b*d*e + a*e^2, 0] \&\& \text{EqQ}[c*d^2 - a*e^2, 0] \&\& \text{EqQ}[B*d + A*e, 0]$

Rule 203

$\text{Int}[(a_)+(b_)*(x_)^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(1*\text{ArcTan}[\text{Rt}[b, 2]*x]/\text{Rt}[a, 2])]/(\text{Rt}[a, 2]*\text{Rt}[b, 2]), x] /; \text{FreeQ}\{a, b, x\} \&\& \text{PosQ}[a/b] \&\& (\text{GtQ}[a, 0] \parallel \text{GtQ}[b, 0])$

Rubi steps

$$\begin{aligned}
\int \frac{1}{(1+x^2)^3 \sqrt{1+x^2+x^4}} dx &= \frac{x\sqrt{1+x^2+x^4}}{4(1+x^2)^2} - \frac{1}{4} \int \frac{-3+2x^2-x^4}{(1+x^2)^2 \sqrt{1+x^2+x^4}} dx \\
&= \frac{x\sqrt{1+x^2+x^4}}{4(1+x^2)^2} + \frac{3x\sqrt{1+x^2+x^4}}{4(1+x^2)} + \frac{1}{8} \int \frac{-10x^2-6x^4}{(1+x^2)\sqrt{1+x^2+x^4}} dx \\
&= \frac{x\sqrt{1+x^2+x^4}}{4(1+x^2)^2} + \frac{3x\sqrt{1+x^2+x^4}}{4(1+x^2)} + \frac{1}{8} \int \frac{x^2(-10-6x^2)}{(1+x^2)\sqrt{1+x^2+x^4}} dx \\
&= \frac{x\sqrt{1+x^2+x^4}}{4(1+x^2)^2} + \frac{3x\sqrt{1+x^2+x^4}}{4(1+x^2)} + \frac{1}{8} \int \frac{-6-10x^2}{(1+x^2)\sqrt{1+x^2+x^4}} dx + \frac{3}{4} \int \frac{1-x^2}{\sqrt{1+x^2+x^4}} dx \\
&= \frac{x\sqrt{1+x^2+x^4}}{4(1+x^2)^2} + \frac{3(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}} E\left(2 \tan^{-1}(x) \middle| \frac{1}{4}\right)}{4\sqrt{1+x^2+x^4}} + \frac{1}{4} \int \frac{1-x^2}{(1+x^2)\sqrt{1+x^2+x^4}} dx \\
&= \frac{x\sqrt{1+x^2+x^4}}{4(1+x^2)^2} + \frac{3(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}} E\left(2 \tan^{-1}(x) \middle| \frac{1}{4}\right)}{4\sqrt{1+x^2+x^4}} - \frac{(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}} F\left(2 \tan^{-1}(x) \middle| \frac{1}{4}\right)}{2\sqrt{1+x^2+x^4}} \\
&= \frac{x\sqrt{1+x^2+x^4}}{4(1+x^2)^2} + \frac{1}{4} \tan^{-1}\left(\frac{x}{\sqrt{1+x^2+x^4}}\right) + \frac{3(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}} E\left(2 \tan^{-1}(x) \middle| \frac{1}{4}\right)}{4\sqrt{1+x^2+x^4}} - \dots
\end{aligned}$$

Mathematica [C] time = 0.331736, size = 235, normalized size = 1.65

$$-2(-1)^{2/3} \sqrt{\sqrt[3]{-1}x^2 + 1} \sqrt{1 - (-1)^{2/3}x^2} \text{EllipticF}\left(i \sinh^{-1}\left((-1)^{5/6}x\right), (-1)^{2/3}\right) - 3\sqrt[3]{-1} \sqrt{\sqrt[3]{-1}x^2 + 1} \sqrt{1 - (-1)^{2/3}x^2} \left(E\left(i \sinh^{-1}\left((-1)^{5/6}x\right), (-1)^{2/3}\right)\right)$$

Antiderivative was successfully verified.

[In] Integrate[1/((1 + x^2)^3*Sqrt[1 + x^2 + x^4]),x]

[Out] ((x*(4 + 3*x^2)*(1 + x^2 + x^4))/(1 + x^2)^2 - 3*(-1)^(1/3)*Sqrt[1 + (-1)^(1/3)*x^2]*Sqrt[1 - (-1)^(2/3)*x^2]*(EllipticE[I*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)] - EllipticF[I*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)]) - 2*(-1)^(2/3)*Sqrt[1 + (-1)^(1/3)*x^2]*Sqrt[1 - (-1)^(2/3)*x^2]*EllipticF[I*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)] - 2*(-1)^(2/3)*Sqrt[1 + (-1)^(1/3)*x^2]*Sqrt[1 - (-1)^(2/3)*x^2]*EllipticPi[(-1)^(1/3), (-I)*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)]/(4*Sqrt[1 + x^2 + x^4])

Maple [C] time = 0.021, size = 418, normalized size = 2.9

$$\frac{x}{4(x^2+1)^2} \sqrt{x^4+x^2+1} + \frac{3x}{4x^2+4} \sqrt{x^4+x^2+1} - \frac{1}{\sqrt{-2+2i\sqrt{3}}} \sqrt{1+\frac{x^2}{2}-\frac{i}{2}x^2\sqrt{3}} \sqrt{1+\frac{x^2}{2}+\frac{i}{2}x^2\sqrt{3}} \text{EllipticF}\left(x\sqrt{\frac{x^2+1}{x^4+x^2+1}}, \frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^2+1)^3/(x^4+x^2+1)^(1/2),x)

```
[Out] 1/4*x*(x^4+x^2+1)^(1/2)/(x^2+1)^2+3/4*x*(x^4+x^2+1)^(1/2)/(x^2+1)-1/(-2+2*I
*3^(1/2))^(1/2)*(1+1/2*x^2-1/2*I*x^2*3^(1/2))^(1/2)*(1+1/2*x^2+1/2*I*x^2*3^
(1/2))^(1/2)/(x^4+x^2+1)^(1/2)*EllipticF(1/2*x*(-2+2*I*3^(1/2))^(1/2),1/2*(
-2+2*I*3^(1/2))^(1/2))+3/(-2+2*I*3^(1/2))^(1/2)*(1+1/2*x^2-1/2*I*x^2*3^(1/2
))^(1/2)*(1+1/2*x^2+1/2*I*x^2*3^(1/2))^(1/2)/(x^4+x^2+1)^(1/2)/(I*3^(1/2)+1
)*EllipticF(1/2*x*(-2+2*I*3^(1/2))^(1/2),1/2*(-2+2*I*3^(1/2))^(1/2))-3/(-2+
2*I*3^(1/2))^(1/2)*(1+1/2*x^2-1/2*I*x^2*3^(1/2))^(1/2)*(1+1/2*x^2+1/2*I*x^2
*3^(1/2))^(1/2)/(x^4+x^2+1)^(1/2)/(I*3^(1/2)+1)*EllipticE(1/2*x*(-2+2*I*3^(
1/2))^(1/2),1/2*(-2+2*I*3^(1/2))^(1/2))+1/2/(-1/2+1/2*I*3^(1/2))^(1/2)*(1+1
/2*x^2-1/2*I*x^2*3^(1/2))^(1/2)*(1+1/2*x^2+1/2*I*x^2*3^(1/2))^(1/2)/(x^4+x^
2+1)^(1/2)*EllipticPi((-1/2+1/2*I*3^(1/2))^(1/2)*x,-1/(-1/2+1/2*I*3^(1/2)),
(-1/2-1/2*I*3^(1/2))^(1/2)/(-1/2+1/2*I*3^(1/2))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^4 + x^2 + 1}(x^2 + 1)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^3/(x^4+x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(x^4 + x^2 + 1)*(x^2 + 1)^3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{x^4 + x^2 + 1}}{x^{10} + 4x^8 + 7x^6 + 7x^4 + 4x^2 + 1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^3/(x^4+x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^4 + x^2 + 1)/(x^10 + 4*x^8 + 7*x^6 + 7*x^4 + 4*x^2 + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{(x^2 - x + 1)(x^2 + x + 1)}(x^2 + 1)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2+1)**3/(x**4+x**2+1)**(1/2),x)
```

```
[Out] Integral(1/(sqrt((x**2 - x + 1)*(x**2 + x + 1))*(x**2 + 1)**3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^4 + x^2 + 1}(x^2 + 1)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^3/(x^4+x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(x^4 + x^2 + 1)*(x^2 + 1)^3), x)
```

$$3.238 \quad \int \frac{(1+x^2)^3}{(1+x^2+x^4)^{3/2}} dx$$

Optimal. Leaf size=144

$$\frac{(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}\text{EllipticF}\left(2\tan^{-1}(x),\frac{1}{4}\right)}{\sqrt{x^4+x^2+1}} + \frac{2\sqrt{x^4+x^2+1}x}{3(x^2+1)} - \frac{(1-x^2)x}{3\sqrt{x^4+x^2+1}} - \frac{2(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}E\left(2\tan^{-1}(x)\middle|\frac{1}{4}\right)}{3\sqrt{x^4+x^2+1}}$$

[Out] $-(x*(1-x^2))/(3*\text{Sqrt}[1+x^2+x^4]) + (2*x*\text{Sqrt}[1+x^2+x^4])/(3*(1+x^2)) - (2*(1+x^2)*\text{Sqrt}[(1+x^2+x^4)/(1+x^2)^2]*\text{EllipticE}[2*\text{ArcTan}[x], 1/4])/(3*\text{Sqrt}[1+x^2+x^4]) + ((1+x^2)*\text{Sqrt}[(1+x^2+x^4)/(1+x^2)^2]*\text{EllipticF}[2*\text{ArcTan}[x], 1/4])/\text{Sqrt}[1+x^2+x^4]$

Rubi [A] time = 0.0445113, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$, Rules used = {1205, 1197, 1103, 1195}

$$\frac{2\sqrt{x^4+x^2+1}x}{3(x^2+1)} - \frac{(1-x^2)x}{3\sqrt{x^4+x^2+1}} + \frac{(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}F\left(2\tan^{-1}(x)\middle|\frac{1}{4}\right)}{\sqrt{x^4+x^2+1}} - \frac{2(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}E\left(2\tan^{-1}(x)\middle|\frac{1}{4}\right)}{3\sqrt{x^4+x^2+1}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(1+x^2)^3/(1+x^2+x^4)^{(3/2)}, x]$

[Out] $-(x*(1-x^2))/(3*\text{Sqrt}[1+x^2+x^4]) + (2*x*\text{Sqrt}[1+x^2+x^4])/(3*(1+x^2)) - (2*(1+x^2)*\text{Sqrt}[(1+x^2+x^4)/(1+x^2)^2]*\text{EllipticE}[2*\text{ArcTan}[x], 1/4])/(3*\text{Sqrt}[1+x^2+x^4]) + ((1+x^2)*\text{Sqrt}[(1+x^2+x^4)/(1+x^2)^2]*\text{EllipticF}[2*\text{ArcTan}[x], 1/4])/\text{Sqrt}[1+x^2+x^4]$

Rule 1205

$\text{Int}[(d_+ + (e_+)(x_+)^2)^{(q_+)}((a_+ + (b_+)(x_+)^2 + (c_+)(x_+)^4)^{(p_+)}, x_+ \text{Symbol}] \rightarrow \text{With}[\{f = \text{Coeff}[\text{PolynomialRemainder}[(d + e*x^2)^q, a + b*x^2 + c*x^4, x], x, 0], g = \text{Coeff}[\text{PolynomialRemainder}[(d + e*x^2)^q, a + b*x^2 + c*x^4, x], x, 2]\}, \text{Simp}[(x*(a + b*x^2 + c*x^4)^{(p+1)}*(a*b*g - f*(b^2 - 2*a*c) - c*(b*f - 2*a*g)*x^2)/(2*a*(p+1)*(b^2 - 4*a*c)), x] + \text{Dist}[1/(2*a*(p+1)*(b^2 - 4*a*c)), \text{Int}[(a + b*x^2 + c*x^4)^{(p+1)}*\text{ExpandToSum}[2*a*(p+1)*(b^2 - 4*a*c)*\text{PolynomialQuotient}[(d + e*x^2)^q, a + b*x^2 + c*x^4, x] + b^2*f*(2*p+3) - 2*a*c*f*(4*p+5) - a*b*g + c*(4*p+7)*(b*f - 2*a*g)*x^2, x], x], x] /; \text{FreeQ}[\{a, b, c, d, e\}, x] \&\amp; \text{NeQ}[b^2 - 4*a*c, 0] \&\amp; \text{NeQ}[c*d^2 - b*d*e + a*e^2, 0] \&\amp; \text{IGtQ}[q, 1] \&\amp; \text{LtQ}[p, -1]$

Rule 1197

$\text{Int}[(d_+ + (e_+)(x_+)^2)/\text{Sqrt}[(a_+ + (b_+)(x_+)^2 + (c_+)(x_+)^4], x_+ \text{Symbol}] \rightarrow \text{With}[\{q = \text{Rt}[c/a, 2]\}, \text{Dist}[(e + d*q)/q, \text{Int}[1/\text{Sqrt}[a + b*x^2 + c*x^4], x], x] - \text{Dist}[e/q, \text{Int}[(1 - q*x^2)/\text{Sqrt}[a + b*x^2 + c*x^4], x], x] /; \text{NeQ}[e + d*q, 0] /; \text{FreeQ}[\{a, b, c, d, e\}, x] \&\amp; \text{NeQ}[b^2 - 4*a*c, 0] \&\amp; \text{PosQ}[c/a]$

Rule 1103


```
Int[1/Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4], x_Symbol] := With[{q = Rt[c/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_)*(x_)^2)/Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4], x_Symbol] := With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c*x^4]), x] /; EqQ[e + d*q^2, 0] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\begin{aligned} \int \frac{(1+x^2)^3}{(1+x^2+x^4)^{3/2}} dx &= -\frac{x(1-x^2)}{3\sqrt{1+x^2+x^4}} + \frac{1}{3} \int \frac{4+2x^2}{\sqrt{1+x^2+x^4}} dx \\ &= -\frac{x(1-x^2)}{3\sqrt{1+x^2+x^4}} - \frac{2}{3} \int \frac{1-x^2}{\sqrt{1+x^2+x^4}} dx + 2 \int \frac{1}{\sqrt{1+x^2+x^4}} dx \\ &= -\frac{x(1-x^2)}{3\sqrt{1+x^2+x^4}} + \frac{2x\sqrt{1+x^2+x^4}}{3(1+x^2)} - \frac{2(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}} E\left(2 \tan^{-1}(x) \middle| \frac{1}{4}\right)}{3\sqrt{1+x^2+x^4}} + \frac{(1+x^2)\sqrt{\frac{1+x^2}{(1+x^2+x^4)}}}{\sqrt{1+x^2+x^4}} \end{aligned}$$

Mathematica [F] time = 0, size = 0, normalized size = 0.

\$Aborted

Verification is Not applicable to the result.

[In] Integrate[(1 + x^2)^3/(1 + x^2 + x^4)^(3/2), x]

[Out] \$Aborted

Maple [C] time = 0.033, size = 268, normalized size = 1.9

$$-4 \frac{-x/6 + 1/6 x^3}{\sqrt{x^4 + x^2 + 1}} + \frac{8}{3\sqrt{-2 + 2i\sqrt{3}}} \sqrt{1 - \left(-\frac{1}{2} + \frac{i}{2}\sqrt{3}\right)x^2} \sqrt{1 - \left(-\frac{1}{2} - \frac{i}{2}\sqrt{3}\right)x^2} \text{EllipticF}\left(\frac{x\sqrt{-2 + 2i\sqrt{3}}}{2}, \frac{\sqrt{-2 + 2i\sqrt{3}}}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2+1)^3/(x^4+x^2+1)^(3/2), x)

[Out] $-4*(-1/6*x+1/6*x^3)/(x^4+x^2+1)^{(1/2)}+8/3/(-2+2*I*3^{(1/2)})^{(1/2)}*(1-(-1/2+1/2*I*3^{(1/2)})*x^2)^{(1/2)}*(1-(-1/2-1/2*I*3^{(1/2)})*x^2)^{(1/2)}/(x^4+x^2+1)^{(1/2)}*EllipticF(1/2*x*(-2+2*I*3^{(1/2)})^{(1/2)}, 1/2*(-2+2*I*3^{(1/2)})^{(1/2)})-8/3/(-2+2*I*3^{(1/2)})^{(1/2)}*(1-(-1/2+1/2*I*3^{(1/2)})*x^2)^{(1/2)}*(1-(-1/2-1/2*I*3^{(1/2)})*x^2)^{(1/2)}/(x^4+x^2+1)^{(1/2)}/(I*3^{(1/2)}+1)*(EllipticF(1/2*x*(-2+2*I*3^{(1/2)})^{(1/2)})^{(1/2)}, 1/2*(-2+2*I*3^{(1/2)})^{(1/2)})-EllipticE(1/2*x*(-2+2*I*3^{(1/2)})^{(1/2)})^{(1/2)}$

$(1/2), 1/2*(-2+2*I*3^(1/2))^(1/2)) - 6*(1/6*x^3+1/3*x)/(x^4+x^2+1)^(1/2) - 6*(-1/3*x^3-1/6*x)/(x^4+x^2+1)^(1/2)$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(x^2 + 1)^3}{(x^4 + x^2 + 1)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+1)^3/(x^4+x^2+1)^(3/2),x, algorithm="maxima")

[Out] integrate((x^2 + 1)^3/(x^4 + x^2 + 1)^(3/2), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(x^6 + 3x^4 + 3x^2 + 1)\sqrt{x^4 + x^2 + 1}}{x^8 + 2x^6 + 3x^4 + 2x^2 + 1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+1)^3/(x^4+x^2+1)^(3/2),x, algorithm="fricas")

[Out] integral((x^6 + 3*x^4 + 3*x^2 + 1)*sqrt(x^4 + x^2 + 1)/(x^8 + 2*x^6 + 3*x^4 + 2*x^2 + 1), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(x^2 + 1)^3}{((x^2 - x + 1)(x^2 + x + 1))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**2+1)**3/(x**4+x**2+1)**(3/2),x)

[Out] Integral((x**2 + 1)**3/((x**2 - x + 1)*(x**2 + x + 1))**3/2, x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(x^2 + 1)^3}{(x^4 + x^2 + 1)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+1)^3/(x^4+x^2+1)^(3/2),x, algorithm="giac")

[Out] integrate((x^2 + 1)^3/(x^4 + x^2 + 1)^(3/2), x)

$$3.239 \quad \int \frac{(1+x^2)^2}{(1+x^2+x^4)^{3/2}} dx$$

Optimal. Leaf size=98

$$-\frac{2\sqrt{x^4+x^2+1}x}{3(x^2+1)} + \frac{(2x^2+1)x}{3\sqrt{x^4+x^2+1}} + \frac{2(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}E\left(2\tan^{-1}(x)\middle|\frac{1}{4}\right)}{3\sqrt{x^4+x^2+1}}$$

[Out] (x*(1 + 2*x^2))/(3*Sqrt[1 + x^2 + x^4]) - (2*x*Sqrt[1 + x^2 + x^4])/(3*(1 + x^2)) + (2*(1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*EllipticE[2*ArcTan[x], 1/4])/(3*Sqrt[1 + x^2 + x^4])

Rubi [A] time = 0.0248185, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$, Rules used = {1205, 1195}

$$-\frac{2\sqrt{x^4+x^2+1}x}{3(x^2+1)} + \frac{(2x^2+1)x}{3\sqrt{x^4+x^2+1}} + \frac{2(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}E\left(2\tan^{-1}(x)\middle|\frac{1}{4}\right)}{3\sqrt{x^4+x^2+1}}$$

Antiderivative was successfully verified.

[In] Int[(1 + x^2)^2/(1 + x^2 + x^4)^(3/2), x]

[Out] (x*(1 + 2*x^2))/(3*Sqrt[1 + x^2 + x^4]) - (2*x*Sqrt[1 + x^2 + x^4])/(3*(1 + x^2)) + (2*(1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*EllipticE[2*ArcTan[x], 1/4])/(3*Sqrt[1 + x^2 + x^4])

Rule 1205

Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{f = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 + c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*g - f*(b^2 - 2*a*c) - c*(b*f - 2*a*g)*x^2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[(d + e*x^2)^q, a + b*x^2 + c*x^4, x] + b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g + c*(4*p + 7)*(b*f - 2*a*g)*x^2, x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1] && LtQ[p, -1]

Rule 1195

Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c*x^4]), x] /; EqQ[e + d*q^2, 0] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]

Rubi steps

$$\int \frac{(1+x^2)^2}{(1+x^2+x^4)^{3/2}} dx = \frac{x(1+2x^2)}{3\sqrt{1+x^2+x^4}} + \frac{1}{3} \int \frac{2-2x^2}{\sqrt{1+x^2+x^4}} dx$$

$$= \frac{x(1+2x^2)}{3\sqrt{1+x^2+x^4}} - \frac{2x\sqrt{1+x^2+x^4}}{3(1+x^2)} + \frac{2(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}} E\left(2 \tan^{-1}(x) \middle| \frac{1}{4}\right)}{3\sqrt{1+x^2+x^4}}$$

Mathematica [F] time = 0, size = 0, normalized size = 0.

\$Aborted

Verification is Not applicable to the result.

[In] Integrate[(1 + x^2)^2/(1 + x^2 + x^4)^(3/2), x]

[Out] \$Aborted

Maple [C] time = 0.007, size = 268, normalized size = 2.7

$$-2 \frac{1/6 x^3 + x/3}{\sqrt{x^4 + x^2 + 1}} + \frac{4}{3 \sqrt{-2 + 2i\sqrt{3}}} \sqrt{1 - \left(-\frac{1}{2} + \frac{i}{2}\sqrt{3}\right) x^2} \sqrt{1 - \left(-\frac{1}{2} - \frac{i}{2}\sqrt{3}\right) x^2} \text{EllipticF}\left(\frac{x\sqrt{-2 + 2i\sqrt{3}}}{2}, \frac{\sqrt{-2 + 2i\sqrt{3}}}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2+1)^2/(x^4+x^2+1)^(3/2), x)

[Out] $-2*(1/6*x^3+1/3*x)/(x^4+x^2+1)^{(1/2)}+4/3/(-2+2*I*3^{(1/2)})^{(1/2)}*(1-(-1/2+1/2*I*3^{(1/2)})*x^2)^{(1/2)}*(1-(-1/2-1/2*I*3^{(1/2)})*x^2)^{(1/2)}/(x^4+x^2+1)^{(1/2)}* \text{EllipticF}(1/2*x*(-2+2*I*3^{(1/2)})^{(1/2)}, 1/2*(-2+2*I*3^{(1/2)})^{(1/2)})+8/3/(-2+2*I*3^{(1/2)})^{(1/2)}*(1-(-1/2+1/2*I*3^{(1/2)})*x^2)^{(1/2)}*(1-(-1/2-1/2*I*3^{(1/2)})*x^2)^{(1/2)}/(x^4+x^2+1)^{(1/2)}/(I*3^{(1/2)}+1)*(\text{EllipticF}(1/2*x*(-2+2*I*3^{(1/2)})^{(1/2)}, 1/2*(-2+2*I*3^{(1/2)})^{(1/2)}) - \text{EllipticE}(1/2*x*(-2+2*I*3^{(1/2)})^{(1/2)}, 1/2*(-2+2*I*3^{(1/2)})^{(1/2)}) - 4*(-1/3*x^3-1/6*x)/(x^4+x^2+1)^{(1/2)} - 2*(-1/6*x+1/6*x^3)/(x^4+x^2+1)^{(1/2)}$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(x^2+1)^2}{(x^4+x^2+1)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+1)^2/(x^4+x^2+1)^(3/2), x, algorithm="maxima")

[Out] integrate((x^2 + 1)^2/(x^4 + x^2 + 1)^(3/2), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(x^4 + 2x^2 + 1)\sqrt{x^4 + x^2 + 1}}{x^8 + 2x^6 + 3x^4 + 2x^2 + 1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+1)^2/(x^4+x^2+1)^(3/2),x, algorithm="fricas")

[Out] integral((x^4 + 2*x^2 + 1)*sqrt(x^4 + x^2 + 1)/(x^8 + 2*x^6 + 3*x^4 + 2*x^2 + 1), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(x^2 + 1)^2}{((x^2 - x + 1)(x^2 + x + 1))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**2+1)**2/(x**4+x**2+1)**(3/2),x)

[Out] Integral((x**2 + 1)**2/((x**2 - x + 1)*(x**2 + x + 1))**(3/2), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(x^2 + 1)^2}{(x^4 + x^2 + 1)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+1)^2/(x^4+x^2+1)^(3/2),x, algorithm="giac")

[Out] integrate((x^2 + 1)^2/(x^4 + x^2 + 1)^(3/2), x)

$$3.240 \quad \int \frac{1+x^2}{(1+x^2+x^4)^{3/2}} dx$$

Optimal. Leaf size=96

$$-\frac{\sqrt{x^4+x^2+1}x}{3(x^2+1)} + \frac{(x^2+2)x}{3\sqrt{x^4+x^2+1}} + \frac{(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}E\left(2\tan^{-1}(x)\middle|\frac{1}{4}\right)}{3\sqrt{x^4+x^2+1}}$$

[Out] (x*(2 + x^2))/(3*Sqrt[1 + x^2 + x^4]) - (x*Sqrt[1 + x^2 + x^4])/(3*(1 + x^2)) + ((1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*EllipticE[2*ArcTan[x], 1/4])/(3*Sqrt[1 + x^2 + x^4])

Rubi [A] time = 0.0206474, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1178, 1195}

$$-\frac{\sqrt{x^4+x^2+1}x}{3(x^2+1)} + \frac{(x^2+2)x}{3\sqrt{x^4+x^2+1}} + \frac{(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}E\left(2\tan^{-1}(x)\middle|\frac{1}{4}\right)}{3\sqrt{x^4+x^2+1}}$$

Antiderivative was successfully verified.

[In] Int[(1 + x^2)/(1 + x^2 + x^4)^(3/2), x]

[Out] (x*(2 + x^2))/(3*Sqrt[1 + x^2 + x^4]) - (x*Sqrt[1 + x^2 + x^4])/(3*(1 + x^2)) + ((1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*EllipticE[2*ArcTan[x], 1/4])/(3*Sqrt[1 + x^2 + x^4])

Rule 1178

Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && IntegerQ[2*p]

Rule 1195

Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c*x^4]), x] /; EqQ[e + d*q^2, 0] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]

Rubi steps

$$\int \frac{1+x^2}{(1+x^2+x^4)^{3/2}} dx = \frac{x(2+x^2)}{3\sqrt{1+x^2+x^4}} + \frac{1}{3} \int \frac{1-x^2}{\sqrt{1+x^2+x^4}} dx$$

$$= \frac{x(2+x^2)}{3\sqrt{1+x^2+x^4}} - \frac{x\sqrt{1+x^2+x^4}}{3(1+x^2)} + \frac{(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}} E\left(2 \tan^{-1}(x) \middle| \frac{1}{4}\right)}{3\sqrt{1+x^2+x^4}}$$

Mathematica [F] time = 0, size = 0, normalized size = 0.

\$Aborted

Verification is Not applicable to the result.

[In] Integrate[(1 + x^2)/(1 + x^2 + x^4)^(3/2), x]

[Out] \$Aborted

Maple [C] time = 0.007, size = 247, normalized size = 2.6

$$-2 \frac{-1/3 x^3 - x/6}{\sqrt{x^4 + x^2 + 1}} + \frac{2}{3 \sqrt{-2 + 2i\sqrt{3}}} \sqrt{1 - \left(-\frac{1}{2} + \frac{i}{2}\sqrt{3}\right) x^2} \sqrt{1 - \left(-\frac{1}{2} - \frac{i}{2}\sqrt{3}\right) x^2} \text{EllipticF}\left(\frac{x\sqrt{-2 + 2i\sqrt{3}}}{2}, \frac{\sqrt{-2 + 2i\sqrt{3}}}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2+1)/(x^4+x^2+1)^(3/2), x)

[Out] $-2*(-1/3*x^3-1/6*x)/(x^4+x^2+1)^{(1/2)}+2/3/(-2+2*I*3^{(1/2)})^{(1/2)}*(1-(-1/2+1/2*I*3^{(1/2)})*x^2)^{(1/2)}*(1-(-1/2-1/2*I*3^{(1/2)})*x^2)^{(1/2)}/(x^4+x^2+1)^{(1/2)}*EllipticF(1/2*x*(-2+2*I*3^{(1/2)})^{(1/2)}, 1/2*(-2+2*I*3^{(1/2)})^{(1/2)})+4/3/(-2+2*I*3^{(1/2)})^{(1/2)}*(1-(-1/2+1/2*I*3^{(1/2)})*x^2)^{(1/2)}*(1-(-1/2-1/2*I*3^{(1/2)})*x^2)^{(1/2)}/(x^4+x^2+1)^{(1/2)}/(I*3^{(1/2)}+1)*(EllipticF(1/2*x*(-2+2*I*3^{(1/2)})^{(1/2)}, 1/2*(-2+2*I*3^{(1/2)})^{(1/2)})-EllipticE(1/2*x*(-2+2*I*3^{(1/2)})^{(1/2)}, 1/2*(-2+2*I*3^{(1/2)})^{(1/2)})-2*(-1/6*x+1/6*x^3)/(x^4+x^2+1)^{(1/2)}$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 + 1}{(x^4 + x^2 + 1)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+1)/(x^4+x^2+1)^(3/2), x, algorithm="maxima")

[Out] integrate((x^2 + 1)/(x^4 + x^2 + 1)^(3/2), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{x^4 + x^2 + 1}(x^2 + 1)}{x^8 + 2x^6 + 3x^4 + 2x^2 + 1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+1)/(x^4+x^2+1)^(3/2),x, algorithm="fricas")

[Out] integral(sqrt(x^4 + x^2 + 1)*(x^2 + 1)/(x^8 + 2*x^6 + 3*x^4 + 2*x^2 + 1), x
)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 + 1}{((x^2 - x + 1)(x^2 + x + 1))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**2+1)/(x**4+x**2+1)**(3/2),x)

[Out] Integral((x**2 + 1)/((x**2 - x + 1)*(x**2 + x + 1))**(3/2), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 + 1}{(x^4 + x^2 + 1)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+1)/(x^4+x^2+1)^(3/2),x, algorithm="giac")

[Out] integrate((x^2 + 1)/(x^4 + x^2 + 1)^(3/2), x)

$$3.241 \quad \int \frac{1}{(1+x^2)(1+x^2+x^4)^{3/2}} dx$$

Optimal. Leaf size=166

$$\frac{3(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}\text{EllipticF}\left(2\tan^{-1}(x), \frac{1}{4}\right)}{4\sqrt{x^4+x^2+1}} + \frac{2\sqrt{x^4+x^2+1}x}{3(x^2+1)} - \frac{(2x^2+1)x}{3\sqrt{x^4+x^2+1}} + \frac{1}{2}\tan^{-1}\left(\frac{x}{\sqrt{x^4+x^2+1}}\right) - \frac{2(x^2+1)}{3\sqrt{x^4+x^2+1}}$$

[Out] $-(x*(1+2*x^2))/(3*\text{Sqrt}[1+x^2+x^4]) + (2*x*\text{Sqrt}[1+x^2+x^4])/(3*(1+x^2)) + \text{ArcTan}[x/\text{Sqrt}[1+x^2+x^4]]/2 - (2*(1+x^2)*\text{Sqrt}[(1+x^2+x^4)/(1+x^2)^2]*\text{EllipticE}[2*\text{ArcTan}[x], 1/4])/(3*\text{Sqrt}[1+x^2+x^4]) + (3*(1+x^2)*\text{Sqrt}[(1+x^2+x^4)/(1+x^2)^2]*\text{EllipticF}[2*\text{ArcTan}[x], 1/4])/(4*\text{Sqrt}[1+x^2+x^4])$

Rubi [A] time = 0.103757, antiderivative size = 166, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.4$, Rules used = {1221, 1119, 1197, 1103, 1195, 1210, 1698, 203}

$$\frac{2\sqrt{x^4+x^2+1}x}{3(x^2+1)} - \frac{(2x^2+1)x}{3\sqrt{x^4+x^2+1}} + \frac{1}{2}\tan^{-1}\left(\frac{x}{\sqrt{x^4+x^2+1}}\right) + \frac{3(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}F\left(2\tan^{-1}(x)\middle|\frac{1}{4}\right)}{4\sqrt{x^4+x^2+1}} - \frac{2(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}}{3\sqrt{x^4+x^2+1}}$$

Antiderivative was successfully verified.

[In] Int[1/((1+x^2)*(1+x^2+x^4)^(3/2)),x]

[Out] $-(x*(1+2*x^2))/(3*\text{Sqrt}[1+x^2+x^4]) + (2*x*\text{Sqrt}[1+x^2+x^4])/(3*(1+x^2)) + \text{ArcTan}[x/\text{Sqrt}[1+x^2+x^4]]/2 - (2*(1+x^2)*\text{Sqrt}[(1+x^2+x^4)/(1+x^2)^2]*\text{EllipticE}[2*\text{ArcTan}[x], 1/4])/(3*\text{Sqrt}[1+x^2+x^4]) + (3*(1+x^2)*\text{Sqrt}[(1+x^2+x^4)/(1+x^2)^2]*\text{EllipticF}[2*\text{ArcTan}[x], 1/4])/(4*\text{Sqrt}[1+x^2+x^4])$

Rule 1221

Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_)/((d_) + (e_.)*(x_)^2), x_Symbol] := Dist[1/(c*d^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x^2)*(a + b*x^2 + c*x^4)^p, x], x] + Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[(a + b*x^2 + c*x^4)^(p+1)/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[p + 1/2, 0]

Rule 1119

Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := Simp[(d*(d*x)^(m-1)*(b + 2*c*x^2)*(a + b*x^2 + c*x^4)^(p+1))/(2*(p+1)*(b^2 - 4*a*c)), x] - Dist[d^2/(2*(p+1)*(b^2 - 4*a*c)), Int[(d*x)^(m-2)*(b*(m-1) + 2*c*(m+4*p+5)*x^2)*(a + b*x^2 + c*x^4)^(p+1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && GtQ[m, 1] && LeQ[m, 3] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])

Rule 1197

Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] := With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne

$Q[e + d*q, 0] /; \text{FreeQ}\{a, b, c, d, e\}, x\} \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0] \ \&\& \ \text{PosQ}[c/a]$

Rule 1103

$\text{Int}[1/\text{Sqrt}[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4], x_Symbol] \ :> \ \text{With}\{q = \text{Rt}[c/a, 4]\}, \ \text{Simp}[\frac{(1 + q^2*x^2)*\text{Sqrt}[a + b*x^2 + c*x^4]}{a*(1 + q^2*x^2)^2}] * \text{EllipticF}[2*\text{ArcTan}[q*x], 1/2 - (b*q^2)/(4*c)] / (2*q*\text{Sqrt}[a + b*x^2 + c*x^4]), x] /; \ \text{FreeQ}\{a, b, c\}, x\} \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0] \ \&\& \ \text{PosQ}[c/a]$

Rule 1195

$\text{Int}[\frac{(d_) + (e_)*(x_)^2}{\text{Sqrt}[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4]}, x_Symbol] \ :> \ \text{With}\{q = \text{Rt}[c/a, 4]\}, \ -\text{Simp}[\frac{d*x*\text{Sqrt}[a + b*x^2 + c*x^4]}{a*(1 + q^2*x^2)}, x] + \text{Simp}[\frac{d*(1 + q^2*x^2)*\text{Sqrt}[a + b*x^2 + c*x^4]}{a*(1 + q^2*x^2)^2}] * \text{EllipticE}[2*\text{ArcTan}[q*x], 1/2 - (b*q^2)/(4*c)] / (q*\text{Sqrt}[a + b*x^2 + c*x^4]), x] /; \ \text{EqQ}[e + d*q^2, 0] /; \ \text{FreeQ}\{a, b, c, d, e\}, x\} \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0] \ \&\& \ \text{PosQ}[c/a]$

Rule 1210

$\text{Int}[1/((d_) + (e_)*(x_)^2)*\text{Sqrt}[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4]], x_Symbol] \ :> \ \text{Dist}[1/(2*d), \ \text{Int}[1/\text{Sqrt}[a + b*x^2 + c*x^4], x], x] + \text{Dist}[1/(2*d), \ \text{Int}[\frac{d - e*x^2}{(d + e*x^2)*\text{Sqrt}[a + b*x^2 + c*x^4]}, x], x] /; \ \text{FreeQ}\{a, b, c, d, e\}, x\} \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0] \ \&\& \ \text{NeQ}[c*d^2 - b*d*e + a*e^2, 0] \ \&\& \ \text{EqQ}[c*d^2 - a*e^2, 0]$

Rule 1698

$\text{Int}[\frac{(A_) + (B_)*(x_)^2}{((d_) + (e_)*(x_)^2)*\text{Sqrt}[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4]}], x_Symbol] \ :> \ \text{Dist}[A, \ \text{Subst}[\text{Int}[1/(d - (b*d - 2*a*e)*x^2), x], x, x/\text{Sqrt}[a + b*x^2 + c*x^4]], x] /; \ \text{FreeQ}\{a, b, c, d, e, A, B\}, x\} \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0] \ \&\& \ \text{NeQ}[c*d^2 - b*d*e + a*e^2, 0] \ \&\& \ \text{EqQ}[c*d^2 - a*e^2, 0] \ \&\& \ \text{EqQ}[B*d + A*e, 0]$

Rule 203

$\text{Int}[\frac{(a_) + (b_)*(x_)^2}{(a_ + b_*x^2)^2}, x_Symbol] \ :> \ \text{Simp}[\frac{1*\text{ArcTan}[\text{Rt}[b, 2]*x]/\text{Rt}[a, 2]]}{\text{Rt}[a, 2]*\text{Rt}[b, 2]}, x] /; \ \text{FreeQ}\{a, b\}, x\} \ \&\& \ \text{PosQ}[a/b] \ \&\& \ (\text{GtQ}[a, 0] \ || \ \text{GtQ}[b, 0])$

Rubi steps

$$\begin{aligned} \int \frac{1}{(1+x^2)(1+x^2+x^4)^{3/2}} dx &= - \int \frac{x^2}{(1+x^2+x^4)^{3/2}} dx + \int \frac{1}{(1+x^2)\sqrt{1+x^2+x^4}} dx \\ &= -\frac{x(1+2x^2)}{3\sqrt{1+x^2+x^4}} + \frac{1}{3} \int \frac{1+2x^2}{\sqrt{1+x^2+x^4}} dx + \frac{1}{2} \int \frac{1}{\sqrt{1+x^2+x^4}} dx + \frac{1}{2} \int \frac{1-x^2}{(1+x^2)\sqrt{1+x^2+x^4}} dx \\ &= -\frac{x(1+2x^2)}{3\sqrt{1+x^2+x^4}} + \frac{(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}} F\left(2 \tan^{-1}(x) \middle| \frac{1}{4}\right)}{4\sqrt{1+x^2+x^4}} + \frac{1}{2} \text{Subst}\left(\int \frac{1}{1+x^2} dx, x, \frac{x}{\sqrt{1+x^2+x^4}}\right) \\ &= -\frac{x(1+2x^2)}{3\sqrt{1+x^2+x^4}} + \frac{2x\sqrt{1+x^2+x^4}}{3(1+x^2)} + \frac{1}{2} \tan^{-1}\left(\frac{x}{\sqrt{1+x^2+x^4}}\right) - \frac{2(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}} E}{3\sqrt{1+x^2+x^4}} \end{aligned}$$

Mathematica [C] time = 0.213119, size = 204, normalized size = 1.23

$$\frac{\sqrt[3]{-1}(\sqrt[3]{-1}-2)\sqrt{\sqrt[3]{-1}x^2+1}\sqrt{1-(-1)^{2/3}x^2}\text{EllipticF}\left(i\sinh^{-1}\left((-1)^{5/6}x\right),(-1)^{2/3}\right)-2x^3+2\sqrt[3]{-1}\sqrt{\sqrt[3]{-1}x^2+1}\sqrt{1-(-1)^{2/3}x^2}}{3\sqrt{x^4+1}}$$

Antiderivative was successfully verified.

[In] Integrate[1/((1 + x^2)*(1 + x^2 + x^4)^(3/2)),x]

[Out] $(-x - 2x^3 + 2(-1)^{1/3}\sqrt{1 + (-1)^{1/3}x^2}\sqrt{1 - (-1)^{2/3}x^2})\text{EllipticE}[I\text{ArcSinh}[(-1)^{5/6}x], (-1)^{2/3}] + (-1)^{1/3}(-2 + (-1)^{1/3})\sqrt{1 + (-1)^{1/3}x^2}\sqrt{1 - (-1)^{2/3}x^2}\text{EllipticF}[I\text{ArcSinh}[(-1)^{5/6}x], (-1)^{2/3}] - 3(-1)^{2/3}\sqrt{1 + (-1)^{1/3}x^2}\sqrt{1 - (-1)^{2/3}x^2}\text{EllipticPi}[(-1)^{1/3}, (-I)\text{ArcSinh}[(-1)^{5/6}x], (-1)^{2/3}]/(3\sqrt{1 + x^2 + x^4})$

Maple [C] time = 0.017, size = 398, normalized size = 2.4

$$-2\frac{1/3x^3+x/6}{\sqrt{x^4+x^2+1}}+\frac{2}{3\sqrt{-2+2i\sqrt{3}}}\sqrt{1+\frac{x^2}{2}-\frac{i}{2}x^2\sqrt{3}}\sqrt{1+\frac{x^2}{2}+\frac{i}{2}x^2\sqrt{3}}\text{EllipticF}\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2},\frac{\sqrt{-2+2i\sqrt{3}}}{2}\right)\sqrt{-2+2i\sqrt{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^2+1)/(x^4+x^2+1)^(3/2),x)

[Out] $-2*(1/3*x^3+1/6*x)/(x^4+x^2+1)^{(1/2)}+2/3/(-2+2*I*3^{(1/2)})^{(1/2)}*(1+1/2*x^2-1/2*I*x^2*3^{(1/2)})^{(1/2)}*(1+1/2*x^2+1/2*I*x^2*3^{(1/2)})^{(1/2)}/(x^4+x^2+1)^{(1/2)}*\text{EllipticF}(1/2*x*(-2+2*I*3^{(1/2)})^{(1/2)},1/2*(-2+2*I*3^{(1/2)})^{(1/2)})-8/3/(-2+2*I*3^{(1/2)})^{(1/2)}*(1+1/2*x^2-1/2*I*x^2*3^{(1/2)})^{(1/2)}*(1+1/2*x^2+1/2*I*x^2*3^{(1/2)})^{(1/2)}/(x^4+x^2+1)^{(1/2)}/(I*3^{(1/2)}+1)*\text{EllipticF}(1/2*x*(-2+2*I*3^{(1/2)})^{(1/2)},1/2*(-2+2*I*3^{(1/2)})^{(1/2)})+8/3/(-2+2*I*3^{(1/2)})^{(1/2)}*(1+1/2*x^2-1/2*I*x^2*3^{(1/2)})^{(1/2)}*(1+1/2*x^2+1/2*I*x^2*3^{(1/2)})^{(1/2)}/(x^4+x^2+1)^{(1/2)}/(I*3^{(1/2)}+1)*\text{EllipticE}(1/2*x*(-2+2*I*3^{(1/2)})^{(1/2)},1/2*(-2+2*I*3^{(1/2)})^{(1/2)})+1/(-1/2+1/2*I*3^{(1/2)})^{(1/2)}*(1+1/2*x^2-1/2*I*x^2*3^{(1/2)})^{(1/2)}*(1+1/2*x^2+1/2*I*x^2*3^{(1/2)})^{(1/2)}/(x^4+x^2+1)^{(1/2)}*\text{EllipticPi}((-1/2+1/2*I*3^{(1/2)})^{(1/2)}*x,-1/(-1/2+1/2*I*3^{(1/2)}),(-1/2-1/2*I*3^{(1/2)})^{(1/2)})/(-1/2+1/2*I*3^{(1/2)})^{(1/2)}$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x^4+x^2+1)^{\frac{3}{2}}(x^2+1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^2+1)/(x^4+x^2+1)^(3/2),x, algorithm="maxima")

[Out] integrate(1/((x^4 + x^2 + 1)^(3/2)*(x^2 + 1)), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{x^4 + x^2 + 1}}{x^{10} + 3x^8 + 5x^6 + 5x^4 + 3x^2 + 1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^2+1)/(x^4+x^2+1)^(3/2),x, algorithm="fricas")

[Out] integral(sqrt(x^4 + x^2 + 1)/(x^10 + 3*x^8 + 5*x^6 + 5*x^4 + 3*x^2 + 1), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{((x^2 - x + 1)(x^2 + x + 1))^{\frac{3}{2}}(x^2 + 1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x**2+1)/(x**4+x**2+1)**(3/2),x)

[Out] Integral(1/(((x**2 - x + 1)*(x**2 + x + 1))**(3/2)*(x**2 + 1)), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x^4 + x^2 + 1)^{\frac{3}{2}}(x^2 + 1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^2+1)/(x^4+x^2+1)^(3/2),x, algorithm="giac")

[Out] integrate(1/((x^4 + x^2 + 1)^(3/2)*(x^2 + 1)), x)

$$3.242 \quad \int \frac{1}{(1+x^2)^2(1+x^2+x^4)^{3/2}} dx$$

Optimal. Leaf size=111

$$\frac{\sqrt{x^4+x^2+1}x}{3(x^2+1)} - \frac{(x^2+2)x}{3\sqrt{x^4+x^2+1}} + \tan^{-1}\left(\frac{x}{\sqrt{x^4+x^2+1}}\right) + \frac{(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}E\left(2\tan^{-1}(x)\middle|\frac{1}{4}\right)}{6\sqrt{x^4+x^2+1}}$$

[Out] $-(x*(2 + x^2))/(3*\text{Sqrt}[1 + x^2 + x^4]) + (x*\text{Sqrt}[1 + x^2 + x^4])/(3*(1 + x^2)) + \text{ArcTan}[x/\text{Sqrt}[1 + x^2 + x^4]] + ((1 + x^2)*\text{Sqrt}[(1 + x^2 + x^4)/(1 + x^2)^2]*\text{EllipticE}[2*\text{ArcTan}[x], 1/4])/(6*\text{Sqrt}[1 + x^2 + x^4])$

Rubi [A] time = 0.278156, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 11, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.55$, Rules used = {1228, 1178, 1195, 1223, 1712, 12, 1317, 1103, 1698, 203, 1210}

$$\frac{\sqrt{x^4+x^2+1}x}{3(x^2+1)} - \frac{(x^2+2)x}{3\sqrt{x^4+x^2+1}} + \tan^{-1}\left(\frac{x}{\sqrt{x^4+x^2+1}}\right) + \frac{(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}E\left(2\tan^{-1}(x)\middle|\frac{1}{4}\right)}{6\sqrt{x^4+x^2+1}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[1/((1 + x^2)^2*(1 + x^2 + x^4)^{(3/2)}), x]$

[Out] $-(x*(2 + x^2))/(3*\text{Sqrt}[1 + x^2 + x^4]) + (x*\text{Sqrt}[1 + x^2 + x^4])/(3*(1 + x^2)) + \text{ArcTan}[x/\text{Sqrt}[1 + x^2 + x^4]] + ((1 + x^2)*\text{Sqrt}[(1 + x^2 + x^4)/(1 + x^2)^2]*\text{EllipticE}[2*\text{ArcTan}[x], 1/4])/(6*\text{Sqrt}[1 + x^2 + x^4])$

Rule 1228

$\text{Int}[(d + (e \cdot x)^2)^q \cdot ((a + (b \cdot x)^2 + (c \cdot x)^4)^p), x_Symbol] \rightarrow \text{Module}\{aa, bb, cc\}, \text{Int}[\text{ExpandIntegrand}[1/\text{Sqrt}[aa + bb*x^2 + c*x^4], (d + e*x^2)^q \cdot (aa + bb*x^2 + cc*x^4)^{(p + 1/2)}, x] /. \{aa \rightarrow a, bb \rightarrow b, cc \rightarrow c\}, x] /; \text{FreeQ}\{a, b, c, d, e\}, x] \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& \text{NeQ}[c*d^2 - b*d*e + a*e^2, 0] \&\& \text{ILtQ}[q, 0] \&\& \text{IntegerQ}[p + 1/2]$

Rule 1178

$\text{Int}[(d + (e \cdot x)^2) \cdot ((a + (b \cdot x)^2 + (c \cdot x)^4)^p), x_Symbol] \rightarrow \text{Simp}[(x \cdot (a \cdot b \cdot e - d \cdot (b^2 - 2 \cdot a \cdot c)) - c \cdot (b \cdot d - 2 \cdot a \cdot e) \cdot x^2) \cdot (a + b \cdot x^2 + c \cdot x^4)^{(p + 1)} / (2 \cdot a \cdot (p + 1) \cdot (b^2 - 4 \cdot a \cdot c)), x] + \text{Dist}[1 / (2 \cdot a \cdot (p + 1) \cdot (b^2 - 4 \cdot a \cdot c)), \text{Int}[\text{Simp}[(2 \cdot p + 3) \cdot d \cdot b^2 - a \cdot b \cdot e - 2 \cdot a \cdot c \cdot d \cdot (4 \cdot p + 5) + (4 \cdot p + 7) \cdot (d \cdot b - 2 \cdot a \cdot e) \cdot c \cdot x^2, x] \cdot (a + b \cdot x^2 + c \cdot x^4)^{(p + 1)}, x], x] /; \text{FreeQ}\{a, b, c, d, e\}, x] \&\& \text{NeQ}[b^2 - 4 \cdot a \cdot c, 0] \&\& \text{NeQ}[c \cdot d^2 - b \cdot d \cdot e + a \cdot e^2, 0] \&\& \text{LtQ}[p, -1] \&\& \text{IntegerQ}[2 \cdot p]$

Rule 1195

$\text{Int}[(d + (e \cdot x)^2) / \text{Sqrt}[(a + (b \cdot x)^2 + (c \cdot x)^4)], x_Symbol] \rightarrow \text{With}\{q = \text{Rt}[c/a, 4]\}, -\text{Simp}[(d \cdot x \cdot \text{Sqrt}[a + b \cdot x^2 + c \cdot x^4]) / (a \cdot (1 + q^2 \cdot x^2)), x] + \text{Simp}[(d \cdot (1 + q^2 \cdot x^2) \cdot \text{Sqrt}[(a + b \cdot x^2 + c \cdot x^4) / (a \cdot (1 + q^2 \cdot x^2)^2)] \cdot \text{EllipticE}[2 \cdot \text{ArcTan}[q \cdot x], 1/2 - (b \cdot q^2) / (4 \cdot c)]) / (q \cdot \text{Sqrt}[a + b \cdot x^2 + c \cdot x^4]), x] /; \text{EqQ}[e + d \cdot q^2, 0] /; \text{FreeQ}\{a, b, c, d, e\}, x] \&\& \text{NeQ}[b^2 -$

$4*a*c, 0] \&\& \text{PosQ}[c/a]$

Rule 1223

$\text{Int}[\frac{(d + e*x^2)^q}{\sqrt{a + b*x^2 + c*x^4}}, x_Symbol] \rightarrow -\text{Simp}[\frac{e^{2*x}*(d + e*x^2)^{q+1}*\sqrt{a + b*x^2 + c*x^4}}{2*d*(q+1)*(c*d^2 - b*d*e + a*e^2)}, x] + \text{Dist}[\frac{1}{2*d*(q+1)*(c*d^2 - b*d*e + a*e^2)}, \text{Int}[\frac{(d + e*x^2)^{q+1}*\text{Simp}[a*e^2*(2*q+3) + 2*d*(c*d - b*e)*(q+1) - 2*e*(c*d*(q+1) - b*e*(q+2))*x^2 + c*e^2*(2*q+5)*x^4, x]}{\sqrt{a + b*x^2 + c*x^4}}, x], x] /; \text{FreeQ}[\{a, b, c, d, e\}, x] \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& \text{ILtQ}[q, -1]$

Rule 1712

$\text{Int}[\frac{P4x}{(d + e*x^2)*\sqrt{a + b*x^2 + c*x^4}}, x_Symbol] \rightarrow \text{With}[\{A = \text{Coeff}[P4x, x, 0], B = \text{Coeff}[P4x, x, 2], C = \text{Coeff}[P4x, x, 4]\}, -\text{Dist}[C/e^2, \text{Int}[(d - e*x^2)/\sqrt{a + b*x^2 + c*x^4}], x], x] + \text{Dist}[1/e^2, \text{Int}[(C*d^2 + A*e^2 + B*e^2*x^2)/((d + e*x^2)*\sqrt{a + b*x^2 + c*x^4}), x], x] /; \text{FreeQ}[\{a, b, c, d, e\}, x] \&\& \text{PolyQ}[P4x, x^2, 2] \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& \text{NeQ}[c*d^2 - b*d*e + a*e^2, 0] \&\& \text{EqQ}[c*d^2 - a*e^2, 0]$

Rule 12

$\text{Int}[a*(u), x_Symbol] \rightarrow \text{Dist}[a, \text{Int}[u, x], x] /; \text{FreeQ}[a, x] \&\& \text{!MatchQ}[u, (b_)*(v_)] /; \text{FreeQ}[b, x]$

Rule 1317

$\text{Int}[\frac{x^2}{(d + e*x^2)*\sqrt{a + b*x^2 + c*x^4}}, x_Symbol] \rightarrow \text{Dist}[d/(2*d*e), \text{Int}[1/\sqrt{a + b*x^2 + c*x^4}], x], x] - \text{Dist}[d/(2*d*e), \text{Int}[(d - e*x^2)/((d + e*x^2)*\sqrt{a + b*x^2 + c*x^4}), x], x] /; \text{FreeQ}[\{a, b, c, d, e\}, x] \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& \text{NeQ}[c*d^2 - b*d*e + a*e^2, 0] \&\& \text{PosQ}[c/a] \&\& \text{EqQ}[c*d^2 - a*e^2, 0]$

Rule 1103

$\text{Int}[1/\sqrt{a + b*x^2 + c*x^4}, x_Symbol] \rightarrow \text{With}[\{q = \text{Rt}[c/a, 4]\}, \text{Simp}[\frac{(1 + q^2*x^2)*\sqrt{a + b*x^2 + c*x^4}}{(a*(1 + q^2*x^2)^2)]*\text{EllipticF}[2*\text{ArcTan}[q*x], 1/2 - (b*q^2)/(4*c)]/(2*q*\sqrt{a + b*x^2 + c*x^4}), x] /; \text{FreeQ}[\{a, b, c\}, x] \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& \text{PosQ}[c/a]$

Rule 1698

$\text{Int}[\frac{(A + B*x^2)}{(d + e*x^2)*\sqrt{a + b*x^2 + c*x^4}}, x_Symbol] \rightarrow \text{Dist}[A, \text{Subst}[\text{Int}[1/(d - (b*d - 2*a*e)*x^2)], x], x, x/\sqrt{a + b*x^2 + c*x^4}], x] /; \text{FreeQ}[\{a, b, c, d, e, A, B\}, x] \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& \text{NeQ}[c*d^2 - b*d*e + a*e^2, 0] \&\& \text{EqQ}[c*d^2 - a*e^2, 0] \&\& \text{EqQ}[B*d + A*e, 0]$

Rule 203

$\text{Int}[\frac{(a + b*x^2)^{-1}}{\sqrt{a + b*x^2 + c*x^4}}, x_Symbol] \rightarrow \text{Simp}[\frac{(1*\text{ArcTan}[\text{Rt}[b, 2]*x]/\text{Rt}[a, 2])}{\text{Rt}[a, 2]*\text{Rt}[b, 2]}, x] /; \text{FreeQ}[\{a, b\}, x] \&\& \text{PosQ}[a/b] \&\& (\text{GtQ}[a, 0] \parallel \text{GtQ}[b, 0])$

Rule 1210

$\text{Int}[1/((d + e*x^2)*\sqrt{a + b*x^2 + c*x^4}), x_Symbol] \rightarrow \text{Dist}[1/(2*d), \text{Int}[1/\sqrt{a + b*x^2 + c*x^4}], x], x] + \text{Dist}[1/(2*d$

), Int[(d - e*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[c*d^2 - a*e^2, 0]

Rubi steps

$$\begin{aligned}
 \int \frac{1}{(1+x^2)^2(1+x^2+x^4)^{3/2}} dx &= \int \left(\frac{-1-x^2}{(1+x^2+x^4)^{3/2}} + \frac{1}{(1+x^2)^2\sqrt{1+x^2+x^4}} + \frac{1}{(1+x^2)\sqrt{1+x^2+x^4}} \right) dx \\
 &= \int \frac{-1-x^2}{(1+x^2+x^4)^{3/2}} dx + \int \frac{1}{(1+x^2)^2\sqrt{1+x^2+x^4}} dx + \int \frac{1}{(1+x^2)\sqrt{1+x^2+x^4}} dx \\
 &= -\frac{x(2+x^2)}{3\sqrt{1+x^2+x^4}} + \frac{x\sqrt{1+x^2+x^4}}{2(1+x^2)} + \frac{1}{3} \int \frac{-1+x^2}{\sqrt{1+x^2+x^4}} dx + \frac{1}{2} \int \frac{1}{\sqrt{1+x^2+x^4}} dx \\
 &= -\frac{x(2+x^2)}{3\sqrt{1+x^2+x^4}} + \frac{5x\sqrt{1+x^2+x^4}}{6(1+x^2)} - \frac{(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}} E\left(2\tan^{-1}(x)\middle|\frac{1}{4}\right)}{3\sqrt{1+x^2+x^4}} + \frac{(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}} H\left(\frac{x}{\sqrt{1+x^2+x^4}}\right)}{6\sqrt{1+x^2+x^4}} \\
 &= -\frac{x(2+x^2)}{3\sqrt{1+x^2+x^4}} + \frac{x\sqrt{1+x^2+x^4}}{3(1+x^2)} + \frac{1}{2} \tan^{-1}\left(\frac{x}{\sqrt{1+x^2+x^4}}\right) + \frac{(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}} H\left(\frac{x}{\sqrt{1+x^2+x^4}}\right)}{6\sqrt{1+x^2+x^4}} \\
 &= -\frac{x(2+x^2)}{3\sqrt{1+x^2+x^4}} + \frac{x\sqrt{1+x^2+x^4}}{3(1+x^2)} + \frac{1}{2} \tan^{-1}\left(\frac{x}{\sqrt{1+x^2+x^4}}\right) + \frac{(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}} H\left(\frac{x}{\sqrt{1+x^2+x^4}}\right)}{6\sqrt{1+x^2+x^4}} \\
 &= -\frac{x(2+x^2)}{3\sqrt{1+x^2+x^4}} + \frac{x\sqrt{1+x^2+x^4}}{3(1+x^2)} + \tan^{-1}\left(\frac{x}{\sqrt{1+x^2+x^4}}\right) + \frac{(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}} E\left(2\tan^{-1}(x)\middle|\frac{1}{4}\right)}{6\sqrt{1+x^2+x^4}}
 \end{aligned}$$

Mathematica [C] time = 0.379126, size = 168, normalized size = 1.51

$$\frac{-\sqrt[3]{-1}(x^2+1)\sqrt{\sqrt[3]{-1}x^2+1}\sqrt{1-(-1)^{2/3}x^2}\left((5\sqrt[3]{-1}-1)\operatorname{EllipticF}\left(i\sinh^{-1}\left((-1)^{5/6}x\right),(-1)^{2/3}\right)+E\left(i\sinh^{-1}\left((-1)^{5/6}x\right),(-1)^{2/3}\right)\right)}{6(x^2+1)\sqrt{x^4+x^2+1}}$$

Antiderivative was successfully verified.

[In] Integrate[1/((1 + x^2)^2*(1 + x^2 + x^4)^(3/2)),x]

[Out] (-2*x*(1 + x^2)*(2 + x^2) + 3*x*(1 + x^2 + x^4) - (-1)^(1/3)*(1 + x^2)*Sqrt[1 + (-1)^(1/3)*x^2]*Sqrt[1 - (-1)^(2/3)*x^2]*(EllipticE[I*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)] + (-1 + 5*(-1)^(1/3))*EllipticF[I*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)] + 12*(-1)^(1/3)*EllipticPi[(-1)^(1/3), (-I)*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)])/(6*(1 + x^2)*Sqrt[1 + x^2 + x^4])

Maple [C] time = 0.025, size = 419, normalized size = 3.8

$$\frac{x}{2x^2+2}\sqrt{x^4+x^2+1} - 2\frac{1/6x^3+x/3}{\sqrt{x^4+x^2+1}} - \frac{5}{3\sqrt{-2+2i\sqrt{3}}}\sqrt{1+\frac{x^2}{2}-\frac{i}{2}x^2\sqrt{3}}\sqrt{1+\frac{x^2}{2}+\frac{i}{2}x^2\sqrt{3}}\text{EllipticF}\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^2+1)^2/(x^4+x^2+1)^(3/2),x)

[Out] 1/2*x*(x^4+x^2+1)^(1/2)/(x^2+1)-2*(1/6*x^3+1/3*x)/(x^4+x^2+1)^(1/2)-5/3/(-2+2*I*3^(1/2))^(1/2)*(1+1/2*x^2-1/2*I*x^2*3^(1/2))^(1/2)*(1+1/2*x^2+1/2*I*x^2*3^(1/2))^(1/2)/(x^4+x^2+1)^(1/2)*EllipticF(1/2*x*(-2+2*I*3^(1/2))^(1/2),1/2*(-2+2*I*3^(1/2))^(1/2))+2/3/(-2+2*I*3^(1/2))^(1/2)*(1+1/2*x^2-1/2*I*x^2*3^(1/2))^(1/2)*(1+1/2*x^2+1/2*I*x^2*3^(1/2))^(1/2)/(x^4+x^2+1)^(1/2)/(I*3^(1/2)+1)*EllipticF(1/2*x*(-2+2*I*3^(1/2))^(1/2),1/2*(-2+2*I*3^(1/2))^(1/2))-2/3/(-2+2*I*3^(1/2))^(1/2)*(1+1/2*x^2-1/2*I*x^2*3^(1/2))^(1/2)*(1+1/2*x^2+1/2*I*x^2*3^(1/2))^(1/2)/(x^4+x^2+1)^(1/2)/(I*3^(1/2)+1)*EllipticE(1/2*x*(-2+2*I*3^(1/2))^(1/2),1/2*(-2+2*I*3^(1/2))^(1/2))+2/(-1/2+1/2*I*3^(1/2))^(1/2)*(1+1/2*x^2-1/2*I*x^2*3^(1/2))^(1/2)*(1+1/2*x^2+1/2*I*x^2*3^(1/2))^(1/2)/(x^4+x^2+1)^(1/2)*EllipticPi((-1/2+1/2*I*3^(1/2))^(1/2)*x,-1/(-1/2+1/2*I*3^(1/2)),(-1/2-1/2*I*3^(1/2))^(1/2)/(-1/2+1/2*I*3^(1/2))^(1/2))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x^4+x^2+1)^{\frac{3}{2}}(x^2+1)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^2+1)^2/(x^4+x^2+1)^(3/2),x, algorithm="maxima")

[Out] integrate(1/((x^4 + x^2 + 1)^(3/2)*(x^2 + 1)^2), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{x^4+x^2+1}}{x^{12}+4x^{10}+8x^8+10x^6+8x^4+4x^2+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^2+1)^2/(x^4+x^2+1)^(3/2),x, algorithm="fricas")

[Out] integral(sqrt(x^4 + x^2 + 1)/(x^12 + 4*x^10 + 8*x^8 + 10*x^6 + 8*x^4 + 4*x^2 + 1), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{((x^2-x+1)(x^2+x+1))^{\frac{3}{2}}(x^2+1)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x**2+1)**2/(x**4+x**2+1)**(3/2),x)

[Out] Integral(1/(((x**2 - x + 1)*(x**2 + x + 1))**(3/2)*(x**2 + 1)**2), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x^4 + x^2 + 1)^{\frac{3}{2}}(x^2 + 1)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^2+1)^2/(x^4+x^2+1)^(3/2),x, algorithm="giac")

[Out] integrate(1/((x^4 + x^2 + 1)^(3/2)*(x^2 + 1)^2), x)

$$3.243 \quad \int \frac{1}{(1+x^2)^3 (1+x^2+x^4)^{3/2}} dx$$

Optimal. Leaf size=190

$$\frac{5(x^2+1) \sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}} \text{EllipticF}\left(2 \tan^{-1}(x), \frac{1}{4}\right)}{4\sqrt{x^4+x^2+1}} - \frac{\sqrt{x^4+x^2+1}x}{3(x^2+1)} + \frac{\sqrt{x^4+x^2+1}x}{4(x^2+1)^2} - \frac{(1-x^2)x}{3\sqrt{x^4+x^2+1}} + \frac{3}{4} \tan^{-1}\left(\frac{x}{\sqrt{x^4+x^2+1}}\right)$$

[Out] $-(x*(1-x^2))/(3*\text{Sqrt}[1+x^2+x^4]) + (x*\text{Sqrt}[1+x^2+x^4])/(4*(1+x^2)^2) - (x*\text{Sqrt}[1+x^2+x^4])/(3*(1+x^2)) + (3*\text{ArcTan}[x/\text{Sqrt}[1+x^2+x^4]])/4 + (19*(1+x^2)*\text{Sqrt}[(1+x^2+x^4)/(1+x^2)^2]*\text{EllipticE}[2*\text{ArcTan}[x], 1/4])/(12*\text{Sqrt}[1+x^2+x^4]) - (5*(1+x^2)*\text{Sqrt}[(1+x^2+x^4)/(1+x^2)^2]*\text{EllipticF}[2*\text{ArcTan}[x], 1/4])/(4*\text{Sqrt}[1+x^2+x^4])$

Rubi [A] time = 0.571493, antiderivative size = 190, normalized size of antiderivative = 1., number of steps used = 23, number of rules used = 14, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.7$, Rules used = {1228, 1092, 1197, 1103, 1195, 1223, 1696, 1593, 1712, 1700, 1698, 203, 12, 1317}

$$-\frac{\sqrt{x^4+x^2+1}x}{3(x^2+1)} + \frac{\sqrt{x^4+x^2+1}x}{4(x^2+1)^2} - \frac{(1-x^2)x}{3\sqrt{x^4+x^2+1}} + \frac{3}{4} \tan^{-1}\left(\frac{x}{\sqrt{x^4+x^2+1}}\right) - \frac{5(x^2+1) \sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}} F\left(2 \tan^{-1}(x) \middle| \frac{1}{4}\right)}{4\sqrt{x^4+x^2+1}} +$$

Antiderivative was successfully verified.

[In] $\text{Int}[1/((1+x^2)^3*(1+x^2+x^4)^{(3/2)}), x]$

[Out] $-(x*(1-x^2))/(3*\text{Sqrt}[1+x^2+x^4]) + (x*\text{Sqrt}[1+x^2+x^4])/(4*(1+x^2)^2) - (x*\text{Sqrt}[1+x^2+x^4])/(3*(1+x^2)) + (3*\text{ArcTan}[x/\text{Sqrt}[1+x^2+x^4]])/4 + (19*(1+x^2)*\text{Sqrt}[(1+x^2+x^4)/(1+x^2)^2]*\text{EllipticE}[2*\text{ArcTan}[x], 1/4])/(12*\text{Sqrt}[1+x^2+x^4]) - (5*(1+x^2)*\text{Sqrt}[(1+x^2+x^4)/(1+x^2)^2]*\text{EllipticF}[2*\text{ArcTan}[x], 1/4])/(4*\text{Sqrt}[1+x^2+x^4])$

Rule 1228

$\text{Int}[(d_ + (e_)*(x_)^2)^{(q_)}*((a_ + (b_)*(x_)^2 + (c_)*(x_)^4)^{(p_)}], x_Symbol] \rightarrow \text{Module}\{aa, bb, cc\}, \text{Int}[\text{ExpandIntegrand}[1/\text{Sqrt}[aa + bb*x^2 + cc*x^4], (d + e*x^2)^q*(aa + bb*x^2 + cc*x^4)^{(p + 1/2)}, x] /. \{aa \rightarrow a, bb \rightarrow b, cc \rightarrow c\}, x] /; \text{FreeQ}\{a, b, c, d, e\}, x] \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& \text{NeQ}[c*d^2 - b*d*e + a*e^2, 0] \&\& \text{ILtQ}[q, 0] \&\& \text{IntegerQ}[p + 1/2]$

Rule 1092

$\text{Int}[(a_ + (b_)*(x_)^2 + (c_)*(x_)^4)^{(p_)}], x_Symbol] \rightarrow -\text{Simp}[(x*(b^2 - 2*a*c + b*c*x^2)*(a + b*x^2 + c*x^4)^{(p + 1)})/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + \text{Dist}[1/(2*a*(p + 1)*(b^2 - 4*a*c)), \text{Int}[(b^2 - 2*a*c + 2*(p + 1)*(b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^{(p + 1)}, x], x] /; \text{FreeQ}\{a, b, c\}, x] \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& \text{LtQ}[p, -1] \&\& \text{IntegerQ}[2*p]$

Rule 1197

$\text{Int}[(d_ + (e_)*(x_)^2)/\text{Sqrt}[(a_ + (b_)*(x_)^2 + (c_)*(x_)^4)], x_Symbol] \rightarrow \text{With}\{q = \text{Rt}[c/a, 2]\}, \text{Dist}[(e + d*q)/q, \text{Int}[1/\text{Sqrt}[a + b*x^2 + c*x^4]$

], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; NeQ[e + d*q, 0] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]

Rule 1103

Int[1/Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4], x_Symbol] := With[{q = Rt[c/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)]/(2*q*Sqrt[a + b*x^2 + c*x^4]), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]

Rule 1195

Int[((d_) + (e_)*(x_)^2)/Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4], x_Symbol] := With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)]/(q*Sqrt[a + b*x^2 + c*x^4]), x] /; EqQ[e + d*q^2, 0] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]

Rule 1223

Int[((d_) + (e_)*(x_)^2)^(q_)/Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4], x_Symbol] := -Simp[(e^2*x*(d + e*x^2)^(q + 1)*Sqrt[a + b*x^2 + c*x^4])/(2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2)), Int[((d + e*x^2)^(q + 1)*Simp[a*e^2*(2*q + 3) + 2*d*(c*d - b*e)*(q + 1) - 2*e*(c*d*(q + 1) - b*e*(q + 2))*x^2 + c*e^2*(2*q + 5)*x^4, x])/Sqrt[a + b*x^2 + c*x^4], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && ILtQ[q, -1]

Rule 1696

Int[(P4x_)*((d_) + (e_)*(x_)^2)^(q_)/Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4], x_Symbol] := With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[P4x, x, 4]}, -Simp[((C*d^2 - B*d*e + A*e^2)*x*(d + e*x^2)^(q + 1)*Sqrt[a + b*x^2 + c*x^4])/(2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2)), Int[((d + e*x^2)^(q + 1)*Simp[a*d*(C*d - B*e) + A*(a*e^2*(2*q + 3) + 2*d*(c*d - b*e)*(q + 1)) - 2*((B*d - A*e)*(b*e*(q + 2) - c*d*(q + 1)) - C*d*(b*d + a*e*(q + 1)))*x^2 + c*(C*d^2 - B*d*e + A*e^2)*(2*q + 5)*x^4, x])/Sqrt[a + b*x^2 + c*x^4], x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[P4x, x^2] && LeQ[Expon[P4x, x], 4] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, -1]

Rule 1593

Int[(u_)*((a_)*(x_)^(p_) + (b_)*(x_)^(q_))^(n_), x_Symbol] := Int[u*x^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rule 1712

Int[(P4x_)/(((d_) + (e_)*(x_)^2)*Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4]), x_Symbol] := With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[P4x, x, 4]}, -Dist[C/e^2, Int[(d - e*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] + Dist[1/e^2, Int[(C*d^2 + A*e^2 + B*e^2*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[P4x, x^2, 2] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[c*d^2 - a*e^2, 0]

Rule 1700

```
Int[((A_) + (B_)*(x_)^2)/(((d_) + (e_)*(x_)^2)*Sqrt[(a_) + (b_)*(x_)^2 +
(c_)*(x_)^4]), x_Symbol] := Dist[(B*d + A*e)/(2*d*e), Int[1/Sqrt[a + b*x^
2 + c*x^4], x], x] - Dist[(B*d - A*e)/(2*d*e), Int[(d - e*x^2)/((d + e*x^2)
*Sqrt[a + b*x^2 + c*x^4]), x], x] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[c*d^2 - a*e^2, 0]
&& NeQ[B*d + A*e, 0]
```

Rule 1698

```
Int[((A_) + (B_)*(x_)^2)/(((d_) + (e_)*(x_)^2)*Sqrt[(a_) + (b_)*(x_)^2 +
(c_)*(x_)^4]), x_Symbol] := Dist[A, Subst[Int[1/(d - (b*d - 2*a*e)*x^2),
x], x, x/Sqrt[a + b*x^2 + c*x^4]], x] /; FreeQ[{a, b, c, d, e, A, B}, x] &&
NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[c*d^2 - a*e^2,
0] && EqQ[B*d + A*e, 0]
```

Rule 203

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_)] /; FreeQ[b, x]
```

Rule 1317

```
Int[(x_)^2/(((d_) + (e_)*(x_)^2)*Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4])
, x_Symbol] := Dist[d/(2*d*e), Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist
[d/(2*d*e), Int[(d - e*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x] /
; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && PosQ[c/a] && EqQ[c*d^2 - a*e^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(1+x^2)^3(1+x^2+x^4)^{3/2}} dx &= \int \left(-\frac{1}{(1+x^2+x^4)^{3/2}} + \frac{1}{(1+x^2)^3\sqrt{1+x^2+x^4}} + \frac{1}{(1+x^2)^2\sqrt{1+x^2+x^4}} \right) dx \\
&= -\int \frac{1}{(1+x^2+x^4)^{3/2}} dx + \int \frac{1}{(1+x^2)^3\sqrt{1+x^2+x^4}} dx + \int \frac{1}{(1+x^2)^2\sqrt{1+x^2+x^4}} dx \\
&= -\frac{x(1-x^2)}{3\sqrt{1+x^2+x^4}} + \frac{x\sqrt{1+x^2+x^4}}{4(1+x^2)^2} + \frac{x\sqrt{1+x^2+x^4}}{2(1+x^2)} - \frac{1}{4} \int \frac{-3+2x^2-x^4}{(1+x^2)^2\sqrt{1+x^2+x^4}} dx \\
&= -\frac{x(1-x^2)}{3\sqrt{1+x^2+x^4}} + \frac{x\sqrt{1+x^2+x^4}}{4(1+x^2)^2} + \frac{5x\sqrt{1+x^2+x^4}}{4(1+x^2)} + \frac{1}{8} \int \frac{-10x^2-6x^4}{(1+x^2)\sqrt{1+x^2+x^4}} dx \\
&= -\frac{x(1-x^2)}{3\sqrt{1+x^2+x^4}} + \frac{x\sqrt{1+x^2+x^4}}{4(1+x^2)^2} + \frac{5x\sqrt{1+x^2+x^4}}{12(1+x^2)} + \frac{5(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}} E\left(2 \operatorname{atan}\left(\frac{x}{\sqrt{1+x^2+x^4}}\right)\right)}{6\sqrt{1+x^2+x^4}} \\
&= -\frac{x(1-x^2)}{3\sqrt{1+x^2+x^4}} + \frac{x\sqrt{1+x^2+x^4}}{4(1+x^2)^2} + \frac{5x\sqrt{1+x^2+x^4}}{12(1+x^2)} + \frac{5(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}} E\left(2 \operatorname{atan}\left(\frac{x}{\sqrt{1+x^2+x^4}}\right)\right)}{6\sqrt{1+x^2+x^4}} \\
&= -\frac{x(1-x^2)}{3\sqrt{1+x^2+x^4}} + \frac{x\sqrt{1+x^2+x^4}}{4(1+x^2)^2} - \frac{x\sqrt{1+x^2+x^4}}{3(1+x^2)} + \frac{19(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}} E\left(2 \operatorname{atan}\left(\frac{x}{\sqrt{1+x^2+x^4}}\right)\right)}{12\sqrt{1+x^2+x^4}} \\
&= -\frac{x(1-x^2)}{3\sqrt{1+x^2+x^4}} + \frac{x\sqrt{1+x^2+x^4}}{4(1+x^2)^2} - \frac{x\sqrt{1+x^2+x^4}}{3(1+x^2)} + \frac{1}{2} \operatorname{atan}^{-1}\left(\frac{x}{\sqrt{1+x^2+x^4}}\right) + \frac{19}{12} \operatorname{atan}^{-1}\left(\frac{x}{\sqrt{1+x^2+x^4}}\right) \\
&= -\frac{x(1-x^2)}{3\sqrt{1+x^2+x^4}} + \frac{x\sqrt{1+x^2+x^4}}{4(1+x^2)^2} - \frac{x\sqrt{1+x^2+x^4}}{3(1+x^2)} + \frac{3}{4} \operatorname{atan}^{-1}\left(\frac{x}{\sqrt{1+x^2+x^4}}\right) + \frac{19}{12} \operatorname{atan}^{-1}\left(\frac{x}{\sqrt{1+x^2+x^4}}\right)
\end{aligned}$$

Mathematica [C] time = 0.336364, size = 192, normalized size = 1.01

$$\frac{-\sqrt[3]{-1}\sqrt{\sqrt[3]{-1}x^2+1}\sqrt{1-(-1)^{2/3}x^2}(x^2+1)^2((-9+10i\sqrt{3})\operatorname{EllipticF}(i\sinh^{-1}((-1)^{5/6}x),(-1)^{2/3})+19E(i\sinh^{-1}((-1)^{5/6}x),(-1)^{2/3}))}{12(x^2+1)\sqrt{1+x^2+x^4}}$$

Antiderivative was successfully verified.

[In] Integrate[1/((1+x^2)^3*(1+x^2+x^4)^(3/2)),x]

[Out] (4*x*(-1+x^2)*(1+x^2)^2+3*x*(1+x^2+x^4)+15*x*(1+x^2)*(1+x^2+x^4)-(-1)^(1/3)*(1+x^2)^2*Sqrt[1+(-1)^(1/3)*x^2]*Sqrt[1-(-1)^(2/3)*x^2]*(19*EllipticE[I*ArcSinh[(-1)^(5/6)*x],(-1)^(2/3)]+(-9+(10*I)*Sqrt[3])*EllipticF[I*ArcSinh[(-1)^(5/6)*x],(-1)^(2/3)]+18*(-1)^(1/3)*EllipticPi[(-1)^(1/3),(-I)*ArcSinh[(-1)^(5/6)*x],(-1)^(2/3)])/(12*(1+x^2)^2*Sqrt[1+x^2+x^4])

Maple [C] time = 0.024, size = 439, normalized size = 2.3

$$\frac{x}{4(x^2+1)^2}\sqrt{x^4+x^2+1} + \frac{5x}{4x^2+4}\sqrt{x^4+x^2+1} - 2\frac{x/6-1/6x^3}{\sqrt{x^4+x^2+1}} - \frac{10}{3\sqrt{-2+2i\sqrt{3}}}\sqrt{1+\frac{x^2}{2}-\frac{i}{2}x^2\sqrt{3}}\sqrt{1+\frac{x^2}{2}+\frac{i}{2}x^2\sqrt{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^2+1)^3/(x^4+x^2+1)^(3/2), x)

[Out] $\frac{1}{4}x(x^4+x^2+1)^{1/2}/(x^2+1)^{5/2} + \frac{5}{4}x(x^4+x^2+1)^{1/2}/(x^2+1)^{5/2} - 2\frac{x/6-1/6x^3}{\sqrt{x^4+x^2+1}} - \frac{10}{3\sqrt{-2+2i\sqrt{3}}}\sqrt{1+\frac{x^2}{2}-\frac{i}{2}x^2\sqrt{3}}\sqrt{1+\frac{x^2}{2}+\frac{i}{2}x^2\sqrt{3}}$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x^4+x^2+1)^{3/2}(x^2+1)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^2+1)^3/(x^4+x^2+1)^(3/2), x, algorithm="maxima")

[Out] integrate(1/((x^4 + x^2 + 1)^(3/2)*(x^2 + 1)^3), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{x^4+x^2+1}}{x^{14}+5x^{12}+12x^{10}+18x^8+18x^6+12x^4+5x^2+1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^2+1)^3/(x^4+x^2+1)^(3/2), x, algorithm="fricas")

[Out] integral(sqrt(x^4 + x^2 + 1)/(x^14 + 5*x^12 + 12*x^10 + 18*x^8 + 18*x^6 + 12*x^4 + 5*x^2 + 1), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{((x^2-x+1)(x^2+x+1))^{3/2}(x^2+1)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x**2+1)**3/(x**4+x**2+1)**(3/2),x)

[Out] Integral(1/(((x**2 - x + 1)*(x**2 + x + 1))**(3/2)*(x**2 + 1)**3), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x^4 + x^2 + 1)^{\frac{3}{2}}(x^2 + 1)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^2+1)^3/(x^4+x^2+1)^(3/2),x, algorithm="giac")

[Out] integrate(1/((x^4 + x^2 + 1)^(3/2)*(x^2 + 1)^3), x)

3.244 $\int (d + ex^2)^4 (a + bx^2 + cx^4) dx$

Optimal. Leaf size=135

$$\frac{1}{9}e^2x^9(eae + 4bd) + 6cd^2 + \frac{1}{5}d^2x^5(6ae^2 + 4bde + cd^2) + \frac{2}{7}dex^7(e(2ae + 3bd) + 2cd^2) + \frac{1}{3}d^3x^3(4ae + bd) + ad^4x + \frac{1}{11}e^3$$

[Out] a*d^4*x + (d^3*(b*d + 4*a*e)*x^3)/3 + (d^2*(c*d^2 + 4*b*d*e + 6*a*e^2)*x^5)/5 + (2*d*e*(2*c*d^2 + e*(3*b*d + 2*a*e))*x^7)/7 + (e^2*(6*c*d^2 + e*(4*b*d + a*e))*x^9)/9 + (e^3*(4*c*d + b*e)*x^11)/11 + (c*e^4*x^13)/13

Rubi [A] time = 0.125944, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {1153}

$$\frac{1}{9}e^2x^9(eae + 4bd) + 6cd^2 + \frac{1}{5}d^2x^5(6ae^2 + 4bde + cd^2) + \frac{2}{7}dex^7(e(2ae + 3bd) + 2cd^2) + \frac{1}{3}d^3x^3(4ae + bd) + ad^4x + \frac{1}{11}e^3$$

Antiderivative was successfully verified.

[In] Int[(d + e*x^2)^4*(a + b*x^2 + c*x^4),x]

[Out] a*d^4*x + (d^3*(b*d + 4*a*e)*x^3)/3 + (d^2*(c*d^2 + 4*b*d*e + 6*a*e^2)*x^5)/5 + (2*d*e*(2*c*d^2 + e*(3*b*d + 2*a*e))*x^7)/7 + (e^2*(6*c*d^2 + e*(4*b*d + a*e))*x^9)/9 + (e^3*(4*c*d + b*e)*x^11)/11 + (c*e^4*x^13)/13

Rule 1153

Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]

Rubi steps

$$\begin{aligned} \int (d + ex^2)^4 (a + bx^2 + cx^4) dx &= \int (ad^4 + d^3(bd + 4ae)x^2 + d^2(cd^2 + 4bde + 6ae^2)x^4 + 2de(2cd^2 + e(3bd + 2ae))x^6 + \\ &= ad^4x + \frac{1}{3}d^3(bd + 4ae)x^3 + \frac{1}{5}d^2(cd^2 + 4bde + 6ae^2)x^5 + \frac{2}{7}de(2cd^2 + e(3bd + 2ae))x^7 \end{aligned}$$

Mathematica [A] time = 0.0384811, size = 135, normalized size = 1.

$$\frac{1}{9}e^2x^9(ae^2 + 4bde + 6cd^2) + \frac{2}{7}dex^7(2ae^2 + 3bde + 2cd^2) + \frac{1}{5}d^2x^5(6ae^2 + 4bde + cd^2) + \frac{1}{3}d^3x^3(4ae + bd) + ad^4x + \frac{1}{11}e^3$$

Antiderivative was successfully verified.

[In] Integrate[(d + e*x^2)^4*(a + b*x^2 + c*x^4),x]

[Out] a*d^4*x + (d^3*(b*d + 4*a*e)*x^3)/3 + (d^2*(c*d^2 + 4*b*d*e + 6*a*e^2)*x^5)/5 + (2*d*e*(2*c*d^2 + 3*b*d*e + 2*a*e^2)*x^7)/7 + (e^2*(6*c*d^2 + 4*b*d*e + a*e^2)*x^9)/9 + (e^3*(4*c*d + b*e)*x^11)/11 + (c*e^4*x^13)/13

Maple [A] time = 0., size = 136, normalized size = 1.

$$\frac{ce^4x^{13}}{13} + \frac{(e^4b + 4de^3c)x^{11}}{11} + \frac{(e^4a + 4de^3b + 6d^2e^2c)x^9}{9} + \frac{(4de^3a + 6d^2e^2b + 4d^3ec)x^7}{7} + \frac{(6d^2e^2a + 4d^3eb + d^4c)x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x^2+d)^4*(c*x^4+b*x^2+a), x)

[Out] 1/13*c*e^4*x^13+1/11*(b*e^4+4*c*d*e^3)*x^11+1/9*(a*e^4+4*b*d*e^3+6*c*d^2*e^2)*x^9+1/7*(4*a*d*e^3+6*b*d^2*e^2+4*c*d^3*e)*x^7+1/5*(6*a*d^2*e^2+4*b*d^3*e+c*d^4)*x^5+1/3*(4*a*d^3*e+b*d^4)*x^3+a*d^4*x

Maxima [A] time = 0.973295, size = 182, normalized size = 1.35

$$\frac{1}{13} ce^4x^{13} + \frac{1}{11} (4cde^3 + be^4)x^{11} + \frac{1}{9} (6cd^2e^2 + 4bde^3 + ae^4)x^9 + \frac{2}{7} (2cd^3e + 3bd^2e^2 + 2ade^3)x^7 + ad^4x + \frac{1}{5} (cd^4 + 4bd^3e)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)^4*(c*x^4+b*x^2+a), x, algorithm="maxima")

[Out] 1/13*c*e^4*x^13 + 1/11*(4*c*d*e^3 + b*e^4)*x^11 + 1/9*(6*c*d^2*e^2 + 4*b*d*e^3 + a*e^4)*x^9 + 2/7*(2*c*d^3*e + 3*b*d^2*e^2 + 2*a*d*e^3)*x^7 + a*d^4*x + 1/5*(c*d^4 + 4*b*d^3*e + 6*a*d^2*e^2)*x^5 + 1/3*(b*d^4 + 4*a*d^3*e)*x^3

Fricas [A] time = 1.34899, size = 355, normalized size = 2.63

$$\frac{1}{13} x^{13} e^4 c + \frac{4}{11} x^{11} e^3 d c + \frac{1}{11} x^{11} e^4 b + \frac{2}{3} x^9 e^2 d^2 c + \frac{4}{9} x^9 e^3 d b + \frac{1}{9} x^9 e^4 a + \frac{4}{7} x^7 e d^3 c + \frac{6}{7} x^7 e^2 d^2 b + \frac{4}{7} x^7 e^3 d a + \frac{1}{5} x^5 d^4 c + \frac{4}{5} x^5 d^3 e$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)^4*(c*x^4+b*x^2+a), x, algorithm="fricas")

[Out] 1/13*x^13*e^4*c + 4/11*x^11*e^3*d*c + 1/11*x^11*e^4*b + 2/3*x^9*e^2*d^2*c + 4/9*x^9*e^3*d*b + 1/9*x^9*e^4*a + 4/7*x^7*e*d^3*c + 6/7*x^7*e^2*d^2*b + 4/7*x^7*e^3*d*a + 1/5*x^5*d^4*c + 4/5*x^5*e*d^3*b + 6/5*x^5*e^2*d^2*a + 1/3*x^3*d^4*b + 4/3*x^3*e*d^3*a + x*d^4*a

Sympy [A] time = 0.093139, size = 156, normalized size = 1.16

$$ad^4x + \frac{ce^4x^{13}}{13} + x^{11} \left(\frac{be^4}{11} + \frac{4cde^3}{11} \right) + x^9 \left(\frac{ae^4}{9} + \frac{4bde^3}{9} + \frac{2cd^2e^2}{3} \right) + x^7 \left(\frac{4ade^3}{7} + \frac{6bd^2e^2}{7} + \frac{4cd^3e}{7} \right) + x^5 \left(\frac{6ad^2e^2}{5} + \frac{4bd^3e}{5} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x**2+d)**4*(c*x**4+b*x**2+a), x)

[Out] a*d**4*x + c*e**4*x**13/13 + x**11*(b*e**4/11 + 4*c*d*e**3/11) + x**9*(a*e**4/9 + 4*b*d*e**3/9 + 2*c*d**2*e**2/3) + x**7*(4*a*d*e**3/7 + 6*b*d**2*e**2/7 + 4*c*d**3*e/7) + x**5*(6*a*d**2*e**2/5 + 4*b*d**3*e/5 + c*d**4/5) + x**

$$3*(4*a*d**3*e/3 + b*d**4/3)$$

Giac [A] time = 1.20255, size = 192, normalized size = 1.42

$$\frac{1}{13} cx^{13}e^4 + \frac{4}{11} cdx^{11}e^3 + \frac{1}{11} bx^{11}e^4 + \frac{2}{3} cd^2x^9e^2 + \frac{4}{9} bdx^9e^3 + \frac{4}{7} cd^3x^7e + \frac{1}{9} ax^9e^4 + \frac{6}{7} bd^2x^7e^2 + \frac{1}{5} cd^4x^5 + \frac{4}{7} adx^7e^3 + \frac{4}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)^4*(c*x^4+b*x^2+a),x, algorithm="giac")

[Out] 1/13*c*x^13*e^4 + 4/11*c*d*x^11*e^3 + 1/11*b*x^11*e^4 + 2/3*c*d^2*x^9*e^2 + 4/9*b*d*x^9*e^3 + 4/7*c*d^3*x^7*e + 1/9*a*x^9*e^4 + 6/7*b*d^2*x^7*e^2 + 1/5*c*d^4*x^5 + 4/7*a*d*x^7*e^3 + 4/5*b*d^3*x^5*e + 6/5*a*d^2*x^5*e^2 + 1/3*b*d^4*x^3 + 4/3*a*d^3*x^3*e + a*d^4*x

3.245 $\int (d + ex^2)^3 (a + bx^2 + cx^4) dx$

Optimal. Leaf size=103

$$\frac{1}{7}ex^7(eae + 3bd) + 3cd^2 + \frac{1}{5}dx^5(3e(ae + bd) + cd^2) + \frac{1}{3}d^2x^3(3ae + bd) + ad^3x + \frac{1}{9}e^2x^9(be + 3cd) + \frac{1}{11}ce^3x^{11}$$

[Out] a*d^3*x + (d^2*(b*d + 3*a*e)*x^3)/3 + (d*(c*d^2 + 3*e*(b*d + a*e))*x^5)/5 + (e*(3*c*d^2 + e*(3*b*d + a*e))*x^7)/7 + (e^2*(3*c*d + b*e)*x^9)/9 + (c*e^3*x^11)/11

Rubi [A] time = 0.0954183, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {1153}

$$\frac{1}{7}ex^7(eae + 3bd) + 3cd^2 + \frac{1}{5}dx^5(3e(ae + bd) + cd^2) + \frac{1}{3}d^2x^3(3ae + bd) + ad^3x + \frac{1}{9}e^2x^9(be + 3cd) + \frac{1}{11}ce^3x^{11}$$

Antiderivative was successfully verified.

[In] Int[(d + e*x^2)^3*(a + b*x^2 + c*x^4),x]

[Out] a*d^3*x + (d^2*(b*d + 3*a*e)*x^3)/3 + (d*(c*d^2 + 3*e*(b*d + a*e))*x^5)/5 + (e*(3*c*d^2 + e*(3*b*d + a*e))*x^7)/7 + (e^2*(3*c*d + b*e)*x^9)/9 + (c*e^3*x^11)/11

Rule 1153

Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]

Rubi steps

$$\begin{aligned} \int (d + ex^2)^3 (a + bx^2 + cx^4) dx &= \int (ad^3 + d^2(bd + 3ae)x^2 + d(cd^2 + 3e(bd + ae))x^4 + e(3cd^2 + e(3bd + ae))x^6 + e^2x^8 + e^3cx^{10}) dx \\ &= ad^3x + \frac{1}{3}d^2(bd + 3ae)x^3 + \frac{1}{5}d(cd^2 + 3e(bd + ae))x^5 + \frac{1}{7}e(3cd^2 + e(3bd + ae))x^7 + \frac{1}{9}e^2x^9 + \frac{1}{11}e^3cx^{11} \end{aligned}$$

Mathematica [A] time = 0.0286084, size = 104, normalized size = 1.01

$$\frac{1}{7}ex^7(ae^2 + 3bde + 3cd^2) + \frac{1}{5}dx^5(3ae^2 + 3bde + cd^2) + \frac{1}{3}d^2x^3(3ae + bd) + ad^3x + \frac{1}{9}e^2x^9(be + 3cd) + \frac{1}{11}ce^3x^{11}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e*x^2)^3*(a + b*x^2 + c*x^4),x]

[Out] a*d^3*x + (d^2*(b*d + 3*a*e)*x^3)/3 + (d*(c*d^2 + 3*b*d*e + 3*a*e^2)*x^5)/5 + (e*(3*c*d^2 + 3*b*d*e + a*e^2)*x^7)/7 + (e^2*(3*c*d + b*e)*x^9)/9 + (c*e^3*x^11)/11

Maple [A] time = 0., size = 103, normalized size = 1.

$$\frac{ce^3x^{11}}{11} + \frac{(e^3b + 3de^2c)x^9}{9} + \frac{(ae^3 + 3de^2b + 3cd^2e)x^7}{7} + \frac{(3de^2a + 3d^2eb + d^3c)x^5}{5} + \frac{(3d^2ea + d^3b)x^3}{3} + ad^3x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x^2+d)^3*(c*x^4+b*x^2+a),x)

[Out] 1/11*c*e^3*x^11+1/9*(b*e^3+3*c*d*e^2)*x^9+1/7*(a*e^3+3*b*d*e^2+3*c*d^2*e)*x^7+1/5*(3*a*d*e^2+3*b*d^2*e+c*d^3)*x^5+1/3*(3*a*d^2*e+b*d^3)*x^3+a*d^3*x

Maxima [A] time = 0.982227, size = 138, normalized size = 1.34

$$\frac{1}{11}ce^3x^{11} + \frac{1}{9}(3cde^2 + be^3)x^9 + \frac{1}{7}(3cd^2e + 3bde^2 + ae^3)x^7 + \frac{1}{5}(cd^3 + 3bd^2e + 3ade^2)x^5 + ad^3x + \frac{1}{3}(bd^3 + 3ad^2e)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)^3*(c*x^4+b*x^2+a),x, algorithm="maxima")

[Out] 1/11*c*e^3*x^11 + 1/9*(3*c*d*e^2 + b*e^3)*x^9 + 1/7*(3*c*d^2*e + 3*b*d*e^2 + a*e^3)*x^7 + 1/5*(c*d^3 + 3*b*d^2*e + 3*a*d*e^2)*x^5 + a*d^3*x + 1/3*(b*d^3 + 3*a*d^2*e)*x^3

Fricas [A] time = 1.38829, size = 263, normalized size = 2.55

$$\frac{1}{11}x^{11}e^3c + \frac{1}{3}x^9e^2dc + \frac{1}{9}x^9e^3b + \frac{3}{7}x^7ed^2c + \frac{3}{7}x^7e^2db + \frac{1}{7}x^7e^3a + \frac{1}{5}x^5d^3c + \frac{3}{5}x^5ed^2b + \frac{3}{5}x^5e^2da + \frac{1}{3}x^3d^3b + x^3ed^2a + xd^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)^3*(c*x^4+b*x^2+a),x, algorithm="fricas")

[Out] 1/11*x^11*e^3*c + 1/3*x^9*e^2*d*c + 1/9*x^9*e^3*b + 3/7*x^7*e*d^2*c + 3/7*x^7*e^2*d*b + 1/7*x^7*e^3*a + 1/5*x^5*d^3*c + 3/5*x^5*e*d^2*b + 3/5*x^5*e^2*d*a + 1/3*x^3*d^3*b + x^3*e*d^2*a + x*d^3*a

Sympy [A] time = 0.083789, size = 112, normalized size = 1.09

$$ad^3x + \frac{ce^3x^{11}}{11} + x^9\left(\frac{be^3}{9} + \frac{cde^2}{3}\right) + x^7\left(\frac{ae^3}{7} + \frac{3bde^2}{7} + \frac{3cd^2e}{7}\right) + x^5\left(\frac{3ade^2}{5} + \frac{3bd^2e}{5} + \frac{cd^3}{5}\right) + x^3\left(ad^2e + \frac{bd^3}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x**2+d)**3*(c*x**4+b*x**2+a),x)

[Out] a*d**3*x + c*e**3*x**11/11 + x**9*(b*e**3/9 + c*d*e**2/3) + x**7*(a*e**3/7 + 3*b*d*e**2/7 + 3*c*d**2*e/7) + x**5*(3*a*d*e**2/5 + 3*b*d**2*e/5 + c*d**3/5) + x**3*(a*d**2*e + b*d**3/3)

Giac [A] time = 1.13618, size = 146, normalized size = 1.42

$$\frac{1}{11} cx^{11}e^3 + \frac{1}{3} cdx^9e^2 + \frac{1}{9} bx^9e^3 + \frac{3}{7} cd^2x^7e + \frac{3}{7} bdx^7e^2 + \frac{1}{5} cd^3x^5 + \frac{1}{7} ax^7e^3 + \frac{3}{5} bd^2x^5e + \frac{3}{5} adx^5e^2 + \frac{1}{3} bd^3x^3 + ad^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)^3*(c*x^4+b*x^2+a),x, algorithm="giac")

[Out] 1/11*c*x^11*e^3 + 1/3*c*d*x^9*e^2 + 1/9*b*x^9*e^3 + 3/7*c*d^2*x^7*e + 3/7*b*d*x^7*e^2 + 1/5*c*d^3*x^5 + 1/7*a*x^7*e^3 + 3/5*b*d^2*x^5*e + 3/5*a*d*x^5*e^2 + 1/3*b*d^3*x^3 + a*d^2*x^3*e + a*d^3*x

3.246 $\int (d + ex^2)^2 (a + bx^2 + cx^4) dx$

Optimal. Leaf size=73

$$\frac{1}{5}x^5(e(ae + 2bd) + cd^2) + \frac{1}{3}dx^3(2ae + bd) + ad^2x + \frac{1}{7}ex^7(be + 2cd) + \frac{1}{9}ce^2x^9$$

[Out] $a*d^2*x + (d*(b*d + 2*a*e)*x^3)/3 + ((c*d^2 + e*(2*b*d + a*e))*x^5)/5 + (e*(2*c*d + b*e)*x^7)/7 + (c*e^2*x^9)/9$

Rubi [A] time = 0.0596723, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {1153}

$$\frac{1}{5}x^5(e(ae + 2bd) + cd^2) + \frac{1}{3}dx^3(2ae + bd) + ad^2x + \frac{1}{7}ex^7(be + 2cd) + \frac{1}{9}ce^2x^9$$

Antiderivative was successfully verified.

[In] Int[(d + e*x^2)^2*(a + b*x^2 + c*x^4), x]

[Out] $a*d^2*x + (d*(b*d + 2*a*e)*x^3)/3 + ((c*d^2 + e*(2*b*d + a*e))*x^5)/5 + (e*(2*c*d + b*e)*x^7)/7 + (c*e^2*x^9)/9$

Rule 1153

Int[((d_) + (e_)*(x_)^2)^(q_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]

Rubi steps

$$\begin{aligned} \int (d + ex^2)^2 (a + bx^2 + cx^4) dx &= \int (ad^2 + d(bd + 2ae)x^2 + (cd^2 + e(2bd + ae))x^4 + e(2cd + be)x^6 + ce^2x^8) dx \\ &= ad^2x + \frac{1}{3}d(bd + 2ae)x^3 + \frac{1}{5}(cd^2 + e(2bd + ae))x^5 + \frac{1}{7}e(2cd + be)x^7 + \frac{1}{9}ce^2x^9 \end{aligned}$$

Mathematica [A] time = 0.0197905, size = 73, normalized size = 1.

$$\frac{1}{5}x^5(ae^2 + 2bde + cd^2) + \frac{1}{3}dx^3(2ae + bd) + ad^2x + \frac{1}{7}ex^7(be + 2cd) + \frac{1}{9}ce^2x^9$$

Antiderivative was successfully verified.

[In] Integrate[(d + e*x^2)^2*(a + b*x^2 + c*x^4), x]

[Out] $a*d^2*x + (d*(b*d + 2*a*e)*x^3)/3 + ((c*d^2 + 2*b*d*e + a*e^2)*x^5)/5 + (e*(2*c*d + b*e)*x^7)/7 + (c*e^2*x^9)/9$

Maple [A] time = 0., size = 70, normalized size = 1.

$$\frac{ce^2x^9}{9} + \frac{(e^2b + 2dec)x^7}{7} + \frac{(ae^2 + 2deb + cd^2)x^5}{5} + \frac{(2dea + bd^2)x^3}{3} + ad^2x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*x^2+d)^2*(c*x^4+b*x^2+a),x)`

[Out] $\frac{1}{9}c^2e^2x^9 + \frac{1}{7}(b^2e^2 + 2c^2d^2e)x^7 + \frac{1}{5}(cd^2 + 2bde + ae^2)x^5 + ad^2x + \frac{1}{3}(bd^2 + 2ade)x^3$

Maxima [A] time = 0.969031, size = 93, normalized size = 1.27

$$\frac{1}{9}ce^2x^9 + \frac{1}{7}(2cde + be^2)x^7 + \frac{1}{5}(cd^2 + 2bde + ae^2)x^5 + ad^2x + \frac{1}{3}(bd^2 + 2ade)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x^2+d)^2*(c*x^4+b*x^2+a),x, algorithm="maxima")`

[Out] $\frac{1}{9}c^2e^2x^9 + \frac{1}{7}(2c^2d^2e + b^2e^2)x^7 + \frac{1}{5}(cd^2 + 2b^2d^2e + a^2e^2)x^5 + ad^2x + \frac{1}{3}(bd^2 + 2a^2d^2e)x^3$

Fricas [A] time = 1.36765, size = 185, normalized size = 2.53

$$\frac{1}{9}x^9e^2c + \frac{2}{7}x^7edc + \frac{1}{7}x^7e^2b + \frac{1}{5}x^5d^2c + \frac{2}{5}x^5edb + \frac{1}{5}x^5e^2a + \frac{1}{3}x^3d^2b + \frac{2}{3}x^3eda + xd^2a$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x^2+d)^2*(c*x^4+b*x^2+a),x, algorithm="fricas")`

[Out] $\frac{1}{9}x^9e^2c + \frac{2}{7}x^7e^2d^2c + \frac{1}{7}x^7e^2b^2 + \frac{1}{5}x^5d^2c + \frac{2}{5}x^5e^2d^2c + \frac{2}{5}x^5e^2d^2b + \frac{1}{5}x^5e^2d^2a + \frac{1}{3}x^3d^2b + \frac{2}{3}x^3e^2d^2a + xd^2a$

Sympy [A] time = 0.078493, size = 78, normalized size = 1.07

$$ad^2x + \frac{ce^2x^9}{9} + x^7\left(\frac{be^2}{7} + \frac{2cde}{7}\right) + x^5\left(\frac{ae^2}{5} + \frac{2bde}{5} + \frac{cd^2}{5}\right) + x^3\left(\frac{2ade}{3} + \frac{bd^2}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x**2+d)**2*(c*x**4+b*x**2+a),x)`

[Out] $a*d**2*x + c*e**2*x**9/9 + x**7*(b*e**2/7 + 2*c*d*e/7) + x**5*(a*e**2/5 + 2*b*d*e/5 + c*d**2/5) + x**3*(2*a*d*e/3 + b*d**2/3)$

Giac [A] time = 1.1569, size = 103, normalized size = 1.41

$$\frac{1}{9}cx^9e^2 + \frac{2}{7}cdx^7e + \frac{1}{7}bx^7e^2 + \frac{1}{5}cd^2x^5 + \frac{2}{5}bdx^5e + \frac{1}{5}ax^5e^2 + \frac{1}{3}bd^2x^3 + \frac{2}{3}adx^3e + ad^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/9*c*x^9*e^2 + 2/7*c*d*x^7*e + 1/7*b*x^7*e^2 + 1/5*c*d^2*x^5 + 2/5*b*d*x^5  
*e + 1/5*a*x^5*e^2 + 1/3*b*d^2*x^3 + 2/3*a*d*x^3*e + a*d^2*x
```


3.247 $\int (d + ex^2)(a + bx^2 + cx^4) dx$

Optimal. Leaf size=42

$$\frac{1}{3}x^3(ae + bd) + adx + \frac{1}{5}x^5(be + cd) + \frac{1}{7}cex^7$$

[Out] a*d*x + ((b*d + a*e)*x^3)/3 + ((c*d + b*e)*x^5)/5 + (c*e*x^7)/7

Rubi [A] time = 0.0273412, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.05$, Rules used = {1153}

$$\frac{1}{3}x^3(ae + bd) + adx + \frac{1}{5}x^5(be + cd) + \frac{1}{7}cex^7$$

Antiderivative was successfully verified.

[In] Int[(d + e*x^2)*(a + b*x^2 + c*x^4), x]

[Out] a*d*x + ((b*d + a*e)*x^3)/3 + ((c*d + b*e)*x^5)/5 + (c*e*x^7)/7

Rule 1153

Int[((d_) + (e_)*(x_)^2)^(q_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]

Rubi steps

$$\begin{aligned} \int (d + ex^2)(a + bx^2 + cx^4) dx &= \int (ad + (bd + ae)x^2 + (cd + be)x^4 + cex^6) dx \\ &= adx + \frac{1}{3}(bd + ae)x^3 + \frac{1}{5}(cd + be)x^5 + \frac{1}{7}cex^7 \end{aligned}$$

Mathematica [A] time = 0.0081182, size = 42, normalized size = 1.

$$\frac{1}{3}x^3(ae + bd) + adx + \frac{1}{5}x^5(be + cd) + \frac{1}{7}cex^7$$

Antiderivative was successfully verified.

[In] Integrate[(d + e*x^2)*(a + b*x^2 + c*x^4), x]

[Out] a*d*x + ((b*d + a*e)*x^3)/3 + ((c*d + b*e)*x^5)/5 + (c*e*x^7)/7

Maple [A] time = 0., size = 37, normalized size = 0.9

$$adx + \frac{(ae + bd)x^3}{3} + \frac{(be + cd)x^5}{5} + \frac{cex^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*x^2+d)*(c*x^4+b*x^2+a),x)`

[Out] `a*d*x+1/3*(a*e+b*d)*x^3+1/5*(b*e+c*d)*x^5+1/7*c*e*x^7`

Maxima [A] time = 0.950216, size = 49, normalized size = 1.17

$$\frac{1}{7}cex^7 + \frac{1}{5}(cd + be)x^5 + \frac{1}{3}(bd + ae)x^3 + adx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x^2+d)*(c*x^4+b*x^2+a),x, algorithm="maxima")`

[Out] `1/7*c*e*x^7 + 1/5*(c*d + b*e)*x^5 + 1/3*(b*d + a*e)*x^3 + a*d*x`

Fricas [A] time = 1.38201, size = 104, normalized size = 2.48

$$\frac{1}{7}x^7ec + \frac{1}{5}x^5dc + \frac{1}{5}x^5eb + \frac{1}{3}x^3db + \frac{1}{3}x^3ea + xda$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x^2+d)*(c*x^4+b*x^2+a),x, algorithm="fricas")`

[Out] `1/7*x^7*e*c + 1/5*x^5*d*c + 1/5*x^5*e*b + 1/3*x^3*d*b + 1/3*x^3*e*a + x*d*a`

Sympy [A] time = 0.063286, size = 39, normalized size = 0.93

$$adx + \frac{cex^7}{7} + x^5\left(\frac{be}{5} + \frac{cd}{5}\right) + x^3\left(\frac{ae}{3} + \frac{bd}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x**2+d)*(c*x**4+b*x**2+a),x)`

[Out] `a*d*x + c*e*x**7/7 + x**5*(b*e/5 + c*d/5) + x**3*(a*e/3 + b*d/3)`

Giac [A] time = 1.13787, size = 58, normalized size = 1.38

$$\frac{1}{7}cx^7e + \frac{1}{5}cdx^5 + \frac{1}{5}bx^5e + \frac{1}{3}bdx^3 + \frac{1}{3}ax^3e + adx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x^2+d)*(c*x^4+b*x^2+a),x, algorithm="giac")`

[Out] `1/7*c*x^7*e + 1/5*c*d*x^5 + 1/5*b*x^5*e + 1/3*b*d*x^3 + 1/3*a*x^3*e + a*d*x`

$$3.248 \quad \int \frac{a+bx^2+cx^4}{d+ex^2} dx$$

Optimal. Leaf size=66

$$\frac{\tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)(ae^2 - bde + cd^2)}{\sqrt{de}^{5/2}} - \frac{x(cd - be)}{e^2} + \frac{cx^3}{3e}$$

[Out] -(((c*d - b*e)*x)/e^2) + (c*x^3)/(3*e) + ((c*d^2 - b*d*e + a*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[d]*e^(5/2))

Rubi [A] time = 0.0446231, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1153, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)(ae^2 - bde + cd^2)}{\sqrt{de}^{5/2}} - \frac{x(cd - be)}{e^2} + \frac{cx^3}{3e}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x^2 + c*x^4)/(d + e*x^2), x]

[Out] -(((c*d - b*e)*x)/e^2) + (c*x^3)/(3*e) + ((c*d^2 - b*d*e + a*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[d]*e^(5/2))

Rule 1153

Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rubi steps

$$\begin{aligned} \int \frac{a + bx^2 + cx^4}{d + ex^2} dx &= \int \left(-\frac{cd - be}{e^2} + \frac{cx^2}{e} + \frac{cd^2 - bde + ae^2}{e^2(d + ex^2)} \right) dx \\ &= -\frac{(cd - be)x}{e^2} + \frac{cx^3}{3e} + \frac{(cd^2 - bde + ae^2) \int \frac{1}{d + ex^2} dx}{e^2} \\ &= -\frac{(cd - be)x}{e^2} + \frac{cx^3}{3e} + \frac{(cd^2 - bde + ae^2) \tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)}{\sqrt{de}^{5/2}} \end{aligned}$$

Mathematica [A] time = 0.0512229, size = 65, normalized size = 0.98

$$\frac{\tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)(ae^2 - bde + cd^2)}{\sqrt{de}^{5/2}} + \frac{x(be - cd)}{e^2} + \frac{cx^3}{3e}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x^2 + c*x^4)/(d + e*x^2),x]

[Out] $\frac{(-(c*d) + b*e)*x}{e^2} + \frac{(c*x^3)}{(3*e)} + \frac{((c*d^2 - b*d*e + a*e^2)*\text{ArcTan}[(\text{Sqrt}[e]*x)/\text{Sqrt}[d]])}{(\text{Sqrt}[d]*e^{(5/2)})}$

Maple [A] time = 0.003, size = 84, normalized size = 1.3

$$\frac{cx^3}{3e} + \frac{bx}{e} - \frac{cdx}{e^2} + a \arctan\left(\frac{ex}{\sqrt{de}}\right) \frac{1}{\sqrt{de}} - \frac{bd}{e} \arctan\left(\frac{ex}{\sqrt{de}}\right) \frac{1}{\sqrt{de}} + \frac{cd^2}{e^2} \arctan\left(\frac{ex}{\sqrt{de}}\right) \frac{1}{\sqrt{de}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^4+b*x^2+a)/(e*x^2+d),x)

[Out] $\frac{1}{3}c*x^3/e + 1/e*b*x - c*d*x/e^2 + 1/(d*e)^{(1/2)}*\arctan(x*e/(d*e)^{(1/2)})*a - 1/e/(d*e)^{(1/2)}*\arctan(x*e/(d*e)^{(1/2)})*d*b + 1/e^2/(d*e)^{(1/2)}*\arctan(x*e/(d*e)^{(1/2)})*c*d^2$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d),x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [A] time = 1.63718, size = 347, normalized size = 5.26

$$\left[\frac{2cde^2x^3 - 3(cd^2 - bde + ae^2)\sqrt{-de} \log\left(\frac{ex^2 - 2\sqrt{-de}x - d}{ex^2 + d}\right) - 6(cd^2e - bde^2)x}{6de^3}, \frac{cde^2x^3 + 3(cd^2 - bde + ae^2)\sqrt{de} \arctan\left(\frac{\sqrt{de}x}{d}\right)}{3de^3} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d),x, algorithm="fricas")

[Out] $\left[\frac{1}{6}*(2*c*d*e^2*x^3 - 3*(c*d^2 - b*d*e + a*e^2)*\text{sqrt}(-d*e)*\log((e*x^2 - 2*\text{sqrt}(-d*e)*x - d)/(e*x^2 + d)) - 6*(c*d^2*e - b*d*e^2)*x)/(d*e^3), \frac{1}{3}*(c*d*e^2*x^3 + 3*(c*d^2 - b*d*e + a*e^2)*\text{sqrt}(d*e)*\arctan(\text{sqrt}(d*e)*x/d) - 3*(c*d^2*e - b*d*e^2)*x)/(d*e^3) \right]$

Sympy [B] time = 0.640437, size = 117, normalized size = 1.77

$$\frac{cx^3}{3e} - \frac{\sqrt{-\frac{1}{de^5}}(ae^2 - bde + cd^2) \log\left(-de^2\sqrt{-\frac{1}{de^5}} + x\right)}{2} + \frac{\sqrt{-\frac{1}{de^5}}(ae^2 - bde + cd^2) \log\left(de^2\sqrt{-\frac{1}{de^5}} + x\right)}{2} + \frac{x(be - cd)}{e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x**4+b*x**2+a)/(e*x**2+d),x)

[Out] c*x**3/(3*e) - sqrt(-1/(d*e**5))*(a*e**2 - b*d*e + c*d**2)*log(-d*e**2*sqrt(-1/(d*e**5)) + x)/2 + sqrt(-1/(d*e**5))*(a*e**2 - b*d*e + c*d**2)*log(d*e**2*sqrt(-1/(d*e**5)) + x)/2 + x*(b*e - c*d)/e**2

Giac [A] time = 1.26151, size = 76, normalized size = 1.15

$$\frac{(cd^2 - bde + ae^2) \arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right) e^{\left(-\frac{5}{2}\right)}}{\sqrt{d}} + \frac{1}{3} (cx^3e^2 - 3cdxe + 3bxe^2)e^{(-3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d),x, algorithm="giac")

[Out] (c*d^2 - b*d*e + a*e^2)*arctan(x*e^(1/2)/sqrt(d))*e^(-5/2)/sqrt(d) + 1/3*(c*x^3*e^2 - 3*c*d*x*e + 3*b*x*e^2)*e^(-3)

$$3.249 \quad \int \frac{a+bx^2+cx^4}{(d+ex^2)^2} dx$$

Optimal. Leaf size=83

$$-\frac{\tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)(3cd^2 - e(ae + bd))}{2d^{3/2}e^{5/2}} + \frac{x\left(a + \frac{d(cd-be)}{e^2}\right)}{2d(d+ex^2)} + \frac{cx}{e^2}$$

[Out] (c*x)/e^2 + ((a + (d*(c*d - b*e))/e^2)*x)/(2*d*(d + e*x^2)) - ((3*c*d^2 - e*(b*d + a*e))*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(3/2)*e^(5/2))

Rubi [A] time = 0.0934114, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {1157, 388, 205}

$$-\frac{\tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)(3cd^2 - e(ae + bd))}{2d^{3/2}e^{5/2}} + \frac{x\left(a + \frac{d(cd-be)}{e^2}\right)}{2d(d+ex^2)} + \frac{cx}{e^2}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x^2 + c*x^4)/(d + e*x^2)^2,x]

[Out] (c*x)/e^2 + ((a + (d*(c*d - b*e))/e^2)*x)/(2*d*(d + e*x^2)) - ((3*c*d^2 - e*(b*d + a*e))*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(3/2)*e^(5/2))

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] := With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2,
x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x],
x, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned} \int \frac{a + bx^2 + cx^4}{(d + ex^2)^2} dx &= \frac{\left(a + \frac{d(cd-be)}{e^2}\right)x}{2d(d + ex^2)} - \int \frac{\frac{cd^2 - e(bd+ae)}{e^2} - \frac{2cdx^2}{e}}{d+ex^2} dx \\ &= \frac{cx}{e^2} + \frac{\left(a + \frac{d(cd-be)}{e^2}\right)x}{2d(d + ex^2)} - \frac{(3cd^2 - e(bd + ae)) \int \frac{1}{d+ex^2} dx}{2de^2} \\ &= \frac{cx}{e^2} + \frac{\left(a + \frac{d(cd-be)}{e^2}\right)x}{2d(d + ex^2)} - \frac{(3cd^2 - e(bd + ae)) \tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)}{2d^{3/2}e^{5/2}} \end{aligned}$$

Mathematica [A] time = 0.0557215, size = 88, normalized size = 1.06

$$\frac{x(ae^2 - bde + cd^2)}{2de^2(d + ex^2)} - \frac{\tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)(-ae^2 - bde + 3cd^2)}{2d^{3/2}e^{5/2}} + \frac{cx}{e^2}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x^2 + c*x^4)/(d + e*x^2)^2,x]

[Out] (c*x)/e^2 + ((c*d^2 - b*d*e + a*e^2)*x)/(2*d*e^2*(d + e*x^2)) - ((3*c*d^2 - b*d*e - a*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(3/2)*e^(5/2))

Maple [A] time = 0.008, size = 118, normalized size = 1.4

$$\frac{cx}{e^2} + \frac{xa}{2d(ex^2 + d)} - \frac{bx}{2e(ex^2 + d)} + \frac{dxc}{2e^2(ex^2 + d)} + \frac{a}{2d} \arctan\left(ex \frac{1}{\sqrt{de}}\right) \frac{1}{\sqrt{de}} + \frac{b}{2e} \arctan\left(ex \frac{1}{\sqrt{de}}\right) \frac{1}{\sqrt{de}} - \frac{3cd}{2e^2} \arctan\left(ex \frac{1}{\sqrt{de}}\right) \frac{1}{\sqrt{de}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^4+b*x^2+a)/(e*x^2+d)^2,x)

[Out] c*x/e^2+1/2/d*x/(e*x^2+d)*a-1/2/e*x/(e*x^2+d)*b+1/2/e^2*d*x/(e*x^2+d)*c+1/2/d/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))*a+1/2/e/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))*b-3/2/e^2*d/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))*c

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^2,x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [A] time = 1.62845, size = 541, normalized size = 6.52

$$\left[\frac{4cd^2e^2x^3 + (3cd^3 - bd^2e - ade^2 + (3cd^2e - bde^2 - ae^3)x^2)\sqrt{-de} \log\left(\frac{ex^2 - 2\sqrt{-dex-d}}{ex^2+d}\right) + 2(3cd^3e - bd^2e^2 + ade^3)x}{4(d^2e^4x^2 + d^3e^3)}, \frac{2cd^2e^2}{4(d^2e^4x^2 + d^3e^3)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^2,x, algorithm="fricas")

[Out] [1/4*(4*c*d^2*e^2*x^3 + (3*c*d^3 - b*d^2*e - a*d*e^2 + (3*c*d^2*e - b*d*e^2 - a*e^3)*x^2)*sqrt(-d*e)*log((e*x^2 - 2*sqrt(-d*e)*x - d)/(e*x^2 + d)) + 2*(3*c*d^3*e - b*d^2*e^2 + a*d*e^3)*x)/(d^2*e^4*x^2 + d^3*e^3), 1/2*(2*c*d^2*e^2*x^3 - (3*c*d^3 - b*d^2*e - a*d*e^2 + (3*c*d^2*e - b*d*e^2 - a*e^3)*x^2)*sqrt(d*e)*arctan(sqrt(d*e)*x/d) + (3*c*d^3*e - b*d^2*e^2 + a*d*e^3)*x)/(d^2*e^4*x^2 + d^3*e^3)]

Sympy [B] time = 1.06323, size = 153, normalized size = 1.84

$$\frac{cx}{e^2} + \frac{x(ae^2 - bde + cd^2)}{2d^2e^2 + 2de^3x^2} - \frac{\sqrt{-\frac{1}{d^3e^5}}(ae^2 + bde - 3cd^2) \log\left(-d^2e^2\sqrt{-\frac{1}{d^3e^5}} + x\right)}{4} + \frac{\sqrt{-\frac{1}{d^3e^5}}(ae^2 + bde - 3cd^2) \log\left(d^2e^2\sqrt{-\frac{1}{d^3e^5}} + x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x**4+b*x**2+a)/(e*x**2+d)**2,x)

[Out] c*x/e**2 + x*(a*e**2 - b*d*e + c*d**2)/(2*d**2*e**2 + 2*d*e**3*x**2) - sqrt(-1/(d**3*e**5))*(a*e**2 + b*d*e - 3*c*d**2)*log(-d**2*e**2*sqrt(-1/(d**3*e**5)) + x)/4 + sqrt(-1/(d**3*e**5))*(a*e**2 + b*d*e - 3*c*d**2)*log(d**2*e**2*sqrt(-1/(d**3*e**5)) + x)/4

Giac [A] time = 1.1552, size = 101, normalized size = 1.22

$$cxe^{(-2)} - \frac{(3cd^2 - bde - ae^2) \arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right) e^{\left(-\frac{5}{2}\right)}}{2d^{\frac{3}{2}}} + \frac{(cd^2x - bdx + axe^2)e^{(-2)}}{2(x^2e + d)d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^2,x, algorithm="giac")

[Out] c*x*e^(-2) - 1/2*(3*c*d^2 - b*d*e - a*e^2)*arctan(x*e^(1/2)/sqrt(d))*e^(-5/2)/d^(3/2) + 1/2*(c*d^2*x - b*d*x*e + a*x*e^2)*e^(-2)/((x^2*e + d)*d)

$$3.250 \quad \int \frac{a+bx^2+cx^4}{(d+ex^2)^3} dx$$

Optimal. Leaf size=115

$$-\frac{x(5cd^2 - e(3ae + bd))}{8d^2e^2(d + ex^2)} + \frac{\tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)(e(3ae + bd) + 3cd^2)}{8d^{5/2}e^{5/2}} + \frac{x\left(a + \frac{d(cd-be)}{e^2}\right)}{4d(d + ex^2)^2}$$

[Out] ((a + (d*(c*d - b*e))/e^2)*x)/(4*d*(d + e*x^2)^2) - ((5*c*d^2 - e*(b*d + 3*a*e))*x)/(8*d^2*e^2*(d + e*x^2)) + ((3*c*d^2 + e*(b*d + 3*a*e))*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(8*d^(5/2)*e^(5/2))

Rubi [A] time = 0.107324, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {1157, 385, 205}

$$-\frac{x(5cd^2 - e(3ae + bd))}{8d^2e^2(d + ex^2)} + \frac{\tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)(e(3ae + bd) + 3cd^2)}{8d^{5/2}e^{5/2}} + \frac{x\left(a + \frac{d(cd-be)}{e^2}\right)}{4d(d + ex^2)^2}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x^2 + c*x^4)/(d + e*x^2)^3,x]

[Out] ((a + (d*(c*d - b*e))/e^2)*x)/(4*d*(d + e*x^2)^2) - ((5*c*d^2 - e*(b*d + 3*a*e))*x)/(8*d^2*e^2*(d + e*x^2)) + ((3*c*d^2 + e*(b*d + 3*a*e))*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(8*d^(5/2)*e^(5/2))

Rule 1157

Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]

Rule 385

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n + p, 0])

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rubi steps

$$\begin{aligned}
\int \frac{a + bx^2 + cx^4}{(d + ex^2)^3} dx &= \frac{\left(a + \frac{d(cd-be)}{e^2}\right)x}{4d(d + ex^2)^2} - \frac{\int \frac{-3a + \frac{d(cd-be)}{e^2} - \frac{4cdx^2}{e}}{(d+ex^2)^2} dx}{4d} \\
&= \frac{\left(a + \frac{d(cd-be)}{e^2}\right)x}{4d(d + ex^2)^2} - \frac{(5cd^2 - e(bd + 3ae))x}{8d^2e^2(d + ex^2)} - \frac{\left(-\frac{4cd^2}{e} + e\left(-3a + \frac{d(cd-be)}{e^2}\right)\right) \int \frac{1}{d+ex^2} dx}{8d^2e} \\
&= \frac{\left(a + \frac{d(cd-be)}{e^2}\right)x}{4d(d + ex^2)^2} - \frac{(5cd^2 - e(bd + 3ae))x}{8d^2e^2(d + ex^2)} + \frac{(3cd^2 + e(bd + 3ae)) \tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)}{8d^{5/2}e^{5/2}}
\end{aligned}$$

Mathematica [A] time = 0.0972002, size = 110, normalized size = 0.96

$$\frac{x \left(e \left(a e \left(5 d + 3 e x^2 \right) + b d \left(e x^2 - d \right) \right) - c d^2 \left(3 d + 5 e x^2 \right) \right)}{8 d^2 e^2 \left(d + e x^2 \right)^2} + \frac{\tan^{-1}\left(\frac{\sqrt{e x}}{\sqrt{d}}\right) \left(e \left(3 a e + b d \right) + 3 c d^2 \right)}{8 d^{5/2} e^{5/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x^2 + c*x^4)/(d + e*x^2)^3,x]

[Out] (x*(-(c*d^2*(3*d + 5*e*x^2)) + e*(b*d*(-d + e*x^2) + a*e*(5*d + 3*e*x^2))))/(8*d^2*e^2*(d + e*x^2)^2) + ((3*c*d^2 + e*(b*d + 3*a*e))*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(8*d^(5/2)*e^(5/2))

Maple [A] time = 0.008, size = 131, normalized size = 1.1

$$\frac{1}{(ex^2 + d)^2} \left(\frac{(3ae^2 + deb - 5cd^2)x^3}{8d^2e} + \frac{(5ae^2 - deb - 3cd^2)x}{8e^2d} \right) + \frac{3a}{8d^2} \arctan\left(ex \frac{1}{\sqrt{de}}\right) \frac{1}{\sqrt{de}} + \frac{b}{8de} \arctan\left(ex \frac{1}{\sqrt{de}}\right) \frac{1}{\sqrt{de}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^4+b*x^2+a)/(e*x^2+d)^3,x)

[Out] (1/8*(3*a*e^2+b*d*e-5*c*d^2)/d^2/e*x^3+1/8*(5*a*e^2-b*d*e-3*c*d^2)/e^2/d*x)/(e*x^2+d)^2+3/8/d^2/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))*a+1/8/d/e/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))*b+3/8/e^2/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))*c

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^3,x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [A] time = 1.57256, size = 813, normalized size = 7.07

$$\frac{2(5cd^3e^2 - bd^2e^3 - 3ade^4)x^3 + (3cd^4 + bd^3e + 3ad^2e^2 + (3cd^2e^2 + bde^3 + 3ae^4)x^4 + 2(3cd^3e + bd^2e^2 + 3ade^3)x^2)}{16(d^3e^5x^4 + 2d^4e^4x^2 + d^5e^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^3,x, algorithm="fricas")

[Out] [-1/16*(2*(5*c*d^3*e^2 - b*d^2*e^3 - 3*a*d*e^4)*x^3 + (3*c*d^4 + b*d^3*e + 3*a*d^2*e^2 + (3*c*d^2*e^2 + b*d*e^3 + 3*a*e^4)*x^4 + 2*(3*c*d^3*e + b*d^2*e^2 + 3*a*d*e^3)*x^2)*sqrt(-d*e)*log((e*x^2 - 2*sqrt(-d*e)*x - d)/(e*x^2 + d)) + 2*(3*c*d^4*e + b*d^3*e^2 - 5*a*d^2*e^3)*x/(d^3*e^5*x^4 + 2*d^4*e^4*x^2 + d^5*e^3), -1/8*((5*c*d^3*e^2 - b*d^2*e^3 - 3*a*d*e^4)*x^3 - (3*c*d^4 + b*d^3*e + 3*a*d^2*e^2 + (3*c*d^2*e^2 + b*d*e^3 + 3*a*e^4)*x^4 + 2*(3*c*d^3*e + b*d^2*e^2 + 3*a*d*e^3)*x^2)*sqrt(d*e)*arctan(sqrt(d*e)*x/d) + (3*c*d^4*e + b*d^3*e^2 - 5*a*d^2*e^3)*x/(d^3*e^5*x^4 + 2*d^4*e^4*x^2 + d^5*e^3)]

Sympy [A] time = 1.80764, size = 196, normalized size = 1.7

$$\frac{\sqrt{-\frac{1}{d^5e^5}}(3ae^2 + bde + 3cd^2) \log\left(-d^3e^2\sqrt{-\frac{1}{d^5e^5}} + x\right)}{16} + \frac{\sqrt{-\frac{1}{d^5e^5}}(3ae^2 + bde + 3cd^2) \log\left(d^3e^2\sqrt{-\frac{1}{d^5e^5}} + x\right)}{16} + \frac{x^3(3ae^3 + bde^2 + 3cd^2)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x**4+b*x**2+a)/(e*x**2+d)**3,x)

[Out] -sqrt(-1/(d**5*e**5))*(3*a*e**2 + b*d*e + 3*c*d**2)*log(-d**3*e**2*sqrt(-1/(d**5*e**5)) + x)/16 + sqrt(-1/(d**5*e**5))*(3*a*e**2 + b*d*e + 3*c*d**2)*log(d**3*e**2*sqrt(-1/(d**5*e**5)) + x)/16 + (x**3*(3*a*e**3 + b*d*e**2 - 5*c*d**2*e) + x*(5*a*d*e**2 - b*d**2*e - 3*c*d**3))/(8*d**4*e**2 + 16*d**3*e**2*x**2 + 8*d**2*e**4*x**4)

Giac [A] time = 1.13708, size = 136, normalized size = 1.18

$$\frac{(3cd^2 + bde + 3ae^2) \arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right) e^{\left(-\frac{5}{2}\right)}}{8d^{\frac{5}{2}}} - \frac{(5cd^2x^3e - bdx^3e^2 + 3cd^3x - 3ax^3e^3 + bd^2xe - 5adxe^2)e^{(-2)}}{8(x^2e + d)^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^3,x, algorithm="giac")

[Out] 1/8*(3*c*d^2 + b*d*e + 3*a*e^2)*arctan(x*e^(1/2)/sqrt(d))*e^(-5/2)/d^(5/2) - 1/8*(5*c*d^2*x^3*e - b*d*x^3*e^2 + 3*c*d^3*x - 3*a*x^3*e^3 + b*d^2*x*e - 5*a*d*x*e^2)*e^(-2)/((x^2*e + d)^2*d^2)

$$3.251 \quad \int \frac{a+bx^2+cx^4}{(d+ex^2)^4} dx$$

Optimal. Leaf size=150

$$\frac{x(e(5ae+bd)+cd^2)}{16d^3e^2(d+ex^2)} - \frac{x(7cd^2-e(5ae+bd))}{24d^2e^2(d+ex^2)^2} + \frac{\tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)(e(5ae+bd)+cd^2)}{16d^{7/2}e^{5/2}} + \frac{x\left(a+\frac{d(cd-be)}{e^2}\right)}{6d(d+ex^2)^3}$$

[Out] ((a + (d*(c*d - b*e))/e^2)*x)/(6*d*(d + e*x^2)^3) - ((7*c*d^2 - e*(b*d + 5*a*e))*x)/(24*d^2*e^2*(d + e*x^2)^2) + ((c*d^2 + e*(b*d + 5*a*e))*x)/(16*d^3*e^2*(d + e*x^2)) + ((c*d^2 + e*(b*d + 5*a*e))*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(16*d^(7/2)*e^(5/2))

Rubi [A] time = 0.205394, antiderivative size = 150, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1157, 385, 199, 205}

$$\frac{x(e(5ae+bd)+cd^2)}{16d^3e^2(d+ex^2)} - \frac{x(7cd^2-e(5ae+bd))}{24d^2e^2(d+ex^2)^2} + \frac{\tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)(e(5ae+bd)+cd^2)}{16d^{7/2}e^{5/2}} + \frac{x\left(a+\frac{d(cd-be)}{e^2}\right)}{6d(d+ex^2)^3}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x^2 + c*x^4)/(d + e*x^2)^4, x]

[Out] ((a + (d*(c*d - b*e))/e^2)*x)/(6*d*(d + e*x^2)^3) - ((7*c*d^2 - e*(b*d + 5*a*e))*x)/(24*d^2*e^2*(d + e*x^2)^2) + ((c*d^2 + e*(b*d + 5*a*e))*x)/(16*d^3*e^2*(d + e*x^2)) + ((c*d^2 + e*(b*d + 5*a*e))*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(16*d^(7/2)*e^(5/2))

Rule 1157

Int[((d_) + (e_)*(x_)^2)^(q_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] := With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]

Rule 385

Int[((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_)), x_Symbol] := -Simp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n + p, 0])

Rule 199

Int[((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := -Simp[(x*(a + b*x^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (IntegerQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin

ator[p + 1/n] < Denominator[p])

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rubi steps

$$\begin{aligned} \int \frac{a + bx^2 + cx^4}{(d + ex^2)^4} dx &= \frac{\left(a + \frac{d(cd-be)}{e^2}\right)x}{6d(d + ex^2)^3} - \frac{\int \frac{-5a + \frac{d(cd-be)}{e^2} - \frac{6cdx^2}{e}}{(d+ex^2)^3} dx}{6d} \\ &= \frac{\left(a + \frac{d(cd-be)}{e^2}\right)x}{6d(d + ex^2)^3} - \frac{(7cd^2 - e(bd + 5ae))x}{24d^2e^2(d + ex^2)^2} + \frac{(cd^2 + e(bd + 5ae)) \int \frac{1}{(d+ex^2)^2} dx}{8d^2e^2} \\ &= \frac{\left(a + \frac{d(cd-be)}{e^2}\right)x}{6d(d + ex^2)^3} - \frac{(7cd^2 - e(bd + 5ae))x}{24d^2e^2(d + ex^2)^2} + \frac{(cd^2 + e(bd + 5ae))x}{16d^3e^2(d + ex^2)} + \frac{(cd^2 + e(bd + 5ae)) \int \frac{1}{d+ex^2}}{16d^3e^2} \\ &= \frac{\left(a + \frac{d(cd-be)}{e^2}\right)x}{6d(d + ex^2)^3} - \frac{(7cd^2 - e(bd + 5ae))x}{24d^2e^2(d + ex^2)^2} + \frac{(cd^2 + e(bd + 5ae))x}{16d^3e^2(d + ex^2)} + \frac{(cd^2 + e(bd + 5ae)) \tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)}{16d^{7/2}e^{5/2}} \end{aligned}$$

Mathematica [A] time = 0.13153, size = 142, normalized size = 0.95

$$\frac{x \left(e \left(ae \left(33d^2 + 40dex^2 + 15e^2x^4 \right) + bd \left(-3d^2 + 8dex^2 + 3e^2x^4 \right) \right) + cd^2 \left(-3d^2 - 8dex^2 + 3e^2x^4 \right) \right)}{48d^3e^2 \left(d + ex^2 \right)^3} + \frac{\tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right) \left(e(5ae) \right)}{16d^{7/2}e^{5/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x^2 + c*x^4)/(d + e*x^2)^4,x]

[Out] (x*(c*d^2*(-3*d^2 - 8*d*e*x^2 + 3*e^2*x^4) + e*(b*d*(-3*d^2 + 8*d*e*x^2 + 3*e^2*x^4) + a*e*(33*d^2 + 40*d*e*x^2 + 15*e^2*x^4)))/(48*d^3*e^2*(d + e*x^2)^3) + ((c*d^2 + e*(b*d + 5*a*e))*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(16*d^(7/2)*e^(5/2))

Maple [A] time = 0.01, size = 158, normalized size = 1.1

$$\frac{1}{(ex^2 + d)^3} \left(\frac{(5ae^2 + deb + cd^2)x^5}{16d^3} + \frac{(5ae^2 + deb - cd^2)x^3}{6d^2e} + \frac{(11ae^2 - deb - cd^2)x}{16de^2} \right) + \frac{5a}{16d^3} \arctan\left(ex \frac{1}{\sqrt{de}}\right) \frac{1}{\sqrt{de}} +$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^4+b*x^2+a)/(e*x^2+d)^4,x)

[Out] (1/16*(5*a*e^2+b*d*e+c*d^2)/d^3*x^5+1/6*(5*a*e^2+b*d*e-c*d^2)/d^2/e*x^3+1/16*(11*a*e^2-b*d*e-c*d^2)/d/e^2*x)/(e*x^2+d)^3+5/16/d^3/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))*a+1/16/d^2/e/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))*b+1/16/d/e

$$\sqrt{2/(d*e)^{(1/2)}*\arctan(x*e/(d*e)^{(1/2)})}*c$$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^4,x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [A] time = 1.67425, size = 1091, normalized size = 7.27

$$\frac{6(cd^3e^3 + bd^2e^4 + 5ade^5)x^5 - 16(cd^4e^2 - bd^3e^3 - 5ad^2e^4)x^3 - 3((cd^2e^3 + bde^4 + 5ae^5)x^6 + cd^5 + bd^4e + 5ad^3e^2 + 3(cd^4e^6x^6 + 3d^5e^5x^4 + \dots))}{96(d^4e^6x^6 + 3d^5e^5x^4 + \dots)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^4,x, algorithm="fricas")

[Out] [1/96*(6*(c*d^3*e^3 + b*d^2*e^4 + 5*a*d*e^5)*x^5 - 16*(c*d^4*e^2 - b*d^3*e^3 - 5*a*d^2*e^4)*x^3 - 3*((c*d^2*e^3 + b*d*e^4 + 5*a*e^5)*x^6 + c*d^5 + b*d^4*e + 5*a*d^3*e^2 + 3*(c*d^3*e^2 + b*d^2*e^3 + 5*a*d*e^4)*x^4 + 3*(c*d^4*e + b*d^3*e^2 + 5*a*d^2*e^3)*x^2)*sqrt(-d*e)*log((e*x^2 - 2*sqrt(-d*e)*x - d)/(e*x^2 + d)) - 6*(c*d^5*e + b*d^4*e^2 - 11*a*d^3*e^3)*x/(d^4*e^6*x^6 + 3*d^5*e^5*x^4 + 3*d^6*e^4*x^2 + d^7*e^3), 1/48*(3*(c*d^3*e^3 + b*d^2*e^4 + 5*a*d*e^5)*x^5 - 8*(c*d^4*e^2 - b*d^3*e^3 - 5*a*d^2*e^4)*x^3 + 3*((c*d^2*e^3 + b*d*e^4 + 5*a*e^5)*x^6 + c*d^5 + b*d^4*e + 5*a*d^3*e^2 + 3*(c*d^3*e^2 + b*d^2*e^3 + 5*a*d*e^4)*x^4 + 3*(c*d^4*e + b*d^3*e^2 + 5*a*d^2*e^3)*x^2)*sqrt(d*e)*arctan(sqrt(d*e)*x/d) - 3*(c*d^5*e + b*d^4*e^2 - 11*a*d^3*e^3)*x/(d^4*e^6*x^6 + 3*d^5*e^5*x^4 + 3*d^6*e^4*x^2 + d^7*e^3)]

Sympy [A] time = 3.32007, size = 241, normalized size = 1.61

$$\frac{\sqrt{-\frac{1}{d^7e^5}}(5ae^2 + bde + cd^2) \log\left(-d^4e^2\sqrt{-\frac{1}{d^7e^5}} + x\right)}{32} + \frac{\sqrt{-\frac{1}{d^7e^5}}(5ae^2 + bde + cd^2) \log\left(d^4e^2\sqrt{-\frac{1}{d^7e^5}} + x\right)}{32} + \frac{x^5(15ae^4 + 3bd^3e^3 + 3cd^2e^2) + x^3(40ad^3e^3 + 8bd^2e^2 - 8cd^3e) + x(33ad^2e^2 - 3bd^3e - 3cd^4)}{(48d^6e^2 + 144d^5e^3x^2 + 144d^4e^4x^4 + 48d^3e^5x^6)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x**4+b*x**2+a)/(e*x**2+d)**4,x)

[Out] -sqrt(-1/(d**7*e**5))*(5*a*e**2 + b*d*e + c*d**2)*log(-d**4*e**2*sqrt(-1/(d**7*e**5)) + x)/32 + sqrt(-1/(d**7*e**5))*(5*a*e**2 + b*d*e + c*d**2)*log(d**4*e**2*sqrt(-1/(d**7*e**5)) + x)/32 + (x**5*(15*a*e**4 + 3*b*d*e**3 + 3*c*d**2*e**2) + x**3*(40*a*d*e**3 + 8*b*d**2*e**2 - 8*c*d**3*e) + x*(33*a*d**2*e**2 - 3*b*d**3*e - 3*c*d**4))/(48*d**6*e**2 + 144*d**5*e**3*x**2 + 144*d**4*e**4*x**4 + 48*d**3*e**5*x**6)

Giac [A] time = 1.17237, size = 181, normalized size = 1.21

$$\frac{(cd^2 + bde + 5ae^2) \arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right) e^{\left(-\frac{5}{2}\right)}}{16d^{\frac{7}{2}}} + \frac{(3cd^2x^5e^2 + 3bdx^5e^3 - 8cd^3x^3e + 15ax^5e^4 + 8bd^2x^3e^2 - 3cd^4x + 40adx^3e^3)}{48(x^2e + d)^3d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^4,x, algorithm="giac")

[Out] 1/16*(c*d^2 + b*d*e + 5*a*e^2)*arctan(x*e^(1/2)/sqrt(d))*e^(-5/2)/d^(7/2) +
 1/48*(3*c*d^2*x^5*e^2 + 3*b*d*x^5*e^3 - 8*c*d^3*x^3*e + 15*a*x^5*e^4 + 8*b
 *d^2*x^3*e^2 - 3*c*d^4*x + 40*a*d*x^3*e^3 - 3*b*d^3*x*e + 33*a*d^2*x*e^2)*e
 ^(-2)/((x^2*e + d)^3*d^3)

3.252 $\int (d + ex^2)^3 (a + bx^2 + cx^4)^2 dx$

Optimal. Leaf size=223

$$\frac{1}{7}x^7 (a^2e^3 + 6abde^2 + 6acd^2e + 3b^2d^2e + 2bcd^3) + a^2d^3x + \frac{1}{11}ex^{11} (2ce(ae + 3bd) + b^2e^2 + 3c^2d^2) + \frac{1}{5}dx^5 (6abde + a(3ae$$

[Out] $a^2d^3x + (a*d^2*(2*b*d + 3*a*e)*x^3)/3 + (d*(b^2*d^2 + 6*a*b*d*e + a*(2*c*d^2 + 3*a*e^2))*x^5)/5 + ((2*b*c*d^3 + 3*b^2*d^2*e + 6*a*c*d^2*e + 6*a*b*d*e^2 + a^2*e^3)*x^7)/7 + ((c^2*d^3 + 6*c*d*e*(b*d + a*e) + b*e^2*(3*b*d + 2*a*e))*x^9)/9 + (e*(3*c^2*d^2 + b^2*e^2 + 2*c*e*(3*b*d + a*e))*x^{11})/11 + (c*e^2*(3*c*d + 2*b*e)*x^{13})/13 + (c^2*e^3*x^{15})/15$

Rubi [A] time = 0.198799, antiderivative size = 223, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = {1153}

$$\frac{1}{7}x^7 (a^2e^3 + 6abde^2 + 6acd^2e + 3b^2d^2e + 2bcd^3) + a^2d^3x + \frac{1}{11}ex^{11} (2ce(ae + 3bd) + b^2e^2 + 3c^2d^2) + \frac{1}{5}dx^5 (6abde + a(3ae$$

Antiderivative was successfully verified.

[In] Int[(d + e*x^2)^3*(a + b*x^2 + c*x^4)^2,x]

[Out] $a^2d^3x + (a*d^2*(2*b*d + 3*a*e)*x^3)/3 + (d*(b^2*d^2 + 6*a*b*d*e + a*(2*c*d^2 + 3*a*e^2))*x^5)/5 + ((2*b*c*d^3 + 3*b^2*d^2*e + 6*a*c*d^2*e + 6*a*b*d*e^2 + a^2*e^3)*x^7)/7 + ((c^2*d^3 + 6*c*d*e*(b*d + a*e) + b*e^2*(3*b*d + 2*a*e))*x^9)/9 + (e*(3*c^2*d^2 + b^2*e^2 + 2*c*e*(3*b*d + a*e))*x^{11})/11 + (c*e^2*(3*c*d + 2*b*e)*x^{13})/13 + (c^2*e^3*x^{15})/15$

Rule 1153

Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]

Rubi steps

$$\begin{aligned} \int (d + ex^2)^3 (a + bx^2 + cx^4)^2 dx &= \int (a^2d^3 + ad^2(2bd + 3ae)x^2 + d(b^2d^2 + 6abde + a(2cd^2 + 3ae^2))x^4 + (2bcd^3 + 3b^2d^2e + a^2e^3)x^6 + (2cd^2 + 3ae^2)ex^8 + (b^2d^2 + 6abde + a(2cd^2 + 3ae^2))e^2x^{10} + (2bcd^3 + 3b^2d^2e + a^2e^3)e^2x^{12} + (2cd^2 + 3ae^2)e^3x^{14} + a^2e^3e^3x^{16}) dx \\ &= a^2d^3x + \frac{1}{3}ad^2(2bd + 3ae)x^3 + \frac{1}{5}d(b^2d^2 + 6abde + a(2cd^2 + 3ae^2))x^5 + \frac{1}{7}(2bcd^3 + 3b^2d^2e + a^2e^3)x^7 + \frac{1}{9}(2cd^2 + 3ae^2)ex^9 + \frac{1}{11}(b^2d^2 + 6abde + a(2cd^2 + 3ae^2))e^2x^{11} + \frac{1}{13}(2bcd^3 + 3b^2d^2e + a^2e^3)e^2x^{13} + \frac{1}{15}(2cd^2 + 3ae^2)e^3x^{15} + \frac{1}{17}a^2e^3e^3x^{17} \end{aligned}$$

Mathematica [A] time = 0.0900365, size = 223, normalized size = 1.

$$\frac{1}{7}x^7 (a^2e^3 + 6abde^2 + 6acd^2e + 3b^2d^2e + 2bcd^3) + a^2d^3x + \frac{1}{11}ex^{11} (2ce(ae + 3bd) + b^2e^2 + 3c^2d^2) + \frac{1}{5}dx^5 (6abde + a(3ae$$

Antiderivative was successfully verified.

[In] Integrate[(d + e*x^2)^3*(a + b*x^2 + c*x^4)^2,x]

[Out] $a^2d^3x + (a*d^2*(2*b*d + 3*a*e)*x^3)/3 + (d*(b^2*d^2 + 6*a*b*d*e + a*(2*c*d^2 + 3*a*e^2))*x^5)/5 + ((2*b*c*d^3 + 3*b^2*d^2*e + 6*a*c*d^2*e + 6*a*b*d$

$$d \cdot e^2 + a^2 \cdot e^3) \cdot x^7) / 7 + ((c^2 \cdot d^3 + 6 \cdot c \cdot d \cdot e \cdot (b \cdot d + a \cdot e) + b \cdot e^2 \cdot (3 \cdot b \cdot d + 2 \cdot a \cdot e)) \cdot x^9) / 9 + (e \cdot (3 \cdot c^2 \cdot d^2 + b^2 \cdot e^2 + 2 \cdot c \cdot e \cdot (3 \cdot b \cdot d + a \cdot e)) \cdot x^{11}) / 11 + (c \cdot e^2 \cdot (3 \cdot c \cdot d + 2 \cdot b \cdot e) \cdot x^{13}) / 13 + (c^2 \cdot e^3 \cdot x^{15}) / 15$$

Maple [A] time = 0.002, size = 219, normalized size = 1.

$$\frac{c^2 e^3 x^{15}}{15} + \frac{(2 e^3 b c + 3 d e^2 c^2) x^{13}}{13} + \frac{(3 d^2 e c^2 + 6 d e^2 b c + e^3 (2 a c + b^2)) x^{11}}{11} + \frac{(c^2 d^3 + 6 d^2 e b c + 3 d e^2 (2 a c + b^2) + 2 e^3 a^2) x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x^2+d)^3*(c*x^4+b*x^2+a)^2,x)

[Out] 1/15*c^2*e^3*x^15+1/13*(2*b*c*e^3+3*c^2*d*e^2)*x^13+1/11*(3*d^2*e*c^2+6*d*e^2*b*c+e^3*(2*a*c+b^2))*x^11+1/9*(c^2*d^3+6*d^2*e*b*c+3*d*e^2*(2*a*c+b^2)+2*e^3*a*b)*x^9+1/7*(2*b*c*d^3+3*d^2*e*(2*a*c+b^2)+6*a*b*d*e^2+a^2*e^3)*x^7+1/5*(d^3*(2*a*c+b^2)+6*d^2*e*a*b+3*d*e^2*a^2)*x^5+1/3*(3*a^2*d^2*e+2*a*b*d^3)*x^3+a^2*d^3*x

Maxima [A] time = 0.953543, size = 294, normalized size = 1.32

$$\frac{1}{15} c^2 e^3 x^{15} + \frac{1}{13} (3 c^2 d e^2 + 2 b c e^3) x^{13} + \frac{1}{11} (3 c^2 d^2 e + 6 b c d e^2 + (b^2 + 2 a c) e^3) x^{11} + \frac{1}{9} (c^2 d^3 + 6 b c d^2 e + 2 a b e^3 + 3 (b^2 + 2 a c) d^2 e) x^9 + \frac{1}{7} (2 b c d^3 + 6 a b d^2 e + a^2 e^3) x^7 + \frac{1}{5} (d^3 (2 a c + b^2) + 6 d^2 e a b + 3 d e^2 a^2) x^5 + \frac{1}{3} (3 a^2 d^2 e + 2 a b d^3) x^3 + a^2 d^3 x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)^3*(c*x^4+b*x^2+a)^2,x, algorithm="maxima")

[Out] 1/15*c^2*e^3*x^15 + 1/13*(3*c^2*d*e^2 + 2*b*c*e^3)*x^13 + 1/11*(3*c^2*d^2*e + 6*b*c*d*e^2 + (b^2 + 2*a*c)*e^3)*x^11 + 1/9*(c^2*d^3 + 6*b*c*d^2*e + 2*a*b*e^3 + 3*(b^2 + 2*a*c)*d^2*e)*x^9 + 1/7*(2*b*c*d^3 + 6*a*b*d^2*e + a^2*e^3 + 3*(b^2 + 2*a*c)*d^2*e)*x^7 + a^2*d^3*x + 1/5*(6*a*b*d^2*e + 3*a^2*d^2*e^2 + (b^2 + 2*a*c)*d^3)*x^5 + 1/3*(2*a*b*d^3 + 3*a^2*d^2*e)*x^3

Fricas [A] time = 1.37177, size = 620, normalized size = 2.78

$$\frac{1}{15} x^{15} e^3 c^2 + \frac{3}{13} x^{13} e^2 d c^2 + \frac{2}{13} x^{13} e^3 c b + \frac{3}{11} x^{11} e d^2 c^2 + \frac{6}{11} x^{11} e^2 d c b + \frac{1}{11} x^{11} e^3 b^2 + \frac{2}{11} x^{11} e^3 c a + \frac{1}{9} x^9 d^3 c^2 + \frac{2}{3} x^9 e d^2 c b + \frac{1}{3} x^9 e^2 d^2 c a + \frac{2}{7} x^7 d^3 c^2 + \frac{6}{7} x^7 e d^2 c b + \frac{1}{7} x^7 e^2 d^2 c a + \frac{1}{5} x^5 d^3 c^2 + \frac{2}{5} x^5 e d^2 c b + \frac{1}{3} x^3 d^3 c^2 + x^3 e d^2 c b + a^2 d^3 x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)^3*(c*x^4+b*x^2+a)^2,x, algorithm="fricas")

[Out] 1/15*x^15*e^3*c^2 + 3/13*x^13*e^2*d*c^2 + 2/13*x^13*e^3*c*b + 3/11*x^11*e*d^2*c^2 + 6/11*x^11*e^2*d*c*b + 1/11*x^11*e^3*b^2 + 2/11*x^11*e^3*c*a + 1/9*x^9*d^3*c^2 + 2/3*x^9*e*d^2*c*b + 1/3*x^9*e^2*d*b^2 + 2/3*x^9*e^2*d*c*a + 2/9*x^9*e^3*b*a + 2/7*x^7*d^3*c^2 + 3/7*x^7*e*d^2*b^2 + 6/7*x^7*e*d^2*c*a + 6/7*x^7*e^2*d*b*a + 1/7*x^7*e^3*a^2 + 1/5*x^5*d^3*b^2 + 2/5*x^5*d^3*c*a + 6/5*x^5*e*d^2*b*a + 3/5*x^5*e^2*d*a^2 + 2/3*x^3*d^3*b*a + x^3*e*d^2*a^2 + x^3*d^3*a^2

Sympy [A] time = 0.107188, size = 272, normalized size = 1.22

$$a^2 d^3 x + \frac{c^2 e^3 x^{15}}{15} + x^{13} \left(\frac{2bce^3}{13} + \frac{3c^2 de^2}{13} \right) + x^{11} \left(\frac{2ace^3}{11} + \frac{b^2 e^3}{11} + \frac{6bcde^2}{11} + \frac{3c^2 d^2 e}{11} \right) + x^9 \left(\frac{2abe^3}{9} + \frac{2acde^2}{3} + \frac{b^2 de^2}{3} + \frac{2bca}{3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x**2+d)**3*(c*x**4+b*x**2+a)**2,x)

[Out] a**2*d**3*x + c**2*e**3*x**15/15 + x**13*(2*b*c*e**3/13 + 3*c**2*d*e**2/13) + x**11*(2*a*c*e**3/11 + b**2*e**3/11 + 6*b*c*d*e**2/11 + 3*c**2*d**2*e/11) + x**9*(2*a*b*e**3/9 + 2*a*c*d*e**2/3 + b**2*d*e**2/3 + 2*b*c*d**2*e/3 + c**2*d**3/9) + x**7*(a**2*e**3/7 + 6*a*b*d*e**2/7 + 6*a*c*d**2*e/7 + 3*b**2*d**2*e/7 + 2*b*c*d**3/7) + x**5*(3*a**2*d*e**2/5 + 6*a*b*d**2*e/5 + 2*a*c*d**3/5 + b**2*d**3/5) + x**3*(a**2*d**2*e + 2*a*b*d**3/3)

Giac [A] time = 1.13954, size = 344, normalized size = 1.54

$$\frac{1}{15} c^2 x^{15} e^3 + \frac{3}{13} c^2 d x^{13} e^2 + \frac{2}{13} b c x^{13} e^3 + \frac{3}{11} c^2 d^2 x^{11} e + \frac{6}{11} b c d x^{11} e^2 + \frac{1}{9} c^2 d^3 x^9 + \frac{1}{11} b^2 x^{11} e^3 + \frac{2}{11} a c x^{11} e^3 + \frac{2}{3} b c d^2 x^9 e +$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)^3*(c*x^4+b*x^2+a)^2,x, algorithm="giac")

[Out] 1/15*c^2*x^15*e^3 + 3/13*c^2*d*x^13*e^2 + 2/13*b*c*x^13*e^3 + 3/11*c^2*d^2*x^11*e + 6/11*b*c*d*x^11*e^2 + 1/9*c^2*d^3*x^9 + 1/11*b^2*x^11*e^3 + 2/11*a*c*x^11*e^3 + 2/3*b*c*d^2*x^9*e + 1/3*b^2*d*x^9*e^2 + 2/3*a*c*d*x^9*e^2 + 2/7*b*c*d^3*x^7 + 2/9*a*b*x^9*e^3 + 3/7*b^2*d^2*x^7*e + 6/7*a*c*d^2*x^7*e + 6/7*a*b*d*x^7*e^2 + 1/5*b^2*d^3*x^5 + 2/5*a*c*d^3*x^5 + 1/7*a^2*x^7*e^3 + 6/5*a*b*d^2*x^5*e + 3/5*a^2*d*x^5*e^2 + 2/3*a*b*d^3*x^3 + a^2*d^2*x^3*e + a^2*d^3*x

3.253 $\int (d + ex^2)^2 (a + bx^2 + cx^4)^2 dx$

Optimal. Leaf size=155

$$a^2d^2x + \frac{1}{9}x^9(2ce(ae + 2bd) + b^2e^2 + c^2d^2) + \frac{2}{7}x^7(abe^2 + 2acde + b^2de + bcd^2) + \frac{1}{5}x^5(4abde + a(ae^2 + 2cd^2) + b^2d^2)$$

```
[Out] a^2*d^2*x + (2*a*d*(b*d + a*e)*x^3)/3 + ((b^2*d^2 + 4*a*b*d*e + a*(2*c*d^2 + a*e^2))*x^5)/5 + (2*(b*c*d^2 + b^2*d*e + 2*a*c*d*e + a*b*e^2)*x^7)/7 + ((c^2*d^2 + b^2*e^2 + 2*c*e*(2*b*d + a*e))*x^9)/9 + (2*c*e*(c*d + b*e)*x^11)/11 + (c^2*e^2*x^13)/13
```

Rubi [A] time = 0.141042, antiderivative size = 155, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = {1153}

$$a^2d^2x + \frac{1}{9}x^9(2ce(ae + 2bd) + b^2e^2 + c^2d^2) + \frac{2}{7}x^7(abe^2 + 2acde + b^2de + bcd^2) + \frac{1}{5}x^5(4abde + a(ae^2 + 2cd^2) + b^2d^2)$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^2*(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] a^2*d^2*x + (2*a*d*(b*d + a*e)*x^3)/3 + ((b^2*d^2 + 4*a*b*d*e + a*(2*c*d^2 + a*e^2))*x^5)/5 + (2*(b*c*d^2 + b^2*d*e + 2*a*c*d*e + a*b*e^2)*x^7)/7 + ((c^2*d^2 + b^2*e^2 + 2*c*e*(2*b*d + a*e))*x^9)/9 + (2*c*e*(c*d + b*e)*x^11)/11 + (c^2*e^2*x^13)/13
```

Rule 1153

```
Int[((d_) + (e_)*(x_)^2)^(q_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\begin{aligned} \int (d + ex^2)^2 (a + bx^2 + cx^4)^2 dx &= \int (a^2d^2 + 2ad(bd + ae)x^2 + (b^2d^2 + 4abde + a(2cd^2 + ae^2))x^4 + 2(bcd^2 + b^2de \\ &\quad + a^2d^2)x + \frac{2}{3}ad(bd + ae)x^3 + \frac{1}{5}(b^2d^2 + 4abde + a(2cd^2 + ae^2))x^5 + \frac{2}{7}(bcd^2 + b^2de \end{aligned}$$

Mathematica [A] time = 0.0536867, size = 156, normalized size = 1.01

$$\frac{1}{5}x^5(a^2e^2 + 4abde + 2acd^2 + b^2d^2) + a^2d^2x + \frac{1}{9}x^9(2ace^2 + b^2e^2 + 4bcde + c^2d^2) + \frac{2}{7}x^7(abe^2 + 2acde + b^2de + bcd^2)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^2*(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] a^2*d^2*x + (2*a*d*(b*d + a*e)*x^3)/3 + ((b^2*d^2 + 2*a*c*d^2 + 4*a*b*d*e + a^2*e^2)*x^5)/5 + (2*(b*c*d^2 + b^2*d*e + 2*a*c*d*e + a*b*e^2)*x^7)/7 + ((c^2*d^2 + 4*b*c*d*e + b^2*e^2 + 2*a*c*e^2)*x^9)/9 + (2*c*e*(c*d + b*e)*x^11
```

) / 11 + (c²e²x¹³) / 13

Maple [A] time = 0.001, size = 155, normalized size = 1.

$$\frac{c^2 e^2 x^{13}}{13} + \frac{(2 e^2 b c + 2 d e c^2) x^{11}}{11} + \frac{(c^2 d^2 + 4 d e b c + e^2 (2 a c + b^2)) x^9}{9} + \frac{(2 b c d^2 + 2 d e (2 a c + b^2) + 2 a b e^2) x^7}{7} + \frac{(d^2 (2 a c + b^2) + 2 a b d e) x^5}{5} + \frac{a^2 d^2 x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x^2+d)^2*(c*x^4+b*x^2+a)^2,x)

[Out] 1/13*c^2*e^2*x^13+1/11*(2*b*c*e^2+2*c^2*d*e)*x^11+1/9*(c^2*d^2+4*d*e*b*c+e^2*(2*a*c+b^2))*x^9+1/7*(2*b*c*d^2+2*d*e*(2*a*c+b^2)+2*a*b*e^2)*x^7+1/5*(d^2*(2*a*c+b^2)+4*a*b*d*e+e^2*a^2)*x^5+1/3*(2*a^2*d*e+2*a*b*d^2)*x^3+a^2*d^2*x

Maxima [A] time = 0.975113, size = 198, normalized size = 1.28

$$\frac{1}{13} c^2 e^2 x^{13} + \frac{2}{11} (c^2 d e + b c e^2) x^{11} + \frac{1}{9} (c^2 d^2 + 4 b c d e + (b^2 + 2 a c) e^2) x^9 + \frac{2}{7} (b c d^2 + a b e^2 + (b^2 + 2 a c) d e) x^7 + \frac{1}{5} (4 a b d e + a^2 d^2) x^5 + \frac{2}{3} a^2 d^2 x^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)^2*(c*x^4+b*x^2+a)^2,x, algorithm="maxima")

[Out] 1/13*c^2*e^2*x^13 + 2/11*(c^2*d*e + b*c*e^2)*x^11 + 1/9*(c^2*d^2 + 4*b*c*d*e + (b^2 + 2*a*c)*e^2)*x^9 + 2/7*(b*c*d^2 + a*b*e^2 + (b^2 + 2*a*c)*d*e)*x^7 + 1/5*(4*a*b*d*e + a^2*d^2 + (b^2 + 2*a*c)*d^2)*x^5 + a^2*d^2*x + 2/3*(a*b*d^2 + a^2*d*e)*x^3

Fricas [A] time = 1.43315, size = 436, normalized size = 2.81

$$\frac{1}{13} x^{13} e^2 c^2 + \frac{2}{11} x^{11} e d c^2 + \frac{2}{11} x^{11} e^2 c b + \frac{1}{9} x^9 d^2 c^2 + \frac{4}{9} x^9 e d c b + \frac{1}{9} x^9 e^2 b^2 + \frac{2}{9} x^9 e^2 c a + \frac{2}{7} x^7 d^2 c b + \frac{2}{7} x^7 e d b^2 + \frac{4}{7} x^7 e d c a + \frac{2}{7} x^7 e^2 a^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)^2*(c*x^4+b*x^2+a)^2,x, algorithm="fricas")

[Out] 1/13*x^13*e^2*c^2 + 2/11*x^11*e*d*c^2 + 2/11*x^11*e^2*c*b + 1/9*x^9*d^2*c^2 + 4/9*x^9*e*d*c*b + 1/9*x^9*e^2*b^2 + 2/9*x^9*e^2*c*a + 2/7*x^7*d^2*c*b + 2/7*x^7*e*d*b^2 + 4/7*x^7*e*d*c*a + 2/7*x^7*e^2*b*a + 1/5*x^5*d^2*b^2 + 2/5*x^5*d^2*c*a + 4/5*x^5*e*d*b*a + 1/5*x^5*e^2*a^2 + 2/3*x^3*d^2*b*a + 2/3*x^3*e*d*a^2 + x*d^2*a^2

Sympy [A] time = 0.09538, size = 192, normalized size = 1.24

$$a^2 d^2 x + \frac{c^2 e^2 x^{13}}{13} + x^{11} \left(\frac{2 b c e^2}{11} + \frac{2 c^2 d e}{11} \right) + x^9 \left(\frac{2 a c e^2}{9} + \frac{b^2 e^2}{9} + \frac{4 b c d e}{9} + \frac{c^2 d^2}{9} \right) + x^7 \left(\frac{2 a b e^2}{7} + \frac{4 a c d e}{7} + \frac{2 b^2 d e}{7} + \frac{2 b c d^2}{7} \right) + \frac{1}{5} (4 a b d e + a^2 d^2) x^5 + \frac{2}{3} a^2 d^2 x^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x**2+d)**2*(c*x**4+b*x**2+a)**2,x)

[Out] a**2*d**2*x + c**2*e**2*x**13/13 + x**11*(2*b*c*e**2/11 + 2*c**2*d*e/11) + x**9*(2*a*c*e**2/9 + b**2*e**2/9 + 4*b*c*d*e/9 + c**2*d**2/9) + x**7*(2*a*b*e**2/7 + 4*a*c*d*e/7 + 2*b**2*d*e/7 + 2*b*c*d**2/7) + x**5*(a**2*e**2/5 + 4*a*b*d*e/5 + 2*a*c*d**2/5 + b**2*d**2/5) + x**3*(2*a**2*d*e/3 + 2*a*b*d**2/3)

Giac [A] time = 1.13493, size = 244, normalized size = 1.57

$$\frac{1}{13} c^2 x^{13} e^2 + \frac{2}{11} c^2 d x^{11} e + \frac{2}{11} b c x^{11} e^2 + \frac{1}{9} c^2 d^2 x^9 + \frac{4}{9} b c d x^9 e + \frac{1}{9} b^2 x^9 e^2 + \frac{2}{9} a c x^9 e^2 + \frac{2}{7} b c d^2 x^7 + \frac{2}{7} b^2 d x^7 e + \frac{4}{7} a c d x^7 e$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)^2*(c*x^4+b*x^2+a)^2,x, algorithm="giac")

[Out] 1/13*c^2*x^13*e^2 + 2/11*c^2*d*x^11*e + 2/11*b*c*x^11*e^2 + 1/9*c^2*d^2*x^9 + 4/9*b*c*d*x^9*e + 1/9*b^2*x^9*e^2 + 2/9*a*c*x^9*e^2 + 2/7*b*c*d^2*x^7 + 2/7*b^2*d*x^7*e + 4/7*a*c*d*x^7*e + 2/7*a*b*x^7*e^2 + 1/5*b^2*d^2*x^5 + 2/5*a*c*d^2*x^5 + 4/5*a*b*d*x^5*e + 1/5*a^2*x^5*e^2 + 2/3*a*b*d^2*x^3 + 2/3*a^2*d*x^3*e + a^2*d^2*x

3.254 $\int (d + ex^2)(a + bx^2 + cx^4)^2 dx$

Optimal. Leaf size=96

$$a^2 dx + \frac{1}{7}x^7(2ace + b^2e + 2bcd) + \frac{1}{5}x^5(2abe + 2acd + b^2d) + \frac{1}{3}ax^3(ae + 2bd) + \frac{1}{9}cx^9(2be + cd) + \frac{1}{11}c^2ex^{11}$$

[Out] $a^2*d*x + (a*(2*b*d + a*e)*x^3)/3 + ((b^2*d + 2*a*c*d + 2*a*b*e)*x^5)/5 + ((2*b*c*d + b^2*e + 2*a*c*e)*x^7)/7 + (c*(c*d + 2*b*e)*x^9)/9 + (c^2*e*x^{11})/11$

Rubi [A] time = 0.06776, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {1153}

$$a^2 dx + \frac{1}{7}x^7(2ace + b^2e + 2bcd) + \frac{1}{5}x^5(2abe + 2acd + b^2d) + \frac{1}{3}ax^3(ae + 2bd) + \frac{1}{9}cx^9(2be + cd) + \frac{1}{11}c^2ex^{11}$$

Antiderivative was successfully verified.

[In] Int[(d + e*x^2)*(a + b*x^2 + c*x^4)^2,x]

[Out] $a^2*d*x + (a*(2*b*d + a*e)*x^3)/3 + ((b^2*d + 2*a*c*d + 2*a*b*e)*x^5)/5 + ((2*b*c*d + b^2*e + 2*a*c*e)*x^7)/7 + (c*(c*d + 2*b*e)*x^9)/9 + (c^2*e*x^{11})/11$

Rule 1153

Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]

Rubi steps

$$\begin{aligned} \int (d + ex^2)(a + bx^2 + cx^4)^2 dx &= \int (a^2d + a(2bd + ae)x^2 + (b^2d + 2acd + 2abe)x^4 + (2bcd + b^2e + 2ace)x^6 + c(cd + 2bd + a^2e)x^8 + c^2ex^{10}) dx \\ &= a^2dx + \frac{1}{3}a(2bd + ae)x^3 + \frac{1}{5}(b^2d + 2acd + 2abe)x^5 + \frac{1}{7}(2bcd + b^2e + 2ace)x^7 + \frac{1}{9}c(cd + 2bd + a^2e)x^9 + \frac{1}{11}c^2ex^{11} \end{aligned}$$

Mathematica [A] time = 0.0239447, size = 96, normalized size = 1.

$$a^2 dx + \frac{1}{7}x^7(2ace + b^2e + 2bcd) + \frac{1}{5}x^5(2abe + 2acd + b^2d) + \frac{1}{3}ax^3(ae + 2bd) + \frac{1}{9}cx^9(2be + cd) + \frac{1}{11}c^2ex^{11}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e*x^2)*(a + b*x^2 + c*x^4)^2,x]

[Out] $a^2*d*x + (a*(2*b*d + a*e)*x^3)/3 + ((b^2*d + 2*a*c*d + 2*a*b*e)*x^5)/5 + ((2*b*c*d + b^2*e + 2*a*c*e)*x^7)/7 + (c*(c*d + 2*b*e)*x^9)/9 + (c^2*e*x^{11})/11$

Maple [A] time = 0., size = 91, normalized size = 1.

$$\frac{c^2ex^{11}}{11} + \frac{(2ebc + dc^2)x^9}{9} + \frac{(2bcd + e(2ac + b^2))x^7}{7} + \frac{(d(2ac + b^2) + 2abe)x^5}{5} + \frac{(ea^2 + 2dab)x^3}{3} + a^2dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x^2+d)*(c*x^4+b*x^2+a)^2,x)

[Out] 1/11*c^2*e*x^11+1/9*(2*b*c*e+c^2*d)*x^9+1/7*(2*b*c*d+e*(2*a*c+b^2))*x^7+1/5*(d*(2*a*c+b^2)+2*a*b*e)*x^5+1/3*(a^2*e+2*a*b*d)*x^3+a^2*d*x

Maxima [A] time = 0.966369, size = 122, normalized size = 1.27

$$\frac{1}{11}c^2ex^{11} + \frac{1}{9}(c^2d + 2bce)x^9 + \frac{1}{7}(2bcd + (b^2 + 2ac)e)x^7 + \frac{1}{5}(2abe + (b^2 + 2ac)d)x^5 + a^2dx + \frac{1}{3}(2abd + a^2e)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)*(c*x^4+b*x^2+a)^2,x, algorithm="maxima")

[Out] 1/11*c^2*e*x^11 + 1/9*(c^2*d + 2*b*c*e)*x^9 + 1/7*(2*b*c*d + (b^2 + 2*a*c)*e)*x^7 + 1/5*(2*a*b*e + (b^2 + 2*a*c)*d)*x^5 + a^2*d*x + 1/3*(2*a*b*d + a^2*e)*x^3

Fricas [A] time = 1.33555, size = 252, normalized size = 2.62

$$\frac{1}{11}x^{11}ec^2 + \frac{1}{9}x^9dc^2 + \frac{2}{9}x^9ecb + \frac{2}{7}x^7dcb + \frac{1}{7}x^7eb^2 + \frac{2}{7}x^7eca + \frac{1}{5}x^5db^2 + \frac{2}{5}x^5dca + \frac{2}{5}x^5eba + \frac{2}{3}x^3dba + \frac{1}{3}x^3ea^2 + xda^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)*(c*x^4+b*x^2+a)^2,x, algorithm="fricas")

[Out] 1/11*x^11*e*c^2 + 1/9*x^9*d*c^2 + 2/9*x^9*e*c*b + 2/7*x^7*d*c*b + 1/7*x^7*e*b^2 + 2/7*x^7*e*c*a + 1/5*x^5*d*b^2 + 2/5*x^5*d*c*a + 2/5*x^5*e*b*a + 2/3*x^3*d*b*a + 1/3*x^3*e*a^2 + x*d*a^2

Sympy [A] time = 0.080717, size = 107, normalized size = 1.11

$$a^2dx + \frac{c^2ex^{11}}{11} + x^9\left(\frac{2bce}{9} + \frac{c^2d}{9}\right) + x^7\left(\frac{2ace}{7} + \frac{b^2e}{7} + \frac{2bcd}{7}\right) + x^5\left(\frac{2abe}{5} + \frac{2acd}{5} + \frac{b^2d}{5}\right) + x^3\left(\frac{a^2e}{3} + \frac{2abd}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x**2+d)*(c*x**4+b*x**2+a)**2,x)

[Out] a**2*d*x + c**2*e*x**11/11 + x**9*(2*b*c*e/9 + c**2*d/9) + x**7*(2*a*c*e/7 + b**2*e/7 + 2*b*c*d/7) + x**5*(2*a*b*e/5 + 2*a*c*d/5 + b**2*d/5) + x**3*(a**2*e/3 + 2*a*b*d/3)

Giac [A] time = 1.18721, size = 143, normalized size = 1.49

$$\frac{1}{11} c^2 x^{11} e + \frac{1}{9} c^2 d x^9 + \frac{2}{9} b c x^9 e + \frac{2}{7} b c d x^7 + \frac{1}{7} b^2 x^7 e + \frac{2}{7} a c x^7 e + \frac{1}{5} b^2 d x^5 + \frac{2}{5} a c d x^5 + \frac{2}{5} a b x^5 e + \frac{2}{3} a b d x^3 + \frac{1}{3} a^2 x^3 e + a^2 d x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)*(c*x^4+b*x^2+a)^2,x, algorithm="giac")

[Out] 1/11*c^2*x^11*e + 1/9*c^2*d*x^9 + 2/9*b*c*x^9*e + 2/7*b*c*d*x^7 + 1/7*b^2*x^7*e + 2/7*a*c*x^7*e + 1/5*b^2*d*x^5 + 2/5*a*c*d*x^5 + 2/5*a*b*x^5*e + 2/3*a*b*d*x^3 + 1/3*a^2*x^3*e + a^2*d*x

3.255 $\int (a + bx^2 + cx^4)^2 dx$

Optimal. Leaf size=49

$$a^2x + \frac{1}{5}x^5(2ac + b^2) + \frac{2}{3}abx^3 + \frac{2}{7}bcx^7 + \frac{c^2x^9}{9}$$

[Out] $a^2x + (2abx^3)/3 + ((b^2 + 2ac)x^5)/5 + (2bcx^7)/7 + (c^2x^9)/9$

Rubi [A] time = 0.0246358, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {1090}

$$a^2x + \frac{1}{5}x^5(2ac + b^2) + \frac{2}{3}abx^3 + \frac{2}{7}bcx^7 + \frac{c^2x^9}{9}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x^2 + c*x^4)^2,x]

[Out] $a^2x + (2abx^3)/3 + ((b^2 + 2ac)x^5)/5 + (2bcx^7)/7 + (c^2x^9)/9$

Rule 1090

Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0]

Rubi steps

$$\begin{aligned} \int (a + bx^2 + cx^4)^2 dx &= \int \left(a^2 + 2abx^2 + b^2 \left(1 + \frac{2ac}{b^2} \right) x^4 + 2bcx^6 + c^2x^8 \right) dx \\ &= a^2x + \frac{2}{3}abx^3 + \frac{1}{5}(b^2 + 2ac)x^5 + \frac{2}{7}bcx^7 + \frac{c^2x^9}{9} \end{aligned}$$

Mathematica [A] time = 0.0055303, size = 49, normalized size = 1.

$$a^2x + \frac{1}{5}x^5(2ac + b^2) + \frac{2}{3}abx^3 + \frac{2}{7}bcx^7 + \frac{c^2x^9}{9}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x^2 + c*x^4)^2,x]

[Out] $a^2x + (2abx^3)/3 + ((b^2 + 2ac)x^5)/5 + (2bcx^7)/7 + (c^2x^9)/9$

Maple [A] time = 0., size = 42, normalized size = 0.9

$$a^2x + \frac{2abx^3}{3} + \frac{(2ac + b^2)x^5}{5} + \frac{2bcx^7}{7} + \frac{c^2x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c*x^4+b*x^2+a)^2,x)`

[Out] $a^2x + \frac{2}{3}abx^3 + \frac{1}{5}(2ac + b^2)x^5 + \frac{2}{7}bcx^7 + \frac{1}{9}c^2x^9$

Maxima [A] time = 0.963936, size = 61, normalized size = 1.24

$$\frac{1}{9}c^2x^9 + \frac{2}{7}bcx^7 + \frac{1}{5}b^2x^5 + a^2x + \frac{2}{15}(3cx^5 + 5bx^3)a$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x^4+b*x^2+a)^2,x, algorithm="maxima")`

[Out] $\frac{1}{9}c^2x^9 + \frac{2}{7}bcx^7 + \frac{1}{5}b^2x^5 + a^2x + \frac{2}{15}(3cx^5 + 5bx^3)a$

Fricas [A] time = 1.34705, size = 104, normalized size = 2.12

$$\frac{1}{9}x^9c^2 + \frac{2}{7}x^7cb + \frac{1}{5}x^5b^2 + \frac{2}{5}x^5ca + \frac{2}{3}x^3ba + xa^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x^4+b*x^2+a)^2,x, algorithm="fricas")`

[Out] $\frac{1}{9}x^9c^2 + \frac{2}{7}x^7cb + \frac{1}{5}x^5b^2 + \frac{2}{5}x^5ca + \frac{2}{3}x^3ba + xa^2$

Sympy [A] time = 0.06829, size = 48, normalized size = 0.98

$$a^2x + \frac{2abx^3}{3} + \frac{2bcx^7}{7} + \frac{c^2x^9}{9} + x^5\left(\frac{2ac}{5} + \frac{b^2}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x**4+b*x**2+a)**2,x)`

[Out] $a^2x + \frac{2abx^3}{3} + \frac{2bcx^7}{7} + \frac{c^2x^9}{9} + x^5\left(\frac{2ac}{5} + \frac{b^2}{5}\right)$

Giac [A] time = 1.10676, size = 58, normalized size = 1.18

$$\frac{1}{9}c^2x^9 + \frac{2}{7}bcx^7 + \frac{1}{5}b^2x^5 + \frac{2}{5}acx^5 + \frac{2}{3}abx^3 + a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x^4+b*x^2+a)^2,x, algorithm="giac")`

[Out] $\frac{1}{9}c^2x^9 + \frac{2}{7}bcx^7 + \frac{1}{5}b^2x^5 + \frac{2}{5}acx^5 + \frac{2}{3}abx^3 + a^2x$

$$3.256 \quad \int \frac{(a+bx^2+cx^4)^2}{d+ex^2} dx$$

Optimal. Leaf size=143

$$\frac{x^3(-2ce(bd-ae)+b^2e^2+c^2d^2)}{3e^3} - \frac{x(cd-be)(cd^2-e(bd-2ae))}{e^4} + \frac{\tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)(ae^2-bde+cd^2)^2}{\sqrt{de}^{9/2}} - \frac{cx^5(cd-2be)}{5e^2} +$$

[Out] -(((c*d - b*e)*(c*d^2 - e*(b*d - 2*a*e))*x)/e^4) + ((c^2*d^2 + b^2*e^2 - 2*c*e*(b*d - a*e))*x^3)/(3*e^3) - (c*(c*d - 2*b*e)*x^5)/(5*e^2) + (c^2*x^7)/(7*e) + ((c*d^2 - b*d*e + a*e^2)^2*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[d]*e^(9/2))

Rubi [A] time = 0.140219, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1153, 205}

$$\frac{x^3(-2ce(bd-ae)+b^2e^2+c^2d^2)}{3e^3} - \frac{x(cd-be)(cd^2-e(bd-2ae))}{e^4} + \frac{\tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)(ae^2-bde+cd^2)^2}{\sqrt{de}^{9/2}} - \frac{cx^5(cd-2be)}{5e^2} +$$

Antiderivative was successfully verified.

[In] Int[(a + b*x^2 + c*x^4)^2/(d + e*x^2), x]

[Out] -(((c*d - b*e)*(c*d^2 - e*(b*d - 2*a*e))*x)/e^4) + ((c^2*d^2 + b^2*e^2 - 2*c*e*(b*d - a*e))*x^3)/(3*e^3) - (c*(c*d - 2*b*e)*x^5)/(5*e^2) + (c^2*x^7)/(7*e) + ((c*d^2 - b*d*e + a*e^2)^2*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[d]*e^(9/2))

Rule 1153

Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rubi steps

$$\begin{aligned} \int \frac{(a+bx^2+cx^4)^2}{d+ex^2} dx &= \int \left(-\frac{(cd-be)(cd^2-e(bd-2ae))}{e^4} + \frac{(c^2d^2+b^2e^2-2ce(bd-ae))x^2}{e^3} - \frac{c(cd-2be)x^4}{e^2} + \frac{c^2x^6}{e} \right. \\ &= -\frac{(cd-be)(cd^2-e(bd-2ae))x}{e^4} + \frac{(c^2d^2+b^2e^2-2ce(bd-ae))x^3}{3e^3} - \frac{c(cd-2be)x^5}{5e^2} + \frac{c^2x^7}{7e} + \\ &= -\frac{(cd-be)(cd^2-e(bd-2ae))x}{e^4} + \frac{(c^2d^2+b^2e^2-2ce(bd-ae))x^3}{3e^3} - \frac{c(cd-2be)x^5}{5e^2} + \frac{c^2x^7}{7e} + \end{aligned}$$

Mathematica [A] time = 0.0649803, size = 144, normalized size = 1.01

$$\frac{x^3(2ace^2 + b^2e^2 - 2bcde + c^2d^2)}{3e^3} + \frac{x(be - cd)(2ae^2 - bde + cd^2)}{e^4} + \frac{\tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)(ae^2 - bde + cd^2)^2}{\sqrt{de}e^{9/2}} + \frac{cx^5(2be - cd)}{5e^2} + \frac{c^2d^3x}{7e^4}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x^2 + c*x^4)^2/(d + e*x^2), x]

[Out] ((-(c*d) + b*e)*(c*d^2 - b*d*e + 2*a*e^2)*x)/e^4 + ((c^2*d^2 - 2*b*c*d*e + b^2*e^2 + 2*a*c*e^2)*x^3)/(3*e^3) + (c*(-(c*d) + 2*b*e)*x^5)/(5*e^2) + (c^2*x^7)/(7*e) + ((c*d^2 - b*d*e + a*e^2)^2*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[d]*e^(9/2))

Maple [B] time = 0.005, size = 267, normalized size = 1.9

$$\frac{c^2x^7}{7e} + \frac{2x^5bc}{5e} - \frac{dc^2x^5}{5e^2} + \frac{2x^3ac}{3e} + \frac{x^3b^2}{3e} - \frac{2x^3bcd}{3e^2} + \frac{x^3c^2d^2}{3e^3} + 2\frac{abx}{e} - 2\frac{acd}{e^2} - \frac{b^2dx}{e^2} + 2\frac{bcd^2x}{e^3} - \frac{c^2d^3x}{e^4} + a^2 \arctan\left(\frac{e}{d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^4+b*x^2+a)^2/(e*x^2+d), x)

[Out] 1/7*c^2*x^7/e+2/5/e*x^5*b*c-1/5*c^2*d*x^5/e^2+2/3/e*x^3*a*c+1/3/e*x^3*b^2-2/3/e^2*x^3*b*c*d+1/3/e^3*x^3*c^2*d^2+2/e*a*b*x-2/e^2*a*c*d*x-1/e^2*b^2*d*x+2/e^3*d^2*b*c*x-1/e^4*c^2*d^3*x+1/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))*a^2-2/e/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))*a*b*d+2/e^2/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))*a*c*d^2+1/e^2/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))*b^2*d^2-2/e^3/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))*b*c*d^3+1/e^4/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))*c^2*d^4

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^2+a)^2/(e*x^2+d), x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [A] time = 1.61883, size = 892, normalized size = 6.24

$$\frac{30c^2de^4x^7 - 42(c^2d^2e^3 - 2bcde^4)x^5 + 70(c^2d^3e^2 - 2bcd^2e^3 + (b^2 + 2ac)de^4)x^3 - 105(c^2d^4 - 2bcd^3e - 2abde^3 + a^2e^4) + 105a^2d^3e^2}{210de^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^2+a)^2/(e*x^2+d), x, algorithm="fricas")

[Out] $\left[\frac{1}{210} (30c^2d^2e^4x^7 - 42(c^2d^2e^3 - 2b^2cd^2e^4)x^5 + 70(c^2d^3e^2 - 2b^2cd^2e^3 + (b^2 + 2a^2c)d^2e^4)x^3 - 105(c^2d^4 - 2b^2cd^3e - 2a^2bd^2e^3 + a^2e^4 + (b^2 + 2a^2c)d^2e^2)\sqrt{-de})\log\left(\frac{ex^2 - 2\sqrt{-de}x - d}{ex^2 + d}\right) - 210(c^2d^4e - 2b^2cd^3e^2 - 2a^2bd^2e^3 + (b^2 + 2a^2c)d^2e^3)x\right] / (de^5), \frac{1}{105} (15c^2d^2e^4x^7 - 21(c^2d^2e^3 - 2b^2cd^2e^4)x^5 + 35(c^2d^3e^2 - 2b^2cd^2e^3 + (b^2 + 2a^2c)d^2e^4)x^3 + 105(c^2d^4 - 2b^2cd^3e - 2a^2bd^2e^3 + a^2e^4 + (b^2 + 2a^2c)d^2e^2)\sqrt{de})\arctan\left(\frac{\sqrt{de}x}{d}\right) - 105(c^2d^4e - 2b^2cd^3e^2 - 2a^2bd^2e^3 + (b^2 + 2a^2c)d^2e^3)x\right] / (de^5)]$

Sympy [B] time = 1.26761, size = 366, normalized size = 2.56

$$\frac{c^2x^7}{7e} - \frac{\sqrt{-\frac{1}{de^9}}(ae^2 - bde + cd^2)^2 \log\left(-\frac{de^4\sqrt{-\frac{1}{de^9}}(ae^2 - bde + cd^2)^2}{a^2e^4 - 2abde^3 + 2acd^2e^2 + b^2d^2e^2 - 2bcd^3e + c^2d^4} + x\right)}{2} + \frac{\sqrt{-\frac{1}{de^9}}(ae^2 - bde + cd^2)^2 \log\left(\frac{ae^2 - bde + cd^2}{a^2e^4 - 2abde^3 + 2acd^2e^2 + b^2d^2e^2 - 2bcd^3e + c^2d^4}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x**4+b*x**2+a)**2/(e*x**2+d),x)

[Out] $c^2x^7/(7e) - \sqrt{-1/(de^9)}(ae^2 - bde + cd^2)^2 \log(-de^4\sqrt{-1/(de^9)}(ae^2 - bde + cd^2)^2/(a^2e^4 - 2a^2bd^2e^3 + 2a^2cd^2e^2 + b^2d^2e^2 - 2b^2cd^3e + c^2d^4) + x)/2 + \sqrt{-1/(de^9)}(ae^2 - bde + cd^2)^2 \log(de^4\sqrt{-1/(de^9)}(ae^2 - bde + cd^2)^2/(a^2e^4 - 2a^2bd^2e^3 + 2a^2cd^2e^2 + b^2d^2e^2 - 2b^2cd^3e + c^2d^4) + x)/2 + x^5(2b^2c^2e - c^2d)/(5e^2) + x^3(2a^2c^2e^2 + b^2d^2e^2 - 2b^2cd^2e + c^2d^2e^2)/(3e^3) + x(2a^2bd^2e^3 - 2a^2cd^2e^2 - b^2d^2e^2 + 2b^2cd^2e - c^2d^2e^2)/e^4$

Giac [A] time = 1.1435, size = 250, normalized size = 1.75

$$\frac{(c^2d^4 - 2bcd^3e + b^2d^2e^2 + 2acd^2e^2 - 2abde^3 + a^2e^4) \arctan\left(\frac{xe^2}{\sqrt{d}}\right) e^{\left(-\frac{9}{2}\right)}}{\sqrt{d}} + \frac{1}{105} (15c^2x^7e^6 - 21c^2dx^5e^5 + 42bcx^5e^6 + \dots)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^2+a)^2/(e*x^2+d),x, algorithm="giac")

[Out] $(c^2d^4 - 2b^2cd^3e + b^2d^2e^2 + 2a^2cd^2e^2 - 2a^2bd^2e^3 + a^2e^4)\arctan(xe^{1/2}/\sqrt{d})e^{(-9/2)}/\sqrt{d} + 1/105(15c^2x^7e^6 - 21c^2d^2x^5e^5 + 42b^2cx^5e^6 + 35c^2d^2x^3e^4 - 70b^2cd^2x^3e^5 - 105c^2d^3x^3e^3 + 35b^2x^3e^6 + 70a^2cx^3e^6 + 210b^2cd^2x^4 - 105b^2d^2x^4e^5 - 210a^2cd^2x^4e^5 + 210a^2bd^2x^4e^6)e^{(-7)}$

$$3.257 \quad \int \frac{(a+bx^2+cx^4)^2}{(d+ex^2)^2} dx$$

Optimal. Leaf size=166

$$\frac{x(-2ce(2bd - ae) + b^2e^2 + 3c^2d^2)}{e^4} + \frac{x(ae^2 - bde + cd^2)^2}{2de^4(d + ex^2)} - \frac{\tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)(ae^2 - bde + cd^2)(7cd^2 - e(ae + 3bd))}{2d^{3/2}e^{9/2}} - \frac{2cx^3(cd^2 - bde + ae^2)}{3e^4}$$

[Out] $((3c^2d^2 + b^2e^2 - 2c^2e(2bd - ae))x)/e^4 - (2c^2(c^2d - b^2e)x^3)/(3e^3) + (c^2x^5)/(5e^2) + ((c^2d^2 - b^2de + a^2e^2)^2x)/(2d^2e^4(d + ex^2)) - ((c^2d^2 - b^2de + a^2e^2)(7cd^2 - e(ae + 3bd))\text{ArcTan}[\sqrt{ex}/\sqrt{d}])/(2d^{3/2}e^{9/2})$

Rubi [A] time = 0.297636, antiderivative size = 166, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1157, 1810, 205}

$$\frac{x(-2ce(2bd - ae) + b^2e^2 + 3c^2d^2)}{e^4} + \frac{x(ae^2 - bde + cd^2)^2}{2de^4(d + ex^2)} - \frac{\tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)(ae^2 - bde + cd^2)(7cd^2 - e(ae + 3bd))}{2d^{3/2}e^{9/2}} - \frac{2cx^3(cd^2 - bde + ae^2)}{3e^4}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x^2 + c*x^4)^2/(d + e*x^2)^2,x]

[Out] $((3c^2d^2 + b^2e^2 - 2c^2e(2bd - ae))x)/e^4 - (2c^2(c^2d - b^2e)x^3)/(3e^3) + (c^2x^5)/(5e^2) + ((c^2d^2 - b^2de + a^2e^2)^2x)/(2d^2e^4(d + ex^2)) - ((c^2d^2 - b^2de + a^2e^2)(7cd^2 - e(ae + 3bd))\text{ArcTan}[\sqrt{ex}/\sqrt{d}])/(2d^{3/2}e^{9/2})$

Rule 1157

Int[((d_) + (e_)*(x_)^2)^(q_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] := With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]

Rule 1810

Int[(Pq_)*((a_) + (b_)*(x_)^2)^(p_), x_Symbol] := Int[ExpandIntegrand[Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rule 205

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rubi steps

$$\begin{aligned}
\int \frac{(a + bx^2 + cx^4)^2}{(d + ex^2)^2} dx &= \frac{(cd^2 - bde + ae^2)^2 x}{2de^4 (d + ex^2)} - \frac{\int \frac{\frac{c^2 d^4 - 2cd^2 e(bd - ae) + e^2(b^2 d^2 - 2abde - a^2 e^2)}{e^4} - \frac{2d(c^2 d^2 + b^2 e^2 - 2ce(bd - ae))x^2}{e^3} + \frac{2cd(cd - 2be)x^4}{e^2} - \frac{2c^2 dx^6}{e}}{d + ex^2}}{2d} \\
&= \frac{(cd^2 - bde + ae^2)^2 x}{2de^4 (d + ex^2)} - \frac{\int \left(-\frac{2d(3c^2 d^2 + b^2 e^2 - 2ce(2bd - ae))}{e^4} + \frac{4cd(cd - be)x^2}{e^3} - \frac{2c^2 dx^4}{e^2} + \frac{7c^2 d^4 - 10bcd^3 e + 3b^2 d^2 e^2}{e^4} \right)}{2d} \\
&= \frac{(3c^2 d^2 + b^2 e^2 - 2ce(2bd - ae))x}{e^4} - \frac{2c(cd - be)x^3}{3e^3} + \frac{c^2 x^5}{5e^2} + \frac{(cd^2 - bde + ae^2)^2 x}{2de^4 (d + ex^2)} - \frac{(7c^2 d^4 - 10bcd^3 e + 3b^2 d^2 e^2)}{e^4} \\
&= \frac{(3c^2 d^2 + b^2 e^2 - 2ce(2bd - ae))x}{e^4} - \frac{2c(cd - be)x^3}{3e^3} + \frac{c^2 x^5}{5e^2} + \frac{(cd^2 - bde + ae^2)^2 x}{2de^4 (d + ex^2)} - \frac{(7c^2 d^4 - 10bcd^3 e + 3b^2 d^2 e^2)}{e^4}
\end{aligned}$$

Mathematica [A] time = 0.102643, size = 183, normalized size = 1.1

$$\frac{\tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)\left(-e^2(a^2 e^2 + 2abde - 3b^2 d^2) + 2cd^2 e(3ae - 5bd) + 7c^2 d^4\right)}{2d^{3/2}e^{9/2}} + \frac{x(2ce(ae - 2bd) + b^2 e^2 + 3c^2 d^2)}{e^4} + \frac{x(eae - 2bd)}{2de^4}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x^2 + c*x^4)^2/(d + e*x^2)^2,x]

[Out] ((3*c^2*d^2 + b^2*e^2 + 2*c*e*(-2*b*d + a*e))*x)/e^4 + (2*c*(-(c*d) + b*e)*x^3)/(3*e^3) + (c^2*x^5)/(5*e^2) + ((c*d^2 + e*(-(b*d) + a*e))^2*x)/(2*d*e^4*(d + e*x^2)) - ((7*c^2*d^4 + 2*c*d^2*e*(-5*b*d + 3*a*e) - e^2*(-3*b^2*d^2 + 2*a*b*d*e + a^2*e^2))*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(3/2)*e^(9/2))

Maple [B] time = 0.013, size = 320, normalized size = 1.9

$$\frac{c^2 x^5}{5e^2} + \frac{2x^3 bc}{3e^2} - \frac{2dc^2 x^3}{3e^3} + 2\frac{acx}{e^2} + \frac{b^2 x}{e^2} - 4\frac{bcdx}{e^3} + 3\frac{c^2 d^2 x}{e^4} + \frac{a^2 x}{2d(ex^2 + d)} - \frac{xab}{e(ex^2 + d)} + \frac{adxc}{e^2(ex^2 + d)} + \frac{dxb^2}{2e^2(ex^2 + d)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^4+b*x^2+a)^2/(e*x^2+d)^2,x)

[Out] 1/5*c^2*x^5/e^2+2/3/e^2*x^3*b*c-2/3*c^2*d*x^3/e^3+2/e^2*a*c*x+1/e^2*b^2*x-4/e^3*b*c*d*x+3/e^4*c^2*d^2*x+1/2/d*x/(e*x^2+d)*a^2-1/e*x/(e*x^2+d)*a*b+1/e^2*d*x/(e*x^2+d)*a*c+1/2/e^2*d*x/(e*x^2+d)*b^2-1/e^3*d^2*x/(e*x^2+d)*b*c+1/2/e^4*d^3*x/(e*x^2+d)*c^2+1/2/d/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))*a^2+1/e/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))*a*b-3/e^2*d/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))*a*c-3/2/e^2*d/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))*b^2+5/e^3*d^2/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))*b*c-7/2/e^4*d^3/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))*c^2

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/(e*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.71495, size = 1272, normalized size = 7.66

$$\left[\frac{12c^2d^2e^4x^7 - 4(7c^2d^3e^3 - 10bcd^2e^4)x^5 + 20(7c^2d^4e^2 - 10bcd^3e^3 + 3(b^2 + 2ac)d^2e^4)x^3 + 15(7c^2d^5 - 10bcd^4e - 2abd^3e^2 - 10b^2cd^2e^2 + 3a^2d^2e^2)x + 15(7c^2d^5 - 10bcd^4e - 2abd^3e^2 - 10b^2cd^2e^2 + 3a^2d^2e^2)}{\dots} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/(e*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] [1/60*(12*c^2*d^2*e^4*x^7 - 4*(7*c^2*d^3*e^3 - 10*b*c*d^2*e^4)*x^5 + 20*(7*c^2*d^4*e^2 - 10*b*c*d^3*e^3 + 3*(b^2 + 2*a*c)*d^2*e^4)*x^3 + 15*(7*c^2*d^5 - 10*b*c*d^4*e - 2*a*b*d^2*e^3 - a^2*d*e^4 + 3*(b^2 + 2*a*c)*d^3*e^2 + (7*c^2*d^4*e - 10*b*c*d^3*e^2 - 2*a*b*d*e^4 - a^2*e^5 + 3*(b^2 + 2*a*c)*d^2*e^3)*x^2)*sqrt(-d*e)*log((e*x^2 - 2*sqrt(-d*e)*x - d)/(e*x^2 + d)) + 30*(7*c^2*d^5*e - 10*b*c*d^4*e^2 - 2*a*b*d^2*e^4 + a^2*d*e^5 + 3*(b^2 + 2*a*c)*d^3*e^3)*x)/(d^2*e^6*x^2 + d^3*e^5), 1/30*(6*c^2*d^2*e^4*x^7 - 2*(7*c^2*d^3*e^3 - 10*b*c*d^2*e^4)*x^5 + 10*(7*c^2*d^4*e^2 - 10*b*c*d^3*e^3 + 3*(b^2 + 2*a*c)*d^2*e^4)*x^3 - 15*(7*c^2*d^5 - 10*b*c*d^4*e - 2*a*b*d^2*e^3 - a^2*d*e^4 + 3*(b^2 + 2*a*c)*d^3*e^2 + (7*c^2*d^4*e - 10*b*c*d^3*e^2 - 2*a*b*d*e^4 - a^2*e^5 + 3*(b^2 + 2*a*c)*d^2*e^3)*x^2)*sqrt(d*e)*arctan(sqrt(d*e)*x/d) + 15*(7*c^2*d^5*e - 10*b*c*d^4*e^2 - 2*a*b*d^2*e^4 + a^2*d*e^5 + 3*(b^2 + 2*a*c)*d^3*e^3)*x)/(d^2*e^6*x^2 + d^3*e^5)]
```

Sympy [B] time = 3.27443, size = 479, normalized size = 2.89

$$\frac{c^2x^5}{5e^2} + \frac{x(a^2e^4 - 2abde^3 + 2acd^2e^2 + b^2d^2e^2 - 2bcd^3e + c^2d^4)}{2d^2e^4 + 2de^5x^2} - \frac{\sqrt{-\frac{1}{d^3e^9}}(ae^2 - bde + cd^2)(ae^2 + 3bde - 7cd^2) \log\left(-\frac{d^2e^4}{a^2e^4 + \dots}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**2/(e*x**2+d)**2,x)
```

```
[Out] c**2*x**5/(5*e**2) + x*(a**2*e**4 - 2*a*b*d*e**3 + 2*a*c*d**2*e**2 + b**2*d**2*e**2 - 2*b*c*d**3*e + c**2*d**4)/(2*d**2*e**4 + 2*d*e**5*x**2) - sqrt(-1/(d**3*e**9))*(a*e**2 - b*d*e + c*d**2)*(a*e**2 + 3*b*d*e - 7*c*d**2)*log(-d**2*e**4*sqrt(-1/(d**3*e**9))*(a*e**2 - b*d*e + c*d**2)*(a*e**2 + 3*b*d*e - 7*c*d**2)/(a**2*e**4 + 2*a*b*d*e**3 - 6*a*c*d**2*e**2 - 3*b**2*d**2*e**2 + 10*b*c*d**3*e - 7*c**2*d**4) + x)/4 + sqrt(-1/(d**3*e**9))*(a*e**2 - b*d*e + c*d**2)*(a*e**2 + 3*b*d*e - 7*c*d**2)*log(d**2*e**4*sqrt(-1/(d**3*e**9))*(a*e**2 - b*d*e + c*d**2)*(a*e**2 + 3*b*d*e - 7*c*d**2)/(a**2*e**4 + 2*a*b*d*e**3 - 6*a*c*d**2*e**2 - 3*b**2*d**2*e**2 + 10*b*c*d**3*e - 7*c**2*d**4) + x)/4 + x**3*(2*b*c*e - 2*c**2*d)/(3*e**3) + x*(2*a*c*e**2 + b**2*e**2 - 4*b*c*d*e + 3*c**2*d**2)/e**4
```

Giac [A] time = 1.12264, size = 279, normalized size = 1.68

$$\frac{1}{15} (3c^2x^5e^8 - 10c^2dx^3e^7 + 10bcx^3e^8 + 45c^2d^2xe^6 - 60bcdxe^7 + 15b^2xe^8 + 30acxe^8)e^{(-10)} - \frac{(7c^2d^4 - 10bcd^3e + 3b^2d^2e^2 + 6a^2cd^2e^2 - 2abde^3 - a^2e^4) \arctan(xe^{1/2}/\sqrt{d})e^{(-9/2)}/d^{(3/2)} + 1/2(c^2d^4x - 2b^2cd^3xe + b^2d^2x^2e^2 + 2ac^2d^2xe^2 - 2ab^2d^2xe^3 + a^2x^2e^4)e^{(-4)}}{(x^2e + d)*d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^2+a)^2/(e*x^2+d)^2,x, algorithm="giac")

[Out] 1/15*(3*c^2*x^5*e^8 - 10*c^2*d*x^3*e^7 + 10*b*c*x^3*e^8 + 45*c^2*d^2*x*e^6 - 60*b*c*d*x*e^7 + 15*b^2*x*e^8 + 30*a*c*x*e^8)*e^(-10) - 1/2*(7*c^2*d^4 - 10*b*c*d^3*e + 3*b^2*d^2*e^2 + 6*a*c*d^2*e^2 - 2*a*b*d*e^3 - a^2*e^4)*arctan(x*e^(1/2)/sqrt(d))*e^(-9/2)/d^(3/2) + 1/2*(c^2*d^4*x - 2*b*c*d^3*x*e + b^2*d^2*x*e^2 + 2*a*c*d^2*x*e^2 - 2*a*b*d*x*e^3 + a^2*x*e^4)*e^(-4)/((x^2*e + d)*d)

$$3.258 \quad \int \frac{(a+bx^2+cx^4)^2}{(d+ex^2)^3} dx$$

Optimal. Leaf size=201

$$\frac{\tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)\left(e^2(3a^2e^2+2abde+3b^2d^2)-6cd^2e(5bd-ae)+35c^2d^4\right)}{8d^{5/2}e^{9/2}} - \frac{x(-3ae^2-5bde+13cd^2)(ae^2-bde+cd^2)}{8d^2e^4(d+ex^2)} + \dots$$

[Out] $-\left(\frac{c(3cd-2be)x}{e^4} + \frac{c^2x^3}{3e^3} + \frac{(cd^2-bde+ae^2)^2x}{4de^4(d+ex^2)^2} - \frac{(13cd^2-5bde-3ae^2)(cd^2-bde+ae^2)x}{8d^2e^4(d+ex^2)} + \frac{(35c^2d^4-6cd^2e(5bd-ae)+e^2(3b^2d^2+2abde+3a^2e^2))\text{ArcTan}\left[\frac{\sqrt{e}x}{\sqrt{d}}\right]}{8d^{5/2}e^{9/2}}\right)$

Rubi [A] time = 0.418775, antiderivative size = 201, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1157, 1814, 1153, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)\left(e^2(3a^2e^2+2abde+3b^2d^2)-6cd^2e(5bd-ae)+35c^2d^4\right)}{8d^{5/2}e^{9/2}} - \frac{x(-3ae^2-5bde+13cd^2)(ae^2-bde+cd^2)}{8d^2e^4(d+ex^2)} + \dots$$

Antiderivative was successfully verified.

[In] Int[(a + b*x^2 + c*x^4)^2/(d + e*x^2)^3,x]

[Out] $-\left(\frac{c(3cd-2be)x}{e^4} + \frac{c^2x^3}{3e^3} + \frac{(cd^2-bde+ae^2)^2x}{4de^4(d+ex^2)^2} - \frac{(13cd^2-5bde-3ae^2)(cd^2-bde+ae^2)x}{8d^2e^4(d+ex^2)} + \frac{(35c^2d^4-6cd^2e(5bd-ae)+e^2(3b^2d^2+2abde+3a^2e^2))\text{ArcTan}\left[\frac{\sqrt{e}x}{\sqrt{d}}\right]}{8d^{5/2}e^{9/2}}\right)$

Rule 1157

Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]

Rule 1814

Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] := With[{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]

Rule 1153

Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],

x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rubi steps

$$\int \frac{(a + bx^2 + cx^4)^2}{(d + ex^2)^3} dx = \frac{(cd^2 - bde + ae^2)^2 x}{4de^4 (d + ex^2)^2} - \frac{\int \frac{\frac{(cd^2 - bde - ae^2)(cd^2 - bde + 3ae^2)}{e^4} - \frac{4d(c^2d^2 + b^2e^2 - 2ce(bd - ae))x^2}{e^3} + \frac{4cd(cd - 2be)x^4}{e^2} - \frac{4c^2dx^6}{e}}{(d + ex^2)^2} dx}{4d}$$

$$= \frac{(cd^2 - bde + ae^2)^2 x}{4de^4 (d + ex^2)^2} - \frac{(13cd^2 - 5bde - 3ae^2)(cd^2 - bde + ae^2)x}{8d^2e^4 (d + ex^2)} + \frac{\int \frac{11c^2d^4 - 2cd^2e(7bd - 3ae) + e^2(3b^2d^2 - 2ce^2)}{e^4} dx}{e^4}$$

$$= \frac{(cd^2 - bde + ae^2)^2 x}{4de^4 (d + ex^2)^2} - \frac{(13cd^2 - 5bde - 3ae^2)(cd^2 - bde + ae^2)x}{8d^2e^4 (d + ex^2)} + \frac{\int \left(-\frac{8cd^2(3cd - 2be)}{e^4} + \frac{8c^2d^2x}{e^3} \right) dx}{e^3}$$

$$= -\frac{c(3cd - 2be)x}{e^4} + \frac{c^2x^3}{3e^3} + \frac{(cd^2 - bde + ae^2)^2 x}{4de^4 (d + ex^2)^2} - \frac{(13cd^2 - 5bde - 3ae^2)(cd^2 - bde + ae^2)x}{8d^2e^4 (d + ex^2)}$$

$$= -\frac{c(3cd - 2be)x}{e^4} + \frac{c^2x^3}{3e^3} + \frac{(cd^2 - bde + ae^2)^2 x}{4de^4 (d + ex^2)^2} - \frac{(13cd^2 - 5bde - 3ae^2)(cd^2 - bde + ae^2)x}{8d^2e^4 (d + ex^2)}$$

Mathematica [A] time = 0.114594, size = 217, normalized size = 1.08

$$\frac{x(e^2(-3a^2e^2 - 2abde + 5b^2d^2) - 2cd^2e(9bd - 5ae) + 13c^2d^4)}{8d^2e^4(d + ex^2)} + \frac{\tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)(e^2(3a^2e^2 + 2abde + 3b^2d^2) + 6cd^2e(ae^2 + b^2d))}{8d^{5/2}e^{9/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x^2 + c*x^4)^2/(d + e*x^2)^3,x]

[Out] (c*(-3*c*d + 2*b*e)*x)/e^4 + (c^2*x^3)/(3*e^3) + ((c*d^2 + e*(-(b*d) + a*e))^2*x)/(4*d*e^4*(d + e*x^2)^2) - ((13*c^2*d^4 - 2*c*d^2*e*(9*b*d - 5*a*e) + e^2*(5*b^2*d^2 - 2*a*b*d*e - 3*a^2*e^2))*x)/(8*d^2*e^4*(d + e*x^2)) + ((35*c^2*d^4 + 6*c*d^2*e*(-5*b*d + a*e) + e^2*(3*b^2*d^2 + 2*a*b*d*e + 3*a^2*e^2))*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(8*d^(5/2)*e^(9/2))

Maple [B] time = 0.012, size = 402, normalized size = 2.

$$\frac{c^2x^3}{3e^3} + 2\frac{bcx}{e^3} - 3\frac{dc^2x}{e^4} + \frac{3a^2ex^3}{8(ex^2 + d)^2d^2} + \frac{abx^3}{4(ex^2 + d)^2d} - \frac{5x^3ac}{4e(ex^2 + d)^2} - \frac{5x^3b^2}{8e(ex^2 + d)^2} + \frac{9x^3bcd}{4e^2(ex^2 + d)^2} - \frac{13cd^2e}{8e^3(e^2 + b^2d)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^4+b*x^2+a)^2/(e*x^2+d)^3,x)

```
[Out] 1/3*c^2*x^3/e^3+2*c/e^3*b*x-3*c^2*d*x/e^4+3/8*e/(e*x^2+d)^2/d^2*x^3*a^2+1/4/(e*x^2+d)^2/d*x^3*a*b-5/4/e/(e*x^2+d)^2*x^3*a*c-5/8/e/(e*x^2+d)^2*x^3*b^2+9/4/e^2/(e*x^2+d)^2*x^3*b*c*d-13/8/e^3/(e*x^2+d)^2*x^3*c^2*d^2+5/8/(e*x^2+d)^2/d*x*a^2-1/4/e/(e*x^2+d)^2*a*b*x-3/4/e^2/(e*x^2+d)^2*a*c*d*x-3/8/e^2/(e*x^2+d)^2*b^2*d*x+7/4/e^3/(e*x^2+d)^2*d^2*b*c*x-11/8/e^4/(e*x^2+d)^2*c^2*d^3*x+3/8/d^2/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))*a^2+1/4/e/d/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))*a*b+3/4/e^2/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))*a*c+3/8/e^2/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))*b^2-15/4/e^3*d/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))*b*c+35/8/e^4*d^2/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))*c^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/(e*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.72383, size = 1694, normalized size = 8.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/(e*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] [1/48*(16*c^2*d^3*e^4*x^7 - 16*(7*c^2*d^4*e^3 - 6*b*c*d^3*e^4)*x^5 - 2*(175*c^2*d^5*e^2 - 150*b*c*d^4*e^3 - 6*a*b*d^2*e^5 - 9*a^2*d*e^6 + 15*(b^2 + 2*a*c)*d^3*e^4)*x^3 - 3*(35*c^2*d^6 - 30*b*c*d^5*e + 2*a*b*d^3*e^3 + 3*a^2*d^2*e^4 + 3*(b^2 + 2*a*c)*d^4*e^2 + (35*c^2*d^4*e^2 - 30*b*c*d^3*e^3 + 2*a*b*d*e^5 + 3*a^2*e^6 + 3*(b^2 + 2*a*c)*d^2*e^4)*x^4 + 2*(35*c^2*d^5*e - 30*b*c*d^4*e^2 + 2*a*b*d^2*e^4 + 3*a^2*d*e^5 + 3*(b^2 + 2*a*c)*d^3*e^3)*x^2)*sqrt(-d*e)*log((e*x^2 - 2*sqrt(-d*e)*x - d)/(e*x^2 + d)) - 6*(35*c^2*d^6*e - 30*b*c*d^5*e^2 + 2*a*b*d^3*e^4 - 5*a^2*d^2*e^5 + 3*(b^2 + 2*a*c)*d^4*e^3)*x)/(d^3*e^7*x^4 + 2*d^4*e^6*x^2 + d^5*e^5), 1/24*(8*c^2*d^3*e^4*x^7 - 8*(7*c^2*d^4*e^3 - 6*b*c*d^3*e^4)*x^5 - (175*c^2*d^5*e^2 - 150*b*c*d^4*e^3 - 6*a*b*d^2*e^5 - 9*a^2*d*e^6 + 15*(b^2 + 2*a*c)*d^3*e^4)*x^3 + 3*(35*c^2*d^6 - 30*b*c*d^5*e + 2*a*b*d^3*e^3 + 3*a^2*d^2*e^4 + 3*(b^2 + 2*a*c)*d^4*e^2 + (35*c^2*d^4*e^2 - 30*b*c*d^3*e^3 + 2*a*b*d*e^5 + 3*a^2*e^6 + 3*(b^2 + 2*a*c)*d^2*e^4)*x^4 + 2*(35*c^2*d^5*e - 30*b*c*d^4*e^2 + 2*a*b*d^2*e^4 + 3*a^2*d*e^5 + 3*(b^2 + 2*a*c)*d^3*e^3)*x^2)*sqrt(d*e)*arctan(sqrt(d*e)*x/d) - 3*(35*c^2*d^6*e - 30*b*c*d^5*e^2 + 2*a*b*d^3*e^4 - 5*a^2*d^2*e^5 + 3*(b^2 + 2*a*c)*d^4*e^3)*x)/(d^3*e^7*x^4 + 2*d^4*e^6*x^2 + d^5*e^5)]
```

Sympy [A] time = 17.1049, size = 396, normalized size = 1.97

$$\frac{c^2 x^3}{3e^3} - \frac{\sqrt{-\frac{1}{d^5 e^9}} (3a^2 e^4 + 2abde^3 + 6acd^2 e^2 + 3b^2 d^2 e^2 - 30bcd^3 e + 35c^2 d^4) \log\left(-d^3 e^4 \sqrt{-\frac{1}{d^5 e^9}} + x\right)}{16} + \frac{\sqrt{-\frac{1}{d^5 e^9}} (3a^2 e^4 + 2abde^3 + 6acd^2 e^2 + 3b^2 d^2 e^2 - 30bcd^3 e + 35c^2 d^4)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x**4+b*x**2+a)**2/(e*x**2+d)**3,x)

[Out] $c^2 x^3 / (3 e^3) - \sqrt{-1 / (d^5 e^9)} (3 a^2 e^4 + 2 a b d e^3 + 6 a^2 c d^2 e^2 + 3 b^2 d^2 e^2 - 30 b^2 c d^3 e + 35 c^2 d^4) \log(-d^3 e^4 \sqrt{-1 / (d^5 e^9)} + x) / 16 + \sqrt{-1 / (d^5 e^9)} (3 a^2 e^4 + 2 a b d e^3 + 6 a^2 c d^2 e^2 + 3 b^2 d^2 e^2 - 30 b^2 c d^3 e + 35 c^2 d^4) \log(d^3 e^4 \sqrt{-1 / (d^5 e^9)} + x) / 16 + (x^3 (3 a^2 e^5 + 2 a b d e^4 - 10 a^2 c d^2 e^3 - 5 b^2 d^2 e^3 + 18 b^2 c d^3 e^2 - 13 c^2 d^4 e) + x (5 a^2 d e^4 - 2 a b d^2 e^3 - 6 a^2 c d^3 e^2 - 3 b^2 d^3 e^2 + 14 b^2 c d^4 e - 11 c^2 d^5)) / (8 d^4 e^4 + 16 d^3 e^5 x^2 + 8 d^2 e^6 x^4) + x (2 b^2 c e - 3 c^2 d) / e^4$

Giac [A] time = 1.1325, size = 329, normalized size = 1.64

$$\frac{1}{3} (c^2 x^3 e^6 - 9 c^2 d x e^5 + 6 b c x e^6) e^{(-9)} + \frac{(35 c^2 d^4 - 30 b c d^3 e + 3 b^2 d^2 e^2 + 6 a c d^2 e^2 + 2 a b d e^3 + 3 a^2 e^4) \arctan\left(\frac{1}{\sqrt{d}}\right) e^{(-\frac{9}{2})}}{8 d^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^2+a)^2/(e*x^2+d)^3,x, algorithm="giac")

[Out] $1/3 (c^2 x^3 e^6 - 9 c^2 d x e^5 + 6 b^2 c x e^6) e^{(-9)} + 1/8 (35 c^2 d^4 - 30 b^2 c d^3 e + 3 b^2 d^2 e^2 + 6 a^2 c d^2 e^2 + 2 a b d e^3 + 3 a^2 e^4) \arctan(x e^{(1/2)} / \sqrt{d}) e^{(-9/2)} / d^{(5/2)} - 1/8 (13 c^2 d^4 x^3 e - 18 b^2 c d^3 x^3 e^2 + 11 c^2 d^5 x + 5 b^2 d^2 x^3 e^3 + 10 a^2 c d^2 x^3 e^3 - 14 b^2 c d^4 x e - 2 a b d^2 x^3 e^4 + 3 b^2 d^3 x e^2 + 6 a^2 c d^3 x e^2 - 3 a^2 x^3 e^5 + 2 a b d^2 x e^3 - 5 a^2 d x e^4) e^{(-4)} / ((x^2 e + d)^2 d^2)$

$$3.259 \quad \int \frac{(a+bx^2+cx^4)^2}{(d+ex^2)^4} dx$$

Optimal. Leaf size=250

$$\frac{x(e^2(5a^2e^2 + 2abde + b^2d^2) - 2cd^2e(11bd - ae) + 29c^2d^4)}{16d^3e^4(d + ex^2)} - \frac{\tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)(-e^2(5a^2e^2 + 2abde + b^2d^2) - 2cd^2e(ae + 5bd))}{16d^{7/2}e^{9/2}}$$

[Out] (c^2*x)/e^4 + ((c*d^2 - b*d*e + a*e^2)^2*x)/(6*d*e^4*(d + e*x^2)^3) - ((19*c*d^2 - 7*b*d*e - 5*a*e^2)*(c*d^2 - b*d*e + a*e^2)*x)/(24*d^2*e^4*(d + e*x^2)^2) + ((29*c^2*d^4 - 2*c*d^2*e*(11*b*d - a*e) + e^2*(b^2*d^2 + 2*a*b*d*e + 5*a^2*e^2))*x)/(16*d^3*e^4*(d + e*x^2)) - ((35*c^2*d^4 - 2*c*d^2*e*(5*b*d + a*e) - e^2*(b^2*d^2 + 2*a*b*d*e + 5*a^2*e^2))*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(16*d^(7/2)*e^(9/2))

Rubi [A] time = 0.542902, antiderivative size = 250, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1157, 1814, 388, 205}

$$\frac{x(e^2(5a^2e^2 + 2abde + b^2d^2) - 2cd^2e(11bd - ae) + 29c^2d^4)}{16d^3e^4(d + ex^2)} - \frac{\tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)(-e^2(5a^2e^2 + 2abde + b^2d^2) - 2cd^2e(ae + 5bd))}{16d^{7/2}e^{9/2}}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x^2 + c*x^4)^2/(d + e*x^2)^4, x]

[Out] (c^2*x)/e^4 + ((c*d^2 - b*d*e + a*e^2)^2*x)/(6*d*e^4*(d + e*x^2)^3) - ((19*c*d^2 - 7*b*d*e - 5*a*e^2)*(c*d^2 - b*d*e + a*e^2)*x)/(24*d^2*e^4*(d + e*x^2)^2) + ((29*c^2*d^4 - 2*c*d^2*e*(11*b*d - a*e) + e^2*(b^2*d^2 + 2*a*b*d*e + 5*a^2*e^2))*x)/(16*d^3*e^4*(d + e*x^2)) - ((35*c^2*d^4 - 2*c*d^2*e*(5*b*d + a*e) - e^2*(b^2*d^2 + 2*a*b*d*e + 5*a^2*e^2))*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(16*d^(7/2)*e^(9/2))

Rule 1157

Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]

Rule 1814

Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] := With[{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Sim
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{(a + bx^2 + cx^4)^2}{(d + ex^2)^4} dx = \frac{(cd^2 - bde + ae^2)^2 x}{6de^4 (d + ex^2)^3} - \frac{\int \frac{c^2 d^4 - 2cd^2 e(bd - ae) + e^2(b^2 d^2 - 2abde - 5a^2 e^2) - 6d(c^2 d^2 + b^2 e^2 - 2ce(bd - ae))x^2 + 6cd(cd - 2be)x^4 - 6c^2 dx^6}{e^4} \frac{e^3}{(d + ex^2)^3} + \frac{6c^2 dx^6}{e}}{6d}$$

$$= \frac{(cd^2 - bde + ae^2)^2 x}{6de^4 (d + ex^2)^3} - \frac{(19cd^2 - 7bde - 5ae^2)(cd^2 - bde + ae^2)x}{24d^2 e^4 (d + ex^2)^2} + \frac{\int \frac{3(5c^2 d^4 - 2cd^2 e(3bd - ae) + e^2(b^2 d^2 - 2abde - 5a^2 e^2))x^2 + 6cd(cd - 2be)x^4 - 6c^2 dx^6}{e^4}}{16a}$$

$$= \frac{(cd^2 - bde + ae^2)^2 x}{6de^4 (d + ex^2)^3} - \frac{(19cd^2 - 7bde - 5ae^2)(cd^2 - bde + ae^2)x}{24d^2 e^4 (d + ex^2)^2} + \frac{(29c^2 d^4 - 2cd^2 e(11bd - 13bd) + 7ae^2 - 13bd) + 19c^2 d^4}{16a}$$

$$= \frac{c^2 x}{e^4} + \frac{(cd^2 - bde + ae^2)^2 x}{6de^4 (d + ex^2)^3} - \frac{(19cd^2 - 7bde - 5ae^2)(cd^2 - bde + ae^2)x}{24d^2 e^4 (d + ex^2)^2} + \frac{(29c^2 d^4 - 2cd^2 e(11bd - 13bd) + 7ae^2 - 13bd) + 19c^2 d^4}{16a}$$

$$= \frac{c^2 x}{e^4} + \frac{(cd^2 - bde + ae^2)^2 x}{6de^4 (d + ex^2)^3} - \frac{(19cd^2 - 7bde - 5ae^2)(cd^2 - bde + ae^2)x}{24d^2 e^4 (d + ex^2)^2} + \frac{(29c^2 d^4 - 2cd^2 e(11bd - 13bd) + 7ae^2 - 13bd) + 19c^2 d^4}{16a}$$

Mathematica [A] time = 0.149606, size = 267, normalized size = 1.07

$$\frac{x(e^2(5a^2e^2 + 2abde + b^2d^2) + 2cd^2e(ae - 11bd) + 29c^2d^4)}{16d^3e^4(d + ex^2)} - \frac{x(e^2(-5a^2e^2 - 2abde + 7b^2d^2) + 2cd^2e(7ae - 13bd) + 19c^2d^4)}{24d^2e^4(d + ex^2)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^2/(d + e*x^2)^4, x]
```

```
[Out] (c^2*x)/e^4 + ((c*d^2 + e*(-(b*d) + a*e))^2*x)/(6*d*e^4*(d + e*x^2)^3) - ((
19*c^2*d^4 + 2*c*d^2*e*(-13*b*d + 7*a*e) + e^2*(7*b^2*d^2 - 2*a*b*d*e - 5*a
^2*e^2))*x)/(24*d^2*e^4*(d + e*x^2)^2) + ((29*c^2*d^4 + 2*c*d^2*e*(-11*b*d
+ a*e) + e^2*(b^2*d^2 + 2*a*b*d*e + 5*a^2*e^2))*x)/(16*d^3*e^4*(d + e*x^2))
- (((35*c^2*d^4 - 2*c*d^2*e*(5*b*d + a*e) - e^2*(b^2*d^2 + 2*a*b*d*e + 5*a^
2*e^2))*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(16*d^(7/2)*e^(9/2))
```

Maple [B] time = 0.011, size = 506, normalized size = 2.

$$\frac{29x^5c^2d}{16e^2(ex^2 + d)^3} + \frac{5a^2ex^3}{6(ex^2 + d)^3d^2} - \frac{x^3ac}{3e(ex^2 + d)^3} + \frac{17x^3c^2d^2}{6e^3(ex^2 + d)^3} - \frac{abx}{8e(ex^2 + d)^3} - \frac{b^2dx}{16e^2(ex^2 + d)^3} + \frac{19c^2d^3}{16e^4(ex^2 + d)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c*x^4+b*x^2+a)^2/(e*x^2+d)^4,x)`

[Out]
$$\frac{29}{16} \frac{e^2}{(e^2x+d)^3} x^5 c^2 d + \frac{5}{6} \frac{e}{(e^2x+d)^3} d^2 x^3 a^2 - \frac{1}{3} \frac{e}{(e^2x+d)^3} x^3 a c + \frac{17}{6} \frac{e^3}{(e^2x+d)^3} x^3 c^2 d^2 - \frac{1}{8} \frac{e}{(e^2x+d)^3} a b x - \frac{1}{16} \frac{e^2}{(e^2x+d)^3} b^2 d x + \frac{19}{16} \frac{e^4}{(e^2x+d)^3} c^2 d^3 x + \frac{1}{8} \frac{e}{(e^2x+d)^3} d x^5 a c + \frac{1}{3} \frac{e}{(e^2x+d)^3} d x^3 a b - \frac{35}{16} \frac{e^4 d}{(d e)^{1/2}} \arctan\left(\frac{x e}{(d e)^{1/2}}\right) c^2 + \frac{5}{16} \frac{e^2}{(e^2x+d)^3} d^3 x^5 a^2 - \frac{11}{8} \frac{e}{(e^2x+d)^3} x^5 b c + \frac{1}{16} \frac{e^2 d}{(d e)^{1/2}} \arctan\left(\frac{x e}{(d e)^{1/2}}\right) b^2 + \frac{5}{8} \frac{e^3}{(d e)^{1/2}} \arctan\left(\frac{x e}{(d e)^{1/2}}\right) b c + \frac{1}{8} \frac{e}{(e^2x+d)^3} d^2 x^5 a b - \frac{5}{3} \frac{e^2}{(e^2x+d)^3} x^3 b c d - \frac{1}{8} \frac{e^2}{(e^2x+d)^3} a c d x - \frac{5}{8} \frac{e^3}{(e^2x+d)^3} d^2 b c x + \frac{1}{8} \frac{e}{d^2} \frac{d}{(d e)^{1/2}} \arctan\left(\frac{x e}{(d e)^{1/2}}\right) a b + \frac{1}{8} \frac{e^2 d}{(d e)^{1/2}} \arctan\left(\frac{x e}{(d e)^{1/2}}\right) a c + \frac{1}{16} \frac{e}{(e^2x+d)^3} d x^5 b^2 + \frac{11}{16} \frac{e}{(e^2x+d)^3} d x a^2 - \frac{1}{6} \frac{e}{(e^2x+d)^3} x^3 b^2 + \frac{5}{16} \frac{e}{d^3} \frac{d}{(d e)^{1/2}} \arctan\left(\frac{x e}{(d e)^{1/2}}\right) a^2 + c^2 x / e^4$$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x^4+b*x^2+a)^2/(e*x^2+d)^4,x, algorithm="maxima")`

[Out] Exception raised: ValueError

Fricas [B] time = 1.68979, size = 2130, normalized size = 8.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x^4+b*x^2+a)^2/(e*x^2+d)^4,x, algorithm="fricas")`

[Out]
$$\left[\frac{1}{96} (96 c^2 d^4 e^4 x^7 + 6 (77 c^2 d^5 e^3 - 22 b c d^4 e^4 + 2 a b d^2 e^6 + 5 a^2 d e^7 + (b^2 + 2 a c) d^3 e^5) x^5 + 16 (35 c^2 d^6 e^2 - 10 b c d^5 e^3 + 2 a b d^3 e^5 + 5 a^2 d^2 e^6 - (b^2 + 2 a c) d^4 e^4) x^3 + 3 (35 c^2 d^7 - 10 b c d^6 e - 2 a b d^4 e^3 - 5 a^2 d^3 e^4 - (b^2 + 2 a c) d^5 e^2 + (35 c^2 d^4 e^3 - 10 b c d^3 e^4 - 2 a b d e^6 - 5 a^2 e^7 - (b^2 + 2 a c) d^2 e^5) x^6 + 3 (35 c^2 d^5 e^2 - 10 b c d^4 e^3 - 2 a b d^2 e^5 - 5 a^2 d e^6 - (b^2 + 2 a c) d^3 e^4) x^4 + 3 (35 c^2 d^6 e - 10 b c d^5 e^2 - 2 a b d^3 e^4 - 5 a^2 d^2 e^5 - (b^2 + 2 a c) d^4 e^3) x^2) \sqrt{-d e} \right] \log\left(\frac{e x^2 - 2 \sqrt{-d e} x - d}{e x^2 + d}\right) + 6 (35 c^2 d^7 e - 10 b c d^6 e^2 - 2 a b d^4 e^4 + 11 a^2 d^3 e^5 - (b^2 + 2 a c) d^5 e^3) x / (d^4 e^8 x^6 + 3 d^5 e^7 x^4 + 3 d^6 e^6 x^2 + d^7 e^5), \frac{1}{48} (48 c^2 d^4 e^4 x^7 + 3 (77 c^2 d^5 e^3 - 22 b c d^4 e^4 + 2 a b d^2 e^6 + 5 a^2 d e^7 + (b^2 + 2 a c) d^3 e^5) x^5 + 8 (35 c^2 d^6 e^2 - 10 b c d^5 e^3 + 2 a b d^3 e^5 + 5 a^2 d^2 e^6 - (b^2 + 2 a c) d^4 e^4) x^3 - 3 (35 c^2 d^7 - 10 b c d^6 e - 2 a b d^4 e^3 - 5 a^2 d^3 e^4 - (b^2 + 2 a c) d^5 e^2 + (35 c^2 d^4 e^3 - 10 b c d^3 e^4 - 2 a b d e^6 - 5 a^2 e^7 - (b^2 + 2 a c) d^2 e^5) x^6 + 3 (35 c^2 d^5 e^2 - 10 b c d^4 e^3 - 2 a b d^2 e^5 - 5 a^2 d e^6 - (b^2 + 2 a c) d^3 e^4) x^4 + 3 (35 c^2 d^6 e - 10 b c d^5 e^2 - 2 a b d^3 e^4 - 5 a^2 d^2 e^5 - (b^2 + 2 a c) d^4 e^3) x^2) \sqrt{d e} \arctan\left(\frac{\sqrt{d e} x}{d}\right) + 3 (35 c^2 d^7 e - 10 b c d^6 e^2 - 2 a b d^4 e^4 + 11 a^2 d^3 e^5 - (b^2 + 2$$

$$*a*c)*d^5*e^3)*x)/(d^4*e^8*x^6 + 3*d^5*e^7*x^4 + 3*d^6*e^6*x^2 + d^7*e^5)]$$

Sympy [A] time = 110.179, size = 457, normalized size = 1.83

$$\frac{c^2x}{e^4} - \frac{\sqrt{-\frac{1}{d^7e^9}}(5a^2e^4 + 2abde^3 + 2acd^2e^2 + b^2d^2e^2 + 10bcd^3e - 35c^2d^4) \log\left(-d^4e^4\sqrt{-\frac{1}{d^7e^9}} + x\right)}{32} + \frac{\sqrt{-\frac{1}{d^7e^9}}(5a^2e^4 + 2a$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x**4+b*x**2+a)**2/(e*x**2+d)**4,x)

[Out] c**2*x/e**4 - sqrt(-1/(d**7*e**9))*(5*a**2*e**4 + 2*a*b*d*e**3 + 2*a*c*d**2*e**2 + b**2*d**2*e**2 + 10*b*c*d**3*e - 35*c**2*d**4)*log(-d**4*e**4*sqrt(-1/(d**7*e**9)) + x)/32 + sqrt(-1/(d**7*e**9))*(5*a**2*e**4 + 2*a*b*d*e**3 + 2*a*c*d**2*e**2 + b**2*d**2*e**2 + 10*b*c*d**3*e - 35*c**2*d**4)*log(d**4*e**4*sqrt(-1/(d**7*e**9)) + x)/32 + (x**5*(15*a**2*e**6 + 6*a*b*d*e**5 + 6*a*c*d**2*e**4 + 3*b**2*d**2*e**4 - 66*b*c*d**3*e**3 + 87*c**2*d**4*e**2) + x**3*(40*a**2*d*e**5 + 16*a*b*d**2*e**4 - 16*a*c*d**3*e**3 - 8*b**2*d**3*e**3 - 80*b*c*d**4*e**2 + 136*c**2*d**5*e) + x*(33*a**2*d**2*e**4 - 6*a*b*d**3*e**3 - 6*a*c*d**4*e**2 - 3*b**2*d**4*e**2 - 30*b*c*d**5*e + 57*c**2*d**6)))/(48*d**6*e**4 + 144*d**5*e**5*x**2 + 144*d**4*e**6*x**4 + 48*d**3*e**7*x**6)

Giac [A] time = 1.11388, size = 400, normalized size = 1.6

$$c^2xe^{(-4)} - \frac{(35c^2d^4 - 10bcd^3e - b^2d^2e^2 - 2acd^2e^2 - 2abde^3 - 5a^2e^4) \arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right) e^{\left(-\frac{9}{2}\right)}}{16d^{\frac{7}{2}}} + \frac{(87c^2d^4x^5e^2 - 66bcd^3x^5e^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^2+a)^2/(e*x^2+d)^4,x, algorithm="giac")

[Out] c^2*x*e^(-4) - 1/16*(35*c^2*d^4 - 10*b*c*d^3*e - b^2*d^2*e^2 - 2*a*c*d^2*e^2 - 2*a*b*d*e^3 - 5*a^2*e^4)*arctan(x*e^(1/2)/sqrt(d))*e^(-9/2)/d^(7/2) + 1/48*(87*c^2*d^4*x^5*e^2 - 66*b*c*d^3*x^5*e^3 + 136*c^2*d^5*x^3*e + 3*b^2*d^2*x^5*e^4 + 6*a*c*d^2*x^5*e^4 - 80*b*c*d^4*x^3*e^2 + 57*c^2*d^6*x + 6*a*b*d*x^5*e^5 - 8*b^2*d^3*x^3*e^3 - 16*a*c*d^3*x^3*e^3 - 30*b*c*d^5*x*e + 15*a^2*x^5*e^6 + 16*a*b*d^2*x^3*e^4 - 3*b^2*d^4*x*e^2 - 6*a*c*d^4*x*e^2 + 40*a^2*d*x^3*e^5 - 6*a*b*d^3*x*e^3 + 33*a^2*d^2*x*e^4)*e^(-4)/((x^2*e + d)^3*d^3)

$$3.260 \quad \int \frac{(a+bx^2+cx^4)^2}{(d+ex^2)^5} dx$$

Optimal. Leaf size=317

$$\frac{x(-e^2(35a^2e^2 + 10abde + 3b^2d^2) - 2cd^2e(3ae + 5bd) + 93c^2d^4)}{128d^4e^4(d+ex^2)} + \frac{x(e^2(35a^2e^2 + 10abde + 3b^2d^2) - 2cd^2e(59bd - 3ae))}{192d^3e^4(d+ex^2)^2}$$

[Out] $((c*d^2 - b*d*e + a*e^2)^2*x)/(8*d*e^4*(d + e*x^2)^4) - ((25*c*d^2 - 9*b*d*e - 7*a*e^2)*(c*d^2 - b*d*e + a*e^2)*x)/(48*d^2*e^4*(d + e*x^2)^3) + ((163*c^2*d^4 - 2*c*d^2*e*(59*b*d - 3*a*e) + e^2*(3*b^2*d^2 + 10*a*b*d*e + 35*a^2*e^2))*x)/(192*d^3*e^4*(d + e*x^2)^2) - ((93*c^2*d^4 - 2*c*d^2*e*(5*b*d + 3*a*e) - e^2*(3*b^2*d^2 + 10*a*b*d*e + 35*a^2*e^2))*x)/(128*d^4*e^4*(d + e*x^2)^2) + ((35*c^2*d^4 + 2*c*d^2*e*(5*b*d + 3*a*e) + e^2*(3*b^2*d^2 + 10*a*b*d*e + 35*a^2*e^2))*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(128*d^(9/2)*e^(9/2))$

Rubi [A] time = 0.649839, antiderivative size = 317, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1157, 1814, 385, 205}

$$\frac{x(-e^2(35a^2e^2 + 10abde + 3b^2d^2) - 2cd^2e(3ae + 5bd) + 93c^2d^4)}{128d^4e^4(d+ex^2)} + \frac{x(e^2(35a^2e^2 + 10abde + 3b^2d^2) - 2cd^2e(59bd - 3ae))}{192d^3e^4(d+ex^2)^2}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x^2 + c*x^4)^2/(d + e*x^2)^5, x]

[Out] $((c*d^2 - b*d*e + a*e^2)^2*x)/(8*d*e^4*(d + e*x^2)^4) - ((25*c*d^2 - 9*b*d*e - 7*a*e^2)*(c*d^2 - b*d*e + a*e^2)*x)/(48*d^2*e^4*(d + e*x^2)^3) + ((163*c^2*d^4 - 2*c*d^2*e*(59*b*d - 3*a*e) + e^2*(3*b^2*d^2 + 10*a*b*d*e + 35*a^2*e^2))*x)/(192*d^3*e^4*(d + e*x^2)^2) - ((93*c^2*d^4 - 2*c*d^2*e*(5*b*d + 3*a*e) - e^2*(3*b^2*d^2 + 10*a*b*d*e + 35*a^2*e^2))*x)/(128*d^4*e^4*(d + e*x^2)^2) + ((35*c^2*d^4 + 2*c*d^2*e*(5*b*d + 3*a*e) + e^2*(3*b^2*d^2 + 10*a*b*d*e + 35*a^2*e^2))*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(128*d^(9/2)*e^(9/2))$

Rule 1157

Int[((d_) + (e_)*(x_)^2)^(q_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] := With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]

Rule 1814

Int[(Pq_)*((a_) + (b_)*(x_)^2)^(p_), x_Symbol] := With[{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := -Simp[(b*c - a*d)*x*(a + b*x^n)^(p + 1)/(a*b*n*(p + 1)), x] - Dist[(a*d - b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n + p, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{(a + bx^2 + cx^4)^2}{(d + ex^2)^5} dx = \frac{(cd^2 - bde + ae^2)^2 x}{8de^4 (d + ex^2)^4} - \frac{\int \frac{c^2 d^4 - 2cd^2 e(bd - ae) + e^2(b^2 d^2 - 2abde - 7a^2 e^2) - 8d(c^2 d^2 + b^2 e^2 - 2ce(bd - ae))x^2 + 8cd(cd - 2be)x^4 - 8c^2 dx^6}{e^4} dx}{8d}$$

$$= \frac{(cd^2 - bde + ae^2)^2 x}{8de^4 (d + ex^2)^4} - \frac{(25cd^2 - 9bde - 7ae^2)(cd^2 - bde + ae^2)x}{48d^2 e^4 (d + ex^2)^3} + \frac{\int \frac{19c^2 d^4 - 2cd^2 e(11bd - 3ae) + e^2(3b^2 d^2 - 2abde - 7a^2 e^2) - 8d(c^2 d^2 + b^2 e^2 - 2ce(bd - ae))x^2 + 8cd(cd - 2be)x^4 - 8c^2 dx^6}{e^4} dx}{48d^2 e^4}$$

$$= \frac{(cd^2 - bde + ae^2)^2 x}{8de^4 (d + ex^2)^4} - \frac{(25cd^2 - 9bde - 7ae^2)(cd^2 - bde + ae^2)x}{48d^2 e^4 (d + ex^2)^3} + \frac{(163c^2 d^4 - 2cd^2 e(59bd - 17ae) + e^2(3b^2 d^2 - 2abde - 7a^2 e^2))x}{48d^2 e^4 (d + ex^2)^3}$$

$$= \frac{(cd^2 - bde + ae^2)^2 x}{8de^4 (d + ex^2)^4} - \frac{(25cd^2 - 9bde - 7ae^2)(cd^2 - bde + ae^2)x}{48d^2 e^4 (d + ex^2)^3} + \frac{(163c^2 d^4 - 2cd^2 e(59bd - 17ae) + e^2(3b^2 d^2 - 2abde - 7a^2 e^2))x}{48d^2 e^4 (d + ex^2)^3}$$

$$= \frac{(cd^2 - bde + ae^2)^2 x}{8de^4 (d + ex^2)^4} - \frac{(25cd^2 - 9bde - 7ae^2)(cd^2 - bde + ae^2)x}{48d^2 e^4 (d + ex^2)^3} + \frac{(163c^2 d^4 - 2cd^2 e(59bd - 17ae) + e^2(3b^2 d^2 - 2abde - 7a^2 e^2))x}{48d^2 e^4 (d + ex^2)^3}$$

Mathematica [A] time = 0.228926, size = 345, normalized size = 1.09

$$\frac{8d^{5/2}\sqrt{ex}(e^2(-7a^2e^2 - 2abde + 9b^2d^2) + 2cd^2e(9ae - 17bd) + 25c^2d^4)}{(d + ex^2)^3} + \frac{2d^{3/2}\sqrt{ex}(e^2(35a^2e^2 + 10abde + 3b^2d^2) + 2cd^2e(3ae - 59bd) + 163c^2d^4)}{(d + ex^2)^2} - \frac{3\sqrt{d}\sqrt{ex}(-e^2(35a^2e^2 + 10abde + 3b^2d^2) + 2cd^2e(3ae - 59bd) + 163c^2d^4)}{(d + ex^2)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^2/(d + e*x^2)^5, x]
```

```
[Out] ((48*d^(7/2)*Sqrt[e]*(c*d^2 + e*(-(b*d) + a*e))^2*x)/(d + e*x^2)^4 - (8*d^(5/2)*Sqrt[e]*(25*c^2*d^4 + 2*c*d^2*e*(-17*b*d + 9*a*e) + e^2*(9*b^2*d^2 - 2*a*b*d*e - 7*a^2*e^2))*x)/(d + e*x^2)^3 + (2*d^(3/2)*Sqrt[e]*(163*c^2*d^4 + 2*c*d^2*e*(-59*b*d + 3*a*e) + e^2*(3*b^2*d^2 + 10*a*b*d*e + 35*a^2*e^2))*x)/(d + e*x^2)^2 - (3*Sqrt[d]*Sqrt[e]*(93*c^2*d^4 - 2*c*d^2*e*(5*b*d + 3*a*e) - e^2*(3*b^2*d^2 + 10*a*b*d*e + 35*a^2*e^2))*x)/(d + e*x^2) + 3*(35*c^2*d^4 + 2*c*d^2*e*(5*b*d + 3*a*e) + e^2*(3*b^2*d^2 + 10*a*b*d*e + 35*a^2*e^2))*ArcTan[(Sqrt[e]*x)/Sqrt[d]]/(384*d^(9/2)*e^(9/2))
```

Maple [A] time = 0.012, size = 412, normalized size = 1.3

$$\frac{1}{(ex^2 + d)^4} \left(\frac{(35a^2e^4 + 10dabe^3 + 6acd^2e^2 + 3b^2d^2e^2 + 10bcd^3e - 93c^2d^4)x^7}{128d^4e} + \frac{(385a^2e^4 + 110dabe^3 + 66acd^2e^2 + 33c^2d^4)}{384d^3e^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^4+b*x^2+a)^2/(e*x^2+d)^5,x)

[Out] (1/128*(35*a^2*e^4+10*a*b*d*e^3+6*a*c*d^2*e^2+3*b^2*d^2*e^2+10*b*c*d^3*e-93*c^2*d^4)/d^4/e*x^7+1/384*(385*a^2*e^4+110*a*b*d*e^3+66*a*c*d^2*e^2+33*b^2*d^2*e^2-146*b*c*d^3*e-511*c^2*d^4)/d^3/e^2*x^5+1/384*(511*a^2*e^4+146*a*b*d*e^3-66*a*c*d^2*e^2-33*b^2*d^2*e^2-110*b*c*d^3*e-385*c^2*d^4)/d^2/e^3*x^3+1/128*(93*a^2*e^4-10*a*b*d*e^3-6*a*c*d^2*e^2-3*b^2*d^2*e^2-10*b*c*d^3*e-35*c^2*d^4)/e^4/d*x)/(e*x^2+d)^4+35/128/d^4/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))*a^2+5/64/d^3/e/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))*a*b+3/64/d^2/e^2/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))*a*c+3/128/d^2/e^2/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))*b^2+5/64/d/e^3/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))*b*c+35/128/e^4/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))*c^2

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^2+a)^2/(e*x^2+d)^5,x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [B] time = 1.77739, size = 2766, normalized size = 8.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^2+a)^2/(e*x^2+d)^5,x, algorithm="fricas")

[Out] [-1/768*(6*(93*c^2*d^5*e^4 - 10*b*c*d^4*e^5 - 10*a*b*d^2*e^7 - 35*a^2*d*e^8 - 3*(b^2 + 2*a*c)*d^3*e^6)*x^7 + 2*(511*c^2*d^6*e^3 + 146*b*c*d^5*e^4 - 110*a*b*d^3*e^6 - 385*a^2*d^2*e^7 - 33*(b^2 + 2*a*c)*d^4*e^5)*x^5 + 2*(385*c^2*d^7*e^2 + 110*b*c*d^6*e^3 - 146*a*b*d^4*e^5 - 511*a^2*d^3*e^6 + 33*(b^2 + 2*a*c)*d^5*e^4)*x^3 + 3*(35*c^2*d^8 + 10*b*c*d^7*e + 10*a*b*d^5*e^3 + 35*a^2*d^4*e^4 + 3*(b^2 + 2*a*c)*d^6*e^2 + (35*c^2*d^4*e^4 + 10*b*c*d^3*e^5 + 10*a*b*d*e^7 + 35*a^2*e^8 + 3*(b^2 + 2*a*c)*d^2*e^6)*x^8 + 4*(35*c^2*d^5*e^3 + 10*b*c*d^4*e^4 + 10*a*b*d^2*e^6 + 35*a^2*d*e^7 + 3*(b^2 + 2*a*c)*d^3*e^5)*x^6 + 6*(35*c^2*d^6*e^2 + 10*b*c*d^5*e^3 + 10*a*b*d^3*e^5 + 35*a^2*d^2*e^6 + 3*(b^2 + 2*a*c)*d^4*e^4)*x^4 + 4*(35*c^2*d^7*e + 10*b*c*d^6*e^2 + 10*a*b*d^4*e^4 + 35*a^2*d^3*e^5 + 3*(b^2 + 2*a*c)*d^5*e^3)*x^2)*sqrt(-d*e)*log((e*x^2 - 2*sqrt(-d*e)*x - d)/(e*x^2 + d)) + 6*(35*c^2*d^8*e + 10*b*c*d^7*e^2 + 10*a*b*d^5*e^4 - 93*a^2*d^4*e^5 + 3*(b^2 + 2*a*c)*d^6*e^3)*x)/(d^5*e^9*x^8 + 4*d^6*e^8*x^6 + 6*d^7*e^7*x^4 + 4*d^8*e^6*x^2 + d^9*e^5), -1/384*(3*(93*c^2*d^5*e^4 - 10*b*c*d^4*e^5 - 10*a*b*d^2*e^7 - 35*a^2*d*e^8 - 3*(b^2 + 2

```
*a*c)*d^3*e^6)*x^7 + (511*c^2*d^6*e^3 + 146*b*c*d^5*e^4 - 110*a*b*d^3*e^6 -
385*a^2*d^2*e^7 - 33*(b^2 + 2*a*c)*d^4*e^5)*x^5 + (385*c^2*d^7*e^2 + 110*b
*c*d^6*e^3 - 146*a*b*d^4*e^5 - 511*a^2*d^3*e^6 + 33*(b^2 + 2*a*c)*d^5*e^4)*
x^3 - 3*(35*c^2*d^8 + 10*b*c*d^7*e + 10*a*b*d^5*e^3 + 35*a^2*d^4*e^4 + 3*(b
^2 + 2*a*c)*d^6*e^2 + (35*c^2*d^4*e^4 + 10*b*c*d^3*e^5 + 10*a*b*d*e^7 + 35*
a^2*e^8 + 3*(b^2 + 2*a*c)*d^2*e^6)*x^8 + 4*(35*c^2*d^5*e^3 + 10*b*c*d^4*e^4
+ 10*a*b*d^2*e^6 + 35*a^2*d*e^7 + 3*(b^2 + 2*a*c)*d^3*e^5)*x^6 + 6*(35*c^2
*d^6*e^2 + 10*b*c*d^5*e^3 + 10*a*b*d^3*e^5 + 35*a^2*d^2*e^6 + 3*(b^2 + 2*a*
c)*d^4*e^4)*x^4 + 4*(35*c^2*d^7*e + 10*b*c*d^6*e^2 + 10*a*b*d^4*e^4 + 35*a^
2*d^3*e^5 + 3*(b^2 + 2*a*c)*d^5*e^3)*x^2)*sqrt(d*e)*arctan(sqrt(d*e)*x/d) +
3*(35*c^2*d^8*e + 10*b*c*d^7*e^2 + 10*a*b*d^5*e^4 - 93*a^2*d^4*e^5 + 3*(b^
2 + 2*a*c)*d^6*e^3)*x)/(d^5*e^9*x^8 + 4*d^6*e^8*x^6 + 6*d^7*e^7*x^4 + 4*d^8
*e^6*x^2 + d^9*e^5)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x**4+b*x**2+a)**2/(e*x**2+d)**5,x)

[Out] Timed out

Giac [A] time = 1.16862, size = 491, normalized size = 1.55

$$\frac{(35c^2d^4 + 10bcd^3e + 3b^2d^2e^2 + 6acd^2e^2 + 10abde^3 + 35a^2e^4) \arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right) e^{\left(-\frac{9}{2}\right)}}{128d^{\frac{9}{2}}} - \frac{(279c^2d^4x^7e^3 - 30bcd^3x^7e^4 + 511c^2d^4x^7e^3 - 30b^2cd^3x^7e^4 + 511c^2d^5x^5e^2 - 9b^2d^2x^7e^5 - 18a^2cd^2x^7e^5 + 146b^2cd^4x^5e^3 + 385c^2d^6x^3e - 30a^2bd^2x^7e^6 - 33b^2d^3x^5e^4 - 66a^2cd^3x^5e^4 + 110b^2cd^5x^3e^2 + 105c^2d^7x - 105a^2d^7x^7e^7 - 110a^2bd^2x^5e^5 + 33b^2d^4x^3e^3 + 66a^2cd^4x^3e^3 + 30b^2cd^6x^2e - 385a^2d^2x^5e^6 - 146a^2bd^3x^3e^4 + 9b^2d^5x^2e^2 + 18a^2cd^5x^2e^2 - 511a^2d^2x^3e^5 + 30a^2bd^4x^2e^3 - 279a^2d^3x^2e^4) e^{-4}}{(x^2e + d)^4d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^2+a)^2/(e*x^2+d)^5,x, algorithm="giac")

[Out] 1/128*(35*c^2*d^4 + 10*b*c*d^3*e + 3*b^2*d^2*e^2 + 6*a*c*d^2*e^2 + 10*a*b*d
*e^3 + 35*a^2*e^4)*arctan(x*e^(1/2)/sqrt(d))*e^(-9/2)/d^(9/2) - 1/384*(279*
c^2*d^4*x^7*e^3 - 30*b*c*d^3*x^7*e^4 + 511*c^2*d^5*x^5*e^2 - 9*b^2*d^2*x^7*
e^5 - 18*a*c*d^2*x^7*e^5 + 146*b*c*d^4*x^5*e^3 + 385*c^2*d^6*x^3*e - 30*a*b
*d*x^7*e^6 - 33*b^2*d^3*x^5*e^4 - 66*a*c*d^3*x^5*e^4 + 110*b*c*d^5*x^3*e^2
+ 105*c^2*d^7*x - 105*a^2*x^7*e^7 - 110*a*b*d^2*x^5*e^5 + 33*b^2*d^4*x^3*e^
3 + 66*a*c*d^4*x^3*e^3 + 30*b*c*d^6*x^2*e - 385*a^2*d^2*x^5*e^6 - 146*a*b*d^3*x
^3*e^4 + 9*b^2*d^5*x^2*e^2 + 18*a*c*d^5*x^2*e^2 - 511*a^2*d^2*x^3*e^5 + 30*a*b
d^4*x^2*e^3 - 279*a^2*d^3*x^2*e^4)*e^(-4)/((x^2*e + d)^4*d^4)

$$3.261 \quad \int \frac{a+bx^2+cx^4}{(d+ex^2)^2} dx$$

Optimal. Leaf size=83

$$-\frac{\tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)(3cd^2 - e(ae + bd))}{2d^{3/2}e^{5/2}} + \frac{x\left(a + \frac{d(cd-be)}{e^2}\right)}{2d(d+ex^2)} + \frac{cx}{e^2}$$

[Out] (c*x)/e^2 + ((a + (d*(c*d - b*e))/e^2)*x)/(2*d*(d + e*x^2)) - ((3*c*d^2 - e*(b*d + a*e))*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(3/2)*e^(5/2))

Rubi [A] time = 0.0933651, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {1157, 388, 205}

$$-\frac{\tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)(3cd^2 - e(ae + bd))}{2d^{3/2}e^{5/2}} + \frac{x\left(a + \frac{d(cd-be)}{e^2}\right)}{2d(d+ex^2)} + \frac{cx}{e^2}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x^2 + c*x^4)/(d + e*x^2)^2,x]

[Out] (c*x)/e^2 + ((a + (d*(c*d - b*e))/e^2)*x)/(2*d*(d + e*x^2)) - ((3*c*d^2 - e*(b*d + a*e))*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(3/2)*e^(5/2))

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] := With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2,
x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x],
x, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned} \int \frac{a + bx^2 + cx^4}{(d + ex^2)^2} dx &= \frac{\left(a + \frac{d(cd-be)}{e^2}\right)x}{2d(d + ex^2)} - \int \frac{\frac{cd^2 - e(bd+ae)}{e^2} - \frac{2cdx^2}{e}}{d+ex^2} dx \\ &= \frac{cx}{e^2} + \frac{\left(a + \frac{d(cd-be)}{e^2}\right)x}{2d(d + ex^2)} - \frac{(3cd^2 - e(bd + ae)) \int \frac{1}{d+ex^2} dx}{2de^2} \\ &= \frac{cx}{e^2} + \frac{\left(a + \frac{d(cd-be)}{e^2}\right)x}{2d(d + ex^2)} - \frac{(3cd^2 - e(bd + ae)) \tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)}{2d^{3/2}e^{5/2}} \end{aligned}$$

Mathematica [A] time = 0.0472368, size = 88, normalized size = 1.06

$$\frac{x(ae^2 - bde + cd^2)}{2de^2(d + ex^2)} - \frac{\tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)(-ae^2 - bde + 3cd^2)}{2d^{3/2}e^{5/2}} + \frac{cx}{e^2}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x^2 + c*x^4)/(d + e*x^2)^2,x]

[Out] (c*x)/e^2 + ((c*d^2 - b*d*e + a*e^2)*x)/(2*d*e^2*(d + e*x^2)) - ((3*c*d^2 - b*d*e - a*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(3/2)*e^(5/2))

Maple [A] time = 0., size = 118, normalized size = 1.4

$$\frac{cx}{e^2} + \frac{xa}{2d(ex^2 + d)} - \frac{bx}{2e(ex^2 + d)} + \frac{dxc}{2e^2(ex^2 + d)} + \frac{a}{2d} \arctan\left(ex \frac{1}{\sqrt{de}}\right) \frac{1}{\sqrt{de}} + \frac{b}{2e} \arctan\left(ex \frac{1}{\sqrt{de}}\right) \frac{1}{\sqrt{de}} - \frac{3cd}{2e^2} \arctan\left(ex \frac{1}{\sqrt{de}}\right) \frac{1}{\sqrt{de}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^4+b*x^2+a)/(e*x^2+d)^2,x)

[Out] c*x/e^2+1/2/d*x/(e*x^2+d)*a-1/2/e*x/(e*x^2+d)*b+1/2/e^2*d*x/(e*x^2+d)*c+1/2/d/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))*a+1/2/e/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))*b-3/2/e^2*d/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))*c

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^2,x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [A] time = 1.65278, size = 541, normalized size = 6.52

$$\left[\frac{4cd^2e^2x^3 + (3cd^3 - bd^2e - ade^2 + (3cd^2e - bde^2 - ae^3)x^2)\sqrt{-de} \log\left(\frac{ex^2 - 2\sqrt{-dex-d}}{ex^2+d}\right) + 2(3cd^3e - bd^2e^2 + ade^3)x}{4(d^2e^4x^2 + d^3e^3)}, \frac{2cd^2e^2}{4(d^2e^4x^2 + d^3e^3)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^2,x, algorithm="fricas")

[Out] [1/4*(4*c*d^2*e^2*x^3 + (3*c*d^3 - b*d^2*e - a*d*e^2 + (3*c*d^2*e - b*d*e^2 - a*e^3)*x^2)*sqrt(-d*e)*log((e*x^2 - 2*sqrt(-d*e)*x - d)/(e*x^2 + d)) + 2*(3*c*d^3*e - b*d^2*e^2 + a*d*e^3)*x)/(d^2*e^4*x^2 + d^3*e^3), 1/2*(2*c*d^2*e^2*x^3 - (3*c*d^3 - b*d^2*e - a*d*e^2 + (3*c*d^2*e - b*d*e^2 - a*e^3)*x^2)*sqrt(d*e)*arctan(sqrt(d*e)*x/d) + (3*c*d^3*e - b*d^2*e^2 + a*d*e^3)*x)/(d^2*e^4*x^2 + d^3*e^3)]

Sympy [B] time = 1.03622, size = 153, normalized size = 1.84

$$\frac{cx}{e^2} + \frac{x(ae^2 - bde + cd^2)}{2d^2e^2 + 2de^3x^2} - \frac{\sqrt{-\frac{1}{d^3e^5}}(ae^2 + bde - 3cd^2) \log\left(-d^2e^2\sqrt{-\frac{1}{d^3e^5}} + x\right)}{4} + \frac{\sqrt{-\frac{1}{d^3e^5}}(ae^2 + bde - 3cd^2) \log\left(d^2e^2\sqrt{-\frac{1}{d^3e^5}} + x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x**4+b*x**2+a)/(e*x**2+d)**2,x)

[Out] c*x/e**2 + x*(a*e**2 - b*d*e + c*d**2)/(2*d**2*e**2 + 2*d*e**3*x**2) - sqrt(-1/(d**3*e**5))*(a*e**2 + b*d*e - 3*c*d**2)*log(-d**2*e**2*sqrt(-1/(d**3*e**5)) + x)/4 + sqrt(-1/(d**3*e**5))*(a*e**2 + b*d*e - 3*c*d**2)*log(d**2*e**2*sqrt(-1/(d**3*e**5)) + x)/4

Giac [A] time = 1.15544, size = 101, normalized size = 1.22

$$cxe^{(-2)} - \frac{(3cd^2 - bde - ae^2) \arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right) e^{\left(-\frac{5}{2}\right)}}{2d^{\frac{3}{2}}} + \frac{(cd^2x - bdx + axe^2)e^{(-2)}}{2(x^2e + d)d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^2,x, algorithm="giac")

[Out] c*x*e^(-2) - 1/2*(3*c*d^2 - b*d*e - a*e^2)*arctan(x*e^(1/2)/sqrt(d))*e^(-5/2)/d^(3/2) + 1/2*(c*d^2*x - b*d*x*e + a*x*e^2)*e^(-2)/((x^2*e + d)*d)

$$3.262 \quad \int \frac{a+x^2(b+cx^2)}{(d+ex^2)^2} dx$$

Optimal. Leaf size=83

$$-\frac{\tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)(3cd^2 - e(ae + bd))}{2d^{3/2}e^{5/2}} + \frac{x\left(a + \frac{d(cd-be)}{e^2}\right)}{2d(d+ex^2)} + \frac{cx}{e^2}$$

[Out] (c*x)/e^2 + ((a + (d*(c*d - b*e))/e^2)*x)/(2*d*(d + e*x^2)) - ((3*c*d^2 - e*(b*d + a*e))*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(3/2)*e^(5/2))

Rubi [A] time = 0.084017, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.13$, Rules used = {1814, 388, 205}

$$-\frac{\tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)(3cd^2 - e(ae + bd))}{2d^{3/2}e^{5/2}} + \frac{x\left(a + \frac{d(cd-be)}{e^2}\right)}{2d(d+ex^2)} + \frac{cx}{e^2}$$

Antiderivative was successfully verified.

[In] Int[(a + x^2*(b + c*x^2))/(d + e*x^2)^2,x]

[Out] (c*x)/e^2 + ((a + (d*(c*d - b*e))/e^2)*x)/(2*d*(d + e*x^2)) - ((3*c*d^2 - e*(b*d + a*e))*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(3/2)*e^(5/2))

Rule 1814

Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]

Rule 388

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rubi steps

$$\begin{aligned} \int \frac{a + x^2(b + cx^2)}{(d + ex^2)^2} dx &= \frac{\left(a + \frac{d(cd-be)}{e^2}\right)x}{2d(d + ex^2)} - \int \frac{\frac{cd^2 - e(bd+ae)}{e^2} - \frac{2cdx^2}{e}}{d+ex^2} dx \\ &= \frac{cx}{e^2} + \frac{\left(a + \frac{d(cd-be)}{e^2}\right)x}{2d(d + ex^2)} - \frac{(3cd^2 - e(bd + ae)) \int \frac{1}{d+ex^2} dx}{2de^2} \\ &= \frac{cx}{e^2} + \frac{\left(a + \frac{d(cd-be)}{e^2}\right)x}{2d(d + ex^2)} - \frac{(3cd^2 - e(bd + ae)) \tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)}{2d^{3/2}e^{5/2}} \end{aligned}$$

Mathematica [A] time = 0.0168342, size = 88, normalized size = 1.06

$$\frac{x(ae^2 - bde + cd^2)}{2de^2(d + ex^2)} - \frac{\tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)(-ae^2 - bde + 3cd^2)}{2d^{3/2}e^{5/2}} + \frac{cx}{e^2}$$

Antiderivative was successfully verified.

[In] Integrate[(a + x^2*(b + c*x^2))/(d + e*x^2)^2,x]

[Out] (c*x)/e^2 + ((c*d^2 - b*d*e + a*e^2)*x)/(2*d*e^2*(d + e*x^2)) - ((3*c*d^2 - b*d*e - a*e^2)*ArcTan[Sqrt[e]*x/Sqrt[d]])/(2*d^(3/2)*e^(5/2))

Maple [A] time = 0.007, size = 118, normalized size = 1.4

$$\frac{cx}{e^2} + \frac{xa}{2d(ex^2 + d)} - \frac{bx}{2e(ex^2 + d)} + \frac{dxc}{2e^2(ex^2 + d)} + \frac{a}{2d} \arctan\left(ex \frac{1}{\sqrt{de}}\right) \frac{1}{\sqrt{de}} + \frac{b}{2e} \arctan\left(ex \frac{1}{\sqrt{de}}\right) \frac{1}{\sqrt{de}} - \frac{3cd}{2e^2} \arctan\left(ex \frac{1}{\sqrt{de}}\right) \frac{1}{\sqrt{de}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+x^2*(c*x^2+b))/(e*x^2+d)^2,x)

[Out] c*x/e^2+1/2/d*x/(e*x^2+d)*a-1/2/e*x/(e*x^2+d)*b+1/2/e^2*d*x/(e*x^2+d)*c+1/2/d/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))*a+1/2/e/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))*b-3/2/e^2*d/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))*c

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+x^2*(c*x^2+b))/(e*x^2+d)^2,x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [A] time = 1.58515, size = 541, normalized size = 6.52

$$\left[\frac{4cd^2e^2x^3 + (3cd^3 - bd^2e - ade^2 + (3cd^2e - bde^2 - ae^3)x^2)\sqrt{-de} \log\left(\frac{ex^2 - 2\sqrt{-dex} - d}{ex^2 + d}\right) + 2(3cd^3e - bd^2e^2 + ade^3)x}{4(d^2e^4x^2 + d^3e^3)}, \dots \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+x^2*(c*x^2+b))/(e*x^2+d)^2,x, algorithm="fricas")

[Out] [1/4*(4*c*d^2*e^2*x^3 + (3*c*d^3 - b*d^2*e - a*d*e^2 + (3*c*d^2*e - b*d*e^2 - a*e^3)*x^2)*sqrt(-d*e)*log((e*x^2 - 2*sqrt(-d*e)*x - d)/(e*x^2 + d)) + 2*(3*c*d^3*e - b*d^2*e^2 + a*d*e^3)*x)/(d^2*e^4*x^2 + d^3*e^3), 1/2*(2*c*d^2*e^2*x^3 - (3*c*d^3 - b*d^2*e - a*d*e^2 + (3*c*d^2*e - b*d*e^2 - a*e^3)*x^2)*sqrt(d*e)*arctan(sqrt(d*e)*x/d) + (3*c*d^3*e - b*d^2*e^2 + a*d*e^3)*x)/(d^2*e^4*x^2 + d^3*e^3)]

Sympy [B] time = 1.06997, size = 153, normalized size = 1.84

$$\frac{cx}{e^2} + \frac{x(ae^2 - bde + cd^2)}{2d^2e^2 + 2de^3x^2} - \frac{\sqrt{-\frac{1}{d^3e^5}}(ae^2 + bde - 3cd^2) \log\left(-d^2e^2\sqrt{-\frac{1}{d^3e^5}} + x\right)}{4} + \frac{\sqrt{-\frac{1}{d^3e^5}}(ae^2 + bde - 3cd^2) \log\left(d^2e^2\sqrt{-\frac{1}{d^3e^5}} + x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+x**2*(c*x**2+b))/(e*x**2+d)**2,x)

[Out] c*x/e**2 + x*(a*e**2 - b*d*e + c*d**2)/(2*d**2*e**2 + 2*d*e**3*x**2) - sqrt(-1/(d**3*e**5))*(a*e**2 + b*d*e - 3*c*d**2)*log(-d**2*e**2*sqrt(-1/(d**3*e**5)) + x)/4 + sqrt(-1/(d**3*e**5))*(a*e**2 + b*d*e - 3*c*d**2)*log(d**2*e**2*sqrt(-1/(d**3*e**5)) + x)/4

Giac [A] time = 1.19264, size = 101, normalized size = 1.22

$$cxe^{(-2)} - \frac{(3cd^2 - bde - ae^2) \arctan\left(\frac{xe^{1/2}}{\sqrt{d}}\right) e^{(-5/2)}}{2d^{3/2}} + \frac{(cd^2x - bdx + axe^2)e^{(-2)}}{2(x^2e + d)d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+x^2*(c*x^2+b))/(e*x^2+d)^2,x, algorithm="giac")

[Out] c*x*e^(-2) - 1/2*(3*c*d^2 - b*d*e - a*e^2)*arctan(x*e^(1/2)/sqrt(d))*e^(-5/2)/d^(3/2) + 1/2*(c*d^2*x - b*d*x*e + a*x*e^2)*e^(-2)/((x^2*e + d)*d)

$$3.263 \quad \int \frac{(d+ex^2)^4}{a+bx^2+cx^4} dx$$

Optimal. Leaf size=459

$$\frac{\left(\frac{2c^2e^2(a^2e^2+6abde+3b^2d^2)-4b^2ce^3(ae+bd)-4c^3d^2e(3ae+bd)+b^4e^4+2c^4d^4}{\sqrt{b^2-4ac}} + e(2cd - be)(-2ce(ae + bd) + b^2e^2 + 2c^2d^2) \right) \tan^{-1} \left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b-\sqrt{b^2-4ac}}} \right)}{\sqrt{2}c^{7/2}\sqrt{b-\sqrt{b^2-4ac}}}$$

[Out] (e^2*(6*c^2*d^2 + b^2*e^2 - c*e*(4*b*d + a*e))*x)/c^3 + (e^3*(4*c*d - b*e)*x^3)/(3*c^2) + (e^4*x^5)/(5*c) + ((e*(2*c*d - b*e)*(2*c^2*d^2 + b^2*e^2 - 2*c*e*(b*d + a*e)) + (2*c^4*d^4 + b^4*e^4 - 4*b^2*c*e^3*(b*d + a*e) - 4*c^3*d^2*e*(b*d + 3*a*e) + 2*c^2*e^2*(3*b^2*d^2 + 6*a*b*d*e + a^2*e^2))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]])/(Sqrt[2]*c^(7/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((e*(2*c*d - b*e)*(2*c^2*d^2 + b^2*e^2 - 2*c*e*(b*d + a*e)) - (2*c^4*d^4 + b^4*e^4 - 4*b^2*c*e^3*(b*d + a*e) - 4*c^3*d^2*e*(b*d + 3*a*e) + 2*c^2*e^2*(3*b^2*d^2 + 6*a*b*d*e + a^2*e^2))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]])/(Sqrt[2]*c^(7/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])

Rubi [A] time = 1.537, antiderivative size = 459, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1170, 1166, 205}

$$\frac{\left(\frac{2c^2e^2(a^2e^2+6abde+3b^2d^2)-4b^2ce^3(ae+bd)-4c^3d^2e(3ae+bd)+b^4e^4+2c^4d^4}{\sqrt{b^2-4ac}} + e(2cd - be)(-2ce(ae + bd) + b^2e^2 + 2c^2d^2) \right) \tan^{-1} \left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b-\sqrt{b^2-4ac}}} \right)}{\sqrt{2}c^{7/2}\sqrt{b-\sqrt{b^2-4ac}}}$$

Antiderivative was successfully verified.

[In] Int[(d + e*x^2)^4/(a + b*x^2 + c*x^4), x]

[Out] (e^2*(6*c^2*d^2 + b^2*e^2 - c*e*(4*b*d + a*e))*x)/c^3 + (e^3*(4*c*d - b*e)*x^3)/(3*c^2) + (e^4*x^5)/(5*c) + ((e*(2*c*d - b*e)*(2*c^2*d^2 + b^2*e^2 - 2*c*e*(b*d + a*e)) + (2*c^4*d^4 + b^4*e^4 - 4*b^2*c*e^3*(b*d + a*e) - 4*c^3*d^2*e*(b*d + 3*a*e) + 2*c^2*e^2*(3*b^2*d^2 + 6*a*b*d*e + a^2*e^2))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]])/(Sqrt[2]*c^(7/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((e*(2*c*d - b*e)*(2*c^2*d^2 + b^2*e^2 - 2*c*e*(b*d + a*e)) - (2*c^4*d^4 + b^4*e^4 - 4*b^2*c*e^3*(b*d + a*e) - 4*c^3*d^2*e*(b*d + 3*a*e) + 2*c^2*e^2*(3*b^2*d^2 + 6*a*b*d*e + a^2*e^2))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]])/(Sqrt[2]*c^(7/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])

Rule 1170

Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] := Int[ExpandIntegrand[(d + e*x^2)^q/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[q]

Rule 1166

Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2

- q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rubi steps

$$\begin{aligned} \int \frac{(d+ex^2)^4}{a+bx^2+cx^4} dx &= \int \left(\frac{e^2(6c^2d^2+b^2e^2-ce(4bd+ae))}{c^3} + \frac{e^3(4cd-be)x^2}{c^2} + \frac{e^4x^4}{c} + \frac{c^3d^4-6ac^2d^2e^2-ab^2e^4+ace^3}{a+bx^2+cx^4} \right) dx \\ &= \frac{e^2(6c^2d^2+b^2e^2-ce(4bd+ae))x}{c^3} + \frac{e^3(4cd-be)x^3}{3c^2} + \frac{e^4x^5}{5c} + \frac{\int \frac{c^3d^4-6ac^2d^2e^2-ab^2e^4+ace^3(4bd+ae)+e^2(a+bx^2+cx^4)}{a+bx^2+cx^4} dx}{c^3} \\ &= \frac{e^2(6c^2d^2+b^2e^2-ce(4bd+ae))x}{c^3} + \frac{e^3(4cd-be)x^3}{3c^2} + \frac{e^4x^5}{5c} + \frac{\left(e(2cd-be)(2c^2d^2+b^2e^2-2ce(4bd+ae)) \right)}{c^3} \\ &= \frac{e^2(6c^2d^2+b^2e^2-ce(4bd+ae))x}{c^3} + \frac{e^3(4cd-be)x^3}{3c^2} + \frac{e^4x^5}{5c} + \frac{\left(e(2cd-be)(2c^2d^2+b^2e^2-2ce(4bd+ae)) \right)}{c^3} \end{aligned}$$

Mathematica [A] time = 0.710884, size = 570, normalized size = 1.24

$$\frac{e^2x(-ce(ae+4bd)+b^2e^2+6c^2d^2)}{c^3} + \frac{\left(2c^2e^2(-3bd(d\sqrt{b^2-4ac}-2ae)) + ae(ae-2d\sqrt{b^2-4ac}) + 3b^2d^2 \right) + 4c^3d^2e(d\sqrt{b^2-4ac}-2ae)}{c^3}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e*x^2)^4/(a + b*x^2 + c*x^4), x]

[Out] (e^2*(6*c^2*d^2 + b^2*e^2 - c*e*(4*b*d + a*e))*x)/c^3 + (e^3*(4*c*d - b*e)*x^3)/(3*c^2) + (e^4*x^5)/(5*c) + ((2*c^4*d^4 + b^3*(b - Sqrt[b^2 - 4*a*c]))*e^4 + 4*c^3*d^2*e*(-(b*d) + Sqrt[b^2 - 4*a*c]*d - 3*a*e) + 2*b*c*e^3*(-2*b^2*d + 2*b*Sqrt[b^2 - 4*a*c]*d - 2*a*b*e + a*Sqrt[b^2 - 4*a*c]*e) + 2*c^2*e^2*(3*b^2*d^2 - 3*b*d*(Sqrt[b^2 - 4*a*c]*d - 2*a*e) + a*e*(-2*Sqrt[b^2 - 4*a*c]*d + a*e)))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]])/(Sqrt[2]*c^(7/2)*Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - ((2*c^4*d^4 + b^3*(b + Sqrt[b^2 - 4*a*c]))*e^4 - 4*c^3*d^2*e*(b*d + Sqrt[b^2 - 4*a*c]*d + 3*a*e) - 2*b*c*e^3*(2*b^2*d + a*Sqrt[b^2 - 4*a*c]*e + 2*b*(Sqrt[b^2 - 4*a*c]*d + a*e)) + 2*c^2*e^2*(3*b^2*d^2 + a*e*(2*Sqrt[b^2 - 4*a*c]*d + a*e) + 3*b*d*(Sqrt[b^2 - 4*a*c]*d + 2*a*e)))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]])/(Sqrt[2]*c^(7/2)*Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]])

Maple [B] time = 0.062, size = 1888, normalized size = 4.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((e*x^2+d)^4/(c*x^4+b*x^2+a),x)$

[Out] $\frac{1}{5}e^4x^5/c - \frac{1}{3}e^4/c^2x^3b - e^4/c^2ax - 6/c/(-4ac+b^2)^{1/2}2^{1/2}/$
 $((-4ac+b^2)^{1/2}-b)c^{1/2}\text{arctanh}(cx^2^{1/2}/(((-4ac+b^2)^{1/2}-b)$
 $)c^{1/2})abde^3 - 6/c/(-4ac+b^2)^{1/2}2^{1/2}/((b+(-4ac+b^2)^{1/2})$
 $)c^{1/2}\text{arctan}(cx^2^{1/2}/((b+(-4ac+b^2)^{1/2})c^{1/2}))abde^3 - 1$
 $/2/c^3/(-4ac+b^2)^{1/2}2^{1/2}/(((-4ac+b^2)^{1/2}-b)c^{1/2}\text{arctanh}($
 $cx^2^{1/2}/(((-4ac+b^2)^{1/2}-b)c^{1/2}))b^4e^4 + 2/c^2^{1/2}/(((-4ac$
 $+b^2)^{1/2}-b)c^{1/2}\text{arctanh}(cx^2^{1/2}/(((-4ac+b^2)^{1/2}-b)c^{1/2}$
 $))a^2de^3 - 1/c/(-4ac+b^2)^{1/2}2^{1/2}/((b+(-4ac+b^2)^{1/2})c^{1/2}$
 $\text{arctan}(cx^2^{1/2}/((b+(-4ac+b^2)^{1/2})c^{1/2}))a^2e^4 - 1/2/c^3/(-4ac$
 $+b^2)^{1/2}2^{1/2}/((b+(-4ac+b^2)^{1/2})c^{1/2}\text{arctan}(cx^2^{1/2}/(($
 $b+(-4ac+b^2)^{1/2})c^{1/2}))b^4e^4 + 3/c^2^{1/2}/(((-4ac+b^2)^{1/2}-b)$
 $)c^{1/2}\text{arctanh}(cx^2^{1/2}/(((-4ac+b^2)^{1/2}-b)c^{1/2}))b^2d^2e^2 - 1$
 $/c/(-4ac+b^2)^{1/2}2^{1/2}/(((-4ac+b^2)^{1/2}-b)c^{1/2}\text{arctanh}(cx^*$
 $2^{1/2}/(((-4ac+b^2)^{1/2}-b)c^{1/2}))a^2e^4 - 2/c^22^{1/2}/(((-4ac+b$
 $^2)^{1/2}-b)c^{1/2}\text{arctanh}(cx^2^{1/2}/(((-4ac+b^2)^{1/2}-b)c^{1/2}))$
 $b^2de^3 - 1/c^22^{1/2}/(((-4ac+b^2)^{1/2}-b)c^{1/2}\text{arctanh}(cx^2^{1/2}/$
 $(((-4ac+b^2)^{1/2}-b)c^{1/2}))ab^4e^4 + 1/c^22^{1/2}/((b+(-4ac+b^2)$
 $^{1/2})c^{1/2}\text{arctan}(cx^2^{1/2}/((b+(-4ac+b^2)^{1/2})c^{1/2}))ab^4e^4 - 2/c^2^{1/2}/$
 $((b+(-4ac+b^2)^{1/2})c^{1/2}\text{arctan}(cx^2^{1/2}/((b+(-4ac+b^2)^{1/2})c^{1/2}))$
 $a^2de^3 - c/(-4ac+b^2)^{1/2}2^{1/2}/(((-4ac+b^2)^{1/2}-b)c^{1/2}\text{arctanh}(cx^*$
 $2^{1/2}/(((-4ac+b^2)^{1/2}-b)c^{1/2}))d^4 + 1/2/c^32^{1/2}/(((-4ac+b^2)^{1/2}-b)c^{1/2}$
 $\text{arctanh}(cx^2^{1/2}/(((-4ac+b^2)^{1/2}-b)c^{1/2}))b^3e^4 - 1/2/c^32^{1/2}/((b+(-4ac+b^2)$
 $^{1/2})c^{1/2}\text{arctan}(cx^2^{1/2}/((b+(-4ac+b^2)^{1/2})c^{1/2}))b^3e^4 -$
 $c/(-4ac+b^2)^{1/2}2^{1/2}/((b+(-4ac+b^2)^{1/2})c^{1/2}\text{arctan}(cx^2^{1/2}/$
 $((b+(-4ac+b^2)^{1/2})c^{1/2}))d^4 + 4/3de^3x^3/c + e^4/c^3b^2x + 6$
 $e^2/cd^2x + 2/c^2/(-4ac+b^2)^{1/2}2^{1/2}/(((-4ac+b^2)^{1/2}-b)c^{1/2}$
 $\text{arctanh}(cx^2^{1/2}/(((-4ac+b^2)^{1/2}-b)c^{1/2}))ab^2e^4 + 2/c^2/(-4ac+b^2)$
 $^{1/2}2^{1/2}/(((-4ac+b^2)^{1/2}-b)c^{1/2}\text{arctanh}(cx^2^{1/2}/(((-4ac+b^2)$
 $^{1/2}-b)c^{1/2}))b^3de^3 + 2/c^2/(-4ac+b^2)^{1/2}2^{1/2}/((b+(-4ac+b^2)$
 $^{1/2})c^{1/2}\text{arctan}(cx^2^{1/2}/((b+(-4ac+b^2)^{1/2})c^{1/2}))b^3d$
 $e^3 - 3/c/(-4ac+b^2)^{1/2}2^{1/2}/((b+(-4ac+b^2)^{1/2})c^{1/2}\text{arctan}$
 $(cx^2^{1/2}/((b+(-4ac+b^2)^{1/2})c^{1/2}))b^2d^2e^2 - 3/c/(-4ac+b^2)$
 $^{1/2}2^{1/2}/(((-4ac+b^2)^{1/2}-b)c^{1/2}\text{arctanh}(cx^2^{1/2}/(((-4ac+b^2)$
 $^{1/2}-b)c^{1/2}))b^2d^2e^2 + 2^{1/2}/((b+(-4ac+b^2)^{1/2})c^{1/2}\text{arctan}(cx^*$
 $2^{1/2}/((b+(-4ac+b^2)^{1/2})c^{1/2}))e^2d^3 - 2^{1/2}/(((-4ac+b^2)^{1/2}-b)c^{1/2}$
 $\text{arctanh}(cx^2^{1/2}/(((-4ac+b^2)^{1/2}-b)c^{1/2}))e^2d^3 - 4e^3/c^2b^2dx + 2/c^22^{1/2}/((b+(-4ac+b^2)$
 $^{1/2})c^{1/2}\text{arctan}(cx^2^{1/2}/((b+(-4ac+b^2)^{1/2})c^{1/2}))b^2de^3 - 3/c^*$
 $2^{1/2}/((b+(-4ac+b^2)^{1/2})c^{1/2}\text{arctan}(cx^2^{1/2}/((b+(-4ac+b^2)$
 $^{1/2})c^{1/2}))b^2d^2e^2 + 6/(-4ac+b^2)^{1/2}2^{1/2}/((b+(-4ac+b^2)$
 $^{1/2})c^{1/2}\text{arctan}(cx^2^{1/2}/((b+(-4ac+b^2)^{1/2})c^{1/2}))a^2d^2e^2 + 2/(-4ac+b^2)$
 $^{1/2}2^{1/2}/((b+(-4ac+b^2)^{1/2})c^{1/2}\text{arctan}(cx^2^{1/2}/((b+(-4ac+b^2)$
 $^{1/2})c^{1/2}))e^2d^3b + 6/(-4ac+b^2)^{1/2}2^{1/2}/(((-4ac+b^2)^{1/2}-b)c^{1/2}$
 $\text{arctanh}(cx^2^{1/2}/(((-4ac+b^2)^{1/2}-b)c^{1/2}))a^2d^2e^2 + 2/(-4ac+b^2)$
 $^{1/2}2^{1/2}/(((-4ac+b^2)^{1/2}-b)c^{1/2}\text{arctanh}(cx^2^{1/2}/(((-4ac+b^2)^{1/2}-b)c^{1/2}))e^2d^3*$
 b

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)^4/(c*x^4+b*x^2+a),x, algorithm="maxima")

[Out] Timed out

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)^4/(c*x^4+b*x^2+a),x, algorithm="fricas")

[Out] Timed out

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x**2+d)**4/(c*x**4+b*x**2+a),x)

[Out] Timed out

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)^4/(c*x^4+b*x^2+a),x, algorithm="giac")

[Out] Exception raised: NotImplementedError

$$3.264 \quad \int \frac{(d+ex^2)^3}{a+bx^2+cx^4} dx$$

Optimal. Leaf size=316

$$\frac{\left(e(-ce(ae+3bd)+b^2e^2+3c^2d^2) + \frac{(2cd-be)(-ce(3ae+bd)+b^2e^2+c^2d^2)}{\sqrt{b^2-4ac}} \right) \tan^{-1} \left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b-\sqrt{b^2-4ac}}} \right) + \frac{\left(e(-ce(ae+3bd)+b^2e^2+3c^2d^2) \right)}{\sqrt{2}c^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}}{\sqrt{2}c^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}$$

```
[Out] (e^2*(3*c*d - b*e)*x)/c^2 + (e^3*x^3)/(3*c) + ((e*(3*c^2*d^2 + b^2*e^2 - c*
e*(3*b*d + a*e)) + ((2*c*d - b*e)*(c^2*d^2 + b^2*e^2 - c*e*(b*d + 3*a*e)))/
Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])
/(Sqrt[2]*c^(5/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((e*(3*c^2*d^2 + b^2*e^2 -
c*e*(3*b*d + a*e)) - ((2*c*d - b*e)*(c^2*d^2 + b^2*e^2 - c*e*(b*d + 3*a*e)
))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]
]])/(Sqrt[2]*c^(5/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rubi [A] time = 0.785581, antiderivative size = 316, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1170, 1166, 205}

$$\frac{\left(e(-ce(ae+3bd)+b^2e^2+3c^2d^2) + \frac{(2cd-be)(-ce(3ae+bd)+b^2e^2+c^2d^2)}{\sqrt{b^2-4ac}} \right) \tan^{-1} \left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b-\sqrt{b^2-4ac}}} \right) + \frac{\left(e(-ce(ae+3bd)+b^2e^2+3c^2d^2) \right)}{\sqrt{2}c^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}}{\sqrt{2}c^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^3/(a + b*x^2 + c*x^4), x]
```

```
[Out] (e^2*(3*c*d - b*e)*x)/c^2 + (e^3*x^3)/(3*c) + ((e*(3*c^2*d^2 + b^2*e^2 - c*
e*(3*b*d + a*e)) + ((2*c*d - b*e)*(c^2*d^2 + b^2*e^2 - c*e*(b*d + 3*a*e)))/
Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])
/(Sqrt[2]*c^(5/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((e*(3*c^2*d^2 + b^2*e^2 -
c*e*(3*b*d + a*e)) - ((2*c*d - b*e)*(c^2*d^2 + b^2*e^2 - c*e*(b*d + 3*a*e)
))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]
]])/(Sqrt[2]*c^(5/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 1170

```
Int[((d_) + (e_)*(x_)^2)^(q_)/((a_) + (b_)*(x_)^2 + (c_)*(x_)^4), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x^2)^q/(a + b*x^2 + c*x^4), x], x] /; FreeQ[
{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
0] && IntegerQ[q]
```

Rule 1166

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (b_)*(x_)^2 + (c_)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
- q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[
c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rubi steps

$$\int \frac{(d + ex^2)^3}{a + bx^2 + cx^4} dx = \int \left(\frac{e^2(3cd - be)}{c^2} + \frac{e^3x^2}{c} + \frac{c^2d^3 - 3acde^2 + abe^3 + e(3c^2d^2 + b^2e^2 - ce(3bd + ae))x^2}{c^2(a + bx^2 + cx^4)} \right) dx$$

$$= \frac{e^2(3cd - be)x}{c^2} + \frac{e^3x^3}{3c} + \frac{\int \frac{c^2d^3 - 3acde^2 + abe^3 + e(3c^2d^2 + b^2e^2 - ce(3bd + ae))x^2}{a + bx^2 + cx^4} dx}{c^2}$$

$$= \frac{e^2(3cd - be)x}{c^2} + \frac{e^3x^3}{3c} + \frac{\left(e(3c^2d^2 + b^2e^2 - ce(3bd + ae)) - \frac{(2cd - be)(c^2d^2 + b^2e^2 - ce(bd + 3ae))}{\sqrt{b^2 - 4ac}} \right) \int \frac{\frac{b}{2} + \frac{1}{2}\sqrt{b^2 - 4ac}}{a + bx^2 + cx^4} dx}{2c^2}$$

$$= \frac{e^2(3cd - be)x}{c^2} + \frac{e^3x^3}{3c} + \frac{\left(e(3c^2d^2 + b^2e^2 - ce(3bd + ae)) + \frac{(2cd - be)(c^2d^2 + b^2e^2 - ce(bd + 3ae))}{\sqrt{b^2 - 4ac}} \right) \tan^{-1} \left(\frac{\frac{b}{2} + \frac{1}{2}\sqrt{b^2 - 4ac}}{\sqrt{a + bx^2 + cx^4}} \right)}{\sqrt{2c^5/2} \sqrt{b - \sqrt{b^2 - 4ac}}}$$

Mathematica [A] time = 0.574659, size = 402, normalized size = 1.27

$$\frac{3\sqrt{2}(3c^2de(d\sqrt{b^2-4ac}-2ae-bd)+ce^2(-3bd\sqrt{b^2-4ac}-ae\sqrt{b^2-4ac}+3abe+3b^2d)+b^2e^3(\sqrt{b^2-4ac}-b)+2c^3d^3)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{3\sqrt{2}(3c^2de(d\sqrt{b^2-4ac}+2ae+bd)+ce^2(-3bd\sqrt{b^2-4ac}+ae\sqrt{b^2-4ac}-3abe-3b^2d)-b^2e^3(\sqrt{b^2-4ac}+b)+2c^3d^3)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{6c^{5/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e*x^2)^3/(a + b*x^2 + c*x^4), x]

[Out] (6*Sqrt[c]*e^2*(3*c*d - b*e)*x + 2*c^(3/2)*e^3*x^3 + (3*Sqrt[2]*(2*c^3*d^3 + b^2*(-b + Sqrt[b^2 - 4*a*c]))*e^3 + 3*c^2*d*e*(-(b*d) + Sqrt[b^2 - 4*a*c]*d - 2*a*e) + c*e^2*(3*b^2*d - 3*b*Sqrt[b^2 - 4*a*c]*d + 3*a*b*e - a*Sqrt[b^2 - 4*a*c]*e))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]]/(Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (3*Sqrt[2]*(-2*c^3*d^3 + b^2*(b + Sqrt[b^2 - 4*a*c])*e^3 + 3*c^2*d*e*(b*d + Sqrt[b^2 - 4*a*c]*d + 2*a*e) - c*e^2*(3*b^2*d + a*Sqrt[b^2 - 4*a*c]*e + 3*b*(Sqrt[b^2 - 4*a*c]*d + a*e)))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]]/(Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]])/(6*c^(5/2))

Maple [B] time = 0.036, size = 1211, normalized size = 3.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x^2+d)^3/(c*x^4+b*x^2+a), x)

[Out] 1/3*e^3*x^3/c-e^3/c^2*b*x+3*d*e^2*x/c+1/2/c*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*a*e^3-1/2/c^2*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*b^2*e^3+3/2/c*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*d*e^2*b-3/2*2^(1/2)

)/(((−4*a*c+b^2)^(1/2)−b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((−4*a*c+b^2)^(1/2)−b)*c)^(1/2))*d^2*e−3/2/c/(−4*a*c+b^2)^(1/2)*2^(1/2)/(((−4*a*c+b^2)^(1/2)−b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((−4*a*c+b^2)^(1/2)−b)*c)^(1/2))*e^3*a*b+3/(−4*a*c+b^2)^(1/2)*2^(1/2)/(((−4*a*c+b^2)^(1/2)−b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((−4*a*c+b^2)^(1/2)−b)*c)^(1/2))*a*d*e^2+1/2/c^2/(−4*a*c+b^2)^(1/2)*2^(1/2)/(((−4*a*c+b^2)^(1/2)−b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((−4*a*c+b^2)^(1/2)−b)*c)^(1/2))*b^3*e^3−3/2/c/(−4*a*c+b^2)^(1/2)*2^(1/2)/(((−4*a*c+b^2)^(1/2)−b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((−4*a*c+b^2)^(1/2)−b)*c)^(1/2))*d^2*e^2*b^2+3/2/(−4*a*c+b^2)^(1/2)*2^(1/2)/(((−4*a*c+b^2)^(1/2)−b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((−4*a*c+b^2)^(1/2)−b)*c)^(1/2))*d^2*e*b−c/(−4*a*c+b^2)^(1/2)*2^(1/2)/(((−4*a*c+b^2)^(1/2)−b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((−4*a*c+b^2)^(1/2)−b)*c)^(1/2))*d^3−1/2/c*2^(1/2)/((b+(−4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(−4*a*c+b^2)^(1/2))*c)^(1/2))*a*e^3+1/2/c^2*2^(1/2)/((b+(−4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(−4*a*c+b^2)^(1/2))*c)^(1/2))*b^2*e^3−3/2/c*2^(1/2)/((b+(−4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(−4*a*c+b^2)^(1/2))*c)^(1/2))*d*e^2*b+3/2*2^(1/2)/((b+(−4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(−4*a*c+b^2)^(1/2))*c)^(1/2))*d^2*e−3/2/c/(−4*a*c+b^2)^(1/2)*2^(1/2)/((b+(−4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(−4*a*c+b^2)^(1/2))*c)^(1/2))*a*d*e^2+1/2/c^2/(−4*a*c+b^2)^(1/2)*2^(1/2)/((b+(−4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(−4*a*c+b^2)^(1/2))*c)^(1/2))*b^3*e^3−3/2/c/(−4*a*c+b^2)^(1/2)*2^(1/2)/((b+(−4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(−4*a*c+b^2)^(1/2))*c)^(1/2))*d*e^2*b^2+3/2/(−4*a*c+b^2)^(1/2)*2^(1/2)/((b+(−4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(−4*a*c+b^2)^(1/2))*c)^(1/2))*d^2*e*b−c/(−4*a*c+b^2)^(1/2)*2^(1/2)/((b+(−4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(−4*a*c+b^2)^(1/2))*c)^(1/2))*d^3

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{ce^3x^3 + 3(3cde^2 - be^3)x}{3c^2} - \int \frac{c^2d^3 - 3acde^2 + abe^3 + (3c^2d^2e - 3bcde + (b^2 - ac)e^3)x^2}{cx^4 + bx^2 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)^3/(c*x^4+b*x^2+a),x, algorithm="maxima")

[Out] 1/3*(c*e^3*x^3 + 3*(3*c*d*e^2 - b*e^3)*x)/c^2 - integrate(-(c^2*d^3 - 3*a*c*d*e^2 + a*b*e^3 + (3*c^2*d^2*e - 3*b*c*d*e^2 + (b^2 - a*c)*e^3)*x^2)/(c*x^4 + b*x^2 + a), x)/c^2

Fricas [B] time = 152.596, size = 19539, normalized size = 61.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)^3/(c*x^4+b*x^2+a),x, algorithm="fricas")

[Out] 1/6*(2*c*e^3*x^3 + 3*sqrt(1/2)*c^2*sqrt(-(b*c^5*d^6 - 12*a*c^5*d^5*e + 15*a*b*c^4*d^4*e^2 - 20*(a*b^2*c^3 - 2*a^2*c^4)*d^3*e^3 + 15*(a*b^3*c^2 - 3*a^2*b*c^3)*d^2*e^4 - 6*(a*b^4*c - 4*a^2*b^2*c^2 + 2*a^3*c^3)*d*e^5 + (a*b^5 - 5*a^2*b^3*c + 5*a^3*b*c^2)*e^6 + (a*b^2*c^5 - 4*a^2*c^6)*sqrt((c^10*d^12 -

$$\begin{aligned}
& 30*a*c^9*d^{10}*e^2 + 40*a*b*c^8*d^9*e^3 - 15*(2*a*b^2*c^7 - 17*a^2*c^8)*d^8* \\
& e^4 + 12*(a*b^3*c^6 - 52*a^2*b*c^7)*d^7*e^5 - 2*(a*b^4*c^5 - 428*a^2*b^2*c^ \\
& 6 + 226*a^3*c^7)*d^6*e^6 - 60*(13*a^2*b^3*c^5 - 16*a^3*b*c^6)*d^5*e^7 + 15* \\
& (33*a^2*b^4*c^4 - 68*a^3*b^2*c^5 + 17*a^4*c^6)*d^4*e^8 - 20*(11*a^2*b^5*c^3 \\
& - 33*a^3*b^3*c^4 + 20*a^4*b*c^5)*d^3*e^9 + 6*(11*a^2*b^6*c^2 - 44*a^3*b^4* \\
& c^3 + 44*a^4*b^2*c^4 - 5*a^5*c^5)*d^2*e^{10} - 12*(a^2*b^7*c - 5*a^3*b^5*c^2 \\
& + 7*a^4*b^3*c^3 - 2*a^5*b*c^4)*d*e^{11} + (a^2*b^8 - 6*a^3*b^6*c + 11*a^4*b^4* \\
& c^2 - 6*a^5*b^2*c^3 + a^6*c^4)*e^{12}/(a^2*b^2*c^{10} - 4*a^3*c^{11}))/ (a*b^2* \\
& c^5 - 4*a^2*c^6))*\log(-2*(c^8*d^{12} - 3*b*c^7*d^{11}*e + 3*(b^2*c^6 - 4*a*c^7) \\
& *d^{10}*e^2 - (b^3*c^5 - 59*a*b*c^6)*d^9*e^3 - 9*(13*a*b^2*c^5 + 3*a^2*c^6)*d \\
& ^8*e^4 + 18*(7*a*b^3*c^4 + 5*a^2*b*c^5)*d^7*e^5 - 42*(2*a*b^4*c^3 + 3*a^2*b \\
& ^2*c^4)*d^6*e^6 + 18*(2*a*b^5*c^2 + 6*a^2*b^3*c^3 - a^3*b*c^4)*d^5*e^7 - 9* \\
& (a*b^6*c + 7*a^2*b^4*c^2 - 2*a^3*b^2*c^3 - 3*a^4*c^4)*d^4*e^8 + (a*b^7 + 21 \\
& *a^2*b^5*c + 10*a^3*b^3*c^2 - 55*a^4*b*c^3)*d^3*e^9 - 3*(a^2*b^6 + 4*a^3*b^4 \\
& *c - 9*a^4*b^2*c^2 - 4*a^5*c^3)*d^2*e^{10} + 3*(a^3*b^5 - a^4*b^3*c - 3*a^5* \\
& b*c^2)*d*e^{11} - (a^4*b^4 - 3*a^5*b^2*c + a^6*c^2)*e^{12})*x + \text{sqrt}(1/2)*((b^2 \\
& *c^7 - 4*a*c^8)*d^9 - 18*(a*b^2*c^6 - 4*a^2*c^7)*d^7*e^2 + 21*(a*b^3*c^5 - \\
& 4*a^2*b*c^6)*d^6*e^3 - 15*(a*b^4*c^4 - 8*a^2*b^2*c^5 + 16*a^3*c^6)*d^5*e^4 \\
& + 3*(2*a*b^5*c^3 - 37*a^2*b^3*c^4 + 116*a^3*b*c^5)*d^4*e^5 - (a*b^6*c^2 - 7 \\
& 2*a^2*b^4*c^3 + 318*a^3*b^2*c^4 - 184*a^4*c^5)*d^3*e^6 - 3*(11*a^2*b^5*c^2 \\
& - 61*a^3*b^3*c^3 + 68*a^4*b*c^4)*d^2*e^7 + 3*(3*a^2*b^6*c - 19*a^3*b^4*c^2 \\
& + 29*a^4*b^2*c^3 - 4*a^5*c^4)*d*e^8 - (a^2*b^7 - 7*a^3*b^5*c + 13*a^4*b^3*c \\
& ^2 - 4*a^5*b*c^3)*e^9 - ((a*b^3*c^7 - 4*a^2*b*c^8)*d^3 - 6*(a^2*b^2*c^7 - 4 \\
& *a^3*c^8)*d^2*e + 3*(a^2*b^3*c^6 - 4*a^3*b*c^7)*d*e^2 - (a^2*b^4*c^5 - 6*a^ \\
& 3*b^2*c^6 + 8*a^4*c^7)*e^3)*\text{sqrt}((c^{10}*d^{12} - 30*a*c^9*d^{10}*e^2 + 40*a*b*c^ \\
& 8*d^9*e^3 - 15*(2*a*b^2*c^7 - 17*a^2*c^8)*d^8*e^4 + 12*(a*b^3*c^6 - 52*a^2* \\
& b*c^7)*d^7*e^5 - 2*(a*b^4*c^5 - 428*a^2*b^2*c^6 + 226*a^3*c^7)*d^6*e^6 - 60 \\
& *(13*a^2*b^3*c^5 - 16*a^3*b*c^6)*d^5*e^7 + 15*(33*a^2*b^4*c^4 - 68*a^3*b^2* \\
& c^5 + 17*a^4*c^6)*d^4*e^8 - 20*(11*a^2*b^5*c^3 - 33*a^3*b^3*c^4 + 20*a^4*b* \\
& c^5)*d^3*e^9 + 6*(11*a^2*b^6*c^2 - 44*a^3*b^4*c^3 + 44*a^4*b^2*c^4 - 5*a^5* \\
& c^5)*d^2*e^{10} - 12*(a^2*b^7*c - 5*a^3*b^5*c^2 + 7*a^4*b^3*c^3 - 2*a^5*b*c^4) \\
& *d*e^{11} + (a^2*b^8 - 6*a^3*b^6*c + 11*a^4*b^4*c^2 - 6*a^5*b^2*c^3 + a^6*c^4) \\
& *e^{12}/(a^2*b^2*c^{10} - 4*a^3*c^{11}))*\text{sqrt}(-(b*c^5*d^6 - 12*a*c^5*d^5*e + \\
& 15*a*b*c^4*d^4*e^2 - 20*(a*b^2*c^3 - 2*a^2*c^4)*d^3*e^3 + 15*(a*b^3*c^2 - 3 \\
& *a^2*b*c^3)*d^2*e^4 - 6*(a*b^4*c - 4*a^2*b^2*c^2 + 2*a^3*c^3)*d*e^5 + (a*b^ \\
& 5 - 5*a^2*b^3*c + 5*a^3*b*c^2)*e^6 + (a*b^2*c^5 - 4*a^2*c^6)*\text{sqrt}((c^{10}*d^{1 \\
& 2} - 30*a*c^9*d^{10}*e^2 + 40*a*b*c^8*d^9*e^3 - 15*(2*a*b^2*c^7 - 17*a^2*c^8)* \\
& d^8*e^4 + 12*(a*b^3*c^6 - 52*a^2*b*c^7)*d^7*e^5 - 2*(a*b^4*c^5 - 428*a^2*b^ \\
& 2*c^6 + 226*a^3*c^7)*d^6*e^6 - 60*(13*a^2*b^3*c^5 - 16*a^3*b*c^6)*d^5*e^7 + \\
& 15*(33*a^2*b^4*c^4 - 68*a^3*b^2*c^5 + 17*a^4*c^6)*d^4*e^8 - 20*(11*a^2*b^5 \\
& *c^3 - 33*a^3*b^3*c^4 + 20*a^4*b*c^5)*d^3*e^9 + 6*(11*a^2*b^6*c^2 - 44*a^3* \\
& b^4*c^3 + 44*a^4*b^2*c^4 - 5*a^5*c^5)*d^2*e^{10} - 12*(a^2*b^7*c - 5*a^3*b^5* \\
& c^2 + 7*a^4*b^3*c^3 - 2*a^5*b*c^4)*d*e^{11} + (a^2*b^8 - 6*a^3*b^6*c + 11*a^4 \\
& *b^4*c^2 - 6*a^5*b^2*c^3 + a^6*c^4)*e^{12}/(a^2*b^2*c^{10} - 4*a^3*c^{11}))/ (a \\
& *b^2*c^5 - 4*a^2*c^6)) - 3*\text{sqrt}(1/2)*c^2*\text{sqrt}(-(b*c^5*d^6 - 12*a*c^5*d^5*e \\
& + 15*a*b*c^4*d^4*e^2 - 20*(a*b^2*c^3 - 2*a^2*c^4)*d^3*e^3 + 15*(a*b^3*c^2 - \\
& 3*a^2*b*c^3)*d^2*e^4 - 6*(a*b^4*c - 4*a^2*b^2*c^2 + 2*a^3*c^3)*d*e^5 + (a* \\
& b^5 - 5*a^2*b^3*c + 5*a^3*b*c^2)*e^6 + (a*b^2*c^5 - 4*a^2*c^6)*\text{sqrt}((c^{10}*d \\
& ^{12} - 30*a*c^9*d^{10}*e^2 + 40*a*b*c^8*d^9*e^3 - 15*(2*a*b^2*c^7 - 17*a^2*c^8) \\
&)*d^8*e^4 + 12*(a*b^3*c^6 - 52*a^2*b*c^7)*d^7*e^5 - 2*(a*b^4*c^5 - 428*a^2* \\
& b^2*c^6 + 226*a^3*c^7)*d^6*e^6 - 60*(13*a^2*b^3*c^5 - 16*a^3*b*c^6)*d^5*e^7 \\
& + 15*(33*a^2*b^4*c^4 - 68*a^3*b^2*c^5 + 17*a^4*c^6)*d^4*e^8 - 20*(11*a^2*b^ \\
& ^5*c^3 - 33*a^3*b^3*c^4 + 20*a^4*b*c^5)*d^3*e^9 + 6*(11*a^2*b^6*c^2 - 44*a^ \\
& 3*b^4*c^3 + 44*a^4*b^2*c^4 - 5*a^5*c^5)*d^2*e^{10} - 12*(a^2*b^7*c - 5*a^3*b^ \\
& 5*c^2 + 7*a^4*b^3*c^3 - 2*a^5*b*c^4)*d*e^{11} + (a^2*b^8 - 6*a^3*b^6*c + 11*a^ \\
& ^4*b^4*c^2 - 6*a^5*b^2*c^3 + a^6*c^4)*e^{12}/(a^2*b^2*c^{10} - 4*a^3*c^{11}))/ (\\
& a*b^2*c^5 - 4*a^2*c^6))*\log(-2*(c^8*d^{12} - 3*b*c^7*d^{11}*e + 3*(b^2*c^6 - 4* \\
& a*c^7)*d^{10}*e^2 - (b^3*c^5 - 59*a*b*c^6)*d^9*e^3 - 9*(13*a*b^2*c^5 + 3*a^2* \\
& c^6)*d^8*e^4 + 18*(7*a*b^3*c^4 + 5*a^2*b*c^5)*d^7*e^5 - 42*(2*a*b^4*c^3 + 3
\end{aligned}$$

$$\begin{aligned}
& *a^2*b^2*c^4)*d^6*e^6 + 18*(2*a*b^5*c^2 + 6*a^2*b^3*c^3 - a^3*b*c^4)*d^5*e^7 - 9*(a*b^6*c + 7*a^2*b^4*c^2 - 2*a^3*b^2*c^3 - 3*a^4*c^4)*d^4*e^8 + (a*b^7 + 21*a^2*b^5*c + 10*a^3*b^3*c^2 - 55*a^4*b*c^3)*d^3*e^9 - 3*(a^2*b^6 + 4*a^3*b^4*c - 9*a^4*b^2*c^2 - 4*a^5*c^3)*d^2*e^10 + 3*(a^3*b^5 - a^4*b^3*c - 3*a^5*b*c^2)*d*e^11 - (a^4*b^4 - 3*a^5*b^2*c + a^6*c^2)*e^12)*x - \text{sqrt}(1/2) \\
& *((b^2*c^7 - 4*a*c^8)*d^9 - 18*(a*b^2*c^6 - 4*a^2*c^7)*d^7*e^2 + 21*(a*b^3*c^5 - 4*a^2*b*c^6)*d^6*e^3 - 15*(a*b^4*c^4 - 8*a^2*b^2*c^5 + 16*a^3*c^6)*d^5*e^4 + 3*(2*a*b^5*c^3 - 37*a^2*b^3*c^4 + 116*a^3*b*c^5)*d^4*e^5 - (a*b^6*c^2 - 72*a^2*b^4*c^3 + 318*a^3*b^2*c^4 - 184*a^4*c^5)*d^3*e^6 - 3*(11*a^2*b^5*c^2 - 61*a^3*b^3*c^3 + 68*a^4*b*c^4)*d^2*e^7 + 3*(3*a^2*b^6*c - 19*a^3*b^4*c^2 + 29*a^4*b^2*c^3 - 4*a^5*c^4)*d*e^8 - (a^2*b^7 - 7*a^3*b^5*c + 13*a^4*b^3*c^2 - 4*a^5*b*c^3)*e^9 - ((a*b^3*c^7 - 4*a^2*b*c^8)*d^3 - 6*(a^2*b^2*c^7 - 4*a^3*c^8)*d^2*e + 3*(a^2*b^3*c^6 - 4*a^3*b*c^7)*d*e^2 - (a^2*b^4*c^5 - 6*a^3*b^2*c^6 + 8*a^4*c^7)*e^3)*\text{sqrt}((c^10*d^12 - 30*a*c^9*d^10*e^2 + 40*a*b*c^8*d^9*e^3 - 15*(2*a*b^2*c^7 - 17*a^2*c^8)*d^8*e^4 + 12*(a*b^3*c^6 - 52*a^2*b*c^7)*d^7*e^5 - 2*(a*b^4*c^5 - 428*a^2*b^2*c^6 + 226*a^3*c^7)*d^6*e^6 - 60*(13*a^2*b^3*c^5 - 16*a^3*b*c^6)*d^5*e^7 + 15*(33*a^2*b^4*c^4 - 68*a^3*b^2*c^5 + 17*a^4*c^6)*d^4*e^8 - 20*(11*a^2*b^5*c^3 - 33*a^3*b^3*c^4 + 20*a^4*b*c^5)*d^3*e^9 + 6*(11*a^2*b^6*c^2 - 44*a^3*b^4*c^3 + 44*a^4*b^2*c^4 - 5*a^5*c^5)*d^2*e^10 - 12*(a^2*b^7*c - 5*a^3*b^5*c^2 + 7*a^4*b^3*c^3 - 2*a^5*b*c^4)*d*e^11 + (a^2*b^8 - 6*a^3*b^6*c + 11*a^4*b^4*c^2 - 6*a^5*b^2*c^3 + a^6*c^4)*e^12)/(a^2*b^2*c^10 - 4*a^3*c^11))*\text{sqrt}(-(b*c^5*d^6 - 12*a*c^5*d^5*e + 15*a*b*c^4*d^4*e^2 - 20*(a*b^2*c^3 - 2*a^2*c^4)*d^3*e^3 + 15*(a*b^3*c^2 - 3*a^2*b*c^3)*d^2*e^4 - 6*(a*b^4*c - 4*a^2*b^2*c^2 + 2*a^3*c^3)*d*e^5 + (a*b^5 - 5*a^2*b^3*c + 5*a^3*b*c^2)*e^6 + (a*b^2*c^5 - 4*a^2*c^6)*\text{sqrt}((c^10*d^12 - 30*a*c^9*d^10*e^2 + 40*a*b*c^8*d^9*e^3 - 15*(2*a*b^2*c^7 - 17*a^2*c^8)*d^8*e^4 + 12*(a*b^3*c^6 - 52*a^2*b*c^7)*d^7*e^5 - 2*(a*b^4*c^5 - 428*a^2*b^2*c^6 + 226*a^3*c^7)*d^6*e^6 - 60*(13*a^2*b^3*c^5 - 16*a^3*b*c^6)*d^5*e^7 + 15*(33*a^2*b^4*c^4 - 68*a^3*b^2*c^5 + 17*a^4*c^6)*d^4*e^8 - 20*(11*a^2*b^5*c^3 - 33*a^3*b^3*c^4 + 20*a^4*b*c^5)*d^3*e^9 + 6*(11*a^2*b^6*c^2 - 44*a^3*b^4*c^3 + 44*a^4*b^2*c^4 - 5*a^5*c^5)*d^2*e^10 - 12*(a^2*b^7*c - 5*a^3*b^5*c^2 + 7*a^4*b^3*c^3 - 2*a^5*b*c^4)*d*e^11 + (a^2*b^8 - 6*a^3*b^6*c + 11*a^4*b^4*c^2 - 6*a^5*b^2*c^3 + a^6*c^4)*e^12)/(a^2*b^2*c^10 - 4*a^3*c^11)))/((a*b^2*c^5 - 4*a^2*c^6))) + 3*\text{sqrt}(1/2)*c^2*\text{sqrt}(-(b*c^5*d^6 - 12*a*c^5*d^5*e + 15*a*b*c^4*d^4*e^2 - 20*(a*b^2*c^3 - 2*a^2*c^4)*d^3*e^3 + 15*(a*b^3*c^2 - 3*a^2*b*c^3)*d^2*e^4 - 6*(a*b^4*c - 4*a^2*b^2*c^2 + 2*a^3*c^3)*d*e^5 + (a*b^5 - 5*a^2*b^3*c + 5*a^3*b*c^2)*e^6 - (a*b^2*c^5 - 4*a^2*c^6)*\text{sqrt}((c^10*d^12 - 30*a*c^9*d^10*e^2 + 40*a*b*c^8*d^9*e^3 - 15*(2*a*b^2*c^7 - 17*a^2*c^8)*d^8*e^4 + 12*(a*b^3*c^6 - 52*a^2*b*c^7)*d^7*e^5 - 2*(a*b^4*c^5 - 428*a^2*b^2*c^6 + 226*a^3*c^7)*d^6*e^6 - 60*(13*a^2*b^3*c^5 - 16*a^3*b*c^6)*d^5*e^7 + 15*(33*a^2*b^4*c^4 - 68*a^3*b^2*c^5 + 17*a^4*c^6)*d^4*e^8 - 20*(11*a^2*b^5*c^3 - 33*a^3*b^3*c^4 + 20*a^4*b*c^5)*d^3*e^9 + 6*(11*a^2*b^6*c^2 - 44*a^3*b^4*c^3 + 44*a^4*b^2*c^4 - 5*a^5*c^5)*d^2*e^10 - 12*(a^2*b^7*c - 5*a^3*b^5*c^2 + 7*a^4*b^3*c^3 - 2*a^5*b*c^4)*d*e^11 + (a^2*b^8 - 6*a^3*b^6*c + 11*a^4*b^4*c^2 - 6*a^5*b^2*c^3 + a^6*c^4)*e^12)/(a^2*b^2*c^10 - 4*a^3*c^11)))/((a*b^2*c^5 - 4*a^2*c^6)))*\log(-2*(c^8*d^12 - 3*b*c^7*d^11*e + 3*(b^2*c^6 - 4*a*c^7)*d^10*e^2 - (b^3*c^5 - 59*a*b*c^6)*d^9*e^3 - 9*(13*a*b^2*c^5 + 3*a^2*c^6)*d^8*e^4 + 18*(7*a*b^3*c^4 + 5*a^2*b*c^5)*d^7*e^5 - 42*(2*a*b^4*c^3 + 3*a^2*b^2*c^4)*d^6*e^6 + 18*(2*a*b^5*c^2 + 6*a^2*b^3*c^3 - a^3*b*c^4)*d^5*e^7 - 9*(a*b^6*c + 7*a^2*b^4*c^2 - 2*a^3*b^2*c^3 - 3*a^4*c^4)*d^4*e^8 + (a*b^7 + 21*a^2*b^5*c + 10*a^3*b^3*c^2 - 55*a^4*b*c^3)*d^3*e^9 - 3*(a^2*b^6 + 4*a^3*b^4*c - 9*a^4*b^2*c^2 - 4*a^5*c^3)*d^2*e^10 + 3*(a^3*b^5 - a^4*b^3*c - 3*a^5*b*c^2)*d*e^11 - (a^4*b^4 - 3*a^5*b^2*c + a^6*c^2)*e^12)*x + \text{sqrt}(1/2)*((b^2*c^7 - 4*a*c^8)*d^9 - 18*(a*b^2*c^6 - 4*a^2*c^7)*d^7*e^2 + 21*(a*b^3*c^5 - 4*a^2*b*c^6)*d^6*e^3 - 15*(a*b^4*c^4 - 8*a^2*b^2*c^5 + 16*a^3*c^6)*d^5*e^4 + 3*(2*a*b^5*c^3 - 37*a^2*b^3*c^4 + 116*a^3*b*c^5)*d^4*e^5 - (a*b^6*c^2 - 72*a^2*b^4*c^3 + 318*a^3*b^2*c^4 - 184*a^4*c^5)*d^3*e^6 - 3*(11*a^2*b^5*c^2 - 61*a^3*b^3*c^3 + 68*a^4*b*c^4)*d^2*e^7 + 3*(3*a^2*b^6*c - 19*a^3*b^4*c^2 + 29*a^4*b^2*c^3 - 4*a^5*c^4)*d*e^8 - (a^2*b^7 - 7*a^3*b^5*c +
\end{aligned}$$

$$\begin{aligned}
& 13a^4b^3c^2 - 4a^5bc^3)e^9 + ((a^2b^3c^7 - 4a^2b^3c^8)d^3 - 6(a^2 \\
& b^2c^7 - 4a^3c^8)d^2e + 3(a^2b^3c^6 - 4a^3b^3c^7)d^2e^2 - (a^2b^4 \\
& 4c^5 - 6a^3b^2c^6 + 8a^4c^7)e^3)\sqrt{(c^{10}d^{12} - 30a^2c^9d^{10}e^2 \\
& + 40a^2b^3c^8d^9e^3 - 15(2a^2b^2c^7 - 17a^2c^8)d^8e^4 + 12(a^2b^3c^6 \\
& - 52a^2b^3c^7)d^7e^5 - 2(a^2b^4c^5 - 428a^2b^2c^6 + 226a^3c^7)d^6e^6 \\
& - 60(13a^2b^3c^5 - 16a^3b^3c^6)d^5e^7 + 15(33a^2b^4c^4 - 68a^3b^2c^5 \\
& + 17a^4c^6)d^4e^8 - 20(11a^2b^5c^3 - 33a^3b^3c^4 + 20a^4b^3c^5)d^3e^9 \\
& + 6(11a^2b^6c^2 - 44a^3b^4c^3 + 44a^4b^2c^4 - 5a^5c^5)d^2e^{10} - 12(a^2b^7c \\
& - 5a^3b^5c^2 + 7a^4b^3c^3 - 2a^5b^3c^4)d^2e^{11} + (a^2b^8 - 6a^3b^6c \\
& + 11a^4b^4c^2 - 6a^5b^2c^3 + a^6c^4)e^{12})/(a^2b^2c^{10} - 4a^3c^{11}))\sqrt{-(b^2c^5d^6 - 12a^2 \\
& c^5d^5e + 15a^2b^3c^4d^4e^2 - 20(a^2b^2c^3 - 2a^2c^4)d^3e^3 + 15(a \\
& b^3c^2 - 3a^2b^3c^3)d^2e^4 - 6(a^2b^4c - 4a^2b^2c^2 + 2a^3c^3)d \\
& e^5 + (a^2b^5 - 5a^2b^3c + 5a^3b^3c^2)e^6 - (a^2b^2c^5 - 4a^2c^6)\sqrt{ \\
& \sqrt{(c^{10}d^{12} - 30a^2c^9d^{10}e^2 + 40a^2b^3c^8d^9e^3 - 15(2a^2b^2c^7 - 17a^2c^8)d^8e^4 \\
& + 12(a^2b^3c^6 - 52a^2b^3c^7)d^7e^5 - 2(a^2b^4c^5 - 428a^2b^2c^6 + 226a^3c^7)d^6e^6 \\
& - 60(13a^2b^3c^5 - 16a^3b^3c^6)d^5e^7 + 15(33a^2b^4c^4 - 68a^3b^2c^5 + 17a^4c^6)d^4e^8 \\
& - 20(11a^2b^5c^3 - 33a^3b^3c^4 + 20a^4b^3c^5)d^3e^9 + 6(11a^2b^6c^2 - 44a^3b^4c^3 \\
& + 44a^4b^2c^4 - 5a^5c^5)d^2e^{10} - 12(a^2b^7c - 5a^3b^5c^2 + 7a^4b^3c^3 - 2a^5b^3c^4)d^2e^{11} \\
& + (a^2b^8 - 6a^3b^6c + 11a^4b^4c^2 - 6a^5b^2c^3 + a^6c^4)e^{12})/(a^2b^2c^{10} - 4a^3c^{11})) \\
&)/(a^2b^2c^5 - 4a^2c^6)) - 3\sqrt{1/2}c^2\sqrt{-(b^2c^5d^6 - 12a^2c^5d^5e + 15a^2b^3c^4d^4e^2 \\
& - 20(a^2b^2c^3 - 2a^2c^4)d^3e^3 + 15(a^2b^3c^2 - 3a^2b^3c^3)d^2e^4 - 6(a^2b^4c - 4a^2b^2c^2 \\
& + 2a^3c^3)d^2e^5 + (a^2b^5 - 5a^2b^3c + 5a^3b^3c^2)e^6 - (a^2b^2c^5 - 4a^2c^6)\sqrt{ \\
& \sqrt{(c^{10}d^{12} - 30a^2c^9d^{10}e^2 + 40a^2b^3c^8d^9e^3 - 15(2a^2b^2c^7 - 17a^2c^8)d^8e^4 \\
& + 12(a^2b^3c^6 - 52a^2b^3c^7)d^7e^5 - 2(a^2b^4c^5 - 428a^2b^2c^6 + 226a^3c^7)d^6e^6 \\
& - 60(13a^2b^3c^5 - 16a^3b^3c^6)d^5e^7 + 15(33a^2b^4c^4 - 68a^3b^2c^5 + 17a^4c^6)d^4e^8 \\
& - 20(11a^2b^5c^3 - 33a^3b^3c^4 + 20a^4b^3c^5)d^3e^9 + 6(11a^2b^6c^2 - 44a^3b^4c^3 \\
& + 44a^4b^2c^4 - 5a^5c^5)d^2e^{10} - 12(a^2b^7c - 5a^3b^5c^2 + 7a^4b^3c^3 - 2a^5b^3c^4)d^2e^{11} \\
& + (a^2b^8 - 6a^3b^6c + 11a^4b^4c^2 - 6a^5b^2c^3 + a^6c^4)e^{12})/(a^2b^2c^{10} - 4a^3c^{11})) \\
&)/(a^2b^2c^5 - 4a^2c^6))\log(-2(c^8d^{12} - 3b^2c^7d^{11}e + 3(b^2c^6 - 4a^2c^7)d^{10}e^2 \\
& - (b^3c^5 - 59a^2b^3c^6)d^9e^3 - 9(13a^2b^2c^5 + 3a^2c^6)d^8e^4 + 18(7a^2b^3c^4 + 5a^2b^3c^5)d^7e^5 \\
& - 42(2a^2b^4c^3 + 3a^2b^2c^4)d^6e^6 + 18(2a^2b^5c^2 + 6a^2b^3c^3 - a^3b^5c^4)d^5e^7 \\
& - 9(a^2b^6c + 7a^2b^4c^2 - 2a^3b^2c^3 - 3a^4c^4)d^4e^8 + (a^2b^7 + 21a^2b^5c + 10a^3b^3c^2 \\
& - 55a^4b^3c^3)d^3e^9 - 3(a^2b^6 + 4a^3b^4c - 9a^4b^2c^2 - 4a^5c^3)d^2e^{10} + 3(a^3b^5 - a^4b^3c \\
& - 3a^5b^3c^2)d^2e^{11} - (a^4b^4 - 3a^5b^2c + a^6c^2)e^{12})\sqrt{1/2}((b^2c^7 - 4a^2c^8)d^9 \\
& - 18(a^2b^2c^6 - 4a^2c^7)d^7e^2 + 21(a^2b^3c^5 - 4a^2b^3c^6)d^6e^3 - 15(a^2b^4c^4 - 8a^2b^2c^5 \\
& + 16a^3c^6)d^5e^4 + 3(2a^2b^5c^3 - 37a^2b^3c^4 + 116a^3b^3c^5)d^4e^5 - (a^2b^6c^2 - 72a^2b^4c^3 \\
& + 318a^3b^2c^4 - 184a^4c^5)d^3e^6 - 3(11a^2b^5c^2 - 61a^3b^3c^3 + 68a^4b^3c^4)d^2e^7 \\
& + 3(3a^2b^6c - 19a^3b^4c^2 + 29a^4b^2c^3 - 4a^5c^4)d^2e^8 - (a^2b^7 - 7a^3b^5c \\
& + 13a^4b^3c^2 - 4a^5b^3c^3)e^9 + ((a^2b^3c^7 - 4a^2b^3c^8)d^3 - 6(a^2b^2c^7 - 4a^3c^8)d^2e \\
& + 3(a^2b^3c^6 - 4a^3b^3c^7)d^2e^2 - (a^2b^4c^5 - 6a^3b^2c^6 + 8a^4c^7)e^3)\sqrt{(c^{10}d^{12} - 30a^2c^9d^{10}e^2 \\
& + 40a^2b^3c^8d^9e^3 - 15(2a^2b^2c^7 - 17a^2c^8)d^8e^4 + 12(a^2b^3c^6 - 52a^2b^3c^7)d^7e^5 \\
& - 2(a^2b^4c^5 - 428a^2b^2c^6 + 226a^3c^7)d^6e^6 - 60(13a^2b^3c^5 - 16a^3b^3c^6)d^5e^7 \\
& + 15(33a^2b^4c^4 - 68a^3b^2c^5 + 17a^4c^6)d^4e^8 - 20(11a^2b^5c^3 - 33a^3b^3c^4 + 20a^4b^3c^5)d^3e^9 \\
& + 6(11a^2b^6c^2 - 44a^3b^4c^3 + 44a^4b^2c^4 - 5a^5c^5)d^2e^{10} - 12(a^2b^7c - 5a^3b^5c^2 + 7a^4b^3c^3 \\
& - 2a^5b^3c^4)d^2e^{11} + (a^2b^8 - 6a^3b^6c + 11a^4b^4c^2 - 6a^5b^2c^3 + a^6c^4)e^{12})/(a^2b^2c^{10} - 4a^3c^{11}))\sqrt{-(b^2c^5d^6 -
\end{aligned}$$

$$12*a*c^5*d^5*e + 15*a*b*c^4*d^4*e^2 - 20*(a*b^2*c^3 - 2*a^2*c^4)*d^3*e^3 + 15*(a*b^3*c^2 - 3*a^2*b*c^3)*d^2*e^4 - 6*(a*b^4*c - 4*a^2*b^2*c^2 + 2*a^3*c^3)*d*e^5 + (a*b^5 - 5*a^2*b^3*c + 5*a^3*b*c^2)*e^6 - (a*b^2*c^5 - 4*a^2*c^6)*sqrt((c^10*d^12 - 30*a*c^9*d^10*e^2 + 40*a*b*c^8*d^9*e^3 - 15*(2*a*b^2*c^7 - 17*a^2*c^8)*d^8*e^4 + 12*(a*b^3*c^6 - 52*a^2*b*c^7)*d^7*e^5 - 2*(a*b^4*c^5 - 428*a^2*b^2*c^6 + 226*a^3*c^7)*d^6*e^6 - 60*(13*a^2*b^3*c^5 - 16*a^3*b*c^6)*d^5*e^7 + 15*(33*a^2*b^4*c^4 - 68*a^3*b^2*c^5 + 17*a^4*c^6)*d^4*e^8 - 20*(11*a^2*b^5*c^3 - 33*a^3*b^3*c^4 + 20*a^4*b*c^5)*d^3*e^9 + 6*(11*a^2*b^6*c^2 - 44*a^3*b^4*c^3 + 44*a^4*b^2*c^4 - 5*a^5*c^5)*d^2*e^10 - 12*(a^2*b^7*c - 5*a^3*b^5*c^2 + 7*a^4*b^3*c^3 - 2*a^5*b*c^4)*d*e^11 + (a^2*b^8 - 6*a^3*b^6*c + 11*a^4*b^4*c^2 - 6*a^5*b^2*c^3 + a^6*c^4)*e^12)/(a^2*b^2*c^10 - 4*a^3*c^11))/(a*b^2*c^5 - 4*a^2*c^6)) + 6*(3*c*d*e^2 - b*e^3)*x)/c^2$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x**2+d)**3/(c*x**4+b*x**2+a),x)

[Out] Timed out

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)^3/(c*x^4+b*x^2+a),x, algorithm="giac")

[Out] Exception raised: NotImplementedError

$$3.265 \quad \int \frac{(d+ex^2)^2}{a+bx^2+cx^4} dx$$

Optimal. Leaf size=238

$$\frac{\left(\frac{-2ce(ae+bd)+b^2e^2+2c^2d^2}{\sqrt{b^2-4ac}} + e(2cd - be)\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\left(e(2cd - be) - \frac{-2ce(ae+bd)+b^2e^2+2c^2d^2}{\sqrt{b^2-4ac}}\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}c^{3/2}\sqrt{\sqrt{b^2-4ac}+b}} + e$$

```
[Out] (e^2*x)/c + ((e*(2*c*d - b*e) + (2*c^2*d^2 + b^2*e^2 - 2*c*e*(b*d + a*e))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]])/(Sqrt[2]*c^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((e*(2*c*d - b*e) - (2*c^2*d^2 + b^2*e^2 - 2*c*e*(b*d + a*e))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]])/(Sqrt[2]*c^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rubi [A] time = 0.635092, antiderivative size = 238, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1170, 1166, 205}

$$\frac{\left(\frac{-2ce(ae+bd)+b^2e^2+2c^2d^2}{\sqrt{b^2-4ac}} + e(2cd - be)\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\left(e(2cd - be) - \frac{-2ce(ae+bd)+b^2e^2+2c^2d^2}{\sqrt{b^2-4ac}}\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}c^{3/2}\sqrt{\sqrt{b^2-4ac}+b}} + e$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^2/(a + b*x^2 + c*x^4), x]
```

```
[Out] (e^2*x)/c + ((e*(2*c*d - b*e) + (2*c^2*d^2 + b^2*e^2 - 2*c*e*(b*d + a*e))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]])/(Sqrt[2]*c^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((e*(2*c*d - b*e) - (2*c^2*d^2 + b^2*e^2 - 2*c*e*(b*d + a*e))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]])/(Sqrt[2]*c^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 1170

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x^2)^q/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[q]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol]
:> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol]
:> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{(d + ex^2)^2}{a + bx^2 + cx^4} dx &= \int \left(\frac{e^2}{c} + \frac{cd^2 - ae^2 + e(2cd - be)x^2}{c(a + bx^2 + cx^4)} \right) dx \\
&= \frac{e^2 x}{c} + \frac{\int \frac{cd^2 - ae^2 + e(2cd - be)x^2}{a + bx^2 + cx^4} dx}{c} \\
&= \frac{e^2 x}{c} + \frac{\left(e(2cd - be) - \frac{2c^2 d^2 + b^2 e^2 - 2ce(bd + ae)}{\sqrt{b^2 - 4ac}} \right) \int \frac{1}{\frac{b}{2} + \frac{1}{2}\sqrt{b^2 - 4ac} + cx^2} dx}{2c} + \frac{\left(e(2cd - be) + \frac{2c^2 d^2 + b^2 e^2 - 2ce(bd + ae)}{\sqrt{b^2 - 4ac}} \right)}{2c} \\
&= \frac{e^2 x}{c} + \frac{\left(e(2cd - be) + \frac{2c^2 d^2 + b^2 e^2 - 2ce(bd + ae)}{\sqrt{b^2 - 4ac}} \right) \tan^{-1} \left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b - \sqrt{b^2 - 4ac}}} \right)}{\sqrt{2}c^{3/2}\sqrt{b - \sqrt{b^2 - 4ac}}} + \frac{\left(e(2cd - be) - \frac{2c^2 d^2 + b^2 e^2 - 2ce(bd + ae)}{\sqrt{b^2 - 4ac}} \right)}{\sqrt{2}c^{3/2}\sqrt{b + \sqrt{b^2 - 4ac}}}
\end{aligned}$$

Mathematica [A] time = 0.332266, size = 269, normalized size = 1.13

$$\frac{\sqrt{2}(-2ce(-d\sqrt{b^2-4ac}+ae+bd)+be^2(b-\sqrt{b^2-4ac})+2c^2d^2)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{2}(-2ce(d\sqrt{b^2-4ac}+ae+bd)+be^2(\sqrt{b^2-4ac}+b)+2c^2d^2)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}}{2c^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e*x^2)^2/(a + b*x^2 + c*x^4), x]

[Out] (2*Sqrt[c]*e^2*x + (Sqrt[2]*(2*c^2*d^2 + b*(b - Sqrt[b^2 - 4*a*c]))*e^2 - 2*c*e*(b*d - Sqrt[b^2 - 4*a*c]*d + a*e))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[2]*(2*c^2*d^2 + b*(b + Sqrt[b^2 - 4*a*c]))*e^2 - 2*c*e*(b*d + Sqrt[b^2 - 4*a*c]*d + a*e))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]])/(2*c^(3/2))

Maple [B] time = 0.027, size = 695, normalized size = 2.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x^2+d)^2/(c*x^4+b*x^2+a), x)

[Out] e^2*x/c+1/2/c*2^(1/2)/((((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*e^2*b-2^(1/2)/((((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/((((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*d*e+1/(-4*a*c+b^2)^(1/2)*2^(1/2)/((((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/((((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*a*e^2-1/2/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/((((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*b^2*e^2+1/(-4*a*c+b^2)^(1/2)*2^(1/2)/((((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/((((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*d*e*b-c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/((((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*d^2-1/2/c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*e^2*b+2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))

$$\frac{1}{2}d^2e + \frac{1}{(-4ac+b^2)^{1/2}} \frac{2^{1/2}}{((b+(-4ac+b^2)^{1/2})c)^{1/2}} \arctan\left(\frac{cx^2}{(b+(-4ac+b^2)^{1/2})c}\right) + \frac{a^2e^{2-1/2}}{(-4ac+b^2)^{1/2}} \frac{2^{1/2}}{((b+(-4ac+b^2)^{1/2})c)^{1/2}} \arctan\left(\frac{cx^2}{(b+(-4ac+b^2)^{1/2})c}\right) + \frac{b^2e^2}{(-4ac+b^2)^{1/2}} \frac{2^{1/2}}{((b+(-4ac+b^2)^{1/2})c)^{1/2}} \arctan\left(\frac{cx^2}{(b+(-4ac+b^2)^{1/2})c}\right) + \frac{e^2b-c}{(-4ac+b^2)^{1/2}} \frac{2^{1/2}}{((b+(-4ac+b^2)^{1/2})c)^{1/2}} \arctan\left(\frac{cx^2}{(b+(-4ac+b^2)^{1/2})c}\right) + d^2$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{e^2x}{c} - \int \frac{cd^2 - ae^2 + (2cde - be^2)x^2}{cx^4 + bx^2 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)^2/(c*x^4+b*x^2+a),x, algorithm="maxima")

[Out] e^2*x/c - integrate(-(c*d^2 - a*e^2 + (2*c*d*e - b*e^2)*x^2)/(c*x^4 + b*x^2 + a), x)/c

Fricas [B] time = 13.6026, size = 9230, normalized size = 38.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)^2/(c*x^4+b*x^2+a),x, algorithm="fricas")

[Out] $\frac{1}{2}(2e^2x - \sqrt{1/2}c\sqrt{-(b^3c^3d^4 - 8a^3c^3d^3e + 6a^2b^2c^2d^2e^2 - 4(a^2b^2c - 2a^2c^2)d^2e^3 + (ab^3 - 3a^2b^2c)e^4 + (ab^2c^3 - 4a^2c^4)\sqrt{(c^6d^8 - 12a^5c^5d^6e^2 + 8a^4b^2c^4d^5e^3 - 48a^2b^2c^3d^3e^5 - 2(a^2b^2c^3 - 19a^2c^4)d^4e^4 + 4(7a^2b^2c^2 - 3a^3c^3)d^2e^6 - 8(a^2b^3c - a^3b^2c^2)d^2e^7 + (a^2b^4 - 2a^3b^2c + a^4c^2)e^8)/(a^2b^2c^6 - 4a^3c^7))})/(ab^2c^3 - 4a^2c^4)) \log(2 * (c^5d^8 - 2b^2c^4d^7e + 14a^4b^2c^3d^5e^3 + (b^2c^3 - 4a^3c^4)d^6e^2 - 5(3a^2b^2c^2 + 2a^2c^3)d^4e^4 + 6(ab^3c + 3a^2b^2c^2)d^3e^5 - (ab^4 + 9a^2b^2c + 4a^3c^2)d^2e^6 + 2(a^2b^3 + a^3b^2c)d^2e^7 - (a^3b^2 - a^4c)e^8) * x + \sqrt{1/2} * ((b^2c^4 - 4a^3c^5)d^6 - 7(ab^2c^3 - 4a^2c^4)d^4e^2 + 4(ab^3c^2 - 4a^2b^2c^3)d^3e^3 - (ab^4c - 11a^2b^2c^2 + 28a^3c^3)d^2e^4 - 4(a^2b^3c - 4a^3b^2c^2)d^2e^5 + (a^2b^4 - 5a^3b^2c + 4a^4c^2)e^6 - ((ab^3c^4 - 4a^2b^2c^5)d^2 - 4(a^2b^2c^4 - 4a^3c^5)d^2e + (a^2b^3c^3 - 4a^3b^2c^4)e^2) * \sqrt{(c^6d^8 - 12a^5c^5d^6e^2 + 8a^4b^2c^4d^5e^3 - 48a^2b^2c^3d^3e^5 - 2(ab^2c^3 - 19a^2c^4)d^4e^4 + 4(7a^2b^2c^2 - 3a^3c^3)d^2e^6 - 8(a^2b^3c - a^3b^2c^2)d^2e^7 + (a^2b^4 - 2a^3b^2c + a^4c^2)e^8)/(a^2b^2c^6 - 4a^3c^7))}) * \sqrt{-(b^3c^3d^4 - 8a^3c^3d^3e + 6a^2b^2c^2d^2e^2 - 4(ab^2c - 2a^2c^2)d^2e^3 + (ab^3 - 3a^2b^2c)e^4 + (ab^2c^3 - 4a^2c^4)\sqrt{(c^6d^8 - 12a^5c^5d^6e^2 + 8a^4b^2c^4d^5e^3 - 48a^2b^2c^3d^3e^5 - 2(ab^2c^3 - 19a^2c^4)d^4e^4 + 4(7a^2b^2c^2 - 3a^3c^3)d^2e^6 - 8(a^2b^3c - a^3b^2c^2)d^2e^7 + (a^2b^4 - 2a^3b^2c + a^4c^2)e^8)/(a^2b^2c^6 - 4a^3c^7))}) + \sqrt{1/2} * c * \sqrt{-(b^3c^3d^4 - 8a^3c^3d^3e + 6a^2b^2c^2d^2e^2 - 4(ab^2c - 2a^2c^2)d^2e^3 + (ab^3 - 3a^2b^2c)e^4 + (ab^2c^3 - 4a^2c^4)\sqrt{(c^6d^8 - 12a^5c^5d^6e^2 + 8a^4b^2c^4d^5e^3 - 48a^2b^2c^3d^3e^5 - 2(ab^2c^3 - 19a^2c^4)d^4e^4 + 4(7a^2b^2c^2 - 3a^3c^3)d^2e^6 - 8(a^2b^3c - a^3b^2c^2)d^2e^7 + (a^2b^4 - 2a^3b^2c + a^4c^2)e^8)/(a^2b^2c^6 - 4a^3c^7))})$

$$\frac{c^2 e^8}{(a^2 b^2 c^6 - 4 a^3 c^7)} \sqrt{-(b^3 c^4 d^4 - 8 a^3 c^3 d^3 e + 6 a^2 b^2 c^2 d^2 e^2 - 4 (a^2 b^2 c - 2 a^2 c^2) d e^3 + (a^3 b^3 - 3 a^2 b^2 c) e^4 - (a^2 b^2 c^3 - 4 a^2 c^4) \sqrt{(c^6 d^8 - 12 a^5 c^5 d^6 e^2 + 8 a^4 b^3 c^4 d^5 e^3 - 48 a^2 b^2 c^3 d^3 e^5 - 2 (a^2 b^2 c^3 - 19 a^2 c^4) d^4 e^4 + 4 (7 a^2 b^2 c^2 - 3 a^3 c^3) d^2 e^6 - 8 (a^2 b^3 c - a^3 b^2 c^2) d e^7 + (a^2 b^4 - 2 a^3 b^2 c + a^4 c^2) e^8})} / (a^2 b^2 c^6 - 4 a^3 c^7)} / (a^2 b^2 c^3 - 4 a^2 c^4) / c$$

Sympy [B] time = 45.2101, size = 920, normalized size = 3.87

$$\text{RootSum}\left(t^4 (256 a^3 c^5 - 128 a^2 b^2 c^4 + 16 a b^4 c^3) + t^2 (48 a^3 b c^2 e^4 - 128 a^3 c^3 d e^3 - 28 a^2 b^3 c e^4 + 96 a^2 b^2 c^2 d e^3 - 96 a^2 b c^3 d^2 e^2) + e^8\right) / (a^2 b^2 c^6 - 4 a^3 c^7) / (a^2 b^2 c^3 - 4 a^2 c^4) / c$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x**2+d)**2/(c*x**4+b*x**2+a),x)

[Out] RootSum(_t**4*(256*a**3*c**5 - 128*a**2*b**2*c**4 + 16*a*b**4*c**3) + _t**2*(48*a**3*b*c**2*e**4 - 128*a**3*c**3*d*e**3 - 28*a**2*b**3*c*e**4 + 96*a**2*b**2*c**2*d*e**3 - 96*a**2*b**c**3*d**2*e**2 + 128*a**2*c**4*d**3*e + 4*a*b**5*e**4 - 16*a*b**4*c*d*e**3 + 24*a*b**3*c**2*d**2*e**2 - 32*a*b**2*c**3*d**3*e - 16*a*b*c**4*d**4 + 4*b**3*c**3*d**4) + a**4*e**8 - 4*a**3*b*d*e**7 + 4*a**3*c*d**2*e**6 + 6*a**2*b**2*d**2*e**6 - 12*a**2*b*c*d**3*e**5 + 6*a**2*c**2*d**4*e**4 - 4*a*b**3*d**3*e**5 + 12*a*b**2*c*d**4*e**4 - 12*a*b*c**2*d**5*e**3 + 4*a*c**3*d**6*e**2 + b**4*d**4*e**4 - 4*b**3*c*d**5*e**3 + 6*b**2*c**2*d**6*e**2 - 4*b*c**3*d**7*e + c**4*d**8, Lambda(_t, _t*log(x + (32*_t**3*a**3*b*c**4*e**2 - 128*_t**3*a**3*c**5*d*e - 8*_t**3*a**2*b**3*c**3*e**2 + 32*_t**3*a**2*b**2*c**4*d*e + 32*_t**3*a**2*b*c**5*d**2 - 8*_t**3*a*b**3*c**4*d**2 - 4*_t**4*c**2*e**6 + 8*_t**4*a**3*b**2*c*e**6 - 36*_t**4*b*c**2*d*e**5 + 60*_t**4*a**3*c**3*d**2*e**4 - 2*_t**4*a**2*b**4*e**6 + 12*_t**4*a**2*b**3*c*d*e**5 - 30*_t**4*a**2*b**2*c**2*d**2*e**4 + 40*_t**4*a**2*b*c**3*d**3*e**3 - 60*_t**4*a**2*c**4*d**4*e**2 + 12*_t**4*a*b*c**4*d**5*e + 4*_t**4*a*c**5*d**6 - 2*_t**4*b**2*c**4*d**6) / (a**4*c*e**8 - a**3*b**2*e**8 + 2*a**3*b*c*d*e**7 - 4*a**3*c**2*d**2*e**6 + 2*a**2*b**3*d*e**7 - 9*a**2*b**2*c*d**2*e**6 + 18*a**2*b*c**2*d**3*e**5 - 10*a**2*c**3*d**4*e**4 - a*b**4*d**2*e**6 + 6*a*b**3*c*d**3*e**5 - 15*a*b**2*c**2*d**4*e**4 + 14*a*b*c**3*d**5*e**3 - 4*a*c**4*d**6*e**2 + b**2*c**3*d**6*e**2 - 2*b*c**4*d**7*e + c**5*d**8))) + e**2*x/c

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)^2/(c*x^4+b*x^2+a),x, algorithm="giac")

[Out] Exception raised: NotImplementedError

$$3.266 \quad \int \frac{d+ex^2}{a+bx^2+cx^4} dx$$

Optimal. Leaf size=174

$$\frac{\left(\frac{2cd-be}{\sqrt{b^2-4ac}} + e\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\left(e - \frac{2cd-be}{\sqrt{b^2-4ac}}\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}$$

[Out] ((e + (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]])/(Sqrt[2]*Sqrt[c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((e - (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]])/(Sqrt[2]*Sqrt[c]*Sqrt[b + Sqrt[b^2 - 4*a*c]])

Rubi [A] time = 0.201606, antiderivative size = 174, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1166, 205}

$$\frac{\left(\frac{2cd-be}{\sqrt{b^2-4ac}} + e\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\left(e - \frac{2cd-be}{\sqrt{b^2-4ac}}\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}$$

Antiderivative was successfully verified.

[In] Int[(d + e*x^2)/(a + b*x^2 + c*x^4), x]

[Out] ((e + (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]])/(Sqrt[2]*Sqrt[c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((e - (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]])/(Sqrt[2]*Sqrt[c]*Sqrt[b + Sqrt[b^2 - 4*a*c]])

Rule 1166

Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rubi steps

$$\begin{aligned} \int \frac{d+ex^2}{a+bx^2+cx^4} dx &= \frac{1}{2} \left(e - \frac{2cd-be}{\sqrt{b^2-4ac}} \right) \int \frac{1}{\frac{b}{2} + \frac{1}{2}\sqrt{b^2-4ac} + cx^2} dx + \frac{1}{2} \left(e + \frac{2cd-be}{\sqrt{b^2-4ac}} \right) \int \frac{1}{\frac{b}{2} - \frac{1}{2}\sqrt{b^2-4ac} + cx^2} dx \\ &= \frac{\left(e + \frac{2cd-be}{\sqrt{b^2-4ac}} \right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\left(e - \frac{2cd-be}{\sqrt{b^2-4ac}} \right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b+\sqrt{b^2-4ac}}} \end{aligned}$$

Mathematica [A] time = 0.145205, size = 172, normalized size = 0.99

$$\frac{\frac{e\left(\sqrt{b^2-4ac}-b\right)+2cd}{\sqrt{b-\sqrt{b^2-4ac}}}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b-\sqrt{b^2-4ac}}}\right)+\frac{e\left(\sqrt{b^2-4ac}+b\right)-2cd}{\sqrt{\sqrt{b^2-4ac}+b}}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e*x^2)/(a + b*x^2 + c*x^4), x]

[Out] (((2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]])/Sqrt[b - Sqrt[b^2 - 4*a*c]] + ((-2*c*d + (b + Sqrt[b^2 - 4*a*c])*e)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]])/Sqrt[b + Sqrt[b^2 - 4*a*c]])/(Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c])

Maple [B] time = 0.019, size = 328, normalized size = 1.9

$$-\frac{\sqrt{2}e}{2}\operatorname{Arctanh}\left(cx\sqrt{2}\frac{1}{\sqrt{(\sqrt{-4ac+b^2}-b)c}}\right)\frac{1}{\sqrt{(\sqrt{-4ac+b^2}-b)c}}+\frac{b\sqrt{2}e}{2}\operatorname{Arctanh}\left(cx\sqrt{2}\frac{1}{\sqrt{(\sqrt{-4ac+b^2}-b)c}}\right)\frac{1}{\sqrt{(\sqrt{-4ac+b^2}-b)c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x^2+d)/(c*x^4+b*x^2+a), x)

[Out] -1/2*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*e+1/2/((-4*a*c+b^2)^(1/2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*b*e-c/((-4*a*c+b^2)^(1/2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*d+1/2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*e+1/2/((-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b*e-c/((-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*d

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex^2 + d}{cx^4 + bx^2 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)/(c*x^4+b*x^2+a), x, algorithm="maxima")

[Out] integrate((e*x^2 + d)/(c*x^4 + b*x^2 + a), x)

Fricas [B] time = 2.28824, size = 3055, normalized size = 17.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="fricas")

[Out] $\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-(b*c*d^2 - 4*a*c*d*e + a*b*e^2 + (a*b^2*c - 4*a^2*c^2)*\sqrt{(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a^2*b^2*c^2 - 4*a^3*c^3))}/(a*b^2*c - 4*a^2*c^2)}*\log(-2*(c^2*d^4 - b*c*d^3*e + a*b*d*e^3 - a^2*e^4)*x + \sqrt{\frac{1}{2}}*((b^2*c - 4*a*c^2)*d^3 - (a*b^2 - 4*a^2*c)*d*e^2 - ((a*b^3*c - 4*a^2*b*c^2)*d - 2*(a^2*b^2*c - 4*a^3*c^2)*e)*\sqrt{(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a^2*b^2*c^2 - 4*a^3*c^3))}*\sqrt{-(b*c*d^2 - 4*a*c*d*e + a*b*e^2 + (a*b^2*c - 4*a^2*c^2)*\sqrt{(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a^2*b^2*c^2 - 4*a^3*c^3))})/(a*b^2*c - 4*a^2*c^2)} - \frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-(b*c*d^2 - 4*a*c*d*e + a*b*e^2 + (a*b^2*c - 4*a^2*c^2)*\sqrt{(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a^2*b^2*c^2 - 4*a^3*c^3))}*\sqrt{(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a^2*b^2*c^2 - 4*a^3*c^3))}/(a*b^2*c - 4*a^2*c^2)}*\log(-2*(c^2*d^4 - b*c*d^3*e + a*b*d*e^3 - a^2*e^4)*x - \sqrt{\frac{1}{2}}*((b^2*c - 4*a*c^2)*d^3 - (a*b^2 - 4*a^2*c)*d*e^2 - ((a*b^3*c - 4*a^2*b*c^2)*d - 2*(a^2*b^2*c - 4*a^3*c^2)*e)*\sqrt{(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a^2*b^2*c^2 - 4*a^3*c^3))}*\sqrt{-(b*c*d^2 - 4*a*c*d*e + a*b*e^2 + (a*b^2*c - 4*a^2*c^2)*\sqrt{(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a^2*b^2*c^2 - 4*a^3*c^3))})/(a*b^2*c - 4*a^2*c^2)} + \frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-(b*c*d^2 - 4*a*c*d*e + a*b*e^2 - (a*b^2*c - 4*a^2*c^2)*\sqrt{(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a^2*b^2*c^2 - 4*a^3*c^3))}*\sqrt{(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a^2*b^2*c^2 - 4*a^3*c^3))}/(a*b^2*c - 4*a^2*c^2)}*\log(-2*(c^2*d^4 - b*c*d^3*e + a*b*d*e^3 - a^2*e^4)*x + \sqrt{\frac{1}{2}}*((b^2*c - 4*a*c^2)*d^3 - (a*b^2 - 4*a^2*c)*d*e^2 + ((a*b^3*c - 4*a^2*b*c^2)*d - 2*(a^2*b^2*c - 4*a^3*c^2)*e)*\sqrt{(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a^2*b^2*c^2 - 4*a^3*c^3))}*\sqrt{-(b*c*d^2 - 4*a*c*d*e + a*b*e^2 - (a*b^2*c - 4*a^2*c^2)*\sqrt{(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a^2*b^2*c^2 - 4*a^3*c^3))})/(a*b^2*c - 4*a^2*c^2)} - \frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-(b*c*d^2 - 4*a*c*d*e + a*b*e^2 - (a*b^2*c - 4*a^2*c^2)*\sqrt{(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a^2*b^2*c^2 - 4*a^3*c^3))}*\sqrt{(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a^2*b^2*c^2 - 4*a^3*c^3))}/(a*b^2*c - 4*a^2*c^2)}*\log(-2*(c^2*d^4 - b*c*d^3*e + a*b*d*e^3 - a^2*e^4)*x - \sqrt{\frac{1}{2}}*((b^2*c - 4*a*c^2)*d^3 - (a*b^2 - 4*a^2*c)*d*e^2 + ((a*b^3*c - 4*a^2*b*c^2)*d - 2*(a^2*b^2*c - 4*a^3*c^2)*e)*\sqrt{(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a^2*b^2*c^2 - 4*a^3*c^3))}*\sqrt{-(b*c*d^2 - 4*a*c*d*e + a*b*e^2 - (a*b^2*c - 4*a^2*c^2)*\sqrt{(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a^2*b^2*c^2 - 4*a^3*c^3))})/(a*b^2*c - 4*a^2*c^2))$

Sympy [A] time = 4.92538, size = 314, normalized size = 1.8

$$\text{RootSum}\left(t^4(256a^3c^3 - 128a^2b^2c^2 + 16ab^4c) + t^2(-16a^2bce^2 + 64a^2c^2de + 4ab^3e^2 - 16ab^2cde - 16abc^2d^2 + 4b^3cd^2) + a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x**2+d)/(c*x**4+b*x**2+a),x)

[Out] $\text{RootSum}(_t**4*(256*a**3*c**3 - 128*a**2*b**2*c**2 + 16*a*b**4*c) + _t**2*(-16*a**2*b*c*e**2 + 64*a**2*c**2*d*e + 4*a*b**3*e**2 - 16*a*b**2*c*d*e - 16*a*b*c**2*d**2 + 4*b**3*c*d**2) + a**2*e**4 - 2*a*b*d*e**3 + 2*a*c*d**2*e**2 + b**2*d**2*e**2 - 2*b*c*d**3*e + c**2*d**4, \text{Lambda}(_t, _t*\log(x + (64*_t**3*a**3*c**2*e - 16*_t**3*a**2*b**2*c*e - 32*_t**3*a**2*b*c**2*d + 8*_t**3*a*b**3*c*d - 2*_t*a**2*b*e**3 + 12*_t*a**2*c*d*e**2 - 6*_t*a*b*c*d**2*e - 4*_t*a*c**2*d**3 + 2*_t*b**2*c*d**3)/(a**2*e**4 - a*b*d*e**3 + b*c*d**3*e - c**2*d**4))))$

Giac [C] time = 2.49077, size = 6479, normalized size = 37.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="giac")

[Out] 1/2*(3*((a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)*sqrt(b^2 -
4*a*c)*b)*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))^2
*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))^3*e*sin(5/4*pi + 1
/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))) - ((a*c^3)^(3/4)*b^2 - 4*
(a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*cosh(1/2*imag_part(a
rcsin(1/2*sqrt(a*c)*b/(a*abs(c))))^3*e*sin(5/4*pi + 1/2*real_part(arcsin(1
/2*sqrt(a*c)*b/(a*abs(c))))^3 - 9*((a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c
+ (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*cos(5/4*pi + 1/2*real_part(arcsin(1/2
*sqrt(a*c)*b/(a*abs(c))))^2*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*a
bs(c))))^2*e*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))
))*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))) + 3*((a*c^3)^(3/
4)*b^2 - 4*(a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*cosh(1/2*
imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))^2*e*sin(5/4*pi + 1/2*real_pa
rt(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))^3*sinh(1/2*imag_part(arcsin(1/2*sq
rt(a*c)*b/(a*abs(c)))) + 9*((a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c + (a*c^
3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*
c)*b/(a*abs(c))))^2*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))
)*e*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))*sinh(1/
2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))^2 - 3*((a*c^3)^(3/4)*b^2 -
4*(a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*cosh(1/2*imag_par
t(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))*e*sin(5/4*pi + 1/2*real_part(arcsin(
1/2*sqrt(a*c)*b/(a*abs(c))))^3*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(
a*abs(c))))^2 - 3*((a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)
*sqrt(b^2 - 4*a*c)*b)*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*
abs(c))))^2*e*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))
))*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))^3 + ((a*c^3)^(3
/4)*b^2 - 4*(a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*e*sin(5/
4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))^3*sinh(1/2*imag_p
art(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))^3 + ((a*c^3)^(1/4)*b^2*c^2 - 4*(a*
c^3)^(1/4)*a*c^3 + (a*c^3)^(1/4)*sqrt(b^2 - 4*a*c)*b*c^2)*d*cosh(1/2*imag_p
art(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))*sin(5/4*pi + 1/2*real_part(arcsin(
1/2*sqrt(a*c)*b/(a*abs(c)))) - ((a*c^3)^(1/4)*b^2*c^2 - 4*(a*c^3)^(1/4)*a*
c^3 + (a*c^3)^(1/4)*sqrt(b^2 - 4*a*c)*b*c^2)*d*sin(5/4*pi + 1/2*real_part(a
rcsin(1/2*sqrt(a*c)*b/(a*abs(c))))*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)
*b/(a*abs(c)))))*arctan(-((a/c)^(1/4)*cos(5/4*pi + 1/2*arcsin(1/2*sqrt(a*c)
)*b/(a*abs(c))) - x)/((a/c)^(1/4)*sin(5/4*pi + 1/2*arcsin(1/2*sqrt(a*c)*b/
(a*abs(c)))))/(a*b^2*c^3 - 4*a^2*c^4) + 1/2*(3*((a*c^3)^(3/4)*b^2 - 4*(a*c
^3)^(3/4)*a*c + (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*cos(1/4*pi + 1/2*real_pa
rt(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))^2*cosh(1/2*imag_part(arcsin(1/2*sq
rt(a*c)*b/(a*abs(c))))^3*e*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*
b/(a*abs(c)))) - ((a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)*
sqrt(b^2 - 4*a*c)*b)*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))
)^3*e*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))^3 - 9
*((a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)
*b)*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))^2*cosh(
1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))^2*e*sin(1/4*pi + 1/2*rea
l_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))*sinh(1/2*imag_part(arcsin(1/2*s
qrt(a*c)*b/(a*abs(c)))) + 3*((a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c + (a*
c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(
a*abs(c))))^2*e*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)
))))^3*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))) + 9*((a*c^3
```


$$\begin{aligned}
&)*\cos(1/4*\pi + 1/2*\text{real_part}(\arcsin(1/2*\sqrt{a*c})*b/(a*\text{abs}(c))))^3*\cosh(1/2* \\
& \text{imag_part}(\arcsin(1/2*\sqrt{a*c})*b/(a*\text{abs}(c))))^3*e - 3*((a*c^3)^{(3/4)}*b^2 \\
& - 4*(a*c^3)^{(3/4)}*a*c + (a*c^3)^{(3/4)}*\sqrt{b^2 - 4*a*c})*b*\cos(1/4*\pi + 1/2* \\
& \text{real_part}(\arcsin(1/2*\sqrt{a*c})*b/(a*\text{abs}(c))))*\cosh(1/2*\text{imag_part}(\arcsin(\\
& 1/2*\sqrt{a*c})*b/(a*\text{abs}(c))))^3*e*\sin(1/4*\pi + 1/2*\text{real_part}(\arcsin(1/2*\sqrt{a*c})* \\
& b/(a*\text{abs}(c))))^2 - 3*((a*c^3)^{(3/4)}*b^2 - 4*(a*c^3)^{(3/4)}*a*c + (a*c^3)^{(3/4)}* \\
& \sqrt{b^2 - 4*a*c})*b*\cos(1/4*\pi + 1/2*\text{real_part}(\arcsin(1/2*\sqrt{a*c})*b/(a*\text{abs}(c))))^3* \\
& \cosh(1/2*\text{imag_part}(\arcsin(1/2*\sqrt{a*c})*b/(a*\text{abs}(c))))^2*e*\sinh(1/2*\text{imag_part}(\arcsin(1/2*\sqrt{a*c})* \\
& b/(a*\text{abs}(c)))) + 9*((a*c^3)^{(3/4)}*b^2 - 4*(a*c^3)^{(3/4)}*a*c + (a*c^3)^{(3/4)}* \\
& \sqrt{b^2 - 4*a*c})*b*\cos(1/4*\pi + 1/2*\text{real_part}(\arcsin(1/2*\sqrt{a*c})*b/(a*\text{abs}(c))))* \\
& \cosh(1/2*\text{imag_part}(\arcsin(1/2*\sqrt{a*c})*b/(a*\text{abs}(c))))^2*e*\sin(1/4*\pi + 1/2*\text{real_part}(\arcsin(1/2*\sqrt{a*c})* \\
& b/(a*\text{abs}(c))))^2*\sinh(1/2*\text{imag_part}(\arcsin(1/2*\sqrt{a*c})*b/(a*\text{abs}(c)))) + 3*((a*c^3)^{(3/4)}*b^2 - 4*(a*c^3)^{(3/4)}*a*c + (a*c^3)^{(3/4)}* \\
& \sqrt{b^2 - 4*a*c})*b*\cos(1/4*\pi + 1/2*\text{real_part}(\arcsin(1/2*\sqrt{a*c})*b/(a*\text{abs}(c))))^3*\cosh(1/2*\text{imag_part}(\arcsin(1/2*\sqrt{a*c})*b/(a*\text{abs}(c))))*e*\sinh(1/2*\text{imag_part}(\arcsin(1/2*\sqrt{a*c})*b/(a*\text{abs}(c))))^2 - 9*((a*c^3)^{(3/4)}*b^2 - 4*(a*c^3)^{(3/4)}*a*c + (a*c^3)^{(3/4)}* \\
& \sqrt{b^2 - 4*a*c})*b*\cos(1/4*\pi + 1/2*\text{real_part}(\arcsin(1/2*\sqrt{a*c})*b/(a*\text{abs}(c))))*\cosh(1/2*\text{imag_part}(\arcsin(1/2*\sqrt{a*c})*b/(a*\text{abs}(c))))*e*\sin(1/4*\pi + 1/2*\text{real_part}(\arcsin(1/2*\sqrt{a*c})*b/(a*\text{abs}(c))))^2*\sinh(1/2*\text{imag_part}(\arcsin(1/2*\sqrt{a*c})*b/(a*\text{abs}(c))))^2 - ((a*c^3)^{(3/4)}*b^2 - 4*(a*c^3)^{(3/4)}*a*c + (a*c^3)^{(3/4)}* \\
& \sqrt{b^2 - 4*a*c})*b*\cos(1/4*\pi + 1/2*\text{real_part}(\arcsin(1/2*\sqrt{a*c})*b/(a*\text{abs}(c))))^3*e*\sinh(1/2*\text{imag_part}(\arcsin(1/2*\sqrt{a*c})*b/(a*\text{abs}(c))))^3 + 3*((a*c^3)^{(3/4)}*b^2 - 4*(a*c^3)^{(3/4)}*a*c + (a*c^3)^{(3/4)}* \\
& \sqrt{b^2 - 4*a*c})*b*\cos(1/4*\pi + 1/2*\text{real_part}(\arcsin(1/2*\sqrt{a*c})*b/(a*\text{abs}(c))))*e*\sin(1/4*\pi + 1/2*\text{real_part}(\arcsin(1/2*\sqrt{a*c})*b/(a*\text{abs}(c))))^2*\sinh(1/2*\text{imag_part}(\arcsin(1/2*\sqrt{a*c})*b/(a*\text{abs}(c))))^3 + ((a*c^3)^{(1/4)}*b^2*c^2 - 4*(a*c^3)^{(1/4)}*a*c^3 + (a*c^3)^{(1/4)}* \\
& \sqrt{b^2 - 4*a*c})*b*c^2)*d*\cos(1/4*\pi + 1/2*\text{real_part}(\arcsin(1/2*\sqrt{a*c})*b/(a*\text{abs}(c))))*\cosh(1/2*\text{imag_part}(\arcsin(1/2*\sqrt{a*c})*b/(a*\text{abs}(c)))) - ((a*c^3)^{(1/4)}*b^2*c^2 - 4*(a*c^3)^{(1/4)}*a*c^3 + (a*c^3)^{(1/4)}* \\
& \sqrt{b^2 - 4*a*c})*b*c^2)*d*\cos(1/4*\pi + 1/2*\text{real_part}(\arcsin(1/2*\sqrt{a*c})*b/(a*\text{abs}(c))))*\sinh(1/2*\text{imag_part}(\arcsin(1/2*\sqrt{a*c})*b/(a*\text{abs}(c)))))*\log(-2*x*(a/c)^{(1/4)}*\cos(1/4*\pi + 1/2*\arcsin(1/2*\sqrt{a*c})*b/(a*\text{abs}(c)))) + x^2 + \sqrt{a/c})/(a*b^2*c^3 - 4*a^2*c^4)
\end{aligned}$$

$$3.267 \quad \int \frac{1}{a+bx^2+cx^4} dx$$

Optimal. Leaf size=150

$$\frac{\sqrt{2}\sqrt{c} \tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{2}\sqrt{c} \tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}$$

[Out] (Sqrt[2]*Sqrt[c]*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[2]*Sqrt[c]*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]])

Rubi [A] time = 0.0982543, antiderivative size = 150, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1093, 205}

$$\frac{\sqrt{2}\sqrt{c} \tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{2}\sqrt{c} \tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x^2 + c*x^4)^(-1), x]

[Out] (Sqrt[2]*Sqrt[c]*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[2]*Sqrt[c]*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]])

Rule 1093

Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rubi steps

$$\begin{aligned} \int \frac{1}{a+bx^2+cx^4} dx &= \frac{c \int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2} dx}{\sqrt{b^2-4ac}} - \frac{c \int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2} dx}{\sqrt{b^2-4ac}} \\ &= \frac{\sqrt{2}\sqrt{c} \tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{2}\sqrt{c} \tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}} \end{aligned}$$

Mathematica [A] time = 0.0863509, size = 129, normalized size = 0.86

$$\frac{\sqrt{2}\sqrt{c} \left(\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{\sqrt{b^2-4ac}+b}} \right)}{\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x^2 + c*x^4)^(-1), x]

[Out] (Sqrt[2]*Sqrt[c]*(ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]]/Sqrt[b - Sqrt[b^2 - 4*a*c]] - ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]]/Sqrt[b + Sqrt[b^2 - 4*a*c]])/Sqrt[b^2 - 4*a*c]

Maple [A] time = 0.013, size = 116, normalized size = 0.8

$$-c\sqrt{2}\operatorname{Artanh}\left(cx\sqrt{2}\frac{1}{\sqrt{(\sqrt{-4ac+b^2}-b)c}}\right)\frac{1}{\sqrt{-4ac+b^2}}\frac{1}{\sqrt{(\sqrt{-4ac+b^2}-b)c}} - c\sqrt{2}\operatorname{arctan}\left(cx\sqrt{2}\frac{1}{\sqrt{(b+\sqrt{-4ac}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(c*x^4+b*x^2+a), x)

[Out] -c/(-4*a*c+b^2)^(1/2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))-c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{cx^4 + bx^2 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x^4+b*x^2+a), x, algorithm="maxima")

[Out] integrate(1/(c*x^4 + b*x^2 + a), x)

Fricas [B] time = 2.06454, size = 1323, normalized size = 8.82

$$-\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{\frac{b+\frac{ab^2-4a^2c}{\sqrt{a^2b^2-4a^3c}}}{ab^2-4a^2c}}\log\left(2cx+\sqrt{\frac{1}{2}}\left(b^2-4ac-\frac{ab^3-4a^2bc}{\sqrt{a^2b^2-4a^3c}}\right)\sqrt{\frac{b+\frac{ab^2-4a^2c}{\sqrt{a^2b^2-4a^3c}}}{ab^2-4a^2c}}\right)+\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{\frac{b+\frac{ab^2-4a^2c}{\sqrt{a^2b^2-4a^3c}}}{ab^2-4a^2c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x^4+b*x^2+a),x, algorithm="fricas")

[Out]
$$\begin{aligned} & -1/2*\sqrt{1/2}*\sqrt{-(b + (a*b^2 - 4*a^2*c))/\sqrt{a^2*b^2 - 4*a^3*c}}/(a*b^2 - 4*a^2*c)) * \log(2*c*x + \sqrt{1/2}*(b^2 - 4*a*c - (a*b^3 - 4*a^2*b*c)/\sqrt{a^2*b^2 - 4*a^3*c})) * \sqrt{-(b + (a*b^2 - 4*a^2*c))/\sqrt{a^2*b^2 - 4*a^3*c}}/(a*b^2 - 4*a^2*c)) \\ & + 1/2*\sqrt{1/2}*\sqrt{-(b + (a*b^2 - 4*a^2*c))/\sqrt{a^2*b^2 - 4*a^3*c}}/(a*b^2 - 4*a^2*c)) * \log(2*c*x - \sqrt{1/2}*(b^2 - 4*a*c - (a*b^3 - 4*a^2*b*c)/\sqrt{a^2*b^2 - 4*a^3*c})) * \sqrt{-(b + (a*b^2 - 4*a^2*c))/\sqrt{a^2*b^2 - 4*a^3*c}}/(a*b^2 - 4*a^2*c)) \\ & - 1/2*\sqrt{1/2}*\sqrt{-(b - (a*b^2 - 4*a^2*c))/\sqrt{a^2*b^2 - 4*a^3*c}}/(a*b^2 - 4*a^2*c)) * \log(2*c*x + \sqrt{1/2}*(b^2 - 4*a*c + (a*b^3 - 4*a^2*b*c)/\sqrt{a^2*b^2 - 4*a^3*c})) * \sqrt{-(b - (a*b^2 - 4*a^2*c))/\sqrt{a^2*b^2 - 4*a^3*c}}/(a*b^2 - 4*a^2*c)) \\ & + 1/2*\sqrt{1/2}*\sqrt{-(b - (a*b^2 - 4*a^2*c))/\sqrt{a^2*b^2 - 4*a^3*c}}/(a*b^2 - 4*a^2*c)) * \log(2*c*x - \sqrt{1/2}*(b^2 - 4*a*c + (a*b^3 - 4*a^2*b*c)/\sqrt{a^2*b^2 - 4*a^3*c})) * \sqrt{-(b - (a*b^2 - 4*a^2*c))/\sqrt{a^2*b^2 - 4*a^3*c}}/(a*b^2 - 4*a^2*c)) \end{aligned}$$

Sympy [A] time = 0.905584, size = 87, normalized size = 0.58

$$\text{RootSum}\left(t^4(256a^3c^2 - 128a^2b^2c + 16ab^4) + t^2(-16abc + 4b^3) + c, \left(t \mapsto t \log\left(x + \frac{32t^3a^2bc - 8t^3ab^3 + 4tac - 2tb^2}{c}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x**4+b*x**2+a),x)

[Out]
$$\text{RootSum}(_t**4*(256*a**3*c**2 - 128*a**2*b**2*c + 16*a*b**4) + _t**2*(-16*a*b*c + 4*b**3) + c, \text{Lambda}(_t, _t*\log(x + (32*_t**3*a**2*b*c - 8*_t**3*a*b**3 + 4*_t*a*c - 2*_t*b**2)/c)))$$

Giac [C] time = 1.41836, size = 1365, normalized size = 9.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x^4+b*x^2+a),x, algorithm="giac")

[Out]
$$\begin{aligned} & 1/2*(((a*c^3)^{(1/4)}*b^2 - 4*(a*c^3)^{(1/4)}*a*c + (a*c^3)^{(1/4)}*\sqrt{b^2 - 4*a*c})*b) * \cosh(1/2*\text{imag_part}(\arcsin(1/2*\sqrt{a*c})*b/(a*\text{abs}(c)))) * \sin(5/4*\pi + 1/2*\text{real_part}(\arcsin(1/2*\sqrt{a*c})*b/(a*\text{abs}(c)))) - ((a*c^3)^{(1/4)}*b^2 - 4*(a*c^3)^{(1/4)}*a*c + (a*c^3)^{(1/4)}*\sqrt{b^2 - 4*a*c})*b) * \sin(5/4*\pi + 1/2*\text{real_part}(\arcsin(1/2*\sqrt{a*c})*b/(a*\text{abs}(c)))) * \sinh(1/2*\text{imag_part}(\arcsin(1/2*\sqrt{a*c})*b/(a*\text{abs}(c)))) * \arctan(-((a/c)^{(1/4)}*\cos(5/4*\pi + 1/2*\arcsin(1/2*\sqrt{a*c})*b/(a*\text{abs}(c)))) - x)/((a/c)^{(1/4)}*\sin(5/4*\pi + 1/2*\arcsin(1/2*\sqrt{a*c})*b/(a*\text{abs}(c)))))/(a*b^2*c - 4*a^2*c^2) + 1/2*(((a*c^3)^{(1/4)}*b^2 - 4*(a*c^3)^{(1/4)}*a*c + (a*c^3)^{(1/4)}*\sqrt{b^2 - 4*a*c})*b) * \cosh(1/2*\text{imag_part}(\arcsin(1/2*\sqrt{a*c})*b/(a*\text{abs}(c)))) * \sin(1/4*\pi + 1/2*\text{real_part}(\arcsin(1/2*\sqrt{a*c})*b/(a*\text{abs}(c)))) - ((a*c^3)^{(1/4)}*b^2 - 4*(a*c^3)^{(1/4)}*a*c + (a*c^3)^{(1/4)}*\sqrt{b^2 - 4*a*c})*b) * \sin(1/4*\pi + 1/2*\text{real_part}(\arcsin(1/2*\sqrt{a*c})*b/(a*\text{abs}(c)))) * \sinh(1/2*\text{imag_part}(\arcsin(1/2*\sqrt{a*c})*b/(a*\text{abs}(c)))) * \arctan(-((a/c)^{(1/4)}*\cos(1/4*\pi + 1/2*\arcsin(1/2*\sqrt{a*c})*b/(a*\text{abs}(c)))) - x)/((a/c)^{(1/4)}*\sin(1/4*\pi + 1/2*\arcsin(1/2*\sqrt{a*c})*b/(a*\text{abs}(c)))))/(a*b^2*c - 4*a^2*c^2) - 1/4*(((a*c^3)^{(1/4)}*b^2 - 4*(a*c^3)^{(1/4)}*a*c + (a*c^3)^{(1/4)}*\sqrt{b^2 - 4*a*c})*b) * \cosh(1/2*\text{imag_part}(\arcsin(1/2*\sqrt{a*c})*b/(a*\text{abs}(c)))) * \sin(5/4*\pi + 1/2*\text{real_part}(\arcsin(1/2*\sqrt{a*c})*b/(a*\text{abs}(c)))) - ((a*c^3)^{(1/4)}*b^2 - 4*(a*c^3)^{(1/4)}*a*c + (a*c^3)^{(1/4)}*\sqrt{b^2 - 4*a*c})*b) * \sin(5/4*\pi + 1/2*\text{real_part}(\arcsin(1/2*\sqrt{a*c})*b/(a*\text{abs}(c)))) * \sinh(1/2*\text{imag_part}(\arcsin(1/2*\sqrt{a*c})*b/(a*\text{abs}(c)))) * \arctan(-((a/c)^{(1/4)}*\cos(5/4*\pi + 1/2*\arcsin(1/2*\sqrt{a*c})*b/(a*\text{abs}(c)))) - x)/((a/c)^{(1/4)}*\sin(5/4*\pi + 1/2*\arcsin(1/2*\sqrt{a*c})*b/(a*\text{abs}(c)))))/(a*b^2*c - 4*a^2*c^2) \end{aligned}$$

```

*c^3)^(1/4)*sqrt(b^2 - 4*a*c)*b)*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt
(a*c)*b/(a*abs(c)))))*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))
)) - ((a*c^3)^(1/4)*b^2 - 4*(a*c^3)^(1/4)*a*c + (a*c^3)^(1/4)*sqrt(b^2 - 4*
a*c)*b)*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))))*sin
h(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))))*log(-2*x*(a/c)^(1/4)*
cos(5/4*pi + 1/2*arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))) + x^2 + sqrt(a/c))/(a*
b^2*c - 4*a^2*c^2) - 1/4*(((a*c^3)^(1/4)*b^2 - 4*(a*c^3)^(1/4)*a*c + (a*c^3
)^(1/4)*sqrt(b^2 - 4*a*c)*b)*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c
)*b/(a*abs(c))))))*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))) -
((a*c^3)^(1/4)*b^2 - 4*(a*c^3)^(1/4)*a*c + (a*c^3)^(1/4)*sqrt(b^2 - 4*a*c
)*b)*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))))*sinh(1/
2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))))*log(-2*x*(a/c)^(1/4)*cos(
1/4*pi + 1/2*arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))) + x^2 + sqrt(a/c))/(a*b^2*
c - 4*a^2*c^2)

```

$$3.268 \quad \int \frac{1}{(d+ex^2)(a+bx^2+cx^4)} dx$$

Optimal. Leaf size=254

$$\frac{\sqrt{c} \left(e - \frac{2cd-be}{\sqrt{b^2-4ac}} \right) \tan^{-1} \left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b-\sqrt{b^2-4ac}}} \right) - \sqrt{c} \left(\frac{2cd-be}{\sqrt{b^2-4ac}} + e \right) \tan^{-1} \left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{\sqrt{b^2-4ac}+b}} \right) + \frac{e^{3/2} \tan^{-1} \left(\frac{\sqrt{ex}}{\sqrt{d}} \right)}{\sqrt{d} (ae^2 - bde + cd^2)}}{\sqrt{2}\sqrt{b-\sqrt{b^2-4ac}} (ae^2 - bde + cd^2) - \sqrt{2}\sqrt{\sqrt{b^2-4ac}+b} (ae^2 - bde + cd^2) + \sqrt{d} (ae^2 - bde + cd^2)}$$

[Out] -((Sqrt[c]*(e - (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]])/(Sqrt[2]*Sqrt[b - Sqrt[b^2 - 4*a*c]]*(c*d^2 - b*d*e + a*e^2)) - (Sqrt[c]*(e + (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]])/(Sqrt[2]*Sqrt[b + Sqrt[b^2 - 4*a*c]]*(c*d^2 - b*d*e + a*e^2)) + (e^(3/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[d]*(c*d^2 - b*d*e + a*e^2))

Rubi [A] time = 0.585595, antiderivative size = 254, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1170, 205, 1166}

$$\frac{\sqrt{c} \left(e - \frac{2cd-be}{\sqrt{b^2-4ac}} \right) \tan^{-1} \left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b-\sqrt{b^2-4ac}}} \right) - \sqrt{c} \left(\frac{2cd-be}{\sqrt{b^2-4ac}} + e \right) \tan^{-1} \left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{\sqrt{b^2-4ac}+b}} \right) + \frac{e^{3/2} \tan^{-1} \left(\frac{\sqrt{ex}}{\sqrt{d}} \right)}{\sqrt{d} (ae^2 - bde + cd^2)}}{\sqrt{2}\sqrt{b-\sqrt{b^2-4ac}} (ae^2 - bde + cd^2) - \sqrt{2}\sqrt{\sqrt{b^2-4ac}+b} (ae^2 - bde + cd^2) + \sqrt{d} (ae^2 - bde + cd^2)}$$

Antiderivative was successfully verified.

[In] Int[1/((d + e*x^2)*(a + b*x^2 + c*x^4)),x]

[Out] -((Sqrt[c]*(e - (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]])/(Sqrt[2]*Sqrt[b - Sqrt[b^2 - 4*a*c]]*(c*d^2 - b*d*e + a*e^2)) - (Sqrt[c]*(e + (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]])/(Sqrt[2]*Sqrt[b + Sqrt[b^2 - 4*a*c]]*(c*d^2 - b*d*e + a*e^2)) + (e^(3/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[d]*(c*d^2 - b*d*e + a*e^2))

Rule 1170

Int[((d_) + (e_)*(x_)^2)^(q_)/((a_) + (b_)*(x_)^2 + (c_)*(x_)^4), x_Symbol] := Int[ExpandIntegrand[(d + e*x^2)^q/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[q]

Rule 205

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 1166

Int[((d_) + (e_)*(x_)^2)/((a_) + (b_)*(x_)^2 + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]

Rubi steps

$$\int \frac{1}{(d + ex^2)(a + bx^2 + cx^4)} dx = \int \left(\frac{e^2}{(cd^2 - bde + ae^2)(d + ex^2)} + \frac{cd - be - cex^2}{(cd^2 - bde + ae^2)(a + bx^2 + cx^4)} \right) dx$$

$$= \frac{\int \frac{cd - be - cex^2}{a + bx^2 + cx^4} dx}{cd^2 - bde + ae^2} + \frac{e^2 \int \frac{1}{d + ex^2} dx}{cd^2 - bde + ae^2}$$

$$= \frac{e^{3/2} \tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)}{\sqrt{d}(cd^2 - bde + ae^2)} - \frac{\left(c\left(e - \frac{2cd - be}{\sqrt{b^2 - 4ac}}\right)\right) \int \frac{1}{\frac{b}{2} - \frac{1}{2}\sqrt{b^2 - 4ac} + cx^2} dx}{2(cd^2 - bde + ae^2)} - \frac{\left(c\left(e + \frac{2cd - be}{\sqrt{b^2 - 4ac}}\right)\right) \int \frac{1}{\frac{b}{2} + \frac{1}{2}\sqrt{b^2 - 4ac} + cx^2} dx}{2(cd^2 - bde + ae^2)}$$

$$= -\frac{\sqrt{c}\left(e - \frac{2cd - be}{\sqrt{b^2 - 4ac}}\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b - \sqrt{b^2 - 4ac}}}\right)}{\sqrt{2}\sqrt{b - \sqrt{b^2 - 4ac}}(cd^2 - bde + ae^2)} - \frac{\sqrt{c}\left(e + \frac{2cd - be}{\sqrt{b^2 - 4ac}}\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b + \sqrt{b^2 - 4ac}}}\right)}{\sqrt{2}\sqrt{b + \sqrt{b^2 - 4ac}}(cd^2 - bde + ae^2)} + \dots$$

Mathematica [A] time = 0.286987, size = 274, normalized size = 1.08

$$\frac{\sqrt{c}\left(e\sqrt{b^2 - 4ac} + be - 2cd\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b - \sqrt{b^2 - 4ac}}}\right)}{\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{b - \sqrt{b^2 - 4ac}}(-ae^2 + bde - cd^2)} + \frac{\sqrt{c}\left(e\sqrt{b^2 - 4ac} - be + 2cd\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{\sqrt{b^2 - 4ac} + b}}\right)}{\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{\sqrt{b^2 - 4ac} + b}(-ae^2 + bde - cd^2)} + \frac{e^{3/2} \tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)}{\sqrt{d}(ae^2 - bde + cd^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (Sqrt[c]*(-2*c*d + b*e + Sqrt[b^2 - 4*a*c]*e)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]]/(Sqrt[2]*Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]*(-(c*d^2) + b*d*e - a*e^2)) + (Sqrt[c]*(2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]]/(Sqrt[2]*Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]*(-(c*d^2) + b*d*e - a*e^2)) + (e^(3/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[d]*(c*d^2 - b*d*e + a*e^2))
```

Maple [B] time = 0.023, size = 480, normalized size = 1.9

$$\frac{c\sqrt{2}e}{2ae^2 - 2deb + 2cd^2} \operatorname{Arctanh}\left(cx\sqrt{2}\frac{1}{\sqrt{(\sqrt{-4ac + b^2} - b)c}}\right) \frac{1}{\sqrt{(\sqrt{-4ac + b^2} - b)c}} + \frac{c\sqrt{2}be}{2ae^2 - 2deb + 2cd^2} \operatorname{Arctanh}\left(\dots\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d)/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/2/(a*e^2-b*d*e+c*d^2)*c*2^(1/2)/((((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/((((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)))+e*1/2/(a*e^2-b*d*e+c*d^2)*c/((-4*a*c+b^2)^(1/2)*2^(1/2)/((((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/((((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)))+b*e-1/(a*e^2-b*d*e+c*d^2)*c^2/((-4*a*c+b^2)^(1/2)*2^(1/2)/((((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/((((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)))+d-1/2/(a*e^2-b*d*e+c*d^2)*c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))+e*1/2/(a*e^2-b*d*e+c*d^2)*c/((-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))+b*e-1
```

$$\frac{(a^2e^2 - bde + cd^2)c^2 / (-4ac + b^2)^{1/2} \cdot 2^{1/2} / ((b + (-4ac + b^2)^{1/2})^{1/2}) \cdot c^{1/2} \cdot \arctan(c \cdot x \cdot 2^{1/2} / ((b + (-4ac + b^2)^{1/2})^{1/2}) \cdot c^{1/2}) \cdot d + e^2 / (a^2e^2 - bde + cd^2) / (de)^{1/2} \cdot \arctan(xe / (de)^{1/2})}{1}$$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="fricas")

[Out] Timed out

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x**2+d)/(c*x**4+b*x**2+a),x)

[Out] Timed out

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="giac")

[Out] Exception raised: NotImplementedError

$$3.269 \quad \int \frac{1}{(d+ex^2)^2(a+bx^2+cx^4)} dx$$

Optimal. Leaf size=429

$$\frac{\sqrt{c} \left(-2ce \left(d\sqrt{b^2 - 4ac} + ae + bd \right) + be^2 \left(\sqrt{b^2 - 4ac} + b \right) + 2c^2d^2 \right) \tan^{-1} \left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b - \sqrt{b^2 - 4ac}}} \right) - \sqrt{c} \left(-2ce \left(-d\sqrt{b^2 - 4ac} + ae + bd \right) + be^2 \left(\sqrt{b^2 - 4ac} + b \right) + 2c^2d^2 \right) \tan^{-1} \left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b - \sqrt{b^2 - 4ac}}} \right)}{\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{b - \sqrt{b^2 - 4ac}}(ae^2 - bde + cd^2)^2} - \frac{\sqrt{c} \left(-2ce \left(-d\sqrt{b^2 - 4ac} + ae + bd \right) + be^2 \left(\sqrt{b^2 - 4ac} + b \right) + 2c^2d^2 \right) \tan^{-1} \left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b - \sqrt{b^2 - 4ac}}} \right)}{\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{b - \sqrt{b^2 - 4ac}}(ae^2 - bde + cd^2)^2}$$

```
[Out] (e^2*x)/(2*d*(c*d^2 - b*d*e + a*e^2)*(d + e*x^2)) + (Sqrt[c]*(2*c^2*d^2 + b*(b + Sqrt[b^2 - 4*a*c]))*e^2 - 2*c*e*(b*d + Sqrt[b^2 - 4*a*c]*d + a*e))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]]/(Sqrt[2]*Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]*(c*d^2 - b*d*e + a*e^2)^2) - (Sqrt[c]*(2*c^2*d^2 + b*(b - Sqrt[b^2 - 4*a*c]))*e^2 - 2*c*e*(b*d - Sqrt[b^2 - 4*a*c]*d + a*e))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]]/(Sqrt[2]*Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]*(c*d^2 - b*d*e + a*e^2)^2) + (e^(3/2)*(2*c*d - b*e)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[d]*(c*d^2 - b*d*e + a*e^2)^2) + (e^(3/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(3/2)*(c*d^2 - b*d*e + a*e^2))
```

Rubi [A] time = 1.4145, antiderivative size = 429, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1170, 199, 205, 1166}

$$\frac{\sqrt{c} \left(-2ce \left(d\sqrt{b^2 - 4ac} + ae + bd \right) + be^2 \left(\sqrt{b^2 - 4ac} + b \right) + 2c^2d^2 \right) \tan^{-1} \left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b - \sqrt{b^2 - 4ac}}} \right) - \sqrt{c} \left(-2ce \left(-d\sqrt{b^2 - 4ac} + ae + bd \right) + be^2 \left(\sqrt{b^2 - 4ac} + b \right) + 2c^2d^2 \right) \tan^{-1} \left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b - \sqrt{b^2 - 4ac}}} \right)}{\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{b - \sqrt{b^2 - 4ac}}(ae^2 - bde + cd^2)^2} - \frac{\sqrt{c} \left(-2ce \left(-d\sqrt{b^2 - 4ac} + ae + bd \right) + be^2 \left(\sqrt{b^2 - 4ac} + b \right) + 2c^2d^2 \right) \tan^{-1} \left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b - \sqrt{b^2 - 4ac}}} \right)}{\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{b - \sqrt{b^2 - 4ac}}(ae^2 - bde + cd^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x^2)^2*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (e^2*x)/(2*d*(c*d^2 - b*d*e + a*e^2)*(d + e*x^2)) + (Sqrt[c]*(2*c^2*d^2 + b*(b + Sqrt[b^2 - 4*a*c]))*e^2 - 2*c*e*(b*d + Sqrt[b^2 - 4*a*c]*d + a*e))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]]/(Sqrt[2]*Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]*(c*d^2 - b*d*e + a*e^2)^2) - (Sqrt[c]*(2*c^2*d^2 + b*(b - Sqrt[b^2 - 4*a*c]))*e^2 - 2*c*e*(b*d - Sqrt[b^2 - 4*a*c]*d + a*e))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]]/(Sqrt[2]*Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]*(c*d^2 - b*d*e + a*e^2)^2) + (e^(3/2)*(2*c*d - b*e)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[d]*(c*d^2 - b*d*e + a*e^2)^2) + (e^(3/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(3/2)*(c*d^2 - b*d*e + a*e^2))
```

Rule 1170

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x^2)^q/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[q]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> -Simp[(x*(a + b*x^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^(p_)]
```

```
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (IntegerQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denominator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{(d + ex^2)^2 (a + bx^2 + cx^4)} dx = \int \left(\frac{e^2}{(cd^2 - bde + ae^2)(d + ex^2)^2} - \frac{e^2(-2cd + be)}{(cd^2 - bde + ae^2)^2 (d + ex^2)} + \frac{c^2d^2 + b^2e^2 - ce(2bd + ae)}{(cd^2 - bde + ae^2)^2} \right) dx$$

$$= \frac{\int \frac{c^2d^2 + b^2e^2 - ce(2bd + ae) - ce(2cd - be)x^2}{a + bx^2 + cx^4} dx}{(cd^2 - bde + ae^2)^2} + \frac{(e^2(2cd - be)) \int \frac{1}{d + ex^2} dx}{(cd^2 - bde + ae^2)^2} + \frac{e^2 \int \frac{1}{(d + ex^2)^2} dx}{cd^2 - bde + ae^2}$$

$$= \frac{e^2x}{2d(cd^2 - bde + ae^2)(d + ex^2)} + \frac{e^{3/2}(2cd - be) \tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)}{\sqrt{d}(cd^2 - bde + ae^2)^2} + \frac{e^2 \int \frac{1}{d + ex^2} dx}{2d(cd^2 - bde + ae^2)} + \dots$$

$$= \frac{e^2x}{2d(cd^2 - bde + ae^2)(d + ex^2)} + \frac{\sqrt{c}\left(2c^2d^2 + b(b + \sqrt{b^2 - 4ac})e^2 - 2ce(bd + \sqrt{b^2 - 4ac})\right)}{\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{b - \sqrt{b^2 - 4ac}}(cd^2 - bde + ae^2)^2}$$

Mathematica [A] time = 0.825196, size = 354, normalized size = 0.83

$$\frac{\sqrt{2}\sqrt{c}\left(-2ce\left(d\sqrt{b^2-4ac}+ae+bd\right)+be^2\left(\sqrt{b^2-4ac}+b\right)+2c^2d^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\sqrt{2}\sqrt{c}\left(2ce\left(-d\sqrt{b^2-4ac}+ae+bd\right)+be^2\left(\sqrt{b^2-4ac}-b\right)-2c^2d^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}$$

$$2(e(ae - bd) + cd^2)^2$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x^2)^2*(a + b*x^2 + c*x^4)), x]
```

```
[Out] ((e^2*(c*d^2 + e*(-(b*d) + a*e))*x)/(d*(d + e*x^2)) + (Sqrt[2]*Sqrt[c]*(2*c^2*d^2 + b*(b + Sqrt[b^2 - 4*a*c]))*e^2 - 2*c*e*(b*d + Sqrt[b^2 - 4*a*c]*d + a*e))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]]/(Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*Sqrt[c]*(-2*c^2*d^2 + b*(-b + Sqrt[b^2 - 4*a*c]))*e^2 + 2*c*e*(b*d - Sqrt[b^2 - 4*a*c]*d + a*e))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]]/(Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + (e^(3/2)*(5*c*d^2 + e*(-3*b*d + a*e))*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/d^(3/2))/(2*(c*d^2 + e*(-(b*d) + a*e))^2)
```

Maple [B] time = 0.03, size = 1141, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\int (1/(e*x^2+d)^2/(c*x^4+b*x^2+a), x)$

[Out]
$$-1/2/(a*e^2-b*d*e+c*d^2)^2*c^2^{(1/2)/(((-4*a*c+b^2)^{(1/2)}-b)*c)^{(1/2)}*\arctan h(c*x*2^{(1/2)/(((-4*a*c+b^2)^{(1/2)}-b)*c)^{(1/2)}}*e^2*b+1/(a*e^2-b*d*e+c*d^2)^2*c^2*2^{(1/2)/(((-4*a*c+b^2)^{(1/2)}-b)*c)^{(1/2)}*\arctanh(c*x*2^{(1/2)/(((-4*a*c+b^2)^{(1/2)}-b)*c)^{(1/2)}}*d*e+1/(a*e^2-b*d*e+c*d^2)^2*c^2/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)/(((-4*a*c+b^2)^{(1/2)}-b)*c)^{(1/2)}*\arctanh(c*x*2^{(1/2)/(((-4*a*c+b^2)^{(1/2)}-b)*c)^{(1/2)}}*a*e^2-1/2/(a*e^2-b*d*e+c*d^2)^2*c/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)/(((-4*a*c+b^2)^{(1/2)}-b)*c)^{(1/2)}*\arctanh(c*x*2^{(1/2)/(((-4*a*c+b^2)^{(1/2)}-b)*c)^{(1/2)}}*b^2*e^2+1/(a*e^2-b*d*e+c*d^2)^2*c^2/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)/(((-4*a*c+b^2)^{(1/2)}-b)*c)^{(1/2)}*\arctanh(c*x*2^{(1/2)/(((-4*a*c+b^2)^{(1/2)}-b)*c)^{(1/2)}}*d*e*b-1/(a*e^2-b*d*e+c*d^2)^2*c^3/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)/(((-4*a*c+b^2)^{(1/2)}-b)*c)^{(1/2)}*\arctanh(c*x*2^{(1/2)/(((-4*a*c+b^2)^{(1/2)}-b)*c)^{(1/2)}}*d^2+1/2/(a*e^2-b*d*e+c*d^2)^2*c^2^{(1/2)/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}}*e^2*b-1/(a*e^2-b*d*e+c*d^2)^2*c^2*2^{(1/2)/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}}*\arctan(c*x*2^{(1/2)/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}}*d*e+1/(a*e^2-b*d*e+c*d^2)^2*c^2/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}}*\arctan(c*x*2^{(1/2)/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}}*a*e^2-1/2/(a*e^2-b*d*e+c*d^2)^2*c/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}}*\arctan(c*x*2^{(1/2)/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}}*b^2*e^2+1/(a*e^2-b*d*e+c*d^2)^2*c^2/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}}*\arctan(c*x*2^{(1/2)/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}}*d*e*b-1/(a*e^2-b*d*e+c*d^2)^2*c^3/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}}*\arctan(c*x*2^{(1/2)/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}}*d^2+1/2*e^4/(a*e^2-b*d*e+c*d^2)^2/d*x/(e*x^2+d)*a-1/2*e^3/(a*e^2-b*d*e+c*d^2)^2*x/(e*x^2+d)*b+1/2*e^2/(a*e^2-b*d*e+c*d^2)^2*d*x/(e*x^2+d)*c+1/2*e^4/(a*e^2-b*d*e+c*d^2)^2/d/(d*e)^{(1/2)}*\arctan(x*e/(d*e)^{(1/2)})*a-3/2*e^3/(a*e^2-b*d*e+c*d^2)^2/(d*e)^{(1/2)}*\arctan(x*e/(d*e)^{(1/2)})*b+5/2*e^2/(a*e^2-b*d*e+c*d^2)^2*d/(d*e)^{(1/2)}*\arctan(x*e/(d*e)^{(1/2)})*c$$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(1/(e*x^2+d)^2/(c*x^4+b*x^2+a), x, \text{algorithm}="maxima")$

[Out] Exception raised: ValueError

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^2/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)**2/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^2/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.270
$$\int \frac{(d+ex^2)^3}{(a+bx^2+cx^4)^2} dx$$

Optimal. Leaf size=563

$$\frac{\left(-b^2\left(ae^3\sqrt{b^2-4ac}-3acde^2+c^2d^3\right)+6ac\left(ae^2+cd^2\right)\left(e\sqrt{b^2-4ac}+2cd\right)-bc\left(cd^2\left(d\sqrt{b^2-4ac}+12ae\right)+ae^2\left(3d\right.\right.\right.}{2\sqrt{2}ac^{3/2}\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}$$

```
[Out] (x*(c*(b^2*d^3 - 2*a*d*(c*d^2 - 3*a*e^2) - (a*b*e*(3*c*d^2 + a*e^2))/c) - (a*b^2*e^3 + 2*a*c*e*(3*c*d^2 - a*e^2) - b*c*d*(c*d^2 + 3*a*e^2))*x^2))/(2*a*c*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - ((a*b^3*e^3 + 6*a*c*(2*c*d + Sqrt[b^2 - 4*a*c]*e)*(c*d^2 + a*e^2) - b^2*(c^2*d^3 - 3*a*c*d*e^2 + a*Sqrt[b^2 - 4*a*c]*e^3) - b*c*(a*e^2*(3*Sqrt[b^2 - 4*a*c]*d + 8*a*e) + c*d^2*(Sqrt[b^2 - 4*a*c]*d + 12*a*e)))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*c^(3/2)*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((a*b^3*e^3 + 6*a*c*(2*c*d - Sqrt[b^2 - 4*a*c]*e)*(c*d^2 + a*e^2) - b^2*(c^2*d^3 - 3*a*c*d*e^2 - a*Sqrt[b^2 - 4*a*c]*e^3) + b*c*(c*d^2*(Sqrt[b^2 - 4*a*c]*d - 12*a*e) + a*e^2*(3*Sqrt[b^2 - 4*a*c]*d - 8*a*e)))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*c^(3/2)*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rubi [A] time = 3.51858, antiderivative size = 563, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1205, 1166, 205}

$$\frac{\left(-b^2\left(ae^3\sqrt{b^2-4ac}-3acde^2+c^2d^3\right)+6ac\left(ae^2+cd^2\right)\left(e\sqrt{b^2-4ac}+2cd\right)-bc\left(cd^2\left(d\sqrt{b^2-4ac}+12ae\right)+ae^2\left(3d\right.\right.\right.}{2\sqrt{2}ac^{3/2}\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^3/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (x*(c*(b^2*d^3 - 2*a*d*(c*d^2 - 3*a*e^2) - (a*b*e*(3*c*d^2 + a*e^2))/c) - (a*b^2*e^3 + 2*a*c*e*(3*c*d^2 - a*e^2) - b*c*d*(c*d^2 + 3*a*e^2))*x^2))/(2*a*c*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - ((a*b^3*e^3 + 6*a*c*(2*c*d + Sqrt[b^2 - 4*a*c]*e)*(c*d^2 + a*e^2) - b^2*(c^2*d^3 - 3*a*c*d*e^2 + a*Sqrt[b^2 - 4*a*c]*e^3) - b*c*(a*e^2*(3*Sqrt[b^2 - 4*a*c]*d + 8*a*e) + c*d^2*(Sqrt[b^2 - 4*a*c]*d + 12*a*e)))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*c^(3/2)*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((a*b^3*e^3 + 6*a*c*(2*c*d - Sqrt[b^2 - 4*a*c]*e)*(c*d^2 + a*e^2) - b^2*(c^2*d^3 - 3*a*c*d*e^2 - a*Sqrt[b^2 - 4*a*c]*e^3) + b*c*(c*d^2*(Sqrt[b^2 - 4*a*c]*d - 12*a*e) + a*e^2*(3*Sqrt[b^2 - 4*a*c]*d - 8*a*e)))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*c^(3/2)*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 1205

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> With[{f = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 + c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*g - f*(b^2 - 2*
```

```
a*c) - c*(b*f - 2*a*g)*x^2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[(d + e*x^2)^q, a + b*x^2 + c*x^4, x] + b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g + c*(4*p + 7)*(b*f - 2*a*g)*x^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1] && LtQ[p, -1]
```

Rule 1166

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (b_)*(x_)^2 + (c_)*(x_)^4), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{(d + ex^2)^3}{(a + bx^2 + cx^4)^2} dx = \frac{x \left(c \left(b^2 d^3 - 2ad(cd^2 - 3ae^2) - \frac{abe(3cd^2 + ae^2)}{c} \right) - (ab^2e^3 + 2ace(3cd^2 - ae^2) - bcd(cd^2 + 3ae^2)) \right) x^2}{2ac(b^2 - 4ac)(a + bx^2 + cx^4)}$$

$$= \frac{x \left(c \left(b^2 d^3 - 2ad(cd^2 - 3ae^2) - \frac{abe(3cd^2 + ae^2)}{c} \right) - (ab^2e^3 + 2ace(3cd^2 - ae^2) - bcd(cd^2 + 3ae^2)) \right) x^2}{2ac(b^2 - 4ac)(a + bx^2 + cx^4)}$$

$$= \frac{x \left(c \left(b^2 d^3 - 2ad(cd^2 - 3ae^2) - \frac{abe(3cd^2 + ae^2)}{c} \right) - (ab^2e^3 + 2ace(3cd^2 - ae^2) - bcd(cd^2 + 3ae^2)) \right) x^2}{2ac(b^2 - 4ac)(a + bx^2 + cx^4)}$$

Mathematica [A] time = 1.71249, size = 540, normalized size = 0.96

$$\frac{2\sqrt{c}x(b(-a^2e^3 - 3acde(d - ex^2) + c^2d^3x^2) + b^2(cd^3 - ae^3x^2) + 2ac(ae^2(3d + ex^2) - cd^2(d + 3ex^2)))}{(b^2 - 4ac)(a + bx^2 + cx^4)} + \frac{\sqrt{2}(b^2(ae^3\sqrt{b^2 - 4ac} - 3acde^2 + c^2d^3) - 6ac(ae^2 + cd^2)(e\sqrt{b^2 - 4ac} + 2cd))}{(b^2 - 4ac)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^3/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] ((2*Sqrt[c]*x*(b^2*(c*d^3 - a*e^3*x^2) + b*(-(a^2*e^3) + c^2*d^3*x^2 - 3*a*c*d*e*(d - e*x^2)) + 2*a*c*(a*e^2*(3*d + e*x^2) - c*d^2*(d + 3*e*x^2))))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt[2]*(-(a*b^3*e^3) - 6*a*c*(2*c*d + Sqrt[b^2 - 4*a*c]*e)*(c*d^2 + a*e^2) + b^2*(c^2*d^3 - 3*a*c*d*e^2 + a*Sqrt[b^2 - 4*a*c]*e^3) + b*c*(a*e^2*(3*Sqrt[b^2 - 4*a*c]*d + 8*a*e) + c*d^2*(Sqrt[b^2 - 4*a*c]*d + 12*a*e))))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]]/((b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*(a*b^3*e^3 + 6*a*c*(2*c*d - Sqrt[b^2 - 4*a*c]*e)*(c*d^2 + a*e^2) + b^2*(-(c^2*d^3) + 3*a*c*d*e^2 + a*Sqrt[b^2 - 4*a*c]*e^3) + b*c*(c*d^2*(Sqrt[b^2 - 4*a*c]*d - 12*a*e) + a*e^2*(3*Sqrt[b^2 - 4*a*c]*d - 8*a*e))))*ArcTan[(Sqrt[2]*S
```

$$\frac{\sqrt{c}x/\sqrt{b + \sqrt{b^2 - 4ac}}}{((b^2 - 4ac)^{3/2}\sqrt{b + \sqrt{b^2 - 4ac}})/(4ac^{3/2})}$$

Maple [B] time = 0.047, size = 1846, normalized size = 3.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x^2+d)^3/(c*x^4+b*x^2+a)^2,x)

[Out]
$$\begin{aligned} & -3a/(4ac-b^2)*c/(-4ac+b^2)^{(1/2)}*2^{(1/2)}/(((-4ac+b^2)^{(1/2)}-b)*c)^{(1/2)} \\ & * \operatorname{arctanh}(c*x^2^{(1/2)}/(((-4ac+b^2)^{(1/2)}-b)*c)^{(1/2)})*d*e^2+3/(4ac-b^2)*c/(-4ac+b^2)^{(1/2)}*2^{(1/2)}/(((-4ac+b^2)^{(1/2)}-b)*c)^{(1/2)} \\ & * \operatorname{arctanh}(c*x^2^{(1/2)}/(((-4ac+b^2)^{(1/2)}-b)*c)^{(1/2)})*b*d^2*e+1/4/a/(4ac-b^2)*c/(-4ac+b^2)^{(1/2)}*2^{(1/2)}/(((-4ac+b^2)^{(1/2)}-b)*c)^{(1/2)} \\ & * \operatorname{arctanh}(c*x^2^{(1/2)}/(((-4ac+b^2)^{(1/2)}-b)*c)^{(1/2)})/(((-4ac+b^2)^{(1/2)}-b)*c)^{(1/2)}*b^2*d^3-3a/(4ac-b^2)*c/(-4ac+b^2)^{(1/2)}*2^{(1/2)}/((b+(-4ac+b^2)^{(1/2)})*c)^{(1/2)} \\ & * \operatorname{arctan}(c*x^2^{(1/2)}/((b+(-4ac+b^2)^{(1/2)})*c)^{(1/2)})*d*e^2+3/(4ac-b^2)*c/(-4ac+b^2)^{(1/2)}*2^{(1/2)}/((b+(-4ac+b^2)^{(1/2)})*c)^{(1/2)} \\ & * \operatorname{arctan}(c*x^2^{(1/2)}/((b+(-4ac+b^2)^{(1/2)})*c)^{(1/2)})/(((-4ac+b^2)^{(1/2)}-b)*c)^{(1/2)}*b*d^2*e+1/4/a/(4ac-b^2)*c/(-4ac+b^2)^{(1/2)}*2^{(1/2)}/((b+(-4ac+b^2)^{(1/2)})*c)^{(1/2)} \\ & * \operatorname{arctan}(c*x^2^{(1/2)}/((b+(-4ac+b^2)^{(1/2)})*c)^{(1/2)})*b^2*d^3+(-1/2*(2a^2*c*e^3-ab^2*e^3+3a*b*c*d*e^2-6a*c^2*d^2*e+b*c^2*d^3)/a/c/(4ac-b^2)*x^3+1/2/c*(a^2*b*e^3-6a^2*c*d*e^2+3a*b*c*d^2*e+2a*c^2*d^3-b^2*c*d^3)/(4ac-b^2)/a*x)/(c*x^4+b*x^2+a)+1/4/a/(4ac-b^2)*c*2^{(1/2)}/(((-4ac+b^2)^{(1/2)}-b)*c)^{(1/2)} \\ & * \operatorname{arctanh}(c*x^2^{(1/2)}/(((-4ac+b^2)^{(1/2)}-b)*c)^{(1/2)})*b*d^3+2a/(4ac-b^2)/(-4ac+b^2)^{(1/2)}*2^{(1/2)}/(((-4ac+b^2)^{(1/2)}-b)*c)^{(1/2)} \\ & * \operatorname{arctanh}(c*x^2^{(1/2)}/(((-4ac+b^2)^{(1/2)}-b)*c)^{(1/2)})*b^3*e^3-3/4/(4ac-b^2)/(-4ac+b^2)^{(1/2)}*2^{(1/2)}/(((-4ac+b^2)^{(1/2)}-b)*c)^{(1/2)} \\ & * \operatorname{arctanh}(c*x^2^{(1/2)}/(((-4ac+b^2)^{(1/2)}-b)*c)^{(1/2)})*b^2*d*e^2-1/4/a/(4ac-b^2)*c*2^{(1/2)}/((b+(-4ac+b^2)^{(1/2)})*c)^{(1/2)} \\ & * \operatorname{arctan}(c*x^2^{(1/2)}/((b+(-4ac+b^2)^{(1/2)})*c)^{(1/2)})*b*d^3+2a/(4ac-b^2)/(-4ac+b^2)^{(1/2)}*2^{(1/2)}/((b+(-4ac+b^2)^{(1/2)})*c)^{(1/2)} \\ & * \operatorname{arctan}(c*x^2^{(1/2)}/((b+(-4ac+b^2)^{(1/2)})*c)^{(1/2)})*b^3*e^3-3/4/(4ac-b^2)/(-4ac+b^2)^{(1/2)}*2^{(1/2)}/((b+(-4ac+b^2)^{(1/2)})*c)^{(1/2)} \\ & * \operatorname{arctan}(c*x^2^{(1/2)}/((b+(-4ac+b^2)^{(1/2)})*c)^{(1/2)})*b^2*d*e^2-3/2*a/(4ac-b^2)*2^{(1/2)}/(((-4ac+b^2)^{(1/2)}-b)*c)^{(1/2)} \\ & * \operatorname{arctanh}(c*x^2^{(1/2)}/(((-4ac+b^2)^{(1/2)}-b)*c)^{(1/2)})*e^3+3/2*a/(4ac-b^2)*2^{(1/2)}/((b+(-4ac+b^2)^{(1/2)})*c)^{(1/2)} \\ & * \operatorname{arctan}(c*x^2^{(1/2)}/((b+(-4ac+b^2)^{(1/2)})*c)^{(1/2)})*e^3+1/4/(4ac-b^2)/c*2^{(1/2)}/(((-4ac+b^2)^{(1/2)}-b)*c)^{(1/2)} \\ & * \operatorname{arctanh}(c*x^2^{(1/2)}/(((-4ac+b^2)^{(1/2)}-b)*c)^{(1/2)})*b^2*e^3+3/4/(4ac-b^2)*2^{(1/2)}/(((-4ac+b^2)^{(1/2)}-b)*c)^{(1/2)} \\ & * \operatorname{arctanh}(c*x^2^{(1/2)}/(((-4ac+b^2)^{(1/2)}-b)*c)^{(1/2)})*b*d*e^2-3/2/(4ac-b^2)*c*2^{(1/2)}/(((-4ac+b^2)^{(1/2)}-b)*c)^{(1/2)} \\ & * \operatorname{arctanh}(c*x^2^{(1/2)}/(((-4ac+b^2)^{(1/2)}-b)*c)^{(1/2)})*d^2*e-3/(4ac-b^2)*c^2/(-4ac+b^2)^{(1/2)}*2^{(1/2)}/((b+(-4ac+b^2)^{(1/2)})*c)^{(1/2)} \\ & * \operatorname{arctan}(c*x^2^{(1/2)}/((b+(-4ac+b^2)^{(1/2)})*c)^{(1/2)})*b^2*e^3-3/4/(4ac-b^2)*2^{(1/2)}/((b+(-4ac+b^2)^{(1/2)})*c)^{(1/2)} \\ & * \operatorname{arctan}(c*x^2^{(1/2)}/((b+(-4ac+b^2)^{(1/2)})*c)^{(1/2)})*b*d*e^2+3/2/(4ac-b^2)*c*2^{(1/2)}/((b+(-4ac+b^2)^{(1/2)})*c)^{(1/2)} \\ & * \operatorname{arctan}(c*x^2^{(1/2)}/((b+(-4ac+b^2)^{(1/2)})*c)^{(1/2)})*d^2*e-3/(4ac-b^2)*c^2/(-4ac+b^2)^{(1/2)}*2^{(1/2)}/((b+(-4ac+b^2)^{(1/2)})*c)^{(1/2)} \\ & * \operatorname{arctan}(c*x^2^{(1/2)}/((b+(-4ac+b^2)^{(1/2)})*c)^{(1/2)})*d^3 \end{aligned}$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{(bc^2d^3 - 6ac^2d^2e + 3abcde^2 - (ab^2 - 2a^2c)e^3)x^3 - (3abcd^2e - 6a^2cde^2 + a^2be^3 - (b^2c - 2ac^2)d^3)x - \int \frac{3abcd^2e - 6a^2cde^2}{2(a^2b^2c - 4a^3c^2 + (ab^2c^2 - 4a^2c^3)x^4 + (ab^3c - 4a^2bc^2)x^2)} dx}{2(a^2b^2c - 4a^3c^2 + (ab^2c^2 - 4a^2c^3)x^4 + (ab^3c - 4a^2bc^2)x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)^3/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")

[Out] 1/2*((b*c^2*d^3 - 6*a*c^2*d^2*e + 3*a*b*c*d*e^2 - (a*b^2 - 2*a^2*c)*e^3)*x^3 - (3*a*b*c*d^2*e - 6*a^2*c*d*e^2 + a^2*b*e^3 - (b^2*c - 2*a*c^2)*d^3)*x)/(a^2*b^2*c - 4*a^3*c^2 + (a*b^2*c^2 - 4*a^2*c^3)*x^4 + (a*b^3*c - 4*a^2*b*c^2)*x^2) - 1/2*integrate(-(3*a*b*c*d^2*e - 6*a^2*c*d*e^2 + a^2*b*e^3 + (b^2*c - 6*a*c^2)*d^3 + (b*c^2*d^3 - 6*a*c^2*d^2*e + 3*a*b*c*d*e^2 + (a*b^2 - 6*a^2*c)*e^3)*x^2)/(c*x^4 + b*x^2 + a), x)/(a*b^2*c - 4*a^2*c^2)

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)^3/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")

[Out] Timed out

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x**2+d)**3/(c*x**4+b*x**2+a)**2,x)

[Out] Timed out

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)^3/(c*x^4+b*x^2+a)^2,x, algorithm="giac")

[Out] Exception raised: NotImplementedError

$$3.271 \quad \int \frac{(d+ex^2)^2}{(a+bx^2+cx^4)^2} dx$$

Optimal. Leaf size=386

$$\frac{x^2 (abe^2 - 4acde + bcd^2) - 2abde - 2a(cd^2 - ae^2) + b^2d^2}{2a(b^2 - 4ac)(a + bx^2 + cx^4)} + \frac{\left(\frac{b^2(cd^2 - ae^2) + 8abcde - 4ac(ae^2 + 3cd^2)}{\sqrt{b^2 - 4ac}} + abe^2 - 4acde + bcd^2\right) \sqrt{b - \sqrt{b^2 - 4ac}}}{2\sqrt{2a}\sqrt{c}(b^2 - 4ac)\sqrt{b - \sqrt{b^2 - 4ac}}}$$

```
[Out] (x*(b^2*d^2 - 2*a*b*d*e - 2*a*(c*d^2 - a*e^2) + (b*c*d^2 - 4*a*c*d*e + a*b*
e^2)*x^2))/(2*a*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + ((b*c*d^2 - 4*a*c*d*e
+ a*b*e^2 + (8*a*b*c*d*e + b^2*(c*d^2 - a*e^2) - 4*a*c*(3*c*d^2 + a*e^2))/S
qrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]])/
(2*Sqrt[2]*a*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((b*c*d^2
- 4*a*c*d*e + a*b*e^2 - (8*a*b*c*d*e + b^2*(c*d^2 - a*e^2) - 4*a*c*(3*c*d^
2 + a*e^2))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2
- 4*a*c]])/(2*Sqrt[2]*a*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 - 4*a*c]]
)
```

Rubi [A] time = 2.07855, antiderivative size = 386, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1205, 1166, 205}

$$\frac{x^2 (abe^2 - 4acde + bcd^2) - 2abde - 2a(cd^2 - ae^2) + b^2d^2}{2a(b^2 - 4ac)(a + bx^2 + cx^4)} + \frac{\left(\frac{b^2(cd^2 - ae^2) + 8abcde - 4ac(ae^2 + 3cd^2)}{\sqrt{b^2 - 4ac}} + abe^2 - 4acde + bcd^2\right) \sqrt{b - \sqrt{b^2 - 4ac}}}{2\sqrt{2a}\sqrt{c}(b^2 - 4ac)\sqrt{b - \sqrt{b^2 - 4ac}}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^2/(a + b*x^2 + c*x^4)^2, x]
```

```
[Out] (x*(b^2*d^2 - 2*a*b*d*e - 2*a*(c*d^2 - a*e^2) + (b*c*d^2 - 4*a*c*d*e + a*b*
e^2)*x^2))/(2*a*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + ((b*c*d^2 - 4*a*c*d*e
+ a*b*e^2 + (8*a*b*c*d*e + b^2*(c*d^2 - a*e^2) - 4*a*c*(3*c*d^2 + a*e^2))/S
qrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]])/
(2*Sqrt[2]*a*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((b*c*d^2
- 4*a*c*d*e + a*b*e^2 - (8*a*b*c*d*e + b^2*(c*d^2 - a*e^2) - 4*a*c*(3*c*d^
2 + a*e^2))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2
- 4*a*c]])/(2*Sqrt[2]*a*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 - 4*a*c]]
)
```

Rule 1205

```
Int[((d_) + (e_)*(x_)^2)^(q_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x
_Symbol] :> With[{f = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*g - f*(b^2 - 2*
a*c) - c*(b*f - 2*a*g)*x^2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*
(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p
+ 1)*(b^2 - 4*a*c)*PolynomialQuotient[(d + e*x^2)^q, a + b*x^2 + c*x^4, x]
+ b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g + c*(4*p + 7)*(b*f - 2*a*g)*x
^2, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[
c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1] && LtQ[p, -1]
```

Rule 1166

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (b_)*(x_)^2 + (c_)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
- q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{(d + ex^2)^2}{(a + bx^2 + cx^4)^2} dx = \frac{x(b^2d^2 - 2abde - 2a(cd^2 - ae^2) + (bcd^2 - 4acde + abe^2)x^2)}{2a(b^2 - 4ac)(a + bx^2 + cx^4)} - \frac{\int \frac{-b^2d^2 - 2abde + 2a(3cd^2 + ae^2) + (-bcd^2 + abe^2)x^2}{a + bx^2 + cx^4} dx}{2a(b^2 - 4ac)}$$

$$= \frac{x(b^2d^2 - 2abde - 2a(cd^2 - ae^2) + (bcd^2 - 4acde + abe^2)x^2)}{2a(b^2 - 4ac)(a + bx^2 + cx^4)} + \frac{\left(bcd^2 - 4acde + abe^2 - \frac{8abcde + b^3d}{4a}\right)}{2a(b^2 - 4ac)}$$

$$= \frac{x(b^2d^2 - 2abde - 2a(cd^2 - ae^2) + (bcd^2 - 4acde + abe^2)x^2)}{2a(b^2 - 4ac)(a + bx^2 + cx^4)} + \frac{\left(bcd^2 - 4acde + abe^2 + \frac{8abcde + b^3d}{4a}\right)}{2\sqrt{2}a\sqrt{c}(b^2 - 4ac)}$$

Mathematica [A] time = 1.18703, size = 415, normalized size = 1.08

$$\frac{2x(2a^2e^2 + abe(ex^2 - 2d) - 2acd(d + 2ex^2) + b^2d^2 + bcd^2x^2)}{(b^2 - 4ac)(a + bx^2 + cx^4)} + \frac{\sqrt{2}(b^2(cd^2 - ae^2) - 4ac(e(d\sqrt{b^2 - 4ac} + ae) + 3cd^2) + b(cd(d\sqrt{b^2 - 4ac} + 8ae) + ae^2\sqrt{b^2 - 4ac})) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b - \sqrt{b^2 - 4ac}}}\right)}{\sqrt{c}(b^2 - 4ac)^{3/2}\sqrt{b - \sqrt{b^2 - 4ac}}}$$

4a

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^2/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] ((2*x*(b^2*d^2 + 2*a^2*e^2 + b*c*d^2*x^2 + a*b*e*(-2*d + e*x^2) - 2*a*c*d*(
d + 2*e*x^2)))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt[2]*(b^2*(c*d^2 -
a*e^2) - 4*a*c*(3*c*d^2 + e*(Sqrt[b^2 - 4*a*c]*d + a*e)) + b*(a*Sqrt[b^2 -
4*a*c]*e^2 + c*d*(Sqrt[b^2 - 4*a*c]*d + 8*a*e)))*ArcTan[(Sqrt[2]*Sqrt[c]*x
)/Sqrt[b - Sqrt[b^2 - 4*a*c]])/(Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[
b^2 - 4*a*c]]) + (Sqrt[2]*(b^2*(-(c*d^2) + a*e^2) + b*(a*Sqrt[b^2 - 4*a*c]*
e^2 + c*d*(Sqrt[b^2 - 4*a*c]*d - 8*a*e)) + 4*a*c*(3*c*d^2 + e*(-(Sqrt[b^2 -
4*a*c]*d) + a*e)))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]])
)/(Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(4*a)
```

Maple [B] time = 0.039, size = 1223, normalized size = 3.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x^2+d)^2/(c*x^4+b*x^2+a)^2,x)

[Out]
$$\begin{aligned} & (-1/2/a*(a*b*e^2-4*a*c*d*e+b*c*d^2)/(4*a*c-b^2)*x^3-1/2*(2*a^2*e^2-2*a*b*d* \\ & e-2*a*c*d^2+b^2*d^2)/a/(4*a*c-b^2)*x)/(c*x^4+b*x^2+a)+1/4/(4*a*c-b^2)*2^{(1/2)} \\ & /(((-4*a*c+b^2)^{(1/2)}-b)*c)^{(1/2)}*\operatorname{arctanh}(c*x*2^{(1/2)}/(((-4*a*c+b^2)^{(1/2)} \\ &)-b)*c)^{(1/2)})*b*e^2-1/(4*a*c-b^2)*c*2^{(1/2)}/(((-4*a*c+b^2)^{(1/2)}-b)*c)^{(1/2)} \\ & *2^{(1/2)}*\operatorname{arctanh}(c*x*2^{(1/2)}/(((-4*a*c+b^2)^{(1/2)}-b)*c)^{(1/2)})*d*e+1/4/a/(4*a*c-b \\ & ^2)*c*2^{(1/2)}/(((-4*a*c+b^2)^{(1/2)}-b)*c)^{(1/2)}*\operatorname{arctanh}(c*x*2^{(1/2)}/(((-4*a* \\ & c+b^2)^{(1/2)}-b)*c)^{(1/2)})*b*d^2-a/(4*a*c-b^2)*c/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/ \\ & (((-4*a*c+b^2)^{(1/2)}-b)*c)^{(1/2)}*\operatorname{arctanh}(c*x*2^{(1/2)}/(((-4*a*c+b^2)^{(1/2)}-b \\ &)*c)^{(1/2)})*e^2-1/4/(4*a*c-b^2)/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/(((-4*a*c+b^2)^{(1/2)} \\ &)-b)*c)^{(1/2)}*\operatorname{arctanh}(c*x*2^{(1/2)}/(((-4*a*c+b^2)^{(1/2)}-b)*c)^{(1/2)})*b^2* \\ & e^2+2/(4*a*c-b^2)*c/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/(((-4*a*c+b^2)^{(1/2)}-b)*c)^{(1/2)} \\ & *2^{(1/2)}*\operatorname{arctanh}(c*x*2^{(1/2)}/(((-4*a*c+b^2)^{(1/2)}-b)*c)^{(1/2)})*b*d*e-3/(4*a*c-b \\ & ^2)*c^2/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/(((-4*a*c+b^2)^{(1/2)}-b)*c)^{(1/2)}*\operatorname{arctanh} \\ & (c*x*2^{(1/2)}/(((-4*a*c+b^2)^{(1/2)}-b)*c)^{(1/2)})*d^2+1/4/a/(4*a*c-b^2)*c/(-4* \\ & a*c+b^2)^{(1/2)}*2^{(1/2)}/(((-4*a*c+b^2)^{(1/2)}-b)*c)^{(1/2)}*\operatorname{arctanh}(c*x*2^{(1/2)} \\ & /(((-4*a*c+b^2)^{(1/2)}-b)*c)^{(1/2)})*b^2*d^2-1/4/(4*a*c-b^2)*2^{(1/2)}/((b+(-4* \\ & a*c+b^2)^{(1/2)})*c)^{(1/2)}*\operatorname{arctan}(c*x*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)} \\ &))*b*e^2+1/(4*a*c-b^2)*c*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\operatorname{arctan}(c* \\ & x*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)})*d*e-1/4/a/(4*a*c-b^2)*c*2^{(1/2)} \\ & /((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\operatorname{arctan}(c*x*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)} \\ &)*c)^{(1/2)})*b*d^2-a/(4*a*c-b^2)*c/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)} \\ &)*c)^{(1/2)}*\operatorname{arctan}(c*x*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)})*e^2 \\ & -1/4/(4*a*c-b^2)/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)} \\ & *2^{(1/2)}*\operatorname{arctan}(c*x*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)})*b^2*e^2+2/(4*a*c-b^2) \\ & *c/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\operatorname{arctan}(c*x* \\ & 2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)})*b*d*e-3/(4*a*c-b^2)*c^2/(-4*a*c+b \\ & ^2)^{(1/2)}*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\operatorname{arctan}(c*x*2^{(1/2)}/((b+ \\ & (-4*a*c+b^2)^{(1/2)})*c)^{(1/2)})*d^2+1/4/a/(4*a*c-b^2)*c/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)} \\ & /((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\operatorname{arctan}(c*x*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)} \\ &)*c)^{(1/2)})*b^2*d^2 \end{aligned}$$

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)^2/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")

[Out] Timed out

Fricas [B] time = 40.5869, size = 14976, normalized size = 38.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)^2/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")

[Out]
$$\begin{aligned} & 1/4*(2*(b*c*d^2 - 4*a*c*d*e + a*b*e^2)*x^3 + \operatorname{sqrt}(1/2)*((a*b^2*c - 4*a^2*c^2) \\ & *x^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2)*\operatorname{sqrt}(-((b^5*c - 15*a* \\ & b^3*c^2 + 60*a^2*b*c^3)*d^4 + 4*(a*b^4*c - 6*a^2*b^2*c^2 - 24*a^3*c^3)*d^3* \end{aligned}$$

$$\begin{aligned}
& e - 2*(a^2*b^3*c - 52*a^3*b*c^2)*d^2*e^2 - 8*(3*a^3*b^2*c + 4*a^4*c^2)*d*e^3 \\
& + (a^3*b^3 + 12*a^4*b*c)*e^4 + (a^3*b^6*c - 12*a^4*b^4*c^2 + 48*a^5*b^2*c^3 - 64*a^6*c^4)*\sqrt{-(16*a^3*b*c^2*d^5*e^3 + 8*a^4*b*c*d^3*e^5 - 4*a^5*c*d^2*e^6 - a^6*e^8 - (b^4*c^2 - 18*a*b^2*c^3 + 81*a^2*c^4)*d^8 - 8*(a*b^3*c^2 - 9*a^2*b*c^3)*d^7*e - 12*(a^2*b^2*c^2 + 3*a^3*c^3)*d^6*e^2 + 2*(a^3*b^2*c - 11*a^4*c^2)*d^4*e^4)/(a^6*b^6*c^2 - 12*a^7*b^4*c^3 + 48*a^8*b^2*c^4 - 64*a^9*c^5)))/((5*b^4*c^3 - 81*a*b^2*c^4 + 324*a^2*c^5)*d^8 - 2*(3*b^5*c^2 - 65*a*b^3*c^3 + 324*a^2*b*c^4)*d^7*e + (b^6*c - 51*a*b^4*c^2 + 336*a^2*b^2*c^3 + 432*a^3*c^4)*d^6*e^2 + 2*(3*a*b^5*c - 27*a^2*b^3*c^2 - 244*a^3*b*c^3)*d^5*e^3 + (3*a^2*b^4*c + 150*a^3*b^2*c^2 + 152*a^4*c^3)*d^4*e^4 - 10*(a^3*b^3*c + 12*a^4*b*c^2)*d^3*e^5 - (a^3*b^4 - 24*a^4*b^2*c - 48*a^5*c^2)*d^2*e^6 - 2*(a^4*b^3 + 12*a^5*b*c)*d*e^7 + (3*a^5*b^2 + 4*a^6*c)*e^8)*x + 1/2*\sqrt{1/2}*((b^8*c - 23*a*b^6*c^2 + 190*a^2*b^4*c^3 - 672*a^3*b^2*c^4 + 864*a^4*c^5)*d^6 + 6*(a*b^7*c - 15*a^2*b^5*c^2 + 72*a^3*b^3*c^3 - 112*a^4*b*c^4)*d^5*e + 2*(2*a^2*b^6*c - a^3*b^4*c^2 - 88*a^4*b^2*c^3 + 240*a^5*c^4)*d^4*e^2 - 12*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^3*e^3 - (a^3*b^6 - 18*a^4*b^4*c + 96*a^5*b^2*c^2 - 160*a^6*c^3)*d^2*e^4 - 2*(a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*d*e^5 + 2*(a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*e^6 - ((a^3*b^9*c - 20*a^4*b^7*c^2 + 144*a^5*b^5*c^3 - 448*a^6*b^3*c^4 + 512*a^7*b*c^5)*d^2 + 2*(a^4*b^8*c - 8*a^5*b^6*c^2 + 128*a^7*b^2*c^4 - 256*a^8*c^5)*d*e - 4*(a^5*b^7*c - 12*a^6*b^5*c^2 + 48*a^7*b^3*c^3 - 64*a^8*b*c^4)*e^2)*\sqrt{-(16*a^3*b*c^2*d^5*e^3 + 8*a^4*b*c*d^3*e^5 - 4*a^5*c*d^2*e^6 - a^6*e^8 - (b^4*c^2 - 18*a*b^2*c^3 + 81*a^2*c^4)*d^8 - 8*(a*b^3*c^2 - 9*a^2*b*c^3)*d^7*e - 12*(a^2*b^2*c^2 + 3*a^3*c^3)*d^6*e^2 + 2*(a^3*b^2*c - 11*a^4*c^2)*d^4*e^4)/(a^6*b^6*c^2 - 12*a^7*b^4*c^3 + 48*a^8*b^2*c^4 - 64*a^9*c^5)))*\sqrt{-((b^5*c - 15*a*b^3*c^2 + 60*a^2*b*c^3)*d^4 + 4*(a*b^4*c - 6*a^2*b^2*c^2 - 24*a^3*c^3)*d^3*e - 2*(a^2*b^3*c - 52*a^3*b*c^2)*d^2*e^2 - 8*(3*a^3*b^2*c + 4*a^4*c^2)*d*e^3 + (a^3*b^3 + 12*a^4*b*c)*e^4 + (a^3*b^6*c - 12*a^4*b^4*c^2 + 48*a^5*b^2*c^3 - 64*a^6*c^4)*\sqrt{-(16*a^3*b*c^2*d^5*e^3 + 8*a^4*b*c*d^3*e^5 - 4*a^5*c*d^2*e^6 - a^6*e^8 - (b^4*c^2 - 18*a*b^2*c^3 + 81*a^2*c^4)*d^8 - 8*(a*b^3*c^2 - 9*a^2*b*c^3)*d^7*e - 12*(a^2*b^2*c^2 + 3*a^3*c^3)*d^6*e^2 + 2*(a^3*b^2*c - 11*a^4*c^2)*d^4*e^4)/(a^6*b^6*c^2 - 12*a^7*b^4*c^3 + 48*a^8*b^2*c^4 - 64*a^9*c^5)))/((5*b^4*c^3 - 81*a*b^2*c^4 + 324*a^2*c^5)*d^8 - 2*(3*b^5*c^2 - 65*a*b^3*c^3 + 324*a^2*b*c^4)*d^7*e + (b^6*c - 51*a*b^4*c^2 + 336*a^2*b^2*c^3 + 432*a^3*c^4)*d^6*e^2 + 2*(3*a*b^5*c - 27*a^2*b^3*c^2 - 244*a^3*b*c^3)*d^5*e^3 + (3*a^2*b^4*c + 150*a^3*b^2*c^2 + 152*a^4*c^3)*d^4*e^4 - 10*(a^3*b^3*c + 12*a^4*b*c^2)*d^3*e^5 - (a^3*b^4 - 24*a^4*b^2*c - 48*a^5*c^2)*d^2*e^6 - 2*(a^4*b^3 + 12*a^5*b*c)*d*e^7 + (3*a^5*b^2 + 4*a^6*c)*e^8)*x - 1/2*\sqrt{1/2}*((b^8*c - 23*a*b^6*c^2 + 190*a^2*b^4*c^3 - 672*a^3*b^2*c^4 + 864*a^4*c^5)*d^6 + 6*(a*b^7*c - 15*a^2*b^5*c^2 + 72*a^3*b^3*c^3 - 112*a^4*b*c^4)*d^5*e + 2*(2*a^2*b^6*c - a^3*b^4*c^2 - 88*a^4*b^2*c^3 + 240*a^5*c^4)*d^4*e^2 - 12*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^3*e^3 - (a^3*b^6 - 18*a^4*b^4*c + 96*a^5*b^2*c^2 - 160*a^6*c^3)*d^2*e^4 - 2*(a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*d*e^5 + 2*(a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*e^6 - ((a^3*b^9*c - 20*a^4*b^7*c^2 + 144*a^5*b^5*c^3 - 448*a^6*b^3*c^4 + 512*a^7*b*c^5)*d^2 + 2*(a^4*b^8*c - 8*a^5*b^6*c^2 + 128*a^7*b^2*c^4 - 256*a^8*c^5)*d*e - 4*(a^5*b^7*c - 12*a^6*b^5*c^2 + 48*a^7*b^3*c^3 - 64*a^8*b*c^4)*e^2)*\sqrt{-(16*a^3*b*c^2*d^5*e^3 + 8*a^4*b*c*d^3*e^5 - 4*a^5*c*d^2}
\end{aligned}$$

$$\begin{aligned} &^2*b^2*c^3 + 432*a^3*c^4)*d^6*e^2 + 2*(3*a*b^5*c - 27*a^2*b^3*c^2 - 244*a^3 \\ &*b*c^3)*d^5*e^3 + (3*a^2*b^4*c + 150*a^3*b^2*c^2 + 152*a^4*c^3)*d^4*e^4 - 1 \\ &0*(a^3*b^3*c + 12*a^4*b*c^2)*d^3*e^5 - (a^3*b^4 - 24*a^4*b^2*c - 48*a^5*c^2 \\ &)*d^2*e^6 - 2*(a^4*b^3 + 12*a^5*b*c)*d*e^7 + (3*a^5*b^2 + 4*a^6*c)*e^8)*x - \\ &1/2*sqrt(1/2)*((b^8*c - 23*a*b^6*c^2 + 190*a^2*b^4*c^3 - 672*a^3*b^2*c^4 + \\ &864*a^4*c^5)*d^6 + 6*(a*b^7*c - 15*a^2*b^5*c^2 + 72*a^3*b^3*c^3 - 112*a^4* \\ &b*c^4)*d^5*e + 2*(2*a^2*b^6*c - a^3*b^4*c^2 - 88*a^4*b^2*c^3 + 240*a^5*c^4) \\ &*d^4*e^2 - 12*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^3*e^3 - (a^3*b^6 \\ &- 18*a^4*b^4*c + 96*a^5*b^2*c^2 - 160*a^6*c^3)*d^2*e^4 - 2*(a^4*b^5 - 8*a^ \\ &5*b^3*c + 16*a^6*b*c^2)*d*e^5 + 2*(a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*e^6 \\ &+ ((a^3*b^9*c - 20*a^4*b^7*c^2 + 144*a^5*b^5*c^3 - 448*a^6*b^3*c^4 + 512*a^ \\ &7*b*c^5)*d^2 + 2*(a^4*b^8*c - 8*a^5*b^6*c^2 + 128*a^7*b^2*c^4 - 256*a^8*c^5 \\ &)*d*e - 4*(a^5*b^7*c - 12*a^6*b^5*c^2 + 48*a^7*b^3*c^3 - 64*a^8*b*c^4)*e^2) \\ &*sqrt(-(16*a^3*b*c^2*d^5*e^3 + 8*a^4*b*c*d^3*e^5 - 4*a^5*c*d^2*e^6 - a^6*e^ \\ &8 - (b^4*c^2 - 18*a*b^2*c^3 + 81*a^2*c^4)*d^8 - 8*(a*b^3*c^2 - 9*a^2*b*c^3) \\ &*d^7*e - 12*(a^2*b^2*c^2 + 3*a^3*c^3)*d^6*e^2 + 2*(a^3*b^2*c - 11*a^4*c^2)* \\ &d^4*e^4)/(a^6*b^6*c^2 - 12*a^7*b^4*c^3 + 48*a^8*b^2*c^4 - 64*a^9*c^5))*sqrt \\ &t(-(b^5*c - 15*a*b^3*c^2 + 60*a^2*b*c^3)*d^4 + 4*(a*b^4*c - 6*a^2*b^2*c^2 \\ &- 24*a^3*c^3)*d^3*e - 2*(a^2*b^3*c - 52*a^3*b*c^2)*d^2*e^2 - 8*(3*a^3*b^2*c \\ &+ 4*a^4*c^2)*d*e^3 + (a^3*b^3 + 12*a^4*b*c)*e^4 - (a^3*b^6*c - 12*a^4*b^4* \\ &c^2 + 48*a^5*b^2*c^3 - 64*a^6*c^4)*sqrt(-(16*a^3*b*c^2*d^5*e^3 + 8*a^4*b*c* \\ &d^3*e^5 - 4*a^5*c*d^2*e^6 - a^6*e^8 - (b^4*c^2 - 18*a*b^2*c^3 + 81*a^2*c^4) \\ &*d^8 - 8*(a*b^3*c^2 - 9*a^2*b*c^3)*d^7*e - 12*(a^2*b^2*c^2 + 3*a^3*c^3)*d^6 \\ &*e^2 + 2*(a^3*b^2*c - 11*a^4*c^2)*d^4*e^4)/(a^6*b^6*c^2 - 12*a^7*b^4*c^3 + \\ &48*a^8*b^2*c^4 - 64*a^9*c^5)))/(a^3*b^6*c - 12*a^4*b^4*c^2 + 48*a^5*b^2*c^3 \\ &- 64*a^6*c^4)) - 2*(2*a*b*d*e - 2*a^2*e^2 - (b^2 - 2*a*c)*d^2)*x)/((a*b^2 \\ &*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2) \end{aligned}$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x**2+d)**2/(c*x**4+b*x**2+a)**2,x)

[Out] Timed out

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)^2/(c*x^4+b*x^2+a)^2,x, algorithm="giac")

[Out] Exception raised: NotImplementedError

$$3.272 \quad \int \frac{d+ex^2}{(a+bx^2+cx^4)^2} dx$$

Optimal. Leaf size=293

$$\frac{x(cx^2(bd-2ae)-abe-2acd+b^2d)}{2a(b^2-4ac)(a+bx^2+cx^4)} + \frac{\sqrt{c}\left(\frac{4abe-12acd+b^2d}{\sqrt{b^2-4ac}} - 2ae + bd\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a(b^2-4ac)\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\sqrt{c}\left(-\frac{4abe-12acd+b^2d}{\sqrt{b^2-4ac}} - 2ae\right)}{2\sqrt{2}a(b^2-4ac)}$$

```
[Out] (x*(b^2*d - 2*a*c*d - a*b*e + c*(b*d - 2*a*e)*x^2))/(2*a*(b^2 - 4*a*c)*(a +
b*x^2 + c*x^4)) + (Sqrt[c]*(b*d - 2*a*e + (b^2*d - 12*a*c*d + 4*a*b*e)/Sqr
t[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]]/(2
*Sqrt[2]*a*(b^2 - 4*a*c)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[c]*(b*d - 2*a
*e - (b^2*d - 12*a*c*d + 4*a*b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c
]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]]/(2*Sqrt[2]*a*(b^2 - 4*a*c)*Sqrt[b + Sqrt
[b^2 - 4*a*c]])
```

Rubi [A] time = 0.789019, antiderivative size = 293, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {1178, 1166, 205}

$$\frac{x(cx^2(bd-2ae)-abe-2acd+b^2d)}{2a(b^2-4ac)(a+bx^2+cx^4)} + \frac{\sqrt{c}\left(\frac{4abe-12acd+b^2d}{\sqrt{b^2-4ac}} - 2ae + bd\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a(b^2-4ac)\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\sqrt{c}\left(-\frac{4abe-12acd+b^2d}{\sqrt{b^2-4ac}} - 2ae\right)}{2\sqrt{2}a(b^2-4ac)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)/(a + b*x^2 + c*x^4)^2, x]
```

```
[Out] (x*(b^2*d - 2*a*c*d - a*b*e + c*(b*d - 2*a*e)*x^2))/(2*a*(b^2 - 4*a*c)*(a +
b*x^2 + c*x^4)) + (Sqrt[c]*(b*d - 2*a*e + (b^2*d - 12*a*c*d + 4*a*b*e)/Sqr
t[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]]/(2
*Sqrt[2]*a*(b^2 - 4*a*c)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[c]*(b*d - 2*a
*e - (b^2*d - 12*a*c*d + 4*a*b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c
]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]]/(2*Sqrt[2]*a*(b^2 - 4*a*c)*Sqrt[b + Sqrt
[b^2 - 4*a*c]])
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] := Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 +
c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2
- 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
- q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
```

$Q[cd^2 - ae^2, 0] \ \&\& \ \text{PosQ}[b^2 - 4ac]$

Rule 205

$\text{Int}[(a + (b \cdot x^2)^{-1}), x_Symbol] \rightarrow \text{Simp}[(\text{Rt}[a/b, 2] \cdot \text{ArcTan}[x/\text{Rt}[a/b, 2]])/a, x] \ /; \ \text{FreeQ}\{a, b, x\} \ \&\& \ \text{PosQ}[a/b]$

Rubi steps

$$\int \frac{d + ex^2}{(a + bx^2 + cx^4)^2} dx = \frac{x(b^2d - 2acd - abe + c(bd - 2ae)x^2)}{2a(b^2 - 4ac)(a + bx^2 + cx^4)} - \frac{\int \frac{-b^2d + 6acd - abe - c(bd - 2ae)x^2}{a + bx^2 + cx^4} dx}{2a(b^2 - 4ac)}$$

$$= \frac{x(b^2d - 2acd - abe + c(bd - 2ae)x^2)}{2a(b^2 - 4ac)(a + bx^2 + cx^4)} + \frac{\left(c\left(bd - 2ae - \frac{b^2d - 12acd + 4abe}{\sqrt{b^2 - 4ac}}\right)\right) \int \frac{1}{\frac{b}{2} + \frac{1}{2}\sqrt{b^2 - 4ac} + cx^2} dx}{4a(b^2 - 4ac)} + \dots$$

$$= \frac{x(b^2d - 2acd - abe + c(bd - 2ae)x^2)}{2a(b^2 - 4ac)(a + bx^2 + cx^4)} + \frac{\sqrt{c}\left(bd - 2ae + \frac{b^2d - 12acd + 4abe}{\sqrt{b^2 - 4ac}}\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b - \sqrt{b^2 - 4ac}}}\right)}{2\sqrt{2}a(b^2 - 4ac)\sqrt{b - \sqrt{b^2 - 4ac}}} + \dots$$

Mathematica [A] time = 0.806411, size = 310, normalized size = 1.06

$$\frac{2x(b(cd x^2 - ae) - 2ac(d + ex^2) + b^2d)}{(b^2 - 4ac)(a + bx^2 + cx^4)} + \frac{\sqrt{2}\sqrt{c}\left(b(d\sqrt{b^2 - 4ac} + 4ae) - 2a(e\sqrt{b^2 - 4ac} + 6cd) + b^2d\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b - \sqrt{b^2 - 4ac}}}\right)}{(b^2 - 4ac)^{3/2}\sqrt{b - \sqrt{b^2 - 4ac}}} + \frac{\sqrt{2}\sqrt{c}\left(bd\sqrt{b^2 - 4ac} - 2ae\sqrt{b^2 - 4ac} - 4abe + 12acd\right)}{(b^2 - 4ac)^{3/2}\sqrt{b^2 - 4ac}}$$

4a

Antiderivative was successfully verified.

[In] Integrate[(d + e*x^2)/(a + b*x^2 + c*x^4)^2,x]

[Out] $((2*x*(b^2*d + b*(-(a*e) + c*d*x^2) - 2*a*c*(d + e*x^2)))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (\text{Sqrt}[2]*\text{Sqrt}[c]*(b^2*d + b*(\text{Sqrt}[b^2 - 4*a*c]*d + 4*a*e) - 2*a*(6*c*d + \text{Sqrt}[b^2 - 4*a*c]*e))*\text{ArcTan}[(\text{Sqrt}[2]*\text{Sqrt}[c]*x)/\text{Sqrt}[b - \text{Sqrt}[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^{(3/2)}*\text{Sqrt}[b - \text{Sqrt}[b^2 - 4*a*c]]) + (\text{Sqrt}[2]*\text{Sqrt}[c]*(-b^2*d) + 12*a*c*d + b*\text{Sqrt}[b^2 - 4*a*c]*d - 4*a*b*e - 2*a*\text{Sqrt}[b^2 - 4*a*c]*e)*\text{ArcTan}[(\text{Sqrt}[2]*\text{Sqrt}[c]*x)/\text{Sqrt}[b + \text{Sqrt}[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^{(3/2)}*\text{Sqrt}[b + \text{Sqrt}[b^2 - 4*a*c]]))/(4*a)$

Maple [B] time = 0.08, size = 1761, normalized size = 6.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x^2+d)/(c*x^4+b*x^2+a)^2,x)

[Out] $-1/4/(4*a*c - b^2)*(-4*a*c + b^2)^{(1/2)}/a*x/(x^2 + 1/2*b/c - 1/2*(-4*a*c + b^2)^{(1/2)}/c)*d + 1/2/(4*a*c - b^2)*x/(x^2 + 1/2*b/c - 1/2*(-4*a*c + b^2)^{(1/2)}/c)*e - 1/4/(4*a*c - b^2)/a*x/(x^2 + 1/2*b/c - 1/2*(-4*a*c + b^2)^{(1/2)}/c)*b*d - 12*c^3/(4*a*c - b^2)/(-4*a*c + b^2)^{(1/2)}/(4*a*c + 3*b^2)*2^{(1/2)}/(((-4*a*c + b^2)^{(1/2)} - b)*c)^{(1/2)}*arctanh(c*x^2^{(1/2)}/(((-4*a*c + b^2)^{(1/2)} - b)*c)^{(1/2)})*d*a - 8*c^2/(4*a*c - b^2)/(-4$

$$\begin{aligned} & *a*c+b^2)^{(1/2)}/(4*a*c+3*b^2)*2^{(1/2)}/(((-4*a*c+b^2)^{(1/2)}-b)*c)^{(1/2)}*\arctan h(c*x*2^{(1/2)}/(((-4*a*c+b^2)^{(1/2)}-b)*c)^{(1/2)})*b^2*d+3/4*c/(4*a*c-b^2)/ \\ & (-4*a*c+b^2)^{(1/2)}/a/(4*a*c+3*b^2)*2^{(1/2)}/(((-4*a*c+b^2)^{(1/2)}-b)*c)^{(1/2)}* \\ & \arctanh(c*x*2^{(1/2)}/(((-4*a*c+b^2)^{(1/2)}-b)*c)^{(1/2)})*b^4*d-2*c^2/(4*a*c-b^2) \\ &)*a/(4*a*c+3*b^2)*2^{(1/2)}/(((-4*a*c+b^2)^{(1/2)}-b)*c)^{(1/2)}*\arctanh(c*x*2^{(1/2)}/ \\ & (((-4*a*c+b^2)^{(1/2)}-b)*c)^{(1/2)})*e^{-3/2}*c/(4*a*c-b^2)/(4*a*c+3*b^2)*2^{(1/2)}/ \\ & (((-4*a*c+b^2)^{(1/2)}-b)*c)^{(1/2)}*\arctanh(c*x*2^{(1/2)}/(((-4*a*c+b^2)^{(1/2)}-b)*c)^{(1/2)})*b \\ & ^2*e+c^2/(4*a*c-b^2)/(4*a*c+3*b^2)*2^{(1/2)}/(((-4*a*c+b^2)^{(1/2)}-b)*c)^{(1/2)}*\arctanh(c*x*2^{(1/2)}/ \\ & (((-4*a*c+b^2)^{(1/2)}-b)*c)^{(1/2)})*b^2*d+3/4*c/(4*a*c-b^2)/a/(4*a*c+3*b^2)*2^{(1/2)}/ \\ & (((-4*a*c+b^2)^{(1/2)}-b)*c)^{(1/2)}*\arctanh(c*x*2^{(1/2)}/(((-4*a*c+b^2)^{(1/2)}-b)*c)^{(1/2)})*b \\ & ^3*d+4*c^2/(4*a*c-b^2)/(-4*a*c+b^2)^{(1/2)}*a/(4*a*c+3*b^2)*2^{(1/2)}/(((-4*a*c+b^2)^{(1/2)}-b)*c)^{(1/2)} \\ & *\arctanh(c*x*2^{(1/2)}/(((-4*a*c+b^2)^{(1/2)}-b)*c)^{(1/2)})*b*e+3*c/(4*a*c-b^2)/(-4*a*c+b^2)^{(1/2)}/ \\ & (4*a*c+3*b^2)*2^{(1/2)}/(((-4*a*c+b^2)^{(1/2)}-b)*c)^{(1/2)}*\arctanh(c*x*2^{(1/2)}/(((-4*a*c+b^2)^{(1/2)}-b)*c)^{(1/2)})*b^3*e+1/4/ \\ & (4*a*c-b^2)*(-4*a*c+b^2)^{(1/2)}/a*x/(x^2+1/2*(-4*a*c+b^2)^{(1/2)}/c+1/2*b/c)*d+1/2/(4*a*c-b^2)*x/ \\ & (x^2+1/2*(-4*a*c+b^2)^{(1/2)}/c+1/2*b/c)*e-1/4/(4*a*c-b^2)/a*x/(x^2+1/2*(-4*a*c+b^2)^{(1/2)}/c+1/2*b/c)*b*d-12*c^3/ \\ & (4*a*c-b^2)/(-4*a*c+b^2)^{(1/2)}/(4*a*c+3*b^2)*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\arctan(c*x*2^{(1/2)}/ \\ & ((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)})*d*a-8*c^2/(4*a*c-b^2)/(-4*a*c+b^2)^{(1/2)}/(4*a*c+3*b^2)*2^{(1/2)}/ \\ & ((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\arctan(c*x*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)})*b^2*d+3/4*c/ \\ & (4*a*c-b^2)/(-4*a*c+b^2)^{(1/2)}/a/(4*a*c+3*b^2)*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\arctan(c*x*2^{(1/2)}/ \\ & ((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)})*b^4*d+2*c^2/(4*a*c-b^2)*a/(4*a*c+3*b^2)*2^{(1/2)}/ \\ & ((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\arctan(c*x*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)})*e+3/2*c/ \\ & (4*a*c-b^2)/(4*a*c+3*b^2)*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\arctan(c*x*2^{(1/2)}/ \\ & ((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)})*b^2*e-c^2/(4*a*c-b^2)/(4*a*c+3*b^2)*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)} \\ & *\arctan(c*x*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)})*b*d-3/4*c/(4*a*c-b^2)/a/(4*a*c+3*b^2)*2^{(1/2)}/ \\ & ((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\arctan(c*x*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)})*b^3*d+4*c^2/ \\ & (4*a*c-b^2)/(-4*a*c+b^2)^{(1/2)}*a/(4*a*c+3*b^2)*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\arctan(c*x*2^{(1/2)}/ \\ & ((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)})*b*e+3*c/(4*a*c-b^2)/(-4*a*c+b^2)^{(1/2)}/(4*a*c+3*b^2)*2^{(1/2)}/ \\ & ((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\arctan(c*x*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)})*b^3*e \end{aligned}$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{(bcd - 2ace)x^3 - (abe - (b^2 - 2ac)d)x}{2((ab^2c - 4a^2c^2)x^4 + a^2b^2 - 4a^3c + (ab^3 - 4a^2bc)x^2)} - \frac{-\int \frac{abe+(bcd-2ace)x^2+(b^2-6ac)d}{cx^4+bx^2+a} dx}{2(ab^2 - 4a^2c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")

[Out] 1/2*((b*c*d - 2*a*c*e)*x^3 - (a*b*e - (b^2 - 2*a*c)*d)*x)/((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2) - 1/2*integrate(-(a*b*e + (b*c*d - 2*a*c*e)*x^2 + (b^2 - 6*a*c)*d)/(c*x^4 + b*x^2 + a), x)/(a*b^2 - 4*a^2*c)

Fricas [B] time = 6.94566, size = 9543, normalized size = 32.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

$$\begin{aligned}
& b^2c^3 + 864a^4c^4)d^3 + 3*(a*b^7 - 15a^2b^5c + 72a^3b^3c^2 - 112 \\
& *a^4b*c^3)*d^2e + 3*(a^2b^6 - 10a^3b^4c + 32a^4b^2c^2 - 32a^5c^3) \\
&)*d^2e + (a^3b^5 - 8a^4b^3c + 16a^5b*c^2)*e^3 + ((a^3b^9 - 20a^4b \\
& ^7c + 144a^5b^5c^2 - 448a^6b^3c^3 + 512a^7b*c^4)*d + (a^4b^8 - 8* \\
& a^5b^6c + 128a^7b^2c^3 - 256a^8c^4)*e)*\text{sqrt}((4a^3b*d*e^3 + a^4e^4 \\
& + (b^4 - 18a*b^2c + 81a^2c^2)*d^4 + 4*(a*b^3 - 9a^2b*c)*d^3e + 6*(a \\
& ^2b^2 - 3a^3c)*d^2e^2)/(a^6b^6 - 12a^7b^4c + 48a^8b^2c^2 - 64a^9c^3)) \\
&)*\text{sqrt}(-((b^5 - 15a*b^3c + 60a^2b*c^2)*d^2 + 2*(a*b^4 - 6a^2b^2c \\
& - 24a^3c^2)*d^2e + (a^2b^3 + 12a^3b*c)*e^2 - (a^3b^6 - 12a^4b^4c \\
& + 48a^5b^2c^2 - 64a^6c^3)*\text{sqrt}((4a^3b*d*e^3 + a^4e^4 + (b^4 - 18* \\
& a*b^2c + 81a^2c^2)*d^4 + 4*(a*b^3 - 9a^2b*c)*d^3e + 6*(a^2b^2 - 3a^ \\
& 3c)*d^2e^2)/(a^6b^6 - 12a^7b^4c + 48a^8b^2c^2 - 64a^9c^3)))/(a^3 \\
& *b^6 - 12a^4b^4c + 48a^5b^2c^2 - 64a^6c^3)) + \text{sqrt}(1/2)*((a*b^2c \\
& - 4a^2c^2)*x^4 + a^2b^2 - 4a^3c + (a*b^3 - 4a^2b*c)*x^2)*\text{sqrt}(-((b^5 \\
& - 15a*b^3c + 60a^2b*c^2)*d^2 + 2*(a*b^4 - 6a^2b^2c - 24a^3c^2)*d* \\
& e + (a^2b^3 + 12a^3b*c)*e^2 - (a^3b^6 - 12a^4b^4c + 48a^5b^2c^2 - \\
& 64a^6c^3)*\text{sqrt}((4a^3b*d*e^3 + a^4e^4 + (b^4 - 18a*b^2c + 81a^2c^2) \\
&)*d^4 + 4*(a*b^3 - 9a^2b*c)*d^3e + 6*(a^2b^2 - 3a^3c)*d^2e^2)/(a^6b \\
& ^6 - 12a^7b^4c + 48a^8b^2c^2 - 64a^9c^3)))/(a^3b^6 - 12a^4b^4c \\
& + 48a^5b^2c^2 - 64a^6c^3))*\log(-((5b^4c^2 - 81a*b^2c^3 + 324a^2c^ \\
& ^4)*d^4 - (3b^5c - 65a*b^3c^2 + 324a^2b*c^3)*d^3e - 3*(3a*b^4c - 2 \\
& 8a^2b^2c^2)*d^2e^2 - (9a^2b^3c - 20a^3b*c^2)*d^2e^3 - (3a^3b^2c \\
& + 4a^4c^2)*e^4)*x - 1/2*\text{sqrt}(1/2)*((b^8 - 23a*b^6c + 190a^2b^4c^2 - \\
& 672a^3b^2c^3 + 864a^4c^4)*d^3 + 3*(a*b^7 - 15a^2b^5c + 72a^3b^3c^ \\
& ^2 - 112a^4b*c^3)*d^2e + 3*(a^2b^6 - 10a^3b^4c + 32a^4b^2c^2 - 32 \\
& *a^5c^3)*d^2e + (a^3b^5 - 8a^4b^3c + 16a^5b*c^2)*e^3 + ((a^3b^9 - \\
& 20a^4b^7c + 144a^5b^5c^2 - 448a^6b^3c^3 + 512a^7b*c^4)*d + (a^4b \\
& ^8 - 8a^5b^6c + 128a^7b^2c^3 - 256a^8c^4)*e)*\text{sqrt}((4a^3b*d*e^3 + \\
& a^4e^4 + (b^4 - 18a*b^2c + 81a^2c^2)*d^4 + 4*(a*b^3 - 9a^2b*c)*d^3* \\
& e + 6*(a^2b^2 - 3a^3c)*d^2e^2)/(a^6b^6 - 12a^7b^4c + 48a^8b^2c^2 \\
& - 64a^9c^3))*\text{sqrt}(-((b^5 - 15a*b^3c + 60a^2b*c^2)*d^2 + 2*(a*b^4 - \\
& 6a^2b^2c - 24a^3c^2)*d^2e + (a^2b^3 + 12a^3b*c)*e^2 - (a^3b^6 - 12* \\
& a^4b^4c + 48a^5b^2c^2 - 64a^6c^3)*\text{sqrt}((4a^3b*d*e^3 + a^4e^4 + (b \\
& ^4 - 18a*b^2c + 81a^2c^2)*d^4 + 4*(a*b^3 - 9a^2b*c)*d^3e + 6*(a^2b^ \\
& 2 - 3a^3c)*d^2e^2)/(a^6b^6 - 12a^7b^4c + 48a^8b^2c^2 - 64a^9c^3 \\
&)))/(a^3b^6 - 12a^4b^4c + 48a^5b^2c^2 - 64a^6c^3)) - 2*(a*b*e - (\\
& b^2 - 2a*c)*d)*x)/((a*b^2c - 4a^2c^2)*x^4 + a^2b^2 - 4a^3c + (a*b^3 \\
& - 4a^2b*c)*x^2)
\end{aligned}$$

Sympy [B] time = 60.2997, size = 1180, normalized size = 4.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x**2+d)/(c*x**4+b*x**2+a)**2,x)

[Out] (x**3*(2*a*c*e - b*c*d) + x*(a*b*e + 2*a*c*d - b**2*d))/(8*a**3*c - 2*a**2*b**2 + x**4*(8*a**2*c**2 - 2*a*b**2*c) + x**2*(8*a**2*b*c - 2*a*b**3)) + Ro
otSum(_t**4*(1048576*a**9*c**6 - 1572864*a**8*b**2*c**5 + 983040*a**7*b**4*c**4 - 327680*a**6*b**6*c**3 + 61440*a**5*b**8*c**2 - 6144*a**4*b**10*c + 2
56*a**3*b**12) + _t**2*(-12288*a**6*b*c**4*e**2 + 49152*a**6*c**5*d*e + 819
2*a**5*b**3*c**3*e**2 - 24576*a**5*b**2*c**4*d*e - 61440*a**5*b*c**5*d**2 -
1536*a**4*b**5*c**2*e**2 - 2048*a**4*b**4*c**3*d*e + 61440*a**4*b**3*c**4*d
2 + 3072*a3*b**6*c**2*d*e - 24064*a**3*b**5*c**3*d**2 + 16*a**2*b**9*e
2 - 576*a2*b**8*c*d*e + 4608*a**2*b**7*c**2*d**2 + 32*a*b**10*d*e - 432
*a*b**9*c*d**2 + 16*b**11*d**2) + 16*a**4*c**3*e**4 + 24*a**3*b**2*c**2*e**

```

4 - 224*a**3*b*c**3*d*e**3 + 288*a**3*c**4*d**2*e**2 + 9*a**2*b**4*c*e**4 -
144*a**2*b**3*c**2*d*e**3 + 960*a**2*b**2*c**3*d**2*e**2 - 2016*a**2*b*c**
4*d**3*e + 1296*a**2*c**5*d**4 + 18*a*b**5*c*d*e**3 - 198*a*b**4*c**2*d**2*
e**2 + 496*a*b**3*c**3*d**3*e - 360*a*b**2*c**4*d**4 + 9*b**6*c*d**2*e**2 -
30*b**5*c**2*d**3*e + 25*b**4*c**3*d**4, Lambda(_t, _t*log(x + (16384*_t**
3*a**8*c**4*e - 8192*_t**3*a**7*b**2*c**3*e - 32768*_t**3*a**7*b*c**4*d + 2
8672*_t**3*a**6*b**3*c**3*d + 512*_t**3*a**5*b**6*c*e - 9216*_t**3*a**5*b**
5*c**2*d - 64*_t**3*a**4*b**8*e + 1280*_t**3*a**4*b**7*c*d - 64*_t**3*a**3*
b**9*d - 128*_t*a**5*b*c**2*e**3 + 576*_t*a**5*c**3*d*e**2 - 16*_t*a**4*b**
3*c*e**3 + 192*_t*a**4*b**2*c**2*d*e**2 - 576*_t*a**4*b*c**3*d**2*e - 1728*_
t*a**4*c**4*d**3 - 4*_t*a**3*b**5*e**3 + 60*_t*a**3*b**4*c*d*e**2 - 528*_t
a**3*b**3*c**2*d**2*e + 2304*_t*a**3*b**2*c**3*d**3 - 12*_t*a**2*b**6*d*e
*2 + 168*_t*a**2*b**5*c*d**2*e - 740*_t*a**2*b**4*c**2*d**3 - 12*_t*a*b**7*
d**2*e + 92*_t*a*b**6*c*d**3 - 4*_t*b**8*d**3)/(4*a**4*c**2*e**4 + 3*a**3*b
**2*c*e**4 - 20*a**3*b*c**2*d*e**3 + 9*a**2*b**3*c*d*e**3 - 84*a**2*b**2*c*
**2*d**2*e**2 + 324*a**2*b*c**3*d**3*e - 324*a**2*c**4*d**4 + 9*a*b**4*c*d**
2*e**2 - 65*a*b**3*c**2*d**3*e + 81*a*b**2*c**3*d**4 + 3*b**5*c*d**3*e - 5*
b**4*c**2*d**4))))

```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")

[Out] Exception raised: NotImplementedError

$$3.273 \quad \int \frac{1}{(a+bx^2+cx^4)^2} dx$$

Optimal. Leaf size=252

$$\frac{x(-2ac + b^2 + bcx^2)}{2a(b^2 - 4ac)(a + bx^2 + cx^4)} + \frac{\sqrt{c}(b\sqrt{b^2 - 4ac} - 12ac + b^2) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b - \sqrt{b^2 - 4ac}}}\right)}{2\sqrt{2}a(b^2 - 4ac)^{3/2}\sqrt{b - \sqrt{b^2 - 4ac}}} - \frac{\sqrt{c}(-b\sqrt{b^2 - 4ac} - 12ac + b^2) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b + \sqrt{b^2 - 4ac}}}\right)}{2\sqrt{2}a(b^2 - 4ac)^{3/2}\sqrt{b + \sqrt{b^2 - 4ac}}}$$

```
[Out] (x*(b^2 - 2*a*c + b*c*x^2))/(2*a*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt[c]*(b^2 - 12*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]])/(2*Sqrt[2]*a*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[c]*(b^2 - 12*a*c - b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]])/(2*Sqrt[2]*a*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rubi [A] time = 0.516592, antiderivative size = 252, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {1092, 1166, 205}

$$\frac{x(-2ac + b^2 + bcx^2)}{2a(b^2 - 4ac)(a + bx^2 + cx^4)} + \frac{\sqrt{c}(b\sqrt{b^2 - 4ac} - 12ac + b^2) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b - \sqrt{b^2 - 4ac}}}\right)}{2\sqrt{2}a(b^2 - 4ac)^{3/2}\sqrt{b - \sqrt{b^2 - 4ac}}} - \frac{\sqrt{c}(-b\sqrt{b^2 - 4ac} - 12ac + b^2) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b + \sqrt{b^2 - 4ac}}}\right)}{2\sqrt{2}a(b^2 - 4ac)^{3/2}\sqrt{b + \sqrt{b^2 - 4ac}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^(-2), x]
```

```
[Out] (x*(b^2 - 2*a*c + b*c*x^2))/(2*a*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt[c]*(b^2 - 12*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]])/(2*Sqrt[2]*a*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[c]*(b^2 - 12*a*c - b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]])/(2*Sqrt[2]*a*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 1092

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> -Simp[(x*(b^2 - 2*a*c + b*c*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(b^2 - 2*a*c + 2*(p + 1)*(b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rubi steps

$$\int \frac{1}{(a + bx^2 + cx^4)^2} dx = \frac{x(b^2 - 2ac + bcx^2)}{2a(b^2 - 4ac)(a + bx^2 + cx^4)} - \frac{\int \frac{b^2 - 2ac - 2(b^2 - 4ac) - bcx^2}{a + bx^2 + cx^4} dx}{2a(b^2 - 4ac)}$$

$$= \frac{x(b^2 - 2ac + bcx^2)}{2a(b^2 - 4ac)(a + bx^2 + cx^4)} - \frac{\left(c(b^2 - 12ac - b\sqrt{b^2 - 4ac})\right) \int \frac{1}{\frac{b}{2} + \frac{1}{2}\sqrt{b^2 - 4ac} + cx^2} dx}{4a(b^2 - 4ac)^{3/2}} + \frac{c(b^2 - 12ac - b\sqrt{b^2 - 4ac})}{2\sqrt{2}a(b^2 - 4ac)^{3/2}\sqrt{b - \sqrt{b^2 - 4ac}}} - \frac{\sqrt{c}(b^2 - 12ac + b\sqrt{b^2 - 4ac}) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b - \sqrt{b^2 - 4ac}}}\right)}{2\sqrt{2}a(b^2 - 4ac)^{3/2}\sqrt{b - \sqrt{b^2 - 4ac}}} - \frac{\sqrt{c}(b^2 - 12ac - b\sqrt{b^2 - 4ac}) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b + \sqrt{b^2 - 4ac}}}\right)}{2\sqrt{2}a(b^2 - 4ac)^{3/2}\sqrt{b + \sqrt{b^2 - 4ac}}}$$

Mathematica [A] time = 0.443417, size = 243, normalized size = 0.96

$$\frac{2x(-2ac + b^2 + bcx^2)}{(b^2 - 4ac)(a + bx^2 + cx^4)} + \frac{\sqrt{2}\sqrt{c}(b\sqrt{b^2 - 4ac} - 12ac + b^2) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b - \sqrt{b^2 - 4ac}}}\right)}{(b^2 - 4ac)^{3/2}\sqrt{b - \sqrt{b^2 - 4ac}}} + \frac{\sqrt{2}\sqrt{c}(b\sqrt{b^2 - 4ac} + 12ac - b^2) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b^2 - 4ac + b}}}\right)}{(b^2 - 4ac)^{3/2}\sqrt{b^2 - 4ac + b}}$$

4a

Antiderivative was successfully verified.

[In] Integrate[(a + b*x^2 + c*x^4)^(-2), x]

[Out] ((2*x*(b^2 - 2*a*c + b*c*x^2))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt[2]*Sqrt[c]*(b^2 - 12*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*Sqrt[c]*(-b^2 + 12*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(4*a)

Maple [B] time = 0.054, size = 733, normalized size = 2.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(c*x^4+b*x^2+a)^2, x)

[Out] -1/4/(4*a*c-b^2)/a*x/(x^2+1/2*b/c-1/2*(-4*a*c+b^2)^(1/2)/c)*b+c/(-4*a*c+b^2)^(1/2)/(4*a*c-b^2)*x/(x^2+1/2*b/c-1/2*(-4*a*c+b^2)^(1/2)/c)-1/4/(-4*a*c+b^2)^(1/2)/(4*a*c-b^2)/a*x/(x^2+1/2*b/c-1/2*(-4*a*c+b^2)^(1/2)/c)*b^2+1/4*c/(4*a*c-b^2)/a^2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x^2^(1/2)/((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*b-3*c^2/(-4*a*c+b^2)^(1/2)/(4*a*c-b^2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x^2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))+1/4*c/(-4*a*c+b^2)^(1/2)/(4*a*c-b^2)/a^2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x^2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*b^2-1/4/(4*a*c-b^2)/a*x/(x^2+1/2*(-4*a*c+b^2)^(1/2)/c+1/2*b/c)*b-c/(-4*a*c+b^2)^(1/2)/(4*a*c-b^2)*x/(x^2+1/2*(-4*a*c+b^2)^(1/2)/c+1/2*b/c)+1/4/(-4*a*

$$\frac{48a^8b^2c^2 - 64a^9c^3}{(a^3b^6 - 12a^4b^4c + 48a^5b^2c^2 - 64a^6c^3)} \log\left(\frac{(5b^4c^2 - 81ab^2c^3 + 324a^2c^4)x + \frac{1}{2}\sqrt{\frac{1}{2}}(b^8 - 23a^2b^6c + 190a^2b^4c^2 - 672a^3b^2c^3 + 864a^4c^4 + (a^3b^9 - 20a^4b^7c + 144a^5b^5c^2 - 448a^6b^3c^3 + 512a^7b^2c^4))\sqrt{\frac{1}{2}}(b^4 - 18ab^2c + 81a^2c^2)}{(a^6b^6 - 12a^7b^4c + 48a^8b^2c^2 - 64a^9c^3)}\right) \sqrt{\frac{1}{2}} \frac{(a^3b^6 - 12a^4b^4c + 48a^5b^2c^2 - 64a^6c^3)\sqrt{\frac{1}{2}}(b^4 - 18ab^2c + 81a^2c^2)}{(a^6b^6 - 12a^7b^4c + 48a^8b^2c^2 - 64a^9c^3)}}{(a^3b^6 - 12a^4b^4c + 48a^5b^2c^2 - 64a^6c^3)} - \sqrt{\frac{1}{2}} \frac{((ab^2c - 4a^2c^2)x^4 + a^2b^2 - 4a^3c + (ab^3 - 4a^2bc)x^2)\sqrt{\frac{1}{2}}(b^5 - 15ab^3c + 60a^2b^2c^2 - (a^3b^6 - 12a^4b^4c + 48a^5b^2c^2 - 64a^6c^3))\sqrt{\frac{1}{2}}(b^4 - 18ab^2c + 81a^2c^2)}{(a^6b^6 - 12a^7b^4c + 48a^8b^2c^2 - 64a^9c^3)}}{(a^3b^6 - 12a^4b^4c + 48a^5b^2c^2 - 64a^6c^3)} \log\left(\frac{(5b^4c^2 - 81ab^2c^3 + 324a^2c^4)x - \frac{1}{2}\sqrt{\frac{1}{2}}(b^8 - 23a^2b^6c + 190a^2b^4c^2 - 672a^3b^2c^3 + 864a^4c^4 + (a^3b^9 - 20a^4b^7c + 144a^5b^5c^2 - 448a^6b^3c^3 + 512a^7b^2c^4))\sqrt{\frac{1}{2}}(b^4 - 18ab^2c + 81a^2c^2)}{(a^6b^6 - 12a^7b^4c + 48a^8b^2c^2 - 64a^9c^3)}}{(a^3b^6 - 12a^4b^4c + 48a^5b^2c^2 - 64a^6c^3)}\right) \sqrt{\frac{1}{2}} \frac{(a^3b^6 - 12a^4b^4c + 48a^5b^2c^2 - 64a^6c^3)\sqrt{\frac{1}{2}}(b^4 - 18ab^2c + 81a^2c^2)}{(a^6b^6 - 12a^7b^4c + 48a^8b^2c^2 - 64a^9c^3)}}{(a^3b^6 - 12a^4b^4c + 48a^5b^2c^2 - 64a^6c^3)} + 2(b^2 - 2ac)x / ((ab^2c - 4a^2c^2)x^4 + a^2b^2 - 4a^3c + (ab^3 - 4a^2bc)x^2)$$

Sympy [A] time = 4.4144, size = 394, normalized size = 1.56

$$\frac{bcx^3 + x(-2ac + b^2)}{8a^3c - 2a^2b^2 + x^4(8a^2c^2 - 2ab^2c) + x^2(8a^2bc - 2ab^3)} + \text{RootSum}\left(t^4(1048576a^9c^6 - 1572864a^8b^2c^5 + 983040a^7b^4c^4 - 327680a^6b^6c^3 + 61440a^5b^8c^2 - 6144a^4b^{10}c + 256a^3b^{12}) + t^2(-61440a^5b^8c^2 + 61440a^4b^6c^3 - 24064a^3b^5c^4 + 4608a^2b^4c^5 - 432ab^3c^6 + 16b^2c^7) + 1296a^2c^5 - 360ab^2c^4 + 25b^4c^3, \text{Lambda}(t, t \log(x + (32768t^3a^7b^2c^4 - 28672t^3a^6b^3c^3 + 9216t^3a^5b^4c^2 - 1280t^3a^4b^5c + 64t^3a^3b^6 + 1728t^2a^4c^4 - 2304t^2a^3b^2c^3 + 740t^2a^2b^3c^2 - 92t^2ab^4c + 4t^2b^5)/(324a^2c^4 - 81ab^2c^3 + 5b^4c^2)))\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x**4+b*x**2+a)**2,x)

[Out] $-(b*c*x**3 + x*(-2*a*c + b**2))/(8*a**3*c - 2*a**2*b**2 + x**4*(8*a**2*c**2 - 2*a*b**2*c) + x**2*(8*a**2*b*c - 2*a*b**3)) + \text{RootSum}(_t**4*(1048576*a**9*c**6 - 1572864*a**8*b**2*c**5 + 983040*a**7*b**4*c**4 - 327680*a**6*b**6*c**3 + 61440*a**5*b**8*c**2 - 6144*a**4*b**10*c + 256*a**3*b**12) + _t**2*(-61440*a**5*b**8*c**2 + 61440*a**4*b**6*c**3 - 24064*a**3*b**5*c**4 + 4608*a**2*b**4*c**5 - 432*a*b**9*c + 16*b**11) + 1296*a**2*c**5 - 360*a*b**2*c**4 + 25*b**4*c**3, \text{Lambda}(_t, _t*\log(x + (32768*_t**3*a**7*b*c**4 - 28672*_t**3*a**6*b**3*c**3 + 9216*_t**3*a**5*b**5*c**2 - 1280*_t**3*a**4*b**7*c + 64*_t**3*a**3*b**9 + 1728*_t*a**4*c**4 - 2304*_t*a**3*b**2*c**3 + 740*_t*a**2*b**4*c**2 - 92*_t*a*b**6*c + 4*_t*b**8)/(324*a**2*c**4 - 81*a*b**2*c**3 + 5*b**4*c**2))))$

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x^4+b*x^2+a)^2,x, algorithm="giac")

[Out] Exception raised: NotImplementedError

$$3.274 \quad \int \frac{1}{(d+ex^2)(a+bx^2+cx^4)^2} dx$$

Optimal. Leaf size=660

$$\frac{x \left(cx^2 (2ace + b^2(-e) + bcd) + 3abce - 2ac^2d + b^2cd + b^3(-e) \right)}{2a (b^2 - 4ac) (a + bx^2 + cx^4) (ae^2 - bde + cd^2)} + \frac{\sqrt{c} \left(\frac{8abce - 12ac^2d + b^2cd + b^3(-e)}{\sqrt{b^2 - 4ac}} + 2ace + b^2(-e) + bcd \right) \tan^{-1} \left(\frac{\sqrt{c} (x \sqrt{a + bx^2 + cx^4})}{\sqrt{b - \sqrt{b^2 - 4ac}}} \right)}{2\sqrt{2a} (b^2 - 4ac) \sqrt{b - \sqrt{b^2 - 4ac}} (ae^2 - bde + cd^2)}$$

[Out] (x*(b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*c*e + c*(b*c*d - b^2*e + 2*a*c*e)*x^2)/(2*a*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)*(a + b*x^2 + c*x^4)) - (Sqrt[c]*e^2*(e - (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]])/(Sqrt[2]*Sqrt[b - Sqrt[b^2 - 4*a*c]]*(c*d^2 - b*d*e + a*e^2)^2) + (Sqrt[c]*(b*c*d - b^2*e + 2*a*c*e + (b^2*c*d - 12*a*c^2*d - b^3*e + 8*a*b*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]])/(2*Sqrt[2]*a*(b^2 - 4*a*c)*Sqrt[b - Sqrt[b^2 - 4*a*c]]*(c*d^2 - b*d*e + a*e^2)) - (Sqrt[c]*e^2*(e + (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]])/(Sqrt[2]*Sqrt[b + Sqrt[b^2 - 4*a*c]]*(c*d^2 - b*d*e + a*e^2)^2) + (Sqrt[c]*(b*c*d - b^2*e + 2*a*c*e - (b^2*c*d - 12*a*c^2*d - b^3*e + 8*a*b*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]])/(2*Sqrt[2]*a*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 - 4*a*c]]*(c*d^2 - b*d*e + a*e^2)) + (e^(7/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[d]*(c*d^2 - b*d*e + a*e^2)^2)

Rubi [A] time = 2.87252, antiderivative size = 660, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1238, 205, 1178, 1166}

$$\frac{x \left(cx^2 (2ace + b^2(-e) + bcd) + 3abce - 2ac^2d + b^2cd + b^3(-e) \right)}{2a (b^2 - 4ac) (a + bx^2 + cx^4) (ae^2 - bde + cd^2)} + \frac{\sqrt{c} \left(\frac{8abce - 12ac^2d + b^2cd + b^3(-e)}{\sqrt{b^2 - 4ac}} + 2ace + b^2(-e) + bcd \right) \tan^{-1} \left(\frac{\sqrt{c} (x \sqrt{a + bx^2 + cx^4})}{\sqrt{b - \sqrt{b^2 - 4ac}}} \right)}{2\sqrt{2a} (b^2 - 4ac) \sqrt{b - \sqrt{b^2 - 4ac}} (ae^2 - bde + cd^2)}$$

Antiderivative was successfully verified.

[In] Int[1/((d + e*x^2)*(a + b*x^2 + c*x^4)^2), x]

[Out] (x*(b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*c*e + c*(b*c*d - b^2*e + 2*a*c*e)*x^2)/(2*a*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)*(a + b*x^2 + c*x^4)) - (Sqrt[c]*e^2*(e - (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]])/(Sqrt[2]*Sqrt[b - Sqrt[b^2 - 4*a*c]]*(c*d^2 - b*d*e + a*e^2)^2) + (Sqrt[c]*(b*c*d - b^2*e + 2*a*c*e + (b^2*c*d - 12*a*c^2*d - b^3*e + 8*a*b*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]])/(2*Sqrt[2]*a*(b^2 - 4*a*c)*Sqrt[b - Sqrt[b^2 - 4*a*c]]*(c*d^2 - b*d*e + a*e^2)) - (Sqrt[c]*e^2*(e + (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]])/(Sqrt[2]*Sqrt[b + Sqrt[b^2 - 4*a*c]]*(c*d^2 - b*d*e + a*e^2)^2) + (Sqrt[c]*(b*c*d - b^2*e + 2*a*c*e - (b^2*c*d - 12*a*c^2*d - b^3*e + 8*a*b*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]])/(2*Sqrt[2]*a*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 - 4*a*c]]*(c*d^2 - b*d*e + a*e^2)) + (e^(7/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[d]*(c*d^2 - b*d*e + a*e^2)^2)

Rule 1238

Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x], x]

```
;/ FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b^2 - 4*a*c, 0] && ((IntegerQ[p]
&& IntegerQ[q]) || IGtQ[p, 0] || IGtQ[q, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] := Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 +
c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2
- 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
- q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{(d + ex^2)(a + bx^2 + cx^4)^2} dx = \int \left(\frac{e^4}{(cd^2 - bde + ae^2)^2 (d + ex^2)} + \frac{cd - be - cex^2}{(cd^2 - bde + ae^2)(a + bx^2 + cx^4)^2} - \frac{e^2(-c}{(cd^2 - bde + ae^2)^2} \right) dx$$

$$= -\frac{e^2 \int \frac{-cd+be+cex^2}{a+bx^2+cx^4} dx}{(cd^2 - bde + ae^2)^2} + \frac{e^4 \int \frac{1}{d+ex^2} dx}{(cd^2 - bde + ae^2)^2} + \frac{\int \frac{cd-be-cex^2}{(a+bx^2+cx^4)^2} dx}{cd^2 - bde + ae^2}$$

$$= \frac{x(b^2cd - 2ac^2d - b^3e + 3abce + c(bcd - b^2e + 2ace)x^2)}{2a(b^2 - 4ac)(cd^2 - bde + ae^2)(a + bx^2 + cx^4)} + \frac{e^{7/2} \tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)}{\sqrt{d}(cd^2 - bde + ae^2)^2} - \frac{\int \frac{cd-be-cex^2}{(a+bx^2+cx^4)^2} dx}{cd^2 - bde + ae^2}$$

$$= \frac{x(b^2cd - 2ac^2d - b^3e + 3abce + c(bcd - b^2e + 2ace)x^2)}{2a(b^2 - 4ac)(cd^2 - bde + ae^2)(a + bx^2 + cx^4)} - \frac{\sqrt{ce^2}\left(e - \frac{2cd-be}{\sqrt{b^2-4ac}}\right) \tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b-\sqrt{b^2-4ac}}(cd^2 - bde + ae^2)}$$

$$= \frac{x(b^2cd - 2ac^2d - b^3e + 3abce + c(bcd - b^2e + 2ace)x^2)}{2a(b^2 - 4ac)(cd^2 - bde + ae^2)(a + bx^2 + cx^4)} - \frac{\sqrt{ce^2}\left(e - \frac{2cd-be}{\sqrt{b^2-4ac}}\right) \tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b-\sqrt{b^2-4ac}}(cd^2 - bde + ae^2)}$$

Mathematica [A] time = 3.04297, size = 708, normalized size = 1.07

$$\frac{\sqrt{2}\sqrt{c}\left(b^2\left(-cde\left(2d\sqrt{b^2-4ac}+3ae\right)-3ae^3\sqrt{b^2-4ac}+c^2d^3\right)+2ac\left(cde\left(d\sqrt{b^2-4ac}-14ae\right)+5ae^3\sqrt{b^2-4ac}-6c^2d^3\right)+bc\left(cd^2\left(d\sqrt{b^2-4ac}+20ae\right)+ae^2\left(16ae-d\sqrt{b^2-4ac}\right)\right)+b^3e^3\right)}{a(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}$$

Antiderivative was successfully verified.

[In] Integrate[1/((d + e*x^2)*(a + b*x^2 + c*x^4)^2), x]

[Out]
$$\frac{\left((2(c d^2 + e(-b d) + a e)) x (b^3 e - b c (3 a e + c d x^2) + 2 a^2 c^2 (d - e x^2) + b^2 c (-d + e x^2)) \right) / (a (-b^2 + 4 a c) (a + b x^2 + c x^4)) + (\sqrt{2} \sqrt{c} (b^4 d e^2 + 2 a c (-6 c^2 d^3 + 5 a \sqrt{b^2 - 4 a c}) e^3 + c d e (\sqrt{b^2 - 4 a c} d - 14 a e)) + b^3 e (-2 c d^2 + e (\sqrt{b^2 - 4 a c} d - 3 a e)) + b^2 (c^2 d^3 - 3 a \sqrt{b^2 - 4 a c}) e^3 - c d e (2 \sqrt{b^2 - 4 a c} d + 3 a e)) + b c (a e^2 (-\sqrt{b^2 - 4 a c} d + 16 a e) + c d^2 (\sqrt{b^2 - 4 a c} d + 20 a e)) \operatorname{ArcTan}[\sqrt{2} \sqrt{c} x / \sqrt{b - \sqrt{b^2 - 4 a c}}]}{a (b^2 - 4 a c)^{3/2} \sqrt{b - \sqrt{b^2 - 4 a c}}} + (\sqrt{2} \sqrt{c} (-b^4 d e^2 - b^2 (c^2 d^3 + 3 a \sqrt{b^2 - 4 a c}) e^3 + c d e (2 \sqrt{b^2 - 4 a c} d - 3 a e)) + b^3 e (2 c d^2 + e (\sqrt{b^2 - 4 a c} d + 3 a e)) + 2 a c (6 c^2 d^3 + 5 a \sqrt{b^2 - 4 a c}) e^3 + c d e (\sqrt{b^2 - 4 a c} d + 14 a e)) + b c (c d^2 (\sqrt{b^2 - 4 a c} d - 20 a e) - a e^2 (\sqrt{b^2 - 4 a c} d + 16 a e)) \operatorname{ArcTan}[\sqrt{2} \sqrt{c} x / \sqrt{b + \sqrt{b^2 - 4 a c}}]}{a (b^2 - 4 a c)^{3/2} \sqrt{b + \sqrt{b^2 - 4 a c}}} + (4 e^{7/2} \operatorname{ArcTan}[\sqrt{e} x / \sqrt{d}] / \sqrt{d}) / (4 (c d^2 + e(-b d) + a e))^2$$

Maple [B] time = 0.074, size = 3841, normalized size = 5.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(e*x^2+d)/(c*x^4+b*x^2+a)^2, x)

[Out]
$$\frac{1/4 (a e^2 - b d e + c d^2)^2 / a (4 a c - b^2) c / (-4 a c + b^2)^{1/2} 2^{1/2} / (((-4 a c + b^2)^{1/2} - b) c)^{1/2} \operatorname{arctanh}(c x^2^{1/2} / (((-4 a c + b^2)^{1/2} - b) c)^{1/2}) + b^4 d e^2 + 1/4 (a e^2 - b d e + c d^2)^2 / a (4 a c - b^2) c / (-4 a c + b^2)^{1/2} 2^{1/2} / ((b + (-4 a c + b^2)^{1/2}) c)^{1/2} \operatorname{arctan}(c x^2^{1/2} / ((b + (-4 a c + b^2)^{1/2}) c)^{1/2}) + b^4 d e^2 - 1/2 (a e^2 - b d e + c d^2)^2 / a (4 a c - b^2) c^2 / (-4 a c + b^2)^{1/2} 2^{1/2} / ((b + (-4 a c + b^2)^{1/2}) c)^{1/2} \operatorname{arctan}(c x^2^{1/2} / ((b + (-4 a c + b^2)^{1/2}) c)^{1/2}) + b^3 d^2 e - 1/2 (a e^2 - b d e + c d^2)^2 / a (4 a c - b^2) c^2 / (-4 a c + b^2)^{1/2} 2^{1/2} / (((-4 a c + b^2)^{1/2} - b) c)^{1/2} \operatorname{arctanh}(c x^2^{1/2} / (((-4 a c + b^2)^{1/2} - b) c)^{1/2}) + b^3 d^2 e + 1/4 (a e^2 - b d e + c d^2)^2 / (c x^4 + b x^2 + a) / a (4 a c - b^2) x b^3 c d^2 e + 1/4 (a e^2 - b d e + c d^2)^2 / (4 a c - b^2) c^2 2^{1/2} / ((b + (-4 a c + b^2)^{1/2}) c)^{1/2} \operatorname{arctan}(c x^2^{1/2} / ((b + (-4 a c + b^2)^{1/2}) c)^{1/2}) + b d e^2 + 1/4 (a e^2 - b d e + c d^2)^2 / a (4 a c - b^2) c^3 2^{1/2} / (((-4 a c + b^2)^{1/2} - b) c)^{1/2} \operatorname{arctanh}(c x^2^{1/2} / (((-4 a c + b^2)^{1/2} - b) c)^{1/2}) + b d^3 - 1/4 (a e^2 - b d e + c d^2)^2 / a (4 a c - b^2) c^3 2^{1/2} / ((b + (-4 a c + b^2)^{1/2}) c)^{1/2} \operatorname{arctan}(c x^2^{1/2} / ((b + (-4 a c + b^2)^{1/2}) c)^{1/2}) + b d^3 - 3/4 (a e^2 - b d e + c d^2)^2 / (4 a c - b^2) c / (-4 a c + b^2)^{1/2} 2^{1/2} / (((-4 a c + b^2)^{1/2} - b) c)^{1/2} \operatorname{arctanh}(c x^2^{1/2} / (((-4 a c + b^2)^{1/2} - b) c)^{1/2}) + b^3 e^3 - 3/4 (a e^2 - b d e + c d^2)^2 / (4 a c - b^2) c / (-4 a c + b^2)^{1/2} 2^{1/2} / ((b + (-4 a c + b^2)^{1/2}) c)^{1/2} \operatorname{arctan}(c x^2^{1/2} / ((b + (-4 a c + b^2)^{1/2}) c)^{1/2}) + b^3 e^3 - 1/4 (a e^2 - b d e + c d^2)^2 / (4 a c - b^2) c^2 2^{1/2} / (((-4 a c + b^2)^{1/2} - b) c)^{1/2} \operatorname{arctanh}(c x^2^{1/2} / (((-4 a c + b^2)^{1/2} - b) c)^{1/2}) + b d e^2 - 1/2 (a e^2 - b d e + c d^2)^2 / (c x^4 + b x^2 + a) c / a (4 a c - b^2) x^3 b^3 d e^2 + 1/4 (a e^2 - b d e + c d^2)^2 / (c x^4 + b x^2 + a) c^2 / a (4 a c - b^2) x^3 b^2 d^2 e + e^4 / (a e^2 - b d e + c d^2)^2 / (d e)^{1/2} \operatorname{arctan}(x e / (d e)^{1/2}) - 7 / (a e^2 - b d e + c d^2)^2 a / (4 a c - b^2) c^3 / (-4 a c + b^2)^{1/2} 2^{1/2} / (((-4 a c + b^2)^{1/2} - b) c)^{1/2} \operatorname{arctanh}(c x^2^{1/2} / (((-4 a c + b^2)^{1/2} - b) c)^{1/2}) + d e^2 + 1/2 (a e^2 - b d e + c d^2)^2 / a (4 a c - b^2) c^2 2^{1/2} / ((b + (-4 a c + b^2)^{1/2}) c)^{1/2} \operatorname{arctan}(c x^2^{1/2} / ((b + (-4 a c + b^2)^{1/2}) c)^{1/2}) + b^2 d^2 e + 1/4 (a e^2 - b d e + c d^2)^2 / a (4 a c - b^2) c^2 2^{1/2} / (((-4 a c + b^2)^{1/2} - b) c)^{1/2} \operatorname{arctanh}(c x^2^{1/2} / (((-4 a c + b^2)^{1/2} - b) c)^{1/2})$$

$$\begin{aligned} & /2)/(((-4ac+b^2)^{1/2}-b)c)^{1/2})b^3d^2e^{2+1/4}/(a^2e-bde+cd^2)^2/a \\ & / (4ac-b^2)c^3/(-4ac+b^2)^{1/2}2^{1/2}/(((-4ac+b^2)^{1/2}-b)c)^{1/2} \\ &)\operatorname{arctanh}(cx^2^{1/2}/(((-4ac+b^2)^{1/2}-b)c)^{1/2})b^2d^3-1/4/(a^2e- \\ & bde+cd^2)^2/a/(4ac-b^2)c^2^{1/2}/((b+(-4ac+b^2)^{1/2})c)^{1/2}\operatorname{arc} \\ & \tan(cx^2^{1/2}/((b+(-4ac+b^2)^{1/2})c)^{1/2})b^3d^2e^{2+1/4}/(a^2e-bd^* \\ & e+cd^2)^2/a/(4ac-b^2)c^3/(-4ac+b^2)^{1/2}2^{1/2}/((b+(-4ac+b^2)^{1/2} \\ &)c)^{1/2}\operatorname{arctan}(cx^2^{1/2}/((b+(-4ac+b^2)^{1/2})c)^{1/2})b^2d^3+ \\ & 4/(a^2e-bde+cd^2)^2a/(4ac-b^2)c^2/(-4ac+b^2)^{1/2}2^{1/2}/((b+(- \\ & 4ac+b^2)^{1/2})c)^{1/2}\operatorname{arctan}(cx^2^{1/2}/((b+(-4ac+b^2)^{1/2})c)^{1/2} \\ &)b^2e^3-7/(a^2e-bde+cd^2)^2a/(4ac-b^2)c^3/(-4ac+b^2)^{1/2}2^{1/2} \\ & /((b+(-4ac+b^2)^{1/2})c)^{1/2}\operatorname{arctan}(cx^2^{1/2}/((b+(-4ac+b^2)^{1/2} \\ &)c)^{1/2})d^2e-1/2/(a^2e-bde+cd^2)^2/a/(4ac-b^2)c^22^{1/2}/(\\ & ((-4ac+b^2)^{1/2}-b)c)^{1/2}\operatorname{arctanh}(cx^2^{1/2}/(((-4ac+b^2)^{1/2}-b) \\ & c)^{1/2})b^2d^2e-3/4/(a^2e-bde+cd^2)^2/(4ac-b^2)c^2/(-4ac+b^2) \\ & ^{1/2}2^{1/2}/(((-4ac+b^2)^{1/2}-b)c)^{1/2}\operatorname{arctanh}(cx^2^{1/2}/(((-4ac \\ & +b^2)^{1/2}-b)c)^{1/2})b^2d^2e^{2+5}/(a^2e-bde+cd^2)^2/(4ac-b^2)c^ \\ & 3/(-4ac+b^2)^{1/2}2^{1/2}/(((-4ac+b^2)^{1/2}-b)c)^{1/2}\operatorname{arctanh}(cx^2 \\ & ^{1/2}/(((-4ac+b^2)^{1/2}-b)c)^{1/2})b^2d^2e+4/(a^2e-bde+cd^2)^2a/ \\ & (4ac-b^2)c^2/(-4ac+b^2)^{1/2}2^{1/2}/(((-4ac+b^2)^{1/2}-b)c)^{1/2} \\ & *\operatorname{arctanh}(cx^2^{1/2}/(((-4ac+b^2)^{1/2}-b)c)^{1/2})b^2e^3-3/4/(a^2e-bd \\ & e+cd^2)^2/(4ac-b^2)c^2/(-4ac+b^2)^{1/2}2^{1/2}/((b+(-4ac+b^2)^{1/2} \\ &)c)^{1/2}\operatorname{arctan}(cx^2^{1/2}/((b+(-4ac+b^2)^{1/2})c)^{1/2})b^2d^2e^2 \\ & +5/(a^2e-bde+cd^2)^2/(4ac-b^2)c^3/(-4ac+b^2)^{1/2}2^{1/2}/((b+(-4 \\ & ac+b^2)^{1/2})c)^{1/2}\operatorname{arctan}(cx^2^{1/2}/((b+(-4ac+b^2)^{1/2})c)^{1/2} \\ &)b^2d^2e+3/4/(a^2e-bde+cd^2)^2/(4ac-b^2)c^2^{1/2}/((b+(-4ac+b^2) \\ &)^{1/2})c)^{1/2}\operatorname{arctan}(cx^2^{1/2}/((b+(-4ac+b^2)^{1/2})c)^{1/2})b^2* \\ & e^3-3/(a^2e-bde+cd^2)^2/(4ac-b^2)c^4/(-4ac+b^2)^{1/2}2^{1/2}/((b+ \\ & (-4ac+b^2)^{1/2})c)^{1/2}\operatorname{arctan}(cx^2^{1/2}/((b+(-4ac+b^2)^{1/2})c)^{1/2} \\ &)d^3+5/2/(a^2e-bde+cd^2)^2a/(4ac-b^2)c^22^{1/2}/(((-4ac+b^ \\ & 2)^{1/2}-b)c)^{1/2}\operatorname{arctanh}(cx^2^{1/2}/(((-4ac+b^2)^{1/2}-b)c)^{1/2})* \\ & e^3-5/2/(a^2e-bde+cd^2)^2a/(4ac-b^2)c^22^{1/2}/((b+(-4ac+b^2)^{1/2} \\ &)c)^{1/2}\operatorname{arctan}(cx^2^{1/2}/((b+(-4ac+b^2)^{1/2})c)^{1/2})e^3+1/2/ \\ & (a^2e-bde+cd^2)^2/(cx^4+bx^2+a)c^2/(4ac-b^2)x^3bde^2-1/2/(a^2e \\ & -bde+cd^2)^2/(cx^4+bx^2+a)c^3/a/(4ac-b^2)x^3bd^3-3/2/(a^2e-bd \\ & e+cd^2)^2/(cx^4+bx^2+a)a/(4ac-b^2)xb^2e^3c+1/(a^2e-bde+cd^2)^2 \\ & / (cx^4+bx^2+a)a/(4ac-b^2)xc^2de^2+1/(a^2e-bde+cd^2)^2/(cx^4+b \\ & x^2+a)/(4ac-b^2)xb^2c^2de^2-5/2/(a^2e-bde+cd^2)^2/(cx^4+bx^2+a) \\ & / (4ac-b^2)xb^2c^2d^2e-1/2/(a^2e-bde+cd^2)^2/(cx^4+bx^2+a)/a/(4a \\ & ac-b^2)xb^4de^2-1/2/(a^2e-bde+cd^2)^2/(cx^4+bx^2+a)/a/(4ac-b^2) \\ & *xb^2c^2d^3+1/2/(a^2e-bde+cd^2)^2/(4ac-b^2)c^32^{1/2}/(((-4ac+ \\ & b^2)^{1/2}-b)c)^{1/2}\operatorname{arctanh}(cx^2^{1/2}/(((-4ac+b^2)^{1/2}-b)c)^{1/2} \\ &)d^2e-1/2/(a^2e-bde+cd^2)^2/(4ac-b^2)c^32^{1/2}/((b+(-4ac+b^2)^{1/2} \\ &)c)^{1/2}\operatorname{arctan}(cx^2^{1/2}/((b+(-4ac+b^2)^{1/2})c)^{1/2})d^2e- \\ & 3/4/(a^2e-bde+cd^2)^2/(4ac-b^2)c^2^{1/2}/(((-4ac+b^2)^{1/2}-b)c)^{1/2} \\ & *\operatorname{arctanh}(cx^2^{1/2}/(((-4ac+b^2)^{1/2}-b)c)^{1/2})b^2e^3-3/(a^2e \\ & -bde+cd^2)^2/(4ac-b^2)c^4/(-4ac+b^2)^{1/2}2^{1/2}/(((-4ac+b^2)^{1/2} \\ & -b)c)^{1/2}\operatorname{arctanh}(cx^2^{1/2}/(((-4ac+b^2)^{1/2}-b)c)^{1/2})d^3 \\ & +1/2/(a^2e-bde+cd^2)^2/(cx^4+bx^2+a)/(4ac-b^2)xb^3e^3+1/(a^2e-b \\ & de+cd^2)^2/(cx^4+bx^2+a)/(4ac-b^2)xc^3d^3-1/(a^2e-bde+cd^2)^2 \\ & / (cx^4+bx^2+a)c^2a/(4ac-b^2)x^3e^3+1/2/(a^2e-bde+cd^2)^2/(cx^4 \\ & +bx^2+a)c/(4ac-b^2)x^3b^2e^3-1/(a^2e-bde+cd^2)^2/(cx^4+bx^2+a) \\ & c^3/(4ac-b^2)x^3d^2e \end{aligned}$$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")

[Out] Timed out

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x**2+d)/(c*x**4+b*x**2+a)**2,x)

[Out] Timed out

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")

[Out] Exception raised: NotImplementedError

3.275 $\int \frac{1}{(d+ex^2)^2(a+bx^2+cx^4)^2} dx$

Optimal. Leaf size=1077

result too large to display

```
[Out] (e^4*x)/(2*d*(c*d^2 - b*d*e + a*e^2)^2*(d + e*x^2)) + (x*(a*b*c*e*(2*c*d - b*e) + (b^2 - 2*a*c)*(c^2*d^2 + b^2*e^2 - c*e*(2*b*d + a*e)) - c*(2*b^2*c*d *e - 4*a*c^2*d*e - b^3*e^2 - b*c*(c*d^2 - 3*a*e^2))*x^2)/(2*a*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)^2*(a + b*x^2 + c*x^4)) + (Sqrt[2]*Sqrt[c]*e^2*(3*c^2*d^2 + b*(b + Sqrt[b^2 - 4*a*c])*e^2 - c*e*(3*b*d + 2*Sqrt[b^2 - 4*a*c]*d + a*e))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]]/(Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]*(c*d^2 - b*d*e + a*e^2)^3) + (Sqrt[c]*(b^4*e^2 - b^3*e*(2*c*d - Sqrt[b^2 - 4*a*c]*e) - 4*a*c^2*(3*c*d^2 - e*(Sqrt[b^2 - 4*a*c]*d + 3*a*e)) + b^2*c*(c*d^2 - e*(2*Sqrt[b^2 - 4*a*c]*d + 9*a*e)) - b*c*(3*a*Sqrt[b^2 - 4*a*c]*e^2 - c*d*(Sqrt[b^2 - 4*a*c]*d + 16*a*e)))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]]/(2*Sqrt[2]*a*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]*(c*d^2 - b*d*e + a*e^2)^2) - (Sqrt[2]*Sqrt[c]*e^2*(3*c^2*d^2 + b*(b - Sqrt[b^2 - 4*a*c])*e^2 - c*e*(3*b*d - 2*Sqrt[b^2 - 4*a*c]*d + a*e))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]]/(Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]*(c*d^2 - b*d*e + a*e^2)^3) - (Sqrt[c]*(b^4*e^2 - b^3*e*(2*c*d + Sqrt[b^2 - 4*a*c]*e) + b*c*(3*a*Sqrt[b^2 - 4*a*c]*e^2 - c*d*(Sqrt[b^2 - 4*a*c]*d - 16*a*e)) + b^2*c*(c*d^2 + e*(2*Sqrt[b^2 - 4*a*c]*d - 9*a*e)) - 4*a*c^2*(3*c*d^2 + e*(Sqrt[b^2 - 4*a*c]*d - 3*a*e)))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]]/(2*Sqrt[2]*a*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]*(c*d^2 - b*d*e + a*e^2)^2) + (2*e^(7/2)*(2*c*d - b*e)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[d]*(c*d^2 - b*d*e + a*e^2)^3) + (e^(7/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(3/2)*(c*d^2 - b*d*e + a*e^2)^2)
```

Rubi [A] time = 12.6389, antiderivative size = 1077, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {1238, 199, 205, 1178, 1166}

$$\frac{xe^4}{2d(cd^2 - bed + ae^2)^2(ex^2 + d)} + \frac{\tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)e^{7/2}}{2d^{3/2}(cd^2 - bed + ae^2)^2} + \frac{2(2cd - be)\tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)e^{7/2}}{\sqrt{d}(cd^2 - bed + ae^2)^3} + \frac{\sqrt{2}\sqrt{c}\left(3c^2d^2 + b\left(b + \sqrt{b^2 - 4ac}\right)\right)}{\sqrt{b^2 - 4ac}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x^2)^2*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] (e^4*x)/(2*d*(c*d^2 - b*d*e + a*e^2)^2*(d + e*x^2)) + (x*(a*b*c*e*(2*c*d - b*e) + (b^2 - 2*a*c)*(c^2*d^2 + b^2*e^2 - c*e*(2*b*d + a*e)) - c*(2*b^2*c*d *e - 4*a*c^2*d*e - b^3*e^2 - b*c*(c*d^2 - 3*a*e^2))*x^2)/(2*a*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)^2*(a + b*x^2 + c*x^4)) + (Sqrt[2]*Sqrt[c]*e^2*(3*c^2*d^2 + b*(b + Sqrt[b^2 - 4*a*c])*e^2 - c*e*(3*b*d + 2*Sqrt[b^2 - 4*a*c]*d + a*e))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]]/(Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]*(c*d^2 - b*d*e + a*e^2)^3) + (Sqrt[c]*(b^4*e^2 - b^3*e*(2*c*d - Sqrt[b^2 - 4*a*c]*e) - 4*a*c^2*(3*c*d^2 - e*(Sqrt[b^2 - 4*a*c]*d + 3*a*e)) + b^2*c*(c*d^2 - e*(2*Sqrt[b^2 - 4*a*c]*d + 9*a*e)) - b*c*(3*a*Sqrt[b^2 - 4*a*c]*e^2 - c*d*(Sqrt[b^2 - 4*a*c]*d + 16*a*e)))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]]/(2*Sqrt[2]*a*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]*(c*d^2 - b*d*e + a*e^2)^2) - (
```

$$\frac{\sqrt{2}\sqrt{c}e^{2(3c^2d^2 + b(b - \sqrt{b^2 - 4ac})e^2 - ce(3bd - 2\sqrt{b^2 - 4ac}d + ae))}\operatorname{ArcTan}\left[\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b + \sqrt{b^2 - 4ac}}}\right]}{(\sqrt{b^2 - 4ac}\sqrt{b + \sqrt{b^2 - 4ac}})(cd^2 - bde + ae^2)^3} - \frac{(\sqrt{c}(b^4e^2 - b^3e(2cd + \sqrt{b^2 - 4ac}e)) + b^2c(3a\sqrt{b^2 - 4ac}e^2 - cd(\sqrt{b^2 - 4ac}d - 16ae)) + b^2c^2(cd^2 + e(2\sqrt{b^2 - 4ac}d - 9ae)) - 4ac^2(3cd^2 + e(\sqrt{b^2 - 4ac}d - 3ae)))\operatorname{ArcTan}\left[\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b + \sqrt{b^2 - 4ac}}}\right]}{(2\sqrt{2}a(b^2 - 4ac)^{3/2}\sqrt{b + \sqrt{b^2 - 4ac}})(cd^2 - bde + ae^2)^2} + \frac{(2e^{7/2}(2cd - be)\operatorname{ArcTan}\left[\frac{\sqrt{e}x}{\sqrt{d}}\right])}{(\sqrt{d}(cd^2 - bde + ae^2)^3)} + \frac{(e^{7/2}\operatorname{ArcTan}\left[\frac{\sqrt{e}x}{\sqrt{d}}\right])}{(2d^{3/2}(cd^2 - bde + ae^2)^2)}$$
Rule 1238

$$\operatorname{Int}[(d + e)(x^2)^q((a + b)(x^2 + c)(x^4)^p), x_Symbol] \rightarrow \operatorname{Int}[\operatorname{ExpandIntegrand}[(d + ex^2)^q(a + bx^2 + cx^4)^p, x], x] /; \operatorname{FreeQ}\{a, b, c, d, e, p, q\}, x \&\& \operatorname{NeQ}[b^2 - 4ac, 0] \&\& ((\operatorname{IntegerQ}[p] \&\& \operatorname{IntegerQ}[q]) \mid\mid \operatorname{IGtQ}[p, 0] \mid\mid \operatorname{IGtQ}[q, 0])$$
Rule 199

$$\operatorname{Int}[(a + b)(x^n)^p, x_Symbol] \rightarrow -\operatorname{Simp}[(x(a + bx^n)^{p+1})/(a^n(p+1)), x] + \operatorname{Dist}[(n(p+1) + 1)/(a^n(p+1)), \operatorname{Int}[(a + bx^n)^{p+1}, x], x] /; \operatorname{FreeQ}\{a, b\}, x \&\& \operatorname{IGtQ}[n, 0] \&\& \operatorname{LtQ}[p, -1] \&\& (\operatorname{IntegerQ}[2p] \mid\mid (n == 2 \&\& \operatorname{IntegerQ}[4p]) \mid\mid (n == 2 \&\& \operatorname{IntegerQ}[3p]) \mid\mid \operatorname{Denominator}[p + 1/n] < \operatorname{Denominator}[p])$$
Rule 205

$$\operatorname{Int}[(a + b)(x^2)^{-1}, x_Symbol] \rightarrow \operatorname{Simp}[(\operatorname{Rt}[a/b, 2] \operatorname{ArcTan}[x/\operatorname{Rt}[a/b, 2]])/a, x] /; \operatorname{FreeQ}\{a, b\}, x \&\& \operatorname{PosQ}[a/b]$$
Rule 1178

$$\operatorname{Int}[(d + e)(x^2)((a + b)(x^2 + c)(x^4)^p), x_Symbol] \rightarrow \operatorname{Simp}[(x(ab^2e - d(b^2 - 2ac) - c(bd - 2ae)x^2)(a + bx^2 + cx^4)^{p+1})/(2a(p+1)(b^2 - 4ac)), x] + \operatorname{Dist}[1/(2a(p+1)(b^2 - 4ac)), \operatorname{Int}[\operatorname{Simp}[(2p+3)d^2b^2 - ab^2e - 2acd(4p+5) + (4p+7)(db - 2ae)cx^2, x](a + bx^2 + cx^4)^{p+1}, x], x] /; \operatorname{FreeQ}\{a, b, c, d, e\}, x \&\& \operatorname{NeQ}[b^2 - 4ac, 0] \&\& \operatorname{NeQ}[cd^2 - bde + ae^2, 0] \&\& \operatorname{LtQ}[p, -1] \&\& \operatorname{IntegerQ}[2p]$$
Rule 1166

$$\operatorname{Int}[(d + e)(x^2)/((a + b)(x^2 + c)(x^4)), x_Symbol] \rightarrow \operatorname{With}\{q = \operatorname{Rt}[b^2 - 4ac, 2]\}, \operatorname{Dist}[e/2 + (2cd - be)/(2q), \operatorname{Int}[1/(b/2 - q/2 + cx^2), x], x] + \operatorname{Dist}[e/2 - (2cd - be)/(2q), \operatorname{Int}[1/(b/2 + q/2 + cx^2), x], x]] /; \operatorname{FreeQ}\{a, b, c, d, e\}, x \&\& \operatorname{NeQ}[b^2 - 4ac, 0] \&\& \operatorname{NeQ}[cd^2 - ae^2, 0] \&\& \operatorname{PosQ}[b^2 - 4ac]$$
Rubi steps

$$\int \frac{1}{(d + ex^2)^2 (a + bx^2 + cx^4)^2} dx = \int \left(\frac{e^4}{(cd^2 - bde + ae^2)^2 (d + ex^2)^2} - \frac{2e^4(-2cd + be)}{(cd^2 - bde + ae^2)^3 (d + ex^2)} + \frac{c^2d^2 + b^2e^2 - ce(2bd + ae)}{(cd^2 - bde + ae^2)^3} \right) dx$$

$$= \frac{e^2 \int \frac{3c^2d^2 + 2b^2e^2 - ce(5bd + ae) - 2ce(2cd - be)x^2}{a + bx^2 + cx^4} dx}{(cd^2 - bde + ae^2)^3} + \frac{(2e^4(2cd - be)) \int \frac{1}{d + ex^2} dx}{(cd^2 - bde + ae^2)^3} + \frac{\int \frac{c^2d^2 + b^2e^2 - ce(2bd + ae)}{a + bx^2 + cx^4} dx}{(cd^2 - bde + ae^2)^3}$$

$$= \frac{e^4x}{2d(cd^2 - bde + ae^2)^2 (d + ex^2)} + \frac{x(abce(2cd - be) + (b^2 - 2ac)(c^2d^2 + b^2e^2 - ce(2bd + ae)))}{2a(b^2 - 4ac)(cd^2 - bde + ae^2)^2}$$

$$= \frac{e^4x}{2d(cd^2 - bde + ae^2)^2 (d + ex^2)} + \frac{x(abce(2cd - be) + (b^2 - 2ac)(c^2d^2 + b^2e^2 - ce(2bd + ae)))}{2a(b^2 - 4ac)(cd^2 - bde + ae^2)^2}$$

$$= \frac{e^4x}{2d(cd^2 - bde + ae^2)^2 (d + ex^2)} + \frac{x(abce(2cd - be) + (b^2 - 2ac)(c^2d^2 + b^2e^2 - ce(2bd + ae)))}{2a(b^2 - 4ac)(cd^2 - bde + ae^2)^2}$$

Mathematica [A] time = 6.28867, size = 1020, normalized size = 0.95

$$\frac{1}{4} \left(\frac{2xe^4}{d(cd^2 + e(ae - bd))^2 (ex^2 + d)} + \frac{2(9cd^2 + e(ae - 5bd)) \tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right) e^{7/2}}{d^{3/2}(cd^2 + e(ae - bd))^3} - \frac{2x(e^2b^4 + ce(ex^2 - 2d)b^3 + c(cd(d - 2ex^2) + b^2))}{a(4ac - b^2)} \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x^2)^2*(a + b*x^2 + c*x^4)^2), x]
```

```
[Out] ((2*e^4*x)/(d*(c*d^2 + e*(-(b*d) + a*e))^2*(d + e*x^2)) - (2*x*(b^4*e^2 + b^3*c*e*(-2*d + e*x^2) + 2*a*c^2*(a*e^2 - c*d*(d - 2*e*x^2)) + b^2*c*(-4*a*e^2 + c*d*(d - 2*e*x^2)) + b*c^2*(c*d^2*x^2 - 3*a*e*(-2*d + e*x^2))))/(a*(-b^2 + 4*a*c)*(c*d^2 + e*(-(b*d) + a*e))^2*(a + b*x^2 + c*x^4)) + (Sqrt[2]*Sqrt[c]*(b^5*d*e^3 + b^3*e*(c*d - Sqrt[b^2 - 4*a*c]*e)*(3*c*d^2 + 5*a*e^2) + b^4*e^2*(-3*c*d^2 + e*(Sqrt[b^2 - 4*a*c]*d - 5*a*e)) - 4*a*c^2*(-3*c^2*d^4 + c*d^2*e*(Sqrt[b^2 - 4*a*c]*d - 12*a*e) + a*e^3*(9*Sqrt[b^2 - 4*a*c]*d + 7*a*e)) - b*c*(-19*a^2*Sqrt[b^2 - 4*a*c]*e^4 + 2*a*c*d*e^2*(-3*Sqrt[b^2 - 4*a*c]*d + 26*a*e) + c^2*d^3*(Sqrt[b^2 - 4*a*c]*d + 28*a*e)) + b^2*c*(-(c^2*d^4) + 3*c*d^2*e*(Sqrt[b^2 - 4*a*c]*d + 4*a*e) + a*e^3*(7*Sqrt[b^2 - 4*a*c]*d + 29*a*e)))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]])/(a*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]*(-(c*d^2) + e*(b*d - a*e))^3) - (Sqrt[2]*Sqrt[c]*(b^5*d*e^3 + b^3*e*(c*d + Sqrt[b^2 - 4*a*c]*e)*(3*c*d^2 + 5*a*e^2) - b^2*c*(c^2*d^4 + a*e^3*(7*Sqrt[b^2 - 4*a*c]*d - 29*a*e) + 3*c*d^2*e*(Sqrt[b^2 - 4*a*c]*d - 4*a*e)) - b^4*e^2*(3*c*d^2 + e*(Sqrt[b^2 - 4*a*c]*d + 5*a*e)) + 4*a*c^2*(3*c^2*d^4 + a*e^3*(9*Sqrt[b^2 - 4*a*c]*d - 7*a*e) + c*d^2*e*(Sqrt[b^2 - 4*a*c]*d + 12*a*e)) + b*c*(-19*a^2*Sqrt[b^2 - 4*a*c]*e^4 + c^2*d^3*(Sqrt[b^2 - 4*a*c]*d - 28*a*e) - 2*a*c*d*e^2*(3*Sqrt[b^2 - 4*a*c]*d + 26*a*e)))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]])/(a*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]*(-(c*d^2) + e*(b*d - a*e))^3) + (2*e^(7/2)*(9*c*d^2 + e*(-5*b*d + a*e))*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(d^(3/2)*(c*d^2 + e*(-(b*d) + a*e))^3))/4
```


Maple [B] time = 0.082, size = 5709, normalized size = 5.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(1/(e*x^2+d)^2/(c*x^4+b*x^2+a)^2,x)$

[Out] result too large to display

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(1/(e*x^2+d)^2/(c*x^4+b*x^2+a)^2,x, \text{algorithm}="maxima")$

[Out] Exception raised: ValueError

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(1/(e*x^2+d)^2/(c*x^4+b*x^2+a)^2,x, \text{algorithm}="fricas")$

[Out] Timed out

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(1/(e*x**2+d)**2/(c*x**4+b*x**2+a)**2,x)$

[Out] Timed out

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(1/(e*x^2+d)^2/(c*x^4+b*x^2+a)^2,x, \text{algorithm}="giac")$

[Out] Exception raised: NotImplementedError

3.276 $\int (d + ex^2)^{5/2} (a + bx^2 + cx^4) dx$

Optimal. Leaf size=215

$$\frac{x(d + ex^2)^{5/2} (80ae^2 - 10bde + 3cd^2)}{480e^2} + \frac{dx(d + ex^2)^{3/2} (80ae^2 - 10bde + 3cd^2)}{384e^2} + \frac{d^2x\sqrt{d + ex^2} (80ae^2 - 10bde + 3cd^2)}{256e^2} +$$

[Out] $(d^2*(3*c*d^2 - 10*b*d*e + 80*a*e^2)*x*\text{Sqrt}[d + e*x^2])/(256*e^2) + (d*(3*c*d^2 - 10*b*d*e + 80*a*e^2)*x*(d + e*x^2)^{(3/2)})/(384*e^2) + ((3*c*d^2 - 10*b*d*e + 80*a*e^2)*x*(d + e*x^2)^{(5/2)})/(480*e^2) - ((3*c*d - 10*b*e)*x*(d + e*x^2)^{(7/2)})/(80*e^2) + (c*x^3*(d + e*x^2)^{(7/2)})/(10*e) + (d^3*(3*c*d^2 - 10*b*d*e + 80*a*e^2)*\text{ArcTanh}[(\text{Sqrt}[e]*x)/\text{Sqrt}[d + e*x^2]])/(256*e^{(5/2)})$

Rubi [A] time = 0.160864, antiderivative size = 215, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {1159, 388, 195, 217, 206}

$$\frac{x(d + ex^2)^{5/2} (80ae^2 - 10bde + 3cd^2)}{480e^2} + \frac{dx(d + ex^2)^{3/2} (80ae^2 - 10bde + 3cd^2)}{384e^2} + \frac{d^2x\sqrt{d + ex^2} (80ae^2 - 10bde + 3cd^2)}{256e^2} +$$

Antiderivative was successfully verified.

[In] Int[(d + e*x^2)^(5/2)*(a + b*x^2 + c*x^4),x]

[Out] $(d^2*(3*c*d^2 - 10*b*d*e + 80*a*e^2)*x*\text{Sqrt}[d + e*x^2])/(256*e^2) + (d*(3*c*d^2 - 10*b*d*e + 80*a*e^2)*x*(d + e*x^2)^{(3/2)})/(384*e^2) + ((3*c*d^2 - 10*b*d*e + 80*a*e^2)*x*(d + e*x^2)^{(5/2)})/(480*e^2) - ((3*c*d - 10*b*e)*x*(d + e*x^2)^{(7/2)})/(80*e^2) + (c*x^3*(d + e*x^2)^{(7/2)})/(10*e) + (d^3*(3*c*d^2 - 10*b*d*e + 80*a*e^2)*\text{ArcTanh}[(\text{Sqrt}[e]*x)/\text{Sqrt}[d + e*x^2]])/(256*e^{(5/2)})$

Rule 1159

Int[((d_) + (e_.)*(x_)^(q_))*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := Simp[(c^p*x^(4*p - 1)*(d + e*x^2)^(q + 1))/(e*(4*p + 2*q + 1)), x] + Dist[1/(e*(4*p + 2*q + 1)), Int[(d + e*x^2)^q*ExpandToSum[e*(4*p + 2*q + 1)*(a + b*x^2 + c*x^4)^p - d*c^p*(4*p - 1)*x^(4*p - 2) - e*c^p*(4*p + 2*q + 1)*x^(4*p), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && !LtQ[q, -1]

Rule 388

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Simp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]

Rule 195

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(x*(a + b*x^n)^p)/(n*p + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] && IntegerQ[4*p])) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n], Denominator[p]]

Rule 217

`Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]`

Rule 206

`Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])`

Rubi steps

$$\begin{aligned} \int (d + ex^2)^{5/2} (a + bx^2 + cx^4) dx &= \frac{cx^3 (d + ex^2)^{7/2}}{10e} + \frac{\int (d + ex^2)^{5/2} (10ae - (3cd - 10be)x^2) dx}{10e} \\ &= -\frac{(3cd - 10be)x (d + ex^2)^{7/2}}{80e^2} + \frac{cx^3 (d + ex^2)^{7/2}}{10e} - \frac{1}{80} \left(-80a - \frac{d(3cd - 10be)}{e^2} \right) \int (d + ex^2)^{3/2} dx \\ &= \frac{1}{480} \left(80a + \frac{d(3cd - 10be)}{e^2} \right) x (d + ex^2)^{5/2} - \frac{(3cd - 10be)x (d + ex^2)^{7/2}}{80e^2} + \frac{cx^3 (d + ex^2)^{7/2}}{10e} \\ &= \frac{1}{384} d \left(80a + \frac{d(3cd - 10be)}{e^2} \right) x (d + ex^2)^{3/2} + \frac{1}{480} \left(80a + \frac{d(3cd - 10be)}{e^2} \right) x (d + ex^2)^{5/2} \\ &= \frac{1}{256} d^2 \left(80a + \frac{d(3cd - 10be)}{e^2} \right) x \sqrt{d + ex^2} + \frac{1}{384} d \left(80a + \frac{d(3cd - 10be)}{e^2} \right) x (d + ex^2)^{3/2} \\ &= \frac{1}{256} d^2 \left(80a + \frac{d(3cd - 10be)}{e^2} \right) x \sqrt{d + ex^2} + \frac{1}{384} d \left(80a + \frac{d(3cd - 10be)}{e^2} \right) x (d + ex^2)^{3/2} \\ &= \frac{1}{256} d^2 \left(80a + \frac{d(3cd - 10be)}{e^2} \right) x \sqrt{d + ex^2} + \frac{1}{384} d \left(80a + \frac{d(3cd - 10be)}{e^2} \right) x (d + ex^2)^{3/2} \end{aligned}$$

Mathematica [A] time = 0.402441, size = 190, normalized size = 0.88

$$\frac{\sqrt{d + ex^2} \left(\sqrt{ex} (10e (8ae (33d^2 + 26dex^2 + 8e^2x^4) + b (118d^2ex^2 + 15d^3 + 136de^2x^4 + 48e^3x^6)) + c (744d^2e^2x^4 + 30d^3e^2x^2 + 1008d^2e^3x^6 + 384e^4x^8) + 10e (8ae (33d^2 + 26d^2ex^2 + 8e^2x^4) + b (15d^3 + 118d^2ex^2 + 136d^2e^2x^4 + 48e^3x^6))) + (15d^{5/2} (3cd^2 + 10e(-bd) + 8ae)) \operatorname{ArcSinh}[\frac{\sqrt{e}x}{\sqrt{d}}] / \sqrt{1 + (ex^2/d)} \right)}{3840e^{5/2}}$$

Antiderivative was successfully verified.

[In] `Integrate[(d + e*x^2)^(5/2)*(a + b*x^2 + c*x^4), x]`

[Out] `(Sqrt[d + e*x^2]*(Sqrt[e]**(c*(-45*d^4 + 30*d^3*e*x^2 + 744*d^2*e^2*x^4 + 1008*d*e^3*x^6 + 384*e^4*x^8) + 10*e*(8*a*e*(33*d^2 + 26*d^2*e*x^2 + 8*e^2*x^4) + b*(15*d^3 + 118*d^2*e*x^2 + 136*d^2*e^2*x^4 + 48*e^3*x^6))) + (15*d^(5/2)*(3*c*d^2 + 10*e*(-(b*d) + 8*a*e))*ArcSinh[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[1 + (e*x^2)/d])/(3840*e^(5/2))`

Maple [A] time = 0.008, size = 283, normalized size = 1.3

$$\frac{cx^3}{10e} (ex^2 + d)^{7/2} - \frac{3cdx}{80e^2} (ex^2 + d)^{7/2} + \frac{cd^2x}{160e^2} (ex^2 + d)^{5/2} + \frac{cd^3x}{128e^2} (ex^2 + d)^{3/2} + \frac{3cd^4x}{256e^2} \sqrt{ex^2 + d} + \frac{3cd^5}{256} \ln(x\sqrt{e} + \sqrt{ex^2 + d})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*x^2+d)^(5/2)*(c*x^4+b*x^2+a),x)`

[Out] $\frac{1}{10}c x^3 (e x^2+d)^{7/2} / e - \frac{3}{80}c d / e^2 x (e x^2+d)^{7/2} + \frac{1}{160}c d^2 / e^2 x (e x^2+d)^{5/2} + \frac{1}{128}c d^3 / e^2 x (e x^2+d)^{3/2} + \frac{3}{256}c d^4 / e^2 x (e x^2+d)^{1/2} + \frac{3}{256}c d^5 / e^{5/2} \ln(x e^{1/2} + (e x^2+d)^{1/2}) + \frac{1}{8}b x (e x^2+d)^{7/2} / e - \frac{1}{48}b d / e x (e x^2+d)^{5/2} - \frac{5}{192}b d^2 / e x (e x^2+d)^{3/2} - \frac{5}{128}b d^3 / e x (e x^2+d)^{1/2} - \frac{5}{128}b d^4 / e^{3/2} \ln(x e^{1/2} + (e x^2+d)^{1/2}) + \frac{1}{6}a x (e x^2+d)^{5/2} + \frac{5}{24}a d x (e x^2+d)^{3/2} + \frac{5}{16}a d^2 x (e x^2+d)^{1/2} + \frac{5}{16}a d^3 / e^{1/2} \ln(x e^{1/2} + (e x^2+d)^{1/2})$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x^2+d)^(5/2)*(c*x^4+b*x^2+a),x, algorithm="maxima")`

[Out] Exception raised: ValueError

Fricas [A] time = 5.98159, size = 882, normalized size = 4.1

$$\frac{15(3cd^5 - 10bd^4e + 80ad^3e^2)\sqrt{e} \log(-2ex^2 - 2\sqrt{ex^2+d}\sqrt{ex-d}) + 2(384ce^5x^9 + 48(21cde^4 + 10be^5)x^7 + 8(93cd^2e^3 + 170bde^4 + 80ae^5)x^5 + 10(3cd^3e^2 + 118bd^2e^3 + 208ade^4)x^3 - 15(3cd^4e - 10bd^3e^2 - 176ad^2e^3)x)\sqrt{e^2+d}}{7680e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x^2+d)^(5/2)*(c*x^4+b*x^2+a),x, algorithm="fricas")`

[Out] $\frac{1}{7680} (15(3cd^5 - 10bd^4e + 80ad^3e^2)\sqrt{e} \log(-2ex^2 - 2\sqrt{ex^2+d}\sqrt{ex-d}) + 2(384ce^5x^9 + 48(21cde^4 + 10be^5)x^7 + 8(93cd^2e^3 + 170bde^4 + 80ae^5)x^5 + 10(3cd^3e^2 + 118bd^2e^3 + 208ade^4)x^3 - 15(3cd^4e - 10bd^3e^2 - 176ad^2e^3)x)\sqrt{e^2+d}) / e^3 - \frac{1}{3840} (15(3cd^5 - 10bd^4e + 80ad^3e^2)\sqrt{-e} \arctan(\sqrt{-e}x/\sqrt{ex^2+d}) - (384ce^5x^9 + 48(21cde^4 + 10be^5)x^7 + 8(93cd^2e^3 + 170bde^4 + 80ae^5)x^5 + 10(3cd^3e^2 + 118bd^2e^3 + 208ade^4)x^3 - 15(3cd^4e - 10bd^3e^2 - 176ad^2e^3)x)\sqrt{e^2+d}) / e^3$

Sympy [B] time = 55.0993, size = 505, normalized size = 2.35

$$\frac{ad^5x\sqrt{1+\frac{ex^2}{d}}}{2} + \frac{3ad^5x}{16\sqrt{1+\frac{ex^2}{d}}} + \frac{35ad^3ex^3}{48\sqrt{1+\frac{ex^2}{d}}} + \frac{17a\sqrt{de^2}x^5}{24\sqrt{1+\frac{ex^2}{d}}} + \frac{5ad^3 \operatorname{asinh}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)}{16\sqrt{e}} + \frac{ae^3x^7}{6\sqrt{d}\sqrt{1+\frac{ex^2}{d}}} + \frac{5bd^7x}{128e\sqrt{1+\frac{ex^2}{d}}} + \frac{133}{384}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x**2+d)**(5/2)*(c*x**4+b*x**2+a),x)`

```
[Out] a*d**(5/2)*x*sqrt(1 + e*x**2/d)/2 + 3*a*d**(5/2)*x/(16*sqrt(1 + e*x**2/d))
+ 35*a*d**(3/2)*e*x**3/(48*sqrt(1 + e*x**2/d)) + 17*a*sqrt(d)*e**2*x**5/(24
*sqrt(1 + e*x**2/d)) + 5*a*d**3*asinh(sqrt(e)*x/sqrt(d))/(16*sqrt(e)) + a*e
**3*x**7/(6*sqrt(d)*sqrt(1 + e*x**2/d)) + 5*b*d**(7/2)*x/(128*e*sqrt(1 + e
*x**2/d)) + 133*b*d**(5/2)*x**3/(384*sqrt(1 + e*x**2/d)) + 127*b*d**(3/2)*e
*x**5/(192*sqrt(1 + e*x**2/d)) + 23*b*sqrt(d)*e**2*x**7/(48*sqrt(1 + e*x**2/
d)) - 5*b*d**4*asinh(sqrt(e)*x/sqrt(d))/(128*e**(3/2)) + b*e**3*x**9/(8*sqr
t(d)*sqrt(1 + e*x**2/d)) - 3*c*d**(9/2)*x/(256*e**2*sqrt(1 + e*x**2/d)) - c
*d**(7/2)*x**3/(256*e*sqrt(1 + e*x**2/d)) + 129*c*d**(5/2)*x**5/(640*sqrt(1
+ e*x**2/d)) + 73*c*d**(3/2)*e*x**7/(160*sqrt(1 + e*x**2/d)) + 29*c*sqrt(d
)*e**2*x**9/(80*sqrt(1 + e*x**2/d)) + 3*c*d**5*asinh(sqrt(e)*x/sqrt(d))/(25
6*e**(5/2)) + c*e**3*x**11/(10*sqrt(d)*sqrt(1 + e*x**2/d))
```

Giac [A] time = 1.21445, size = 243, normalized size = 1.13

$$-\frac{1}{256} (3cd^5 - 10bd^4e + 80ad^3e^2)e^{(-\frac{5}{2})} \log\left(\left| -xe^{\frac{1}{2}} + \sqrt{x^2e + d} \right|\right) + \frac{1}{3840} (2(4(6(8cx^2e^2 + (21cde^9 + 10be^{10})e^{(-8)})x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(5/2)*(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/256*(3*c*d^5 - 10*b*d^4*e + 80*a*d^3*e^2)*e^(-5/2)*log(abs(-x*e^(1/2) +
sqrt(x^2*e + d))) + 1/3840*(2*(4*(6*(8*c*x^2*e^2 + (21*c*d*e^9 + 10*b*e^10)
*e^(-8))*x^2 + (93*c*d^2*e^8 + 170*b*d*e^9 + 80*a*e^10)*e^(-8))*x^2 + 5*(3*
c*d^3*e^7 + 118*b*d^2*e^8 + 208*a*d*e^9)*e^(-8))*x^2 - 15*(3*c*d^4*e^6 - 10
*b*d^3*e^7 - 176*a*d^2*e^8)*e^(-8))*sqrt(x^2*e + d)*x
```

3.277 $\int (d + ex^2)^{3/2} (a + bx^2 + cx^4) dx$

Optimal. Leaf size=175

$$\frac{x(d + ex^2)^{3/2} (48ae^2 - 8bde + 3cd^2)}{192e^2} + \frac{dx\sqrt{d + ex^2} (48ae^2 - 8bde + 3cd^2)}{128e^2} + \frac{d^2 \tanh^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d+ex^2}}\right) (48ae^2 - 8bde + 3cd^2)}{128e^{5/2}}$$

[Out] (d*(3*c*d^2 - 8*b*d*e + 48*a*e^2)*x*Sqrt[d + e*x^2])/(128*e^2) + ((3*c*d^2 - 8*b*d*e + 48*a*e^2)*x*(d + e*x^2)^(3/2))/(192*e^2) - ((3*c*d - 8*b*e)*x*(d + e*x^2)^(5/2))/(48*e^2) + (c*x^3*(d + e*x^2)^(5/2))/(8*e) + (d^2*(3*c*d^2 - 8*b*d*e + 48*a*e^2)*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(128*e^(5/2))

Rubi [A] time = 0.121627, antiderivative size = 175, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {1159, 388, 195, 217, 206}

$$\frac{x(d + ex^2)^{3/2} (48ae^2 - 8bde + 3cd^2)}{192e^2} + \frac{dx\sqrt{d + ex^2} (48ae^2 - 8bde + 3cd^2)}{128e^2} + \frac{d^2 \tanh^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d+ex^2}}\right) (48ae^2 - 8bde + 3cd^2)}{128e^{5/2}}$$

Antiderivative was successfully verified.

[In] Int[(d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4),x]

[Out] (d*(3*c*d^2 - 8*b*d*e + 48*a*e^2)*x*Sqrt[d + e*x^2])/(128*e^2) + ((3*c*d^2 - 8*b*d*e + 48*a*e^2)*x*(d + e*x^2)^(3/2))/(192*e^2) - ((3*c*d - 8*b*e)*x*(d + e*x^2)^(5/2))/(48*e^2) + (c*x^3*(d + e*x^2)^(5/2))/(8*e) + (d^2*(3*c*d^2 - 8*b*d*e + 48*a*e^2)*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(128*e^(5/2))

Rule 1159

Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] := Simp[(c^p*x^(4*p - 1)*(d + e*x^2)^(q + 1))/(e*(4*p + 2*q + 1)), x] + Dist[1/(e*(4*p + 2*q + 1)), Int[(d + e*x^2)^q*ExpandToSum[e*(4*p + 2*q + 1)*(a + b*x^2 + c*x^4)^p - d*c^p*(4*p - 1)*x^(4*p - 2) - e*c^p*(4*p + 2*q + 1)*x^(4*p), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && !LtQ[q, -1]

Rule 388

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Simp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]

Rule 195

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(x*(a + b*x^n)^p)/(n*p + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] && IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n], Denominator[p]])

Rule 217

`Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]`

Rule 206

`Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])`

Rubi steps

$$\begin{aligned}
 \int (d + ex^2)^{3/2} (a + bx^2 + cx^4) dx &= \frac{cx^3 (d + ex^2)^{5/2}}{8e} + \frac{\int (d + ex^2)^{3/2} (8ae - (3cd - 8be)x^2) dx}{8e} \\
 &= -\frac{(3cd - 8be)x (d + ex^2)^{5/2}}{48e^2} + \frac{cx^3 (d + ex^2)^{5/2}}{8e} - \frac{1}{48} \left(-48a - \frac{d(3cd - 8be)}{e^2} \right) \int (d + ex^2)^{1/2} dx \\
 &= \frac{1}{192} \left(48a + \frac{d(3cd - 8be)}{e^2} \right) x (d + ex^2)^{3/2} - \frac{(3cd - 8be)x (d + ex^2)^{5/2}}{48e^2} + \frac{cx^3 (d + ex^2)^{5/2}}{8e} \\
 &= \frac{1}{128} d \left(48a + \frac{d(3cd - 8be)}{e^2} \right) x \sqrt{d + ex^2} + \frac{1}{192} \left(48a + \frac{d(3cd - 8be)}{e^2} \right) x (d + ex^2)^{3/2} \\
 &= \frac{1}{128} d \left(48a + \frac{d(3cd - 8be)}{e^2} \right) x \sqrt{d + ex^2} + \frac{1}{192} \left(48a + \frac{d(3cd - 8be)}{e^2} \right) x (d + ex^2)^{3/2} \\
 &= \frac{1}{128} d \left(48a + \frac{d(3cd - 8be)}{e^2} \right) x \sqrt{d + ex^2} + \frac{1}{192} \left(48a + \frac{d(3cd - 8be)}{e^2} \right) x (d + ex^2)^{3/2}
 \end{aligned}$$

Mathematica [A] time = 0.32968, size = 157, normalized size = 0.9

$$\frac{\sqrt{d + ex^2} \left(\sqrt{ex} (8e (6ae (5d + 2ex^2) + b (3d^2 + 14dex^2 + 8e^2x^4)) + c (6d^2ex^2 - 9d^3 + 72de^2x^4 + 48e^3x^6)) + \frac{3d^{3/2} \sinh^{-1}\left(\frac{x\sqrt{e}}{\sqrt{d}}\right)}{384e^{5/2}} \right)}{384e^{5/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4), x]

[Out] (Sqrt[d + e*x^2]*(Sqrt[e]**(c*(-9*d^3 + 6*d^2*e*x^2 + 72*d*e^2*x^4 + 48*e^3*x^6) + 8*e*(6*a*e*(5*d + 2*e*x^2) + b*(3*d^2 + 14*d*e*x^2 + 8*e^2*x^4))) + (3*d^(3/2)*(3*c*d^2 + 8*e*(-(b*d) + 6*a*e))*ArcSinh[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[1 + (e*x^2)/d])/(384*e^(5/2))

Maple [A] time = 0.008, size = 229, normalized size = 1.3

$$\frac{cx^3}{8e} (ex^2 + d)^{\frac{5}{2}} - \frac{cdx}{16e^2} (ex^2 + d)^{\frac{5}{2}} + \frac{cd^2x}{64e^2} (ex^2 + d)^{\frac{3}{2}} + \frac{3cd^3x}{128e^2} \sqrt{ex^2 + d} + \frac{3cd^4}{128} \ln(x\sqrt{e} + \sqrt{ex^2 + d}) e^{-\frac{5}{2}} + \frac{bx}{6e} (ex^2 + d)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x^2+d)^(3/2)*(c*x^4+b*x^2+a), x)

[Out] $\frac{1}{8}c*x^3*(e*x^2+d)^{(5/2)}/e-1/16*c*d/e^2*x*(e*x^2+d)^{(5/2)}+1/64*c*d^2/e^2*x*(e*x^2+d)^{(3/2)}+3/128*c*d^3/e^2*x*(e*x^2+d)^{(1/2)}+3/128*c*d^4/e^{(5/2)}*\ln(x*e^{(1/2)}+(e*x^2+d)^{(1/2)})+1/6*b*x*(e*x^2+d)^{(5/2)}/e-1/24*b*d/e*x*(e*x^2+d)^{(3/2)}-1/16*b*d^2/e*x*(e*x^2+d)^{(1/2)}-1/16*b*d^3/e^{(3/2)}*\ln(x*e^{(1/2)}+(e*x^2+d)^{(1/2)})+1/4*a*x*(e*x^2+d)^{(3/2)}+3/8*a*d*x*(e*x^2+d)^{(1/2)}+3/8*a*d^2/e^{(1/2)}*\ln(x*e^{(1/2)}+(e*x^2+d)^{(1/2)})$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x^2+d)^(3/2)*(c*x^4+b*x^2+a),x, algorithm="maxima")`

[Out] Exception raised: ValueError

Fricas [A] time = 4.94813, size = 701, normalized size = 4.01

$$\frac{3(3cd^4 - 8bd^3e + 48ad^2e^2)\sqrt{e} \log\left(-2ex^2 - 2\sqrt{ex^2 + d}\sqrt{ex - d}\right) + 2(48ce^4x^7 + 8(9cde^3 + 8be^4)x^5 + 2(3cd^2e^2 + 56bd^3e))\sqrt{e}}{768e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x^2+d)^(3/2)*(c*x^4+b*x^2+a),x, algorithm="fricas")`

[Out] $\frac{1}{768}*(3*(3*c*d^4 - 8*b*d^3*e + 48*a*d^2*e^2)*\sqrt{e}*\log(-2*e*x^2 - 2*\sqrt{e*x^2 + d}*\sqrt{e*x - d}) + 2*(48*c*e^4*x^7 + 8*(9*c*d*e^3 + 8*b*e^4)*x^5 + 2*(3*c*d^2*e^2 + 56*b*d^3*e + 48*a*d^2*e^2)*\sqrt{e})*\sqrt{e} - 1/384*(3*(3*c*d^4 - 8*b*d^3*e + 48*a*d^2*e^2)*\sqrt{-e}*\arctan(\sqrt{-e}*x/\sqrt{e*x^2 + d}) - (48*c*e^4*x^7 + 8*(9*c*d*e^3 + 8*b*e^4)*x^5 + 2*(3*c*d^2*e^2 + 56*b*d^3*e + 48*a*d^2*e^2)*\sqrt{e})*\sqrt{e})/\sqrt{e}$

Sympy [B] time = 28.1383, size = 413, normalized size = 2.36

$$\frac{ad^{\frac{3}{2}}x\sqrt{1 + \frac{ex^2}{d}}}{2} + \frac{ad^{\frac{3}{2}}x}{8\sqrt{1 + \frac{ex^2}{d}}} + \frac{3a\sqrt{d}ex^3}{8\sqrt{1 + \frac{ex^2}{d}}} + \frac{3ad^2 \operatorname{asinh}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)}{8\sqrt{e}} + \frac{ae^2x^5}{4\sqrt{d}\sqrt{1 + \frac{ex^2}{d}}} + \frac{bd^{\frac{5}{2}}x}{16e\sqrt{1 + \frac{ex^2}{d}}} + \frac{17bd^{\frac{3}{2}}x^3}{48\sqrt{1 + \frac{ex^2}{d}}} + \frac{11b\sqrt{d}}{24\sqrt{1 + \frac{ex^2}{d}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x**2+d)**(3/2)*(c*x**4+b*x**2+a),x)`

[Out] $a*d^{(3/2)}*x*\sqrt{1 + e*x**2/d}/2 + a*d^{(3/2)}*x/(8*\sqrt{1 + e*x**2/d}) + 3*a*\sqrt{d}*e*x**3/(8*\sqrt{1 + e*x**2/d}) + 3*a*d**2*asinh(\sqrt{e}*x/\sqrt{d})/(8*\sqrt{e}) + a*e**2*x**5/(4*\sqrt{d}*\sqrt{1 + e*x**2/d}) + b*d**(5/2)*x/(16*e*\sqrt{1 + e*x**2/d}) + 17*b*d**(3/2)*x**3/(48*\sqrt{1 + e*x**2/d}) + 11*b*\sqrt{d}*e*x**5/(24*\sqrt{1 + e*x**2/d}) - b*d**3*asinh(\sqrt{e}*x/\sqrt{d})/(16*e**(3/2)) + b*e**2*x**7/(6*\sqrt{d}*\sqrt{1 + e*x**2/d}) - 3*c*d**(7/2)*x$

$$\begin{aligned} &/ (128e^{**2}\sqrt{1 + e^{**2}/d}) - cd^{**}(5/2)x^{**3}/(128e\sqrt{1 + e^{**2}/d}) \\ &+ 13cd^{**}(3/2)x^{**5}/(64\sqrt{1 + e^{**2}/d}) + 5c\sqrt{d}e^{**7}/(16\sqrt{1 + e^{**2}/d}) \\ &+ 3cd^{**4}\operatorname{asinh}(\sqrt{e}x/\sqrt{d})/(128e^{**}(5/2)) + ce^{**2}x^{**9}/(8\sqrt{d}\sqrt{1 + e^{**2}/d}) \end{aligned}$$

Giac [A] time = 1.1349, size = 196, normalized size = 1.12

$$-\frac{1}{128} (3cd^4 - 8bd^3e + 48ad^2e^2)e^{\left(-\frac{5}{2}\right)} \log\left(\left|-xe^{\frac{1}{2}} + \sqrt{x^2e + d}\right|\right) + \frac{1}{384} \left(2\left(4\left(6cx^2e + (9cde^6 + 8be^7)e^{(-6)}\right)x^2 + (3cd^2e\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)^(3/2)*(c*x^4+b*x^2+a),x, algorithm="giac")

[Out] -1/128*(3*c*d^4 - 8*b*d^3*e + 48*a*d^2*e^2)*e^(-5/2)*log(abs(-x*e^(1/2) + sqrt(x^2*e + d))) + 1/384*(2*(4*(6*c*x^2*e + (9*c*d*e^6 + 8*b*e^7)*e^(-6))*x^2 + (3*c*d^2*e^5 + 56*b*d*e^6 + 48*a*e^7)*e^(-6))*x^2 - 3*(3*c*d^3*e^4 - 8*b*d^2*e^5 - 80*a*d*e^6)*e^(-6))*sqrt(x^2*e + d)*x

3.278 $\int \sqrt{d + ex^2} (a + bx^2 + cx^4) dx$

Optimal. Leaf size=132

$$\frac{x\sqrt{d+ex^2}(8ae^2-2bde+cd^2)}{16e^2} + \frac{d \tanh^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d+ex^2}}\right)(8ae^2-2bde+cd^2)}{16e^{5/2}} - \frac{x(d+ex^2)^{3/2}(cd-2be)}{8e^2} + \frac{cx^3(d+ex^2)^{3/2}}{6e}$$

[Out] $((c*d^2 - 2*b*d*e + 8*a*e^2)*x*\text{Sqrt}[d + e*x^2])/(16*e^2) - ((c*d - 2*b*e)*x*(d + e*x^2)^{(3/2)})/(8*e^2) + (c*x^3*(d + e*x^2)^{(3/2)})/(6*e) + (d*(c*d^2 - 2*b*d*e + 8*a*e^2)*\text{ArcTanh}[(\text{Sqrt}[e]*x)/\text{Sqrt}[d + e*x^2]])/(16*e^{(5/2)})$

Rubi [A] time = 0.109218, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {1159, 388, 195, 217, 206}

$$\frac{x\sqrt{d+ex^2}(8ae^2-2bde+cd^2)}{16e^2} + \frac{d \tanh^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d+ex^2}}\right)(8ae^2-2bde+cd^2)}{16e^{5/2}} - \frac{x(d+ex^2)^{3/2}(cd-2be)}{8e^2} + \frac{cx^3(d+ex^2)^{3/2}}{6e}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Sqrt}[d + e*x^2]*(a + b*x^2 + c*x^4), x]$

[Out] $((c*d^2 - 2*b*d*e + 8*a*e^2)*x*\text{Sqrt}[d + e*x^2])/(16*e^2) - ((c*d - 2*b*e)*x*(d + e*x^2)^{(3/2)})/(8*e^2) + (c*x^3*(d + e*x^2)^{(3/2)})/(6*e) + (d*(c*d^2 - 2*b*d*e + 8*a*e^2)*\text{ArcTanh}[(\text{Sqrt}[e]*x)/\text{Sqrt}[d + e*x^2]])/(16*e^{(5/2)})$

Rule 1159

$\text{Int}[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x]$ $\rightarrow \text{Simp}[(c^p*x^{(4*p-1)}*(d + e*x^2)^{(q+1)})/(e*(4*p + 2*q + 1)), x] + \text{Dist}[1/(e*(4*p + 2*q + 1)), \text{Int}[(d + e*x^2)^q*\text{ExpandToSum}[e*(4*p + 2*q + 1)*(a + b*x^2 + c*x^4)^p - d*c^p*(4*p - 1)*x^{(4*p-2)} - e*c^p*(4*p + 2*q + 1)*x^{(4*p)}, x], x] /;$ $\text{FreeQ}\{a, b, c, d, e, q\}, x$ && $\text{NeQ}[b^2 - 4*a*c, 0]$ && $\text{NeQ}[c*d^2 - b*d*e + a*e^2, 0]$ && $\text{IGtQ}[p, 0]$ && $\text{!LtQ}[q, -1]$

Rule 388

$\text{Int}[(a + b*x^n)^p*(c + d*x^n), x]$ $\rightarrow \text{Simp}[(d*x*(a + b*x^n)^{(p+1)})/(b*(n*(p+1) + 1)), x] - \text{Dist}[(a*d - b*c*(n*(p+1) + 1))/(b*(n*(p+1) + 1)), \text{Int}[(a + b*x^n)^p, x], x] /;$ $\text{FreeQ}\{a, b, c, d, n\}, x$ && $\text{NeQ}[b*c - a*d, 0]$ && $\text{NeQ}[n*(p+1) + 1, 0]$

Rule 195

$\text{Int}[(a + b*x^n)^p, x]$ $\rightarrow \text{Simp}[(x*(a + b*x^n)^p)/(n*p + 1), x] + \text{Dist}[(a*n*p)/(n*p + 1), \text{Int}[(a + b*x^n)^{(p-1)}, x], x] /;$ $\text{FreeQ}\{a, b\}, x$ && $\text{IGtQ}[n, 0]$ && $\text{GtQ}[p, 0]$ && $(\text{IntegerQ}[2*p] \parallel (\text{EqQ}[n, 2] \&\& \text{IntegerQ}[4*p]) \parallel (\text{EqQ}[n, 2] \&\& \text{IntegerQ}[3*p])) \parallel \text{LtQ}[\text{Denominator}[p + 1/n], \text{Denominator}[p]]$

Rule 217

$\text{Int}[1/\text{Sqrt}[a + b*x^2], x]$ $\rightarrow \text{Subst}[\text{Int}[1/(1 - b*x^2), x], x, x/\text{Sqrt}[a + b*x^2]] /;$ $\text{FreeQ}\{a, b\}, x$ && $\text{!GtQ}[a, 0]$

Rule 206

$\text{Int}[(a + b \cdot x^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(1 \cdot \text{ArcTanh}[\text{Rt}[-b, 2] \cdot x] / \text{Rt}[a, 2]) / (\text{Rt}[a, 2] \cdot \text{Rt}[-b, 2]), x] /;$ FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned} \int \sqrt{d + ex^2} (a + bx^2 + cx^4) dx &= \frac{cx^3 (d + ex^2)^{3/2}}{6e} + \frac{\int \sqrt{d + ex^2} (6ae - 3(cd - 2be)x^2) dx}{6e} \\ &= -\frac{(cd - 2be)x (d + ex^2)^{3/2}}{8e^2} + \frac{cx^3 (d + ex^2)^{3/2}}{6e} + \frac{1}{8} \left(8a + \frac{d(cd - 2be)}{e^2} \right) \int \sqrt{d + ex^2} dx \\ &= \frac{1}{16} \left(8a + \frac{d(cd - 2be)}{e^2} \right) x \sqrt{d + ex^2} - \frac{(cd - 2be)x (d + ex^2)^{3/2}}{8e^2} + \frac{cx^3 (d + ex^2)^{3/2}}{6e} + \frac{1}{16} \int \sqrt{d + ex^2} dx \\ &= \frac{1}{16} \left(8a + \frac{d(cd - 2be)}{e^2} \right) x \sqrt{d + ex^2} - \frac{(cd - 2be)x (d + ex^2)^{3/2}}{8e^2} + \frac{cx^3 (d + ex^2)^{3/2}}{6e} + \frac{1}{16} \int \sqrt{d + ex^2} dx \\ &= \frac{1}{16} \left(8a + \frac{d(cd - 2be)}{e^2} \right) x \sqrt{d + ex^2} - \frac{(cd - 2be)x (d + ex^2)^{3/2}}{8e^2} + \frac{cx^3 (d + ex^2)^{3/2}}{6e} + \frac{d}{16} \int \sqrt{d + ex^2} dx \end{aligned}$$

Mathematica [A] time = 0.237715, size = 121, normalized size = 0.92

$$\frac{\sqrt{d + ex^2} \left(\sqrt{ex} (6e (4ae + b (d + 2ex^2))) + c (-3d^2 + 2dex^2 + 8e^2x^4) + \frac{3\sqrt{d} \sinh^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right) (8ae^2 - 2bde + cd^2)}{\sqrt{\frac{ex^2}{d} + 1}} \right)}{48e^{5/2}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[d + e*x^2]*(a + b*x^2 + c*x^4), x]

[Out] (Sqrt[d + e*x^2]*(Sqrt[e]*x*(c*(-3*d^2 + 2*d*e*x^2 + 8*e^2*x^4) + 6*e*(4*a*e + b*(d + 2*e*x^2))) + (3*Sqrt[d]*(c*d^2 - 2*b*d*e + 8*a*e^2)*ArcSinh[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[1 + (e*x^2)/d])/ (48*e^(5/2))

Maple [A] time = 0.01, size = 175, normalized size = 1.3

$$\frac{cx^3}{6e} (ex^2 + d)^{\frac{3}{2}} - \frac{cdx}{8e^2} (ex^2 + d)^{\frac{3}{2}} + \frac{cd^2x}{16e^2} \sqrt{ex^2 + d} + \frac{cd^3}{16} \ln(x\sqrt{e} + \sqrt{ex^2 + d}) e^{-\frac{5}{2}} + \frac{bx}{4e} (ex^2 + d)^{\frac{3}{2}} - \frac{bdx}{8e} \sqrt{ex^2 + d} - \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x^2+d)^(1/2)*(c*x^4+b*x^2+a), x)

[Out] 1/6*c*x^3*(e*x^2+d)^(3/2)/e-1/8*c*d/e^2*x*(e*x^2+d)^(3/2)+1/16*c*d^2/e^2*x*(e*x^2+d)^(1/2)+1/16*c*d^3/e^(5/2)*ln(x*e^(1/2)+(e*x^2+d)^(1/2))+1/4*b*x*(e*x^2+d)^(3/2)/e-1/8*b*d/e*x*(e*x^2+d)^(1/2)-1/8*b*d^2/e^(3/2)*ln(x*e^(1/2)+(e*x^2+d)^(1/2))+1/2*a*x*(e*x^2+d)^(1/2)+1/2*a*d/e^(1/2)*ln(x*e^(1/2)+(e*x^2+d)^(1/2))

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)^(1/2)*(c*x^4+b*x^2+a),x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [A] time = 4.96313, size = 533, normalized size = 4.04

$$\frac{3(cd^3 - 2bd^2e + 8ade^2)\sqrt{e} \log\left(-2ex^2 - 2\sqrt{ex^2 + d}\sqrt{ex - d}\right) + 2(8ce^3x^5 + 2(cde^2 + 6be^3)x^3 - 3(cd^2e - 2bde^2 - 8ae^3))}{96e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)^(1/2)*(c*x^4+b*x^2+a),x, algorithm="fricas")

[Out] [1/96*(3*(c*d^3 - 2*b*d^2*e + 8*a*d*e^2)*sqrt(e)*log(-2*e*x^2 - 2*sqrt(e*x^2 + d)*sqrt(e)*x - d) + 2*(8*c*e^3*x^5 + 2*(c*d*e^2 + 6*b*e^3)*x^3 - 3*(c*d^2*e - 2*b*d*e^2 - 8*a*e^3)*x)*sqrt(e*x^2 + d))/e^3, -1/48*(3*(c*d^3 - 2*b*d^2*e + 8*a*d*e^2)*sqrt(-e)*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)) - (8*c*e^3*x^5 + 2*(c*d*e^2 + 6*b*e^3)*x^3 - 3*(c*d^2*e - 2*b*d*e^2 - 8*a*e^3)*x)*sqrt(e*x^2 + d))/e^3]

Sympy [B] time = 11.0742, size = 272, normalized size = 2.06

$$\frac{a\sqrt{d}x\sqrt{1 + \frac{ex^2}{d}}}{2} + \frac{ad \operatorname{asinh}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)}{2\sqrt{e}} + \frac{bd^{\frac{3}{2}}x}{8e\sqrt{1 + \frac{ex^2}{d}}} + \frac{3b\sqrt{d}x^3}{8\sqrt{1 + \frac{ex^2}{d}}} - \frac{bd^2 \operatorname{asinh}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)}{8e^{\frac{3}{2}}} + \frac{bex^5}{4\sqrt{d}\sqrt{1 + \frac{ex^2}{d}}} - \frac{cd^{\frac{5}{2}}x}{16e^2\sqrt{1 + \frac{ex^2}{d}}} - \frac{c}{48e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x**2+d)**(1/2)*(c*x**4+b*x**2+a),x)

[Out] a*sqrt(d)*x*sqrt(1 + e*x**2/d)/2 + a*d*asinh(sqrt(e)*x/sqrt(d))/(2*sqrt(e)) + b*d**(3/2)*x/(8*e*sqrt(1 + e*x**2/d)) + 3*b*sqrt(d)*x**3/(8*sqrt(1 + e*x**2/d)) - b*d**2*asinh(sqrt(e)*x/sqrt(d))/(8*e**(3/2)) + b*e*x**5/(4*sqrt(d)*sqrt(1 + e*x**2/d)) - c*d**(5/2)*x/(16*e**2*sqrt(1 + e*x**2/d)) - c*d**(3/2)*x**3/(48*e*sqrt(1 + e*x**2/d)) + 5*c*sqrt(d)*x**5/(24*sqrt(1 + e*x**2/d)) + c*d**3*asinh(sqrt(e)*x/sqrt(d))/(16*e**(5/2)) + c*e*x**7/(6*sqrt(d)*sqrt(1 + e*x**2/d))

Giac [A] time = 1.13256, size = 143, normalized size = 1.08

$$-\frac{1}{16}(cd^3 - 2bd^2e + 8ade^2)e^{\left(-\frac{5}{2}\right)} \log\left(\left|-xe^{\frac{1}{2}} + \sqrt{x^2e + d}\right|\right) + \frac{1}{48}\left(2\left(4cx^2 + (cde^3 + 6be^4)e^{(-4)}\right)x^2 - 3\left(cd^2e^2 - 2bde^3 - 8ae^3\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(1/2)*(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/16*(c*d^3 - 2*b*d^2*e + 8*a*d*e^2)*e^(-5/2)*log(abs(-x*e^(1/2) + sqrt(x^2*e + d))) + 1/48*(2*(4*c*x^2 + (c*d*e^3 + 6*b*e^4)*e^(-4))*x^2 - 3*(c*d^2*e^2 - 2*b*d*e^3 - 8*a*e^4)*e^(-4))*sqrt(x^2*e + d)*x
```

$$3.279 \quad \int \frac{a+bx^2+cx^4}{\sqrt{d+ex^2}} dx$$

Optimal. Leaf size=97

$$\frac{\tanh^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d+ex^2}}\right)(8ae^2 - 4bde + 3cd^2)}{8e^{5/2}} - \frac{x\sqrt{d+ex^2}(3cd - 4be)}{8e^2} + \frac{cx^3\sqrt{d+ex^2}}{4e}$$

[Out] -((3*c*d - 4*b*e)*x*Sqrt[d + e*x^2])/(8*e^2) + (c*x^3*Sqrt[d + e*x^2])/(4*e) + ((3*c*d^2 - 4*b*d*e + 8*a*e^2)*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(8*e^(5/2))

Rubi [A] time = 0.0613475, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1159, 388, 217, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d+ex^2}}\right)(8ae^2 - 4bde + 3cd^2)}{8e^{5/2}} - \frac{x\sqrt{d+ex^2}(3cd - 4be)}{8e^2} + \frac{cx^3\sqrt{d+ex^2}}{4e}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x^2 + c*x^4)/Sqrt[d + e*x^2], x]

[Out] -((3*c*d - 4*b*e)*x*Sqrt[d + e*x^2])/(8*e^2) + (c*x^3*Sqrt[d + e*x^2])/(4*e) + ((3*c*d^2 - 4*b*d*e + 8*a*e^2)*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(8*e^(5/2))

Rule 1159

Int[((d_) + (e_)*(x_)^2)^(q_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] :> Simp[(c^p*x^(4*p - 1)*(d + e*x^2)^(q + 1))/(e*(4*p + 2*q + 1)), x] + Dist[1/(e*(4*p + 2*q + 1)), Int[(d + e*x^2)^q*ExpandToSum[e*(4*p + 2*q + 1)*(a + b*x^2 + c*x^4)^p - d*c^p*(4*p - 1)*x^(4*p - 2) - e*c^p*(4*p + 2*q + 1)*x^(4*p)], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && !LtQ[q, -1]

Rule 388

Int[((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_)), x_Symbol] :> Simp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]

Rule 217

Int[1/Sqrt[(a_) + (b_)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned}
\int \frac{a + bx^2 + cx^4}{\sqrt{d + ex^2}} dx &= \frac{cx^3\sqrt{d + ex^2}}{4e} + \frac{\int \frac{4ae - (3cd - 4be)x^2}{\sqrt{d + ex^2}} dx}{4e} \\
&= -\frac{(3cd - 4be)x\sqrt{d + ex^2}}{8e^2} + \frac{cx^3\sqrt{d + ex^2}}{4e} - \frac{1}{8} \left(-8a - \frac{d(3cd - 4be)}{e^2} \right) \int \frac{1}{\sqrt{d + ex^2}} dx \\
&= -\frac{(3cd - 4be)x\sqrt{d + ex^2}}{8e^2} + \frac{cx^3\sqrt{d + ex^2}}{4e} - \frac{1}{8} \left(-8a - \frac{d(3cd - 4be)}{e^2} \right) \text{Subst} \left(\int \frac{1}{1 - ex^2} dx, x, \frac{\sqrt{d + ex^2}}{\sqrt{d}} \right) \\
&= -\frac{(3cd - 4be)x\sqrt{d + ex^2}}{8e^2} + \frac{cx^3\sqrt{d + ex^2}}{4e} + \frac{(3cd^2 - 4bde + 8ae^2) \tanh^{-1} \left(\frac{\sqrt{ex}}{\sqrt{d + ex^2}} \right)}{8e^{5/2}}
\end{aligned}$$

Mathematica [A] time = 0.0629485, size = 82, normalized size = 0.85

$$\frac{\tanh^{-1} \left(\frac{\sqrt{ex}}{\sqrt{d + ex^2}} \right) (8ae^2 - 4bde + 3cd^2) + \sqrt{ex}\sqrt{d + ex^2} (4be - 3cd + 2cex^2)}{8e^{5/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x^2 + c*x^4)/Sqrt[d + e*x^2], x]

[Out] (Sqrt[e]*x*Sqrt[d + e*x^2]*(-3*c*d + 4*b*e + 2*c*e*x^2) + (3*c*d^2 - 4*b*d*e + 8*a*e^2)*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(8*e^(5/2))

Maple [A] time = 0.007, size = 122, normalized size = 1.3

$$\frac{cx^3}{4e} \sqrt{ex^2 + d} - \frac{3cdx}{8e^2} \sqrt{ex^2 + d} + \frac{3cd^2}{8} \ln(x\sqrt{e} + \sqrt{ex^2 + d}) e^{-\frac{5}{2}} + \frac{bx}{2e} \sqrt{ex^2 + d} - \frac{bd}{2} \ln(x\sqrt{e} + \sqrt{ex^2 + d}) e^{-\frac{3}{2}} + a \ln(x\sqrt{e} + \sqrt{ex^2 + d})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^4+b*x^2+a)/(e*x^2+d)^(1/2), x)

[Out] 1/4*c*x^3*(e*x^2+d)^(1/2)/e-3/8*c*d/e^2*x*(e*x^2+d)^(1/2)+3/8*c*d^2/e^(5/2)*ln(x*e^(1/2)+(e*x^2+d)^(1/2))+1/2*b*x/e*(e*x^2+d)^(1/2)-1/2*b*d/e^(3/2)*ln(x*e^(1/2)+(e*x^2+d)^(1/2))+a*ln(x*e^(1/2)+(e*x^2+d)^(1/2))/e^(1/2)

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^(1/2), x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [A] time = 4.69738, size = 408, normalized size = 4.21

$$\left[\frac{(3cd^2 - 4bde + 8ae^2)\sqrt{e} \log\left(-2ex^2 - 2\sqrt{ex^2 + d}\sqrt{ex - d}\right) + 2(2ce^2x^3 - (3cde - 4be^2)x)\sqrt{ex^2 + d}}{16e^3}, \dots \right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*((3*c*d^2 - 4*b*d*e + 8*a*e^2)*sqrt(e)*log(-2*e*x^2 - 2*sqrt(e*x^2 + d)*sqrt(e)*x - d) + 2*(2*c*e^2*x^3 - (3*c*d*e - 4*b*e^2)*x)*sqrt(e*x^2 + d))/e^3, -1/8*((3*c*d^2 - 4*b*d*e + 8*a*e^2)*sqrt(-e)*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)) - (2*c*e^2*x^3 - (3*c*d*e - 4*b*e^2)*x)*sqrt(e*x^2 + d))/e^3]
```

Sympy [A] time = 6.37356, size = 230, normalized size = 2.37

$$a \begin{cases} \frac{\sqrt{-\frac{d}{e}} \operatorname{asin}\left(x\sqrt{-\frac{e}{d}}\right)}{\sqrt{d}} & \text{for } d > 0 \wedge e < 0 \\ \frac{\sqrt{\frac{d}{e}} \operatorname{asinh}\left(x\sqrt{\frac{e}{d}}\right)}{\sqrt{d}} & \text{for } d > 0 \wedge e > 0 \\ \frac{\sqrt{-\frac{d}{e}} \operatorname{acosh}\left(x\sqrt{-\frac{e}{d}}\right)}{\sqrt{-d}} & \text{for } e > 0 \wedge d < 0 \end{cases} + \frac{b\sqrt{dx}\sqrt{1 + \frac{ex^2}{d}}}{2e} - \frac{bd \operatorname{asinh}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)}{2e^{\frac{3}{2}}} - \frac{3cd^{\frac{3}{2}}x}{8e^2\sqrt{1 + \frac{ex^2}{d}}} - \frac{c\sqrt{dx}^3}{8e\sqrt{1 + \frac{ex^2}{d}}} + \frac{3cd^2 \operatorname{asinh}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)}{8e^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)/(e*x**2+d)**(1/2),x)
```

```
[Out] a*Piecewise((sqrt(-d/e)*asin(x*sqrt(-e/d))/sqrt(d), (d > 0) & (e < 0)), (sqrt(d/e)*asinh(x*sqrt(e/d))/sqrt(d), (d > 0) & (e > 0)), (sqrt(-d/e)*acosh(x*sqrt(-e/d))/sqrt(-d), (e > 0) & (d < 0))) + b*sqrt(d)*x*sqrt(1 + e*x**2/d)/(2*e) - b*d*asinh(sqrt(e)*x/sqrt(d))/(2*e**(3/2)) - 3*c*d**(3/2)*x/(8*e**2*sqrt(1 + e*x**2/d)) - c*sqrt(d)*x**3/(8*e*sqrt(1 + e*x**2/d)) + 3*c*d**2*a*sinh(sqrt(e)*x/sqrt(d))/(8*e**(5/2)) + c*x**5/(4*sqrt(d)*sqrt(1 + e*x**2/d))
```

Giac [A] time = 1.17638, size = 107, normalized size = 1.1

$$-\frac{1}{8}(3cd^2 - 4bde + 8ae^2)e^{\left(-\frac{5}{2}\right)} \log\left(\left|-xe^{\frac{1}{2}} + \sqrt{x^2e + d}\right|\right) + \frac{1}{8}(2cx^2e^{(-1)} - (3cde - 4be^2)e^{(-3)})\sqrt{x^2e + dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] -1/8*(3*c*d^2 - 4*b*d*e + 8*a*e^2)*e^(-5/2)*log(abs(-x*e^(1/2) + sqrt(x^2*e + d))) + 1/8*(2*c*x^2*e^(-1) - (3*c*d*e - 4*b*e^2)*e^(-3))*sqrt(x^2*e + d)*x
```


$$3.280 \quad \int \frac{a+bx^2+cx^4}{(d+ex^2)^{3/2}} dx$$

Optimal. Leaf size=89

$$\frac{x \left(a + \frac{d(cd-be)}{e^2} \right)}{d\sqrt{d+ex^2}} - \frac{(3cd-2be) \tanh^{-1} \left(\frac{\sqrt{ex}}{\sqrt{d+ex^2}} \right)}{2e^{5/2}} + \frac{cx\sqrt{d+ex^2}}{2e^2}$$

[Out] ((a + (d*(c*d - b*e))/e^2)*x)/(d*Sqrt[d + e*x^2]) + (c*x*Sqrt[d + e*x^2])/(2*e^2) - ((3*c*d - 2*b*e)*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(2*e^(5/2))

Rubi [A] time = 0.0726533, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1157, 388, 217, 206}

$$\frac{x \left(a + \frac{d(cd-be)}{e^2} \right)}{d\sqrt{d+ex^2}} - \frac{(3cd-2be) \tanh^{-1} \left(\frac{\sqrt{ex}}{\sqrt{d+ex^2}} \right)}{2e^{5/2}} + \frac{cx\sqrt{d+ex^2}}{2e^2}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x^2 + c*x^4)/(d + e*x^2)^(3/2), x]

[Out] ((a + (d*(c*d - b*e))/e^2)*x)/(d*Sqrt[d + e*x^2]) + (c*x*Sqrt[d + e*x^2])/(2*e^2) - ((3*c*d - 2*b*e)*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(2*e^(5/2))

Rule 1157

Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]

Rule 388

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]

Rule 217

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned}
\int \frac{a + bx^2 + cx^4}{(d + ex^2)^{3/2}} dx &= \frac{\left(a + \frac{d(cd-be)}{e^2}\right)x}{d\sqrt{d + ex^2}} - \frac{\int \frac{\frac{d(cd-be) - cd^2}{e^2} \frac{cdx^2}{e}}{\sqrt{d+ex^2}} dx}{d} \\
&= \frac{\left(a + \frac{d(cd-be)}{e^2}\right)x}{d\sqrt{d + ex^2}} + \frac{cx\sqrt{d + ex^2}}{2e^2} - \frac{(3cd - 2be) \int \frac{1}{\sqrt{d+ex^2}} dx}{2e^2} \\
&= \frac{\left(a + \frac{d(cd-be)}{e^2}\right)x}{d\sqrt{d + ex^2}} + \frac{cx\sqrt{d + ex^2}}{2e^2} - \frac{(3cd - 2be) \operatorname{Subst}\left(\int \frac{1}{1-ex^2} dx, x, \frac{x}{\sqrt{d+ex^2}}\right)}{2e^2} \\
&= \frac{\left(a + \frac{d(cd-be)}{e^2}\right)x}{d\sqrt{d + ex^2}} + \frac{cx\sqrt{d + ex^2}}{2e^2} - \frac{(3cd - 2be) \tanh^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d+ex^2}}\right)}{2e^{5/2}}
\end{aligned}$$

Mathematica [A] time = 0.107454, size = 98, normalized size = 1.1

$$\frac{\sqrt{ex} \left(2e(ae - bd) + cd(3d + ex^2)\right) - d^{3/2} \sqrt{\frac{ex^2}{d} + 1} (3cd - 2be) \sinh^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)}{2de^{5/2} \sqrt{d + ex^2}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x^2 + c*x^4)/(d + e*x^2)^(3/2), x]

[Out] (Sqrt[e]*x*(2*e*(-(b*d) + a*e) + c*d*(3*d + e*x^2)) - d^(3/2)*(3*c*d - 2*b*e)*Sqrt[1 + (e*x^2)/d]*ArcSinh[(Sqrt[e]*x)/Sqrt[d]])/(2*d*e^(5/2)*Sqrt[d + e*x^2])

Maple [A] time = 0.007, size = 112, normalized size = 1.3

$$\frac{cx^3}{2e} \frac{1}{\sqrt{ex^2 + d}} + \frac{3cdx}{2e^2} \frac{1}{\sqrt{ex^2 + d}} - \frac{3cd}{2} \ln\left(x\sqrt{e} + \sqrt{ex^2 + d}\right) e^{-\frac{5}{2}} - \frac{bx}{e} \frac{1}{\sqrt{ex^2 + d}} + b \ln\left(x\sqrt{e} + \sqrt{ex^2 + d}\right) e^{-\frac{3}{2}} + \frac{ax}{d} \frac{1}{\sqrt{ex^2 + d}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^4+b*x^2+a)/(e*x^2+d)^(3/2), x)

[Out] 1/2*c*x^3/e/(e*x^2+d)^(1/2)+3/2*c*d/e^2*x/(e*x^2+d)^(1/2)-3/2*c*d/e^(5/2)*ln(x*e^(1/2)+(e*x^2+d)^(1/2))-b*x/e/(e*x^2+d)^(1/2)+b/e^(3/2)*ln(x*e^(1/2)+(e*x^2+d)^(1/2))+a*x/d/(e*x^2+d)^(1/2)

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^(3/2), x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [A] time = 4.29752, size = 552, normalized size = 6.2

$$\left[\frac{(3cd^3 - 2bd^2e + (3cd^2e - 2bde^2)x^2)\sqrt{e}\log(-2ex^2 - 2\sqrt{ex^2 + d}\sqrt{ex} - d) - 2(cde^2x^3 + (3cd^2e - 2bde^2 + 2ae^3)x)}{4(d^4x^2 + d^2e^3)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^(3/2),x, algorithm="fricas")

[Out] [-1/4*((3*c*d^3 - 2*b*d^2*e + (3*c*d^2*e - 2*b*d*e^2)*x^2)*sqrt(e)*log(-2*e*x^2 - 2*sqrt(e*x^2 + d)*sqrt(e)*x - d) - 2*(c*d*e^2*x^3 + (3*c*d^2*e - 2*b*d*e^2 + 2*a*e^3)*x)*sqrt(e*x^2 + d))/(d*e^4*x^2 + d^2*e^3), 1/2*((3*c*d^3 - 2*b*d^2*e + (3*c*d^2*e - 2*b*d*e^2)*x^2)*sqrt(-e)*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)) + (c*d*e^2*x^3 + (3*c*d^2*e - 2*b*d*e^2 + 2*a*e^3)*x)*sqrt(e*x^2 + d))/(d*e^4*x^2 + d^2*e^3)]

Sympy [A] time = 7.82472, size = 134, normalized size = 1.51

$$\frac{ax}{d^{\frac{3}{2}}\sqrt{1 + \frac{ex^2}{d}}} + b \left(\frac{\operatorname{asinh}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)}{e^{\frac{3}{2}}} - \frac{x}{\sqrt{de}\sqrt{1 + \frac{ex^2}{d}}} \right) + c \left(\frac{3\sqrt{d}x}{2e^2\sqrt{1 + \frac{ex^2}{d}}} - \frac{3d \operatorname{asinh}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)}{2e^{\frac{5}{2}}} + \frac{x^3}{2\sqrt{de}\sqrt{1 + \frac{ex^2}{d}}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x**4+b*x**2+a)/(e*x**2+d)**(3/2),x)

[Out] a*x/(d**(3/2)*sqrt(1 + e*x**2/d)) + b*(asinh(sqrt(e)*x/sqrt(d))/e**(3/2) - x/(sqrt(d)*e*sqrt(1 + e*x**2/d))) + c*(3*sqrt(d)*x/(2*e**2*sqrt(1 + e*x**2/d)) - 3*d*asinh(sqrt(e)*x/sqrt(d))/(2*e**(5/2)) + x**3/(2*sqrt(d)*e*sqrt(1 + e*x**2/d)))

Giac [A] time = 1.21141, size = 108, normalized size = 1.21

$$\frac{1}{2}(3cd - 2be)e^{\left(-\frac{5}{2}\right)} \log\left(\left|-xe^{\frac{1}{2}} + \sqrt{x^2e + d}\right|\right) + \frac{\left(cx^2e^{(-1)} + \frac{(3cd^2e - 2bde^2 + 2ae^3)e^{(-3)}}{d}\right)x}{2\sqrt{x^2e + d}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^(3/2),x, algorithm="giac")

[Out] 1/2*(3*c*d - 2*b*e)*e^(-5/2)*log(abs(-x*e^(1/2) + sqrt(x^2*e + d))) + 1/2*(c*x^2*e^(-1) + (3*c*d^2*e - 2*b*d*e^2 + 2*a*e^3)*e^(-3)/d)*x/sqrt(x^2*e + d)

$$3.281 \quad \int \frac{a+bx^2+cx^4}{(d+ex^2)^{5/2}} dx$$

Optimal. Leaf size=101

$$-\frac{x(4cd^2 - e(2ae + bd))}{3d^2e^2\sqrt{d+ex^2}} + \frac{x\left(a + \frac{d(cd-be)}{e^2}\right)}{3d(d+ex^2)^{3/2}} + \frac{c \tanh^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d+ex^2}}\right)}{e^{5/2}}$$

[Out] ((a + (d*(c*d - b*e))/e^2)*x)/(3*d*(d + e*x^2)^(3/2)) - ((4*c*d^2 - e*(b*d + 2*a*e))*x)/(3*d^2*e^2*sqrt[d + e*x^2]) + (c*ArcTanh[(sqrt[e]*x)/sqrt[d + e*x^2]])/e^(5/2)

Rubi [A] time = 0.0713267, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1157, 385, 217, 206}

$$-\frac{x(4cd^2 - e(2ae + bd))}{3d^2e^2\sqrt{d+ex^2}} + \frac{x\left(a + \frac{d(cd-be)}{e^2}\right)}{3d(d+ex^2)^{3/2}} + \frac{c \tanh^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d+ex^2}}\right)}{e^{5/2}}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x^2 + c*x^4)/(d + e*x^2)^(5/2), x]

[Out] ((a + (d*(c*d - b*e))/e^2)*x)/(3*d*(d + e*x^2)^(3/2)) - ((4*c*d^2 - e*(b*d + 2*a*e))*x)/(3*d^2*e^2*sqrt[d + e*x^2]) + (c*ArcTanh[(sqrt[e]*x)/sqrt[d + e*x^2]])/e^(5/2)

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] := With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2,
x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x,
0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
p, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

$Q[a, 0] \mid\mid LtQ[b, 0])$

Rubi steps

$$\begin{aligned} \int \frac{a + bx^2 + cx^4}{(d + ex^2)^{5/2}} dx &= \frac{\left(a + \frac{d(cd-be)}{e^2}\right)x}{3d(d + ex^2)^{3/2}} - \frac{\int \frac{-2a + \frac{d(cd-be)}{e^2} - \frac{3cdx^2}{e}}{(d+ex^2)^{3/2}} dx}{3d} \\ &= \frac{\left(a + \frac{d(cd-be)}{e^2}\right)x}{3d(d + ex^2)^{3/2}} - \frac{(4cd^2 - e(bd + 2ae))x}{3d^2e^2\sqrt{d + ex^2}} + \frac{c \int \frac{1}{\sqrt{d+ex^2}} dx}{e^2} \\ &= \frac{\left(a + \frac{d(cd-be)}{e^2}\right)x}{3d(d + ex^2)^{3/2}} - \frac{(4cd^2 - e(bd + 2ae))x}{3d^2e^2\sqrt{d + ex^2}} + \frac{c \operatorname{Subst}\left(\int \frac{1}{1-ex^2} dx, x, \frac{x}{\sqrt{d+ex^2}}\right)}{e^2} \\ &= \frac{\left(a + \frac{d(cd-be)}{e^2}\right)x}{3d(d + ex^2)^{3/2}} - \frac{(4cd^2 - e(bd + 2ae))x}{3d^2e^2\sqrt{d + ex^2}} + \frac{c \tanh^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d+ex^2}}\right)}{e^{5/2}} \end{aligned}$$

Mathematica [A] time = 0.193527, size = 112, normalized size = 1.11

$$\frac{\sqrt{ex} \left(e^2 (3ad + 2aex^2 + bdx^2) - cd^2 (3d + 4ex^2) \right) + 3cd^{5/2} (d + ex^2) \sqrt{\frac{ex^2}{d} + 1} \sinh^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)}{3d^2e^{5/2} (d + ex^2)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x^2 + c*x^4)/(d + e*x^2)^(5/2), x]

[Out] (Sqrt[e]*x*(-(c*d^2*(3*d + 4*e*x^2)) + e^2*(3*a*d + b*d*x^2 + 2*a*e*x^2)) + 3*c*d^(5/2)*(d + e*x^2)*Sqrt[1 + (e*x^2)/d]*ArcSinh[(Sqrt[e]*x)/Sqrt[d]])/(3*d^2*e^(5/2)*(d + e*x^2)^(3/2))

Maple [A] time = 0.009, size = 124, normalized size = 1.2

$$-\frac{cx^3}{3e} (ex^2 + d)^{-\frac{3}{2}} - \frac{cx}{e^2} \frac{1}{\sqrt{ex^2 + d}} + c \ln\left(x\sqrt{e} + \sqrt{ex^2 + d}\right) e^{-\frac{5}{2}} - \frac{bx}{3e} (ex^2 + d)^{-\frac{3}{2}} + \frac{bx}{3de} \frac{1}{\sqrt{ex^2 + d}} + \frac{ax}{3d} (ex^2 + d)^{-\frac{3}{2}} +$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^4+b*x^2+a)/(e*x^2+d)^(5/2), x)

[Out] -1/3*c*x^3/e/(e*x^2+d)^(3/2)-c/e^2*x/(e*x^2+d)^(1/2)+c/e^(5/2)*ln(x*e^(1/2)+(e*x^2+d)^(1/2))-1/3*b/e*x/(e*x^2+d)^(3/2)+1/3*b/d/e*x/(e*x^2+d)^(1/2)+1/3*a*x/d/(e*x^2+d)^(3/2)+2/3*a/d^2*x/(e*x^2+d)^(1/2)

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^(5/2),x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [A] time = 4.88764, size = 612, normalized size = 6.06

$$\left[\frac{3(cd^2e^2x^4 + 2cd^3ex^2 + cd^4)\sqrt{e} \log\left(-2ex^2 - 2\sqrt{ex^2 + d}\sqrt{ex - d}\right) - 2\left((4cd^2e^2 - bde^3 - 2ae^4)x^3 + 3(cd^3e - ade^3)x\right)\sqrt{ex^2 + d}}{6(d^2e^5x^4 + 2d^3e^4x^2 + d^4e^3)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^(5/2),x, algorithm="fricas")

[Out] [1/6*(3*(c*d^2*e^2*x^4 + 2*c*d^3*e*x^2 + c*d^4)*sqrt(e)*log(-2*e*x^2 - 2*sqrt(e*x^2 + d)*sqrt(e)*x - d) - 2*((4*c*d^2*e^2 - b*d*e^3 - 2*a*e^4)*x^3 + 3*(c*d^3*e - a*d*e^3)*x)*sqrt(e*x^2 + d))/(d^2*e^5*x^4 + 2*d^3*e^4*x^2 + d^4*e^3), -1/3*(3*(c*d^2*e^2*x^4 + 2*c*d^3*e*x^2 + c*d^4)*sqrt(-e)*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)) + ((4*c*d^2*e^2 - b*d*e^3 - 2*a*e^4)*x^3 + 3*(c*d^3*e - a*d*e^3)*x)*sqrt(e*x^2 + d))/(d^2*e^5*x^4 + 2*d^3*e^4*x^2 + d^4*e^3)]

Sympy [B] time = 18.4519, size = 450, normalized size = 4.46

$$a \left(\frac{3dx}{3d^{\frac{7}{2}}\sqrt{1 + \frac{ex^2}{d}} + 3d^{\frac{5}{2}}ex^2\sqrt{1 + \frac{ex^2}{d}}} + \frac{2ex^3}{3d^{\frac{7}{2}}\sqrt{1 + \frac{ex^2}{d}} + 3d^{\frac{5}{2}}ex^2\sqrt{1 + \frac{ex^2}{d}}} \right) + \frac{bx^3}{3d^{\frac{5}{2}}\sqrt{1 + \frac{ex^2}{d}} + 3d^{\frac{3}{2}}ex^2\sqrt{1 + \frac{ex^2}{d}}} + c \left(\frac{3d^{\frac{39}{2}}}{3d^{\frac{39}{2}}e^{\frac{27}{2}}\sqrt{1 + \frac{ex^2}{d}}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x**4+b*x**2+a)/(e*x**2+d)**(5/2),x)

[Out] a*(3*d*x/(3*d**(7/2)*sqrt(1 + e*x**2/d) + 3*d**(5/2)*e*x**2*sqrt(1 + e*x**2/d)) + 2*e*x**3/(3*d**(7/2)*sqrt(1 + e*x**2/d) + 3*d**(5/2)*e*x**2*sqrt(1 + e*x**2/d))) + b*x**3/(3*d**(5/2)*sqrt(1 + e*x**2/d) + 3*d**(3/2)*e*x**2*sqrt(1 + e*x**2/d)) + c*(3*d**(39/2)*e**11*sqrt(1 + e*x**2/d)*asinh(sqrt(e)*x/sqrt(d))/(3*d**(39/2)*e**(27/2)*sqrt(1 + e*x**2/d) + 3*d**(37/2)*e**(29/2)*x**2*sqrt(1 + e*x**2/d)) + 3*d**(37/2)*e**12*x**2*sqrt(1 + e*x**2/d)*asinh(sqrt(e)*x/sqrt(d))/(3*d**(39/2)*e**(27/2)*sqrt(1 + e*x**2/d) + 3*d**(37/2)*e**(29/2)*x**2*sqrt(1 + e*x**2/d)) - 3*d**19*e**(23/2)*x/(3*d**(39/2)*e**(27/2)*sqrt(1 + e*x**2/d) + 3*d**(37/2)*e**(29/2)*x**2*sqrt(1 + e*x**2/d)) - 4*d**18*e**(25/2)*x**3/(3*d**(39/2)*e**(27/2)*sqrt(1 + e*x**2/d) + 3*d**(37/2)*e**(29/2)*x**2*sqrt(1 + e*x**2/d)))

Giac [A] time = 1.12376, size = 119, normalized size = 1.18

$$-ce^{\left(-\frac{5}{2}\right)} \log\left(\left|-xe^{\frac{1}{2}} + \sqrt{x^2e + d}\right|\right) - \frac{\left(\frac{(4cd^2e^2 - bde^3 - 2ae^4)x^2e^{(-3)}}{d^2} + \frac{3(cd^3e - ade^3)e^{(-3)}}{d^2}\right)x}{3(x^2e + d)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^(5/2),x, algorithm="giac")
```

```
[Out] -c*e^(-5/2)*log(abs(-x*e^(1/2) + sqrt(x^2*e + d))) - 1/3*((4*c*d^2*e^2 - b*  
d*e^3 - 2*a*e^4)*x^2*e^(-3)/d^2 + 3*(c*d^3*e - a*d*e^3)*e^(-3)/d^2)*x/(x^2*  
e + d)^(3/2)
```

$$3.282 \quad \int \frac{a+bx^2+cx^4}{(d+ex^2)^{7/2}} dx$$

Optimal. Leaf size=86

$$\frac{x^5(2e(4ae+bd)+3cd^2)}{15d^3(d+ex^2)^{5/2}} + \frac{x^3(4ae+bd)}{3d^2(d+ex^2)^{5/2}} + \frac{ax}{d(d+ex^2)^{5/2}}$$

[Out] (a*x)/(d*(d + e*x^2)^(5/2)) + ((b*d + 4*a*e)*x^3)/(3*d^2*(d + e*x^2)^(5/2)) + ((3*c*d^2 + 2*e*(b*d + 4*a*e))*x^5)/(15*d^3*(d + e*x^2)^(5/2))

Rubi [A] time = 0.107052, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1155, 1803, 12, 264}

$$\frac{x^5(2e(4ae+bd)+3cd^2)}{15d^3(d+ex^2)^{5/2}} + \frac{x^3(4ae+bd)}{3d^2(d+ex^2)^{5/2}} + \frac{ax}{d(d+ex^2)^{5/2}}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x^2 + c*x^4)/(d + e*x^2)^(7/2), x]

[Out] (a*x)/(d*(d + e*x^2)^(5/2)) + ((b*d + 4*a*e)*x^3)/(3*d^2*(d + e*x^2)^(5/2)) + ((3*c*d^2 + 2*e*(b*d + 4*a*e))*x^5)/(15*d^3*(d + e*x^2)^(5/2))

Rule 1155

Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] := Simp[(a^p*x*(d + e*x^2)^(q + 1))/d, x] + Dist[1/d, Int[x^2*(d + e*x^2)^q*(d*PolynomialQuotient[(a + b*x^2 + c*x^4)^p - a^p, x^2, x] - e*a^p*(2*q + 3)), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && ILtQ[q + 1/2, 0] && LtQ[4*p + 2*q + 1, 0]

Rule 1803

Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] := With[{A = Coef f[Pq, x, 0], Q = PolynomialQuotient[Pq - Coeff[Pq, x, 0], x^2, x]}, Simp[(A*x^(m + 1)*(a + b*x^2)^(p + 1))/(a*(m + 1)), x] + Dist[1/(a*(m + 1)), Int[x^(m + 2)*(a + b*x^2)^p*(a*(m + 1)*Q - A*b*(m + 2*(p + 1) + 1)), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x^2] && IntegerQ[m/2] && ILtQ[(m + 1)/2 + p, 0] && LtQ[m + Expon[Pq, x] + 2*p + 1, 0]

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match Q[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 264

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[((c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]

Rubi steps

$$\begin{aligned}
\int \frac{a + bx^2 + cx^4}{(d + ex^2)^{7/2}} dx &= \frac{ax}{d(d + ex^2)^{5/2}} + \frac{\int \frac{x^2(4ae + d(b + cx^2))}{(d + ex^2)^{7/2}} dx}{d} \\
&= \frac{ax}{d(d + ex^2)^{5/2}} + \frac{(bd + 4ae)x^3}{3d^2(d + ex^2)^{5/2}} + \frac{\int \frac{(3cd^2 + 2e(bd + 4ae))x^4}{(d + ex^2)^{7/2}} dx}{3d^2} \\
&= \frac{ax}{d(d + ex^2)^{5/2}} + \frac{(bd + 4ae)x^3}{3d^2(d + ex^2)^{5/2}} + \frac{1}{3} \left(3c + \frac{2e(bd + 4ae)}{d^2} \right) \int \frac{x^4}{(d + ex^2)^{7/2}} dx \\
&= \frac{ax}{d(d + ex^2)^{5/2}} + \frac{(bd + 4ae)x^3}{3d^2(d + ex^2)^{5/2}} + \frac{(3cd^2 + 2e(bd + 4ae))x^5}{15d^3(d + ex^2)^{5/2}}
\end{aligned}$$

Mathematica [A] time = 0.051325, size = 67, normalized size = 0.78

$$\frac{a(15d^2x + 20dex^3 + 8e^2x^5) + dx^3(5bd + 2bex^2 + 3cdx^2)}{15d^3(d + ex^2)^{5/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x^2 + c*x^4)/(d + e*x^2)^(7/2), x]

[Out] (d*x^3*(5*b*d + 3*c*d*x^2 + 2*b*e*x^2) + a*(15*d^2*x + 20*d*e*x^3 + 8*e^2*x^5))/(15*d^3*(d + e*x^2)^(5/2))

Maple [A] time = 0.006, size = 66, normalized size = 0.8

$$\frac{x(8ae^2x^4 + 2bdex^4 + 3cd^2x^4 + 20adex^2 + 5bd^2x^2 + 15ad^2)}{15d^3} (ex^2 + d)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^4+b*x^2+a)/(e*x^2+d)^(7/2), x)

[Out] 1/15*x*(8*a*e^2*x^4+2*b*d*e*x^4+3*c*d^2*x^4+20*a*d*e*x^2+5*b*d^2*x^2+15*a*d^2)/(e*x^2+d)^(5/2)/d^3

Maxima [B] time = 0.952352, size = 234, normalized size = 2.72

$$-\frac{cx^3}{2(ex^2 + d)^{\frac{5}{2}}e} + \frac{8ax}{15\sqrt{ex^2 + dd^3}} + \frac{4ax}{15(ex^2 + d)^{\frac{3}{2}}d^2} + \frac{ax}{5(ex^2 + d)^{\frac{5}{2}}d} + \frac{cx}{10(ex^2 + d)^{\frac{3}{2}}e^2} + \frac{cx}{5\sqrt{ex^2 + dde^2}} - \frac{3cdx}{10(ex^2 + d)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^(7/2), x, algorithm="maxima")

[Out] $-1/2*c*x^3/((e*x^2 + d)^{(5/2)}*e) + 8/15*a*x/(sqrt(e*x^2 + d)*d^3) + 4/15*a*x/((e*x^2 + d)^{(3/2)}*d^2) + 1/5*a*x/((e*x^2 + d)^{(5/2)}*d) + 1/10*c*x/((e*x^2 + d)^{(3/2)}*e^2) + 1/5*c*x/(sqrt(e*x^2 + d)*d*e^2) - 3/10*c*d*x/((e*x^2 + d)^{(5/2)}*e^2) - 1/5*b*x/((e*x^2 + d)^{(5/2)}*e) + 2/15*b*x/(sqrt(e*x^2 + d)*d^2*e) + 1/15*b*x/((e*x^2 + d)^{(3/2)}*d*e)$

Fricas [A] time = 4.71996, size = 198, normalized size = 2.3

$$\frac{\left(\left(3cd^2 + 2bde + 8ae^2\right)x^5 + 15ad^2x + 5\left(bd^2 + 4ade\right)x^3\right)\sqrt{ex^2 + d}}{15\left(d^3e^3x^6 + 3d^4e^2x^4 + 3d^5ex^2 + d^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^(7/2),x, algorithm="fricas")

[Out] $1/15*\left(\left(3*c*d^2 + 2*b*d*e + 8*a*e^2\right)*x^5 + 15*a*d^2*x + 5*\left(b*d^2 + 4*a*d*e\right)*x^3\right)*sqrt(e*x^2 + d)/(d^3*e^3*x^6 + 3*d^4*e^2*x^4 + 3*d^5*e*x^2 + d^6)$

Sympy [B] time = 56.603, size = 639, normalized size = 7.43

$$a \left(\frac{15d^5x}{15d^{\frac{17}{2}}\sqrt{1 + \frac{ex^2}{d}} + 45d^{\frac{15}{2}}ex^2\sqrt{1 + \frac{ex^2}{d}} + 45d^{\frac{13}{2}}e^2x^4\sqrt{1 + \frac{ex^2}{d}} + 15d^{\frac{11}{2}}e^3x^6\sqrt{1 + \frac{ex^2}{d}}} + \frac{15d^5x}{15d^{\frac{17}{2}}\sqrt{1 + \frac{ex^2}{d}} + 45d^{\frac{15}{2}}ex^2\sqrt{1 + \frac{ex^2}{d}}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x**4+b*x**2+a)/(e*x**2+d)**(7/2),x)

[Out] $a*(15*d**5*x/(15*d**(17/2)*sqrt(1 + e*x**2/d) + 45*d**(15/2)*e*x**2*sqrt(1 + e*x**2/d) + 45*d**(13/2)*e**2*x**4*sqrt(1 + e*x**2/d) + 15*d**(11/2)*e**3*x**6*sqrt(1 + e*x**2/d)) + 35*d**4*e*x**3/(15*d**(17/2)*sqrt(1 + e*x**2/d) + 45*d**(15/2)*e*x**2*sqrt(1 + e*x**2/d) + 45*d**(13/2)*e**2*x**4*sqrt(1 + e*x**2/d) + 15*d**(11/2)*e**3*x**6*sqrt(1 + e*x**2/d)) + 28*d**3*e**2*x**5/(15*d**(17/2)*sqrt(1 + e*x**2/d) + 45*d**(15/2)*e*x**2*sqrt(1 + e*x**2/d) + 45*d**(13/2)*e**2*x**4*sqrt(1 + e*x**2/d) + 15*d**(11/2)*e**3*x**6*sqrt(1 + e*x**2/d)) + 8*d**2*e**3*x**7/(15*d**(17/2)*sqrt(1 + e*x**2/d) + 45*d**(15/2)*e*x**2*sqrt(1 + e*x**2/d) + 45*d**(13/2)*e**2*x**4*sqrt(1 + e*x**2/d) + 15*d**(11/2)*e**3*x**6*sqrt(1 + e*x**2/d)) + b*(5*d*x**3/(15*d**(9/2)*sqrt(1 + e*x**2/d) + 30*d**(7/2)*e*x**2*sqrt(1 + e*x**2/d) + 15*d**(5/2)*e**2*x**4*sqrt(1 + e*x**2/d)) + 2*e*x**5/(15*d**(9/2)*sqrt(1 + e*x**2/d) + 30*d**(7/2)*e*x**2*sqrt(1 + e*x**2/d) + 15*d**(5/2)*e**2*x**4*sqrt(1 + e*x**2/d)) + c*x**5/(5*d**(7/2)*sqrt(1 + e*x**2/d) + 10*d**(5/2)*e*x**2*sqrt(1 + e*x**2/d) + 5*d**(3/2)*e**2*x**4*sqrt(1 + e*x**2/d))$

Giac [A] time = 1.16173, size = 101, normalized size = 1.17

$$\frac{\left(x^2\left(\frac{(3cd^2e^2+2bde^3+8ae^4)x^2e^{(-2)}}{d^3} + \frac{5(bd^2e^2+4ade^3)e^{(-2)}}{d^3}\right) + \frac{15a}{d}\right)x}{15\left(x^2e + d\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^(7/2),x, algorithm="giac")
```

```
[Out] 1/15*(x^2*((3*c*d^2*e^2 + 2*b*d*e^3 + 8*a*e^4)*x^2*e^(-2)/d^3 + 5*(b*d^2*e^2 + 4*a*d*e^3)*e^(-2)/d^3) + 15*a/d)*x/(x^2*e + d)^(5/2)
```

$$3.283 \quad \int \frac{a+bx^2+cx^4}{(d+ex^2)^{9/2}} dx$$

Optimal. Leaf size=126

$$\frac{2ex^7(4e(6ae+bd)+3cd^2)}{105d^4(d+ex^2)^{7/2}} + \frac{x^5(4e(6ae+bd)+3cd^2)}{15d^3(d+ex^2)^{7/2}} + \frac{x^3(6ae+bd)}{3d^2(d+ex^2)^{7/2}} + \frac{ax}{d(d+ex^2)^{7/2}}$$

[Out] (a*x)/(d*(d + e*x^2)^(7/2)) + ((b*d + 6*a*e)*x^3)/(3*d^2*(d + e*x^2)^(7/2)) + ((3*c*d^2 + 4*e*(b*d + 6*a*e))*x^5)/(15*d^3*(d + e*x^2)^(7/2)) + (2*e*(3*c*d^2 + 4*e*(b*d + 6*a*e))*x^7)/(105*d^4*(d + e*x^2)^(7/2))

Rubi [A] time = 0.146389, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {1155, 1803, 12, 271, 264}

$$\frac{2ex^7(4e(6ae+bd)+3cd^2)}{105d^4(d+ex^2)^{7/2}} + \frac{x^5(4e(6ae+bd)+3cd^2)}{15d^3(d+ex^2)^{7/2}} + \frac{x^3(6ae+bd)}{3d^2(d+ex^2)^{7/2}} + \frac{ax}{d(d+ex^2)^{7/2}}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x^2 + c*x^4)/(d + e*x^2)^(9/2), x]

[Out] (a*x)/(d*(d + e*x^2)^(7/2)) + ((b*d + 6*a*e)*x^3)/(3*d^2*(d + e*x^2)^(7/2)) + ((3*c*d^2 + 4*e*(b*d + 6*a*e))*x^5)/(15*d^3*(d + e*x^2)^(7/2)) + (2*e*(3*c*d^2 + 4*e*(b*d + 6*a*e))*x^7)/(105*d^4*(d + e*x^2)^(7/2))

Rule 1155

Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := Simp[(a^p*x*(d + e*x^2)^(q + 1))/d, x] + Dist[1/d, Int[x^2*(d + e*x^2)^q*(d*PolynomialQuotient[(a + b*x^2 + c*x^4)^p - a^p, x^2, x] - e*a^p*(2*q + 3)), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && ILtQ[q + 1/2, 0] && LtQ[4*p + 2*q + 1, 0]

Rule 1803

Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] := With[{A = Coef[Pq, x, 0], Q = PolynomialQuotient[Pq - Coef[Pq, x, 0], x^2, x]}, Simp[(A*x^(m + 1)*(a + b*x^2)^(p + 1))/(a*(m + 1)), x] + Dist[1/(a*(m + 1)), Int[x^(m + 2)*(a + b*x^2)^p*(a*(m + 1)*Q - A*b*(m + 2*(p + 1) + 1)), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x^2] && IntegerQ[m/2] && ILtQ[(m + 1)/2 + p, 0] && LtQ[m + Expon[Pq, x] + 2*p + 1, 0]

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 271

Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(x^(m + 1)*(a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m + 1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL

tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]

Rule 264

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[((c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]

Rubi steps

$$\begin{aligned} \int \frac{a + bx^2 + cx^4}{(d + ex^2)^{9/2}} dx &= \frac{ax}{d(d + ex^2)^{7/2}} + \frac{\int \frac{x^2(6ae + d(bc + cx^2))}{(d + ex^2)^{9/2}} dx}{d} \\ &= \frac{ax}{d(d + ex^2)^{7/2}} + \frac{(bd + 6ae)x^3}{3d^2(d + ex^2)^{7/2}} + \frac{\int \frac{(3cd^2 + 4e(bd + 6ae))x^4}{(d + ex^2)^{9/2}} dx}{3d^2} \\ &= \frac{ax}{d(d + ex^2)^{7/2}} + \frac{(bd + 6ae)x^3}{3d^2(d + ex^2)^{7/2}} + \frac{1}{3} \left(3c + \frac{4e(bd + 6ae)}{d^2} \right) \int \frac{x^4}{(d + ex^2)^{9/2}} dx \\ &= \frac{ax}{d(d + ex^2)^{7/2}} + \frac{(bd + 6ae)x^3}{3d^2(d + ex^2)^{7/2}} + \frac{(3cd^2 + 4e(bd + 6ae))x^5}{15d^3(d + ex^2)^{7/2}} + \frac{(2e(3cd^2 + 4e(bd + 6ae))) \int \frac{x^4}{(d + ex^2)^{9/2}} dx}{15d^3} \\ &= \frac{ax}{d(d + ex^2)^{7/2}} + \frac{(bd + 6ae)x^3}{3d^2(d + ex^2)^{7/2}} + \frac{(3cd^2 + 4e(bd + 6ae))x^5}{15d^3(d + ex^2)^{7/2}} + \frac{2e(3cd^2 + 4e(bd + 6ae))x^7}{105d^4(d + ex^2)^{7/2}} \end{aligned}$$

Mathematica [A] time = 0.094898, size = 101, normalized size = 0.8

$$\frac{3a(70d^2ex^3 + 35d^3x + 56de^2x^5 + 16e^3x^7) + dx^3(b(35d^2 + 28dex^2 + 8e^2x^4) + 3cdx^2(7d + 2ex^2))}{105d^4(d + ex^2)^{7/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x^2 + c*x^4)/(d + e*x^2)^(9/2), x]

[Out] (3*a*(35*d^3*x + 70*d^2*e*x^3 + 56*d*e^2*x^5 + 16*e^3*x^7) + d*x^3*(3*c*d*x^2*(7*d + 2*e*x^2) + b*(35*d^2 + 28*d*e*x^2 + 8*e^2*x^4)))/(105*d^4*(d + e*x^2)^(7/2))

Maple [A] time = 0.006, size = 100, normalized size = 0.8

$$\frac{x(48ae^3x^6 + 8bde^2x^6 + 6cd^2ex^6 + 168ade^2x^4 + 28bd^2ex^4 + 21cd^3x^4 + 210ad^2ex^2 + 35bd^3x^2 + 105ad^3)}{105d^4}(ex^2 + d)^{-7/2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^4+b*x^2+a)/(e*x^2+d)^(9/2), x)

[Out] 1/105*x*(48*a*e^3*x^6+8*b*d*e^2*x^6+6*c*d^2*e*x^6+168*a*d*e^2*x^4+28*b*d^2*e*x^4+21*c*d^3*x^4+210*a*d^2*e*x^2+35*b*d^3*x^2+105*a*d^3)/(e*x^2+d)^(7/2)/

d^4

Maxima [B] time = 0.99571, size = 306, normalized size = 2.43

$$-\frac{cx^3}{4(ex^2+d)^{\frac{7}{2}}e} + \frac{16ax}{35\sqrt{ex^2+d}d^4} + \frac{8ax}{35(ex^2+d)^{\frac{3}{2}}d^3} + \frac{6ax}{35(ex^2+d)^{\frac{5}{2}}d^2} + \frac{ax}{7(ex^2+d)^{\frac{7}{2}}d} + \frac{3cx}{140(ex^2+d)^{\frac{5}{2}}e^2} + \frac{2cx}{35\sqrt{ex^2+d}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^(9/2),x, algorithm="maxima")

[Out] $-1/4*c*x^3/((e*x^2+d)^{(7/2)}*e) + 16/35*a*x/(sqrt(e*x^2+d)*d^4) + 8/35*a*x/((e*x^2+d)^{(3/2)}*d^3) + 6/35*a*x/((e*x^2+d)^{(5/2)}*d^2) + 1/7*a*x/((e*x^2+d)^{(7/2)}*d) + 3/140*c*x/((e*x^2+d)^{(5/2)}*e^2) + 2/35*c*x/(sqrt(e*x^2+d)*d^2*e^2) + 1/35*c*x/((e*x^2+d)^{(3/2)}*d*e^2) - 3/28*c*d*x/((e*x^2+d)^{(7/2)}*e^2) - 1/7*b*x/((e*x^2+d)^{(7/2)}*e) + 8/105*b*x/(sqrt(e*x^2+d)*d^3*e) + 4/105*b*x/((e*x^2+d)^{(3/2)}*d^2*e) + 1/35*b*x/((e*x^2+d)^{(5/2)}*d*e)$

Fricas [A] time = 5.13565, size = 294, normalized size = 2.33

$$\frac{(2(3cd^2e + 4bde^2 + 24ae^3)x^7 + 7(3cd^3 + 4bd^2e + 24ade^2)x^5 + 105ad^3x + 35(bd^3 + 6ad^2e)x^3)\sqrt{ex^2+d}}{105(d^4e^4x^8 + 4d^5e^3x^6 + 6d^6e^2x^4 + 4d^7ex^2 + d^8)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^(9/2),x, algorithm="fricas")

[Out] $1/105*(2*(3*c*d^2*e + 4*b*d*e^2 + 24*a*e^3)*x^7 + 7*(3*c*d^3 + 4*b*d^2*e + 24*a*d*e^2)*x^5 + 105*a*d^3*x + 35*(b*d^3 + 6*a*d^2*e)*x^3)*sqrt(e*x^2+d)/(d^4*e^4*x^8 + 4*d^5*e^3*x^6 + 6*d^6*e^2*x^4 + 4*d^7*e*x^2 + d^8)$

Sympy [B] time = 145.542, size = 1989, normalized size = 15.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x**4+b*x**2+a)/(e*x**2+d)**(9/2),x)

[Out] $a*(35*d**14*x/(35*d**(37/2)*sqrt(1 + e*x**2/d) + 210*d**(35/2)*e*x**2*sqrt(1 + e*x**2/d) + 525*d**(33/2)*e**2*x**4*sqrt(1 + e*x**2/d) + 700*d**(31/2)*e**3*x**6*sqrt(1 + e*x**2/d) + 525*d**(29/2)*e**4*x**8*sqrt(1 + e*x**2/d) + 210*d**(27/2)*e**5*x**10*sqrt(1 + e*x**2/d) + 35*d**(25/2)*e**6*x**12*sqrt(1 + e*x**2/d)) + 175*d**13*e*x**3/(35*d**(37/2)*sqrt(1 + e*x**2/d) + 210*d**(35/2)*e*x**2*sqrt(1 + e*x**2/d) + 525*d**(33/2)*e**2*x**4*sqrt(1 + e*x**2/d) + 700*d**(31/2)*e**3*x**6*sqrt(1 + e*x**2/d) + 525*d**(29/2)*e**4*x**8*sqrt(1 + e*x**2/d) + 210*d**(27/2)*e**5*x**10*sqrt(1 + e*x**2/d) + 35*d**(25/2)*e**6*x**12*sqrt(1 + e*x**2/d)) + 371*d**12*e**2*x**5/(35*d**(37/2)*sqrt(1 + e*x**2/d) + 210*d**(35/2)*e*x**2*sqrt(1 + e*x**2/d) + 525*d**(33/2)*e**2*x**4*sqrt(1 + e*x**2/d) + 700*d**(31/2)*e**3*x**6*sqrt(1 + e*x**2/d) +$

$525*d^{29/2}*e^{4*x^8*\sqrt{1 + e*x^2/d}} + 210*d^{27/2}*e^{5*x^{10}*\sqrt{1 + e*x^2/d}} + 35*d^{25/2}*e^{6*x^{12}*\sqrt{1 + e*x^2/d}} + 429*d^{11}*e^{3*x^7/(35*d^{37/2}*\sqrt{1 + e*x^2/d}} + 210*d^{35/2}*e^{x^2*\sqrt{1 + e*x^2/d}} + 525*d^{33/2}*e^{2*x^4*\sqrt{1 + e*x^2/d}} + 700*d^{31/2}*e^{3*x^6*\sqrt{1 + e*x^2/d}} + 525*d^{29/2}*e^{4*x^8*\sqrt{1 + e*x^2/d}} + 210*d^{27/2}*e^{5*x^{10}*\sqrt{1 + e*x^2/d}} + 35*d^{25/2}*e^{6*x^{12}*\sqrt{1 + e*x^2/d}} + 286*d^{10}*e^{4*x^9/(35*d^{37/2}*\sqrt{1 + e*x^2/d}} + 210*d^{35/2}*e^{x^2*\sqrt{1 + e*x^2/d}} + 525*d^{33/2}*e^{2*x^4*\sqrt{1 + e*x^2/d}} + 700*d^{31/2}*e^{3*x^6*\sqrt{1 + e*x^2/d}} + 525*d^{29/2}*e^{4*x^8*\sqrt{1 + e*x^2/d}} + 210*d^{27/2}*e^{5*x^{10}*\sqrt{1 + e*x^2/d}} + 35*d^{25/2}*e^{6*x^{12}*\sqrt{1 + e*x^2/d}} + 104*d^9*e^{5*x^{11}/(35*d^{37/2}*\sqrt{1 + e*x^2/d}} + 210*d^{35/2}*e^{x^2*\sqrt{1 + e*x^2/d}} + 525*d^{33/2}*e^{2*x^4*\sqrt{1 + e*x^2/d}} + 700*d^{31/2}*e^{3*x^6*\sqrt{1 + e*x^2/d}} + 525*d^{29/2}*e^{4*x^8*\sqrt{1 + e*x^2/d}} + 210*d^{27/2}*e^{5*x^{10}*\sqrt{1 + e*x^2/d}} + 35*d^{25/2}*e^{6*x^{12}*\sqrt{1 + e*x^2/d}} + 16*d^8*e^{6*x^{13}/(35*d^{37/2}*\sqrt{1 + e*x^2/d}} + 210*d^{35/2}*e^{x^2*\sqrt{1 + e*x^2/d}} + 525*d^{33/2}*e^{2*x^4*\sqrt{1 + e*x^2/d}} + 700*d^{31/2}*e^{3*x^6*\sqrt{1 + e*x^2/d}} + 525*d^{29/2}*e^{4*x^8*\sqrt{1 + e*x^2/d}} + 210*d^{27/2}*e^{5*x^{10}*\sqrt{1 + e*x^2/d}} + 35*d^{25/2}*e^{6*x^{12}*\sqrt{1 + e*x^2/d}} + b*(35*d^5*x^3/(105*d^{19/2}*\sqrt{1 + e*x^2/d}} + 420*d^{17/2}*e^{x^2*\sqrt{1 + e*x^2/d}} + 630*d^{15/2}*e^{2*x^4*\sqrt{1 + e*x^2/d}} + 420*d^{13/2}*e^{3*x^6*\sqrt{1 + e*x^2/d}} + 105*d^{11/2}*e^{4*x^8*\sqrt{1 + e*x^2/d}} + 63*d^4*e^{x^5/(105*d^{19/2}*\sqrt{1 + e*x^2/d}} + 420*d^{17/2}*e^{x^2*\sqrt{1 + e*x^2/d}} + 630*d^{15/2}*e^{2*x^4*\sqrt{1 + e*x^2/d}} + 420*d^{13/2}*e^{3*x^6*\sqrt{1 + e*x^2/d}} + 105*d^{11/2}*e^{4*x^8*\sqrt{1 + e*x^2/d}} + 36*d^3*e^{2*x^7/(105*d^{19/2}*\sqrt{1 + e*x^2/d}} + 420*d^{17/2}*e^{x^2*\sqrt{1 + e*x^2/d}} + 630*d^{15/2}*e^{2*x^4*\sqrt{1 + e*x^2/d}} + 420*d^{13/2}*e^{3*x^6*\sqrt{1 + e*x^2/d}} + 105*d^{11/2}*e^{4*x^8*\sqrt{1 + e*x^2/d}} + 8*d^2*e^{3*x^9/(105*d^{19/2}*\sqrt{1 + e*x^2/d}} + 420*d^{17/2}*e^{x^2*\sqrt{1 + e*x^2/d}} + 630*d^{15/2}*e^{2*x^4*\sqrt{1 + e*x^2/d}} + 420*d^{13/2}*e^{3*x^6*\sqrt{1 + e*x^2/d}} + 105*d^{11/2}*e^{4*x^8*\sqrt{1 + e*x^2/d}})) + c*(7*d*x^5/(35*d^{11/2}*\sqrt{1 + e*x^2/d}} + 105*d^{9/2}*e^{x^2*\sqrt{1 + e*x^2/d}} + 105*d^{7/2}*e^{2*x^4*\sqrt{1 + e*x^2/d}} + 35*d^{5/2}*e^{3*x^6*\sqrt{1 + e*x^2/d}} + 2*e^{x^7/(35*d^{11/2}*\sqrt{1 + e*x^2/d}} + 105*d^{9/2}*e^{x^2*\sqrt{1 + e*x^2/d}} + 105*d^{7/2}*e^{2*x^4*\sqrt{1 + e*x^2/d}} + 35*d^{5/2}*e^{3*x^6*\sqrt{1 + e*x^2/d}}))$

Giac [A] time = 1.17566, size = 153, normalized size = 1.21

$$\frac{\left(\left(x^2 \left(\frac{2(3cd^2e^4 + 4bde^5 + 24ae^6)x^2e^{(-3)}}{d^4} + \frac{7(3cd^3e^3 + 4bd^2e^4 + 24ade^5)e^{(-3)}}{d^4} \right) + \frac{35(bd^3e^3 + 6ad^2e^4)e^{(-3)}}{d^4} \right) x^2 + \frac{105a}{d} \right) x}{105(x^2e + d)^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^(9/2),x, algorithm="giac")

[Out] 1/105*((x^2*(2*(3*c*d^2*e^4 + 4*b*d*e^5 + 24*a*e^6)*x^2*e^(-3)/d^4 + 7*(3*c*d^3*e^3 + 4*b*d^2*e^4 + 24*a*d*e^5)*e^(-3)/d^4) + 35*(b*d^3*e^3 + 6*a*d^2*e^4)*e^(-3)/d^4)*x^2 + 105*a/d)*x/(x^2*e + d)^(7/2)

$$3.284 \quad \int \frac{a+bx^2+cx^4}{(d+ex^2)^{11/2}} dx$$

Optimal. Leaf size=165

$$\frac{8e^2x^9(2e(8ae+bd)+cd^2)}{315d^5(d+ex^2)^{9/2}} + \frac{4ex^7(2e(8ae+bd)+cd^2)}{35d^4(d+ex^2)^{9/2}} + \frac{x^5(2e(8ae+bd)+cd^2)}{5d^3(d+ex^2)^{9/2}} + \frac{x^3(8ae+bd)}{3d^2(d+ex^2)^{9/2}} + \frac{ax}{d(d+ex^2)^{9/2}}$$

[Out] (a*x)/(d*(d + e*x^2)^(9/2)) + ((b*d + 8*a*e)*x^3)/(3*d^2*(d + e*x^2)^(9/2)) + ((c*d^2 + 2*e*(b*d + 8*a*e))*x^5)/(5*d^3*(d + e*x^2)^(9/2)) + (4*e*(c*d^2 + 2*e*(b*d + 8*a*e))*x^7)/(35*d^4*(d + e*x^2)^(9/2)) + (8*e^2*(c*d^2 + 2*e*(b*d + 8*a*e))*x^9)/(315*d^5*(d + e*x^2)^(9/2))

Rubi [A] time = 0.210036, antiderivative size = 164, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {1155, 1803, 12, 271, 264}

$$\frac{8e^2x^9(2e(8ae+bd)+cd^2)}{315d^5(d+ex^2)^{9/2}} + \frac{4ex^7(2e(8ae+bd)+cd^2)}{35d^4(d+ex^2)^{9/2}} + \frac{x^5\left(\frac{2e(8ae+bd)}{d^2} + c\right)}{5d(d+ex^2)^{9/2}} + \frac{x^3(8ae+bd)}{3d^2(d+ex^2)^{9/2}} + \frac{ax}{d(d+ex^2)^{9/2}}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x^2 + c*x^4)/(d + e*x^2)^(11/2), x]

[Out] (a*x)/(d*(d + e*x^2)^(9/2)) + ((b*d + 8*a*e)*x^3)/(3*d^2*(d + e*x^2)^(9/2)) + ((c + (2*e*(b*d + 8*a*e))/d^2)*x^5)/(5*d*(d + e*x^2)^(9/2)) + (4*e*(c*d^2 + 2*e*(b*d + 8*a*e))*x^7)/(35*d^4*(d + e*x^2)^(9/2)) + (8*e^2*(c*d^2 + 2*e*(b*d + 8*a*e))*x^9)/(315*d^5*(d + e*x^2)^(9/2))

Rule 1155

Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] := Simp[(a^p*x*(d + e*x^2)^(q+1))/d, x] + Dist[1/d, Int[x^2*(d + e*x^2)^q*(d*PolynomialQuotient[(a + b*x^2 + c*x^4)^p - a^p, x^2, x] - e*a^p*(2*q + 3)), x, x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && ILtQ[q + 1/2, 0] && LtQ[4*p + 2*q + 1, 0]

Rule 1803

Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] := With[{A = Coef[Pq, x, 0], Q = PolynomialQuotient[Pq - Coef[Pq, x, 0], x^2, x]}, Simp[(A*x^(m+1)*(a + b*x^2)^(p+1))/(a*(m+1)), x] + Dist[1/(a*(m+1)), Int[x^(m+2)*(a + b*x^2)^p*(a*(m+1)*Q - A*b*(m+2*(p+1)+1)), x, x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x^2] && IntegerQ[m/2] && ILtQ[(m+1)/2 + p, 0] && LtQ[m + Expon[Pq, x] + 2*p + 1, 0]

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 271


```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned} \int \frac{a + bx^2 + cx^4}{(d + ex^2)^{11/2}} dx &= \frac{ax}{d(d + ex^2)^{9/2}} + \frac{\int \frac{x^2(8ae + d(b + cx^2))}{(d + ex^2)^{11/2}} dx}{d} \\ &= \frac{ax}{d(d + ex^2)^{9/2}} + \frac{(bd + 8ae)x^3}{3d^2(d + ex^2)^{9/2}} + \frac{\int \frac{(3cd^2 + 6e(bd + 8ae))x^4}{(d + ex^2)^{11/2}} dx}{3d^2} \\ &= \frac{ax}{d(d + ex^2)^{9/2}} + \frac{(bd + 8ae)x^3}{3d^2(d + ex^2)^{9/2}} + \left(c + \frac{2e(bd + 8ae)}{d^2}\right) \int \frac{x^4}{(d + ex^2)^{11/2}} dx \\ &= \frac{ax}{d(d + ex^2)^{9/2}} + \frac{(bd + 8ae)x^3}{3d^2(d + ex^2)^{9/2}} + \frac{\left(c + \frac{2e(bd + 8ae)}{d^2}\right)x^5}{5d(d + ex^2)^{9/2}} + \frac{\left(4e\left(c + \frac{2e(bd + 8ae)}{d^2}\right)\right) \int \frac{x^6}{(d + ex^2)^{11/2}} dx}{5d} \\ &= \frac{ax}{d(d + ex^2)^{9/2}} + \frac{(bd + 8ae)x^3}{3d^2(d + ex^2)^{9/2}} + \frac{\left(c + \frac{2e(bd + 8ae)}{d^2}\right)x^5}{5d(d + ex^2)^{9/2}} + \frac{4e\left(c + \frac{2e(bd + 8ae)}{d^2}\right)x^7}{35d^2(d + ex^2)^{9/2}} + \frac{\left(8e^2\left(c + \frac{2e(bd + 8ae)}{d^2}\right)\right) \int \frac{x^8}{(d + ex^2)^{11/2}} dx}{315d^3} \\ &= \frac{ax}{d(d + ex^2)^{9/2}} + \frac{(bd + 8ae)x^3}{3d^2(d + ex^2)^{9/2}} + \frac{\left(c + \frac{2e(bd + 8ae)}{d^2}\right)x^5}{5d(d + ex^2)^{9/2}} + \frac{4e\left(c + \frac{2e(bd + 8ae)}{d^2}\right)x^7}{35d^2(d + ex^2)^{9/2}} + \frac{8e^2\left(c + \frac{2e(bd + 8ae)}{d^2}\right) \int \frac{x^8}{(d + ex^2)^{11/2}} dx}{315d^3} \end{aligned}$$

Mathematica [A] time = 0.118709, size = 132, normalized size = 0.8

$$\frac{a(1008d^2e^2x^5 + 840d^3ex^3 + 315d^4x + 576de^3x^7 + 128e^4x^9) + dx^3(b(126d^2ex^2 + 105d^3 + 72de^2x^4 + 16e^3x^6) + cdx^2)}{315d^5(d + ex^2)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)/(d + e*x^2)^(11/2), x]
```

```
[Out] (a*(315*d^4*x + 840*d^3*e*x^3 + 1008*d^2*e^2*x^5 + 576*d*e^3*x^7 + 128*e^4*
x^9) + d*x^3*(c*d*x^2*(63*d^2 + 36*d*e*x^2 + 8*e^2*x^4) + b*(105*d^3 + 126*
d^2*e*x^2 + 72*d*e^2*x^4 + 16*e^3*x^6)))/(315*d^5*(d + e*x^2)^(9/2))
```

Maple [A] time = 0.005, size = 136, normalized size = 0.8

$$\frac{x(128ae^4x^8 + 16bde^3x^8 + 8cd^2e^2x^8 + 576ade^3x^6 + 72bd^2e^2x^6 + 36cd^3ex^6 + 1008ad^2e^2x^4 + 126bd^3ex^4 + 63cd^4x^4)}{315d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c*x^4+b*x^2+a)/(e*x^2+d)^(11/2),x)`

[Out] $\frac{1}{315}x(128ae^4x^8+16bde^3x^8+8cd^2e^2x^8+576ad^3e^3x^6+72bd^2e^2x^6+36cd^3e^3x^6+1008ad^2e^2x^4+126bd^3e^3x^4+63cd^4x^4+840ad^3e^3x^2+105bd^4x^2+315ad^4)/(e*x^2+d)^(9/2)/d^5$

Maxima [A] time = 0.989696, size = 379, normalized size = 2.3

$$-\frac{cx^3}{6(ex^2+d)^{\frac{9}{2}}e} + \frac{128ax}{315\sqrt{ex^2+dd^5}} + \frac{64ax}{315(ex^2+d)^{\frac{3}{2}}d^4} + \frac{16ax}{105(ex^2+d)^{\frac{5}{2}}d^3} + \frac{8ax}{63(ex^2+d)^{\frac{7}{2}}d^2} + \frac{ax}{9(ex^2+d)^{\frac{9}{2}}d} + \frac{c}{126(ex^2+d)^{\frac{11}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x^4+b*x^2+a)/(e*x^2+d)^(11/2),x, algorithm="maxima")`

[Out] $-\frac{1}{6}cx^3/((e*x^2+d)^(9/2)*e) + \frac{128}{315}ax/(\sqrt{e*x^2+d}*d^5) + \frac{64}{315}ax/((e*x^2+d)^(3/2)*d^4) + \frac{16}{105}ax/((e*x^2+d)^(5/2)*d^3) + \frac{8}{63}ax/((e*x^2+d)^(7/2)*d^2) + \frac{1}{9}ax/((e*x^2+d)^(9/2)*d) + \frac{1}{126}cx/((e*x^2+d)^(7/2)*e^2) + \frac{8}{315}cx/(\sqrt{e*x^2+d}*d^3*e^2) + \frac{4}{315}cx/((e*x^2+d)^(3/2)*d^2*e^2) + \frac{1}{105}cx/((e*x^2+d)^(5/2)*d*e^2) - \frac{1}{18}cdx/((e*x^2+d)^(9/2)*e^2) - \frac{1}{9}bx/((e*x^2+d)^(9/2)*e) + \frac{16}{315}bx/(\sqrt{e*x^2+d}*d^4*e) + \frac{8}{315}bx/((e*x^2+d)^(3/2)*d^3*e) + \frac{2}{105}bx/((e*x^2+d)^(5/2)*d^2*e) + \frac{1}{63}bx/((e*x^2+d)^(7/2)*d*e)$

Fricas [A] time = 6.21639, size = 386, normalized size = 2.34

$$\frac{(8(cd^2e^2+2bde^3+16ae^4)x^9+36(cd^3e+2bd^2e^2+16ade^3)x^7+315ad^4x+63(cd^4+2bd^3e+16ad^2e^2)x^5+105(bd^4-315(d^5e^5x^{10}+5d^6e^4x^8+10d^7e^3x^6+10d^8e^2x^4+5d^9ex^2+d^{10})))}{315(d^5e^5x^{10}+5d^6e^4x^8+10d^7e^3x^6+10d^8e^2x^4+5d^9ex^2+d^{10})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x^4+b*x^2+a)/(e*x^2+d)^(11/2),x, algorithm="fricas")`

[Out] $\frac{1}{315}(8(c*d^2*e^2+2*b*d*e^3+16*a*e^4)*x^9+36(c*d^3*e+2*b*d^2*e^2+16*a*d*e^3)*x^7+315*a*d^4*x+63(c*d^4+2*b*d^3*e+16*a*d^2*e^2)*x^5+105*(b*d^4+8*a*d^3*e)*x^3)*\sqrt{e*x^2+d}/(d^5*e^5*x^{10}+5*d^6*e^4*x^8+10*d^7*e^3*x^6+10*d^8*e^2*x^4+5*d^9*e*x^2+d^{10})$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x**4+b*x**2+a)/(e*x**2+d)**(11/2),x)`

[Out] Timed out

Giac [A] time = 1.14314, size = 200, normalized size = 1.21

$$\frac{\left(\left(4x^2\left(\frac{2(cd^2e^6+2bde^7+16ae^8)x^2e^{-4}}{d^5} + \frac{9(cd^3e^5+2bd^2e^6+16ade^7)e^{-4}}{d^5}\right) + \frac{63(cd^4e^4+2bd^3e^5+16ad^2e^6)e^{-4}}{d^5}\right)x^2 + \frac{105(bd^4e^4+8ad^3e^5)e^{-4}}{d^5}\right)x^2}{315(x^2e+d)^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^(11/2),x, algorithm="giac")

[Out] 1/315*((4*x^2*(2*(c*d^2*e^6 + 2*b*d*e^7 + 16*a*e^8)*x^2*e^(-4)/d^5 + 9*(c*d^3*e^5 + 2*b*d^2*e^6 + 16*a*d*e^7)*e^(-4)/d^5) + 63*(c*d^4*e^4 + 2*b*d^3*e^5 + 16*a*d^2*e^6)*e^(-4)/d^5)*x^2 + 105*(b*d^4*e^4 + 8*a*d^3*e^5)*e^(-4)/d^5)*x^2 + 315*a/d)*x/(x^2*e + d)^(9/2)

$$3.285 \quad \int \frac{a+bx^2+cx^4}{(d+ex^2)^{13/2}} dx$$

Optimal. Leaf size=210

$$\frac{16e^3x^{11}(8e(10ae+bd)+3cd^2)}{3465d^6(d+ex^2)^{11/2}} + \frac{8e^2x^9(8e(10ae+bd)+3cd^2)}{315d^5(d+ex^2)^{11/2}} + \frac{2ex^7(8e(10ae+bd)+3cd^2)}{35d^4(d+ex^2)^{11/2}} + \frac{x^5(8e(10ae+bd)+3cd^2)}{15d^3(d+ex^2)^{11/2}}$$

[Out] (a*x)/(d*(d + e*x^2)^(11/2)) + ((b*d + 10*a*e)*x^3)/(3*d^2*(d + e*x^2)^(11/2)) + ((3*c*d^2 + 8*e*(b*d + 10*a*e))*x^5)/(15*d^3*(d + e*x^2)^(11/2)) + (2*e*(3*c*d^2 + 8*e*(b*d + 10*a*e))*x^7)/(35*d^4*(d + e*x^2)^(11/2)) + (8*e^2*(3*c*d^2 + 8*e*(b*d + 10*a*e))*x^9)/(315*d^5*(d + e*x^2)^(11/2)) + (16*e^3*(3*c*d^2 + 8*e*(b*d + 10*a*e))*x^11)/(3465*d^6*(d + e*x^2)^(11/2))

Rubi [A] time = 0.221537, antiderivative size = 210, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {1155, 1803, 12, 271, 264}

$$\frac{16e^3x^{11}(8e(10ae+bd)+3cd^2)}{3465d^6(d+ex^2)^{11/2}} + \frac{8e^2x^9(8e(10ae+bd)+3cd^2)}{315d^5(d+ex^2)^{11/2}} + \frac{2ex^7(8e(10ae+bd)+3cd^2)}{35d^4(d+ex^2)^{11/2}} + \frac{x^5(8e(10ae+bd)+3cd^2)}{15d^3(d+ex^2)^{11/2}}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x^2 + c*x^4)/(d + e*x^2)^(13/2), x]

[Out] (a*x)/(d*(d + e*x^2)^(11/2)) + ((b*d + 10*a*e)*x^3)/(3*d^2*(d + e*x^2)^(11/2)) + ((3*c*d^2 + 8*e*(b*d + 10*a*e))*x^5)/(15*d^3*(d + e*x^2)^(11/2)) + (2*e*(3*c*d^2 + 8*e*(b*d + 10*a*e))*x^7)/(35*d^4*(d + e*x^2)^(11/2)) + (8*e^2*(3*c*d^2 + 8*e*(b*d + 10*a*e))*x^9)/(315*d^5*(d + e*x^2)^(11/2)) + (16*e^3*(3*c*d^2 + 8*e*(b*d + 10*a*e))*x^11)/(3465*d^6*(d + e*x^2)^(11/2))

Rule 1155

Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] := Simp[(a^p*x*(d + e*x^2)^(q + 1))/d, x] + Dist[1/d, Int[x^2*(d + e*x^2)^q*(d*PolynomialQuotient[(a + b*x^2 + c*x^4)^p - a^p, x^2, x] - e*a^p*(2*q + 3)), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && ILtQ[q + 1/2, 0] && LtQ[4*p + 2*q + 1, 0]

Rule 1803

Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] := With[{A = Coef[Pq, x, 0], Q = PolynomialQuotient[Pq - Coef[Pq, x, 0], x^2, x]}, Simp[(A*x^(m + 1)*(a + b*x^2)^(p + 1))/(a*(m + 1)), x] + Dist[1/(a*(m + 1)), Int[x^(m + 2)*(a + b*x^2)^p*(a*(m + 1)*Q - A*b*(m + 2*(p + 1) + 1)), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x^2] && IntegerQ[m/2] && ILtQ[(m + 1)/2 + p, 0] && LtQ[m + Expon[Pq, x] + 2*p + 1, 0]

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned} \int \frac{a + bx^2 + cx^4}{(d + ex^2)^{13/2}} dx &= \frac{ax}{d(d + ex^2)^{11/2}} + \frac{\int \frac{x^2(10ae + d(b + cx^2))}{(d + ex^2)^{13/2}} dx}{d} \\ &= \frac{ax}{d(d + ex^2)^{11/2}} + \frac{(bd + 10ae)x^3}{3d^2(d + ex^2)^{11/2}} + \frac{\int \frac{(3cd^2 + 8e(bd + 10ae))x^4}{(d + ex^2)^{13/2}} dx}{3d^2} \\ &= \frac{ax}{d(d + ex^2)^{11/2}} + \frac{(bd + 10ae)x^3}{3d^2(d + ex^2)^{11/2}} + \frac{1}{3} \left(3c + \frac{8e(bd + 10ae)}{d^2} \right) \int \frac{x^4}{(d + ex^2)^{13/2}} dx \\ &= \frac{ax}{d(d + ex^2)^{11/2}} + \frac{(bd + 10ae)x^3}{3d^2(d + ex^2)^{11/2}} + \frac{(3cd^2 + 8e(bd + 10ae))x^5}{15d^3(d + ex^2)^{11/2}} + \frac{(2e(3cd^2 + 8e(bd + 10ae)))}{5d^3} \\ &= \frac{ax}{d(d + ex^2)^{11/2}} + \frac{(bd + 10ae)x^3}{3d^2(d + ex^2)^{11/2}} + \frac{(3cd^2 + 8e(bd + 10ae))x^5}{15d^3(d + ex^2)^{11/2}} + \frac{2e(3cd^2 + 8e(bd + 10ae))x^7}{35d^4(d + ex^2)^{11/2}} \\ &= \frac{ax}{d(d + ex^2)^{11/2}} + \frac{(bd + 10ae)x^3}{3d^2(d + ex^2)^{11/2}} + \frac{(3cd^2 + 8e(bd + 10ae))x^5}{15d^3(d + ex^2)^{11/2}} + \frac{2e(3cd^2 + 8e(bd + 10ae))x^7}{35d^4(d + ex^2)^{11/2}} \\ &= \frac{ax}{d(d + ex^2)^{11/2}} + \frac{(bd + 10ae)x^3}{3d^2(d + ex^2)^{11/2}} + \frac{(3cd^2 + 8e(bd + 10ae))x^5}{15d^3(d + ex^2)^{11/2}} + \frac{2e(3cd^2 + 8e(bd + 10ae))x^7}{35d^4(d + ex^2)^{11/2}} \end{aligned}$$

Mathematica [A] time = 0.145272, size = 167, normalized size = 0.8

$$\frac{5a(3168d^2e^3x^7 + 3696d^3e^2x^5 + 2310d^4ex^3 + 693d^5x + 1408de^4x^9 + 256e^5x^{11}) + dx^3(b(1584d^2e^2x^4 + 1848d^3ex^2 + 1152d^4 + 704d^5e^3x^6 + 128e^4x^8))}{3465d^6(d + ex^2)^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)/(d + e*x^2)^(13/2), x]
```

```
[Out] (5*a*(693*d^5*x + 2310*d^4*e*x^3 + 3696*d^3*e^2*x^5 + 3168*d^2*e^3*x^7 + 14
08*d*e^4*x^9 + 256*e^5*x^11) + d*x^3*(3*c*d*x^2*(231*d^3 + 198*d^2*e*x^2 +
88*d*e^2*x^4 + 16*e^3*x^6) + b*(1155*d^4 + 1848*d^3*e*x^2 + 1584*d^2*e^2*x^
4 + 704*d*e^3*x^6 + 128*e^4*x^8)))/(3465*d^6*(d + e*x^2)^(11/2))
```

Maple [A] time = 0.006, size = 172, normalized size = 0.8

$$\frac{x(1280ae^5x^{10} + 128bde^4x^{10} + 48cd^2e^3x^{10} + 7040ade^4x^8 + 704bd^2e^3x^8 + 264cd^3e^2x^8 + 15840ad^2e^3x^6 + 1584bd^3e^2x^6 + 3465d^6)}{3465d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^4+b*x^2+a)/(e*x^2+d)^(13/2),x)

[Out] 1/3465*x*(1280*a*e^5*x^10+128*b*d*e^4*x^10+48*c*d^2*e^3*x^10+7040*a*d*e^4*x^8+704*b*d^2*e^3*x^8+264*c*d^3*e^2*x^8+15840*a*d^2*e^3*x^6+1584*b*d^3*e^2*x^6+594*c*d^4*e*x^6+18480*a*d^3*e^2*x^4+1848*b*d^4*e*x^4+693*c*d^5*x^4+11550*a*d^4*e*x^2+1155*b*d^5*x^2+3465*a*d^5)/(e*x^2+d)^(11/2)/d^6

Maxima [A] time = 0.989096, size = 452, normalized size = 2.15

$$-\frac{cx^3}{8(ex^2+d)^{\frac{11}{2}}e} + \frac{256ax}{693\sqrt{ex^2+dd^6}} + \frac{128ax}{693(ex^2+d)^{\frac{3}{2}}d^5} + \frac{32ax}{231(ex^2+d)^{\frac{5}{2}}d^4} + \frac{80ax}{693(ex^2+d)^{\frac{7}{2}}d^3} + \frac{10ax}{99(ex^2+d)^{\frac{9}{2}}d^2} + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^(13/2),x, algorithm="maxima")

[Out] -1/8*c*x^3/((e*x^2 + d)^(11/2)*e) + 256/693*a*x/(sqrt(e*x^2 + d)*d^6) + 128/693*a*x/((e*x^2 + d)^(3/2)*d^5) + 32/231*a*x/((e*x^2 + d)^(5/2)*d^4) + 80/693*a*x/((e*x^2 + d)^(7/2)*d^3) + 10/99*a*x/((e*x^2 + d)^(9/2)*d^2) + 1/11*a*x/((e*x^2 + d)^(11/2)*d) + 1/264*c*x/((e*x^2 + d)^(9/2)*e^2) + 16/1155*c*x/(sqrt(e*x^2 + d)*d^4*e^2) + 8/1155*c*x/((e*x^2 + d)^(3/2)*d^3*e^2) + 2/385*c*x/((e*x^2 + d)^(5/2)*d^2*e^2) + 1/231*c*x/((e*x^2 + d)^(7/2)*d*e^2) - 3/88*c*d*x/((e*x^2 + d)^(11/2)*e^2) - 1/11*b*x/((e*x^2 + d)^(11/2)*e) + 128/3465*b*x/(sqrt(e*x^2 + d)*d^5*e) + 64/3465*b*x/((e*x^2 + d)^(3/2)*d^4*e) + 16/1155*b*x/((e*x^2 + d)^(5/2)*d^3*e) + 8/693*b*x/((e*x^2 + d)^(7/2)*d^2*e) + 1/99*b*x/((e*x^2 + d)^(9/2)*d*e)

Fricas [A] time = 7.45462, size = 502, normalized size = 2.39

$$\frac{(16(3cd^2e^3 + 8bde^4 + 80ae^5)x^{11} + 88(3cd^3e^2 + 8bd^2e^3 + 80ade^4)x^9 + 198(3cd^4e + 8bd^3e^2 + 80ad^2e^3)x^7 + 3465ad^5x^5 + 3465(d^6e^6x^{12} + 6d^7e^5x^{10} + 15d^8e^4x^8 + 20d^9e^3x^6 + 15d^{10}e^2x^4 + \dots))}{3465(d^6e^6x^{12} + 6d^7e^5x^{10} + 15d^8e^4x^8 + 20d^9e^3x^6 + 15d^{10}e^2x^4 + \dots)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^(13/2),x, algorithm="fricas")

[Out] 1/3465*(16*(3*c*d^2*e^3 + 8*b*d*e^4 + 80*a*e^5)*x^11 + 88*(3*c*d^3*e^2 + 8*b*d^2*e^3 + 80*a*d^2*e^3)*x^9 + 198*(3*c*d^4*e + 8*b*d^3*e^2 + 80*a*d^2*e^3)*x^7 + 3465*a*d^5*x + 231*(3*c*d^5 + 8*b*d^4*e + 80*a*d^3*e^2)*x^5 + 1155*(b*d^5 + 10*a*d^4*e)*x^3)*sqrt(e*x^2 + d)/(d^6*e^6*x^12 + 6*d^7*e^5*x^10 + 15*d^8*e^4*x^8 + 20*d^9*e^3*x^6 + 15*d^10*e^2*x^4 + 6*d^11*e*x^2 + d^12)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x**4+b*x**2+a)/(e*x**2+d)**(13/2),x)

[Out] Timed out

Giac [A] time = 1.19784, size = 255, normalized size = 1.21

$$\frac{\left(\left(2\left(4x^2\left(\frac{2(3cd^2e^8+8bde^9+80ae^{10})x^2e^{(-5)}}{d^6} + \frac{11(3cd^3e^7+8bd^2e^8+80ade^9)e^{(-5)}}{d^6}\right) + \frac{99(3cd^4e^6+8bd^3e^7+80ad^2e^8)e^{(-5)}}{d^6}\right)\right)x^2 + \frac{231(3cd^5e^5+8bd^4e^6+80ad^3e^7)e^{(-5)}}{d^6}\right)}{3465(x^2e+d)^{\frac{11}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^(13/2),x, algorithm="giac")

[Out] 1/3465*((2*(4*x^2*(2*(3*c*d^2*e^8 + 8*b*d*e^9 + 80*a*e^10)*x^2*e^(-5)/d^6 + 11*(3*c*d^3*e^7 + 8*b*d^2*e^8 + 80*a*d*e^9)*e^(-5)/d^6) + 99*(3*c*d^4*e^6 + 8*b*d^3*e^7 + 80*a*d^2*e^8)*e^(-5)/d^6)*x^2 + 231*(3*c*d^5*e^5 + 8*b*d^4*e^6 + 80*a*d^3*e^7)*e^(-5)/d^6)*x^2 + 1155*(b*d^5*e^5 + 10*a*d^4*e^6)*e^(-5)/d^6)*x/(x^2*e + d)^(11/2)

3.286 $\int (7 + 5x^2)^3 \sqrt{2 + 3x^2 + x^4} dx$

Optimal. Leaf size=193

$$\frac{2945\sqrt{2}(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\text{EllipticF}\left(\tan^{-1}(x), \frac{1}{2}\right)}{21\sqrt{x^4+3x^2+2}} + \frac{125}{9}(x^4+3x^2+2)^{3/2}x^3 + \frac{275}{7}(x^4+3x^2+2)^{3/2}x + \frac{1}{21}(757x^2+2608)\sqrt{x^4+3x^2+2}$$

```
[Out] (577*x*(2 + x^2))/(3*Sqrt[2 + 3*x^2 + x^4]) + (x*(2608 + 757*x^2)*Sqrt[2 + 3*x^2 + x^4])/21 + (275*x*(2 + 3*x^2 + x^4)^(3/2))/7 + (125*x^3*(2 + 3*x^2 + x^4)^(3/2))/9 - (577*Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x], 1/2])/(3*Sqrt[2 + 3*x^2 + x^4]) + (2945*Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(21*Sqrt[2 + 3*x^2 + x^4])
```

Rubi [A] time = 0.0987772, antiderivative size = 193, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$, Rules used = {1206, 1679, 1176, 1189, 1099, 1135}

$$\frac{125}{9}(x^4+3x^2+2)^{3/2}x^3 + \frac{275}{7}(x^4+3x^2+2)^{3/2}x + \frac{1}{21}(757x^2+2608)\sqrt{x^4+3x^2+2} + \frac{577(x^2+2)x}{3\sqrt{x^4+3x^2+2}} + \frac{2945\sqrt{2}(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\text{EllipticF}\left(\tan^{-1}(x), \frac{1}{2}\right)}{21\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Int[(7 + 5*x^2)^3*Sqrt[2 + 3*x^2 + x^4], x]
```

```
[Out] (577*x*(2 + x^2))/(3*Sqrt[2 + 3*x^2 + x^4]) + (x*(2608 + 757*x^2)*Sqrt[2 + 3*x^2 + x^4])/21 + (275*x*(2 + 3*x^2 + x^4)^(3/2))/7 + (125*x^3*(2 + 3*x^2 + x^4)^(3/2))/9 - (577*Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x], 1/2])/(3*Sqrt[2 + 3*x^2 + x^4]) + (2945*Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(21*Sqrt[2 + 3*x^2 + x^4])
```

Rule 1206

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := Simp[(e^q*x^(2*q - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(4*p + 2*q + 1)), x] + Dist[1/(c*(4*p + 2*q + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*p + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1]
```

Rule 1679

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := With[{q = Expon[Pq, x^2], e = Coeff[Pq, x^2, Expon[Pq, x^2]]}, Simp[(e*x^(2*q - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(2*q + 4*p + 1)), x] + Dist[1/(c*(2*q + 4*p + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*q + 4*p + 1)*Pq - a*e*(2*q - 3)*x^(2*q - 4) - b*e*(2*q + 2*p - 1)*x^(2*q - 2) - c*e*(2*q + 4*p + 1)*x^(2*q), x], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && !LtQ[p, -1]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := Simp[(x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*(a + b*x^2 + c*x^4)^p)/(c*(4*p + 1)*(4*p + 3)), x] + Dist[(2*p)/(c*(4*p + 1)*(4*p + 3)),
```



```
Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) -
b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1189

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol]
:> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a]
|| PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1099

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[((2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]*EllipticF[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)/(b + q)])/(2*
a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1135

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
Rt[b^2 - 4*a*c, 2]}, Simp[(x*(b + q + 2*c*x^2))/(2*c*Sqrt[a + b*x^2 + c*x^
4]), x] - Simp[(Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q)
)*x^2)/(2*a + (b + q)*x^2)]*EllipticE[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)
/(b + q)])/(2*c*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] && !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int (7 + 5x^2)^3 \sqrt{2 + 3x^2 + x^4} dx &= \frac{125}{9}x^3(2 + 3x^2 + x^4)^{3/2} + \frac{1}{9} \int \sqrt{2 + 3x^2 + x^4} (3087 + 5865x^2 + 2475x^4) dx \\
&= \frac{275}{7}x(2 + 3x^2 + x^4)^{3/2} + \frac{125}{9}x^3(2 + 3x^2 + x^4)^{3/2} + \frac{1}{63} \int (16659 + 11355x^2) \sqrt{2 + 3x^2 + x^4} dx \\
&= \frac{1}{21}x(2608 + 757x^2) \sqrt{2 + 3x^2 + x^4} + \frac{275}{7}x(2 + 3x^2 + x^4)^{3/2} + \frac{125}{9}x^3(2 + 3x^2 + x^4)^{3/2} \\
&= \frac{1}{21}x(2608 + 757x^2) \sqrt{2 + 3x^2 + x^4} + \frac{275}{7}x(2 + 3x^2 + x^4)^{3/2} + \frac{125}{9}x^3(2 + 3x^2 + x^4)^{3/2} \\
&= \frac{577x(2 + x^2)}{3\sqrt{2 + 3x^2 + x^4}} + \frac{1}{21}x(2608 + 757x^2) \sqrt{2 + 3x^2 + x^4} + \frac{275}{7}x(2 + 3x^2 + x^4)^{3/2} + \frac{125}{9}x^3(2 + 3x^2 + x^4)^{3/2}
\end{aligned}$$

Mathematica [C] time = 0.100681, size = 119, normalized size = 0.62

$$\frac{-5553i\sqrt{x^2 + 1}\sqrt{x^2 + 2}\text{EllipticF}\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right), 2\right) + 875x^{11} + 7725x^9 + 28496x^7 + 57312x^5 + 61214x^3 - 12117i\sqrt{x^2 + 2}}{63\sqrt{x^4 + 3x^2 + 2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(7 + 5*x^2)^3*Sqrt[2 + 3*x^2 + x^4], x]
```

```
[Out] (25548*x + 61214*x^3 + 57312*x^5 + 28496*x^7 + 7725*x^9 + 875*x^11 - (12117
*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticE[I*ArcSinh[x/Sqrt[2]], 2] - (5553*
```

$I) \sqrt{1+x^2} \sqrt{2+x^2} \operatorname{EllipticF}\left[\operatorname{ArcSinh}\left[\frac{x}{\sqrt{2}}\right], 2\right] / (63 \sqrt{2+3x^2+x^4})$

Maple [C] time = 0.027, size = 172, normalized size = 0.9

$$\frac{125x^7}{9} \sqrt{x^4+3x^2+2} + \frac{1700x^5}{21} \sqrt{x^4+3x^2+2} + \frac{11446x^3}{63} \sqrt{x^4+3x^2+2} + \frac{4258x}{21} \sqrt{x^4+3x^2+2} - \frac{2945i}{21} \sqrt{2} \operatorname{EllipticF}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((5*x^2+7)^3*(x^4+3*x^2+2)^(1/2), x)

[Out] $125/9x^7(x^4+3x^2+2)^{1/2} + 1700/21x^5(x^4+3x^2+2)^{1/2} + 11446/63x^3(x^4+3x^2+2)^{1/2} + 4258/21x(x^4+3x^2+2)^{1/2} - 2945/21i \sqrt{2} \operatorname{EllipticF}\left(\frac{1}{2} \operatorname{ArcSinh}\left(\frac{x}{\sqrt{2}}\right), 2\right) + 577/6i \sqrt{2} \operatorname{EllipticE}\left(\frac{1}{2} \operatorname{ArcSinh}\left(\frac{x}{\sqrt{2}}\right), 2\right) - \operatorname{EllipticE}\left(\frac{1}{2} \operatorname{ArcSinh}\left(\frac{x}{\sqrt{2}}\right), 2\right)$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{x^4+3x^2+2} (5x^2+7)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^2+7)^3*(x^4+3*x^2+2)^(1/2), x, algorithm="maxima")

[Out] integrate(sqrt(x^4 + 3*x^2 + 2)*(5*x^2 + 7)^3, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\left(125x^6 + 525x^4 + 735x^2 + 343\right) \sqrt{x^4 + 3x^2 + 2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^2+7)^3*(x^4+3*x^2+2)^(1/2), x, algorithm="fricas")

[Out] integral((125*x^6 + 525*x^4 + 735*x^2 + 343)*sqrt(x^4 + 3*x^2 + 2), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{(x^2+1)(x^2+2)} (5x^2+7)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x**2+7)**3*(x**4+3*x**2+2)**(1/2), x)

[Out] Integral(sqrt((x**2 + 1)*(x**2 + 2))*(5*x**2 + 7)**3, x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{x^4 + 3x^2 + 2}(5x^2 + 7)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^3*(x^4+3*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^4 + 3*x^2 + 2)*(5*x^2 + 7)^3, x)
```

3.287 $\int (7 + 5x^2)^2 \sqrt{2 + 3x^2 + x^4} dx$

Optimal. Leaf size=168

$$\frac{472\sqrt{2}(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\text{EllipticF}\left(\tan^{-1}(x), \frac{1}{2}\right)}{21\sqrt{x^4+3x^2+2}} + \frac{25}{7}x(x^4+3x^2+2)^{3/2} + \frac{1}{21}x(114x^2+407)\sqrt{x^4+3x^2+2} + \frac{31x(x^2+2)}{\sqrt{x^4+3x^2+2}}$$

```
[Out] (31*x*(2 + x^2))/Sqrt[2 + 3*x^2 + x^4] + (x*(407 + 114*x^2)*Sqrt[2 + 3*x^2 + x^4])/21 + (25*x*(2 + 3*x^2 + x^4)^(3/2))/7 - (31*Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x], 1/2])/Sqrt[2 + 3*x^2 + x^4] + (472*Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(21*Sqrt[2 + 3*x^2 + x^4])
```

Rubi [A] time = 0.0677099, antiderivative size = 168, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {1206, 1176, 1189, 1099, 1135}

$$\frac{25}{7}x(x^4+3x^2+2)^{3/2} + \frac{1}{21}x(114x^2+407)\sqrt{x^4+3x^2+2} + \frac{31x(x^2+2)}{\sqrt{x^4+3x^2+2}} + \frac{472\sqrt{2}(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}F\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{21\sqrt{x^4+3x^2+2}} - \dots$$

Antiderivative was successfully verified.

```
[In] Int[(7 + 5*x^2)^2*Sqrt[2 + 3*x^2 + x^4], x]
```

```
[Out] (31*x*(2 + x^2))/Sqrt[2 + 3*x^2 + x^4] + (x*(407 + 114*x^2)*Sqrt[2 + 3*x^2 + x^4])/21 + (25*x*(2 + 3*x^2 + x^4)^(3/2))/7 - (31*Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x], 1/2])/Sqrt[2 + 3*x^2 + x^4] + (472*Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(21*Sqrt[2 + 3*x^2 + x^4])
```

Rule 1206

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := Simp[(e^q*x^(2*q - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(4*p + 2*q + 1)), x] + Dist[1/(c*(4*p + 2*q + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*p + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := Simp[(x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*(a + b*x^2 + c*x^4)^p)/(c*(4*p + 1)*(4*p + 3)), x] + Dist[(2*p)/(c*(4*p + 1)*(4*p + 3)), Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) - b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1189

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
```

] || PosQ[(b - q)/a] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]

Rule 1099

Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[((2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q)*x^2)/(2*a + (b + q)*x^2)]*EllipticF[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)/(b + q)])/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] && !(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]

Rule 1135

Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(x*(b + q + 2*c*x^2))/(2*c*Sqrt[a + b*x^2 + c*x^4]), x] - Simp[(Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q)*x^2)/(2*a + (b + q)*x^2)]*EllipticE[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)/(b + q)])/(2*c*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] && !(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]

Rubi steps

$$\begin{aligned} \int (7 + 5x^2)^2 \sqrt{2 + 3x^2 + x^4} dx &= \frac{25}{7}x(2 + 3x^2 + x^4)^{3/2} + \frac{1}{7} \int (293 + 190x^2) \sqrt{2 + 3x^2 + x^4} dx \\ &= \frac{1}{21}x(407 + 114x^2) \sqrt{2 + 3x^2 + x^4} + \frac{25}{7}x(2 + 3x^2 + x^4)^{3/2} + \frac{1}{105} \int \frac{4720 + 3255x^2}{\sqrt{2 + 3x^2 + x^4}} dx \\ &= \frac{1}{21}x(407 + 114x^2) \sqrt{2 + 3x^2 + x^4} + \frac{25}{7}x(2 + 3x^2 + x^4)^{3/2} + 31 \int \frac{x^2}{\sqrt{2 + 3x^2 + x^4}} dx \\ &= \frac{31x(2 + x^2)}{\sqrt{2 + 3x^2 + x^4}} + \frac{1}{21}x(407 + 114x^2) \sqrt{2 + 3x^2 + x^4} + \frac{25}{7}x(2 + 3x^2 + x^4)^{3/2} - \frac{31\sqrt{2}}{\sqrt{2 + 3x^2 + x^4}} \end{aligned}$$

Mathematica [C] time = 0.0788284, size = 114, normalized size = 0.68

$$\frac{-293i\sqrt{x^2 + 1}\sqrt{x^2 + 2}\text{EllipticF}\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right), 2\right) + 75x^9 + 564x^7 + 1724x^5 + 2349x^3 - 651i\sqrt{x^2 + 1}\sqrt{x^2 + 2}E\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right), 2\right)}{21\sqrt{x^4 + 3x^2 + 2}}$$

Antiderivative was successfully verified.

[In] Integrate[(7 + 5*x^2)^2*Sqrt[2 + 3*x^2 + x^4], x]

[Out] (1114*x + 2349*x^3 + 1724*x^5 + 564*x^7 + 75*x^9 - (651*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticE[I*ArcSinh[x/Sqrt[2]], 2] - (293*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticF[I*ArcSinh[x/Sqrt[2]], 2])/(21*Sqrt[2 + 3*x^2 + x^4])

Maple [C] time = 0.007, size = 155, normalized size = 0.9

$$\frac{25x^5}{7}\sqrt{x^4 + 3x^2 + 2} + \frac{113x^3}{7}\sqrt{x^4 + 3x^2 + 2} + \frac{557x}{21}\sqrt{x^4 + 3x^2 + 2} - \frac{472i}{21}\sqrt{2}\text{EllipticF}\left(\frac{i}{2}x\sqrt{2}, \sqrt{2}\right)\sqrt{2x^2 + 4}\sqrt{x^2 + 2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((5*x^2+7)^2*(x^4+3*x^2+2)^(1/2),x)`

[Out] $25/7*x^5*(x^4+3*x^2+2)^{(1/2)}+113/7*x^3*(x^4+3*x^2+2)^{(1/2)}+557/21*x*(x^4+3*x^2+2)^{(1/2)}-472/21*I*2^{(1/2)}*(2*x^2+4)^{(1/2)}*(x^2+1)^{(1/2)}/(x^4+3*x^2+2)^{(1/2)}*EllipticF(1/2*I*x*2^{(1/2)},2^{(1/2)})+31/2*I*2^{(1/2)}*(2*x^2+4)^{(1/2)}*(x^2+1)^{(1/2)}/(x^4+3*x^2+2)^{(1/2)}*(EllipticF(1/2*I*x*2^{(1/2)},2^{(1/2)})-EllipticE(1/2*I*x*2^{(1/2)},2^{(1/2)}))$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{x^4 + 3x^2 + 2}(5x^2 + 7)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((5*x^2+7)^2*(x^4+3*x^2+2)^(1/2),x, algorithm="maxima")`

[Out] `integrate(sqrt(x^4 + 3*x^2 + 2)*(5*x^2 + 7)^2, x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(25x^4 + 70x^2 + 49\right)\sqrt{x^4 + 3x^2 + 2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((5*x^2+7)^2*(x^4+3*x^2+2)^(1/2),x, algorithm="fricas")`

[Out] `integral((25*x^4 + 70*x^2 + 49)*sqrt(x^4 + 3*x^2 + 2), x)`

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{(x^2 + 1)(x^2 + 2)}(5x^2 + 7)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((5*x**2+7)**2*(x**4+3*x**2+2)**(1/2),x)`

[Out] `Integral(sqrt((x**2 + 1)*(x**2 + 2))*(5*x**2 + 7)**2, x)`

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{x^4 + 3x^2 + 2}(5x^2 + 7)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((5*x^2+7)^2*(x^4+3*x^2+2)^(1/2),x, algorithm="giac")`

[Out] `integrate(sqrt(x^4 + 3*x^2 + 2)*(5*x^2 + 7)^2, x)`

3.288 $\int (7 + 5x^2) \sqrt{2 + 3x^2 + x^4} dx$

Optimal. Leaf size=149

$$\frac{11\sqrt{2}(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\text{EllipticF}\left(\tan^{-1}(x), \frac{1}{2}\right)}{3\sqrt{x^4+3x^2+2}} + \frac{5x(x^2+2)}{\sqrt{x^4+3x^2+2}} + \frac{1}{3}x(3x^2+10)\sqrt{x^4+3x^2+2} - \frac{5\sqrt{2}(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}}{\sqrt{x^4+3x^2+2}}$$

```
[Out] (5*x*(2 + x^2))/Sqrt[2 + 3*x^2 + x^4] + (x*(10 + 3*x^2)*Sqrt[2 + 3*x^2 + x^4])/3 - (5*Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x], 1/2])/Sqrt[2 + 3*x^2 + x^4] + (11*Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(3*Sqrt[2 + 3*x^2 + x^4])
```

Rubi [A] time = 0.0475073, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1176, 1189, 1099, 1135}

$$\frac{5x(x^2+2)}{\sqrt{x^4+3x^2+2}} + \frac{1}{3}x(3x^2+10)\sqrt{x^4+3x^2+2} + \frac{11\sqrt{2}(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}F\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{3\sqrt{x^4+3x^2+2}} - \frac{5\sqrt{2}(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}E\left(\tan^{-1}(x)\right)}{\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Int[(7 + 5*x^2)*Sqrt[2 + 3*x^2 + x^4], x]
```

```
[Out] (5*x*(2 + x^2))/Sqrt[2 + 3*x^2 + x^4] + (x*(10 + 3*x^2)*Sqrt[2 + 3*x^2 + x^4])/3 - (5*Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x], 1/2])/Sqrt[2 + 3*x^2 + x^4] + (11*Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(3*Sqrt[2 + 3*x^2 + x^4])
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*(a + b*x^2 + c*x^4)^p)/(c*(4*p + 1)*(4*p + 3)), x] + Dist[(2*p)/(c*(4*p + 1)*(4*p + 3)), Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) - b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1189

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a] || PosQ[(b - q)/a] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1099

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[((2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q)*x^2)/(2*a + (b + q)*x^2)]*EllipticF[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)/(b + q)])/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] && ! (PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)]) /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1135

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] := With[{q =
  Rt[b^2 - 4*a*c, 2]}, Simp[(x*(b + q + 2*c*x^2))/(2*c*Sqrt[a + b*x^2 + c*x^
  4]), x] - Simp[(Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q)
  )*x^2]/(2*a + (b + q)*x^2)]*EllipticE[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)
  /(b + q)]/(2*c*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] && !(PosQ[
  (b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
  c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned} \int (7 + 5x^2) \sqrt{2 + 3x^2 + x^4} dx &= \frac{1}{3}x(10 + 3x^2) \sqrt{2 + 3x^2 + x^4} + \frac{1}{15} \int \frac{110 + 75x^2}{\sqrt{2 + 3x^2 + x^4}} dx \\ &= \frac{1}{3}x(10 + 3x^2) \sqrt{2 + 3x^2 + x^4} + 5 \int \frac{x^2}{\sqrt{2 + 3x^2 + x^4}} dx + \frac{22}{3} \int \frac{1}{\sqrt{2 + 3x^2 + x^4}} dx \\ &= \frac{5x(2 + x^2)}{\sqrt{2 + 3x^2 + x^4}} + \frac{1}{3}x(10 + 3x^2) \sqrt{2 + 3x^2 + x^4} - \frac{5\sqrt{2}(1 + x^2) \sqrt{\frac{2+x^2}{1+x^2}} E\left(\tan^{-1}(x) \middle| \frac{1}{2}\right)}{\sqrt{2 + 3x^2 + x^4}} + \end{aligned}$$

Mathematica [C] time = 0.0621387, size = 109, normalized size = 0.73

$$\frac{-7i\sqrt{x^2+1}\sqrt{x^2+2}\text{EllipticF}\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right), 2\right) + 3x^7 + 19x^5 + 36x^3 - 15i\sqrt{x^2+1}\sqrt{x^2+2}E\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right) \middle| 2\right) + 20x}{3\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

[In] Integrate[(7 + 5*x^2)*Sqrt[2 + 3*x^2 + x^4], x]

```
[Out] (20*x + 36*x^3 + 19*x^5 + 3*x^7 - (15*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticE[I*ArcSinh[x/Sqrt[2]], 2] - (7*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticF[I*ArcSinh[x/Sqrt[2]], 2])/(3*Sqrt[2 + 3*x^2 + x^4])
```

Maple [C] time = 0.007, size = 137, normalized size = 0.9

$$x^3\sqrt{x^4+3x^2+2} + \frac{10x}{3}\sqrt{x^4+3x^2+2} - \frac{11i}{3}\sqrt{2}\text{EllipticF}\left(\frac{i}{2}x\sqrt{2}, \sqrt{2}\right)\sqrt{2x^2+4}\sqrt{x^2+1} - \frac{1}{\sqrt{x^4+3x^2+2}} + \frac{5i}{2}\sqrt{2}\left(\text{EllipticE}\left(\frac{i}{2}x\sqrt{2}, \sqrt{2}\right) - \text{EllipticF}\left(\frac{i}{2}x\sqrt{2}, \sqrt{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((5*x^2+7)*(x^4+3*x^2+2)^(1/2), x)

```
[Out] x^3*(x^4+3*x^2+2)^(1/2)+10/3*x*(x^4+3*x^2+2)^(1/2)-11/3*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticF(1/2*I*x*2^(1/2), 2^(1/2))+5/2*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*(EllipticF(1/2*I*x*2^(1/2), 2^(1/2))-EllipticE(1/2*I*x*2^(1/2), 2^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{x^4 + 3x^2 + 2}(5x^2 + 7) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^2+7)*(x^4+3*x^2+2)^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(x^4 + 3*x^2 + 2)*(5*x^2 + 7), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{x^4 + 3x^2 + 2}(5x^2 + 7), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^2+7)*(x^4+3*x^2+2)^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(x^4 + 3*x^2 + 2)*(5*x^2 + 7), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{(x^2 + 1)(x^2 + 2)}(5x^2 + 7) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x**2+7)*(x**4+3*x**2+2)**(1/2),x)

[Out] Integral(sqrt((x**2 + 1)*(x**2 + 2))*(5*x**2 + 7), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{x^4 + 3x^2 + 2}(5x^2 + 7) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^2+7)*(x^4+3*x^2+2)^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(x^4 + 3*x^2 + 2)*(5*x^2 + 7), x)

3.289 $\int \sqrt{2 + 3x^2 + x^4} dx$

Optimal. Leaf size=141

$$\frac{2\sqrt{2}(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\text{EllipticF}\left(\tan^{-1}(x), \frac{1}{2}\right)}{3\sqrt{x^4+3x^2+2}} + \frac{1}{3}\sqrt{x^4+3x^2+2}x + \frac{(x^2+2)x}{\sqrt{x^4+3x^2+2}} - \frac{\sqrt{2}(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}E\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{\sqrt{x^4+3x^2+2}}$$

[Out] (x*(2 + x^2))/Sqrt[2 + 3*x^2 + x^4] + (x*Sqrt[2 + 3*x^2 + x^4])/3 - (Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x], 1/2])/Sqrt[2 + 3*x^2 + x^4] + (2*Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(3*Sqrt[2 + 3*x^2 + x^4])

Rubi [A] time = 0.0423545, antiderivative size = 141, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {1091, 1189, 1099, 1135}

$$\frac{1}{3}\sqrt{x^4+3x^2+2}x + \frac{(x^2+2)x}{\sqrt{x^4+3x^2+2}} + \frac{2\sqrt{2}(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}F\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{3\sqrt{x^4+3x^2+2}} - \frac{\sqrt{2}(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}E\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[2 + 3*x^2 + x^4], x]

[Out] (x*(2 + x^2))/Sqrt[2 + 3*x^2 + x^4] + (x*Sqrt[2 + 3*x^2 + x^4])/3 - (Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x], 1/2])/Sqrt[2 + 3*x^2 + x^4] + (2*Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(3*Sqrt[2 + 3*x^2 + x^4])

Rule 1091

Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := Simp[(x*(a + b*x^2 + c*x^4)^p)/(4*p + 1), x] + Dist[(2*p)/(4*p + 1), Int[(2*a + b*x^2)*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[2*p]

Rule 1189

Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a] || PosQ[(b - q)/a] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]

Rule 1099

Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[((2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q)*x^2)/(2*a + (b + q)*x^2)]*EllipticF[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)/(b + q)])/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] && !(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)]) /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]

Rule 1135

Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(x*(b + q + 2*c*x^2))/(2*c*Sqrt[a + b*x^2 + c*x^4]), x]

4]), x] - Simp[(Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q)*x^2)/(2*a + (b + q)*x^2)]*EllipticE[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)/(b + q)])/(2*c*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] && !(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)]]) /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]

Rubi steps

$$\begin{aligned}\int \sqrt{2 + 3x^2 + x^4} dx &= \frac{1}{3}x\sqrt{2 + 3x^2 + x^4} + \frac{1}{3} \int \frac{4 + 3x^2}{\sqrt{2 + 3x^2 + x^4}} dx \\ &= \frac{1}{3}x\sqrt{2 + 3x^2 + x^4} + \frac{4}{3} \int \frac{1}{\sqrt{2 + 3x^2 + x^4}} dx + \int \frac{x^2}{\sqrt{2 + 3x^2 + x^4}} dx \\ &= \frac{x(2 + x^2)}{\sqrt{2 + 3x^2 + x^4}} + \frac{1}{3}x\sqrt{2 + 3x^2 + x^4} - \frac{\sqrt{2}(1 + x^2)\sqrt{\frac{2+x^2}{1+x^2}}E\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{\sqrt{2 + 3x^2 + x^4}} + \frac{2\sqrt{2}(1 + x^2)\sqrt{\frac{2+x^2}{1+x^2}}}{3\sqrt{2 + 3x^2 + x^4}}\end{aligned}$$

Mathematica [C] time = 0.0343925, size = 102, normalized size = 0.72

$$\frac{-i\sqrt{x^2 + 1}\sqrt{x^2 + 2}\text{EllipticF}\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right), 2\right) + x^5 + 3x^3 - 3i\sqrt{x^2 + 1}\sqrt{x^2 + 2}E\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right) + 2x}{3\sqrt{x^4 + 3x^2 + 2}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[2 + 3*x^2 + x^4], x]

[Out] (2*x + 3*x^3 + x^5 - (3*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticE[I*ArcSinh[x/Sqrt[2]], 2] - I*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticF[I*ArcSinh[x/Sqrt[2]], 2])/(3*Sqrt[2 + 3*x^2 + x^4])

Maple [C] time = 0.004, size = 121, normalized size = 0.9

$$\frac{x}{3}\sqrt{x^4 + 3x^2 + 2} - \frac{2i}{3}\sqrt{2}\text{EllipticF}\left(\frac{i}{2}x\sqrt{2}, \sqrt{2}\right)\sqrt{2x^2 + 4}\sqrt{x^2 + 1} - \frac{1}{\sqrt{x^4 + 3x^2 + 2}} + \frac{i}{2}\sqrt{2}\left(\text{EllipticF}\left(\frac{i}{2}x\sqrt{2}, \sqrt{2}\right) - E\left(\frac{i}{2}x\sqrt{2}, \sqrt{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^4+3*x^2+2)^(1/2), x)

[Out] 1/3*x*(x^4+3*x^2+2)^(1/2)-2/3*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticF(1/2*I*x*2^(1/2), 2^(1/2))+1/2*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*(EllipticF(1/2*I*x*2^(1/2), 2^(1/2))-EllipticE(1/2*I*x*2^(1/2), 2^(1/2)))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{x^4 + 3x^2 + 2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+3*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^4 + 3*x^2 + 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{x^4 + 3x^2 + 2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+3*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^4 + 3*x^2 + 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{x^4 + 3x^2 + 2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+3*x**2+2)**(1/2),x)
```

```
[Out] Integral(sqrt(x**4 + 3*x**2 + 2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{x^4 + 3x^2 + 2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+3*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^4 + 3*x^2 + 2), x)
```

$$3.290 \quad \int \frac{\sqrt{2+3x^2+x^4}}{7+5x^2} dx$$

Optimal. Leaf size=178

$$\frac{(x^2+1)\sqrt{\frac{x^2+2}{2x^2+2}}\text{EllipticF}\left(\tan^{-1}(x), \frac{1}{2}\right)}{5\sqrt{x^4+3x^2+2}} + \frac{x(x^2+2)}{5\sqrt{x^4+3x^2+2}} - \frac{\sqrt{2}(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}E\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{5\sqrt{x^4+3x^2+2}} + \frac{3(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\Pi}{35\sqrt{2}\sqrt{x^4+3x^2+2}}$$

[Out] (x*(2 + x^2))/(5*Sqrt[2 + 3*x^2 + x^4]) - (Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x], 1/2])/(5*Sqrt[2 + 3*x^2 + x^4]) + ((1 + x^2)*Sqrt[(2 + x^2)/(2 + 2*x^2)]*EllipticF[ArcTan[x], 1/2])/(5*Sqrt[2 + 3*x^2 + x^4]) + (3*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticPi[2/7, ArcTan[x], 1/2])/(35*Sqrt[2]*Sqrt[2 + 3*x^2 + x^4])

Rubi [A] time = 0.123788, antiderivative size = 232, normalized size of antiderivative = 1.3, number of steps used = 8, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {1208, 1189, 1099, 1135, 1214, 1456, 539}

$$\frac{x(x^2+2)}{5\sqrt{x^4+3x^2+2}} + \frac{4\sqrt{2}(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}F\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{25\sqrt{x^4+3x^2+2}} - \frac{3(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}F\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{25\sqrt{2}\sqrt{x^4+3x^2+2}} - \frac{\sqrt{2}(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}E\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{5\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[2 + 3*x^2 + x^4]/(7 + 5*x^2), x]

[Out] (x*(2 + x^2))/(5*Sqrt[2 + 3*x^2 + x^4]) - (Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x], 1/2])/(5*Sqrt[2 + 3*x^2 + x^4]) - (3*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(25*Sqrt[2]*Sqrt[2 + 3*x^2 + x^4]) + (4*Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(25*Sqrt[2 + 3*x^2 + x^4]) + (3*(2 + x^2)*EllipticPi[2/7, ArcTan[x], 1/2])/(35*Sqrt[2]*Sqrt[(2 + x^2)/(1 + x^2)]*Sqrt[2 + 3*x^2 + x^4])

Rule 1208

Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_)/((d_) + (e_.)*(x_)^2), x_Symbol] := -Dist[(e^2)^(-1), Int[(c*d - b*e - c*e*x^2)*(a + b*x^2 + c*x^4)^(p-1), x], x] + Dist[(c*d^2 - b*d*e + a*e^2)/e^2, Int[(a + b*x^2 + c*x^4)^(p-1)/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p + 1/2, 0]

Rule 1189

Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a] || PosQ[(b - q)/a] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]

Rule 1099

Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[((2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q)*x^2)/(2*a + (b + q)*x^2)]*EllipticF[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)/(b + q)])/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] && !(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[

{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]

Rule 1135

Int[(x_)^2/Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4], x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(x*(b + q + 2*c*x^2))/(2*c*Sqrt[a + b*x^2 + c*x^4]), x] - Simp[(Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q)*x^2)/(2*a + (b + q)*x^2)]*EllipticE[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)/(b + q))]/(2*c*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] && !(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]

Rule 1214

Int[1/(((d_) + (e_)*(x_)^2)*Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4]), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/(2*c*d - e*(b - q)), Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[e/(2*c*d - e*(b - q)), Int[(b - q + 2*c*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0] && !LtQ[c, 0]

Rule 1456

Int[((d_) + (e_)*(x_)^(n_))^(q_)*((f_) + (g_)*(x_)^(n_))^(r_)*((a_) + (b_)*(x_)^(n_) + (c_)*(x_)^(n2_))^(p_), x_Symbol] := Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d + (c*x^n)/e)^FracPart[p]), Int[(d + e*x^n)^(p + q)*(f + g*x^n)^r*(a/d + (c*x^n)/e)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q, r}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && !IntegerQ[p]

Rule 539

Int[Sqrt[(c_) + (d_)*(x_)^2]/(((a_) + (b_)*(x_)^2)*Sqrt[(e_) + (f_)*(x_)^2]), x_Symbol] := Simp[(c*Sqrt[e + f*x^2]*EllipticPi[1 - (b*c)/(a*d), ArcTan[Rt[d/c, 2]*x], 1 - (c*f)/(d*e)]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(e + f*x^2))/(e*(c + d*x^2))]), x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[d/c]

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{2+3x^2+x^4}}{7+5x^2} dx &= -\left(\frac{1}{25} \int \frac{-8-5x^2}{\sqrt{2+3x^2+x^4}} dx\right) - \frac{6}{25} \int \frac{1}{(7+5x^2)\sqrt{2+3x^2+x^4}} dx \\ &= -\left(\frac{3}{25} \int \frac{1}{\sqrt{2+3x^2+x^4}} dx\right) + \frac{1}{5} \int \frac{x^2}{\sqrt{2+3x^2+x^4}} dx + \frac{3}{10} \int \frac{2+2x^2}{(7+5x^2)\sqrt{2+3x^2+x^4}} dx + \frac{8}{25} \\ &= \frac{x(2+x^2)}{5\sqrt{2+3x^2+x^4}} - \frac{\sqrt{2}(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}E\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{5\sqrt{2+3x^2+x^4}} - \frac{3(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}F\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{25\sqrt{2}\sqrt{2+3x^2+x^4}} + \frac{4\sqrt{2}(1+x^2)}{25\sqrt{2+3x^2+x^4}} \\ &= \frac{x(2+x^2)}{5\sqrt{2+3x^2+x^4}} - \frac{\sqrt{2}(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}E\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{5\sqrt{2+3x^2+x^4}} - \frac{3(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}F\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{25\sqrt{2}\sqrt{2+3x^2+x^4}} + \frac{4\sqrt{2}(1+x^2)}{25\sqrt{2+3x^2+x^4}} \end{aligned}$$

Mathematica [C] time = 0.141391, size = 90, normalized size = 0.51

$$\frac{i\sqrt{x^2+1}\sqrt{x^2+2}\left(21\text{EllipticF}\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right), 2\right) + 35E\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right) - 6\Pi\left(\frac{10}{7}; i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)\right)}{175\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[2 + 3*x^2 + x^4]/(7 + 5*x^2),x]

[Out] ((-I/175)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*(35*EllipticE[I*ArcSinh[x/Sqrt[2]]], 2] + 21*EllipticF[I*ArcSinh[x/Sqrt[2]], 2] - 6*EllipticPi[10/7, I*ArcSinh[x/Sqrt[2]], 2])/Sqrt[2 + 3*x^2 + x^4]

Maple [C] time = 0.026, size = 138, normalized size = 0.8

$$-\frac{3i}{50}\sqrt{2}\text{EllipticF}\left(\frac{i}{2}x\sqrt{2}, \sqrt{2}\right)\sqrt{2x^2+4}\sqrt{x^2+1}\frac{1}{\sqrt{x^4+3x^2+2}} - \frac{i}{10}\sqrt{2}\text{EllipticE}\left(\frac{i}{2}x\sqrt{2}, \sqrt{2}\right)\sqrt{2x^2+4}\sqrt{x^2+1}\frac{1}{\sqrt{x^4+3x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^4+3*x^2+2)^(1/2)/(5*x^2+7),x)

[Out] -3/50*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticF(1/2*I*x*2^(1/2),2^(1/2))-1/10*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticE(1/2*I*x*2^(1/2),2^(1/2))+6/175*I*2^(1/2)*(1+1/2*x^2)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticPi(1/2*I*x*2^(1/2),10/7,2^(1/2))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^4 + 3x^2 + 2}}{5x^2 + 7} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4+3*x^2+2)^(1/2)/(5*x^2+7),x, algorithm="maxima")

[Out] integrate(sqrt(x^4 + 3*x^2 + 2)/(5*x^2 + 7), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{x^4 + 3x^2 + 2}}{5x^2 + 7}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4+3*x^2+2)^(1/2)/(5*x^2+7),x, algorithm="fricas")

[Out] integral(sqrt(x^4 + 3*x^2 + 2)/(5*x^2 + 7), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{(x^2 + 1)(x^2 + 2)}}{5x^2 + 7} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**4+3*x**2+2)**(1/2)/(5*x**2+7),x)

[Out] Integral(sqrt((x**2 + 1)*(x**2 + 2))/(5*x**2 + 7), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^4 + 3x^2 + 2}}{5x^2 + 7} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4+3*x^2+2)^(1/2)/(5*x^2+7),x, algorithm="giac")

[Out] integrate(sqrt(x^4 + 3*x^2 + 2)/(5*x^2 + 7), x)

$$3.291 \quad \int \frac{\sqrt{2+3x^2+x^4}}{(7+5x^2)^2} dx$$

Optimal. Leaf size=209

$$\frac{3(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\text{EllipticF}\left(\tan^{-1}(x), \frac{1}{2}\right)}{140\sqrt{2}\sqrt{x^4+3x^2+2}} + \frac{\sqrt{x^4+3x^2+2}x}{14(5x^2+7)} - \frac{(x^2+2)x}{70\sqrt{x^4+3x^2+2}} + \frac{(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}E\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{35\sqrt{2}\sqrt{x^4+3x^2+2}} - \frac{(x^2+2)\sqrt{\frac{x^2+2}{x^2+1}}E\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{90\sqrt{2}\sqrt{x^4+3x^2+2}}$$

```
[Out] -(x*(2 + x^2))/(70*Sqrt[2 + 3*x^2 + x^4]) + (x*Sqrt[2 + 3*x^2 + x^4])/(14*(7 + 5*x^2)) + ((1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x], 1/2])/
(35*Sqrt[2]*Sqrt[2 + 3*x^2 + x^4]) + (3*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(140*Sqrt[2]*Sqrt[2 + 3*x^2 + x^4]) - ((2 + x^2)*EllipticPi[2/7, ArcTan[x], 1/2])/(980*Sqrt[2]*Sqrt[(2 + x^2)/(1 + x^2)]*Sqrt[2 + 3*x^2 + x^4])
```

Rubi [A] time = 0.123326, antiderivative size = 209, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {1226, 1189, 1099, 1135, 1214, 1456, 539}

$$\frac{\sqrt{x^4+3x^2+2}x}{14(5x^2+7)} - \frac{(x^2+2)x}{70\sqrt{x^4+3x^2+2}} + \frac{3(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}F\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{140\sqrt{2}\sqrt{x^4+3x^2+2}} + \frac{(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}E\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{35\sqrt{2}\sqrt{x^4+3x^2+2}} - \frac{(x^2+2)\sqrt{\frac{x^2+2}{x^2+1}}E\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{980\sqrt{2}\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 + 3*x^2 + x^4]/(7 + 5*x^2)^2, x]
```

```
[Out] -(x*(2 + x^2))/(70*Sqrt[2 + 3*x^2 + x^4]) + (x*Sqrt[2 + 3*x^2 + x^4])/(14*(7 + 5*x^2)) + ((1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x], 1/2])/
(35*Sqrt[2]*Sqrt[2 + 3*x^2 + x^4]) + (3*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(140*Sqrt[2]*Sqrt[2 + 3*x^2 + x^4]) - ((2 + x^2)*EllipticPi[2/7, ArcTan[x], 1/2])/(980*Sqrt[2]*Sqrt[(2 + x^2)/(1 + x^2)]*Sqrt[2 + 3*x^2 + x^4])
```

Rule 1226

```
Int[Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]/((d_) + (e_.)*(x_)^2)^2, x_Symbol]
:> Simp[(x*Sqrt[a + b*x^2 + c*x^4])/(2*d*(d + e*x^2)), x] + (Dist[c/(2*d*e^2), Int[(d - e*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[(c*d^2 - a*e^2)/(2*d*e^2), Int[1/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]) /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0]
```

Rule 1189

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol]
:> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a] || PosQ[(b - q)/a] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1099

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol]
:> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[((2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q)*x^2)/(2*a +
```

$(b + q)x^2] * \text{EllipticF}[\text{ArcTan}[\text{Rt}[(b + q)/(2a), 2]x], (2q)/(b + q)] / (2a * \text{Rt}[(b + q)/(2a), 2] * \text{Sqrt}[a + bx^2 + cx^4]), x] /; \text{PosQ}[(b + q)/a] \&\& \text{!(PosQ}[(b - q)/a] \&\& \text{SimplerSqrtQ}[(b - q)/(2a), (b + q)/(2a)])] /; \text{FreeQ}[\{a, b, c\}, x] \&\& \text{GtQ}[b^2 - 4ac, 0]$

Rule 1135

$\text{Int}[(x)^2 / \text{Sqrt}[(a) + (b) * (x)^2 + (c) * (x)^4], x_Symbol] :> \text{With}[\{q = \text{Rt}[b^2 - 4ac, 2]\}, \text{Simp}[(x * (b + q + 2cx^2)) / (2c * \text{Sqrt}[a + bx^2 + cx^4]), x] - \text{Simp}[(\text{Rt}[(b + q)/(2a), 2] * (2a + (b + q)x^2) * \text{Sqrt}[(2a + (b - q)x^2]) / (2a + (b + q)x^2)] * \text{EllipticE}[\text{ArcTan}[\text{Rt}[(b + q)/(2a), 2]x], (2q)/(b + q)]) / (2c * \text{Sqrt}[a + bx^2 + cx^4]), x] /; \text{PosQ}[(b + q)/a] \&\& \text{!(PosQ}[(b - q)/a] \&\& \text{SimplerSqrtQ}[(b - q)/(2a), (b + q)/(2a)])] /; \text{FreeQ}[\{a, b, c\}, x] \&\& \text{GtQ}[b^2 - 4ac, 0]$

Rule 1214

$\text{Int}[1 / (((d) + (e) * (x)^2) * \text{Sqrt}[(a) + (b) * (x)^2 + (c) * (x)^4]), x_Symbol] :> \text{With}[\{q = \text{Rt}[b^2 - 4ac, 2]\}, \text{Dist}[(2c) / (2cd - e(b - q)), \text{Int}[1 / \text{Sqrt}[a + bx^2 + cx^4], x], x] - \text{Dist}[e / (2cd - e(b - q)), \text{Int}[(b - q + 2cx^2) / ((d + ex^2) * \text{Sqrt}[a + bx^2 + cx^4]), x], x]] /; \text{FreeQ}[\{a, b, c, d, e\}, x] \&\& \text{GtQ}[b^2 - 4ac, 0] \&\& \text{!LtQ}[c, 0]$

Rule 1456

$\text{Int}[(d) + (e) * (x)^n]^q * ((f) + (g) * (x)^n)^r * (a) + (b) * (x)^n + (c) * (x)^{2n}]^p, x_Symbol] :> \text{Dist}[(a + bx^n + cx^{2n})^{\text{FracPart}[p]} / ((d + ex^n)^{\text{FracPart}[p]} * (a/d + (cx^n)/e)^{\text{FracPart}[p]}), \text{Int}[(d + ex^n)^{p+q} * (f + gx^n)^r * (a/d + (cx^n)/e)^p, x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, g, n, p, q, r\}, x] \&\& \text{EqQ}[n^2, 2n] \&\& \text{NeQ}[b^2 - 4ac, 0] \&\& \text{EqQ}[c*d^2 - b*d*e + a*e^2, 0] \&\& \text{!IntegerQ}[p]$

Rule 539

$\text{Int}[\text{Sqrt}[(c) + (d) * (x)^2] / (((a) + (b) * (x)^2) * \text{Sqrt}[(e) + (f) * (x)^2]), x_Symbol] :> \text{Simp}[(c * \text{Sqrt}[e + fx^2] * \text{EllipticPi}[1 - (b*c)/(a*d), \text{ArcTan}[\text{Rt}[d/c, 2]x], 1 - (c*f)/(d*e)]) / (a * \text{Rt}[d/c, 2] * \text{Sqrt}[c + dx^2] * \text{Sqrt}[(c * (e + fx^2)) / (e * (c + dx^2))]), x] /; \text{FreeQ}[\{a, b, c, d, e, f\}, x] \&\& \text{PosQ}[d/c]$

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{2+3x^2+x^4}}{(7+5x^2)^2} dx &= \frac{x\sqrt{2+3x^2+x^4}}{14(7+5x^2)} + \frac{1}{350} \int \frac{7-5x^2}{\sqrt{2+3x^2+x^4}} dx + \frac{1}{350} \int \frac{1}{(7+5x^2)\sqrt{2+3x^2+x^4}} dx \\ &= \frac{x\sqrt{2+3x^2+x^4}}{14(7+5x^2)} + \frac{1}{700} \int \frac{1}{\sqrt{2+3x^2+x^4}} dx - \frac{1}{280} \int \frac{2+2x^2}{(7+5x^2)\sqrt{2+3x^2+x^4}} dx - \frac{1}{70} \int \frac{1}{\sqrt{2+3x^2+x^4}} dx \\ &= -\frac{x(2+x^2)}{70\sqrt{2+3x^2+x^4}} + \frac{x\sqrt{2+3x^2+x^4}}{14(7+5x^2)} + \frac{(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}} E\left(\tan^{-1}(x) \middle| \frac{1}{2}\right)}{35\sqrt{2}\sqrt{2+3x^2+x^4}} + \frac{3(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}} F\left(\tan^{-1}(x) \middle| \frac{1}{2}\right)}{140\sqrt{2}\sqrt{2+3x^2+x^4}} \\ &= -\frac{x(2+x^2)}{70\sqrt{2+3x^2+x^4}} + \frac{x\sqrt{2+3x^2+x^4}}{14(7+5x^2)} + \frac{(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}} E\left(\tan^{-1}(x) \middle| \frac{1}{2}\right)}{35\sqrt{2}\sqrt{2+3x^2+x^4}} + \frac{3(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}} F\left(\tan^{-1}(x) \middle| \frac{1}{2}\right)}{140\sqrt{2}\sqrt{2+3x^2+x^4}} \end{aligned}$$

Mathematica [C] time = 0.268229, size = 208, normalized size = 1.

$$\frac{-84i\sqrt{x^2+1}\sqrt{x^2+2}(5x^2+7)\operatorname{EllipticF}\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right),2\right)+175x^5+525x^3+35i\sqrt{x^2+1}\sqrt{x^2+2}(5x^2+7)E\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\right)}{2450(5x^2+7)\sqrt{x}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[2 + 3*x^2 + x^4]/(7 + 5*x^2)^2,x]

[Out] (350*x + 525*x^3 + 175*x^5 + (35*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*(7 + 5*x^2)*EllipticE[I*ArcSinh[x/Sqrt[2]], 2] - (84*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*(7 + 5*x^2)*EllipticF[I*ArcSinh[x/Sqrt[2]], 2] - (7*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticPi[10/7, I*ArcSinh[x/Sqrt[2]], 2] - (5*I)*x^2*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticPi[10/7, I*ArcSinh[x/Sqrt[2]], 2])/(2450*(7 + 5*x^2)*Sqrt[2 + 3*x^2 + x^4])

Maple [C] time = 0.02, size = 162, normalized size = 0.8

$$\frac{x}{70x^2+98}\sqrt{x^4+3x^2+2}-\frac{3i}{175}\sqrt{2}\operatorname{EllipticF}\left(\frac{i}{2}x\sqrt{2},\sqrt{2}\right)\sqrt{2x^2+4}\sqrt{x^2+1}\frac{1}{\sqrt{x^4+3x^2+2}}+\frac{i}{140}\sqrt{2}\operatorname{EllipticE}\left(\frac{i}{2}x\sqrt{2},\sqrt{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^4+3*x^2+2)^(1/2)/(5*x^2+7)^2,x)

[Out] 1/14*x*(x^4+3*x^2+2)^(1/2)/(5*x^2+7)-3/175*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticF(1/2*I*x*2^(1/2),2^(1/2))+1/140*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticE(1/2*I*x*2^(1/2),2^(1/2))-1/2450*I*2^(1/2)*(1+1/2*x^2)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticPi(1/2*I*x*2^(1/2),10/7,2^(1/2))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^4+3x^2+2}}{(5x^2+7)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4+3*x^2+2)^(1/2)/(5*x^2+7)^2,x, algorithm="maxima")

[Out] integrate(sqrt(x^4 + 3*x^2 + 2)/(5*x^2 + 7)^2, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\sqrt{x^4+3x^2+2}}{25x^4+70x^2+49},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4+3*x^2+2)^(1/2)/(5*x^2+7)^2,x, algorithm="fricas")

[Out] `integral(sqrt(x^4 + 3*x^2 + 2)/(25*x^4 + 70*x^2 + 49), x)`

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{(x^2 + 1)(x^2 + 2)}}{(5x^2 + 7)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x**4+3*x**2+2)**(1/2)/(5*x**2+7)**2,x)`

[Out] `Integral(sqrt((x**2 + 1)*(x**2 + 2))/(5*x**2 + 7)**2, x)`

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^4 + 3x^2 + 2}}{(5x^2 + 7)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^4+3*x^2+2)^(1/2)/(5*x^2+7)^2,x, algorithm="giac")`

[Out] `integrate(sqrt(x^4 + 3*x^2 + 2)/(5*x^2 + 7)^2, x)`

$$3.292 \quad \int \frac{\sqrt{2+3x^2+x^4}}{(7+5x^2)^3} dx$$

Optimal. Leaf size=237

$$\frac{81(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\text{EllipticF}\left(\tan^{-1}(x), \frac{1}{2}\right)}{7840\sqrt{2}\sqrt{x^4+3x^2+2}} + \frac{11\sqrt{x^4+3x^2+2}x}{2352(5x^2+7)} + \frac{\sqrt{x^4+3x^2+2}}{28(5x^2+7)^2} - \frac{11(x^2+2)x}{11760\sqrt{x^4+3x^2+2}} + \frac{11(x^2+1)}{5880\sqrt{2}\sqrt{x^4+3x^2+2}}$$

```
[Out] (-11*x*(2 + x^2))/(11760*Sqrt[2 + 3*x^2 + x^4]) + (x*Sqrt[2 + 3*x^2 + x^4])
/(28*(7 + 5*x^2)^2) + (11*x*Sqrt[2 + 3*x^2 + x^4])/(2352*(7 + 5*x^2)) + (11
*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x], 1/2])/(5880*Sqrt[
2]*Sqrt[2 + 3*x^2 + x^4]) + (81*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*Ellipti
cF[ArcTan[x], 1/2])/(7840*Sqrt[2]*Sqrt[2 + 3*x^2 + x^4]) - (1201*(2 + x^2)*
EllipticPi[2/7, ArcTan[x], 1/2])/(164640*Sqrt[2]*Sqrt[(2 + x^2)/(1 + x^2)]*
Sqrt[2 + 3*x^2 + x^4])
```

Rubi [A] time = 0.595241, antiderivative size = 237, normalized size of antiderivative = 1., number of steps used = 25, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {1228, 1223, 1696, 1716, 1189, 1099, 1135, 1214, 1456, 539}

$$\frac{11\sqrt{x^4+3x^2+2}x}{2352(5x^2+7)} + \frac{\sqrt{x^4+3x^2+2}x}{28(5x^2+7)^2} - \frac{11(x^2+2)x}{11760\sqrt{x^4+3x^2+2}} + \frac{81(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}F\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{7840\sqrt{2}\sqrt{x^4+3x^2+2}} + \frac{11(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}}{5880\sqrt{2}\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 + 3*x^2 + x^4]/(7 + 5*x^2)^3, x]
```

```
[Out] (-11*x*(2 + x^2))/(11760*Sqrt[2 + 3*x^2 + x^4]) + (x*Sqrt[2 + 3*x^2 + x^4])
/(28*(7 + 5*x^2)^2) + (11*x*Sqrt[2 + 3*x^2 + x^4])/(2352*(7 + 5*x^2)) + (11
*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x], 1/2])/(5880*Sqrt[
2]*Sqrt[2 + 3*x^2 + x^4]) + (81*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*Ellipti
cF[ArcTan[x], 1/2])/(7840*Sqrt[2]*Sqrt[2 + 3*x^2 + x^4]) - (1201*(2 + x^2)*
EllipticPi[2/7, ArcTan[x], 1/2])/(164640*Sqrt[2]*Sqrt[(2 + x^2)/(1 + x^2)]*
Sqrt[2 + 3*x^2 + x^4])
```

Rule 1228

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Module[{aa, bb, cc}, Int[ExpandIntegrand[1/Sqrt[aa + bb*x^2 + c
c*x^4], (d + e*x^2)^q*(aa + bb*x^2 + cc*x^4)^(p + 1/2), x] /. {aa -> a, bb
-> b, cc -> c}, x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, 0] && IntegerQ[p + 1/2]
```

Rule 1223

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x
_Symbol] :> -Simp[(e^2*x*(d + e*x^2)^(q + 1)*Sqrt[a + b*x^2 + c*x^4])/(2*d*(
q + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(2*d*(q + 1)*(c*d^2 - b*d*e +
a*e^2)), Int[((d + e*x^2)^(q + 1)*Simp[a*e^2*(2*q + 3) + 2*d*(c*d - b*e)*(q
+ 1) - 2*e*(c*d*(q + 1) - b*e*(q + 2))*x^2 + c*e^2*(2*q + 5)*x^4, x])/Sqrt
[a + b*x^2 + c*x^4], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c
, 0] && ILtQ[q, -1]
```

Rule 1696

```
Int[((P4x_)*((d_) + (e_)*(x_)^2)^(q_))/Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4], x_Symbol] := With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[P4x, x, 4]}, -Simp[((C*d^2 - B*d*e + A*e^2)*x*(d + e*x^2)^(q + 1)*Sqrt[a + b*x^2 + c*x^4]/(2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2)), Int[((d + e*x^2)^(q + 1)*Simp[a*d*(C*d - B*e) + A*(a*e^2*(2*q + 3) + 2*d*(c*d - b*e)*(q + 1)) - 2*((B*d - A*e)*(b*e*(q + 2) - c*d*(q + 1)) - C*d*(b*d + a*e*(q + 1)))*x^2 + c*(C*d^2 - B*d*e + A*e^2)*(2*q + 5)*x^4, x])/Sqrt[a + b*x^2 + c*x^4], x], x] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[P4x, x^2] && LeQ[Expon[P4x, x], 4] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, -1]
```

Rule 1716

```
Int[(P4x_)/(((d_) + (e_)*(x_)^2)*Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4]), x_Symbol] := With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[P4x, x, 4]}, -Dist[(e^2)^(-1), Int[(C*d - B*e - C*e*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] + Dist[(C*d^2 - B*d*e + A*e^2)/e^2, Int[1/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[P4x, x^2, 2] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0]
```

Rule 1189

```
Int[((d_) + (e_)*(x_)^2)/Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4], x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a] || PosQ[(b - q)/a] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1099

```
Int[1/Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4], x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[((2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q)*x^2)/(2*a + (b + q)*x^2)]*EllipticF[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)/(b + q)])/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] && !(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)]) /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1135

```
Int[(x_)^2/Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4], x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(x*(b + q + 2*c*x^2))/(2*c*Sqrt[a + b*x^2 + c*x^4]), x] - Simp[(Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q)*x^2)/(2*a + (b + q)*x^2)]*EllipticE[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)/(b + q)])/(2*c*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] && !(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)]) /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1214

```
Int[1/(((d_) + (e_)*(x_)^2)*Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4]), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/(2*c*d - e*(b - q)), Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[e/(2*c*d - e*(b - q)), Int[(b - q + 2*c*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0] && !LtQ[c, 0]
```

Rule 1456

```
Int[((d_) + (e_)*(x_)^(n_))^(q_)*((f_) + (g_)*(x_)^(n_))^(r_)*((a_) + (
```

```
b_.*(x_)^(n_) + (c_.*(x_)^(n2_))^(p_), x_Symbol] := Dist[(a + b*x^n + c*x
^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d + (c*x^n)/e)^FracPart[p])
, Int[(d + e*x^n)^(p + q)*(f + g*x^n)^r*(a/d + (c*x^n)/e)^p, x], x] /; Free
Q[{a, b, c, d, e, f, g, n, p, q, r}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && !IntegerQ[p]
```

Rule 539

```
Int[Sqrt[(c_) + (d_.*(x_)^2)]/(((a_) + (b_.*(x_)^2)*Sqrt[(e_) + (f_.*(x_)
^2)], x_Symbol] := Simp[(c*Sqrt[e + f*x^2]*EllipticPi[1 - (b*c)/(a*d), ArcT
an[Rt[d/c, 2]*x], 1 - (c*f)/(d*e)]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c
*(e + f*x^2))/(e*(c + d*x^2)]), x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

Rubi steps

$$\int \frac{\sqrt{2 + 3x^2 + x^4}}{(7 + 5x^2)^3} dx = \int \left(-\frac{6}{25(7 + 5x^2)^3 \sqrt{2 + 3x^2 + x^4}} + \frac{1}{25(7 + 5x^2)^2 \sqrt{2 + 3x^2 + x^4}} + \frac{1}{25(7 + 5x^2) \sqrt{2 + 3x^2 + x^4}} \right) dx$$

$$= \frac{1}{25} \int \frac{1}{(7 + 5x^2)^2 \sqrt{2 + 3x^2 + x^4}} dx + \frac{1}{25} \int \frac{1}{(7 + 5x^2) \sqrt{2 + 3x^2 + x^4}} dx - \frac{6}{25} \int \frac{1}{(7 + 5x^2)^3 \sqrt{2 + 3x^2 + x^4}} dx$$

$$= \frac{x\sqrt{2 + 3x^2 + x^4}}{28(7 + 5x^2)^2} - \frac{x\sqrt{2 + 3x^2 + x^4}}{84(7 + 5x^2)} + \frac{\int \frac{62 + 70x^2 + 25x^4}{(7 + 5x^2)\sqrt{2 + 3x^2 + x^4}} dx}{2100} - \frac{1}{700} \int \frac{74 - 10x^2 - 25x^4}{(7 + 5x^2)^2 \sqrt{2 + 3x^2 + x^4}} dx$$

$$= \frac{x\sqrt{2 + 3x^2 + x^4}}{28(7 + 5x^2)^2} + \frac{11x\sqrt{2 + 3x^2 + x^4}}{2352(7 + 5x^2)} + \frac{(1 + x^2) \sqrt{\frac{2+x^2}{1+x^2}} F\left(\tan^{-1}(x) \middle| \frac{1}{2}\right)}{50\sqrt{2}\sqrt{2 + 3x^2 + x^4}} - \frac{\int \frac{2838 + 2310x^2 + 975x^4}{(7 + 5x^2)\sqrt{2 + 3x^2 + x^4}} dx}{58800}$$

$$= \frac{x\sqrt{2 + 3x^2 + x^4}}{28(7 + 5x^2)^2} + \frac{11x\sqrt{2 + 3x^2 + x^4}}{2352(7 + 5x^2)} + \frac{(1 + x^2) \sqrt{\frac{2+x^2}{1+x^2}} F\left(\tan^{-1}(x) \middle| \frac{1}{2}\right)}{50\sqrt{2}\sqrt{2 + 3x^2 + x^4}} - \frac{(2 + x^2) \Pi\left(\frac{2}{7}; \tan^{-1}(x) \middle| \frac{1}{2}\right)}{70\sqrt{2}\sqrt{\frac{2+x^2}{1+x^2}}\sqrt{2 + 3x^2 + x^4}}$$

$$= \frac{x(2 + x^2)}{420\sqrt{2 + 3x^2 + x^4}} + \frac{x\sqrt{2 + 3x^2 + x^4}}{28(7 + 5x^2)^2} + \frac{11x\sqrt{2 + 3x^2 + x^4}}{2352(7 + 5x^2)} - \frac{(1 + x^2) \sqrt{\frac{2+x^2}{1+x^2}} E\left(\tan^{-1}(x) \middle| \frac{1}{2}\right)}{210\sqrt{2}\sqrt{2 + 3x^2 + x^4}} + \frac{11(1 + x^2) \sqrt{\frac{2+x^2}{1+x^2}} E\left(\tan^{-1}(x) \middle| \frac{1}{2}\right)}{5880\sqrt{2}\sqrt{2 + 3x^2 + x^4}}$$

$$= -\frac{11x(2 + x^2)}{11760\sqrt{2 + 3x^2 + x^4}} + \frac{x\sqrt{2 + 3x^2 + x^4}}{28(7 + 5x^2)^2} + \frac{11x\sqrt{2 + 3x^2 + x^4}}{2352(7 + 5x^2)} + \frac{11(1 + x^2) \sqrt{\frac{2+x^2}{1+x^2}} E\left(\tan^{-1}(x) \middle| \frac{1}{2}\right)}{5880\sqrt{2}\sqrt{2 + 3x^2 + x^4}}$$

$$= -\frac{11x(2 + x^2)}{11760\sqrt{2 + 3x^2 + x^4}} + \frac{x\sqrt{2 + 3x^2 + x^4}}{28(7 + 5x^2)^2} + \frac{11x\sqrt{2 + 3x^2 + x^4}}{2352(7 + 5x^2)} + \frac{11(1 + x^2) \sqrt{\frac{2+x^2}{1+x^2}} E\left(\tan^{-1}(x) \middle| \frac{1}{2}\right)}{5880\sqrt{2}\sqrt{2 + 3x^2 + x^4}}$$

Mathematica [C] time = 0.341789, size = 174, normalized size = 0.73

$$\frac{-434i\sqrt{x^2 + 1}\sqrt{x^2 + 2}\text{EllipticF}\left(i \sinh^{-1}\left(\frac{x}{\sqrt{2}}\right), 2\right) + \frac{1925x(x^4 + 3x^2 + 2)}{5x^2 + 7} + \frac{14700x(x^4 + 3x^2 + 2)}{(5x^2 + 7)^2} + 385i\sqrt{x^2 + 1}\sqrt{x^2 + 2}E\left(i \sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\right)}{411600\sqrt{x^4 + 3x^2 + 2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2 + 3*x^2 + x^4]/(7 + 5*x^2)^3, x]
```

```
[Out] ((14700*x*(2 + 3*x^2 + x^4))/(7 + 5*x^2)^2 + (1925*x*(2 + 3*x^2 + x^4))/(7 + 5*x^2) + (385*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticE[I*ArcSinh[x/Sqrt[2]], 2] - (434*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticF[I*ArcSinh[x/Sqrt[2]], 2] - (1201*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticPi[10/7, I*ArcSinh[x/Sqrt[2]], 2])/(411600*Sqrt[2 + 3*x^2 + x^4])
```

Maple [C] time = 0.018, size = 186, normalized size = 0.8

$$\frac{x}{28(5x^2+7)^2}\sqrt{x^4+3x^2+2} + \frac{11x}{11760x^2+16464}\sqrt{x^4+3x^2+2} - \frac{31i}{58800}\sqrt{2}\text{EllipticF}\left(\frac{i}{2}x\sqrt{2}, \sqrt{2}\right)\sqrt{2x^2+4}\sqrt{x^2+1} - \frac{1201i}{411600}\sqrt{2}\text{EllipticPi}\left(\frac{10}{7}, I\text{ArcSinh}\left[\frac{x}{\sqrt{2}}\right], 2\right)\sqrt{x^4+3x^2+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+3*x^2+2)^(1/2)/(5*x^2+7)^3,x)
```

```
[Out] 1/28*x*(x^4+3*x^2+2)^(1/2)/(5*x^2+7)^2+11/2352*x*(x^4+3*x^2+2)^(1/2)/(5*x^2+7)-31/58800*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticF(1/2*I*x*2^(1/2),2^(1/2))+11/23520*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticE(1/2*I*x*2^(1/2),2^(1/2))-1201/411600*I*2^(1/2)*(1+1/2*x^2)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticPi(1/2*I*x*2^(1/2),10/7,2^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^4 + 3x^2 + 2}}{(5x^2 + 7)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+3*x^2+2)^(1/2)/(5*x^2+7)^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^4 + 3*x^2 + 2)/(5*x^2 + 7)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{x^4 + 3x^2 + 2}}{125x^6 + 525x^4 + 735x^2 + 343}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+3*x^2+2)^(1/2)/(5*x^2+7)^3,x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^4 + 3*x^2 + 2)/(125*x^6 + 525*x^4 + 735*x^2 + 343), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{(x^2 + 1)(x^2 + 2)}}{(5x^2 + 7)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**4+3*x**2+2)**(1/2)/(5*x**2+7)**3,x)

[Out] Integral(sqrt((x**2 + 1)*(x**2 + 2))/(5*x**2 + 7)**3, x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^4 + 3x^2 + 2}}{(5x^2 + 7)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4+3*x^2+2)^(1/2)/(5*x^2+7)^3,x, algorithm="giac")

[Out] integrate(sqrt(x^4 + 3*x^2 + 2)/(5*x^2 + 7)^3, x)

3.293 $\int (7 + 5x^2)^3 (2 + 3x^2 + x^4)^{3/2} dx$

Optimal. Leaf size=219

$$\frac{1171349\sqrt{2}(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\text{EllipticF}\left(\tan^{-1}(x), \frac{1}{2}\right)}{5005\sqrt{x^4+3x^2+2}} + \frac{125}{13}(x^4+3x^2+2)^{5/2}x^3 + \frac{3825}{143}(x^4+3x^2+2)^{5/2}x + \frac{(65345x^2+208212)(x^4+3x^2+2)^{3/2}}{3003} + \frac{(297911x^2+1032541)}{5005}$$

```
[Out] (20884*x*(2 + x^2))/(65*Sqrt[2 + 3*x^2 + x^4]) + (x*(1032541 + 297911*x^2)*
Sqrt[2 + 3*x^2 + x^4])/5005 + (x*(208212 + 65345*x^2)*(2 + 3*x^2 + x^4)^(3/
2))/3003 + (3825*x*(2 + 3*x^2 + x^4)^(5/2))/143 + (125*x^3*(2 + 3*x^2 + x^4
)^(5/2))/13 - (20884*Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[
ArcTan[x], 1/2])/(65*Sqrt[2 + 3*x^2 + x^4]) + (1171349*Sqrt[2]*(1 + x^2)*Sqr
t[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(5005*Sqrt[2 + 3*x^2 + x
^4])
```

Rubi [A] time = 0.12395, antiderivative size = 219, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$, Rules used = {1206, 1679, 1176, 1189, 1099, 1135}

$$\frac{125}{13}(x^4+3x^2+2)^{5/2}x^3 + \frac{3825}{143}(x^4+3x^2+2)^{5/2}x + \frac{(65345x^2+208212)(x^4+3x^2+2)^{3/2}}{3003} + \frac{(297911x^2+1032541)}{5005}$$

Antiderivative was successfully verified.

```
[In] Int[(7 + 5*x^2)^3*(2 + 3*x^2 + x^4)^(3/2), x]
```

```
[Out] (20884*x*(2 + x^2))/(65*Sqrt[2 + 3*x^2 + x^4]) + (x*(1032541 + 297911*x^2)*
Sqrt[2 + 3*x^2 + x^4])/5005 + (x*(208212 + 65345*x^2)*(2 + 3*x^2 + x^4)^(3/
2))/3003 + (3825*x*(2 + 3*x^2 + x^4)^(5/2))/143 + (125*x^3*(2 + 3*x^2 + x^4
)^(5/2))/13 - (20884*Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[
ArcTan[x], 1/2])/(65*Sqrt[2 + 3*x^2 + x^4]) + (1171349*Sqrt[2]*(1 + x^2)*Sq
rt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(5005*Sqrt[2 + 3*x^2 + x
^4])
```

Rule 1206

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] := Simp[(e^q*x^(2*q - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(4*p + 2*
q + 1)), x] + Dist[1/(c*(4*p + 2*q + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandT
oSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*p
+ 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0] && IGtQ[q, 1]
```

Rule 1679

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := With[{q =
Expon[Pq, x^2], e = Coeff[Pq, x^2, Expon[Pq, x^2]]}, Simp[(e*x^(2*q - 3)*(
a + b*x^2 + c*x^4)^(p + 1))/(c*(2*q + 4*p + 1)), x] + Dist[1/(c*(2*q + 4*p
+ 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*q + 4*p + 1)*Pq - a*e*(2*
q - 3)*x^(2*q - 4) - b*e*(2*q + 2*p - 1)*x^(2*q - 2) - c*e*(2*q + 4*p + 1)*
x^(2*q), x], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && Expon[P
q, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && !LtQ[p, -1]
```

Rule 1176

```
Int[((d_) + (e_)*(x_)^2)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol]
:> Simp[(x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*(a + b*x^2 + c*x^4)^p)/(c*(4*p + 1)*(4*p + 3)), x] + Dist[(2*p)/(c*(4*p + 1)*(4*p + 3)), Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) - b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1189

```
Int[((d_) + (e_)*(x_)^2)/Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4], x_Symbol]
:> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a] || PosQ[(b - q)/a] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1099

```
Int[1/Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4], x_Symbol]
:> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[((2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q)*x^2)/(2*a + (b + q)*x^2)]*EllipticF[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)/(b + q)])/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] && !(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1135

```
Int[(x_)^2/Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4], x_Symbol]
:> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(x*(b + q + 2*c*x^2))/(2*c*Sqrt[a + b*x^2 + c*x^4]), x] - Simp[(Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q)*x^2)/(2*a + (b + q)*x^2)]*EllipticE[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)/(b + q)])/(2*c*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] && !(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned} \int (7 + 5x^2)^3 (2 + 3x^2 + x^4)^{3/2} dx &= \frac{125}{13} x^3 (2 + 3x^2 + x^4)^{5/2} + \frac{1}{13} \int (2 + 3x^2 + x^4)^{3/2} (4459 + 8805x^2 + 3825x^4) dx \\ &= \frac{3825}{143} x (2 + 3x^2 + x^4)^{5/2} + \frac{125}{13} x^3 (2 + 3x^2 + x^4)^{5/2} + \frac{1}{143} \int (41399 + 28005x^2) dx \\ &= \frac{x(208212 + 65345x^2)(2 + 3x^2 + x^4)^{3/2}}{3003} + \frac{3825}{143} x (2 + 3x^2 + x^4)^{5/2} + \frac{125}{13} x^3 (2 + 3x^2 + x^4)^{5/2} \\ &= \frac{x(1032541 + 297911x^2)\sqrt{2 + 3x^2 + x^4}}{5005} + \frac{x(208212 + 65345x^2)(2 + 3x^2 + x^4)^{5/2}}{3003} \\ &= \frac{x(1032541 + 297911x^2)\sqrt{2 + 3x^2 + x^4}}{5005} + \frac{x(208212 + 65345x^2)(2 + 3x^2 + x^4)^{5/2}}{3003} \\ &= \frac{20884x(2 + x^2)}{65\sqrt{2 + 3x^2 + x^4}} + \frac{x(1032541 + 297911x^2)\sqrt{2 + 3x^2 + x^4}}{5005} + \frac{x(208212 + 65345x^2)(2 + 3x^2 + x^4)^{5/2}}{3003} \end{aligned}$$

Mathematica [F] time = 0, size = 0, normalized size = 0.

\$Aborted

Verification is Not applicable to the result.

[In] Integrate[(7 + 5*x^2)^3*(2 + 3*x^2 + x^4)^(3/2),x]

[Out] \$Aborted

Maple [C] time = 0.017, size = 206, normalized size = 0.9

$$\frac{10067363x^3}{15015}\sqrt{x^4+3x^2+2} + \frac{2262081x}{5005}\sqrt{x^4+3x^2+2} + \frac{131810x^7}{429}\sqrt{x^4+3x^2+2} + \frac{598324x^5}{1001}\sqrt{x^4+3x^2+2} + \frac{12079}{143}\sqrt{x^4+3x^2+2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((5*x^2+7)^3*(x^4+3*x^2+2)^(3/2),x)

[Out] 10067363/15015*x^3*(x^4+3*x^2+2)^(1/2)+2262081/5005*x*(x^4+3*x^2+2)^(1/2)+131810/429*x^7*(x^4+3*x^2+2)^(1/2)+598324/1001*x^5*(x^4+3*x^2+2)^(1/2)+12079/143*x^9*(x^4+3*x^2+2)^(1/2)+125/13*x^11*(x^4+3*x^2+2)^(1/2)-1171349/5005*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticF(1/2*I*x*2^(1/2),2^(1/2))+10442/65*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*(EllipticF(1/2*I*x*2^(1/2),2^(1/2))-EllipticE(1/2*I*x*2^(1/2),2^(1/2)))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (x^4 + 3x^2 + 2)^{\frac{3}{2}} (5x^2 + 7)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^2+7)^3*(x^4+3*x^2+2)^(3/2),x, algorithm="maxima")

[Out] integrate((x^4 + 3*x^2 + 2)^(3/2)*(5*x^2 + 7)^3, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(125x^{10} + 900x^8 + 2560x^6 + 3598x^4 + 2499x^2 + 686\right)\sqrt{x^4 + 3x^2 + 2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^2+7)^3*(x^4+3*x^2+2)^(3/2),x, algorithm="fricas")

[Out] integral((125*x^10 + 900*x^8 + 2560*x^6 + 3598*x^4 + 2499*x^2 + 686)*sqrt(x^4 + 3*x^2 + 2), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int ((x^2 + 1)(x^2 + 2))^{\frac{3}{2}} (5x^2 + 7)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x**2+7)**3*(x**4+3*x**2+2)**(3/2),x)

[Out] Integral(((x**2 + 1)*(x**2 + 2))**(3/2)*(5*x**2 + 7)**3, x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (x^4 + 3x^2 + 2)^{\frac{3}{2}} (5x^2 + 7)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^2+7)^3*(x^4+3*x^2+2)^(3/2),x, algorithm="giac")

[Out] integrate((x^4 + 3*x^2 + 2)^(3/2)*(5*x^2 + 7)^3, x)

3.294 $\int (7 + 5x^2)^2 (2 + 3x^2 + x^4)^{3/2} dx$

Optimal. Leaf size=198

$$\frac{13879\sqrt{2}(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\text{EllipticF}\left(\tan^{-1}(x), \frac{1}{2}\right)}{385\sqrt{x^4+3x^2+2}} + \frac{25}{11}x(x^4+3x^2+2)^{5/2} + \frac{1}{693}x(2240x^2+7281)(x^4+3x^2+2)^{3/2} + \dots$$

```
[Out] (742*x*(2 + x^2))/(15*Sqrt[2 + 3*x^2 + x^4]) + (x*(36783 + 10643*x^2)*Sqrt[2 + 3*x^2 + x^4])/1155 + (x*(7281 + 2240*x^2)*(2 + 3*x^2 + x^4)^(3/2))/693 + (25*x*(2 + 3*x^2 + x^4)^(5/2))/11 - (742*Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x], 1/2])/(15*Sqrt[2 + 3*x^2 + x^4]) + (13879*Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(385*Sqrt[2 + 3*x^2 + x^4])
```

Rubi [A] time = 0.0859281, antiderivative size = 198, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {1206, 1176, 1189, 1099, 1135}

$$\frac{25}{11}x(x^4+3x^2+2)^{5/2} + \frac{1}{693}x(2240x^2+7281)(x^4+3x^2+2)^{3/2} + \frac{x(10643x^2+36783)\sqrt{x^4+3x^2+2}}{1155} + \frac{742x(x^2+2)}{15\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Int[(7 + 5*x^2)^2*(2 + 3*x^2 + x^4)^(3/2), x]
```

```
[Out] (742*x*(2 + x^2))/(15*Sqrt[2 + 3*x^2 + x^4]) + (x*(36783 + 10643*x^2)*Sqrt[2 + 3*x^2 + x^4])/1155 + (x*(7281 + 2240*x^2)*(2 + 3*x^2 + x^4)^(3/2))/693 + (25*x*(2 + 3*x^2 + x^4)^(5/2))/11 - (742*Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x], 1/2])/(15*Sqrt[2 + 3*x^2 + x^4]) + (13879*Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(385*Sqrt[2 + 3*x^2 + x^4])
```

Rule 1206

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := Simp[(e^q*x^(2*q - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(4*p + 2*q + 1)), x] + Dist[1/(c*(4*p + 2*q + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*p + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := Simp[(x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*(a + b*x^2 + c*x^4)^p)/(c*(4*p + 1)*(4*p + 3)), x] + Dist[(2*p)/(c*(4*p + 1)*(4*p + 3)), Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) - b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1189

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol]
:= With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a]
] || PosQ[(b - q)/a] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1099

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]},
Simp[((2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q)*x^2)/(2*a + (b + q)*x^2)]*
EllipticF[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)/(b + q)])/(2*a*Rt[(b + q)/(2*a), 2]*
Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] && !(PosQ[(b - q)/a] &&
SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1135

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]},
Simp[(x*(b + q + 2*c*x^2))/(2*c*Sqrt[a + b*x^2 + c*x^4]), x] - Simp[(Rt[(b + q)/(2*a), 2]*
(2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q)*x^2)/(2*a + (b + q)*x^2)]*
EllipticE[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)/(b + q)])/(2*c*Sqrt[a + b*x^2 + c*x^4]), x] /;
PosQ[(b + q)/a] && !(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /;
FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned} \int (7 + 5x^2)^2 (2 + 3x^2 + x^4)^{3/2} dx &= \frac{25}{11}x(2 + 3x^2 + x^4)^{5/2} + \frac{1}{11} \int (489 + 320x^2)(2 + 3x^2 + x^4)^{3/2} dx \\ &= \frac{1}{693}x(7281 + 2240x^2)(2 + 3x^2 + x^4)^{3/2} + \frac{25}{11}x(2 + 3x^2 + x^4)^{5/2} + \frac{1}{231} \int (1568x^2 - 1155) \sqrt{2 + 3x^2 + x^4} dx \\ &= \frac{x(36783 + 10643x^2)\sqrt{2 + 3x^2 + x^4}}{1155} + \frac{1}{693}x(7281 + 2240x^2)(2 + 3x^2 + x^4)^{3/2} - \frac{1155x}{231} \\ &= \frac{x(36783 + 10643x^2)\sqrt{2 + 3x^2 + x^4}}{1155} + \frac{1}{693}x(7281 + 2240x^2)(2 + 3x^2 + x^4)^{3/2} - \frac{5x}{7} \\ &= \frac{742x(2 + x^2)}{15\sqrt{2 + 3x^2 + x^4}} + \frac{x(36783 + 10643x^2)\sqrt{2 + 3x^2 + x^4}}{1155} + \frac{1}{693}x(7281 + 2240x^2)(2 + 3x^2 + x^4)^{3/2} \end{aligned}$$

Mathematica [F] time = 0, size = 0, normalized size = 0.

\$Aborted

Verification is Not applicable to the result.

[In] Integrate[(7 + 5*x^2)^2*(2 + 3*x^2 + x^4)^(3/2), x]

[Out] \$Aborted

Maple [C] time = 0.008, size = 189, normalized size = 1.

$$\frac{25x^9}{11}\sqrt{x^4 + 3x^2 + 2} + \frac{1670x^7}{99}\sqrt{x^4 + 3x^2 + 2} + \frac{11492x^5}{231}\sqrt{x^4 + 3x^2 + 2} + \frac{258044x^3}{3465}\sqrt{x^4 + 3x^2 + 2} + \frac{23851x}{385}\sqrt{x^4 + 3x^2 + 2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((5*x^2+7)^2*(x^4+3*x^2+2)^(3/2),x)`

[Out] $25/11*x^9*(x^4+3*x^2+2)^{(1/2)}+1670/99*x^7*(x^4+3*x^2+2)^{(1/2)}+11492/231*x^5*(x^4+3*x^2+2)^{(1/2)}+258044/3465*x^3*(x^4+3*x^2+2)^{(1/2)}+23851/385*x*(x^4+3*x^2+2)^{(1/2)}+371/15*I*2^{(1/2)}*(2*x^2+4)^{(1/2)}*(x^2+1)^{(1/2)}/(x^4+3*x^2+2)^{(1/2)}*(\text{EllipticF}(1/2*I*x*2^{(1/2)},2^{(1/2)})-\text{EllipticE}(1/2*I*x*2^{(1/2)},2^{(1/2)}))-13879/385*I*2^{(1/2)}*(2*x^2+4)^{(1/2)}*(x^2+1)^{(1/2)}/(x^4+3*x^2+2)^{(1/2)}*\text{EllipticF}(1/2*I*x*2^{(1/2)},2^{(1/2)})$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (x^4 + 3x^2 + 2)^{\frac{3}{2}} (5x^2 + 7)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((5*x^2+7)^2*(x^4+3*x^2+2)^(3/2),x, algorithm="maxima")`

[Out] `integrate((x^4 + 3*x^2 + 2)^(3/2)*(5*x^2 + 7)^2, x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(25x^8 + 145x^6 + 309x^4 + 287x^2 + 98\right)\sqrt{x^4 + 3x^2 + 2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((5*x^2+7)^2*(x^4+3*x^2+2)^(3/2),x, algorithm="fricas")`

[Out] `integral((25*x^8 + 145*x^6 + 309*x^4 + 287*x^2 + 98)*sqrt(x^4 + 3*x^2 + 2), x)`

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int ((x^2 + 1)(x^2 + 2))^{\frac{3}{2}} (5x^2 + 7)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((5*x**2+7)**2*(x**4+3*x**2+2)**(3/2),x)`

[Out] `Integral(((x**2 + 1)*(x**2 + 2))**(3/2)*(5*x**2 + 7)**2, x)`

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (x^4 + 3x^2 + 2)^{\frac{3}{2}} (5x^2 + 7)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.


```
[In] integrate((5*x^2+7)^2*(x^4+3*x^2+2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((x^4 + 3*x^2 + 2)^(3/2)*(5*x^2 + 7)^2, x)
```

3.295 $\int (7 + 5x^2) (2 + 3x^2 + x^4)^{3/2} dx$

Optimal. Leaf size=179

$$\frac{197\sqrt{2}(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\text{EllipticF}\left(\tan^{-1}(x), \frac{1}{2}\right)}{35\sqrt{x^4+3x^2+2}} + \frac{1}{63}x(35x^2+108)(x^4+3x^2+2)^{3/2} + \frac{1}{105}x(149x^2+519)\sqrt{x^4+3x^2+2}$$

```
[Out] (116*x*(2 + x^2))/(15*Sqrt[2 + 3*x^2 + x^4]) + (x*(519 + 149*x^2)*Sqrt[2 + 3*x^2 + x^4])/105 + (x*(108 + 35*x^2)*(2 + 3*x^2 + x^4)^(3/2))/63 - (116*Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x], 1/2])/(15*Sqrt[2 + 3*x^2 + x^4]) + (197*Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(35*Sqrt[2 + 3*x^2 + x^4])
```

Rubi [A] time = 0.0639407, antiderivative size = 179, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1176, 1189, 1099, 1135}

$$\frac{1}{63}x(35x^2+108)(x^4+3x^2+2)^{3/2} + \frac{1}{105}x(149x^2+519)\sqrt{x^4+3x^2+2} + \frac{116x(x^2+2)}{15\sqrt{x^4+3x^2+2}} + \frac{197\sqrt{2}(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}F\left(\tan^{-1}(x), \frac{1}{2}\right)}{35\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Int[(7 + 5*x^2)*(2 + 3*x^2 + x^4)^(3/2), x]
```

```
[Out] (116*x*(2 + x^2))/(15*Sqrt[2 + 3*x^2 + x^4]) + (x*(519 + 149*x^2)*Sqrt[2 + 3*x^2 + x^4])/105 + (x*(108 + 35*x^2)*(2 + 3*x^2 + x^4)^(3/2))/63 - (116*Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x], 1/2])/(15*Sqrt[2 + 3*x^2 + x^4]) + (197*Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(35*Sqrt[2 + 3*x^2 + x^4])
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := Simp[(x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*(a + b*x^2 + c*x^4)^p)/(c*(4*p + 1)*(4*p + 3)), x] + Dist[(2*p)/(c*(4*p + 1)*(4*p + 3)), Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) - b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1189

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a] || PosQ[(b - q)/a] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1099

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[((2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q)*x^2)/(2*a + (b + q)*x^2)]*EllipticF[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)/(b + q)])/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] && !(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
```

{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]

Rule 1135

Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(x*(b + q + 2*c*x^2))/(2*c*Sqrt[a + b*x^2 + c*x^4]), x] - Simp[(Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q)*x^2)/(2*a + (b + q)*x^2)]*EllipticE[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)/(b + q)])/(2*c*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] && !(PosQ[(b - q)/a] && SimplersqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]

Rubi steps

$$\begin{aligned} \int (7 + 5x^2)(2 + 3x^2 + x^4)^{3/2} dx &= \frac{1}{63}x(108 + 35x^2)(2 + 3x^2 + x^4)^{3/2} + \frac{1}{21} \int (222 + 149x^2) \sqrt{2 + 3x^2 + x^4} dx \\ &= \frac{1}{105}x(519 + 149x^2) \sqrt{2 + 3x^2 + x^4} + \frac{1}{63}x(108 + 35x^2)(2 + 3x^2 + x^4)^{3/2} + \frac{1}{315} \int (222 + 149x^2) \sqrt{2 + 3x^2 + x^4} dx \\ &= \frac{1}{105}x(519 + 149x^2) \sqrt{2 + 3x^2 + x^4} + \frac{1}{63}x(108 + 35x^2)(2 + 3x^2 + x^4)^{3/2} + \frac{116}{15} \int \sqrt{2 + 3x^2 + x^4} dx \\ &= \frac{116x(2 + x^2)}{15\sqrt{2 + 3x^2 + x^4}} + \frac{1}{105}x(519 + 149x^2) \sqrt{2 + 3x^2 + x^4} + \frac{1}{63}x(108 + 35x^2)(2 + 3x^2 + x^4)^{3/2} \end{aligned}$$

Mathematica [F] time = 0, size = 0, normalized size = 0.

\$Aborted

Verification is Not applicable to the result.

[In] Integrate[(7 + 5*x^2)*(2 + 3*x^2 + x^4)^(3/2), x]

[Out] \$Aborted

Maple [C] time = 0.005, size = 172, normalized size = 1.

$$\frac{5x^7}{9} \sqrt{x^4 + 3x^2 + 2} + \frac{71x^5}{21} \sqrt{x^4 + 3x^2 + 2} + \frac{2417x^3}{315} \sqrt{x^4 + 3x^2 + 2} + \frac{293x}{35} \sqrt{x^4 + 3x^2 + 2} - \frac{197i}{35} \sqrt{2} \text{EllipticF}\left(\frac{i}{2}x, \sqrt{x^4 + 3x^2 + 2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((5*x^2+7)*(x^4+3*x^2+2)^(3/2), x)

[Out] 5/9*x^7*(x^4+3*x^2+2)^(1/2)+71/21*x^5*(x^4+3*x^2+2)^(1/2)+2417/315*x^3*(x^4+3*x^2+2)^(1/2)+293/35*x*(x^4+3*x^2+2)^(1/2)-197/35*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticF(1/2*I*x*2^(1/2), 2^(1/2))+58/15*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*(EllipticF(1/2*I*x*2^(1/2), 2^(1/2))-EllipticE(1/2*I*x*2^(1/2), 2^(1/2)))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (x^4 + 3x^2 + 2)^{\frac{3}{2}} (5x^2 + 7) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^2+7)*(x^4+3*x^2+2)^(3/2),x, algorithm="maxima")

[Out] integrate((x^4 + 3*x^2 + 2)^(3/2)*(5*x^2 + 7), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(5x^6 + 22x^4 + 31x^2 + 14\right)\sqrt{x^4 + 3x^2 + 2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^2+7)*(x^4+3*x^2+2)^(3/2),x, algorithm="fricas")

[Out] integral((5*x^6 + 22*x^4 + 31*x^2 + 14)*sqrt(x^4 + 3*x^2 + 2), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left((x^2 + 1)(x^2 + 2) \right)^{\frac{3}{2}} (5x^2 + 7) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x**2+7)*(x**4+3*x**2+2)**(3/2),x)

[Out] Integral(((x**2 + 1)*(x**2 + 2))**(3/2)*(5*x**2 + 7), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (x^4 + 3x^2 + 2)^{\frac{3}{2}} (5x^2 + 7) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^2+7)*(x^4+3*x^2+2)^(3/2),x, algorithm="giac")

[Out] integrate((x^4 + 3*x^2 + 2)^(3/2)*(5*x^2 + 7), x)

3.296 $\int (2 + 3x^2 + x^4)^{3/2} dx$

Optimal. Leaf size=172

$$\frac{31\sqrt{2}(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\text{EllipticF}\left(\tan^{-1}(x), \frac{1}{2}\right)}{35\sqrt{x^4+3x^2+2}} + \frac{1}{7}x(x^4+3x^2+2)^{3/2} + \frac{1}{35}x(9x^2+29)\sqrt{x^4+3x^2+2} + \frac{6x(x^2+2)}{5\sqrt{x^4+3x^2+2}}$$

```
[Out] (6*x*(2 + x^2))/(5*Sqrt[2 + 3*x^2 + x^4]) + (x*(29 + 9*x^2)*Sqrt[2 + 3*x^2 + x^4])/35 + (x*(2 + 3*x^2 + x^4)^(3/2))/7 - (6*Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x], 1/2])/(5*Sqrt[2 + 3*x^2 + x^4]) + (31*Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(35*Sqrt[2 + 3*x^2 + x^4])
```

Rubi [A] time = 0.058111, antiderivative size = 172, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {1091, 1176, 1189, 1099, 1135}

$$\frac{1}{7}x(x^4+3x^2+2)^{3/2} + \frac{1}{35}x(9x^2+29)\sqrt{x^4+3x^2+2} + \frac{6x(x^2+2)}{5\sqrt{x^4+3x^2+2}} + \frac{31\sqrt{2}(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{35\sqrt{x^4+3x^2+2}} - \frac{6x(x^2+2)}{5\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x^2 + x^4)^(3/2), x]
```

```
[Out] (6*x*(2 + x^2))/(5*Sqrt[2 + 3*x^2 + x^4]) + (x*(29 + 9*x^2)*Sqrt[2 + 3*x^2 + x^4])/35 + (x*(2 + 3*x^2 + x^4)^(3/2))/7 - (6*Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x], 1/2])/(5*Sqrt[2 + 3*x^2 + x^4]) + (31*Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(35*Sqrt[2 + 3*x^2 + x^4])
```

Rule 1091

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(x*(a + b*x^2 + c*x^4)^p)/(4*p + 1), x] + Dist[(2*p)/(4*p + 1), Int[(2*a + b*x^2)*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*(a + b*x^2 + c*x^4)^p)/(c*(4*p + 1)*(4*p + 3)), x] + Dist[(2*p)/(c*(4*p + 1)*(4*p + 3)), Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) - b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1189

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a] || PosQ[(b - q)/a] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1099

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[((2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q)*x^2)/(2*a + (b + q)*x^2)]*EllipticF[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)/(b + q)])/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] && !(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1135

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(x*(b + q + 2*c*x^2))/(2*c*Sqrt[a + b*x^2 + c*x^4]), x] - Simp[(Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q)*x^2)/(2*a + (b + q)*x^2)]*EllipticE[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)/(b + q)])/(2*c*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] && !(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned} \int (2 + 3x^2 + x^4)^{3/2} dx &= \frac{1}{7}x(2 + 3x^2 + x^4)^{3/2} + \frac{3}{7} \int (4 + 3x^2) \sqrt{2 + 3x^2 + x^4} dx \\ &= \frac{1}{35}x(29 + 9x^2) \sqrt{2 + 3x^2 + x^4} + \frac{1}{7}x(2 + 3x^2 + x^4)^{3/2} + \frac{1}{35} \int \frac{62 + 42x^2}{\sqrt{2 + 3x^2 + x^4}} dx \\ &= \frac{1}{35}x(29 + 9x^2) \sqrt{2 + 3x^2 + x^4} + \frac{1}{7}x(2 + 3x^2 + x^4)^{3/2} + \frac{6}{5} \int \frac{x^2}{\sqrt{2 + 3x^2 + x^4}} dx + \frac{62}{35} \int \frac{1}{\sqrt{2 + 3x^2 + x^4}} dx \\ &= \frac{6x(2 + x^2)}{5\sqrt{2 + 3x^2 + x^4}} + \frac{1}{35}x(29 + 9x^2) \sqrt{2 + 3x^2 + x^4} + \frac{1}{7}x(2 + 3x^2 + x^4)^{3/2} - \frac{6\sqrt{2}(1 + x^2) \sqrt{\frac{2+x^2}{1+x^2}}}{5\sqrt{2 + 3x^2 + x^4}} \end{aligned}$$

Mathematica [C] time = 0.0421048, size = 114, normalized size = 0.66

$$\frac{-20i\sqrt{x^2 + 1}\sqrt{x^2 + 2}\text{EllipticF}\left(i \sinh^{-1}\left(\frac{x}{\sqrt{2}}\right), 2\right) + 5x^9 + 39x^7 + 121x^5 + 165x^3 - 42i\sqrt{x^2 + 1}\sqrt{x^2 + 2}E\left(i \sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\right)}{35\sqrt{x^4 + 3x^2 + 2}}$$

Antiderivative was successfully verified.

[In] Integrate[(2 + 3*x^2 + x^4)^(3/2), x]

[Out] (78*x + 165*x^3 + 121*x^5 + 39*x^7 + 5*x^9 - (42*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticE[I*ArcSinh[x/Sqrt[2]], 2] - (20*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticF[I*ArcSinh[x/Sqrt[2]], 2])/(35*Sqrt[2 + 3*x^2 + x^4])

Maple [C] time = 0.003, size = 155, normalized size = 0.9

$$\frac{x^5}{7} \sqrt{x^4 + 3x^2 + 2} + \frac{24x^3}{35} \sqrt{x^4 + 3x^2 + 2} + \frac{39x}{35} \sqrt{x^4 + 3x^2 + 2} - \frac{31i}{35} \sqrt{2} \text{EllipticF}\left(\frac{i}{2}x\sqrt{2}, \sqrt{2}\right) \sqrt{2x^2 + 4}\sqrt{x^2 + 1} - \frac{6\sqrt{2}(1 + x^2) \sqrt{\frac{2+x^2}{1+x^2}}}{5\sqrt{2 + 3x^2 + x^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^4+3*x^2+2)^(3/2), x)

```
[Out] 1/7*x^5*(x^4+3*x^2+2)^(1/2)+24/35*x^3*(x^4+3*x^2+2)^(1/2)+39/35*x*(x^4+3*x^2+2)^(1/2)-31/35*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticF(1/2*I*x*2^(1/2),2^(1/2))+3/5*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*(EllipticF(1/2*I*x*2^(1/2),2^(1/2))-EllipticE(1/2*I*x*2^(1/2),2^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (x^4 + 3x^2 + 2)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+3*x^2+2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((x^4 + 3*x^2 + 2)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(x^4 + 3x^2 + 2\right)^{\frac{3}{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+3*x^2+2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((x^4 + 3*x^2 + 2)^(3/2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (x^4 + 3x^2 + 2)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+3*x**2+2)**(3/2),x)
```

```
[Out] Integral((x**4 + 3*x**2 + 2)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (x^4 + 3x^2 + 2)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+3*x^2+2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((x^4 + 3*x^2 + 2)^(3/2), x)
```

$$3.297 \quad \int \frac{(2+3x^2+x^4)^{3/2}}{7+5x^2} dx$$

Optimal. Leaf size=207

$$\frac{56\sqrt{2}(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\text{EllipticF}\left(\tan^{-1}(x), \frac{1}{2}\right)}{375\sqrt{x^4+3x^2+2}} + \frac{24x(x^2+2)}{125\sqrt{x^4+3x^2+2}} + \frac{1}{75}x(3x^2+11)\sqrt{x^4+3x^2+2} - \frac{24\sqrt{2}(x^2+1)\sqrt{\frac{x^2}{x^2+1}}}{125\sqrt{x^4+3x^2+2}}$$

[Out] (24*x*(2 + x^2))/(125*Sqrt[2 + 3*x^2 + x^4]) + (x*(11 + 3*x^2)*Sqrt[2 + 3*x^2 + x^4])/75 - (24*Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x], 1/2])/(125*Sqrt[2 + 3*x^2 + x^4]) + (56*Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(375*Sqrt[2 + 3*x^2 + x^4]) - (9*Sqrt[2]*(2 + x^2)*EllipticPi[2/7, ArcTan[x], 1/2])/(875*Sqrt[(2 + x^2)/(1 + x^2)]*Sqrt[2 + 3*x^2 + x^4])

Rubi [A] time = 0.199018, antiderivative size = 207, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1208, 1176, 1189, 1099, 1135, 1214, 1456, 539}

$$\frac{24x(x^2+2)}{125\sqrt{x^4+3x^2+2}} + \frac{1}{75}x(3x^2+11)\sqrt{x^4+3x^2+2} + \frac{56\sqrt{2}(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{375\sqrt{x^4+3x^2+2}} - \frac{24\sqrt{2}(x^2+1)\sqrt{\frac{x^2}{x^2+1}}E\left(\tan^{-1}(x)|\frac{1}{2}\right)}{125\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

[In] Int[(2 + 3*x^2 + x^4)^(3/2)/(7 + 5*x^2), x]

[Out] (24*x*(2 + x^2))/(125*Sqrt[2 + 3*x^2 + x^4]) + (x*(11 + 3*x^2)*Sqrt[2 + 3*x^2 + x^4])/75 - (24*Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x], 1/2])/(125*Sqrt[2 + 3*x^2 + x^4]) + (56*Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(375*Sqrt[2 + 3*x^2 + x^4]) - (9*Sqrt[2]*(2 + x^2)*EllipticPi[2/7, ArcTan[x], 1/2])/(875*Sqrt[(2 + x^2)/(1 + x^2)]*Sqrt[2 + 3*x^2 + x^4])

Rule 1208

Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_)/((d_) + (e_.)*(x_)^2), x_Symbol] := -Dist[(e^2)^(-1), Int[(c*d - b*e - c*e*x^2)*(a + b*x^2 + c*x^4)^(p - 1), x], x] + Dist[(c*d^2 - b*d*e + a*e^2)/e^2, Int[(a + b*x^2 + c*x^4)^(p - 1)/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p + 1/2, 0]

Rule 1176

Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := Simp[(x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*(a + b*x^2 + c*x^4)^p)/(c*(4*p + 1)*(4*p + 3)), x] + Dist[(2*p)/(c*(4*p + 1)*(4*p + 3)), Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) - b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]

Rule 1189


```
Int[((d_) + (e_)*(x_)^2)/Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4], x_Symbol]
:> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a]
] || PosQ[(b - q)/a] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1099

```
Int[1/Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4], x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]},
Simp[((2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q)*x^2)/(2*a + (b + q)*x^2)]*
EllipticF[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)/(b + q)])/(2*a*Rt[(b + q)/(2*a), 2]*
Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] && !(PosQ[(b - q)/a] &&
SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1135

```
Int[(x_)^2/Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4], x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]},
Simp[(x*(b + q + 2*c*x^2))/(2*c*Sqrt[a + b*x^2 + c*x^4]), x] - Simp[(Rt[(b + q)/(2*a), 2]*
(2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q)*x^2)/(2*a + (b + q)*x^2)]*
EllipticE[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)/(b + q)])/(2*c*Sqrt[a + b*x^2 + c*x^4]),
x] /; PosQ[(b + q)/a] && !(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /;
FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1214

```
Int[1/(((d_) + (e_)*(x_)^2)*Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4]), x_Symbol]
:> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/(2*c*d - e*(b - q)), Int[1/Sqrt[a + b*x^2 + c*x^4],
x], x] - Dist[e/(2*c*d - e*(b - q)), Int[(b - q + 2*c*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]),
x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0] && !LtQ[c, 0]
```

Rule 1456

```
Int[((d_) + (e_)*(x_)^(n_))^(q_)*((f_) + (g_)*(x_)^(n_))^(r_)*((a_) + (b_)*(x_)^(n_) + (c_)*(x_)^(n2_))^(p_), x_Symbol]
:> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d + (c*x^n)/e)^FracPart[p]),
Int[(d + e*x^n)^(p + q)*(f + g*x^n)^r*(a/d + (c*x^n)/e)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q, r}, x]
&& EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && !IntegerQ[p]
```

Rule 539

```
Int[Sqrt[(c_) + (d_)*(x_)^2]/(((a_) + (b_)*(x_)^2)*Sqrt[(e_) + (f_)*(x_)^2]), x_Symbol]
:> Simp[(c*Sqrt[e + f*x^2]*EllipticPi[1 - (b*c)/(a*d), ArcTan[Rt[d/c, 2]*x], 1 - (c*f)/(d*e)]/(a*e*Rt[d/c, 2]*
Sqrt[c + d*x^2]*Sqrt[(c*(e + f*x^2))/(e*(c + d*x^2))]), x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[d/c]
```

Rubi steps

$$\begin{aligned}
\int \frac{(2+3x^2+x^4)^{3/2}}{7+5x^2} dx &= -\left(\frac{1}{25} \int (-8-5x^2) \sqrt{2+3x^2+x^4} dx\right) - \frac{6}{25} \int \frac{\sqrt{2+3x^2+x^4}}{7+5x^2} dx \\
&= \frac{1}{75} x(11+3x^2) \sqrt{2+3x^2+x^4} - \frac{1}{375} \int \frac{-130-90x^2}{\sqrt{2+3x^2+x^4}} dx + \frac{6}{625} \int \frac{-8-5x^2}{\sqrt{2+3x^2+x^4}} dx + \frac{36}{625} \int \frac{1}{\sqrt{2+3x^2+x^4}} dx \\
&= \frac{1}{75} x(11+3x^2) \sqrt{2+3x^2+x^4} + \frac{18}{625} \int \frac{1}{\sqrt{2+3x^2+x^4}} dx - \frac{6}{125} \int \frac{x^2}{\sqrt{2+3x^2+x^4}} dx - \frac{9}{125} \int \frac{1}{\sqrt{2+3x^2+x^4}} dx \\
&= \frac{24x(2+x^2)}{125\sqrt{2+3x^2+x^4}} + \frac{1}{75} x(11+3x^2) \sqrt{2+3x^2+x^4} - \frac{24\sqrt{2}(1+x^2) \sqrt{\frac{2+x^2}{1+x^2}} E\left(\tan^{-1}(x) \middle| \frac{1}{2}\right)}{125\sqrt{2+3x^2+x^4}} + \frac{36}{625} \int \frac{1}{\sqrt{2+3x^2+x^4}} dx \\
&= \frac{24x(2+x^2)}{125\sqrt{2+3x^2+x^4}} + \frac{1}{75} x(11+3x^2) \sqrt{2+3x^2+x^4} - \frac{24\sqrt{2}(1+x^2) \sqrt{\frac{2+x^2}{1+x^2}} E\left(\tan^{-1}(x) \middle| \frac{1}{2}\right)}{125\sqrt{2+3x^2+x^4}} + \frac{36}{625} \int \frac{1}{\sqrt{2+3x^2+x^4}} dx
\end{aligned}$$

Mathematica [C] time = 0.175629, size = 148, normalized size = 0.71

$$\frac{-1022i\sqrt{x^2+1}\sqrt{x^2+2}\text{EllipticF}\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right), 2\right) + 525x^7 + 3500x^5 + 6825x^3 - 2520i\sqrt{x^2+1}\sqrt{x^2+2}E\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\right)}{13125\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

[In] Integrate[(2 + 3*x^2 + x^4)^(3/2)/(7 + 5*x^2), x]

[Out] (3850*x + 6825*x^3 + 3500*x^5 + 525*x^7 - (2520*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticE[I*ArcSinh[x/Sqrt[2]], 2] - (1022*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticF[I*ArcSinh[x/Sqrt[2]], 2] - (108*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticPi[10/7, I*ArcSinh[x/Sqrt[2]], 2])/(13125*Sqrt[2 + 3*x^2 + x^4])

Maple [C] time = 0.014, size = 170, normalized size = 0.8

$$\frac{x^3}{25} \sqrt{x^4+3x^2+2} + \frac{11x}{75} \sqrt{x^4+3x^2+2} - \frac{73i}{1875} \sqrt{2} \text{EllipticF}\left(\frac{i}{2}x\sqrt{2}, \sqrt{2}\right) \sqrt{2x^2+4}\sqrt{x^2+1} \frac{1}{\sqrt{x^4+3x^2+2}} - \frac{12i}{125} \sqrt{2} \text{EllipticE}\left(\frac{i}{2}x\sqrt{2}, \sqrt{2}\right) \sqrt{2x^2+4}\sqrt{x^2+1} \frac{1}{\sqrt{x^4+3x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^4+3*x^2+2)^(3/2)/(5*x^2+7), x)

[Out] 1/25*x^3*(x^4+3*x^2+2)^(1/2)+11/75*x*(x^4+3*x^2+2)^(1/2)-73/1875*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticF(1/2*I*x*2^(1/2), 2^(1/2))-12/125*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticE(1/2*I*x*2^(1/2), 2^(1/2))-36/4375*I*2^(1/2)*(1+1/2*x^2)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticPi(1/2*I*x*2^(1/2), 10/7, 2^(1/2))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(x^4+3x^2+2)^{\frac{3}{2}}}{5x^2+7} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4+3*x^2+2)^(3/2)/(5*x^2+7),x, algorithm="maxima")

[Out] integrate((x^4 + 3*x^2 + 2)^(3/2)/(5*x^2 + 7), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(x^4 + 3x^2 + 2)^{\frac{3}{2}}}{5x^2 + 7}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4+3*x^2+2)^(3/2)/(5*x^2+7),x, algorithm="fricas")

[Out] integral((x^4 + 3*x^2 + 2)^(3/2)/(5*x^2 + 7), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{((x^2 + 1)(x^2 + 2))^{\frac{3}{2}}}{5x^2 + 7} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**4+3*x**2+2)**(3/2)/(5*x**2+7),x)

[Out] Integral(((x**2 + 1)*(x**2 + 2))**(3/2)/(5*x**2 + 7), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(x^4 + 3x^2 + 2)^{\frac{3}{2}}}{5x^2 + 7} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4+3*x^2+2)^(3/2)/(5*x^2+7),x, algorithm="giac")

[Out] integrate((x^4 + 3*x^2 + 2)^(3/2)/(5*x^2 + 7), x)

$$3.298 \quad \int \frac{(2+3x^2+x^4)^{3/2}}{(7+5x^2)^2} dx$$

Optimal. Leaf size=222

$$\frac{59(x^2+1)\sqrt{\frac{x^2+2}{2x^2+2}}\text{EllipticF}\left(\tan^{-1}(x), \frac{1}{2}\right)}{1050\sqrt{x^4+3x^2+2}} - \frac{3\sqrt{x^4+3x^2+2}x}{175(5x^2+7)} + \frac{1}{75}\sqrt{x^4+3x^2+2}x + \frac{9(x^2+2)x}{175\sqrt{x^4+3x^2+2}} - \frac{9\sqrt{2}(x^2+1)}{175}$$

[Out] (9*x*(2 + x^2))/(175*Sqrt[2 + 3*x^2 + x^4]) + (x*Sqrt[2 + 3*x^2 + x^4])/75 - (3*x*Sqrt[2 + 3*x^2 + x^4])/(175*(7 + 5*x^2)) - (9*Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x], 1/2])/(175*Sqrt[2 + 3*x^2 + x^4]) + (59*(1 + x^2)*Sqrt[(2 + x^2)/(2 + 2*x^2)]*EllipticF[ArcTan[x], 1/2])/(1050*Sqrt[2 + 3*x^2 + x^4]) + (9*(1 + x^2)*Sqrt[(2 + x^2)/(2 + 2*x^2)]*EllipticPi[2/7, ArcTan[x], 1/2])/(2450*Sqrt[2 + 3*x^2 + x^4])

Rubi [A] time = 0.444338, antiderivative size = 333, normalized size of antiderivative = 1.5, number of steps used = 21, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {1228, 1099, 1135, 1122, 1189, 1223, 1716, 1214, 1456, 539}

$$-\frac{3\sqrt{x^4+3x^2+2}x}{175(5x^2+7)} + \frac{1}{75}\sqrt{x^4+3x^2+2}x + \frac{9(x^2+2)x}{175\sqrt{x^4+3x^2+2}} + \frac{44\sqrt{2}(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{1875\sqrt{x^4+3x^2+2}} + \frac{81(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}}{8750\sqrt{2}\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

[In] Int[(2 + 3*x^2 + x^4)^(3/2)/(7 + 5*x^2)^2,x]

[Out] (9*x*(2 + x^2))/(175*Sqrt[2 + 3*x^2 + x^4]) + (x*Sqrt[2 + 3*x^2 + x^4])/75 - (3*x*Sqrt[2 + 3*x^2 + x^4])/(175*(7 + 5*x^2)) - (9*Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x], 1/2])/(175*Sqrt[2 + 3*x^2 + x^4]) + (81*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(8750*Sqrt[2]*Sqrt[2 + 3*x^2 + x^4]) + (44*Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(1875*Sqrt[2 + 3*x^2 + x^4]) - (39*(2 + x^2)*EllipticPi[2/7, ArcTan[x], 1/2])/(12250*Sqrt[2]*Sqrt[(2 + x^2)/(1 + x^2)]*Sqrt[2 + 3*x^2 + x^4]) + (3*Sqrt[2]*(2 + x^2)*EllipticPi[2/7, ArcTan[x], 1/2])/(875*Sqrt[(2 + x^2)/(1 + x^2)]*Sqrt[2 + 3*x^2 + x^4])

Rule 1228

Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := Module[{aa, bb, cc}, Int[ExpandIntegrand[1/Sqrt[aa + bb*x^2 + cc*x^4], (d + e*x^2)^q*(aa + bb*x^2 + cc*x^4)^(p + 1/2), x] /. {aa -> a, bb -> b, cc -> c}, x] /. FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, 0] && IntegerQ[p + 1/2]

Rule 1099

Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[((2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q)*x^2)/(2*a + (b + q)*x^2)]*EllipticF[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)/(b + q)])/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]), x] /. PosQ[(b + q)/a] && !(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /. FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]

Rule 1135

Int[(x_)^2/Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4], x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(x*(b + q + 2*c*x^2))/(2*c*Sqrt[a + b*x^2 + c*x^4]), x] - Simp[(Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q)*x^2)/(2*a + (b + q)*x^2)]*EllipticE[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)/(b + q)])/(2*c*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] && !(PosQ[(b - q)/a] && SimplersqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]

Rule 1122

Int[((d_)*(x_))^(m_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] := Simp[(d^3*(d*x)^(m - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(m + 4*p + 1)), x] - Dist[d^4/(c*(m + 4*p + 1)), Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m + 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])

Rule 1189

Int[((d_) + (e_)*(x_)^2)/Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4], x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a] || PosQ[(b - q)/a] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]

Rule 1223

Int[((d_) + (e_)*(x_)^2)^(q_)/Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4], x_Symbol] := -Simp[(e^2*x*(d + e*x^2)^(q + 1)*Sqrt[a + b*x^2 + c*x^4])/(2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2)), Int[((d + e*x^2)^(q + 1)*Simp[a*e^2*(2*q + 3) + 2*d*(c*d - b*e)*(q + 1) - 2*e*(c*d*(q + 1) - b*e*(q + 2))*x^2 + c*e^2*(2*q + 5)*x^4, x])/Sqrt[a + b*x^2 + c*x^4], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && ILtQ[q, -1]

Rule 1716

Int[(P4x_)/(((d_) + (e_)*(x_)^2)*Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4]), x_Symbol] := With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[P4x, x, 4]}, -Dist[(e^2)^(-1), Int[(C*d - B*e - C*e*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] + Dist[(C*d^2 - B*d*e + A*e^2)/e^2, Int[1/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[P4x, x^2, 2] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0]

Rule 1214

Int[1/(((d_) + (e_)*(x_)^2)*Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4]), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/(2*c*d - e*(b - q)), Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[e/(2*c*d - e*(b - q)), Int[(b - q + 2*c*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0] && !LtQ[c, 0]

Rule 1456

Int[((d_) + (e_)*(x_)^(n_))^(q_)*((f_) + (g_)*(x_)^(n_))^(r_)*((a_) + (b_)*(x_)^(n_) + (c_)*(x_)^(n2_))^(p_), x_Symbol] := Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d + (c*x^n)/e)^FracPart[p]), Int[(d + e*x^n)^(p + q)*(f + g*x^n)^r*(a/d + (c*x^n)/e)^p, x], x] /; Free

Q[{a, b, c, d, e, f, g, n, p, q, r}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && !IntegerQ[p]

Rule 539

Int[Sqrt[(c_) + (d_)*(x_)^2]/(((a_) + (b_)*(x_)^2)*Sqrt[(e_) + (f_)*(x_)^2]), x_Symbol] :> Simp[(c*Sqrt[e + f*x^2]*EllipticPi[1 - (b*c)/(a*d), ArcTan[Rt[d/c, 2]*x], 1 - (c*f)/(d*e)]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(e + f*x^2))/(e*(c + d*x^2))]), x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[d/c]

Rubi steps

$$\int \frac{(2 + 3x^2 + x^4)^{3/2}}{(7 + 5x^2)^2} dx = \int \left(\frac{52}{625\sqrt{2 + 3x^2 + x^4}} + \frac{16x^2}{125\sqrt{2 + 3x^2 + x^4}} + \frac{x^4}{25\sqrt{2 + 3x^2 + x^4}} + \frac{36}{625(7 + 5x^2)^2\sqrt{2 + 3x^2 + x^4}} \right) dx$$

$$= -\left(\frac{12}{625} \int \frac{1}{(7 + 5x^2)\sqrt{2 + 3x^2 + x^4}} dx \right) + \frac{1}{25} \int \frac{x^4}{\sqrt{2 + 3x^2 + x^4}} dx + \frac{36}{625} \int \frac{1}{(7 + 5x^2)^2\sqrt{2 + 3x^2 + x^4}} dx$$

$$= \frac{16x(2 + x^2)}{125\sqrt{2 + 3x^2 + x^4}} + \frac{1}{75}x\sqrt{2 + 3x^2 + x^4} - \frac{3x\sqrt{2 + 3x^2 + x^4}}{175(7 + 5x^2)} - \frac{16\sqrt{2}(1 + x^2)\sqrt{\frac{2+x^2}{1+x^2}}E\left(\tan^{-1}\left(\frac{x}{\sqrt{2+x^2}}\right)\right)}{125\sqrt{2 + 3x^2 + x^4}}$$

$$= \frac{16x(2 + x^2)}{125\sqrt{2 + 3x^2 + x^4}} + \frac{1}{75}x\sqrt{2 + 3x^2 + x^4} - \frac{3x\sqrt{2 + 3x^2 + x^4}}{175(7 + 5x^2)} - \frac{16\sqrt{2}(1 + x^2)\sqrt{\frac{2+x^2}{1+x^2}}E\left(\tan^{-1}\left(\frac{x}{\sqrt{2+x^2}}\right)\right)}{125\sqrt{2 + 3x^2 + x^4}}$$

$$= \frac{6x(2 + x^2)}{125\sqrt{2 + 3x^2 + x^4}} + \frac{1}{75}x\sqrt{2 + 3x^2 + x^4} - \frac{3x\sqrt{2 + 3x^2 + x^4}}{175(7 + 5x^2)} - \frac{6\sqrt{2}(1 + x^2)\sqrt{\frac{2+x^2}{1+x^2}}E\left(\tan^{-1}\left(\frac{x}{\sqrt{2+x^2}}\right)\right)}{125\sqrt{2 + 3x^2 + x^4}}$$

$$= \frac{9x(2 + x^2)}{175\sqrt{2 + 3x^2 + x^4}} + \frac{1}{75}x\sqrt{2 + 3x^2 + x^4} - \frac{3x\sqrt{2 + 3x^2 + x^4}}{175(7 + 5x^2)} - \frac{9\sqrt{2}(1 + x^2)\sqrt{\frac{2+x^2}{1+x^2}}E\left(\tan^{-1}\left(\frac{x}{\sqrt{2+x^2}}\right)\right)}{175\sqrt{2 + 3x^2 + x^4}}$$

$$= \frac{9x(2 + x^2)}{175\sqrt{2 + 3x^2 + x^4}} + \frac{1}{75}x\sqrt{2 + 3x^2 + x^4} - \frac{3x\sqrt{2 + 3x^2 + x^4}}{175(7 + 5x^2)} - \frac{9\sqrt{2}(1 + x^2)\sqrt{\frac{2+x^2}{1+x^2}}E\left(\tan^{-1}\left(\frac{x}{\sqrt{2+x^2}}\right)\right)}{175\sqrt{2 + 3x^2 + x^4}}$$

Mathematica [C] time = 0.29148, size = 213, normalized size = 0.96

$$\frac{-182i\sqrt{x^2 + 1}\sqrt{x^2 + 2}(5x^2 + 7)\text{EllipticF}\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right), 2\right) + 1225x^7 + 5075x^5 + 6650x^3 - 945i\sqrt{x^2 + 1}\sqrt{x^2 + 2}(5x^2 + 7)}{18375(5x^2 + 7)\sqrt{2 + 3x^2 + x^4}}$$

Antiderivative was successfully verified.

[In] Integrate[(2 + 3*x^2 + x^4)^(3/2)/(7 + 5*x^2)^2,x]

[Out] (2800*x + 6650*x^3 + 5075*x^5 + 1225*x^7 - (945*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*(7 + 5*x^2)*EllipticE[I*ArcSinh[x/Sqrt[2]], 2] - (182*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*(7 + 5*x^2)*EllipticF[I*ArcSinh[x/Sqrt[2]], 2] + (189*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticPi[10/7, I*ArcSinh[x/Sqrt[2]], 2] + (135*I)*x^2*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticPi[10/7, I*ArcSinh[x/Sqrt[2]], 2])/(18375*(7 + 5*x^2)*Sqrt[2 + 3*x^2 + x^4])

Maple [C] time = 0.019, size = 177, normalized size = 0.8

$$-\frac{3x}{875x^2 + 1225}\sqrt{x^4 + 3x^2 + 2} + \frac{x}{75}\sqrt{x^4 + 3x^2 + 2} - \frac{13i}{2625}\sqrt{2}\operatorname{EllipticF}\left(\frac{i}{2}x\sqrt{2}, \sqrt{2}\right)\sqrt{2x^2 + 4}\sqrt{x^2 + 1} - \frac{1}{\sqrt{x^4 + 3x^2 + 2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^4+3*x^2+2)^(3/2)/(5*x^2+7)^2,x)

[Out] -3/175*x*(x^4+3*x^2+2)^(1/2)/(5*x^2+7)+1/75*x*(x^4+3*x^2+2)^(1/2)-13/2625*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticF(1/2*I*x*2^(1/2),2^(1/2))-9/350*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticE(1/2*I*x*2^(1/2),2^(1/2))+9/6125*I*2^(1/2)*(1+1/2*x^2)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticPi(1/2*I*x*2^(1/2),10/7,2^(1/2))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(x^4 + 3x^2 + 2)^{\frac{3}{2}}}{(5x^2 + 7)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4+3*x^2+2)^(3/2)/(5*x^2+7)^2,x, algorithm="maxima")

[Out] integrate((x^4 + 3*x^2 + 2)^(3/2)/(5*x^2 + 7)^2, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{(x^4 + 3x^2 + 2)^{\frac{3}{2}}}{25x^4 + 70x^2 + 49}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4+3*x^2+2)^(3/2)/(5*x^2+7)^2,x, algorithm="fricas")

[Out] integral((x^4 + 3*x^2 + 2)^(3/2)/(25*x^4 + 70*x^2 + 49), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{((x^2 + 1)(x^2 + 2))^{\frac{3}{2}}}{(5x^2 + 7)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**4+3*x**2+2)**(3/2)/(5*x**2+7)**2,x)

[Out] Integral(((x**2 + 1)*(x**2 + 2))**(3/2)/(5*x**2 + 7)**2, x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(x^4 + 3x^2 + 2)^{\frac{3}{2}}}{(5x^2 + 7)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4+3*x^2+2)^(3/2)/(5*x^2+7)^2,x, algorithm="giac")

[Out] integrate((x^4 + 3*x^2 + 2)^(3/2)/(5*x^2 + 7)^2, x)

$$3.299 \quad \int \frac{(2+3x^2+x^4)^{3/2}}{(7+5x^2)^3} dx$$

Optimal. Leaf size=231

$$\frac{5(x^2+1)\sqrt{\frac{x^2+2}{2x^2+2}}\text{EllipticF}\left(\tan^{-1}(x), \frac{1}{2}\right)}{784\sqrt{x^4+3x^2+2}} + \frac{17\sqrt{x^4+3x^2+2}x}{9800(5x^2+7)} - \frac{3\sqrt{x^4+3x^2+2}x}{350(5x^2+7)^2} + \frac{3(x^2+2)x}{392\sqrt{x^4+3x^2+2}} - \frac{3(x^2+1)}{1960\sqrt{x^4+3x^2+2}}$$

[Out] (3*x*(2 + x^2))/(392*Sqrt[2 + 3*x^2 + x^4]) - (3*x*Sqrt[2 + 3*x^2 + x^4])/(350*(7 + 5*x^2)^2) + (17*x*Sqrt[2 + 3*x^2 + x^4])/(9800*(7 + 5*x^2)) - (3*(1 + x^2)*Sqrt[(2 + x^2)/(2 + 2*x^2)]*EllipticE[ArcTan[x], 1/2])/(196*Sqrt[2 + 3*x^2 + x^4]) + (5*(1 + x^2)*Sqrt[(2 + x^2)/(2 + 2*x^2)]*EllipticF[ArcTan[x], 1/2])/(784*Sqrt[2 + 3*x^2 + x^4]) + (141*(2 + x^2)*EllipticPi[2/7, ArcTan[x], 1/2])/(27440*Sqrt[2]*Sqrt[(2 + x^2)/(1 + x^2)]*Sqrt[2 + 3*x^2 + x^4])

Rubi [A] time = 0.667752, antiderivative size = 288, normalized size of antiderivative = 1.25, number of steps used = 27, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {1228, 1099, 1135, 1223, 1696, 1716, 1189, 1214, 1456, 539}

$$\frac{17\sqrt{x^4+3x^2+2}x}{9800(5x^2+7)} - \frac{3\sqrt{x^4+3x^2+2}x}{350(5x^2+7)^2} + \frac{3(x^2+2)x}{392\sqrt{x^4+3x^2+2}} + \frac{5(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}F\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{784\sqrt{2}\sqrt{x^4+3x^2+2}} - \frac{6\sqrt{2}(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}}{875\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

[In] Int[(2 + 3*x^2 + x^4)^(3/2)/(7 + 5*x^2)^3,x]

[Out] (3*x*(2 + x^2))/(392*Sqrt[2 + 3*x^2 + x^4]) - (3*x*Sqrt[2 + 3*x^2 + x^4])/(350*(7 + 5*x^2)^2) + (17*x*Sqrt[2 + 3*x^2 + x^4])/(9800*(7 + 5*x^2)) - (39*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x], 1/2])/(24500*Sqrt[2]*Sqrt[2 + 3*x^2 + x^4]) - (6*Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x], 1/2])/(875*Sqrt[2 + 3*x^2 + x^4]) + (5*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(784*Sqrt[2]*Sqrt[2 + 3*x^2 + x^4]) + (141*(2 + x^2)*EllipticPi[2/7, ArcTan[x], 1/2])/(27440*Sqrt[2]*Sqrt[(2 + x^2)/(1 + x^2)]*Sqrt[2 + 3*x^2 + x^4])

Rule 1228

Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{aa, bb, cc}, Int[ExpandIntegrand[1/Sqrt[aa + bb*x^2 + cc*x^4], (d + e*x^2)^q*(aa + bb*x^2 + cc*x^4)^(p + 1/2), x] /. {aa -> a, bb -> b, cc -> c}, x] /. FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, 0] && IntegerQ[p + 1/2]

Rule 1099

Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[((2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q)*x^2)/(2*a + (b + q)*x^2)]*EllipticF[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)/(b + q)])/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]), x] /. PosQ[(b + q)/a] && !(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /. FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]

Rule 1135

```
Int[(x_)^2/Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4], x_Symbol] := With[{q =
  Rt[b^2 - 4*a*c, 2]}, Simp[(x*(b + q + 2*c*x^2))/(2*c*Sqrt[a + b*x^2 + c*x^
  4]), x] - Simp[(Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q)
  )*x^2]/(2*a + (b + q)*x^2)]*EllipticE[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)
  /(b + q)]/(2*c*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] && !(PosQ[
  (b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
  c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1223

```
Int[((d_) + (e_)*(x_)^2)^(q_)/Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4], x_
Symbol] := -Simp[(e^2*x*(d + e*x^2)^(q + 1)*Sqrt[a + b*x^2 + c*x^4]/(2*d*(
q + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(2*d*(q + 1)*(c*d^2 - b*d*e +
a*e^2)), Int[((d + e*x^2)^(q + 1)*Simp[a*e^2*(2*q + 3) + 2*d*(c*d - b*e)*(q
+ 1) - 2*e*(c*d*(q + 1) - b*e*(q + 2))*x^2 + c*e^2*(2*q + 5)*x^4, x])/Sqrt
[a + b*x^2 + c*x^4], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c
, 0] && ILtQ[q, -1]
```

Rule 1696

```
Int[((P4x_)*((d_) + (e_)*(x_)^2)^(q_))/Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x
_)^4], x_Symbol] := With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = C
oeff[P4x, x, 4]}, -Simp[((C*d^2 - B*d*e + A*e^2)*x*(d + e*x^2)^(q + 1)*Sqrt
[a + b*x^2 + c*x^4]/(2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(2*
d*(q + 1)*(c*d^2 - b*d*e + a*e^2)), Int[((d + e*x^2)^(q + 1)*Simp[a*d*(C*d
- B*e) + A*(a*e^2*(2*q + 3) + 2*d*(c*d - b*e)*(q + 1)) - 2*((B*d - A*e)*(b
e*(q + 2) - c*d*(q + 1)) - C*d*(b*d + a*e*(q + 1)))*x^2 + c*(C*d^2 - B*d*e
+ A*e^2)*(2*q + 5)*x^4, x])/Sqrt[a + b*x^2 + c*x^4], x], x]] /; FreeQ[{a, b
, c, d, e}, x] && PolyQ[P4x, x^2] && LeQ[Expon[P4x, x], 4] && NeQ[b^2 - 4*a
*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, -1]
```

Rule 1716

```
Int[(P4x_)/(((d_) + (e_)*(x_)^2)*Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4])
, x_Symbol] := With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[
P4x, x, 4]}, -Dist[(e^2)^(-1), Int[(C*d - B*e - C*e*x^2)/Sqrt[a + b*x^2 + c
*x^4], x], x] + Dist[(C*d^2 - B*d*e + A*e^2)/e^2, Int[1/((d + e*x^2)*Sqrt[a
+ b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[P4x, x^2,
2] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 -
a*e^2, 0]
```

Rule 1189

```
Int[((d_) + (e_)*(x_)^2)/Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4], x_Symbo
l] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1214

```
Int[1/(((d_) + (e_)*(x_)^2)*Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4]), x_S
ymbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/(2*c*d - e*(b - q)), In
t[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[e/(2*c*d - e*(b - q)), Int[(b -
q + 2*c*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
c, d, e}, x] && GtQ[b^2 - 4*a*c, 0] && !LtQ[c, 0]
```

Rule 1456

```
Int[((d_) + (e_)*(x_)^(n_))^(q_)*((f_) + (g_)*(x_)^(n_))^(r_)*((a_) + (
b_)*(x_)^(n_) + (c_)*(x_)^(n2_))^(p_), x_Symbol] :> Dist[(a + b*x^n + c*x
^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d + (c*x^n)/e)^FracPart[p])
, Int[(d + e*x^n)^(p + q)*(f + g*x^n)^r*(a/d + (c*x^n)/e)^p, x], x] /; Free
Q[{a, b, c, d, e, f, g, n, p, q, r}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && !IntegerQ[p]
```

Rule 539

```
Int[Sqrt[(c_) + (d_)*(x_)^2]/(((a_) + (b_)*(x_)^2)*Sqrt[(e_) + (f_)*(x_)
^2]), x_Symbol] :> Simp[(c*Sqrt[e + f*x^2]*EllipticPi[1 - (b*c)/(a*d), ArcT
an[Rt[d/c, 2]*x], 1 - (c*f)/(d*e)]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c
*(e + f*x^2))/(e*(c + d*x^2)]), x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

Rubi steps

$$\int \frac{(2 + 3x^2 + x^4)^{3/2}}{(7 + 5x^2)^3} dx = \int \left(\frac{9}{625\sqrt{2 + 3x^2 + x^4}} + \frac{x^2}{125\sqrt{2 + 3x^2 + x^4}} + \frac{36}{625(7 + 5x^2)^3\sqrt{2 + 3x^2 + x^4}} - \frac{1}{625(7 + 5x^2)\sqrt{2 + 3x^2 + x^4}} \right) dx$$

$$= \frac{1}{125} \int \frac{x^2}{\sqrt{2 + 3x^2 + x^4}} dx + \frac{9}{625} \int \frac{1}{\sqrt{2 + 3x^2 + x^4}} dx - \frac{11}{625} \int \frac{1}{(7 + 5x^2)\sqrt{2 + 3x^2 + x^4}} dx$$

$$= \frac{x(2 + x^2)}{125\sqrt{2 + 3x^2 + x^4}} - \frac{3x\sqrt{2 + 3x^2 + x^4}}{350(7 + 5x^2)^2} + \frac{x\sqrt{2 + 3x^2 + x^4}}{175(7 + 5x^2)} - \frac{\sqrt{2}(1 + x^2)\sqrt{\frac{2+x^2}{1+x^2}}E(\tan^{-1}(x))}{125\sqrt{2 + 3x^2 + x^4}}$$

$$= \frac{x(2 + x^2)}{125\sqrt{2 + 3x^2 + x^4}} - \frac{3x\sqrt{2 + 3x^2 + x^4}}{350(7 + 5x^2)^2} + \frac{17x\sqrt{2 + 3x^2 + x^4}}{9800(7 + 5x^2)} - \frac{\sqrt{2}(1 + x^2)\sqrt{\frac{2+x^2}{1+x^2}}E(\tan^{-1}(x))}{125\sqrt{2 + 3x^2 + x^4}}$$

$$= \frac{x(2 + x^2)}{125\sqrt{2 + 3x^2 + x^4}} - \frac{3x\sqrt{2 + 3x^2 + x^4}}{350(7 + 5x^2)^2} + \frac{17x\sqrt{2 + 3x^2 + x^4}}{9800(7 + 5x^2)} - \frac{\sqrt{2}(1 + x^2)\sqrt{\frac{2+x^2}{1+x^2}}E(\tan^{-1}(x))}{125\sqrt{2 + 3x^2 + x^4}}$$

$$= \frac{6x(2 + x^2)}{875\sqrt{2 + 3x^2 + x^4}} - \frac{3x\sqrt{2 + 3x^2 + x^4}}{350(7 + 5x^2)^2} + \frac{17x\sqrt{2 + 3x^2 + x^4}}{9800(7 + 5x^2)} - \frac{6\sqrt{2}(1 + x^2)\sqrt{\frac{2+x^2}{1+x^2}}E(\tan^{-1}(x))}{875\sqrt{2 + 3x^2 + x^4}}$$

$$= \frac{3x(2 + x^2)}{392\sqrt{2 + 3x^2 + x^4}} - \frac{3x\sqrt{2 + 3x^2 + x^4}}{350(7 + 5x^2)^2} + \frac{17x\sqrt{2 + 3x^2 + x^4}}{9800(7 + 5x^2)} - \frac{39(1 + x^2)\sqrt{\frac{2+x^2}{1+x^2}}E(\tan^{-1}(x))}{24500\sqrt{2}\sqrt{2 + 3x^2 + x^4}}$$

$$= \frac{3x(2 + x^2)}{392\sqrt{2 + 3x^2 + x^4}} - \frac{3x\sqrt{2 + 3x^2 + x^4}}{350(7 + 5x^2)^2} + \frac{17x\sqrt{2 + 3x^2 + x^4}}{9800(7 + 5x^2)} - \frac{39(1 + x^2)\sqrt{\frac{2+x^2}{1+x^2}}E(\tan^{-1}(x))}{24500\sqrt{2}\sqrt{2 + 3x^2 + x^4}}$$

Mathematica [C] time = 0.375343, size = 174, normalized size = 0.75

$$\frac{-406i\sqrt{x^2 + 1}\sqrt{x^2 + 2}\text{EllipticF}\left(i \sinh^{-1}\left(\frac{x}{\sqrt{2}}\right), 2\right) + \frac{119x(x^4 + 3x^2 + 2)}{5x^2 + 7} - \frac{588x(x^4 + 3x^2 + 2)}{(5x^2 + 7)^2} - 525i\sqrt{x^2 + 1}\sqrt{x^2 + 2}E\left(i \sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\right)}{68600\sqrt{x^4 + 3x^2 + 2}}$$

Antiderivative was successfully verified.

[In] Integrate[(2 + 3*x^2 + x^4)^(3/2)/(7 + 5*x^2)^3,x]

[Out] ((-588*x*(2 + 3*x^2 + x^4))/(7 + 5*x^2)^2 + (119*x*(2 + 3*x^2 + x^4))/(7 + 5*x^2) - (525*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticE[I*ArcSinh[x/Sqrt[2]], 2] - (406*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticF[I*ArcSinh[x/Sqrt[2]], 2] + (141*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticPi[10/7, I*ArcSinh[x/Sqrt[2]], 2])/(68600*Sqrt[2 + 3*x^2 + x^4])

Maple [C] time = 0.02, size = 186, normalized size = 0.8

$$-\frac{3x}{350(5x^2+7)^2}\sqrt{x^4+3x^2+2} + \frac{17x}{49000x^2+68600}\sqrt{x^4+3x^2+2} - \frac{29i}{9800}\sqrt{2}\text{EllipticF}\left(\frac{i}{2}x\sqrt{2},\sqrt{2}\right)\sqrt{2x^2+4}\sqrt{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^4+3*x^2+2)^(3/2)/(5*x^2+7)^3,x)

[Out] -3/350*x*(x^4+3*x^2+2)^(1/2)/(5*x^2+7)^2+17/9800*x*(x^4+3*x^2+2)^(1/2)/(5*x^2+7)-29/9800*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticF(1/2*I*x*2^(1/2),2^(1/2))-3/784*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticE(1/2*I*x*2^(1/2),2^(1/2))+141/68600*I*2^(1/2)*(1+1/2*x^2)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticPi(1/2*I*x*2^(1/2),10/7,2^(1/2))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(x^4 + 3x^2 + 2)^{\frac{3}{2}}}{(5x^2 + 7)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4+3*x^2+2)^(3/2)/(5*x^2+7)^3,x, algorithm="maxima")

[Out] integrate((x^4 + 3*x^2 + 2)^(3/2)/(5*x^2 + 7)^3, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(x^4 + 3x^2 + 2)^{\frac{3}{2}}}{125x^6 + 525x^4 + 735x^2 + 343}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4+3*x^2+2)^(3/2)/(5*x^2+7)^3,x, algorithm="fricas")

[Out] integral((x^4 + 3*x^2 + 2)^(3/2)/(125*x^6 + 525*x^4 + 735*x^2 + 343), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{((x^2 + 1)(x^2 + 2))^{\frac{3}{2}}}{(5x^2 + 7)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**4+3*x**2+2)**(3/2)/(5*x**2+7)**3,x)

[Out] Integral(((x**2 + 1)*(x**2 + 2))**(3/2)/(5*x**2 + 7)**3, x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(x^4 + 3x^2 + 2)^{\frac{3}{2}}}{(5x^2 + 7)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4+3*x^2+2)^(3/2)/(5*x^2+7)^3,x, algorithm="giac")

[Out] integrate((x^4 + 3*x^2 + 2)^(3/2)/(5*x^2 + 7)^3, x)

$$3.300 \quad \int \frac{(7+5x^2)^3}{\sqrt{2+3x^2+x^4}} dx$$

Optimal. Leaf size=157

$$\frac{193(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\text{EllipticF}\left(\tan^{-1}(x), \frac{1}{2}\right)}{\sqrt{2}\sqrt{x^4+3x^2+2}} + 25\sqrt{x^4+3x^2+2}x^3 + 75\sqrt{x^4+3x^2+2}x + \frac{135(x^2+2)x}{\sqrt{x^4+3x^2+2}} - \frac{135\sqrt{2}(x^2+1)}{\sqrt{x^4+3x^2+2}}$$

[Out] (135*x*(2 + x^2))/Sqrt[2 + 3*x^2 + x^4] + 75*x*Sqrt[2 + 3*x^2 + x^4] + 25*x^3*Sqrt[2 + 3*x^2 + x^4] - (135*Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x], 1/2])/Sqrt[2 + 3*x^2 + x^4] + (193*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(Sqrt[2]*Sqrt[2 + 3*x^2 + x^4])

Rubi [A] time = 0.0835597, antiderivative size = 157, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {1206, 1679, 1189, 1099, 1135}

$$25\sqrt{x^4+3x^2+2}x^3 + 75\sqrt{x^4+3x^2+2}x + \frac{135(x^2+2)x}{\sqrt{x^4+3x^2+2}} + \frac{193(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}F\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^4+3x^2+2}} - \frac{135\sqrt{2}(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}}{\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

[In] Int[(7 + 5*x^2)^3/Sqrt[2 + 3*x^2 + x^4], x]

[Out] (135*x*(2 + x^2))/Sqrt[2 + 3*x^2 + x^4] + 75*x*Sqrt[2 + 3*x^2 + x^4] + 25*x^3*Sqrt[2 + 3*x^2 + x^4] - (135*Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x], 1/2])/Sqrt[2 + 3*x^2 + x^4] + (193*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(Sqrt[2]*Sqrt[2 + 3*x^2 + x^4])

Rule 1206

Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := Simp[(e^q*x^(2*q - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(4*p + 2*q + 1)), x] + Dist[1/(c*(4*p + 2*q + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*p + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1]

Rule 1679

Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := With[{q = Expon[Pq, x^2], e = Coeff[Pq, x^2, Expon[Pq, x^2]]}, Simp[(e*x^(2*q - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(2*q + 4*p + 1)), x] + Dist[1/(c*(2*q + 4*p + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*q + 4*p + 1)*Pq - a*e*(2*q - 3)*x^(2*q - 4) - b*e*(2*q + 2*p - 1)*x^(2*q - 2) - c*e*(2*q + 4*p + 1)*x^(2*q), x], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && !LtQ[p, -1]

Rule 1189

Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],

$x], x] + \text{Dist}[e, \text{Int}[x^2/\text{Sqrt}[a + b*x^2 + c*x^4], x], x] /; \text{PosQ}[(b + q)/a]$
 $] || \text{PosQ}[(b - q)/a] /; \text{FreeQ}[\{a, b, c, d, e\}, x] \&\& \text{GtQ}[b^2 - 4*a*c, 0]$

Rule 1099

$\text{Int}[1/\text{Sqrt}[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4], x_Symbol] := \text{With}[\{q = \text{Rt}[b^2 - 4*a*c, 2]\}, \text{Simp}[\{(2*a + (b + q)*x^2)*\text{Sqrt}[(2*a + (b - q)*x^2)/(2*a + (b + q)*x^2)]*\text{EllipticF}[\text{ArcTan}[\text{Rt}[(b + q)/(2*a), 2]*x], (2*q)/(b + q)]\}/(2*a*\text{Rt}[(b + q)/(2*a), 2]*\text{Sqrt}[a + b*x^2 + c*x^4]), x] /; \text{PosQ}[(b + q)/a] \&\& !(\text{PosQ}[(b - q)/a] \&\& \text{SimplerSqrtQ}[(b - q)/(2*a), (b + q)/(2*a)])] /; \text{FreeQ}[\{a, b, c\}, x] \&\& \text{GtQ}[b^2 - 4*a*c, 0]$

Rule 1135

$\text{Int}[(x_)^2/\text{Sqrt}[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4], x_Symbol] := \text{With}[\{q = \text{Rt}[b^2 - 4*a*c, 2]\}, \text{Simp}[(x*(b + q + 2*c*x^2))/(2*c*\text{Sqrt}[a + b*x^2 + c*x^4]), x] - \text{Simp}[(\text{Rt}[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*\text{Sqrt}[(2*a + (b - q)*x^2)/(2*a + (b + q)*x^2)]*\text{EllipticE}[\text{ArcTan}[\text{Rt}[(b + q)/(2*a), 2]*x], (2*q)/(b + q)]\}/(2*c*\text{Sqrt}[a + b*x^2 + c*x^4]), x] /; \text{PosQ}[(b + q)/a] \&\& !(\text{PosQ}[(b - q)/a] \&\& \text{SimplerSqrtQ}[(b - q)/(2*a), (b + q)/(2*a)])] /; \text{FreeQ}[\{a, b, c\}, x] \&\& \text{GtQ}[b^2 - 4*a*c, 0]$

Rubi steps

$$\begin{aligned} \int \frac{(7 + 5x^2)^3}{\sqrt{2 + 3x^2 + x^4}} dx &= 25x^3\sqrt{2 + 3x^2 + x^4} + \frac{1}{5} \int \frac{1715 + 2925x^2 + 1125x^4}{\sqrt{2 + 3x^2 + x^4}} dx \\ &= 75x\sqrt{2 + 3x^2 + x^4} + 25x^3\sqrt{2 + 3x^2 + x^4} + \frac{1}{15} \int \frac{2895 + 2025x^2}{\sqrt{2 + 3x^2 + x^4}} dx \\ &= 75x\sqrt{2 + 3x^2 + x^4} + 25x^3\sqrt{2 + 3x^2 + x^4} + 135 \int \frac{x^2}{\sqrt{2 + 3x^2 + x^4}} dx + 193 \int \frac{1}{\sqrt{2 + 3x^2 + x^4}} dx \\ &= \frac{135x(2 + x^2)}{\sqrt{2 + 3x^2 + x^4}} + 75x\sqrt{2 + 3x^2 + x^4} + 25x^3\sqrt{2 + 3x^2 + x^4} - \frac{135\sqrt{2}(1 + x^2)\sqrt{\frac{2+x^2}{1+x^2}}E\left(\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)\right)}{\sqrt{2 + 3x^2 + x^4}} \end{aligned}$$

Mathematica [C] time = 0.102239, size = 106, normalized size = 0.68

$$\frac{-58i\sqrt{x^2 + 1}\sqrt{x^2 + 2}\text{EllipticF}\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right), 2\right) + 25x(x^6 + 6x^4 + 11x^2 + 6) - 135i\sqrt{x^2 + 1}\sqrt{x^2 + 2}E\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\right)}{\sqrt{x^4 + 3x^2 + 2}}$$

Antiderivative was successfully verified.

[In] Integrate[(7 + 5*x^2)^3/Sqrt[2 + 3*x^2 + x^4], x]

[Out] (25*x*(6 + 11*x^2 + 6*x^4 + x^6) - (135*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticE[I*ArcSinh[x/Sqrt[2]], 2] - (58*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticF[I*ArcSinh[x/Sqrt[2]], 2])/Sqrt[2 + 3*x^2 + x^4]

Maple [C] time = 0.016, size = 138, normalized size = 0.9

$$25x^3\sqrt{x^4 + 3x^2 + 2} + 75x\sqrt{x^4 + 3x^2 + 2} - \frac{193i}{2}\sqrt{2}\text{EllipticF}\left(\frac{i}{2}x\sqrt{2}, \sqrt{2}\right)\sqrt{2x^2 + 4}\sqrt{x^2 + 1} - \frac{1}{\sqrt{x^4 + 3x^2 + 2}} + \frac{135i}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((5*x^2+7)^3/(x^4+3*x^2+2)^(1/2),x)`

[Out] `25*x^3*(x^4+3*x^2+2)^(1/2)+75*x*(x^4+3*x^2+2)^(1/2)-193/2*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticF(1/2*I*x*2^(1/2),2^(1/2))+135/2*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*(EllipticF(1/2*I*x*2^(1/2),2^(1/2))-EllipticE(1/2*I*x*2^(1/2),2^(1/2)))`

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(5x^2 + 7)^3}{\sqrt{x^4 + 3x^2 + 2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((5*x^2+7)^3/(x^4+3*x^2+2)^(1/2),x, algorithm="maxima")`

[Out] `integrate((5*x^2 + 7)^3/sqrt(x^4 + 3*x^2 + 2), x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{125x^6 + 525x^4 + 735x^2 + 343}{\sqrt{x^4 + 3x^2 + 2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((5*x^2+7)^3/(x^4+3*x^2+2)^(1/2),x, algorithm="fricas")`

[Out] `integral((125*x^6 + 525*x^4 + 735*x^2 + 343)/sqrt(x^4 + 3*x^2 + 2), x)`

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(5x^2 + 7)^3}{\sqrt{(x^2 + 1)(x^2 + 2)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((5*x**2+7)**3/(x**4+3*x**2+2)**(1/2),x)`

[Out] `Integral((5*x**2 + 7)**3/sqrt((x**2 + 1)*(x**2 + 2)), x)`

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(5x^2 + 7)^3}{\sqrt{x^4 + 3x^2 + 2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.


```
[In] integrate((5*x^2+7)^3/(x^4+3*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((5*x^2 + 7)^3/sqrt(x^4 + 3*x^2 + 2), x)
```

$$3.301 \quad \int \frac{(7+5x^2)^2}{\sqrt{2+3x^2+x^4}} dx$$

Optimal. Leaf size=142

$$\frac{97(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\text{EllipticF}\left(\tan^{-1}(x), \frac{1}{2}\right)}{3\sqrt{2}\sqrt{x^4+3x^2+2}} + \frac{25\sqrt{x^4+3x^2+2}}{3} + \frac{20(x^2+2)x}{\sqrt{x^4+3x^2+2}} - \frac{20\sqrt{2}(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}E\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{\sqrt{x^4+3x^2+2}}$$

[Out] (20*x*(2 + x^2))/Sqrt[2 + 3*x^2 + x^4] + (25*x*Sqrt[2 + 3*x^2 + x^4])/3 - (20*Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x], 1/2])/Sqrt[2 + 3*x^2 + x^4] + (97*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(3*Sqrt[2]*Sqrt[2 + 3*x^2 + x^4])

Rubi [A] time = 0.0543072, antiderivative size = 142, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1206, 1189, 1099, 1135}

$$\frac{25\sqrt{x^4+3x^2+2}}{3} + \frac{20(x^2+2)x}{\sqrt{x^4+3x^2+2}} + \frac{97(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}F\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{3\sqrt{2}\sqrt{x^4+3x^2+2}} - \frac{20\sqrt{2}(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}E\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

[In] Int[(7 + 5*x^2)^2/Sqrt[2 + 3*x^2 + x^4], x]

[Out] (20*x*(2 + x^2))/Sqrt[2 + 3*x^2 + x^4] + (25*x*Sqrt[2 + 3*x^2 + x^4])/3 - (20*Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x], 1/2])/Sqrt[2 + 3*x^2 + x^4] + (97*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(3*Sqrt[2]*Sqrt[2 + 3*x^2 + x^4])

Rule 1206

Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(e^q*x^(2*q - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(4*p + 2*q + 1)), x] + Dist[1/(c*(4*p + 2*q + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandT oSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*p + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1]

Rule 1189

Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a] || PosQ[(b - q)/a] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]

Rule 1099

Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[((2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q)*x^2)/(2*a + (b + q)*x^2)]*EllipticF[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)/(b + q)])/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] && !(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)]) /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]

Rule 1135

Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(x*(b + q + 2*c*x^2))/(2*c*Sqrt[a + b*x^2 + c*x^4]), x] - Simp[(Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q)*x^2)/(2*a + (b + q)*x^2)]*EllipticE[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)/(b + q)])/(2*c*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] && !(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]

Rubi steps

$$\begin{aligned} \int \frac{(7 + 5x^2)^2}{\sqrt{2 + 3x^2 + x^4}} dx &= \frac{25}{3}x\sqrt{2 + 3x^2 + x^4} + \frac{1}{3} \int \frac{97 + 60x^2}{\sqrt{2 + 3x^2 + x^4}} dx \\ &= \frac{25}{3}x\sqrt{2 + 3x^2 + x^4} + 20 \int \frac{x^2}{\sqrt{2 + 3x^2 + x^4}} dx + \frac{97}{3} \int \frac{1}{\sqrt{2 + 3x^2 + x^4}} dx \\ &= \frac{20x(2 + x^2)}{\sqrt{2 + 3x^2 + x^4}} + \frac{25}{3}x\sqrt{2 + 3x^2 + x^4} - \frac{20\sqrt{2}(1 + x^2)\sqrt{\frac{2+x^2}{1+x^2}}E\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{\sqrt{2 + 3x^2 + x^4}} + \frac{97(1 + x^2)\sqrt{\frac{2}{1+x^2}}}{3\sqrt{2}\sqrt{2}} \end{aligned}$$

Mathematica [C] time = 0.0879792, size = 104, normalized size = 0.73

$$\frac{-37i\sqrt{x^2+1}\sqrt{x^2+2}\text{EllipticF}\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right), 2\right) + 25x(x^4 + 3x^2 + 2) - 60i\sqrt{x^2+1}\sqrt{x^2+2}E\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)}{3\sqrt{x^4 + 3x^2 + 2}}$$

Antiderivative was successfully verified.

[In] Integrate[(7 + 5*x^2)^2/Sqrt[2 + 3*x^2 + x^4], x]

[Out] (25*x*(2 + 3*x^2 + x^4) - (60*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticE[I*ArcSinh[x/Sqrt[2]], 2] - (37*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticF[I*ArcSinh[x/Sqrt[2]], 2])/(3*Sqrt[2 + 3*x^2 + x^4])

Maple [C] time = 0.007, size = 121, normalized size = 0.9

$$\frac{25x}{3}\sqrt{x^4 + 3x^2 + 2} - \frac{97i}{6}\sqrt{2}\text{EllipticF}\left(\frac{i}{2}x\sqrt{2}, \sqrt{2}\right)\sqrt{2x^2 + 4}\sqrt{x^2 + 1} - \frac{1}{\sqrt{x^4 + 3x^2 + 2}} + 10i\sqrt{2}\left(\text{EllipticF}\left(\frac{i}{2}x\sqrt{2}, \sqrt{2}\right) - \text{EllipticE}\left(\frac{i}{2}x\sqrt{2}, \sqrt{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((5*x^2+7)^2/(x^4+3*x^2+2)^(1/2), x)

[Out] 25/3*x*(x^4+3*x^2+2)^(1/2)-97/6*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticF(1/2*I*x*2^(1/2), 2^(1/2))+10*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*(EllipticF(1/2*I*x*2^(1/2), 2^(1/2))-EllipticE(1/2*I*x*2^(1/2), 2^(1/2)))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(5x^2 + 7)^2}{\sqrt{x^4 + 3x^2 + 2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^2+7)^2/(x^4+3*x^2+2)^(1/2),x, algorithm="maxima")

[Out] integrate((5*x^2 + 7)^2/sqrt(x^4 + 3*x^2 + 2), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{25x^4 + 70x^2 + 49}{\sqrt{x^4 + 3x^2 + 2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^2+7)^2/(x^4+3*x^2+2)^(1/2),x, algorithm="fricas")

[Out] integral((25*x^4 + 70*x^2 + 49)/sqrt(x^4 + 3*x^2 + 2), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(5x^2 + 7)^2}{\sqrt{(x^2 + 1)(x^2 + 2)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x**2+7)**2/(x**4+3*x**2+2)**(1/2),x)

[Out] Integral((5*x**2 + 7)**2/sqrt((x**2 + 1)*(x**2 + 2)), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(5x^2 + 7)^2}{\sqrt{x^4 + 3x^2 + 2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^2+7)^2/(x^4+3*x^2+2)^(1/2),x, algorithm="giac")

[Out] integrate((5*x^2 + 7)^2/sqrt(x^4 + 3*x^2 + 2), x)

$$3.302 \quad \int \frac{7+5x^2}{\sqrt{2+3x^2+x^4}} dx$$

Optimal. Leaf size=121

$$\frac{7(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\text{EllipticF}\left(\tan^{-1}(x), \frac{1}{2}\right)}{\sqrt{2}\sqrt{x^4+3x^2+2}} + \frac{5x(x^2+2)}{\sqrt{x^4+3x^2+2}} - \frac{5\sqrt{2}(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}E\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{\sqrt{x^4+3x^2+2}}$$

[Out] (5*x*(2 + x^2))/Sqrt[2 + 3*x^2 + x^4] - (5*Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x], 1/2])/Sqrt[2 + 3*x^2 + x^4] + (7*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(Sqrt[2]*Sqrt[2 + 3*x^2 + x^4])

Rubi [A] time = 0.0324471, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {1189, 1099, 1135}

$$\frac{5x(x^2+2)}{\sqrt{x^4+3x^2+2}} + \frac{7(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}F\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^4+3x^2+2}} - \frac{5\sqrt{2}(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}E\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

[In] Int[(7 + 5*x^2)/Sqrt[2 + 3*x^2 + x^4], x]

[Out] (5*x*(2 + x^2))/Sqrt[2 + 3*x^2 + x^4] - (5*Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x], 1/2])/Sqrt[2 + 3*x^2 + x^4] + (7*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(Sqrt[2]*Sqrt[2 + 3*x^2 + x^4])

Rule 1189

Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a] || PosQ[(b - q)/a] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]

Rule 1099

Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[((2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q)*x^2)/(2*a + (b + q)*x^2)]*EllipticF[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)/(b + q)])/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] && !(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]

Rule 1135

Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(x*(b + q + 2*c*x^2))/(2*c*Sqrt[a + b*x^2 + c*x^4]), x] - Simp[(Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q)*x^2)/(2*a + (b + q)*x^2)]*EllipticE[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)/(b + q)])/(2*c*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] && !(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]

Rubi steps

$$\int \frac{7 + 5x^2}{\sqrt{2 + 3x^2 + x^4}} dx = 5 \int \frac{x^2}{\sqrt{2 + 3x^2 + x^4}} dx + 7 \int \frac{1}{\sqrt{2 + 3x^2 + x^4}} dx$$

$$= \frac{5x(2 + x^2)}{\sqrt{2 + 3x^2 + x^4}} - \frac{5\sqrt{2}(1 + x^2)\sqrt{\frac{2+x^2}{1+x^2}}E\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{\sqrt{2 + 3x^2 + x^4}} + \frac{7(1 + x^2)\sqrt{\frac{2+x^2}{1+x^2}}F\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{\sqrt{2}\sqrt{2 + 3x^2 + x^4}}$$

Mathematica [C] time = 0.0608012, size = 69, normalized size = 0.57

$$\frac{i\sqrt{x^2+1}\sqrt{x^2+2}\left(2\text{EllipticF}\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right),2\right)+5E\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)\right)}{\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

[In] Integrate[(7 + 5*x^2)/Sqrt[2 + 3*x^2 + x^4], x]

[Out] ((-1)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*(5*EllipticE[I*ArcSinh[x/Sqrt[2]], 2] + 2*EllipticF[I*ArcSinh[x/Sqrt[2]], 2]))/Sqrt[2 + 3*x^2 + x^4]

Maple [C] time = 0.004, size = 106, normalized size = 0.9

$$\frac{5i}{2}\sqrt{2}\left(\text{EllipticF}\left(\frac{i}{2}x\sqrt{2},\sqrt{2}\right)-\text{EllipticE}\left(\frac{i}{2}x\sqrt{2},\sqrt{2}\right)\right)\sqrt{2x^2+4}\sqrt{x^2+1}\frac{1}{\sqrt{x^4+3x^2+2}}-\frac{7i}{2}\sqrt{2}\text{EllipticF}\left(\frac{i}{2}x\sqrt{2},\sqrt{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((5*x^2+7)/(x^4+3*x^2+2)^(1/2), x)

[Out] 5/2*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*(EllipticF(1/2*I*x*2^(1/2),2^(1/2))-EllipticE(1/2*I*x*2^(1/2),2^(1/2)))-7/2*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticF(1/2*I*x*2^(1/2),2^(1/2))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{5x^2 + 7}{\sqrt{x^4 + 3x^2 + 2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^2+7)/(x^4+3*x^2+2)^(1/2), x, algorithm="maxima")

[Out] integrate((5*x^2 + 7)/sqrt(x^4 + 3*x^2 + 2), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{5x^2 + 7}{\sqrt{x^4 + 3x^2 + 2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)/(x^4+3*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((5*x^2 + 7)/sqrt(x^4 + 3*x^2 + 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{5x^2 + 7}{\sqrt{(x^2 + 1)(x^2 + 2)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+7)/(x**4+3*x**2+2)**(1/2),x)
```

```
[Out] Integral((5*x**2 + 7)/sqrt((x**2 + 1)*(x**2 + 2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{5x^2 + 7}{\sqrt{x^4 + 3x^2 + 2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)/(x^4+3*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((5*x^2 + 7)/sqrt(x^4 + 3*x^2 + 2), x)
```

$$3.303 \quad \int \frac{1}{\sqrt{2+3x^2+x^4}} dx$$

Optimal. Leaf size=48

$$\frac{(x^2 + 1) \sqrt{\frac{x^2+2}{x^2+1}} \operatorname{EllipticF}\left(\tan^{-1}(x), \frac{1}{2}\right)}{\sqrt{2}\sqrt{x^4 + 3x^2 + 2}}$$

[Out] ((1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(Sqrt[2]*Sqrt[2 + 3*x^2 + x^4])

Rubi [A] time = 0.0060736, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {1099}

$$\frac{(x^2 + 1) \sqrt{\frac{x^2+2}{x^2+1}} F\left(\tan^{-1}(x) \middle| \frac{1}{2}\right)}{\sqrt{2}\sqrt{x^4 + 3x^2 + 2}}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[2 + 3*x^2 + x^4], x]

[Out] ((1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(Sqrt[2]*Sqrt[2 + 3*x^2 + x^4])

Rule 1099

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[((2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q)*x^2)/(2*a + (b + q)*x^2)]*EllipticF[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)/(b + q)])/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] && !(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{2+3x^2+x^4}} dx = \frac{(1+x^2) \sqrt{\frac{2+x^2}{1+x^2}} F\left(\tan^{-1}(x) \middle| \frac{1}{2}\right)}{\sqrt{2}\sqrt{2+3x^2+x^4}}$$

Mathematica [C] time = 0.0149432, size = 50, normalized size = 1.04

$$\frac{i\sqrt{x^2+1}\sqrt{x^2+2}\operatorname{EllipticF}\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right), 2\right)}{\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[2 + 3*x^2 + x^4], x]

[Out] ((-I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticF[I*ArcSinh[x/Sqrt[2]], 2])/Sqrt[2 + 3*x^2 + x^4]

Maple [C] time = 0.003, size = 46, normalized size = 1.

$$-\frac{i}{2}\sqrt{2}\operatorname{EllipticF}\left(\frac{i}{2}x\sqrt{2}, \sqrt{2}\right)\sqrt{2x^2+4}\sqrt{x^2+1}\frac{1}{\sqrt{x^4+3x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^4+3*x^2+2)^(1/2), x)

[Out] -1/2*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticF(1/2*I*x*2^(1/2), 2^(1/2))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^4+3x^2+2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^4+3*x^2+2)^(1/2), x, algorithm="maxima")

[Out] integrate(1/sqrt(x^4 + 3*x^2 + 2), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{1}{\sqrt{x^4+3x^2+2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^4+3*x^2+2)^(1/2), x, algorithm="fricas")

[Out] integral(1/sqrt(x^4 + 3*x^2 + 2), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^4+3x^2+2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x**4+3*x**2+2)**(1/2), x)

[Out] Integral(1/sqrt(x**4 + 3*x**2 + 2), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^4+3x^2+2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+3*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(x^4 + 3*x^2 + 2), x)
```

$$3.304 \quad \int \frac{1}{(7+5x^2)\sqrt{2+3x^2+x^4}} dx$$

Optimal. Leaf size=106

$$\frac{(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\text{EllipticF}\left(\tan^{-1}(x), \frac{1}{2}\right)}{2\sqrt{2}\sqrt{x^4+3x^2+2}} - \frac{5(x^2+2)\Pi\left(\frac{2}{7}; \tan^{-1}(x)\middle|\frac{1}{2}\right)}{14\sqrt{2}\sqrt{\frac{x^2+2}{x^2+1}}\sqrt{x^4+3x^2+2}}$$

[Out] $((1+x^2)*\text{Sqrt}[(2+x^2)/(1+x^2)]*\text{EllipticF}[\text{ArcTan}[x], 1/2])/(2*\text{Sqrt}[2]*\text{Sqrt}[2+3*x^2+x^4]) - (5*(2+x^2)*\text{EllipticPi}[2/7, \text{ArcTan}[x], 1/2])/(14*\text{Sqrt}[2]*\text{Sqrt}[(2+x^2)/(1+x^2)]*\text{Sqrt}[2+3*x^2+x^4])$

Rubi [A] time = 0.0722979, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1214, 1099, 1456, 539}

$$\frac{(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}F\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{2\sqrt{2}\sqrt{x^4+3x^2+2}} - \frac{5(x^2+2)\Pi\left(\frac{2}{7}; \tan^{-1}(x)\middle|\frac{1}{2}\right)}{14\sqrt{2}\sqrt{\frac{x^2+2}{x^2+1}}\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

[In] Int[1/((7 + 5*x^2)*Sqrt[2 + 3*x^2 + x^4]),x]

[Out] $((1+x^2)*\text{Sqrt}[(2+x^2)/(1+x^2)]*\text{EllipticF}[\text{ArcTan}[x], 1/2])/(2*\text{Sqrt}[2]*\text{Sqrt}[2+3*x^2+x^4]) - (5*(2+x^2)*\text{EllipticPi}[2/7, \text{ArcTan}[x], 1/2])/(14*\text{Sqrt}[2]*\text{Sqrt}[(2+x^2)/(1+x^2)]*\text{Sqrt}[2+3*x^2+x^4])$

Rule 1214

Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/(2*c*d - e*(b - q)), Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[e/(2*c*d - e*(b - q)), Int[(b - q + 2*c*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0] && !LtQ[c, 0]

Rule 1099

Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[((2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q)*x^2)/(2*a + (b + q)*x^2)]*EllipticF[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)/(b + q)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] && !(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]

Rule 1456

Int[((d_) + (e_.)*(x_)^(n_))^(q_.)*((f_) + (g_.)*(x_)^(n_))^(r_.)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(2*n_))^(p_), x_Symbol] := Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d + (c*x^n)/e)^FracPart[p]), Int[(d + e*x^n)^(p + q)*(f + g*x^n)^r*(a/d + (c*x^n)/e)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q, r}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && !IntegerQ[p]

Rule 539

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)^2]), x_Symbol] := Simp[(c*Sqrt[e + f*x^2]*EllipticPi[1 - (b*c)/(a*d), ArcTan[Rt[d/c, 2]*x], 1 - (c*f)/(d*e)])/((a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(e + f*x^2))/(e*(c + d*x^2))]), x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[d/c]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{(7+5x^2)\sqrt{2+3x^2+x^4}} dx &= \frac{1}{2} \int \frac{1}{\sqrt{2+3x^2+x^4}} dx - \frac{5}{4} \int \frac{2+2x^2}{(7+5x^2)\sqrt{2+3x^2+x^4}} dx \\ &= \frac{(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}} F\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{2\sqrt{2}\sqrt{2+3x^2+x^4}} - \frac{\left(5\sqrt{1+\frac{x^2}{2}}\sqrt{2+2x^2}\right) \int \frac{\sqrt{2+2x^2}}{\sqrt{1+\frac{x^2}{2}}(7+5x^2)} dx}{4\sqrt{2+3x^2+x^4}} \\ &= \frac{(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}} F\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{2\sqrt{2}\sqrt{2+3x^2+x^4}} - \frac{5(2+x^2)\Pi\left(\frac{2}{7}; \tan^{-1}(x)\middle|\frac{1}{2}\right)}{14\sqrt{2}\sqrt{\frac{2+x^2}{1+x^2}}\sqrt{2+3x^2+x^4}} \end{aligned}$$

Mathematica [C] time = 0.09398, size = 55, normalized size = 0.52

$$\frac{i\sqrt{x^2+1}\sqrt{x^2+2}\Pi\left(\frac{10}{7}; i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)}{7\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((7 + 5*x^2)*Sqrt[2 + 3*x^2 + x^4]),x]
```

```
[Out] ((-I/7)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticPi[10/7, I*ArcSinh[x/Sqrt[2]], 2])/Sqrt[2 + 3*x^2 + x^4]
```

Maple [C] time = 0.012, size = 47, normalized size = 0.4

$$-\frac{i}{7}\sqrt{2}\sqrt{1+\frac{x^2}{2}}\sqrt{x^2+1}\text{EllipticPi}\left(\frac{i}{2}x\sqrt{2}, \frac{10}{7}, \sqrt{2}\right)\frac{1}{\sqrt{x^4+3x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5*x^2+7)/(x^4+3*x^2+2)^(1/2),x)
```

```
[Out] -1/7*I*2^(1/2)*(1+1/2*x^2)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticPi(1/2*I*x*2^(1/2),10/7,2^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^4+3x^2+2}(5x^2+7)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(5*x^2+7)/(x^4+3*x^2+2)^(1/2),x, algorithm="maxima")

[Out] integrate(1/(sqrt(x^4 + 3*x^2 + 2)*(5*x^2 + 7)), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{x^4 + 3x^2 + 2}}{5x^6 + 22x^4 + 31x^2 + 14}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(5*x^2+7)/(x^4+3*x^2+2)^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(x^4 + 3*x^2 + 2)/(5*x^6 + 22*x^4 + 31*x^2 + 14), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{(x^2 + 1)(x^2 + 2)}(5x^2 + 7)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(5*x**2+7)/(x**4+3*x**2+2)**(1/2),x)

[Out] Integral(1/(sqrt((x**2 + 1)*(x**2 + 2))*(5*x**2 + 7)), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^4 + 3x^2 + 2}(5x^2 + 7)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(5*x^2+7)/(x^4+3*x^2+2)^(1/2),x, algorithm="giac")

[Out] integrate(1/(sqrt(x^4 + 3*x^2 + 2)*(5*x^2 + 7)), x)

$$3.305 \quad \int \frac{1}{(7+5x^2)^2 \sqrt{2+3x^2+x^4}} dx$$

Optimal. Leaf size=209

$$\frac{9(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\text{EllipticF}\left(\tan^{-1}(x), \frac{1}{2}\right)}{56\sqrt{2}\sqrt{x^4+3x^2+2}} - \frac{25\sqrt{x^4+3x^2+2}x}{84(5x^2+7)} + \frac{5(x^2+2)x}{84\sqrt{x^4+3x^2+2}} - \frac{5(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}E\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{42\sqrt{2}\sqrt{x^4+3x^2+2}} - \frac{65(x^2+2)\text{EllipticPi}\left[\frac{2}{7}, \text{ArcTan}[x], \frac{1}{2}\right]}{1176\sqrt{2}\sqrt{x^4+3x^2+2}}$$

[Out] (5*x*(2 + x^2))/(84*Sqrt[2 + 3*x^2 + x^4]) - (25*x*Sqrt[2 + 3*x^2 + x^4])/(84*(7 + 5*x^2)) - (5*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x], 1/2])/(42*Sqrt[2]*Sqrt[2 + 3*x^2 + x^4]) + (9*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(56*Sqrt[2]*Sqrt[2 + 3*x^2 + x^4]) - (65*(2 + x^2)*EllipticPi[2/7, ArcTan[x], 1/2])/(1176*Sqrt[2]*Sqrt[(2 + x^2)/(1 + x^2)]*Sqrt[2 + 3*x^2 + x^4])

Rubi [A] time = 0.187819, antiderivative size = 209, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1223, 1716, 1189, 1099, 1135, 1214, 1456, 539}

$$-\frac{25\sqrt{x^4+3x^2+2}x}{84(5x^2+7)} + \frac{5(x^2+2)x}{84\sqrt{x^4+3x^2+2}} + \frac{9(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}F\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{56\sqrt{2}\sqrt{x^4+3x^2+2}} - \frac{5(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}E\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{42\sqrt{2}\sqrt{x^4+3x^2+2}} - \frac{65(x^2+2)\text{EllipticPi}\left[\frac{2}{7}, \text{ArcTan}[x], \frac{1}{2}\right]}{1176\sqrt{2}\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

[In] Int[1/((7 + 5*x^2)^2*Sqrt[2 + 3*x^2 + x^4]),x]

[Out] (5*x*(2 + x^2))/(84*Sqrt[2 + 3*x^2 + x^4]) - (25*x*Sqrt[2 + 3*x^2 + x^4])/(84*(7 + 5*x^2)) - (5*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x], 1/2])/(42*Sqrt[2]*Sqrt[2 + 3*x^2 + x^4]) + (9*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(56*Sqrt[2]*Sqrt[2 + 3*x^2 + x^4]) - (65*(2 + x^2)*EllipticPi[2/7, ArcTan[x], 1/2])/(1176*Sqrt[2]*Sqrt[(2 + x^2)/(1 + x^2)]*Sqrt[2 + 3*x^2 + x^4])

Rule 1223

Int[((d_) + (e_)*(x_)^2)^(q_)/Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4], x_Symbol] := -Simp[(e^2*x*(d + e*x^2)^(q + 1)*Sqrt[a + b*x^2 + c*x^4])/(2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2)), Int[((d + e*x^2)^(q + 1)*Simp[a*e^2*(2*q + 3) + 2*d*(c*d - b*e)*(q + 1) - 2*e*(c*d*(q + 1) - b*e*(q + 2))*x^2 + c*e^2*(2*q + 5)*x^4, x])/Sqrt[a + b*x^2 + c*x^4], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && ILtQ[q, -1]

Rule 1716

Int[(P4x_)/(((d_) + (e_)*(x_)^2)*Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4]), x_Symbol] := With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[P4x, x, 4]}, -Dist[(e^2)^(-1), Int[(C*d - B*e - C*e*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] + Dist[(C*d^2 - B*d*e + A*e^2)/e^2, Int[1/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[P4x, x^2, 2] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0]

Rule 1189

```
Int[((d_) + (e_)*(x_)^2)/Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4], x_Symbol]
:> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a]
] || PosQ[(b - q)/a] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1099

```
Int[1/Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[((2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]*EllipticF[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)/(b + q)])/(2*
a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1135

```
Int[(x_)^2/Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4], x_Symbol] :> With[{q =
Rt[b^2 - 4*a*c, 2]}, Simp[(x*(b + q + 2*c*x^2))/(2*c*Sqrt[a + b*x^2 + c*x^
4]), x] - Simp[(Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q)
)*x^2)/(2*a + (b + q)*x^2)]*EllipticE[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)
/(b + q)])/(2*c*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] && !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1214

```
Int[1/(((d_) + (e_)*(x_)^2)*Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/(2*c*d - e*(b - q)), In
t[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[e/(2*c*d - e*(b - q)), Int[(b -
q + 2*c*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x] /; FreeQ[{a, b,
c, d, e}, x] && GtQ[b^2 - 4*a*c, 0] && !LtQ[c, 0]
```

Rule 1456

```
Int[((d_) + (e_)*(x_)^(n_))^(q_)*((f_) + (g_)*(x_)^(n_))^(r_)*((a_) + (
b_)*(x_)^(n_) + (c_)*(x_)^(n2_))^(p_), x_Symbol] :> Dist[(a + b*x^n + c*x
^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d + (c*x^n)/e)^FracPart[p])
, Int[(d + e*x^n)^(p + q)*(f + g*x^n)^r*(a/d + (c*x^n)/e)^p, x], x] /; Free
Q[{a, b, c, d, e, f, g, n, p, q, r}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && !IntegerQ[p]
```

Rule 539

```
Int[Sqrt[(c_) + (d_)*(x_)^2]/(((a_) + (b_)*(x_)^2)*Sqrt[(e_) + (f_)*(x_)
^2]), x_Symbol] :> Simp[(c*Sqrt[e + f*x^2]*EllipticPi[1 - (b*c)/(a*d), ArcT
an[Rt[d/c, 2]*x], 1 - (c*f)/(d*e)]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c
*(e + f*x^2))/(e*(c + d*x^2))]), x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(7+5x^2)^2 \sqrt{2+3x^2+x^4}} dx &= -\frac{25x\sqrt{2+3x^2+x^4}}{84(7+5x^2)} + \frac{1}{84} \int \frac{62+70x^2+25x^4}{(7+5x^2)\sqrt{2+3x^2+x^4}} dx \\
&= -\frac{25x\sqrt{2+3x^2+x^4}}{84(7+5x^2)} - \frac{\int \frac{-175-125x^2}{\sqrt{2+3x^2+x^4}} dx}{2100} + \frac{13}{84} \int \frac{1}{(7+5x^2)\sqrt{2+3x^2+x^4}} dx \\
&= -\frac{25x\sqrt{2+3x^2+x^4}}{84(7+5x^2)} + \frac{5}{84} \int \frac{x^2}{\sqrt{2+3x^2+x^4}} dx + \frac{13}{168} \int \frac{1}{\sqrt{2+3x^2+x^4}} dx + \frac{1}{12} \int \frac{1}{7+5x^2} dx \\
&= \frac{5x(2+x^2)}{84\sqrt{2+3x^2+x^4}} - \frac{25x\sqrt{2+3x^2+x^4}}{84(7+5x^2)} - \frac{5(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}} E\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{42\sqrt{2}\sqrt{2+3x^2+x^4}} + \frac{9(1+x^2)}{56\sqrt{2}\sqrt{2+3x^2+x^4}} \\
&= \frac{5x(2+x^2)}{84\sqrt{2+3x^2+x^4}} - \frac{25x\sqrt{2+3x^2+x^4}}{84(7+5x^2)} - \frac{5(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}} E\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{42\sqrt{2}\sqrt{2+3x^2+x^4}} + \frac{9(1+x^2)}{56\sqrt{2}\sqrt{2+3x^2+x^4}}
\end{aligned}$$

Mathematica [C] time = 0.257523, size = 208, normalized size = 1.

$$\frac{-14i\sqrt{x^2+1}\sqrt{x^2+2}(5x^2+7)\text{EllipticF}\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right),2\right)-175x^5-525x^3-35i\sqrt{x^2+1}\sqrt{x^2+2}(5x^2+7)E\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\right)}{588(5x^2+7)\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/((7 + 5*x^2)^2*Sqrt[2 + 3*x^2 + x^4]),x]

[Out] (-350*x - 525*x^3 - 175*x^5 - (35*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*(7 + 5*x^2)*EllipticE[I*ArcSinh[x/Sqrt[2]], 2] - (14*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*(7 + 5*x^2)*EllipticF[I*ArcSinh[x/Sqrt[2]], 2] - (91*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticPi[10/7, I*ArcSinh[x/Sqrt[2]], 2] - (65*I)*x^2*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticPi[10/7, I*ArcSinh[x/Sqrt[2]], 2])/(588*(7 + 5*x^2)*Sqrt[2 + 3*x^2 + x^4])

Maple [C] time = 0.016, size = 162, normalized size = 0.8

$$-\frac{25x}{420x^2+588}\sqrt{x^4+3x^2+2}-\frac{i}{84}\sqrt{2}\text{EllipticF}\left(\frac{i}{2}x\sqrt{2},\sqrt{2}\right)\sqrt{2x^2+4}\sqrt{x^2+1}\frac{1}{\sqrt{x^4+3x^2+2}}-\frac{5i}{168}\sqrt{2}\text{EllipticE}\left(\frac{i}{2}x\sqrt{2},\sqrt{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(5*x^2+7)^2/(x^4+3*x^2+2)^(1/2),x)

[Out] -25/84*x*(x^4+3*x^2+2)^(1/2)/(5*x^2+7)-1/84*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticF(1/2*I*x*2^(1/2),2^(1/2))-5/168*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticE(1/2*I*x*2^(1/2),2^(1/2))-13/588*I*2^(1/2)*(1+1/2*x^2)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticPi(1/2*I*x*2^(1/2),10/7,2^(1/2))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^4 + 3x^2 + 2}(5x^2 + 7)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(5*x^2+7)^2/(x^4+3*x^2+2)^(1/2),x, algorithm="maxima")

[Out] integrate(1/(sqrt(x^4 + 3*x^2 + 2)*(5*x^2 + 7)^2), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{x^4 + 3x^2 + 2}}{25x^8 + 145x^6 + 309x^4 + 287x^2 + 98}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(5*x^2+7)^2/(x^4+3*x^2+2)^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(x^4 + 3*x^2 + 2)/(25*x^8 + 145*x^6 + 309*x^4 + 287*x^2 + 98), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{(x^2 + 1)(x^2 + 2)}(5x^2 + 7)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(5*x**2+7)**2/(x**4+3*x**2+2)**(1/2),x)

[Out] Integral(1/(sqrt((x**2 + 1)*(x**2 + 2))*(5*x**2 + 7)**2), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^4 + 3x^2 + 2}(5x^2 + 7)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(5*x^2+7)^2/(x^4+3*x^2+2)^(1/2),x, algorithm="giac")

[Out] integrate(1/(sqrt(x^4 + 3*x^2 + 2)*(5*x^2 + 7)^2), x)

$$3.306 \quad \int \frac{1}{(7+5x^2)^3 \sqrt{2+3x^2+x^4}} dx$$

Optimal. Leaf size=237

$$\frac{631(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\text{EllipticF}\left(\tan^{-1}(x), \frac{1}{2}\right)}{9408\sqrt{2}\sqrt{x^4+3x^2+2}} - \frac{325\sqrt{x^4+3x^2+2}x}{4704(5x^2+7)} - \frac{25\sqrt{x^4+3x^2+2}x}{168(5x^2+7)^2} + \frac{65(x^2+2)x}{4704\sqrt{x^4+3x^2+2}} - \frac{65(x^2+2)}{2352\sqrt{2}\sqrt{x^4+3x^2+2}}$$

```
[Out] (65*x*(2 + x^2))/(4704*Sqrt[2 + 3*x^2 + x^4]) - (25*x*Sqrt[2 + 3*x^2 + x^4])/(168*(7 + 5*x^2)^2) - (325*x*Sqrt[2 + 3*x^2 + x^4])/(4704*(7 + 5*x^2)) - (65*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x], 1/2])/(2352*Sqrt[2]*Sqrt[2 + 3*x^2 + x^4]) + (631*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(9408*Sqrt[2]*Sqrt[2 + 3*x^2 + x^4]) - (2525*(2 + x^2)*EllipticPi[2/7, ArcTan[x], 1/2])/(65856*Sqrt[2]*Sqrt[(2 + x^2)/(1 + x^2)])*Sqrt[2 + 3*x^2 + x^4]
```

Rubi [A] time = 0.249866, antiderivative size = 237, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {1223, 1696, 1716, 1189, 1099, 1135, 1214, 1456, 539}

$$-\frac{325\sqrt{x^4+3x^2+2}x}{4704(5x^2+7)} - \frac{25\sqrt{x^4+3x^2+2}x}{168(5x^2+7)^2} + \frac{65(x^2+2)x}{4704\sqrt{x^4+3x^2+2}} + \frac{631(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}F\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{9408\sqrt{2}\sqrt{x^4+3x^2+2}} - \frac{65(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}}{2352\sqrt{2}\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((7 + 5*x^2)^3*Sqrt[2 + 3*x^2 + x^4]), x]
```

```
[Out] (65*x*(2 + x^2))/(4704*Sqrt[2 + 3*x^2 + x^4]) - (25*x*Sqrt[2 + 3*x^2 + x^4])/(168*(7 + 5*x^2)^2) - (325*x*Sqrt[2 + 3*x^2 + x^4])/(4704*(7 + 5*x^2)) - (65*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x], 1/2])/(2352*Sqrt[2]*Sqrt[2 + 3*x^2 + x^4]) + (631*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(9408*Sqrt[2]*Sqrt[2 + 3*x^2 + x^4]) - (2525*(2 + x^2)*EllipticPi[2/7, ArcTan[x], 1/2])/(65856*Sqrt[2]*Sqrt[(2 + x^2)/(1 + x^2)])*Sqrt[2 + 3*x^2 + x^4]
```

Rule 1223

```
Int[((d_) + (e_)*(x_)^2)^(q_)/Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4], x_
Symbol] := -Simp[(e^2*x*(d + e*x^2)^(q + 1)*Sqrt[a + b*x^2 + c*x^4])/(2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2)), Int[((d + e*x^2)^(q + 1)*Simp[a*e^2*(2*q + 3) + 2*d*(c*d - b*e)*(q + 1) - 2*e*(c*d*(q + 1) - b*e*(q + 2))*x^2 + c*e^2*(2*q + 5)*x^4, x])/Sqrt[a + b*x^2 + c*x^4], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && ILtQ[q, -1]
```

Rule 1696

```
Int[((P4x_)*((d_) + (e_)*(x_)^2)^(q_))/Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4], x_Symbol] := With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[P4x, x, 4]}, -Simp[((C*d^2 - B*d*e + A*e^2)*x*(d + e*x^2)^(q + 1)*Sqrt[a + b*x^2 + c*x^4])/(2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2)), Int[((d + e*x^2)^(q + 1)*Simp[a*d*(C*d - B*e) + A*(a*e^2*(2*q + 3) + 2*d*(c*d - b*e)*(q + 1)) - 2*((B*d - A*e)*(b*e*(q + 2) - c*d*(q + 1)) - C*d*(b*d + a*e*(q + 1)))*x^2 + c*(C*d^2 - B*d*e
```

+ A*e^2)*(2*q + 5)*x^4, x])/Sqrt[a + b*x^2 + c*x^4], x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[P4x, x^2] && LeQ[Expon[P4x, x], 4] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, -1]

Rule 1716

Int[(P4x_)/(((d_) + (e_)*(x_)^2)*Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4]), x_Symbol] := With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[P4x, x, 4]}, -Dist[(e^2)^(-1), Int[(C*d - B*e - C*e*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] + Dist[(C*d^2 - B*d*e + A*e^2)/e^2, Int[1/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[P4x, x^2, 2] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0]

Rule 1189

Int[((d_) + (e_)*(x_)^2)/Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4], x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]

Rule 1099

Int[1/Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4], x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[((2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q)*x^2)/(2*a + (b + q)*x^2)]*EllipticF[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)/(b + q)])/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] && !(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]

Rule 1135

Int[(x_)^2/Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4], x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(x*(b + q + 2*c*x^2))/(2*c*Sqrt[a + b*x^2 + c*x^4]), x] - Simp[(Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q)*x^2)/(2*a + (b + q)*x^2)]*EllipticE[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)/(b + q)])/(2*c*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] && !(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]

Rule 1214

Int[1/(((d_) + (e_)*(x_)^2)*Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4]), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/(2*c*d - e*(b - q)), Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[e/(2*c*d - e*(b - q)), Int[(b - q + 2*c*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0] && !LtQ[c, 0]

Rule 1456

Int[((d_) + (e_)*(x_)^(n_))^(q_)*((f_) + (g_)*(x_)^(n_))^(r_)*((a_) + (b_)*(x_)^(n_) + (c_)*(x_)^(2*n_))^(p_), x_Symbol] := Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d + (c*x^n)/e)^FracPart[p]), Int[(d + e*x^n)^(p + q)*(f + g*x^n)^r*(a/d + (c*x^n)/e)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q, r}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && !IntegerQ[p]

Rule 539

```
Int[Sqrt[(c_) + (d_)*(x_)^2]/(((a_) + (b_)*(x_)^2)*Sqrt[(e_) + (f_)*(x_)^2]), x_Symbol] := Simp[(c*Sqrt[e + f*x^2]*EllipticPi[1 - (b*c)/(a*d), ArcTan[Rt[d/c, 2]*x], 1 - (c*f)/(d*e)])/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(e + f*x^2))/(e*(c + d*x^2))]), x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[d/c]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{(7 + 5x^2)^3 \sqrt{2 + 3x^2 + x^4}} dx &= -\frac{25x\sqrt{2 + 3x^2 + x^4}}{168(7 + 5x^2)^2} + \frac{1}{168} \int \frac{74 - 10x^2 - 25x^4}{(7 + 5x^2)^2 \sqrt{2 + 3x^2 + x^4}} dx \\ &= -\frac{25x\sqrt{2 + 3x^2 + x^4}}{168(7 + 5x^2)^2} - \frac{325x\sqrt{2 + 3x^2 + x^4}}{4704(7 + 5x^2)} + \frac{\int \frac{2838 + 2310x^2 + 975x^4}{(7 + 5x^2)\sqrt{2 + 3x^2 + x^4}} dx}{14112} \\ &= -\frac{25x\sqrt{2 + 3x^2 + x^4}}{168(7 + 5x^2)^2} - \frac{325x\sqrt{2 + 3x^2 + x^4}}{4704(7 + 5x^2)} - \frac{\int \frac{-4725 - 4875x^2}{\sqrt{2 + 3x^2 + x^4}} dx}{352800} + \frac{505 \int \frac{1}{(7 + 5x^2)\sqrt{2 + 3x^2 + x^4}} dx}{4704} \\ &= -\frac{25x\sqrt{2 + 3x^2 + x^4}}{168(7 + 5x^2)^2} - \frac{325x\sqrt{2 + 3x^2 + x^4}}{4704(7 + 5x^2)} + \frac{3}{224} \int \frac{1}{\sqrt{2 + 3x^2 + x^4}} dx + \frac{65 \int \frac{x^2}{\sqrt{2 + 3x^2 + x^4}} dx}{4704} \\ &= \frac{65x(2 + x^2)}{4704\sqrt{2 + 3x^2 + x^4}} - \frac{25x\sqrt{2 + 3x^2 + x^4}}{168(7 + 5x^2)^2} - \frac{325x\sqrt{2 + 3x^2 + x^4}}{4704(7 + 5x^2)} - \frac{65(1 + x^2)\sqrt{\frac{2+x^2}{1+x^2}} E}{2352\sqrt{2}\sqrt{2 + 3x^2 + x^4}} \\ &= \frac{65x(2 + x^2)}{4704\sqrt{2 + 3x^2 + x^4}} - \frac{25x\sqrt{2 + 3x^2 + x^4}}{168(7 + 5x^2)^2} - \frac{325x\sqrt{2 + 3x^2 + x^4}}{4704(7 + 5x^2)} - \frac{65(1 + x^2)\sqrt{\frac{2+x^2}{1+x^2}} E}{2352\sqrt{2}\sqrt{2 + 3x^2 + x^4}} \end{aligned}$$

Mathematica [C] time = 0.334159, size = 186, normalized size = 0.78

$$\frac{14i\sqrt{x^2 + 1}\sqrt{x^2 + 2}(5x^2 + 7)^2 \operatorname{EllipticF}\left(i \sinh^{-1}\left(\frac{x}{\sqrt{2}}\right), 2\right) - 175x(65x^6 + 314x^4 + 487x^2 + 238) - 455i\sqrt{x^2 + 1}\sqrt{x^2 + 2}}{32928(5x^2 + 7)^2 \sqrt{x^4 + 3x^2 + 2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((7 + 5*x^2)^3*Sqrt[2 + 3*x^2 + x^4]), x]
```

```
[Out] (-175*x*(238 + 487*x^2 + 314*x^4 + 65*x^6) - (455*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*(7 + 5*x^2)^2*EllipticE[I*ArcSinh[x/Sqrt[2]]], 2) + (14*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*(7 + 5*x^2)^2*EllipticF[I*ArcSinh[x/Sqrt[2]], 2] - (505*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*(7 + 5*x^2)^2*EllipticPi[10/7, I*ArcSinh[x/Sqrt[2]], 2))/(32928*(7 + 5*x^2)^2*Sqrt[2 + 3*x^2 + x^4])
```

Maple [C] time = 0.017, size = 186, normalized size = 0.8

$$-\frac{25x}{168(5x^2 + 7)^2} \sqrt{x^4 + 3x^2 + 2} - \frac{325x}{23520x^2 + 32928} \sqrt{x^4 + 3x^2 + 2} + \frac{i}{4704} \sqrt{2} \operatorname{EllipticF}\left(\frac{i}{2}x\sqrt{2}, \sqrt{2}\right) \sqrt{2x^2 + 4}\sqrt{x^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(5*x^2+7)^3/(x^4+3*x^2+2)^(1/2),x)

[Out]
$$-25/168*x*(x^4+3*x^2+2)^{(1/2)}/(5*x^2+7)^2-325/4704*x*(x^4+3*x^2+2)^{(1/2)}/(5*x^2+7)+1/4704*I*2^{(1/2)}*(2*x^2+4)^{(1/2)}*(x^2+1)^{(1/2)}/(x^4+3*x^2+2)^{(1/2)}*EllipticF(1/2*I*x*2^{(1/2)},2^{(1/2)})-65/9408*I*2^{(1/2)}*(2*x^2+4)^{(1/2)}*(x^2+1)^{(1/2)}/(x^4+3*x^2+2)^{(1/2)}*EllipticE(1/2*I*x*2^{(1/2)},2^{(1/2)})-505/32928*I*2^{(1/2)}*(1+1/2*x^2)^{(1/2)}*(x^2+1)^{(1/2)}/(x^4+3*x^2+2)^{(1/2)}*EllipticPi(1/2*I*x*2^{(1/2)},10/7,2^{(1/2)})$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^4 + 3x^2 + 2}(5x^2 + 7)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(5*x^2+7)^3/(x^4+3*x^2+2)^(1/2),x, algorithm="maxima")

[Out] integrate(1/(sqrt(x^4 + 3*x^2 + 2)*(5*x^2 + 7)^3), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{x^4 + 3x^2 + 2}}{125x^{10} + 900x^8 + 2560x^6 + 3598x^4 + 2499x^2 + 686}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(5*x^2+7)^3/(x^4+3*x^2+2)^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(x^4 + 3*x^2 + 2)/(125*x^10 + 900*x^8 + 2560*x^6 + 3598*x^4 + 2499*x^2 + 686), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{(x^2 + 1)(x^2 + 2)}(5x^2 + 7)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(5*x**2+7)**3/(x**4+3*x**2+2)**(1/2),x)

[Out] Integral(1/(sqrt((x**2 + 1)*(x**2 + 2))*(5*x**2 + 7)**3), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^4 + 3x^2 + 2}(5x^2 + 7)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+7)^3/(x^4+3*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(x^4 + 3*x^2 + 2)*(5*x^2 + 7)^3), x)
```

$$3.307 \quad \int \frac{(7+5x^2)^5}{(2+3x^2+x^4)^{3/2}} dx$$

Optimal. Leaf size=189

$$\frac{15383(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\text{EllipticF}\left(\tan^{-1}(x), \frac{1}{2}\right)}{3\sqrt{2}\sqrt{x^4+3x^2+2}} + 625\sqrt{x^4+3x^2+2}x^3 + \frac{5000}{3}\sqrt{x^4+3x^2+2}x + \frac{7679(x^2+2)x}{2\sqrt{x^4+3x^2+2}} - \frac{(179x^2+115)x}{2\sqrt{x^4+3x^2+2}}$$

```
[Out] (7679*x*(2 + x^2))/(2*Sqrt[2 + 3*x^2 + x^4]) - (x*(115 + 179*x^2))/(2*Sqrt[2 + 3*x^2 + x^4]) + (5000*x*Sqrt[2 + 3*x^2 + x^4])/3 + 625*x^3*Sqrt[2 + 3*x^2 + x^4] - (7679*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x], 1/2])/(Sqrt[2]*Sqrt[2 + 3*x^2 + x^4]) + (15383*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(3*Sqrt[2]*Sqrt[2 + 3*x^2 + x^4])
```

Rubi [A] time = 0.112963, antiderivative size = 189, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {1205, 1679, 1189, 1099, 1135}

$$625\sqrt{x^4+3x^2+2}x^3 + \frac{5000}{3}\sqrt{x^4+3x^2+2}x + \frac{7679(x^2+2)x}{2\sqrt{x^4+3x^2+2}} - \frac{(179x^2+115)x}{2\sqrt{x^4+3x^2+2}} + \frac{15383(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}F\left(\tan^{-1}(x), \frac{1}{2}\right)}{3\sqrt{2}\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Int[(7 + 5*x^2)^5/(2 + 3*x^2 + x^4)^(3/2), x]
```

```
[Out] (7679*x*(2 + x^2))/(2*Sqrt[2 + 3*x^2 + x^4]) - (x*(115 + 179*x^2))/(2*Sqrt[2 + 3*x^2 + x^4]) + (5000*x*Sqrt[2 + 3*x^2 + x^4])/3 + 625*x^3*Sqrt[2 + 3*x^2 + x^4] - (7679*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x], 1/2])/(Sqrt[2]*Sqrt[2 + 3*x^2 + x^4]) + (15383*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(3*Sqrt[2]*Sqrt[2 + 3*x^2 + x^4])
```

Rule 1205

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{f = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 + c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*g - f*(b^2 - 2*a*c) - c*(b*f - 2*a*g)*x^2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[(d + e*x^2)^q, a + b*x^2 + c*x^4, x] + b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g + c*(4*p + 7)*(b*f - 2*a*g)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1] && LtQ[p, -1]
```

Rule 1679

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{q = Expon[Pq, x^2], e = Coeff[Pq, x^2, Expon[Pq, x^2]]}, Simp[(e*x^(2*q - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(2*q + 4*p + 1)), x] + Dist[1/(c*(2*q + 4*p + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*q + 4*p + 1)*Pq - a*e*(2*q - 3)*x^(2*q - 4) - b*e*(2*q + 2*p - 1)*x^(2*q - 2) - c*e*(2*q + 4*p + 1)*x^(2*q), x], x], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && !LtQ[p, -1]
```

Rule 1189

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol]
:> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a]
] || PosQ[(b - q)/a] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1099

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]},
Simp[((2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q)*x^2)/(2*a + (b + q)*x^2)]*
EllipticF[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)/(b + q)])/(2*a*Rt[(b + q)/(2*a), 2]*
Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] && !(PosQ[(b - q)/a] &&
SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1135

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]},
Simp[(x*(b + q + 2*c*x^2))/(2*c*Sqrt[a + b*x^2 + c*x^4]), x] - Simp[(Rt[(b + q)/(2*a), 2]*
(2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q)*x^2)/(2*a + (b + q)*x^2)]*
EllipticE[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)/(b + q)])/(2*c*Sqrt[a + b*x^2 + c*x^4]), x] /;
PosQ[(b + q)/a] && !(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /;
FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{(7 + 5x^2)^5}{(2 + 3x^2 + x^4)^{3/2}} dx &= -\frac{x(115 + 179x^2)}{2\sqrt{2 + 3x^2 + x^4}} - \frac{1}{2} \int \frac{-16922 - 35179x^2 - 25000x^4 - 6250x^6}{\sqrt{2 + 3x^2 + x^4}} dx \\ &= -\frac{x(115 + 179x^2)}{2\sqrt{2 + 3x^2 + x^4}} + 625x^3\sqrt{2 + 3x^2 + x^4} - \frac{1}{10} \int \frac{-84610 - 138395x^2 - 50000x^4}{\sqrt{2 + 3x^2 + x^4}} dx \\ &= -\frac{x(115 + 179x^2)}{2\sqrt{2 + 3x^2 + x^4}} + \frac{5000}{3}x\sqrt{2 + 3x^2 + x^4} + 625x^3\sqrt{2 + 3x^2 + x^4} - \frac{1}{30} \int \frac{-153830 - 115185x^2}{\sqrt{2 + 3x^2 + x^4}} dx \\ &= -\frac{x(115 + 179x^2)}{2\sqrt{2 + 3x^2 + x^4}} + \frac{5000}{3}x\sqrt{2 + 3x^2 + x^4} + 625x^3\sqrt{2 + 3x^2 + x^4} + \frac{7679}{2} \int \frac{x^2}{\sqrt{2 + 3x^2 + x^4}} dx \\ &= \frac{7679x(2 + x^2)}{2\sqrt{2 + 3x^2 + x^4}} - \frac{x(115 + 179x^2)}{2\sqrt{2 + 3x^2 + x^4}} + \frac{5000}{3}x\sqrt{2 + 3x^2 + x^4} + 625x^3\sqrt{2 + 3x^2 + x^4} - \frac{7679(1 + x^2)}{2\sqrt{2 + 3x^2 + x^4}} \end{aligned}$$

Mathematica [F] time = 0, size = 0, normalized size = 0.

\$Aborted

Verification is Not applicable to the result.

[In] Integrate[(7 + 5*x^2)^5/(2 + 3*x^2 + x^4)^(3/2), x]

[Out] \$Aborted

Maple [C] time = 0.031, size = 274, normalized size = 1.5

$$-6250 \frac{17/2 x^3 + 9x}{\sqrt{x^4 + 3x^2 + 2}} + 625 x^3 \sqrt{x^4 + 3x^2 + 2} + \frac{5000x}{3} \sqrt{x^4 + 3x^2 + 2} + \frac{7679i}{4} \sqrt{2} \left(\text{EllipticF} \left(\frac{i}{2} x \sqrt{2}, \sqrt{2} \right) - \text{EllipticE} \left(\frac{i}{2} x \sqrt{2}, \sqrt{2} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((5*x^2+7)^5/(x^4+3*x^2+2)^(3/2), x)

[Out] -6250*(17/2*x^3+9*x)/(x^4+3*x^2+2)^(1/2)+625*x^3*(x^4+3*x^2+2)^(1/2)+5000/3*x*(x^4+3*x^2+2)^(1/2)+7679/4*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*(EllipticF(1/2*I*x*2^(1/2),2^(1/2))-EllipticE(1/2*I*x*2^(1/2),2^(1/2)))-15383/6*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticF(1/2*I*x*2^(1/2),2^(1/2))-43750*(-9/2*x^3-5*x)/(x^4+3*x^2+2)^(1/2)-122500*(5/2*x^3+3*x)/(x^4+3*x^2+2)^(1/2)-171500*(-3/2*x^3-2*x)/(x^4+3*x^2+2)^(1/2)-120050*(x^3+3/2*x)/(x^4+3*x^2+2)^(1/2)-33614*(-3/4*x^3-5/4*x)/(x^4+3*x^2+2)^(1/2)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(5x^2 + 7)^5}{(x^4 + 3x^2 + 2)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^2+7)^5/(x^4+3*x^2+2)^(3/2), x, algorithm="maxima")

[Out] integrate((5*x^2 + 7)^5/(x^4 + 3*x^2 + 2)^(3/2), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral} \left(\frac{(3125x^{10} + 21875x^8 + 61250x^6 + 85750x^4 + 60025x^2 + 16807)\sqrt{x^4 + 3x^2 + 2}}{x^8 + 6x^6 + 13x^4 + 12x^2 + 4}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^2+7)^5/(x^4+3*x^2+2)^(3/2), x, algorithm="fricas")

[Out] integral((3125*x^10 + 21875*x^8 + 61250*x^6 + 85750*x^4 + 60025*x^2 + 16807)*sqrt(x^4 + 3*x^2 + 2)/(x^8 + 6*x^6 + 13*x^4 + 12*x^2 + 4), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(5x^2 + 7)^5}{((x^2 + 1)(x^2 + 2))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x**2+7)**5/(x**4+3*x**2+2)**(3/2), x)

[Out] Integral((5*x**2 + 7)**5/((x**2 + 1)*(x**2 + 2))**(3/2), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(5x^2 + 7)^5}{(x^4 + 3x^2 + 2)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^2+7)^5/(x^4+3*x^2+2)^(3/2),x, algorithm="giac")

[Out] integrate((5*x^2 + 7)^5/(x^4 + 3*x^2 + 2)^(3/2), x)

$$3.308 \quad \int \frac{(7+5x^2)^4}{(2+3x^2+x^4)^{3/2}} dx$$

Optimal. Leaf size=170

$$\frac{1067\sqrt{2}(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\text{EllipticF}\left(\tan^{-1}(x), \frac{1}{2}\right)}{3\sqrt{x^4+3x^2+2}} + \frac{625\sqrt{x^4+3x^2+2x}}{3} + \frac{637(x^2+2)x}{2\sqrt{x^4+3x^2+2}} + \frac{(113x^2+145)x}{2\sqrt{x^4+3x^2+2}} - \frac{637(x^2+1)\sqrt{2}\sqrt{x^4+3x^2+2}}{\sqrt{2}\sqrt{x^4+3x^2+2}}$$

```
[Out] (637*x*(2 + x^2))/(2*Sqrt[2 + 3*x^2 + x^4]) + (x*(145 + 113*x^2))/(2*Sqrt[2 + 3*x^2 + x^4]) + (625*x*Sqrt[2 + 3*x^2 + x^4])/3 - (637*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x], 1/2])/(Sqrt[2]*Sqrt[2 + 3*x^2 + x^4]) + (1067*Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(3*Sqrt[2 + 3*x^2 + x^4])
```

Rubi [A] time = 0.0830864, antiderivative size = 170, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {1205, 1679, 1189, 1099, 1135}

$$\frac{625\sqrt{x^4+3x^2+2x}}{3} + \frac{637(x^2+2)x}{2\sqrt{x^4+3x^2+2}} + \frac{(113x^2+145)x}{2\sqrt{x^4+3x^2+2}} + \frac{1067\sqrt{2}(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\text{F}\left(\tan^{-1}(x)|\frac{1}{2}\right)}{3\sqrt{x^4+3x^2+2}} - \frac{637(x^2+1)\sqrt{2}\sqrt{x^4+3x^2+2}}{\sqrt{2}\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Int[(7 + 5*x^2)^4/(2 + 3*x^2 + x^4)^(3/2), x]
```

```
[Out] (637*x*(2 + x^2))/(2*Sqrt[2 + 3*x^2 + x^4]) + (x*(145 + 113*x^2))/(2*Sqrt[2 + 3*x^2 + x^4]) + (625*x*Sqrt[2 + 3*x^2 + x^4])/3 - (637*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x], 1/2])/(Sqrt[2]*Sqrt[2 + 3*x^2 + x^4]) + (1067*Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(3*Sqrt[2 + 3*x^2 + x^4])
```

Rule 1205

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{f = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 + c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*g - f*(b^2 - 2*a*c) - c*(b*f - 2*a*g)*x^2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[(d + e*x^2)^q, a + b*x^2 + c*x^4, x] + b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g + c*(4*p + 7)*(b*f - 2*a*g)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1] && LtQ[p, -1]
```

Rule 1679

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{q = Expon[Pq, x^2], e = Coeff[Pq, x^2, Expon[Pq, x^2]]}, Simp[(e*x^(2*q - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(2*q + 4*p + 1)), x] + Dist[1/(c*(2*q + 4*p + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*q + 4*p + 1)*Pq - a*e*(2*q - 3)*x^(2*q - 4) - b*e*(2*q + 2*p - 1)*x^(2*q - 2) - c*e*(2*q + 4*p + 1)*x^(2*q), x], x], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && !LtQ[p, -1]
```

Rule 1189

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol]
:> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a]
] || PosQ[(b - q)/a] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1099

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]},
Simp[((2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q)*x^2)/(2*a + (b + q)*x^2)]*
EllipticF[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)/(b + q)])/(2*a*Rt[(b + q)/(2*a), 2]*
Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] && !(PosQ[(b - q)/a] &&
SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1135

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]},
Simp[(x*(b + q + 2*c*x^2))/(2*c*Sqrt[a + b*x^2 + c*x^4]), x] - Simp[(Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*
Sqrt[(2*a + (b - q)*x^2)/(2*a + (b + q)*x^2)]*EllipticE[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)/(b + q)])/(2*c*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] && !(PosQ[(b - q)/a] &&
SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{(7 + 5x^2)^4}{(2 + 3x^2 + x^4)^{3/2}} dx &= \frac{x(145 + 113x^2)}{2\sqrt{2 + 3x^2 + x^4}} - \frac{1}{2} \int \frac{-2256 - 3137x^2 - 1250x^4}{\sqrt{2 + 3x^2 + x^4}} dx \\ &= \frac{x(145 + 113x^2)}{2\sqrt{2 + 3x^2 + x^4}} + \frac{625}{3} x\sqrt{2 + 3x^2 + x^4} - \frac{1}{6} \int \frac{-4268 - 1911x^2}{\sqrt{2 + 3x^2 + x^4}} dx \\ &= \frac{x(145 + 113x^2)}{2\sqrt{2 + 3x^2 + x^4}} + \frac{625}{3} x\sqrt{2 + 3x^2 + x^4} + \frac{637}{2} \int \frac{x^2}{\sqrt{2 + 3x^2 + x^4}} dx + \frac{2134}{3} \int \frac{1}{\sqrt{2 + 3x^2 + x^4}} dx \\ &= \frac{637x(2 + x^2)}{2\sqrt{2 + 3x^2 + x^4}} + \frac{x(145 + 113x^2)}{2\sqrt{2 + 3x^2 + x^4}} + \frac{625}{3} x\sqrt{2 + 3x^2 + x^4} - \frac{637(1 + x^2)\sqrt{\frac{2+x^2}{1+x^2}} E\left(\tan^{-1}(x)\right)\frac{1}{2}}{\sqrt{2}\sqrt{2 + 3x^2 + x^4}} \end{aligned}$$

Mathematica [F] time = 0, size = 0, normalized size = 0.

\$Aborted

Verification is Not applicable to the result.

[In] Integrate[(7 + 5*x^2)^4/(2 + 3*x^2 + x^4)^(3/2), x]

[Out] \$Aborted

Maple [C] time = 0.007, size = 234, normalized size = 1.4

$$-1250 \frac{-9/2 x^3 - 5x}{\sqrt{x^4 + 3x^2 + 2}} + \frac{625x}{3} \sqrt{x^4 + 3x^2 + 2} - \frac{1067i}{3} \sqrt{2} \text{EllipticF}\left(\frac{i}{2}x\sqrt{2}, \sqrt{2}\right) \sqrt{2x^2 + 4}\sqrt{x^2 + 1} \frac{1}{\sqrt{x^4 + 3x^2 + 2}} + \frac{637}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((5*x^2+7)^4/(x^4+3*x^2+2)^(3/2),x)`

[Out] $-1250*(-9/2*x^3-5*x)/(x^4+3*x^2+2)^{(1/2)}+625/3*x*(x^4+3*x^2+2)^{(1/2)}-1067/3*I*2^{(1/2)}*(2*x^2+4)^{(1/2)}*(x^2+1)^{(1/2)}/(x^4+3*x^2+2)^{(1/2)}*EllipticF(1/2*I*x*2^{(1/2)},2^{(1/2)})+637/4*I*2^{(1/2)}*(2*x^2+4)^{(1/2)}*(x^2+1)^{(1/2)}/(x^4+3*x^2+2)^{(1/2)}*(EllipticF(1/2*I*x*2^{(1/2)},2^{(1/2)})-EllipticE(1/2*I*x*2^{(1/2)},2^{(1/2)}))-7000*(5/2*x^3+3*x)/(x^4+3*x^2+2)^{(1/2)}-14700*(-3/2*x^3-2*x)/(x^4+3*x^2+2)^{(1/2)}-13720*(x^3+3/2*x)/(x^4+3*x^2+2)^{(1/2)}-4802*(-3/4*x^3-5/4*x)/(x^4+3*x^2+2)^{(1/2)}$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(5x^2 + 7)^4}{(x^4 + 3x^2 + 2)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((5*x^2+7)^4/(x^4+3*x^2+2)^(3/2),x, algorithm="maxima")`

[Out] `integrate((5*x^2 + 7)^4/(x^4 + 3*x^2 + 2)^(3/2), x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(625x^8 + 3500x^6 + 7350x^4 + 6860x^2 + 2401)\sqrt{x^4 + 3x^2 + 2}}{x^8 + 6x^6 + 13x^4 + 12x^2 + 4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((5*x^2+7)^4/(x^4+3*x^2+2)^(3/2),x, algorithm="fricas")`

[Out] `integral((625*x^8 + 3500*x^6 + 7350*x^4 + 6860*x^2 + 2401)*sqrt(x^4 + 3*x^2 + 2)/(x^8 + 6*x^6 + 13*x^4 + 12*x^2 + 4), x)`

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(5x^2 + 7)^4}{((x^2 + 1)(x^2 + 2))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((5*x**2+7)**4/(x**4+3*x**2+2)**(3/2),x)`

[Out] `Integral((5*x**2 + 7)**4/((x**2 + 1)*(x**2 + 2))**3/2, x)`

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(5x^2 + 7)^4}{(x^4 + 3x^2 + 2)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^4/(x^4+3*x^2+2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((5*x^2 + 7)^4/(x^4 + 3*x^2 + 2)^(3/2), x)
```

$$3.309 \quad \int \frac{(7+5x^2)^3}{(2+3x^2+x^4)^{3/2}} dx$$

Optimal. Leaf size=149

$$\frac{169(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\text{EllipticF}\left(\tan^{-1}(x), \frac{1}{2}\right)}{\sqrt{2}\sqrt{x^4+3x^2+2}} + \frac{x(5-11x^2)}{2\sqrt{x^4+3x^2+2}} + \frac{261x(x^2+2)}{2\sqrt{x^4+3x^2+2}} - \frac{261(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}E\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^4+3x^2+2}}$$

```
[Out] (x*(5 - 11*x^2))/(2*Sqrt[2 + 3*x^2 + x^4]) + (261*x*(2 + x^2))/(2*Sqrt[2 + 3*x^2 + x^4]) - (261*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x], 1/2])/(Sqrt[2]*Sqrt[2 + 3*x^2 + x^4]) + (169*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(Sqrt[2]*Sqrt[2 + 3*x^2 + x^4])
```

Rubi [A] time = 0.0522078, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1205, 1189, 1099, 1135}

$$\frac{x(5-11x^2)}{2\sqrt{x^4+3x^2+2}} + \frac{261x(x^2+2)}{2\sqrt{x^4+3x^2+2}} + \frac{169(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}F\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^4+3x^2+2}} - \frac{261(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}E\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Int[(7 + 5*x^2)^3/(2 + 3*x^2 + x^4)^(3/2), x]
```

```
[Out] (x*(5 - 11*x^2))/(2*Sqrt[2 + 3*x^2 + x^4]) + (261*x*(2 + x^2))/(2*Sqrt[2 + 3*x^2 + x^4]) - (261*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x], 1/2])/(Sqrt[2]*Sqrt[2 + 3*x^2 + x^4]) + (169*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(Sqrt[2]*Sqrt[2 + 3*x^2 + x^4])
```

Rule 1205

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := With[{f = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 + c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*g - f*(b^2 - 2*a*c) - c*(b*f - 2*a*g)*x^2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[(d + e*x^2)^q, a + b*x^2 + c*x^4, x] + b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g + c*(4*p + 7)*(b*f - 2*a*g)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1] && LtQ[p, -1]
```

Rule 1189

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a] || PosQ[(b - q)/a] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1099

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[((2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q)*x^2)/(2*a + (b + q)*x^2)]*EllipticF[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)/(b + q)])/(2*
```

```
a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4], x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)]]) /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1135

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] := With[{q =
Rt[b^2 - 4*a*c, 2]}, Simp[(x*(b + q + 2*c*x^2))/(2*c*Sqrt[a + b*x^2 + c*x^
4]), x] - Simp[(Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q)
)*x^2]/(2*a + (b + q)*x^2)]*EllipticE[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)
/(b + q)]/(2*c*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] && !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)]]) /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{(7 + 5x^2)^3}{(2 + 3x^2 + x^4)^{3/2}} dx &= \frac{x(5 - 11x^2)}{2\sqrt{2 + 3x^2 + x^4}} - \frac{1}{2} \int \frac{-338 - 261x^2}{\sqrt{2 + 3x^2 + x^4}} dx \\ &= \frac{x(5 - 11x^2)}{2\sqrt{2 + 3x^2 + x^4}} + \frac{261}{2} \int \frac{x^2}{\sqrt{2 + 3x^2 + x^4}} dx + 169 \int \frac{1}{\sqrt{2 + 3x^2 + x^4}} dx \\ &= \frac{x(5 - 11x^2)}{2\sqrt{2 + 3x^2 + x^4}} + \frac{261x(2 + x^2)}{2\sqrt{2 + 3x^2 + x^4}} - \frac{261(1 + x^2)\sqrt{\frac{2+x^2}{1+x^2}}E\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{\sqrt{2}\sqrt{2 + 3x^2 + x^4}} + \frac{169(1 + x^2)\sqrt{\frac{2+x^2}{1+x^2}}}{\sqrt{2}\sqrt{2 + 3x^2 + x^4}} \end{aligned}$$

Mathematica [F] time = 0, size = 0, normalized size = 0.

\$Aborted

Verification is Not applicable to the result.

[In] Integrate[(7 + 5*x^2)^3/(2 + 3*x^2 + x^4)^(3/2), x]

[Out] \$Aborted

Maple [C] time = 0.006, size = 196, normalized size = 1.3

$$-250 \frac{5/2 x^3 + 3x}{\sqrt{x^4 + 3x^2 + 2}} - \frac{169i}{2} \sqrt{2} \text{EllipticF}\left(\frac{i}{2}x\sqrt{2}, \sqrt{2}\right) \sqrt{2x^2 + 4}\sqrt{x^2 + 1} \frac{1}{\sqrt{x^4 + 3x^2 + 2}} + \frac{261i}{4} \sqrt{2} \left(\text{EllipticF}\left(\frac{i}{2}x\sqrt{2}, \sqrt{2}\right) \sqrt{2x^2 + 4}\sqrt{x^2 + 1} - \text{EllipticE}\left(\frac{i}{2}x\sqrt{2}, \sqrt{2}\right) \sqrt{2x^2 + 4}\sqrt{x^2 + 1} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((5*x^2+7)^3/(x^4+3*x^2+2)^(3/2), x)

[Out] $-250*(5/2*x^3+3*x)/(x^4+3*x^2+2)^{(1/2)} - 169/2*I*2^{(1/2)}*(2*x^2+4)^{(1/2)}*(x^2+1)^{(1/2)}/(x^4+3*x^2+2)^{(1/2)}*EllipticF(1/2*I*x*2^{(1/2)}, 2^{(1/2)}) + 261/4*I*2^{(1/2)}*(2*x^2+4)^{(1/2)}*(x^2+1)^{(1/2)}/(x^4+3*x^2+2)^{(1/2)}*(EllipticF(1/2*I*x*2^{(1/2)}, 2^{(1/2)}) - EllipticE(1/2*I*x*2^{(1/2)}, 2^{(1/2)})) - 1050*(-3/2*x^3-2*x)/(x^4+3*x^2+2)^{(1/2)} - 1470*(x^3+3/2*x)/(x^4+3*x^2+2)^{(1/2)} - 686*(-3/4*x^3-5/4*x)/(x^4+3*x^2+2)^{(1/2)}$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(5x^2 + 7)^3}{(x^4 + 3x^2 + 2)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^2+7)^3/(x^4+3*x^2+2)^(3/2),x, algorithm="maxima")

[Out] integrate((5*x^2 + 7)^3/(x^4 + 3*x^2 + 2)^(3/2), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(125x^6 + 525x^4 + 735x^2 + 343)\sqrt{x^4 + 3x^2 + 2}}{x^8 + 6x^6 + 13x^4 + 12x^2 + 4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^2+7)^3/(x^4+3*x^2+2)^(3/2),x, algorithm="fricas")

[Out] integral((125*x^6 + 525*x^4 + 735*x^2 + 343)*sqrt(x^4 + 3*x^2 + 2)/(x^8 + 6*x^6 + 13*x^4 + 12*x^2 + 4), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(5x^2 + 7)^3}{((x^2 + 1)(x^2 + 2))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x**2+7)**3/(x**4+3*x**2+2)**(3/2),x)

[Out] Integral((5*x**2 + 7)**3/((x**2 + 1)*(x**2 + 2))**3/2, x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(5x^2 + 7)^3}{(x^4 + 3x^2 + 2)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^2+7)^3/(x^4+3*x^2+2)^(3/2),x, algorithm="giac")

[Out] integrate((5*x^2 + 7)^3/(x^4 + 3*x^2 + 2)^(3/2), x)

$$3.310 \quad \int \frac{(7+5x^2)^2}{(2+3x^2+x^4)^{3/2}} dx$$

Optimal. Leaf size=149

$$\frac{6\sqrt{2}(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\text{EllipticF}\left(\tan^{-1}(x), \frac{1}{2}\right)}{\sqrt{x^4+3x^2+2}} - \frac{17x(x^2+2)}{2\sqrt{x^4+3x^2+2}} + \frac{x(17x^2+25)}{2\sqrt{x^4+3x^2+2}} + \frac{17(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}E\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^4+3x^2+2}}$$

[Out] $(-17*x*(2 + x^2))/(2*\text{Sqrt}[2 + 3*x^2 + x^4]) + (x*(25 + 17*x^2))/(2*\text{Sqrt}[2 + 3*x^2 + x^4]) + (17*(1 + x^2)*\text{Sqrt}[(2 + x^2)/(1 + x^2)]*\text{EllipticE}[\text{ArcTan}[x], 1/2])/(\text{Sqrt}[2]*\text{Sqrt}[2 + 3*x^2 + x^4]) + (6*\text{Sqrt}[2]*(1 + x^2)*\text{Sqrt}[(2 + x^2)/(1 + x^2)]*\text{EllipticF}[\text{ArcTan}[x], 1/2])/ \text{Sqrt}[2 + 3*x^2 + x^4]$

Rubi [A] time = 0.0511373, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1205, 1189, 1099, 1135}

$$-\frac{17x(x^2+2)}{2\sqrt{x^4+3x^2+2}} + \frac{x(17x^2+25)}{2\sqrt{x^4+3x^2+2}} + \frac{6\sqrt{2}(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}F\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{\sqrt{x^4+3x^2+2}} + \frac{17(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}E\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(7 + 5*x^2)^2/(2 + 3*x^2 + x^4)^{(3/2)}, x]$

[Out] $(-17*x*(2 + x^2))/(2*\text{Sqrt}[2 + 3*x^2 + x^4]) + (x*(25 + 17*x^2))/(2*\text{Sqrt}[2 + 3*x^2 + x^4]) + (17*(1 + x^2)*\text{Sqrt}[(2 + x^2)/(1 + x^2)]*\text{EllipticE}[\text{ArcTan}[x], 1/2])/(\text{Sqrt}[2]*\text{Sqrt}[2 + 3*x^2 + x^4]) + (6*\text{Sqrt}[2]*(1 + x^2)*\text{Sqrt}[(2 + x^2)/(1 + x^2)]*\text{EllipticF}[\text{ArcTan}[x], 1/2])/ \text{Sqrt}[2 + 3*x^2 + x^4]$

Rule 1205

$\text{Int}[(d + (e_*)*(x_)^2)^{(q_*)}*((a_) + (b_*)*(x_)^2 + (c_*)*(x_)^4)^{(p_*)}, x_Symbol] \rightarrow \text{With}[\{f = \text{Coeff}[\text{PolynomialRemainder}[(d + e*x^2)^q, a + b*x^2 + c*x^4, x], x, 0], g = \text{Coeff}[\text{PolynomialRemainder}[(d + e*x^2)^q, a + b*x^2 + c*x^4, x], x, 2]\}, \text{Simp}[(x*(a + b*x^2 + c*x^4)^{(p+1)}*(a*b*g - f*(b^2 - 2*a*c) - c*(b*f - 2*a*g)*x^2))/(2*a*(p+1)*(b^2 - 4*a*c)), x] + \text{Dist}[1/(2*a*(p+1)*(b^2 - 4*a*c)), \text{Int}[(a + b*x^2 + c*x^4)^{(p+1)}*\text{ExpandToSum}[2*a*(p+1)*(b^2 - 4*a*c)*\text{PolynomialQuotient}[(d + e*x^2)^q, a + b*x^2 + c*x^4, x] + b^2*f*(2*p+3) - 2*a*c*f*(4*p+5) - a*b*g + c*(4*p+7)*(b*f - 2*a*g)*x^2, x], x], x] /; \text{FreeQ}[\{a, b, c, d, e\}, x] \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& \text{NeQ}[c*d^2 - b*d*e + a*e^2, 0] \&\& \text{IGtQ}[q, 1] \&\& \text{LtQ}[p, -1]$

Rule 1189

$\text{Int}[(d + (e_*)*(x_)^2)/\text{Sqrt}[(a_) + (b_*)*(x_)^2 + (c_*)*(x_)^4], x_Symbol] \rightarrow \text{With}[\{q = \text{Rt}[b^2 - 4*a*c, 2]\}, \text{Dist}[d, \text{Int}[1/\text{Sqrt}[a + b*x^2 + c*x^4], x], x] + \text{Dist}[e, \text{Int}[x^2/\text{Sqrt}[a + b*x^2 + c*x^4], x], x] /; \text{PosQ}[(b + q)/a] || \text{PosQ}[(b - q)/a] /; \text{FreeQ}[\{a, b, c, d, e\}, x] \&\& \text{GtQ}[b^2 - 4*a*c, 0]$

Rule 1099

$\text{Int}[1/\text{Sqrt}[(a_) + (b_*)*(x_)^2 + (c_*)*(x_)^4], x_Symbol] \rightarrow \text{With}[\{q = \text{Rt}[b^2 - 4*a*c, 2]\}, \text{Simp}[(2*a + (b + q)*x^2)*\text{Sqrt}[(2*a + (b - q)*x^2)/(2*a + (b + q)*x^2)]*\text{EllipticF}[\text{ArcTan}[\text{Rt}[(b + q)/(2*a), 2]*x], (2*q)/(b + q)]/(2*$

```
a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4], x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1135

```
Int[(x_)^2/Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4], x_Symbol] := With[{q =
Rt[b^2 - 4*a*c, 2]}, Simp[(x*(b + q + 2*c*x^2))/(2*c*Sqrt[a + b*x^2 + c*x^
4]), x] - Simp[(Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q)
*x^2)/(2*a + (b + q)*x^2)]*EllipticE[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)
/(b + q)])/(2*c*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] && !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{(7 + 5x^2)^2}{(2 + 3x^2 + x^4)^{3/2}} dx &= \frac{x(25 + 17x^2)}{2\sqrt{2 + 3x^2 + x^4}} - \frac{1}{2} \int \frac{-24 + 17x^2}{\sqrt{2 + 3x^2 + x^4}} dx \\ &= \frac{x(25 + 17x^2)}{2\sqrt{2 + 3x^2 + x^4}} - \frac{17}{2} \int \frac{x^2}{\sqrt{2 + 3x^2 + x^4}} dx + 12 \int \frac{1}{\sqrt{2 + 3x^2 + x^4}} dx \\ &= -\frac{17x(2 + x^2)}{2\sqrt{2 + 3x^2 + x^4}} + \frac{x(25 + 17x^2)}{2\sqrt{2 + 3x^2 + x^4}} + \frac{17(1 + x^2)\sqrt{\frac{2+x^2}{1+x^2}}E\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{\sqrt{2}\sqrt{2 + 3x^2 + x^4}} + \frac{6\sqrt{2}(1 + x^2)\sqrt{\sqrt{2 + 3x^2 + x^4}}}{\sqrt{2 + 3x^2 + x^4}} \end{aligned}$$

Mathematica [F] time = 0, size = 0, normalized size = 0.

\$Aborted

Verification is Not applicable to the result.

[In] Integrate[(7 + 5*x^2)^2/(2 + 3*x^2 + x^4)^(3/2), x]

[Out] \$Aborted

Maple [C] time = 0.007, size = 173, normalized size = 1.2

$$-50 \frac{-3/2 x^3 - 2x}{\sqrt{x^4 + 3x^2 + 2}} - 6i\sqrt{2}\text{EllipticF}\left(\frac{i}{2}x\sqrt{2}, \sqrt{2}\right)\sqrt{2x^2 + 4}\sqrt{x^2 + 1} \frac{1}{\sqrt{x^4 + 3x^2 + 2}} - \frac{17i}{4}\sqrt{2}\left(\text{EllipticF}\left(\frac{i}{2}x\sqrt{2}, \sqrt{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((5*x^2+7)^2/(x^4+3*x^2+2)^(3/2), x)

[Out] $-50*(-3/2*x^3-2*x)/(x^4+3*x^2+2)^{(1/2)}-6*I*2^{(1/2)}*(2*x^2+4)^{(1/2)}*(x^2+1)^{(1/2)}/(x^4+3*x^2+2)^{(1/2)}*EllipticF(1/2*I*x*2^{(1/2)}, 2^{(1/2)})-17/4*I*2^{(1/2)}*(2*x^2+4)^{(1/2)}*(x^2+1)^{(1/2)}/(x^4+3*x^2+2)^{(1/2)}*(EllipticF(1/2*I*x*2^{(1/2)}, 2^{(1/2)})-EllipticE(1/2*I*x*2^{(1/2)}, 2^{(1/2)}))-140*(x^3+3/2*x)/(x^4+3*x^2+2)^{(1/2)}-98*(-3/4*x^3-5/4*x)/(x^4+3*x^2+2)^{(1/2)}$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(5x^2 + 7)^2}{(x^4 + 3x^2 + 2)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^2+7)^2/(x^4+3*x^2+2)^(3/2),x, algorithm="maxima")

[Out] integrate((5*x^2 + 7)^2/(x^4 + 3*x^2 + 2)^(3/2), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(25x^4 + 70x^2 + 49)\sqrt{x^4 + 3x^2 + 2}}{x^8 + 6x^6 + 13x^4 + 12x^2 + 4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^2+7)^2/(x^4+3*x^2+2)^(3/2),x, algorithm="fricas")

[Out] integral((25*x^4 + 70*x^2 + 49)*sqrt(x^4 + 3*x^2 + 2)/(x^8 + 6*x^6 + 13*x^4 + 12*x^2 + 4), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(5x^2 + 7)^2}{((x^2 + 1)(x^2 + 2))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x**2+7)**2/(x**4+3*x**2+2)**(3/2),x)

[Out] Integral((5*x**2 + 7)**2/((x**2 + 1)*(x**2 + 2))**3/2, x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(5x^2 + 7)^2}{(x^4 + 3x^2 + 2)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^2+7)^2/(x^4+3*x^2+2)^(3/2),x, algorithm="giac")

[Out] integrate((5*x^2 + 7)^2/(x^4 + 3*x^2 + 2)^(3/2), x)

$$3.311 \quad \int \frac{7+5x^2}{(2+3x^2+x^4)^{3/2}} dx$$

Optimal. Leaf size=145

$$\frac{(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\text{EllipticF}\left(\tan^{-1}(x), \frac{1}{2}\right)}{\sqrt{2}\sqrt{x^4+3x^2+2}} - \frac{x(x^2+2)}{2\sqrt{x^4+3x^2+2}} + \frac{x(x^2+5)}{2\sqrt{x^4+3x^2+2}} + \frac{(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}E\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^4+3x^2+2}}$$

```
[Out] -(x*(2 + x^2))/(2*Sqrt[2 + 3*x^2 + x^4]) + (x*(5 + x^2))/(2*Sqrt[2 + 3*x^2 + x^4]) + ((1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x], 1/2])/(Sqrt[2]*Sqrt[2 + 3*x^2 + x^4]) + ((1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(Sqrt[2]*Sqrt[2 + 3*x^2 + x^4])
```

Rubi [A] time = 0.0414866, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1178, 1189, 1099, 1135}

$$-\frac{x(x^2+2)}{2\sqrt{x^4+3x^2+2}} + \frac{x(x^2+5)}{2\sqrt{x^4+3x^2+2}} + \frac{(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}F\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^4+3x^2+2}} + \frac{(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}E\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Int[(7 + 5*x^2)/(2 + 3*x^2 + x^4)^(3/2), x]
```

```
[Out] -(x*(2 + x^2))/(2*Sqrt[2 + 3*x^2 + x^4]) + (x*(5 + x^2))/(2*Sqrt[2 + 3*x^2 + x^4]) + ((1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x], 1/2])/(Sqrt[2]*Sqrt[2 + 3*x^2 + x^4]) + ((1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(Sqrt[2]*Sqrt[2 + 3*x^2 + x^4])
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1189

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol]
:> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a] || PosQ[(b - q)/a] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1099

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol]
:> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[((2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q)*x^2)/(2*a + (b + q)*x^2)]*EllipticF[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)/(b + q)])/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] && !(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1135

```
Int[(x_)^2/Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4], x_Symbol] := With[{q =
  Rt[b^2 - 4*a*c, 2]}, Simp[(x*(b + q + 2*c*x^2))/(2*c*Sqrt[a + b*x^2 + c*x^
  4]), x] - Simp[(Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q
  )*x^2)/(2*a + (b + q)*x^2)]*EllipticE[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)
  /(b + q))]/(2*c*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] && !(PosQ[
  (b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
  c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{7 + 5x^2}{(2 + 3x^2 + x^4)^{3/2}} dx &= \frac{x(5 + x^2)}{2\sqrt{2 + 3x^2 + x^4}} - \frac{1}{2} \int \frac{-2 + x^2}{\sqrt{2 + 3x^2 + x^4}} dx \\ &= \frac{x(5 + x^2)}{2\sqrt{2 + 3x^2 + x^4}} - \frac{1}{2} \int \frac{x^2}{\sqrt{2 + 3x^2 + x^4}} dx + \int \frac{1}{\sqrt{2 + 3x^2 + x^4}} dx \\ &= -\frac{x(2 + x^2)}{2\sqrt{2 + 3x^2 + x^4}} + \frac{x(5 + x^2)}{2\sqrt{2 + 3x^2 + x^4}} + \frac{(1 + x^2)\sqrt{\frac{2+x^2}{1+x^2}}E\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{\sqrt{2}\sqrt{2 + 3x^2 + x^4}} + \frac{(1 + x^2)\sqrt{\frac{2+x^2}{1+x^2}}F\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{\sqrt{2}\sqrt{2 + 3x^2 + x^4}} \end{aligned}$$

Mathematica [F] time = 0, size = 0, normalized size = 0.

\$Aborted

Verification is Not applicable to the result.

[In] Integrate[(7 + 5*x^2)/(2 + 3*x^2 + x^4)^(3/2), x]

[Out] \$Aborted

Maple [C] time = 0.004, size = 150, normalized size = 1.

$$-10 \frac{x^3 + 3/2 x}{\sqrt{x^4 + 3x^2 + 2}} - \frac{i}{2} \sqrt{2} \text{EllipticF}\left(\frac{i}{2} x \sqrt{2}, \sqrt{2}\right) \sqrt{2x^2 + 4} \sqrt{x^2 + 1} \frac{1}{\sqrt{x^4 + 3x^2 + 2}} - \frac{i}{4} \sqrt{2} \left(\text{EllipticF}\left(\frac{i}{2} x \sqrt{2}, \sqrt{2}\right) - \text{EllipticE}\left(\frac{i}{2} x \sqrt{2}, \sqrt{2}\right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((5*x^2+7)/(x^4+3*x^2+2)^(3/2), x)

[Out] $-10*(x^3+3/2*x)/(x^4+3*x^2+2)^{(1/2)} - 1/2*I*2^{(1/2)}*(2*x^2+4)^{(1/2)}*(x^2+1)^{(1/2)}/(x^4+3*x^2+2)^{(1/2)} * \text{EllipticF}(1/2*I*x*2^{(1/2)}, 2^{(1/2)}) - 1/4*I*2^{(1/2)}*(2*x^2+4)^{(1/2)}*(x^2+1)^{(1/2)}/(x^4+3*x^2+2)^{(1/2)} * (\text{EllipticF}(1/2*I*x*2^{(1/2)}, 2^{(1/2)}) - \text{EllipticE}(1/2*I*x*2^{(1/2)}, 2^{(1/2)})) - 14*(-3/4*x^3-5/4*x)/(x^4+3*x^2+2)^{(1/2)}$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{5x^2 + 7}{(x^4 + 3x^2 + 2)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^2+7)/(x^4+3*x^2+2)^(3/2),x, algorithm="maxima")

[Out] integrate((5*x^2 + 7)/(x^4 + 3*x^2 + 2)^(3/2), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{x^4 + 3x^2 + 2}(5x^2 + 7)}{x^8 + 6x^6 + 13x^4 + 12x^2 + 4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^2+7)/(x^4+3*x^2+2)^(3/2),x, algorithm="fricas")

[Out] integral(sqrt(x^4 + 3*x^2 + 2)*(5*x^2 + 7)/(x^8 + 6*x^6 + 13*x^4 + 12*x^2 + 4), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{5x^2 + 7}{((x^2 + 1)(x^2 + 2))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x**2+7)/(x**4+3*x**2+2)**(3/2),x)

[Out] Integral((5*x**2 + 7)/((x**2 + 1)*(x**2 + 2))**(3/2), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{5x^2 + 7}{(x^4 + 3x^2 + 2)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^2+7)/(x^4+3*x^2+2)^(3/2),x, algorithm="giac")

[Out] integrate((5*x^2 + 7)/(x^4 + 3*x^2 + 2)^(3/2), x)

$$3.312 \quad \int \frac{1}{(2+3x^2+x^4)^{3/2}} dx$$

Optimal. Leaf size=149

$$\frac{\sqrt{2}(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\text{EllipticF}\left(\tan^{-1}(x), \frac{1}{2}\right)}{\sqrt{x^4+3x^2+2}} - \frac{3x(x^2+2)}{2\sqrt{x^4+3x^2+2}} + \frac{x(3x^2+5)}{2\sqrt{x^4+3x^2+2}} + \frac{3(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}E\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^4+3x^2+2}}$$

[Out] (-3*x*(2 + x^2))/(2*Sqrt[2 + 3*x^2 + x^4]) + (x*(5 + 3*x^2))/(2*Sqrt[2 + 3*x^2 + x^4]) + (3*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x], 1/2])/(Sqrt[2]*Sqrt[2 + 3*x^2 + x^4]) - (Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/Sqrt[2 + 3*x^2 + x^4]

Rubi [A] time = 0.0417569, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {1092, 1189, 1099, 1135}

$$-\frac{3x(x^2+2)}{2\sqrt{x^4+3x^2+2}} + \frac{x(3x^2+5)}{2\sqrt{x^4+3x^2+2}} - \frac{\sqrt{2}(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}F\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{\sqrt{x^4+3x^2+2}} + \frac{3(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}E\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

[In] Int[(2 + 3*x^2 + x^4)^(-3/2), x]

[Out] (-3*x*(2 + x^2))/(2*Sqrt[2 + 3*x^2 + x^4]) + (x*(5 + 3*x^2))/(2*Sqrt[2 + 3*x^2 + x^4]) + (3*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x], 1/2])/(Sqrt[2]*Sqrt[2 + 3*x^2 + x^4]) - (Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/Sqrt[2 + 3*x^2 + x^4]

Rule 1092

Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := -Simp[(x*(b^2 - 2*a*c + b*c*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(b^2 - 2*a*c + 2*(p + 1)*(b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]

Rule 1189

Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a] || PosQ[(b - q)/a] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]

Rule 1099

Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[((2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q)*x^2)/(2*a + (b + q)*x^2)]*EllipticF[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)/(b + q)])/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] && !(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)]) /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]

Rule 1135


```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] := With[{q =
  Rt[b^2 - 4*a*c, 2]}, Simp[(x*(b + q + 2*c*x^2))/(2*c*Sqrt[a + b*x^2 + c*x^
  4]), x] - Simp[(Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q
  )*x^2)/(2*a + (b + q)*x^2)]*EllipticE[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)
  /(b + q)])/(2*c*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] && !(PosQ[
  (b - q)/a] && SimplersqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
  c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{(2 + 3x^2 + x^4)^{3/2}} dx &= \frac{x(5 + 3x^2)}{2\sqrt{2 + 3x^2 + x^4}} - \frac{1}{2} \int \frac{4 + 3x^2}{\sqrt{2 + 3x^2 + x^4}} dx \\ &= \frac{x(5 + 3x^2)}{2\sqrt{2 + 3x^2 + x^4}} - \frac{3}{2} \int \frac{x^2}{\sqrt{2 + 3x^2 + x^4}} dx - 2 \int \frac{1}{\sqrt{2 + 3x^2 + x^4}} dx \\ &= -\frac{3x(2 + x^2)}{2\sqrt{2 + 3x^2 + x^4}} + \frac{x(5 + 3x^2)}{2\sqrt{2 + 3x^2 + x^4}} + \frac{3(1 + x^2)\sqrt{\frac{2+x^2}{1+x^2}}E\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{\sqrt{2}\sqrt{2 + 3x^2 + x^4}} - \frac{\sqrt{2}(1 + x^2)\sqrt{\frac{2+x^2}{1+x^2}}}{\sqrt{2 + 3x^2 + x^4}} \end{aligned}$$

Mathematica [C] time = 0.0399735, size = 99, normalized size = 0.66

$$\frac{i\sqrt{x^2+1}\sqrt{x^2+2}\text{EllipticF}\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right), 2\right) + 3x^3 + 3i\sqrt{x^2+1}\sqrt{x^2+2}E\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right) + 5x}{2\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x^2 + x^4)^(-3/2), x]
```

```
[Out] (5*x + 3*x^3 + (3*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticE[I*ArcSinh[x/Sqrt
[2]], 2] + I*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticF[I*ArcSinh[x/Sqrt[2]], 2]
)/(2*Sqrt[2 + 3*x^2 + x^4])
```

Maple [C] time = 0.003, size = 129, normalized size = 0.9

$$-2 \frac{-3/4 x^3 - 5/4 x}{\sqrt{x^4 + 3x^2 + 2}} + i\sqrt{2}\text{EllipticF}\left(\frac{i}{2}x\sqrt{2}, \sqrt{2}\right)\sqrt{2x^2 + 4}\sqrt{x^2 + 1} \frac{1}{\sqrt{x^4 + 3x^2 + 2}} - \frac{3i}{4}\sqrt{2}\left(\text{EllipticF}\left(\frac{i}{2}x\sqrt{2}, \sqrt{2}\right) - \text{EllipticE}\left(\frac{i}{2}x\sqrt{2}, \sqrt{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4+3*x^2+2)^(3/2), x)
```

```
[Out] -2*(-3/4*x^3-5/4*x)/(x^4+3*x^2+2)^(1/2)+I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(
1/2)/(x^4+3*x^2+2)^(1/2)*EllipticF(1/2*I*x*2^(1/2), 2^(1/2))-3/4*I*2^(1/2)*
(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*(EllipticF(1/2*I*x*2^(1/2)
, 2^(1/2))-EllipticE(1/2*I*x*2^(1/2), 2^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x^4 + 3x^2 + 2)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^4+3*x^2+2)^(3/2),x, algorithm="maxima")

[Out] integrate((x^4 + 3*x^2 + 2)^(-3/2), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{x^4 + 3x^2 + 2}}{x^8 + 6x^6 + 13x^4 + 12x^2 + 4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^4+3*x^2+2)^(3/2),x, algorithm="fricas")

[Out] integral(sqrt(x^4 + 3*x^2 + 2)/(x^8 + 6*x^6 + 13*x^4 + 12*x^2 + 4), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x^4 + 3x^2 + 2)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x**4+3*x**2+2)**(3/2),x)

[Out] Integral((x**4 + 3*x**2 + 2)**(-3/2), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x^4 + 3x^2 + 2)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^4+3*x^2+2)^(3/2),x, algorithm="giac")

[Out] integrate((x^4 + 3*x^2 + 2)^(-3/2), x)

$$3.313 \quad \int \frac{1}{(7+5x^2)(2+3x^2+x^4)^{3/2}} dx$$

Optimal. Leaf size=173

$$\frac{9(x^2+1)\sqrt{\frac{x^2+2}{2x^2+2}}\text{EllipticF}\left(\tan^{-1}(x), \frac{1}{2}\right)}{4\sqrt{x^4+3x^2+2}} + \frac{x}{6\sqrt{x^4+3x^2+2}} + \frac{\sqrt{2}(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}E\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{3\sqrt{x^4+3x^2+2}} + \frac{125(x^2+1)\sqrt{\frac{3}{x}}}{84\sqrt{2}}$$

[Out] $x/(6*\text{Sqrt}[2 + 3*x^2 + x^4]) + (\text{Sqrt}[2]*(1 + x^2)*\text{Sqrt}[(2 + x^2)/(1 + x^2)]* \text{EllipticE}[\text{ArcTan}[x], 1/2])/(3*\text{Sqrt}[2 + 3*x^2 + x^4]) - (9*(1 + x^2)*\text{Sqrt}[(2 + x^2)/(2 + 2*x^2)]*\text{EllipticF}[\text{ArcTan}[x], 1/2])/(4*\text{Sqrt}[2 + 3*x^2 + x^4]) + (125*(1 + x^2)*\text{Sqrt}[(2 + x^2)/(1 + x^2)]*\text{EllipticPi}[2/7, \text{ArcTan}[x], 1/2])/(84*\text{Sqrt}[2]*\text{Sqrt}[2 + 3*x^2 + x^4])$

Rubi [A] time = 0.140555, antiderivative size = 207, normalized size of antiderivative = 1.2, number of steps used = 9, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1221, 1178, 1189, 1099, 1135, 1214, 1456, 539}

$$\frac{x(x^2+2)}{3\sqrt{x^4+3x^2+2}} + \frac{x(2x^2+5)}{6\sqrt{x^4+3x^2+2}} - \frac{9(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}F\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{4\sqrt{2}\sqrt{x^4+3x^2+2}} + \frac{\sqrt{2}(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}E\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{3\sqrt{x^4+3x^2+2}} + \frac{125(x^2+1)\sqrt{\frac{3}{x}}}{84\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[1/((7 + 5*x^2)*(2 + 3*x^2 + x^4)^(3/2)),x]

[Out] $-(x*(2 + x^2))/(3*\text{Sqrt}[2 + 3*x^2 + x^4]) + (x*(5 + 2*x^2))/(6*\text{Sqrt}[2 + 3*x^2 + x^4]) + (\text{Sqrt}[2]*(1 + x^2)*\text{Sqrt}[(2 + x^2)/(1 + x^2)]*\text{EllipticE}[\text{ArcTan}[x], 1/2])/(3*\text{Sqrt}[2 + 3*x^2 + x^4]) - (9*(1 + x^2)*\text{Sqrt}[(2 + x^2)/(1 + x^2)]*\text{EllipticF}[\text{ArcTan}[x], 1/2])/(4*\text{Sqrt}[2]*\text{Sqrt}[2 + 3*x^2 + x^4]) + (125*(2 + x^2)*\text{EllipticPi}[2/7, \text{ArcTan}[x], 1/2])/(84*\text{Sqrt}[2]*\text{Sqrt}[(2 + x^2)/(1 + x^2)]*\text{Sqrt}[2 + 3*x^2 + x^4])$

Rule 1221

Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_)/((d_) + (e_.)*(x_)^2), x_Symbol] :> Dist[1/(c*d^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x^2)*(a + b*x^2 + c*x^4)^p, x], x] + Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[(a + b*x^2 + c*x^4)^(p + 1)/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[p + 1/2, 0]

Rule 1178

Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && IntegerQ[2*p]

Rule 1189

Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],

$x], x] + \text{Dist}[e, \text{Int}[x^2/\text{Sqrt}[a + b*x^2 + c*x^4], x], x] /; \text{PosQ}[(b + q)/a]$
 $] || \text{PosQ}[(b - q)/a] /; \text{FreeQ}\{a, b, c, d, e\}, x \ \&\& \ \text{GtQ}[b^2 - 4*a*c, 0]$

Rule 1099

$\text{Int}[1/\text{Sqrt}[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4], x_Symbol] := \text{With}\{q = \text{Rt}[b^2 - 4*a*c, 2]\}, \text{Simp}[(2*a + (b + q)*x^2)*\text{Sqrt}[(2*a + (b - q)*x^2)/(2*a + (b + q)*x^2)]*\text{EllipticF}[\text{ArcTan}[\text{Rt}[(b + q)/(2*a), 2]*x], (2*q)/(b + q)]/(2*a*\text{Rt}[(b + q)/(2*a), 2]*\text{Sqrt}[a + b*x^2 + c*x^4]), x] /; \text{PosQ}[(b + q)/a] \ \&\& \ !(\text{PosQ}[(b - q)/a] \ \&\& \ \text{SimplerSqrtQ}[(b - q)/(2*a), (b + q)/(2*a)])] /; \text{FreeQ}\{a, b, c\}, x \ \&\& \ \text{GtQ}[b^2 - 4*a*c, 0]$

Rule 1135

$\text{Int}[(x_)^2/\text{Sqrt}[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4], x_Symbol] := \text{With}\{q = \text{Rt}[b^2 - 4*a*c, 2]\}, \text{Simp}[(x*(b + q + 2*c*x^2))/(2*c*\text{Sqrt}[a + b*x^2 + c*x^4]), x] - \text{Simp}[(\text{Rt}[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*\text{Sqrt}[(2*a + (b - q)*x^2)/(2*a + (b + q)*x^2)]*\text{EllipticE}[\text{ArcTan}[\text{Rt}[(b + q)/(2*a), 2]*x], (2*q)/(b + q)]/(2*c*\text{Sqrt}[a + b*x^2 + c*x^4]), x] /; \text{PosQ}[(b + q)/a] \ \&\& \ !(\text{PosQ}[(b - q)/a] \ \&\& \ \text{SimplerSqrtQ}[(b - q)/(2*a), (b + q)/(2*a)])] /; \text{FreeQ}\{a, b, c\}, x \ \&\& \ \text{GtQ}[b^2 - 4*a*c, 0]$

Rule 1214

$\text{Int}[1/(((d_) + (e_)*(x_)^2)*\text{Sqrt}[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4]), x_Symbol] := \text{With}\{q = \text{Rt}[b^2 - 4*a*c, 2]\}, \text{Dist}[(2*c)/(2*c*d - e*(b - q)), \text{Int}[1/\text{Sqrt}[a + b*x^2 + c*x^4], x], x] - \text{Dist}[e/(2*c*d - e*(b - q)), \text{Int}[(b - q + 2*c*x^2)/((d + e*x^2)*\text{Sqrt}[a + b*x^2 + c*x^4]), x], x] /; \text{FreeQ}\{a, b, c, d, e\}, x \ \&\& \ \text{GtQ}[b^2 - 4*a*c, 0] \ \&\& \ !\text{LtQ}[c, 0]$

Rule 1456

$\text{Int}(((d_) + (e_)*(x_)^{(n_)})^{(q_)}*((f_) + (g_)*(x_)^{(n_)})^{(r_)}*((a_) + (b_)*(x_)^{(n_) + (c_)*(x_)^{(n2_)})^{(p_)}, x_Symbol] := \text{Dist}[(a + b*x^n + c*x^{(2*n)})^{(p)}*\text{FracPart}[p]/((d + e*x^n)^{p/q}*(a/d + (c*x^n)/e)^{p/r}], \text{Int}[(d + e*x^n)^{(p + q)}*(f + g*x^n)^r*(a/d + (c*x^n)/e)^p, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, g, n, p, q, r\}, x \ \&\& \ \text{EqQ}[n2, 2*n] \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0] \ \&\& \ \text{EqQ}[c*d^2 - b*d*e + a*e^2, 0] \ \&\& \ !\text{IntegerQ}[p]$

Rule 539

$\text{Int}[\text{Sqrt}[(c_) + (d_)*(x_)^2]/(((a_) + (b_)*(x_)^2)*\text{Sqrt}[(e_) + (f_)*(x_)^2]), x_Symbol] := \text{Simp}[(c*\text{Sqrt}[e + f*x^2]*\text{EllipticPi}[1 - (b*c)/(a*d), \text{ArcTan}[\text{Rt}[d/c, 2]*x], 1 - (c*f)/(d*e)]/(a*e*\text{Rt}[d/c, 2]*\text{Sqrt}[c + d*x^2]*\text{Sqrt}[(c*(e + f*x^2))/(e*(c + d*x^2))]), x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x \ \&\& \ \text{PosQ}[d/c]$

Rubi steps

$$\begin{aligned}
\int \frac{1}{(7+5x^2)(2+3x^2+x^4)^{3/2}} dx &= -\left(\frac{1}{6} \int \frac{-8-5x^2}{(2+3x^2+x^4)^{3/2}} dx\right) - \frac{25}{6} \int \frac{1}{(7+5x^2)\sqrt{2+3x^2+x^4}} dx \\
&= \frac{x(5+2x^2)}{6\sqrt{2+3x^2+x^4}} + \frac{1}{12} \int \frac{-2-4x^2}{\sqrt{2+3x^2+x^4}} dx - \frac{25}{12} \int \frac{1}{\sqrt{2+3x^2+x^4}} dx + \frac{125}{24} \int \frac{1}{(7+5x^2)\sqrt{2+3x^2+x^4}} dx \\
&= \frac{x(5+2x^2)}{6\sqrt{2+3x^2+x^4}} - \frac{25(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}} F\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{12\sqrt{2}\sqrt{2+3x^2+x^4}} - \frac{1}{6} \int \frac{1}{\sqrt{2+3x^2+x^4}} dx - \frac{125}{24} \int \frac{1}{(7+5x^2)\sqrt{2+3x^2+x^4}} dx \\
&= -\frac{x(2+x^2)}{3\sqrt{2+3x^2+x^4}} + \frac{x(5+2x^2)}{6\sqrt{2+3x^2+x^4}} + \frac{\sqrt{2}(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}} E\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{3\sqrt{2+3x^2+x^4}} - \frac{9(1+x^2)}{24} \int \frac{1}{(7+5x^2)\sqrt{2+3x^2+x^4}} dx
\end{aligned}$$

Mathematica [C] time = 0.17013, size = 138, normalized size = 0.8

$$\frac{-7i\sqrt{x^2+1}\sqrt{x^2+2}\text{EllipticF}\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right),2\right)+14x^3+14i\sqrt{x^2+1}\sqrt{x^2+2}E\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)+25i\sqrt{x^2+1}\sqrt{x^2+2}\text{EllipticE}\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)}{42\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/((7 + 5*x^2)*(2 + 3*x^2 + x^4)^(3/2)), x]

[Out] (35*x + 14*x^3 + (14*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticE[I*ArcSinh[x/Sqrt[2]], 2] - (7*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticF[I*ArcSinh[x/Sqrt[2]], 2] + (25*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticPi[10/7, I*ArcSinh[x/Sqrt[2]], 2])/(42*Sqrt[2 + 3*x^2 + x^4])

Maple [C] time = 0.015, size = 161, normalized size = 0.9

$$-2 \frac{1}{\sqrt{x^4+3x^2+2}} \left(-\frac{1}{6}x^3 - \frac{5x}{12}\right) - \frac{i}{12}\sqrt{2}\text{EllipticF}\left(\frac{i}{2}x\sqrt{2}, \sqrt{2}\right)\sqrt{2x^2+4}\sqrt{x^2+1} \frac{1}{\sqrt{x^4+3x^2+2}} + \frac{i}{6}\sqrt{2}\text{EllipticE}\left(\frac{i}{2}x\sqrt{2}, \sqrt{2}\right)\sqrt{2x^2+4}\sqrt{x^2+1} \frac{1}{\sqrt{x^4+3x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(5*x^2+7)/(x^4+3*x^2+2)^(3/2), x)

[Out] -2*(-1/6*x^3-5/12*x)/(x^4+3*x^2+2)^(1/2)-1/12*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticF(1/2*I*x*2^(1/2), 2^(1/2))+1/6*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticE(1/2*I*x*2^(1/2), 2^(1/2))+25/42*I*2^(1/2)*(1+1/2*x^2)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticPi(1/2*I*x*2^(1/2), 10/7, 2^(1/2))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x^4+3x^2+2)^{\frac{3}{2}}(5x^2+7)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(5*x^2+7)/(x^4+3*x^2+2)^(3/2),x, algorithm="maxima")

[Out] integrate(1/((x^4 + 3*x^2 + 2)^(3/2)*(5*x^2 + 7)), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{x^4 + 3x^2 + 2}}{5x^{10} + 37x^8 + 107x^6 + 151x^4 + 104x^2 + 28}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(5*x^2+7)/(x^4+3*x^2+2)^(3/2),x, algorithm="fricas")

[Out] integral(sqrt(x^4 + 3*x^2 + 2)/(5*x^10 + 37*x^8 + 107*x^6 + 151*x^4 + 104*x^2 + 28), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{((x^2 + 1)(x^2 + 2))^{\frac{3}{2}}(5x^2 + 7)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(5*x**2+7)/(x**4+3*x**2+2)**(3/2),x)

[Out] Integral(1/(((x**2 + 1)*(x**2 + 2))**(3/2)*(5*x**2 + 7)), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x^4 + 3x^2 + 2)^{\frac{3}{2}}(5x^2 + 7)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(5*x^2+7)/(x^4+3*x^2+2)^(3/2),x, algorithm="giac")

[Out] integrate(1/((x^4 + 3*x^2 + 2)^(3/2)*(5*x^2 + 7)), x)

$$3.314 \quad \int \frac{1}{(7+5x^2)^2(2+3x^2+x^4)^{3/2}} dx$$

Optimal. Leaf size=235

$$\frac{463(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\text{EllipticF}\left(\tan^{-1}(x), \frac{1}{2}\right)}{336\sqrt{2}\sqrt{x^4+3x^2+2}} + \frac{625\sqrt{x^4+3x^2+2}x}{504(5x^2+7)} - \frac{31(x^2+2)x}{56\sqrt{x^4+3x^2+2}} + \frac{(11x^2+20)x}{36\sqrt{x^4+3x^2+2}} + \frac{31(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}}{28\sqrt{2}\sqrt{x^4+3x^2+2}}$$

[Out] (-31*x*(2 + x^2))/(56*Sqrt[2 + 3*x^2 + x^4]) + (x*(20 + 11*x^2))/(36*Sqrt[2 + 3*x^2 + x^4]) + (625*x*Sqrt[2 + 3*x^2 + x^4])/(504*(7 + 5*x^2)) + (31*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x], 1/2])/(28*Sqrt[2]*Sqrt[2 + 3*x^2 + x^4]) - (463*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(336*Sqrt[2]*Sqrt[2 + 3*x^2 + x^4]) + (375*(2 + x^2)*EllipticPi[2/7, ArcTan[x], 1/2])/(784*Sqrt[2]*Sqrt[(2 + x^2)/(1 + x^2)]*Sqrt[2 + 3*x^2 + x^4])

Rubi [A] time = 0.429144, antiderivative size = 235, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {1228, 1178, 1189, 1099, 1135, 1223, 1716, 1214, 1456, 539}

$$\frac{625\sqrt{x^4+3x^2+2}x}{504(5x^2+7)} - \frac{31(x^2+2)x}{56\sqrt{x^4+3x^2+2}} + \frac{(11x^2+20)x}{36\sqrt{x^4+3x^2+2}} - \frac{463(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}F\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{336\sqrt{2}\sqrt{x^4+3x^2+2}} + \frac{31(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}}{28\sqrt{2}\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

[In] Int[1/((7 + 5*x^2)^2*(2 + 3*x^2 + x^4)^(3/2)),x]

[Out] (-31*x*(2 + x^2))/(56*Sqrt[2 + 3*x^2 + x^4]) + (x*(20 + 11*x^2))/(36*Sqrt[2 + 3*x^2 + x^4]) + (625*x*Sqrt[2 + 3*x^2 + x^4])/(504*(7 + 5*x^2)) + (31*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x], 1/2])/(28*Sqrt[2]*Sqrt[2 + 3*x^2 + x^4]) - (463*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(336*Sqrt[2]*Sqrt[2 + 3*x^2 + x^4]) + (375*(2 + x^2)*EllipticPi[2/7, ArcTan[x], 1/2])/(784*Sqrt[2]*Sqrt[(2 + x^2)/(1 + x^2)]*Sqrt[2 + 3*x^2 + x^4])

Rule 1228

Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{aa, bb, cc}, Int[ExpandIntegrand[1/Sqrt[aa + bb*x^2 + cc*x^4], (d + e*x^2)^q*(aa + bb*x^2 + cc*x^4)^(p + 1/2), x] /. {aa -> a, bb -> b, cc -> c}, x] /. FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, 0] && IntegerQ[p + 1/2]

Rule 1178

Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(x*(a*b*e - d*(b^2 - 2*a*c)) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /. FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && IntegerQ[2*p]

Rule 1189

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol]
:> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a]
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1099

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]},
Simp[((2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q)*x^2)/(2*a + (b + q)*x^2)]*
EllipticF[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)/(b + q)])/(2*a*Rt[(b + q)/(2*a), 2]*
Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] && !(PosQ[(b - q)/a] &&
SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1135

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]},
Simp[(x*(b + q + 2*c*x^2))/(2*c*Sqrt[a + b*x^2 + c*x^4]), x] - Simp[(Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*
Sqrt[(2*a + (b - q)*x^2)/(2*a + (b + q)*x^2)]*EllipticE[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)/(b + q)])/(2*c*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] && !(PosQ[(b - q)/a] &&
SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1223

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol]
:> -Simp[(e^2*x*(d + e*x^2)^(q + 1)*Sqrt[a + b*x^2 + c*x^4])/(2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2)), Int[((d + e*x^2)^(q + 1)*Simp[a*e^2*(2*q + 3) + 2*d*(c*d - b*e)*(q + 1) - 2*e*(c*d*(q + 1) - b*e*(q + 2))*x^2 + c*e^2*(2*q + 5)*x^4, x])/Sqrt[a + b*x^2 + c*x^4], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && ILtQ[q, -1]
```

Rule 1716

```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_Symbol]
:> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[P4x, x, 4]}, -Dist[(e^2)^(-1), Int[(C*d - B*e - C*e*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] + Dist[(C*d^2 - B*d*e + A*e^2)/e^2, Int[1/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[P4x, x^2, 2] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0]
```

Rule 1214

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_Symbol]
:> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/(2*c*d - e*(b - q)), Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[e/(2*c*d - e*(b - q)), Int[(b - q + 2*c*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0] && !LtQ[c, 0]
```

Rule 1456

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((f_) + (g_.)*(x_)^(n_))^(r_)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d + (c*x^n)/e)^FracPart[p]), Int[(d + e*x^n)^(p + q)*(f + g*x^n)^r*(a/d + (c*x^n)/e)^p, x], x] /; Free
```


$Q[\{a, b, c, d, e, f, g, n, p, q, r\}, x] \ \&\& \ \text{EqQ}[n^2, 2*n] \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0] \ \&\& \ \text{EqQ}[c*d^2 - b*d*e + a*e^2, 0] \ \&\& \ !\text{IntegerQ}[p]$

Rule 539

$\text{Int}[\text{Sqrt}[(c_) + (d_)*(x_)^2]/(((a_) + (b_)*(x_)^2)*\text{Sqrt}[(e_) + (f_)*(x_)^2]), x_Symbol] \ :> \ \text{Simp}[(c*\text{Sqrt}[e + f*x^2]*\text{EllipticPi}[1 - (b*c)/(a*d), \text{ArcTan}[\text{Rt}[d/c, 2]*x], 1 - (c*f)/(d*e)]/(a*e*\text{Rt}[d/c, 2]*\text{Sqrt}[c + d*x^2]*\text{Sqrt}[(c*(e + f*x^2))/(e*(c + d*x^2)]), x] \ /; \ \text{FreeQ}[\{a, b, c, d, e, f\}, x] \ \&\& \ \text{PosQ}[d/c]$

Rubi steps

$$\begin{aligned} \int \frac{1}{(7+5x^2)^2(2+3x^2+x^4)^{3/2}} dx &= \int \left(\frac{14+5x^2}{36(2+3x^2+x^4)^{3/2}} - \frac{25}{6(7+5x^2)^2\sqrt{2+3x^2+x^4}} - \frac{25}{36(7+5x^2)\sqrt{2+3x^2+x^4}} \right) dx \\ &= \frac{1}{36} \int \frac{14+5x^2}{(2+3x^2+x^4)^{3/2}} dx - \frac{25}{36} \int \frac{1}{(7+5x^2)\sqrt{2+3x^2+x^4}} dx - \frac{25}{6} \int \frac{1}{(7+5x^2)\sqrt{2+3x^2+x^4}} dx \\ &= \frac{x(20+11x^2)}{36\sqrt{2+3x^2+x^4}} + \frac{625x\sqrt{2+3x^2+x^4}}{504(7+5x^2)} - \frac{1}{72} \int \frac{26+22x^2}{\sqrt{2+3x^2+x^4}} dx - \frac{25}{504} \int \frac{1}{(7+5x^2)\sqrt{2+3x^2+x^4}} dx \\ &= \frac{x(20+11x^2)}{36\sqrt{2+3x^2+x^4}} + \frac{625x\sqrt{2+3x^2+x^4}}{504(7+5x^2)} - \frac{25(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}F\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{72\sqrt{2}\sqrt{2+3x^2+x^4}} + \frac{25}{504} \int \frac{1}{(7+5x^2)\sqrt{2+3x^2+x^4}} dx \\ &= -\frac{11x(2+x^2)}{36\sqrt{2+3x^2+x^4}} + \frac{x(20+11x^2)}{36\sqrt{2+3x^2+x^4}} + \frac{625x\sqrt{2+3x^2+x^4}}{504(7+5x^2)} + \frac{11(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}}{18\sqrt{2}\sqrt{2+3x^2+x^4}} \\ &= -\frac{31x(2+x^2)}{56\sqrt{2+3x^2+x^4}} + \frac{x(20+11x^2)}{36\sqrt{2+3x^2+x^4}} + \frac{625x\sqrt{2+3x^2+x^4}}{504(7+5x^2)} + \frac{31(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}}{28\sqrt{2}\sqrt{2+3x^2+x^4}} \\ &= -\frac{31x(2+x^2)}{56\sqrt{2+3x^2+x^4}} + \frac{x(20+11x^2)}{36\sqrt{2+3x^2+x^4}} + \frac{625x\sqrt{2+3x^2+x^4}}{504(7+5x^2)} + \frac{31(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}}{28\sqrt{2}\sqrt{2+3x^2+x^4}} \end{aligned}$$

Mathematica [C] time = 0.274219, size = 208, normalized size = 0.89

$$\frac{182i\sqrt{x^2+1}\sqrt{x^2+2}(5x^2+7)\text{EllipticF}\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right), 2\right) + 3255x^5 + 10157x^3 + 651i\sqrt{x^2+1}\sqrt{x^2+2}(5x^2+7)E\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\right)}{1176(5x^2+7)}$$

Antiderivative was successfully verified.

[In] Integrate[1/((7 + 5*x^2)^2*(2 + 3*x^2 + x^4)^(3/2)), x]

[Out] (7490*x + 10157*x^3 + 3255*x^5 + (651*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*(7 + 5*x^2)*EllipticE[I*ArcSinh[x/Sqrt[2]], 2] + (182*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*(7 + 5*x^2)*EllipticF[I*ArcSinh[x/Sqrt[2]], 2] + (1575*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticPi[10/7, I*ArcSinh[x/Sqrt[2]], 2] + (1125*I)*x^2*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticPi[10/7, I*ArcSinh[x/Sqrt[2]], 2])/(1176*(7 + 5*x^2)*Sqrt[2 + 3*x^2 + x^4])

Maple [C] time = 0.019, size = 185, normalized size = 0.8

$$\frac{625x}{2520x^2 + 3528} \sqrt{x^4 + 3x^2 + 2} - 2 \frac{1}{\sqrt{x^4 + 3x^2 + 2}} \left(-\frac{11x^3}{72} - \frac{5x}{18} \right) + \frac{13i}{168} \sqrt{2} \text{EllipticF} \left(\frac{i}{2} x \sqrt{2}, \sqrt{2} \right) \sqrt{2x^2 + 4} \sqrt{x^2 + 1} \frac{1}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(5*x^2+7)^2/(x^4+3*x^2+2)^(3/2),x)

[Out] 625/504*x*(x^4+3*x^2+2)^(1/2)/(5*x^2+7)-2*(-11/72*x^3-5/18*x)/(x^4+3*x^2+2)^(1/2)+13/168*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticF(1/2*I*x*2^(1/2),2^(1/2))+31/112*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticE(1/2*I*x*2^(1/2),2^(1/2))+75/392*I*2^(1/2)*(1+1/2*x^2)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticPi(1/2*I*x*2^(1/2),10/7,2^(1/2))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x^4 + 3x^2 + 2)^{\frac{3}{2}} (5x^2 + 7)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(5*x^2+7)^2/(x^4+3*x^2+2)^(3/2),x, algorithm="maxima")

[Out] integrate(1/((x^4 + 3*x^2 + 2)^(3/2)*(5*x^2 + 7)^2), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral} \left(\frac{\sqrt{x^4 + 3x^2 + 2}}{25x^{12} + 220x^{10} + 794x^8 + 1504x^6 + 1577x^4 + 868x^2 + 196}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(5*x^2+7)^2/(x^4+3*x^2+2)^(3/2),x, algorithm="fricas")

[Out] integral(sqrt(x^4 + 3*x^2 + 2)/(25*x^12 + 220*x^10 + 794*x^8 + 1504*x^6 + 1577*x^4 + 868*x^2 + 196), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{((x^2 + 1)(x^2 + 2))^{\frac{3}{2}} (5x^2 + 7)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(5*x**2+7)**2/(x**4+3*x**2+2)**(3/2),x)

[Out] Integral(1/(((x**2 + 1)*(x**2 + 2))**(3/2)*(5*x**2 + 7)**2), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x^4 + 3x^2 + 2)^{\frac{3}{2}}(5x^2 + 7)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(5*x^2+7)^2/(x^4+3*x^2+2)^(3/2),x, algorithm="giac")

[Out] integrate(1/((x^4 + 3*x^2 + 2)^(3/2)*(5*x^2 + 7)^2), x)

$$3.315 \quad \int \frac{1}{(7+5x^2)^3(2+3x^2+x^4)^{3/2}} dx$$

Optimal. Leaf size=263

$$\frac{49907(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\text{EllipticF}\left(\tan^{-1}(x), \frac{1}{2}\right)}{56448\sqrt{2}\sqrt{x^4+3x^2+2}} + \frac{41875\sqrt{x^4+3x^2+2}x}{84672(5x^2+7)} + \frac{625\sqrt{x^4+3x^2+2}x}{1008(5x^2+7)^2} - \frac{5797(x^2+2)x}{28224\sqrt{x^4+3x^2+2}} +$$

```
[Out] (-5797*x*(2 + x^2))/(28224*Sqrt[2 + 3*x^2 + x^4]) + (x*(50 + 23*x^2))/(216*
Sqrt[2 + 3*x^2 + x^4]) + (625*x*Sqrt[2 + 3*x^2 + x^4])/(1008*(7 + 5*x^2)^2)
+ (41875*x*Sqrt[2 + 3*x^2 + x^4])/(84672*(7 + 5*x^2)) + (5797*(1 + x^2)*Sq
rt[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x], 1/2])/(14112*Sqrt[2]*Sqrt[2 +
3*x^2 + x^4]) - (49907*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan
[x], 1/2])/(56448*Sqrt[2]*Sqrt[2 + 3*x^2 + x^4]) + (192625*(2 + x^2)*Ellipt
icPi[2/7, ArcTan[x], 1/2])/(395136*Sqrt[2]*Sqrt[(2 + x^2)/(1 + x^2)]*Sqrt[2
+ 3*x^2 + x^4])
```

Rubi [A] time = 0.760469, antiderivative size = 263, normalized size of antiderivative = 1., number of steps used = 29, number of rules used = 11, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.458$, Rules used = {1228, 1178, 1189, 1099, 1135, 1223, 1696, 1716, 1214, 1456, 539}

$$\frac{41875\sqrt{x^4+3x^2+2}x}{84672(5x^2+7)} + \frac{625\sqrt{x^4+3x^2+2}x}{1008(5x^2+7)^2} - \frac{5797(x^2+2)x}{28224\sqrt{x^4+3x^2+2}} + \frac{(23x^2+50)x}{216\sqrt{x^4+3x^2+2}} - \frac{49907(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}F\left(\tan^{-1}(x), \frac{1}{2}\right)}{56448\sqrt{2}\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((7 + 5*x^2)^3*(2 + 3*x^2 + x^4)^(3/2)), x]
```

```
[Out] (-5797*x*(2 + x^2))/(28224*Sqrt[2 + 3*x^2 + x^4]) + (x*(50 + 23*x^2))/(216*
Sqrt[2 + 3*x^2 + x^4]) + (625*x*Sqrt[2 + 3*x^2 + x^4])/(1008*(7 + 5*x^2)^2)
+ (41875*x*Sqrt[2 + 3*x^2 + x^4])/(84672*(7 + 5*x^2)) + (5797*(1 + x^2)*Sq
rt[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x], 1/2])/(14112*Sqrt[2]*Sqrt[2 +
3*x^2 + x^4]) - (49907*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan
[x], 1/2])/(56448*Sqrt[2]*Sqrt[2 + 3*x^2 + x^4]) + (192625*(2 + x^2)*Ellipt
icPi[2/7, ArcTan[x], 1/2])/(395136*Sqrt[2]*Sqrt[(2 + x^2)/(1 + x^2)]*Sqrt[2
+ 3*x^2 + x^4])
```

Rule 1228

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] := Module[{aa, bb, cc}, Int[ExpandIntegrand[1/Sqrt[aa + bb*x^2 + c
*x^4], (d + e*x^2)^q*(aa + bb*x^2 + cc*x^4)^(p + 1/2), x] /. {aa -> a, bb
-> b, cc -> c}, x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, 0] && IntegerQ[p + 1/2]
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] := Simp[(x*(a*b*e - d*(b^2 - 2*a*c)) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 +
c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2
- 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
```

LtQ[p, -1] && IntegerQ[2*p]

Rule 1189

Int[((d_) + (e_)*(x_)^2)/Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4], x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a] || PosQ[(b - q)/a] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]

Rule 1099

Int[1/Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4], x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[((2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q)*x^2)/(2*a + (b + q)*x^2)]*EllipticF[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)/(b + q)])/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] && !(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]

Rule 1135

Int[(x_)^2/Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4], x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(x*(b + q + 2*c*x^2))/(2*c*Sqrt[a + b*x^2 + c*x^4]), x] - Simp[(Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q)*x^2)/(2*a + (b + q)*x^2)]*EllipticE[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)/(b + q)])/(2*c*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] && !(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]

Rule 1223

Int[((d_) + (e_)*(x_)^2)^(q_)/Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4], x_Symbol] := -Simp[(e^2*x*(d + e*x^2)^(q + 1)*Sqrt[a + b*x^2 + c*x^4])/(2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2)), Int[((d + e*x^2)^(q + 1)*Simp[a*e^2*(2*q + 3) + 2*d*(c*d - b*e)*(q + 1) - 2*e*(c*d*(q + 1) - b*e*(q + 2))*x^2 + c*e^2*(2*q + 5)*x^4, x])/Sqrt[a + b*x^2 + c*x^4], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && ILtQ[q, -1]

Rule 1696

Int[((P4x_)*((d_) + (e_)*(x_)^2)^(q_))/Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4], x_Symbol] := With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[P4x, x, 4]}, -Simp[((C*d^2 - B*d*e + A*e^2)*x*(d + e*x^2)^(q + 1)*Sqrt[a + b*x^2 + c*x^4])/(2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2)), Int[((d + e*x^2)^(q + 1)*Simp[a*d*(C*d - B*e) + A*(a*e^2*(2*q + 3) + 2*d*(c*d - b*e)*(q + 1)) - 2*((B*d - A*e)*(b*e*(q + 2) - c*d*(q + 1)) - C*d*(b*d + a*e*(q + 1)))*x^2 + c*(C*d^2 - B*d*e + A*e^2)*(2*q + 5)*x^4, x])/Sqrt[a + b*x^2 + c*x^4], x], x] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[P4x, x^2] && LeQ[Expon[P4x, x], 4] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, -1]

Rule 1716

Int[(P4x_)/(((d_) + (e_)*(x_)^2)*Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4]), x_Symbol] := With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[P4x, x, 4]}, -Dist[(e^2)^(-1), Int[(C*d - B*e - C*e*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] + Dist[(C*d^2 - B*d*e + A*e^2)/e^2, Int[1/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[P4x, x^2, 2] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 -

$a \cdot e^2, 0]$

Rule 1214

Int[1/(((d_) + (e_)*(x_)^2)*Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/(2*c*d - e*(b - q)), Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[e/(2*c*d - e*(b - q)), Int[(b - q + 2*c*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0] && !LtQ[c, 0]

Rule 1456

Int[((d_) + (e_)*(x_)^(n_))^(q_)*((f_) + (g_)*(x_)^(n_))^(r_)*((a_) + (b_)*(x_)^(n_) + (c_)*(x_)^(n2_))^(p_), x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d + (c*x^n)/e)^FracPart[p]), Int[(d + e*x^n)^(p + q)*(f + g*x^n)^r*(a/d + (c*x^n)/e)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q, r}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && !IntegerQ[p]

Rule 539

Int[Sqrt[(c_) + (d_)*(x_)^2]/(((a_) + (b_)*(x_)^2)*Sqrt[(e_) + (f_)*(x_)^2]), x_Symbol] :> Simp[(c*Sqrt[e + f*x^2]*EllipticPi[1 - (b*c)/(a*d), ArcTan[Rt[d/c, 2]*x], 1 - (c*f)/(d*e)]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(e + f*x^2))/(e*(c + d*x^2))]), x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[d/c]

Rubi steps

$$\begin{aligned}
\int \frac{1}{(7+5x^2)^3(2+3x^2+x^4)^{3/2}} dx &= \int \left(-\frac{-62-35x^2}{216(2+3x^2+x^4)^{3/2}} - \frac{25}{6(7+5x^2)^3\sqrt{2+3x^2+x^4}} - \frac{25}{36(7+5x^2)^2\sqrt{2+3x^2+x^4}} \right) dx \\
&= -\left(\frac{1}{216} \int \frac{-62-35x^2}{(2+3x^2+x^4)^{3/2}} dx \right) - \frac{25}{36} \int \frac{1}{(7+5x^2)^2\sqrt{2+3x^2+x^4}} dx - \frac{175}{216} \int \frac{1}{(7+5x^2)\sqrt{2+3x^2+x^4}} dx \\
&= \frac{x(50+23x^2)}{216\sqrt{2+3x^2+x^4}} + \frac{625x\sqrt{2+3x^2+x^4}}{1008(7+5x^2)^2} + \frac{625x\sqrt{2+3x^2+x^4}}{3024(7+5x^2)} + \frac{1}{432} \int \frac{-38}{\sqrt{2+3x^2+x^4}} dx \\
&= \frac{x(50+23x^2)}{216\sqrt{2+3x^2+x^4}} + \frac{625x\sqrt{2+3x^2+x^4}}{1008(7+5x^2)^2} + \frac{41875x\sqrt{2+3x^2+x^4}}{84672(7+5x^2)} - \frac{175(1+x^2)}{432\sqrt{2+3x^2+x^4}} \\
&= -\frac{23x(2+x^2)}{216\sqrt{2+3x^2+x^4}} + \frac{x(50+23x^2)}{216\sqrt{2+3x^2+x^4}} + \frac{625x\sqrt{2+3x^2+x^4}}{1008(7+5x^2)^2} + \frac{41875x\sqrt{2+3x^2+x^4}}{84672(7+5x^2)} \\
&= -\frac{149x(2+x^2)}{1008\sqrt{2+3x^2+x^4}} + \frac{x(50+23x^2)}{216\sqrt{2+3x^2+x^4}} + \frac{625x\sqrt{2+3x^2+x^4}}{1008(7+5x^2)^2} + \frac{41875x\sqrt{2+3x^2+x^4}}{84672(7+5x^2)} \\
&= -\frac{5797x(2+x^2)}{28224\sqrt{2+3x^2+x^4}} + \frac{x(50+23x^2)}{216\sqrt{2+3x^2+x^4}} + \frac{625x\sqrt{2+3x^2+x^4}}{1008(7+5x^2)^2} + \frac{41875x\sqrt{2+3x^2+x^4}}{84672(7+5x^2)} \\
&= -\frac{5797x(2+x^2)}{28224\sqrt{2+3x^2+x^4}} + \frac{x(50+23x^2)}{216\sqrt{2+3x^2+x^4}} + \frac{625x\sqrt{2+3x^2+x^4}}{1008(7+5x^2)^2} + \frac{41875x\sqrt{2+3x^2+x^4}}{84672(7+5x^2)}
\end{aligned}$$

Mathematica [C] time = 0.488673, size = 159, normalized size = 0.6

$$\frac{-742i\sqrt{x^2+1}\sqrt{x^2+2}\text{EllipticF}\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right),2\right) + \frac{7x(144925x^6+698290x^4+1089803x^2+550550)}{(5x^2+7)^2} + 40579i\sqrt{x^2+1}\sqrt{x^2+2}E\left(\frac{x}{\sqrt{2}}\right)}{197568\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/((7+5*x^2)^3*(2+3*x^2+x^4)^(3/2)),x]

[Out] ((7*x*(550550+1089803*x^2+698290*x^4+144925*x^6))/(7+5*x^2)^2+(40579*I)*Sqrt[1+x^2]*Sqrt[2+x^2]*EllipticE[I*ArcSinh[x/Sqrt[2]],2]- (742*I)*Sqrt[1+x^2]*Sqrt[2+x^2]*EllipticF[I*ArcSinh[x/Sqrt[2]],2]+(38525*I)*Sqrt[1+x^2]*Sqrt[2+x^2]*EllipticPi[10/7,I*ArcSinh[x/Sqrt[2]],2])/(197568*Sqrt[2+3*x^2+x^4])

Maple [C] time = 0.023, size = 209, normalized size = 0.8

$$\frac{625x}{1008(5x^2+7)^2}\sqrt{x^4+3x^2+2} + \frac{41875x}{423360x^2+592704}\sqrt{x^4+3x^2+2} - 2\frac{1}{\sqrt{x^4+3x^2+2}}\left(-\frac{23x^3}{432}-\frac{25x}{216}\right) - \frac{53i}{28224}\sqrt{x^4+3x^2+2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(5*x^2+7)^3/(x^4+3*x^2+2)^(3/2),x)

[Out] 625/1008*x*(x^4+3*x^2+2)^(1/2)/(5*x^2+7)^2+41875/84672*x*(x^4+3*x^2+2)^(1/2)/(5*x^2+7)-2*(-23/432*x^3-25/216*x)/(x^4+3*x^2+2)^(1/2)-53/28224*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticF(1/2*I*x*2^(1/2),2^(1/2))+5797/56448*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticE(1/2*I*x*2^(1/2),2^(1/2))+38525/197568*I*2^(1/2)*(1+1/2*x^2)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticPi(1/2*I*x*2^(1/2),10/7,2^(1/2))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x^4 + 3x^2 + 2)^{\frac{3}{2}}(5x^2 + 7)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(5*x^2+7)^3/(x^4+3*x^2+2)^(3/2),x, algorithm="maxima")

[Out] integrate(1/((x^4 + 3*x^2 + 2)^(3/2)*(5*x^2 + 7)^3), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{x^4 + 3x^2 + 2}}{125x^{14} + 1275x^{12} + 5510x^{10} + 13078x^8 + 18413x^6 + 15379x^4 + 7056x^2 + 1372}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(5*x^2+7)^3/(x^4+3*x^2+2)^(3/2),x, algorithm="fricas")

[Out] integral(sqrt(x^4 + 3*x^2 + 2)/(125*x^14 + 1275*x^12 + 5510*x^10 + 13078*x^8 + 18413*x^6 + 15379*x^4 + 7056*x^2 + 1372), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{((x^2 + 1)(x^2 + 2))^{\frac{3}{2}}(5x^2 + 7)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(5*x**2+7)**3/(x**4+3*x**2+2)**(3/2),x)

[Out] Integral(1/(((x**2 + 1)*(x**2 + 2))**(3/2)*(5*x**2 + 7)**3), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x^4 + 3x^2 + 2)^{\frac{3}{2}}(5x^2 + 7)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+7)^3/(x^4+3*x^2+2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((x^4 + 3*x^2 + 2)^(3/2))*(5*x^2 + 7)^3), x)
```

3.316 $\int (7 + 5x^2)^4 \sqrt{2 + x^2 - x^4} dx$

Optimal. Leaf size=116

$$-\frac{539419}{77} \text{EllipticF}\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right), -2\right) - \frac{625}{11} (-x^4 + x^2 + 2)^{3/2} x^5 - \frac{14500}{33} (-x^4 + x^2 + 2)^{3/2} x^3 - \frac{116100}{77} (-x^4 + x^2 + 2)^{3/2} x$$

[Out] (x*(177953 + 717372*x^2)*Sqrt[2 + x^2 - x^4])/231 - (116100*x*(2 + x^2 - x^4)^(3/2))/77 - (14500*x^3*(2 + x^2 - x^4)^(3/2))/33 - (625*x^5*(2 + x^2 - x^4)^(3/2))/11 + (3764813*EllipticE[ArcSin[x/Sqrt[2]], -2])/231 - (539419*EllipticF[ArcSin[x/Sqrt[2]], -2])/77

Rubi [A] time = 0.111749, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {1206, 1679, 1176, 1180, 524, 424, 419}

$$-\frac{625}{11} (-x^4 + x^2 + 2)^{3/2} x^5 - \frac{14500}{33} (-x^4 + x^2 + 2)^{3/2} x^3 - \frac{116100}{77} (-x^4 + x^2 + 2)^{3/2} x + \frac{1}{231} (717372x^2 + 177953) \sqrt{-x^4}$$

Antiderivative was successfully verified.

[In] Int[(7 + 5*x^2)^4*Sqrt[2 + x^2 - x^4],x]

[Out] (x*(177953 + 717372*x^2)*Sqrt[2 + x^2 - x^4])/231 - (116100*x*(2 + x^2 - x^4)^(3/2))/77 - (14500*x^3*(2 + x^2 - x^4)^(3/2))/33 - (625*x^5*(2 + x^2 - x^4)^(3/2))/11 + (3764813*EllipticE[ArcSin[x/Sqrt[2]], -2])/231 - (539419*EllipticF[ArcSin[x/Sqrt[2]], -2])/77

Rule 1206

Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(e^q*x^(2*q - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(4*p + 2*q + 1)), x] + Dist[1/(c*(4*p + 2*q + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*p + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1]

Rule 1679

Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{q = Expon[Pq, x^2], e = Coeff[Pq, x^2, Expon[Pq, x^2]]}, Simp[(e*x^(2*q - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(2*q + 4*p + 1)), x] + Dist[1/(c*(2*q + 4*p + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*q + 4*p + 1)*Pq - a*e*(2*q - 3)*x^(2*q - 4) - b*e*(2*q + 2*p - 1)*x^(2*q - 2) - c*e*(2*q + 4*p + 1)*x^(2*q), x], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && !LtQ[p, -1]

Rule 1176

Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*(a + b*x^2 + c*x^4)^p)/(c*(4*p + 1)*(4*p + 3)), x] + Dist[(2*p)/(c*(4*p + 1)*(4*p + 3)), Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) - b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&

GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]

Rule 1180

Int[((d_) + (e_)*(x_)^2)/Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4], x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]

Rule 524

Int[((e_) + (f_)*(x_)^(n_))/(Sqrt[(a_) + (b_)*(x_)^(n_)]*Sqrt[(c_) + (d_)*(x_)^(n_)]), x_Symbol] := Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n], x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && !(EqQ[n, 2] && ((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && (!GtQ[c, 0] || SimplerSqrtQ[-(b/a), -(d/c)]))))))

Rule 424

Int[Sqrt[(a_) + (b_)*(x_)^2]/Sqrt[(c_) + (d_)*(x_)^2], x_Symbol] := Simp[Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)]/(Sqrt[c]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]

Rule 419

Int[1/(Sqrt[(a_) + (b_)*(x_)^2]*Sqrt[(c_) + (d_)*(x_)^2]), x_Symbol] := Simp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)]/(Sqrt[a]*Sqrt[c]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0] && !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])

Rubi steps

$$\begin{aligned} \int (7 + 5x^2)^4 \sqrt{2 + x^2 - x^4} dx &= -\frac{625}{11}x^5(2 + x^2 - x^4)^{3/2} - \frac{1}{11} \int \sqrt{2 + x^2 - x^4} (-26411 - 75460x^2 - 87100x^4 - 43500x^6) dx \\ &= -\frac{14500}{33}x^3(2 + x^2 - x^4)^{3/2} - \frac{625}{11}x^5(2 + x^2 - x^4)^{3/2} + \frac{1}{99} \int \sqrt{2 + x^2 - x^4} (237699 + 116100x^2 + 116100x^4 + 116100x^6) dx \\ &= -\frac{116100}{77}x(2 + x^2 - x^4)^{3/2} - \frac{14500}{33}x^3(2 + x^2 - x^4)^{3/2} - \frac{625}{11}x^5(2 + x^2 - x^4)^{3/2} - \frac{1}{69} \int \sqrt{2 + x^2 - x^4} (177953 + 717372x^2) dx \\ &= \frac{1}{231}x(177953 + 717372x^2)\sqrt{2 + x^2 - x^4} - \frac{116100}{77}x(2 + x^2 - x^4)^{3/2} - \frac{14500}{33}x^3(2 + x^2 - x^4)^{3/2} \\ &= \frac{1}{231}x(177953 + 717372x^2)\sqrt{2 + x^2 - x^4} - \frac{116100}{77}x(2 + x^2 - x^4)^{3/2} - \frac{14500}{33}x^3(2 + x^2 - x^4)^{3/2} \\ &= \frac{1}{231}x(177953 + 717372x^2)\sqrt{2 + x^2 - x^4} - \frac{116100}{77}x(2 + x^2 - x^4)^{3/2} - \frac{14500}{33}x^3(2 + x^2 - x^4)^{3/2} \\ &= \frac{1}{231}x(177953 + 717372x^2)\sqrt{2 + x^2 - x^4} - \frac{116100}{77}x(2 + x^2 - x^4)^{3/2} - \frac{14500}{33}x^3(2 + x^2 - x^4)^{3/2} \end{aligned}$$

Mathematica [C] time = 0.125528, size = 112, normalized size = 0.97

$$\frac{-4838091i\sqrt{-2x^4 + 2x^2 + 4}\text{EllipticF}\left(i\sinh^{-1}(x), -\frac{1}{2}\right) - 13125x^{13} - 75250x^{11} - 105925x^9 + 231228x^7 + 1125819x^5}{231\sqrt{-x^4 + x^2 + 2}}$$

Antiderivative was successfully verified.

[In] Integrate[(7 + 5*x^2)^4*Sqrt[2 + x^2 - x^4],x]

[Out] (-1037294*x - 186503*x^3 + 1125819*x^5 + 231228*x^7 - 105925*x^9 - 75250*x^11 - 13125*x^13 + (3764813*I)*Sqrt[4 + 2*x^2 - 2*x^4]*EllipticE[I*ArcSinh[x], -1/2] - (4838091*I)*Sqrt[4 + 2*x^2 - 2*x^4]*EllipticF[I*ArcSinh[x], -1/2])/(231*Sqrt[2 + x^2 - x^4])

Maple [A] time = 0.032, size = 193, normalized size = 1.7

$$-\frac{518647x}{231}\sqrt{-x^4+x^2+2} + \frac{20050x^5}{21}\sqrt{-x^4+x^2+2} + \frac{166072x^3}{231}\sqrt{-x^4+x^2+2} + \frac{625x^9}{11}\sqrt{-x^4+x^2+2} + \frac{12625x^7}{33}\sqrt{-x^4+x^2+2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((5*x^2+7)^4*(-x^4+x^2+2)^(1/2),x)

[Out] -518647/231*x*(-x^4+x^2+2)^(1/2)+20050/21*x^5*(-x^4+x^2+2)^(1/2)+166072/231*x^3*(-x^4+x^2+2)^(1/2)+625/11*x^9*(-x^4+x^2+2)^(1/2)+12625/33*x^7*(-x^4+x^2+2)^(1/2)-3764813/462*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*(EllipticF(1/2*x*2^(1/2),I*2^(1/2))-EllipticE(1/2*x*2^(1/2),I*2^(1/2)))+1073278/231*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticF(1/2*x*2^(1/2),I*2^(1/2))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-x^4 + x^2 + 2}(5x^2 + 7)^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^2+7)^4*(-x^4+x^2+2)^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(-x^4 + x^2 + 2)*(5*x^2 + 7)^4, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(625x^8 + 3500x^6 + 7350x^4 + 6860x^2 + 2401\right)\sqrt{-x^4 + x^2 + 2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^2+7)^4*(-x^4+x^2+2)^(1/2),x, algorithm="fricas")

[Out] integral((625*x^8 + 3500*x^6 + 7350*x^4 + 6860*x^2 + 2401)*sqrt(-x^4 + x^2 + 2), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-(x^2 - 2)(x^2 + 1)}(5x^2 + 7)^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x**2+7)**4*(-x**4+x**2+2)**(1/2),x)

[Out] Integral(sqrt(-(x**2 - 2)*(x**2 + 1))*(5*x**2 + 7)**4, x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-x^4 + x^2 + 2} (5x^2 + 7)^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^2+7)^4*(-x^4+x^2+2)^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(-x^4 + x^2 + 2)*(5*x^2 + 7)^4, x)

3.317 $\int (7 + 5x^2)^3 \sqrt{2 + x^2 - x^4} dx$

Optimal. Leaf size=95

$$-\frac{8735}{21} \text{EllipticF}\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right), -2\right) - \frac{125}{9} (-x^4 + x^2 + 2)^{3/2} x^3 - \frac{1825}{21} (-x^4 + x^2 + 2)^{3/2} x + \frac{1}{63} (14691x^2 + 5956) \sqrt{-x^4 + x^2 + 2}$$

[Out] (x*(5956 + 14691*x^2)*Sqrt[2 + x^2 - x^4])/63 - (1825*x*(2 + x^2 - x^4)^(3/2))/21 - (125*x^3*(2 + x^2 - x^4)^(3/2))/9 + (79411*EllipticE[ArcSin[x/Sqrt[2]], -2])/63 - (8735*EllipticF[ArcSin[x/Sqrt[2]], -2])/21

Rubi [A] time = 0.0870409, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {1206, 1679, 1176, 1180, 524, 424, 419}

$$-\frac{125}{9} (-x^4 + x^2 + 2)^{3/2} x^3 - \frac{1825}{21} (-x^4 + x^2 + 2)^{3/2} x + \frac{1}{63} (14691x^2 + 5956) \sqrt{-x^4 + x^2 + 2} - \frac{8735}{21} F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\right) - 2$$

Antiderivative was successfully verified.

[In] Int[(7 + 5*x^2)^3*Sqrt[2 + x^2 - x^4],x]

[Out] (x*(5956 + 14691*x^2)*Sqrt[2 + x^2 - x^4])/63 - (1825*x*(2 + x^2 - x^4)^(3/2))/21 - (125*x^3*(2 + x^2 - x^4)^(3/2))/9 + (79411*EllipticE[ArcSin[x/Sqrt[2]], -2])/63 - (8735*EllipticF[ArcSin[x/Sqrt[2]], -2])/21

Rule 1206

Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := Simp[(e^q*x^(2*q - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(4*p + 2*q + 1)), x] + Dist[1/(c*(4*p + 2*q + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*p + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1]

Rule 1679

Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := With[{q = Expon[Pq, x^2], e = Coeff[Pq, x^2, Expon[Pq, x^2]]}, Simp[(e*x^(2*q - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(2*q + 4*p + 1)), x] + Dist[1/(c*(2*q + 4*p + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*q + 4*p + 1)*Pq - a*e*(2*q - 3)*x^(2*q - 4) - b*e*(2*q + 2*p - 1)*x^(2*q - 2) - c*e*(2*q + 4*p + 1)*x^(2*q), x], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && !LtQ[p, -1]

Rule 1176

Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := Simp[(x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*(a + b*x^2 + c*x^4)^p)/(c*(4*p + 1)*(4*p + 3)), x] + Dist[(2*p)/(c*(4*p + 1)*(4*p + 3)), Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) - b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]

Rule 1180

```
Int[((d_) + (e_)*(x_)^2)/Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4], x_Symbol]
:> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt
[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x] /; FreeQ[{a, b, c, d, e}
, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rule 524

```
Int[((e_) + (f_)*(x_)^(n_))/(Sqrt[(a_) + (b_)*(x_)^(n_)]*Sqrt[(c_) + (d_
)*(x_)^(n_)]), x_Symbol] := Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] && !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && (!GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 424

```
Int[Sqrt[(a_) + (b_)*(x_)^2]/Sqrt[(c_) + (d_)*(x_)^2], x_Symbol] := Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_)*(x_)^2]*Sqrt[(c_) + (d_)*(x_)^2]), x_Symbol] := S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] && !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\begin{aligned}
\int (7 + 5x^2)^3 \sqrt{2 + x^2 - x^4} dx &= -\frac{125}{9}x^3(2 + x^2 - x^4)^{3/2} - \frac{1}{9} \int (-3087 - 7365x^2 - 5475x^4) \sqrt{2 + x^2 - x^4} dx \\
&= -\frac{1825}{21}x(2 + x^2 - x^4)^{3/2} - \frac{125}{9}x^3(2 + x^2 - x^4)^{3/2} + \frac{1}{63} \int (32559 + 73455x^2) \sqrt{2 + x^2 - x^4} dx \\
&= \frac{1}{63}x(5956 + 14691x^2) \sqrt{2 + x^2 - x^4} - \frac{1825}{21}x(2 + x^2 - x^4)^{3/2} - \frac{125}{9}x^3(2 + x^2 - x^4)^{3/2} \\
&= \frac{1}{63}x(5956 + 14691x^2) \sqrt{2 + x^2 - x^4} - \frac{1825}{21}x(2 + x^2 - x^4)^{3/2} - \frac{125}{9}x^3(2 + x^2 - x^4)^{3/2} \\
&= \frac{1}{63}x(5956 + 14691x^2) \sqrt{2 + x^2 - x^4} - \frac{1825}{21}x(2 + x^2 - x^4)^{3/2} - \frac{125}{9}x^3(2 + x^2 - x^4)^{3/2} \\
&= \frac{1}{63}x(5956 + 14691x^2) \sqrt{2 + x^2 - x^4} - \frac{1825}{21}x(2 + x^2 - x^4)^{3/2} - \frac{125}{9}x^3(2 + x^2 - x^4)^{3/2}
\end{aligned}$$

Mathematica [C] time = 0.101818, size = 107, normalized size = 1.13

$$\frac{-106014i\sqrt{-2x^4 + 2x^2 + 4}\text{EllipticF}\left(i\sinh^{-1}(x), -\frac{1}{2}\right) - 875x^{11} - 3725x^9 - 1116x^7 + 21660x^5 + 9938x^3 + 79411i\sqrt{-2x^4 + 2x^2 + 4}}{63\sqrt{-x^4 + x^2 + 2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(7 + 5*x^2)^3*Sqrt[2 + x^2 - x^4],x]
```

```
[Out] (-9988*x + 9938*x^3 + 21660*x^5 - 1116*x^7 - 3725*x^9 - 875*x^11 + (79411*I)
)*Sqrt[4 + 2*x^2 - 2*x^4]*EllipticE[I*ArcSinh[x], -1/2] - (106014*I)*Sqrt[4
```

+ 2*x^2 - 2*x^4]*EllipticF[I*ArcSinh[x], -1/2])/(63*Sqrt[2 + x^2 - x^4])

Maple [B] time = 0.008, size = 176, normalized size = 1.9

$$\frac{125x^7}{9}\sqrt{-x^4+x^2+2} + \frac{4600x^5}{63}\sqrt{-x^4+x^2+2} + \frac{7466x^3}{63}\sqrt{-x^4+x^2+2} - \frac{4994x}{63}\sqrt{-x^4+x^2+2} + \frac{26603\sqrt{2}}{63}\sqrt{-2x^2+2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((5*x^2+7)^3*(-x^4+x^2+2)^(1/2),x)

[Out] 125/9*x^7*(-x^4+x^2+2)^(1/2)+4600/63*x^5*(-x^4+x^2+2)^(1/2)+7466/63*x^3*(-x^4+x^2+2)^(1/2)-4994/63*x*(-x^4+x^2+2)^(1/2)+26603/63*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticF(1/2*x*2^(1/2),I*2^(1/2))-79411/126*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*(EllipticF(1/2*x*2^(1/2),I*2^(1/2))-EllipticE(1/2*x*2^(1/2),I*2^(1/2)))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-x^4 + x^2 + 2} (5x^2 + 7)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^2+7)^3*(-x^4+x^2+2)^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(-x^4 + x^2 + 2)*(5*x^2 + 7)^3, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(125x^6 + 525x^4 + 735x^2 + 343\right)\sqrt{-x^4 + x^2 + 2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^2+7)^3*(-x^4+x^2+2)^(1/2),x, algorithm="fricas")

[Out] integral((125*x^6 + 525*x^4 + 735*x^2 + 343)*sqrt(-x^4 + x^2 + 2), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-(x^2 - 2)(x^2 + 1)} (5x^2 + 7)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x**2+7)**3*(-x**4+x**2+2)**(1/2),x)

[Out] Integral(sqrt(-(x**2 - 2)*(x**2 + 1))*(5*x**2 + 7)**3, x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-x^4 + x^2 + 2} (5x^2 + 7)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^3*(-x^4+x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-x^4 + x^2 + 2)*(5*x^2 + 7)^3, x)
```

3.318 $\int (7 + 5x^2)^2 \sqrt{2 + x^2 - x^4} dx$

Optimal. Leaf size=74

$$-\frac{79}{7}\text{EllipticF}\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right), -2\right) - \frac{25}{7}x(-x^4 + x^2 + 2)^{3/2} + \frac{1}{21}x(354x^2 + 275)\sqrt{-x^4 + x^2 + 2} + \frac{2045}{21}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\right) -$$

```
[Out] (x*(275 + 354*x^2)*Sqrt[2 + x^2 - x^4])/21 - (25*x*(2 + x^2 - x^4)^(3/2))/7
+ (2045*EllipticE[ArcSin[x/Sqrt[2]], -2])/21 - (79*EllipticF[ArcSin[x/Sqrt
[2]], -2])/7
```

Rubi [A] time = 0.0606281, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$, Rules used = {1206, 1176, 1180, 524, 424, 419}

$$-\frac{25}{7}x(-x^4 + x^2 + 2)^{3/2} + \frac{1}{21}x(354x^2 + 275)\sqrt{-x^4 + x^2 + 2} - \frac{79}{7}F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\right) - 2 + \frac{2045}{21}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\right) - 2$$

Antiderivative was successfully verified.

```
[In] Int[(7 + 5*x^2)^2*Sqrt[2 + x^2 - x^4], x]
```

```
[Out] (x*(275 + 354*x^2)*Sqrt[2 + x^2 - x^4])/21 - (25*x*(2 + x^2 - x^4)^(3/2))/7
+ (2045*EllipticE[ArcSin[x/Sqrt[2]], -2])/21 - (79*EllipticF[ArcSin[x/Sqrt
[2]], -2])/7
```

Rule 1206

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] := Simp[(e^q*x^(2*q - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(4*p + 2*
q + 1)), x] + Dist[1/(c*(4*p + 2*q + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandT
oSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*p
+ 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0] && IGtQ[q, 1]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] := Simp[(x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*(a + b*x^2 + c
*x^4)^p)/(c*(4*p + 1)*(4*p + 3)), x] + Dist[(2*p)/(c*(4*p + 1)*(4*p + 3)),
Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) -
b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol]
:= With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt
[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e},
x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] := Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
```

```
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x]
]; FreeQ[{a, b, c, d, e, f, n}, x] && !(EqQ[n, 2] && ((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && (!GtQ[c, 0] || SimplerSqrtQ[-(b/a), -(d/c)]))))))
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] := Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c), 2]), x]
]; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] := S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt[-(d/c), 2]), x]
]; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0] && !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\begin{aligned} \int (7 + 5x^2)^2 \sqrt{2 + x^2 - x^4} dx &= -\frac{25}{7}x(2 + x^2 - x^4)^{3/2} - \frac{1}{7} \int (-393 - 590x^2) \sqrt{2 + x^2 - x^4} dx \\ &= \frac{1}{21}x(275 + 354x^2) \sqrt{2 + x^2 - x^4} - \frac{25}{7}x(2 + x^2 - x^4)^{3/2} + \frac{1}{105} \int \frac{9040 + 10225x^2}{\sqrt{2 + x^2 - x^4}} dx \\ &= \frac{1}{21}x(275 + 354x^2) \sqrt{2 + x^2 - x^4} - \frac{25}{7}x(2 + x^2 - x^4)^{3/2} + \frac{2}{105} \int \frac{9040 + 10225x^2}{\sqrt{4 - 2x^2}\sqrt{2 + 2x^2}} dx \\ &= \frac{1}{21}x(275 + 354x^2) \sqrt{2 + x^2 - x^4} - \frac{25}{7}x(2 + x^2 - x^4)^{3/2} - \frac{158}{7} \int \frac{1}{\sqrt{4 - 2x^2}\sqrt{2 + 2x^2}} dx \\ &= \frac{1}{21}x(275 + 354x^2) \sqrt{2 + x^2 - x^4} - \frac{25}{7}x(2 + x^2 - x^4)^{3/2} + \frac{2045}{21} E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right) \middle| -2\right) \end{aligned}$$

Mathematica [C] time = 0.0893145, size = 102, normalized size = 1.38

$$\frac{-2949i\sqrt{-2x^4 + 2x^2 + 4}\text{EllipticF}\left(i\sinh^{-1}(x), -\frac{1}{2}\right) - 75x^9 - 204x^7 + 304x^5 + 683x^3 + 2045i\sqrt{-2x^4 + 2x^2 + 4}E\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right) \middle| -2\right)}{21\sqrt{-x^4 + x^2 + 2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(7 + 5*x^2)^2*Sqrt[2 + x^2 - x^4], x]
```

```
[Out] (250*x + 683*x^3 + 304*x^5 - 204*x^7 - 75*x^9 + (2045*I)*Sqrt[4 + 2*x^2 - 2*x^4]*EllipticE[I*ArcSinh[x], -1/2] - (2949*I)*Sqrt[4 + 2*x^2 - 2*x^4]*EllipticF[I*ArcSinh[x], -1/2])/(21*Sqrt[2 + x^2 - x^4])
```

Maple [B] time = 0.009, size = 159, normalized size = 2.2

$$\frac{25x^5}{7} \sqrt{-x^4 + x^2 + 2} + \frac{93x^3}{7} \sqrt{-x^4 + x^2 + 2} + \frac{125x}{21} \sqrt{-x^4 + x^2 + 2} + \frac{904\sqrt{2}}{21} \sqrt{-2x^2 + 4} \sqrt{x^2 + 1} \text{EllipticF}\left(\frac{x\sqrt{2}}{2}, i\sqrt{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+7)^2*(-x^4+x^2+2)^(1/2), x)
```

```
[Out] 25/7*x^5*(-x^4+x^2+2)^(1/2)+93/7*x^3*(-x^4+x^2+2)^(1/2)+125/21*x*(-x^4+x^2+2)^(1/2)+904/21*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticF(1/2*x*2^(1/2),I*2^(1/2))-2045/42*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*(EllipticF(1/2*x*2^(1/2),I*2^(1/2))-EllipticE(1/2*x*2^(1/2),I*2^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-x^4 + x^2 + 2} (5x^2 + 7)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^2*(-x^4+x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-x^4 + x^2 + 2)*(5*x^2 + 7)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(25x^4 + 70x^2 + 49\right)\sqrt{-x^4 + x^2 + 2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^2*(-x^4+x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((25*x^4 + 70*x^2 + 49)*sqrt(-x^4 + x^2 + 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-(x^2 - 2)(x^2 + 1)} (5x^2 + 7)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+7)**2*(-x**4+x**2+2)**(1/2),x)
```

```
[Out] Integral(sqrt(-(x**2 - 2)*(x**2 + 1))*(5*x**2 + 7)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-x^4 + x^2 + 2} (5x^2 + 7)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^2*(-x^4+x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-x^4 + x^2 + 2)*(5*x^2 + 7)^2, x)
```

3.319 $\int (7 + 5x^2) \sqrt{2 + x^2 - x^4} dx$

Optimal. Leaf size=46

$$3\text{EllipticF}\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right), -2\right) + x\sqrt{-x^4 + x^2 + 2}(x^2 + 2) + 7E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right) \middle| -2\right)$$

[Out] x*(2 + x^2)*Sqrt[2 + x^2 - x^4] + 7*EllipticE[ArcSin[x/Sqrt[2]], -2] + 3*EllipticF[ArcSin[x/Sqrt[2]], -2]

Rubi [A] time = 0.0460943, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {1176, 1180, 524, 424, 419}

$$x\sqrt{-x^4 + x^2 + 2}(x^2 + 2) + 3F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right) \middle| -2\right) + 7E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right) \middle| -2\right)$$

Antiderivative was successfully verified.

[In] Int[(7 + 5*x^2)*Sqrt[2 + x^2 - x^4], x]

[Out] x*(2 + x^2)*Sqrt[2 + x^2 - x^4] + 7*EllipticE[ArcSin[x/Sqrt[2]], -2] + 3*EllipticF[ArcSin[x/Sqrt[2]], -2]

Rule 1176

Int[((d_) + (e_)*(x_)^2)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] :> Simp[(x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*(a + b*x^2 + c*x^4)^p)/(c*(4*p + 1)*(4*p + 3)), x] + Dist[(2*p)/(c*(4*p + 1)*(4*p + 3)), Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) - b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]

Rule 1180

Int[((d_) + (e_)*(x_)^2)/Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4], x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]

Rule 524

Int[((e_) + (f_)*(x_)^(n_))/(Sqrt[(a_) + (b_)*(x_)^(n_)]*Sqrt[(c_) + (d_)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n], x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && !(EqQ[n, 2] && ((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && (!GtQ[c, 0] || SimplerSqrtQ[-(b/a), -(d/c)]))))))

Rule 424

Int[Sqrt[(a_) + (b_)*(x_)^2]/Sqrt[(c_) + (d_)*(x_)^2], x_Symbol] :> Simp[(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] && !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\begin{aligned} \int (7 + 5x^2) \sqrt{2 + x^2 - x^4} dx &= x(2 + x^2) \sqrt{2 + x^2 - x^4} - \frac{1}{15} \int \frac{-150 - 105x^2}{\sqrt{2 + x^2 - x^4}} dx \\ &= x(2 + x^2) \sqrt{2 + x^2 - x^4} - \frac{2}{15} \int \frac{-150 - 105x^2}{\sqrt{4 - 2x^2} \sqrt{2 + 2x^2}} dx \\ &= x(2 + x^2) \sqrt{2 + x^2 - x^4} + 6 \int \frac{1}{\sqrt{4 - 2x^2} \sqrt{2 + 2x^2}} dx + 7 \int \frac{\sqrt{2 + 2x^2}}{\sqrt{4 - 2x^2}} dx \\ &= x(2 + x^2) \sqrt{2 + x^2 - x^4} + 7E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right) \middle| -2\right) + 3F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right) \middle| -2\right) \end{aligned}$$

Mathematica [C] time = 0.0809684, size = 94, normalized size = 2.04

$$\frac{-12i\sqrt{-2x^4 + 2x^2 + 4}\text{EllipticF}\left(i\sinh^{-1}(x), -\frac{1}{2}\right) - x^7 - x^5 + 4x^3 + 7i\sqrt{-2x^4 + 2x^2 + 4}E\left(i\sinh^{-1}(x) \middle| -\frac{1}{2}\right) + 4x}{\sqrt{-x^4 + x^2 + 2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(7 + 5*x^2)*Sqrt[2 + x^2 - x^4], x]
```

```
[Out] (4*x + 4*x^3 - x^5 - x^7 + (7*I)*Sqrt[4 + 2*x^2 - 2*x^4]*EllipticE[I*ArcSin
h[x], -1/2] - (12*I)*Sqrt[4 + 2*x^2 - 2*x^4]*EllipticF[I*ArcSinh[x], -1/2])
/Sqrt[2 + x^2 - x^4]
```

Maple [B] time = 0.006, size = 141, normalized size = 3.1

$$x^3\sqrt{-x^4 + x^2 + 2} + 2x\sqrt{-x^4 + x^2 + 2} + 5\frac{\sqrt{2}\sqrt{-2x^2 + 4}\sqrt{x^2 + 1}\text{EllipticF}\left(\frac{1}{2}x\sqrt{2}, i\sqrt{2}\right)}{\sqrt{-x^4 + x^2 + 2}} - \frac{7\sqrt{2}}{2}\sqrt{-2x^2 + 4}\sqrt{x^2 + 1} \left(E \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+7)*(-x^4+x^2+2)^(1/2), x)
```

```
[Out] x^3*(-x^4+x^2+2)^(1/2)+2*x*(-x^4+x^2+2)^(1/2)+5*2^(1/2)*(-2*x^2+4)^(1/2)*(x
^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticF(1/2*x*2^(1/2), I*2^(1/2))-7/2*2^(1/
2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*(EllipticF(1/2*x*2^(1/
2), I*2^(1/2))-EllipticE(1/2*x*2^(1/2), I*2^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-x^4 + x^2 + 2}(5x^2 + 7) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^2+7)*(-x^4+x^2+2)^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(-x^4 + x^2 + 2)*(5*x^2 + 7), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{-x^4 + x^2 + 2}(5x^2 + 7), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^2+7)*(-x^4+x^2+2)^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(-x^4 + x^2 + 2)*(5*x^2 + 7), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-(x^2 - 2)(x^2 + 1)}(5x^2 + 7) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x**2+7)*(-x**4+x**2+2)**(1/2),x)

[Out] Integral(sqrt(-(x**2 - 2)*(x**2 + 1))*(5*x**2 + 7), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-x^4 + x^2 + 2}(5x^2 + 7) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^2+7)*(-x^4+x^2+2)^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(-x^4 + x^2 + 2)*(5*x^2 + 7), x)

3.320 $\int \sqrt{2 + x^2 - x^4} dx$

Optimal. Leaf size=44

$$\text{EllipticF}\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right), -2\right) + \frac{1}{3}\sqrt{-x^4 + x^2 + 2x} + \frac{1}{3}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right) \middle| -2\right)$$

[Out] (x*Sqrt[2 + x^2 - x^4])/3 + EllipticE[ArcSin[x/Sqrt[2]], -2]/3 + EllipticF[ArcSin[x/Sqrt[2]], -2]

Rubi [A] time = 0.0401104, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {1091, 1180, 524, 424, 419}

$$\frac{1}{3}\sqrt{-x^4 + x^2 + 2x} + F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right) \middle| -2\right) + \frac{1}{3}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right) \middle| -2\right)$$

Antiderivative was successfully verified.

[In] Int[Sqrt[2 + x^2 - x^4], x]

[Out] (x*Sqrt[2 + x^2 - x^4])/3 + EllipticE[ArcSin[x/Sqrt[2]], -2]/3 + EllipticF[ArcSin[x/Sqrt[2]], -2]

Rule 1091

Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := Simp[(x*(a + b*x^2 + c*x^4)^p)/(4*p + 1), x] + Dist[(2*p)/(4*p + 1), Int[(2*a + b*x^2)*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[2*p]

Rule 1180

Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]

Rule 524

Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.)*(x_)^(n_)]), x_Symbol] := Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n], x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && !(EqQ[n, 2] && ((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && (!GtQ[c, 0] || Simpler SqrtQ[-(b/a), -(d/c)]))))))

Rule 424

Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] := Simp[(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]

Rule 419

Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] := Simp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt

$[-(d/c), 2]), x] /; \text{FreeQ}\{a, b, c, d, x\} \ \&\& \ \text{NegQ}[d/c] \ \&\& \ \text{GtQ}[c, 0] \ \&\& \ \text{GtQ}[a, 0] \ \&\& \ !(\text{NegQ}[b/a] \ \&\& \ \text{SimplerSqrtQ}[-(b/a), -(d/c)])$

Rubi steps

$$\begin{aligned} \int \sqrt{2+x^2-x^4} dx &= \frac{1}{3}x\sqrt{2+x^2-x^4} + \frac{1}{3} \int \frac{4+x^2}{\sqrt{2+x^2-x^4}} dx \\ &= \frac{1}{3}x\sqrt{2+x^2-x^4} + \frac{2}{3} \int \frac{4+x^2}{\sqrt{4-2x^2}\sqrt{2+2x^2}} dx \\ &= \frac{1}{3}x\sqrt{2+x^2-x^4} + \frac{1}{3} \int \frac{\sqrt{2+2x^2}}{\sqrt{4-2x^2}} dx + 2 \int \frac{1}{\sqrt{4-2x^2}\sqrt{2+2x^2}} dx \\ &= \frac{1}{3}x\sqrt{2+x^2-x^4} + \frac{1}{3}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle| -2\right) + F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle| -2\right) \end{aligned}$$

Mathematica [C] time = 0.0450932, size = 90, normalized size = 2.05

$$\frac{-3i\sqrt{-2x^4+2x^2+4}\text{EllipticF}\left(i\sinh^{-1}(x), -\frac{1}{2}\right) - x^5 + x^3 + i\sqrt{-2x^4+2x^2+4}E\left(i\sinh^{-1}(x)\middle| -\frac{1}{2}\right) + 2x}{3\sqrt{-x^4+x^2+2}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[2 + x^2 - x^4], x]

[Out] $(2x + x^3 - x^5 + I\text{Sqrt}[4 + 2x^2 - 2x^4]\text{EllipticE}[I\text{ArcSinh}[x], -1/2] - (3I)\text{Sqrt}[4 + 2x^2 - 2x^4]\text{EllipticF}[I\text{ArcSinh}[x], -1/2]) / (3\text{Sqrt}[2 + x^2 - x^4])$

Maple [B] time = 0.003, size = 125, normalized size = 2.8

$$\frac{x}{3}\sqrt{-x^4+x^2+2} + \frac{2\sqrt{2}}{3}\sqrt{-2x^2+4}\sqrt{x^2+1}\text{EllipticF}\left(\frac{x\sqrt{2}}{2}, i\sqrt{2}\right) \frac{1}{\sqrt{-x^4+x^2+2}} - \frac{\sqrt{2}}{6}\sqrt{-2x^2+4}\sqrt{x^2+1}\left(\text{EllipticE}\left(\frac{x\sqrt{2}}{2}, i\sqrt{2}\right) - \frac{1}{\sqrt{-x^4+x^2+2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-x^4+x^2+2)^(1/2), x)

[Out] $1/3*x*(-x^4+x^2+2)^(1/2)+2/3*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*\text{EllipticF}(1/2*x*2^(1/2), I*2^(1/2))-1/6*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*(\text{EllipticE}(1/2*x*2^(1/2), I*2^(1/2))-\text{EllipticE}(1/2*x*2^(1/2), I*2^(1/2)))$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-x^4+x^2+2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^4+x^2+2)^(1/2), x, algorithm="maxima")

[Out] integrate(sqrt(-x^4 + x^2 + 2), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{-x^4 + x^2 + 2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^4+x^2+2)^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(-x^4 + x^2 + 2), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-x^4 + x^2 + 2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x**4+x**2+2)**(1/2),x)

[Out] Integral(sqrt(-x**4 + x**2 + 2), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-x^4 + x^2 + 2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^4+x^2+2)^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(-x^4 + x^2 + 2), x)

$$3.321 \quad \int \frac{\sqrt{2+x^2-x^4}}{7+5x^2} dx$$

Optimal. Leaf size=46

$$\frac{17}{25} \text{EllipticF}\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right), -2\right) - \frac{1}{5} E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right) \middle| -2\right) - \frac{34}{175} \Pi\left(-\frac{10}{7}; \sin^{-1}\left(\frac{x}{\sqrt{2}}\right) \middle| -2\right)$$

[Out] -EllipticE[ArcSin[x/Sqrt[2]], -2]/5 + (17*EllipticF[ArcSin[x/Sqrt[2]], -2])/25 - (34*EllipticPi[-10/7, ArcSin[x/Sqrt[2]], -2])/175

Rubi [A] time = 0.079296, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {1208, 1180, 524, 424, 419, 1212, 537}

$$\frac{17}{25} F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right) \middle| -2\right) - \frac{1}{5} E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right) \middle| -2\right) - \frac{34}{175} \Pi\left(-\frac{10}{7}; \sin^{-1}\left(\frac{x}{\sqrt{2}}\right) \middle| -2\right)$$

Antiderivative was successfully verified.

[In] Int[Sqrt[2 + x^2 - x^4]/(7 + 5*x^2), x]

[Out] -EllipticE[ArcSin[x/Sqrt[2]], -2]/5 + (17*EllipticF[ArcSin[x/Sqrt[2]], -2])/25 - (34*EllipticPi[-10/7, ArcSin[x/Sqrt[2]], -2])/175

Rule 1208

Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_)/((d_) + (e_.)*(x_)^2), x_Symbol] := -Dist[(e^2)^(-1), Int[(c*d - b*e - c*e*x^2)*(a + b*x^2 + c*x^4)^(p - 1), x], x] + Dist[(c*d^2 - b*d*e + a*e^2)/e^2, Int[(a + b*x^2 + c*x^4)^(p - 1)/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p + 1/2, 0]

Rule 1180

Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]

Rule 524

Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.)*(x_)^(n_)]), x_Symbol] := Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n], x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && !(EqQ[n, 2] && ((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && (!GtQ[c, 0] || SimplerSqrtQ[-(b/a), -(d/c)]))))))

Rule 424

Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] := Simp[(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)]/(Sqrt[c]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]

Rule 419

```
Int[1/(Sqrt[(a_) + (b_)*(x_)^2]*Sqrt[(c_) + (d_)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)]/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] && !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 1212

```
Int[1/(((d_) + (e_)*(x_)^2)*Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/((d + e*x^2
)*Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c,
d, e}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rule 537

```
Int[1/(((a_) + (b_)*(x_)^2)*Sqrt[(c_) + (d_)*(x_)^2]*Sqrt[(e_) + (f_)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)]/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] && !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] && !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{2+x^2-x^4}}{7+5x^2} dx &= -\left(\frac{1}{25} \int \frac{-12+5x^2}{\sqrt{2+x^2-x^4}} dx\right) - \frac{34}{25} \int \frac{1}{(7+5x^2)\sqrt{2+x^2-x^4}} dx \\ &= -\left(\frac{2}{25} \int \frac{-12+5x^2}{\sqrt{4-2x^2}\sqrt{2+2x^2}} dx\right) - \frac{68}{25} \int \frac{1}{\sqrt{4-2x^2}\sqrt{2+2x^2}(7+5x^2)} dx \\ &= -\frac{34}{175} \Pi\left(-\frac{10}{7}; \sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\right) - 2 - \frac{1}{5} \int \frac{\sqrt{2+2x^2}}{\sqrt{4-2x^2}} dx + \frac{34}{25} \int \frac{1}{\sqrt{4-2x^2}\sqrt{2+2x^2}} dx \\ &= -\frac{1}{5} E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\right) - 2 + \frac{17}{25} F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\right) - 2 - \frac{34}{175} \Pi\left(-\frac{10}{7}; \sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\right) - 2 \end{aligned}$$

Mathematica [C] time = 0.129506, size = 51, normalized size = 1.11

$$-\frac{1}{175}i\sqrt{2}\left(7\text{EllipticF}\left(i\sinh^{-1}(x), -\frac{1}{2}\right) + 35E\left(i\sinh^{-1}(x)\middle|-\frac{1}{2}\right) - 17\Pi\left(\frac{5}{7}; i\sinh^{-1}(x)\middle|-\frac{1}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2 + x^2 - x^4]/(7 + 5*x^2), x]
```

```
[Out] (-I/175)*Sqrt[2]*(35*EllipticE[I*ArcSinh[x], -1/2] + 7*EllipticF[I*ArcSinh[
x], -1/2] - 17*EllipticPi[5/7, I*ArcSinh[x], -1/2])
```

Maple [B] time = 0.013, size = 141, normalized size = 3.1

$$\frac{17\sqrt{2}}{50}\sqrt{-2x^2+4}\sqrt{x^2+1}\text{EllipticF}\left(\frac{x\sqrt{2}}{2}, i\sqrt{2}\right)\frac{1}{\sqrt{-x^4+x^2+2}} - \frac{\sqrt{2}}{10}\sqrt{-2x^2+4}\sqrt{x^2+1}\text{EllipticE}\left(\frac{x\sqrt{2}}{2}, i\sqrt{2}\right)\frac{1}{\sqrt{-x^4+}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^4+x^2+2)^(1/2)/(5*x^2+7), x)
```

```
[Out] 17/50*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticF(1/2*x*2^(1/2),I*2^(1/2))-1/10*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticE(1/2*x*2^(1/2),I*2^(1/2))-34/175*2^(1/2)*(1-1/2*x^2)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticPi(1/2*x*2^(1/2),-10/7,I*2^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-x^4 + x^2 + 2}}{5x^2 + 7} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+x^2+2)^(1/2)/(5*x^2+7),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-x^4 + x^2 + 2)/(5*x^2 + 7), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-x^4 + x^2 + 2}}{5x^2 + 7}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+x^2+2)^(1/2)/(5*x^2+7),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-x^4 + x^2 + 2)/(5*x^2 + 7), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-(x^2 - 2)(x^2 + 1)}}{5x^2 + 7} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**4+x**2+2)**(1/2)/(5*x**2+7),x)
```

```
[Out] Integral(sqrt(-(x**2 - 2)*(x**2 + 1))/(5*x**2 + 7), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-x^4 + x^2 + 2}}{5x^2 + 7} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+x^2+2)^(1/2)/(5*x^2+7),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-x^4 + x^2 + 2)/(5*x^2 + 7), x)
```

$$3.322 \quad \int \frac{\sqrt{2+x^2-x^4}}{(7+5x^2)^2} dx$$

Optimal. Leaf size=74

$$-\frac{6}{175} \text{EllipticF}\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right), -2\right) + \frac{\sqrt{-x^4+x^2+2x}}{14(5x^2+7)} + \frac{1}{70} E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right) \middle| -2\right) + \frac{99\pi\left(-\frac{10}{7}; \sin^{-1}\left(\frac{x}{\sqrt{2}}\right) \middle| -2\right)}{2450}$$

[Out] (x*Sqrt[2 + x^2 - x^4])/(14*(7 + 5*x^2)) + EllipticE[ArcSin[x/Sqrt[2]], -2]/70 - (6*EllipticF[ArcSin[x/Sqrt[2]], -2])/175 + (99*EllipticPi[-10/7, ArcSin[x/Sqrt[2]], -2])/2450

Rubi [A] time = 0.0795178, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {1226, 1180, 524, 424, 419, 1212, 537}

$$\frac{\sqrt{-x^4+x^2+2x}}{14(5x^2+7)} - \frac{6}{175} F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right) \middle| -2\right) + \frac{1}{70} E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right) \middle| -2\right) + \frac{99\pi\left(-\frac{10}{7}; \sin^{-1}\left(\frac{x}{\sqrt{2}}\right) \middle| -2\right)}{2450}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[2 + x^2 - x^4]/(7 + 5*x^2)^2,x]

[Out] (x*Sqrt[2 + x^2 - x^4])/(14*(7 + 5*x^2)) + EllipticE[ArcSin[x/Sqrt[2]], -2]/70 - (6*EllipticF[ArcSin[x/Sqrt[2]], -2])/175 + (99*EllipticPi[-10/7, ArcSin[x/Sqrt[2]], -2])/2450

Rule 1226

Int[Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]/((d_) + (e_.)*(x_)^2)^2, x_Symbol] :> Simp[(x*Sqrt[a + b*x^2 + c*x^4])/(2*d*(d + e*x^2)), x] + (Dist[c/(2*d*e^2), Int[(d - e*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[(c*d^2 - a*e^2)/(2*d*e^2), Int[1/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]) /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0]

Rule 1180

Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]

Rule 524

Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n], x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && !(EqQ[n, 2] && ((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && (!GtQ[c, 0] || SimplerSqrtQ[-(b/a), -(d/c)]))))))

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] := Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)]/(Sqrt[c]*Rt[-(d/c)
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] := S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)]/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] && !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 1212

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/((d + e*x^2)
)*Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c,
d, e}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] := Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)]/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] && !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] && !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{2+x^2-x^4}}{(7+5x^2)^2} dx &= \frac{x\sqrt{2+x^2-x^4}}{14(7+5x^2)} - \frac{1}{350} \int \frac{7-5x^2}{\sqrt{2+x^2-x^4}} dx + \frac{99}{350} \int \frac{1}{(7+5x^2)\sqrt{2+x^2-x^4}} dx \\ &= \frac{x\sqrt{2+x^2-x^4}}{14(7+5x^2)} - \frac{1}{175} \int \frac{7-5x^2}{\sqrt{4-2x^2}\sqrt{2+2x^2}} dx + \frac{99}{175} \int \frac{1}{\sqrt{4-2x^2}\sqrt{2+2x^2}(7+5x^2)} dx \\ &= \frac{x\sqrt{2+x^2-x^4}}{14(7+5x^2)} + \frac{99\Pi\left(-\frac{10}{7}; \sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\right) - 2}{2450} + \frac{1}{70} \int \frac{\sqrt{2+2x^2}}{\sqrt{4-2x^2}} dx - \frac{12}{175} \int \frac{1}{\sqrt{4-2x^2}\sqrt{2+2x^2}} dx \\ &= \frac{x\sqrt{2+x^2-x^4}}{14(7+5x^2)} + \frac{1}{70} E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\right) - 2 - \frac{6}{175} F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\right) - 2 + \frac{99\Pi\left(-\frac{10}{7}; \sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\right) - 2}{2450} \end{aligned}$$

Mathematica [C] time = 0.276638, size = 196, normalized size = 2.65

$$\frac{-21i\sqrt{2}(5x^2+7)\sqrt{-x^4+x^2+2}\operatorname{EllipticF}\left(i\sinh^{-1}(x), -\frac{1}{2}\right) - 350x^5 + 350x^3 + 70i\sqrt{2}(5x^2+7)\sqrt{-x^4+x^2+2}E\left(i\sinh^{-1}(x), -\frac{1}{2}\right)}{4900(5x^2+7)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2 + x^2 - x^4]/(7 + 5*x^2)^2, x]
```

```
[Out] (700*x + 350*x^3 - 350*x^5 + (70*I)*Sqrt[2]*(7 + 5*x^2)*Sqrt[2 + x^2 - x^4]
*EllipticE[I*ArcSinh[x], -1/2] - (21*I)*Sqrt[2]*(7 + 5*x^2)*Sqrt[2 + x^2 -
x^4]*EllipticF[I*ArcSinh[x], -1/2] - (693*I)*Sqrt[2]*Sqrt[2 + x^2 - x^4]*E1
llipticPi[5/7, I*ArcSinh[x], -1/2] - (495*I)*Sqrt[2]*x^2*Sqrt[2 + x^2 - x^4]
*EllipticPi[5/7, I*ArcSinh[x], -1/2))/(4900*(7 + 5*x^2)*Sqrt[2 + x^2 - x^4]
```

)

Maple [B] time = 0.02, size = 165, normalized size = 2.2

$$\frac{x}{70x^2+98}\sqrt{-x^4+x^2+2}-\frac{3\sqrt{2}}{175}\sqrt{-2x^2+4}\sqrt{x^2+1}\operatorname{EllipticF}\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)\frac{1}{\sqrt{-x^4+x^2+2}}+\frac{\sqrt{2}}{140}\sqrt{-2x^2+4}\sqrt{x^2+1}\operatorname{EllipticE}\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-x^4+x^2+2)^(1/2)/(5*x^2+7)^2,x)

[Out] 1/14*x*(-x^4+x^2+2)^(1/2)/(5*x^2+7)-3/175*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticF(1/2*x*2^(1/2),I*2^(1/2))+1/140*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticE(1/2*x*2^(1/2),I*2^(1/2))+99/2450*2^(1/2)*(1-1/2*x^2)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticPi(1/2*x*2^(1/2),-10/7,I*2^(1/2))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-x^4+x^2+2}}{(5x^2+7)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^4+x^2+2)^(1/2)/(5*x^2+7)^2,x, algorithm="maxima")

[Out] integrate(sqrt(-x^4 + x^2 + 2)/(5*x^2 + 7)^2, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\sqrt{-x^4+x^2+2}}{25x^4+70x^2+49},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^4+x^2+2)^(1/2)/(5*x^2+7)^2,x, algorithm="fricas")

[Out] integral(sqrt(-x^4 + x^2 + 2)/(25*x^4 + 70*x^2 + 49), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-(x^2-2)(x^2+1)}}{(5x^2+7)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x**4+x**2+2)**(1/2)/(5*x**2+7)**2,x)


```
[Out] Integral(sqrt(-(x**2 - 2)*(x**2 + 1))/(5*x**2 + 7)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-x^4 + x^2 + 2}}{(5x^2 + 7)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+x^2+2)^(1/2)/(5*x^2+7)^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(-x^4 + x^2 + 2)/(5*x^2 + 7)^2, x)
```

$$3.323 \quad \int \frac{\sqrt{2+x^2-x^4}}{(7+5x^2)^3} dx$$

Optimal. Leaf size=102

$$-\frac{269\text{EllipticF}\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right), -2\right)}{166600} - \frac{31\sqrt{-x^4+x^2+2x}}{13328(5x^2+7)} + \frac{\sqrt{-x^4+x^2+2x}}{28(5x^2+7)^2} - \frac{31E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle| -2\right)}{66640} + \frac{16601\Pi\left(-\frac{10}{7}; \sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\right)}{2332400}$$

[Out] (x*Sqrt[2 + x^2 - x^4])/(28*(7 + 5*x^2)^2) - (31*x*Sqrt[2 + x^2 - x^4])/(13328*(7 + 5*x^2)) - (31*EllipticE[ArcSin[x/Sqrt[2]], -2])/66640 - (269*EllipticF[ArcSin[x/Sqrt[2]], -2])/166600 + (16601*EllipticPi[-10/7, ArcSin[x/Sqrt[2]], -2])/2332400

Rubi [A] time = 0.41383, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 21, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {1228, 1223, 1696, 1716, 1180, 524, 424, 419, 1212, 537}

$$-\frac{31\sqrt{-x^4+x^2+2x}}{13328(5x^2+7)} + \frac{\sqrt{-x^4+x^2+2x}}{28(5x^2+7)^2} - \frac{269F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle| -2\right)}{166600} - \frac{31E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle| -2\right)}{66640} + \frac{16601\Pi\left(-\frac{10}{7}; \sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\right)}{2332400}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[2 + x^2 - x^4]/(7 + 5*x^2)^3, x]

[Out] (x*Sqrt[2 + x^2 - x^4])/(28*(7 + 5*x^2)^2) - (31*x*Sqrt[2 + x^2 - x^4])/(13328*(7 + 5*x^2)) - (31*EllipticE[ArcSin[x/Sqrt[2]], -2])/66640 - (269*EllipticF[ArcSin[x/Sqrt[2]], -2])/166600 + (16601*EllipticPi[-10/7, ArcSin[x/Sqrt[2]], -2])/2332400

Rule 1228

Int[((d_) + (e_)*(x_)^2)^(q_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] := Module[{aa, bb, cc}, Int[ExpandIntegrand[1/Sqrt[aa + bb*x^2 + cc*x^4], (d + e*x^2)^q*(aa + bb*x^2 + cc*x^4)^(p + 1/2), x] /. {aa -> a, bb -> b, cc -> c}, x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, 0] && IntegerQ[p + 1/2]

Rule 1223

Int[((d_) + (e_)*(x_)^2)^(q_)/Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4], x_Symbol] := -Simp[(e^2*x*(d + e*x^2)^(q + 1)*Sqrt[a + b*x^2 + c*x^4])/(2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2)), Int[((d + e*x^2)^(q + 1)*Simp[a*e^2*(2*q + 3) + 2*d*(c*d - b*e)*(q + 1) - 2*e*(c*d*(q + 1) - b*e*(q + 2))*x^2 + c*e^2*(2*q + 5)*x^4, x])/Sqrt[a + b*x^2 + c*x^4], x, x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && ILtQ[q, -1]

Rule 1696

Int[((P4x_)*((d_) + (e_)*(x_)^2)^(q_))/Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4], x_Symbol] := With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[P4x, x, 4]}, -Simp[(C*d^2 - B*d*e + A*e^2)*x*(d + e*x^2)^(q + 1)*Sqrt[a + b*x^2 + c*x^4]/(2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2)), Int[((d + e*x^2)^(q + 1)*Simp[a*d*(C*d

- B*e) + A*(a*e^2*(2*q + 3) + 2*d*(c*d - b*e)*(q + 1)) - 2*((B*d - A*e)*(b*e*(q + 2) - c*d*(q + 1)) - C*d*(b*d + a*e*(q + 1)))*x^2 + c*(C*d^2 - B*d*e + A*e^2)*(2*q + 5)*x^4, x))/Sqrt[a + b*x^2 + c*x^4], x] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[P4x, x^2] && LeQ[Expon[P4x, x], 4] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, -1]

Rule 1716

Int[(P4x_)/(((d_) + (e_)*(x_)^2)*Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4]), x_Symbol] := With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[P4x, x, 4]}, -Dist[(e^2)^(-1), Int[(C*d - B*e - C*e*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] + Dist[(C*d^2 - B*d*e + A*e^2)/e^2, Int[1/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[P4x, x^2, 2] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0]

Rule 1180

Int[((d_) + (e_)*(x_)^2)/Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4], x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]

Rule 524

Int[((e_) + (f_)*(x_)^(n_))/(Sqrt[(a_) + (b_)*(x_)^(n_)]*Sqrt[(c_) + (d_)*(x_)^(n_)]), x_Symbol] := Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n], x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && !(EqQ[n, 2] && ((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && (!GtQ[c, 0] || SimplerSqrtQ[-(b/a), -(d/c)]))))))

Rule 424

Int[Sqrt[(a_) + (b_)*(x_)^2]/Sqrt[(c_) + (d_)*(x_)^2], x_Symbol] := Simp[(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]

Rule 419

Int[1/(Sqrt[(a_) + (b_)*(x_)^2]*Sqrt[(c_) + (d_)*(x_)^2]), x_Symbol] := Simp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0] && !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])

Rule 1212

Int[1/(((d_) + (e_)*(x_)^2)*Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4]), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/((d + e*x^2)*Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]

Rule 537

Int[1/(((a_) + (b_)*(x_)^2)*Sqrt[(c_) + (d_)*(x_)^2]*Sqrt[(e_) + (f_)*(x_)^2]), x_Symbol] := Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d, e, f}, x] && !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] && !(!GtQ[f/e, 0]

&& SimplerSqrtQ[-(f/e), -(d/c)]

Rubi steps

$$\begin{aligned}
 \int \frac{\sqrt{2+x^2-x^4}}{(7+5x^2)^3} dx &= \int \left(-\frac{34}{25(7+5x^2)^3 \sqrt{2+x^2-x^4}} + \frac{19}{25(7+5x^2)^2 \sqrt{2+x^2-x^4}} - \frac{1}{25(7+5x^2) \sqrt{2+x^2-x^4}} \right) dx \\
 &= -\left(\frac{1}{25} \int \frac{1}{(7+5x^2) \sqrt{2+x^2-x^4}} dx \right) + \frac{19}{25} \int \frac{1}{(7+5x^2)^2 \sqrt{2+x^2-x^4}} dx - \frac{34}{25} \int \frac{1}{(7+5x^2)^3 \sqrt{2+x^2-x^4}} dx \\
 &= \frac{x\sqrt{2+x^2-x^4}}{28(7+5x^2)^2} - \frac{19x\sqrt{2+x^2-x^4}}{476(7+5x^2)} - \frac{1}{700} \int \frac{186-190x^2+25x^4}{(7+5x^2)^2 \sqrt{2+x^2-x^4}} dx + \frac{19}{11900} \int \frac{118-70x^2-25x^4}{(7+5x^2)\sqrt{2+x^2-x^4}} dx \\
 &= \frac{x\sqrt{2+x^2-x^4}}{28(7+5x^2)^2} - \frac{31x\sqrt{2+x^2-x^4}}{13328(7+5x^2)} - \frac{1}{175} \Pi\left(-\frac{10}{7}; \sin^{-1}\left(\frac{x}{\sqrt{2}}\right) \middle| -2\right) - \frac{\int \frac{37698-32690x^2-12525x^4}{(7+5x^2)\sqrt{2+x^2-x^4}} dx}{333200} \\
 &= \frac{x\sqrt{2+x^2-x^4}}{28(7+5x^2)^2} - \frac{31x\sqrt{2+x^2-x^4}}{13328(7+5x^2)} - \frac{1}{175} \Pi\left(-\frac{10}{7}; \sin^{-1}\left(\frac{x}{\sqrt{2}}\right) \middle| -2\right) + \frac{\int \frac{75775+62625x^2}{\sqrt{2+x^2-x^4}} dx}{8330000} - \frac{19}{333200} \int \frac{1}{\sqrt{2+x^2-x^4}} dx \\
 &= \frac{x\sqrt{2+x^2-x^4}}{28(7+5x^2)^2} - \frac{31x\sqrt{2+x^2-x^4}}{13328(7+5x^2)} + \frac{2697\Pi\left(-\frac{10}{7}; \sin^{-1}\left(\frac{x}{\sqrt{2}}\right) \middle| -2\right)}{83300} + \frac{\int \frac{75775+62625x^2}{\sqrt{4-2x^2}\sqrt{2+2x^2}} dx}{4165000} - \frac{19}{333200} \int \frac{1}{\sqrt{2+x^2-x^4}} dx \\
 &= \frac{x\sqrt{2+x^2-x^4}}{28(7+5x^2)^2} - \frac{31x\sqrt{2+x^2-x^4}}{13328(7+5x^2)} - \frac{19E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right) \middle| -2\right)}{2380} - \frac{19F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right) \middle| -2\right)}{5950} + \frac{16601\Pi\left(-\frac{10}{7}; \sin^{-1}\left(\frac{x}{\sqrt{2}}\right) \middle| -2\right)}{333200} \\
 &= \frac{x\sqrt{2+x^2-x^4}}{28(7+5x^2)^2} - \frac{31x\sqrt{2+x^2-x^4}}{13328(7+5x^2)} - \frac{31E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right) \middle| -2\right)}{66640} - \frac{269F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right) \middle| -2\right)}{166600} + \frac{16601\Pi\left(-\frac{10}{7}; \sin^{-1}\left(\frac{x}{\sqrt{2}}\right) \middle| -2\right)}{333200}
 \end{aligned}$$

Mathematica [C] time = 0.35361, size = 244, normalized size = 2.39

$$7021i\sqrt{2}(5x^2+7)^2\sqrt{-x^4+x^2+2}\text{EllipticF}\left(i\sinh^{-1}(x),-\frac{1}{2}\right)+54250x^7-144900x^5-17850x^3-2170i\sqrt{2}(5x^2+7)^2\sqrt{-x^4+x^2+2}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[2 + x^2 - x^4]/(7 + 5*x^2)^3, x]

[Out] (181300*x - 17850*x^3 - 144900*x^5 + 54250*x^7 - (2170*I)*Sqrt[2]*(7 + 5*x^2)^2*Sqrt[2 + x^2 - x^4]*EllipticE[I*ArcSinh[x], -1/2] + (7021*I)*Sqrt[2]*(7 + 5*x^2)^2*Sqrt[2 + x^2 - x^4]*EllipticF[I*ArcSinh[x], -1/2] - (813449*I)*Sqrt[2]*Sqrt[2 + x^2 - x^4]*EllipticPi[5/7, I*ArcSinh[x], -1/2] - (1162070*I)*Sqrt[2]*x^2*Sqrt[2 + x^2 - x^4]*EllipticPi[5/7, I*ArcSinh[x], -1/2] - (415025*I)*Sqrt[2]*x^4*Sqrt[2 + x^2 - x^4]*EllipticPi[5/7, I*ArcSinh[x], -1/2])/(4664800*(7 + 5*x^2)^2*Sqrt[2 + x^2 - x^4])

Maple [A] time = 0.021, size = 189, normalized size = 1.9

$$\frac{x}{28(5x^2+7)^2}\sqrt{-x^4+x^2+2}-\frac{31x}{66640x^2+93296}\sqrt{-x^4+x^2+2}-\frac{269\sqrt{2}}{333200}\sqrt{-2x^2+4}\sqrt{x^2+1}\text{EllipticF}\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)\sqrt{-x^4+x^2+2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-x^4+x^2+2)^(1/2)/(5*x^2+7)^3,x)

[Out] 1/28*x*(-x^4+x^2+2)^(1/2)/(5*x^2+7)^2-31/13328*x*(-x^4+x^2+2)^(1/2)/(5*x^2+7)-269/333200*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticF(1/2*x*2^(1/2),I*2^(1/2))-31/133280*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticE(1/2*x*2^(1/2),I*2^(1/2))+16601/2332400*2^(1/2)*(1-1/2*x^2)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticPi(1/2*x*2^(1/2),-10/7,I*2^(1/2))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-x^4 + x^2 + 2}}{(5x^2 + 7)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^4+x^2+2)^(1/2)/(5*x^2+7)^3,x, algorithm="maxima")

[Out] integrate(sqrt(-x^4 + x^2 + 2)/(5*x^2 + 7)^3, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-x^4 + x^2 + 2}}{125x^6 + 525x^4 + 735x^2 + 343}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^4+x^2+2)^(1/2)/(5*x^2+7)^3,x, algorithm="fricas")

[Out] integral(sqrt(-x^4 + x^2 + 2)/(125*x^6 + 525*x^4 + 735*x^2 + 343), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-(x^2 - 2)(x^2 + 1)}}{(5x^2 + 7)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x**4+x**2+2)**(1/2)/(5*x**2+7)**3,x)

[Out] Integral(sqrt(-(x**2 - 2)*(x**2 + 1))/(5*x**2 + 7)**3, x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-x^4 + x^2 + 2}}{(5x^2 + 7)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+x^2+2)^(1/2)/(5*x^2+7)^3,x, algorithm="giac")
```

```
[Out] integrate(sqrt(-x^4 + x^2 + 2)/(5*x^2 + 7)^3, x)
```

$$3.324 \quad \int (7 + 5x^2)^4 (2 + x^2 - x^4)^{3/2} dx$$

Optimal. Leaf size=142

$$\frac{50794416 \operatorname{EllipticF}\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right), -2\right)}{5005} - \frac{125}{3} (-x^4 + x^2 + 2)^{5/2} x^5 - \frac{11750}{39} (-x^4 + x^2 + 2)^{5/2} x^3 - \frac{132300}{143} (-x^4 + x^2 + 2)^{5/2} x - \frac{(69817 - 1581440x^2)(2 + x^2 - x^4)^{3/2}}{1001} - \frac{(132300x(2 + x^2 - x^4)^{5/2})}{143} - \frac{(11750x^3(2 + x^2 - x^4)^{5/2})}{39} - \frac{(125x^5(2 + x^2 - x^4)^{5/2})}{3} + \frac{(124141422 \operatorname{EllipticE}[\operatorname{ArcSin}[x/\sqrt{2}], -2])}{5005} - \frac{(50794416 \operatorname{EllipticF}[\operatorname{ArcSin}[x/\sqrt{2}], -2])}{5005}$$

[Out] (3*x*(2193559 + 7837383*x^2)*Sqrt[2 + x^2 - x^4])/5005 - (x*(69817 - 1581440*x^2)*(2 + x^2 - x^4)^(3/2))/1001 - (132300*x*(2 + x^2 - x^4)^(5/2))/143 - (11750*x^3*(2 + x^2 - x^4)^(5/2))/39 - (125*x^5*(2 + x^2 - x^4)^(5/2))/3 + (124141422*EllipticE[ArcSin[x/Sqrt[2]], -2])/5005 - (50794416*EllipticF[ArcSin[x/Sqrt[2]], -2])/5005

Rubi [A] time = 0.131612, antiderivative size = 142, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {1206, 1679, 1176, 1180, 524, 424, 419}

$$-\frac{125}{3} (-x^4 + x^2 + 2)^{5/2} x^5 - \frac{11750}{39} (-x^4 + x^2 + 2)^{5/2} x^3 - \frac{132300}{143} (-x^4 + x^2 + 2)^{5/2} x - \frac{(69817 - 1581440x^2)(-x^4 + x^2 + 2)^{3/2}}{1001}$$

Antiderivative was successfully verified.

[In] Int[(7 + 5*x^2)^4*(2 + x^2 - x^4)^(3/2), x]

[Out] (3*x*(2193559 + 7837383*x^2)*Sqrt[2 + x^2 - x^4])/5005 - (x*(69817 - 1581440*x^2)*(2 + x^2 - x^4)^(3/2))/1001 - (132300*x*(2 + x^2 - x^4)^(5/2))/143 - (11750*x^3*(2 + x^2 - x^4)^(5/2))/39 - (125*x^5*(2 + x^2 - x^4)^(5/2))/3 + (124141422*EllipticE[ArcSin[x/Sqrt[2]], -2])/5005 - (50794416*EllipticF[ArcSin[x/Sqrt[2]], -2])/5005

Rule 1206

Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(e^q*x^(2*q - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(4*p + 2*q + 1)), x] + Dist[1/(c*(4*p + 2*q + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*p + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1]

Rule 1679

Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{q = Expon[Pq, x^2], e = Coeff[Pq, x^2, Expon[Pq, x^2]]}, Simp[(e*x^(2*q - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(2*q + 4*p + 1)), x] + Dist[1/(c*(2*q + 4*p + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*q + 4*p + 1)*Pq - a*e*(2*q - 3)*x^(2*q - 4) - b*e*(2*q + 2*p - 1)*x^(2*q - 2) - c*e*(2*q + 4*p + 1)*x^(2*q), x], x], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && !LtQ[p, -1]

Rule 1176

Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*(a + b*x^2 + c*x^4)^p)/(c*(4*p + 1)*(4*p + 3)), x] + Dist[(2*p)/(c*(4*p + 1)*(4*p + 3)),

```
Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) -
  b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1180

```
Int[((d_) + (e_)*(x_)^2)/Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4], x_Symbol]
:> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt
[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e},
x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rule 524

```
Int[((e_) + (f_)*(x_)^(n_))/(Sqrt[(a_) + (b_)*(x_)^(n_)]*Sqrt[(c_) + (d_
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && (!GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 424

```
Int[Sqrt[(a_) + (b_)*(x_)^2]/Sqrt[(c_) + (d_)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d))]/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_)*(x_)^2]*Sqrt[(c_) + (d_)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d))]/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] && !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\begin{aligned}
\int (7 + 5x^2)^4 (2 + x^2 - x^4)^{3/2} dx &= -\frac{125}{3}x^5 (2 + x^2 - x^4)^{5/2} - \frac{1}{15} \int (2 + x^2 - x^4)^{3/2} (-36015 - 102900x^2 - 116500x^4 - 5 \\
&= -\frac{11750}{39}x^3 (2 + x^2 - x^4)^{5/2} - \frac{125}{3}x^5 (2 + x^2 - x^4)^{5/2} + \frac{1}{195} \int (2 + x^2 - x^4)^{3/2} (46819 \\
&= -\frac{132300}{143}x (2 + x^2 - x^4)^{5/2} - \frac{11750}{39}x^3 (2 + x^2 - x^4)^{5/2} - \frac{125}{3}x^5 (2 + x^2 - x^4)^{5/2} - \int (\\
&= -\frac{x(69817 - 1581440x^2)(2 + x^2 - x^4)^{3/2}}{1001} - \frac{132300}{143}x (2 + x^2 - x^4)^{5/2} - \frac{11750}{39}x^3 (2 + \\
&= \frac{3x(2193559 + 7837383x^2)\sqrt{2 + x^2 - x^4}}{5005} - \frac{x(69817 - 1581440x^2)(2 + x^2 - x^4)^{3/2}}{1001} \\
&= \frac{3x(2193559 + 7837383x^2)\sqrt{2 + x^2 - x^4}}{5005} - \frac{x(69817 - 1581440x^2)(2 + x^2 - x^4)^{3/2}}{1001} \\
&= \frac{3x(2193559 + 7837383x^2)\sqrt{2 + x^2 - x^4}}{5005} - \frac{x(69817 - 1581440x^2)(2 + x^2 - x^4)^{3/2}}{1001} \\
&= \frac{3x(2193559 + 7837383x^2)\sqrt{2 + x^2 - x^4}}{5005} - \frac{x(69817 - 1581440x^2)(2 + x^2 - x^4)^{3/2}}{1001}
\end{aligned}$$

Mathematica [F] time = 0, size = 0, normalized size = 0.

\$Aborted

Verification is Not applicable to the result.

[In] Integrate[(7 + 5*x^2)^4*(2 + x^2 - x^4)^(3/2), x]

[Out] \$Aborted

Maple [A] time = 0.023, size = 227, normalized size = 1.6

$$-\frac{12639493x}{5005}\sqrt{-x^4+x^2+2}-\frac{125x^{13}}{3}\sqrt{-x^4+x^2+2}+\frac{833561x^5}{273}\sqrt{-x^4+x^2+2}-\frac{8500x^{11}}{39}\sqrt{-x^4+x^2+2}+\frac{432713}{150}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((5*x^2+7)^4*(-x^4+x^2+2)^(3/2), x)

[Out] -12639493/5005*x*(-x^4+x^2+2)^(1/2)-125/3*x^13*(-x^4+x^2+2)^(1/2)+833561/273*x^5*(-x^4+x^2+2)^(1/2)-8500/39*x^11*(-x^4+x^2+2)^(1/2)+43271392/15015*x^3*(-x^4+x^2+2)^(1/2)-84775/429*x^9*(-x^4+x^2+2)^(1/2)+432290/429*x^7*(-x^4+x^2+2)^(1/2)-62070711/5005*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*(EllipticF(1/2*x*2^(1/2), I*2^(1/2))-EllipticE(1/2*x*2^(1/2), I*2^(1/2)))+36673503/5005*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticF(1/2*x*2^(1/2), I*2^(1/2))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (-x^4 + x^2 + 2)^{\frac{3}{2}} (5x^2 + 7)^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^2+7)^4*(-x^4+x^2+2)^(3/2), x, algorithm="maxima")

[Out] integrate((-x^4 + x^2 + 2)^(3/2)*(5*x^2 + 7)^4, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\left(625x^{12} + 2875x^{10} + 2600x^8 - 7490x^6 - 19159x^4 - 16121x^2 - 4802\right)\sqrt{-x^4 + x^2 + 2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^2+7)^4*(-x^4+x^2+2)^(3/2), x, algorithm="fricas")

[Out] integral(-(625*x^12 + 2875*x^10 + 2600*x^8 - 7490*x^6 - 19159*x^4 - 16121*x^2 - 4802)*sqrt(-x^4 + x^2 + 2), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-(x^2 - 2)(x^2 + 1) \right)^{\frac{3}{2}} (5x^2 + 7)^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x**2+7)**4*(-x**4+x**2+2)**(3/2),x)

[Out] Integral((-x**2 - 2)*(x**2 + 1))**(3/2)*(5*x**2 + 7)**4, x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-x^4 + x^2 + 2 \right)^{\frac{3}{2}} (5x^2 + 7)^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^2+7)^4*(-x^4+x^2+2)^(3/2),x, algorithm="giac")

[Out] integrate((-x^4 + x^2 + 2)^(3/2)*(5*x^2 + 7)^4, x)

3.325 $\int (7 + 5x^2)^3 (2 + x^2 - x^4)^{3/2} dx$

Optimal. Leaf size=121

$$-\frac{3199778 \operatorname{EllipticF}\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right), -2\right)}{5005} - \frac{125}{13} (-x^4 + x^2 + 2)^{5/2} x^3 - \frac{7825}{143} (-x^4 + x^2 + 2)^{5/2} x + \frac{(374045x^2 + 33792)}{3003}$$

[Out] (x*(2512273 + 5712051*x^2)*Sqrt[2 + x^2 - x^4])/15015 + (x*(33792 + 374045*x^2)*(2 + x^2 - x^4)^(3/2))/3003 - (7825*x*(2 + x^2 - x^4)^(5/2))/143 - (125*x^3*(2 + x^2 - x^4)^(5/2))/13 + (31072528*EllipticE[ArcSin[x/Sqrt[2]]], -2)/15015 - (3199778*EllipticF[ArcSin[x/Sqrt[2]]], -2)/5005

Rubi [A] time = 0.101206, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {1206, 1679, 1176, 1180, 524, 424, 419}

$$-\frac{125}{13} (-x^4 + x^2 + 2)^{5/2} x^3 - \frac{7825}{143} (-x^4 + x^2 + 2)^{5/2} x + \frac{(374045x^2 + 33792)(-x^4 + x^2 + 2)^{3/2} x}{3003} + \frac{(5712051x^2 + 2512273)\sqrt{2 + x^2 - x^4}}{15015}$$

Antiderivative was successfully verified.

[In] Int[(7 + 5*x^2)^3*(2 + x^2 - x^4)^(3/2), x]

[Out] (x*(2512273 + 5712051*x^2)*Sqrt[2 + x^2 - x^4])/15015 + (x*(33792 + 374045*x^2)*(2 + x^2 - x^4)^(3/2))/3003 - (7825*x*(2 + x^2 - x^4)^(5/2))/143 - (125*x^3*(2 + x^2 - x^4)^(5/2))/13 + (31072528*EllipticE[ArcSin[x/Sqrt[2]]], -2)/15015 - (3199778*EllipticF[ArcSin[x/Sqrt[2]]], -2)/5005

Rule 1206

Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(e^q*x^(2*q - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(4*p + 2*q + 1)), x] + Dist[1/(c*(4*p + 2*q + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*p + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1]

Rule 1679

Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{q = Expon[Pq, x^2], e = Coeff[Pq, x^2, Expon[Pq, x^2]]}, Simp[(e*x^(2*q - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(2*q + 4*p + 1)), x] + Dist[1/(c*(2*q + 4*p + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*q + 4*p + 1)*Pq - a*e*(2*q - 3)*x^(2*q - 4) - b*e*(2*q + 2*p - 1)*x^(2*q - 2) - c*e*(2*q + 4*p + 1)*x^(2*q), x], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && !LtQ[p, -1]

Rule 1176

Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*(a + b*x^2 + c*x^4)^p)/(c*(4*p + 1)*(4*p + 3)), x] + Dist[(2*p)/(c*(4*p + 1)*(4*p + 3)), Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) - b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x] /; FreeQ[{a,

b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]

Rule 1180

Int[((d_) + (e_)*(x_)^2)/Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4], x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]

Rule 524

Int[((e_) + (f_)*(x_)^(n_))/(Sqrt[(a_) + (b_)*(x_)^(n_)]*Sqrt[(c_) + (d_)*(x_)^(n_)]), x_Symbol] := Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n], x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && !(EqQ[n, 2] && ((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && (!GtQ[c, 0] || SimplerSqrtQ[-(b/a), -(d/c)]))))))

Rule 424

Int[Sqrt[(a_) + (b_)*(x_)^2]/Sqrt[(c_) + (d_)*(x_)^2], x_Symbol] := Simp[Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)]/(Sqrt[c]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]

Rule 419

Int[1/(Sqrt[(a_) + (b_)*(x_)^2]*Sqrt[(c_) + (d_)*(x_)^2]), x_Symbol] := Simp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)]/(Sqrt[a]*Sqrt[c]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0] && !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])

Rubi steps

$$\begin{aligned}
 \int (7 + 5x^2)^3 (2 + x^2 - x^4)^{3/2} dx &= -\frac{125}{13}x^3 (2 + x^2 - x^4)^{5/2} - \frac{1}{13} \int (-4459 - 10305x^2 - 7825x^4) (2 + x^2 - x^4)^{3/2} dx \\
 &= -\frac{7825}{143}x (2 + x^2 - x^4)^{5/2} - \frac{125}{13}x^3 (2 + x^2 - x^4)^{5/2} + \frac{1}{143} \int (64699 + 160305x^2) (2 + x^2 - x^4)^{3/2} dx \\
 &= \frac{x(33792 + 374045x^2) (2 + x^2 - x^4)^{3/2}}{3003} - \frac{7825}{143}x (2 + x^2 - x^4)^{5/2} - \frac{125}{13}x^3 (2 + x^2 - x^4)^{5/2} \\
 &= \frac{x(2512273 + 5712051x^2) \sqrt{2 + x^2 - x^4}}{15015} + \frac{x(33792 + 374045x^2) (2 + x^2 - x^4)^{3/2}}{3003} - \frac{7825}{143}x (2 + x^2 - x^4)^{5/2} - \frac{125}{13}x^3 (2 + x^2 - x^4)^{5/2} \\
 &= \frac{x(2512273 + 5712051x^2) \sqrt{2 + x^2 - x^4}}{15015} + \frac{x(33792 + 374045x^2) (2 + x^2 - x^4)^{3/2}}{3003} - \frac{7825}{143}x (2 + x^2 - x^4)^{5/2} - \frac{125}{13}x^3 (2 + x^2 - x^4)^{5/2} \\
 &= \frac{x(2512273 + 5712051x^2) \sqrt{2 + x^2 - x^4}}{15015} + \frac{x(33792 + 374045x^2) (2 + x^2 - x^4)^{3/2}}{3003} - \frac{7825}{143}x (2 + x^2 - x^4)^{5/2} - \frac{125}{13}x^3 (2 + x^2 - x^4)^{5/2} \\
 &= \frac{x(2512273 + 5712051x^2) \sqrt{2 + x^2 - x^4}}{15015} + \frac{x(33792 + 374045x^2) (2 + x^2 - x^4)^{3/2}}{3003} - \frac{7825}{143}x (2 + x^2 - x^4)^{5/2} - \frac{125}{13}x^3 (2 + x^2 - x^4)^{5/2}
 \end{aligned}$$

Mathematica [F] time = 0, size = 0, normalized size = 0.

\$Aborted

Verification is Not applicable to the result.

[In] Integrate[(7 + 5*x^2)^3*(2 + x^2 - x^4)^(3/2), x]

[Out] \$Aborted

Maple [A] time = 0.007, size = 210, normalized size = 1.7

$$-\frac{436307x}{15015}\sqrt{-x^4+x^2+2} + \frac{65248x^5}{273}\sqrt{-x^4+x^2+2} - \frac{125x^{11}}{13}\sqrt{-x^4+x^2+2} + \frac{5757461x^3}{15015}\sqrt{-x^4+x^2+2} - \frac{5075x^9}{143}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((5*x^2+7)^3*(-x^4+x^2+2)^(3/2), x)

[Out] -436307/15015*x*(-x^4+x^2+2)^(1/2)+65248/273*x^5*(-x^4+x^2+2)^(1/2)-125/13*x^11*(-x^4+x^2+2)^(1/2)+5757461/15015*x^3*(-x^4+x^2+2)^(1/2)-5075/143*x^9*(-x^4+x^2+2)^(1/2)+5890/429*x^7*(-x^4+x^2+2)^(1/2)-15536264/15015*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*(EllipticF(1/2*x*2^(1/2), I*2^(1/2))-EllipticE(1/2*x*2^(1/2), I*2^(1/2)))+10736597/15015*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticF(1/2*x*2^(1/2), I*2^(1/2))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (-x^4 + x^2 + 2)^{\frac{3}{2}} (5x^2 + 7)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^2+7)^3*(-x^4+x^2+2)^(3/2), x, algorithm="maxima")

[Out] integrate((-x^4 + x^2 + 2)^(3/2)*(5*x^2 + 7)^3, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\left(125x^{10} + 400x^8 - 40x^6 - 1442x^4 - 1813x^2 - 686\right)\sqrt{-x^4 + x^2 + 2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^2+7)^3*(-x^4+x^2+2)^(3/2), x, algorithm="fricas")

[Out] integral(-(125*x^10 + 400*x^8 - 40*x^6 - 1442*x^4 - 1813*x^2 - 686)*sqrt(-x^4 + x^2 + 2), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-\left(x^2 - 2\right)\left(x^2 + 1\right)\right)^{\frac{3}{2}} \left(5x^2 + 7\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+7)**3*(-x**4+x**2+2)**(3/2),x)
```

```
[Out] Integral((-x**2 - 2)*(x**2 + 1))**(3/2)*(5*x**2 + 7)**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (-x^4 + x^2 + 2)^{\frac{3}{2}} (5x^2 + 7)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^3*(-x^4+x^2+2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((-x^4 + x^2 + 2)^(3/2)*(5*x^2 + 7)^3, x)
```

$$3.326 \quad \int (7 + 5x^2)^2 (2 + x^2 - x^4)^{3/2} dx$$

Optimal. Leaf size=100

$$-\frac{3392}{165} \text{EllipticF}\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right), -2\right) - \frac{25}{11}x(-x^4 + x^2 + 2)^{5/2} + \frac{1}{99}x(920x^2 + 363)(-x^4 + x^2 + 2)^{3/2} + \frac{1}{495}x(14889x^2$$

[Out] (x*(11497 + 14889*x^2)*Sqrt[2 + x^2 - x^4])/495 + (x*(363 + 920*x^2)*(2 + x^2 - x^4)^(3/2))/99 - (25*x*(2 + x^2 - x^4)^(5/2))/11 + (85942*EllipticE[ArcSin[x/Sqrt[2]], -2])/495 - (3392*EllipticF[ArcSin[x/Sqrt[2]], -2])/165

Rubi [A] time = 0.0738041, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$, Rules used = {1206, 1176, 1180, 524, 424, 419}

$$-\frac{25}{11}x(-x^4 + x^2 + 2)^{5/2} + \frac{1}{99}x(920x^2 + 363)(-x^4 + x^2 + 2)^{3/2} + \frac{1}{495}x(14889x^2 + 11497)\sqrt{-x^4 + x^2 + 2} - \frac{3392}{165}F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right), -2\right)$$

Antiderivative was successfully verified.

[In] Int[(7 + 5*x^2)^2*(2 + x^2 - x^4)^(3/2), x]

[Out] (x*(11497 + 14889*x^2)*Sqrt[2 + x^2 - x^4])/495 + (x*(363 + 920*x^2)*(2 + x^2 - x^4)^(3/2))/99 - (25*x*(2 + x^2 - x^4)^(5/2))/11 + (85942*EllipticE[ArcSin[x/Sqrt[2]], -2])/495 - (3392*EllipticF[ArcSin[x/Sqrt[2]], -2])/165

Rule 1206

Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(e^q*x^(2*q - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(4*p + 2*q + 1)), x] + Dist[1/(c*(4*p + 2*q + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandT oSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*p + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1]

Rule 1176

Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*(a + b*x^2 + c*x^4)^p)/(c*(4*p + 1)*(4*p + 3)), x] + Dist[(2*p)/(c*(4*p + 1)*(4*p + 3)), Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) - b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]

Rule 1180

Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]

Rule 524

Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],

```
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] && !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && (!GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] && !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\begin{aligned}
\int (7 + 5x^2)^2 (2 + x^2 - x^4)^{3/2} dx &= -\frac{25}{11}x(2 + x^2 - x^4)^{5/2} - \frac{1}{11} \int (-589 - 920x^2)(2 + x^2 - x^4)^{3/2} dx \\
&= \frac{1}{99}x(363 + 920x^2)(2 + x^2 - x^4)^{3/2} - \frac{25}{11}x(2 + x^2 - x^4)^{5/2} + \frac{1}{231} \int (23044 + 34741x^2) \sqrt{2 + x^2 - x^4} dx \\
&= \frac{1}{495}x(11497 + 14889x^2) \sqrt{2 + x^2 - x^4} + \frac{1}{99}x(363 + 920x^2)(2 + x^2 - x^4)^{3/2} - \frac{25}{11}x(2 + x^2 - x^4)^{5/2} \\
&= \frac{1}{495}x(11497 + 14889x^2) \sqrt{2 + x^2 - x^4} + \frac{1}{99}x(363 + 920x^2)(2 + x^2 - x^4)^{3/2} - \frac{25}{11}x(2 + x^2 - x^4)^{5/2} \\
&= \frac{1}{495}x(11497 + 14889x^2) \sqrt{2 + x^2 - x^4} + \frac{1}{99}x(363 + 920x^2)(2 + x^2 - x^4)^{3/2} - \frac{25}{11}x(2 + x^2 - x^4)^{5/2} \\
&= \frac{1}{495}x(11497 + 14889x^2) \sqrt{2 + x^2 - x^4} + \frac{1}{99}x(363 + 920x^2)(2 + x^2 - x^4)^{3/2} - \frac{25}{11}x(2 + x^2 - x^4)^{5/2}
\end{aligned}$$

Mathematica [F] time = 0, size = 0, normalized size = 0.

\$Aborted

Verification is Not applicable to the result.

[In] Integrate[(7 + 5*x^2)^2*(2 + x^2 - x^4)^(3/2), x]

[Out] \$Aborted

Maple [B] time = 0.008, size = 193, normalized size = 1.9

$$-\frac{25x^9}{11}\sqrt{-x^4+x^2+2} - \frac{470x^7}{99}\sqrt{-x^4+x^2+2} + \frac{112x^5}{9}\sqrt{-x^4+x^2+2} + \frac{21404x^3}{495}\sqrt{-x^4+x^2+2} + \frac{10627x}{495}\sqrt{-x^4+x^2+2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((5*x^2+7)^2*(-x^4+x^2+2)^(3/2), x)

[Out] $-25/11*x^9*(-x^4+x^2+2)^{(1/2)}-470/99*x^7*(-x^4+x^2+2)^{(1/2)}+112/9*x^5*(-x^4+x^2+2)^{(1/2)}+21404/495*x^3*(-x^4+x^2+2)^{(1/2)}+10627/495*x*(-x^4+x^2+2)^{(1/2)}+37883/495*2^{(1/2)}*(-2*x^2+4)^{(1/2)}*(x^2+1)^{(1/2)}/(-x^4+x^2+2)^{(1/2)}*EllipticF(1/2*x*2^{(1/2)},I*2^{(1/2)})-42971/495*2^{(1/2)}*(-2*x^2+4)^{(1/2)}*(x^2+1)^{(1/2)}/(-x^4+x^2+2)^{(1/2)}*(EllipticF(1/2*x*2^{(1/2)},I*2^{(1/2)})-EllipticE(1/2*x*2^{(1/2)},I*2^{(1/2)}))$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (-x^4 + x^2 + 2)^{\frac{3}{2}} (5x^2 + 7)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^2+7)^2*(-x^4+x^2+2)^(3/2),x, algorithm="maxima")

[Out] integrate((-x^4 + x^2 + 2)^(3/2)*(5*x^2 + 7)^2, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\left(25x^8 + 45x^6 - 71x^4 - 189x^2 - 98\right)\sqrt{-x^4 + x^2 + 2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^2+7)^2*(-x^4+x^2+2)^(3/2),x, algorithm="fricas")

[Out] integral(-(25*x^8 + 45*x^6 - 71*x^4 - 189*x^2 - 98)*sqrt(-x^4 + x^2 + 2), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-\left(x^2 - 2\right)\left(x^2 + 1\right)\right)^{\frac{3}{2}} \left(5x^2 + 7\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x**2+7)**2*(-x**4+x**2+2)**(3/2),x)

[Out] Integral((-x**2 - 2)*(x**2 + 1))**3/2*(5*x**2 + 7)**2, x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (-x^4 + x^2 + 2)^{\frac{3}{2}} (5x^2 + 7)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^2+7)^2*(-x^4+x^2+2)^(3/2),x, algorithm="giac")

[Out] integrate((-x^4 + x^2 + 2)^(3/2)*(5*x^2 + 7)^2, x)

3.327 $\int (7 + 5x^2) (2 + x^2 - x^4)^{3/2} dx$

Optimal. Leaf size=81

$$\frac{418}{105} \text{EllipticF}\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right), -2\right) + \frac{1}{63}x(35x^2 + 48)(-x^4 + x^2 + 2)^{3/2} + \frac{1}{315}x(669x^2 + 1087)\sqrt{-x^4 + x^2 + 2} + \frac{4432}{315}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right), -2\right)$$

[Out] (x*(1087 + 669*x^2)*Sqrt[2 + x^2 - x^4])/315 + (x*(48 + 35*x^2)*(2 + x^2 - x^4)^(3/2))/63 + (4432*EllipticE[ArcSin[x/Sqrt[2]], -2])/315 + (418*EllipticF[ArcSin[x/Sqrt[2]], -2])/105

Rubi [A] time = 0.0554957, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {1176, 1180, 524, 424, 419}

$$\frac{1}{63}x(35x^2 + 48)(-x^4 + x^2 + 2)^{3/2} + \frac{1}{315}x(669x^2 + 1087)\sqrt{-x^4 + x^2 + 2} + \frac{418}{105}F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right), -2\right) + \frac{4432}{315}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right), -2\right)$$

Antiderivative was successfully verified.

[In] Int[(7 + 5*x^2)*(2 + x^2 - x^4)^(3/2),x]

[Out] (x*(1087 + 669*x^2)*Sqrt[2 + x^2 - x^4])/315 + (x*(48 + 35*x^2)*(2 + x^2 - x^4)^(3/2))/63 + (4432*EllipticE[ArcSin[x/Sqrt[2]], -2])/315 + (418*EllipticF[ArcSin[x/Sqrt[2]], -2])/105

Rule 1176

Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := Simp[(x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*(a + b*x^2 + c*x^4)^p)/(c*(4*p + 1)*(4*p + 3)), x] + Dist[(2*p)/(c*(4*p + 1)*(4*p + 3)), Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) - b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]

Rule 1180

Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]

Rule 524

Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.)*(x_)^(n_)]), x_Symbol] := Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n], x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && !(EqQ[n, 2] && ((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && (!GtQ[c, 0] || SimplerSqrtQ[-(b/a), -(d/c)]))))))

Rule 424

Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] := Simp[(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/Sqrt[c]*Rt[-(d/c

), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]

Rule 419

Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> Simp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0] && !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])

Rubi steps

$$\begin{aligned} \int (7 + 5x^2)(2 + x^2 - x^4)^{3/2} dx &= \frac{1}{63}x(48 + 35x^2)(2 + x^2 - x^4)^{3/2} - \frac{1}{21} \int (-262 - 223x^2) \sqrt{2 + x^2 - x^4} dx \\ &= \frac{1}{315}x(1087 + 669x^2) \sqrt{2 + x^2 - x^4} + \frac{1}{63}x(48 + 35x^2)(2 + x^2 - x^4)^{3/2} + \frac{1}{315} \int 56 \sqrt{2 + x^2 - x^4} dx \\ &= \frac{1}{315}x(1087 + 669x^2) \sqrt{2 + x^2 - x^4} + \frac{1}{63}x(48 + 35x^2)(2 + x^2 - x^4)^{3/2} + \frac{2}{315} \int \sqrt{2 + x^2 - x^4} dx \\ &= \frac{1}{315}x(1087 + 669x^2) \sqrt{2 + x^2 - x^4} + \frac{1}{63}x(48 + 35x^2)(2 + x^2 - x^4)^{3/2} + \frac{836}{105} \int \sqrt{2 + x^2 - x^4} dx \\ &= \frac{1}{315}x(1087 + 669x^2) \sqrt{2 + x^2 - x^4} + \frac{1}{63}x(48 + 35x^2)(2 + x^2 - x^4)^{3/2} + \frac{4432}{315}E\left(\sin^{-1}\left(\frac{\sqrt{2+x^2-x^4}}{\sqrt{2-x^4}}\right), \frac{1}{2}\right) \end{aligned}$$

Mathematica [F] time = 0, size = 0, normalized size = 0.

\$Aborted

Verification is Not applicable to the result.

[In] Integrate[(7 + 5*x^2)*(2 + x^2 - x^4)^(3/2), x]

[Out] \$Aborted

Maple [B] time = 0.006, size = 176, normalized size = 2.2

$$-\frac{5x^7}{9}\sqrt{-x^4+x^2+2} - \frac{13x^5}{63}\sqrt{-x^4+x^2+2} + \frac{1259x^3}{315}\sqrt{-x^4+x^2+2} + \frac{1567x}{315}\sqrt{-x^4+x^2+2} + \frac{2843\sqrt{2}}{315}\sqrt{-2x^2+4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((5*x^2+7)*(-x^4+x^2+2)^(3/2), x)

[Out] $-5/9*x^7*(-x^4+x^2+2)^{(1/2)} - 13/63*x^5*(-x^4+x^2+2)^{(1/2)} + 1259/315*x^3*(-x^4+x^2+2)^{(1/2)} + 1567/315*x*(-x^4+x^2+2)^{(1/2)} + 2843/315*2^{(1/2)}*(-2*x^2+4)^{(1/2)}*(x^2+1)^{(1/2)}/(-x^4+x^2+2)^{(1/2)}*EllipticF(1/2*x*2^{(1/2)}, I*2^{(1/2)}) - 2216/315*2^{(1/2)}*(-2*x^2+4)^{(1/2)}*(x^2+1)^{(1/2)}/(-x^4+x^2+2)^{(1/2)}*(EllipticF(1/2*x*2^{(1/2)}, I*2^{(1/2)}) - EllipticE(1/2*x*2^{(1/2)}, I*2^{(1/2)}))$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (-x^4 + x^2 + 2)^{\frac{3}{2}} (5x^2 + 7) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^2+7)*(-x^4+x^2+2)^(3/2),x, algorithm="maxima")

[Out] integrate((-x^4 + x^2 + 2)^(3/2)*(5*x^2 + 7), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\left(5x^6 + 2x^4 - 17x^2 - 14\right)\sqrt{-x^4 + x^2 + 2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^2+7)*(-x^4+x^2+2)^(3/2),x, algorithm="fricas")

[Out] integral(-(5*x^6 + 2*x^4 - 17*x^2 - 14)*sqrt(-x^4 + x^2 + 2), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-\left(x^2 - 2\right)\left(x^2 + 1\right)\right)^{\frac{3}{2}} \left(5x^2 + 7\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x**2+7)*(-x**4+x**2+2)**(3/2),x)

[Out] Integral((-x**2 - 2)*(x**2 + 1))**(3/2)*(5*x**2 + 7), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-x^4 + x^2 + 2\right)^{\frac{3}{2}} \left(5x^2 + 7\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^2+7)*(-x^4+x^2+2)^(3/2),x, algorithm="giac")

[Out] integrate((-x^4 + x^2 + 2)^(3/2)*(5*x^2 + 7), x)

3.328 $\int (2 + x^2 - x^4)^{3/2} dx$

Optimal. Leaf size=74

$$\frac{48}{35} \text{EllipticF}\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right), -2\right) + \frac{1}{7}x(-x^4 + x^2 + 2)^{3/2} + \frac{1}{35}x(3x^2 + 19)\sqrt{-x^4 + x^2 + 2} + \frac{34}{35}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right) \middle| -2\right)$$

[Out] (x*(19 + 3*x^2)*Sqrt[2 + x^2 - x^4])/35 + (x*(2 + x^2 - x^4)^(3/2))/7 + (34*EllipticE[ArcSin[x/Sqrt[2]], -2])/35 + (48*EllipticF[ArcSin[x/Sqrt[2]], -2])/35

Rubi [A] time = 0.0516522, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {1091, 1176, 1180, 524, 424, 419}

$$\frac{1}{7}x(-x^4 + x^2 + 2)^{3/2} + \frac{1}{35}x(3x^2 + 19)\sqrt{-x^4 + x^2 + 2} + \frac{48}{35}F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right) \middle| -2\right) + \frac{34}{35}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right) \middle| -2\right)$$

Antiderivative was successfully verified.

[In] Int[(2 + x^2 - x^4)^(3/2), x]

[Out] (x*(19 + 3*x^2)*Sqrt[2 + x^2 - x^4])/35 + (x*(2 + x^2 - x^4)^(3/2))/7 + (34*EllipticE[ArcSin[x/Sqrt[2]], -2])/35 + (48*EllipticF[ArcSin[x/Sqrt[2]], -2])/35

Rule 1091

Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(x*(a + b*x^2 + c*x^4)^p)/(4*p + 1), x] + Dist[(2*p)/(4*p + 1), Int[(2*a + b*x^2)*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[2*p]

Rule 1176

Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*(a + b*x^2 + c*x^4)^p)/(c*(4*p + 1)*(4*p + 3)), x] + Dist[(2*p)/(c*(4*p + 1)*(4*p + 3)), Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) - b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]

Rule 1180

Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]

Rule 524

Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n], x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && !(EqQ[n, 2] && ((PosQ[b/a] && PosQ

[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && (!GtQ[c, 0] || SimplerSqrtQ[-(b/a), -(d/c)]))))))

Rule 424

Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]

Rule 419

Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> Simp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0] && !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])

Rubi steps

$$\begin{aligned} \int (2 + x^2 - x^4)^{3/2} dx &= \frac{1}{7}x(2 + x^2 - x^4)^{3/2} + \frac{3}{7} \int (4 + x^2) \sqrt{2 + x^2 - x^4} dx \\ &= \frac{1}{35}x(19 + 3x^2) \sqrt{2 + x^2 - x^4} + \frac{1}{7}x(2 + x^2 - x^4)^{3/2} - \frac{1}{35} \int \frac{-82 - 34x^2}{\sqrt{2 + x^2 - x^4}} dx \\ &= \frac{1}{35}x(19 + 3x^2) \sqrt{2 + x^2 - x^4} + \frac{1}{7}x(2 + x^2 - x^4)^{3/2} - \frac{2}{35} \int \frac{-82 - 34x^2}{\sqrt{4 - 2x^2}\sqrt{2 + 2x^2}} dx \\ &= \frac{1}{35}x(19 + 3x^2) \sqrt{2 + x^2 - x^4} + \frac{1}{7}x(2 + x^2 - x^4)^{3/2} + \frac{34}{35} \int \frac{\sqrt{2 + 2x^2}}{\sqrt{4 - 2x^2}} dx + \frac{96}{35} \int \frac{1}{\sqrt{4 - 2x^2}\sqrt{2 + 2x^2}} dx \\ &= \frac{1}{35}x(19 + 3x^2) \sqrt{2 + x^2 - x^4} + \frac{1}{7}x(2 + x^2 - x^4)^{3/2} + \frac{34}{35}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\right) - 2 + \frac{48}{35}F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\right) \end{aligned}$$

Mathematica [C] time = 0.0533497, size = 102, normalized size = 1.38

$$\frac{-75i\sqrt{-2x^4 + 2x^2 + 4}\text{EllipticF}\left(i \sinh^{-1}(x), -\frac{1}{2}\right) + 5x^9 - 13x^7 - 31x^5 + 45x^3 + 34i\sqrt{-2x^4 + 2x^2 + 4}E\left(i \sinh^{-1}(x) \mid -\frac{1}{2}\right)}{35\sqrt{-x^4 + x^2 + 2}}$$

Antiderivative was successfully verified.

[In] Integrate[(2 + x^2 - x^4)^(3/2), x]

[Out] (58*x + 45*x^3 - 31*x^5 - 13*x^7 + 5*x^9 + (34*I)*Sqrt[4 + 2*x^2 - 2*x^4]*EllipticE[I*ArcSinh[x], -1/2] - (75*I)*Sqrt[4 + 2*x^2 - 2*x^4]*EllipticF[I*ArcSinh[x], -1/2])/(35*Sqrt[2 + x^2 - x^4])

Maple [B] time = 0.003, size = 159, normalized size = 2.2

$$-\frac{x^5}{7}\sqrt{-x^4 + x^2 + 2} + \frac{8x^3}{35}\sqrt{-x^4 + x^2 + 2} + \frac{29x}{35}\sqrt{-x^4 + x^2 + 2} + \frac{41\sqrt{2}}{35}\sqrt{-2x^2 + 4}\sqrt{x^2 + 1}\text{EllipticF}\left(\frac{x\sqrt{2}}{2}, i\sqrt{2}\right)\frac{1}{\sqrt{-x^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-x^4+x^2+2)^(3/2), x)

```
[Out] -1/7*x^5*(-x^4+x^2+2)^(1/2)+8/35*x^3*(-x^4+x^2+2)^(1/2)+29/35*x*(-x^4+x^2+2)^(1/2)+41/35*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticF(1/2*x*2^(1/2),I*2^(1/2))-17/35*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*(EllipticF(1/2*x*2^(1/2),I*2^(1/2))-EllipticE(1/2*x*2^(1/2),I*2^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (-x^4 + x^2 + 2)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+x^2+2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((-x^4 + x^2 + 2)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(-x^4 + x^2 + 2\right)^{\frac{3}{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+x^2+2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((-x^4 + x^2 + 2)^(3/2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (-x^4 + x^2 + 2)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**4+x**2+2)**(3/2),x)
```

```
[Out] Integral((-x**4 + x**2 + 2)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (-x^4 + x^2 + 2)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+x^2+2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((-x^4 + x^2 + 2)^(3/2), x)
```

$$3.329 \quad \int \frac{(2+x^2-x^4)^{3/2}}{7+5x^2} dx$$

Optimal. Leaf size=72

$$-\frac{178}{625}\text{EllipticF}\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right), -2\right) + \frac{1}{75}x\sqrt{-x^4+x^2+2}(13-3x^2) + \frac{92}{375}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle| -2\right) + \frac{1156\Pi\left(-\frac{10}{7}; \sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\right)}{4375}$$

[Out] (x*(13 - 3*x^2)*Sqrt[2 + x^2 - x^4])/75 + (92*EllipticE[ArcSin[x/Sqrt[2]], -2])/375 - (178*EllipticF[ArcSin[x/Sqrt[2]], -2])/625 + (1156*EllipticPi[-10/7, ArcSin[x/Sqrt[2]], -2])/4375

Rubi [A] time = 0.136052, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1208, 1176, 1180, 524, 424, 419, 1212, 537}

$$\frac{1}{75}x\sqrt{-x^4+x^2+2}(13-3x^2) - \frac{178}{625}F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle| -2\right) + \frac{92}{375}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle| -2\right) + \frac{1156\Pi\left(-\frac{10}{7}; \sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\right)}{4375}$$

Antiderivative was successfully verified.

[In] Int[(2 + x^2 - x^4)^(3/2)/(7 + 5*x^2), x]

[Out] (x*(13 - 3*x^2)*Sqrt[2 + x^2 - x^4])/75 + (92*EllipticE[ArcSin[x/Sqrt[2]], -2])/375 - (178*EllipticF[ArcSin[x/Sqrt[2]], -2])/625 + (1156*EllipticPi[-10/7, ArcSin[x/Sqrt[2]], -2])/4375

Rule 1208

Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_)/((d_) + (e_.)*(x_)^2), x_Symbol] := -Dist[(e^2)^(-1), Int[(c*d - b*e - c*e*x^2)*(a + b*x^2 + c*x^4)^(p-1), x], x] + Dist[(c*d^2 - b*d*e + a*e^2)/e^2, Int[(a + b*x^2 + c*x^4)^(p-1)/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p + 1/2, 0]

Rule 1176

Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := Simp[(x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*(a + b*x^2 + c*x^4)^p)/(c*(4*p + 1)*(4*p + 3)), x] + Dist[(2*p)/(c*(4*p + 1)*(4*p + 3)), Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) - b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p-1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]

Rule 1180

Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]

Rule 524

Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.)*(x_)^(n_)]), x_Symbol] := Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],


```
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] && !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && (!GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] := Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)]/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] := S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)]/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] && !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 1212

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/((d + e*x^2
)*Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c,
d, e}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] := Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)]/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] && !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] && !(GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rubi steps

$$\begin{aligned}
\int \frac{(2+x^2-x^4)^{3/2}}{7+5x^2} dx &= -\left(\frac{1}{25} \int (-12+5x^2) \sqrt{2+x^2-x^4} dx\right) - \frac{34}{25} \int \frac{\sqrt{2+x^2-x^4}}{7+5x^2} dx \\
&= \frac{1}{75} x (13-3x^2) \sqrt{2+x^2-x^4} + \frac{1}{375} \int \frac{230-10x^2}{\sqrt{2+x^2-x^4}} dx + \frac{34}{625} \int \frac{-12+5x^2}{\sqrt{2+x^2-x^4}} dx + \frac{1156}{625} \int \frac{1}{\sqrt{2+x^2-x^4}} dx \\
&= \frac{1}{75} x (13-3x^2) \sqrt{2+x^2-x^4} + \frac{2}{375} \int \frac{230-10x^2}{\sqrt{4-2x^2}\sqrt{2+2x^2}} dx + \frac{68}{625} \int \frac{-12+5x^2}{\sqrt{4-2x^2}\sqrt{2+2x^2}} dx \\
&= \frac{1}{75} x (13-3x^2) \sqrt{2+x^2-x^4} + \frac{1156\Pi\left(-\frac{10}{7}; \sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\right) - 2}{4375} - \frac{2}{75} \int \frac{\sqrt{2+2x^2}}{\sqrt{4-2x^2}} dx + \frac{34}{125} \int \frac{1}{\sqrt{2+x^2-x^4}} dx \\
&= \frac{1}{75} x (13-3x^2) \sqrt{2+x^2-x^4} + \frac{92}{375} E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\right) - 2 - \frac{178}{625} F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\right) - 2 + \frac{1156\Pi\left(-\frac{10}{7}; \sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\right) - 2}{4375}
\end{aligned}$$

Mathematica [C] time = 0.195584, size = 130, normalized size = 1.81

$$\frac{-2961i\sqrt{-2x^4+2x^2+4}\operatorname{EllipticF}\left(i\sinh^{-1}(x), -\frac{1}{2}\right) + 525x^7 - 2800x^5 + 1225x^3 + 3220i\sqrt{-2x^4+2x^2+4}E\left(i\sinh^{-1}(x)\right)}{13125\sqrt{-x^4+x^2+2}}$$

Antiderivative was successfully verified.

[In] Integrate[(2 + x^2 - x^4)^(3/2)/(7 + 5*x^2),x]

[Out] (4550*x + 1225*x^3 - 2800*x^5 + 525*x^7 + (3220*I)*Sqrt[4 + 2*x^2 - 2*x^4]*EllipticE[I*ArcSinh[x], -1/2] - (2961*I)*Sqrt[4 + 2*x^2 - 2*x^4]*EllipticF[I*ArcSinh[x], -1/2] - (1734*I)*Sqrt[4 + 2*x^2 - 2*x^4]*EllipticPi[5/7, I*ArcSinh[x], -1/2))/(13125*Sqrt[2 + x^2 - x^4])

Maple [B] time = 0.015, size = 173, normalized size = 2.4

$$-\frac{x^3}{25}\sqrt{-x^4+x^2+2} + \frac{13x}{75}\sqrt{-x^4+x^2+2} - \frac{89\sqrt{2}}{625}\sqrt{-2x^2+4}\sqrt{x^2+1}\text{EllipticF}\left(\frac{x\sqrt{2}}{2}, i\sqrt{2}\right) \frac{1}{\sqrt{-x^4+x^2+2}} + \frac{46\sqrt{2}}{375}\sqrt{-x^4+x^2+2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-x^4+x^2+2)^(3/2)/(5*x^2+7),x)

[Out] -1/25*x^3*(-x^4+x^2+2)^(1/2)+13/75*x*(-x^4+x^2+2)^(1/2)-89/625*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticF(1/2*x*2^(1/2),I*2^(1/2))+46/375*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticE(1/2*x*2^(1/2),I*2^(1/2))+1156/4375*2^(1/2)*(1-1/2*x^2)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticPi(1/2*x*2^(1/2),-10/7,I*2^(1/2))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(-x^4 + x^2 + 2)^{\frac{3}{2}}}{5x^2 + 7} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^4+x^2+2)^(3/2)/(5*x^2+7),x, algorithm="maxima")

[Out] integrate((-x^4 + x^2 + 2)^(3/2)/(5*x^2 + 7), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(-x^4 + x^2 + 2)^{\frac{3}{2}}}{5x^2 + 7}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^4+x^2+2)^(3/2)/(5*x^2+7),x, algorithm="fricas")

[Out] integral((-x^4 + x^2 + 2)^(3/2)/(5*x^2 + 7), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{-(x^2 - 2)(x^2 + 1)^{\frac{3}{2}}}{5x^2 + 7} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x**4+x**2+2)**(3/2)/(5*x**2+7),x)

[Out] Integral((-x**2 - 2)*(x**2 + 1)**(3/2)/(5*x**2 + 7), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(-x^4 + x^2 + 2)^{\frac{3}{2}}}{5x^2 + 7} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^4+x^2+2)^(3/2)/(5*x^2+7),x, algorithm="giac")

[Out] integrate((-x^4 + x^2 + 2)^(3/2)/(5*x^2 + 7), x)

$$3.330 \quad \int \frac{(2+x^2-x^4)^{3/2}}{(7+5x^2)^2} dx$$

Optimal. Leaf size=93

$$\frac{458}{875} \text{EllipticF}\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right), -2\right) - \frac{17\sqrt{-x^4+x^2+2x}}{175(5x^2+7)} - \frac{1}{75}\sqrt{-x^4+x^2+2x} - \frac{97}{525}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle| -2\right) - \frac{1241\Pi\left(-\frac{10}{7}; \sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\right)}{6125}$$

[Out] $-(x*\text{Sqrt}[2 + x^2 - x^4])/75 - (17*x*\text{Sqrt}[2 + x^2 - x^4])/(175*(7 + 5*x^2)) - (97*\text{EllipticE}[\text{ArcSin}[x/\text{Sqrt}[2]], -2])/525 + (458*\text{EllipticF}[\text{ArcSin}[x/\text{Sqrt}[2]], -2])/875 - (1241*\text{EllipticPi}[-10/7, \text{ArcSin}[x/\text{Sqrt}[2]], -2])/6125$

Rubi [A] time = 0.320221, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 21, number of rules used = 13, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.542$, Rules used = {1228, 1095, 419, 1132, 493, 424, 1122, 1180, 1223, 1716, 524, 1212, 537}

$$-\frac{17\sqrt{-x^4+x^2+2x}}{175(5x^2+7)} - \frac{1}{75}\sqrt{-x^4+x^2+2x} + \frac{458}{875}F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle| -2\right) - \frac{97}{525}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle| -2\right) - \frac{1241\Pi\left(-\frac{10}{7}; \sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\right)}{6125}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(2 + x^2 - x^4)^{(3/2)}/(7 + 5*x^2)^2, x]$

[Out] $-(x*\text{Sqrt}[2 + x^2 - x^4])/75 - (17*x*\text{Sqrt}[2 + x^2 - x^4])/(175*(7 + 5*x^2)) - (97*\text{EllipticE}[\text{ArcSin}[x/\text{Sqrt}[2]], -2])/525 + (458*\text{EllipticF}[\text{ArcSin}[x/\text{Sqrt}[2]], -2])/875 - (1241*\text{EllipticPi}[-10/7, \text{ArcSin}[x/\text{Sqrt}[2]], -2])/6125$

Rule 1228

$\text{Int}[(d + (e \cdot x^2)^q) \cdot ((a + (b \cdot x^2 + c \cdot x^4)^p), x_Symbol] :> \text{Module}\{aa, bb, cc\}, \text{Int}[\text{ExpandIntegrand}[1/\text{Sqrt}[aa + bb*x^2 + c*x^4], (d + e*x^2)^q*(aa + bb*x^2 + cc*x^4)^{(p + 1/2)}, x] /. \{aa \to a, bb \to b, cc \to c\}, x] /. \text{FreeQ}\{a, b, c, d, e\}, x] \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& \text{NeQ}[c*d^2 - b*d*e + a*e^2, 0] \&\& \text{ILtQ}[q, 0] \&\& \text{IntegerQ}[p + 1/2]$

Rule 1095

$\text{Int}[1/\text{Sqrt}[(a + (b \cdot x^2 + c \cdot x^4)], x_Symbol] :> \text{With}\{q = \text{Rt}[b^2 - 4*a*c, 2]\}, \text{Dist}[2*\text{Sqrt}[-c], \text{Int}[1/(\text{Sqrt}[b + q + 2*c*x^2]*\text{Sqrt}[-b + q - 2*c*x^2]), x], x] /. \text{FreeQ}\{a, b, c\}, x] \&\& \text{GtQ}[b^2 - 4*a*c, 0] \&\& \text{LtQ}[c, 0]$

Rule 419

$\text{Int}[1/(\text{Sqrt}[(a + (b \cdot x^2 + c \cdot x^4)]*\text{Sqrt}[(c + (d \cdot x^2)]), x_Symbol] :> \text{Simp}[(1*\text{EllipticF}[\text{ArcSin}[\text{Rt}[-(d/c), 2]*x], (b*c)/(a*d)]/(\text{Sqrt}[a]*\text{Sqrt}[c]*\text{Rt}[-(d/c), 2]), x] /. \text{FreeQ}\{a, b, c, d\}, x] \&\& \text{NegQ}[d/c] \&\& \text{GtQ}[c, 0] \&\& \text{GtQ}[a, 0] \&\& !(\text{NegQ}[b/a] \&\& \text{SimplerSqrtQ}[-(b/a), -(d/c)])]$

Rule 1132

$\text{Int}[(x^2/\text{Sqrt}[(a + (b \cdot x^2 + c \cdot x^4)], x_Symbol] :> \text{With}\{q = \text{Rt}[b^2 - 4*a*c, 2]\}, \text{Dist}[2*\text{Sqrt}[-c], \text{Int}[x^2/(\text{Sqrt}[b + q + 2*c*x^2]*\text{Sqrt}[-b + q - 2*c*x^2]), x], x] /. \text{FreeQ}\{a, b, c\}, x] \&\& \text{GtQ}[b^2 - 4*a*c, 0] \&\& \text{LtQ}[c, 0]$

$-b + q - 2*c*x^2$), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]

Rule 493

Int[(x_)^(n_)/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.)*(x_)^(n_)]), x_Symbol] := Dist[1/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n], x], x] - Dist[a/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && (EqQ[n, 2] || EqQ[n, 4]) && !(EqQ[n, 2] && SimplerSqrtQ[-(b/a), -(d/c)])

Rule 424

Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] := Simp[(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]

Rule 1122

Int[((d_.)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := Simp[(d^3*(d*x)^(m-3)*(a + b*x^2 + c*x^4)^(p+1))/(c*(m+4*p+1)), x] - Dist[d^4/(c*(m+4*p+1)), Int[(d*x)^(m-4)*Simp[a*(m-3) + b*(m+2*p-1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m+4*p+1, 0] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])

Rule 1180

Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]

Rule 1223

Int[((d_) + (e_.)*(x_)^2)^(q_)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] := -Simp[(e^2*x*(d + e*x^2)^(q+1)*Sqrt[a + b*x^2 + c*x^4])/(2*d*(q+1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(2*d*(q+1)*(c*d^2 - b*d*e + a*e^2)), Int[((d + e*x^2)^(q+1)*Simp[a*e^2*(2*q+3) + 2*d*(c*d - b*e)*(q+1) - 2*e*(c*d*(q+1) - b*e*(q+2))*x^2 + c*e^2*(2*q+5)*x^4, x])/Sqrt[a + b*x^2 + c*x^4], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && ILtQ[q, -1]

Rule 1716

Int[(P4x)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_Symbol] := With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[P4x, x, 4]}, -Dist[(e^2)^(-1), Int[(C*d - B*e - C*e*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] + Dist[(C*d^2 - B*d*e + A*e^2)/e^2, Int[1/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[P4x, x^2, 2] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0]

Rule 524

Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.)*(x_)^(n_)]), x_Symbol] := Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n], x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && !(EqQ[n, 2] && ((PosQ[b/a] && PosQ

```
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && (!GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 1212

```
Int[1/(((d_) + (e_)*(x_)^2)*Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/((d + e*x^2
)*Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c,
d, e}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rule 537

```
Int[1/(((a_) + (b_)*(x_)^2)*Sqrt[(c_) + (d_)*(x_)^2]*Sqrt[(e_) + (f_)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)]/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] && !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] && !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rubi steps

$$\begin{aligned} \int \frac{(2+x^2-x^4)^{3/2}}{(7+5x^2)^2} dx &= \int \left(\frac{212}{625\sqrt{2+x^2-x^4}} - \frac{24x^2}{125\sqrt{2+x^2-x^4}} + \frac{x^4}{25\sqrt{2+x^2-x^4}} + \frac{1156}{625(7+5x^2)^2\sqrt{2+x^2-x^4}} - \frac{6}{125\sqrt{2+x^2-x^4}} \right) dx \\ &= \frac{1}{25} \int \frac{x^4}{\sqrt{2+x^2-x^4}} dx - \frac{24}{125} \int \frac{x^2}{\sqrt{2+x^2-x^4}} dx + \frac{212}{625} \int \frac{1}{\sqrt{2+x^2-x^4}} dx + \frac{1156}{625} \int \frac{1}{(7+5x^2)^2\sqrt{2+x^2-x^4}} dx \\ &= -\frac{1}{75}x\sqrt{2+x^2-x^4} - \frac{17x\sqrt{2+x^2-x^4}}{175(7+5x^2)} + \frac{17}{4375} \int \frac{118-70x^2-25x^4}{(7+5x^2)\sqrt{2+x^2-x^4}} dx + \frac{1}{75} \int \frac{2+2x^2}{\sqrt{2+x^2-x^4}} dx - \frac{48}{125} \int \frac{1}{\sqrt{2+x^2-x^4}} dx \\ &= -\frac{1}{75}x\sqrt{2+x^2-x^4} - \frac{17x\sqrt{2+x^2-x^4}}{175(7+5x^2)} + \frac{212}{625} F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\right) - 2 - \frac{1292\Pi\left(-\frac{10}{7}; \sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\right)}{4375} \\ &= -\frac{1}{75}x\sqrt{2+x^2-x^4} - \frac{17x\sqrt{2+x^2-x^4}}{175(7+5x^2)} - \frac{62}{375} E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\right) - 2 + \frac{332}{625} F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\right) - 2 \\ &= -\frac{1}{75}x\sqrt{2+x^2-x^4} - \frac{17x\sqrt{2+x^2-x^4}}{175(7+5x^2)} - \frac{62}{375} E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\right) - 2 + \frac{332}{625} F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\right) - 2 \\ &= -\frac{1}{75}x\sqrt{2+x^2-x^4} - \frac{17x\sqrt{2+x^2-x^4}}{175(7+5x^2)} - \frac{97}{525} E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\right) - 2 + \frac{458}{875} F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\right) - 2 \end{aligned}$$

Mathematica [C] time = 0.313982, size = 201, normalized size = 2.16

$$\frac{567i\sqrt{2}(5x^2+7)\sqrt{-x^4+x^2+2}\operatorname{EllipticF}\left(i\sinh^{-1}(x), -\frac{1}{2}\right) + 2450x^7 + 4550x^5 - 11900x^3 - 6790i\sqrt{2}(5x^2+7)\sqrt{-x^4+x^2+2}}{36750}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x^2 - x^4)^(3/2)/(7 + 5*x^2)^2, x]
```

```
[Out] (-14000*x - 11900*x^3 + 4550*x^5 + 2450*x^7 - (6790*I)*Sqrt[2]*(7 + 5*x^2)*
Sqrt[2 + x^2 - x^4]*EllipticE[I*ArcSinh[x], -1/2] + (567*I)*Sqrt[2]*(7 + 5*
x^2)*Sqrt[2 + x^2 - x^4]*EllipticF[I*ArcSinh[x], -1/2] + (26061*I)*Sqrt[2]*
```

$\text{Sqrt}[2 + x^2 - x^4] * \text{EllipticPi}[5/7, I * \text{ArcSinh}[x], -1/2] + (18615 * I) * \text{Sqrt}[2 * x^2 * \text{Sqrt}[2 + x^2 - x^4] * \text{EllipticPi}[5/7, I * \text{ArcSinh}[x], -1/2]] / (36750 * (7 + 5 * x^2) * \text{Sqrt}[2 + x^2 - x^4])$

Maple [B] time = 0.021, size = 180, normalized size = 1.9

$$-\frac{17x}{875x^2 + 1225} \sqrt{-x^4 + x^2 + 2} - \frac{x}{75} \sqrt{-x^4 + x^2 + 2} + \frac{229\sqrt{2}}{875} \sqrt{-2x^2 + 4} \sqrt{x^2 + 1} \text{EllipticF}\left(\frac{x\sqrt{2}}{2}, i\sqrt{2}\right) \frac{1}{\sqrt{-x^4 + x^2 + 2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-x^4+x^2+2)^(3/2)/(5*x^2+7)^2,x)

[Out] -17/175*x*(-x^4+x^2+2)^(1/2)/(5*x^2+7)-1/75*x*(-x^4+x^2+2)^(1/2)+229/875*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticF(1/2*x*2^(1/2),I*2^(1/2))-97/1050*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticE(1/2*x*2^(1/2),I*2^(1/2))-1241/6125*2^(1/2)*(1-1/2*x^2)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticPi(1/2*x*2^(1/2),-10/7,I*2^(1/2))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(-x^4 + x^2 + 2)^{\frac{3}{2}}}{(5x^2 + 7)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^4+x^2+2)^(3/2)/(5*x^2+7)^2,x, algorithm="maxima")

[Out] integrate((-x^4 + x^2 + 2)^(3/2)/(5*x^2 + 7)^2, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(-x^4 + x^2 + 2)^{\frac{3}{2}}}{25x^4 + 70x^2 + 49}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^4+x^2+2)^(3/2)/(5*x^2+7)^2,x, algorithm="fricas")

[Out] integral((-x^4 + x^2 + 2)^(3/2)/(25*x^4 + 70*x^2 + 49), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{-(x^2 - 2)(x^2 + 1)^{\frac{3}{2}}}{(5x^2 + 7)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x**4+x**2+2)**(3/2)/(5*x**2+7)**2,x)

[Out] Integral((-x**2 - 2)*(x**2 + 1)**(3/2)/(5*x**2 + 7)**2, x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(-x^4 + x^2 + 2)^{\frac{3}{2}}}{(5x^2 + 7)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^4+x^2+2)^(3/2)/(5*x^2+7)^2,x, algorithm="giac")

[Out] integrate((-x^4 + x^2 + 2)^(3/2)/(5*x^2 + 7)^2, x)

$$3.331 \quad \int \frac{(2+x^2-x^4)^{3/2}}{(7+5x^2)^3} dx$$

Optimal. Leaf size=102

$$-\frac{1251 \operatorname{EllipticF}\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right), -2\right)}{24500} + \frac{563\sqrt{-x^4+x^2+2x}}{9800(5x^2+7)} - \frac{17\sqrt{-x^4+x^2+2x}}{350(5x^2+7)^2} + \frac{191E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle| -2\right)}{9800} + \frac{9879\Pi\left(-\frac{10}{7}; \sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\right)}{343000}$$

[Out] (-17*x*Sqrt[2 + x^2 - x^4])/(350*(7 + 5*x^2)^2) + (563*x*Sqrt[2 + x^2 - x^4])/(9800*(7 + 5*x^2)) + (191*EllipticE[ArcSin[x/Sqrt[2]], -2])/9800 - (1251*EllipticF[ArcSin[x/Sqrt[2]], -2])/24500 + (9879*EllipticPi[-10/7, ArcSin[x/Sqrt[2]], -2])/343000

Rubi [A] time = 0.497346, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 27, number of rules used = 13, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.542$, Rules used = {1228, 1095, 419, 1132, 493, 424, 1223, 1696, 1716, 1180, 524, 1212, 537}

$$\frac{563\sqrt{-x^4+x^2+2x}}{9800(5x^2+7)} - \frac{17\sqrt{-x^4+x^2+2x}}{350(5x^2+7)^2} - \frac{1251F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle| -2\right)}{24500} + \frac{191E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle| -2\right)}{9800} + \frac{9879\Pi\left(-\frac{10}{7}; \sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\right)}{343000}$$

Antiderivative was successfully verified.

[In] Int[(2 + x^2 - x^4)^(3/2)/(7 + 5*x^2)^3, x]

[Out] (-17*x*Sqrt[2 + x^2 - x^4])/(350*(7 + 5*x^2)^2) + (563*x*Sqrt[2 + x^2 - x^4])/(9800*(7 + 5*x^2)) + (191*EllipticE[ArcSin[x/Sqrt[2]], -2])/9800 - (1251*EllipticF[ArcSin[x/Sqrt[2]], -2])/24500 + (9879*EllipticPi[-10/7, ArcSin[x/Sqrt[2]], -2])/343000

Rule 1228

Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{aa, bb, cc}, Int[ExpandIntegrand[1/Sqrt[aa + bb*x^2 + cc*x^4], (d + e*x^2)^q*(aa + bb*x^2 + cc*x^4)^(p + 1/2), x] /. {aa -> a, bb -> b, cc -> c}, x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, 0] && IntegerQ[p + 1/2]

Rule 1095

Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]

Rule 419

Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> Simp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)]/(Sqrt[a]*Sqrt[c]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0] && !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])

Rule 1132

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] := With[{q =
  Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[x^2/(Sqrt[b + q + 2*c*x^2]*Sqrt[
  -b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] &
& LtQ[c, 0]
```

Rule 493

```
Int[(x_)^(n_)/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.)*(x_)^(n_)]),
x_Symbol] := Dist[1/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n], x], x] - Dist[a
/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d},
x] && NeQ[b*c - a*d, 0] && (EqQ[n, 2] || EqQ[n, 4]) && !(EqQ[n, 2] && Simp
lerSqrtQ[-(b/a), -(d/c)])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] := Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d))]/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 1223

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_
Symbol] := -Simp[(e^2*x*(d + e*x^2)^(q + 1)*Sqrt[a + b*x^2 + c*x^4])/(2*d*(
q + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(2*d*(q + 1)*(c*d^2 - b*d*e +
a*e^2)), Int[((d + e*x^2)^(q + 1)*Simp[a*e^2*(2*q + 3) + 2*d*(c*d - b*e)*(q
+ 1) - 2*e*(c*d*(q + 1) - b*e*(q + 2))*x^2 + c*e^2*(2*q + 5)*x^4, x])/Sqrt
[a + b*x^2 + c*x^4], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c
, 0] && ILtQ[q, -1]
```

Rule 1696

```
Int[((P4x_)*((d_) + (e_.)*(x_)^2)^(q_))/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x
_)^4], x_Symbol] := With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = C
oeff[P4x, x, 4]}, -Simp[((C*d^2 - B*d*e + A*e^2)*x*(d + e*x^2)^(q + 1)*Sqrt
[a + b*x^2 + c*x^4])/(2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(2*
d*(q + 1)*(c*d^2 - b*d*e + a*e^2)), Int[((d + e*x^2)^(q + 1)*Simp[a*d*(C*d
- B*e) + A*(a*e^2*(2*q + 3) + 2*d*(c*d - b*e)*(q + 1)) - 2*((B*d - A*e)*(b
e*(q + 2) - c*d*(q + 1)) - C*d*(b*d + a*e*(q + 1)))*x^2 + c*(C*d^2 - B*d*e
+ A*e^2)*(2*q + 5)*x^4, x])/Sqrt[a + b*x^2 + c*x^4], x], x]] /; FreeQ[{a, b
, c, d, e}, x] && PolyQ[P4x, x^2] && LeQ[Expon[P4x, x], 4] && NeQ[b^2 - 4*a
*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, -1]
```

Rule 1716

```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] := With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[
P4x, x, 4]}, -Dist[(e^2)^(-1), Int[(C*d - B*e - C*e*x^2)/Sqrt[a + b*x^2 + c
*x^4], x], x] + Dist[(C*d^2 - B*d*e + A*e^2)/e^2, Int[1/((d + e*x^2)*Sqrt[a
+ b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[P4x, x^2,
2] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 -
a*e^2, 0]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt
[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}
, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] := Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] && !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && (!GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 1212

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/((d + e*x^2
)*Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x] /; FreeQ[{a, b, c,
d, e}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] := Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] && !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] && !(GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rubi steps

$$\begin{aligned} \int \frac{(2+x^2-x^4)^{3/2}}{(7+5x^2)^3} dx &= \int \left(-\frac{31}{625\sqrt{2+x^2-x^4}} + \frac{x^2}{125\sqrt{2+x^2-x^4}} + \frac{1156}{625(7+5x^2)^3\sqrt{2+x^2-x^4}} - \frac{1299}{625(7+5x^2)^2} \right) dx \\ &= \frac{1}{125} \int \frac{x^2}{\sqrt{2+x^2-x^4}} dx - \frac{31}{625} \int \frac{1}{\sqrt{2+x^2-x^4}} dx + \frac{429}{625} \int \frac{1}{(7+5x^2)\sqrt{2+x^2-x^4}} dx + \frac{1156}{625} \int \frac{1}{(7+5x^2)^2\sqrt{2+x^2-x^4}} dx \\ &= -\frac{17x\sqrt{2+x^2-x^4}}{350(7+5x^2)^2} + \frac{19x\sqrt{2+x^2-x^4}}{175(7+5x^2)} + \frac{17}{8750} \int \frac{186-190x^2+25x^4}{(7+5x^2)^2\sqrt{2+x^2-x^4}} dx - \frac{19}{4375} \int \frac{118-70x^2-25x^4}{(7+5x^2)\sqrt{2+x^2-x^4}} dx \\ &= -\frac{17x\sqrt{2+x^2-x^4}}{350(7+5x^2)^2} + \frac{563x\sqrt{2+x^2-x^4}}{9800(7+5x^2)} - \frac{31}{625} F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right) \middle| -2\right) + \frac{429\Pi\left(-\frac{10}{7}; \sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\right)}{4375} \\ &= -\frac{17x\sqrt{2+x^2-x^4}}{350(7+5x^2)^2} + \frac{563x\sqrt{2+x^2-x^4}}{9800(7+5x^2)} + \frac{1}{125} E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right) \middle| -2\right) - \frac{36}{625} F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right) \middle| -2\right) \\ &= -\frac{17x\sqrt{2+x^2-x^4}}{350(7+5x^2)^2} + \frac{563x\sqrt{2+x^2-x^4}}{9800(7+5x^2)} + \frac{1}{125} E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right) \middle| -2\right) - \frac{36}{625} F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right) \middle| -2\right) \\ &= -\frac{17x\sqrt{2+x^2-x^4}}{350(7+5x^2)^2} + \frac{563x\sqrt{2+x^2-x^4}}{9800(7+5x^2)} + \frac{26}{875} E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right) \middle| -2\right) - \frac{214F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right) \middle| -2\right)}{4375} \\ &= -\frac{17x\sqrt{2+x^2-x^4}}{350(7+5x^2)^2} + \frac{563x\sqrt{2+x^2-x^4}}{9800(7+5x^2)} + \frac{191E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right) \middle| -2\right)}{9800} - \frac{1251F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right) \middle| -2\right)}{24500} \end{aligned}$$

Mathematica [C] time = 0.422615, size = 244, normalized size = 2.39

$$-2541i\sqrt{2}(5x^2+7)^2\sqrt{-x^4+x^2+2}\operatorname{EllipticF}\left(i\sinh^{-1}(x),-\frac{1}{2}\right)-197050x^7-45500x^5+636650x^3+13370i\sqrt{2}(5x^2+7)$$

Antiderivative was successfully verified.

[In] Integrate[(2 + x^2 - x^4)^(3/2)/(7 + 5*x^2)^3,x]

[Out] (485100*x + 636650*x^3 - 45500*x^5 - 197050*x^7 + (13370*I)*Sqrt[2]*(7 + 5*x^2)^2*Sqrt[2 + x^2 - x^4]*EllipticE[I*ArcSinh[x], -1/2] - (2541*I)*Sqrt[2]*(7 + 5*x^2)^2*Sqrt[2 + x^2 - x^4]*EllipticF[I*ArcSinh[x], -1/2] - (484071*I)*Sqrt[2]*Sqrt[2 + x^2 - x^4]*EllipticPi[5/7, I*ArcSinh[x], -1/2] - (691530*I)*Sqrt[2]*x^2*Sqrt[2 + x^2 - x^4]*EllipticPi[5/7, I*ArcSinh[x], -1/2] - (246975*I)*Sqrt[2]*x^4*Sqrt[2 + x^2 - x^4]*EllipticPi[5/7, I*ArcSinh[x], -1/2])/(686000*(7 + 5*x^2)^2*Sqrt[2 + x^2 - x^4])

Maple [A] time = 0.018, size = 189, normalized size = 1.9

$$-\frac{17x}{350(5x^2+7)^2}\sqrt{-x^4+x^2+2}+\frac{563x}{49000x^2+68600}\sqrt{-x^4+x^2+2}-\frac{1251\sqrt{2}}{49000}\sqrt{-2x^2+4}\sqrt{x^2+1}\operatorname{EllipticF}\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-x^4+x^2+2)^(3/2)/(5*x^2+7)^3,x)

[Out] -17/350*x*(-x^4+x^2+2)^(1/2)/(5*x^2+7)^2+563/9800*x*(-x^4+x^2+2)^(1/2)/(5*x^2+7)-1251/49000*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticF(1/2*x*2^(1/2),I*2^(1/2))+191/19600*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticE(1/2*x*2^(1/2),I*2^(1/2))+9879/343000*2^(1/2)*(1-1/2*x^2)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticPi(1/2*x*2^(1/2),-10/7,I*2^(1/2))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(-x^4+x^2+2)^{\frac{3}{2}}}{(5x^2+7)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^4+x^2+2)^(3/2)/(5*x^2+7)^3,x, algorithm="maxima")

[Out] integrate((-x^4 + x^2 + 2)^(3/2)/(5*x^2 + 7)^3, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{(-x^4+x^2+2)^{\frac{3}{2}}}{125x^6+525x^4+735x^2+343},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^4+x^2+2)^(3/2)/(5*x^2+7)^3,x, algorithm="fricas")

[Out] integral((-x^4 + x^2 + 2)^(3/2)/(125*x^6 + 525*x^4 + 735*x^2 + 343), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-\left(x^2 - 2\right)\left(x^2 + 1\right)\right)^{\frac{3}{2}}}{\left(5x^2 + 7\right)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x**4+x**2+2)**(3/2)/(5*x**2+7)**3,x)

[Out] Integral((-x**2 - 2)*(x**2 + 1)**(3/2)/(5*x**2 + 7)**3, x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-x^4 + x^2 + 2\right)^{\frac{3}{2}}}{\left(5x^2 + 7\right)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^4+x^2+2)^(3/2)/(5*x^2+7)^3,x, algorithm="giac")

[Out] integrate((-x^4 + x^2 + 2)^(3/2)/(5*x^2 + 7)^3, x)

$$3.332 \quad \int \frac{(7+5x^2)^3}{\sqrt{2+x^2-x^4}} dx$$

Optimal. Leaf size=65

$$-542\text{EllipticF}\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right), -2\right) - 25\sqrt{-x^4+x^2+2x^3} - \frac{625}{3}\sqrt{-x^4+x^2+2x} + \frac{3905}{3}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle| -2\right)$$

[Out] (-625*x*Sqrt[2 + x^2 - x^4])/3 - 25*x^3*Sqrt[2 + x^2 - x^4] + (3905*EllipticE[ArcSin[x/Sqrt[2]], -2])/3 - 542*EllipticF[ArcSin[x/Sqrt[2]], -2]

Rubi [A] time = 0.0763707, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$, Rules used = {1206, 1679, 1180, 524, 424, 419}

$$-25\sqrt{-x^4+x^2+2x^3} - \frac{625}{3}\sqrt{-x^4+x^2+2x} - 542F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle| -2\right) + \frac{3905}{3}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle| -2\right)$$

Antiderivative was successfully verified.

[In] Int[(7 + 5*x^2)^3/Sqrt[2 + x^2 - x^4], x]

[Out] (-625*x*Sqrt[2 + x^2 - x^4])/3 - 25*x^3*Sqrt[2 + x^2 - x^4] + (3905*EllipticE[ArcSin[x/Sqrt[2]], -2])/3 - 542*EllipticF[ArcSin[x/Sqrt[2]], -2]

Rule 1206

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[(e^q*x^(2*q - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(4*p + 2*q + 1)), x]
+ Dist[1/(c*(4*p + 2*q + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*q + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1]
```

Rule 1679

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> With[{q = Expon[Pq, x^2], e = Coeff[Pq, x^2, Expon[Pq, x^2]]}, Simp[(e*x^(2*q - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(2*q + 4*p + 1)), x]
+ Dist[1/(c*(2*q + 4*p + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*q + 4*p + 1)*Pq - a*e*(2*q - 3)*x^(2*q - 4) - b*e*(2*q + 2*p - 1)*x^(2*q - 2) - c*e*(2*q + 4*p + 1)*x^(2*q), x], x] /;
FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && !LtQ[p, -1]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol]
:> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x] /;
FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.)*(x_)^(n_)]), x_Symbol]
:> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
```

```
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x]
]; FreeQ[{a, b, c, d, e, f, n}, x] && !(EqQ[n, 2] && ((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && (!GtQ[c, 0] || SimplerSqrtQ[-(b/a), -(d/c)]))))))
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] := Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)]/(Sqrt[c]*Rt[-(d/c), 2]), x]
]; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] := S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)]/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x]
]; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0] && !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\begin{aligned} \int \frac{(7+5x^2)^3}{\sqrt{2+x^2-x^4}} dx &= -25x^3\sqrt{2+x^2-x^4} - \frac{1}{5} \int \frac{-1715-4425x^2-3125x^4}{\sqrt{2+x^2-x^4}} dx \\ &= -\frac{625}{3}x\sqrt{2+x^2-x^4} - 25x^3\sqrt{2+x^2-x^4} + \frac{1}{15} \int \frac{11395+19525x^2}{\sqrt{2+x^2-x^4}} dx \\ &= -\frac{625}{3}x\sqrt{2+x^2-x^4} - 25x^3\sqrt{2+x^2-x^4} + \frac{2}{15} \int \frac{11395+19525x^2}{\sqrt{4-2x^2}\sqrt{2+2x^2}} dx \\ &= -\frac{625}{3}x\sqrt{2+x^2-x^4} - 25x^3\sqrt{2+x^2-x^4} - 1084 \int \frac{1}{\sqrt{4-2x^2}\sqrt{2+2x^2}} dx + \frac{3905}{3} \int \frac{\sqrt{2+2x^2}}{\sqrt{4-2x^2}} dx \\ &= -\frac{625}{3}x\sqrt{2+x^2-x^4} - 25x^3\sqrt{2+x^2-x^4} + \frac{3905}{3}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\right) - 2 \end{aligned}$$

Mathematica [C] time = 0.112961, size = 97, normalized size = 1.49

$$\frac{-10089i\sqrt{-2x^4+2x^2+4}\operatorname{EllipticF}\left(i\sinh^{-1}(x), -\frac{1}{2}\right) + 150x^7 + 1100x^5 - 1550x^3 + 7810i\sqrt{-2x^4+2x^2+4}E\left(i\sinh^{-1}(x)\right)}{6\sqrt{-x^4+x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(7 + 5*x^2)^3/Sqrt[2 + x^2 - x^4], x]
```

```
[Out] (-2500*x - 1550*x^3 + 1100*x^5 + 150*x^7 + (7810*I)*Sqrt[4 + 2*x^2 - 2*x^4]
*EllipticE[I*ArcSinh[x], -1/2] - (10089*I)*Sqrt[4 + 2*x^2 - 2*x^4]*Elliptic
F[I*ArcSinh[x], -1/2])/(6*Sqrt[2 + x^2 - x^4])
```

Maple [B] time = 0.018, size = 142, normalized size = 2.2

$$-25x^3\sqrt{-x^4+x^2+2} - \frac{625x}{3}\sqrt{-x^4+x^2+2} + \frac{2279\sqrt{2}}{6}\sqrt{-2x^2+4}\sqrt{x^2+1}\operatorname{EllipticF}\left(\frac{x\sqrt{2}}{2}, i\sqrt{2}\right) \frac{1}{\sqrt{-x^4+x^2+2}} - \frac{3}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((5*x^2+7)^3/(-x^4+x^2+2)^(1/2),x)`

[Out] `-25*x^3*(-x^4+x^2+2)^(1/2)-625/3*x*(-x^4+x^2+2)^(1/2)+2279/6*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticF(1/2*x*2^(1/2),I*2^(1/2))-3905/6*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*(EllipticF(1/2*x*2^(1/2),I*2^(1/2))-EllipticE(1/2*x*2^(1/2),I*2^(1/2)))`

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(5x^2 + 7)^3}{\sqrt{-x^4 + x^2 + 2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((5*x^2+7)^3/(-x^4+x^2+2)^(1/2),x, algorithm="maxima")`

[Out] `integrate((5*x^2 + 7)^3/sqrt(-x^4 + x^2 + 2), x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{(125x^6 + 525x^4 + 735x^2 + 343)\sqrt{-x^4 + x^2 + 2}}{x^4 - x^2 - 2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((5*x^2+7)^3/(-x^4+x^2+2)^(1/2),x, algorithm="fricas")`

[Out] `integral(-(125*x^6 + 525*x^4 + 735*x^2 + 343)*sqrt(-x^4 + x^2 + 2)/(x^4 - x^2 - 2), x)`

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(5x^2 + 7)^3}{\sqrt{-(x^2 - 2)(x^2 + 1)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((5*x**2+7)**3/(-x**4+x**2+2)**(1/2),x)`

[Out] `Integral((5*x**2 + 7)**3/sqrt(-(x**2 - 2)*(x**2 + 1)), x)`

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(5x^2 + 7)^3}{\sqrt{-x^4 + x^2 + 2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.


```
[In] integrate((5*x^2+7)^3/(-x^4+x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((5*x^2 + 7)^3/sqrt(-x^4 + x^2 + 2), x)
```

$$3.333 \quad \int \frac{(7+5x^2)^2}{\sqrt{2+x^2-x^4}} dx$$

Optimal. Leaf size=46

$$-21\text{EllipticF}\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right), -2\right) - \frac{25}{3}\sqrt{-x^4+x^2+2x} + \frac{260}{3}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right) \middle| -2\right)$$

[Out] $(-25*x*\text{Sqrt}[2 + x^2 - x^4])/3 + (260*\text{EllipticE}[\text{ArcSin}[x/\text{Sqrt}[2]], -2])/3 - 21*\text{EllipticF}[\text{ArcSin}[x/\text{Sqrt}[2]], -2]$

Rubi [A] time = 0.0510287, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {1206, 1180, 524, 424, 419}

$$-\frac{25}{3}\sqrt{-x^4+x^2+2x} - 21F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right) \middle| -2\right) + \frac{260}{3}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right) \middle| -2\right)$$

Antiderivative was successfully verified.

[In] $\text{Int}[(7 + 5*x^2)^2/\text{Sqrt}[2 + x^2 - x^4], x]$

[Out] $(-25*x*\text{Sqrt}[2 + x^2 - x^4])/3 + (260*\text{EllipticE}[\text{ArcSin}[x/\text{Sqrt}[2]], -2])/3 - 21*\text{EllipticF}[\text{ArcSin}[x/\text{Sqrt}[2]], -2]$

Rule 1206

$\text{Int}[(d_ + (e_)*(x_)^2)^{(q_)}*((a_ + (b_)*(x_)^2 + (c_)*(x_)^4)^{(p_)}], x_Symbol] \rightarrow \text{Simp}[(e^q*x^{2q-3}*(a + b*x^2 + c*x^4)^{p+1})/(c*(4p+2q+1)), x] + \text{Dist}[1/(c*(4p+2q+1)), \text{Int}[(a + b*x^2 + c*x^4)^p*\text{ExpandToSum}[c*(4p+2q+1)*(d + e*x^2)^q - a*(2q-3)*e^q*x^{2q-4} - b*(2p+2q-1)*e^q*x^{2q-2} - c*(4p+2q+1)*e^q*x^{2q}], x], x] /;$
 $\text{FreeQ}\{a, b, c, d, e, p\}, x \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0] \ \&\& \ \text{NeQ}[c*d^2 - b*d*e + a*e^2, 0] \ \&\& \ \text{IGtQ}[q, 1]$

Rule 1180

$\text{Int}[(d_ + (e_)*(x_)^2)/\text{Sqrt}[(a_ + (b_)*(x_)^2 + (c_)*(x_)^4)], x_Symbol] \rightarrow \text{With}\{q = \text{Rt}[b^2 - 4*a*c, 2]\}, \text{Dist}[2*\text{Sqrt}[-c], \text{Int}[(d + e*x^2)/(\text{Sqrt}[b + q + 2*c*x^2]*\text{Sqrt}[-b + q - 2*c*x^2]), x], x] /;$
 $\text{FreeQ}\{a, b, c, d, e, x\} \ \&\& \ \text{GtQ}[b^2 - 4*a*c, 0] \ \&\& \ \text{LtQ}[c, 0]$

Rule 524

$\text{Int}[(e_ + (f_)*(x_)^{(n_)})/(\text{Sqrt}[(a_ + (b_)*(x_)^{(n_)})]*\text{Sqrt}[(c_ + (d_)*(x_)^{(n_)})]), x_Symbol] \rightarrow \text{Dist}[f/b, \text{Int}[\text{Sqrt}[a + b*x^n]/\text{Sqrt}[c + d*x^n], x], x] + \text{Dist}[(b*e - a*f)/b, \text{Int}[1/(\text{Sqrt}[a + b*x^n]*\text{Sqrt}[c + d*x^n]), x], x] /;$
 $\text{FreeQ}\{a, b, c, d, e, f, n\}, x \ \&\& \ !(\text{EqQ}[n, 2] \ \&\& \ ((\text{PosQ}[b/a] \ \&\& \ \text{PosQ}[d/c]) \ || \ (\text{NegQ}[b/a] \ \&\& \ (\text{PosQ}[d/c] \ || \ (\text{GtQ}[a, 0] \ \&\& \ (!\text{GtQ}[c, 0] \ || \ \text{SimplerSqrtQ}[-(b/a), -(d/c)]))))))$

Rule 424

$\text{Int}[\text{Sqrt}[(a_ + (b_)*(x_)^2)/\text{Sqrt}[(c_ + (d_)*(x_)^2)], x_Symbol] \rightarrow \text{Simp}[(\text{Sqrt}[a]*\text{EllipticE}[\text{ArcSin}[\text{Rt}[-(d/c), 2]*x], (b*c)/(a*d))]/(\text{Sqrt}[c]*\text{Rt}[-(d/c)$

), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]

Rule 419

Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> Simp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0] && !(NegQ[b/a] && SimplersqrtQ[-(b/a), -(d/c)])

Rubi steps

$$\begin{aligned} \int \frac{(7+5x^2)^2}{\sqrt{2+x^2-x^4}} dx &= -\frac{25}{3}x\sqrt{2+x^2-x^4} - \frac{1}{3} \int \frac{-197-260x^2}{\sqrt{2+x^2-x^4}} dx \\ &= -\frac{25}{3}x\sqrt{2+x^2-x^4} - \frac{2}{3} \int \frac{-197-260x^2}{\sqrt{4-2x^2}\sqrt{2+2x^2}} dx \\ &= -\frac{25}{3}x\sqrt{2+x^2-x^4} - 42 \int \frac{1}{\sqrt{4-2x^2}\sqrt{2+2x^2}} dx + \frac{260}{3} \int \frac{\sqrt{2+2x^2}}{\sqrt{4-2x^2}} dx \\ &= -\frac{25}{3}x\sqrt{2+x^2-x^4} + \frac{260}{3}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle| -2\right) - 21F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle| -2\right) \end{aligned}$$

Mathematica [C] time = 0.0965935, size = 92, normalized size = 2.

$$\frac{-717i\sqrt{-2x^4+2x^2+4}\text{EllipticF}\left(i\sinh^{-1}(x), -\frac{1}{2}\right) + 50x^5 - 50x^3 + 520i\sqrt{-2x^4+2x^2+4}E\left(i\sinh^{-1}(x)\middle| -\frac{1}{2}\right) - 100x}{6\sqrt{-x^4+x^2+2}}$$

Antiderivative was successfully verified.

[In] Integrate[(7 + 5*x^2)^2/Sqrt[2 + x^2 - x^4], x]

[Out] (-100*x - 50*x^3 + 50*x^5 + (520*I)*Sqrt[4 + 2*x^2 - 2*x^4]*EllipticE[I*ArcSinh[x], -1/2] - (717*I)*Sqrt[4 + 2*x^2 - 2*x^4]*EllipticF[I*ArcSinh[x], -1/2])/(6*Sqrt[2 + x^2 - x^4])

Maple [B] time = 0.007, size = 125, normalized size = 2.7

$$-\frac{25x}{3}\sqrt{-x^4+x^2+2} + \frac{197\sqrt{2}}{6}\sqrt{-2x^2+4}\sqrt{x^2+1}\text{EllipticF}\left(\frac{x\sqrt{2}}{2}, i\sqrt{2}\right) \frac{1}{\sqrt{-x^4+x^2+2}} - \frac{130\sqrt{2}}{3}\sqrt{-2x^2+4}\sqrt{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((5*x^2+7)^2/(-x^4+x^2+2)^(1/2), x)

[Out] -25/3*x*(-x^4+x^2+2)^(1/2)+197/6*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticF(1/2*x*2^(1/2), I*2^(1/2))-130/3*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*(EllipticF(1/2*x*2^(1/2), I*2^(1/2))-EllipticE(1/2*x*2^(1/2), I*2^(1/2)))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(5x^2 + 7)^2}{\sqrt{-x^4 + x^2 + 2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^2+7)^2/(-x^4+x^2+2)^(1/2),x, algorithm="maxima")

[Out] integrate((5*x^2 + 7)^2/sqrt(-x^4 + x^2 + 2), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{(25x^4 + 70x^2 + 49)\sqrt{-x^4 + x^2 + 2}}{x^4 - x^2 - 2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^2+7)^2/(-x^4+x^2+2)^(1/2),x, algorithm="fricas")

[Out] integral(-(25*x^4 + 70*x^2 + 49)*sqrt(-x^4 + x^2 + 2)/(x^4 - x^2 - 2), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(5x^2 + 7)^2}{\sqrt{-(x^2 - 2)(x^2 + 1)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x**2+7)**2/(-x**4+x**2+2)**(1/2),x)

[Out] Integral((5*x**2 + 7)**2/sqrt(-(x**2 - 2)*(x**2 + 1)), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(5x^2 + 7)^2}{\sqrt{-x^4 + x^2 + 2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^2+7)^2/(-x^4+x^2+2)^(1/2),x, algorithm="giac")

[Out] integrate((5*x^2 + 7)^2/sqrt(-x^4 + x^2 + 2), x)

$$3.334 \quad \int \frac{7+5x^2}{\sqrt{2+x^2-x^4}} dx$$

Optimal. Leaf size=25

$$2\text{EllipticF}\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right), -2\right) + 5E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right) \middle| -2\right)$$

[Out] 5*EllipticE[ArcSin[x/Sqrt[2]], -2] + 2*EllipticF[ArcSin[x/Sqrt[2]], -2]

Rubi [A] time = 0.0379229, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1180, 524, 424, 419}

$$2F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right) \middle| -2\right) + 5E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right) \middle| -2\right)$$

Antiderivative was successfully verified.

[In] Int[(7 + 5*x^2)/Sqrt[2 + x^2 - x^4], x]

[Out] 5*EllipticE[ArcSin[x/Sqrt[2]], -2] + 2*EllipticF[ArcSin[x/Sqrt[2]], -2]

Rule 1180

Int[((d_) + (e_)*(x_)^2)/Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4], x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]

Rule 524

Int[((e_) + (f_)*(x_)^(n_))/(Sqrt[(a_) + (b_)*(x_)^(n_)]*Sqrt[(c_) + (d_)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n], x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && !(EqQ[n, 2] && ((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && (!GtQ[c, 0] || SimplerSqrtQ[-(b/a), -(d/c)]))))))

Rule 424

Int[Sqrt[(a_) + (b_)*(x_)^2]/Sqrt[(c_) + (d_)*(x_)^2], x_Symbol] :> Simp[(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]

Rule 419

Int[1/(Sqrt[(a_) + (b_)*(x_)^2]*Sqrt[(c_) + (d_)*(x_)^2]), x_Symbol] :> Simp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0] && !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])

Rubi steps

$$\begin{aligned} \int \frac{7+5x^2}{\sqrt{2+x^2-x^4}} dx &= 2 \int \frac{7+5x^2}{\sqrt{4-2x^2}\sqrt{2+2x^2}} dx \\ &= 4 \int \frac{1}{\sqrt{4-2x^2}\sqrt{2+2x^2}} dx + 5 \int \frac{\sqrt{2+2x^2}}{\sqrt{4-2x^2}} dx \\ &= 5E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle| -2\right) + 2F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle| -2\right) \end{aligned}$$

Mathematica [C] time = 0.0587153, size = 34, normalized size = 1.36

$$\frac{i\left(10E\left(i\sinh^{-1}(x)\middle| -\frac{1}{2}\right) - 17\text{EllipticF}\left(i\sinh^{-1}(x), -\frac{1}{2}\right)\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Integrate[(7 + 5*x^2)/Sqrt[2 + x^2 - x^4], x]

[Out] (I*(10*EllipticE[I*ArcSinh[x], -1/2] - 17*EllipticF[I*ArcSinh[x], -1/2]))/Sqrt[2]

Maple [B] time = 0.005, size = 110, normalized size = 4.4

$$-\frac{5\sqrt{2}}{2}\sqrt{-2x^2+4}\sqrt{x^2+1}\left(\text{EllipticF}\left(\frac{x\sqrt{2}}{2}, i\sqrt{2}\right) - \text{EllipticE}\left(\frac{x\sqrt{2}}{2}, i\sqrt{2}\right)\right)\frac{1}{\sqrt{-x^4+x^2+2}} + \frac{7\sqrt{2}}{2}\sqrt{-2x^2+4}\sqrt{x^2+1}E$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((5*x^2+7)/(-x^4+x^2+2)^(1/2), x)

[Out] -5/2*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*(EllipticF(1/2*x*2^(1/2), I*2^(1/2))-EllipticE(1/2*x*2^(1/2), I*2^(1/2)))+7/2*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticF(1/2*x*2^(1/2), I*2^(1/2))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{5x^2+7}{\sqrt{-x^4+x^2+2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^2+7)/(-x^4+x^2+2)^(1/2), x, algorithm="maxima")

[Out] integrate((5*x^2 + 7)/sqrt(-x^4 + x^2 + 2), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-x^4+x^2+2}(5x^2+7)}{x^4-x^2-2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)/(-x^4+x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-x^4 + x^2 + 2)*(5*x^2 + 7)/(x^4 - x^2 - 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{5x^2 + 7}{\sqrt{-(x^2 - 2)(x^2 + 1)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+7)/(-x**4+x**2+2)**(1/2),x)
```

```
[Out] Integral((5*x**2 + 7)/sqrt(-(x**2 - 2)*(x**2 + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{5x^2 + 7}{\sqrt{-x^4 + x^2 + 2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)/(-x^4+x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((5*x^2 + 7)/sqrt(-x^4 + x^2 + 2), x)
```

$$3.335 \quad \int \frac{1}{\sqrt{2+x^2-x^4}} dx$$

Optimal. Leaf size=10

$$\text{EllipticF}\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right), -2\right)$$

[Out] EllipticF[ArcSin[x/Sqrt[2]], -2]

Rubi [A] time = 0.0105507, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1095, 419}

$$F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle| -2\right)$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[2 + x^2 - x^4], x]

[Out] EllipticF[ArcSin[x/Sqrt[2]], -2]

Rule 1095

Int[1/Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4], x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]

Rule 419

Int[1/(Sqrt[(a_) + (b_)*(x_)^2]*Sqrt[(c_) + (d_)*(x_)^2]), x_Symbol] := Simp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)]/(Sqrt[a]*Sqrt[c]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0] && !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{2+x^2-x^4}} dx &= 2 \int \frac{1}{\sqrt{4-2x^2}\sqrt{2+2x^2}} dx \\ &= F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle| -2\right) \end{aligned}$$

Mathematica [C] time = 0.0152104, size = 19, normalized size = 1.9

$$\frac{i\text{EllipticF}\left(i\sinh^{-1}(x), -\frac{1}{2}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[2 + x^2 - x^4], x]

[Out] $((-I)*\text{EllipticF}[I*\text{ArcSinh}[x], -1/2])/ \text{Sqrt}[2]$

Maple [B] time = 0.004, size = 47, normalized size = 4.7

$$\frac{\sqrt{2}}{2} \sqrt{-2x^2 + 4} \sqrt{x^2 + 1} \text{EllipticF}\left(\frac{x\sqrt{2}}{2}, i\sqrt{2}\right) \frac{1}{\sqrt{-x^4 + x^2 + 2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(-x^4+x^2+2)^(1/2), x)`

[Out] $1/2*2^{(1/2)}*(-2*x^2+4)^{(1/2)}*(x^2+1)^{(1/2)}/(-x^4+x^2+2)^{(1/2)}*\text{EllipticF}(1/2*x*2^{(1/2)}, I*2^{(1/2)})$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-x^4 + x^2 + 2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(-x^4+x^2+2)^(1/2), x, algorithm="maxima")`

[Out] `integrate(1/sqrt(-x^4 + x^2 + 2), x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-x^4 + x^2 + 2}}{x^4 - x^2 - 2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(-x^4+x^2+2)^(1/2), x, algorithm="fricas")`

[Out] `integral(-sqrt(-x^4 + x^2 + 2)/(x^4 - x^2 - 2), x)`

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-x^4 + x^2 + 2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(-x**4+x**2+2)**(1/2), x)`

[Out] `Integral(1/sqrt(-x**4 + x**2 + 2), x)`

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-x^4 + x^2 + 2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^4+x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-x^4 + x^2 + 2), x)
```

$$3.336 \quad \int \frac{1}{(7+5x^2)\sqrt{2+x^2-x^4}} dx$$

Optimal. Leaf size=17

$$\frac{1}{7}\Pi\left(-\frac{10}{7}; \sin^{-1}\left(\frac{x}{\sqrt{2}}\right) \middle| -2\right)$$

[Out] EllipticPi[-10/7, ArcSin[x/Sqrt[2]], -2]/7

Rubi [A] time = 0.0330613, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1212, 537}

$$\frac{1}{7}\Pi\left(-\frac{10}{7}; \sin^{-1}\left(\frac{x}{\sqrt{2}}\right) \middle| -2\right)$$

Antiderivative was successfully verified.

[In] Int[1/((7 + 5*x^2)*Sqrt[2 + x^2 - x^4]),x]

[Out] EllipticPi[-10/7, ArcSin[x/Sqrt[2]], -2]/7

Rule 1212

Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/((d + e*x^2)*Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]

Rule 537

Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d, e, f}, x] && !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] && !(!GtQ[f/e, 0] && SimplerSqrtQ[-(f/e), -(d/c)])

Rubi steps

$$\begin{aligned} \int \frac{1}{(7+5x^2)\sqrt{2+x^2-x^4}} dx &= 2 \int \frac{1}{\sqrt{4-2x^2}\sqrt{2+2x^2}(7+5x^2)} dx \\ &= \frac{1}{7}\Pi\left(-\frac{10}{7}; \sin^{-1}\left(\frac{x}{\sqrt{2}}\right) \middle| -2\right) \end{aligned}$$

Mathematica [C] time = 0.0972269, size = 24, normalized size = 1.41

$$-\frac{i\Pi\left(\frac{5}{7}; i \sinh^{-1}(x) \middle| -\frac{1}{2}\right)}{7\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/((7 + 5*x^2)*Sqrt[2 + x^2 - x^4]),x]

[Out] ((-I/7)*EllipticPi[5/7, I*ArcSinh[x], -1/2])/Sqrt[2]

Maple [B] time = 0.01, size = 48, normalized size = 2.8

$$\frac{\sqrt{2}}{7} \sqrt{1 - \frac{x^2}{2}} \sqrt{x^2 + 1} \text{EllipticPi} \left(\frac{x\sqrt{2}}{2}, -\frac{10}{7}, i\sqrt{2} \right) \frac{1}{\sqrt{-x^4 + x^2 + 2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(5*x^2+7)/(-x^4+x^2+2)^(1/2),x)

[Out] 1/7*2^(1/2)*(1-1/2*x^2)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticPi(1/2*x*2^(1/2),-10/7,I*2^(1/2))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-x^4 + x^2 + 2}(5x^2 + 7)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(5*x^2+7)/(-x^4+x^2+2)^(1/2),x, algorithm="maxima")

[Out] integrate(1/(sqrt(-x^4 + x^2 + 2)*(5*x^2 + 7)), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral} \left(-\frac{\sqrt{-x^4 + x^2 + 2}}{5x^6 + 2x^4 - 17x^2 - 14}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(5*x^2+7)/(-x^4+x^2+2)^(1/2),x, algorithm="fricas")

[Out] integral(-sqrt(-x^4 + x^2 + 2)/(5*x^6 + 2*x^4 - 17*x^2 - 14), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-(x^2 - 2)(x^2 + 1)(5x^2 + 7)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(5*x**2+7)/(-x**4+x**2+2)**(1/2),x)

[Out] Integral(1/(sqrt(-(x**2 - 2)*(x**2 + 1))*(5*x**2 + 7)), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-x^4 + x^2 + 2}(5x^2 + 7)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(5*x^2+7)/(-x^4+x^2+2)^(1/2),x, algorithm="giac")

[Out] integrate(1/(sqrt(-x^4 + x^2 + 2)*(5*x^2 + 7)), x)

$$3.337 \quad \int \frac{1}{(7+5x^2)^2 \sqrt{2+x^2-x^4}} dx$$

Optimal. Leaf size=74

$$-\frac{1}{238} \text{EllipticF}\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right), -2\right) - \frac{25\sqrt{-x^4+x^2+2x}}{476(5x^2+7)} - \frac{5}{476} E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right) \middle| -2\right) + \frac{167\pi\left(-\frac{10}{7}; \sin^{-1}\left(\frac{x}{\sqrt{2}}\right) \middle| -2\right)}{3332}$$

[Out] (-25*x*Sqrt[2 + x^2 - x^4])/(476*(7 + 5*x^2)) - (5*EllipticE[ArcSin[x/Sqrt[2]], -2])/476 - EllipticF[ArcSin[x/Sqrt[2]], -2]/238 + (167*EllipticPi[-10/7, ArcSin[x/Sqrt[2]], -2])/3332

Rubi [A] time = 0.133883, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1223, 1716, 1180, 524, 424, 419, 1212, 537}

$$-\frac{25\sqrt{-x^4+x^2+2x}}{476(5x^2+7)} - \frac{1}{238} F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right) \middle| -2\right) - \frac{5}{476} E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right) \middle| -2\right) + \frac{167\pi\left(-\frac{10}{7}; \sin^{-1}\left(\frac{x}{\sqrt{2}}\right) \middle| -2\right)}{3332}$$

Antiderivative was successfully verified.

[In] Int[1/((7 + 5*x^2)^2*Sqrt[2 + x^2 - x^4]),x]

[Out] (-25*x*Sqrt[2 + x^2 - x^4])/(476*(7 + 5*x^2)) - (5*EllipticE[ArcSin[x/Sqrt[2]], -2])/476 - EllipticF[ArcSin[x/Sqrt[2]], -2]/238 + (167*EllipticPi[-10/7, ArcSin[x/Sqrt[2]], -2])/3332

Rule 1223

Int[((d_) + (e_.)*(x_)^2)^(q_)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] := -Simp[(e^2*x*(d + e*x^2)^(q + 1)*Sqrt[a + b*x^2 + c*x^4])/(2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2)), Int[((d + e*x^2)^(q + 1)*Simp[a*e^2*(2*q + 3) + 2*d*(c*d - b*e)*(q + 1) - 2*e*(c*d*(q + 1) - b*e*(q + 2))*x^2 + c*e^2*(2*q + 5)*x^4, x])/Sqrt[a + b*x^2 + c*x^4], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && ILtQ[q, -1]

Rule 1716

Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_Symbol] := With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[P4x, x, 4]}, -Dist[(e^2)^(-1), Int[(C*d - B*e - C*e*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] + Dist[(C*d^2 - B*d*e + A*e^2)/e^2, Int[1/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[P4x, x^2, 2] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0]

Rule 1180

Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]

Rule 524

```
Int[((e_) + (f_)*(x_)^(n_))/(Sqrt[(a_) + (b_)*(x_)^(n_)]*Sqrt[(c_) + (d_)
]*(x_)^(n_)], x_Symbol] := Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] && !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && (!GtQ[c, 0] || Simplr
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 424

```
Int[Sqrt[(a_) + (b_)*(x_)^2]/Sqrt[(c_) + (d_)*(x_)^2], x_Symbol] := Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_)*(x_)^2]*Sqrt[(c_) + (d_)*(x_)^2]), x_Symbol] := S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] && !(NegQ[b/a] && SimplrSqrtQ[-(b/a), -(d/c)])
```

Rule 1212

```
Int[1/(((d_) + (e_)*(x_)^2)*Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4]), x_S
ymbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/((d + e*x^2
)*Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x] /; FreeQ[{a, b, c,
d, e}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rule 537

```
Int[1/(((a_) + (b_)*(x_)^2)*Sqrt[(c_) + (d_)*(x_)^2]*Sqrt[(e_) + (f_)*(x
_)^2]), x_Symbol] := Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] && !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] && !( !GtQ[f/e, 0]
&& SimplrSqrtQ[-(f/e), -(d/c)])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(7+5x^2)^2 \sqrt{2+x^2-x^4}} dx &= -\frac{25x\sqrt{2+x^2-x^4}}{476(7+5x^2)} + \frac{1}{476} \int \frac{118-70x^2-25x^4}{(7+5x^2)\sqrt{2+x^2-x^4}} dx \\
&= -\frac{25x\sqrt{2+x^2-x^4}}{476(7+5x^2)} - \frac{\int \frac{175+125x^2}{\sqrt{2+x^2-x^4}} dx}{11900} + \frac{167}{476} \int \frac{1}{(7+5x^2)\sqrt{2+x^2-x^4}} dx \\
&= -\frac{25x\sqrt{2+x^2-x^4}}{476(7+5x^2)} - \frac{\int \frac{175+125x^2}{\sqrt{4-2x^2}\sqrt{2+2x^2}} dx}{5950} + \frac{167}{238} \int \frac{1}{\sqrt{4-2x^2}\sqrt{2+2x^2}(7+5x^2)} dx \\
&= -\frac{25x\sqrt{2+x^2-x^4}}{476(7+5x^2)} + \frac{167\text{Pi}\left(-\frac{10}{7}; \sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\right) - 2}{3332} - \frac{1}{119} \int \frac{1}{\sqrt{4-2x^2}\sqrt{2+2x^2}} dx \\
&= -\frac{25x\sqrt{2+x^2-x^4}}{476(7+5x^2)} - \frac{5}{476} E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\right) - 2 - \frac{1}{238} F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\right) - 2 + \frac{167\text{Pi}}{3332}
\end{aligned}$$

Mathematica [C] time = 0.278775, size = 196, normalized size = 2.65

$$\frac{119i\sqrt{2}(5x^2+7)\sqrt{-x^4+x^2+2}\operatorname{EllipticF}\left(i\sinh^{-1}(x),-\frac{1}{2}\right)+350x^5-350x^3-70i\sqrt{2}(5x^2+7)\sqrt{-x^4+x^2+2}E\left(i\sinh^{-1}(x),-\frac{1}{2}\right)}{6664(5x^2+7)\sqrt{-x^4+x^2+2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/((7 + 5*x^2)^2*Sqrt[2 + x^2 - x^4]),x]

[Out] (-700*x - 350*x^3 + 350*x^5 - (70*I)*Sqrt[2]*(7 + 5*x^2)*Sqrt[2 + x^2 - x^4])*EllipticE[I*ArcSinh[x], -1/2] + (119*I)*Sqrt[2]*(7 + 5*x^2)*Sqrt[2 + x^2 - x^4]*EllipticF[I*ArcSinh[x], -1/2] - (1169*I)*Sqrt[2]*Sqrt[2 + x^2 - x^4]*EllipticPi[5/7, I*ArcSinh[x], -1/2] - (835*I)*Sqrt[2]*x^2*Sqrt[2 + x^2 - x^4]*EllipticPi[5/7, I*ArcSinh[x], -1/2]/(6664*(7 + 5*x^2)*Sqrt[2 + x^2 - x^4])

Maple [B] time = 0.016, size = 165, normalized size = 2.2

$$-\frac{25x}{2380x^2+3332}\sqrt{-x^4+x^2+2}-\frac{\sqrt{2}}{476}\sqrt{-2x^2+4}\sqrt{x^2+1}\operatorname{EllipticF}\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)\frac{1}{\sqrt{-x^4+x^2+2}}-\frac{5\sqrt{2}}{952}\sqrt{-2x^2+4}\sqrt{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(5*x^2+7)^2/(-x^4+x^2+2)^(1/2),x)

[Out] -25/476*x*(-x^4+x^2+2)^(1/2)/(5*x^2+7)-1/476*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticF(1/2*x*2^(1/2),I*2^(1/2))-5/952*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticE(1/2*x*2^(1/2),I*2^(1/2))+167/3332*2^(1/2)*(1-1/2*x^2)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticPi(1/2*x*2^(1/2),-10/7,I*2^(1/2))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-x^4+x^2+2}(5x^2+7)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(5*x^2+7)^2/(-x^4+x^2+2)^(1/2),x, algorithm="maxima")

[Out] integrate(1/(sqrt(-x^4 + x^2 + 2)*(5*x^2 + 7)^2), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(-\frac{\sqrt{-x^4+x^2+2}}{25x^8+45x^6-71x^4-189x^2-98},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(5*x^2+7)^2/(-x^4+x^2+2)^(1/2),x, algorithm="fricas")

[Out] integral(-sqrt(-x⁴ + x² + 2)/(25*x⁸ + 45*x⁶ - 71*x⁴ - 189*x² - 98), x
)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-(x^2 - 2)(x^2 + 1)}(5x^2 + 7)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(5*x**2+7)**2/(-x**4+x**2+2)**(1/2), x)

[Out] Integral(1/(sqrt(-(x**2 - 2)*(x**2 + 1))*(5*x**2 + 7)**2), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-x^4 + x^2 + 2}(5x^2 + 7)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(5*x^2+7)^2/(-x^4+x^2+2)^(1/2), x, algorithm="giac")

[Out] integrate(1/(sqrt(-x⁴ + x² + 2)*(5*x² + 7)²), x)

$$3.338 \quad \int \frac{1}{(7+5x^2)^3 \sqrt{2+x^2-x^4}} dx$$

Optimal. Leaf size=102

$$-\frac{263 \operatorname{EllipticF}\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right), -2\right)}{226576} - \frac{12525 \sqrt{-x^4 + x^2 + 2x}}{453152(5x^2 + 7)} - \frac{25 \sqrt{-x^4 + x^2 + 2x}}{952(5x^2 + 7)^2} - \frac{2505 E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right) \middle| -2\right)}{453152} + \frac{58915 \Pi\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right) \middle| -2\right)}{3172064}$$

[Out] (-25*x*Sqrt[2 + x^2 - x^4])/(952*(7 + 5*x^2)^2) - (12525*x*Sqrt[2 + x^2 - x^4])/(453152*(7 + 5*x^2)) - (2505*EllipticE[ArcSin[x/Sqrt[2]], -2])/453152 - (263*EllipticF[ArcSin[x/Sqrt[2]], -2])/226576 + (58915*EllipticPi[-10/7, ArcSin[x/Sqrt[2]], -2])/3172064

Rubi [A] time = 0.191768, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {1223, 1696, 1716, 1180, 524, 424, 419, 1212, 537}

$$-\frac{12525 \sqrt{-x^4 + x^2 + 2x}}{453152(5x^2 + 7)} - \frac{25 \sqrt{-x^4 + x^2 + 2x}}{952(5x^2 + 7)^2} - \frac{263 F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right) \middle| -2\right)}{226576} - \frac{2505 E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right) \middle| -2\right)}{453152} + \frac{58915 \Pi\left(-\frac{10}{7}; \sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\right)}{3172064}$$

Antiderivative was successfully verified.

[In] Int[1/((7 + 5*x^2)^3*Sqrt[2 + x^2 - x^4]),x]

[Out] (-25*x*Sqrt[2 + x^2 - x^4])/(952*(7 + 5*x^2)^2) - (12525*x*Sqrt[2 + x^2 - x^4])/(453152*(7 + 5*x^2)) - (2505*EllipticE[ArcSin[x/Sqrt[2]], -2])/453152 - (263*EllipticF[ArcSin[x/Sqrt[2]], -2])/226576 + (58915*EllipticPi[-10/7, ArcSin[x/Sqrt[2]], -2])/3172064

Rule 1223

Int[((d_) + (e_)*(x_)^2)^(q_)/Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4], x_Symbol] := -Simp[(e^2*x*(d + e*x^2)^(q + 1)*Sqrt[a + b*x^2 + c*x^4])/(2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2)), Int[((d + e*x^2)^(q + 1)*Simp[a*e^2*(2*q + 3) + 2*d*(c*d - b*e)*(q + 1) - 2*e*(c*d*(q + 1) - b*e*(q + 2))*x^2 + c*e^2*(2*q + 5)*x^4, x])/Sqrt[a + b*x^2 + c*x^4], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && ILtQ[q, -1]

Rule 1696

Int[((P4x_)*((d_) + (e_)*(x_)^2)^(q_))/Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4], x_Symbol] := With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[P4x, x, 4]}, -Simp[((C*d^2 - B*d*e + A*e^2)*x*(d + e*x^2)^(q + 1)*Sqrt[a + b*x^2 + c*x^4])/(2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2)), Int[((d + e*x^2)^(q + 1)*Simp[a*d*(C*d - B*e) + A*(a*e^2*(2*q + 3) + 2*d*(c*d - b*e)*(q + 1)) - 2*((B*d - A*e)*(b*e*(q + 2) - c*d*(q + 1)) - C*d*(b*d + a*e*(q + 1)))*x^2 + c*(C*d^2 - B*d*e + A*e^2)*(2*q + 5)*x^4, x])/Sqrt[a + b*x^2 + c*x^4], x], x] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[P4x, x^2] && LeQ[Expon[P4x, x], 4] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, -1]

Rule 1716

```
Int[(P4x_)/(((d_) + (e_)*(x_)^2)*Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4])
, x_Symbol] := With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[
P4x, x, 4]}, -Dist[(e^2)^(-1), Int[(C*d - B*e - C*e*x^2)/Sqrt[a + b*x^2 + c
*x^4], x], x] + Dist[(C*d^2 - B*d*e + A*e^2)/e^2, Int[1/((d + e*x^2)*Sqrt[a
+ b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[P4x, x^2,
2] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 -
a*e^2, 0]
```

Rule 1180

```
Int[((d_) + (e_)*(x_)^2)/Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4], x_Symbo
l] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt
[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}
, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rule 524

```
Int[((e_) + (f_)*(x_)^(n_))/(Sqrt[(a_) + (b_)*(x_)^(n_)]*Sqrt[(c_) + (d_
)*(x_)^(n_)]), x_Symbol] := Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] && !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && (!GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 424

```
Int[Sqrt[(a_) + (b_)*(x_)^2]/Sqrt[(c_) + (d_)*(x_)^2], x_Symbol] := Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)]/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_)*(x_)^2]*Sqrt[(c_) + (d_)*(x_)^2]), x_Symbol] := S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)]/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] && !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 1212

```
Int[1/(((d_) + (e_)*(x_)^2)*Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4]), x_S
ymbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/((d + e*x^2
)*Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c,
d, e}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rule 537

```
Int[1/(((a_) + (b_)*(x_)^2)*Sqrt[(c_) + (d_)*(x_)^2]*Sqrt[(e_) + (f_)*(x
_)^2]), x_Symbol] := Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)]/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] && !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] && !(GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(7+5x^2)^3 \sqrt{2+x^2-x^4}} dx &= -\frac{25x\sqrt{2+x^2-x^4}}{952(7+5x^2)^2} + \frac{1}{952} \int \frac{186-190x^2+25x^4}{(7+5x^2)^2 \sqrt{2+x^2-x^4}} dx \\
&= -\frac{25x\sqrt{2+x^2-x^4}}{952(7+5x^2)^2} - \frac{12525x\sqrt{2+x^2-x^4}}{453152(7+5x^2)} + \frac{\int \frac{37698-32690x^2-12525x^4}{(7+5x^2)\sqrt{2+x^2-x^4}} dx}{453152} \\
&= -\frac{25x\sqrt{2+x^2-x^4}}{952(7+5x^2)^2} - \frac{12525x\sqrt{2+x^2-x^4}}{453152(7+5x^2)} - \frac{\int \frac{75775+62625x^2}{\sqrt{2+x^2-x^4}} dx}{11328800} + \frac{58915 \int \frac{1}{(7+5x^2)\sqrt{2+x^2-x^4}} dx}{453152} \\
&= -\frac{25x\sqrt{2+x^2-x^4}}{952(7+5x^2)^2} - \frac{12525x\sqrt{2+x^2-x^4}}{453152(7+5x^2)} - \frac{\int \frac{75775+62625x^2}{\sqrt{4-2x^2}\sqrt{2+2x^2}} dx}{5664400} + \frac{58915 \int \frac{1}{\sqrt{4-2x^2}\sqrt{2+2x^2}} dx}{226576} \\
&= -\frac{25x\sqrt{2+x^2-x^4}}{952(7+5x^2)^2} - \frac{12525x\sqrt{2+x^2-x^4}}{453152(7+5x^2)} + \frac{58915\pi \left(-\frac{10}{7}; \sin^{-1}\left(\frac{x}{\sqrt{2}}\right) \middle| -2\right)}{3172064} - \frac{263 \int \frac{1}{\sqrt{4-2x^2}\sqrt{2+2x^2}} dx}{226576} \\
&= -\frac{25x\sqrt{2+x^2-x^4}}{952(7+5x^2)^2} - \frac{12525x\sqrt{2+x^2-x^4}}{453152(7+5x^2)} - \frac{2505E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right) \middle| -2\right)}{453152} - \frac{263F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right) \middle| -2\right)}{226576}
\end{aligned}$$

Mathematica [C] time = 0.405084, size = 108, normalized size = 1.06

$$\frac{56287i\sqrt{2}\text{EllipticF}\left(i\sinh^{-1}(x), -\frac{1}{2}\right) + \frac{350x(2505x^6+1478x^4-8993x^2-7966)}{(5x^2+7)^2\sqrt{-x^4+x^2+2}} - 35070i\sqrt{2}E\left(i\sinh^{-1}(x) \middle| -\frac{1}{2}\right) - 58915i\sqrt{2}\pi\left(\frac{5}{7}; i\right)}{6344128}$$

Antiderivative was successfully verified.

[In] Integrate[1/((7 + 5*x^2)^3*Sqrt[2 + x^2 - x^4]),x]

[Out] ((350*x*(-7966 - 8993*x^2 + 1478*x^4 + 2505*x^6))/((7 + 5*x^2)^2*Sqrt[2 + x^2 - x^4]) - (35070*I)*Sqrt[2]*EllipticE[I*ArcSinh[x], -1/2] + (56287*I)*Sqrt[2]*EllipticF[I*ArcSinh[x], -1/2] - (58915*I)*Sqrt[2]*EllipticPi[5/7, I*ArcSinh[x], -1/2])/6344128

Maple [A] time = 0.019, size = 189, normalized size = 1.9

$$-\frac{25x}{952(5x^2+7)^2}\sqrt{-x^4+x^2+2} - \frac{12525x}{2265760x^2+3172064}\sqrt{-x^4+x^2+2} - \frac{263\sqrt{2}}{453152}\sqrt{-2x^2+4}\sqrt{x^2+1}\text{EllipticF}\left(\frac{x\sqrt{2}}{2}, i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(5*x^2+7)^3/(-x^4+x^2+2)^(1/2),x)

[Out] -25/952*x*(-x^4+x^2+2)^(1/2)/(5*x^2+7)^2-12525/453152*x*(-x^4+x^2+2)^(1/2)/(5*x^2+7)-263/453152*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticF(1/2*x*2^(1/2),I*2^(1/2))-2505/906304*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticE(1/2*x*2^(1/2),I*2^(1/2))+58915/3172064*2^(1/2)*(1-1/2*x^2)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticPi(1/2*x*2^(1/2),-10/7,I*2^(1/2))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-x^4 + x^2 + 2}(5x^2 + 7)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(5*x^2+7)^3/(-x^4+x^2+2)^(1/2),x, algorithm="maxima")

[Out] integrate(1/(sqrt(-x^4 + x^2 + 2)*(5*x^2 + 7)^3), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-x^4 + x^2 + 2}}{125x^{10} + 400x^8 - 40x^6 - 1442x^4 - 1813x^2 - 686}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(5*x^2+7)^3/(-x^4+x^2+2)^(1/2),x, algorithm="fricas")

[Out] integral(-sqrt(-x^4 + x^2 + 2)/(125*x^10 + 400*x^8 - 40*x^6 - 1442*x^4 - 1813*x^2 - 686), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-(x^2 - 2)(x^2 + 1)}(5x^2 + 7)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(5*x**2+7)**3/(-x**4+x**2+2)**(1/2),x)

[Out] Integral(1/(sqrt(-(x**2 - 2)*(x**2 + 1))*(5*x**2 + 7)**3), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-x^4 + x^2 + 2}(5x^2 + 7)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(5*x^2+7)^3/(-x^4+x^2+2)^(1/2),x, algorithm="giac")

[Out] integrate(1/(sqrt(-x^4 + x^2 + 2)*(5*x^2 + 7)^3), x)

$$3.339 \quad \int \frac{(7+5x^2)^5}{(2+x^2-x^4)^{3/2}} dx$$

Optimal. Leaf size=93

$$\frac{627857}{6} \text{EllipticF}\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right), -2\right) + 625\sqrt{-x^4+x^2+2}x^3 + \frac{27500}{3}\sqrt{-x^4+x^2+2}x + \frac{(1419793x^2+1419985)x}{18\sqrt{-x^4+x^2+2}} - \frac{3482293}{18}$$

[Out] (x*(1419985 + 1419793*x^2))/(18*Sqrt[2 + x^2 - x^4]) + (27500*x*Sqrt[2 + x^2 - x^4])/3 + 625*x^3*Sqrt[2 + x^2 - x^4] - (3482293*EllipticE[ArcSin[x/Sqrt[2]], -2])/18 + (627857*EllipticF[ArcSin[x/Sqrt[2]], -2])/6

Rubi [A] time = 0.100598, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$, Rules used = {1205, 1679, 1180, 524, 424, 419}

$$625\sqrt{-x^4+x^2+2}x^3 + \frac{27500}{3}\sqrt{-x^4+x^2+2}x + \frac{(1419793x^2+1419985)x}{18\sqrt{-x^4+x^2+2}} + \frac{627857}{6}F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle| -2\right) - \frac{3482293}{18}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle| -2\right)$$

Antiderivative was successfully verified.

[In] Int[(7 + 5*x^2)^5/(2 + x^2 - x^4)^(3/2),x]

[Out] (x*(1419985 + 1419793*x^2))/(18*Sqrt[2 + x^2 - x^4]) + (27500*x*Sqrt[2 + x^2 - x^4])/3 + 625*x^3*Sqrt[2 + x^2 - x^4] - (3482293*EllipticE[ArcSin[x/Sqrt[2]], -2])/18 + (627857*EllipticF[ArcSin[x/Sqrt[2]], -2])/6

Rule 1205

Int[((d_) + (e_)*(x_)^2)^(q_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] :> With[{f = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 + c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*g - f*(b^2 - 2*a*c) - c*(b*f - 2*a*g)*x^2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[(d + e*x^2)^q, a + b*x^2 + c*x^4, x] + b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g + c*(4*p + 7)*(b*f - 2*a*g)*x^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1] && LtQ[p, -1]

Rule 1679

Int[(Pq)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] :> With[{q = Expon[Pq, x^2], e = Coeff[Pq, x^2, Expon[Pq, x^2]]}, Simp[(e*x^(2*q - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(2*q + 4*p + 1)), x] + Dist[1/(c*(2*q + 4*p + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*q + 4*p + 1)*Pq - a*e*(2*q - 3)*x^(2*q - 4) - b*e*(2*q + 2*p - 1)*x^(2*q - 2) - c*e*(2*q + 4*p + 1)*x^(2*q), x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && !LtQ[p, -1]

Rule 1180

Int[((d_) + (e_)*(x_)^2)/Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4], x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}

, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)], x_Symbol] := Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] && !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && (!GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] := Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] := S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] && !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\begin{aligned} \int \frac{(7 + 5x^2)^5}{(2 + x^2 - x^4)^{3/2}} dx &= \frac{x(1419985 + 1419793x^2)}{18\sqrt{2 + x^2 - x^4}} - \frac{1}{18} \int \frac{1268722 + 3084793x^2 + 450000x^4 + 56250x^6}{\sqrt{2 + x^2 - x^4}} dx \\ &= \frac{x(1419985 + 1419793x^2)}{18\sqrt{2 + x^2 - x^4}} + 625x^3\sqrt{2 + x^2 - x^4} + \frac{1}{90} \int \frac{-6343610 - 15761465x^2 - 2475000x^4}{\sqrt{2 + x^2 - x^4}} dx \\ &= \frac{x(1419985 + 1419793x^2)}{18\sqrt{2 + x^2 - x^4}} + \frac{27500}{3}x\sqrt{2 + x^2 - x^4} + 625x^3\sqrt{2 + x^2 - x^4} - \frac{1}{270} \int \frac{23980830 - 15761465x^2 - 2475000x^4}{\sqrt{2 + x^2 - x^4}} dx \\ &= \frac{x(1419985 + 1419793x^2)}{18\sqrt{2 + x^2 - x^4}} + \frac{27500}{3}x\sqrt{2 + x^2 - x^4} + 625x^3\sqrt{2 + x^2 - x^4} - \frac{1}{135} \int \frac{23980830 - 15761465x^2 - 2475000x^4}{\sqrt{4 - 2x^2 - x^4}} dx \\ &= \frac{x(1419985 + 1419793x^2)}{18\sqrt{2 + x^2 - x^4}} + \frac{27500}{3}x\sqrt{2 + x^2 - x^4} + 625x^3\sqrt{2 + x^2 - x^4} - \frac{3482293}{18} \int \frac{\sqrt{2 + x^2 - x^4}}{\sqrt{4 - 2x^2 - x^4}} dx \\ &= \frac{x(1419985 + 1419793x^2)}{18\sqrt{2 + x^2 - x^4}} + \frac{27500}{3}x\sqrt{2 + x^2 - x^4} + 625x^3\sqrt{2 + x^2 - x^4} - \frac{3482293}{18} E\left(\sin^{-1}\left(\frac{\sqrt{2 + x^2 - x^4}}{\sqrt{4 - 2x^2 - x^4}}\right)\right) \end{aligned}$$

Mathematica [F] time = 0, size = 0, normalized size = 0.

\$Aborted

Verification is Not applicable to the result.

[In] Integrate[(7 + 5*x^2)^5/(2 + x^2 - x^4)^(3/2), x]

[Out] \$Aborted

Maple [B] time = 0.033, size = 280, normalized size = 3.

$$6250 \frac{1}{\sqrt{-x^4 + x^2 + 2}} \left(\frac{17x^3}{18} + \frac{7x}{9} \right) + 625x^3 \sqrt{-x^4 + x^2 + 2} + \frac{27500x}{3} \sqrt{-x^4 + x^2 + 2} - \frac{799361\sqrt{2}}{18} \sqrt{-2x^2 + 4} \sqrt{x^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((5*x^2+7)^5/(-x^4+x^2+2)^(3/2), x)

[Out] 6250*(17/18*x^3+7/9*x)/(-x^4+x^2+2)^(1/2)+625*x^3*(-x^4+x^2+2)^(1/2)+27500/3*x*(-x^4+x^2+2)^(1/2)-799361/18*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticF(1/2*x*2^(1/2), I*2^(1/2))+3482293/36*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*(EllipticF(1/2*x*2^(1/2), I*2^(1/2))-EllipticE(1/2*x*2^(1/2), I*2^(1/2)))+43750*(7/18*x^3+5/9*x)/(-x^4+x^2+2)^(1/2)+122500*(5/18*x^3+1/9*x)/(-x^4+x^2+2)^(1/2)+171500*(1/18*x^3+2/9*x)/(-x^4+x^2+2)^(1/2)+120050*(1/9*x^3-1/18*x)/(-x^4+x^2+2)^(1/2)+33614*(5/36*x-1/36*x^3)/(-x^4+x^2+2)^(1/2)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(5x^2 + 7)^5}{(-x^4 + x^2 + 2)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^2+7)^5/(-x^4+x^2+2)^(3/2), x, algorithm="maxima")

[Out] integrate((5*x^2 + 7)^5/(-x^4 + x^2 + 2)^(3/2), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral} \left(\frac{(3125x^{10} + 21875x^8 + 61250x^6 + 85750x^4 + 60025x^2 + 16807)\sqrt{-x^4 + x^2 + 2}}{x^8 - 2x^6 - 3x^4 + 4x^2 + 4}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^2+7)^5/(-x^4+x^2+2)^(3/2), x, algorithm="fricas")

[Out] integral((3125*x^10 + 21875*x^8 + 61250*x^6 + 85750*x^4 + 60025*x^2 + 16807)*sqrt(-x^4 + x^2 + 2)/(x^8 - 2*x^6 - 3*x^4 + 4*x^2 + 4), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(5x^2 + 7)^5}{(-(x^2 - 2)(x^2 + 1))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x**2+7)**5/(-x**4+x**2+2)**(3/2),x)

[Out] Integral((5*x**2 + 7)**5/(-(x**2 - 2)*(x**2 + 1))**(3/2), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(5x^2 + 7)^5}{(-x^4 + x^2 + 2)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^2+7)^5/(-x^4+x^2+2)^(3/2),x, algorithm="giac")

[Out] integrate((5*x^2 + 7)^5/(-x^4 + x^2 + 2)^(3/2), x)

$$3.340 \quad \int \frac{(7+5x^2)^4}{(2+x^2-x^4)^{3/2}} dx$$

Optimal. Leaf size=74

$$\frac{31921}{6} \text{EllipticF}\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right), -2\right) + \frac{625}{3} \sqrt{-x^4 + x^2 + 2x} + \frac{(83489x^2 + 83585)x}{18\sqrt{-x^4 + x^2 + 2}} - \frac{165239}{18} E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right) \middle| -2\right)$$

[Out] (x*(83585 + 83489*x^2))/(18*Sqrt[2 + x^2 - x^4]) + (625*x*Sqrt[2 + x^2 - x^4])/3 - (165239*EllipticE[ArcSin[x/Sqrt[2]], -2])/18 + (31921*EllipticF[ArcSin[x/Sqrt[2]], -2])/6

Rubi [A] time = 0.0761503, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$, Rules used = {1205, 1679, 1180, 524, 424, 419}

$$\frac{625}{3} \sqrt{-x^4 + x^2 + 2x} + \frac{(83489x^2 + 83585)x}{18\sqrt{-x^4 + x^2 + 2}} + \frac{31921}{6} F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right) \middle| -2\right) - \frac{165239}{18} E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right) \middle| -2\right)$$

Antiderivative was successfully verified.

[In] Int[(7 + 5*x^2)^4/(2 + x^2 - x^4)^(3/2), x]

[Out] (x*(83585 + 83489*x^2))/(18*Sqrt[2 + x^2 - x^4]) + (625*x*Sqrt[2 + x^2 - x^4])/3 - (165239*EllipticE[ArcSin[x/Sqrt[2]], -2])/18 + (31921*EllipticF[ArcSin[x/Sqrt[2]], -2])/6

Rule 1205

Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{f = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 + c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*g - f*(b^2 - 2*a*c) - c*(b*f - 2*a*g)*x^2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[(d + e*x^2)^q, a + b*x^2 + c*x^4, x] + b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g + c*(4*p + 7)*(b*f - 2*a*g)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1] && LtQ[p, -1]

Rule 1679

Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{q = Expon[Pq, x^2], e = Coeff[Pq, x^2, Expon[Pq, x^2]]}, Simp[(e*x^(2*q - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(2*q + 4*p + 1)), x] + Dist[1/(c*(2*q + 4*p + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*q + 4*p + 1)*Pq - a*e*(2*q - 3)*x^(2*q - 4) - b*e*(2*q + 2*p - 1)*x^(2*q - 2) - c*e*(2*q + 4*p + 1)*x^(2*q), x], x], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && !LtQ[p, -1]

Rule 1180

Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x] /; FreeQ[{a, b, c, d, e}

, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]

Rule 524

Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.)*(x_)^(n_)]), x_Symbol] := Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n], x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && !(EqQ[n, 2] && ((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && (!GtQ[c, 0] || Simplr SqrtQ[-(b/a), -(d/c)]))))))

Rule 424

Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] := Simp[(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]

Rule 419

Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] := Simp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0] && !(NegQ[b/a] && SimplrSqrtQ[-(b/a), -(d/c)])

Rubi steps

$$\begin{aligned} \int \frac{(7 + 5x^2)^4}{(2 + x^2 - x^4)^{3/2}} dx &= \frac{x(83585 + 83489x^2)}{18\sqrt{2 + x^2 - x^4}} - \frac{1}{18} \int \frac{61976 + 157739x^2 + 11250x^4}{\sqrt{2 + x^2 - x^4}} dx \\ &= \frac{x(83585 + 83489x^2)}{18\sqrt{2 + x^2 - x^4}} + \frac{625}{3}x\sqrt{2 + x^2 - x^4} + \frac{1}{54} \int \frac{-208428 - 495717x^2}{\sqrt{2 + x^2 - x^4}} dx \\ &= \frac{x(83585 + 83489x^2)}{18\sqrt{2 + x^2 - x^4}} + \frac{625}{3}x\sqrt{2 + x^2 - x^4} + \frac{1}{27} \int \frac{-208428 - 495717x^2}{\sqrt{4 - 2x^2}\sqrt{2 + 2x^2}} dx \\ &= \frac{x(83585 + 83489x^2)}{18\sqrt{2 + x^2 - x^4}} + \frac{625}{3}x\sqrt{2 + x^2 - x^4} - \frac{165239}{18} \int \frac{\sqrt{2 + 2x^2}}{\sqrt{4 - 2x^2}} dx + \frac{31921}{3} \int \frac{1}{\sqrt{4 - 2x^2}} dx \\ &= \frac{x(83585 + 83489x^2)}{18\sqrt{2 + x^2 - x^4}} + \frac{625}{3}x\sqrt{2 + x^2 - x^4} - \frac{165239}{18} E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right) \middle| -2\right) + \frac{31921}{6} F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right) \middle| -2\right) \end{aligned}$$

Mathematica [F] time = 0, size = 0, normalized size = 0.

\$Aborted

Verification is Not applicable to the result.

[In] Integrate[(7 + 5*x^2)^4/(2 + x^2 - x^4)^(3/2), x]

[Out] \$Aborted

Maple [B] time = 0.008, size = 240, normalized size = 3.2

$$1250 \frac{1}{\sqrt{-x^4 + x^2 + 2}} \left(\frac{7x^3}{18} + 5/9x \right) + \frac{625x}{3} \sqrt{-x^4 + x^2 + 2} - \frac{17369\sqrt{2}}{9} \sqrt{-2x^2 + 4} \sqrt{x^2 + 1} \text{EllipticF}\left(\frac{x\sqrt{2}}{2}, i\sqrt{2}\right) \frac{1}{\sqrt{-x^4 + x^2 + 2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((5*x^2+7)^4/(-x^4+x^2+2)^(3/2),x)`

[Out] $1250*(7/18*x^3+5/9*x)/(-x^4+x^2+2)^{(1/2)}+625/3*x*(-x^4+x^2+2)^{(1/2)}-17369/9*x^{2^{(1/2)}}*(-2*x^2+4)^{(1/2)}*(x^2+1)^{(1/2)}/(-x^4+x^2+2)^{(1/2)}*EllipticF(1/2*x*2^{(1/2)},I*2^{(1/2)})+165239/36*2^{(1/2)}*(-2*x^2+4)^{(1/2)}*(x^2+1)^{(1/2)}/(-x^4+x^2+2)^{(1/2)}*(EllipticF(1/2*x*2^{(1/2)},I*2^{(1/2)})-EllipticE(1/2*x*2^{(1/2)},I*2^{(1/2)}))+7000*(5/18*x^3+1/9*x)/(-x^4+x^2+2)^{(1/2)}+14700*(1/18*x^3+2/9*x)/(-x^4+x^2+2)^{(1/2)}+13720*(1/9*x^3-1/18*x)/(-x^4+x^2+2)^{(1/2)}+4802*(5/36*x-1/36*x^3)/(-x^4+x^2+2)^{(1/2)}$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(5x^2 + 7)^4}{(-x^4 + x^2 + 2)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((5*x^2+7)^4/(-x^4+x^2+2)^(3/2),x, algorithm="maxima")`

[Out] `integrate((5*x^2 + 7)^4/(-x^4 + x^2 + 2)^(3/2), x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(625x^8 + 3500x^6 + 7350x^4 + 6860x^2 + 2401)\sqrt{-x^4 + x^2 + 2}}{x^8 - 2x^6 - 3x^4 + 4x^2 + 4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((5*x^2+7)^4/(-x^4+x^2+2)^(3/2),x, algorithm="fricas")`

[Out] `integral((625*x^8 + 3500*x^6 + 7350*x^4 + 6860*x^2 + 2401)*sqrt(-x^4 + x^2 + 2)/(x^8 - 2*x^6 - 3*x^4 + 4*x^2 + 4), x)`

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(5x^2 + 7)^4}{(-(x^2 - 2)(x^2 + 1))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((5*x**2+7)**4/(-x**4+x**2+2)**(3/2),x)`

[Out] `Integral((5*x**2 + 7)**4/(-(x**2 - 2)*(x**2 + 1))** (3/2), x)`

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(5x^2 + 7)^4}{(-x^4 + x^2 + 2)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^4/(-x^4+x^2+2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((5*x^2 + 7)^4/(-x^4 + x^2 + 2)^(3/2), x)
```

$$3.341 \quad \int \frac{(7+5x^2)^3}{(2+x^2-x^4)^{3/2}} dx$$

Optimal. Leaf size=55

$$\frac{1763}{6} \text{EllipticF}\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right), -2\right) + \frac{x(4897x^2 + 4945)}{18\sqrt{-x^4 + x^2 + 2}} - \frac{7147}{18} E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right) \middle| -2\right)$$

[Out] (x*(4945 + 4897*x^2))/(18*sqrt[2 + x^2 - x^4]) - (7147*EllipticE[ArcSin[x/Sqrt[2]], -2])/18 + (1763*EllipticF[ArcSin[x/Sqrt[2]], -2])/6

Rubi [A] time = 0.0514476, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {1205, 1180, 524, 424, 419}

$$\frac{x(4897x^2 + 4945)}{18\sqrt{-x^4 + x^2 + 2}} + \frac{1763}{6} F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right) \middle| -2\right) - \frac{7147}{18} E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right) \middle| -2\right)$$

Antiderivative was successfully verified.

[In] Int[(7 + 5*x^2)^3/(2 + x^2 - x^4)^(3/2), x]

[Out] (x*(4945 + 4897*x^2))/(18*sqrt[2 + x^2 - x^4]) - (7147*EllipticE[ArcSin[x/Sqrt[2]], -2])/18 + (1763*EllipticF[ArcSin[x/Sqrt[2]], -2])/6

Rule 1205

Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{f = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 + c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*g - f*(b^2 - 2*a*c) - c*(b*f - 2*a*g)*x^2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[(d + e*x^2)^q, a + b*x^2 + c*x^4, x] + b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g + c*(4*p + 7)*(b*f - 2*a*g)*x^2, x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1] && LtQ[p, -1]

Rule 1180

Int[((d_) + (e_.)*(x_)^2)/sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*sqrt[-c], Int[(d + e*x^2)/(sqrt[b + q + 2*c*x^2]*sqrt[-b + q - 2*c*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]

Rule 524

Int[((e_) + (f_.)*(x_)^(n_))/(sqrt[(a_) + (b_.)*(x_)^(n_)]*sqrt[(c_) + (d_.)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[sqrt[a + b*x^n]/sqrt[c + d*x^n], x], x] + Dist[(b*e - a*f)/b, Int[1/(sqrt[a + b*x^n]*sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && !(EqQ[n, 2] && ((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && (!GtQ[c, 0] || SimplerSqrtQ[-(b/a), -(d/c)]))))))

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] := Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)]/(Sqrt[c]*Rt[-(d/c)
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] := S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)]/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] && !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\begin{aligned} \int \frac{(7+5x^2)^3}{(2+x^2-x^4)^{3/2}} dx &= \frac{x(4945+4897x^2)}{18\sqrt{2+x^2-x^4}} - \frac{1}{18} \int \frac{1858+7147x^2}{\sqrt{2+x^2-x^4}} dx \\ &= \frac{x(4945+4897x^2)}{18\sqrt{2+x^2-x^4}} - \frac{1}{9} \int \frac{1858+7147x^2}{\sqrt{4-2x^2}\sqrt{2+2x^2}} dx \\ &= \frac{x(4945+4897x^2)}{18\sqrt{2+x^2-x^4}} - \frac{7147}{18} \int \frac{\sqrt{2+2x^2}}{\sqrt{4-2x^2}} dx + \frac{1763}{3} \int \frac{1}{\sqrt{4-2x^2}\sqrt{2+2x^2}} dx \\ &= \frac{x(4945+4897x^2)}{18\sqrt{2+x^2-x^4}} - \frac{7147}{18} E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\right) - 2 + \frac{1763}{6} F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\right) - 2 \end{aligned}$$

Mathematica [F] time = 0, size = 0, normalized size = 0.

\$Aborted

Verification is Not applicable to the result.

[In] Integrate[(7 + 5*x^2)^3/(2 + x^2 - x^4)^(3/2), x]

[Out] \$Aborted

Maple [B] time = 0.008, size = 202, normalized size = 3.7

$$250 \frac{1}{\sqrt{-x^4+x^2+2}} \left(\frac{5x^3}{18} + x/9 \right) - \frac{929\sqrt{2}}{18} \sqrt{-2x^2+4} \sqrt{x^2+1} \text{EllipticF}\left(\frac{x\sqrt{2}}{2}, i\sqrt{2}\right) \frac{1}{\sqrt{-x^4+x^2+2}} + \frac{7147\sqrt{2}}{36} \sqrt{-2x^2+4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((5*x^2+7)^3/(-x^4+x^2+2)^(3/2), x)

[Out] 250*(5/18*x^3+1/9*x)/(-x^4+x^2+2)^(1/2)-929/18*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticF(1/2*x*2^(1/2), I*2^(1/2))+7147/36*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*(EllipticF(1/2*x*2^(1/2), I*2^(1/2))-EllipticE(1/2*x*2^(1/2), I*2^(1/2)))+1050*(1/18*x^3+2/9*x)/(-x^4+x^2+2)^(1/2)+1470*(1/9*x^3-1/18*x)/(-x^4+x^2+2)^(1/2)+686*(5/36*x-1/36*x^3)/(-x^4+x^2+2)^(1/2)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(5x^2 + 7)^3}{(-x^4 + x^2 + 2)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^2+7)^3/(-x^4+x^2+2)^(3/2),x, algorithm="maxima")

[Out] integrate((5*x^2 + 7)^3/(-x^4 + x^2 + 2)^(3/2), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(125x^6 + 525x^4 + 735x^2 + 343)\sqrt{-x^4 + x^2 + 2}}{x^8 - 2x^6 - 3x^4 + 4x^2 + 4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^2+7)^3/(-x^4+x^2+2)^(3/2),x, algorithm="fricas")

[Out] integral((125*x^6 + 525*x^4 + 735*x^2 + 343)*sqrt(-x^4 + x^2 + 2)/(x^8 - 2*x^6 - 3*x^4 + 4*x^2 + 4), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(5x^2 + 7)^3}{(-(x^2 - 2)(x^2 + 1))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x**2+7)**3/(-x**4+x**2+2)**(3/2),x)

[Out] Integral((5*x**2 + 7)**3/(-(x**2 - 2)*(x**2 + 1))**3/2, x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(5x^2 + 7)^3}{(-x^4 + x^2 + 2)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^2+7)^3/(-x^4+x^2+2)^(3/2),x, algorithm="giac")

[Out] integrate((5*x^2 + 7)^3/(-x^4 + x^2 + 2)^(3/2), x)

$$3.342 \quad \int \frac{(7+5x^2)^2}{(2+x^2-x^4)^{3/2}} dx$$

Optimal. Leaf size=55

$$\frac{139}{6} \text{EllipticF}\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right), -2\right) + \frac{x(281x^2 + 305)}{18\sqrt{-x^4 + x^2 + 2}} - \frac{281}{18} E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right) \middle| -2\right)$$

[Out] (x*(305 + 281*x^2))/(18*Sqrt[2 + x^2 - x^4]) - (281*EllipticE[ArcSin[x/Sqrt[2]], -2])/18 + (139*EllipticF[ArcSin[x/Sqrt[2]], -2])/6

Rubi [A] time = 0.0511672, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {1205, 1180, 524, 424, 419}

$$\frac{x(281x^2 + 305)}{18\sqrt{-x^4 + x^2 + 2}} + \frac{139}{6} F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right) \middle| -2\right) - \frac{281}{18} E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right) \middle| -2\right)$$

Antiderivative was successfully verified.

[In] Int[(7 + 5*x^2)^2/(2 + x^2 - x^4)^(3/2), x]

[Out] (x*(305 + 281*x^2))/(18*Sqrt[2 + x^2 - x^4]) - (281*EllipticE[ArcSin[x/Sqrt[2]], -2])/18 + (139*EllipticF[ArcSin[x/Sqrt[2]], -2])/6

Rule 1205

Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{f = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 + c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*g - f*(b^2 - 2*a*c) - c*(b*f - 2*a*g)*x^2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[(d + e*x^2)^q, a + b*x^2 + c*x^4, x] + b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g + c*(4*p + 7)*(b*f - 2*a*g)*x^2, x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1] && LtQ[p, -1]

Rule 1180

Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]

Rule 524

Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n], x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && !(EqQ[n, 2] && ((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && (!GtQ[c, 0] || SimplerSqrtQ[-(b/a), -(d/c)]))))))

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] && !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\begin{aligned} \int \frac{(7+5x^2)^2}{(2+x^2-x^4)^{3/2}} dx &= \frac{x(305+281x^2)}{18\sqrt{2+x^2-x^4}} - \frac{1}{18} \int \frac{-136+281x^2}{\sqrt{2+x^2-x^4}} dx \\ &= \frac{x(305+281x^2)}{18\sqrt{2+x^2-x^4}} - \frac{1}{9} \int \frac{-136+281x^2}{\sqrt{4-2x^2}\sqrt{2+2x^2}} dx \\ &= \frac{x(305+281x^2)}{18\sqrt{2+x^2-x^4}} - \frac{281}{18} \int \frac{\sqrt{2+2x^2}}{\sqrt{4-2x^2}} dx + \frac{139}{3} \int \frac{1}{\sqrt{4-2x^2}\sqrt{2+2x^2}} dx \\ &= \frac{x(305+281x^2)}{18\sqrt{2+x^2-x^4}} - \frac{281}{18} E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle| -2\right) + \frac{139}{6} F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle| -2\right) \end{aligned}$$

Mathematica [F] time = 0, size = 0, normalized size = 0.

\$Aborted

Verification is Not applicable to the result.

[In] Integrate[(7 + 5*x^2)^2/(2 + x^2 - x^4)^(3/2), x]

[Out] \$Aborted

Maple [B] time = 0.007, size = 179, normalized size = 3.3

$$50 \frac{1/18 x^3 + 2/9 x}{\sqrt{-x^4 + x^2 + 2}} + \frac{34\sqrt{2}}{9} \sqrt{-2x^2 + 4} \sqrt{x^2 + 1} \operatorname{EllipticF}\left(\frac{x\sqrt{2}}{2}, i\sqrt{2}\right) \frac{1}{\sqrt{-x^4 + x^2 + 2}} + \frac{281\sqrt{2}}{36} \sqrt{-2x^2 + 4} \sqrt{x^2 + 1} \left(\operatorname{EllipticE}\left(\frac{x\sqrt{2}}{2}, i\sqrt{2}\right) - \operatorname{EllipticF}\left(\frac{x\sqrt{2}}{2}, i\sqrt{2}\right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((5*x^2+7)^2/(-x^4+x^2+2)^(3/2), x)

[Out] 50*(1/18*x^3+2/9*x)/(-x^4+x^2+2)^(1/2)+34/9*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticF(1/2*x*2^(1/2), I*2^(1/2))+281/36*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*(EllipticF(1/2*x*2^(1/2), I*2^(1/2))-EllipticE(1/2*x*2^(1/2), I*2^(1/2)))+140*(1/9*x^3-1/18*x)/(-x^4+x^2+2)^(1/2)+98*(5/36*x-1/36*x^3)/(-x^4+x^2+2)^(1/2)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(5x^2 + 7)^2}{(-x^4 + x^2 + 2)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^2+7)^2/(-x^4+x^2+2)^(3/2),x, algorithm="maxima")

[Out] integrate((5*x^2 + 7)^2/(-x^4 + x^2 + 2)^(3/2), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(25x^4 + 70x^2 + 49)\sqrt{-x^4 + x^2 + 2}}{x^8 - 2x^6 - 3x^4 + 4x^2 + 4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^2+7)^2/(-x^4+x^2+2)^(3/2),x, algorithm="fricas")

[Out] integral((25*x^4 + 70*x^2 + 49)*sqrt(-x^4 + x^2 + 2)/(x^8 - 2*x^6 - 3*x^4 + 4*x^2 + 4), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(5x^2 + 7)^2}{(-(x^2 - 2)(x^2 + 1))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x**2+7)**2/(-x**4+x**2+2)**(3/2),x)

[Out] Integral((5*x**2 + 7)**2/(-(x**2 - 2)*(x**2 + 1))**(3/2), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(5x^2 + 7)^2}{(-x^4 + x^2 + 2)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^2+7)^2/(-x^4+x^2+2)^(3/2),x, algorithm="giac")

[Out] integrate((5*x^2 + 7)^2/(-x^4 + x^2 + 2)^(3/2), x)

$$3.343 \quad \int \frac{7+5x^2}{(2+x^2-x^4)^{3/2}} dx$$

Optimal. Leaf size=55

$$\frac{17}{6} \text{EllipticF}\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right), -2\right) + \frac{x(13x^2+25)}{18\sqrt{-x^4+x^2+2}} - \frac{13}{18} E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right) \middle| -2\right)$$

[Out] (x*(25 + 13*x^2))/(18*Sqrt[2 + x^2 - x^4]) - (13*EllipticE[ArcSin[x/Sqrt[2]], -2])/18 + (17*EllipticF[ArcSin[x/Sqrt[2]], -2])/6

Rubi [A] time = 0.0453853, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {1178, 1180, 524, 424, 419}

$$\frac{x(13x^2+25)}{18\sqrt{-x^4+x^2+2}} + \frac{17}{6} F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right) \middle| -2\right) - \frac{13}{18} E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right) \middle| -2\right)$$

Antiderivative was successfully verified.

[In] Int[(7 + 5*x^2)/(2 + x^2 - x^4)^(3/2), x]

[Out] (x*(25 + 13*x^2))/(18*Sqrt[2 + x^2 - x^4]) - (13*EllipticE[ArcSin[x/Sqrt[2]], -2])/18 + (17*EllipticF[ArcSin[x/Sqrt[2]], -2])/6

Rule 1178

Int[((d_) + (e_)*(x_)^2)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] := Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && IntegerQ[2*p]

Rule 1180

Int[((d_) + (e_)*(x_)^2)/Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4], x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]

Rule 524

Int[((e_) + (f_)*(x_)^(n_))/(Sqrt[(a_) + (b_)*(x_)^(n_)]*Sqrt[(c_) + (d_)*(x_)^(n_)]), x_Symbol] := Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n], x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && !(EqQ[n, 2] && ((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && (!GtQ[c, 0] || Simpler SqrtQ[-(b/a), -(d/c)]))))))

Rule 424

Int[Sqrt[(a_) + (b_)*(x_)^2]/Sqrt[(c_) + (d_)*(x_)^2], x_Symbol] := Simp[(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c

), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]

Rule 419

Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> Simp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0] && !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])

Rubi steps

$$\begin{aligned} \int \frac{7+5x^2}{(2+x^2-x^4)^{3/2}} dx &= \frac{x(25+13x^2)}{18\sqrt{2+x^2-x^4}} - \frac{1}{18} \int \frac{-38+13x^2}{\sqrt{2+x^2-x^4}} dx \\ &= \frac{x(25+13x^2)}{18\sqrt{2+x^2-x^4}} - \frac{1}{9} \int \frac{-38+13x^2}{\sqrt{4-2x^2}\sqrt{2+2x^2}} dx \\ &= \frac{x(25+13x^2)}{18\sqrt{2+x^2-x^4}} - \frac{13}{18} \int \frac{\sqrt{2+2x^2}}{\sqrt{4-2x^2}} dx + \frac{17}{3} \int \frac{1}{\sqrt{4-2x^2}\sqrt{2+2x^2}} dx \\ &= \frac{x(25+13x^2)}{18\sqrt{2+x^2-x^4}} - \frac{13}{18} E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right) \middle| -2\right) + \frac{17}{6} F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right) \middle| -2\right) \end{aligned}$$

Mathematica [F] time = 0, size = 0, normalized size = 0.

\$Aborted

Verification is Not applicable to the result.

[In] Integrate[(7 + 5*x^2)/(2 + x^2 - x^4)^(3/2), x]

[Out] \$Aborted

Maple [B] time = 0.006, size = 156, normalized size = 2.8

$$10 \frac{1/9 x^3 - x/18}{\sqrt{-x^4 + x^2 + 2}} + \frac{19\sqrt{2}}{18} \sqrt{-2x^2 + 4} \sqrt{x^2 + 1} \text{EllipticF}\left(\frac{x\sqrt{2}}{2}, i\sqrt{2}\right) \frac{1}{\sqrt{-x^4 + x^2 + 2}} + \frac{13\sqrt{2}}{36} \sqrt{-2x^2 + 4} \sqrt{x^2 + 1} \left(\text{EllipticE}\left(\frac{x\sqrt{2}}{2}, i\sqrt{2}\right) - \frac{1}{\sqrt{-x^4 + x^2 + 2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((5*x^2+7)/(-x^4+x^2+2)^(3/2), x)

[Out] 10*(1/9*x^3-1/18*x)/(-x^4+x^2+2)^(1/2)+19/18*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticF(1/2*x*2^(1/2), I*2^(1/2))+13/36*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*(EllipticF(1/2*x*2^(1/2), I*2^(1/2))-EllipticE(1/2*x*2^(1/2), I*2^(1/2)))+14*(5/36*x-1/36*x^3)/(-x^4+x^2+2)^(1/2)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{5x^2 + 7}{(-x^4 + x^2 + 2)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^2+7)/(-x^4+x^2+2)^(3/2),x, algorithm="maxima")

[Out] integrate((5*x^2 + 7)/(-x^4 + x^2 + 2)^(3/2), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-x^4 + x^2 + 2}(5x^2 + 7)}{x^8 - 2x^6 - 3x^4 + 4x^2 + 4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^2+7)/(-x^4+x^2+2)^(3/2),x, algorithm="fricas")

[Out] integral(sqrt(-x^4 + x^2 + 2)*(5*x^2 + 7)/(x^8 - 2*x^6 - 3*x^4 + 4*x^2 + 4), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{5x^2 + 7}{(- (x^2 - 2)(x^2 + 1))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x**2+7)/(-x**4+x**2+2)**(3/2),x)

[Out] Integral((5*x**2 + 7)/(-(x**2 - 2)*(x**2 + 1))**(3/2), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{5x^2 + 7}{(-x^4 + x^2 + 2)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^2+7)/(-x^4+x^2+2)^(3/2),x, algorithm="giac")

[Out] integrate((5*x^2 + 7)/(-x^4 + x^2 + 2)^(3/2), x)

$$3.344 \quad \int \frac{1}{(2+x^2-x^4)^{3/2}} dx$$

Optimal. Leaf size=55

$$\frac{1}{6} \text{EllipticF}\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right), -2\right) + \frac{x(5-x^2)}{18\sqrt{-x^4+x^2+2}} + \frac{1}{18} E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right) \middle| -2\right)$$

[Out] (x*(5 - x^2))/(18*sqrt[2 + x^2 - x^4]) + EllipticE[ArcSin[x/Sqrt[2]], -2]/18 + EllipticF[ArcSin[x/Sqrt[2]], -2]/6

Rubi [A] time = 0.0408684, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {1092, 1180, 524, 424, 419}

$$\frac{x(5-x^2)}{18\sqrt{-x^4+x^2+2}} + \frac{1}{6} F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right) \middle| -2\right) + \frac{1}{18} E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right) \middle| -2\right)$$

Antiderivative was successfully verified.

[In] Int[(2 + x^2 - x^4)^(-3/2), x]

[Out] (x*(5 - x^2))/(18*sqrt[2 + x^2 - x^4]) + EllipticE[ArcSin[x/Sqrt[2]], -2]/18 + EllipticF[ArcSin[x/Sqrt[2]], -2]/6

Rule 1092

Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> -Simp[(x*(b^2 - 2*a*c + b*c*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(b^2 - 2*a*c + 2*(p + 1)*(b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]

Rule 1180

Int[((d_) + (e_.)*(x_)^2)/sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*sqrt[-c], Int[(d + e*x^2)/(sqrt[b + q + 2*c*x^2]*sqrt[-b + q - 2*c*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]

Rule 524

Int[((e_) + (f_.)*(x_)^(n_))/(sqrt[(a_) + (b_.)*(x_)^(n_)]*sqrt[(c_) + (d_.)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[sqrt[a + b*x^n]/sqrt[c + d*x^n], x], x] + Dist[(b*e - a*f)/b, Int[1/(sqrt[a + b*x^n]*sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && !(EqQ[n, 2] && ((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && (!GtQ[c, 0] || SimplerSqrtQ[-(b/a), -(d/c)]))))))

Rule 424

Int[sqrt[(a_) + (b_.)*(x_)^2]/sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[(sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)]/(sqrt[c]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]

Rule 419

```
Int[1/(Sqrt[(a_) + (b_)*(x_)^2]*Sqrt[(c_) + (d_)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] && !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\begin{aligned} \int \frac{1}{(2+x^2-x^4)^{3/2}} dx &= \frac{x(5-x^2)}{18\sqrt{2+x^2-x^4}} - \frac{1}{18} \int \frac{-4-x^2}{\sqrt{2+x^2-x^4}} dx \\ &= \frac{x(5-x^2)}{18\sqrt{2+x^2-x^4}} - \frac{1}{9} \int \frac{-4-x^2}{\sqrt{4-2x^2}\sqrt{2+2x^2}} dx \\ &= \frac{x(5-x^2)}{18\sqrt{2+x^2-x^4}} + \frac{1}{18} \int \frac{\sqrt{2+2x^2}}{\sqrt{4-2x^2}} dx + \frac{1}{3} \int \frac{1}{\sqrt{4-2x^2}\sqrt{2+2x^2}} dx \\ &= \frac{x(5-x^2)}{18\sqrt{2+x^2-x^4}} + \frac{1}{18} E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\right) - 2 + \frac{1}{6} F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\right) - 2 \end{aligned}$$

Mathematica [F] time = 0, size = 0, normalized size = 0.

\$Aborted

Verification is Not applicable to the result.

[In] Integrate[(2 + x^2 - x^4)^(-3/2), x]

[Out] \$Aborted

Maple [B] time = 0.004, size = 133, normalized size = 2.4

$$2 \frac{1}{\sqrt{-x^4+x^2+2}} \left(\frac{5x}{36} - \frac{1}{36}x^3 \right) + \frac{\sqrt{2}}{9} \sqrt{-2x^2+4} \sqrt{x^2+1} \operatorname{EllipticF}\left(\frac{x\sqrt{2}}{2}, i\sqrt{2}\right) \frac{1}{\sqrt{-x^4+x^2+2}} - \frac{\sqrt{2}}{36} \sqrt{-2x^2+4} \sqrt{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(-x^4+x^2+2)^(3/2), x)

[Out] 2*(5/36*x-1/36*x^3)/(-x^4+x^2+2)^(1/2)+1/9*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticF(1/2*x*2^(1/2), I*2^(1/2))-1/36*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*(EllipticF(1/2*x*2^(1/2), I*2^(1/2))-EllipticE(1/2*x*2^(1/2), I*2^(1/2)))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(-x^4+x^2+2)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-x^4+x^2+2)^(3/2), x, algorithm="maxima")

[Out] integrate((-x^4 + x^2 + 2)^(-3/2), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-x^4 + x^2 + 2}}{x^8 - 2x^6 - 3x^4 + 4x^2 + 4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-x^4+x^2+2)^(3/2),x, algorithm="fricas")

[Out] integral(sqrt(-x^4 + x^2 + 2)/(x^8 - 2*x^6 - 3*x^4 + 4*x^2 + 4), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(-x^4 + x^2 + 2)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-x**4+x**2+2)**(3/2),x)

[Out] Integral((-x**4 + x**2 + 2)**(-3/2), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(-x^4 + x^2 + 2)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-x^4+x^2+2)^(3/2),x, algorithm="giac")

[Out] integrate((-x^4 + x^2 + 2)^(-3/2), x)

$$3.345 \quad \int \frac{1}{(7+5x^2)(2+x^2-x^4)^{3/2}} dx$$

Optimal. Leaf size=72

$$\frac{1}{102} \text{EllipticF}\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right), -2\right) + \frac{x(35-16x^2)}{306\sqrt{-x^4+x^2+2}} + \frac{8}{153} E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right) \middle| -2\right) - \frac{25}{238} \Pi\left(-\frac{10}{7}; \sin^{-1}\left(\frac{x}{\sqrt{2}}\right) \middle| -2\right)$$

[Out] (x*(35 - 16*x^2))/(306*Sqrt[2 + x^2 - x^4]) + (8*EllipticE[ArcSin[x/Sqrt[2]], -2])/153 + EllipticF[ArcSin[x/Sqrt[2]], -2]/102 - (25*EllipticPi[-10/7, ArcSin[x/Sqrt[2]], -2])/238

Rubi [A] time = 0.0895718, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1221, 1178, 1180, 524, 424, 419, 1212, 537}

$$\frac{x(35-16x^2)}{306\sqrt{-x^4+x^2+2}} + \frac{1}{102} F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right) \middle| -2\right) + \frac{8}{153} E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right) \middle| -2\right) - \frac{25}{238} \Pi\left(-\frac{10}{7}; \sin^{-1}\left(\frac{x}{\sqrt{2}}\right) \middle| -2\right)$$

Antiderivative was successfully verified.

[In] Int[1/((7 + 5*x^2)*(2 + x^2 - x^4)^(3/2)),x]

[Out] (x*(35 - 16*x^2))/(306*Sqrt[2 + x^2 - x^4]) + (8*EllipticE[ArcSin[x/Sqrt[2]], -2])/153 + EllipticF[ArcSin[x/Sqrt[2]], -2]/102 - (25*EllipticPi[-10/7, ArcSin[x/Sqrt[2]], -2])/238

Rule 1221

Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_)/((d_) + (e_.)*(x_)^2), x_Symbol] := Dist[1/(c*d^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x^2)*(a + b*x^2 + c*x^4)^p, x], x] + Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[(a + b*x^2 + c*x^4)^(p + 1)/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[p + 1/2, 0]

Rule 1178

Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && IntegerQ[2*p]

Rule 1180

Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]

Rule 524

Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.)*(x_)^(n_)]), x_Symbol] := Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],

```
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] && !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && (!GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] := Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)]/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] := S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)]/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] && !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 1212

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/((d + e*x^2
)*Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c,
d, e}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] := Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)]/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] && !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] && !(GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rubi steps

$$\begin{aligned} \int \frac{1}{(7+5x^2)(2+x^2-x^4)^{3/2}} dx &= -\left(\frac{1}{34} \int \frac{-12+5x^2}{(2+x^2-x^4)^{3/2}} dx \right) - \frac{25}{34} \int \frac{1}{(7+5x^2)\sqrt{2+x^2-x^4}} dx \\ &= \frac{x(35-16x^2)}{306\sqrt{2+x^2-x^4}} + \frac{1}{612} \int \frac{38+32x^2}{\sqrt{2+x^2-x^4}} dx - \frac{25}{17} \int \frac{1}{\sqrt{4-2x^2}\sqrt{2+2x^2}(7+5x^2)} dx \\ &= \frac{x(35-16x^2)}{306\sqrt{2+x^2-x^4}} - \frac{25}{238} \Pi\left(-\frac{10}{7}; \sin^{-1}\left(\frac{x}{\sqrt{2}}\right) \middle| -2\right) + \frac{1}{306} \int \frac{38+32x^2}{\sqrt{4-2x^2}\sqrt{2+2x^2}} dx \\ &= \frac{x(35-16x^2)}{306\sqrt{2+x^2-x^4}} - \frac{25}{238} \Pi\left(-\frac{10}{7}; \sin^{-1}\left(\frac{x}{\sqrt{2}}\right) \middle| -2\right) + \frac{1}{51} \int \frac{1}{\sqrt{4-2x^2}\sqrt{2+2x^2}} dx \\ &= \frac{x(35-16x^2)}{306\sqrt{2+x^2-x^4}} + \frac{8}{153} E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right) \middle| -2\right) + \frac{1}{102} F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right) \middle| -2\right) - \frac{25}{238} \Pi\left(\right) \end{aligned}$$

Mathematica [C] time = 0.212313, size = 101, normalized size = 1.4

$$-357i\sqrt{2}\text{EllipticF}\left(i\sinh^{-1}(x), -\frac{1}{2}\right) - \frac{224x^3}{\sqrt{-x^4+x^2+2}} + \frac{490x}{\sqrt{-x^4+x^2+2}} + 224i\sqrt{2}E\left(i\sinh^{-1}(x) \middle| -\frac{1}{2}\right) + 225i\sqrt{2}\Pi\left(\frac{5}{7}; i\sinh^{-1}(x)\right)$$

4284

Antiderivative was successfully verified.

[In] Integrate[1/((7 + 5*x^2)*(2 + x^2 - x^4)^(3/2)),x]

[Out] ((490*x)/Sqrt[2 + x^2 - x^4] - (224*x^3)/Sqrt[2 + x^2 - x^4] + (224*I)*Sqrt[2]*EllipticE[I*ArcSinh[x], -1/2] - (357*I)*Sqrt[2]*EllipticF[I*ArcSinh[x], -1/2] + (225*I)*Sqrt[2]*EllipticPi[5/7, I*ArcSinh[x], -1/2])/4284

Maple [B] time = 0.014, size = 164, normalized size = 2.3

$$2 \frac{1}{\sqrt{-x^4 + x^2 + 2}} \left(-\frac{4x^3}{153} + \frac{35x}{612} \right) + \frac{\sqrt{2}}{204} \sqrt{-2x^2 + 4} \sqrt{x^2 + 1} \text{EllipticF} \left(\frac{x\sqrt{2}}{2}, i\sqrt{2} \right) \frac{1}{\sqrt{-x^4 + x^2 + 2}} + \frac{4\sqrt{2}}{153} \sqrt{-2x^2 + 4} \sqrt{x^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(5*x^2+7)/(-x^4+x^2+2)^(3/2),x)

[Out] 2*(-4/153*x^3+35/612*x)/(-x^4+x^2+2)^(1/2)+1/204*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticF(1/2*x*2^(1/2),I*2^(1/2))+4/153*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticE(1/2*x*2^(1/2),I*2^(1/2))-25/238*2^(1/2)*(1-1/2*x^2)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticPi(1/2*x*2^(1/2),-10/7,I*2^(1/2))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(-x^4 + x^2 + 2)^{\frac{3}{2}} (5x^2 + 7)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(5*x^2+7)/(-x^4+x^2+2)^(3/2),x, algorithm="maxima")

[Out] integrate(1/((-x^4 + x^2 + 2)^(3/2)*(5*x^2 + 7)), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral} \left(\frac{\sqrt{-x^4 + x^2 + 2}}{5x^{10} - 3x^8 - 29x^6 - x^4 + 48x^2 + 28}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(5*x^2+7)/(-x^4+x^2+2)^(3/2),x, algorithm="fricas")

[Out] integral(sqrt(-x^4 + x^2 + 2)/(5*x^10 - 3*x^8 - 29*x^6 - x^4 + 48*x^2 + 28), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(-(x^2 - 2)(x^2 + 1))^{\frac{3}{2}} (5x^2 + 7)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(5*x**2+7)/(-x**4+x**2+2)**(3/2),x)

[Out] Integral(1/((-x**2 - 2)*(x**2 + 1))**(3/2)*(5*x**2 + 7)), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(-x^4 + x^2 + 2)^{\frac{3}{2}}(5x^2 + 7)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(5*x^2+7)/(-x^4+x^2+2)^(3/2),x, algorithm="giac")

[Out] integrate(1/((-x^4 + x^2 + 2)^(3/2)*(5*x^2 + 7)), x)

$$3.346 \quad \int \frac{1}{(7+5x^2)^2(2+x^2-x^4)^{3/2}} dx$$

Optimal. Leaf size=100

$$\frac{89\text{EllipticF}\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right), -2\right)}{24276} + \frac{625\sqrt{-x^4+x^2+2x}}{16184(5x^2+7)} + \frac{(580-287x^2)x}{10404\sqrt{-x^4+x^2+2}} + \frac{5143E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\right|-2)}{145656} - \frac{10825\Pi\left(-\frac{10}{7}\right)}{113288}$$

[Out] (x*(580 - 287*x^2))/(10404*Sqrt[2 + x^2 - x^4]) + (625*x*Sqrt[2 + x^2 - x^4])/(16184*(7 + 5*x^2)) + (5143*EllipticE[ArcSin[x/Sqrt[2]], -2])/145656 + (89*EllipticF[ArcSin[x/Sqrt[2]], -2])/24276 - (10825*EllipticPi[-10/7, ArcSin[x/Sqrt[2]], -2])/113288

Rubi [A] time = 0.296641, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {1228, 1178, 1180, 524, 424, 419, 1223, 1716, 1212, 537}

$$\frac{625\sqrt{-x^4+x^2+2x}}{16184(5x^2+7)} + \frac{(580-287x^2)x}{10404\sqrt{-x^4+x^2+2}} + \frac{89F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\right|-2)}{24276} + \frac{5143E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\right|-2)}{145656} - \frac{10825\Pi\left(-\frac{10}{7}; \sin^{-1}\right)}{113288}$$

Antiderivative was successfully verified.

[In] Int[1/((7 + 5*x^2)^2*(2 + x^2 - x^4)^(3/2)),x]

[Out] (x*(580 - 287*x^2))/(10404*Sqrt[2 + x^2 - x^4]) + (625*x*Sqrt[2 + x^2 - x^4])/(16184*(7 + 5*x^2)) + (5143*EllipticE[ArcSin[x/Sqrt[2]], -2])/145656 + (89*EllipticF[ArcSin[x/Sqrt[2]], -2])/24276 - (10825*EllipticPi[-10/7, ArcSin[x/Sqrt[2]], -2])/113288

Rule 1228

Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := Module[{aa, bb, cc}, Int[ExpandIntegrand[1/Sqrt[aa + bb*x^2 + c*x^4], (d + e*x^2)^q*(aa + bb*x^2 + cc*x^4)^(p + 1/2), x] /. {aa -> a, bb -> b, cc -> c}, x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, 0] && IntegerQ[p + 1/2]

Rule 1178

Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := Simp[(x*(a*b*e - d*(b^2 - 2*a*c)) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && IntegerQ[2*p]

Rule 1180

Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)], x_Symbol] := Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] && !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && (!GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] := Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)]/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] := S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)]/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] && !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 1223

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_
Symbol] := -Simp[(e^2*x*(d + e*x^2)^(q + 1)*Sqrt[a + b*x^2 + c*x^4])/(2*d*(
q + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(2*d*(q + 1)*(c*d^2 - b*d*e +
a*e^2)), Int[((d + e*x^2)^(q + 1)*Simp[a*e^2*(2*q + 3) + 2*d*(c*d - b*e)*(q
+ 1) - 2*e*(c*d*(q + 1) - b*e*(q + 2))*x^2 + c*e^2*(2*q + 5)*x^4, x])/Sqrt
[a + b*x^2 + c*x^4], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c
, 0] && ILtQ[q, -1]
```

Rule 1716

```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] := With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[
P4x, x, 4]}, -Dist[(e^2)^(-1), Int[(C*d - B*e - C*e*x^2)/Sqrt[a + b*x^2 + c
*x^4], x], x] + Dist[(C*d^2 - B*d*e + A*e^2)/e^2, Int[1/((d + e*x^2)*Sqrt[a
+ b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[P4x, x^2,
2] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 -
a*e^2, 0]
```

Rule 1212

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/((d + e*x^2
)*Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c,
d, e}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] := Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)]/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] && !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] && !(!GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(7+5x^2)^2(2+x^2-x^4)^{3/2}} dx &= \int \left(\frac{194-95x^2}{1156(2+x^2-x^4)^{3/2}} - \frac{25}{34(7+5x^2)^2\sqrt{2+x^2-x^4}} - \frac{475}{1156(7+5x^2)\sqrt{2+x^2-x^4}} \right) dx \\
&= \frac{\int \frac{194-95x^2}{(2+x^2-x^4)^{3/2}} dx}{1156} - \frac{475}{1156} \int \frac{1}{(7+5x^2)\sqrt{2+x^2-x^4}} dx - \frac{25}{34} \int \frac{1}{(7+5x^2)^2\sqrt{2+x^2-x^4}} dx \\
&= \frac{x(580-287x^2)}{10404\sqrt{2+x^2-x^4}} + \frac{625x\sqrt{2+x^2-x^4}}{16184(7+5x^2)} - \frac{\int \frac{-586-574x^2}{\sqrt{2+x^2-x^4}} dx}{20808} - \frac{25 \int \frac{118-70x^2-25x^4}{(7+5x^2)\sqrt{2+x^2-x^4}} dx}{16184} \\
&= \frac{x(580-287x^2)}{10404\sqrt{2+x^2-x^4}} + \frac{625x\sqrt{2+x^2-x^4}}{16184(7+5x^2)} - \frac{475\Pi\left(-\frac{10}{7}; \sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\right) - 2}{8092} + \frac{\int \frac{175+125x^2}{\sqrt{2+x^2-x^4}} dx}{16184} \\
&= \frac{x(580-287x^2)}{10404\sqrt{2+x^2-x^4}} + \frac{625x\sqrt{2+x^2-x^4}}{16184(7+5x^2)} - \frac{475\Pi\left(-\frac{10}{7}; \sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\right) - 2}{8092} + \frac{\int \frac{175+125x^2}{\sqrt{4-2x^2-x^4}} dx}{8092} \\
&= \frac{x(580-287x^2)}{10404\sqrt{2+x^2-x^4}} + \frac{625x\sqrt{2+x^2-x^4}}{16184(7+5x^2)} + \frac{287E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\right) - 2}{10404} + \frac{F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\right)}{1734} \\
&= \frac{x(580-287x^2)}{10404\sqrt{2+x^2-x^4}} + \frac{625x\sqrt{2+x^2-x^4}}{16184(7+5x^2)} + \frac{5143E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\right) - 2}{145656} + \frac{89F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\right)}{24270}
\end{aligned}$$

Mathematica [C] time = 0.325327, size = 196, normalized size = 1.96

$$-111741i\sqrt{2}(5x^2+7)\sqrt{-x^4+x^2+2}\text{EllipticF}\left(i\sinh^{-1}(x), -\frac{1}{2}\right) - 360010x^5 + 253386x^3 + 72002i\sqrt{2}(5x^2+7)\sqrt{-x^4+x^2+2}$$

20391

Antiderivative was successfully verified.

[In] Integrate[1/((7+5*x^2)^2*(2+x^2-x^4)^(3/2)),x]

[Out] (953260*x + 253386*x^3 - 360010*x^5 + (72002*I)*Sqrt[2]*(7+5*x^2)*Sqrt[2+x^2-x^4]*EllipticE[I*ArcSinh[x], -1/2] - (111741*I)*Sqrt[2]*(7+5*x^2)*Sqrt[2+x^2-x^4]*EllipticF[I*ArcSinh[x], -1/2] + (681975*I)*Sqrt[2]*Sqrt[2+x^2-x^4]*EllipticPi[5/7, I*ArcSinh[x], -1/2] + (487125*I)*Sqrt[2]*x^2*Sqrt[2+x^2-x^4]*EllipticPi[5/7, I*ArcSinh[x], -1/2])/(2039184*(7+5*x^2)*Sqrt[2+x^2-x^4])

Maple [B] time = 0.02, size = 188, normalized size = 1.9

$$\frac{625x}{80920x^2+113288}\sqrt{-x^4+x^2+2} + 2\frac{1}{\sqrt{-x^4+x^2+2}}\left(-\frac{287x^3}{20808} + \frac{145x}{5202}\right) + \frac{89\sqrt{2}}{48552}\sqrt{-2x^2+4}\sqrt{x^2+1}\text{EllipticF}\left(\frac{x\sqrt{2}}{2}, i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(5*x^2+7)^2/(-x^4+x^2+2)^(3/2),x)

[Out] 625/16184*x*(-x^4+x^2+2)^(1/2)/(5*x^2+7)+2*(-287/20808*x^3+145/5202*x)/(-x^4+x^2+2)^(1/2)+89/48552*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticF(1/2*x*2^(1/2),I*2^(1/2))+5143/291312*2^(1/2)*(-2*x^2+4)^(1/2)

$/2) * (x^2 + 1)^{1/2} / (-x^4 + x^2 + 2)^{1/2} * \text{EllipticE}(1/2 * x^2^{1/2}, I * 2^{1/2}) - 108$
 $25/113288 * 2^{1/2} * (1 - 1/2 * x^2)^{1/2} * (x^2 + 1)^{1/2} / (-x^4 + x^2 + 2)^{1/2} * \text{EllipticPi}(1/2 * x^2^{1/2}, -10/7, I * 2^{1/2})$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(-x^4 + x^2 + 2)^{\frac{3}{2}} (5x^2 + 7)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(5*x^2+7)^2/(-x^4+x^2+2)^(3/2),x, algorithm="maxima")

[Out] integrate(1/((-x^4 + x^2 + 2)^(3/2)*(5*x^2 + 7)^2), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-x^4 + x^2 + 2}}{25x^{12} + 20x^{10} - 166x^8 - 208x^6 + 233x^4 + 476x^2 + 196}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(5*x^2+7)^2/(-x^4+x^2+2)^(3/2),x, algorithm="fricas")

[Out] integral(sqrt(-x^4 + x^2 + 2)/(25*x^12 + 20*x^10 - 166*x^8 - 208*x^6 + 233*x^4 + 476*x^2 + 196), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(-(x^2 - 2)(x^2 + 1))^{\frac{3}{2}} (5x^2 + 7)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(5*x**2+7)**2/(-x**4+x**2+2)**(3/2),x)

[Out] Integral(1/((-x**2 - 2)*(x**2 + 1))**3/2*(5*x**2 + 7)**2), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(-x^4 + x^2 + 2)^{\frac{3}{2}} (5x^2 + 7)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(5*x^2+7)^2/(-x^4+x^2+2)^(3/2),x, algorithm="giac")

[Out] integrate(1/((-x^4 + x^2 + 2)^(3/2)*(5*x^2 + 7)^2), x)

$$3.347 \quad \int \frac{1}{(7+5x^2)^3(2+x^2-x^4)^{3/2}} dx$$

Optimal. Leaf size=128

$$\frac{60409 \operatorname{EllipticF}\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right), -2\right)}{23110752} + \frac{645625\sqrt{-x^4+x^2+2x}}{15407168(5x^2+7)} + \frac{625\sqrt{-x^4+x^2+2x}}{32368(5x^2+7)^2} + \frac{(9830-4909x^2)x}{353736\sqrt{-x^4+x^2+2}} + \frac{3086453}{138664512}$$

[Out] (x*(9830 - 4909*x^2))/(353736*Sqrt[2 + x^2 - x^4]) + (625*x*Sqrt[2 + x^2 - x^4])/(32368*(7 + 5*x^2)^2) + (645625*x*Sqrt[2 + x^2 - x^4])/(15407168*(7 + 5*x^2)) + (3086453*EllipticE[ArcSin[x/Sqrt[2]], -2])/138664512 + (60409*EllipticF[ArcSin[x/Sqrt[2]], -2])/23110752 - (6898575*EllipticPi[-10/7, ArcSin[x/Sqrt[2]], -2])/107850176

Rubi [A] time = 0.570137, antiderivative size = 128, normalized size of antiderivative = 1., number of steps used = 26, number of rules used = 11, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.458$, Rules used = {1228, 1178, 1180, 524, 424, 419, 1223, 1696, 1716, 1212, 537}

$$\frac{645625\sqrt{-x^4+x^2+2x}}{15407168(5x^2+7)} + \frac{625\sqrt{-x^4+x^2+2x}}{32368(5x^2+7)^2} + \frac{(9830-4909x^2)x}{353736\sqrt{-x^4+x^2+2}} + \frac{60409F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\right)-2}{23110752} + \frac{3086453E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\right)}{138664512}$$

Antiderivative was successfully verified.

[In] Int[1/((7 + 5*x^2)^3*(2 + x^2 - x^4)^(3/2)),x]

[Out] (x*(9830 - 4909*x^2))/(353736*Sqrt[2 + x^2 - x^4]) + (625*x*Sqrt[2 + x^2 - x^4])/(32368*(7 + 5*x^2)^2) + (645625*x*Sqrt[2 + x^2 - x^4])/(15407168*(7 + 5*x^2)) + (3086453*EllipticE[ArcSin[x/Sqrt[2]], -2])/138664512 + (60409*EllipticF[ArcSin[x/Sqrt[2]], -2])/23110752 - (6898575*EllipticPi[-10/7, ArcSin[x/Sqrt[2]], -2])/107850176

Rule 1228

Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := Module[{aa, bb, cc}, Int[ExpandIntegrand[1/Sqrt[aa + bb*x^2 + cc*x^4], (d + e*x^2)^q*(aa + bb*x^2 + cc*x^4)^(p + 1/2), x] /. {aa -> a, bb -> b, cc -> c}, x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, 0] && IntegerQ[p + 1/2]

Rule 1178

Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && IntegerQ[2*p]

Rule 1180

Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x] /; FreeQ[{a, b, c, d, e}

, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]

Rule 524

Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.)*(x_)^(n_)]), x_Symbol] := Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n], x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && !(EqQ[n, 2] && ((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && (!GtQ[c, 0] || SimplifierSqrtQ[-(b/a), -(d/c)]))))))

Rule 424

Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] := Simp[(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]

Rule 419

Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] := Simp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0] && !(NegQ[b/a] && SimplifierSqrtQ[-(b/a), -(d/c)])

Rule 1223

Int[((d_) + (e_.)*(x_)^2)^(q_)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] := -Simp[(e^2*x*(d + e*x^2)^(q + 1)*Sqrt[a + b*x^2 + c*x^4])/(2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2)), Int[((d + e*x^2)^(q + 1)*Simp[a*e^2*(2*q + 3) + 2*d*(c*d - b*e)*(q + 1) - 2*e*(c*d*(q + 1) - b*e*(q + 2))*x^2 + c*e^2*(2*q + 5)*x^4, x])/Sqrt[a + b*x^2 + c*x^4], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && ILtQ[q, -1]

Rule 1696

Int[((P4x_)*((d_) + (e_.)*(x_)^2)^(q_))/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] := With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[P4x, x, 4]}, -Simp[((C*d^2 - B*d*e + A*e^2)*x*(d + e*x^2)^(q + 1)*Sqrt[a + b*x^2 + c*x^4])/(2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2)), Int[((d + e*x^2)^(q + 1)*Simp[a*d*(C*d - B*e) + A*(a*e^2*(2*q + 3) + 2*d*(c*d - b*e)*(q + 1)) - 2*((B*d - A*e)*(b*e*(q + 2) - c*d*(q + 1)) - C*d*(b*d + a*e*(q + 1)))*x^2 + c*(C*d^2 - B*d*e + A*e^2)*(2*q + 5)*x^4, x])/Sqrt[a + b*x^2 + c*x^4], x], x] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[P4x, x^2] && LeQ[Expon[P4x, x], 4] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, -1]

Rule 1716

Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_Symbol] := With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[P4x, x, 4]}, -Dist[(e^2)^(-1), Int[(C*d - B*e - C*e*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] + Dist[(C*d^2 - B*d*e + A*e^2)/e^2, Int[1/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[P4x, x^2, 2] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0]

Rule 1212

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_Symbol]
:> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/((d + e*x^2)*Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)^2]), x_Symbol]
:> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x], (c*f)/(d*e))]/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d, e, f}, x] && !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] && !( !GtQ[f/e, 0] && SimplersqrtQ[-(f/e), -(d/c)])
```

Rubi steps

$$\int \frac{1}{(7 + 5x^2)^3 (2 + x^2 - x^4)^{3/2}} dx = \int \left(-\frac{-3278 + 1635x^2}{39304(2 + x^2 - x^4)^{3/2}} - \frac{25}{34(7 + 5x^2)^3 \sqrt{2 + x^2 - x^4}} - \frac{475}{1156(7 + 5x^2)^2 \sqrt{2 + x^2 - x^4}} \right) dx$$

$$= -\frac{\int \frac{-3278+1635x^2}{(2+x^2-x^4)^{3/2}} dx}{39304} - \frac{8175 \int \frac{1}{(7+5x^2)\sqrt{2+x^2-x^4}} dx}{39304} - \frac{475 \int \frac{1}{(7+5x^2)^2 \sqrt{2+x^2-x^4}} dx}{1156} - \frac{25}{34} \int \frac{1}{\sqrt{2+x^2-x^4}} dx$$

$$= \frac{x(9830 - 4909x^2)}{353736\sqrt{2 + x^2 - x^4}} + \frac{625x\sqrt{2 + x^2 - x^4}}{32368(7 + 5x^2)^2} + \frac{11875x\sqrt{2 + x^2 - x^4}}{550256(7 + 5x^2)} + \frac{\int \frac{9842+9818x^2}{\sqrt{2+x^2-x^4}} dx}{707472}$$

$$= \frac{x(9830 - 4909x^2)}{353736\sqrt{2 + x^2 - x^4}} + \frac{625x\sqrt{2 + x^2 - x^4}}{32368(7 + 5x^2)^2} + \frac{645625x\sqrt{2 + x^2 - x^4}}{15407168(7 + 5x^2)} - \frac{8175\Pi\left(-\frac{10}{7}; \sin^{-1}\left(\frac{x\sqrt{2+x^2-x^4}}{7+5x^2}\right)\right)}{2750000}$$

$$= \frac{x(9830 - 4909x^2)}{353736\sqrt{2 + x^2 - x^4}} + \frac{625x\sqrt{2 + x^2 - x^4}}{32368(7 + 5x^2)^2} + \frac{645625x\sqrt{2 + x^2 - x^4}}{15407168(7 + 5x^2)} - \frac{8175\Pi\left(-\frac{10}{7}; \sin^{-1}\left(\frac{x\sqrt{2+x^2-x^4}}{7+5x^2}\right)\right)}{2750000}$$

$$= \frac{x(9830 - 4909x^2)}{353736\sqrt{2 + x^2 - x^4}} + \frac{625x\sqrt{2 + x^2 - x^4}}{32368(7 + 5x^2)^2} + \frac{645625x\sqrt{2 + x^2 - x^4}}{15407168(7 + 5x^2)} + \frac{4909E\left(\sin^{-1}\left(\frac{x\sqrt{2+x^2-x^4}}{7+5x^2}\right)\right)}{353736}$$

$$= \frac{x(9830 - 4909x^2)}{353736\sqrt{2 + x^2 - x^4}} + \frac{625x\sqrt{2 + x^2 - x^4}}{32368(7 + 5x^2)^2} + \frac{645625x\sqrt{2 + x^2 - x^4}}{15407168(7 + 5x^2)} + \frac{90101E\left(\sin^{-1}\left(\frac{x\sqrt{2+x^2-x^4}}{7+5x^2}\right)\right)}{495232}$$

$$= \frac{x(9830 - 4909x^2)}{353736\sqrt{2 + x^2 - x^4}} + \frac{625x\sqrt{2 + x^2 - x^4}}{32368(7 + 5x^2)^2} + \frac{645625x\sqrt{2 + x^2 - x^4}}{15407168(7 + 5x^2)} + \frac{3086453E\left(\sin^{-1}\left(\frac{x\sqrt{2+x^2-x^4}}{7+5x^2}\right)\right)}{1386624}$$

Mathematica [C] time = 0.402093, size = 244, normalized size = 1.91

$$-67352691i\sqrt{2}(5x^2 + 7)^2 \sqrt{-x^4 + x^2 + 2}\text{EllipticF}\left(i \sinh^{-1}(x), -\frac{1}{2}\right) - 1080258550x^7 - 737347940x^5 + 3876617542x^3 + \dots$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((7 + 5*x^2)^3*(2 + x^2 - x^4)^(3/2)),x]
```

```
[Out] (3857257460*x + 3876617542*x^3 - 737347940*x^5 - 1080258550*x^7 + (43210342 *I)*Sqrt[2]*(7 + 5*x^2)^2*Sqrt[2 + x^2 - x^4]*EllipticE[I*ArcSinh[x], -1/2]
```

- (67352691*I)*Sqrt[2]*(7 + 5*x^2)^2*Sqrt[2 + x^2 - x^4]*EllipticF[I*ArcSinh[x], -1/2] + (3042271575*I)*Sqrt[2]*Sqrt[2 + x^2 - x^4]*EllipticPi[5/7, I*ArcSinh[x], -1/2] + (4346102250*I)*Sqrt[2]*x^2*Sqrt[2 + x^2 - x^4]*EllipticPi[5/7, I*ArcSinh[x], -1/2] + (1552179375*I)*Sqrt[2]*x^4*Sqrt[2 + x^2 - x^4]*EllipticPi[5/7, I*ArcSinh[x], -1/2])/((1941303168*(7 + 5*x^2)^2*Sqrt[2 + x^2 - x^4])

Maple [A] time = 0.023, size = 212, normalized size = 1.7

$$\frac{625x}{32368(5x^2+7)^2}\sqrt{-x^4+x^2+2} + \frac{645625x}{77035840x^2+107850176}\sqrt{-x^4+x^2+2} + 2\frac{1}{\sqrt{-x^4+x^2+2}}\left(-\frac{4909x^3}{707472} + \frac{4915}{353736}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(5*x^2+7)^3/(-x^4+x^2+2)^(3/2),x)

[Out] 625/32368*x*(-x^4+x^2+2)^(1/2)/(5*x^2+7)^2+645625/15407168*x*(-x^4+x^2+2)^(1/2)/(5*x^2+7)+2*(-4909/707472*x^3+4915/353736*x)/(-x^4+x^2+2)^(1/2)+60409/46221504*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticF(1/2*x*2^(1/2),I*2^(1/2))+3086453/277329024*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticE(1/2*x*2^(1/2),I*2^(1/2))-6898575/107850176*2^(1/2)*(1-1/2*x^2)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticPi(1/2*x*2^(1/2),-10/7,I*2^(1/2))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(-x^4+x^2+2)^{\frac{3}{2}}(5x^2+7)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(5*x^2+7)^3/(-x^4+x^2+2)^(3/2),x, algorithm="maxima")

[Out] integrate(1/((-x^4 + x^2 + 2)^(3/2)*(5*x^2 + 7)^3), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-x^4+x^2+2}}{125x^{14}+275x^{12}-690x^{10}-2202x^8-291x^6+4011x^4+4312x^2+1372},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(5*x^2+7)^3/(-x^4+x^2+2)^(3/2),x, algorithm="fricas")

[Out] integral(sqrt(-x^4 + x^2 + 2)/(125*x^14 + 275*x^12 - 690*x^10 - 2202*x^8 - 291*x^6 + 4011*x^4 + 4312*x^2 + 1372), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(- (x^2 - 2)(x^2 + 1))^{\frac{3}{2}} (5x^2 + 7)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(5*x**2+7)**3/(-x**4+x**2+2)**(3/2),x)

[Out] Integral(1/((- (x**2 - 2)*(x**2 + 1))**(3/2)*(5*x**2 + 7)**3), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(-x^4 + x^2 + 2)^{\frac{3}{2}} (5x^2 + 7)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(5*x^2+7)^3/(-x^4+x^2+2)^(3/2),x, algorithm="giac")

[Out] integrate(1/((-x^4 + x^2 + 2)^(3/2)*(5*x^2 + 7)^3), x)

3.348 $\int (7 + 5x^2)^4 \sqrt{4 + 3x^2 + x^4} dx$

Optimal. Leaf size=242

$$\frac{33159(x^2 + 2) \sqrt{\frac{x^4 + 3x^2 + 4}{(x^2 + 2)^2}} \text{EllipticF}\left(2 \tan^{-1}\left(\frac{x}{\sqrt{2}}\right), \frac{1}{8}\right)}{11\sqrt{2}\sqrt{x^4 + 3x^2 + 4}} + \frac{625}{11}(x^4 + 3x^2 + 4)^{3/2} x^5 + \frac{23500}{99}(x^4 + 3x^2 + 4)^{3/2} x^3 + \frac{3050}{11}(x^4 + 3x^2 + 4)^{3/2} x + \frac{1}{33}(4516x^2 + 18727)\sqrt{x^4 + 3x^2 + 4}$$

```
[Out] (51665*x*Sqrt[4 + 3*x^2 + x^4])/(33*(2 + x^2)) + (x*(18727 + 4516*x^2)*Sqrt[4 + 3*x^2 + x^4])/33 + (3050*x*(4 + 3*x^2 + x^4)^(3/2))/11 + (23500*x^3*(4 + 3*x^2 + x^4)^(3/2))/99 + (625*x^5*(4 + 3*x^2 + x^4)^(3/2))/11 - (51665*Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE[2*ArcTan[x/Sqrt[2]], 1/8])/(33*Sqrt[4 + 3*x^2 + x^4]) + (33159*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(11*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4])
```

Rubi [A] time = 0.150363, antiderivative size = 242, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$, Rules used = {1206, 1679, 1176, 1197, 1103, 1195}

$$\frac{625}{11}(x^4 + 3x^2 + 4)^{3/2} x^5 + \frac{23500}{99}(x^4 + 3x^2 + 4)^{3/2} x^3 + \frac{3050}{11}(x^4 + 3x^2 + 4)^{3/2} x + \frac{1}{33}(4516x^2 + 18727)\sqrt{x^4 + 3x^2 + 4}$$

Antiderivative was successfully verified.

```
[In] Int[(7 + 5*x^2)^4*Sqrt[4 + 3*x^2 + x^4], x]
```

```
[Out] (51665*x*Sqrt[4 + 3*x^2 + x^4])/(33*(2 + x^2)) + (x*(18727 + 4516*x^2)*Sqrt[4 + 3*x^2 + x^4])/33 + (3050*x*(4 + 3*x^2 + x^4)^(3/2))/11 + (23500*x^3*(4 + 3*x^2 + x^4)^(3/2))/99 + (625*x^5*(4 + 3*x^2 + x^4)^(3/2))/11 - (51665*Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE[2*ArcTan[x/Sqrt[2]], 1/8])/(33*Sqrt[4 + 3*x^2 + x^4]) + (33159*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(11*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4])
```

Rule 1206

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(e^q*x^(2*q - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(4*p + 2*q + 1)), x] + Dist[1/(c*(4*p + 2*q + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*p + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1]
```

Rule 1679

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{q = Expon[Pq, x^2], e = Coeff[Pq, x^2, Expon[Pq, x^2]]}, Simp[(e*x^(2*q - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(2*q + 4*p + 1)), x] + Dist[1/(c*(2*q + 4*p + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*q + 4*p + 1)*Pq - a*e*(2*q - 3)*x^(2*q - 4) - b*e*(2*q + 2*p - 1)*x^(2*q - 2) - c*e*(2*q + 4*p + 1)*x^(2*q), x], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && Expon[Pq, x^2] > 0]
```

$q, x^2] > 1 \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0] \ \&\& \ \text{!LtQ}[p, -1]$

Rule 1176

$\text{Int}[(d_ + (e_)*(x_)^2)*((a_ + (b_)*(x_)^2 + (c_)*(x_)^4)^{(p_)}), x_Symbol] \rightarrow \text{Simp}[(x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*(a + b*x^2 + c*x^4)^p)/(c*(4*p + 1)*(4*p + 3)), x] + \text{Dist}[(2*p)/(c*(4*p + 1)*(4*p + 3)), \text{Int}[\text{Simp}[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) - b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^{(p - 1)}, x], x] /; \text{FreeQ}[\{a, b, c, d, e\}, x] \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0] \ \&\& \ \text{NeQ}[c*d^2 - b*d*e + a*e^2, 0] \ \&\& \ \text{GtQ}[p, 0] \ \&\& \ \text{FractionQ}[p] \ \&\& \ \text{IntegerQ}[2*p]$

Rule 1197

$\text{Int}[(d_ + (e_)*(x_)^2)/\text{Sqrt}[(a_ + (b_)*(x_)^2 + (c_)*(x_)^4)], x_Symbol] \rightarrow \text{With}[\{q = \text{Rt}[c/a, 2]\}, \text{Dist}[(e + d*q)/q, \text{Int}[1/\text{Sqrt}[a + b*x^2 + c*x^4], x], x] - \text{Dist}[e/q, \text{Int}[(1 - q*x^2)/\text{Sqrt}[a + b*x^2 + c*x^4], x], x] /; \text{NeQ}[e + d*q, 0] /; \text{FreeQ}[\{a, b, c, d, e\}, x] \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0] \ \&\& \ \text{PosQ}[c/a]$

Rule 1103

$\text{Int}[1/\text{Sqrt}[(a_ + (b_)*(x_)^2 + (c_)*(x_)^4)], x_Symbol] \rightarrow \text{With}[\{q = \text{Rt}[c/a, 4]\}, \text{Simp}[(1 + q^2*x^2)*\text{Sqrt}[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*\text{EllipticF}[2*\text{ArcTan}[q*x], 1/2 - (b*q^2)/(4*c)]/(2*q*\text{Sqrt}[a + b*x^2 + c*x^4]), x] /; \text{FreeQ}[\{a, b, c\}, x] \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0] \ \&\& \ \text{PosQ}[c/a]$

Rule 1195

$\text{Int}[(d_ + (e_)*(x_)^2)/\text{Sqrt}[(a_ + (b_)*(x_)^2 + (c_)*(x_)^4)], x_Symbol] \rightarrow \text{With}[\{q = \text{Rt}[c/a, 4]\}, -\text{Simp}[(d*x*\text{Sqrt}[a + b*x^2 + c*x^4])/(a*(1 + q^2*x^2)), x] + \text{Simp}[(d*(1 + q^2*x^2)*\text{Sqrt}[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*\text{EllipticE}[2*\text{ArcTan}[q*x], 1/2 - (b*q^2)/(4*c)]/(q*\text{Sqrt}[a + b*x^2 + c*x^4]), x] /; \text{EqQ}[e + d*q^2, 0] /; \text{FreeQ}[\{a, b, c, d, e\}, x] \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0] \ \&\& \ \text{PosQ}[c/a]$

Rubi steps

$$\begin{aligned} \int (7 + 5x^2)^4 \sqrt{4 + 3x^2 + x^4} dx &= \frac{625}{11}x^5(4 + 3x^2 + x^4)^{3/2} + \frac{1}{11} \int \sqrt{4 + 3x^2 + x^4} (26411 + 75460x^2 + 68350x^4 + 23500x^6) dx \\ &= \frac{23500}{99}x^3(4 + 3x^2 + x^4)^{3/2} + \frac{625}{11}x^5(4 + 3x^2 + x^4)^{3/2} + \frac{1}{99} \int \sqrt{4 + 3x^2 + x^4} (237699 + 103500x^2 + 15000x^4) dx \\ &= \frac{3050}{11}x(4 + 3x^2 + x^4)^{3/2} + \frac{23500}{99}x^3(4 + 3x^2 + x^4)^{3/2} + \frac{625}{11}x^5(4 + 3x^2 + x^4)^{3/2} + \frac{1}{693} \int \sqrt{4 + 3x^2 + x^4} (18727 + 4516x^2) dx \\ &= \frac{1}{33}x(18727 + 4516x^2)\sqrt{4 + 3x^2 + x^4} + \frac{3050}{11}x(4 + 3x^2 + x^4)^{3/2} + \frac{23500}{99}x^3(4 + 3x^2 + x^4)^{3/2} + \frac{625}{11}x^5(4 + 3x^2 + x^4)^{3/2} \\ &= \frac{1}{33}x(18727 + 4516x^2)\sqrt{4 + 3x^2 + x^4} + \frac{3050}{11}x(4 + 3x^2 + x^4)^{3/2} + \frac{23500}{99}x^3(4 + 3x^2 + x^4)^{3/2} + \frac{625}{11}x^5(4 + 3x^2 + x^4)^{3/2} \\ &= \frac{51665x\sqrt{4 + 3x^2 + x^4}}{33(2 + x^2)} + \frac{1}{33}x(18727 + 4516x^2)\sqrt{4 + 3x^2 + x^4} + \frac{3050}{11}x(4 + 3x^2 + x^4)^{3/2} + \frac{23500}{99}x^3(4 + 3x^2 + x^4)^{3/2} + \frac{625}{11}x^5(4 + 3x^2 + x^4)^{3/2} \end{aligned}$$

Mathematica [C] time = 0.587995, size = 354, normalized size = 1.46

$$3\sqrt{2}(51665\sqrt{7} - 36253i)\sqrt{\frac{-2ix^2 + \sqrt{7} - 3i}{\sqrt{7} - 3i}}\sqrt{\frac{2ix^2 + \sqrt{7} + 3i}{\sqrt{7} + 3i}}\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{\frac{2i}{\sqrt{7} - 3i}}x\right), \frac{-\sqrt{7} + 3i}{\sqrt{7} + 3i}\right) + 4\sqrt{-\frac{i}{\sqrt{7} - 3i}}x(5625x^{12}$$

Antiderivative was successfully verified.

[In] Integrate[(7 + 5*x^2)^4*Sqrt[4 + 3*x^2 + x^4], x]

[Out] (4*Sqrt[(-I)/(-3*I + Sqrt[7])]*x*(663924 + 1257535*x^2 + 1217475*x^4 + 712748*x^6 + 264075*x^8 + 57250*x^10 + 5625*x^12) - 154995*Sqrt[2]*(3*I + Sqrt[7])*Sqrt[(-3*I + Sqrt[7] - (2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[(3*I + Sqrt[7] + (2*I)*x^2)/(3*I + Sqrt[7])]*EllipticE[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])) + 3*Sqrt[2]*(-36253*I + 51665*Sqrt[7])*Sqrt[(-3*I + Sqrt[7] - (2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[(3*I + Sqrt[7] + (2*I)*x^2)/(3*I + Sqrt[7])]*EllipticF[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7]))/(396*Sqrt[(-I)/(-3*I + Sqrt[7])])*Sqrt[4 + 3*x^2 + x^4))

Maple [C] time = 0.148, size = 292, normalized size = 1.2

$$\frac{625x^9}{11}\sqrt{x^4 + 3x^2 + 4} + \frac{40375x^7}{99}\sqrt{x^4 + 3x^2 + 4} + \frac{189898x^3}{99}\sqrt{x^4 + 3x^2 + 4} + \frac{55327x}{33}\sqrt{x^4 + 3x^2 + 4} + \frac{382496}{33}\sqrt{-6 + 2I^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((5*x^2+7)^4*(x^4+3*x^2+4)^(1/2), x)

[Out] 625/11*x^9*(x^4+3*x^2+4)^(1/2)+40375/99*x^7*(x^4+3*x^2+4)^(1/2)+189898/99*x^3*(x^4+3*x^2+4)^(1/2)+55327/33*x*(x^4+3*x^2+4)^(1/2)+382496/33/(-6+2*I^7^(1/2))^(1/2)*(1-(-3/8+1/8*I^7^(1/2))*x^2)^(1/2)*(1-(-3/8-1/8*I^7^(1/2))*x^2)^(1/2)/(x^4+3*x^2+4)^(1/2)*EllipticF(1/4*x*(-6+2*I^7^(1/2))^(1/2), 1/4*(2+6*I^7^(1/2))^(1/2))-1653280/33/(-6+2*I^7^(1/2))^(1/2)*(1-(-3/8+1/8*I^7^(1/2))*x^2)^(1/2)*(1-(-3/8-1/8*I^7^(1/2))*x^2)^(1/2)/(x^4+3*x^2+4)^(1/2)/(I^7^(1/2)+3)*(EllipticF(1/4*x*(-6+2*I^7^(1/2))^(1/2), 1/4*(2+6*I^7^(1/2))^(1/2))-EllipticE(1/4*x*(-6+2*I^7^(1/2))^(1/2), 1/4*(2+6*I^7^(1/2))^(1/2)))+3650/3*x^5*(x^4+3*x^2+4)^(1/2)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{x^4 + 3x^2 + 4}(5x^2 + 7)^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^2+7)^4*(x^4+3*x^2+4)^(1/2), x, algorithm="maxima")

[Out] integrate(sqrt(x^4 + 3*x^2 + 4)*(5*x^2 + 7)^4, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(625x^8 + 3500x^6 + 7350x^4 + 6860x^2 + 2401\right)\sqrt{x^4 + 3x^2 + 4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^2+7)^4*(x^4+3*x^2+4)^(1/2),x, algorithm="fricas")

[Out] integral((625*x^8 + 3500*x^6 + 7350*x^4 + 6860*x^2 + 2401)*sqrt(x^4 + 3*x^2 + 4), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{(x^2 - x + 2)(x^2 + x + 2)}(5x^2 + 7)^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x**2+7)**4*(x**4+3*x**2+4)**(1/2),x)

[Out] Integral(sqrt((x**2 - x + 2)*(x**2 + x + 2))*(5*x**2 + 7)**4, x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{x^4 + 3x^2 + 4}(5x^2 + 7)^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^2+7)^4*(x^4+3*x^2+4)^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(x^4 + 3*x^2 + 4)*(5*x^2 + 7)^4, x)

$$3.349 \quad \int (7 + 5x^2)^3 \sqrt{4 + 3x^2 + x^4} dx$$

Optimal. Leaf size=221

$$\frac{1301(x^2 + 2) \sqrt{\frac{x^4 + 3x^2 + 4}{(x^2 + 2)^2}} \text{EllipticF}\left(2 \tan^{-1}\left(\frac{x}{\sqrt{2}}\right), \frac{1}{8}\right)}{3\sqrt{2}\sqrt{x^4 + 3x^2 + 4}} + \frac{125}{9}(x^4 + 3x^2 + 4)^{3/2} x^3 + \frac{275}{7}(x^4 + 3x^2 + 4)^{3/2} x + \frac{1}{21}(407x^2 + 1708)\sqrt{x^4 + 3x^2 + 4} + \frac{4717}{21(x^2 + 2)}$$

```
[Out] (4717*x*Sqrt[4 + 3*x^2 + x^4])/(21*(2 + x^2)) + (x*(1708 + 407*x^2)*Sqrt[4 + 3*x^2 + x^4])/21 + (275*x*(4 + 3*x^2 + x^4)^(3/2))/7 + (125*x^3*(4 + 3*x^2 + x^4)^(3/2))/9 - (4717*Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE[2*ArcTan[x/Sqrt[2]], 1/8])/(21*Sqrt[4 + 3*x^2 + x^4]) + (1301*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(3*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4])
```

Rubi [A] time = 0.112281, antiderivative size = 221, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$, Rules used = {1206, 1679, 1176, 1197, 1103, 1195}

$$\frac{125}{9}(x^4 + 3x^2 + 4)^{3/2} x^3 + \frac{275}{7}(x^4 + 3x^2 + 4)^{3/2} x + \frac{1}{21}(407x^2 + 1708)\sqrt{x^4 + 3x^2 + 4} + \frac{4717\sqrt{x^4 + 3x^2 + 4}x}{21(x^2 + 2)} + \frac{4717}{21(x^2 + 2)}$$

Antiderivative was successfully verified.

```
[In] Int[(7 + 5*x^2)^3*Sqrt[4 + 3*x^2 + x^4], x]
```

```
[Out] (4717*x*Sqrt[4 + 3*x^2 + x^4])/(21*(2 + x^2)) + (x*(1708 + 407*x^2)*Sqrt[4 + 3*x^2 + x^4])/21 + (275*x*(4 + 3*x^2 + x^4)^(3/2))/7 + (125*x^3*(4 + 3*x^2 + x^4)^(3/2))/9 - (4717*Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE[2*ArcTan[x/Sqrt[2]], 1/8])/(21*Sqrt[4 + 3*x^2 + x^4]) + (1301*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(3*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4])
```

Rule 1206

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(e^q*x^(2*q - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(4*p + 2*q + 1)), x] + Dist[1/(c*(4*p + 2*q + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*p + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x] /;
```

Rule 1679

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{q = Expon[Pq, x^2], e = Coeff[Pq, x^2, Expon[Pq, x^2]]}, Simp[(e*x^(2*q - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(2*q + 4*p + 1)), x] + Dist[1/(c*(2*q + 4*p + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*q + 4*p + 1)*Pq - a*e*(2*q - 3)*x^(2*q - 4) - b*e*(2*q + 2*p - 1)*x^(2*q - 2) - c*e*(2*q + 4*p + 1)*x^(2*q), x], x] /;
```

Rule 1176

```
Int[((d_) + (e_)*(x_)^2)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol]
:> Simp[(x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*(a + b*x^2 + c*x^4)^p)/(c*(4*p + 1)*(4*p + 3)), x] + Dist[(2*p)/(c*(4*p + 1)*(4*p + 3)),
Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) -
b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1197

```
Int[((d_) + (e_)*(x_)^2)/Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4], x_Symbol]
:> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4],
x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; NeQ[e + d*q, 0] /;
FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4], x_Symbol] :> With[{q = Rt[c/a, 4]},
Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]),
x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_)*(x_)^2)/Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4], x_Symbol]
:> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^2*x^2)),
x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c*x^4]),
x] /; EqQ[e + d*q^2, 0] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
PosQ[c/a]
```

Rubi steps

$$\begin{aligned} \int (7 + 5x^2)^3 \sqrt{4 + 3x^2 + x^4} dx &= \frac{125}{9} x^3 (4 + 3x^2 + x^4)^{3/2} + \frac{1}{9} \int \sqrt{4 + 3x^2 + x^4} (3087 + 5115x^2 + 2475x^4) dx \\ &= \frac{275}{7} x (4 + 3x^2 + x^4)^{3/2} + \frac{125}{9} x^3 (4 + 3x^2 + x^4)^{3/2} + \frac{1}{63} \int (11709 + 6105x^2) \sqrt{4 + 3x^2 + x^4} dx \\ &= \frac{1}{21} x (1708 + 407x^2) \sqrt{4 + 3x^2 + x^4} + \frac{275}{7} x (4 + 3x^2 + x^4)^{3/2} + \frac{125}{9} x^3 (4 + 3x^2 + x^4)^{3/2} \\ &= \frac{1}{21} x (1708 + 407x^2) \sqrt{4 + 3x^2 + x^4} + \frac{275}{7} x (4 + 3x^2 + x^4)^{3/2} + \frac{125}{9} x^3 (4 + 3x^2 + x^4)^{3/2} \\ &= \frac{4717x\sqrt{4 + 3x^2 + x^4}}{21(2 + x^2)} + \frac{1}{21} x (1708 + 407x^2) \sqrt{4 + 3x^2 + x^4} + \frac{275}{7} x (4 + 3x^2 + x^4)^{3/2} + \end{aligned}$$

Mathematica [C] time = 0.515225, size = 349, normalized size = 1.58

$$3\sqrt{2} (4717\sqrt{7} - 3409i) \sqrt{\frac{-2ix^2 + \sqrt{7} - 3i}{\sqrt{7} - 3i}} \sqrt{\frac{2ix^2 + \sqrt{7} + 3i}{\sqrt{7} + 3i}} \text{EllipticF}\left(i \sinh^{-1}\left(\sqrt{\frac{-2i}{\sqrt{7} - 3i}} x\right), \frac{-\sqrt{7} + 3i}{\sqrt{7} + 3i}\right) + 4\sqrt{\frac{i}{\sqrt{7} - 3i}} x (875x^{10} + 7725x^7 + 252\sqrt{-\frac{i}{\sqrt{7}}})$$

Antiderivative was successfully verified.

[In] Integrate[(7 + 5*x^2)^3*Sqrt[4 + 3*x^2 + x^4],x]

[Out] (4*Sqrt[(-I)/(-3*I + Sqrt[7])]*x*(60096 + 93656*x^2 + 71862*x^4 + 30946*x^6 + 7725*x^8 + 875*x^10) - 14151*Sqrt[2]*(3*I + Sqrt[7])*Sqrt[(-3*I + Sqrt[7] - (2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[(3*I + Sqrt[7] + (2*I)*x^2)/(3*I + Sqrt[7])]*EllipticE[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])] + 3*Sqrt[2]*(-3409*I + 4717*Sqrt[7])*Sqrt[(-3*I + Sqrt[7] - (2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[(3*I + Sqrt[7] + (2*I)*x^2)/(3*I + Sqrt[7])]*EllipticF[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])])/(252*Sqrt[(-I)/(-3*I + Sqrt[7])]*Sqrt[4 + 3*x^2 + x^4])

Maple [C] time = 0.011, size = 275, normalized size = 1.2

$$\frac{125x^7}{9}\sqrt{x^4+3x^2+4} + \frac{1700x^5}{21}\sqrt{x^4+3x^2+4} + \frac{12146x^3}{63}\sqrt{x^4+3x^2+4} + \frac{5008x}{21}\sqrt{x^4+3x^2+4} + \frac{35120}{21\sqrt{-6+2i\sqrt{7}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((5*x^2+7)^3*(x^4+3*x^2+4)^(1/2),x)

[Out] 125/9*x^7*(x^4+3*x^2+4)^(1/2)+1700/21*x^5*(x^4+3*x^2+4)^(1/2)+12146/63*x^3*(x^4+3*x^2+4)^(1/2)+5008/21*x*(x^4+3*x^2+4)^(1/2)+35120/21/(-6+2*I*7^(1/2))^(1/2)*(1-(-3/8+1/8*I*7^(1/2))*x^2)^(1/2)*(1-(-3/8-1/8*I*7^(1/2))*x^2)^(1/2)/(x^4+3*x^2+4)^(1/2)*EllipticF(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))-150944/21/(-6+2*I*7^(1/2))^(1/2)*(1-(-3/8+1/8*I*7^(1/2))*x^2)^(1/2)*(1-(-3/8-1/8*I*7^(1/2))*x^2)^(1/2)/(x^4+3*x^2+4)^(1/2)/(I*7^(1/2)+3)*(EllipticF(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))-EllipticE(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2)))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{x^4 + 3x^2 + 4}(5x^2 + 7)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^2+7)^3*(x^4+3*x^2+4)^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(x^4 + 3*x^2 + 4)*(5*x^2 + 7)^3, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(125x^6 + 525x^4 + 735x^2 + 343\right)\sqrt{x^4 + 3x^2 + 4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^2+7)^3*(x^4+3*x^2+4)^(1/2),x, algorithm="fricas")

[Out] integral((125*x^6 + 525*x^4 + 735*x^2 + 343)*sqrt(x^4 + 3*x^2 + 4), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{(x^2 - x + 2)(x^2 + x + 2)} (5x^2 + 7)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x**2+7)**3*(x**4+3*x**2+4)**(1/2),x)

[Out] Integral(sqrt((x**2 - x + 2)*(x**2 + x + 2))*(5*x**2 + 7)**3, x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{x^4 + 3x^2 + 4} (5x^2 + 7)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^2+7)^3*(x^4+3*x^2+4)^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(x^4 + 3*x^2 + 4)*(5*x^2 + 7)^3, x)

3.350 $\int (7 + 5x^2)^2 \sqrt{4 + 3x^2 + x^4} dx$

Optimal. Leaf size=198

$$\frac{81(x^2 + 2) \sqrt{\frac{x^4 + 3x^2 + 4}{(x^2 + 2)^2}} \text{EllipticF}\left(2 \tan^{-1}\left(\frac{x}{\sqrt{2}}\right), \frac{1}{8}\right)}{\sqrt{2}\sqrt{x^4 + 3x^2 + 4}} + \frac{25}{7}x(x^4 + 3x^2 + 4)^{3/2} + \frac{1}{7}x(38x^2 + 119)\sqrt{x^4 + 3x^2 + 4} + \frac{319x}{7}$$

[Out] (319*x*Sqrt[4 + 3*x^2 + x^4])/(7*(2 + x^2)) + (x*(119 + 38*x^2)*Sqrt[4 + 3*x^2 + x^4])/7 + (25*x*(4 + 3*x^2 + x^4)^(3/2))/7 - (319*Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE[2*ArcTan[x/Sqrt[2]], 1/8])/(7*Sqrt[4 + 3*x^2 + x^4]) + (81*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(Sqrt[2]*Sqrt[4 + 3*x^2 + x^4])

Rubi [A] time = 0.0772802, antiderivative size = 198, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {1206, 1176, 1197, 1103, 1195}

$$\frac{25}{7}x(x^4 + 3x^2 + 4)^{3/2} + \frac{1}{7}x(38x^2 + 119)\sqrt{x^4 + 3x^2 + 4} + \frac{319x\sqrt{x^4 + 3x^2 + 4}}{7(x^2 + 2)} + \frac{81(x^2 + 2) \sqrt{\frac{x^4 + 3x^2 + 4}{(x^2 + 2)^2}} F\left(2 \tan^{-1}\left(\frac{x}{\sqrt{2}}\right)\right)}{\sqrt{2}\sqrt{x^4 + 3x^2 + 4}}$$

Antiderivative was successfully verified.

[In] Int[(7 + 5*x^2)^2*Sqrt[4 + 3*x^2 + x^4], x]

[Out] (319*x*Sqrt[4 + 3*x^2 + x^4])/(7*(2 + x^2)) + (x*(119 + 38*x^2)*Sqrt[4 + 3*x^2 + x^4])/7 + (25*x*(4 + 3*x^2 + x^4)^(3/2))/7 - (319*Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE[2*ArcTan[x/Sqrt[2]], 1/8])/(7*Sqrt[4 + 3*x^2 + x^4]) + (81*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(Sqrt[2]*Sqrt[4 + 3*x^2 + x^4])

Rule 1206

Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(e^q*x^(2*q - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(4*p + 2*q + 1)), x] + Dist[1/(c*(4*p + 2*q + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*q + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x] /;

FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1]

Rule 1176

Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*(a + b*x^2 + c*x^4)^p)/(c*(4*p + 1)*(4*p + 3)), x] + Dist[(2*p)/(c*(4*p + 1)*(4*p + 3)), Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) - b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /;

FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol]
:> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol]
:> With[{q = Rt[c/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol]
:> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\begin{aligned} \int (7 + 5x^2)^2 \sqrt{4 + 3x^2 + x^4} dx &= \frac{25}{7}x(4 + 3x^2 + x^4)^{3/2} + \frac{1}{7} \int (243 + 190x^2) \sqrt{4 + 3x^2 + x^4} dx \\ &= \frac{1}{7}x(119 + 38x^2) \sqrt{4 + 3x^2 + x^4} + \frac{25}{7}x(4 + 3x^2 + x^4)^{3/2} + \frac{1}{105} \int \frac{7440 + 4785x^2}{\sqrt{4 + 3x^2 + x^4}} dx \\ &= \frac{1}{7}x(119 + 38x^2) \sqrt{4 + 3x^2 + x^4} + \frac{25}{7}x(4 + 3x^2 + x^4)^{3/2} - \frac{638}{7} \int \frac{1 - \frac{x^2}{2}}{\sqrt{4 + 3x^2 + x^4}} dx + \frac{319\sqrt{7}}{7} \\ &= \frac{319x\sqrt{4 + 3x^2 + x^4}}{7(2 + x^2)} + \frac{1}{7}x(119 + 38x^2) \sqrt{4 + 3x^2 + x^4} + \frac{25}{7}x(4 + 3x^2 + x^4)^{3/2} - \frac{638}{7} \int \frac{1 - \frac{x^2}{2}}{\sqrt{4 + 3x^2 + x^4}} dx + \frac{319\sqrt{7}}{7} \end{aligned}$$

Mathematica [C] time = 0.473367, size = 343, normalized size = 1.73

$$\frac{\sqrt{2} (319\sqrt{7} - 35i) \sqrt{\frac{-2ix^2 + \sqrt{7} - 3i}{\sqrt{7} - 3i}} \sqrt{\frac{2ix^2 + \sqrt{7} + 3i}{\sqrt{7} + 3i}} \text{EllipticF}\left(i \sinh^{-1}\left(\sqrt{\frac{-2i}{\sqrt{7} - 3i}}x\right), \frac{-\sqrt{7} + 3i}{\sqrt{7} + 3i}\right) + 4\sqrt{-\frac{i}{\sqrt{7} - 3i}}x(25x^8 + 188x^6 + 65x^4) + 28\sqrt{-\frac{i}{\sqrt{7} - 3i}}\sqrt{x^4 + 3x^2 + 4}}{28\sqrt{-\frac{i}{\sqrt{7} - 3i}}\sqrt{x^4 + 3x^2 + 4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(7 + 5*x^2)^2*Sqrt[4 + 3*x^2 + x^4],x]
```

```
[Out] (4*Sqrt[(-I)/(-3*I + Sqrt[7])]*x*(876 + 1109*x^2 + 658*x^4 + 188*x^6 + 25*x^8) - 319*Sqrt[2]*(3*I + Sqrt[7])*Sqrt[(-3*I + Sqrt[7] - (2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[(3*I + Sqrt[7] + (2*I)*x^2)/(3*I + Sqrt[7])]*EllipticE[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])] + Sqrt[2]*(-35*I + 319*Sqrt[7])*Sqrt[(-3*I + Sqrt[7] - (2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[(3*I + Sqrt[7] + (2*I)*x^2)/(3*I + Sqrt[7])]*EllipticF[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])])/(28*Sqrt[(-I)/(-3*I + Sqrt[7])]*Sqrt[4 + 3*x^2 + x^4])
```

Maple [C] time = 0.009, size = 258, normalized size = 1.3

$$\frac{25x^5}{7}\sqrt{x^4+3x^2+4} + \frac{113x^3}{7}\sqrt{x^4+3x^2+4} + \frac{219x}{7}\sqrt{x^4+3x^2+4} + \frac{1984}{7\sqrt{-6+2i\sqrt{7}}}\sqrt{1-\left(-\frac{3}{8}+\frac{i}{8}\sqrt{7}\right)x^2}\sqrt{1-\left(-\frac{3}{8}-\frac{i}{8}\sqrt{7}\right)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((5*x^2+7)^2*(x^4+3*x^2+4)^(1/2), x)

[Out] 25/7*x^5*(x^4+3*x^2+4)^(1/2)+113/7*x^3*(x^4+3*x^2+4)^(1/2)+219/7*x*(x^4+3*x^2+4)^(1/2)+1984/7/(-6+2*I*7^(1/2))^(1/2)*(1-(-3/8+1/8*I*7^(1/2))*x^2)^(1/2)*(1-(-3/8-1/8*I*7^(1/2))*x^2)^(1/2)/(x^4+3*x^2+4)^(1/2)*EllipticF(1/4*x*(-6+2*I*7^(1/2))^(1/2), 1/4*(2+6*I*7^(1/2))^(1/2))-10208/7/(-6+2*I*7^(1/2))^(1/2)*(1-(-3/8+1/8*I*7^(1/2))*x^2)^(1/2)*(1-(-3/8-1/8*I*7^(1/2))*x^2)^(1/2)/(x^4+3*x^2+4)^(1/2)/(I*7^(1/2)+3)*(EllipticF(1/4*x*(-6+2*I*7^(1/2))^(1/2), 1/4*(2+6*I*7^(1/2))^(1/2))-EllipticE(1/4*x*(-6+2*I*7^(1/2))^(1/2), 1/4*(2+6*I*7^(1/2))^(1/2)))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{x^4 + 3x^2 + 4}(5x^2 + 7)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^2+7)^2*(x^4+3*x^2+4)^(1/2), x, algorithm="maxima")

[Out] integrate(sqrt(x^4 + 3*x^2 + 4)*(5*x^2 + 7)^2, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(25x^4 + 70x^2 + 49\right)\sqrt{x^4 + 3x^2 + 4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^2+7)^2*(x^4+3*x^2+4)^(1/2), x, algorithm="fricas")

[Out] integral((25*x^4 + 70*x^2 + 49)*sqrt(x^4 + 3*x^2 + 4), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{(x^2 - x + 2)(x^2 + x + 2)}(5x^2 + 7)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x**2+7)**2*(x**4+3*x**2+4)**(1/2), x)

[Out] Integral(sqrt((x**2 - x + 2)*(x**2 + x + 2))*(5*x**2 + 7)**2, x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{x^4 + 3x^2 + 4}(5x^2 + 7)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^2+7)^2*(x^4+3*x^2+4)^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(x^4 + 3*x^2 + 4)*(5*x^2 + 7)^2, x)

3.351 $\int (7 + 5x^2) \sqrt{4 + 3x^2 + x^4} dx$

Optimal. Leaf size=177

$$\frac{49(x^2 + 2) \sqrt{\frac{x^4 + 3x^2 + 4}{(x^2 + 2)^2}} \operatorname{EllipticF}\left(2 \tan^{-1}\left(\frac{x}{\sqrt{2}}\right), \frac{1}{8}\right)}{3\sqrt{2}\sqrt{x^4 + 3x^2 + 4}} + \frac{1}{3}(3x^2 + 10)\sqrt{x^4 + 3x^2 + 4x} + \frac{9\sqrt{x^4 + 3x^2 + 4x}}{x^2 + 2} - \frac{9\sqrt{2}(x^2 + 2)}{\sqrt{x^4 + 3x^2 + 4}}$$

[Out] (9*x*Sqrt[4 + 3*x^2 + x^4])/(2 + x^2) + (x*(10 + 3*x^2)*Sqrt[4 + 3*x^2 + x^4])/3 - (9*Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE[2*ArcTan[x/Sqrt[2]], 1/8])/Sqrt[4 + 3*x^2 + x^4] + (49*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(3*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4])

Rubi [A] time = 0.0539762, antiderivative size = 177, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1176, 1197, 1103, 1195}

$$\frac{1}{3}(3x^2 + 10)\sqrt{x^4 + 3x^2 + 4x} + \frac{9\sqrt{x^4 + 3x^2 + 4x}}{x^2 + 2} + \frac{49(x^2 + 2) \sqrt{\frac{x^4 + 3x^2 + 4}{(x^2 + 2)^2}} F\left(2 \tan^{-1}\left(\frac{x}{\sqrt{2}}\right) \middle| \frac{1}{8}\right)}{3\sqrt{2}\sqrt{x^4 + 3x^2 + 4}} - \frac{9\sqrt{2}(x^2 + 2) \sqrt{\frac{x^4 + 3x^2 + 4}{(x^2 + 2)^2}}}{\sqrt{x^4 + 3x^2 + 4}}$$

Antiderivative was successfully verified.

[In] Int[(7 + 5*x^2)*Sqrt[4 + 3*x^2 + x^4], x]

[Out] (9*x*Sqrt[4 + 3*x^2 + x^4])/(2 + x^2) + (x*(10 + 3*x^2)*Sqrt[4 + 3*x^2 + x^4])/3 - (9*Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE[2*ArcTan[x/Sqrt[2]], 1/8])/Sqrt[4 + 3*x^2 + x^4] + (49*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(3*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4])

Rule 1176

Int[((d_) + (e_)*(x_)^2)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] :> Simp[(x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*(a + b*x^2 + c*x^4)^p)/(c*(4*p + 1)*(4*p + 3)), x] + Dist[(2*p)/(c*(4*p + 1)*(4*p + 3)), Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) - b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]

Rule 1197

Int[((d_) + (e_)*(x_)^2)/Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4], x_Symbol] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; NeQ[e + d*q, 0] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]

Rule 1103

Int[1/Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4], x_Symbol] :> With[{q = Rt[c/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4])

), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]

Rule 1195

Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]

Rubi steps

$$\begin{aligned} \int (7 + 5x^2) \sqrt{4 + 3x^2 + x^4} dx &= \frac{1}{3}x(10 + 3x^2) \sqrt{4 + 3x^2 + x^4} + \frac{1}{15} \int \frac{220 + 135x^2}{\sqrt{4 + 3x^2 + x^4}} dx \\ &= \frac{1}{3}x(10 + 3x^2) \sqrt{4 + 3x^2 + x^4} - 18 \int \frac{1 - \frac{x^2}{2}}{\sqrt{4 + 3x^2 + x^4}} dx + \frac{98}{3} \int \frac{1}{\sqrt{4 + 3x^2 + x^4}} dx \\ &= \frac{9x\sqrt{4 + 3x^2 + x^4}}{2 + x^2} + \frac{1}{3}x(10 + 3x^2) \sqrt{4 + 3x^2 + x^4} - \frac{9\sqrt{2}(2 + x^2) \sqrt{\frac{4 + 3x^2 + x^4}{(2 + x^2)^2}} E\left(2 \tan^{-1}\left(\frac{\sqrt{4 + 3x^2 + x^4}}{2 + x^2}\right)\right)}{\sqrt{4 + 3x^2 + x^4}} \end{aligned}$$

Mathematica [C] time = 0.436806, size = 338, normalized size = 1.91

$$\frac{\sqrt{2}(27\sqrt{7} - 7i) \sqrt{\frac{-2ix^2 + \sqrt{7} - 3i}{\sqrt{7} - 3i}} \sqrt{\frac{2ix^2 + \sqrt{7} + 3i}{\sqrt{7} + 3i}} \text{EllipticF}\left(i \sinh^{-1}\left(\sqrt{\frac{-2i}{\sqrt{7} - 3i}} x\right), \frac{-\sqrt{7} + 3i}{\sqrt{7} + 3i}\right) + 4\sqrt{-\frac{i}{\sqrt{7} - 3i}} x(3x^6 + 19x^4 + 42x^2 + 4)}{12\sqrt{-\frac{i}{\sqrt{7} - 3i}} \sqrt{x^4 + 3x^2 + 4}}$$

Antiderivative was successfully verified.

[In] Integrate[(7 + 5*x^2)*Sqrt[4 + 3*x^2 + x^4], x]

[Out] (4*Sqrt[(-I)/(-3*I + Sqrt[7])]*x*(40 + 42*x^2 + 19*x^4 + 3*x^6) - 27*Sqrt[2]*(3*I + Sqrt[7])*Sqrt[(-3*I + Sqrt[7] - (2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[(3*I + Sqrt[7] + (2*I)*x^2)/(3*I + Sqrt[7])]*EllipticE[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])) + Sqrt[2]*(-7*I + 27*Sqrt[7])*Sqrt[(-3*I + Sqrt[7] - (2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[(3*I + Sqrt[7] + (2*I)*x^2)/(3*I + Sqrt[7])]*EllipticF[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7]))]/(12*Sqrt[(-I)/(-3*I + Sqrt[7])]*Sqrt[4 + 3*x^2 + x^4])

Maple [C] time = 0.007, size = 240, normalized size = 1.4

$$x^3 \sqrt{x^4 + 3x^2 + 4} + \frac{10x}{3} \sqrt{x^4 + 3x^2 + 4} + \frac{176}{3\sqrt{-6 + 2i\sqrt{7}}} \sqrt{1 - \left(-\frac{3}{8} + \frac{i}{8}\sqrt{7}\right)x^2} \sqrt{1 - \left(-\frac{3}{8} - \frac{i}{8}\sqrt{7}\right)x^2} \text{EllipticF}\left(\frac{x\sqrt{-6 + 2i\sqrt{7}}}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((5*x^2+7)*(x^4+3*x^2+4)^(1/2), x)

```
[Out] x^3*(x^4+3*x^2+4)^(1/2)+10/3*x*(x^4+3*x^2+4)^(1/2)+176/3/(-6+2*I*7^(1/2))^(1/2)*(1-(-3/8+1/8*I*7^(1/2))*x^2)^(1/2)*(1-(-3/8-1/8*I*7^(1/2))*x^2)^(1/2)/(x^4+3*x^2+4)^(1/2)*EllipticF(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))-288/(-6+2*I*7^(1/2))^(1/2)*(1-(-3/8+1/8*I*7^(1/2))*x^2)^(1/2)*(1-(-3/8-1/8*I*7^(1/2))*x^2)^(1/2)/(x^4+3*x^2+4)^(1/2)/(I*7^(1/2)+3)*(EllipticF(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))-EllipticE(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{x^4 + 3x^2 + 4}(5x^2 + 7) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)*(x^4+3*x^2+4)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^4 + 3*x^2 + 4)*(5*x^2 + 7), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{x^4 + 3x^2 + 4}(5x^2 + 7), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)*(x^4+3*x^2+4)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^4 + 3*x^2 + 4)*(5*x^2 + 7), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{(x^2 - x + 2)(x^2 + x + 2)}(5x^2 + 7) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+7)*(x**4+3*x**2+4)**(1/2),x)
```

```
[Out] Integral(sqrt((x**2 - x + 2)*(x**2 + x + 2))*(5*x**2 + 7), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{x^4 + 3x^2 + 4}(5x^2 + 7) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)*(x^4+3*x^2+4)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^4 + 3*x^2 + 4)*(5*x^2 + 7), x)
```

3.352 $\int \sqrt{4 + 3x^2 + x^4} dx$

Optimal. Leaf size=169

$$\frac{7(x^2 + 2) \sqrt{\frac{x^4 + 3x^2 + 4}{(x^2 + 2)^2}} \text{EllipticF}\left(2 \tan^{-1}\left(\frac{x}{\sqrt{2}}\right), \frac{1}{8}\right)}{3\sqrt{2}\sqrt{x^4 + 3x^2 + 4}} + \frac{\sqrt{x^4 + 3x^2 + 4}x}{x^2 + 2} + \frac{1}{3}\sqrt{x^4 + 3x^2 + 4}x - \frac{\sqrt{2}(x^2 + 2) \sqrt{\frac{x^4 + 3x^2 + 4}{(x^2 + 2)^2}} E\left(2 \tan^{-1}\left(\frac{x}{\sqrt{2}}\right)\right)}{\sqrt{x^4 + 3x^2 + 4}}$$

[Out] (x*Sqrt[4 + 3*x^2 + x^4])/3 + (x*Sqrt[4 + 3*x^2 + x^4])/(2 + x^2) - (Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE[2*ArcTan[x/Sqrt[2]], 1/8])/Sqrt[4 + 3*x^2 + x^4] + (7*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(3*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4])

Rubi [A] time = 0.0508969, antiderivative size = 169, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {1091, 1197, 1103, 1195}

$$\frac{\sqrt{x^4 + 3x^2 + 4}x}{x^2 + 2} + \frac{1}{3}\sqrt{x^4 + 3x^2 + 4}x + \frac{7(x^2 + 2) \sqrt{\frac{x^4 + 3x^2 + 4}{(x^2 + 2)^2}} F\left(2 \tan^{-1}\left(\frac{x}{\sqrt{2}}\right) \middle| \frac{1}{8}\right)}{3\sqrt{2}\sqrt{x^4 + 3x^2 + 4}} - \frac{\sqrt{2}(x^2 + 2) \sqrt{\frac{x^4 + 3x^2 + 4}{(x^2 + 2)^2}} E\left(2 \tan^{-1}\left(\frac{x}{\sqrt{2}}\right)\right)}{\sqrt{x^4 + 3x^2 + 4}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[4 + 3*x^2 + x^4], x]

[Out] (x*Sqrt[4 + 3*x^2 + x^4])/3 + (x*Sqrt[4 + 3*x^2 + x^4])/(2 + x^2) - (Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE[2*ArcTan[x/Sqrt[2]], 1/8])/Sqrt[4 + 3*x^2 + x^4] + (7*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(3*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4])

Rule 1091

Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := Simp[(x*(a + b*x^2 + c*x^4)^p)/(4*p + 1), x] + Dist[(2*p)/(4*p + 1), Int[(2*a + b*x^2)*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[2*p]

Rule 1197

Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] := With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; NeQ[e + d*q, 0] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]

Rule 1103

Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] := With[{q = Rt[c/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]), x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]

Rule 1195

Int[((d_) + (e_)*(x_)^2)/Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4], x_Symbol] := With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)]/(q*Sqrt[a + b*x^2 + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]

Rubi steps

$$\begin{aligned} \int \sqrt{4 + 3x^2 + x^4} dx &= \frac{1}{3}x\sqrt{4 + 3x^2 + x^4} + \frac{1}{3} \int \frac{8 + 3x^2}{\sqrt{4 + 3x^2 + x^4}} dx \\ &= \frac{1}{3}x\sqrt{4 + 3x^2 + x^4} - 2 \int \frac{1 - \frac{x^2}{2}}{\sqrt{4 + 3x^2 + x^4}} dx + \frac{14}{3} \int \frac{1}{\sqrt{4 + 3x^2 + x^4}} dx \\ &= \frac{1}{3}x\sqrt{4 + 3x^2 + x^4} + \frac{x\sqrt{4 + 3x^2 + x^4}}{2 + x^2} - \frac{\sqrt{2}(2 + x^2) \sqrt{\frac{4 + 3x^2 + x^4}{(2 + x^2)^2}} E\left(2 \tan^{-1}\left(\frac{x}{\sqrt{2}}\right) \middle| \frac{1}{8}\right)}{\sqrt{4 + 3x^2 + x^4}} + \frac{7(2 + x^2)}{\sqrt{4 + 3x^2 + x^4}} \end{aligned}$$

Mathematica [C] time = 0.353942, size = 331, normalized size = 1.96

$$\frac{\sqrt{2}(3\sqrt{7} - 7i) \sqrt{\frac{-2ix^2 + \sqrt{7} - 3i}{\sqrt{7} - 3i}} \sqrt{\frac{2ix^2 + \sqrt{7} + 3i}{\sqrt{7} + 3i}} \text{EllipticF}\left(i \sinh^{-1}\left(\sqrt{\frac{-2i}{\sqrt{7} - 3i}} x\right), \frac{-\sqrt{7} + 3i}{\sqrt{7} + 3i}\right) + 4 \sqrt{\frac{i}{\sqrt{7} - 3i}} x (x^4 + 3x^2 + 4) - 3 \sqrt{\frac{i}{\sqrt{7} - 3i}} \sqrt{x^4 + 3x^2 + 4}}{12 \sqrt{\frac{i}{\sqrt{7} - 3i}} \sqrt{x^4 + 3x^2 + 4}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[4 + 3*x^2 + x^4], x]

[Out] (4*Sqrt[(-1)/(-3*I + Sqrt[7])]*x*(4 + 3*x^2 + x^4) - 3*Sqrt[2]*(3*I + Sqrt[7])*Sqrt[(-3*I + Sqrt[7] - (2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[(3*I + Sqrt[7] + (2*I)*x^2)/(3*I + Sqrt[7])]*EllipticE[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])] + Sqrt[2]*(-7*I + 3*Sqrt[7])*Sqrt[(-3*I + Sqrt[7] - (2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[(3*I + Sqrt[7] + (2*I)*x^2)/(3*I + Sqrt[7])]*EllipticF[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])])/(12*Sqrt[(-1)/(-3*I + Sqrt[7])]*Sqrt[4 + 3*x^2 + x^4])

Maple [C] time = 0.003, size = 224, normalized size = 1.3

$$\frac{x}{3} \sqrt{x^4 + 3x^2 + 4} + \frac{32}{3 \sqrt{-6 + 2i\sqrt{7}}} \sqrt{1 - \left(-\frac{3}{8} + \frac{i}{8}\sqrt{7}\right) x^2} \sqrt{1 - \left(-\frac{3}{8} - \frac{i}{8}\sqrt{7}\right) x^2} \text{EllipticF}\left(\frac{x \sqrt{-6 + 2i\sqrt{7}}}{4}, \frac{\sqrt{2 + 6i\sqrt{7}}}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^4+3*x^2+4)^(1/2), x)

[Out] 1/3*x*(x^4+3*x^2+4)^(1/2)+32/3/(-6+2*I*7^(1/2))^(1/2)*(1-(-3/8+1/8*I*7^(1/2))*x^2)^(1/2)*(1-(-3/8-1/8*I*7^(1/2))*x^2)^(1/2)/(x^4+3*x^2+4)^(1/2)*EllipticF(1/4*x*(-6+2*I*7^(1/2))^(1/2), 1/4*(2+6*I*7^(1/2))^(1/2))-32/(-6+2*I*7^(1/2))^(1/2)

$$\begin{aligned} & /2))^{\frac{1}{2}} * (1 - (-\frac{3}{8} + \frac{1}{8}i\sqrt{7})x^2)^{\frac{1}{2}} * (1 - (-\frac{3}{8} - \frac{1}{8}i\sqrt{7})x^2)^{\frac{1}{2}} \\ & / (x^4 + 3x^2 + 4)^{\frac{1}{2}} / (i\sqrt{7} + 3) * (\text{EllipticF}(\frac{1}{4}x * (-6 + 2i\sqrt{7})^{\frac{1}{2}})^{\frac{1}{2}}, \\ & \frac{1}{4} * (2 + 6i\sqrt{7})^{\frac{1}{2}})^{\frac{1}{2}}) - \text{EllipticE}(\frac{1}{4}x * (-6 + 2i\sqrt{7})^{\frac{1}{2}})^{\frac{1}{2}}, \frac{1}{4} * \\ & (2 + 6i\sqrt{7})^{\frac{1}{2}})^{\frac{1}{2}}) \end{aligned}$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{x^4 + 3x^2 + 4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4+3*x^2+4)^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(x^4 + 3*x^2 + 4), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}(\sqrt{x^4 + 3x^2 + 4}, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4+3*x^2+4)^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(x^4 + 3*x^2 + 4), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{x^4 + 3x^2 + 4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**4+3*x**2+4)**(1/2),x)

[Out] Integral(sqrt(x**4 + 3*x**2 + 4), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{x^4 + 3x^2 + 4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4+3*x^2+4)^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(x^4 + 3*x^2 + 4), x)

$$3.353 \quad \int \frac{\sqrt{4+3x^2+x^4}}{7+5x^2} dx$$

Optimal. Leaf size=322

$$\frac{11\sqrt{2}(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{75\sqrt{x^4+3x^2+4}} + \frac{9(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{25\sqrt{2}\sqrt{x^4+3x^2+4}} + \frac{\sqrt{x^4+3x^2+4}}{5(x^2+2)}$$

```
[Out] (x*Sqrt[4 + 3*x^2 + x^4])/(5*(2 + x^2)) + (Sqrt[11/35]*ArcTan[(2*Sqrt[11/35]*x)/Sqrt[4 + 3*x^2 + x^4]])/5 - (Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE[2*ArcTan[x/Sqrt[2]], 1/8])/(5*Sqrt[4 + 3*x^2 + x^4]) + (9*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(25*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4]) - (11*Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(75*Sqrt[4 + 3*x^2 + x^4]) + (187*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticPi[-9/280, 2*ArcTan[x/Sqrt[2]], 1/8])/(525*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4])
```

Rubi [A] time = 0.151624, antiderivative size = 322, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$, Rules used = {1208, 1197, 1103, 1195, 1216, 1706}

$$\frac{\sqrt{x^4+3x^2+4x}}{5(x^2+2)} + \frac{1}{5}\sqrt{\frac{11}{35}}\tan^{-1}\left(\frac{2\sqrt{\frac{11}{35}}x}{\sqrt{x^4+3x^2+4}}\right) - \frac{11\sqrt{2}(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|\frac{1}{8}\right)}{75\sqrt{x^4+3x^2+4}} + \frac{9(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|\frac{1}{8}\right)}{25\sqrt{2}\sqrt{x^4+3x^2+4}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[4 + 3*x^2 + x^4]/(7 + 5*x^2),x]
```

```
[Out] (x*Sqrt[4 + 3*x^2 + x^4])/(5*(2 + x^2)) + (Sqrt[11/35]*ArcTan[(2*Sqrt[11/35]*x)/Sqrt[4 + 3*x^2 + x^4]])/5 - (Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE[2*ArcTan[x/Sqrt[2]], 1/8])/(5*Sqrt[4 + 3*x^2 + x^4]) + (9*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(25*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4]) - (11*Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(75*Sqrt[4 + 3*x^2 + x^4]) + (187*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticPi[-9/280, 2*ArcTan[x/Sqrt[2]], 1/8])/(525*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4])
```

Rule 1208

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_)/((d_) + (e_.)*(x_)^2), x_Symbol]
:> -Dist[(e^2)^(-1), Int[(c*d - b*e - c*e*x^2)*(a + b*x^2 + c*x^4)^(p - 1), x], x] + Dist[(c*d^2 - b*d*e + a*e^2)/e^2, Int[(a + b*x^2 + c*x^4)^(p - 1)/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p + 1/2, 0]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol]
:> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; NeQ[e + d*q, 0] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
```

c/a]

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] := With[{q = Rt[c/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] := With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c*x^4]), x]] /; EqQ[e + d*q^2, 0] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1216

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_Symbol] := With[{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1706

```
Int(((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_Symbol] := With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*ArcTan[(Rt[-b + (c*d)/e + (a*e)/d, 2]*x)/Sqrt[a + b*x^2 + c*x^4]])/(2*d*e*Rt[-b + (c*d)/e + (a*e)/d, 2]), x] + Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a + b*x^2 + c*x^4))/(a*(A + B*x^2)^2)]*EllipticPi[Cancel[-((B*d - A*e)^2/(4*d*e*A*B))], 2*ArcTan[q*x], 1/2 - (b*A)/(4*a*B)])/(4*d*e*A*q*Sqrt[a + b*x^2 + c*x^4]), x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{4+3x^2+x^4}}{7+5x^2} dx &= -\left(\frac{1}{25} \int \frac{-8-5x^2}{\sqrt{4+3x^2+x^4}} dx\right) + \frac{44}{25} \int \frac{1}{(7+5x^2)\sqrt{4+3x^2+x^4}} dx \\ &= -\left(\frac{2}{5} \int \frac{1-\frac{x^2}{2}}{\sqrt{4+3x^2+x^4}} dx\right) - \frac{44}{75} \int \frac{1}{\sqrt{4+3x^2+x^4}} dx + \frac{18}{25} \int \frac{1}{\sqrt{4+3x^2+x^4}} dx + \frac{88}{15} \int \frac{1}{(7+5x^2)\sqrt{4+3x^2+x^4}} dx \\ &= \frac{x\sqrt{4+3x^2+x^4}}{5(2+x^2)} + \frac{1}{5}\sqrt{\frac{11}{35}} \tan^{-1}\left(\frac{2\sqrt{\frac{11}{35}}x}{\sqrt{4+3x^2+x^4}}\right) - \frac{\sqrt{2}(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}E\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|\frac{1}{8}\right)}{5\sqrt{4+3x^2+x^4}} + \dots \end{aligned}$$

Mathematica [C] time = 0.250902, size = 283, normalized size = 0.88

$$\frac{\sqrt{1 - \frac{2ix^2}{\sqrt{7}-3i}} \sqrt{1 + \frac{2ix^2}{\sqrt{7}+3i}} \left((-35\sqrt{7} + 7i) \operatorname{EllipticF} \left(i \sinh^{-1} \left(\sqrt{\frac{-2i}{\sqrt{7}-3i}} x \right), \frac{-\sqrt{7}+3i}{\sqrt{7}+3i} \right) + 35(\sqrt{7} + 3i) E \left(i \sinh^{-1} \left(\sqrt{\frac{-2i}{\sqrt{7}-3i}} x \right), \frac{-\sqrt{7}+3i}{\sqrt{7}+3i} \right) \right)}{350\sqrt{2} \sqrt{\frac{i}{\sqrt{7}-3i}} \sqrt{x^4 + 3x^2 + 4}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[4 + 3*x^2 + x^4]/(7 + 5*x^2), x]

[Out] $-\left(\operatorname{Sqrt}\left[1 - \frac{(2I)x^2}{-3I + \operatorname{Sqrt}[7]}\right]\right) \operatorname{Sqrt}\left[1 + \frac{(2I)x^2}{3I + \operatorname{Sqrt}[7]}\right] \left((35(3I + \operatorname{Sqrt}[7]) \operatorname{EllipticE}\left[I \operatorname{ArcSinh}\left[\operatorname{Sqrt}\left[\frac{-2I}{-3I + \operatorname{Sqrt}[7]}\right] x\right], \frac{3I - \operatorname{Sqrt}[7]}{3I + \operatorname{Sqrt}[7]}\right] + (7I - 35 \operatorname{Sqrt}[7]) \operatorname{EllipticF}\left[I \operatorname{ArcSinh}\left[\operatorname{Sqrt}\left[\frac{-2I}{-3I + \operatorname{Sqrt}[7]}\right] x\right], \frac{3I - \operatorname{Sqrt}[7]}{3I + \operatorname{Sqrt}[7]}\right] + (88I) \operatorname{EllipticPi}\left[\frac{5(3 + I \operatorname{Sqrt}[7])}{14}, I \operatorname{ArcSinh}\left[\operatorname{Sqrt}\left[\frac{-2I}{-3I + \operatorname{Sqrt}[7]}\right] x\right], \frac{3I - \operatorname{Sqrt}[7]}{3I + \operatorname{Sqrt}[7]}\right] \right) \right) / (350 \operatorname{Sqrt}[2] \operatorname{Sqrt}\left[\frac{i}{-3I + \operatorname{Sqrt}[7]}\right] \operatorname{Sqrt}[4 + 3x^2 + x^4])$

Maple [C] time = 0.051, size = 386, normalized size = 1.2

$$\frac{32}{25 \sqrt{-6 + 2i\sqrt{7}}} \sqrt{1 + \frac{3x^2}{8} - \frac{i}{8} x^2 \sqrt{7}} \sqrt{1 + \frac{3x^2}{8} + \frac{i}{8} x^2 \sqrt{7}} \operatorname{EllipticF} \left(\frac{x \sqrt{-6 + 2i\sqrt{7}}}{4}, \frac{\sqrt{2 + 6i\sqrt{7}}}{4} \right) \frac{1}{\sqrt{x^4 + 3x^2 + 4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^4+3*x^2+4)^(1/2)/(5*x^2+7), x)

[Out] $\frac{32}{25} \frac{(-6+2I\sqrt{7})^{1/2}}{(x^4+3x^2+4)^{1/2}} \frac{(1+3/8x^2-1/8I\sqrt{7}x^2)^{1/2}}{(x^4+3x^2+4)^{1/2}} \operatorname{EllipticF}\left(\frac{1}{4}x\sqrt{-6+2I\sqrt{7}}, \frac{1}{4}\sqrt{2+6I\sqrt{7}}\right) \frac{1}{\sqrt{x^4+3x^2+4}} - \frac{32}{5} \frac{(2+6I\sqrt{7})^{1/2}}{(-6+2I\sqrt{7})^{1/2}} \frac{(1+3/8x^2-1/8I\sqrt{7}x^2)^{1/2}}{(x^4+3x^2+4)^{1/2}} \operatorname{EllipticF}\left(\frac{1}{4}x\sqrt{-6+2I\sqrt{7}}, \frac{1}{4}\sqrt{2+6I\sqrt{7}}\right) \frac{1}{\sqrt{x^4+3x^2+4}} + \frac{32}{5} \frac{(2+6I\sqrt{7})^{1/2}}{(-6+2I\sqrt{7})^{1/2}} \frac{(1+3/8x^2+1/8I\sqrt{7}x^2)^{1/2}}{(x^4+3x^2+4)^{1/2}} \operatorname{EllipticE}\left(\frac{1}{4}x\sqrt{-6+2I\sqrt{7}}, \frac{1}{4}\sqrt{2+6I\sqrt{7}}\right) \frac{1}{\sqrt{x^4+3x^2+4}} + \frac{44}{175} \frac{(-3/8+1/8I\sqrt{7})^{1/2}}{(1+3/8x^2-1/8I\sqrt{7}x^2)^{1/2}} \frac{(1+3/8x^2+1/8I\sqrt{7}x^2)^{1/2}}{(x^4+3x^2+4)^{1/2}} \operatorname{EllipticPi}\left(\frac{-3/8+1/8I\sqrt{7}}{(-3/8+1/8I\sqrt{7})^{1/2}}, \frac{-5/7}{(-3/8+1/8I\sqrt{7})^{1/2}}, \frac{-3/8-1/8I\sqrt{7}}{(-3/8+1/8I\sqrt{7})^{1/2}}\right) \frac{1}{\sqrt{x^4+3x^2+4}}$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^4 + 3x^2 + 4}}{5x^2 + 7} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4+3*x^2+4)^(1/2)/(5*x^2+7), x, algorithm="maxima")

[Out] integrate(sqrt(x^4 + 3*x^2 + 4)/(5*x^2 + 7), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{x^4 + 3x^2 + 4}}{5x^2 + 7}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4+3*x^2+4)^(1/2)/(5*x^2+7),x, algorithm="fricas")

[Out] integral(sqrt(x^4 + 3*x^2 + 4)/(5*x^2 + 7), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{(x^2 - x + 2)(x^2 + x + 2)}}{5x^2 + 7} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**4+3*x**2+4)**(1/2)/(5*x**2+7),x)

[Out] Integral(sqrt((x**2 - x + 2)*(x**2 + x + 2))/(5*x**2 + 7), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^4 + 3x^2 + 4}}{5x^2 + 7} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4+3*x^2+4)^(1/2)/(5*x^2+7),x, algorithm="giac")

[Out] integrate(sqrt(x^4 + 3*x^2 + 4)/(5*x^2 + 7), x)

$$3.354 \quad \int \frac{\sqrt{4+3x^2+x^4}}{(7+5x^2)^2} dx$$

Optimal. Leaf size=284

$$\frac{(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right), \frac{1}{8}\right)}{35\sqrt{2}\sqrt{x^4+3x^2+4}} - \frac{\sqrt{x^4+3x^2+4}x}{70(x^2+2)} + \frac{\sqrt{x^4+3x^2+4}}{14(5x^2+7)} + \frac{51\tan^{-1}\left(\frac{2\sqrt{\frac{11}{35}}x}{\sqrt{x^4+3x^2+4}}\right)}{280\sqrt{385}} + \dots$$

```
[Out] -(x*Sqrt[4 + 3*x^2 + x^4])/(70*(2 + x^2)) + (x*Sqrt[4 + 3*x^2 + x^4])/(14*(7 + 5*x^2)) + (51*ArcTan[(2*Sqrt[11/35]*x)/Sqrt[4 + 3*x^2 + x^4]])/(280*Sqrt[385]) + ((2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE[2*ArcTan[x/Sqrt[2]], 1/8])/(35*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4]) - ((2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(35*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4]) + (289*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticPi[-9/280, 2*ArcTan[x/Sqrt[2]], 1/8])/(9800*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4])
```

Rubi [A] time = 0.150621, antiderivative size = 284, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$, Rules used = {1226, 1197, 1103, 1195, 1216, 1706}

$$-\frac{\sqrt{x^4+3x^2+4}x}{70(x^2+2)} + \frac{\sqrt{x^4+3x^2+4}}{14(5x^2+7)} + \frac{51\tan^{-1}\left(\frac{2\sqrt{\frac{11}{35}}x}{\sqrt{x^4+3x^2+4}}\right)}{280\sqrt{385}} - \frac{(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|\frac{1}{8}\right)}{35\sqrt{2}\sqrt{x^4+3x^2+4}} + \frac{(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\text{EllipticE}\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right), \frac{1}{8}\right)}{35\sqrt{2}\sqrt{x^4+3x^2+4}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[4 + 3*x^2 + x^4]/(7 + 5*x^2)^2,x]
```

```
[Out] -(x*Sqrt[4 + 3*x^2 + x^4])/(70*(2 + x^2)) + (x*Sqrt[4 + 3*x^2 + x^4])/(14*(7 + 5*x^2)) + (51*ArcTan[(2*Sqrt[11/35]*x)/Sqrt[4 + 3*x^2 + x^4]])/(280*Sqrt[385]) + ((2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE[2*ArcTan[x/Sqrt[2]], 1/8])/(35*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4]) - ((2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(35*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4]) + (289*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticPi[-9/280, 2*ArcTan[x/Sqrt[2]], 1/8])/(9800*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4])
```

Rule 1226

```
Int[Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]/((d_) + (e_.)*(x_)^2)^2, x_Symbol] :> Simp[(x*Sqrt[a + b*x^2 + c*x^4])/(2*d*(d + e*x^2)), x] + (Dist[c/(2*d*e^2), Int[(d - e*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[(c*d^2 - a*e^2)/(2*d*e^2), Int[1/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]) /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; NeQ[e + d*q, 0] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
```

c/a]

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] := With[{q = Rt[c/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] := With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c*x^4]), x]] /; EqQ[e + d*q^2, 0] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1216

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_Symbol] := With[{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1706

```
Int(((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_Symbol] := With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*ArcTan[(Rt[-b + (c*d)/e + (a*e)/d, 2]*x)/Sqrt[a + b*x^2 + c*x^4]])/(2*d*e*Rt[-b + (c*d)/e + (a*e)/d, 2]), x] + Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a + b*x^2 + c*x^4))/(a*(A + B*x^2)^2)]*EllipticPi[Cancel[-((B*d - A*e)^2/(4*d*e*A*B))], 2*ArcTan[q*x], 1/2 - (b*A)/(4*a*B)])/(4*d*e*A*q*Sqrt[a + b*x^2 + c*x^4]), x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{4+3x^2+x^4}}{(7+5x^2)^2} dx &= \frac{x\sqrt{4+3x^2+x^4}}{14(7+5x^2)} + \frac{1}{350} \int \frac{7-5x^2}{\sqrt{4+3x^2+x^4}} dx + \frac{51}{350} \int \frac{1}{(7+5x^2)\sqrt{4+3x^2+x^4}} dx \\ &= \frac{x\sqrt{4+3x^2+x^4}}{14(7+5x^2)} - \frac{3}{350} \int \frac{1}{\sqrt{4+3x^2+x^4}} dx + \frac{1}{35} \int \frac{1-\frac{x^2}{2}}{\sqrt{4+3x^2+x^4}} dx - \frac{17}{350} \int \frac{1}{\sqrt{4+3x^2+x^4}} dx \\ &= -\frac{x\sqrt{4+3x^2+x^4}}{70(2+x^2)} + \frac{x\sqrt{4+3x^2+x^4}}{14(7+5x^2)} + \frac{51 \tan^{-1}\left(\frac{2\sqrt{\frac{11}{35}}x}{\sqrt{4+3x^2+x^4}}\right)}{280\sqrt{385}} + \frac{(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}} E\left(2 \tan^{-1}\left(\frac{x}{\sqrt{4+3x^2+x^4}}\right)\right)}{35\sqrt{2}\sqrt{4+3x^2+x^4}} \end{aligned}$$

Mathematica [C] time = 0.770724, size = 481, normalized size = 1.69

$$-98i(5x^2+7)\sqrt{2-\frac{4ix^2}{\sqrt{7-3i}}}\sqrt{1+\frac{2ix^2}{\sqrt{7+3i}}}\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{-\frac{2i}{\sqrt{7-3i}}}x\right),\frac{-\sqrt{7+3i}}{\sqrt{7+3i}}\right)+35(\sqrt{7}+3i)(5x^2+7)\sqrt{2-\frac{4ix^2}{\sqrt{7-3i}}}\sqrt{1+\frac{2ix^2}{\sqrt{7+3i}}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[4 + 3*x^2 + x^4]/(7 + 5*x^2)^2,x]

[Out] (700*Sqrt[(-I)/(-3*I + Sqrt[7])]*x*(4 + 3*x^2 + x^4) + 35*(3*I + Sqrt[7])*(7 + 5*x^2)*Sqrt[2 - ((4*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[1 + ((2*I)*x^2)/(3*I + Sqrt[7])])*(EllipticE[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])) - EllipticF[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])) - (98*I)*(7 + 5*x^2)*Sqrt[2 - ((4*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[1 + ((2*I)*x^2)/(3*I + Sqrt[7])]*EllipticF[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])) - (102*I)*(7 + 5*x^2)*Sqrt[2 - ((4*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[1 + ((2*I)*x^2)/(3*I + Sqrt[7])]*EllipticPi[(5*(3 + I*Sqrt[7]))/14, I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])))/(9800*Sqrt[(-I)/(-3*I + Sqrt[7])]*(7 + 5*x^2)*Sqrt[4 + 3*x^2 + x^4])

Maple [C] time = 0.023, size = 410, normalized size = 1.4

$$\frac{x}{70x^2 + 98} \sqrt{x^4 + 3x^2 + 4} + \frac{2}{25\sqrt{-6 + 2i\sqrt{7}}} \sqrt{1 + \frac{3x^2}{8} - \frac{i}{8}x^2\sqrt{7}} \sqrt{1 + \frac{3x^2}{8} + \frac{i}{8}x^2\sqrt{7}} \text{EllipticF}\left(\frac{x\sqrt{-6 + 2i\sqrt{7}}}{4}, \sqrt{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^4+3*x^2+4)^(1/2)/(5*x^2+7)^2,x)

[Out] 1/14*x*(x^4+3*x^2+4)^(1/2)/(5*x^2+7)+2/25/(-6+2*I*7^(1/2))^(1/2)*(1+3/8*x^2-1/8*I*x^2*7^(1/2))^(1/2)*(1+3/8*x^2+1/8*I*x^2*7^(1/2))^(1/2)/(x^4+3*x^2+4)^(1/2)*EllipticF(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))+16/35/(-6+2*I*7^(1/2))^(1/2)*(1+3/8*x^2-1/8*I*x^2*7^(1/2))^(1/2)*(1+3/8*x^2+1/8*I*x^2*7^(1/2))^(1/2)/(x^4+3*x^2+4)^(1/2)/(I*7^(1/2)+3)*EllipticF(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))-16/35/(-6+2*I*7^(1/2))^(1/2)*(1+3/8*x^2-1/8*I*x^2*7^(1/2))^(1/2)*(1+3/8*x^2+1/8*I*x^2*7^(1/2))^(1/2)/(x^4+3*x^2+4)^(1/2)/(I*7^(1/2)+3)*EllipticE(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))+51/2450/(-3/8+1/8*I*7^(1/2))^(1/2)*(1+3/8*x^2-1/8*I*x^2*7^(1/2))^(1/2)*(1+3/8*x^2+1/8*I*x^2*7^(1/2))^(1/2)/(x^4+3*x^2+4)^(1/2)*EllipticPi((-3/8+1/8*I*7^(1/2))^(1/2)*x,-5/7/(-3/8+1/8*I*7^(1/2)),(-3/8-1/8*I*7^(1/2))^(1/2)/(-3/8+1/8*I*7^(1/2))^(1/2))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^4 + 3x^2 + 4}}{(5x^2 + 7)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4+3*x^2+4)^(1/2)/(5*x^2+7)^2,x, algorithm="maxima")

[Out] integrate(sqrt(x^4 + 3*x^2 + 4)/(5*x^2 + 7)^2, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{x^4 + 3x^2 + 4}}{25x^4 + 70x^2 + 49}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4+3*x^2+4)^(1/2)/(5*x^2+7)^2,x, algorithm="fricas")

[Out] integral(sqrt(x^4 + 3*x^2 + 4)/(25*x^4 + 70*x^2 + 49), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{(x^2 - x + 2)(x^2 + x + 2)}}{(5x^2 + 7)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**4+3*x**2+4)**(1/2)/(5*x**2+7)**2,x)

[Out] Integral(sqrt((x**2 - x + 2)*(x**2 + x + 2))/(5*x**2 + 7)**2, x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^4 + 3x^2 + 4}}{(5x^2 + 7)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4+3*x^2+4)^(1/2)/(5*x^2+7)^2,x, algorithm="giac")

[Out] integrate(sqrt(x^4 + 3*x^2 + 4)/(5*x^2 + 7)^2, x)

$$3.355 \quad \int \frac{\sqrt{4+3x^2+x^4}}{(7+5x^2)^3} dx$$

Optimal. Leaf size=312

$$\frac{23(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}} \operatorname{EllipticF}\left(2 \tan^{-1}\left(\frac{x}{\sqrt{2}}\right), \frac{1}{8}\right)}{2940\sqrt{2}\sqrt{x^4+3x^2+4}} - \frac{139\sqrt{x^4+3x^2+4x}}{86240(x^2+2)} + \frac{139\sqrt{x^4+3x^2+4x}}{17248(5x^2+7)} + \frac{\sqrt{x^4+3x^2+4x}}{28(5x^2+7)^2}$$

```
[Out] (-139*x*Sqrt[4 + 3*x^2 + x^4])/(86240*(2 + x^2)) + (x*Sqrt[4 + 3*x^2 + x^4])/(28*(7 + 5*x^2)^2) + (139*x*Sqrt[4 + 3*x^2 + x^4])/(17248*(7 + 5*x^2)) + (14999*ArcTan[(2*Sqrt[11/35]*x)/Sqrt[4 + 3*x^2 + x^4]])/(344960*Sqrt[385]) + (139*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE[2*ArcTan[x/Sqrt[2]], 1/8])/(43120*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4]) - (23*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(2940*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4]) + (254983*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticPi[-9/280, 2*ArcTan[x/Sqrt[2]], 1/8])/(36220800*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4])
```

Rubi [A] time = 0.710943, antiderivative size = 312, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {1228, 1223, 1696, 1714, 1195, 1708, 1103, 1706, 1216}

$$-\frac{139\sqrt{x^4+3x^2+4x}}{86240(x^2+2)} + \frac{139\sqrt{x^4+3x^2+4x}}{17248(5x^2+7)} + \frac{\sqrt{x^4+3x^2+4x}}{28(5x^2+7)^2} + \frac{14999 \tan^{-1}\left(\frac{2\sqrt{\frac{11}{35}}x}{\sqrt{x^4+3x^2+4}}\right)}{344960\sqrt{385}} - \frac{23(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}} F}{2940\sqrt{2}\sqrt{x^4+3x^2+4}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[4 + 3*x^2 + x^4]/(7 + 5*x^2)^3, x]
```

```
[Out] (-139*x*Sqrt[4 + 3*x^2 + x^4])/(86240*(2 + x^2)) + (x*Sqrt[4 + 3*x^2 + x^4])/(28*(7 + 5*x^2)^2) + (139*x*Sqrt[4 + 3*x^2 + x^4])/(17248*(7 + 5*x^2)) + (14999*ArcTan[(2*Sqrt[11/35]*x)/Sqrt[4 + 3*x^2 + x^4]])/(344960*Sqrt[385]) + (139*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE[2*ArcTan[x/Sqrt[2]], 1/8])/(43120*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4]) - (23*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(2940*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4]) + (254983*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticPi[-9/280, 2*ArcTan[x/Sqrt[2]], 1/8])/(36220800*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4])
```

Rule 1228

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{aa, bb, cc}, Int[ExpandIntegrand[1/Sqrt[aa + bb*x^2 + cc*x^4], (d + e*x^2)^q*(aa + bb*x^2 + cc*x^4)^(p + 1/2), x] /. {aa -> a, bb -> b, cc -> c}, x] /. FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, 0] && IntegerQ[p + 1/2]
```

Rule 1223

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> -Simp[(e^2*x*(d + e*x^2)^(q + 1)*Sqrt[a + b*x^2 + c*x^4])/(2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(2*d*(q + 1)*(c*d^2 - b*d*e +
```

$a e^2$)), Int[((d + e*x^2)^(q + 1)*Simp[a*e^2*(2*q + 3) + 2*d*(c*d - b*e)*(q + 1) - 2*e*(c*d*(q + 1) - b*e*(q + 2))*x^2 + c*e^2*(2*q + 5)*x^4, x])/Sqrt[a + b*x^2 + c*x^4], x, x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && ILtQ[q, -1]

Rule 1696

Int[((P4x_)*((d_) + (e_)*(x_)^2)^(q_))/Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4], x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[P4x, x, 4]}, -Simp[((C*d^2 - B*d*e + A*e^2)*x*(d + e*x^2)^(q + 1)*Sqrt[a + b*x^2 + c*x^4])/(2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2)), Int[((d + e*x^2)^(q + 1)*Simp[a*d*(C*d - B*e) + A*(a*e^2*(2*q + 3) + 2*d*(c*d - b*e)*(q + 1)) - 2*((B*d - A*e)*(b*e*(q + 2) - c*d*(q + 1)) - C*d*(b*d + a*e*(q + 1)))*x^2 + c*(C*d^2 - B*d*e + A*e^2)*(2*q + 5)*x^4, x])/Sqrt[a + b*x^2 + c*x^4], x, x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[P4x, x^2] && LeQ[Expon[P4x, x], 4] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, -1]

Rule 1714

Int[(P4x_)/(((d_) + (e_)*(x_)^2)*Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4]), x_Symbol] :> With[{q = Rt[c/a, 2], A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[P4x, x, 4]}, -Dist[C/(e*q), Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x, x] + Dist[1/(c*e), Int[(A*c*e + a*C*d*q + (B*c*e - C*(c*d - a*e*q))*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x, x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[P4x, x^2, 2] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && !GtQ[b^2 - 4*a*c, 0]

Rule 1195

Int[((d_) + (e_)*(x_)^2)/Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4], x_Symbol] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c*x^4]), x] /; EqQ[e + d*q^2, 0] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]

Rule 1708

Int[((A_) + (B_)*(x_)^2)/(((d_) + (e_)*(x_)^2)*Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4]), x_Symbol] :> With[{q = Rt[c/a, 2]}, Dist[(A*(c*d + a*e*q) - a*B*(e + d*q))/(c*d^2 - a*e^2), Int[1/Sqrt[a + b*x^2 + c*x^4], x, x] + Dist[(a*(B*d - A*e)*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x, x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && NeQ[c*A^2 - a*B^2, 0]

Rule 1103

Int[1/Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4], x_Symbol] :> With[{q = Rt[c/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]), x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]

Rule 1706

Int[((A_) + (B_)*(x_)^2)/(((d_) + (e_)*(x_)^2)*Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*Arc

Tan[(Rt[-b + (c*d)/e + (a*e)/d, 2]*x)/Sqrt[a + b*x^2 + c*x^4]]/(2*d*e*Rt[-b + (c*d)/e + (a*e)/d, 2]), x] + Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a + b*x^2 + c*x^4))/(a*(A + B*x^2)^2)]*EllipticPi[Cancel[-((B*d - A*e)^2/(4*d*e*A*B))], 2*ArcTan[q*x], 1/2 - (b*A)/(4*a*B)])/(4*d*e*A*q*Sqrt[a + b*x^2 + c*x^4]), x] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]

Rule 1216

Int[1/(((d_) + (e_)*(x_)^2)*Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4]), x_Symbol] :> With[{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]

Rubi steps

$$\int \frac{\sqrt{4 + 3x^2 + x^4}}{(7 + 5x^2)^3} dx = \int \left(\frac{44}{25(7 + 5x^2)^3 \sqrt{4 + 3x^2 + x^4}} + \frac{1}{25(7 + 5x^2)^2 \sqrt{4 + 3x^2 + x^4}} + \frac{1}{25(7 + 5x^2) \sqrt{4 + 3x^2 + x^4}} \right) dx$$

$$= \frac{1}{25} \int \frac{1}{(7 + 5x^2)^2 \sqrt{4 + 3x^2 + x^4}} dx + \frac{1}{25} \int \frac{1}{(7 + 5x^2) \sqrt{4 + 3x^2 + x^4}} dx + \frac{44}{25} \int \frac{1}{(7 + 5x^2)^3 \sqrt{4 + 3x^2 + x^4}} dx$$

$$= \frac{x\sqrt{4 + 3x^2 + x^4}}{28(7 + 5x^2)^2} + \frac{x\sqrt{4 + 3x^2 + x^4}}{616(7 + 5x^2)} - \frac{\int \frac{12 + 70x^2 + 25x^4}{(7 + 5x^2)\sqrt{4 + 3x^2 + x^4}} dx}{15400} - \frac{1}{700} \int \frac{-76 - 10x^2 - 25x^4}{(7 + 5x^2)^2 \sqrt{4 + 3x^2 + x^4}} dx$$

$$= \frac{x\sqrt{4 + 3x^2 + x^4}}{28(7 + 5x^2)^2} + \frac{139x\sqrt{4 + 3x^2 + x^4}}{17248(7 + 5x^2)} + \frac{\tan^{-1}\left(\frac{2\sqrt{\frac{11}{35}}x}{\sqrt{4 + 3x^2 + x^4}}\right)}{20\sqrt{385}} - \frac{(2 + x^2)\sqrt{\frac{4 + 3x^2 + x^4}{(2 + x^2)^2}} F\left(2 \tan^{-1}\left(\frac{2\sqrt{\frac{11}{35}}x}{\sqrt{4 + 3x^2 + x^4}}\right)\right)}{150\sqrt{2}\sqrt{4 + 3x^2 + x^4}}$$

$$= -\frac{x\sqrt{4 + 3x^2 + x^4}}{3080(2 + x^2)} + \frac{x\sqrt{4 + 3x^2 + x^4}}{28(7 + 5x^2)^2} + \frac{139x\sqrt{4 + 3x^2 + x^4}}{17248(7 + 5x^2)} + \frac{\tan^{-1}\left(\frac{2\sqrt{\frac{11}{35}}x}{\sqrt{4 + 3x^2 + x^4}}\right)}{20\sqrt{385}} + \frac{(2 + x^2)\sqrt{\frac{4 + 3x^2 + x^4}{(2 + x^2)^2}} F\left(2 \tan^{-1}\left(\frac{2\sqrt{\frac{11}{35}}x}{\sqrt{4 + 3x^2 + x^4}}\right)\right)}{150\sqrt{2}\sqrt{4 + 3x^2 + x^4}}$$

$$= -\frac{139x\sqrt{4 + 3x^2 + x^4}}{86240(2 + x^2)} + \frac{x\sqrt{4 + 3x^2 + x^4}}{28(7 + 5x^2)^2} + \frac{139x\sqrt{4 + 3x^2 + x^4}}{17248(7 + 5x^2)} + \frac{653 \tan^{-1}\left(\frac{2\sqrt{\frac{11}{35}}x}{\sqrt{4 + 3x^2 + x^4}}\right)}{12320\sqrt{385}} + \frac{(2 + x^2)\sqrt{\frac{4 + 3x^2 + x^4}{(2 + x^2)^2}} F\left(2 \tan^{-1}\left(\frac{2\sqrt{\frac{11}{35}}x}{\sqrt{4 + 3x^2 + x^4}}\right)\right)}{12320\sqrt{2}\sqrt{4 + 3x^2 + x^4}}$$

$$= -\frac{139x\sqrt{4 + 3x^2 + x^4}}{86240(2 + x^2)} + \frac{x\sqrt{4 + 3x^2 + x^4}}{28(7 + 5x^2)^2} + \frac{139x\sqrt{4 + 3x^2 + x^4}}{17248(7 + 5x^2)} + \frac{14999 \tan^{-1}\left(\frac{2\sqrt{\frac{11}{35}}x}{\sqrt{4 + 3x^2 + x^4}}\right)}{344960\sqrt{385}} + \frac{(2 + x^2)\sqrt{\frac{4 + 3x^2 + x^4}{(2 + x^2)^2}} F\left(2 \tan^{-1}\left(\frac{2\sqrt{\frac{11}{35}}x}{\sqrt{4 + 3x^2 + x^4}}\right)\right)}{344960\sqrt{2}\sqrt{4 + 3x^2 + x^4}}$$

Mathematica [C] time = 0.670891, size = 308, normalized size = 0.99

$$\frac{700x(695x^2 + 1589)(x^4 + 3x^2 + 4)}{(5x^2 + 7)^2} + i\sqrt{6 + 2i\sqrt{7}}\sqrt{1 - \frac{2ix^2}{\sqrt{7-3i}}}\sqrt{1 + \frac{2ix^2}{\sqrt{7+3i}}}\left((-9597 + 4865i\sqrt{7}) \operatorname{EllipticF}\left(i \sinh^{-1}\left(\sqrt{-\frac{2i}{\sqrt{7-3i}}}x\right)\right)\right)$$

12073600\sqrt{3}

Antiderivative was successfully verified.

[In] Integrate[Sqrt[4 + 3*x^2 + x^4]/(7 + 5*x^2)^3, x]

```
[Out] ((700*x*(1589 + 695*x^2)*(4 + 3*x^2 + x^4))/(7 + 5*x^2)^2 + I*Sqrt[6 + (2*I)*Sqrt[7]]*Sqrt[1 - ((2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[1 + ((2*I)*x^2)/(3*I + Sqrt[7])])*(4865*(3 - I*Sqrt[7])*EllipticE[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])] + (-9597 + (4865*I)*Sqrt[7])*EllipticF[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])] - 29998*EllipticPi[(5*(3 + I*Sqrt[7]))/14, I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])]))/(12073600*Sqrt[4 + 3*x^2 + x^4])
```

Maple [C] time = 0.024, size = 434, normalized size = 1.4

$$\frac{x}{28(5x^2+7)^2}\sqrt{x^4+3x^2+4} + \frac{139x}{86240x^2+120736}\sqrt{x^4+3x^2+4} - \frac{51}{15400\sqrt{-6+2i\sqrt{7}}}\sqrt{1+\frac{3x^2}{8}-\frac{i}{8}x^2\sqrt{7}}\sqrt{1+\frac{3x^2}{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+3*x^2+4)^(1/2)/(5*x^2+7)^3,x)
```

```
[Out] 1/28*x*(x^4+3*x^2+4)^(1/2)/(5*x^2+7)^2+139/17248*x*(x^4+3*x^2+4)^(1/2)/(5*x^2+7)-51/15400/(-6+2*I*7^(1/2))^(1/2)*(1+3/8*x^2-1/8*I*x^2*7^(1/2))^(1/2)*(1+3/8*x^2+1/8*I*x^2*7^(1/2))^(1/2)/(x^4+3*x^2+4)^(1/2)*EllipticF(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))+139/2695/(-6+2*I*7^(1/2))^(1/2)*(1+3/8*x^2-1/8*I*x^2*7^(1/2))^(1/2)*(1+3/8*x^2+1/8*I*x^2*7^(1/2))^(1/2)/(x^4+3*x^2+4)^(1/2)/(I*7^(1/2)+3)*EllipticF(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))-139/2695/(-6+2*I*7^(1/2))^(1/2)*(1+3/8*x^2-1/8*I*x^2*7^(1/2))^(1/2)*(1+3/8*x^2+1/8*I*x^2*7^(1/2))^(1/2)/(x^4+3*x^2+4)^(1/2)/(I*7^(1/2)+3)*EllipticE(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))+14999/3018400/(-3/8+1/8*I*7^(1/2))^(1/2)*(1+3/8*x^2-1/8*I*x^2*7^(1/2))^(1/2)*(1+3/8*x^2+1/8*I*x^2*7^(1/2))^(1/2)/(x^4+3*x^2+4)^(1/2)*EllipticPi((-3/8+1/8*I*7^(1/2))^(1/2)*x,-5/7/(-3/8+1/8*I*7^(1/2)),(-3/8-1/8*I*7^(1/2))^(1/2)/(-3/8+1/8*I*7^(1/2))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^4 + 3x^2 + 4}}{(5x^2 + 7)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+3*x^2+4)^(1/2)/(5*x^2+7)^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^4 + 3*x^2 + 4)/(5*x^2 + 7)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{x^4 + 3x^2 + 4}}{125x^6 + 525x^4 + 735x^2 + 343}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4+3*x^2+4)^(1/2)/(5*x^2+7)^3,x, algorithm="fricas")

[Out] integral(sqrt(x^4 + 3*x^2 + 4)/(125*x^6 + 525*x^4 + 735*x^2 + 343), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{(x^2 - x + 2)(x^2 + x + 2)}}{(5x^2 + 7)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**4+3*x**2+4)**(1/2)/(5*x**2+7)**3,x)

[Out] Integral(sqrt((x**2 - x + 2)*(x**2 + x + 2))/(5*x**2 + 7)**3, x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^4 + 3x^2 + 4}}{(5x^2 + 7)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4+3*x^2+4)^(1/2)/(5*x^2+7)^3,x, algorithm="giac")

[Out] integrate(sqrt(x^4 + 3*x^2 + 4)/(5*x^2 + 7)^3, x)

$$3.356 \quad \int (7 + 5x^2)^4 (4 + 3x^2 + x^4)^{3/2} dx$$

Optimal. Leaf size=268

$$\frac{2383556\sqrt{2}(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{429\sqrt{x^4+3x^2+4}} + \frac{125}{3}(x^4+3x^2+4)^{5/2}x^5 + \frac{2250}{13}(x^4+3x^2+4)^{5/2}x^3 + \frac{92150}{429}(x^4+3x^2+4)^{5/2}x + \frac{(131080x^2+452001)(x^4+3x^2+4)^{5/2}}{1287}$$

```
[Out] (12665086*x*Sqrt[4 + 3*x^2 + x^4])/(2145*(2 + x^2)) + (7*x*(661429 + 174989*x^2)*Sqrt[4 + 3*x^2 + x^4])/2145 + (x*(452001 + 131080*x^2)*(4 + 3*x^2 + x^4)^(3/2))/1287 + (92150*x*(4 + 3*x^2 + x^4)^(5/2))/429 + (2250*x^3*(4 + 3*x^2 + x^4)^(5/2))/13 + (125*x^5*(4 + 3*x^2 + x^4)^(5/2))/3 - (12665086*Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE[2*ArcTan[x/Sqrt[2]], 1/8])/(2145*Sqrt[4 + 3*x^2 + x^4]) + (2383556*Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(429*Sqrt[4 + 3*x^2 + x^4])
```

Rubi [A] time = 0.173873, antiderivative size = 268, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$, Rules used = {1206, 1679, 1176, 1197, 1103, 1195}

$$\frac{125}{3}(x^4+3x^2+4)^{5/2}x^5 + \frac{2250}{13}(x^4+3x^2+4)^{5/2}x^3 + \frac{92150}{429}(x^4+3x^2+4)^{5/2}x + \frac{(131080x^2+452001)(x^4+3x^2+4)^{5/2}}{1287}$$

Antiderivative was successfully verified.

```
[In] Int[(7 + 5*x^2)^4*(4 + 3*x^2 + x^4)^(3/2), x]
```

```
[Out] (12665086*x*Sqrt[4 + 3*x^2 + x^4])/(2145*(2 + x^2)) + (7*x*(661429 + 174989*x^2)*Sqrt[4 + 3*x^2 + x^4])/2145 + (x*(452001 + 131080*x^2)*(4 + 3*x^2 + x^4)^(3/2))/1287 + (92150*x*(4 + 3*x^2 + x^4)^(5/2))/429 + (2250*x^3*(4 + 3*x^2 + x^4)^(5/2))/13 + (125*x^5*(4 + 3*x^2 + x^4)^(5/2))/3 - (12665086*Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE[2*ArcTan[x/Sqrt[2]], 1/8])/(2145*Sqrt[4 + 3*x^2 + x^4]) + (2383556*Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(429*Sqrt[4 + 3*x^2 + x^4])
```

Rule 1206

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := Simp[(e^q*x^(2*q - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(4*p + 2*q + 1)), x] + Dist[1/(c*(4*p + 2*q + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*p + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1]
```

Rule 1679

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := With[{q = Expon[Pq, x^2], e = Coeff[Pq, x^2, Expon[Pq, x^2]]}, Simp[(e*x^(2*q - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(2*q + 4*p + 1)), x] + Dist[1/(c*(2*q + 4*p + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*q + 4*p + 1)*Pq - a*e*(2*
```

$q - 3)x^{(2q - 4)} - b \cdot e \cdot (2q + 2p - 1)x^{(2q - 2)} - c \cdot e \cdot (2q + 4p + 1)x^{(2q)}$, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && !LtQ[p, -1]

Rule 1176

Int[((d_) + (e_)*(x_)^2)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] :> Simp[(x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*(a + b*x^2 + c*x^4)^p)/(c*(4*p + 1)*(4*p + 3)), x] + Dist[(2*p)/(c*(4*p + 1)*(4*p + 3)), Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) - b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]

Rule 1197

Int[((d_) + (e_)*(x_)^2)/Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4], x_Symbol] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; NeQ[e + d*q, 0] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]

Rule 1103

Int[1/Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4], x_Symbol] :> With[{q = Rt[c/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]

Rule 1195

Int[((d_) + (e_)*(x_)^2)/Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4], x_Symbol] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c*x^4]), x] /; EqQ[e + d*q^2, 0] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]

Rubi steps

$$\begin{aligned}
\int (7 + 5x^2)^4 (4 + 3x^2 + x^4)^{3/2} dx &= \frac{125}{3} x^5 (4 + 3x^2 + x^4)^{5/2} + \frac{1}{15} \int (4 + 3x^2 + x^4)^{3/2} (36015 + 102900x^2 + 97750x^4 + \\
&= \frac{2250}{13} x^3 (4 + 3x^2 + x^4)^{5/2} + \frac{125}{3} x^5 (4 + 3x^2 + x^4)^{5/2} + \frac{1}{195} \int (4 + 3x^2 + x^4)^{3/2} (468 \\
&= \frac{92150}{429} x (4 + 3x^2 + x^4)^{5/2} + \frac{2250}{13} x^3 (4 + 3x^2 + x^4)^{5/2} + \frac{125}{3} x^5 (4 + 3x^2 + x^4)^{5/2} + \\
&= \frac{x(452001 + 131080x^2)(4 + 3x^2 + x^4)^{3/2}}{1287} + \frac{92150}{429} x (4 + 3x^2 + x^4)^{5/2} + \frac{2250}{13} x^3 (4 + 3x^2 + x^4)^{5/2} \\
&= \frac{7x(661429 + 174989x^2)\sqrt{4 + 3x^2 + x^4}}{2145} + \frac{x(452001 + 131080x^2)(4 + 3x^2 + x^4)^{3/2}}{1287} \\
&= \frac{7x(661429 + 174989x^2)\sqrt{4 + 3x^2 + x^4}}{2145} + \frac{x(452001 + 131080x^2)(4 + 3x^2 + x^4)^{3/2}}{1287} \\
&= \frac{12665086x\sqrt{4 + 3x^2 + x^4}}{2145(2 + x^2)} + \frac{7x(661429 + 174989x^2)\sqrt{4 + 3x^2 + x^4}}{2145} + \frac{x(452001 + 131080x^2)(4 + 3x^2 + x^4)^{3/2}}{1287}
\end{aligned}$$

Mathematica [F] time = 0, size = 0, normalized size = 0.

\$Aborted

Verification is Not applicable to the result.

[In] Integrate[(7 + 5*x^2)^4*(4 + 3*x^2 + x^4)^(3/2),x]

[Out] \$Aborted

Maple [C] time = 0.039, size = 326, normalized size = 1.2

$$\frac{15015343 x}{2145} \sqrt{x^4 + 3x^2 + 4} + \frac{64070384 x^3}{6435} \sqrt{x^4 + 3x^2 + 4} + \frac{6863530 x^7}{1287} \sqrt{x^4 + 3x^2 + 4} + \frac{356027 x^5}{39} \sqrt{x^4 + 3x^2 + 4} + \frac{5500}{13} x^{11} \sqrt{x^4 + 3x^2 + 4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((5*x^2+7)^4*(x^4+3*x^2+4)^(3/2),x)

[Out] 15015343/2145*x*(x^4+3*x^2+4)^(1/2)+64070384/6435*x^3*(x^4+3*x^2+4)^(1/2)+6863530/1287*x^7*(x^4+3*x^2+4)^(1/2)+356027/39*x^5*(x^4+3*x^2+4)^(1/2)+5500/13*x^11*(x^4+3*x^2+4)^(1/2)+841525/429*x^9*(x^4+3*x^2+4)^(1/2)+125/3*x^13*(x^4+3*x^2+4)^(1/2)+89363792/2145/(-6+2*I*7^(1/2))^(1/2)*(1-(-3/8+1/8*I*7^(1/2))*x^2)^(1/2)*(1-(-3/8-1/8*I*7^(1/2))*x^2)^(1/2)/(x^4+3*x^2+4)^(1/2)*EllipticF(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))-405282752/2145/(-6+2*I*7^(1/2))^(1/2)*(1-(-3/8+1/8*I*7^(1/2))*x^2)^(1/2)*(1-(-3/8-1/8*I*7^(1/2))*x^2)^(1/2)/(x^4+3*x^2+4)^(1/2)/(I*7^(1/2)+3)*(EllipticF(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))-EllipticE(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2)))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (x^4 + 3x^2 + 4)^{\frac{3}{2}} (5x^2 + 7)^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^2+7)^4*(x^4+3*x^2+4)^(3/2),x, algorithm="maxima")

[Out] integrate((x^4 + 3*x^2 + 4)^(3/2)*(5*x^2 + 7)^4, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(625x^{12} + 5375x^{10} + 20350x^8 + 42910x^6 + 52381x^4 + 34643x^2 + 9604\right)\sqrt{x^4 + 3x^2 + 4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^2+7)^4*(x^4+3*x^2+4)^(3/2),x, algorithm="fricas")

[Out] integral((625*x^12 + 5375*x^10 + 20350*x^8 + 42910*x^6 + 52381*x^4 + 34643*x^2 + 9604)*sqrt(x^4 + 3*x^2 + 4), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left((x^2 - x + 2)(x^2 + x + 2) \right)^{\frac{3}{2}} (5x^2 + 7)^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x**2+7)**4*(x**4+3*x**2+4)**(3/2),x)

[Out] Integral(((x**2 - x + 2)*(x**2 + x + 2))**(3/2)*(5*x**2 + 7)**4, x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (x^4 + 3x^2 + 4)^{\frac{3}{2}} (5x^2 + 7)^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^2+7)^4*(x^4+3*x^2+4)^(3/2),x, algorithm="giac")

[Out] integrate((x^4 + 3*x^2 + 4)^(3/2)*(5*x^2 + 7)^4, x)

$$3.357 \quad \int (7 + 5x^2)^3 (4 + 3x^2 + x^4)^{3/2} dx$$

Optimal. Leaf size=247

$$\frac{121826\sqrt{2}(x^2 + 2) \sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}} \text{EllipticF}\left(2 \tan^{-1}\left(\frac{x}{\sqrt{2}}\right), \frac{1}{8}\right)}{143\sqrt{x^4 + 3x^2 + 4}} + \frac{125}{13}(x^4 + 3x^2 + 4)^{5/2} x^3 + \frac{3825}{143}(x^4 + 3x^2 + 4)^{5/2} x + \frac{(15365x^2 + 53504)(x^4 + 3x^2 + 4)^{3/2}}{1001} + \frac{(435441x^2 + 1653701)\sqrt{x^4 + 3x^2 + x^4}}{5005}$$

[Out] (4525662*x*Sqrt[4 + 3*x^2 + x^4])/(5005*(2 + x^2)) + (x*(1653701 + 435441*x^2)*Sqrt[4 + 3*x^2 + x^4])/5005 + (x*(53504 + 15365*x^2)*(4 + 3*x^2 + x^4)^(3/2))/1001 + (3825*x*(4 + 3*x^2 + x^4)^(5/2))/143 + (125*x^3*(4 + 3*x^2 + x^4)^(5/2))/13 - (4525662*Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE[2*ArcTan[x/Sqrt[2]], 1/8])/(5005*Sqrt[4 + 3*x^2 + x^4]) + (121826*Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(143*Sqrt[4 + 3*x^2 + x^4])

Rubi [A] time = 0.12993, antiderivative size = 247, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$, Rules used = {1206, 1679, 1176, 1197, 1103, 1195}

$$\frac{125}{13}(x^4 + 3x^2 + 4)^{5/2} x^3 + \frac{3825}{143}(x^4 + 3x^2 + 4)^{5/2} x + \frac{(15365x^2 + 53504)(x^4 + 3x^2 + 4)^{3/2}}{1001} + \frac{(435441x^2 + 1653701)\sqrt{x^4 + 3x^2 + x^4}}{5005}$$

Antiderivative was successfully verified.

[In] Int[(7 + 5*x^2)^3*(4 + 3*x^2 + x^4)^(3/2),x]

[Out] (4525662*x*Sqrt[4 + 3*x^2 + x^4])/(5005*(2 + x^2)) + (x*(1653701 + 435441*x^2)*Sqrt[4 + 3*x^2 + x^4])/5005 + (x*(53504 + 15365*x^2)*(4 + 3*x^2 + x^4)^(3/2))/1001 + (3825*x*(4 + 3*x^2 + x^4)^(5/2))/143 + (125*x^3*(4 + 3*x^2 + x^4)^(5/2))/13 - (4525662*Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE[2*ArcTan[x/Sqrt[2]], 1/8])/(5005*Sqrt[4 + 3*x^2 + x^4]) + (121826*Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(143*Sqrt[4 + 3*x^2 + x^4])

Rule 1206

Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(e^q*x^(2*q - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(4*p + 2*q + 1)), x] + Dist[1/(c*(4*p + 2*q + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*p + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1]

Rule 1679

Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{q = Expon[Pq, x^2], e = Coeff[Pq, x^2, Expon[Pq, x^2]]}, Simp[(e*x^(2*q - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(2*q + 4*p + 1)), x] + Dist[1/(c*(2*q + 4*p + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*q + 4*p + 1)*Pq - a*e*(2*q - 3)*x^(2*q - 4) - b*e*(2*q + 2*p - 1)*x^(2*q - 2) - c*e*(2*q + 4*p + 1)*x^(2*q), x], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && Expon[P

$q, x^2 > 1 \ \&\& \ \text{NeQ}[b^2 - 4ac, 0] \ \&\& \ !\text{LtQ}[p, -1]$

Rule 1176

$\text{Int}[(d_ + (e_)(x_)^2)((a_ + (b_)(x_)^2 + (c_)(x_)^4)^{p_}), x_Symbol] \rightarrow \text{Simp}[(x(2bep + cd(4p + 3) + ce(4p + 1)x^2)(a + bx^2 + cx^4)^p)/(c(4p + 1)(4p + 3)), x] + \text{Dist}[(2p)/(c(4p + 1)(4p + 3)), \text{Int}[\text{Simp}[2acd(4p + 3) - abe + (2ac^2e(4p + 1) + bcd(4p + 3) - b^2e(2p + 1))x^2, x](a + bx^2 + cx^4)^{p-1}, x], x] /;$ FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4ac, 0] && NeQ[cd^2 - bde + ae^2, 0] && GtQ[p, 0] && FractionQ[p] && IntegerQ[2p]

Rule 1197

$\text{Int}[(d_ + (e_)(x_)^2)/\text{Sqrt}[(a_ + (b_)(x_)^2 + (c_)(x_)^4)], x_Symbol] \rightarrow \text{With}[q = \text{Rt}[c/a, 2], \text{Dist}[(e + dq)/q, \text{Int}[1/\text{Sqrt}[a + bx^2 + cx^4], x], x] - \text{Dist}[e/q, \text{Int}[(1 - qx^2)/\text{Sqrt}[a + bx^2 + cx^4], x], x] /;$ NeQ[e + dq, 0] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4ac, 0] && PosQ[c/a]

Rule 1103

$\text{Int}[1/\text{Sqrt}[(a_ + (b_)(x_)^2 + (c_)(x_)^4)], x_Symbol] \rightarrow \text{With}[q = \text{Rt}[c/a, 4], \text{Simp}[(1 + q^2x^2)\text{Sqrt}[(a + bx^2 + cx^4)/(a(1 + q^2x^2)^2)] * \text{EllipticF}[2\text{ArcTan}[qx], 1/2 - (bq^2)/(4c)]/(2q\text{Sqrt}[a + bx^2 + cx^4]), x] /;$ FreeQ[{a, b, c}, x] && NeQ[b^2 - 4ac, 0] && PosQ[c/a]

Rule 1195

$\text{Int}[(d_ + (e_)(x_)^2)/\text{Sqrt}[(a_ + (b_)(x_)^2 + (c_)(x_)^4)], x_Symbol] \rightarrow \text{With}[q = \text{Rt}[c/a, 4], -\text{Simp}[(dx\text{Sqrt}[a + bx^2 + cx^4])/(a(1 + q^2x^2)), x] + \text{Simp}[(d(1 + q^2x^2)\text{Sqrt}[(a + bx^2 + cx^4)/(a(1 + q^2x^2)^2)] * \text{EllipticE}[2\text{ArcTan}[qx], 1/2 - (bq^2)/(4c)]/(q\text{Sqrt}[a + bx^2 + cx^4]), x] /;$ EqQ[e + dq^2, 0] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4ac, 0] && PosQ[c/a]

Rubi steps

$$\begin{aligned} \int (7 + 5x^2)^3 (4 + 3x^2 + x^4)^{3/2} dx &= \frac{125}{13}x^3(4 + 3x^2 + x^4)^{5/2} + \frac{1}{13} \int (4 + 3x^2 + x^4)^{3/2} (4459 + 8055x^2 + 3825x^4) dx \\ &= \frac{3825}{143}x(4 + 3x^2 + x^4)^{5/2} + \frac{125}{13}x^3(4 + 3x^2 + x^4)^{5/2} + \frac{1}{143} \int (33749 + 19755x^2) \\ &= \frac{x(53504 + 15365x^2)(4 + 3x^2 + x^4)^{3/2}}{1001} + \frac{3825}{143}x(4 + 3x^2 + x^4)^{5/2} + \frac{125}{13}x^3(4 + \\ &= \frac{x(1653701 + 435441x^2)\sqrt{4 + 3x^2 + x^4}}{5005} + \frac{x(53504 + 15365x^2)(4 + 3x^2 + x^4)^{3/2}}{1001} \\ &= \frac{x(1653701 + 435441x^2)\sqrt{4 + 3x^2 + x^4}}{5005} + \frac{x(53504 + 15365x^2)(4 + 3x^2 + x^4)^{3/2}}{1001} \\ &= \frac{4525662x\sqrt{4 + 3x^2 + x^4}}{5005(2 + x^2)} + \frac{x(1653701 + 435441x^2)\sqrt{4 + 3x^2 + x^4}}{5005} + \frac{x(53504 + 15365x^2)(4 + 3x^2 + x^4)^{3/2}}{1001} \end{aligned}$$

Mathematica [F] time = 0, size = 0, normalized size = 0.

\$Aborted

Verification is Not applicable to the result.

[In] Integrate[(7 + 5*x^2)^3*(4 + 3*x^2 + x^4)^(3/2), x]

[Out] \$Aborted

Maple [C] time = 0.007, size = 309, normalized size = 1.3

$$\frac{4865781 x}{5005} \sqrt{x^4 + 3x^2 + 4} + \frac{5528301 x^3}{5005} \sqrt{x^4 + 3x^2 + 4} + \frac{48520 x^7}{143} \sqrt{x^4 + 3x^2 + 4} + \frac{71434 x^5}{91} \sqrt{x^4 + 3x^2 + 4} + \frac{125 x^{11}}{13} \sqrt{x^4 + 3x^2 + 4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((5*x^2+7)^3*(x^4+3*x^2+4)^(3/2), x)

[Out] 4865781/5005*x*(x^4+3*x^2+4)^(1/2)+5528301/5005*x^3*(x^4+3*x^2+4)^(1/2)+48520/143*x^7*(x^4+3*x^2+4)^(1/2)+71434/91*x^5*(x^4+3*x^2+4)^(1/2)+125/13*x^11*(x^4+3*x^2+4)^(1/2)+12075/143*x^9*(x^4+3*x^2+4)^(1/2)+32017264/5005/(-6+2*I*7^(1/2))^(1/2)*(1-(-3/8+1/8*I*7^(1/2))*x^2)^(1/2)*(1-(-3/8-1/8*I*7^(1/2))*x^2)^(1/2)/(x^4+3*x^2+4)^(1/2)*EllipticF(1/4*x*(-6+2*I*7^(1/2))^(1/2), 1/4*(2+6*I*7^(1/2))^(1/2))-144821184/5005/(-6+2*I*7^(1/2))^(1/2)*(1-(-3/8+1/8*I*7^(1/2))*x^2)^(1/2)*(1-(-3/8-1/8*I*7^(1/2))*x^2)^(1/2)/(x^4+3*x^2+4)^(1/2)/(I*7^(1/2)+3)*(EllipticF(1/4*x*(-6+2*I*7^(1/2))^(1/2), 1/4*(2+6*I*7^(1/2))^(1/2))-EllipticE(1/4*x*(-6+2*I*7^(1/2))^(1/2), 1/4*(2+6*I*7^(1/2))^(1/2)))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (x^4 + 3x^2 + 4)^{\frac{3}{2}} (5x^2 + 7)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^2+7)^3*(x^4+3*x^2+4)^(3/2), x, algorithm="maxima")

[Out] integrate((x^4 + 3*x^2 + 4)^(3/2)*(5*x^2 + 7)^3, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(125x^{10} + 900x^8 + 2810x^6 + 4648x^4 + 3969x^2 + 1372\right)\sqrt{x^4 + 3x^2 + 4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^2+7)^3*(x^4+3*x^2+4)^(3/2), x, algorithm="fricas")

[Out] integral((125*x^10 + 900*x^8 + 2810*x^6 + 4648*x^4 + 3969*x^2 + 1372)*sqrt(x^4 + 3*x^2 + 4), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int ((x^2 - x + 2)(x^2 + x + 2))^{\frac{3}{2}} (5x^2 + 7)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x**2+7)**3*(x**4+3*x**2+4)**(3/2),x)

[Out] Integral(((x**2 - x + 2)*(x**2 + x + 2))**(3/2)*(5*x**2 + 7)**3, x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (x^4 + 3x^2 + 4)^{\frac{3}{2}} (5x^2 + 7)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^2+7)^3*(x^4+3*x^2+4)^(3/2),x, algorithm="giac")

[Out] integrate((x^4 + 3*x^2 + 4)^(3/2)*(5*x^2 + 7)^3, x)

3.358 $\int (7 + 5x^2)^2 (4 + 3x^2 + x^4)^{3/2} dx$

Optimal. Leaf size=226

$$\frac{4628\sqrt{2}(x^2 + 2) \sqrt{\frac{x^4 + 3x^2 + 4}{(x^2 + 2)^2}} \text{EllipticF}\left(2 \tan^{-1}\left(\frac{x}{\sqrt{2}}\right), \frac{1}{8}\right)}{33\sqrt{x^4 + 3x^2 + 4}} + \frac{25}{11}x(x^4 + 3x^2 + 4)^{5/2} + \frac{1}{693}x(2240x^2 + 6831)(x^4 + 3x^2 + 4)^{3/2}$$

[Out] (175346*x*Sqrt[4 + 3*x^2 + x^4])/(1155*(2 + x^2)) + (x*(64533 + 18253*x^2)*Sqrt[4 + 3*x^2 + x^4])/1155 + (x*(6831 + 2240*x^2)*(4 + 3*x^2 + x^4)^(3/2))/693 + (25*x*(4 + 3*x^2 + x^4)^(5/2))/11 - (175346*Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE[2*ArcTan[x/Sqrt[2]], 1/8])/(1155*Sqrt[4 + 3*x^2 + x^4]) + (4628*Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(33*Sqrt[4 + 3*x^2 + x^4])

Rubi [A] time = 0.0979694, antiderivative size = 226, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {1206, 1176, 1197, 1103, 1195}

$$\frac{25}{11}x(x^4 + 3x^2 + 4)^{5/2} + \frac{1}{693}x(2240x^2 + 6831)(x^4 + 3x^2 + 4)^{3/2} + \frac{x(18253x^2 + 64533)\sqrt{x^4 + 3x^2 + 4}}{1155} + \frac{175346x\sqrt{x^4 + 3x^2 + 4}}{1155(x^2 + 2)}$$

Antiderivative was successfully verified.

[In] Int[(7 + 5*x^2)^2*(4 + 3*x^2 + x^4)^(3/2), x]

[Out] (175346*x*Sqrt[4 + 3*x^2 + x^4])/(1155*(2 + x^2)) + (x*(64533 + 18253*x^2)*Sqrt[4 + 3*x^2 + x^4])/1155 + (x*(6831 + 2240*x^2)*(4 + 3*x^2 + x^4)^(3/2))/693 + (25*x*(4 + 3*x^2 + x^4)^(5/2))/11 - (175346*Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE[2*ArcTan[x/Sqrt[2]], 1/8])/(1155*Sqrt[4 + 3*x^2 + x^4]) + (4628*Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(33*Sqrt[4 + 3*x^2 + x^4])

Rule 1206

Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(e^q*x^(2*q - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(4*p + 2*q + 1)), x] + Dist[1/(c*(4*p + 2*q + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*p + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x] /;

FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1]

Rule 1176

Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*(a + b*x^2 + c*x^4)^p)/(c*(4*p + 1)*(4*p + 3)), x] + Dist[(2*p)/(c*(4*p + 1)*(4*p + 3)), Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) - b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /;

FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]

Rule 1197

```
Int[((d_) + (e_)*(x_)^2)/Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4], x_Symbol]
:> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; NeQ[e + d*q, 0] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4], x_Symbol] :> With[{q = Rt[c/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]), x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_)*(x_)^2)/Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4], x_Symbol]
:> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c*x^4]), x] /; EqQ[e + d*q^2, 0] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\begin{aligned} \int (7 + 5x^2)^2 (4 + 3x^2 + x^4)^{3/2} dx &= \frac{25}{11}x(4 + 3x^2 + x^4)^{5/2} + \frac{1}{11} \int (439 + 320x^2)(4 + 3x^2 + x^4)^{3/2} dx \\ &= \frac{1}{693}x(6831 + 2240x^2)(4 + 3x^2 + x^4)^{3/2} + \frac{25}{11}x(4 + 3x^2 + x^4)^{5/2} + \frac{1}{231} \int (2776 + 1760x^2)(4 + 3x^2 + x^4)^{1/2} dx \\ &= \frac{x(64533 + 18253x^2)\sqrt{4 + 3x^2 + x^4}}{1155} + \frac{1}{693}x(6831 + 2240x^2)(4 + 3x^2 + x^4)^{3/2} + \frac{1}{231} \int (2776 + 1760x^2)\sqrt{4 + 3x^2 + x^4} dx \\ &= \frac{x(64533 + 18253x^2)\sqrt{4 + 3x^2 + x^4}}{1155} + \frac{1}{693}x(6831 + 2240x^2)(4 + 3x^2 + x^4)^{3/2} + \frac{1}{231} \int (2776 + 1760x^2)\sqrt{4 + 3x^2 + x^4} dx \\ &= \frac{175346x\sqrt{4 + 3x^2 + x^4}}{1155(2 + x^2)} + \frac{x(64533 + 18253x^2)\sqrt{4 + 3x^2 + x^4}}{1155} + \frac{1}{693}x(6831 + 2240x^2)(4 + 3x^2 + x^4)^{3/2} \end{aligned}$$

Mathematica [F] time = 0, size = 0, normalized size = 0.

\$Aborted

Verification is Not applicable to the result.

[In] Integrate[(7 + 5*x^2)^2*(4 + 3*x^2 + x^4)^(3/2), x]

[Out] \$Aborted

Maple [C] time = 0.007, size = 292, normalized size = 1.3

$$\frac{25x^9}{11}\sqrt{x^4 + 3x^2 + 4} + \frac{1670x^7}{99}\sqrt{x^4 + 3x^2 + 4} + \frac{1222x^5}{21}\sqrt{x^4 + 3x^2 + 4} + \frac{391024x^3}{3465}\sqrt{x^4 + 3x^2 + 4} + \frac{50691x}{385}\sqrt{x^4 + 3x^2 + 4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((5*x^2+7)^2*(x^4+3*x^2+4)^(3/2),x)`

[Out] $25/11*x^9*(x^4+3*x^2+4)^{(1/2)}+1670/99*x^7*(x^4+3*x^2+4)^{(1/2)}+1222/21*x^5*(x^4+3*x^2+4)^{(1/2)}+391024/3465*x^3*(x^4+3*x^2+4)^{(1/2)}+50691/385*x*(x^4+3*x^2+4)^{(1/2)}+396304/385/(-6+2*I*7^{(1/2)})^{(1/2)}*(1-(-3/8+1/8*I*7^{(1/2)})*x^2)^{(1/2)}*(1-(-3/8-1/8*I*7^{(1/2)})*x^2)^{(1/2)}/(x^4+3*x^2+4)^{(1/2)}*EllipticF(1/4*x*(-6+2*I*7^{(1/2)})^{(1/2)},1/4*(2+6*I*7^{(1/2)})^{(1/2)})-5611072/1155/(-6+2*I*7^{(1/2)})^{(1/2)}*(1-(-3/8+1/8*I*7^{(1/2)})*x^2)^{(1/2)}*(1-(-3/8-1/8*I*7^{(1/2)})*x^2)^{(1/2)}/(x^4+3*x^2+4)^{(1/2)}/(I*7^{(1/2)}+3)*(EllipticF(1/4*x*(-6+2*I*7^{(1/2)})^{(1/2)},1/4*(2+6*I*7^{(1/2)})^{(1/2)})-EllipticE(1/4*x*(-6+2*I*7^{(1/2)})^{(1/2)},1/4*(2+6*I*7^{(1/2)})^{(1/2)}))$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (x^4 + 3x^2 + 4)^{\frac{3}{2}} (5x^2 + 7)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((5*x^2+7)^2*(x^4+3*x^2+4)^(3/2),x, algorithm="maxima")`

[Out] `integrate((x^4 + 3*x^2 + 4)^(3/2)*(5*x^2 + 7)^2, x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(25x^8 + 145x^6 + 359x^4 + 427x^2 + 196\right)\sqrt{x^4 + 3x^2 + 4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((5*x^2+7)^2*(x^4+3*x^2+4)^(3/2),x, algorithm="fricas")`

[Out] `integral((25*x^8 + 145*x^6 + 359*x^4 + 427*x^2 + 196)*sqrt(x^4 + 3*x^2 + 4), x)`

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int ((x^2 - x + 2)(x^2 + x + 2))^{\frac{3}{2}} (5x^2 + 7)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((5*x**2+7)**2*(x**4+3*x**2+4)**(3/2),x)`

[Out] `Integral(((x**2 - x + 2)*(x**2 + x + 2))**(3/2)*(5*x**2 + 7)**2, x)`

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (x^4 + 3x^2 + 4)^{\frac{3}{2}} (5x^2 + 7)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^2*(x^4+3*x^2+4)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((x^4 + 3*x^2 + 4)^(3/2)*(5*x^2 + 7)^2, x)
```

3.359 $\int (7 + 5x^2) (4 + 3x^2 + x^4)^{3/2} dx$

Optimal. Leaf size=207

$$\frac{74\sqrt{2}(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{3\sqrt{x^4+3x^2+4}} + \frac{1}{63}x(35x^2+108)(x^4+3x^2+4)^{3/2} + \frac{1}{105}x(289x^2+1029)\sqrt{x^4+3x^2+4}$$

[Out] (2798*x*Sqrt[4 + 3*x^2 + x^4])/(105*(2 + x^2)) + (x*(1029 + 289*x^2)*Sqrt[4 + 3*x^2 + x^4])/105 + (x*(108 + 35*x^2)*(4 + 3*x^2 + x^4)^(3/2))/63 - (2798*Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE[2*ArcTan[x/Sqrt[2]], 1/8])/(105*Sqrt[4 + 3*x^2 + x^4]) + (74*Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(3*Sqrt[4 + 3*x^2 + x^4])

Rubi [A] time = 0.0712722, antiderivative size = 207, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1176, 1197, 1103, 1195}

$$\frac{1}{63}x(35x^2+108)(x^4+3x^2+4)^{3/2} + \frac{1}{105}x(289x^2+1029)\sqrt{x^4+3x^2+4} + \frac{2798x\sqrt{x^4+3x^2+4}}{105(x^2+2)} + \frac{74\sqrt{2}(x^2+2)\sqrt{\frac{x^4}{(x^2+2)^2}}}{3\sqrt{x^4+3x^2+4}}$$

Antiderivative was successfully verified.

[In] Int[(7 + 5*x^2)*(4 + 3*x^2 + x^4)^(3/2), x]

[Out] (2798*x*Sqrt[4 + 3*x^2 + x^4])/(105*(2 + x^2)) + (x*(1029 + 289*x^2)*Sqrt[4 + 3*x^2 + x^4])/105 + (x*(108 + 35*x^2)*(4 + 3*x^2 + x^4)^(3/2))/63 - (2798*Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE[2*ArcTan[x/Sqrt[2]], 1/8])/(105*Sqrt[4 + 3*x^2 + x^4]) + (74*Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(3*Sqrt[4 + 3*x^2 + x^4])

Rule 1176

Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := Simp[(x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*(a + b*x^2 + c*x^4)^p)/(c*(4*p + 1)*(4*p + 3)), x] + Dist[(2*p)/(c*(4*p + 1)*(4*p + 3)), Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) - b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]

Rule 1197

Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] := With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; NeQ[e + d*q, 0] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] := With[{q = Rt[c/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] := With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c*x^4]), x] /; EqQ[e + d*q^2, 0] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\begin{aligned} \int (7 + 5x^2)(4 + 3x^2 + x^4)^{3/2} dx &= \frac{1}{63}x(108 + 35x^2)(4 + 3x^2 + x^4)^{3/2} + \frac{1}{21} \int (444 + 289x^2) \sqrt{4 + 3x^2 + x^4} dx \\ &= \frac{1}{105}x(1029 + 289x^2) \sqrt{4 + 3x^2 + x^4} + \frac{1}{63}x(108 + 35x^2)(4 + 3x^2 + x^4)^{3/2} + \frac{1}{315} \int (444 + 289x^2) \sqrt{4 + 3x^2 + x^4} dx \\ &= \frac{1}{105}x(1029 + 289x^2) \sqrt{4 + 3x^2 + x^4} + \frac{1}{63}x(108 + 35x^2)(4 + 3x^2 + x^4)^{3/2} - \frac{559}{105} \int \sqrt{4 + 3x^2 + x^4} dx \\ &= \frac{2798x\sqrt{4 + 3x^2 + x^4}}{105(2 + x^2)} + \frac{1}{105}x(1029 + 289x^2) \sqrt{4 + 3x^2 + x^4} + \frac{1}{63}x(108 + 35x^2)(4 + 3x^2 + x^4)^{3/2} \end{aligned}$$

Mathematica [F] time = 0, size = 0, normalized size = 0.

\$Aborted

Verification is Not applicable to the result.

[In] Integrate[(7 + 5*x^2)*(4 + 3*x^2 + x^4)^(3/2), x]

[Out] \$Aborted

Maple [C] time = 0.007, size = 275, normalized size = 1.3

$$\frac{5x^7}{9}\sqrt{x^4 + 3x^2 + 4} + \frac{71x^5}{21}\sqrt{x^4 + 3x^2 + 4} + \frac{3187x^3}{315}\sqrt{x^4 + 3x^2 + 4} + \frac{583x}{35}\sqrt{x^4 + 3x^2 + 4} + \frac{6352}{35\sqrt{-6 + 2i\sqrt{7}}}\sqrt{1 - \dots}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((5*x^2+7)*(x^4+3*x^2+4)^(3/2), x)

[Out] $5/9*x^7*(x^4+3*x^2+4)^{(1/2)}+71/21*x^5*(x^4+3*x^2+4)^{(1/2)}+3187/315*x^3*(x^4+3*x^2+4)^{(1/2)}+583/35*x*(x^4+3*x^2+4)^{(1/2)}+6352/35/(-6+2*I*7^{(1/2)})^{(1/2)}*(1-(-3/8+1/8*I*7^{(1/2)})*x^2)^{(1/2)}*(1-(-3/8-1/8*I*7^{(1/2)})*x^2)^{(1/2)}/(x^4+3*x^2+4)^{(1/2)}*EllipticF(1/4*x*(-6+2*I*7^{(1/2)})^{(1/2)}, 1/4*(2+6*I*7^{(1/2)})^{(1/2)})-89536/105/(-6+2*I*7^{(1/2)})^{(1/2)}*(1-(-3/8+1/8*I*7^{(1/2)})*x^2)^{(1/2)}*$

$(1 - (-3/8 - 1/8 * I * 7^{(1/2)}) * x^2)^{(1/2)} / (x^4 + 3 * x^2 + 4)^{(1/2)} / (I * 7^{(1/2)} + 3) * (\text{EllipticF}(1/4 * x * (-6 + 2 * I * 7^{(1/2)})^{(1/2)}, 1/4 * (2 + 6 * I * 7^{(1/2)})^{(1/2)}) - \text{EllipticE}(1/4 * x * (-6 + 2 * I * 7^{(1/2)})^{(1/2)}, 1/4 * (2 + 6 * I * 7^{(1/2)})^{(1/2)}))$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (x^4 + 3x^2 + 4)^{\frac{3}{2}} (5x^2 + 7) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^2+7)*(x^4+3*x^2+4)^(3/2),x, algorithm="maxima")

[Out] integrate((x^4 + 3*x^2 + 4)^(3/2)*(5*x^2 + 7), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(5x^6 + 22x^4 + 41x^2 + 28\right)\sqrt{x^4 + 3x^2 + 4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^2+7)*(x^4+3*x^2+4)^(3/2),x, algorithm="fricas")

[Out] integral((5*x^6 + 22*x^4 + 41*x^2 + 28)*sqrt(x^4 + 3*x^2 + 4), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left((x^2 - x + 2)(x^2 + x + 2) \right)^{\frac{3}{2}} (5x^2 + 7) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x**2+7)*(x**4+3*x**2+4)**(3/2),x)

[Out] Integral(((x**2 - x + 2)*(x**2 + x + 2))**(3/2)*(5*x**2 + 7), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (x^4 + 3x^2 + 4)^{\frac{3}{2}} (5x^2 + 7) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^2+7)*(x^4+3*x^2+4)^(3/2),x, algorithm="giac")

[Out] integrate((x^4 + 3*x^2 + 4)^(3/2)*(5*x^2 + 7), x)

3.360 $\int (4 + 3x^2 + x^4)^{3/2} dx$

Optimal. Leaf size=198

$$\frac{4\sqrt{2}(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right), \frac{1}{8}\right)}{\sqrt{x^4+3x^2+4}} + \frac{1}{7}x(x^4+3x^2+4)^{3/2} + \frac{1}{35}x(9x^2+49)\sqrt{x^4+3x^2+4} + \frac{138x}{35\sqrt{x^4+3x^2+4}}$$

```
[Out] (138*x*Sqrt[4 + 3*x^2 + x^4])/(35*(2 + x^2)) + (x*(49 + 9*x^2)*Sqrt[4 + 3*x^2 + x^4])/35 + (x*(4 + 3*x^2 + x^4)^(3/2))/7 - (138*Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE[2*ArcTan[x/Sqrt[2]], 1/8])/(35*Sqrt[4 + 3*x^2 + x^4]) + (4*Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/Sqrt[4 + 3*x^2 + x^4]
```

Rubi [A] time = 0.0689473, antiderivative size = 198, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {1091, 1176, 1197, 1103, 1195}

$$\frac{1}{7}x(x^4+3x^2+4)^{3/2} + \frac{1}{35}x(9x^2+49)\sqrt{x^4+3x^2+4} + \frac{138x\sqrt{x^4+3x^2+4}}{35(x^2+2)} + \frac{4\sqrt{2}(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right), \frac{1}{8}\right)}{\sqrt{x^4+3x^2+4}}$$

Antiderivative was successfully verified.

```
[In] Int[(4 + 3*x^2 + x^4)^(3/2), x]
```

```
[Out] (138*x*Sqrt[4 + 3*x^2 + x^4])/(35*(2 + x^2)) + (x*(49 + 9*x^2)*Sqrt[4 + 3*x^2 + x^4])/35 + (x*(4 + 3*x^2 + x^4)^(3/2))/7 - (138*Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE[2*ArcTan[x/Sqrt[2]], 1/8])/(35*Sqrt[4 + 3*x^2 + x^4]) + (4*Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/Sqrt[4 + 3*x^2 + x^4]
```

Rule 1091

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(x*(a + b*x^2 + c*x^4)^p)/(4*p + 1), x] + Dist[(2*p)/(4*p + 1), Int[(2*a + b*x^2)*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*(a + b*x^2 + c*x^4)^p)/(c*(4*p + 1)*(4*p + 3)), x] + Dist[(2*p)/(c*(4*p + 1)*(4*p + 3)), Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) - b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
```

$Q[e + d*q, 0] /; \text{FreeQ}\{a, b, c, d, e\}, x\} \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0] \ \&\& \ \text{PosQ}[c/a]$

Rule 1103

$\text{Int}[1/\text{Sqrt}[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4], x_Symbol] \ :> \ \text{With}\{q = \text{Rt}[c/a, 4]\}, \ \text{Simp}[\frac{(1 + q^2*x^2)*\text{Sqrt}[a + b*x^2 + c*x^4]}{a*(1 + q^2*x^2)^2}] * \text{EllipticF}[2*\text{ArcTan}[q*x], 1/2 - (b*q^2)/(4*c)] / (2*q*\text{Sqrt}[a + b*x^2 + c*x^4]), x] /; \ \text{FreeQ}\{a, b, c\}, x\} \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0] \ \&\& \ \text{PosQ}[c/a]$

Rule 1195

$\text{Int}[\frac{(d_) + (e_)*(x_)^2}{\text{Sqrt}[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4]}, x_Symbol] \ :> \ \text{With}\{q = \text{Rt}[c/a, 4]\}, \ -\text{Simp}[\frac{d*x*\text{Sqrt}[a + b*x^2 + c*x^4]}{a*(1 + q^2*x^2)}, x] + \text{Simp}[\frac{d*(1 + q^2*x^2)*\text{Sqrt}[a + b*x^2 + c*x^4]}{a*(1 + q^2*x^2)^2}] * \text{EllipticE}[2*\text{ArcTan}[q*x], 1/2 - (b*q^2)/(4*c)] / (q*\text{Sqrt}[a + b*x^2 + c*x^4]), x] /; \ \text{EqQ}[e + d*q^2, 0] /; \ \text{FreeQ}\{a, b, c, d, e\}, x\} \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0] \ \&\& \ \text{PosQ}[c/a]$

Rubi steps

$$\begin{aligned} \int (4 + 3x^2 + x^4)^{3/2} dx &= \frac{1}{7}x(4 + 3x^2 + x^4)^{3/2} + \frac{3}{7} \int (8 + 3x^2) \sqrt{4 + 3x^2 + x^4} dx \\ &= \frac{1}{35}x(49 + 9x^2) \sqrt{4 + 3x^2 + x^4} + \frac{1}{7}x(4 + 3x^2 + x^4)^{3/2} + \frac{1}{35} \int \frac{284 + 138x^2}{\sqrt{4 + 3x^2 + x^4}} dx \\ &= \frac{1}{35}x(49 + 9x^2) \sqrt{4 + 3x^2 + x^4} + \frac{1}{7}x(4 + 3x^2 + x^4)^{3/2} - \frac{276}{35} \int \frac{1 - \frac{x^2}{2}}{\sqrt{4 + 3x^2 + x^4}} dx + 16 \int \frac{1}{\sqrt{4 + 3x^2 + x^4}} dx \\ &= \frac{138x\sqrt{4 + 3x^2 + x^4}}{35(2 + x^2)} + \frac{1}{35}x(49 + 9x^2) \sqrt{4 + 3x^2 + x^4} + \frac{1}{7}x(4 + 3x^2 + x^4)^{3/2} - \frac{138\sqrt{2}(2 + x^2)}{3} \end{aligned}$$

Mathematica [C] time = 0.418245, size = 343, normalized size = 1.73

$$\frac{\sqrt{2}(69\sqrt{7} - 77i) \sqrt{\frac{-2ix^2 + \sqrt{7} - 3i}{\sqrt{7} - 3i}} \sqrt{\frac{2ix^2 + \sqrt{7} + 3i}{\sqrt{7} + 3i}} \text{EllipticF}\left(i \sinh^{-1}\left(\sqrt{\frac{-2i}{\sqrt{7} - 3i}}x\right), \frac{-\sqrt{7} + 3i}{\sqrt{7} + 3i}\right) + 2\sqrt{\frac{i}{\sqrt{7} - 3i}}x(5x^8 + 39x^6 + 161x^4)}{70\sqrt{\frac{i}{\sqrt{7} - 3i}}\sqrt{x^4 + 3x^2 + 4}}$$

Antiderivative was successfully verified.

[In] Integrate[(4 + 3*x^2 + x^4)^(3/2), x]

[Out] (2*Sqrt[(-I)/(-3*I + Sqrt[7])]*x*(276 + 303*x^2 + 161*x^4 + 39*x^6 + 5*x^8) - 69*Sqrt[2]*(3*I + Sqrt[7])*Sqrt[(-3*I + Sqrt[7] - (2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[(3*I + Sqrt[7] + (2*I)*x^2)/(3*I + Sqrt[7])]*EllipticE[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])] + Sqrt[2]*(-77*I + 69*Sqrt[7])*Sqrt[(-3*I + Sqrt[7] - (2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[(3*I + Sqrt[7] + (2*I)*x^2)/(3*I + Sqrt[7])]*EllipticF[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])])/(70*Sqrt[(-I)/(-3*I + Sqrt[7])]*Sqrt[4 + 3*x^2 + x^4])

Maple [C] time = 0.003, size = 258, normalized size = 1.3

$$\frac{x^5}{7}\sqrt{x^4+3x^2+4} + \frac{24x^3}{35}\sqrt{x^4+3x^2+4} + \frac{69x}{35}\sqrt{x^4+3x^2+4} + \frac{1136}{35\sqrt{-6+2i\sqrt{7}}}\sqrt{1-\left(-\frac{3}{8}+\frac{i}{8}\sqrt{7}\right)x^2}\sqrt{1-\left(-\frac{3}{8}-\frac{i}{8}\sqrt{7}\right)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^4+3*x^2+4)^(3/2),x)

[Out] 1/7*x^5*(x^4+3*x^2+4)^(1/2)+24/35*x^3*(x^4+3*x^2+4)^(1/2)+69/35*x*(x^4+3*x^2+4)^(1/2)+1136/35/(-6+2*I*7^(1/2))^(1/2)*(1-(-3/8+1/8*I*7^(1/2))*x^2)^(1/2)*(1-(-3/8-1/8*I*7^(1/2))*x^2)^(1/2)/(x^4+3*x^2+4)^(1/2)*EllipticF(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))-4416/35/(-6+2*I*7^(1/2))^(1/2)*(1-(-3/8+1/8*I*7^(1/2))*x^2)^(1/2)*(1-(-3/8-1/8*I*7^(1/2))*x^2)^(1/2)/(x^4+3*x^2+4)^(1/2)/(I*7^(1/2)+3)*(EllipticF(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))-EllipticE(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2)))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (x^4 + 3x^2 + 4)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4+3*x^2+4)^(3/2),x, algorithm="maxima")

[Out] integrate((x^4 + 3*x^2 + 4)^(3/2), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(x^4 + 3x^2 + 4\right)^{\frac{3}{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4+3*x^2+4)^(3/2),x, algorithm="fricas")

[Out] integral((x^4 + 3*x^2 + 4)^(3/2), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (x^4 + 3x^2 + 4)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**4+3*x**2+4)**(3/2),x)

[Out] Integral((x**4 + 3*x**2 + 4)**(3/2), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (x^4 + 3x^2 + 4)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4+3*x^2+4)^(3/2),x, algorithm="giac")

[Out] integrate((x^4 + 3*x^2 + 4)^(3/2), x)

$$3.361 \quad \int \frac{(4+3x^2+x^4)^{3/2}}{7+5x^2} dx$$

Optimal. Leaf size=284

$$\frac{54\sqrt{2}(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right), \frac{1}{8}\right)}{125\sqrt{x^4+3x^2+4}} + \frac{1}{75}(3x^2+11)\sqrt{x^4+3x^2+4}x + \frac{94\sqrt{x^4+3x^2+4}x}{125(x^2+2)} + \frac{44}{125}\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}$$

```
[Out] (94*x*Sqrt[4 + 3*x^2 + x^4])/(125*(2 + x^2)) + (x*(11 + 3*x^2)*Sqrt[4 + 3*x^2 + x^4])/75 + (44*Sqrt[11/35]*ArcTan[(2*Sqrt[11/35]*x)/Sqrt[4 + 3*x^2 + x^4]])/125 - (94*Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE[2*ArcTan[x/Sqrt[2]], 1/8])/(125*Sqrt[4 + 3*x^2 + x^4]) + (54*Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(125*Sqrt[4 + 3*x^2 + x^4]) + (4114*Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticPi[-9/280, 2*ArcTan[x/Sqrt[2]], 1/8])/(13125*Sqrt[4 + 3*x^2 + x^4])
```

Rubi [A] time = 0.236115, antiderivative size = 284, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {1208, 1176, 1197, 1103, 1195, 1216, 1706}

$$\frac{1}{75}(3x^2+11)\sqrt{x^4+3x^2+4}x + \frac{94\sqrt{x^4+3x^2+4}x}{125(x^2+2)} + \frac{44}{125}\sqrt{\frac{11}{35}}\tan^{-1}\left(\frac{2\sqrt{\frac{11}{35}}x}{\sqrt{x^4+3x^2+4}}\right) + \frac{54\sqrt{2}(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}}{125\sqrt{x^4+3x^2+4}}$$

Antiderivative was successfully verified.

```
[In] Int[(4 + 3*x^2 + x^4)^(3/2)/(7 + 5*x^2), x]
```

```
[Out] (94*x*Sqrt[4 + 3*x^2 + x^4])/(125*(2 + x^2)) + (x*(11 + 3*x^2)*Sqrt[4 + 3*x^2 + x^4])/75 + (44*Sqrt[11/35]*ArcTan[(2*Sqrt[11/35]*x)/Sqrt[4 + 3*x^2 + x^4]])/125 - (94*Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE[2*ArcTan[x/Sqrt[2]], 1/8])/(125*Sqrt[4 + 3*x^2 + x^4]) + (54*Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(125*Sqrt[4 + 3*x^2 + x^4]) + (4114*Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticPi[-9/280, 2*ArcTan[x/Sqrt[2]], 1/8])/(13125*Sqrt[4 + 3*x^2 + x^4])
```

Rule 1208

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_)/((d_) + (e_.)*(x_)^2), x_Symbol] := -Dist[(e^2)^(-1), Int[(c*d - b*e - c*e*x^2)*(a + b*x^2 + c*x^4)^(p - 1), x], x] + Dist[(c*d^2 - b*d*e + a*e^2)/e^2, Int[(a + b*x^2 + c*x^4)^(p - 1)/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p + 1/2, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := Simp[(x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*(a + b*x^2 + c*x^4)^p)/(c*(4*p + 1)*(4*p + 3)), x] + Dist[(2*p)/(c*(4*p + 1)*(4*p + 3)), Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) - b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
```

b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]

Rule 1197

Int[((d_) + (e_)*(x_)^2)/Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4], x_Symbol] :=> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; NeQ[e + d*q, 0] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]

Rule 1103

Int[1/Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4], x_Symbol] :=> With[{q = Rt[c/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]), x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]

Rule 1195

Int[((d_) + (e_)*(x_)^2)/Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4], x_Symbol] :=> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c*x^4]), x] /; EqQ[e + d*q^2, 0] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]

Rule 1216

Int[1/(((d_) + (e_)*(x_)^2)*Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4]), x_Symbol] :=> With[{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]

Rule 1706

Int[((A_) + (B_)*(x_)^2)/(((d_) + (e_)*(x_)^2)*Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4]), x_Symbol] :=> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*ArcTan[(Rt[-b + (c*d)/e + (a*e)/d, 2]*x)/Sqrt[a + b*x^2 + c*x^4]])/(2*d*e*Rt[-b + (c*d)/e + (a*e)/d, 2]), x] + Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a + b*x^2 + c*x^4))/(a*(A + B*x^2)^2)]*EllipticPi[Cancel[-((B*d - A*e)^2/(4*d*e*A*B))], 2*ArcTan[q*x], 1/2 - (b*A)/(4*a*B)]/(4*d*e*A*q*Sqrt[a + b*x^2 + c*x^4]), x] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]

Rubi steps

$$\begin{aligned}
\int \frac{(4 + 3x^2 + x^4)^{3/2}}{7 + 5x^2} dx &= -\left(\frac{1}{25} \int (-8 - 5x^2) \sqrt{4 + 3x^2 + x^4} dx\right) + \frac{44}{25} \int \frac{\sqrt{4 + 3x^2 + x^4}}{7 + 5x^2} dx \\
&= \frac{1}{75} x (11 + 3x^2) \sqrt{4 + 3x^2 + x^4} - \frac{1}{375} \int \frac{-260 - 150x^2}{\sqrt{4 + 3x^2 + x^4}} dx - \frac{44}{625} \int \frac{-8 - 5x^2}{\sqrt{4 + 3x^2 + x^4}} dx + \frac{19}{6} \\
&= \frac{1}{75} x (11 + 3x^2) \sqrt{4 + 3x^2 + x^4} - \frac{88}{125} \int \frac{1 - \frac{x^2}{2}}{\sqrt{4 + 3x^2 + x^4}} dx - \frac{4}{5} \int \frac{1 - \frac{x^2}{2}}{\sqrt{4 + 3x^2 + x^4}} dx - \frac{1936}{9} \\
&= \frac{94x\sqrt{4 + 3x^2 + x^4}}{125(2 + x^2)} + \frac{1}{75} x (11 + 3x^2) \sqrt{4 + 3x^2 + x^4} + \frac{44}{125} \sqrt{\frac{11}{35}} \tan^{-1} \left(\frac{2\sqrt{\frac{11}{35}}x}{\sqrt{4 + 3x^2 + x^4}} \right) - \frac{1936}{9}
\end{aligned}$$

Mathematica [C] time = 0.718348, size = 477, normalized size = 1.68

$$7\sqrt{2}(705\sqrt{7} - 241i) \sqrt{\frac{-2ix^2 + \sqrt{7} - 3i}{\sqrt{7} - 3i}} \sqrt{\frac{2ix^2 + \sqrt{7} + 3i}{\sqrt{7} + 3i}} \text{EllipticF}\left(i \sinh^{-1}\left(\sqrt{\frac{-2i}{\sqrt{7} - 3i}}x\right), \frac{-\sqrt{7} + 3i}{\sqrt{7} + 3i}\right) + 350\sqrt{\frac{i}{\sqrt{7} - 3i}}x(3x^6 + 20x^4)$$

Antiderivative was successfully verified.

[In] Integrate[(4 + 3*x^2 + x^4)^(3/2)/(7 + 5*x^2), x]

[Out] (350*Sqrt[(-I)/(-3*I + Sqrt[7])]*x*(44 + 45*x^2 + 20*x^4 + 3*x^6) - 4935*Sqrt[2]*(3*I + Sqrt[7])*Sqrt[(-3*I + Sqrt[7] - (2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[(3*I + Sqrt[7] + (2*I)*x^2)/(3*I + Sqrt[7])]*EllipticE[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])] + 7*Sqrt[2]*(-241*I + 705*Sqrt[7])*Sqrt[(-3*I + Sqrt[7] - (2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[(3*I + Sqrt[7] + (2*I)*x^2)/(3*I + Sqrt[7])]*EllipticF[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])] - (5808*I)*Sqrt[2]*Sqrt[(-3*I + Sqrt[7] - (2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[(3*I + Sqrt[7] + (2*I)*x^2)/(3*I + Sqrt[7])]*EllipticPi[(5*(3 + I*Sqrt[7]))/14, I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])])/(26250*Sqrt[(-I)/(-3*I + Sqrt[7])]*Sqrt[4 + 3*x^2 + x^4])

Maple [C] time = 0.019, size = 418, normalized size = 1.5

$$\frac{x^3}{25} \sqrt{x^4 + 3x^2 + 4} + \frac{11x}{75} \sqrt{x^4 + 3x^2 + 4} + \frac{9424}{1875 \sqrt{-6 + 2i\sqrt{7}}} \sqrt{1 + \frac{3x^2}{8} - \frac{i}{8}x^2\sqrt{7}} \sqrt{1 + \frac{3x^2}{8} + \frac{i}{8}x^2\sqrt{7}} \text{EllipticF}\left(x\sqrt{\frac{3x^2 + 4}{x^4 + 3x^2 + 4}}, \frac{-6 + 2i\sqrt{7}}{1875}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^4+3*x^2+4)^(3/2)/(5*x^2+7), x)

[Out] 1/25*x^3*(x^4+3*x^2+4)^(1/2)+11/75*x*(x^4+3*x^2+4)^(1/2)+9424/1875/(-6+2*I*sqrt(7)^(1/2))^(1/2)*(1+3/8*x^2-1/8*I*x^2*sqrt(7)^(1/2))^(1/2)*(1+3/8*x^2+1/8*I*x^2*sqrt(7)^(1/2))^(1/2)/(x^4+3*x^2+4)^(1/2)*EllipticF(1/4*x*(-6+2*I*sqrt(7)^(1/2))^(1/2), 1/4*(2+6*I*sqrt(7)^(1/2))^(1/2))-3008/125/(-6+2*I*sqrt(7)^(1/2))^(1/2)*(1+3/8*x^2-1/8*I*x^2*sqrt(7)^(1/2))^(1/2)*(1+3/8*x^2+1/8*I*x^2*sqrt(7)^(1/2))^(1/2)/(x^4+3*x^2+4)^(1/2)/(I*sqrt(7)^(1/2)+3)*EllipticF(1/4*x*(-6+2*I*sqrt(7)^(1/2))^(1/2), 1/4*(2+6*I*sqrt(7)^(1/2))^(1/2))

) + 3008/125/(-6+2*I*7^(1/2))^(1/2)*(1+3/8*x^2-1/8*I*x^2*7^(1/2))^(1/2)*(1+3/8*x^2+1/8*I*x^2*7^(1/2))^(1/2)/(x^4+3*x^2+4)^(1/2)/(I*7^(1/2)+3)*EllipticE(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))+1936/4375/(-3/8+1/8*I*7^(1/2))^(1/2)*(1+3/8*x^2-1/8*I*x^2*7^(1/2))^(1/2)*(1+3/8*x^2+1/8*I*x^2*7^(1/2))^(1/2)/(x^4+3*x^2+4)^(1/2)*EllipticPi((-3/8+1/8*I*7^(1/2))^(1/2)*x,-5/7/(-3/8+1/8*I*7^(1/2)),(-3/8-1/8*I*7^(1/2))^(1/2)/(-3/8+1/8*I*7^(1/2))^(1/2))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(x^4 + 3x^2 + 4)^{\frac{3}{2}}}{5x^2 + 7} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4+3*x^2+4)^(3/2)/(5*x^2+7),x, algorithm="maxima")

[Out] integrate((x^4 + 3*x^2 + 4)^(3/2)/(5*x^2 + 7), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(x^4 + 3x^2 + 4)^{\frac{3}{2}}}{5x^2 + 7}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4+3*x^2+4)^(3/2)/(5*x^2+7),x, algorithm="fricas")

[Out] integral((x^4 + 3*x^2 + 4)^(3/2)/(5*x^2 + 7), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{((x^2 - x + 2)(x^2 + x + 2))^{\frac{3}{2}}}{5x^2 + 7} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**4+3*x**2+4)**(3/2)/(5*x**2+7),x)

[Out] Integral(((x**2 - x + 2)*(x**2 + x + 2))**(3/2)/(5*x**2 + 7), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(x^4 + 3x^2 + 4)^{\frac{3}{2}}}{5x^2 + 7} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+3*x^2+4)^(3/2)/(5*x^2+7),x, algorithm="giac")
```

```
[Out] integrate((x^4 + 3*x^2 + 4)^(3/2)/(5*x^2 + 7), x)
```

$$3.362 \quad \int \frac{(4+3x^2+x^4)^{3/2}}{(7+5x^2)^2} dx$$

Optimal. Leaf size=305

$$\frac{4\sqrt{2}(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right), \frac{1}{8}\right)}{175\sqrt{x^4+3x^2+4}} + \frac{4\sqrt{x^4+3x^2+4}x}{175(x^2+2)} + \frac{22\sqrt{x^4+3x^2+4}}{175(5x^2+7)} + \frac{1}{75}\sqrt{x^4+3x^2+4} + \frac{1}{350}\sqrt{x^4+3x^2+4}$$

[Out] (x*Sqrt[4 + 3*x^2 + x^4])/75 + (4*x*Sqrt[4 + 3*x^2 + x^4])/(175*(2 + x^2)) + (22*x*Sqrt[4 + 3*x^2 + x^4])/(175*(7 + 5*x^2)) + (13*Sqrt[11/35]*ArcTan[(2*Sqrt[11/35]*x)/Sqrt[4 + 3*x^2 + x^4]])/350 - (4*Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE[2*ArcTan[x/Sqrt[2]], 1/8])/(175*Sqrt[4 + 3*x^2 + x^4]) + (4*Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(175*Sqrt[4 + 3*x^2 + x^4]) + (2431*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticPi[-9/280, 2*ArcTan[x/Sqrt[2]], 1/8])/(36750*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4])

Rubi [A] time = 0.534379, antiderivative size = 372, normalized size of antiderivative = 1.22, number of steps used = 19, number of rules used = 11, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.458$, Rules used = {1228, 1103, 1139, 1195, 1122, 1197, 1223, 1714, 1708, 1706, 1216}

$$\frac{4\sqrt{x^4+3x^2+4}x}{175(x^2+2)} + \frac{22\sqrt{x^4+3x^2+4}}{175(5x^2+7)} + \frac{1}{75}\sqrt{x^4+3x^2+4}x + \frac{13}{350}\sqrt{\frac{11}{35}}\tan^{-1}\left(\frac{2\sqrt{\frac{11}{35}}x}{\sqrt{x^4+3x^2+4}}\right) + \frac{4\sqrt{2}(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}}{175\sqrt{x^4+3x^2+4}}$$

Antiderivative was successfully verified.

[In] Int[(4 + 3*x^2 + x^4)^(3/2)/(7 + 5*x^2)^2,x]

[Out] (x*Sqrt[4 + 3*x^2 + x^4])/75 + (4*x*Sqrt[4 + 3*x^2 + x^4])/(175*(2 + x^2)) + (22*x*Sqrt[4 + 3*x^2 + x^4])/(175*(7 + 5*x^2)) + (13*Sqrt[11/35]*ArcTan[(2*Sqrt[11/35]*x)/Sqrt[4 + 3*x^2 + x^4]])/350 - (4*Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE[2*ArcTan[x/Sqrt[2]], 1/8])/(175*Sqrt[4 + 3*x^2 + x^4]) + (4*Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(175*Sqrt[4 + 3*x^2 + x^4]) + (6919*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticPi[-9/280, 2*ArcTan[x/Sqrt[2]], 1/8])/(183750*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4]) + (187*Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticPi[-9/280, 2*ArcTan[x/Sqrt[2]], 1/8])/(13125*Sqrt[4 + 3*x^2 + x^4])

Rule 1228

Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := Module[{aa, bb, cc}, Int[ExpandIntegrand[1/Sqrt[aa + bb*x^2 + c*x^4], (d + e*x^2)^q*(aa + bb*x^2 + cc*x^4)^(p + 1/2), x] /. {aa -> a, bb -> b, cc -> c}, x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, 0] && IntegerQ[p + 1/2]

Rule 1103

Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] := With[{q = Rt[c/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*

EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)]/(2*q*Sqrt[a + b*x^2 + c*x^4]), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]

Rule 1139

Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c/a, 2]}, Dist[1/q, Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]

Rule 1195

Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)]/(q*Sqrt[a + b*x^2 + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]

Rule 1122

Int[((d_.)*(x_)^(m_.))*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(d^3*(d*x)^(m - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(m + 4*p + 1)), x] - Dist[d^4/(c*(m + 4*p + 1)), Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m + 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])

Rule 1197

Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]

Rule 1223

Int[((d_) + (e_.)*(x_)^2)^(q_)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> -Simp[(e^2*x*(d + e*x^2)^(q + 1)*Sqrt[a + b*x^2 + c*x^4])/(2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2)), Int[((d + e*x^2)^(q + 1)*Simp[a*e^2*(2*q + 3) + 2*d*(c*d - b*e)*(q + 1) - 2*e*(c*d*(q + 1) - b*e*(q + 2))*x^2 + c*e^2*(2*q + 5)*x^4, x])/Sqrt[a + b*x^2 + c*x^4], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && ILtQ[q, -1]

Rule 1714

Int[(P4x)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[c/a, 2], A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[P4x, x, 4]}, -Dist[C/(e*q), Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] + Dist[1/(c*e), Int[(A*c*e + a*C*d*q + (B*c*e - C*(c*d - a*e*q))*x^2]/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[P4x, x^2, 2] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && !GtQ[b^2 - 4*a*c, 0]

Rule 1708

```
Int[((A_.) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_Symbol] := With[{q = Rt[c/a, 2]}, Dist[(A*(c*d + a*e*q) - a*B*(e + d*q))/(c*d^2 - a*e^2), Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] + Dist[(a*(B*d - A*e)*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && NeQ[c*A^2 - a*B^2, 0]
```

Rule 1706

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_Symbol] := With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*ArcTan[(Rt[-b + (c*d)/e + (a*e)/d, 2]*x)/Sqrt[a + b*x^2 + c*x^4]])/(2*d*e*Rt[-b + (c*d)/e + (a*e)/d, 2]), x] + Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a + b*x^2 + c*x^4))/(a*(A + B*x^2)^2)]*EllipticPi[Cancel[-((B*d - A*e)^2/(4*d*e*A*B))], 2*ArcTan[q*x], 1/2 - (b*A)/(4*a*B)]/(4*d*e*A*q*Sqrt[a + b*x^2 + c*x^4]), x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rule 1216

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_Symbol] := With[{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rubi steps

$$\begin{aligned} \int \frac{(4 + 3x^2 + x^4)^{3/2}}{(7 + 5x^2)^2} dx &= \int \left(\frac{152}{625\sqrt{4 + 3x^2 + x^4}} + \frac{16x^2}{125\sqrt{4 + 3x^2 + x^4}} + \frac{x^4}{25\sqrt{4 + 3x^2 + x^4}} + \frac{1936}{625(7 + 5x^2)^2\sqrt{4 + 3x^2 + x^4}} \right) dx \\ &= \frac{1}{25} \int \frac{x^4}{\sqrt{4 + 3x^2 + x^4}} dx + \frac{16}{125} \int \frac{x^2}{\sqrt{4 + 3x^2 + x^4}} dx + \frac{88}{625} \int \frac{1}{(7 + 5x^2)\sqrt{4 + 3x^2 + x^4}} dx + \frac{1936}{625} \int \frac{1}{(7 + 5x^2)^2\sqrt{4 + 3x^2 + x^4}} dx \\ &= \frac{1}{75} x\sqrt{4 + 3x^2 + x^4} + \frac{22x\sqrt{4 + 3x^2 + x^4}}{175(7 + 5x^2)} + \frac{38\sqrt{2}(2 + x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}} F\left(2 \tan^{-1}\left(\frac{x}{\sqrt{2}}\right) \middle| \frac{1}{8}\right)}{625\sqrt{4 + 3x^2 + x^4}} - \frac{1936}{625} \int \frac{1}{(7 + 5x^2)^2\sqrt{4 + 3x^2 + x^4}} dx \\ &= \frac{1}{75} x\sqrt{4 + 3x^2 + x^4} + \frac{16x\sqrt{4 + 3x^2 + x^4}}{125(2 + x^2)} + \frac{22x\sqrt{4 + 3x^2 + x^4}}{175(7 + 5x^2)} + \frac{2}{125} \sqrt{\frac{11}{35}} \tan^{-1} \left(\frac{2\sqrt{\frac{11}{35}}x}{\sqrt{4 + 3x^2 + x^4}} \right) \\ &= \frac{1}{75} x\sqrt{4 + 3x^2 + x^4} + \frac{4x\sqrt{4 + 3x^2 + x^4}}{175(2 + x^2)} + \frac{22x\sqrt{4 + 3x^2 + x^4}}{175(7 + 5x^2)} + \frac{2}{125} \sqrt{\frac{11}{35}} \tan^{-1} \left(\frac{2\sqrt{\frac{11}{35}}x}{\sqrt{4 + 3x^2 + x^4}} \right) \\ &= \frac{1}{75} x\sqrt{4 + 3x^2 + x^4} + \frac{4x\sqrt{4 + 3x^2 + x^4}}{175(2 + x^2)} + \frac{22x\sqrt{4 + 3x^2 + x^4}}{175(7 + 5x^2)} + \frac{13}{350} \sqrt{\frac{11}{35}} \tan^{-1} \left(\frac{2\sqrt{\frac{11}{35}}x}{\sqrt{4 + 3x^2 + x^4}} \right) \end{aligned}$$

Mathematica [C] time = 0.585088, size = 309, normalized size = 1.01

$$\frac{175x(7x^2+23)(x^4+3x^2+4)}{5x^2+7} - i\sqrt{6+2i\sqrt{7}}\sqrt{1-\frac{2ix^2}{\sqrt{7}-3i}}\sqrt{1+\frac{2ix^2}{\sqrt{7}+3i}} \left(7(158+15i\sqrt{7})\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{-\frac{2i}{\sqrt{7}-3i}}x\right), \frac{-\sqrt{7}+3i}{\sqrt{7}+3i}\right)\right)$$

$$18375\sqrt{x^4+3x^2+4}$$

Antiderivative was successfully verified.

[In] Integrate[(4 + 3*x^2 + x^4)^(3/2)/(7 + 5*x^2)^2,x]

[Out] ((175*x*(23 + 7*x^2)*(4 + 3*x^2 + x^4))/(7 + 5*x^2) - I*Sqrt[6 + (2*I)*Sqrt[7]]*Sqrt[1 - ((2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[1 + ((2*I)*x^2)/(3*I + Sqrt[7])])*(105*(3 - I*Sqrt[7])*EllipticE[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])) + 7*(158 + (15*I)*Sqrt[7])*EllipticF[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])] + 429*EllipticPi[(5*(3 + I*Sqrt[7]))/14, I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])))/(18375*Sqrt[4 + 3*x^2 + x^4])

Maple [C] time = 0.023, size = 425, normalized size = 1.4

$$\frac{22x}{875x^2+1225}\sqrt{x^4+3x^2+4} + \frac{x}{75}\sqrt{x^4+3x^2+4} + \frac{232}{375\sqrt{-6+2i\sqrt{7}}}\sqrt{1+\frac{3x^2}{8}-\frac{i}{8}x^2\sqrt{7}}\sqrt{1+\frac{3x^2}{8}+\frac{i}{8}x^2\sqrt{7}}\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{-\frac{2i}{\sqrt{7}-3i}}x\right), \frac{-\sqrt{7}+3i}{\sqrt{7}+3i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^4+3*x^2+4)^(3/2)/(5*x^2+7)^2,x)

[Out] 22/175*x*(x^4+3*x^2+4)^(1/2)/(5*x^2+7)+1/75*x*(x^4+3*x^2+4)^(1/2)+232/375/((-6+2*I*7^(1/2))^(1/2)*(1+3/8*x^2-1/8*I*x^2*7^(1/2))^(1/2)*(1+3/8*x^2+1/8*I*x^2*7^(1/2))^(1/2)/(x^4+3*x^2+4)^(1/2)*EllipticF(1/4*x*(-6+2*I*7^(1/2))^(1/2), 1/4*(2+6*I*7^(1/2))^(1/2))-128/175/((-6+2*I*7^(1/2))^(1/2)*(1+3/8*x^2-1/8*I*x^2*7^(1/2))^(1/2)*(1+3/8*x^2+1/8*I*x^2*7^(1/2))^(1/2)/(x^4+3*x^2+4)^(1/2)/(I*7^(1/2)+3)*EllipticF(1/4*x*(-6+2*I*7^(1/2))^(1/2), 1/4*(2+6*I*7^(1/2))^(1/2))+128/175/((-6+2*I*7^(1/2))^(1/2)*(1+3/8*x^2-1/8*I*x^2*7^(1/2))^(1/2)*(1+3/8*x^2+1/8*I*x^2*7^(1/2))^(1/2)/(x^4+3*x^2+4)^(1/2)/(I*7^(1/2)+3)*EllipticE(1/4*x*(-6+2*I*7^(1/2))^(1/2), 1/4*(2+6*I*7^(1/2))^(1/2))+286/6125/((-3/8+1/8*I*7^(1/2))^(1/2)*(1+3/8*x^2-1/8*I*x^2*7^(1/2))^(1/2)*(1+3/8*x^2+1/8*I*x^2*7^(1/2))^(1/2)/(x^4+3*x^2+4)^(1/2)*EllipticPi((-3/8+1/8*I*7^(1/2))^(1/2)*x, -5/7/((-3/8+1/8*I*7^(1/2)), (-3/8-1/8*I*7^(1/2))^(1/2)/(-3/8+1/8*I*7^(1/2))^(1/2))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(x^4 + 3x^2 + 4)^{\frac{3}{2}}}{(5x^2 + 7)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4+3*x^2+4)^(3/2)/(5*x^2+7)^2,x, algorithm="maxima")

[Out] integrate((x^4 + 3*x^2 + 4)^(3/2)/(5*x^2 + 7)^2, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(x^4 + 3x^2 + 4)^{\frac{3}{2}}}{25x^4 + 70x^2 + 49}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4+3*x^2+4)^(3/2)/(5*x^2+7)^2,x, algorithm="fricas")

[Out] integral((x^4 + 3*x^2 + 4)^(3/2)/(25*x^4 + 70*x^2 + 49), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{((x^2 - x + 2)(x^2 + x + 2))^{\frac{3}{2}}}{(5x^2 + 7)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**4+3*x**2+4)**(3/2)/(5*x**2+7)**2,x)

[Out] Integral(((x**2 - x + 2)*(x**2 + x + 2))**(3/2)/(5*x**2 + 7)**2, x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(x^4 + 3x^2 + 4)^{\frac{3}{2}}}{(5x^2 + 7)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4+3*x^2+4)^(3/2)/(5*x^2+7)^2,x, algorithm="giac")

[Out] integrate((x^4 + 3*x^2 + 4)^(3/2)/(5*x^2 + 7)^2, x)

$$3.363 \quad \int \frac{(4+3x^2+x^4)^{3/2}}{(7+5x^2)^3} dx$$

Optimal. Leaf size=440

$$\frac{22\sqrt{2}(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right), \frac{1}{8}\right)}{13125\sqrt{x^4+3x^2+4}} - \frac{817(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right), \frac{1}{8}\right)}{91875\sqrt{2}\sqrt{x^4+3x^2+4}} + \frac{9\sqrt{x^4+3x^2+4}}{1960(x^2+2)}$$

```
[Out] (9*x*Sqrt[4 + 3*x^2 + x^4])/(1960*(2 + x^2)) + (11*x*Sqrt[4 + 3*x^2 + x^4])
/(175*(7 + 5*x^2)^2) + (167*x*Sqrt[4 + 3*x^2 + x^4])/(9800*(7 + 5*x^2)) + (
1347*ArcTan[(2*Sqrt[11/35]*x)/Sqrt[4 + 3*x^2 + x^4]])/(7840*Sqrt[385]) + (1
11*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE[2*ArcTan[x/Sqrt[
2]], 1/8])/(24500*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4]) - (6*Sqrt[2]*(2 + x^2)*Sqr
t[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE[2*ArcTan[x/Sqrt[2]], 1/8])/(875*
Sqrt[4 + 3*x^2 + x^4]) - (817*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]
*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(91875*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4])
- (22*Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*Ar
cTan[x/Sqrt[2]], 1/8])/(13125*Sqrt[4 + 3*x^2 + x^4]) + (7633*(2 + x^2)*Sqrt
[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticPi[-9/280, 2*ArcTan[x/Sqrt[2]], 1/8
])/ (274400*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4])
```

Rubi [A] time = 0.799709, antiderivative size = 440, normalized size of antiderivative = 1., number of steps used = 22, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {1228, 1103, 1139, 1195, 1223, 1696, 1714, 1708, 1706, 1216}

$$\frac{9\sqrt{x^4+3x^2+4x}}{1960(x^2+2)} + \frac{167\sqrt{x^4+3x^2+4x}}{9800(5x^2+7)} + \frac{11\sqrt{x^4+3x^2+4x}}{175(5x^2+7)^2} + \frac{1347 \tan^{-1}\left(\frac{2\sqrt{\frac{11}{35}}x}{\sqrt{x^4+3x^2+4}}\right)}{7840\sqrt{385}} - \frac{22\sqrt{2}(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}}{13125\sqrt{x^4+3x^2+4}}$$

Antiderivative was successfully verified.

```
[In] Int[(4 + 3*x^2 + x^4)^(3/2)/(7 + 5*x^2)^3,x]
```

```
[Out] (9*x*Sqrt[4 + 3*x^2 + x^4])/(1960*(2 + x^2)) + (11*x*Sqrt[4 + 3*x^2 + x^4])
/(175*(7 + 5*x^2)^2) + (167*x*Sqrt[4 + 3*x^2 + x^4])/(9800*(7 + 5*x^2)) + (
1347*ArcTan[(2*Sqrt[11/35]*x)/Sqrt[4 + 3*x^2 + x^4]])/(7840*Sqrt[385]) + (1
11*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE[2*ArcTan[x/Sqrt[
2]], 1/8])/(24500*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4]) - (6*Sqrt[2]*(2 + x^2)*Sqr
t[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE[2*ArcTan[x/Sqrt[2]], 1/8])/(875*
Sqrt[4 + 3*x^2 + x^4]) - (817*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]
*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(91875*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4])
- (22*Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*Ar
cTan[x/Sqrt[2]], 1/8])/(13125*Sqrt[4 + 3*x^2 + x^4]) + (7633*(2 + x^2)*Sqrt
[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticPi[-9/280, 2*ArcTan[x/Sqrt[2]], 1/8
])/ (274400*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4])
```

Rule 1228

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] := Module[{aa, bb, cc}, Int[ExpandIntegrand[1/Sqrt[aa + bb*x^2 + c
c*x^4], (d + e*x^2)^q*(aa + bb*x^2 + cc*x^4)^(p + 1/2), x] /. {aa -> a, bb
-> b, cc -> c}, x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
```

NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, 0] && IntegerQ[p + 1/2]

Rule 1103

Int[1/Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4], x_Symbol] := With[{q = Rt[c/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]

Rule 1139

Int[(x_)^2/Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4], x_Symbol] := With[{q = Rt[c/a, 2]}, Dist[1/q, Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]

Rule 1195

Int[((d_) + (e_)*(x_)^2)/Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4], x_Symbol] := With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c*x^4]), x]] /; EqQ[e + d*q^2, 0] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]

Rule 1223

Int[((d_) + (e_)*(x_)^2)^(q_)/Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4], x_Symbol] := -Simp[(e^2*x*(d + e*x^2)^(q + 1)*Sqrt[a + b*x^2 + c*x^4])/(2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2)), Int[((d + e*x^2)^(q + 1)*Simp[a*e^2*(2*q + 3) + 2*d*(c*d - b*e)*(q + 1) - 2*e*(c*d*(q + 1) - b*e*(q + 2))*x^2 + c*e^2*(2*q + 5)*x^4, x])/Sqrt[a + b*x^2 + c*x^4], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && ILtQ[q, -1]

Rule 1696

Int[(P4x_)*((d_) + (e_)*(x_)^2)^(q_)/Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4], x_Symbol] := With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[P4x, x, 4]}, -Simp[((C*d^2 - B*d*e + A*e^2)*x*(d + e*x^2)^(q + 1)*Sqrt[a + b*x^2 + c*x^4])/(2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2)), Int[((d + e*x^2)^(q + 1)*Simp[a*d*(C*d - B*e) + A*(a*e^2*(2*q + 3) + 2*d*(c*d - b*e)*(q + 1)) - 2*((B*d - A*e)*(b*e*(q + 2) - c*d*(q + 1)) - C*d*(b*d + a*e*(q + 1)))*x^2 + c*(C*d^2 - B*d*e + A*e^2)*(2*q + 5)*x^4, x])/Sqrt[a + b*x^2 + c*x^4], x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[P4x, x^2] && LeQ[Expon[P4x, x], 4] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, -1]

Rule 1714

Int[(P4x_)/(((d_) + (e_)*(x_)^2)*Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4]), x_Symbol] := With[{q = Rt[c/a, 2], A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[P4x, x, 4]}, -Dist[C/(e*q), Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] + Dist[1/(c*e), Int[(A*c*e + a*C*d*q + (B*c*e - C*(c*d - a*e*q))*x^2]/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[P4x, x^2, 2] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && !GtQ[b^2 - 4*a*c, 0]

Rule 1708

Int[((A_) + (B_)*(x_)^2)/(((d_) + (e_)*(x_)^2)*Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4]), x_Symbol] :> With[{q = Rt[c/a, 2]}, Dist[(A*(c*d + a*e*q) - a*B*(e + d*q))/(c*d^2 - a*e^2), Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] + Dist[(a*(B*d - A*e)*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && NeQ[c*A^2 - a*B^2, 0]

Rule 1706

Int[((A_) + (B_)*(x_)^2)/(((d_) + (e_)*(x_)^2)*Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*ArcTan[(Rt[-b + (c*d)/e + (a*e)/d, 2]*x)/Sqrt[a + b*x^2 + c*x^4]]]/(2*d*e*Rt[-b + (c*d)/e + (a*e)/d, 2]), x] + Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a + b*x^2 + c*x^4))/(a*(A + B*x^2)^2)]*EllipticPi[Cancel[-((B*d - A*e)^2/(4*d*e*A*B))], 2*ArcTan[q*x], 1/2 - (b*A)/(4*a*B)]/(4*d*e*A*q*Sqrt[a + b*x^2 + c*x^4]), x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]

Rule 1216

Int[1/(((d_) + (e_)*(x_)^2)*Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4]), x_Symbol] :> With[{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]

Rubi steps

$$\int \frac{(4 + 3x^2 + x^4)^{3/2}}{(7 + 5x^2)^3} dx = \int \left(\frac{9}{625\sqrt{4 + 3x^2 + x^4}} + \frac{x^2}{125\sqrt{4 + 3x^2 + x^4}} + \frac{1936}{625(7 + 5x^2)^3\sqrt{4 + 3x^2 + x^4}} + \frac{8}{625(7 + 5x^2)^2} \right) dx$$

$$= \frac{1}{125} \int \frac{x^2}{\sqrt{4 + 3x^2 + x^4}} dx + \frac{9}{625} \int \frac{1}{\sqrt{4 + 3x^2 + x^4}} dx + \frac{88}{625} \int \frac{1}{(7 + 5x^2)^2\sqrt{4 + 3x^2 + x^4}} dx + \dots$$

$$= \frac{11x\sqrt{4 + 3x^2 + x^4}}{175(7 + 5x^2)^2} + \frac{x\sqrt{4 + 3x^2 + x^4}}{175(7 + 5x^2)} + \frac{9(2 + x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}} F\left(2 \tan^{-1}\left(\frac{x}{\sqrt{2}}\right) \middle| \frac{1}{8}\right)}{1250\sqrt{2}\sqrt{4 + 3x^2 + x^4}} - \frac{\int \frac{12+70x^2}{(7+5x^2)\sqrt{4+3x^2+x^4}} dx}{43} + \dots$$

$$= \frac{x\sqrt{4 + 3x^2 + x^4}}{125(2 + x^2)} + \frac{11x\sqrt{4 + 3x^2 + x^4}}{175(7 + 5x^2)^2} + \frac{167x\sqrt{4 + 3x^2 + x^4}}{9800(7 + 5x^2)} + \frac{89 \tan^{-1}\left(\frac{2\sqrt{\frac{11}{35}}x}{\sqrt{4+3x^2+x^4}}\right)}{500\sqrt{385}} - \frac{\sqrt{2}(2 + x^2)}{43} + \dots$$

$$= \frac{6x\sqrt{4 + 3x^2 + x^4}}{875(2 + x^2)} + \frac{11x\sqrt{4 + 3x^2 + x^4}}{175(7 + 5x^2)^2} + \frac{167x\sqrt{4 + 3x^2 + x^4}}{9800(7 + 5x^2)} + \frac{89 \tan^{-1}\left(\frac{2\sqrt{\frac{11}{35}}x}{\sqrt{4+3x^2+x^4}}\right)}{500\sqrt{385}} - \frac{6\sqrt{2}(2 + x^2)}{43} + \dots$$

$$= \frac{9x\sqrt{4 + 3x^2 + x^4}}{1960(2 + x^2)} + \frac{11x\sqrt{4 + 3x^2 + x^4}}{175(7 + 5x^2)^2} + \frac{167x\sqrt{4 + 3x^2 + x^4}}{9800(7 + 5x^2)} + \frac{3}{175} \sqrt{\frac{11}{35}} \tan^{-1}\left(\frac{2\sqrt{\frac{11}{35}}x}{\sqrt{4 + 3x^2 + x^4}}\right) + \dots$$

$$= \frac{9x\sqrt{4 + 3x^2 + x^4}}{1960(2 + x^2)} + \frac{11x\sqrt{4 + 3x^2 + x^4}}{175(7 + 5x^2)^2} + \frac{167x\sqrt{4 + 3x^2 + x^4}}{9800(7 + 5x^2)} + \frac{3}{175} \sqrt{\frac{11}{35}} \tan^{-1}\left(\frac{2\sqrt{\frac{11}{35}}x}{\sqrt{4 + 3x^2 + x^4}}\right) + \dots$$

Mathematica [C] time = 0.702228, size = 309, normalized size = 0.7

$$\frac{140x(167x^2+357)(x^4+3x^2+4)}{(5x^2+7)^2} - i\sqrt{6+2i\sqrt{7}}\sqrt{1-\frac{2ix^2}{\sqrt{7}-3i}}\sqrt{1+\frac{2ix^2}{\sqrt{7}+3i}} \left(7(103+45i\sqrt{7}) \operatorname{EllipticF}\left(i \sinh^{-1}\left(\sqrt{-\frac{2i}{\sqrt{7}-3i}}x\right), \frac{-\sqrt{7}+3i}{\sqrt{7}+3i}\right)\right)$$

274400√x⁴ + 3x²

Antiderivative was successfully verified.

```
[In] Integrate[(4 + 3*x^2 + x^4)^(3/2)/(7 + 5*x^2)^3,x]
```

```
[Out] ((140*x*(357 + 167*x^2)*(4 + 3*x^2 + x^4))/(7 + 5*x^2)^2 - I*Sqrt[6 + (2*I)*Sqrt[7]]*Sqrt[1 - ((2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[1 + ((2*I)*x^2)/(3*I + Sqrt[7])])*(315*(3 - I*Sqrt[7])*EllipticE[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])] + 7*(103 + (45*I)*Sqrt[7])*EllipticF[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])] + 2694*EllipticPi[(5*(3 + I*Sqrt[7]))/14, I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])])/(274400*Sqrt[4 + 3*x^2 + x^4])
```

Maple [C] time = 0.023, size = 434, normalized size = 1.

$$\frac{11x}{175(5x^2+7)^2}\sqrt{x^4+3x^2+4} + \frac{167x}{49000x^2+68600}\sqrt{x^4+3x^2+4} + \frac{17}{350\sqrt{-6+2i\sqrt{7}}}\sqrt{1+\frac{3x^2}{8}-\frac{i}{8}x^2\sqrt{7}}\sqrt{1+\frac{3x^2}{8}} + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^4+3*x^2+4)^(3/2)/(5*x^2+7)^3,x)`

[Out] $11/175*x*(x^4+3*x^2+4)^{(1/2)}/(5*x^2+7)^2+167/9800*x*(x^4+3*x^2+4)^{(1/2)}/(5*x^2+7)+17/350/(-6+2*I*7^{(1/2)})^{(1/2)}*(1+3/8*x^2-1/8*I*x^2*7^{(1/2)})^{(1/2)}*(1+3/8*x^2+1/8*I*x^2*7^{(1/2)})^{(1/2)}/(x^4+3*x^2+4)^{(1/2)}*EllipticF(1/4*x*(-6+2*I*7^{(1/2)})^{(1/2)},1/4*(2+6*I*7^{(1/2)})^{(1/2)})-36/245/(-6+2*I*7^{(1/2)})^{(1/2)}*(1+3/8*x^2-1/8*I*x^2*7^{(1/2)})^{(1/2)}*(1+3/8*x^2+1/8*I*x^2*7^{(1/2)})^{(1/2)}/(x^4+3*x^2+4)^{(1/2)}/(I*7^{(1/2)}+3)*EllipticF(1/4*x*(-6+2*I*7^{(1/2)})^{(1/2)},1/4*(2+6*I*7^{(1/2)})^{(1/2)})+36/245/(-6+2*I*7^{(1/2)})^{(1/2)}*(1+3/8*x^2-1/8*I*x^2*7^{(1/2)})^{(1/2)}*(1+3/8*x^2+1/8*I*x^2*7^{(1/2)})^{(1/2)}/(x^4+3*x^2+4)^{(1/2)}/(I*7^{(1/2)}+3)*EllipticE(1/4*x*(-6+2*I*7^{(1/2)})^{(1/2)},1/4*(2+6*I*7^{(1/2)})^{(1/2)})+1347/68600/(-3/8+1/8*I*7^{(1/2)})^{(1/2)}*(1+3/8*x^2-1/8*I*x^2*7^{(1/2)})^{(1/2)}*(1+3/8*x^2+1/8*I*x^2*7^{(1/2)})^{(1/2)}/(x^4+3*x^2+4)^{(1/2)}*EllipticPi((-3/8+1/8*I*7^{(1/2)})^{(1/2)}*x,-5/7/(-3/8+1/8*I*7^{(1/2)}),(-3/8-1/8*I*7^{(1/2)})^{(1/2)}/(-3/8+1/8*I*7^{(1/2)})^{(1/2)})$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(x^4 + 3x^2 + 4)^{\frac{3}{2}}}{(5x^2 + 7)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^4+3*x^2+4)^(3/2)/(5*x^2+7)^3,x, algorithm="maxima")`

[Out] `integrate((x^4 + 3*x^2 + 4)^(3/2)/(5*x^2 + 7)^3, x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(x^4 + 3x^2 + 4)^{\frac{3}{2}}}{125x^6 + 525x^4 + 735x^2 + 343}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^4+3*x^2+4)^(3/2)/(5*x^2+7)^3,x, algorithm="fricas")`

[Out] `integral((x^4 + 3*x^2 + 4)^(3/2)/(125*x^6 + 525*x^4 + 735*x^2 + 343), x)`

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{((x^2 - x + 2)(x^2 + x + 2))^{\frac{3}{2}}}{(5x^2 + 7)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x**4+3*x**2+4)**(3/2)/(5*x**2+7)**3,x)`

[Out] Integral(((x**2 - x + 2)*(x**2 + x + 2))**(3/2)/(5*x**2 + 7)**3, x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(x^4 + 3x^2 + 4)^{\frac{3}{2}}}{(5x^2 + 7)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4+3*x^2+4)^(3/2)/(5*x^2+7)^3,x, algorithm="giac")

[Out] integrate((x^4 + 3*x^2 + 4)^(3/2)/(5*x^2 + 7)^3, x)

$$3.364 \quad \int \frac{(7+5x^2)^3}{\sqrt{4+3x^2+x^4}} dx$$

Optimal. Leaf size=187

$$\frac{13(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{2\sqrt{2}\sqrt{x^4+3x^2+4}} + 25\sqrt{x^4+3x^2+4}x^3 - \frac{15\sqrt{x^4+3x^2+4}x}{x^2+2} + 75\sqrt{x^4+3x^2+4}x +$$

[Out] 75*x*Sqrt[4 + 3*x^2 + x^4] + 25*x^3*Sqrt[4 + 3*x^2 + x^4] - (15*x*Sqrt[4 + 3*x^2 + x^4])/(2 + x^2) + (15*Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE[2*ArcTan[x/Sqrt[2]], 1/8])/Sqrt[4 + 3*x^2 + x^4] + (13*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(2*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4])

Rubi [A] time = 0.0927102, antiderivative size = 187, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {1206, 1679, 1197, 1103, 1195}

$$25\sqrt{x^4+3x^2+4}x^3 - \frac{15\sqrt{x^4+3x^2+4}x}{x^2+2} + 75\sqrt{x^4+3x^2+4}x + \frac{13(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{2\sqrt{2}\sqrt{x^4+3x^2+4}} + \frac{15\sqrt{2}(x^2+2)}{2\sqrt{2}\sqrt{x^4+3x^2+4}}$$

Antiderivative was successfully verified.

[In] Int[(7 + 5*x^2)^3/Sqrt[4 + 3*x^2 + x^4], x]

[Out] 75*x*Sqrt[4 + 3*x^2 + x^4] + 25*x^3*Sqrt[4 + 3*x^2 + x^4] - (15*x*Sqrt[4 + 3*x^2 + x^4])/(2 + x^2) + (15*Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE[2*ArcTan[x/Sqrt[2]], 1/8])/Sqrt[4 + 3*x^2 + x^4] + (13*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(2*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4])

Rule 1206

Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(e^q*x^(2*q - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(4*p + 2*q + 1)), x] + Dist[1/(c*(4*p + 2*q + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*p + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1]

Rule 1679

Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{q = Expon[Pq, x^2], e = Coeff[Pq, x^2, Expon[Pq, x^2]]}, Simp[(e*x^(2*q - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(2*q + 4*p + 1)), x] + Dist[1/(c*(2*q + 4*p + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*q + 4*p + 1)*Pq - a*e*(2*q - 3)*x^(2*q - 4) - b*e*(2*q + 2*p - 1)*x^(2*q - 2) - c*e*(2*q + 4*p + 1)*x^(2*q), x], x], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && !LtQ[p, -1]

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol]
:> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; NeQ[e + d*q, 0] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]), x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol]
:> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c*x^4]), x] /; EqQ[e + d*q^2, 0] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\begin{aligned} \int \frac{(7 + 5x^2)^3}{\sqrt{4 + 3x^2 + x^4}} dx &= 25x^3\sqrt{4 + 3x^2 + x^4} + \frac{1}{5} \int \frac{1715 + 2175x^2 + 1125x^4}{\sqrt{4 + 3x^2 + x^4}} dx \\ &= 75x\sqrt{4 + 3x^2 + x^4} + 25x^3\sqrt{4 + 3x^2 + x^4} + \frac{1}{15} \int \frac{645 - 225x^2}{\sqrt{4 + 3x^2 + x^4}} dx \\ &= 75x\sqrt{4 + 3x^2 + x^4} + 25x^3\sqrt{4 + 3x^2 + x^4} + 13 \int \frac{1}{\sqrt{4 + 3x^2 + x^4}} dx + 30 \int \frac{1 - \frac{x^2}{2}}{\sqrt{4 + 3x^2 + x^4}} dx \\ &= 75x\sqrt{4 + 3x^2 + x^4} + 25x^3\sqrt{4 + 3x^2 + x^4} - \frac{15x\sqrt{4 + 3x^2 + x^4}}{2 + x^2} + \frac{15\sqrt{2}(2 + x^2) \sqrt{\frac{4 + 3x^2 + x^4}{(2 + x^2)^2}} E\left(2 \operatorname{arctan}\left(\frac{\sqrt{4 + 3x^2 + x^4}}{2 + x^2}\right)\right)}{\sqrt{4 + 3x^2 + x^4}} \end{aligned}$$

Mathematica [C] time = 0.483693, size = 337, normalized size = 1.8

$$\frac{-\sqrt{2}(15\sqrt{7} + 131i) \sqrt{\frac{-2ix^2 + \sqrt{7} - 3i}{\sqrt{7} - 3i}} \sqrt{\frac{2ix^2 + \sqrt{7} + 3i}{\sqrt{7} + 3i}} \operatorname{EllipticF}\left(i \sinh^{-1}\left(\sqrt{\frac{-2i}{\sqrt{7} - 3i}} x\right), \frac{-\sqrt{7} + 3i}{\sqrt{7} + 3i}\right) + 100 \sqrt{\frac{i}{\sqrt{7} - 3i}} x (x^6 + 6x^4 + 13x^2)}{4 \sqrt{\frac{i}{\sqrt{7} - 3i}} \sqrt{x^4 + 3x^2 + 4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(7 + 5*x^2)^3/Sqrt[4 + 3*x^2 + x^4], x]
```

```
[Out] (100*Sqrt[(-I)/(-3*I + Sqrt[7])]*x*(12 + 13*x^2 + 6*x^4 + x^6) + 15*Sqrt[2]
*(3*I + Sqrt[7])*Sqrt[(-3*I + Sqrt[7] - (2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[(3*I + Sqrt[7] + (2*I)*x^2)/(3*I + Sqrt[7])]*EllipticE[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7]) - Sqrt[2]*(131*I + 15*Sqrt[7])*Sqrt[(-3*I + Sqrt[7] - (2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[(3*I + Sqrt[7] + (2*I)*x^2)/(3*I + Sqrt[7])]*EllipticF[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])]/(4*Sqrt[(-I)/(-3*I + Sqrt[7])]*Sqrt[4 + 3*x^2 + x^4])
```

Maple [C] time = 0.029, size = 241, normalized size = 1.3

$$25x^3\sqrt{x^4+3x^2+4}+75x\sqrt{x^4+3x^2+4}+172\frac{\sqrt{1-(-3/8+i/8\sqrt{7})x^2}\sqrt{1-(-3/8-i/8\sqrt{7})x^2}\text{EllipticF}\left(\frac{1}{4}x\sqrt{-6}\right)}{\sqrt{-6+2i\sqrt{7}}\sqrt{x^4+3x^2+4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((5*x^2+7)^3/(x^4+3*x^2+4)^(1/2), x)

[Out] $25x^3(x^4+3x^2+4)^{1/2}+75x(x^4+3x^2+4)^{1/2}+172/(-6+2I\sqrt{7})^{1/2}(1/2)*(1-(-3/8+1/8I\sqrt{7}))x^2)^{1/2}*(1-(-3/8-1/8I\sqrt{7}))x^2)^{1/2}/(x^4+3x^2+4)^{1/2}*\text{EllipticF}(1/4*x*(-6+2I\sqrt{7})^{1/2}, 1/4*(2+6I\sqrt{7})^{1/2})^{1/2}+480/(-6+2I\sqrt{7})^{1/2}*(1-(-3/8+1/8I\sqrt{7}))x^2)^{1/2}*(1-(-3/8-1/8I\sqrt{7}))x^2)^{1/2}/(x^4+3x^2+4)^{1/2}/(I\sqrt{7}+3)*(\text{EllipticF}(1/4*x*(-6+2I\sqrt{7})^{1/2}, 1/4*(2+6I\sqrt{7})^{1/2})^{1/2})-\text{EllipticE}(1/4*x*(-6+2I\sqrt{7})^{1/2}, 1/4*(2+6I\sqrt{7})^{1/2})^{1/2})$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(5x^2+7)^3}{\sqrt{x^4+3x^2+4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^2+7)^3/(x^4+3*x^2+4)^(1/2), x, algorithm="maxima")

[Out] integrate((5*x^2 + 7)^3/sqrt(x^4 + 3*x^2 + 4), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{125x^6+525x^4+735x^2+343}{\sqrt{x^4+3x^2+4}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^2+7)^3/(x^4+3*x^2+4)^(1/2), x, algorithm="fricas")

[Out] integral((125*x^6 + 525*x^4 + 735*x^2 + 343)/sqrt(x^4 + 3*x^2 + 4), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(5x^2+7)^3}{\sqrt{(x^2-x+2)(x^2+x+2)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x**2+7)**3/(x**4+3*x**2+4)**(1/2), x)

```
[Out] Integral((5*x**2 + 7)**3/sqrt((x**2 - x + 2)*(x**2 + x + 2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(5x^2 + 7)^3}{\sqrt{x^4 + 3x^2 + 4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^3/(x^4+3*x^2+4)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((5*x^2 + 7)^3/sqrt(x^4 + 3*x^2 + 4), x)
```

$$3.365 \quad \int \frac{(7+5x^2)^2}{\sqrt{4+3x^2+x^4}} dx$$

Optimal. Leaf size=170

$$\frac{167(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right), \frac{1}{8}\right)}{6\sqrt{2}\sqrt{x^4+3x^2+4}} + \frac{20\sqrt{x^4+3x^2+4}x}{x^2+2} + \frac{25}{3}\sqrt{x^4+3x^2+4}x - \frac{20\sqrt{2}(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}}{\sqrt{x^4+3x^2+4}}$$

[Out] (25*x*Sqrt[4 + 3*x^2 + x^4])/3 + (20*x*Sqrt[4 + 3*x^2 + x^4])/(2 + x^2) - (20*Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE[2*ArcTan[x/Sqrt[2]], 1/8])/Sqrt[4 + 3*x^2 + x^4] + (167*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(6*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4])

Rubi [A] time = 0.0588587, antiderivative size = 170, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1206, 1197, 1103, 1195}

$$\frac{20\sqrt{x^4+3x^2+4}x}{x^2+2} + \frac{25}{3}\sqrt{x^4+3x^2+4}x + \frac{167(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|\frac{1}{8}\right)}{6\sqrt{2}\sqrt{x^4+3x^2+4}} - \frac{20\sqrt{2}(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}E\left(\frac{x}{\sqrt{2}}\middle|\frac{1}{8}\right)}{\sqrt{x^4+3x^2+4}}$$

Antiderivative was successfully verified.

[In] Int[(7 + 5*x^2)^2/Sqrt[4 + 3*x^2 + x^4], x]

[Out] (25*x*Sqrt[4 + 3*x^2 + x^4])/3 + (20*x*Sqrt[4 + 3*x^2 + x^4])/(2 + x^2) - (20*Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE[2*ArcTan[x/Sqrt[2]], 1/8])/Sqrt[4 + 3*x^2 + x^4] + (167*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(6*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4])

Rule 1206

Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(e^q*x^(2*q - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(4*p + 2*q + 1)), x] + Dist[1/(c*(4*p + 2*q + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*p + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1]

Rule 1197

Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; NeQ[e + d*q, 0] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]

Rule 1103

Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]]*

EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)]/(2*q*Sqrt[a + b*x^2 + c*x^4]), x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]

Rule 1195

Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] := With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c*x^4]), x] /; EqQ[e + d*q^2, 0] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]

Rubi steps

$$\begin{aligned} \int \frac{(7 + 5x^2)^2}{\sqrt{4 + 3x^2 + x^4}} dx &= \frac{25}{3}x\sqrt{4 + 3x^2 + x^4} + \frac{1}{3} \int \frac{47 + 60x^2}{\sqrt{4 + 3x^2 + x^4}} dx \\ &= \frac{25}{3}x\sqrt{4 + 3x^2 + x^4} - 40 \int \frac{1 - \frac{x^2}{2}}{\sqrt{4 + 3x^2 + x^4}} dx + \frac{167}{3} \int \frac{1}{\sqrt{4 + 3x^2 + x^4}} dx \\ &= \frac{25}{3}x\sqrt{4 + 3x^2 + x^4} + \frac{20x\sqrt{4 + 3x^2 + x^4}}{2 + x^2} - \frac{20\sqrt{2}(2 + x^2) \sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}} E\left(2 \tan^{-1}\left(\frac{x}{\sqrt{2}}\right) \middle| \frac{1}{8}\right)}{\sqrt{4 + 3x^2 + x^4}} + \frac{167}{3} \int \frac{1}{\sqrt{4 + 3x^2 + x^4}} dx \end{aligned}$$

Mathematica [C] time = 0.437662, size = 331, normalized size = 1.95

$$\frac{\sqrt{2}(30\sqrt{7} + 43i) \sqrt{\frac{-2ix^2 + \sqrt{7}-3i}{\sqrt{7}-3i}} \sqrt{\frac{2ix^2 + \sqrt{7}+3i}{\sqrt{7}+3i}} \text{EllipticF}\left(i \sinh^{-1}\left(\sqrt{-\frac{2i}{\sqrt{7}-3i}}x\right), \frac{-\sqrt{7}+3i}{\sqrt{7}+3i}\right) + 50\sqrt{-\frac{i}{\sqrt{7}-3i}}x(x^4 + 3x^2 + 4) - 30}{6\sqrt{-\frac{i}{\sqrt{7}-3i}}\sqrt{x^4 + 3x^2 + 4}}$$

Antiderivative was successfully verified.

[In] Integrate[(7 + 5*x^2)^2/Sqrt[4 + 3*x^2 + x^4], x]

[Out] (50*Sqrt[(-I)/(-3*I + Sqrt[7])]*x*(4 + 3*x^2 + x^4) - 30*Sqrt[2]*(3*I + Sqrt[7])*Sqrt[(-3*I + Sqrt[7] - (2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[(3*I + Sqrt[7] + (2*I)*x^2)/(3*I + Sqrt[7])]*EllipticE[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])) + Sqrt[2]*(43*I + 30*Sqrt[7])*Sqrt[(-3*I + Sqrt[7] - (2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[(3*I + Sqrt[7] + (2*I)*x^2)/(3*I + Sqrt[7])]*EllipticF[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])))/(6*Sqrt[(-I)/(-3*I + Sqrt[7])]*Sqrt[4 + 3*x^2 + x^4])

Maple [C] time = 0.007, size = 224, normalized size = 1.3

$$\frac{25x}{3}\sqrt{x^4 + 3x^2 + 4} + \frac{188}{3\sqrt{-6 + 2i\sqrt{7}}}\sqrt{1 - \left(-\frac{3}{8} + \frac{i}{8}\sqrt{7}\right)x^2}\sqrt{1 - \left(-\frac{3}{8} - \frac{i}{8}\sqrt{7}\right)x^2}\text{EllipticF}\left(\frac{x\sqrt{-6 + 2i\sqrt{7}}}{4}, \frac{\sqrt{2 + 6i\sqrt{7}}}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((5*x^2+7)^2/(x^4+3*x^2+4)^(1/2),x)

[Out] $25/3*x*(x^4+3*x^2+4)^{(1/2)}+188/3/(-6+2*I*7^{(1/2)})^{(1/2)}*(1-(-3/8+1/8*I*7^{(1/2)})*x^2)^{(1/2)}*(1-(-3/8-1/8*I*7^{(1/2)})*x^2)^{(1/2)}/(x^4+3*x^2+4)^{(1/2)}*EllipticF(1/4*x*(-6+2*I*7^{(1/2)})^{(1/2)},1/4*(2+6*I*7^{(1/2)})^{(1/2)})-640/(-6+2*I*7^{(1/2)})^{(1/2)}*(1-(-3/8+1/8*I*7^{(1/2)})*x^2)^{(1/2)}*(1-(-3/8-1/8*I*7^{(1/2)})*x^2)^{(1/2)}/(x^4+3*x^2+4)^{(1/2)}/(I*7^{(1/2)}+3)*(EllipticF(1/4*x*(-6+2*I*7^{(1/2)})^{(1/2)},1/4*(2+6*I*7^{(1/2)})^{(1/2)})-EllipticE(1/4*x*(-6+2*I*7^{(1/2)})^{(1/2)},1/4*(2+6*I*7^{(1/2)})^{(1/2)}))$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(5x^2 + 7)^2}{\sqrt{x^4 + 3x^2 + 4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^2+7)^2/(x^4+3*x^2+4)^(1/2),x, algorithm="maxima")

[Out] integrate((5*x^2 + 7)^2/sqrt(x^4 + 3*x^2 + 4), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{25x^4 + 70x^2 + 49}{\sqrt{x^4 + 3x^2 + 4}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^2+7)^2/(x^4+3*x^2+4)^(1/2),x, algorithm="fricas")

[Out] integral((25*x^4 + 70*x^2 + 49)/sqrt(x^4 + 3*x^2 + 4), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(5x^2 + 7)^2}{\sqrt{(x^2 - x + 2)(x^2 + x + 2)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x**2+7)**2/(x**4+3*x**2+4)**(1/2),x)

[Out] Integral((5*x**2 + 7)**2/sqrt((x**2 - x + 2)*(x**2 + x + 2)), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(5x^2 + 7)^2}{\sqrt{x^4 + 3x^2 + 4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^2/(x^4+3*x^2+4)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((5*x^2 + 7)^2/sqrt(x^4 + 3*x^2 + 4), x)
```


$$3.366 \quad \int \frac{7+5x^2}{\sqrt{4+3x^2+x^4}} dx$$

Optimal. Leaf size=151

$$\frac{17(x^2+2) \sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}} \text{EllipticF}\left(2 \tan^{-1}\left(\frac{x}{\sqrt{2}}\right), \frac{1}{8}\right)}{2\sqrt{2}\sqrt{x^4+3x^2+4}} + \frac{5\sqrt{x^4+3x^2+4x}}{x^2+2} - \frac{5\sqrt{2}(x^2+2) \sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}} E\left(2 \tan^{-1}\left(\frac{x}{\sqrt{2}}\right) \middle| \frac{1}{8}\right)}{\sqrt{x^4+3x^2+4}}$$

[Out] (5*x*Sqrt[4 + 3*x^2 + x^4])/(2 + x^2) - (5*Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE[2*ArcTan[x/Sqrt[2]], 1/8])/Sqrt[4 + 3*x^2 + x^4] + (17*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(2*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4])

Rubi [A] time = 0.0361226, antiderivative size = 151, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {1197, 1103, 1195}

$$\frac{5\sqrt{x^4+3x^2+4x}}{x^2+2} + \frac{17(x^2+2) \sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}} F\left(2 \tan^{-1}\left(\frac{x}{\sqrt{2}}\right) \middle| \frac{1}{8}\right)}{2\sqrt{2}\sqrt{x^4+3x^2+4}} - \frac{5\sqrt{2}(x^2+2) \sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}} E\left(2 \tan^{-1}\left(\frac{x}{\sqrt{2}}\right) \middle| \frac{1}{8}\right)}{\sqrt{x^4+3x^2+4}}$$

Antiderivative was successfully verified.

[In] Int[(7 + 5*x^2)/Sqrt[4 + 3*x^2 + x^4], x]

[Out] (5*x*Sqrt[4 + 3*x^2 + x^4])/(2 + x^2) - (5*Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE[2*ArcTan[x/Sqrt[2]], 1/8])/Sqrt[4 + 3*x^2 + x^4] + (17*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(2*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4])

Rule 1197

Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; NeQ[e + d*q, 0] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]

Rule 1103

Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]), x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]

Rule 1195

Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c*x^4]), x] /; EqQ[e + d*q^2, 0] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]

Rubi steps

$$\int \frac{7 + 5x^2}{\sqrt{4 + 3x^2 + x^4}} dx = - \left(10 \int \frac{1 - \frac{x^2}{2}}{\sqrt{4 + 3x^2 + x^4}} dx \right) + 17 \int \frac{1}{\sqrt{4 + 3x^2 + x^4}} dx$$

$$= \frac{5x\sqrt{4 + 3x^2 + x^4}}{2 + x^2} - \frac{5\sqrt{2}(2 + x^2) \sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}} E\left(2 \tan^{-1}\left(\frac{x}{\sqrt{2}}\right) \middle| \frac{1}{8}\right)}{\sqrt{4 + 3x^2 + x^4}} + \frac{17(2 + x^2) \sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}} F\left(2 \tan^{-1}\left(\frac{x}{\sqrt{2}}\right) \middle| \frac{1}{8}\right)}{2\sqrt{2}\sqrt{4 + 3x^2 + x^4}}$$

Mathematica [C] time = 0.177229, size = 214, normalized size = 1.42

$$\frac{\sqrt{1 - \frac{2ix^2}{\sqrt{7}-3i}} \sqrt{1 + \frac{2ix^2}{\sqrt{7}+3i}} \left((5\sqrt{7} + i) \text{EllipticF}\left(i \sinh^{-1}\left(\sqrt{-\frac{2i}{\sqrt{7}-3i}}x\right), \frac{-\sqrt{7}+3i}{\sqrt{7}+3i}\right) - 5(\sqrt{7} + 3i) E\left(i \sinh^{-1}\left(\sqrt{-\frac{2i}{-3i+\sqrt{7}}}x\right) \middle| \frac{3i-7}{3i+7}\right) \right)}{2\sqrt{2} \sqrt{-\frac{i}{\sqrt{7}-3i}} \sqrt{x^4 + 3x^2 + 4}}$$

Antiderivative was successfully verified.

[In] Integrate[(7 + 5*x^2)/Sqrt[4 + 3*x^2 + x^4], x]

[Out] (Sqrt[1 - ((2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[1 + ((2*I)*x^2)/(3*I + Sqrt[7])])*(-5*(3*I + Sqrt[7])*EllipticE[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])) + (I + 5*Sqrt[7])*EllipticF[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])))/(2*Sqrt[2]*Sqrt[(-I)/(-3*I + Sqrt[7])]*Sqrt[4 + 3*x^2 + x^4])

Maple [C] time = 0.007, size = 209, normalized size = 1.4

$$-160 \frac{\sqrt{1 - (-3/8 + i/8\sqrt{7})x^2} \sqrt{1 - (-3/8 - i/8\sqrt{7})x^2} \left(\text{EllipticF}\left(1/4x\sqrt{-6 + 2i\sqrt{7}}, 1/4\sqrt{2 + 6i\sqrt{7}}\right) - \text{EllipticE}\left(1/4x\sqrt{-6 + 2i\sqrt{7}}, 1/4\sqrt{2 + 6i\sqrt{7}}\right) \right)}{\sqrt{-6 + 2i\sqrt{7}} \sqrt{x^4 + 3x^2 + 4} (i\sqrt{7} + 3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((5*x^2+7)/(x^4+3*x^2+4)^(1/2), x)

[Out] -160/(-6+2*I*7^(1/2))^(1/2)*(1-(-3/8+1/8*I*7^(1/2))*x^2)^(1/2)*(1-(-3/8-1/8*I*7^(1/2))*x^2)^(1/2)/(x^4+3*x^2+4)^(1/2)/(I*7^(1/2)+3)*(EllipticF(1/4*x*(sqrt(-6+2*I*7^(1/2)))^(1/2), 1/4*(2+6*I*7^(1/2))^(1/2))-EllipticE(1/4*x*(sqrt(-6+2*I*7^(1/2)))^(1/2), 1/4*(2+6*I*7^(1/2))^(1/2)))+28/(-6+2*I*7^(1/2))^(1/2)*(1-(-3/8+1/8*I*7^(1/2))*x^2)^(1/2)*(1-(-3/8-1/8*I*7^(1/2))*x^2)^(1/2)/(x^4+3*x^2+4)^(1/2)*EllipticF(1/4*x*(sqrt(-6+2*I*7^(1/2)))^(1/2), 1/4*(2+6*I*7^(1/2))^(1/2))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{5x^2 + 7}{\sqrt{x^4 + 3x^2 + 4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^2+7)/(x^4+3*x^2+4)^(1/2),x, algorithm="maxima")

[Out] integrate((5*x^2 + 7)/sqrt(x^4 + 3*x^2 + 4), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{5x^2 + 7}{\sqrt{x^4 + 3x^2 + 4}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^2+7)/(x^4+3*x^2+4)^(1/2),x, algorithm="fricas")

[Out] integral((5*x^2 + 7)/sqrt(x^4 + 3*x^2 + 4), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{5x^2 + 7}{\sqrt{(x^2 - x + 2)(x^2 + x + 2)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x**2+7)/(x**4+3*x**2+4)**(1/2),x)

[Out] Integral((5*x**2 + 7)/sqrt((x**2 - x + 2)*(x**2 + x + 2)), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{5x^2 + 7}{\sqrt{x^4 + 3x^2 + 4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^2+7)/(x^4+3*x^2+4)^(1/2),x, algorithm="giac")

[Out] integrate((5*x^2 + 7)/sqrt(x^4 + 3*x^2 + 4), x)

$$3.367 \quad \int \frac{1}{\sqrt{4+3x^2+x^4}} dx$$

Optimal. Leaf size=64

$$\frac{(x^2 + 2) \sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}} \text{EllipticF}\left(2 \tan^{-1}\left(\frac{x}{\sqrt{2}}\right), \frac{1}{8}\right)}{2\sqrt{2}\sqrt{x^4 + 3x^2 + 4}}$$

[Out] ((2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(2*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4])

Rubi [A] time = 0.0070358, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {1103}

$$\frac{(x^2 + 2) \sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}} F\left(2 \tan^{-1}\left(\frac{x}{\sqrt{2}}\right) \middle| \frac{1}{8}\right)}{2\sqrt{2}\sqrt{x^4 + 3x^2 + 4}}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[4 + 3*x^2 + x^4], x]

[Out] ((2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(2*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4])

Rule 1103

Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]

Rubi steps

$$\int \frac{1}{\sqrt{4+3x^2+x^4}} dx = \frac{(2+x^2) \sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}} F\left(2 \tan^{-1}\left(\frac{x}{\sqrt{2}}\right) \middle| \frac{1}{8}\right)}{2\sqrt{2}\sqrt{4+3x^2+x^4}}$$

Mathematica [C] time = 0.0553322, size = 142, normalized size = 2.22

$$\frac{i \sqrt{1 - \frac{2x^2}{-3-i\sqrt{7}}} \sqrt{1 - \frac{2x^2}{-3+i\sqrt{7}}} \text{EllipticF}\left(i \sinh^{-1}\left(\sqrt{\frac{-2}{-3-i\sqrt{7}}} x\right), \frac{-3-i\sqrt{7}}{-3+i\sqrt{7}}\right)}{\sqrt{2} \sqrt{-\frac{1}{-3-i\sqrt{7}}} \sqrt{x^4 + 3x^2 + 4}}$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[4 + 3*x^2 + x^4], x]

```
[Out] ((-I)*Sqrt[1 - (2*x^2)/(-3 - I*Sqrt[7])]*Sqrt[1 - (2*x^2)/(-3 + I*Sqrt[7])]
*EllipticF[I*ArcSinh[Sqrt[-2/(-3 - I*Sqrt[7])]*x], (-3 - I*Sqrt[7])/(-3 + I
*Sqrt[7])])/(Sqrt[2]*Sqrt[-(-3 - I*Sqrt[7])^(-1)]*Sqrt[4 + 3*x^2 + x^4])
```

Maple [C] time = 0.003, size = 85, normalized size = 1.3

$$4 \frac{\sqrt{1 - (-3/8 + i/8\sqrt{7})x^2} \sqrt{1 - (-3/8 - i/8\sqrt{7})x^2} \text{EllipticF}\left(\frac{1}{4}x\sqrt{-6 + 2i\sqrt{7}}, \frac{1}{4}\sqrt{2 + 6i\sqrt{7}}\right)}{\sqrt{-6 + 2i\sqrt{7}}\sqrt{x^4 + 3x^2 + 4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4+3*x^2+4)^(1/2), x)
```

```
[Out] 4/(-6+2*I*7^(1/2))^(1/2)*(1-(-3/8+1/8*I*7^(1/2))*x^2)^(1/2)*(1-(-3/8-1/8*I*
7^(1/2))*x^2)^(1/2)/(x^4+3*x^2+4)^(1/2)*EllipticF(1/4*x*(-6+2*I*7^(1/2))^(1
/2), 1/4*(2+6*I*7^(1/2))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^4 + 3x^2 + 4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+3*x^2+4)^(1/2), x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(x^4 + 3*x^2 + 4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{x^4 + 3x^2 + 4}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+3*x^2+4)^(1/2), x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(x^4 + 3*x^2 + 4), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^4 + 3x^2 + 4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**4+3*x**2+4)**(1/2), x)
```

[Out] Integral(1/sqrt(x**4 + 3*x**2 + 4), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^4 + 3x^2 + 4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^4+3*x^2+4)^(1/2),x, algorithm="giac")

[Out] integrate(1/sqrt(x^4 + 3*x^2 + 4), x)

$$3.368 \quad \int \frac{1}{(7+5x^2)\sqrt{4+3x^2+x^4}} dx$$

Optimal. Leaf size=168

$$\frac{(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}} \operatorname{EllipticF}\left(2 \tan^{-1}\left(\frac{x}{\sqrt{2}}\right), \frac{1}{8}\right)}{6\sqrt{2}\sqrt{x^4+3x^2+4}} + \frac{1}{4}\sqrt{\frac{5}{77}} \tan^{-1}\left(\frac{2\sqrt{\frac{11}{35}}x}{\sqrt{x^4+3x^2+4}}\right) + \frac{17(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}} \Pi\left(-\frac{9}{280}; \frac{x}{\sqrt{2}}\right)}{84\sqrt{2}\sqrt{x^4+3x^2+4}}$$

```
[Out] (Sqrt[5/77]*ArcTan[(2*Sqrt[11/35]*x)/Sqrt[4 + 3*x^2 + x^4]])/4 - ((2 + x^2)
*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/
(6*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4]) + (17*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2
+ x^2)^2]*EllipticPi[-9/280, 2*ArcTan[x/Sqrt[2]], 1/8])/(84*Sqrt[2]*Sqrt[4
+ 3*x^2 + x^4])
```

Rubi [A] time = 0.0820125, antiderivative size = 168, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1216, 1103, 1706}

$$\frac{1}{4}\sqrt{\frac{5}{77}} \tan^{-1}\left(\frac{2\sqrt{\frac{11}{35}}x}{\sqrt{x^4+3x^2+4}}\right) - \frac{(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}} F\left(2 \tan^{-1}\left(\frac{x}{\sqrt{2}}\right) \middle| \frac{1}{8}\right)}{6\sqrt{2}\sqrt{x^4+3x^2+4}} + \frac{17(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}} \Pi\left(-\frac{9}{280}; 2 \tan^{-1}\left(\frac{x}{\sqrt{2}}\right)\right)}{84\sqrt{2}\sqrt{x^4+3x^2+4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((7 + 5*x^2)*Sqrt[4 + 3*x^2 + x^4]),x]
```

```
[Out] (Sqrt[5/77]*ArcTan[(2*Sqrt[11/35]*x)/Sqrt[4 + 3*x^2 + x^4]])/4 - ((2 + x^2)
*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/
(6*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4]) + (17*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2
+ x^2)^2]*EllipticPi[-9/280, 2*ArcTan[x/Sqrt[2]], 1/8])/(84*Sqrt[2]*Sqrt[4
+ 3*x^2 + x^4])
```

Rule 1216

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_Symbol]
:> With[{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/
Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int
[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)]/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1706

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
(c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*Arc
Tan[(Rt[-b + (c*d)/e + (a*e)/d, 2]*x)/Sqrt[a + b*x^2 + c*x^4]])/(2*d*e*Rt[-
```

$b + (c*d)/e + (a*e)/d, 2]), x] + \text{Simp}[(B*d + A*e)*(A + B*x^2)*\text{Sqrt}[(A^2*(a + b*x^2 + c*x^4))/(a*(A + B*x^2)^2)]*\text{EllipticPi}[\text{Cancel}[-((B*d - A*e)^2/(4*d*e*A*B))], 2*\text{ArcTan}[q*x], 1/2 - (b*A)/(4*a*B)]/(4*d*e*A*q*\text{Sqrt}[a + b*x^2 + c*x^4]), x]] /; \text{FreeQ}\{a, b, c, d, e, A, B\}, x] \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& \text{NeQ}[c*d^2 - b*d*e + a*e^2, 0] \&\& \text{NeQ}[c*d^2 - a*e^2, 0] \&\& \text{PosQ}[c/a] \&\& \text{EqQ}[c*A^2 - a*B^2, 0]$

Rubi steps

$$\int \frac{1}{(7+5x^2)\sqrt{4+3x^2+x^4}} dx = -\left(\frac{1}{3} \int \frac{1}{\sqrt{4+3x^2+x^4}} dx\right) + \frac{10}{3} \int \frac{1+\frac{x^2}{2}}{(7+5x^2)\sqrt{4+3x^2+x^4}} dx$$

$$= \frac{1}{4} \sqrt{\frac{5}{77}} \tan^{-1}\left(\frac{2\sqrt{\frac{11}{35}}x}{\sqrt{4+3x^2+x^4}}\right) - \frac{(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}} F\left(2 \tan^{-1}\left(\frac{x}{\sqrt{2}}\right) \middle| \frac{1}{8}\right)}{6\sqrt{2}\sqrt{4+3x^2+x^4}} + \frac{17(2+x^2)}{\dots}$$

Mathematica [C] time = 0.135387, size = 159, normalized size = 0.95

$$\frac{i\sqrt{1-\frac{2x^2}{-3-i\sqrt{7}}}\sqrt{1-\frac{2x^2}{-3+i\sqrt{7}}}\Pi\left(-\frac{5}{14}(-3-i\sqrt{7}); i \sinh^{-1}\left(\sqrt{-\frac{2}{-3-i\sqrt{7}}}x\right) \middle| \frac{-3-i\sqrt{7}}{-3+i\sqrt{7}}\right)}{7\sqrt{2}\sqrt{-\frac{1}{-3-i\sqrt{7}}}\sqrt{x^4+3x^2+4}}$$

Antiderivative was successfully verified.

[In] Integrate[1/((7 + 5*x^2)*Sqrt[4 + 3*x^2 + x^4]),x]

[Out] ((-I/7)*Sqrt[1 - (2*x^2)/(-3 - I*Sqrt[7])]*Sqrt[1 - (2*x^2)/(-3 + I*Sqrt[7])])*EllipticPi[(-5*(-3 - I*Sqrt[7]))/14, I*ArcSinh[Sqrt[-2/(-3 - I*Sqrt[7])]*x], (-3 - I*Sqrt[7])/(-3 + I*Sqrt[7])]/(Sqrt[2]*Sqrt[-(-3 - I*Sqrt[7])^(-1)]*Sqrt[4 + 3*x^2 + x^4])

Maple [C] time = 0.016, size = 107, normalized size = 0.6

$$\frac{1}{7\sqrt{-3/8+i/8\sqrt{7}}}\sqrt{1+\frac{3x^2}{8}-\frac{i}{8}x^2\sqrt{7}}\sqrt{1+\frac{3x^2}{8}+\frac{i}{8}x^2\sqrt{7}}\text{EllipticPi}\left(\sqrt{-\frac{3}{8}+\frac{i}{8}\sqrt{7}}x, -\frac{5}{-\frac{21}{8}+\frac{7i}{8}\sqrt{7}}\sqrt{\frac{-\frac{3}{8}-\frac{i}{8}\sqrt{7}}{-\frac{3}{8}+\frac{i}{8}\sqrt{7}}}\right)\sqrt{x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(5*x^2+7)/(x^4+3*x^2+4)^(1/2),x)

[Out] 1/7/(-3/8+1/8*I*7^(1/2))^(1/2)*(1+3/8*x^2-1/8*I*x^2*7^(1/2))^(1/2)*(1+3/8*x^2+1/8*I*x^2*7^(1/2))^(1/2)/(x^4+3*x^2+4)^(1/2)*EllipticPi((-3/8+1/8*I*7^(1/2))^(1/2)*x, -5/7/(-3/8+1/8*I*7^(1/2)), (-3/8-1/8*I*7^(1/2))^(1/2)/(-3/8+1/8*I*7^(1/2))^(1/2))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^4+3x^2+4}(5x^2+7)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(5*x^2+7)/(x^4+3*x^2+4)^(1/2),x, algorithm="maxima")

[Out] integrate(1/(sqrt(x^4 + 3*x^2 + 4)*(5*x^2 + 7)), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{x^4 + 3x^2 + 4}}{5x^6 + 22x^4 + 41x^2 + 28}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(5*x^2+7)/(x^4+3*x^2+4)^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(x^4 + 3*x^2 + 4)/(5*x^6 + 22*x^4 + 41*x^2 + 28), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{(x^2 - x + 2)(x^2 + x + 2)}(5x^2 + 7)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(5*x**2+7)/(x**4+3*x**2+4)**(1/2),x)

[Out] Integral(1/(sqrt((x**2 - x + 2)*(x**2 + x + 2))*(5*x**2 + 7)), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^4 + 3x^2 + 4}(5x^2 + 7)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(5*x^2+7)/(x^4+3*x^2+4)^(1/2),x, algorithm="giac")

[Out] integrate(1/(sqrt(x^4 + 3*x^2 + 4)*(5*x^2 + 7)), x)

$$3.369 \quad \int \frac{1}{(7+5x^2)^2 \sqrt{4+3x^2+x^4}} dx$$

Optimal. Leaf size=286

$$\frac{(x^2 + 2) \sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}} \text{EllipticF}\left(2 \tan^{-1}\left(\frac{x}{\sqrt{2}}\right), \frac{1}{8}\right)}{42\sqrt{2}\sqrt{x^4 + 3x^2 + 4}} - \frac{5\sqrt{x^4 + 3x^2 + 4x}}{616(x^2 + 2)} + \frac{25\sqrt{x^4 + 3x^2 + 4x}}{616(5x^2 + 7)} + \frac{37\sqrt{\frac{5}{77}} \tan^{-1}\left(\frac{2\sqrt{\frac{11}{35}}x}{\sqrt{x^4+3x^2+4}}\right)}{2464}$$

[Out] (-5*x*Sqrt[4 + 3*x^2 + x^4])/(616*(2 + x^2)) + (25*x*Sqrt[4 + 3*x^2 + x^4])/(616*(7 + 5*x^2)) + (37*Sqrt[5/77]*ArcTan[(2*Sqrt[11/35]*x)/Sqrt[4 + 3*x^2 + x^4]])/2464 + (5*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE[2*ArcTan[x/Sqrt[2]], 1/8])/(308*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4]) - ((2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(42*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4]) + (629*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticPi[-9/280, 2*ArcTan[x/Sqrt[2]], 1/8])/(51744*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4])

Rubi [A] time = 0.224618, antiderivative size = 286, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$, Rules used = {1223, 1714, 1195, 1708, 1103, 1706}

$$-\frac{5\sqrt{x^4 + 3x^2 + 4x}}{616(x^2 + 2)} + \frac{25\sqrt{x^4 + 3x^2 + 4x}}{616(5x^2 + 7)} + \frac{37\sqrt{\frac{5}{77}} \tan^{-1}\left(\frac{2\sqrt{\frac{11}{35}}x}{\sqrt{x^4+3x^2+4}}\right)}{2464} - \frac{(x^2 + 2) \sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}} F\left(2 \tan^{-1}\left(\frac{x}{\sqrt{2}}\right) \middle| \frac{1}{8}\right)}{42\sqrt{2}\sqrt{x^4 + 3x^2 + 4}} + \frac{5(x^2 + 2)}{2464}$$

Antiderivative was successfully verified.

[In] Int[1/((7 + 5*x^2)^2*Sqrt[4 + 3*x^2 + x^4]),x]

[Out] (-5*x*Sqrt[4 + 3*x^2 + x^4])/(616*(2 + x^2)) + (25*x*Sqrt[4 + 3*x^2 + x^4])/(616*(7 + 5*x^2)) + (37*Sqrt[5/77]*ArcTan[(2*Sqrt[11/35]*x)/Sqrt[4 + 3*x^2 + x^4]])/2464 + (5*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE[2*ArcTan[x/Sqrt[2]], 1/8])/(308*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4]) - ((2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(42*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4]) + (629*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticPi[-9/280, 2*ArcTan[x/Sqrt[2]], 1/8])/(51744*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4])

Rule 1223

Int[((d_) + (e_.)*(x_)^2)^(q_)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] := -Simp[(e^2*x*(d + e*x^2)^(q + 1)*Sqrt[a + b*x^2 + c*x^4])/(2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2)), Int[((d + e*x^2)^(q + 1)*Simp[a*e^2*(2*q + 3) + 2*d*(c*d - b*e)*(q + 1) - 2*e*(c*d*(q + 1) - b*e*(q + 2))*x^2 + c*e^2*(2*q + 5)*x^4, x])/Sqrt[a + b*x^2 + c*x^4], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && ILtQ[q, -1]

Rule 1714

Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_Symbol] := With[{q = Rt[c/a, 2], A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[P4x, x, 4]}, -Dist[C/(e*q), Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] + Dist[1/(c*e), Int[(A*c*e + a*C*d*q + (B*c*e - C*(c*d - a

$*e*q))x^2)/((d + e*x^2)*\text{Sqrt}[a + b*x^2 + c*x^4], x], x]] /; \text{FreeQ}[\{a, b, c, d, e\}, x] \ \&\& \text{PolyQ}[P4x, x^2, 2] \ \&\& \text{NeQ}[b^2 - 4*a*c, 0] \ \&\& \text{NeQ}[c*d^2 - b*d*e + a*e^2, 0] \ \&\& \text{NeQ}[c*d^2 - a*e^2, 0] \ \&\& \text{PosQ}[c/a] \ \&\& \text{!GtQ}[b^2 - 4*a*c, 0]$

Rule 1195

$\text{Int}[(d + (e \cdot x^2)/\text{Sqrt}[a + (b \cdot x^2 + (c \cdot x^4)], x_Symbol] :> \text{With}[\{q = \text{Rt}[c/a, 4]\}, -\text{Simp}[(d*x*\text{Sqrt}[a + b*x^2 + c*x^4])/(a*(1 + q^2*x^2)), x] + \text{Simp}[(d*(1 + q^2*x^2)*\text{Sqrt}[a + b*x^2 + c*x^4])/(a*(1 + q^2*x^2)^2)]*\text{EllipticE}[2*\text{ArcTan}[q*x], 1/2 - (b*q^2)/(4*c)]/(q*\text{Sqrt}[a + b*x^2 + c*x^4]), x] /; \text{EqQ}[e + d*q^2, 0] /; \text{FreeQ}[\{a, b, c, d, e\}, x] \ \&\& \text{NeQ}[b^2 - 4*a*c, 0] \ \&\& \text{PosQ}[c/a]$

Rule 1708

$\text{Int}[(A + (B \cdot x^2)/((d + (e \cdot x^2)*\text{Sqrt}[a + (b \cdot x^2 + (c \cdot x^4)], x_Symbol] :> \text{With}[\{q = \text{Rt}[c/a, 2]\}, \text{Dist}[(A*(c*d + a*e*q) - a*B*(e + d*q))/(c*d^2 - a*e^2), \text{Int}[1/\text{Sqrt}[a + b*x^2 + c*x^4], x], x] + \text{Dist}[(a*(B*d - A*e)*(e + d*q))/(c*d^2 - a*e^2), \text{Int}[(1 + q*x^2)/((d + e*x^2)*\text{Sqrt}[a + b*x^2 + c*x^4]), x], x]] /; \text{FreeQ}[\{a, b, c, d, e, A, B\}, x] \ \&\& \text{NeQ}[b^2 - 4*a*c, 0] \ \&\& \text{NeQ}[c*d^2 - b*d*e + a*e^2, 0] \ \&\& \text{NeQ}[c*d^2 - a*e^2, 0] \ \&\& \text{PosQ}[c/a] \ \&\& \text{NeQ}[c*A^2 - a*B^2, 0]$

Rule 1103

$\text{Int}[1/\text{Sqrt}[a + (b \cdot x^2 + (c \cdot x^4)], x_Symbol] :> \text{With}[\{q = \text{Rt}[c/a, 4]\}, \text{Simp}[(1 + q^2*x^2)*\text{Sqrt}[a + b*x^2 + c*x^4])/(a*(1 + q^2*x^2)^2)]*\text{EllipticF}[2*\text{ArcTan}[q*x], 1/2 - (b*q^2)/(4*c)]/(2*q*\text{Sqrt}[a + b*x^2 + c*x^4]), x]] /; \text{FreeQ}[\{a, b, c\}, x] \ \&\& \text{NeQ}[b^2 - 4*a*c, 0] \ \&\& \text{PosQ}[c/a]$

Rule 1706

$\text{Int}[(A + (B \cdot x^2)/((d + (e \cdot x^2)*\text{Sqrt}[a + (b \cdot x^2 + (c \cdot x^4)], x_Symbol] :> \text{With}[\{q = \text{Rt}[B/A, 2]\}, -\text{Simp}[(B*d - A*e)*\text{ArcTan}[(\text{Rt}[-b + (c*d)/e + (a*e)/d, 2]*x)/\text{Sqrt}[a + b*x^2 + c*x^4]])/(2*d*e*\text{Rt}[-b + (c*d)/e + (a*e)/d, 2]), x] + \text{Simp}[(B*d + A*e)*(A + B*x^2)*\text{Sqrt}[(A^2*(a + b*x^2 + c*x^4))/(a*(A + B*x^2)^2)]*\text{EllipticPi}[\text{Cancel}[-(B*d - A*e)^2/(4*d*e*A*B)], 2*\text{ArcTan}[q*x], 1/2 - (b*A)/(4*a*B)]/(4*d*e*A*q*\text{Sqrt}[a + b*x^2 + c*x^4]), x]] /; \text{FreeQ}[\{a, b, c, d, e, A, B\}, x] \ \&\& \text{NeQ}[b^2 - 4*a*c, 0] \ \&\& \text{NeQ}[c*d^2 - b*d*e + a*e^2, 0] \ \&\& \text{NeQ}[c*d^2 - a*e^2, 0] \ \&\& \text{PosQ}[c/a] \ \&\& \text{EqQ}[c*A^2 - a*B^2, 0]$

Rubi steps

$$\begin{aligned}
\int \frac{1}{(7+5x^2)^2 \sqrt{4+3x^2+x^4}} dx &= \frac{25x\sqrt{4+3x^2+x^4}}{616(7+5x^2)} - \frac{1}{616} \int \frac{12+70x^2+25x^4}{(7+5x^2)\sqrt{4+3x^2+x^4}} dx \\
&= \frac{25x\sqrt{4+3x^2+x^4}}{616(7+5x^2)} - \frac{\int \frac{410+425x^2}{(7+5x^2)\sqrt{4+3x^2+x^4}} dx}{3080} + \frac{5}{308} \int \frac{1-\frac{x^2}{2}}{\sqrt{4+3x^2+x^4}} dx \\
&= -\frac{5x\sqrt{4+3x^2+x^4}}{616(2+x^2)} + \frac{25x\sqrt{4+3x^2+x^4}}{616(7+5x^2)} + \frac{5(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}} E\left(2 \tan^{-1}\left(\frac{x}{\sqrt{2}}\right) \middle| \frac{1}{8}\right)}{308\sqrt{2}\sqrt{4+3x^2+x^4}} \\
&= -\frac{5x\sqrt{4+3x^2+x^4}}{616(2+x^2)} + \frac{25x\sqrt{4+3x^2+x^4}}{616(7+5x^2)} + \frac{37\sqrt{\frac{5}{77}} \tan^{-1}\left(\frac{2\sqrt{\frac{11}{35}}x}{\sqrt{4+3x^2+x^4}}\right)}{2464} + \frac{5(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}}{308}
\end{aligned}$$

Mathematica [C] time = 0.782755, size = 481, normalized size = 1.68

$$98i(5x^2+7)\sqrt{2-\frac{4ix^2}{\sqrt{7-3i}}}\sqrt{1+\frac{2ix^2}{\sqrt{7+3i}}}\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{-\frac{2i}{\sqrt{7-3i}}}x\right),\frac{-\sqrt{7+3i}}{\sqrt{7+3i}}\right)+35(\sqrt{7}+3i)(5x^2+7)\sqrt{2-\frac{4ix^2}{\sqrt{7-3i}}}\sqrt{1+\frac{2ix^2}{\sqrt{7+3i}}}$$

Antiderivative was successfully verified.

[In] Integrate[1/((7+5*x^2)^2*Sqrt[4+3*x^2+x^4]),x]

[Out] (700*Sqrt[(-I)/(-3*I+Sqrt[7])]*x*(4+3*x^2+x^4)+35*(3*I+Sqrt[7])*(7+5*x^2)*Sqrt[2-((4*I)*x^2)/(-3*I+Sqrt[7])]*Sqrt[1+((2*I)*x^2)/(3*I+Sqrt[7])])*(EllipticE[I*ArcSinh[Sqrt[(-2*I)/(-3*I+Sqrt[7])]]*x],(3*I-Sqrt[7])/(3*I+Sqrt[7]))-EllipticF[I*ArcSinh[Sqrt[(-2*I)/(-3*I+Sqrt[7])]]*x],(3*I-Sqrt[7])/(3*I+Sqrt[7]))+(98*I)*(7+5*x^2)*Sqrt[2-((4*I)*x^2)/(-3*I+Sqrt[7])]*Sqrt[1+((2*I)*x^2)/(3*I+Sqrt[7])]*EllipticF[I*ArcSinh[Sqrt[(-2*I)/(-3*I+Sqrt[7])]]*x],(3*I-Sqrt[7])/(3*I+Sqrt[7]))-(74*I)*(7+5*x^2)*Sqrt[2-((4*I)*x^2)/(-3*I+Sqrt[7])]*Sqrt[1+((2*I)*x^2)/(3*I+Sqrt[7])]*EllipticPi[(5*(3+I*Sqrt[7]))/14,I*ArcSinh[Sqrt[(-2*I)/(-3*I+Sqrt[7])]]*x],(3*I-Sqrt[7])/(3*I+Sqrt[7]))/(17248*Sqrt[(-I)/(-3*I+Sqrt[7])]*(7+5*x^2)*Sqrt[4+3*x^2+x^4])

Maple [C] time = 0.02, size = 410, normalized size = 1.4

$$\frac{25x}{3080x^2+4312}\sqrt{x^4+3x^2+4}-\frac{1}{22\sqrt{-6+2i\sqrt{7}}}\sqrt{1+\frac{3x^2}{8}-\frac{i}{8}x^2\sqrt{7}}\sqrt{1+\frac{3x^2}{8}+\frac{i}{8}x^2\sqrt{7}}\text{EllipticF}\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\sqrt{\frac{1}{8}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(5*x^2+7)^2/(x^4+3*x^2+4)^(1/2),x)

[Out] 25/616*x*(x^4+3*x^2+4)^(1/2)/(5*x^2+7)-1/22/(-6+2*I*7^(1/2))^(1/2)*(1+3/8*x^2-1/8*I*x^2*7^(1/2))^(1/2)*(1+3/8*x^2+1/8*I*x^2*7^(1/2))^(1/2)/(x^4+3*x^2+4)^(1/2)*EllipticF(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))+20/77/(-6+2*I*7^(1/2))^(1/2)*(1+3/8*x^2-1/8*I*x^2*7^(1/2))^(1/2)*(1+3/8*x^2

$$+1/8*I*x^2*7^{(1/2)})^{(1/2)}/(x^4+3*x^2+4)^{(1/2)}/(I*7^{(1/2)}+3)*\text{EllipticF}(1/4*x*(-6+2*I*7^{(1/2)})^{(1/2)},1/4*(2+6*I*7^{(1/2)})^{(1/2)})-20/77/(-6+2*I*7^{(1/2)})^{(1/2)}*(1+3/8*x^2-1/8*I*x^2*7^{(1/2)})^{(1/2)}*(1+3/8*x^2+1/8*I*x^2*7^{(1/2)})^{(1/2)})/(x^4+3*x^2+4)^{(1/2)}/(I*7^{(1/2)}+3)*\text{EllipticE}(1/4*x*(-6+2*I*7^{(1/2)})^{(1/2)},1/4*(2+6*I*7^{(1/2)})^{(1/2)})+37/4312/(-3/8+1/8*I*7^{(1/2)})^{(1/2)}*(1+3/8*x^2-1/8*I*x^2*7^{(1/2)})^{(1/2)}*(1+3/8*x^2+1/8*I*x^2*7^{(1/2)})^{(1/2)})/(x^4+3*x^2+4)^{(1/2)}*\text{EllipticPi}((-3/8+1/8*I*7^{(1/2)})^{(1/2)}*x,-5/7/(-3/8+1/8*I*7^{(1/2)}),(-3/8-1/8*I*7^{(1/2)})^{(1/2)})/(-3/8+1/8*I*7^{(1/2)})^{(1/2)}$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^4 + 3x^2 + 4}(5x^2 + 7)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(5*x^2+7)^2/(x^4+3*x^2+4)^(1/2),x, algorithm="maxima")

[Out] integrate(1/(sqrt(x^4 + 3*x^2 + 4)*(5*x^2 + 7)^2), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{x^4 + 3x^2 + 4}}{25x^8 + 145x^6 + 359x^4 + 427x^2 + 196}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(5*x^2+7)^2/(x^4+3*x^2+4)^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(x^4 + 3*x^2 + 4)/(25*x^8 + 145*x^6 + 359*x^4 + 427*x^2 + 196), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{(x^2 - x + 2)(x^2 + x + 2)}(5x^2 + 7)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(5*x**2+7)**2/(x**4+3*x**2+4)**(1/2),x)

[Out] Integral(1/(sqrt((x**2 - x + 2)*(x**2 + x + 2))*(5*x**2 + 7)**2), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^4 + 3x^2 + 4}(5x^2 + 7)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+7)^2/(x^4+3*x^2+4)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(x^4 + 3*x^2 + 4)*(5*x^2 + 7)^2), x)
```

$$3.370 \quad \int \frac{1}{(7+5x^2)^3 \sqrt{4+3x^2+x^4}} dx$$

Optimal. Leaf size=314

$$\frac{(x^2 + 2) \sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}} \text{EllipticF}\left(2 \tan^{-1}\left(\frac{x}{\sqrt{2}}\right), \frac{1}{8}\right)}{8624\sqrt{2}\sqrt{x^4 + 3x^2 + 4}} - \frac{555\sqrt{x^4 + 3x^2 + 4x}}{758912(x^2 + 2)} + \frac{2775\sqrt{x^4 + 3x^2 + 4x}}{758912(5x^2 + 7)} + \frac{25\sqrt{x^4 + 3x^2 + 4x}}{1232(5x^2 + 7)^2}$$

```
[Out] (-555*x*Sqrt[4 + 3*x^2 + x^4])/(758912*(2 + x^2)) + (25*x*Sqrt[4 + 3*x^2 + x^4])/(1232*(7 + 5*x^2)^2) + (2775*x*Sqrt[4 + 3*x^2 + x^4])/(758912*(7 + 5*x^2)) - (3285*Sqrt[5/77]*ArcTan[(2*Sqrt[11/35]*x)/Sqrt[4 + 3*x^2 + x^4]])/3035648 + (555*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE[2*ArcTan[x/Sqrt[2]], 1/8])/(379456*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4]) - ((2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(8624*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4]) - (18615*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticPi[-9/280, 2*ArcTan[x/Sqrt[2]], 1/8])/(21249536*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4])
```

Rubi [A] time = 0.289723, antiderivative size = 314, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {1223, 1696, 1714, 1195, 1708, 1103, 1706}

$$-\frac{555\sqrt{x^4 + 3x^2 + 4x}}{758912(x^2 + 2)} + \frac{2775\sqrt{x^4 + 3x^2 + 4x}}{758912(5x^2 + 7)} + \frac{25\sqrt{x^4 + 3x^2 + 4x}}{1232(5x^2 + 7)^2} - \frac{3285\sqrt{\frac{5}{77}} \tan^{-1}\left(\frac{2\sqrt{\frac{11}{35}}x}{\sqrt{x^4+3x^2+4}}\right)}{3035648} - \frac{(x^2 + 2) \sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}}{8624\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((7 + 5*x^2)^3*Sqrt[4 + 3*x^2 + x^4]),x]
```

```
[Out] (-555*x*Sqrt[4 + 3*x^2 + x^4])/(758912*(2 + x^2)) + (25*x*Sqrt[4 + 3*x^2 + x^4])/(1232*(7 + 5*x^2)^2) + (2775*x*Sqrt[4 + 3*x^2 + x^4])/(758912*(7 + 5*x^2)) - (3285*Sqrt[5/77]*ArcTan[(2*Sqrt[11/35]*x)/Sqrt[4 + 3*x^2 + x^4]])/3035648 + (555*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE[2*ArcTan[x/Sqrt[2]], 1/8])/(379456*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4]) - ((2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(8624*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4]) - (18615*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticPi[-9/280, 2*ArcTan[x/Sqrt[2]], 1/8])/(21249536*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4])
```

Rule 1223

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> -Simp[(e^2*x*(d + e*x^2)^(q + 1)*Sqrt[a + b*x^2 + c*x^4])/(2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2)), Int[((d + e*x^2)^(q + 1)*Simp[a*e^2*(2*q + 3) + 2*d*(c*d - b*e)*(q + 1) - 2*e*(c*d*(q + 1) - b*e*(q + 2))*x^2 + c*e^2*(2*q + 5)*x^4, x])/Sqrt[a + b*x^2 + c*x^4], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && ILtQ[q, -1]
```

Rule 1696

```
Int[((P4x_)*((d_) + (e_.)*(x_)^2)^(q_))/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = C
```

```

oeff[P4x, x, 4]}, -Simp[((C*d^2 - B*d*e + A*e^2)*x*(d + e*x^2)^(q + 1)*Sqrt
[a + b*x^2 + c*x^4])/(2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(2*
d*(q + 1)*(c*d^2 - b*d*e + a*e^2)), Int[((d + e*x^2)^(q + 1)*Simp[a*d*(C*d
- B*e) + A*(a*e^2*(2*q + 3) + 2*d*(c*d - b*e)*(q + 1)) - 2*((B*d - A*e)*(b*
e*(q + 2) - c*d*(q + 1)) - C*d*(b*d + a*e*(q + 1)))*x^2 + c*(C*d^2 - B*d*e
+ A*e^2)*(2*q + 5)*x^4, x])/Sqrt[a + b*x^2 + c*x^4], x], x]] /; FreeQ[{a, b
, c, d, e}, x] && PolyQ[P4x, x^2] && LeQ[Expon[P4x, x], 4] && NeQ[b^2 - 4*a
*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, -1]

```

Rule 1714

```

Int[(P4x_)/(((d_) + (e_)*(x_)^2)*Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4])
, x_Symbol] := With[{q = Rt[c/a, 2], A = Coeff[P4x, x, 0], B = Coeff[P4x, x
, 2], C = Coeff[P4x, x, 4]}, -Dist[C/(e*q), Int[(1 - q*x^2)/Sqrt[a + b*x^2
+ c*x^4], x], x] + Dist[1/(c*e), Int[(A*c*e + a*C*d*q + (B*c*e - C*(c*d - a
*e*q))*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b
, c, d, e}, x] && PolyQ[P4x, x^2, 2] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*
d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && !GtQ[b^2 - 4*a*c,
0]

```

Rule 1195

```

Int[((d_) + (e_)*(x_)^2)/Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4], x_Symbol]
:= With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)]/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]

```

Rule 1708

```

Int[((A_) + (B_)*(x_)^2)/(((d_) + (e_)*(x_)^2)*Sqrt[(a_) + (b_)*(x_)^2
+ (c_)*(x_)^4]), x_Symbol] := With[{q = Rt[c/a, 2]}, Dist[(A*(c*d + a*e*q)
- a*B*(e + d*q))/(c*d^2 - a*e^2), Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] +
Dist[(a*(B*d - A*e)*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0
] && PosQ[c/a] && NeQ[c*A^2 - a*B^2, 0]

```

Rule 1103

```

Int[1/Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4], x_Symbol] := With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)]/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]

```

Rule 1706

```

Int[((A_) + (B_)*(x_)^2)/(((d_) + (e_)*(x_)^2)*Sqrt[(a_) + (b_)*(x_)^2 +
(c_)*(x_)^4]), x_Symbol] := With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*Arc
Tan[(Rt[-b + (c*d)/e + (a*e)/d, 2]*x)/Sqrt[a + b*x^2 + c*x^4]])/(2*d*e*Rt[-
b + (c*d)/e + (a*e)/d, 2]), x] + Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a
+ b*x^2 + c*x^4))/(a*(A + B*x^2)^2)]*EllipticPi[Cancel[-((B*d - A*e)^2/(4*
d*e*A*B))], 2*ArcTan[q*x], 1/2 - (b*A)/(4*a*B)]/(4*d*e*A*q*Sqrt[a + b*x^2
+ c*x^4]), x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]

```

Rubi steps

$$\int \frac{1}{(7 + 5x^2)^3 \sqrt{4 + 3x^2 + x^4}} dx = \frac{25x\sqrt{4 + 3x^2 + x^4}}{1232(7 + 5x^2)^2} - \frac{\int \frac{-76-10x^2-25x^4}{(7+5x^2)^2 \sqrt{4+3x^2+x^4}} dx}{1232}$$

$$= \frac{25x\sqrt{4 + 3x^2 + x^4}}{1232(7 + 5x^2)^2} + \frac{2775x\sqrt{4 + 3x^2 + x^4}}{758912(7 + 5x^2)} + \frac{\int \frac{-4412-4690x^2-2775x^4}{(7+5x^2)\sqrt{4+3x^2+x^4}} dx}{758912}$$

$$= \frac{25x\sqrt{4 + 3x^2 + x^4}}{1232(7 + 5x^2)^2} + \frac{2775x\sqrt{4 + 3x^2 + x^4}}{758912(7 + 5x^2)} + \frac{\int \frac{-60910-31775x^2}{(7+5x^2)\sqrt{4+3x^2+x^4}} dx}{3794560} + \frac{555 \int \frac{1-\frac{x^2}{2}}{\sqrt{4+3x^2}} dx}{3794560}$$

$$= -\frac{555x\sqrt{4 + 3x^2 + x^4}}{758912(2 + x^2)} + \frac{25x\sqrt{4 + 3x^2 + x^4}}{1232(7 + 5x^2)^2} + \frac{2775x\sqrt{4 + 3x^2 + x^4}}{758912(7 + 5x^2)} + \frac{555(2 + x^2)}{3794560}$$

$$= -\frac{555x\sqrt{4 + 3x^2 + x^4}}{758912(2 + x^2)} + \frac{25x\sqrt{4 + 3x^2 + x^4}}{1232(7 + 5x^2)^2} + \frac{2775x\sqrt{4 + 3x^2 + x^4}}{758912(7 + 5x^2)} - \frac{3285\sqrt{\frac{5}{77}} \arctan\left(\frac{x}{\sqrt{77}}\right)}{303}$$

Mathematica [C] time = 0.89413, size = 308, normalized size = 0.98

$$\frac{700x(555x^2+1393)(x^4+3x^2+4)}{(5x^2+7)^2} + i\sqrt{6+2i\sqrt{7}}\sqrt{1-\frac{2ix^2}{\sqrt{7}-3i}}\sqrt{1+\frac{2ix^2}{\sqrt{7}+3i}}\left((-9401+3885i\sqrt{7})\operatorname{EllipticF}\left(i\sinh^{-1}\left(\sqrt{\frac{-2i}{\sqrt{7}-3i}}x\right),\sqrt{7}\right)\right)$$

21249536√x

Antiderivative was successfully verified.

```
[In] Integrate[1/((7 + 5*x^2)^3*Sqrt[4 + 3*x^2 + x^4]),x]
```

```
[Out] ((700*x*(1393 + 555*x^2)*(4 + 3*x^2 + x^4))/(7 + 5*x^2)^2 + I*Sqrt[6 + (2*I)*Sqrt[7]]*Sqrt[1 - ((2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[1 + ((2*I)*x^2)/(3*I + Sqrt[7])]*(3885*(3 - I*Sqrt[7])*EllipticE[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])] + (-9401 + (3885*I)*Sqrt[7])*EllipticF[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])] + 6570*EllipticPi[(5*(3 + I*Sqrt[7]))/14, I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])])/(21249536*Sqrt[4 + 3*x^2 + x^4])
```

Maple [C] time = 0.025, size = 434, normalized size = 1.4

$$\frac{25x}{1232(5x^2+7)^2}\sqrt{x^4+3x^2+4} + \frac{2775x}{3794560x^2+5312384}\sqrt{x^4+3x^2+4} - \frac{23}{27104\sqrt{-6+2i\sqrt{7}}}\sqrt{1+\frac{3x^2}{8}-\frac{i}{8}x^2\sqrt{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5*x^2+7)^3/(x^4+3*x^2+4)^(1/2),x)
```

```
[Out] 25/1232*x*(x^4+3*x^2+4)^(1/2)/(5*x^2+7)^2+2775/758912*x*(x^4+3*x^2+4)^(1/2)/(5*x^2+7)-23/27104/(-6+2*I*7^(1/2))^(1/2)*(1+3/8*x^2-1/8*I*x^2*7^(1/2))^(1/2)*(1+3/8*x^2+1/8*I*x^2*7^(1/2))^(1/2)/(x^4+3*x^2+4)^(1/2)*EllipticF(1/4*x
```

$$\begin{aligned} & *(-6+2*I*7^{(1/2)})^{(1/2)}, 1/4*(2+6*I*7^{(1/2)})^{(1/2)}+555/23716/(-6+2*I*7^{(1/2)})^{(1/2)} \\ & *(1+3/8*x^2-1/8*I*x^2*7^{(1/2)})^{(1/2)}*(1+3/8*x^2+1/8*I*x^2*7^{(1/2)})^{(1/2)} \\ & /((x^4+3*x^2+4)^{(1/2)}*(I*7^{(1/2)}+3)*\text{EllipticF}(1/4*x*(-6+2*I*7^{(1/2)})^{(1/2)}, \\ & 1/4*(2+6*I*7^{(1/2)})^{(1/2)})-555/23716/(-6+2*I*7^{(1/2)})^{(1/2)}*(1+3/8*x^2- \\ & 1/8*I*x^2*7^{(1/2)})^{(1/2)}*(1+3/8*x^2+1/8*I*x^2*7^{(1/2)})^{(1/2)} \\ & /((x^4+3*x^2+4)^{(1/2)}*(I*7^{(1/2)}+3)*\text{EllipticE}(1/4*x*(-6+2*I*7^{(1/2)})^{(1/2)}, \\ & 1/4*(2+6*I*7^{(1/2)})^{(1/2)})-3285/5312384/(-3/8+1/8*I*7^{(1/2)})^{(1/2)}*(1+3/8*x^2- \\ & 1/8*I*x^2*7^{(1/2)})^{(1/2)}*(1+3/8*x^2+1/8*I*x^2*7^{(1/2)})^{(1/2)} \\ & /((x^4+3*x^2+4)^{(1/2)}*\text{EllipticPi}((-3/8+1/8*I*7^{(1/2)})^{(1/2)}*x, \\ & -5/7/(-3/8+1/8*I*7^{(1/2)}), (-3/8-1/8*I*7^{(1/2)})^{(1/2)}) \\ & /(-3/8+1/8*I*7^{(1/2)})^{(1/2)} \end{aligned}$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^4 + 3x^2 + 4}(5x^2 + 7)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(5*x^2+7)^3/(x^4+3*x^2+4)^(1/2),x, algorithm="maxima")

[Out] integrate(1/(sqrt(x^4 + 3*x^2 + 4)*(5*x^2 + 7)^3), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{x^4 + 3x^2 + 4}}{125x^{10} + 900x^8 + 2810x^6 + 4648x^4 + 3969x^2 + 1372}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(5*x^2+7)^3/(x^4+3*x^2+4)^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(x^4 + 3*x^2 + 4)/(125*x^10 + 900*x^8 + 2810*x^6 + 4648*x^4 + 3969*x^2 + 1372), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{(x^2 - x + 2)(x^2 + x + 2)}(5x^2 + 7)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(5*x**2+7)**3/(x**4+3*x**2+4)**(1/2),x)

[Out] Integral(1/(sqrt((x**2 - x + 2)*(x**2 + x + 2))*(5*x**2 + 7)**3), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^4 + 3x^2 + 4}(5x^2 + 7)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+7)^3/(x^4+3*x^2+4)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(x^4 + 3*x^2 + 4)*(5*x^2 + 7)^3), x)
```

$$3.371 \quad \int \frac{(7+5x^2)^5}{(4+3x^2+x^4)^{3/2}} dx$$

Optimal. Leaf size=219

$$\frac{130729(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right), \frac{1}{8}\right)}{12\sqrt{2}\sqrt{x^4+3x^2+4}} + 625\sqrt{x^4+3x^2+4}x^3 - \frac{220779\sqrt{x^4+3x^2+4}x}{28(x^2+2)} + \frac{5000}{3}\sqrt{x^4+3x^2+4}$$

[Out] (x*(99493 + 45779*x^2))/(28*Sqrt[4 + 3*x^2 + x^4]) + (5000*x*Sqrt[4 + 3*x^2 + x^4])/3 + 625*x^3*Sqrt[4 + 3*x^2 + x^4] - (220779*x*Sqrt[4 + 3*x^2 + x^4])/((28*(2 + x^2)) + (220779*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE[2*ArcTan[x/Sqrt[2]], 1/8])/(14*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4]) - (130729*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(12*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4])

Rubi [A] time = 0.121085, antiderivative size = 219, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {1205, 1679, 1197, 1103, 1195}

$$625\sqrt{x^4+3x^2+4}x^3 - \frac{220779\sqrt{x^4+3x^2+4}x}{28(x^2+2)} + \frac{5000}{3}\sqrt{x^4+3x^2+4}x + \frac{(45779x^2+99493)x}{28\sqrt{x^4+3x^2+4}} - \frac{130729(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}}{12\sqrt{2}\sqrt{x^4+3x^2+4}}$$

Antiderivative was successfully verified.

[In] Int[(7 + 5*x^2)^5/(4 + 3*x^2 + x^4)^(3/2), x]

[Out] (x*(99493 + 45779*x^2))/(28*Sqrt[4 + 3*x^2 + x^4]) + (5000*x*Sqrt[4 + 3*x^2 + x^4])/3 + 625*x^3*Sqrt[4 + 3*x^2 + x^4] - (220779*x*Sqrt[4 + 3*x^2 + x^4])/((28*(2 + x^2)) + (220779*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE[2*ArcTan[x/Sqrt[2]], 1/8])/(14*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4]) - (130729*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(12*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4])

Rule 1205

Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{f = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 + c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*g - f*(b^2 - 2*a*c) - c*(b*f - 2*a*g)*x^2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[(d + e*x^2)^q, a + b*x^2 + c*x^4, x] + b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g + c*(4*p + 7)*(b*f - 2*a*g)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1] && LtQ[p, -1]

Rule 1679

Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{q = Expon[Pq, x^2], e = Coeff[Pq, x^2, Expon[Pq, x^2]]}, Simp[(e*x^(2*q - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(2*q + 4*p + 1)), x] + Dist[1/(c*(2*q + 4*p + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*q + 4*p + 1)*Pq - a*e*(2*

$q - 3) * x^{(2*q - 4)} - b * e * (2*q + 2*p - 1) * x^{(2*q - 2)} - c * e * (2*q + 4*p + 1) * x^{(2*q)}$, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && !LtQ[p, -1]

Rule 1197

Int[((d_) + (e_)*(x_)^2)/Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4], x_Symbol] := With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; NeQ[e + d*q, 0] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]

Rule 1103

Int[1/Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4], x_Symbol] := With[{q = Rt[c/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]

Rule 1195

Int[((d_) + (e_)*(x_)^2)/Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4], x_Symbol] := With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c*x^4]), x] /; EqQ[e + d*q^2, 0] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]

Rubi steps

$$\int \frac{(7 + 5x^2)^5}{(4 + 3x^2 + x^4)^{3/2}} dx = \frac{x(99493 + 45779x^2)}{28\sqrt{4 + 3x^2 + x^4}} + \frac{1}{28} \int \frac{18156 + 269221x^2 + 350000x^4 + 87500x^6}{\sqrt{4 + 3x^2 + x^4}} dx$$

$$= \frac{x(99493 + 45779x^2)}{28\sqrt{4 + 3x^2 + x^4}} + 625x^3\sqrt{4 + 3x^2 + x^4} + \frac{1}{140} \int \frac{90780 + 296105x^2 + 700000x^4}{\sqrt{4 + 3x^2 + x^4}} dx$$

$$= \frac{x(99493 + 45779x^2)}{28\sqrt{4 + 3x^2 + x^4}} + \frac{5000}{3}x\sqrt{4 + 3x^2 + x^4} + 625x^3\sqrt{4 + 3x^2 + x^4} + \frac{1}{420} \int \frac{-2527660 - 1000000x^2}{\sqrt{4 + 3x^2 + x^4}} dx$$

$$= \frac{x(99493 + 45779x^2)}{28\sqrt{4 + 3x^2 + x^4}} + \frac{5000}{3}x\sqrt{4 + 3x^2 + x^4} + 625x^3\sqrt{4 + 3x^2 + x^4} + \frac{220779}{14} \int \frac{1 - 3x^2}{\sqrt{4 + 3x^2 + x^4}} dx$$

$$= \frac{x(99493 + 45779x^2)}{28\sqrt{4 + 3x^2 + x^4}} + \frac{5000}{3}x\sqrt{4 + 3x^2 + x^4} + 625x^3\sqrt{4 + 3x^2 + x^4} - \frac{220779x\sqrt{4 + 3x^2 + x^4}}{28(2 + x^2)}$$

Mathematica [C] time = 0.520698, size = 339, normalized size = 1.55

$$-\sqrt{2} (662337\sqrt{7} + 975947i) \sqrt{\frac{-2ix^2 + \sqrt{7} - 3i}{\sqrt{7} - 3i}} \sqrt{\frac{2ix^2 + \sqrt{7} + 3i}{\sqrt{7} + 3i}} \text{EllipticF}\left(i \sinh^{-1}\left(\sqrt{\frac{-2i}{\sqrt{7} - 3i}} x\right), \frac{-\sqrt{7} + 3i}{\sqrt{7} + 3i}\right) + 4 \sqrt{-\frac{i}{\sqrt{7} - 3i}} x (52500 - 336 \sqrt{-\frac{i}{\sqrt{7} - 3i}})$$

Antiderivative was successfully verified.

[In] Integrate[(7 + 5*x^2)^5/(4 + 3*x^2 + x^4)^(3/2), x]

```
[Out] (4*Sqrt[(-I)/(-3*I + Sqrt[7])] * x*(858479 + 767337*x^2 + 297500*x^4 + 52500*x^6) + 662337*Sqrt[2]*(3*I + Sqrt[7])*Sqrt[(-3*I + Sqrt[7] - (2*I)*x^2)/(-3*I + Sqrt[7])] * Sqrt[(3*I + Sqrt[7] + (2*I)*x^2)/(3*I + Sqrt[7])] * EllipticE[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])] * x], (3*I - Sqrt[7])/(3*I + Sqrt[7])] - Sqrt[2]*(975947*I + 662337*Sqrt[7])*Sqrt[(-3*I + Sqrt[7] - (2*I)*x^2)/(-3*I + Sqrt[7])] * Sqrt[(3*I + Sqrt[7] + (2*I)*x^2)/(3*I + Sqrt[7])] * EllipticF[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])] * x], (3*I - Sqrt[7])/(3*I + Sqrt[7])])]/(336*Sqrt[(-I)/(-3*I + Sqrt[7])] * Sqrt[4 + 3*x^2 + x^4])
```

Maple [C] time = 0.048, size = 379, normalized size = 1.7

$$-6250 \frac{1}{\sqrt{x^4 + 3x^2 + 4}} \left(\frac{31x^3}{14} + \frac{18x}{7} \right) + 625x^3 \sqrt{x^4 + 3x^2 + 4} + \frac{5000x}{3} \sqrt{x^4 + 3x^2 + 4} - \frac{505532}{21 \sqrt{-6 + 2i\sqrt{7}}} \sqrt{1 - \left(-\frac{3}{8} + \frac{i}{8} \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+7)^5/(x^4+3*x^2+4)^(3/2),x)
```

```
[Out] -6250*(31/14*x^3+18/7*x)/(x^4+3*x^2+4)^(1/2)+625*x^3*(x^4+3*x^2+4)^(1/2)+5000/3*x*(x^4+3*x^2+4)^(1/2)-505532/21/(-6+2*I*7^(1/2))^(1/2)*(1-(-3/8+1/8*I*7^(1/2))*x^2)^(1/2)*(1-(-3/8-1/8*I*7^(1/2))*x^2)^(1/2)/(x^4+3*x^2+4)^(1/2)*EllipticF(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))+1766232/7/(-6+2*I*7^(1/2))^(1/2)*(1-(-3/8+1/8*I*7^(1/2))*x^2)^(1/2)*(1-(-3/8-1/8*I*7^(1/2))*x^2)^(1/2)/(x^4+3*x^2+4)^(1/2)/(I*7^(1/2)+3)*(EllipticF(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))-EllipticE(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2)))-43750*(-9/14*x^3+2/7*x)/(x^4+3*x^2+4)^(1/2)-122500*(-1/14*x^3-6/7*x)/(x^4+3*x^2+4)^(1/2)-171500*(3/14*x^3+4/7*x)/(x^4+3*x^2+4)^(1/2)-120050*(-1/7*x^3-3/14*x)/(x^4+3*x^2+4)^(1/2)-33614*(1/56*x+3/56*x^3)/(x^4+3*x^2+4)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(5x^2 + 7)^5}{(x^4 + 3x^2 + 4)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^5/(x^4+3*x^2+4)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((5*x^2 + 7)^5/(x^4 + 3*x^2 + 4)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral} \left(\frac{(3125x^{10} + 21875x^8 + 61250x^6 + 85750x^4 + 60025x^2 + 16807)\sqrt{x^4 + 3x^2 + 4}}{x^8 + 6x^6 + 17x^4 + 24x^2 + 16}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^5/(x^4+3*x^2+4)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((3125*x^10 + 21875*x^8 + 61250*x^6 + 85750*x^4 + 60025*x^2 + 16807)
)*sqrt(x^4 + 3*x^2 + 4)/(x^8 + 6*x^6 + 17*x^4 + 24*x^2 + 16), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(5x^2 + 7)^5}{((x^2 - x + 2)(x^2 + x + 2))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+7)**5/(x**4+3*x**2+4)**(3/2),x)
```

```
[Out] Integral((5*x**2 + 7)**5/((x**2 - x + 2)*(x**2 + x + 2))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(5x^2 + 7)^5}{(x^4 + 3x^2 + 4)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^5/(x^4+3*x^2+4)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((5*x^2 + 7)^5/(x^4 + 3*x^2 + 4)^(3/2), x)
```

$$3.372 \quad \int \frac{(7+5x^2)^4}{(4+3x^2+x^4)^{3/2}} dx$$

Optimal. Leaf size=200

$$\frac{4243(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right), \frac{1}{8}\right)}{12\sqrt{2}\sqrt{x^4+3x^2+4}} + \frac{14523\sqrt{x^4+3x^2+4}x}{28(x^2+2)} + \frac{625}{3}\sqrt{x^4+3x^2+4}x + \frac{(2719-4023x^2)}{28\sqrt{x^4+3x^2+4}}$$

[Out] (x*(2719 - 4023*x^2))/(28*Sqrt[4 + 3*x^2 + x^4]) + (625*x*Sqrt[4 + 3*x^2 + x^4])/3 + (14523*x*Sqrt[4 + 3*x^2 + x^4])/(28*(2 + x^2)) - (14523*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE[2*ArcTan[x/Sqrt[2]], 1/8])/(14*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4]) + (4243*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(12*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4])

Rubi [A] time = 0.0892912, antiderivative size = 200, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {1205, 1679, 1197, 1103, 1195}

$$\frac{14523\sqrt{x^4+3x^2+4}x}{28(x^2+2)} + \frac{625}{3}\sqrt{x^4+3x^2+4}x + \frac{(2719-4023x^2)x}{28\sqrt{x^4+3x^2+4}} + \frac{4243(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|\frac{1}{8}\right)}{12\sqrt{2}\sqrt{x^4+3x^2+4}} - \frac{14523(2+x^2)\sqrt{x^4+3x^2+4}}{28\sqrt{x^4+3x^2+4}}$$

Antiderivative was successfully verified.

[In] Int[(7 + 5*x^2)^4/(4 + 3*x^2 + x^4)^(3/2), x]

[Out] (x*(2719 - 4023*x^2))/(28*Sqrt[4 + 3*x^2 + x^4]) + (625*x*Sqrt[4 + 3*x^2 + x^4])/3 + (14523*x*Sqrt[4 + 3*x^2 + x^4])/(28*(2 + x^2)) - (14523*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE[2*ArcTan[x/Sqrt[2]], 1/8])/(14*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4]) + (4243*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(12*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4])

Rule 1205

Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{f = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 + c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*g - f*(b^2 - 2*a*c) - c*(b*f - 2*a*g)*x^2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[(d + e*x^2)^q, a + b*x^2 + c*x^4, x] + b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g + c*(4*p + 7)*(b*f - 2*a*g)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1] && LtQ[p, -1]

Rule 1679

Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{q = Expon[Pq, x^2], e = Coeff[Pq, x^2, Expon[Pq, x^2]]}, Simp[(e*x^(2*q - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(2*q + 4*p + 1)), x] + Dist[1/(c*(2*q + 4*p + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*q + 4*p + 1)*Pq - a*e*(2*

$q - 3)x^{(2q - 4)} - b \cdot e \cdot (2q + 2p - 1)x^{(2q - 2)} - c \cdot e \cdot (2q + 4p + 1)x^{(2q)}$, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && !LtQ[p, -1]

Rule 1197

Int[((d_) + (e_)*(x_)^2)/Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4], x_Symbol] := With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; NeQ[e + d*q, 0] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]

Rule 1103

Int[1/Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4], x_Symbol] := With[{q = Rt[c/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]

Rule 1195

Int[((d_) + (e_)*(x_)^2)/Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4], x_Symbol] := With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c*x^4]), x] /; EqQ[e + d*q^2, 0] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]

Rubi steps

$$\begin{aligned} \int \frac{(7 + 5x^2)^4}{(4 + 3x^2 + x^4)^{3/2}} dx &= \frac{x(2719 - 4023x^2)}{28\sqrt{4 + 3x^2 + x^4}} + \frac{1}{28} \int \frac{14088 + 49523x^2 + 17500x^4}{\sqrt{4 + 3x^2 + x^4}} dx \\ &= \frac{x(2719 - 4023x^2)}{28\sqrt{4 + 3x^2 + x^4}} + \frac{625}{3} x\sqrt{4 + 3x^2 + x^4} + \frac{1}{84} \int \frac{-27736 + 43569x^2}{\sqrt{4 + 3x^2 + x^4}} dx \\ &= \frac{x(2719 - 4023x^2)}{28\sqrt{4 + 3x^2 + x^4}} + \frac{625}{3} x\sqrt{4 + 3x^2 + x^4} + \frac{4243}{6} \int \frac{1}{\sqrt{4 + 3x^2 + x^4}} dx - \frac{14523}{14} \int \frac{1 - \sqrt{4 + 3x^2 + x^4}}{\sqrt{4 + 3x^2 + x^4}} dx \\ &= \frac{x(2719 - 4023x^2)}{28\sqrt{4 + 3x^2 + x^4}} + \frac{625}{3} x\sqrt{4 + 3x^2 + x^4} + \frac{14523x\sqrt{4 + 3x^2 + x^4}}{28(2 + x^2)} - \frac{14523(2 + x^2)\sqrt{4 + 3x^2 + x^4}}{14\sqrt{2}\sqrt{4 + 3x^2 + x^4}} \end{aligned}$$

Mathematica [F] time = 0, size = 0, normalized size = 0.

\$Aborted

Verification is Not applicable to the result.

[In] Integrate[(7 + 5*x^2)^4/(4 + 3*x^2 + x^4)^(3/2), x]

[Out] \$Aborted

Maple [C] time = 0.007, size = 339, normalized size = 1.7

$$-1250 \frac{1}{\sqrt{x^4 + 3x^2 + 4}} \left(-\frac{9x^3}{14} + 2/7x \right) + \frac{625x}{3} \sqrt{x^4 + 3x^2 + 4} - \frac{27736}{21 \sqrt{-6 + 2i\sqrt{7}}} \sqrt{1 - \left(-\frac{3}{8} + \frac{i}{8}\sqrt{7} \right) x^2} \sqrt{1 - \left(-\frac{3}{8} - \frac{i}{8}\sqrt{7} \right) x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((5*x^2+7)^4/(x^4+3*x^2+4)^(3/2),x)

[Out] $-1250 \cdot (-9/14 \cdot x^3 + 2/7 \cdot x) / (x^4 + 3 \cdot x^2 + 4)^{(1/2)} + 625/3 \cdot x \cdot (x^4 + 3 \cdot x^2 + 4)^{(1/2)} - 27736/21 / (-6 + 2 \cdot I \cdot 7^{(1/2)})^{(1/2)} \cdot (1 - (-3/8 + 1/8 \cdot I \cdot 7^{(1/2)}) \cdot x^2)^{(1/2)} \cdot (1 - (-3/8 - 1/8 \cdot I \cdot 7^{(1/2)}) \cdot x^2)^{(1/2)} / (x^4 + 3 \cdot x^2 + 4)^{(1/2)} \cdot \text{EllipticF}(1/4 \cdot x \cdot (-6 + 2 \cdot I \cdot 7^{(1/2)})^{(1/2)}, 1/4 \cdot (2 + 6 \cdot I \cdot 7^{(1/2)})^{(1/2)}) - 116184/7 / (-6 + 2 \cdot I \cdot 7^{(1/2)})^{(1/2)} \cdot (1 - (-3/8 + 1/8 \cdot I \cdot 7^{(1/2)}) \cdot x^2)^{(1/2)} \cdot (1 - (-3/8 - 1/8 \cdot I \cdot 7^{(1/2)}) \cdot x^2)^{(1/2)} / (x^4 + 3 \cdot x^2 + 4)^{(1/2)} / (I \cdot 7^{(1/2)} + 3) \cdot (\text{EllipticF}(1/4 \cdot x \cdot (-6 + 2 \cdot I \cdot 7^{(1/2)})^{(1/2)}, 1/4 \cdot (2 + 6 \cdot I \cdot 7^{(1/2)})^{(1/2)})^{(1/2)}) - \text{EllipticE}(1/4 \cdot x \cdot (-6 + 2 \cdot I \cdot 7^{(1/2)})^{(1/2)}, 1/4 \cdot (2 + 6 \cdot I \cdot 7^{(1/2)})^{(1/2)}) - 7000 \cdot (-1/14 \cdot x^3 - 6/7 \cdot x) / (x^4 + 3 \cdot x^2 + 4)^{(1/2)} - 14700 \cdot (3/14 \cdot x^3 + 4/7 \cdot x) / (x^4 + 3 \cdot x^2 + 4)^{(1/2)} - 13720 \cdot (-1/7 \cdot x^3 - 3/14 \cdot x) / (x^4 + 3 \cdot x^2 + 4)^{(1/2)} - 4802 \cdot (1/56 \cdot x + 3/56 \cdot x^3) / (x^4 + 3 \cdot x^2 + 4)^{(1/2)}$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(5x^2 + 7)^4}{(x^4 + 3x^2 + 4)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^2+7)^4/(x^4+3*x^2+4)^(3/2),x, algorithm="maxima")

[Out] integrate((5*x^2 + 7)^4/(x^4 + 3*x^2 + 4)^(3/2), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral} \left(\frac{(625x^8 + 3500x^6 + 7350x^4 + 6860x^2 + 2401)\sqrt{x^4 + 3x^2 + 4}}{x^8 + 6x^6 + 17x^4 + 24x^2 + 16}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^2+7)^4/(x^4+3*x^2+4)^(3/2),x, algorithm="fricas")

[Out] integral((625*x^8 + 3500*x^6 + 7350*x^4 + 6860*x^2 + 2401)*sqrt(x^4 + 3*x^2 + 4)/(x^8 + 6*x^6 + 17*x^4 + 24*x^2 + 16), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(5x^2 + 7)^4}{((x^2 - x + 2)(x^2 + x + 2))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x**2+7)**4/(x**4+3*x**2+4)**(3/2),x)

[Out] Integral((5*x**2 + 7)**4/((x**2 - x + 2)*(x**2 + x + 2))**3/2, x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(5x^2 + 7)^4}{(x^4 + 3x^2 + 4)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^2+7)^4/(x^4+3*x^2+4)^(3/2),x, algorithm="giac")

[Out] integrate((5*x^2 + 7)^4/(x^4 + 3*x^2 + 4)^(3/2), x)

$$3.373 \quad \int \frac{(7+5x^2)^3}{(4+3x^2+x^4)^{3/2}} dx$$

Optimal. Leaf size=181

$$\frac{973(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right), \frac{1}{8}\right)}{4\sqrt{2}\sqrt{x^4+3x^2+4}} + \frac{4449\sqrt{x^4+3x^2+4x}}{28(x^2+2)} - \frac{(949x^2+2323)x}{28\sqrt{x^4+3x^2+4}} - \frac{4449(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}}{14\sqrt{2}\sqrt{x^4+3x^2+4}}$$

[Out] $-(x*(2323 + 949*x^2))/(28*\text{Sqrt}[4 + 3*x^2 + x^4]) + (4449*x*\text{Sqrt}[4 + 3*x^2 + x^4])/(28*(2 + x^2)) - (4449*(2 + x^2)*\text{Sqrt}[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*\text{EllipticE}[2*\text{ArcTan}[x/\text{Sqrt}[2]], 1/8])/(14*\text{Sqrt}[2]*\text{Sqrt}[4 + 3*x^2 + x^4]) + (973*(2 + x^2)*\text{Sqrt}[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*\text{EllipticF}[2*\text{ArcTan}[x/\text{Sqrt}[2]], 1/8])/(4*\text{Sqrt}[2]*\text{Sqrt}[4 + 3*x^2 + x^4])$

Rubi [A] time = 0.0586473, antiderivative size = 181, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1205, 1197, 1103, 1195}

$$\frac{4449\sqrt{x^4+3x^2+4x}}{28(x^2+2)} - \frac{(949x^2+2323)x}{28\sqrt{x^4+3x^2+4}} + \frac{973(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|\frac{1}{8}\right)}{4\sqrt{2}\sqrt{x^4+3x^2+4}} - \frac{4449(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}E\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|\frac{1}{8}\right)}{14\sqrt{2}\sqrt{x^4+3x^2+4}}$$

Antiderivative was successfully verified.

[In] Int[(7 + 5*x^2)^3/(4 + 3*x^2 + x^4)^(3/2), x]

[Out] $-(x*(2323 + 949*x^2))/(28*\text{Sqrt}[4 + 3*x^2 + x^4]) + (4449*x*\text{Sqrt}[4 + 3*x^2 + x^4])/(28*(2 + x^2)) - (4449*(2 + x^2)*\text{Sqrt}[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*\text{EllipticE}[2*\text{ArcTan}[x/\text{Sqrt}[2]], 1/8])/(14*\text{Sqrt}[2]*\text{Sqrt}[4 + 3*x^2 + x^4]) + (973*(2 + x^2)*\text{Sqrt}[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*\text{EllipticF}[2*\text{ArcTan}[x/\text{Sqrt}[2]], 1/8])/(4*\text{Sqrt}[2]*\text{Sqrt}[4 + 3*x^2 + x^4])$

Rule 1205

Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{f = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 + c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x^4)^(p+1)*(a*b*g - f*(b^2 - 2*a*c) - c*(b*f - 2*a*g)*x^2)/(2*a*(p+1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p+1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*x^4)^(p+1)*ExpandToSum[2*a*(p+1)*(b^2 - 4*a*c)*PolynomialQuotient[(d + e*x^2)^q, a + b*x^2 + c*x^4, x] + b^2*f*(2*p+3) - 2*a*c*f*(4*p+5) - a*b*g + c*(4*p+7)*(b*f - 2*a*g)*x^2, x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1] && LtQ[p, -1]

Rule 1197

Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; NeQ[e + d*q, 0] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] := With[{q = Rt[c/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]), x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] := With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c*x^4]), x] /; EqQ[e + d*q^2, 0] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\begin{aligned} \int \frac{(7 + 5x^2)^3}{(4 + 3x^2 + x^4)^{3/2}} dx &= -\frac{x(2323 + 949x^2)}{28\sqrt{4 + 3x^2 + x^4}} + \frac{1}{28} \int \frac{4724 + 4449x^2}{\sqrt{4 + 3x^2 + x^4}} dx \\ &= -\frac{x(2323 + 949x^2)}{28\sqrt{4 + 3x^2 + x^4}} - \frac{4449}{14} \int \frac{1 - \frac{x^2}{2}}{\sqrt{4 + 3x^2 + x^4}} dx + \frac{973}{2} \int \frac{1}{\sqrt{4 + 3x^2 + x^4}} dx \\ &= -\frac{x(2323 + 949x^2)}{28\sqrt{4 + 3x^2 + x^4}} + \frac{4449x\sqrt{4 + 3x^2 + x^4}}{28(2 + x^2)} - \frac{4449(2 + x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}E\left(2 \tan^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|\frac{1}{8}\right)}{14\sqrt{2}\sqrt{4 + 3x^2 + x^4}} \end{aligned}$$

Mathematica [F] time = 0, size = 0, normalized size = 0.

\$Aborted

Verification is Not applicable to the result.

[In] Integrate[(7 + 5*x^2)^3/(4 + 3*x^2 + x^4)^(3/2), x]

[Out] \$Aborted

Maple [C] time = 0.006, size = 301, normalized size = 1.7

$$-250 \frac{-1/14 x^3 - 6/7 x}{\sqrt{x^4 + 3x^2 + 4}} + \frac{4724}{7\sqrt{-6 + 2i\sqrt{7}}} \sqrt{1 - \left(-\frac{3}{8} + \frac{i}{8}\sqrt{7}\right)x^2} \sqrt{1 - \left(-\frac{3}{8} - \frac{i}{8}\sqrt{7}\right)x^2} \text{EllipticF}\left(\frac{x\sqrt{-6 + 2i\sqrt{7}}}{4}, \frac{\sqrt{2 + 3x^2 + x^4}}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((5*x^2+7)^3/(x^4+3*x^2+4)^(3/2), x)

[Out] -250*(-1/14*x^3-6/7*x)/(x^4+3*x^2+4)^(1/2)+4724/7/(-6+2*I*7^(1/2))^(1/2)*(1-(-3/8+1/8*I*7^(1/2))*x^2)^(1/2)*(1-(-3/8-1/8*I*7^(1/2))*x^2)^(1/2)/(x^4+3*x^2+4)^(1/2)*EllipticF(1/4*x*(-6+2*I*7^(1/2))^(1/2), 1/4*(2+6*I*7^(1/2))^(1/2))-35592/7/(-6+2*I*7^(1/2))^(1/2)*(1-(-3/8+1/8*I*7^(1/2))*x^2)^(1/2)*(1-(-3/8-1/8*I*7^(1/2))*x^2)^(1/2)/(x^4+3*x^2+4)^(1/2)/(I*7^(1/2)+3)*(EllipticF(

$1/4*x*(-6+2*I*7^(1/2))^(1/2), 1/4*(2+6*I*7^(1/2))^(1/2)) - \text{EllipticE}(1/4*x*(-6+2*I*7^(1/2))^(1/2), 1/4*(2+6*I*7^(1/2))^(1/2)) - 1050*(3/14*x^3+4/7*x)/(x^4+3*x^2+4)^(1/2) - 1470*(-1/7*x^3-3/14*x)/(x^4+3*x^2+4)^(1/2) - 686*(1/56*x+3/56*x^3)/(x^4+3*x^2+4)^(1/2)$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(5x^2 + 7)^3}{(x^4 + 3x^2 + 4)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^2+7)^3/(x^4+3*x^2+4)^(3/2),x, algorithm="maxima")

[Out] integrate((5*x^2 + 7)^3/(x^4 + 3*x^2 + 4)^(3/2), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(125x^6 + 525x^4 + 735x^2 + 343)\sqrt{x^4 + 3x^2 + 4}}{x^8 + 6x^6 + 17x^4 + 24x^2 + 16}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^2+7)^3/(x^4+3*x^2+4)^(3/2),x, algorithm="fricas")

[Out] integral((125*x^6 + 525*x^4 + 735*x^2 + 343)*sqrt(x^4 + 3*x^2 + 4)/(x^8 + 6*x^6 + 17*x^4 + 24*x^2 + 16), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(5x^2 + 7)^3}{((x^2 - x + 2)(x^2 + x + 2))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x**2+7)**3/(x**4+3*x**2+4)**(3/2),x)

[Out] Integral((5*x**2 + 7)**3/((x**2 - x + 2)*(x**2 + x + 2))**3/2, x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(5x^2 + 7)^3}{(x^4 + 3x^2 + 4)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^3/(x^4+3*x^2+4)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((5*x^2 + 7)^3/(x^4 + 3*x^2 + 4)^(3/2), x)
```

$$3.374 \quad \int \frac{(7+5x^2)^2}{(4+3x^2+x^4)^{3/2}} dx$$

Optimal. Leaf size=181

$$\frac{9(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right), \frac{1}{8}\right)}{4\sqrt{2}\sqrt{x^4+3x^2+4}} - \frac{113\sqrt{x^4+3x^2+4x}}{28(x^2+2)} - \frac{(9-113x^2)x}{28\sqrt{x^4+3x^2+4}} + \frac{113(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}E\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right), \frac{1}{8}\right)}{14\sqrt{2}\sqrt{x^4+3x^2+4}}$$

[Out] $-(x*(9 - 113*x^2))/(28*\text{Sqrt}[4 + 3*x^2 + x^4]) - (113*x*\text{Sqrt}[4 + 3*x^2 + x^4])/((28*(2 + x^2)) + (113*(2 + x^2)*\text{Sqrt}[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*\text{EllipticE}[2*\text{ArcTan}[x/\text{Sqrt}[2]], 1/8])/(14*\text{Sqrt}[2]*\text{Sqrt}[4 + 3*x^2 + x^4]) + (9*(2 + x^2)*\text{Sqrt}[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*\text{EllipticF}[2*\text{ArcTan}[x/\text{Sqrt}[2]], 1/8])/(4*\text{Sqrt}[2]*\text{Sqrt}[4 + 3*x^2 + x^4])$

Rubi [A] time = 0.0586936, antiderivative size = 181, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1205, 1197, 1103, 1195}

$$-\frac{113\sqrt{x^4+3x^2+4x}}{28(x^2+2)} - \frac{(9-113x^2)x}{28\sqrt{x^4+3x^2+4}} + \frac{9(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right), \frac{1}{8}\right)}{4\sqrt{2}\sqrt{x^4+3x^2+4}} + \frac{113(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}E\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right), \frac{1}{8}\right)}{14\sqrt{2}\sqrt{x^4+3x^2+4}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(7 + 5*x^2)^2/(4 + 3*x^2 + x^4)^{(3/2)}, x]$

[Out] $-(x*(9 - 113*x^2))/(28*\text{Sqrt}[4 + 3*x^2 + x^4]) - (113*x*\text{Sqrt}[4 + 3*x^2 + x^4])/((28*(2 + x^2)) + (113*(2 + x^2)*\text{Sqrt}[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*\text{EllipticE}[2*\text{ArcTan}[x/\text{Sqrt}[2]], 1/8])/(14*\text{Sqrt}[2]*\text{Sqrt}[4 + 3*x^2 + x^4]) + (9*(2 + x^2)*\text{Sqrt}[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*\text{EllipticF}[2*\text{ArcTan}[x/\text{Sqrt}[2]], 1/8])/(4*\text{Sqrt}[2]*\text{Sqrt}[4 + 3*x^2 + x^4])$

Rule 1205

$\text{Int}[(d + (e \cdot x^2)^q) \cdot ((a + (b \cdot x^2 + c) \cdot x^4)^p), x_Symbol] \rightarrow \text{With}[\{f = \text{Coeff}[\text{PolynomialRemainder}[(d + e \cdot x^2)^q, a + b \cdot x^2 + c \cdot x^4, x], x, 0], g = \text{Coeff}[\text{PolynomialRemainder}[(d + e \cdot x^2)^q, a + b \cdot x^2 + c \cdot x^4, x], x, 2]\}, \text{Simp}[(x \cdot (a + b \cdot x^2 + c \cdot x^4)^{p+1} \cdot (a \cdot b \cdot g - f \cdot (b^2 - 2 \cdot a \cdot c) - c \cdot (b \cdot f - 2 \cdot a \cdot g) \cdot x^2)) / (2 \cdot a \cdot (p + 1) \cdot (b^2 - 4 \cdot a \cdot c)), x] + \text{Dist}[1 / (2 \cdot a \cdot (p + 1) \cdot (b^2 - 4 \cdot a \cdot c)), \text{Int}[(a + b \cdot x^2 + c \cdot x^4)^{p+1} \cdot \text{ExpandToSum}[2 \cdot a \cdot (p + 1) \cdot (b^2 - 4 \cdot a \cdot c) \cdot \text{PolynomialQuotient}[(d + e \cdot x^2)^q, a + b \cdot x^2 + c \cdot x^4, x] + b^2 \cdot f \cdot (2 \cdot p + 3) - 2 \cdot a \cdot c \cdot f \cdot (4 \cdot p + 5) - a \cdot b \cdot g + c \cdot (4 \cdot p + 7) \cdot (b \cdot f - 2 \cdot a \cdot g) \cdot x^2, x], x] /; \text{FreeQ}[\{a, b, c, d, e\}, x] \&\amp; \text{NeQ}[b^2 - 4 \cdot a \cdot c, 0] \&\amp; \text{NeQ}[c \cdot d^2 - b \cdot d \cdot e + a \cdot e^2, 0] \&\amp; \text{IGtQ}[q, 1] \&\amp; \text{LtQ}[p, -1]$

Rule 1197

$\text{Int}[(d + (e \cdot x^2) / \text{Sqrt}[a + (b \cdot x^2 + c) \cdot x^4]), x_Symbol] \rightarrow \text{With}[\{q = \text{Rt}[c/a, 2]\}, \text{Dist}[(e + d \cdot q) / q, \text{Int}[1 / \text{Sqrt}[a + b \cdot x^2 + c \cdot x^4], x], x] - \text{Dist}[e / q, \text{Int}[(1 - q \cdot x^2) / \text{Sqrt}[a + b \cdot x^2 + c \cdot x^4], x], x] /; \text{NeQ}[e + d \cdot q, 0] /; \text{FreeQ}[\{a, b, c, d, e\}, x] \&\amp; \text{NeQ}[b^2 - 4 \cdot a \cdot c, 0] \&\amp; \text{PosQ}[c/a]$

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] := With[{q = Rt[c/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]), x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] := With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c*x^4]), x] /; EqQ[e + d*q^2, 0] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\begin{aligned} \int \frac{(7 + 5x^2)^2}{(4 + 3x^2 + x^4)^{3/2}} dx &= -\frac{x(9 - 113x^2)}{28\sqrt{4 + 3x^2 + x^4}} + \frac{1}{28} \int \frac{352 - 113x^2}{\sqrt{4 + 3x^2 + x^4}} dx \\ &= -\frac{x(9 - 113x^2)}{28\sqrt{4 + 3x^2 + x^4}} + \frac{9}{2} \int \frac{1}{\sqrt{4 + 3x^2 + x^4}} dx + \frac{113}{14} \int \frac{1 - \frac{x^2}{2}}{\sqrt{4 + 3x^2 + x^4}} dx \\ &= -\frac{x(9 - 113x^2)}{28\sqrt{4 + 3x^2 + x^4}} - \frac{113x\sqrt{4 + 3x^2 + x^4}}{28(2 + x^2)} + \frac{113(2 + x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}} E\left(2 \tan^{-1}\left(\frac{x}{\sqrt{2}}\right) \middle| \frac{1}{8}\right)}{14\sqrt{2}\sqrt{4 + 3x^2 + x^4}} + \dots \end{aligned}$$

Mathematica [F] time = 0, size = 0, normalized size = 0.

\$Aborted

Verification is Not applicable to the result.

[In] Integrate[(7 + 5*x^2)^2/(4 + 3*x^2 + x^4)^(3/2), x]

[Out] \$Aborted

Maple [C] time = 0.006, size = 278, normalized size = 1.5

$$-50 \frac{3/14 x^3 + 4/7 x}{\sqrt{x^4 + 3x^2 + 4}} + \frac{352}{7\sqrt{-6 + 2i\sqrt{7}}} \sqrt{1 - \left(-\frac{3}{8} + \frac{i}{8}\sqrt{7}\right)x^2} \sqrt{1 - \left(-\frac{3}{8} - \frac{i}{8}\sqrt{7}\right)x^2} \text{EllipticF}\left(\frac{x\sqrt{-6 + 2i\sqrt{7}}}{4}, \frac{\sqrt{2 + 6i\sqrt{7}}}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((5*x^2+7)^2/(x^4+3*x^2+4)^(3/2), x)

[Out] $-50*(3/14*x^3+4/7*x)/(x^4+3*x^2+4)^{(1/2)}+352/7/(-6+2*I*7^{(1/2)})^{(1/2)}*(1-(-3/8+1/8*I*7^{(1/2)})*x^2)^{(1/2)}*(1-(-3/8-1/8*I*7^{(1/2)})*x^2)^{(1/2)}/(x^4+3*x^2+4)^{(1/2)}*EllipticF(1/4*x*(-6+2*I*7^{(1/2)})^{(1/2)}, 1/4*(2+6*I*7^{(1/2)})^{(1/2)})+904/7/(-6+2*I*7^{(1/2)})^{(1/2)}*(1-(-3/8+1/8*I*7^{(1/2)})*x^2)^{(1/2)}*(1-(-3/8-1/8*I*7^{(1/2)})*x^2)^{(1/2)}/(x^4+3*x^2+4)^{(1/2)}/(I*7^{(1/2)}+3)*(EllipticF(1/4*x$

$*(-6+2*I*7^{(1/2)})^{(1/2)}, 1/4*(2+6*I*7^{(1/2)})^{(1/2)}) - \text{EllipticE}(1/4*x*(-6+2*I*7^{(1/2)})^{(1/2)}, 1/4*(2+6*I*7^{(1/2)})^{(1/2)}) - 140*(-1/7*x^3 - 3/14*x)/(x^4+3*x^2+4)^{(1/2)} - 98*(1/56*x+3/56*x^3)/(x^4+3*x^2+4)^{(1/2)}$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(5x^2 + 7)^2}{(x^4 + 3x^2 + 4)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^2+7)^2/(x^4+3*x^2+4)^(3/2),x, algorithm="maxima")

[Out] integrate((5*x^2 + 7)^2/(x^4 + 3*x^2 + 4)^(3/2), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(25x^4 + 70x^2 + 49)\sqrt{x^4 + 3x^2 + 4}}{x^8 + 6x^6 + 17x^4 + 24x^2 + 16}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^2+7)^2/(x^4+3*x^2+4)^(3/2),x, algorithm="fricas")

[Out] integral((25*x^4 + 70*x^2 + 49)*sqrt(x^4 + 3*x^2 + 4)/(x^8 + 6*x^6 + 17*x^4 + 24*x^2 + 16), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(5x^2 + 7)^2}{((x^2 - x + 2)(x^2 + x + 2))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x**2+7)**2/(x**4+3*x**2+4)**(3/2),x)

[Out] Integral((5*x**2 + 7)**2/((x**2 - x + 2)*(x**2 + x + 2))**3/2, x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(5x^2 + 7)^2}{(x^4 + 3x^2 + 4)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^2+7)^2/(x^4+3*x^2+4)^(3/2),x, algorithm="giac")

```
[Out] integrate((5*x^2 + 7)^2/(x^4 + 3*x^2 + 4)^(3/2), x)
```

$$3.375 \quad \int \frac{7+5x^2}{(4+3x^2+x^4)^{3/2}} dx$$

Optimal. Leaf size=181

$$\frac{3(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right), \frac{1}{8}\right)}{4\sqrt{2}\sqrt{x^4+3x^2+4}} - \frac{19\sqrt{x^4+3x^2+4}x}{28(x^2+2)} + \frac{(19x^2+53)x}{28\sqrt{x^4+3x^2+4}} + \frac{19(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}E\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right), \frac{1}{8}\right)}{14\sqrt{2}\sqrt{x^4+3x^2+4}}$$

```
[Out] (x*(53 + 19*x^2))/(28*Sqrt[4 + 3*x^2 + x^4]) - (19*x*Sqrt[4 + 3*x^2 + x^4])
/(28*(2 + x^2)) + (19*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE[2*ArcTan[x/Sqrt[2]], 1/8])/(14*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4]) - (3*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(4*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4])
```

Rubi [A] time = 0.0523354, antiderivative size = 181, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1178, 1197, 1103, 1195}

$$-\frac{19\sqrt{x^4+3x^2+4}x}{28(x^2+2)} + \frac{(19x^2+53)x}{28\sqrt{x^4+3x^2+4}} - \frac{3(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right), \frac{1}{8}\right)}{4\sqrt{2}\sqrt{x^4+3x^2+4}} + \frac{19(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}E\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right), \frac{1}{8}\right)}{14\sqrt{2}\sqrt{x^4+3x^2+4}}$$

Antiderivative was successfully verified.

```
[In] Int[(7 + 5*x^2)/(4 + 3*x^2 + x^4)^(3/2), x]
```

```
[Out] (x*(53 + 19*x^2))/(28*Sqrt[4 + 3*x^2 + x^4]) - (19*x*Sqrt[4 + 3*x^2 + x^4])
/(28*(2 + x^2)) + (19*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE[2*ArcTan[x/Sqrt[2]], 1/8])/(14*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4]) - (3*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(4*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4])
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[(x*(a*b*e - d*(b^2 - 2*a*c)) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 + c*x^4)^(p + 1)]/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol]
:> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; NeQ[e + d*q, 0] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol]
:> With[{q = Rt[c/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
```

EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)]/(2*q*Sqrt[a + b*x^2 + c*x^4]), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]

Rule 1195

Int[((d_) + (e_)*(x_)^2)/Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4], x_Symbol] :=> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)]/(q*Sqrt[a + b*x^2 + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]

Rubi steps

$$\begin{aligned} \int \frac{7 + 5x^2}{(4 + 3x^2 + x^4)^{3/2}} dx &= \frac{x(53 + 19x^2)}{28\sqrt{4 + 3x^2 + x^4}} + \frac{1}{28} \int \frac{-4 - 19x^2}{\sqrt{4 + 3x^2 + x^4}} dx \\ &= \frac{x(53 + 19x^2)}{28\sqrt{4 + 3x^2 + x^4}} + \frac{19}{14} \int \frac{1 - \frac{x^2}{2}}{\sqrt{4 + 3x^2 + x^4}} dx - \frac{3}{2} \int \frac{1}{\sqrt{4 + 3x^2 + x^4}} dx \\ &= \frac{x(53 + 19x^2)}{28\sqrt{4 + 3x^2 + x^4}} - \frac{19x\sqrt{4 + 3x^2 + x^4}}{28(2 + x^2)} + \frac{19(2 + x^2) \sqrt{\frac{4 + 3x^2 + x^4}{(2 + x^2)^2}} E\left(2 \tan^{-1}\left(\frac{x}{\sqrt{2}}\right) \middle| \frac{1}{8}\right)}{14\sqrt{2}\sqrt{4 + 3x^2 + x^4}} - \frac{3(2 + x^2)}{14\sqrt{2}\sqrt{4 + 3x^2 + x^4}} \end{aligned}$$

Mathematica [F] time = 0, size = 0, normalized size = 0.

\$Aborted

Verification is Not applicable to the result.

[In] Integrate[(7 + 5*x^2)/(4 + 3*x^2 + x^4)^(3/2), x]

[Out] \$Aborted

Maple [C] time = 0.004, size = 255, normalized size = 1.4

$$-10 \frac{-1/7 x^3 - 3/14 x}{\sqrt{x^4 + 3x^2 + 4}} - \frac{4}{7\sqrt{-6 + 2i\sqrt{7}}} \sqrt{1 - \left(-\frac{3}{8} + \frac{i}{8}\sqrt{7}\right)x^2} \sqrt{1 - \left(-\frac{3}{8} - \frac{i}{8}\sqrt{7}\right)x^2} \text{EllipticF}\left(\frac{x\sqrt{-6 + 2i\sqrt{7}}}{4}, \frac{\sqrt{2 + 6i\sqrt{7}}}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((5*x^2+7)/(x^4+3*x^2+4)^(3/2), x)

[Out] $-10*(-1/7*x^3-3/14*x)/(x^4+3*x^2+4)^{(1/2)}-4/7/(-6+2*I*7^{(1/2)})^{(1/2)}*(1-(-3/8+1/8*I*7^{(1/2)})*x^2)^{(1/2)}*(1-(-3/8-1/8*I*7^{(1/2)})*x^2)^{(1/2)}/(x^4+3*x^2+4)^{(1/2)}*EllipticF(1/4*x*(-6+2*I*7^{(1/2)})^{(1/2)}, 1/4*(2+6*I*7^{(1/2)})^{(1/2)})+152/7/(-6+2*I*7^{(1/2)})^{(1/2)}*(1-(-3/8+1/8*I*7^{(1/2)})*x^2)^{(1/2)}*(1-(-3/8-1/8*I*7^{(1/2)})*x^2)^{(1/2)}/(x^4+3*x^2+4)^{(1/2)}/(I*7^{(1/2)}+3)*(EllipticF(1/4*x*(-6+2*I*7^{(1/2)})^{(1/2)}, 1/4*(2+6*I*7^{(1/2)})^{(1/2)})-EllipticE(1/4*x*(-6+2*I*7^{(1/2)})^{(1/2)}, 1/4*(2+6*I*7^{(1/2)})^{(1/2)}))-14*(1/56*x+3/56*x^3)/(x^4+3*x^2+4)^{(1/2)}$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{5x^2 + 7}{(x^4 + 3x^2 + 4)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^2+7)/(x^4+3*x^2+4)^(3/2),x, algorithm="maxima")

[Out] integrate((5*x^2 + 7)/(x^4 + 3*x^2 + 4)^(3/2), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{x^4 + 3x^2 + 4}(5x^2 + 7)}{x^8 + 6x^6 + 17x^4 + 24x^2 + 16}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^2+7)/(x^4+3*x^2+4)^(3/2),x, algorithm="fricas")

[Out] integral(sqrt(x^4 + 3*x^2 + 4)*(5*x^2 + 7)/(x^8 + 6*x^6 + 17*x^4 + 24*x^2 + 16), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{5x^2 + 7}{((x^2 - x + 2)(x^2 + x + 2))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x**2+7)/(x**4+3*x**2+4)**(3/2),x)

[Out] Integral((5*x**2 + 7)/((x**2 - x + 2)*(x**2 + x + 2))**(3/2), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{5x^2 + 7}{(x^4 + 3x^2 + 4)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^2+7)/(x^4+3*x^2+4)^(3/2),x, algorithm="giac")

[Out] integrate((5*x^2 + 7)/(x^4 + 3*x^2 + 4)^(3/2), x)

$$3.376 \quad \int \frac{1}{(4+3x^2+x^4)^{3/2}} dx$$

Optimal. Leaf size=181

$$\frac{(x^2+2) \sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}} \operatorname{EllipticF}\left(2 \tan^{-1}\left(\frac{x}{\sqrt{2}}\right), \frac{1}{8}\right)}{4\sqrt{2}\sqrt{x^4+3x^2+4}} + \frac{3\sqrt{x^4+3x^2+4x}}{28(x^2+2)} - \frac{(3x^2+1)x}{28\sqrt{x^4+3x^2+4}} - \frac{3(x^2+2) \sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}} E\left(2 \tan^{-1}\left(\frac{x}{\sqrt{2}}\right)\right)}{14\sqrt{2}\sqrt{x^4+3x^2+4}}$$

```
[Out] -(x*(1 + 3*x^2))/(28*sqrt[4 + 3*x^2 + x^4]) + (3*x*sqrt[4 + 3*x^2 + x^4])/(28*(2 + x^2)) - (3*(2 + x^2)*sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE[2*ArcTan[x/sqrt[2]], 1/8])/(14*sqrt[2]*sqrt[4 + 3*x^2 + x^4]) + ((2 + x^2)*sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/sqrt[2]], 1/8])/(4*sqrt[2]*sqrt[4 + 3*x^2 + x^4])
```

Rubi [A] time = 0.0497633, antiderivative size = 181, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {1092, 1197, 1103, 1195}

$$\frac{3\sqrt{x^4+3x^2+4x}}{28(x^2+2)} - \frac{(3x^2+1)x}{28\sqrt{x^4+3x^2+4}} + \frac{(x^2+2) \sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}} F\left(2 \tan^{-1}\left(\frac{x}{\sqrt{2}}\right) \middle| \frac{1}{8}\right)}{4\sqrt{2}\sqrt{x^4+3x^2+4}} - \frac{3(x^2+2) \sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}} E\left(2 \tan^{-1}\left(\frac{x}{\sqrt{2}}\right)\right)}{14\sqrt{2}\sqrt{x^4+3x^2+4}}$$

Antiderivative was successfully verified.

```
[In] Int[(4 + 3*x^2 + x^4)^(-3/2), x]
```

```
[Out] -(x*(1 + 3*x^2))/(28*sqrt[4 + 3*x^2 + x^4]) + (3*x*sqrt[4 + 3*x^2 + x^4])/(28*(2 + x^2)) - (3*(2 + x^2)*sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE[2*ArcTan[x/sqrt[2]], 1/8])/(14*sqrt[2]*sqrt[4 + 3*x^2 + x^4]) + ((2 + x^2)*sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/sqrt[2]], 1/8])/(4*sqrt[2]*sqrt[4 + 3*x^2 + x^4])
```

Rule 1092

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> -Simp[(x*(b^2 - 2*a*c + b*c*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(b^2 - 2*a*c + 2*(p + 1)*(b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/sqrt[a + b*x^2 + c*x^4], x], x] - Dist[e/q, Int[(1 - q*x^2)/sqrt[a + b*x^2 + c*x^4], x], x] /; NeQ[e + d*q, 0] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1103

```
Int[1/sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c/a, 4]}, Simp[((1 + q^2*x^2)*sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*sqrt[a + b*x^2 + c*x^4])
```

), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]

Rule 1195

Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]

Rubi steps

$$\begin{aligned} \int \frac{1}{(4 + 3x^2 + x^4)^{3/2}} dx &= -\frac{x(1 + 3x^2)}{28\sqrt{4 + 3x^2 + x^4}} + \frac{1}{28} \int \frac{8 + 3x^2}{\sqrt{4 + 3x^2 + x^4}} dx \\ &= -\frac{x(1 + 3x^2)}{28\sqrt{4 + 3x^2 + x^4}} - \frac{3}{14} \int \frac{1 - \frac{x^2}{2}}{\sqrt{4 + 3x^2 + x^4}} dx + \frac{1}{2} \int \frac{1}{\sqrt{4 + 3x^2 + x^4}} dx \\ &= -\frac{x(1 + 3x^2)}{28\sqrt{4 + 3x^2 + x^4}} + \frac{3x\sqrt{4 + 3x^2 + x^4}}{28(2 + x^2)} - \frac{3(2 + x^2)\sqrt{\frac{4 + 3x^2 + x^4}{(2 + x^2)^2}} E\left(2 \tan^{-1}\left(\frac{x}{\sqrt{2}}\right) \middle| \frac{1}{8}\right)}{14\sqrt{2}\sqrt{4 + 3x^2 + x^4}} + \frac{1}{2} \int \frac{1}{\sqrt{4 + 3x^2 + x^4}} dx \end{aligned}$$

Mathematica [C] time = 0.356641, size = 328, normalized size = 1.81

$$\frac{\sqrt{2}(3\sqrt{7} - 7i) \sqrt{\frac{-2ix^2 + \sqrt{7} - 3i}{\sqrt{7} - 3i}} \sqrt{\frac{2ix^2 + \sqrt{7} + 3i}{\sqrt{7} + 3i}} \text{EllipticF}\left(i \sinh^{-1}\left(\sqrt{\frac{-2i}{\sqrt{7} - 3i}} x\right), \frac{-\sqrt{7} + 3i}{\sqrt{7} + 3i}\right) - 4\sqrt{\frac{i}{\sqrt{7} - 3i}} x(3x^2 + 1) - 3\sqrt{2}(\sqrt{7} + 7i)}{112\sqrt{\frac{i}{\sqrt{7} - 3i}} \sqrt{x^4 + 3x^2 + 4}}$$

Antiderivative was successfully verified.

[In] Integrate[(4 + 3*x^2 + x^4)^(-3/2), x]

[Out] (-4*Sqrt[(-I)/(-3*I + Sqrt[7])]*x*(1 + 3*x^2) - 3*Sqrt[2]*(3*I + Sqrt[7])*Sqrt[(-3*I + Sqrt[7] - (2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[(3*I + Sqrt[7] + (2*I)*x^2)/(3*I + Sqrt[7])]*EllipticE[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])] + Sqrt[2]*(-7*I + 3*Sqrt[7])*Sqrt[(-3*I + Sqrt[7] - (2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[(3*I + Sqrt[7] + (2*I)*x^2)/(3*I + Sqrt[7])]*EllipticF[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])])/(112*Sqrt[(-I)/(-3*I + Sqrt[7])]*Sqrt[4 + 3*x^2 + x^4])

Maple [C] time = 0.004, size = 232, normalized size = 1.3

$$-2 \frac{1}{\sqrt{x^4 + 3x^2 + 4}} \left(\frac{x}{56} + \frac{3x^3}{56} \right) + \frac{8}{7\sqrt{-6 + 2i\sqrt{7}}} \sqrt{1 - \left(-\frac{3}{8} + \frac{i}{8}\sqrt{7} \right) x^2} \sqrt{1 - \left(-\frac{3}{8} - \frac{i}{8}\sqrt{7} \right) x^2} \text{EllipticF}\left(\frac{x\sqrt{-6 + 2i\sqrt{7}}}{4}, \frac{1}{8} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^4+3*x^2+4)^(3/2), x)


```
[Out] -2*(1/56*x+3/56*x^3)/(x^4+3*x^2+4)^(1/2)+8/7/(-6+2*I*7^(1/2))^(1/2)*(1-(-3/8+1/8*I*7^(1/2))*x^2)^(1/2)*(1-(-3/8-1/8*I*7^(1/2))*x^2)^(1/2)/(x^4+3*x^2+4)^(1/2)*EllipticF(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))-24/7/(-6+2*I*7^(1/2))^(1/2)*(1-(-3/8+1/8*I*7^(1/2))*x^2)^(1/2)*(1-(-3/8-1/8*I*7^(1/2))*x^2)^(1/2)/(x^4+3*x^2+4)^(1/2)/(I*7^(1/2)+3)*(EllipticF(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))-EllipticE(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x^4 + 3x^2 + 4)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+3*x^2+4)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((x^4 + 3*x^2 + 4)^(-3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{x^4 + 3x^2 + 4}}{x^8 + 6x^6 + 17x^4 + 24x^2 + 16}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+3*x^2+4)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^4 + 3*x^2 + 4)/(x^8 + 6*x^6 + 17*x^4 + 24*x^2 + 16), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x^4 + 3x^2 + 4)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**4+3*x**2+4)**(3/2),x)
```

```
[Out] Integral((x**4 + 3*x**2 + 4)**(-3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x^4 + 3x^2 + 4)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+3*x^2+4)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((x^4 + 3*x^2 + 4)^(-3/2), x)
```

$$3.377 \quad \int \frac{1}{(7+5x^2)(4+3x^2+x^4)^{3/2}} dx$$

Optimal. Leaf size=284

$$\frac{(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}} \operatorname{EllipticF}\left(2 \tan^{-1}\left(\frac{x}{\sqrt{2}}\right), \frac{1}{8}\right)}{12\sqrt{2}\sqrt{x^4+3x^2+4}} + \frac{\sqrt{x^4+3x^2+4x}}{77(x^2+2)} - \frac{(4x^2+13)x}{308\sqrt{x^4+3x^2+4}} + \frac{25}{176}\sqrt{\frac{5}{77}} \tan^{-1}\left(\frac{2\sqrt{x^4+3x^2+4x}}{\sqrt{x^4+3x^2+4}}\right)$$

[Out] $-(x*(13 + 4*x^2))/(308*\operatorname{Sqrt}[4 + 3*x^2 + x^4]) + (x*\operatorname{Sqrt}[4 + 3*x^2 + x^4])/(77*(2 + x^2)) + (25*\operatorname{Sqrt}[5/77]*\operatorname{ArcTan}[(2*\operatorname{Sqrt}[11/35]*x)/\operatorname{Sqrt}[4 + 3*x^2 + x^4]])/176 - (\operatorname{Sqrt}[2]*(2 + x^2)*\operatorname{Sqrt}[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*\operatorname{EllipticE}[2*\operatorname{ArcTan}[x/\operatorname{Sqrt}[2]], 1/8])/(77*\operatorname{Sqrt}[4 + 3*x^2 + x^4]) - ((2 + x^2)*\operatorname{Sqrt}[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*\operatorname{EllipticF}[2*\operatorname{ArcTan}[x/\operatorname{Sqrt}[2]], 1/8])/(12*\operatorname{Sqrt}[2]*\operatorname{Sqrt}[4 + 3*x^2 + x^4]) + (425*(2 + x^2)*\operatorname{Sqrt}[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*\operatorname{EllipticPi}[-9/280, 2*\operatorname{ArcTan}[x/\operatorname{Sqrt}[2]], 1/8])/(3696*\operatorname{Sqrt}[2]*\operatorname{Sqrt}[4 + 3*x^2 + x^4])$

Rubi [A] time = 0.16267, antiderivative size = 284, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {1221, 1178, 1197, 1103, 1195, 1216, 1706}

$$\frac{\sqrt{x^4+3x^2+4x}}{77(x^2+2)} - \frac{(4x^2+13)x}{308\sqrt{x^4+3x^2+4}} + \frac{25}{176}\sqrt{\frac{5}{77}} \tan^{-1}\left(\frac{2\sqrt{\frac{11}{35}}x}{\sqrt{x^4+3x^2+4}}\right) - \frac{(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}} F\left(2 \tan^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|\frac{1}{8}\right)}{12\sqrt{2}\sqrt{x^4+3x^2+4}}$$

Antiderivative was successfully verified.

[In] Int[1/((7 + 5*x^2)*(4 + 3*x^2 + x^4)^(3/2)), x]

[Out] $-(x*(13 + 4*x^2))/(308*\operatorname{Sqrt}[4 + 3*x^2 + x^4]) + (x*\operatorname{Sqrt}[4 + 3*x^2 + x^4])/(77*(2 + x^2)) + (25*\operatorname{Sqrt}[5/77]*\operatorname{ArcTan}[(2*\operatorname{Sqrt}[11/35]*x)/\operatorname{Sqrt}[4 + 3*x^2 + x^4]])/176 - (\operatorname{Sqrt}[2]*(2 + x^2)*\operatorname{Sqrt}[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*\operatorname{EllipticE}[2*\operatorname{ArcTan}[x/\operatorname{Sqrt}[2]], 1/8])/(77*\operatorname{Sqrt}[4 + 3*x^2 + x^4]) - ((2 + x^2)*\operatorname{Sqrt}[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*\operatorname{EllipticF}[2*\operatorname{ArcTan}[x/\operatorname{Sqrt}[2]], 1/8])/(12*\operatorname{Sqrt}[2]*\operatorname{Sqrt}[4 + 3*x^2 + x^4]) + (425*(2 + x^2)*\operatorname{Sqrt}[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*\operatorname{EllipticPi}[-9/280, 2*\operatorname{ArcTan}[x/\operatorname{Sqrt}[2]], 1/8])/(3696*\operatorname{Sqrt}[2]*\operatorname{Sqrt}[4 + 3*x^2 + x^4])$

Rule 1221

Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_)/((d_) + (e_.)*(x_)^2), x_Symbol] :> Dist[1/(c*d^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x^2)*(a + b*x^2 + c*x^4)^p, x], x] + Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[(a + b*x^2 + c*x^4)^(p + 1)/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[p + 1/2, 0]

Rule 1178

Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,

b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && IntegerQ[2*p]

Rule 1197

Int[((d_) + (e_)*(x_)^2)/Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4], x_Symbol] := With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; NeQ[e + d*q, 0] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]

Rule 1103

Int[1/Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4], x_Symbol] := With[{q = Rt[c/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)]/(2*q*Sqrt[a + b*x^2 + c*x^4]), x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]

Rule 1195

Int[((d_) + (e_)*(x_)^2)/Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4], x_Symbol] := With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)]/(q*Sqrt[a + b*x^2 + c*x^4]), x] /; EqQ[e + d*q^2, 0] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]

Rule 1216

Int[1/(((d_) + (e_)*(x_)^2)*Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4]), x_Symbol] := With[{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]

Rule 1706

Int[((A_) + (B_)*(x_)^2)/(((d_) + (e_)*(x_)^2)*Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4]), x_Symbol] := With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*ArcTan[(Rt[-b + (c*d)/e + (a*e)/d, 2]*x)/Sqrt[a + b*x^2 + c*x^4]])/(2*d*e*Rt[-b + (c*d)/e + (a*e)/d, 2]), x] + Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a + b*x^2 + c*x^4))/(a*(A + B*x^2)^2)]*EllipticPi[Cancel[-((B*d - A*e)^2/(4*d*e*A*B))], 2*ArcTan[q*x], 1/2 - (b*A)/(4*a*B)]/(4*d*e*A*q*Sqrt[a + b*x^2 + c*x^4]), x] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]

Rubi steps

$$\begin{aligned}
\int \frac{1}{(7+5x^2)(4+3x^2+x^4)^{3/2}} dx &= \frac{1}{44} \int \frac{-8-5x^2}{(4+3x^2+x^4)^{3/2}} dx + \frac{25}{44} \int \frac{1}{(7+5x^2)\sqrt{4+3x^2+x^4}} dx \\
&= -\frac{x(13+4x^2)}{308\sqrt{4+3x^2+x^4}} + \frac{\int \frac{-4+16x^2}{\sqrt{4+3x^2+x^4}} dx}{1232} - \frac{25}{132} \int \frac{1}{\sqrt{4+3x^2+x^4}} dx + \frac{125}{66} \int \frac{1}{(7+5x^2)\sqrt{4+3x^2+x^4}} dx \\
&= -\frac{x(13+4x^2)}{308\sqrt{4+3x^2+x^4}} + \frac{25}{176} \sqrt{\frac{5}{77}} \tan^{-1} \left(\frac{2\sqrt{\frac{11}{35}}x}{\sqrt{4+3x^2+x^4}} \right) - \frac{25(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}}{264\sqrt{2}\sqrt{4+3x^2+x^4}} \\
&= -\frac{x(13+4x^2)}{308\sqrt{4+3x^2+x^4}} + \frac{x\sqrt{4+3x^2+x^4}}{77(2+x^2)} + \frac{25}{176} \sqrt{\frac{5}{77}} \tan^{-1} \left(\frac{2\sqrt{\frac{11}{35}}x}{\sqrt{4+3x^2+x^4}} \right) - \frac{25(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}}{264\sqrt{2}\sqrt{4+3x^2+x^4}}
\end{aligned}$$

Mathematica [C] time = 0.540215, size = 483, normalized size = 1.7

$$\sqrt{2}(2\sqrt{7}+7i)\sqrt{\frac{-2ix^2+\sqrt{7}-3i}{\sqrt{7}-3i}}\sqrt{\frac{2ix^2+\sqrt{7}+3i}{\sqrt{7}+3i}}\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{\frac{-2i}{\sqrt{7}-3i}}x\right),\frac{-\sqrt{7}+3i}{\sqrt{7}+3i}\right)-8\sqrt{\frac{i}{\sqrt{7}-3i}}x^3-2\sqrt{2}(\sqrt{7}+3i)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/((7 + 5*x^2)*(4 + 3*x^2 + x^4)^(3/2)),x]

[Out] (-26*Sqrt[(-I)/(-3*I + Sqrt[7])]*x - 8*Sqrt[(-I)/(-3*I + Sqrt[7])]*x^3 - 2*Sqrt[2]*(3*I + Sqrt[7])*Sqrt[(-3*I + Sqrt[7] - (2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[(3*I + Sqrt[7] + (2*I)*x^2)/(3*I + Sqrt[7])]*EllipticE[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])] + Sqrt[2]*(7*I + 2*Sqrt[7])*Sqrt[(-3*I + Sqrt[7] - (2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[(3*I + Sqrt[7] + (2*I)*x^2)/(3*I + Sqrt[7])]*EllipticF[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])] - (25*I)*Sqrt[2]*Sqrt[(-3*I + Sqrt[7] - (2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[(3*I + Sqrt[7] + (2*I)*x^2)/(3*I + Sqrt[7])]*EllipticPi[(5*(3 + I*Sqrt[7]))/14, I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])]/(616*Sqrt[(-I)/(-3*I + Sqrt[7])]*Sqrt[4 + 3*x^2 + x^4])

Maple [C] time = 0.018, size = 409, normalized size = 1.4

$$-2\frac{1}{\sqrt{x^4+3x^2+4}}\left(\frac{x^3}{154}+\frac{13x}{616}\right)-\frac{1}{77\sqrt{-6+2i\sqrt{7}}}\sqrt{1+\frac{3x^2}{8}-\frac{i}{8}x^2\sqrt{7}}\sqrt{1+\frac{3x^2}{8}+\frac{i}{8}x^2\sqrt{7}}\text{EllipticF}\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(5*x^2+7)/(x^4+3*x^2+4)^(3/2),x)

[Out] -2*(1/154*x^3+13/616*x)/(x^4+3*x^2+4)^(1/2)-1/77/(-6+2*I*7^(1/2))^(1/2)*(1+3/8*x^2-1/8*I*x^2*7^(1/2))^(1/2)*(1+3/8*x^2+1/8*I*x^2*7^(1/2))^(1/2)/(x^4+3*x^2+4)^(1/2)*EllipticF(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))-32/77/(-6+2*I*7^(1/2))^(1/2)*(1+3/8*x^2-1/8*I*x^2*7^(1/2))^(1/2)*(1+3/8*x^2+1/8*I*x^2*7^(1/2))^(1/2)/(x^4+3*x^2+4)^(1/2)

$$8x^2+1/8I*x^2*7^{(1/2)})^{(1/2)}/(x^4+3*x^2+4)^{(1/2)}/(I*7^{(1/2)}+3)*\text{EllipticF}(1/4*x*(-6+2*I*7^{(1/2)})^{(1/2)},1/4*(2+6*I*7^{(1/2)})^{(1/2)})+32/77/(-6+2*I*7^{(1/2)})^{(1/2)}*(1+3/8*x^2-1/8*I*x^2*7^{(1/2)})^{(1/2)}*(1+3/8*x^2+1/8*I*x^2*7^{(1/2)})^{(1/2)}/(x^4+3*x^2+4)^{(1/2)}/(I*7^{(1/2)}+3)*\text{EllipticE}(1/4*x*(-6+2*I*7^{(1/2)})^{(1/2)},1/4*(2+6*I*7^{(1/2)})^{(1/2)})+25/308/(-3/8+1/8*I*7^{(1/2)})^{(1/2)}*(1+3/8*x^2-1/8*I*x^2*7^{(1/2)})^{(1/2)}*(1+3/8*x^2+1/8*I*x^2*7^{(1/2)})^{(1/2)}/(x^4+3*x^2+4)^{(1/2)}*\text{EllipticPi}((-3/8+1/8*I*7^{(1/2)})^{(1/2)}*x,-5/7/(-3/8+1/8*I*7^{(1/2)}),(-3/8-1/8*I*7^{(1/2)})^{(1/2)}/(-3/8+1/8*I*7^{(1/2)})^{(1/2)})$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x^4 + 3x^2 + 4)^{\frac{3}{2}}(5x^2 + 7)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(5*x^2+7)/(x^4+3*x^2+4)^(3/2),x, algorithm="maxima")

[Out] integrate(1/((x^4 + 3*x^2 + 4)^(3/2)*(5*x^2 + 7)), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{x^4 + 3x^2 + 4}}{5x^{10} + 37x^8 + 127x^6 + 239x^4 + 248x^2 + 112}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(5*x^2+7)/(x^4+3*x^2+4)^(3/2),x, algorithm="fricas")

[Out] integral(sqrt(x^4 + 3*x^2 + 4)/(5*x^10 + 37*x^8 + 127*x^6 + 239*x^4 + 248*x^2 + 112), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{((x^2 - x + 2)(x^2 + x + 2))^{\frac{3}{2}}(5x^2 + 7)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(5*x**2+7)/(x**4+3*x**2+4)**(3/2),x)

[Out] Integral(1/(((x**2 - x + 2)*(x**2 + x + 2))**(3/2)*(5*x**2 + 7)), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x^4 + 3x^2 + 4)^{\frac{3}{2}}(5x^2 + 7)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+7)/(x^4+3*x^2+4)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((x^4 + 3*x^2 + 4)^(3/2)*(5*x^2 + 7)), x)
```

$$3.378 \quad \int \frac{1}{(7+5x^2)^2(4+3x^2+x^4)^{3/2}} dx$$

Optimal. Leaf size=312

$$\frac{2\sqrt{2}(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{231\sqrt{x^4+3x^2+4}} - \frac{199\sqrt{x^4+3x^2+4x}}{27104(x^2+2)} + \frac{625\sqrt{x^4+3x^2+4x}}{27104(5x^2+7)} + \frac{(37x^2+24)x}{13552\sqrt{x^4+3x^2+4}}$$

[Out] (x*(24 + 37*x^2))/(13552*Sqrt[4 + 3*x^2 + x^4]) - (199*x*Sqrt[4 + 3*x^2 + x^4])/(27104*(2 + x^2)) + (625*x*Sqrt[4 + 3*x^2 + x^4])/(27104*(7 + 5*x^2)) + (575*Sqrt[5/77]*ArcTan[(2*Sqrt[11/35]*x)/Sqrt[4 + 3*x^2 + x^4]])/108416 + (199*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE[2*ArcTan[x/Sqrt[2]], 1/8])/(13552*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4]) - (2*Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(231*Sqrt[4 + 3*x^2 + x^4]) + (9775*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticPi[-9/280, 2*ArcTan[x/Sqrt[2]], 1/8])/(2276736*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4])

Rubi [A] time = 0.504976, antiderivative size = 312, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {1228, 1178, 1197, 1103, 1195, 1223, 1714, 1708, 1706, 1216}

$$-\frac{199\sqrt{x^4+3x^2+4x}}{27104(x^2+2)} + \frac{625\sqrt{x^4+3x^2+4x}}{27104(5x^2+7)} + \frac{(37x^2+24)x}{13552\sqrt{x^4+3x^2+4}} + \frac{575\sqrt{\frac{5}{77}}\tan^{-1}\left(\frac{2\sqrt{\frac{11}{35}}x}{\sqrt{x^4+3x^2+4}}\right)}{108416} - \frac{2\sqrt{2}(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}}{231\sqrt{x^4+3x^2+4}}$$

Antiderivative was successfully verified.

[In] Int[1/((7 + 5*x^2)^2*(4 + 3*x^2 + x^4)^(3/2)),x]

[Out] (x*(24 + 37*x^2))/(13552*Sqrt[4 + 3*x^2 + x^4]) - (199*x*Sqrt[4 + 3*x^2 + x^4])/(27104*(2 + x^2)) + (625*x*Sqrt[4 + 3*x^2 + x^4])/(27104*(7 + 5*x^2)) + (575*Sqrt[5/77]*ArcTan[(2*Sqrt[11/35]*x)/Sqrt[4 + 3*x^2 + x^4]])/108416 + (199*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE[2*ArcTan[x/Sqrt[2]], 1/8])/(13552*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4]) - (2*Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(231*Sqrt[4 + 3*x^2 + x^4]) + (9775*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticPi[-9/280, 2*ArcTan[x/Sqrt[2]], 1/8])/(2276736*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4])

Rule 1228

Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := Module[{aa, bb, cc}, Int[ExpandIntegrand[1/Sqrt[aa + bb*x^2 + c*x^4], (d + e*x^2)^q*(aa + bb*x^2 + cc*x^4)^(p + 1/2), x] /. {aa -> a, bb -> b, cc -> c}, x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, 0] && IntegerQ[p + 1/2]

Rule 1178

Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := Simp[(x*(a*b*e - d*(b^2 - 2*a*c)) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7

)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && IntegerQ[2*p]

Rule 1197

Int[((d_) + (e_)*(x_)^2)/Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4], x_Symbol] := With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; NeQ[e + d*q, 0] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]

Rule 1103

Int[1/Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4], x_Symbol] := With[{q = Rt[c/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]), x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]

Rule 1195

Int[((d_) + (e_)*(x_)^2)/Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4], x_Symbol] := With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c*x^4]), x] /; EqQ[e + d*q^2, 0] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]

Rule 1223

Int[((d_) + (e_)*(x_)^2)^(q_)/Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4], x_Symbol] := -Simp[(e^2*x*(d + e*x^2)^(q + 1)*Sqrt[a + b*x^2 + c*x^4])/(2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2)), Int[((d + e*x^2)^(q + 1)*Simp[a*e^2*(2*q + 3) + 2*d*(c*d - b*e)*(q + 1) - 2*e*(c*d*(q + 1) - b*e*(q + 2))*x^2 + c*e^2*(2*q + 5)*x^4, x])/Sqrt[a + b*x^2 + c*x^4], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && ILtQ[q, -1]

Rule 1714

Int[(P4x_)/(((d_) + (e_)*(x_)^2)*Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4]), x_Symbol] := With[{q = Rt[c/a, 2], A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[P4x, x, 4]}, -Dist[C/(e*q), Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] + Dist[1/(c*e), Int[(A*c*e + a*C*d*q + (B*c*e - C*(c*d - a*e*q))*x^2]/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[P4x, x^2, 2] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && !GtQ[b^2 - 4*a*c, 0]

Rule 1708

Int[((A_) + (B_)*(x_)^2)/(((d_) + (e_)*(x_)^2)*Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4]), x_Symbol] := With[{q = Rt[c/a, 2]}, Dist[(A*(c*d + a*e*q) - a*B*(e + d*q))/(c*d^2 - a*e^2), Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] + Dist[(a*(B*d - A*e)*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && NeQ[c*A^2 - a*B^2, 0]

Rule 1706

```
Int[((A_) + (B_)*(x_)^2)/(((d_) + (e_)*(x_)^2)*Sqrt[(a_) + (b_)*(x_)^2 +
(c_)*(x_)^4]), x_Symbol] := With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*Arc
Tan[(Rt[-b + (c*d)/e + (a*e)/d, 2]*x)/Sqrt[a + b*x^2 + c*x^4]])/(2*d*e*Rt[-
b + (c*d)/e + (a*e)/d, 2]), x] + Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a
+ b*x^2 + c*x^4))/(a*(A + B*x^2)^2)]*EllipticPi[Cancel[-((B*d - A*e)^2/(4*
d*e*A*B))], 2*ArcTan[q*x], 1/2 - (b*A)/(4*a*B)])/(4*d*e*A*q*Sqrt[a + b*x^2
+ c*x^4]), x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rule 1216

```
Int[1/(((d_) + (e_)*(x_)^2)*Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4]), x_S
ymbol] := With[{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/
Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int
[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{(7+5x^2)^2(4+3x^2+x^4)^{3/2}} dx &= \int \left(\frac{-36+5x^2}{1936(4+3x^2+x^4)^{3/2}} + \frac{25}{44(7+5x^2)^2\sqrt{4+3x^2+x^4}} - \frac{25}{1936(7+5x^2)\sqrt{4+3x^2+x^4}} \right) dx \\ &= \frac{\int \frac{-36+5x^2}{(4+3x^2+x^4)^{3/2}} dx}{1936} - \frac{25 \int \frac{1}{(7+5x^2)\sqrt{4+3x^2+x^4}} dx}{1936} + \frac{25}{44} \int \frac{1}{(7+5x^2)^2\sqrt{4+3x^2+x^4}} dx \\ &= \frac{x(24+37x^2)}{13552\sqrt{4+3x^2+x^4}} + \frac{625x\sqrt{4+3x^2+x^4}}{27104(7+5x^2)} + \frac{\int \frac{-348-148x^2}{\sqrt{4+3x^2+x^4}} dx}{54208} - \frac{25 \int \frac{12+70x^2+25x^4}{(7+5x^2)\sqrt{4+3x^2+x^4}} dx}{27104} \\ &= \frac{x(24+37x^2)}{13552\sqrt{4+3x^2+x^4}} + \frac{625x\sqrt{4+3x^2+x^4}}{27104(7+5x^2)} - \frac{25\sqrt{\frac{5}{77}} \tan^{-1}\left(\frac{2\sqrt{\frac{11}{35}}x}{\sqrt{4+3x^2+x^4}}\right)}{7744} + \frac{25(2+5x^2)}{7744} \\ &= \frac{x(24+37x^2)}{13552\sqrt{4+3x^2+x^4}} - \frac{199x\sqrt{4+3x^2+x^4}}{27104(2+x^2)} + \frac{625x\sqrt{4+3x^2+x^4}}{27104(7+5x^2)} - \frac{25\sqrt{\frac{5}{77}} \tan^{-1}\left(\frac{2\sqrt{\frac{11}{35}}x}{\sqrt{4+3x^2+x^4}}\right)}{7744} \\ &= \frac{x(24+37x^2)}{13552\sqrt{4+3x^2+x^4}} - \frac{199x\sqrt{4+3x^2+x^4}}{27104(2+x^2)} + \frac{625x\sqrt{4+3x^2+x^4}}{27104(7+5x^2)} + \frac{575\sqrt{\frac{5}{77}} \tan^{-1}\left(\frac{2\sqrt{\frac{11}{35}}x}{\sqrt{4+3x^2+x^4}}\right)}{1084} \end{aligned}$$

Mathematica [C] time = 0.593117, size = 311, normalized size = 1.

$$\frac{28x(995x^4 + 2633x^2 + 2836) + i\sqrt{6+2i\sqrt{7}}(5x^2+7)\sqrt{1-\frac{2ix^2}{\sqrt{7}-3i}}\sqrt{1+\frac{2ix^2}{\sqrt{7}+3i}}\left(7(101+199i\sqrt{7})\operatorname{EllipticF}\left(i\sinh^{-1}\left(\sqrt{\frac{4+3x^2+x^4}{7+5x^2}}\right)\right)\right)}{758912(5x^2+7)}$$

Antiderivative was successfully verified.

[In] Integrate[1/((7 + 5*x^2)^2*(4 + 3*x^2 + x^4)^(3/2)), x]

```
[Out] (28*x*(2836 + 2633*x^2 + 995*x^4) + I*Sqrt[6 + (2*I)*Sqrt[7]]*(7 + 5*x^2)*Sqrt[1 - ((2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[1 + ((2*I)*x^2)/(3*I + Sqrt[7])])*(1393*(3 - I*Sqrt[7])*EllipticE[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])) + 7*(101 + (199*I)*Sqrt[7])*EllipticF[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])) - 1150*EllipticPi[(5*(3 + I*Sqrt[7]))/14, I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])))/(758912*(7 + 5*x^2)*Sqrt[4 + 3*x^2 + x^4])
```

Maple [C] time = 0.025, size = 433, normalized size = 1.4

$$-2 \frac{1}{\sqrt{x^4 + 3x^2 + 4}} \left(-\frac{37x^3}{27104} - \frac{3x}{3388} \right) + \frac{625x}{135520x^2 + 189728} \sqrt{x^4 + 3x^2 + 4} - \frac{349}{6776\sqrt{-6 + 2i\sqrt{7}}} \sqrt{1 + \frac{3x^2}{8} - \frac{i}{8}x^2\sqrt{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5*x^2+7)^2/(x^4+3*x^2+4)^(3/2), x)
```

```
[Out] -2*(-37/27104*x^3-3/3388*x)/(x^4+3*x^2+4)^(1/2)+625/27104*x*(x^4+3*x^2+4)^(1/2)/(5*x^2+7)-349/6776/(-6+2*I*7^(1/2))^(1/2)*(1+3/8*x^2-1/8*I*x^2*7^(1/2))^(1/2)*(1+3/8*x^2+1/8*I*x^2*7^(1/2))^(1/2)/(x^4+3*x^2+4)^(1/2)*EllipticF(1/4*x*(-6+2*I*7^(1/2))^(1/2), 1/4*(2+6*I*7^(1/2))^(1/2))+199/847/(-6+2*I*7^(1/2))^(1/2)*(1+3/8*x^2-1/8*I*x^2*7^(1/2))^(1/2)*(1+3/8*x^2+1/8*I*x^2*7^(1/2))^(1/2)/(x^4+3*x^2+4)^(1/2)/(I*7^(1/2)+3)*EllipticF(1/4*x*(-6+2*I*7^(1/2))^(1/2), 1/4*(2+6*I*7^(1/2))^(1/2))-199/847/(-6+2*I*7^(1/2))^(1/2)*(1+3/8*x^2-1/8*I*x^2*7^(1/2))^(1/2)*(1+3/8*x^2+1/8*I*x^2*7^(1/2))^(1/2)/(x^4+3*x^2+4)^(1/2)/(I*7^(1/2)+3)*EllipticE(1/4*x*(-6+2*I*7^(1/2))^(1/2), 1/4*(2+6*I*7^(1/2))^(1/2))+575/189728/(-3/8+1/8*I*7^(1/2))^(1/2)*(1+3/8*x^2-1/8*I*x^2*7^(1/2))^(1/2)*(1+3/8*x^2+1/8*I*x^2*7^(1/2))^(1/2)/(x^4+3*x^2+4)^(1/2)*EllipticPi((-3/8+1/8*I*7^(1/2))^(1/2)*x, -5/7/(-3/8+1/8*I*7^(1/2)), (-3/8-1/8*I*7^(1/2))^(1/2)/(-3/8+1/8*I*7^(1/2))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x^4 + 3x^2 + 4)^{\frac{3}{2}}(5x^2 + 7)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+7)^2/(x^4+3*x^2+4)^(3/2), x, algorithm="maxima")
```

```
[Out] integrate(1/((x^4 + 3*x^2 + 4)^(3/2)*(5*x^2 + 7)^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{x^4 + 3x^2 + 4}}{25x^{12} + 220x^{10} + 894x^8 + 2084x^6 + 2913x^4 + 2296x^2 + 784}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(5*x^2+7)^2/(x^4+3*x^2+4)^(3/2),x, algorithm="fricas")

[Out] integral(sqrt(x^4 + 3*x^2 + 4)/(25*x^12 + 220*x^10 + 894*x^8 + 2084*x^6 + 2913*x^4 + 2296*x^2 + 784), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{((x^2 - x + 2)(x^2 + x + 2))^{\frac{3}{2}} (5x^2 + 7)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(5*x**2+7)**2/(x**4+3*x**2+4)**(3/2),x)

[Out] Integral(1/(((x**2 - x + 2)*(x**2 + x + 2))**(3/2)*(5*x**2 + 7)**2), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x^4 + 3x^2 + 4)^{\frac{3}{2}} (5x^2 + 7)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(5*x^2+7)^2/(x^4+3*x^2+4)^(3/2),x, algorithm="giac")

[Out] integrate(1/((x^4 + 3*x^2 + 4)^(3/2)*(5*x^2 + 7)^2), x)

$$3.379 \quad \int \frac{1}{(7+5x^2)^3 (4+3x^2+x^4)^{3/2}} dx$$

Optimal. Leaf size=340

$$\frac{843(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{379456\sqrt{2}\sqrt{x^4+3x^2+4}} - \frac{18159\sqrt{x^4+3x^2+4x}}{33392128(x^2+2)} + \frac{51875\sqrt{x^4+3x^2+4x}}{33392128(5x^2+7)} + \frac{625\sqrt{x^4+3x^2+4x}}{54208(5x^2+7)}$$

[Out] (x*(548 + 139*x^2))/(596288*Sqrt[4 + 3*x^2 + x^4]) - (18159*x*Sqrt[4 + 3*x^2 + x^4])/(33392128*(2 + x^2)) + (625*x*Sqrt[4 + 3*x^2 + x^4])/(54208*(7 + 5*x^2)^2) + (51875*x*Sqrt[4 + 3*x^2 + x^4])/(33392128*(7 + 5*x^2)) - (529425*Sqrt[5/77]*ArcTan[(2*Sqrt[11/35]*x)/Sqrt[4 + 3*x^2 + x^4]])/133568512 + (18159*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE[2*ArcTan[x/Sqrt[2]], 1/8])/(16696064*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4]) + (843*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(379456*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4]) - (3000075*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticPi[-9/280, 2*ArcTan[x/Sqrt[2]], 1/8])/(934979584*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4])

Rubi [A] time = 0.871899, antiderivative size = 340, normalized size of antiderivative = 1., number of steps used = 22, number of rules used = 11, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.458$, Rules used = {1228, 1178, 1197, 1103, 1195, 1223, 1696, 1714, 1708, 1706, 1216}

$$-\frac{18159\sqrt{x^4+3x^2+4x}}{33392128(x^2+2)} + \frac{51875\sqrt{x^4+3x^2+4x}}{33392128(5x^2+7)} + \frac{625\sqrt{x^4+3x^2+4x}}{54208(5x^2+7)^2} + \frac{(139x^2+548)x}{596288\sqrt{x^4+3x^2+4}} - \frac{529425\sqrt{\frac{5}{77}}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{133568512}$$

Antiderivative was successfully verified.

[In] Int[1/((7 + 5*x^2)^3*(4 + 3*x^2 + x^4)^(3/2)),x]

[Out] (x*(548 + 139*x^2))/(596288*Sqrt[4 + 3*x^2 + x^4]) - (18159*x*Sqrt[4 + 3*x^2 + x^4])/(33392128*(2 + x^2)) + (625*x*Sqrt[4 + 3*x^2 + x^4])/(54208*(7 + 5*x^2)^2) + (51875*x*Sqrt[4 + 3*x^2 + x^4])/(33392128*(7 + 5*x^2)) - (529425*Sqrt[5/77]*ArcTan[(2*Sqrt[11/35]*x)/Sqrt[4 + 3*x^2 + x^4]])/133568512 + (18159*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE[2*ArcTan[x/Sqrt[2]], 1/8])/(16696064*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4]) + (843*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(379456*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4]) - (3000075*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticPi[-9/280, 2*ArcTan[x/Sqrt[2]], 1/8])/(934979584*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4])

Rule 1228

Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{aa, bb, cc}, Int[ExpandIntegrand[1/Sqrt[aa + bb*x^2 + cc*x^4], (d + e*x^2)^q*(aa + bb*x^2 + cc*x^4)^(p + 1/2), x] /. {aa -> a, bb -> b, cc -> c}, x] /. FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, 0] && IntegerQ[p + 1/2]

Rule 1178

Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 +

```

c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2
- 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]

```

Rule 1197

```

Int[((d_) + (e_)*(x_)^2)/Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4], x_Symbol]
:> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]

```

Rule 1103

```

Int[1/Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]

```

Rule 1195

```

Int[((d_) + (e_)*(x_)^2)/Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4], x_Symbol]
:> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 +
c*x^4]), x] /; EqQ[e + d*q^2, 0] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]

```

Rule 1223

```

Int[((d_) + (e_)*(x_)^2)^(q_)/Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4], x_
Symbol] :> -Simp[(e^2*x*(d + e*x^2)^(q + 1)*Sqrt[a + b*x^2 + c*x^4])/(2*d*(
q + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(2*d*(q + 1)*(c*d^2 - b*d*e +
a*e^2)), Int[((d + e*x^2)^(q + 1)*Simp[a*e^2*(2*q + 3) + 2*d*(c*d - b*e)*(q
+ 1) - 2*e*(c*d*(q + 1) - b*e*(q + 2))*x^2 + c*e^2*(2*q + 5)*x^4, x])/Sqrt
[a + b*x^2 + c*x^4], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c
, 0] && ILtQ[q, -1]

```

Rule 1696

```

Int[((P4x_)*((d_) + (e_)*(x_)^2)^(q_))/Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x
_)^4], x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = C
oeff[P4x, x, 4]}, -Simp[((C*d^2 - B*d*e + A*e^2)*x*(d + e*x^2)^(q + 1)*Sqrt
[a + b*x^2 + c*x^4])/(2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(2*
d*(q + 1)*(c*d^2 - b*d*e + a*e^2)), Int[((d + e*x^2)^(q + 1)*Simp[a*d*(C*d
- B*e) + A*(a*e^2*(2*q + 3) + 2*d*(c*d - b*e)*(q + 1)) - 2*((B*d - A*e)*(b*
e*(q + 2) - c*d*(q + 1) - C*d*(b*d + a*e*(q + 1)))*x^2 + c*(C*d^2 - B*d*e
+ A*e^2)*(2*q + 5)*x^4, x])/Sqrt[a + b*x^2 + c*x^4], x], x]] /; FreeQ[{a, b
, c, d, e}, x] && PolyQ[P4x, x^2] && LeQ[Expon[P4x, x], 4] && NeQ[b^2 - 4*a
*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, -1]

```

Rule 1714

```

Int[(P4x_)/(((d_) + (e_)*(x_)^2)*Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4])
, x_Symbol] :> With[{q = Rt[c/a, 2], A = Coeff[P4x, x, 0], B = Coeff[P4x, x
, 2], C = Coeff[P4x, x, 4]}, -Dist[C/(e*q), Int[(1 - q*x^2)/Sqrt[a + b*x^2
+ c*x^4], x], x] + Dist[1/(c*e), Int[(A*c*e + a*C*d*q + (B*c*e - C*(c*d - a

```

```
*e*q))*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
c, d, e}, x] && PolyQ[P4x, x^2, 2] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*
d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && !GtQ[b^2 - 4*a*c,
0]
```

Rule 1708

```
Int[((A_) + (B_)*(x_)^2)/(((d_) + (e_)*(x_)^2)*Sqrt[(a_) + (b_)*(x_)^2 +
(c_)*(x_)^4]), x_Symbol] := With[{q = Rt[c/a, 2]}, Dist[(A*(c*d + a*e*q)
- a*B*(e + d*q))/(c*d^2 - a*e^2), Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] +
Dist[(a*(B*d - A*e)*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^2)
)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0
] && PosQ[c/a] && NeQ[c*A^2 - a*B^2, 0]
```

Rule 1706

```
Int[((A_) + (B_)*(x_)^2)/(((d_) + (e_)*(x_)^2)*Sqrt[(a_) + (b_)*(x_)^2 +
(c_)*(x_)^4]), x_Symbol] := With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*Arc
Tan[(Rt[-b + (c*d)/e + (a*e)/d, 2]*x)/Sqrt[a + b*x^2 + c*x^4]])/(2*d*e*Rt[-
b + (c*d)/e + (a*e)/d, 2]), x] + Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a
+ b*x^2 + c*x^4))/(a*(A + B*x^2)^2)]*EllipticPi[Cancel[-((B*d - A*e)^2/(4*
d*e*A*B))], 2*ArcTan[q*x], 1/2 - (b*A)/(4*a*B)]/(4*d*e*A*q*Sqrt[a + b*x^2
+ c*x^4]), x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rule 1216

```
Int[1/(((d_) + (e_)*(x_)^2)*Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4]), x_S
ymbol] := With[{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/
Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int
[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(7+5x^2)^3(4+3x^2+x^4)^{3/2}} dx &= \int \left(\frac{388+215x^2}{85184(4+3x^2+x^4)^{3/2}} + \frac{25}{44(7+5x^2)^3\sqrt{4+3x^2+x^4}} - \frac{25}{1936(7+5x^2)^2\sqrt{4+3x^2+x^4}} \right) dx \\
&= \frac{\int \frac{388+215x^2}{(4+3x^2+x^4)^{3/2}} dx}{85184} - \frac{1075 \int \frac{1}{(7+5x^2)\sqrt{4+3x^2+x^4}} dx}{85184} - \frac{25 \int \frac{1}{(7+5x^2)^2\sqrt{4+3x^2+x^4}} dx}{1936} + \frac{25}{44} \int \frac{1}{\sqrt{4+3x^2+x^4}} dx \\
&= \frac{x(548+139x^2)}{596288\sqrt{4+3x^2+x^4}} + \frac{625x\sqrt{4+3x^2+x^4}}{54208(7+5x^2)^2} - \frac{625x\sqrt{4+3x^2+x^4}}{1192576(7+5x^2)} + \frac{\int \frac{524-556x^2}{\sqrt{4+3x^2+x^4}} dx}{2385152} \\
&= \frac{x(548+139x^2)}{596288\sqrt{4+3x^2+x^4}} + \frac{625x\sqrt{4+3x^2+x^4}}{54208(7+5x^2)^2} + \frac{51875x\sqrt{4+3x^2+x^4}}{33392128(7+5x^2)} - \frac{1075\sqrt{5}}{77} \arctan\left(\frac{x\sqrt{5}}{4+3x^2+x^4}\right) \\
&= \frac{x(548+139x^2)}{596288\sqrt{4+3x^2+x^4}} - \frac{153x\sqrt{4+3x^2+x^4}}{1192576(2+x^2)} + \frac{625x\sqrt{4+3x^2+x^4}}{54208(7+5x^2)^2} + \frac{51875x\sqrt{4+3x^2+x^4}}{33392128(7+5x^2)} \\
&= \frac{x(548+139x^2)}{596288\sqrt{4+3x^2+x^4}} - \frac{18159x\sqrt{4+3x^2+x^4}}{33392128(2+x^2)} + \frac{625x\sqrt{4+3x^2+x^4}}{54208(7+5x^2)^2} + \frac{51875x\sqrt{4+3x^2+x^4}}{33392128(7+5x^2)} \\
&= \frac{x(548+139x^2)}{596288\sqrt{4+3x^2+x^4}} - \frac{18159x\sqrt{4+3x^2+x^4}}{33392128(2+x^2)} + \frac{625x\sqrt{4+3x^2+x^4}}{54208(7+5x^2)^2} + \frac{51875x\sqrt{4+3x^2+x^4}}{33392128(7+5x^2)}
\end{aligned}$$

Mathematica [C] time = 0.739348, size = 320, normalized size = 0.94

$$28x(453975x^6 + 2838330x^4 + 5811451x^2 + 4496212) + 3i\sqrt{6+2i\sqrt{7}}\sqrt{1-\frac{2ix^2}{\sqrt{7}-3i}}\sqrt{1+\frac{2ix^2}{\sqrt{7}+3i}}(5x^2+7)^2\left(7i(6053\sqrt{7}+\dots)\right)$$

Antiderivative was successfully verified.

[In] Integrate[1/((7+5*x^2)^3*(4+3*x^2+x^4)^(3/2)),x]

[Out] (28*x*(4496212+5811451*x^2+2838330*x^4+453975*x^6)+(3*I)*Sqrt[6+(2*I)*Sqrt[7]]*(7+5*x^2)^2*Sqrt[1-((2*I)*x^2)/(-3*I+Sqrt[7])]*Sqrt[1+((2*I)*x^2)/(3*I+Sqrt[7])]*(42371*(3-I*Sqrt[7])*EllipticE[I*ArcSinh[Sqrt[(-2*I)/(-3*I+Sqrt[7])]*x],(3*I-Sqrt[7])/(3*I+Sqrt[7])] + (7*I)*(23633*I+6053*Sqrt[7])*EllipticF[I*ArcSinh[Sqrt[(-2*I)/(-3*I+Sqrt[7])]*x],(3*I-Sqrt[7])/(3*I+Sqrt[7])] + 352950*EllipticPi[(5*(3+I*Sqrt[7]))/14,I*ArcSinh[Sqrt[(-2*I)/(-3*I+Sqrt[7])]*x],(3*I-Sqrt[7])/(3*I+Sqrt[7])]))/(934979584*(7+5*x^2)^2*Sqrt[4+3*x^2+x^4])

Maple [C] time = 0.025, size = 457, normalized size = 1.3

$$-2 \frac{1}{\sqrt{x^4+3x^2+4}} \left(-\frac{139x^3}{1192576} - \frac{137x}{298144} \right) + \frac{625x}{54208(5x^2+7)^2} \sqrt{x^4+3x^2+4} + \frac{51875x}{166960640x^2+233744896} \sqrt{x^4+3x^2+4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(5*x^2+7)^3/(x^4+3*x^2+4)^(3/2), x)`

[Out]
$$-2*(-139/1192576*x^3-137/298144*x)/(x^4+3*x^2+4)^{(1/2)}+625/54208*x*(x^4+3*x^2+4)^{(1/2)}/(5*x^2+7)^2+51875/33392128*x*(x^4+3*x^2+4)^{(1/2)}/(5*x^2+7)+1173/1192576/(-6+2*I*7^{(1/2)})^{(1/2)}*(1+3/8*x^2-1/8*I*x^2*7^{(1/2)})^{(1/2)}*(1+3/8*x^2+1/8*I*x^2*7^{(1/2)})^{(1/2)}/(x^4+3*x^2+4)^{(1/2)}*EllipticF(1/4*x*(-6+2*I*7^{(1/2)})^{(1/2)}, 1/4*(2+6*I*7^{(1/2)})^{(1/2)})+18159/1043504/(-6+2*I*7^{(1/2)})^{(1/2)}*(1+3/8*x^2-1/8*I*x^2*7^{(1/2)})^{(1/2)}*(1+3/8*x^2+1/8*I*x^2*7^{(1/2)})^{(1/2)}/(x^4+3*x^2+4)^{(1/2)}/(I*7^{(1/2)}+3)*EllipticF(1/4*x*(-6+2*I*7^{(1/2)})^{(1/2)}, 1/4*(2+6*I*7^{(1/2)})^{(1/2)})-18159/1043504/(-6+2*I*7^{(1/2)})^{(1/2)}*(1+3/8*x^2-1/8*I*x^2*7^{(1/2)})^{(1/2)}*(1+3/8*x^2+1/8*I*x^2*7^{(1/2)})^{(1/2)}/(x^4+3*x^2+4)^{(1/2)}/(I*7^{(1/2)}+3)*EllipticE(1/4*x*(-6+2*I*7^{(1/2)})^{(1/2)}, 1/4*(2+6*I*7^{(1/2)})^{(1/2)})-529425/233744896/(-3/8+1/8*I*7^{(1/2)})^{(1/2)}*(1+3/8*x^2-1/8*I*x^2*7^{(1/2)})^{(1/2)}*(1+3/8*x^2+1/8*I*x^2*7^{(1/2)})^{(1/2)}/(x^4+3*x^2+4)^{(1/2)}*EllipticPi((-3/8+1/8*I*7^{(1/2)})^{(1/2)}*x, -5/7/(-3/8+1/8*I*7^{(1/2)}), (-3/8-1/8*I*7^{(1/2)})^{(1/2)}/(-3/8+1/8*I*7^{(1/2)})^{(1/2)})$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x^4 + 3x^2 + 4)^{\frac{3}{2}}(5x^2 + 7)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(5*x^2+7)^3/(x^4+3*x^2+4)^(3/2), x, algorithm="maxima")`

[Out] `integrate(1/((x^4 + 3*x^2 + 4)^(3/2)*(5*x^2 + 7)^3), x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{x^4 + 3x^2 + 4}}{125x^{14} + 1275x^{12} + 6010x^{10} + 16678x^8 + 29153x^6 + 31871x^4 + 19992x^2 + 5488}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(5*x^2+7)^3/(x^4+3*x^2+4)^(3/2), x, algorithm="fricas")`

[Out] `integral(sqrt(x^4 + 3*x^2 + 4)/(125*x^14 + 1275*x^12 + 6010*x^10 + 16678*x^8 + 29153*x^6 + 31871*x^4 + 19992*x^2 + 5488), x)`

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{((x^2 - x + 2)(x^2 + x + 2))^{\frac{3}{2}}(5x^2 + 7)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(5*x**2+7)**3/(x**4+3*x**2+4)**(3/2), x)`

[Out] Integral(1/(((x**2 - x + 2)*(x**2 + x + 2))**(3/2)*(5*x**2 + 7)**3), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x^4 + 3x^2 + 4)^{\frac{3}{2}}(5x^2 + 7)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(5*x^2+7)^3/(x^4+3*x^2+4)^(3/2),x, algorithm="giac")

[Out] integrate(1/((x^4 + 3*x^2 + 4)^(3/2)*(5*x^2 + 7)^3), x)

3.380 $\int \frac{(d+ex^2)^3}{\sqrt{a+bx^2+cx^4}} dx$

Optimal. Leaf size=467

$$\frac{\sqrt[4]{a}(\sqrt{a} + \sqrt{cx^2}) \sqrt{\frac{a+bx^2+cx^4}{(\sqrt{a}+\sqrt{cx^2})^2}} \left(e(-3ce(3ae + 10bd) + 8b^2e^2 + 45c^2d^2) + \frac{\sqrt{c}(4abe^3-15acde^2+15c^2d^3)}{\sqrt{a}} \right) \text{EllipticF}\left(2 \tan^{-1}\left(\frac{\sqrt{a}}{\sqrt{a+bx^2+cx^4}}\right)\right)}{30c^{11/4}\sqrt{a+bx^2+cx^4}}$$

```
[Out] (e^2*(15*c*d - 4*b*e)*x*Sqrt[a + b*x^2 + c*x^4])/(15*c^2) + (e^3*x^3*Sqrt[a + b*x^2 + c*x^4])/(5*c) + (e*(45*c^2*d^2 + 8*b^2*e^2 - 3*c*e*(10*b*d + 3*a*e))*x*Sqrt[a + b*x^2 + c*x^4])/(15*c^(5/2)*(Sqrt[a] + Sqrt[c]*x^2)) - (a^(1/4)*e*(45*c^2*d^2 + 8*b^2*e^2 - 3*c*e*(10*b*d + 3*a*e))*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(15*c^(11/4)*Sqrt[a + b*x^2 + c*x^4]) + (a^(1/4)*((Sqrt[c]*(15*c^2*d^3 - 15*a*c*d*e^2 + 4*a*b*e^3))/Sqrt[a] + e*(45*c^2*d^2 + 8*b^2*e^2 - 3*c*e*(10*b*d + 3*a*e)))*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(30*c^(11/4)*Sqrt[a + b*x^2 + c*x^4])
```

Rubi [A] time = 0.424683, antiderivative size = 467, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {1206, 1679, 1197, 1103, 1195}

$$\frac{ex\sqrt{a+bx^2+cx^4}(-3ce(3ae+10bd)+8b^2e^2+45c^2d^2)}{15c^{5/2}(\sqrt{a}+\sqrt{cx^2})} + \frac{\sqrt[4]{a}(\sqrt{a}+\sqrt{cx^2})\sqrt{\frac{a+bx^2+cx^4}{(\sqrt{a}+\sqrt{cx^2})^2}}\left(e(-3ce(3ae+10bd)+8b^2e^2)+\frac{\sqrt{c}(4abe^3-15acde^2+15c^2d^3)}{\sqrt{a}}\right)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt{a}}{\sqrt{a+bx^2+cx^4}}\right)\right)}{30c^{11/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^3/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (e^2*(15*c*d - 4*b*e)*x*Sqrt[a + b*x^2 + c*x^4])/(15*c^2) + (e^3*x^3*Sqrt[a + b*x^2 + c*x^4])/(5*c) + (e*(45*c^2*d^2 + 8*b^2*e^2 - 3*c*e*(10*b*d + 3*a*e))*x*Sqrt[a + b*x^2 + c*x^4])/(15*c^(5/2)*(Sqrt[a] + Sqrt[c]*x^2)) - (a^(1/4)*e*(45*c^2*d^2 + 8*b^2*e^2 - 3*c*e*(10*b*d + 3*a*e))*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(15*c^(11/4)*Sqrt[a + b*x^2 + c*x^4]) + (a^(1/4)*((Sqrt[c]*(15*c^2*d^3 - 15*a*c*d*e^2 + 4*a*b*e^3))/Sqrt[a] + e*(45*c^2*d^2 + 8*b^2*e^2 - 3*c*e*(10*b*d + 3*a*e)))*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(30*c^(11/4)*Sqrt[a + b*x^2 + c*x^4])
```

Rule 1206

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(e^q*x^(2*q - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(4*p + 2*q + 1)), x] + Dist[1/(c*(4*p + 2*q + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*q + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1]
```

Rule 1679

```
Int[(Pq_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] := With[{q =
  Expon[Pq, x^2], e = Coeff[Pq, x^2, Expon[Pq, x^2]]}, Simp[(e*x^(2*q - 3)*(
  a + b*x^2 + c*x^4)^(p + 1))/(c*(2*q + 4*p + 1)), x] + Dist[1/(c*(2*q + 4*p
  + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*q + 4*p + 1)*Pq - a*e*(2*
  q - 3)*x^(2*q - 4) - b*e*(2*q + 2*p - 1)*x^(2*q - 2) - c*e*(2*q + 4*p + 1)*
  x^(2*q), x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && Expon[P
  q, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && !LtQ[p, -1]
```

Rule 1197

```
Int[((d_) + (e_)*(x_)^2)/Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4], x_Symbol] := With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; NeQ[e + d*q, 0] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4], x_Symbol] := With[{q = Rt[c/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_)*(x_)^2)/Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4], x_Symbol] := With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c*x^4]), x] /; EqQ[e + d*q^2, 0] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\begin{aligned} \int \frac{(d + ex^2)^3}{\sqrt{a + bx^2 + cx^4}} dx &= \frac{e^3 x^3 \sqrt{a + bx^2 + cx^4}}{5c} + \int \frac{5cd^3 + 3e(5cd^2 - ae^2)x^2 + e^2(15cd - 4be)x^4}{\sqrt{a + bx^2 + cx^4}} dx \\ &= \frac{e^2(15cd - 4be)x\sqrt{a + bx^2 + cx^4}}{15c^2} + \frac{e^3 x^3 \sqrt{a + bx^2 + cx^4}}{5c} + \int \frac{15c^2 d^3 - 15acde^2 + 4abe^3 + e(45c^2 d^2 + 8b^2 e^2 - 3ce(10bd + 3ae))}{\sqrt{a + bx^2 + cx^4}} dx \\ &= \frac{e^2(15cd - 4be)x\sqrt{a + bx^2 + cx^4}}{15c^2} + \frac{e^3 x^3 \sqrt{a + bx^2 + cx^4}}{5c} - \frac{(\sqrt{ae}(45c^2 d^2 + 8b^2 e^2 - 3ce(10bd + 3ae)))}{15c^{5/2}} \\ &= \frac{e^2(15cd - 4be)x\sqrt{a + bx^2 + cx^4}}{15c^2} + \frac{e^3 x^3 \sqrt{a + bx^2 + cx^4}}{5c} + \frac{e(45c^2 d^2 + 8b^2 e^2 - 3ce(10bd + 3ae))}{15c^{5/2}(\sqrt{a} + \sqrt{cx^2})} \end{aligned}$$

Mathematica [C] time = 2.90362, size = 584, normalized size = 1.25

$$-i\sqrt{\frac{\sqrt{b^2 - 4ac + b + 2cx^2}}{\sqrt{b^2 - 4ac + b}}}\sqrt{\frac{-2\sqrt{b^2 - 4ac + 2b + 4cx^2}}{b - \sqrt{b^2 - 4ac}}}\left(15c^2 de\left(3d\sqrt{b^2 - 4ac} - 2ae - 3bd\right) + ce^2\left(-30bd\sqrt{b^2 - 4ac} - 9ae\sqrt{b^2 - 4ac} + 17ab\right)\right)$$

Antiderivative was successfully verified.

[In] Integrate[(d + e*x^2)^3/Sqrt[a + b*x^2 + c*x^4],x]

[Out] $(4*c*\sqrt{c/(b + \sqrt{b^2 - 4*a*c})})*e^2*x*(a + b*x^2 + c*x^4)*(-4*b*e + 3*c*(5*d + e*x^2)) + I*(-b + \sqrt{b^2 - 4*a*c})*e*(45*c^2*d^2 + 8*b^2*e^2 - 3*c*e*(10*b*d + 3*a*e))*\sqrt{(b + \sqrt{b^2 - 4*a*c} + 2*c*x^2)/(b + \sqrt{b^2 - 4*a*c})}*\sqrt{(2*b - 2*\sqrt{b^2 - 4*a*c} + 4*c*x^2)/(b - \sqrt{b^2 - 4*a*c})}*\text{EllipticE}[I*\text{ArcSinh}[\sqrt{2}*\sqrt{c/(b + \sqrt{b^2 - 4*a*c})}]*x], (b + \sqrt{b^2 - 4*a*c})/(b - \sqrt{b^2 - 4*a*c})] - I*(30*c^3*d^3 + 8*b^2*(-b + \sqrt{b^2 - 4*a*c})*e^3 + 15*c^2*d*e*(-3*b*d + 3*\sqrt{b^2 - 4*a*c}*d - 2*a*e) + c*e^2*(30*b^2*d - 30*b*\sqrt{b^2 - 4*a*c}*d + 17*a*b*e - 9*a*\sqrt{b^2 - 4*a*c}*e))*\sqrt{(b + \sqrt{b^2 - 4*a*c} + 2*c*x^2)/(b + \sqrt{b^2 - 4*a*c})}*\sqrt{(2*b - 2*\sqrt{b^2 - 4*a*c} + 4*c*x^2)/(b - \sqrt{b^2 - 4*a*c})}*\text{EllipticF}[I*\text{ArcSinh}[\sqrt{2}*\sqrt{c/(b + \sqrt{b^2 - 4*a*c})}]*x], (b + \sqrt{b^2 - 4*a*c})/(b - \sqrt{b^2 - 4*a*c})]/(60*c^3*\sqrt{c/(b + \sqrt{b^2 - 4*a*c})})*\sqrt{a + b*x^2 + c*x^4})$

Maple [B] time = 0.066, size = 1186, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x^2+d)^3/(c*x^4+b*x^2+a)^(1/2),x)

[Out] $e^3*(1/5/c*x^3*(c*x^4+b*x^2+a)^{(1/2)}-4/15*b/c^2*x*(c*x^4+b*x^2+a)^{(1/2)}+1/15*b/c^2*a^2^{(1/2)}/(((-4*a*c+b^2)^{(1/2)}-b)/a)^{(1/2)}*(4-2*((-4*a*c+b^2)^{(1/2)}-b)/a*x^2)^{(1/2)}*(4+2*(b+(-4*a*c+b^2)^{(1/2)})/a*x^2)^{(1/2)}/(c*x^4+b*x^2+a)^{(1/2)}*\text{EllipticF}(1/2*x^2^{(1/2)}*((-4*a*c+b^2)^{(1/2)}-b)/a)^{(1/2)}, 1/2*(-4+2*b*(b+(-4*a*c+b^2)^{(1/2)})/a/c)^{(1/2)})-1/2*(-3/5/c*a+8/15*b^2/c^2)*a^2^{(1/2)}/(((-4*a*c+b^2)^{(1/2)}-b)/a)^{(1/2)}*(4-2*((-4*a*c+b^2)^{(1/2)}-b)/a*x^2)^{(1/2)}*(4+2*(b+(-4*a*c+b^2)^{(1/2)})/a*x^2)^{(1/2)}/(c*x^4+b*x^2+a)^{(1/2)}/(b+(-4*a*c+b^2)^{(1/2)})*(\text{EllipticF}(1/2*x^2^{(1/2)}*((-4*a*c+b^2)^{(1/2)}-b)/a)^{(1/2)}, 1/2*(-4+2*b*(b+(-4*a*c+b^2)^{(1/2)})/a/c)^{(1/2)})-\text{EllipticE}(1/2*x^2^{(1/2)}*((-4*a*c+b^2)^{(1/2)}-b)/a)^{(1/2)}, 1/2*(-4+2*b*(b+(-4*a*c+b^2)^{(1/2)})/a/c)^{(1/2)})))+3*d*e^2*(1/3/c*x*(c*x^4+b*x^2+a)^{(1/2)}-1/12/c*a^2^{(1/2)}/(((-4*a*c+b^2)^{(1/2)}-b)/a)^{(1/2)}*(4-2*((-4*a*c+b^2)^{(1/2)}-b)/a*x^2)^{(1/2)}*(4+2*(b+(-4*a*c+b^2)^{(1/2)})/a*x^2)^{(1/2)}/(c*x^4+b*x^2+a)^{(1/2)}*\text{EllipticF}(1/2*x^2^{(1/2)}*((-4*a*c+b^2)^{(1/2)}-b)/a)^{(1/2)}, 1/2*(-4+2*b*(b+(-4*a*c+b^2)^{(1/2)})/a/c)^{(1/2)})+1/3*b/c*a^2^{(1/2)}/(((-4*a*c+b^2)^{(1/2)}-b)/a)^{(1/2)}*(4-2*((-4*a*c+b^2)^{(1/2)}-b)/a*x^2)^{(1/2)}*(4+2*(b+(-4*a*c+b^2)^{(1/2)})/a*x^2)^{(1/2)}/(c*x^4+b*x^2+a)^{(1/2)}/(b+(-4*a*c+b^2)^{(1/2)})*(\text{EllipticF}(1/2*x^2^{(1/2)}*((-4*a*c+b^2)^{(1/2)}-b)/a)^{(1/2)}, 1/2*(-4+2*b*(b+(-4*a*c+b^2)^{(1/2)})/a/c)^{(1/2)})-\text{EllipticE}(1/2*x^2^{(1/2)}*((-4*a*c+b^2)^{(1/2)}-b)/a)^{(1/2)}, 1/2*(-4+2*b*(b+(-4*a*c+b^2)^{(1/2)})/a/c)^{(1/2)})))-3/2*d^2*e*a^2^{(1/2)}/(((-4*a*c+b^2)^{(1/2)}-b)/a)^{(1/2)}*(4-2*((-4*a*c+b^2)^{(1/2)}-b)/a*x^2)^{(1/2)}*(4+2*(b+(-4*a*c+b^2)^{(1/2)})/a*x^2)^{(1/2)}/(c*x^4+b*x^2+a)^{(1/2)}/(b+(-4*a*c+b^2)^{(1/2)})*(\text{EllipticF}(1/2*x^2^{(1/2)}*((-4*a*c+b^2)^{(1/2)}-b)/a)^{(1/2)}, 1/2*(-4+2*b*(b+(-4*a*c+b^2)^{(1/2)})/a/c)^{(1/2)})-\text{EllipticE}(1/2*x^2^{(1/2)}*((-4*a*c+b^2)^{(1/2)}-b)/a)^{(1/2)}, 1/2*(-4+2*b*(b+(-4*a*c+b^2)^{(1/2)})/a/c)^{(1/2)})))+1/4*d^3*2^{(1/2)}/(((-4*a*c+b^2)^{(1/2)}-b)/a)^{(1/2)}*(4-2*((-4*a*c+b^2)^{(1/2)}-b)/a*x^2)^{(1/2)}*(4+2*(b+(-4*a*c+b^2)^{(1/2)})/a*x^2)^{(1/2)}/(c*x^4+b*x^2+a)^{(1/2)}*\text{EllipticF}(1/2*x^2^{(1/2)}*((-4*a*c+b^2)^{(1/2)}-b)/a)^{(1/2)}, 1/2*(-4+2*b*(b+(-4*a*c+b^2)^{(1/2)})/a/c)^{(1/2)})$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^2 + d)^3}{\sqrt{cx^4 + bx^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)^3/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")

[Out] integrate((e*x^2 + d)^3/sqrt(c*x^4 + b*x^2 + a), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{e^3x^6 + 3de^2x^4 + 3d^2ex^2 + d^3}{\sqrt{cx^4 + bx^2 + a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)^3/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")

[Out] integral((e^3*x^6 + 3*d*e^2*x^4 + 3*d^2*e*x^2 + d^3)/sqrt(c*x^4 + b*x^2 + a), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(d + ex^2)^3}{\sqrt{a + bx^2 + cx^4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x**2+d)**3/(c*x**4+b*x**2+a)**(1/2),x)

[Out] Integral((d + e*x**2)**3/sqrt(a + b*x**2 + c*x**4), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^2 + d)^3}{\sqrt{cx^4 + bx^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)^3/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")

[Out] integrate((e*x^2 + d)^3/sqrt(c*x^4 + b*x^2 + a), x)

$$3.381 \quad \int \frac{(d+ex^2)^2}{\sqrt{a+bx^2+cx^4}} dx$$

Optimal. Leaf size=356

$$\frac{\sqrt[4]{a}(\sqrt{a} + \sqrt{cx^2}) \sqrt{\frac{a+bx^2+cx^4}{(\sqrt{a}+\sqrt{cx^2})^2}} \left(\frac{\sqrt{c}(3cd^2-ae^2)}{\sqrt{a}} + 2e(3cd-be) \right) \text{EllipticF} \left(2 \tan^{-1} \left(\frac{\sqrt[4]{cx}}{\sqrt[4]{a}} \right), \frac{1}{4} \left(2 - \frac{b}{\sqrt{a}\sqrt{c}} \right) \right)}{6c^{7/4}\sqrt{a+bx^2+cx^4}} + \frac{2ex\sqrt{a+bx^2+cx^4}}{3c^{3/2}(\sqrt{a} + \sqrt{cx^2})}$$

[Out] (e^2*x*Sqrt[a + b*x^2 + c*x^4])/(3*c) + (2*e*(3*c*d - b*e)*x*Sqrt[a + b*x^2 + c*x^4])/(3*c^(3/2)*(Sqrt[a] + Sqrt[c]*x^2)) - (2*a^(1/4)*e*(3*c*d - b*e)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(3*c^(7/4)*Sqrt[a + b*x^2 + c*x^4]) + (a^(1/4)*(2*e*(3*c*d - b*e) + (Sqrt[c]*(3*c*d^2 - a*e^2))/Sqrt[a])*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(6*c^(7/4)*Sqrt[a + b*x^2 + c*x^4])

Rubi [A] time = 0.191376, antiderivative size = 356, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1206, 1197, 1103, 1195}

$$\frac{\sqrt[4]{a}(\sqrt{a} + \sqrt{cx^2}) \sqrt{\frac{a+bx^2+cx^4}{(\sqrt{a}+\sqrt{cx^2})^2}} \left(\frac{\sqrt{c}(3cd^2-ae^2)}{\sqrt{a}} + 2e(3cd-be) \right) F \left(2 \tan^{-1} \left(\frac{\sqrt[4]{cx}}{\sqrt[4]{a}} \right) \middle| \frac{1}{4} \left(2 - \frac{b}{\sqrt{a}\sqrt{c}} \right) \right)}{6c^{7/4}\sqrt{a+bx^2+cx^4}} + \frac{2ex\sqrt{a+bx^2+cx^4}(3cd-be)}{3c^{3/2}(\sqrt{a} + \sqrt{cx^2})}$$

Antiderivative was successfully verified.

[In] Int[(d + e*x^2)^2/Sqrt[a + b*x^2 + c*x^4], x]

[Out] (e^2*x*Sqrt[a + b*x^2 + c*x^4])/(3*c) + (2*e*(3*c*d - b*e)*x*Sqrt[a + b*x^2 + c*x^4])/(3*c^(3/2)*(Sqrt[a] + Sqrt[c]*x^2)) - (2*a^(1/4)*e*(3*c*d - b*e)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(3*c^(7/4)*Sqrt[a + b*x^2 + c*x^4]) + (a^(1/4)*(2*e*(3*c*d - b*e) + (Sqrt[c]*(3*c*d^2 - a*e^2))/Sqrt[a])*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(6*c^(7/4)*Sqrt[a + b*x^2 + c*x^4])

Rule 1206

Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(e^q*x^(2*q - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(4*p + 2*q + 1)), x] + Dist[1/(c*(4*p + 2*q + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*p + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1]

Rule 1197

Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; NeQ[e + d*q, 0] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[

c/a]

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] := With[{q = Rt[c/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]), x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] := With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c*x^4]), x] /; EqQ[e + d*q^2, 0] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{(d + ex^2)^2}{\sqrt{a + bx^2 + cx^4}} dx = \frac{e^2x\sqrt{a + bx^2 + cx^4}}{3c} + \frac{\int \frac{3cd^2 - ae^2 + 2e(3cd - be)x^2}{\sqrt{a + bx^2 + cx^4}} dx}{3c}$$

$$= \frac{e^2x\sqrt{a + bx^2 + cx^4}}{3c} - \frac{(2\sqrt{ae}(3cd - be)) \int \frac{1 - \frac{\sqrt{cx^2}}{\sqrt{a}}}{\sqrt{a + bx^2 + cx^4}} dx}{3c^{3/2}} + \frac{(3cd^2 - ae^2 + \frac{2\sqrt{ae}(3cd - be)}{\sqrt{c}}) \int \frac{1}{\sqrt{a + bx^2 + cx^4}} dx}{3c}$$

$$= \frac{e^2x\sqrt{a + bx^2 + cx^4}}{3c} + \frac{2e(3cd - be)x\sqrt{a + bx^2 + cx^4}}{3c^{3/2}(\sqrt{a} + \sqrt{cx^2})} - \frac{2^4\sqrt{ae}(3cd - be)(\sqrt{a} + \sqrt{cx^2}) \sqrt{\frac{a + bx^2 + cx^4}{(\sqrt{a} + \sqrt{cx^2})^2}}}{3c^{7/4}\sqrt{a + bx^2 + cx^4}}$$

Mathematica [C] time = 1.63391, size = 488, normalized size = 1.37

$$i \sqrt{\frac{\sqrt{b^2 - 4ac + b + 2cx^2}}{\sqrt{b^2 - 4ac + b}}} \sqrt{\frac{-2\sqrt{b^2 - 4ac + 2b + 4cx^2}}{b - \sqrt{b^2 - 4ac}}} \left(ce \left(-3d\sqrt{b^2 - 4ac} + ae + 3bd \right) + be^2 \left(\sqrt{b^2 - 4ac} - b \right) - 3c^2d^2 \right) \text{EllipticF} \left(i \sinh^{-1} \left(\dots \right) \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^2/Sqrt[a + b*x^2 + c*x^4], x]
```

```
[Out] (2*c*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*e^2*x*(a + b*x^2 + c*x^4) - I*(-b + Sqrt[b^2 - 4*a*c])*e*(-3*c*d + b*e)*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]]*x, (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] + I*(-3*c^2*d^2 + b*(-b + Sqrt[b^2 - 4*a*c])*e^2 + c*e*(3*b*d - 3*Sqrt[b^2 - 4*a*c]*d + a*e))*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]]*x, (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])])]/(6*c^2*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[a + b*x^2 + c*x^4])
```


Maple [B] time = 0.009, size = 756, normalized size = 2.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((e*x^2+d)^2/(c*x^4+b*x^2+a)^{(1/2)}, x)$

[Out]
$$e^2*(1/3/c*x*(c*x^4+b*x^2+a)^{(1/2)}-1/12/c*a*2^{(1/2)}/(((-4*a*c+b^2)^{(1/2)}-b)/a)^{(1/2)}*(4-2*((-4*a*c+b^2)^{(1/2)}-b)/a*x^2)^{(1/2)}*(4+2*(b+(-4*a*c+b^2)^{(1/2)})/a*x^2)^{(1/2)}/(c*x^4+b*x^2+a)^{(1/2)}*EllipticF(1/2*x*2^{(1/2)}*((-4*a*c+b^2)^{(1/2)}-b)/a)^{(1/2)}, 1/2*(-4+2*b*(b+(-4*a*c+b^2)^{(1/2)})/a/c)^{(1/2)})+1/3*b/c*a*2^{(1/2)}/(((-4*a*c+b^2)^{(1/2)}-b)/a)^{(1/2)}*(4-2*((-4*a*c+b^2)^{(1/2)}-b)/a*x^2)^{(1/2)}*(4+2*(b+(-4*a*c+b^2)^{(1/2)})/a*x^2)^{(1/2)}/(c*x^4+b*x^2+a)^{(1/2)}/(b+(-4*a*c+b^2)^{(1/2)})*(EllipticF(1/2*x*2^{(1/2)}*((-4*a*c+b^2)^{(1/2)}-b)/a)^{(1/2)}, 1/2*(-4+2*b*(b+(-4*a*c+b^2)^{(1/2)})/a/c)^{(1/2)})-EllipticE(1/2*x*2^{(1/2)}*((-4*a*c+b^2)^{(1/2)}-b)/a)^{(1/2)}, 1/2*(-4+2*b*(b+(-4*a*c+b^2)^{(1/2)})/a/c)^{(1/2)})))-d*e*a*2^{(1/2)}/(((-4*a*c+b^2)^{(1/2)}-b)/a)^{(1/2)}*(4-2*((-4*a*c+b^2)^{(1/2)}-b)/a*x^2)^{(1/2)}*(4+2*(b+(-4*a*c+b^2)^{(1/2)})/a*x^2)^{(1/2)}/(c*x^4+b*x^2+a)^{(1/2)}/(b+(-4*a*c+b^2)^{(1/2)})*(EllipticF(1/2*x*2^{(1/2)}*((-4*a*c+b^2)^{(1/2)}-b)/a)^{(1/2)}, 1/2*(-4+2*b*(b+(-4*a*c+b^2)^{(1/2)})/a/c)^{(1/2)})-EllipticE(1/2*x*2^{(1/2)}*((-4*a*c+b^2)^{(1/2)}-b)/a)^{(1/2)}, 1/2*(-4+2*b*(b+(-4*a*c+b^2)^{(1/2)})/a/c)^{(1/2)})))+1/4*d^2*2^{(1/2)}/(((-4*a*c+b^2)^{(1/2)}-b)/a)^{(1/2)}*(4-2*((-4*a*c+b^2)^{(1/2)}-b)/a*x^2)^{(1/2)}*(4+2*(b+(-4*a*c+b^2)^{(1/2)})/a*x^2)^{(1/2)}/(c*x^4+b*x^2+a)^{(1/2)}*EllipticF(1/2*x*2^{(1/2)}*((-4*a*c+b^2)^{(1/2)}-b)/a)^{(1/2)}, 1/2*(-4+2*b*(b+(-4*a*c+b^2)^{(1/2)})/a/c)^{(1/2)})$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^2 + d)^2}{\sqrt{cx^4 + bx^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}((e*x^2+d)^2/(c*x^4+b*x^2+a)^{(1/2)}, x, \text{algorithm}="maxima")$

[Out] $\text{integrate}((e*x^2 + d)^2/\text{sqrt}(c*x^4 + b*x^2 + a), x)$

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{e^2x^4 + 2dex^2 + d^2}{\sqrt{cx^4 + bx^2 + a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}((e*x^2+d)^2/(c*x^4+b*x^2+a)^{(1/2)}, x, \text{algorithm}="fricas")$

[Out] $\text{integral}((e^2*x^4 + 2*d*e*x^2 + d^2)/\text{sqrt}(c*x^4 + b*x^2 + a), x)$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(d + ex^2)^2}{\sqrt{a + bx^2 + cx^4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x**2+d)**2/(c*x**4+b*x**2+a)**(1/2),x)

[Out] Integral((d + e*x**2)**2/sqrt(a + b*x**2 + c*x**4), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^2 + d)^2}{\sqrt{cx^4 + bx^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)^2/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")

[Out] integrate((e*x^2 + d)^2/sqrt(c*x^4 + b*x^2 + a), x)

$$3.382 \quad \int \frac{d+ex^2}{\sqrt{a+bx^2+cx^4}} dx$$

Optimal. Leaf size=283

$$\frac{\sqrt[4]{a}(\sqrt{a} + \sqrt{cx^2}) \sqrt{\frac{a+bx^2+cx^4}{(\sqrt{a}+\sqrt{cx^2})^2}} \left(\frac{\sqrt{cd}}{\sqrt{a}} + e\right) \text{EllipticF}\left(2 \tan^{-1}\left(\frac{\sqrt[4]{cx}}{\sqrt[4]{a}}\right), \frac{1}{4}\left(2 - \frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2c^{3/4}\sqrt{a+bx^2+cx^4}} - \frac{\sqrt[4]{ae}(\sqrt{a} + \sqrt{cx^2}) \sqrt{\frac{a+bx^2+cx^4}{(\sqrt{a}+\sqrt{cx^2})^2}} E\left(2 \tan^{-1}\left(\frac{\sqrt[4]{cx}}{\sqrt[4]{a}}\right), \frac{1}{4}\left(2 - \frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{c^{3/4}\sqrt{a+bx^2+cx^4}}$$

[Out] (e*x*Sqrt[a + b*x^2 + c*x^4])/(Sqrt[c]*(Sqrt[a] + Sqrt[c]*x^2)) - (a^(1/4)*e*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(c^(3/4)*Sqrt[a + b*x^2 + c*x^4]) + (a^(1/4)*((Sqrt[c]*d)/Sqrt[a] + e)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*c^(3/4)*Sqrt[a + b*x^2 + c*x^4])

Rubi [A] time = 0.0832347, antiderivative size = 283, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1197, 1103, 1195}

$$\frac{\sqrt[4]{a}(\sqrt{a} + \sqrt{cx^2}) \sqrt{\frac{a+bx^2+cx^4}{(\sqrt{a}+\sqrt{cx^2})^2}} \left(\frac{\sqrt{cd}}{\sqrt{a}} + e\right) F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{cx}}{\sqrt[4]{a}}\right), \frac{1}{4}\left(2 - \frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2c^{3/4}\sqrt{a+bx^2+cx^4}} - \frac{\sqrt[4]{ae}(\sqrt{a} + \sqrt{cx^2}) \sqrt{\frac{a+bx^2+cx^4}{(\sqrt{a}+\sqrt{cx^2})^2}} E\left(2 \tan^{-1}\left(\frac{\sqrt[4]{cx}}{\sqrt[4]{a}}\right), \frac{1}{4}\left(2 - \frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{c^{3/4}\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

[In] Int[(d + e*x^2)/Sqrt[a + b*x^2 + c*x^4], x]

[Out] (e*x*Sqrt[a + b*x^2 + c*x^4])/(Sqrt[c]*(Sqrt[a] + Sqrt[c]*x^2)) - (a^(1/4)*e*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(c^(3/4)*Sqrt[a + b*x^2 + c*x^4]) + (a^(1/4)*((Sqrt[c]*d)/Sqrt[a] + e)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*c^(3/4)*Sqrt[a + b*x^2 + c*x^4])

Rule 1197

Int[((d_) + (e_)*(x_)^2)/Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4], x_Symbol] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; NeQ[e + d*q, 0] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]

Rule 1103

Int[1/Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4], x_Symbol] :> With[{q = Rt[c/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]), x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]

Rule 1195

Int[((d_) + (e_)*(x_)^2)/Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4], x_Symbol] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^2*x^2)^2)], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]

2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c*x^4]), x] /; EqQ[e + d*q^2, 0] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]

Rubi steps

$$\int \frac{d + ex^2}{\sqrt{a + bx^2 + cx^4}} dx = -\frac{(\sqrt{ae}) \int \frac{1 - \frac{\sqrt{cx^2}}{\sqrt{a}}}{\sqrt{a+bx^2+cx^4}} dx}{\sqrt{c}} + \left(d + \frac{\sqrt{ae}}{\sqrt{c}}\right) \int \frac{1}{\sqrt{a + bx^2 + cx^4}} dx$$

$$= \frac{ex\sqrt{a + bx^2 + cx^4}}{\sqrt{c}(\sqrt{a} + \sqrt{cx^2})} - \frac{\sqrt[4]{ae}(\sqrt{a} + \sqrt{cx^2}) \sqrt{\frac{a+bx^2+cx^4}{(\sqrt{a}+\sqrt{cx^2})^2}} E\left(2 \tan^{-1}\left(\frac{\sqrt[4]{cx}}{\sqrt[4]{a}}\right) \middle| \frac{1}{4}\left(2 - \frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{c^{3/4}\sqrt{a + bx^2 + cx^4}} + \dots$$

Mathematica [C] time = 0.261149, size = 302, normalized size = 1.07

$$\frac{i\sqrt{\frac{\sqrt{b^2-4ac+b+2cx^2}}{\sqrt{b^2-4ac+b}}}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}} + 1 \left(\left(e(b - \sqrt{b^2 - 4ac}) - 2cd \right) \text{EllipticF}\left(i \sinh^{-1}\left(\sqrt{2}x\sqrt{\frac{c}{\sqrt{b^2-4ac+b}}}\right), \frac{\sqrt{b^2-4ac+b}}{b-\sqrt{b^2-4ac}} \right) + e\left(\sqrt{b^2 - 4ac}\right) \right)}{2\sqrt{2}c\sqrt{\frac{c}{\sqrt{b^2-4ac+b}}}\sqrt{a + bx^2 + cx^4}}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e*x^2)/Sqrt[a + b*x^2 + c*x^4], x]

[Out] ((I/2)*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]])*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*((-b + Sqrt[b^2 - 4*a*c])*e*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])) + (-2*c*d + (b - Sqrt[b^2 - 4*a*c])*e)*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])))/(Sqrt[2]*c*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])])*Sqrt[a + b*x^2 + c*x^4])

Maple [A] time = 0.003, size = 362, normalized size = 1.3

$$-\frac{ae\sqrt{2}}{2} \sqrt{4 - 2\frac{(\sqrt{-4ac + b^2} - b)x^2}{a}} \sqrt{4 + 2\frac{(b + \sqrt{-4ac + b^2})x^2}{a}} \left(\text{EllipticF}\left(\frac{x\sqrt{2}}{2} \sqrt{\frac{1}{a}(\sqrt{-4ac + b^2} - b)}, \frac{1}{2} \sqrt{-4 + 2\frac{b}{\dots}} \right) \dots \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x^2+d)/(c*x^4+b*x^2+a)^(1/2), x)

[Out] -1/2*e*a^2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2)*(4-2*((-4*a*c+b^2)^(1/2)-b)/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2), 1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*x*2^(1/2)*(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2), 1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2)))+1/4*d*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2)*(4-2*((-4*a*c+b^2)^(1/2)-b)/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2), 1/2*(

$$-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex^2 + d}{\sqrt{cx^4 + bx^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")

[Out] integrate((e*x^2 + d)/sqrt(c*x^4 + b*x^2 + a), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{ex^2 + d}{\sqrt{cx^4 + bx^2 + a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")

[Out] integral((e*x^2 + d)/sqrt(c*x^4 + b*x^2 + a), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{d + ex^2}{\sqrt{a + bx^2 + cx^4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x**2+d)/(c*x**4+b*x**2+a)**(1/2),x)

[Out] Integral((d + e*x**2)/sqrt(a + b*x**2 + c*x**4), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex^2 + d}{\sqrt{cx^4 + bx^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")

[Out] integrate((e*x^2 + d)/sqrt(c*x^4 + b*x^2 + a), x)

$$3.383 \quad \int \frac{1}{(d+ex^2)\sqrt{a+bx^2+cx^4}} dx$$

Optimal. Leaf size=401

$$\frac{\sqrt[4]{c}(\sqrt{a} + \sqrt{cx^2}) \sqrt{\frac{a+bx^2+cx^4}{(\sqrt{a}+\sqrt{cx^2})^2}} \text{EllipticF}\left(2 \tan^{-1}\left(\frac{\sqrt[4]{cx}}{\sqrt[4]{a}}\right), \frac{1}{4}\left(2 - \frac{b}{\sqrt{a}\sqrt{c}}\right)\right) a^{3/4}(\sqrt{a} + \sqrt{cx^2}) \sqrt{\frac{a+bx^2+cx^4}{(\sqrt{a}+\sqrt{cx^2})^2}} \left(\frac{\sqrt{cd}}{\sqrt{a}} + e\right)^2 \Pi\left(-\frac{\sqrt{cd}-\sqrt{ae}}{\sqrt{a}}\right)}{2^4 \sqrt[4]{a} \sqrt{a+bx^2+cx^4} (\sqrt{cd}-\sqrt{ae}) - 4^4 \sqrt[4]{cd} \sqrt{a+bx^2+cx^4}}$$

[Out] (Sqrt[e]*ArcTan[(Sqrt[c*d^2 - b*d*e + a*e^2]*x)/(Sqrt[d]*Sqrt[e]*Sqrt[a + b*x^2 + c*x^4]))/(2*Sqrt[d]*Sqrt[c*d^2 - b*d*e + a*e^2]) + (c^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*a^(1/4)*(Sqrt[c]*d - Sqrt[a]*e)*Sqrt[a + b*x^2 + c*x^4]) - (a^(3/4)*((Sqrt[c]*d)/Sqrt[a] + e)^2*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticPi[-(Sqrt[c]*d - Sqrt[a]*e)^2/(4*Sqrt[a]*Sqrt[c]*d*e), 2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(4*c^(1/4)*d*(c*d^2 - a*e^2)*Sqrt[a + b*x^2 + c*x^4])

Rubi [A] time = 0.345737, antiderivative size = 401, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1216, 1103, 1706}

$$\frac{a^{3/4}(\sqrt{a} + \sqrt{cx^2}) \sqrt{\frac{a+bx^2+cx^4}{(\sqrt{a}+\sqrt{cx^2})^2}} \left(\frac{\sqrt{cd}}{\sqrt{a}} + e\right)^2 \Pi\left(-\frac{(\sqrt{cd}-\sqrt{ae})^2}{4\sqrt{a}\sqrt{cde}}; 2 \tan^{-1}\left(\frac{\sqrt[4]{cx}}{\sqrt[4]{a}}\right) \middle| \frac{1}{4}\left(2 - \frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{4^4 \sqrt[4]{cd} \sqrt{a+bx^2+cx^4} (cd^2 - ae^2)} + \frac{\sqrt{e} \tan^{-1}\left(\frac{x\sqrt{ae^2-bde+cd^2}}{\sqrt{d}\sqrt{e}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{d}\sqrt{ae^2-bde+cd^2}}$$

Antiderivative was successfully verified.

[In] Int[1/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]),x]

[Out] (Sqrt[e]*ArcTan[(Sqrt[c*d^2 - b*d*e + a*e^2]*x)/(Sqrt[d]*Sqrt[e]*Sqrt[a + b*x^2 + c*x^4]))/(2*Sqrt[d]*Sqrt[c*d^2 - b*d*e + a*e^2]) + (c^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*a^(1/4)*(Sqrt[c]*d - Sqrt[a]*e)*Sqrt[a + b*x^2 + c*x^4]) - (a^(3/4)*((Sqrt[c]*d)/Sqrt[a] + e)^2*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticPi[-(Sqrt[c]*d - Sqrt[a]*e)^2/(4*Sqrt[a]*Sqrt[c]*d*e), 2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(4*c^(1/4)*d*(c*d^2 - a*e^2)*Sqrt[a + b*x^2 + c*x^4])

Rule 1216

Int[1/(((d_) + (e_)*(x_)^2)*Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4]), x_Symbol] :> With[{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]

Rule 1103

Int[1/Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4], x_Symbol] :> With[{q = Rt[c/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4])

), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]

Rule 1706

Int[((A_) + (B_)*(x_)^2)/(((d_) + (e_)*(x_)^2)*Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4]), x_Symbol] := With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*ArcTan[(Rt[-b + (c*d)/e + (a*e)/d, 2]*x)/Sqrt[a + b*x^2 + c*x^4]])/(2*d*e*Rt[-b + (c*d)/e + (a*e)/d, 2]), x] + Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a + b*x^2 + c*x^4))/(a*(A + B*x^2)^2)])*EllipticPi[Cancel[-((B*d - A*e)^2/(4*d*e*A*B))], 2*ArcTan[q*x], 1/2 - (b*A)/(4*a*B)]/(4*d*e*A*q*Sqrt[a + b*x^2 + c*x^4]), x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]

Rubi steps

$$\int \frac{1}{(d + ex^2)\sqrt{a + bx^2 + cx^4}} dx = \frac{\sqrt{c} \int \frac{1}{\sqrt{a + bx^2 + cx^4}} dx}{\sqrt{cd} - \sqrt{ae}} - \frac{(\sqrt{ae}) \int \frac{1 + \frac{\sqrt{cx^2}}{\sqrt{a}}}{(d + ex^2)\sqrt{a + bx^2 + cx^4}} dx}{\sqrt{cd} - \sqrt{ae}}$$

$$= \frac{\sqrt{e} \tan^{-1}\left(\frac{\sqrt{cd^2 - bde + ae^2}x}{\sqrt{d}\sqrt{e}\sqrt{a + bx^2 + cx^4}}\right)}{2\sqrt{d}\sqrt{cd^2 - bde + ae^2}} + \frac{\sqrt[4]{c}(\sqrt{a} + \sqrt{cx^2})\sqrt{\frac{a + bx^2 + cx^4}{(\sqrt{a} + \sqrt{cx^2})^2}} F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\right)^{\frac{1}{4}}}{2\sqrt[4]{a}(\sqrt{cd} - \sqrt{ae})\sqrt{a + bx^2 + cx^4}}$$

Mathematica [C] time = 0.227594, size = 214, normalized size = 0.53

$$\frac{i\sqrt{\frac{\sqrt{b^2 - 4ac + b + 2cx^2}}{\sqrt{b^2 - 4ac + b}}}\sqrt{\frac{2cx^2}{b - \sqrt{b^2 - 4ac}}} + \Pi\left(\frac{(b + \sqrt{b^2 - 4ac})e}{2cd}; i \sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{b + \sqrt{b^2 - 4ac}}}x\right)\right)^{\frac{b + \sqrt{b^2 - 4ac}}{b - \sqrt{b^2 - 4ac}}}}{\sqrt{2d}\sqrt{\frac{c}{\sqrt{b^2 - 4ac + b}}}\sqrt{a + bx^2 + cx^4}}$$

Antiderivative was successfully verified.

[In] Integrate[1/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x]

[Out] ((-I)*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]])*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]])*EllipticPi[((b + Sqrt[b^2 - 4*a*c])*e)/(2*c*d), I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c]))/(Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])])*d*Sqrt[a + b*x^2 + c*x^4])

Maple [A] time = 0.03, size = 200, normalized size = 0.5

$$\frac{\sqrt{2}}{d}\sqrt{1 - \frac{x^2}{2a}\sqrt{-4ac + b^2} + \frac{bx^2}{2a}\sqrt{1 + \frac{bx^2}{2a} + \frac{x^2}{2a}\sqrt{-4ac + b^2}}}\text{EllipticPi}\left(\frac{x\sqrt{2}}{2}\sqrt{\frac{1}{a}(\sqrt{-4ac + b^2} - b)}, -2\frac{ae}{(\sqrt{-4ac + b^2})}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2), x)

```
[Out] 1/d*2^(1/2)/(1/a*(-4*a*c+b^2)^(1/2)-b/a)^(1/2)*(1-1/2/a*x^2*(-4*a*c+b^2)^(1/2)+1/2/a*x^2*b)^(1/2)*(1+1/2/a*x^2*b+1/2/a*x^2*(-4*a*c+b^2)^(1/2))^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticPi(1/2*x*2^(1/2)*(((4*a*c+b^2)^(1/2)-b)/a)^(1/2),-2/((4*a*c+b^2)^(1/2)-b)*a/d*e,(-1/2*(b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*2^(1/2)/(((4*a*c+b^2)^(1/2)-b)/a)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^4 + bx^2 + a}(ex^2 + d)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(d + ex^2)\sqrt{a + bx^2 + cx^4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(1/((d + e*x**2)*sqrt(a + b*x**2 + c*x**4)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^4 + bx^2 + a}(ex^2 + d)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)), x)
```


$$3.384 \quad \int \frac{1}{(d+ex^2)^2 \sqrt{a+bx^2+cx^4}} dx$$

Optimal. Leaf size=718

$$\frac{\sqrt[4]{c}(\sqrt{a} + \sqrt{cx^2}) \sqrt{\frac{a+bx^2+cx^4}{(\sqrt{a}+\sqrt{cx^2})^2}} \text{EllipticF}\left(2 \tan^{-1}\left(\frac{\sqrt[4]{cx}}{\sqrt[4]{a}}\right), \frac{1}{4}\left(2 - \frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{ad}\sqrt{a+bx^2+cx^4}(\sqrt{cd}-\sqrt{ae})} + \frac{e^2x\sqrt{a+bx^2+cx^4}}{2d(d+ex^2)(ae^2-bde+cd^2)} - \frac{\sqrt{cex}}{2d(\sqrt{a}+\sqrt{cx^2})}$$

```
[Out] -(Sqrt[c]*e*x*Sqrt[a + b*x^2 + c*x^4])/(2*d*(c*d^2 - b*d*e + a*e^2)*(Sqrt[a]
+ Sqrt[c]*x^2)) + (e^2*x*Sqrt[a + b*x^2 + c*x^4])/(2*d*(c*d^2 - b*d*e + a
*e^2)*(d + e*x^2)) + (Sqrt[e]*(3*c*d^2 - e*(2*b*d - a*e))*ArcTan[(Sqrt[c*d^
2 - b*d*e + a*e^2]*x)/(Sqrt[d]*Sqrt[e]*Sqrt[a + b*x^2 + c*x^4])])/(4*d^(3/2
)*(c*d^2 - b*d*e + a*e^2)^(3/2)) + (a^(1/4)*c^(1/4)*e*(Sqrt[a] + Sqrt[c]*x^
2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(
c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*d*(c*d^2 - b*d*e + a*
e^2)*Sqrt[a + b*x^2 + c*x^4]) + (c^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a +
b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^
(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*a^(1/4)*d*(Sqrt[c]*d - Sqrt[a]*e)*
Sqrt[a + b*x^2 + c*x^4]) - ((Sqrt[c]*d + Sqrt[a]*e)*(3*c*d^2 - e*(2*b*d - a
*e))*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^
2)^2]*EllipticPi[-(Sqrt[c]*d - Sqrt[a]*e)^2/(4*Sqrt[a]*Sqrt[c]*d*e), 2*ArcT
an[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(8*a^(1/4)*c^(1/4)*d
^2*(Sqrt[c]*d - Sqrt[a]*e)*(c*d^2 - b*d*e + a*e^2)*Sqrt[a + b*x^2 + c*x^4])
```

Rubi [A] time = 1.08066, antiderivative size = 718, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {1223, 1714, 1195, 1708, 1103, 1706}

$$\frac{e^2x\sqrt{a+bx^2+cx^4}}{2d(d+ex^2)(ae^2-bde+cd^2)} - \frac{\sqrt{cex}\sqrt{a+bx^2+cx^4}}{2d(\sqrt{a}+\sqrt{cx^2})(ae^2-bde+cd^2)} + \frac{\sqrt{e}(3cd^2 - e(2bd - ae)) \tan^{-1}\left(\frac{x\sqrt{ae^2-bde+cd^2}}{\sqrt{d}\sqrt{e}\sqrt{a+bx^2+cx^4}}\right)}{4d^{3/2}(ae^2-bde+cd^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x^2)^2*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] -(Sqrt[c]*e*x*Sqrt[a + b*x^2 + c*x^4])/(2*d*(c*d^2 - b*d*e + a*e^2)*(Sqrt[a]
+ Sqrt[c]*x^2)) + (e^2*x*Sqrt[a + b*x^2 + c*x^4])/(2*d*(c*d^2 - b*d*e + a
*e^2)*(d + e*x^2)) + (Sqrt[e]*(3*c*d^2 - e*(2*b*d - a*e))*ArcTan[(Sqrt[c*d^
2 - b*d*e + a*e^2]*x)/(Sqrt[d]*Sqrt[e]*Sqrt[a + b*x^2 + c*x^4])])/(4*d^(3/2
)*(c*d^2 - b*d*e + a*e^2)^(3/2)) + (a^(1/4)*c^(1/4)*e*(Sqrt[a] + Sqrt[c]*x^
2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(
c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*d*(c*d^2 - b*d*e + a*
e^2)*Sqrt[a + b*x^2 + c*x^4]) + (c^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a +
b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^
(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*a^(1/4)*d*(Sqrt[c]*d - Sqrt[a]*e)*
Sqrt[a + b*x^2 + c*x^4]) - ((Sqrt[c]*d + Sqrt[a]*e)*(3*c*d^2 - e*(2*b*d - a
*e))*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^
2)^2]*EllipticPi[-(Sqrt[c]*d - Sqrt[a]*e)^2/(4*Sqrt[a]*Sqrt[c]*d*e), 2*ArcT
an[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(8*a^(1/4)*c^(1/4)*d
^2*(Sqrt[c]*d - Sqrt[a]*e)*(c*d^2 - b*d*e + a*e^2)*Sqrt[a + b*x^2 + c*x^4])
```

Rule 1223

```
Int[((d_) + (e_.)*(x_)^2)^q/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_
Symbol] := -Simp[(e^2*x*(d + e*x^2)^(q + 1)*Sqrt[a + b*x^2 + c*x^4])/(2*d*(
q + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(2*d*(q + 1)*(c*d^2 - b*d*e +
a*e^2)), Int[((d + e*x^2)^(q + 1)*Simp[a*e^2*(2*q + 3) + 2*d*(c*d - b*e)*(q
+ 1) - 2*e*(c*d*(q + 1) - b*e*(q + 2))*x^2 + c*e^2*(2*q + 5)*x^4, x])/Sqrt
[a + b*x^2 + c*x^4], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c
, 0] && ILtQ[q, -1]
```

Rule 1714

```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] := With[{q = Rt[c/a, 2], A = Coeff[P4x, x, 0], B = Coeff[P4x, x
, 2], C = Coeff[P4x, x, 4]}, -Dist[C/(e*q), Int[(1 - q*x^2)/Sqrt[a + b*x^2
+ c*x^4], x], x] + Dist[1/(c*e), Int[(A*c*e + a*C*d*q + (B*c*e - C*(c*d - a
*e*q))*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
c, d, e}, x] && PolyQ[P4x, x^2, 2] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*
d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && !GtQ[b^2 - 4*a*c,
0]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] := With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rule 1708

```
Int[((A_.) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2
+ (c_.)*(x_)^4]), x_Symbol] := With[{q = Rt[c/a, 2]}, Dist[(A*(c*d + a*e*q)
- a*B*(e + d*q))/(c*d^2 - a*e^2), Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] +
Dist[(a*(B*d - A*e)*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^2
)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0
] && PosQ[c/a] && NeQ[c*A^2 - a*B^2, 0]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] := With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1706

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
(c_.)*(x_)^4]), x_Symbol] := With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*Arc
Tan[(Rt[-b + (c*d)/e + (a*e)/d, 2]*x)/Sqrt[a + b*x^2 + c*x^4]])/(2*d*e*Rt[-
b + (c*d)/e + (a*e)/d, 2]), x] + Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a
+ b*x^2 + c*x^4))/(a*(A + B*x^2)^2)]*EllipticPi[Cancel[-((B*d - A*e)^2/(4*
d*e*A*B))], 2*ArcTan[q*x], 1/2 - (b*A)/(4*a*B)])/(4*d*e*A*q*Sqrt[a + b*x^2
+ c*x^4]), x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(d+ex^2)^2 \sqrt{a+bx^2+cx^4}} dx &= \frac{e^2 x \sqrt{a+bx^2+cx^4}}{2d(cd^2-bde+ae^2)(d+ex^2)} - \frac{\int \frac{-2cd^2+e(2bd-ae)+2cdex^2+ce^2x^4}{(d+ex^2)\sqrt{a+bx^2+cx^4}} dx}{2d(cd^2-bde+ae^2)} \\
&= \frac{e^2 x \sqrt{a+bx^2+cx^4}}{2d(cd^2-bde+ae^2)(d+ex^2)} - \frac{\int \frac{\sqrt{ac}^{3/2}de^2+ce(-2cd^2+e(2bd-ae))+2c^2de^2-ce^2(cd-\sqrt{a}\sqrt{ce})x^2}{(d+ex^2)\sqrt{a+bx^2+cx^4}}}{2cde(cd^2-bde+ae^2)} \\
&= -\frac{\sqrt{ce}x\sqrt{a+bx^2+cx^4}}{2d(cd^2-bde+ae^2)(\sqrt{a}+\sqrt{cx^2})} + \frac{e^2x\sqrt{a+bx^2+cx^4}}{2d(cd^2-bde+ae^2)(d+ex^2)} + \frac{\sqrt[4]{a}\sqrt[4]{ce}(\sqrt{a}+\sqrt{cx^2})}{2d(cd^2-bde+ae^2)(d+ex^2)} \\
&= -\frac{\sqrt{ce}x\sqrt{a+bx^2+cx^4}}{2d(cd^2-bde+ae^2)(\sqrt{a}+\sqrt{cx^2})} + \frac{e^2x\sqrt{a+bx^2+cx^4}}{2d(cd^2-bde+ae^2)(d+ex^2)} + \frac{\sqrt{e}(3cd^2-bde+ae^2)}{2d(cd^2-bde+ae^2)(d+ex^2)}
\end{aligned}$$

Mathematica [C] time = 1.88822, size = 1069, normalized size = 1.49

$$2i\sqrt{2}c\sqrt{\frac{2cx^2+b+\sqrt{b^2-4ac}}{b+\sqrt{b^2-4ac}}}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}}+1(ex^2+d)\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\right),\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)d^2-6i\sqrt{2}c\sqrt{\frac{2cx^2+b+\sqrt{b^2-4ac}}{b+\sqrt{b^2-4ac}}}$$

Antiderivative was successfully verified.

[In] Integrate[1/((d + e*x^2)^2*Sqrt[a + b*x^2 + c*x^4]),x]

[Out] (4*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*d*e^2*x*(a + b*x^2 + c*x^4) + I*Sqrt[2]*(b - Sqrt[b^2 - 4*a*c])*d*e*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*(d + e*x^2)*(EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])) - EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])) + (2*I)*Sqrt[2]*c*d^2*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*(d + e*x^2)*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])) - (6*I)*Sqrt[2]*c*d^2*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*(d + e*x^2)*EllipticPi[((b + Sqrt[b^2 - 4*a*c])*e)/(2*c*d), I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])) + (4*I)*Sqrt[2]*b*d*e*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*(d + e*x^2)*EllipticPi[((b + Sqrt[b^2 - 4*a*c])*e)/(2*c*d), I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])) - (2*I)*Sqrt[2]*a*e^2*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*(d + e*x^2)*EllipticPi[((b + Sqrt[b^2 - 4*a*c])*e)/(2*c*d), I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c]))]/(8*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*d*(c*d^3 + d*e*(-(b*d) + a*e))*(d + e*x^2)*Sqrt[a + b*x^2 + c*x^4])

Maple [A] time = 0.027, size = 1279, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(e*x^2+d)^2/(c*x^4+b*x^2+a)^(1/2),x)`

[Out] $\frac{1}{2}e^{2x}(cx^4+bx^2+a)^{1/2}/d/(ae^2-bd+cd^2)/(e^2+d)-1/8c/(ae^2-bd+cd^2)^2^{1/2}/(1/a(-4ac+b^2)^{1/2}-b/a)^{1/2}(4-2/a^2x^2(-4ac+b^2)^{1/2}+2/a^2x^2b)^{1/2}(4+2/a^2x^2b+2/a^2x^2(-4ac+b^2)^{1/2})^{1/2}/(cx^4+bx^2+a)^{1/2}*\text{EllipticF}(1/2*x^2^{1/2}*(((4ac+b^2)^{1/2}-b)/a)^{1/2},1/2*(-4+2*b*(b+(-4ac+b^2)^{1/2}))/a/c)^{1/2})+1/4*c*e/(ae^2-bd+cd^2)/d*a^2^{1/2}/(1/a(-4ac+b^2)^{1/2}-b/a)^{1/2}(4-2/a^2x^2(-4ac+b^2)^{1/2}+2/a^2x^2b)^{1/2}(4+2/a^2x^2b+2/a^2x^2(-4ac+b^2)^{1/2})^{1/2}/(cx^4+bx^2+a)^{1/2}/(b+(-4ac+b^2)^{1/2})*\text{EllipticF}(1/2*x^2^{1/2}*(((4ac+b^2)^{1/2}-b)/a)^{1/2},1/2*(-4+2*b*(b+(-4ac+b^2)^{1/2}))/a/c)^{1/2})-1/4*c*e/(ae^2-bd+cd^2)/d*a^2^{1/2}/(1/a(-4ac+b^2)^{1/2}-b/a)^{1/2}(4-2/a^2x^2(-4ac+b^2)^{1/2}+2/a^2x^2b)^{1/2}(4+2/a^2x^2b+2/a^2x^2(-4ac+b^2)^{1/2})^{1/2})^{1/2}/(cx^4+bx^2+a)^{1/2}/(b+(-4ac+b^2)^{1/2})*\text{EllipticE}(1/2*x^2^{1/2}*(((4ac+b^2)^{1/2}-b)/a)^{1/2},1/2*(-4+2*b*(b+(-4ac+b^2)^{1/2}))/a/c)^{1/2})+1/2/(ae^2-bd+cd^2)/d^2*e^2*x^{1/2}/(1/a(-4ac+b^2)^{1/2}-b/a)^{1/2}(1-1/2/a^2x^2(-4ac+b^2)^{1/2}+1/2/a^2x^2b)^{1/2}(1+1/2/a^2x^2b+1/2/a^2x^2(-4ac+b^2)^{1/2})^{1/2}/(cx^4+bx^2+a)^{1/2}*\text{EllipticPi}(1/2*x^2^{1/2}*(((4ac+b^2)^{1/2}-b)/a)^{1/2},-2/((4ac+b^2)^{1/2}-b)*a/d*e,(-1/2*(b+(-4ac+b^2)^{1/2}))/a)^{1/2}*2^{1/2}/(((4ac+b^2)^{1/2}-b)/a)^{1/2})*a-1/(ae^2-bd+cd^2)/d*e^2*x^{1/2}/(1/a(-4ac+b^2)^{1/2}-b/a)^{1/2}(1-1/2/a^2x^2(-4ac+b^2)^{1/2}+1/2/a^2x^2b)^{1/2}(1+1/2/a^2x^2b+1/2/a^2x^2(-4ac+b^2)^{1/2})^{1/2}/(cx^4+bx^2+a)^{1/2}*\text{EllipticPi}(1/2*x^2^{1/2}*(((4ac+b^2)^{1/2}-b)/a)^{1/2},-2/((4ac+b^2)^{1/2}-b)*a/d*e,(-1/2*(b+(-4ac+b^2)^{1/2}))/a)^{1/2}*2^{1/2}/(((4ac+b^2)^{1/2}-b)/a)^{1/2})*b+3/2/(ae^2-bd+cd^2)*x^{1/2}/(1/a(-4ac+b^2)^{1/2}-b/a)^{1/2}(1-1/2/a^2x^2(-4ac+b^2)^{1/2}+1/2/a^2x^2b)^{1/2}(1+1/2/a^2x^2b+1/2/a^2x^2(-4ac+b^2)^{1/2})^{1/2}/(cx^4+bx^2+a)^{1/2}*\text{EllipticPi}(1/2*x^2^{1/2}*(((4ac+b^2)^{1/2}-b)/a)^{1/2},-2/((4ac+b^2)^{1/2}-b)*a/d*e,(-1/2*(b+(-4ac+b^2)^{1/2}))/a)^{1/2}*2^{1/2}/(((4ac+b^2)^{1/2}-b)/a)^{1/2})*c$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^4 + bx^2 + a}(ex^2 + d)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(e*x^2+d)^2/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")`

[Out] `integrate(1/(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)^2), x)`

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(e*x^2+d)^2/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")`

[Out] Timed out

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(d + ex^2)^2 \sqrt{a + bx^2 + cx^4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x**2+d)**2/(c*x**4+b*x**2+a)**(1/2), x)

[Out] Integral(1/((d + e*x**2)**2*sqrt(a + b*x**2 + c*x**4)), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^4 + bx^2 + a}(ex^2 + d)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x^2+d)^2/(c*x^4+b*x^2+a)^(1/2), x, algorithm="giac")

[Out] integrate(1/(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)^2), x)

$$3.385 \quad \int \frac{(d+ex^2)^3}{\sqrt{a+bx^2-cx^4}} dx$$

Optimal. Leaf size=553

$$\frac{(b - \sqrt{4ac + b^2}) \sqrt{\sqrt{4ac + b^2} + b} \sqrt{1 - \frac{2cx^2}{b - \sqrt{4ac + b^2}}} \sqrt{1 - \frac{2cx^2}{\sqrt{4ac + b^2} + b}} \left(\frac{2c(4abe^3 + 15acde^2 + 15c^2d^3)}{b - \sqrt{4ac + b^2}} + e(3ce(3ae + 10bd) + 8b^2e^2 + 45c^2d^2) \right)}{30\sqrt{2}c^{7/2}\sqrt{a + bx^2 - cx^4}}$$

[Out] $-(e^2(15c*d + 4b*e)*x*\text{Sqrt}[a + b*x^2 - c*x^4])/(15*c^2) - (e^3*x^3*\text{Sqrt}[a + b*x^2 - c*x^4])/(5*c) - ((b - \text{Sqrt}[b^2 + 4*a*c])*\text{Sqrt}[b + \text{Sqrt}[b^2 + 4*a*c]])*e*(45*c^2*d^2 + 8*b^2*e^2 + 3*c*e*(10*b*d + 3*a*e))*\text{Sqrt}[1 - (2*c*x^2)/(b - \text{Sqrt}[b^2 + 4*a*c])]*\text{Sqrt}[1 - (2*c*x^2)/(b + \text{Sqrt}[b^2 + 4*a*c])]*\text{EllipticE}[\text{ArcSin}[(\text{Sqrt}[2]*\text{Sqrt}[c]*x)/\text{Sqrt}[b + \text{Sqrt}[b^2 + 4*a*c]]], (b + \text{Sqrt}[b^2 + 4*a*c])/(b - \text{Sqrt}[b^2 + 4*a*c])]/(30*\text{Sqrt}[2]*c^{7/2}*\text{Sqrt}[a + b*x^2 - c*x^4]) + ((b - \text{Sqrt}[b^2 + 4*a*c])*\text{Sqrt}[b + \text{Sqrt}[b^2 + 4*a*c]])*((2*c*(15*c^2*d^3 + 15*a*c*d*e^2 + 4*a*b*e^3))/(b - \text{Sqrt}[b^2 + 4*a*c]) + e*(45*c^2*d^2 + 8*b^2*e^2 + 3*c*e*(10*b*d + 3*a*e))*\text{Sqrt}[1 - (2*c*x^2)/(b - \text{Sqrt}[b^2 + 4*a*c])]*\text{Sqrt}[1 - (2*c*x^2)/(b + \text{Sqrt}[b^2 + 4*a*c])]*\text{EllipticF}[\text{ArcSin}[(\text{Sqrt}[2]*\text{Sqrt}[c]*x)/\text{Sqrt}[b + \text{Sqrt}[b^2 + 4*a*c]]], (b + \text{Sqrt}[b^2 + 4*a*c])/(b - \text{Sqrt}[b^2 + 4*a*c])]/(30*\text{Sqrt}[2]*c^{7/2}*\text{Sqrt}[a + b*x^2 - c*x^4])$

Rubi [A] time = 1.27979, antiderivative size = 553, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {1206, 1679, 1202, 524, 424, 419}

$$\frac{(b - \sqrt{4ac + b^2}) \sqrt{\sqrt{4ac + b^2} + b} \sqrt{1 - \frac{2cx^2}{b - \sqrt{4ac + b^2}}} \sqrt{1 - \frac{2cx^2}{\sqrt{4ac + b^2} + b}} \left(\frac{2c(4abe^3 + 15acde^2 + 15c^2d^3)}{b - \sqrt{4ac + b^2}} + e(3ce(3ae + 10bd) + 8b^2e^2 + 45c^2d^2) \right)}{30\sqrt{2}c^{7/2}\sqrt{a + bx^2 - cx^4}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(d + e*x^2)^3/\text{Sqrt}[a + b*x^2 - c*x^4], x]$

[Out] $-(e^2(15c*d + 4b*e)*x*\text{Sqrt}[a + b*x^2 - c*x^4])/(15*c^2) - (e^3*x^3*\text{Sqrt}[a + b*x^2 - c*x^4])/(5*c) - ((b - \text{Sqrt}[b^2 + 4*a*c])*\text{Sqrt}[b + \text{Sqrt}[b^2 + 4*a*c]])*e*(45*c^2*d^2 + 8*b^2*e^2 + 3*c*e*(10*b*d + 3*a*e))*\text{Sqrt}[1 - (2*c*x^2)/(b - \text{Sqrt}[b^2 + 4*a*c])]*\text{Sqrt}[1 - (2*c*x^2)/(b + \text{Sqrt}[b^2 + 4*a*c])]*\text{EllipticE}[\text{ArcSin}[(\text{Sqrt}[2]*\text{Sqrt}[c]*x)/\text{Sqrt}[b + \text{Sqrt}[b^2 + 4*a*c]]], (b + \text{Sqrt}[b^2 + 4*a*c])/(b - \text{Sqrt}[b^2 + 4*a*c])]/(30*\text{Sqrt}[2]*c^{7/2}*\text{Sqrt}[a + b*x^2 - c*x^4]) + ((b - \text{Sqrt}[b^2 + 4*a*c])*\text{Sqrt}[b + \text{Sqrt}[b^2 + 4*a*c]])*((2*c*(15*c^2*d^3 + 15*a*c*d*e^2 + 4*a*b*e^3))/(b - \text{Sqrt}[b^2 + 4*a*c]) + e*(45*c^2*d^2 + 8*b^2*e^2 + 3*c*e*(10*b*d + 3*a*e))*\text{Sqrt}[1 - (2*c*x^2)/(b - \text{Sqrt}[b^2 + 4*a*c])]*\text{Sqrt}[1 - (2*c*x^2)/(b + \text{Sqrt}[b^2 + 4*a*c])]*\text{EllipticF}[\text{ArcSin}[(\text{Sqrt}[2]*\text{Sqrt}[c]*x)/\text{Sqrt}[b + \text{Sqrt}[b^2 + 4*a*c]]], (b + \text{Sqrt}[b^2 + 4*a*c])/(b - \text{Sqrt}[b^2 + 4*a*c])]/(30*\text{Sqrt}[2]*c^{7/2}*\text{Sqrt}[a + b*x^2 - c*x^4])$

Rule 1206

$\text{Int}[(d + e*x^2)^q/\text{Sqrt}[a + b*x^2 - c*x^4], x]$
 $\text{Simp}[(e^q*x^{2q-3}*(a + b*x^2 + c*x^4)^{p+1})/(c*(4p+2q+1)), x] + \text{Dist}[1/(c*(4p+2q+1)), \text{Int}[(a + b*x^2 + c*x^4)^p*\text{ExpandToSum}[c*(4p+2q+1)*(d + e*x^2)^q - a*(2q-3)*e^q*x^{2q-4} - b*(2q+2q-1)*e^q*x^{2q-2} - c*(4p+2q+1)*e^q*x^{2q}], x], x] /;$
 $\text{FreeQ}\{a, b, c, d, e, p\}, x \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0] \ \&\& \ \text{NeQ}[c*d^2 - b*d*e +$

$a*e^2, 0]$ && IGtQ[q, 1]

Rule 1679

Int[(Pq_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] := With[{q = Expon[Pq, x^2], e = Coeff[Pq, x^2, Expon[Pq, x^2]]}, Simp[(e*x^(2*q - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(2*q + 4*p + 1)), x] + Dist[1/(c*(2*q + 4*p + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*q + 4*p + 1)*Pq - a*e*(2*q - 3)*x^(2*q - 4) - b*e*(2*q + 2*p - 1)*x^(2*q - 2) - c*e*(2*q + 4*p + 1)*x^(2*q), x], x], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && !LtQ[p, -1]

Rule 1202

Int[((d_) + (e_)*(x_)^2)/Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4], x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(Sqrt[1 + (2*c*x^2)/(b - q)]*Sqrt[1 + (2*c*x^2)/(b + q)])/Sqrt[a + b*x^2 + c*x^4], Int[(d + e*x^2)/(Sqrt[1 + (2*c*x^2)/(b - q)]*Sqrt[1 + (2*c*x^2)/(b + q)]), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[c/a]

Rule 524

Int[((e_) + (f_)*(x_)^(n_))/(Sqrt[(a_) + (b_)*(x_)^(n_)]*Sqrt[(c_) + (d_)*(x_)^(n_)]), x_Symbol] := Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n], x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && !(EqQ[n, 2] && ((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && (!GtQ[c, 0] || SimplerSqrtQ[-(b/a), -(d/c)]))))))

Rule 424

Int[Sqrt[(a_) + (b_)*(x_)^2]/Sqrt[(c_) + (d_)*(x_)^2], x_Symbol] := Simp[(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]

Rule 419

Int[1/(Sqrt[(a_) + (b_)*(x_)^2]*Sqrt[(c_) + (d_)*(x_)^2]), x_Symbol] := Simp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0] && !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])

Rubi steps

$$\begin{aligned} & (1/2)/a*x^2)^{(1/2)}*(4+2*(b+(4*a*c+b^2)^{(1/2)})/a*x^2)^{(1/2)} / (-c*x^4+b*x^2+a) \\ &)^{(1/2)}*EllipticF(1/2*x^2)^{(1/2)}*((-b+(4*a*c+b^2)^{(1/2)})/a)^{(1/2)}, 1/2*(-4-2* \\ & b*(b+(4*a*c+b^2)^{(1/2)})/a/c)^{(1/2)}-1/2*(3/5/c*a+8/15*b^2/c^2)*a^2)^{(1/2)} / ((\\ & -b+(4*a*c+b^2)^{(1/2)})/a)^{(1/2)}*(4-2*(-b+(4*a*c+b^2)^{(1/2)})/a*x^2)^{(1/2)}*(4+ \\ & 2*(b+(4*a*c+b^2)^{(1/2)})/a*x^2)^{(1/2)} / (-c*x^4+b*x^2+a)^{(1/2)} / (b+(4*a*c+b^2)^{(1/2)}) \\ &)*(EllipticF(1/2*x^2)^{(1/2)}*((-b+(4*a*c+b^2)^{(1/2)})/a)^{(1/2)}, 1/2*(-4-2* \\ & b*(b+(4*a*c+b^2)^{(1/2)})/a/c)^{(1/2)}-EllipticE(1/2*x^2)^{(1/2)}*((-b+(4*a*c+b^2)^{(1/2)}) \\ &)^{(1/2)})/a)^{(1/2)}, 1/2*(-4-2*b*(b+(4*a*c+b^2)^{(1/2)})/a/c)^{(1/2)})))+3*d*e^2*(\\ & -1/3/c*x*(-c*x^4+b*x^2+a)^{(1/2)}+1/12/c*a^2)^{(1/2)} / ((-b+(4*a*c+b^2)^{(1/2)})/a) \\ &)^{(1/2)}*(4-2*(-b+(4*a*c+b^2)^{(1/2)})/a*x^2)^{(1/2)}*(4+2*(b+(4*a*c+b^2)^{(1/2)})/ \\ & a*x^2)^{(1/2)} / (-c*x^4+b*x^2+a)^{(1/2)}*EllipticF(1/2*x^2)^{(1/2)}*((-b+(4*a*c+b^2)^{(1/2)}) \\ &)^{(1/2)})/a)^{(1/2)}, 1/2*(-4-2*b*(b+(4*a*c+b^2)^{(1/2)})/a/c)^{(1/2)}-1/3*b/c*a^2 \\ &)^{(1/2)} / ((-b+(4*a*c+b^2)^{(1/2)})/a)^{(1/2)}*(4-2*(-b+(4*a*c+b^2)^{(1/2)})/a*x^2)^{(1/2)} \\ &)*(4+2*(b+(4*a*c+b^2)^{(1/2)})/a*x^2)^{(1/2)} / (-c*x^4+b*x^2+a)^{(1/2)} / (b+(4*a \\ & *c+b^2)^{(1/2)})*(EllipticF(1/2*x^2)^{(1/2)}*((-b+(4*a*c+b^2)^{(1/2)})/a)^{(1/2)}, 1 \\ & /2*(-4-2*b*(b+(4*a*c+b^2)^{(1/2)})/a/c)^{(1/2)}-EllipticE(1/2*x^2)^{(1/2)}*((-b+(\\ & 4*a*c+b^2)^{(1/2)})/a)^{(1/2)}, 1/2*(-4-2*b*(b+(4*a*c+b^2)^{(1/2)})/a/c)^{(1/2)})))- \\ & 3/2*d^2*e*a^2)^{(1/2)} / ((-b+(4*a*c+b^2)^{(1/2)})/a)^{(1/2)}*(4-2*(-b+(4*a*c+b^2)^{(1/2)}) \\ &)^{(1/2)})/a*x^2)^{(1/2)}*(4+2*(b+(4*a*c+b^2)^{(1/2)})/a*x^2)^{(1/2)} / (-c*x^4+b*x^2+a) \\ &)^{(1/2)} / (b+(4*a*c+b^2)^{(1/2)})*(EllipticF(1/2*x^2)^{(1/2)}*((-b+(4*a*c+b^2)^{(1/2)})/a)^{(1/2)}, 1 \\ & /2*(-4-2*b*(b+(4*a*c+b^2)^{(1/2)})/a/c)^{(1/2)}-EllipticE(1/2*x^2)^{(1/2)}*((-b+(\\ & 4*a*c+b^2)^{(1/2)})/a)^{(1/2)}, 1/2*(-4-2*b*(b+(4*a*c+b^2)^{(1/2)})/a/c)^{(1/2)})))+ \\ & 1/4*d^3*2)^{(1/2)} / ((-b+(4*a*c+b^2)^{(1/2)})/a)^{(1/2)}*(4-2*(-b+(4*a*c+b^2)^{(1/2)}) \\ &)^{(1/2)})/a*x^2)^{(1/2)}*(4+2*(b+(4*a*c+b^2)^{(1/2)})/a*x^2)^{(1/2)} / (-c*x^4+ \\ & b*x^2+a)^{(1/2)}*EllipticF(1/2*x^2)^{(1/2)}*((-b+(4*a*c+b^2)^{(1/2)})/a)^{(1/2)}, 1/2 \\ & *(-4-2*b*(b+(4*a*c+b^2)^{(1/2)})/a/c)^{(1/2)})) \end{aligned}$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^2 + d)^3}{\sqrt{-cx^4 + bx^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)^3/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")

[Out] integrate((e*x^2 + d)^3/sqrt(-c*x^4 + b*x^2 + a), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{(e^3x^6 + 3de^2x^4 + 3d^2ex^2 + d^3)\sqrt{-cx^4 + bx^2 + a}}{cx^4 - bx^2 - a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)^3/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")

[Out] integral(-(e^3*x^6 + 3*d*e^2*x^4 + 3*d^2*e*x^2 + d^3)*sqrt(-c*x^4 + b*x^2 + a)/(c*x^4 - b*x^2 - a), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(d + ex^2)^3}{\sqrt{a + bx^2 - cx^4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x**2+d)**3/(-c*x**4+b*x**2+a)**(1/2),x)

[Out] Integral((d + e*x**2)**3/sqrt(a + b*x**2 - c*x**4), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^2 + d)^3}{\sqrt{-cx^4 + bx^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)^3/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")

[Out] integrate((e*x^2 + d)^3/sqrt(-c*x^4 + b*x^2 + a), x)

$$3.386 \quad \int \frac{(d+ex^2)^2}{\sqrt{a+bx^2-cx^4}} dx$$

Optimal. Leaf size=454

$$\frac{\sqrt{\sqrt{4ac+b^2}+b}\sqrt{1-\frac{2cx^2}{b-\sqrt{4ac+b^2}}}\sqrt{1-\frac{2cx^2}{\sqrt{4ac+b^2}+b}}\left(ce\left(-3d\sqrt{4ac+b^2}+ae+3bd\right)+be^2\left(b-\sqrt{4ac+b^2}\right)+3c^2d^2\right)\text{EllipticE}\left[\frac{\text{ArcSin}\left[\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2+4ac}}}\right]}{\sqrt{b+\sqrt{b^2+4ac}}}\right],\left(b+\sqrt{b^2+4ac}\right)/\left(b-\sqrt{b^2+4ac}\right)}{3\sqrt{2}c^{5/2}\sqrt{a+bx^2-cx^4}}$$

```
[Out] -(e^2*x*Sqrt[a + b*x^2 - c*x^4])/(3*c) - ((b - Sqrt[b^2 + 4*a*c])*Sqrt[b + Sqrt[b^2 + 4*a*c]]*e*(3*c*d + b*e)*Sqrt[1 - (2*c*x^2)/(b - Sqrt[b^2 + 4*a*c]])*Sqrt[1 - (2*c*x^2)/(b + Sqrt[b^2 + 4*a*c]])*EllipticE[ArcSin[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 + 4*a*c]]], (b + Sqrt[b^2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c]))/(3*Sqrt[2]*c^(5/2)*Sqrt[a + b*x^2 - c*x^4]) + (Sqrt[b + Sqrt[b^2 + 4*a*c]]*(3*c^2*d^2 + b*(b - Sqrt[b^2 + 4*a*c])*e^2 + c*e*(3*b*d - 3*Sqrt[b^2 + 4*a*c]*d + a*e))*Sqrt[1 - (2*c*x^2)/(b - Sqrt[b^2 + 4*a*c]])*Sqrt[1 - (2*c*x^2)/(b + Sqrt[b^2 + 4*a*c]])*EllipticF[ArcSin[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 + 4*a*c]]], (b + Sqrt[b^2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c]))/(3*Sqrt[2]*c^(5/2)*Sqrt[a + b*x^2 - c*x^4])
```

Rubi [A] time = 0.793513, antiderivative size = 454, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {1206, 1202, 524, 424, 419}

$$\frac{\sqrt{\sqrt{4ac+b^2}+b}\sqrt{1-\frac{2cx^2}{b-\sqrt{4ac+b^2}}}\sqrt{1-\frac{2cx^2}{\sqrt{4ac+b^2}+b}}\left(ce\left(-3d\sqrt{4ac+b^2}+ae+3bd\right)+be^2\left(b-\sqrt{4ac+b^2}\right)+3c^2d^2\right)F\left(\sin^{-1}\left[\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2+4ac}}}\right],\frac{b+\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)}{3\sqrt{2}c^{5/2}\sqrt{a+bx^2-cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^2/Sqrt[a + b*x^2 - c*x^4],x]
```

```
[Out] -(e^2*x*Sqrt[a + b*x^2 - c*x^4])/(3*c) - ((b - Sqrt[b^2 + 4*a*c])*Sqrt[b + Sqrt[b^2 + 4*a*c]]*e*(3*c*d + b*e)*Sqrt[1 - (2*c*x^2)/(b - Sqrt[b^2 + 4*a*c]])*Sqrt[1 - (2*c*x^2)/(b + Sqrt[b^2 + 4*a*c]])*EllipticE[ArcSin[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 + 4*a*c]]], (b + Sqrt[b^2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c]))/(3*Sqrt[2]*c^(5/2)*Sqrt[a + b*x^2 - c*x^4]) + (Sqrt[b + Sqrt[b^2 + 4*a*c]]*(3*c^2*d^2 + b*(b - Sqrt[b^2 + 4*a*c])*e^2 + c*e*(3*b*d - 3*Sqrt[b^2 + 4*a*c]*d + a*e))*Sqrt[1 - (2*c*x^2)/(b - Sqrt[b^2 + 4*a*c]])*Sqrt[1 - (2*c*x^2)/(b + Sqrt[b^2 + 4*a*c]])*EllipticF[ArcSin[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 + 4*a*c]]], (b + Sqrt[b^2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c]))/(3*Sqrt[2]*c^(5/2)*Sqrt[a + b*x^2 - c*x^4])
```

Rule 1206

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Simp[(e^q*x^(2*q - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(4*p + 2*q + 1)), x] + Dist[1/(c*(4*p + 2*q + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*p + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1]
```

Rule 1202

```
Int[((d_) + (e_)*(x_)^2)/Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4], x_Symbol]
:> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(Sqrt[1 + (2*c*x^2)/(b - q)]*Sqrt[1 + (2*c*x^2)/(b + q)])/Sqrt[a + b*x^2 + c*x^4], Int[(d + e*x^2)/(Sqrt[1 + (2*c*x^2)/(b - q)]*Sqrt[1 + (2*c*x^2)/(b + q)]), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[c/a]
```

Rule 524

```
Int[((e_) + (f_)*(x_)^(n_))/(Sqrt[(a_) + (b_)*(x_)^(n_)]*Sqrt[(c_) + (d_)*(x_)^(n_)]), x_Symbol]
:> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n], x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && !(EqQ[n, 2] && ((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && (!GtQ[c, 0] || SimplerSqrtQ[-(b/a), -(d/c)]))))))
```

Rule 424

```
Int[Sqrt[(a_) + (b_)*(x_)^2]/Sqrt[(c_) + (d_)*(x_)^2], x_Symbol]
:> Simp[(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_)*(x_)^2]*Sqrt[(c_) + (d_)*(x_)^2]), x_Symbol]
:> Simp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0] && !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{(d + ex^2)^2}{\sqrt{a + bx^2 - cx^4}} dx = -\frac{e^2x\sqrt{a + bx^2 - cx^4}}{3c} - \frac{\int \frac{-3cd^2 - ae^2 - 2e(3cd + be)x^2}{\sqrt{a + bx^2 - cx^4}} dx}{3c}$$

$$= -\frac{e^2x\sqrt{a + bx^2 - cx^4}}{3c} - \frac{\left(\sqrt{1 - \frac{2cx^2}{b - \sqrt{b^2 + 4ac}}}\sqrt{1 - \frac{2cx^2}{b + \sqrt{b^2 + 4ac}}}\right) \int \frac{-3cd^2 - ae^2 - 2e(3cd + be)x^2}{\sqrt{1 - \frac{2cx^2}{b - \sqrt{b^2 + 4ac}}}\sqrt{1 - \frac{2cx^2}{b + \sqrt{b^2 + 4ac}}}} dx}{3c\sqrt{a + bx^2 - cx^4}}$$

$$= -\frac{e^2x\sqrt{a + bx^2 - cx^4}}{3c} - \frac{\left((b - \sqrt{b^2 + 4ac})e(3cd + be)\sqrt{1 - \frac{2cx^2}{b - \sqrt{b^2 + 4ac}}}\sqrt{1 - \frac{2cx^2}{b + \sqrt{b^2 + 4ac}}}\right) \int \frac{\sqrt{1 - \frac{2cx^2}{b - \sqrt{b^2 + 4ac}}}}{\sqrt{1 - \frac{2cx^2}{b + \sqrt{b^2 + 4ac}}}} dx}{3c^2\sqrt{a + bx^2 - cx^4}}$$

$$= -\frac{e^2x\sqrt{a + bx^2 - cx^4}}{3c} - \frac{(b - \sqrt{b^2 + 4ac})\sqrt{b + \sqrt{b^2 + 4ac}}e(3cd + be)\sqrt{1 - \frac{2cx^2}{b - \sqrt{b^2 + 4ac}}}\sqrt{1 - \frac{2cx^2}{b + \sqrt{b^2 + 4ac}}}}{3\sqrt{2}c^{5/2}\sqrt{a + bx^2 - cx^4}}$$

Mathematica [C] time = 1.39612, size = 503, normalized size = 1.11

$$i\sqrt{2}\sqrt{\frac{\sqrt{4ac + b^2 + b - 2cx^2}}{\sqrt{4ac + b^2 + b}}}\sqrt{\frac{\sqrt{4ac + b^2 - b + 2cx^2}}{\sqrt{4ac + b^2 - b}}}\left(-ce\left(-3d\sqrt{4ac + b^2} + ae + 3bd\right) + be^2\left(\sqrt{4ac + b^2} - b\right) - 3c^2d^2\right)\text{EllipticF}\left(i\sinh^{-1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^2/Sqrt[a + b*x^2 - c*x^4], x]
```

```
[Out] (2*c*Sqrt[-(c/(b + Sqrt[b^2 + 4*a*c]))]*e^2*x*(-a - b*x^2 + c*x^4) - I*Sqrt[2]*(-b + Sqrt[b^2 + 4*a*c]))*(3*c*d + b*e)*Sqrt[(b + Sqrt[b^2 + 4*a*c] - 2*c*x^2)/(b + Sqrt[b^2 + 4*a*c])]*Sqrt[(-b + Sqrt[b^2 + 4*a*c] + 2*c*x^2)/(-b + Sqrt[b^2 + 4*a*c])]*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[-(c/(b + Sqrt[b^2 + 4*a*c]))]]*x], (b + Sqrt[b^2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c]) + I*Sqrt[2]*(-3*c^2*d^2 + b*(-b + Sqrt[b^2 + 4*a*c])*e^2 - c*e*(3*b*d - 3*Sqrt[b^2 + 4*a*c]*d + a*e))*Sqrt[(b + Sqrt[b^2 + 4*a*c] - 2*c*x^2)/(b + Sqrt[b^2 + 4*a*c])]*Sqrt[(-b + Sqrt[b^2 + 4*a*c] + 2*c*x^2)/(-b + Sqrt[b^2 + 4*a*c])]*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[-(c/(b + Sqrt[b^2 + 4*a*c]))]]*x], (b + Sqrt[b^2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c]))/(6*c^2*Sqrt[-(c/(b + Sqrt[b^2 + 4*a*c]))]*Sqrt[a + b*x^2 - c*x^4])
```

Maple [A] time = 0.01, size = 761, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^2/(-c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] e^2*(-1/3/c*x*(-c*x^4+b*x^2+a)^(1/2)+1/12/c*a*2^(1/2)/((-b+(4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(-c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4-2*b*(b+(4*a*c+b^2)^(1/2))/a/c)^(1/2))-1/3*b/c*a*2^(1/2)/((-b+(4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(-c*x^4+b*x^2+a)^(1/2)/(b+(4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*((-b+(4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4-2*b*(b+(4*a*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*x*2^(1/2)*((-b+(4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4-2*b*(b+(4*a*c+b^2)^(1/2))/a/c)^(1/2))))-d*e*a*2^(1/2)/((-b+(4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(-c*x^4+b*x^2+a)^(1/2)/(b+(4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*((-b+(4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4-2*b*(b+(4*a*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*x*2^(1/2)*((-b+(4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4-2*b*(b+(4*a*c+b^2)^(1/2))/a/c)^(1/2)))+1/4*d^2*2^(1/2)/((-b+(4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(-c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4-2*b*(b+(4*a*c+b^2)^(1/2))/a/c)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^2 + d)^2}{\sqrt{-cx^4 + bx^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)^2/sqrt(-c*x^4 + b*x^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{(e^2x^4 + 2dex^2 + d^2)\sqrt{-cx^4 + bx^2 + a}}{cx^4 - bx^2 - a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)^2/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")

[Out] integral(-(e^2*x^4 + 2*d*e*x^2 + d^2)*sqrt(-c*x^4 + b*x^2 + a)/(c*x^4 - b*x^2 - a), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(d + ex^2)^2}{\sqrt{a + bx^2 - cx^4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x**2+d)**2/(-c*x**4+b*x**2+a)**(1/2),x)

[Out] Integral((d + e*x**2)**2/sqrt(a + b*x**2 - c*x**4), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^2 + d)^2}{\sqrt{-cx^4 + bx^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)^2/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")

[Out] integrate((e*x^2 + d)^2/sqrt(-c*x^4 + b*x^2 + a), x)

$$3.387 \quad \int \frac{d+ex^2}{\sqrt{a+bx^2-cx^4}} dx$$

Optimal. Leaf size=385

$$\frac{\sqrt{\sqrt{4ac+b^2}+b}\sqrt{1-\frac{2cx^2}{b-\sqrt{4ac+b^2}}}\sqrt{1-\frac{2cx^2}{\sqrt{4ac+b^2}+b}}\left(e\left(b-\sqrt{4ac+b^2}\right)+2cd\right)\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{\sqrt{4ac+b^2}+b}}\right),\frac{\sqrt{4ac+b^2}+b}{b-\sqrt{4ac+b^2}}\right)}{2\sqrt{2}c^{3/2}\sqrt{a+bx^2-cx^4}}$$

```
[Out] -((b - Sqrt[b^2 + 4*a*c])*Sqrt[b + Sqrt[b^2 + 4*a*c]]*e*Sqrt[1 - (2*c*x^2)/(b - Sqrt[b^2 + 4*a*c]])*Sqrt[1 - (2*c*x^2)/(b + Sqrt[b^2 + 4*a*c]])*EllipticE[ArcSin[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 + 4*a*c]]], (b + Sqrt[b^2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c])]/(2*Sqrt[2]*c^(3/2)*Sqrt[a + b*x^2 - c*x^4]) + (Sqrt[b + Sqrt[b^2 + 4*a*c]]*(2*c*d + (b - Sqrt[b^2 + 4*a*c])*e)*Sqrt[1 - (2*c*x^2)/(b - Sqrt[b^2 + 4*a*c]])*Sqrt[1 - (2*c*x^2)/(b + Sqrt[b^2 + 4*a*c]])*EllipticF[ArcSin[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 + 4*a*c]]], (b + Sqrt[b^2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c])]/(2*Sqrt[2]*c^(3/2)*Sqrt[a + b*x^2 - c*x^4])
```

Rubi [A] time = 0.337288, antiderivative size = 385, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.16$, Rules used = {1202, 524, 424, 419}

$$\frac{\sqrt{\sqrt{4ac+b^2}+b}\sqrt{1-\frac{2cx^2}{b-\sqrt{4ac+b^2}}}\sqrt{1-\frac{2cx^2}{\sqrt{4ac+b^2}+b}}\left(e\left(b-\sqrt{4ac+b^2}\right)+2cd\right)F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b+\sqrt{b^2+4ac}}}\right)\middle|\frac{b+\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)}{2\sqrt{2}c^{3/2}\sqrt{a+bx^2-cx^4}} - \frac{e\left(b-\sqrt{4ac+b^2}\right)}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)/Sqrt[a + b*x^2 - c*x^4], x]
```

```
[Out] -((b - Sqrt[b^2 + 4*a*c])*Sqrt[b + Sqrt[b^2 + 4*a*c]]*e*Sqrt[1 - (2*c*x^2)/(b - Sqrt[b^2 + 4*a*c]])*Sqrt[1 - (2*c*x^2)/(b + Sqrt[b^2 + 4*a*c]])*EllipticE[ArcSin[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 + 4*a*c]]], (b + Sqrt[b^2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c])]/(2*Sqrt[2]*c^(3/2)*Sqrt[a + b*x^2 - c*x^4]) + (Sqrt[b + Sqrt[b^2 + 4*a*c]]*(2*c*d + (b - Sqrt[b^2 + 4*a*c])*e)*Sqrt[1 - (2*c*x^2)/(b - Sqrt[b^2 + 4*a*c]])*Sqrt[1 - (2*c*x^2)/(b + Sqrt[b^2 + 4*a*c]])*EllipticF[ArcSin[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 + 4*a*c]]], (b + Sqrt[b^2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c])]/(2*Sqrt[2]*c^(3/2)*Sqrt[a + b*x^2 - c*x^4])
```

Rule 1202

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(Sqrt[1 + (2*c*x^2)/(b - q)]*Sqrt[1 + (2*c*x^2)/(b + q)])/Sqrt[a + b*x^2 + c*x^4], Int[(d + e*x^2)/(Sqrt[1 + (2*c*x^2)/(b - q)]*Sqrt[1 + (2*c*x^2)/(b + q)]), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[c/a]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)])*Sqrt[(c_) + (d_.)*(x_)^(n_)], x_Symbol] := Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n], x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && !(EqQ[n, 2] && ((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && (!GtQ[c, 0] || Simpler
```

SqrtQ[-(b/a), -(d/c)])))))

Rule 424

Int[Sqrt[(a_) + (b_)*(x_)^2]/Sqrt[(c_) + (d_)*(x_)^2], x_Symbol] :> Simp[(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]

Rule 419

Int[1/(Sqrt[(a_) + (b_)*(x_)^2]*Sqrt[(c_) + (d_)*(x_)^2]), x_Symbol] :> Simp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0] && !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])

Rubi steps

$$\int \frac{d + ex^2}{\sqrt{a + bx^2 - cx^4}} dx = \frac{\left(\sqrt{1 - \frac{2cx^2}{b - \sqrt{b^2 + 4ac}}}\sqrt{1 - \frac{2cx^2}{b + \sqrt{b^2 + 4ac}}}\right) \int \frac{d + ex^2}{\sqrt{1 - \frac{2cx^2}{b - \sqrt{b^2 + 4ac}}}\sqrt{1 - \frac{2cx^2}{b + \sqrt{b^2 + 4ac}}}} dx}{\sqrt{a + bx^2 - cx^4}}$$

$$= -\frac{\left((b - \sqrt{b^2 + 4ac})\left(-\frac{2cd}{b - \sqrt{b^2 + 4ac}} - e\right)\sqrt{1 - \frac{2cx^2}{b - \sqrt{b^2 + 4ac}}}\sqrt{1 - \frac{2cx^2}{b + \sqrt{b^2 + 4ac}}}\right) \int \frac{1}{\sqrt{1 - \frac{2cx^2}{b - \sqrt{b^2 + 4ac}}}\sqrt{1 - \frac{2cx^2}{b + \sqrt{b^2 + 4ac}}}} dx}{2c\sqrt{a + bx^2 - cx^4}}$$

$$= -\frac{(b - \sqrt{b^2 + 4ac})\sqrt{b + \sqrt{b^2 + 4ac}}\sqrt{1 - \frac{2cx^2}{b - \sqrt{b^2 + 4ac}}}\sqrt{1 - \frac{2cx^2}{b + \sqrt{b^2 + 4ac}}}E\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b + \sqrt{b^2 + 4ac}}}\right)\right)}{2\sqrt{2}c^{3/2}\sqrt{a + bx^2 - cx^4}}$$

Mathematica [C] time = 0.260553, size = 293, normalized size = 0.76

$$\frac{i\sqrt{\frac{2cx^2}{\sqrt{4ac+b^2}-b}} + 1\sqrt{1 - \frac{2cx^2}{\sqrt{4ac+b^2}+b}} \left(e(b - \sqrt{4ac + b^2}) + 2cd \right) \text{EllipticF}\left(i \sinh^{-1}\left(\sqrt{2}x\sqrt{\frac{c}{\sqrt{4ac+b^2}+b}}\right), \frac{\sqrt{4ac+b^2}+b}{b - \sqrt{4ac+b^2}}\right) + e\left(\sqrt{4ac+b^2} - b\right)}{2\sqrt{2}c\sqrt{-\frac{c}{\sqrt{4ac+b^2}+b}}\sqrt{a + bx^2 - cx^4}}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e*x^2)/Sqrt[a + b*x^2 - c*x^4], x]

[Out] ((-I/2)*Sqrt[1 + (2*c*x^2)/(-b + Sqrt[b^2 + 4*a*c])]*Sqrt[1 - (2*c*x^2)/(b + Sqrt[b^2 + 4*a*c])]*((-b + Sqrt[b^2 + 4*a*c])*e*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[-(c/(b + Sqrt[b^2 + 4*a*c])])*x], (b + Sqrt[b^2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c])]) + (2*c*d + (b - Sqrt[b^2 + 4*a*c])*e)*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[-(c/(b + Sqrt[b^2 + 4*a*c])])*x], (b + Sqrt[b^2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c])]))/(Sqrt[2]*c*Sqrt[-(c/(b + Sqrt[b^2 + 4*a*c])])*Sqrt[a + b*x^2 - c*x^4])

Maple [A] time = 0.006, size = 364, normalized size = 1.

$$-\frac{ae\sqrt{2}}{2}\sqrt{4 - 2\frac{(-b + \sqrt{4ac + b^2})x^2}{a}}\sqrt{4 + 2\frac{(b + \sqrt{4ac + b^2})x^2}{a}}\left(\text{EllipticF}\left(\frac{x\sqrt{2}}{2}\sqrt{\frac{1}{a}(-b + \sqrt{4ac + b^2})}, \frac{1}{2}\sqrt{-4 - 2\frac{b}{a}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*x^2+d)/(-c*x^4+b*x^2+a)^(1/2),x)`

[Out]
$$-1/2*e*a*2^{(1/2)}/((-b+(4*a*c+b^2)^{(1/2)})/a)^{(1/2)}*(4-2*(-b+(4*a*c+b^2)^{(1/2)})/a*x^2)^{(1/2)}*(4+2*(b+(4*a*c+b^2)^{(1/2)})/a*x^2)^{(1/2)}/(-c*x^4+b*x^2+a)^{(1/2)}/(b+(4*a*c+b^2)^{(1/2)})*(\text{EllipticF}(1/2*x*2^{(1/2)}*((-b+(4*a*c+b^2)^{(1/2)})/a)^{(1/2)}, 1/2*(-4-2*b*(b+(4*a*c+b^2)^{(1/2)})/a/c)^{(1/2)})-\text{EllipticE}(1/2*x*2^{(1/2)}*((-b+(4*a*c+b^2)^{(1/2)})/a)^{(1/2)}, 1/2*(-4-2*b*(b+(4*a*c+b^2)^{(1/2)})/a/c)^{(1/2)}))+1/4*d*2^{(1/2)}/((-b+(4*a*c+b^2)^{(1/2)})/a)^{(1/2)}*(4-2*(-b+(4*a*c+b^2)^{(1/2)})/a*x^2)^{(1/2)}*(4+2*(b+(4*a*c+b^2)^{(1/2)})/a*x^2)^{(1/2)}/(-c*x^4+b*x^2+a)^{(1/2)}*\text{EllipticF}(1/2*x*2^{(1/2)}*((-b+(4*a*c+b^2)^{(1/2)})/a)^{(1/2)}, 1/2*(-4-2*b*(b+(4*a*c+b^2)^{(1/2)})/a/c)^{(1/2)})$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex^2 + d}{\sqrt{-cx^4 + bx^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x^2+d)/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")`

[Out] `integrate((e*x^2 + d)/sqrt(-c*x^4 + b*x^2 + a), x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-cx^4 + bx^2 + a}(ex^2 + d)}{cx^4 - bx^2 - a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x^2+d)/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")`

[Out] `integral(-sqrt(-c*x^4 + b*x^2 + a)*(e*x^2 + d)/(c*x^4 - b*x^2 - a), x)`

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{d + ex^2}{\sqrt{a + bx^2 - cx^4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x**2+d)/(-c*x**4+b*x**2+a)**(1/2),x)`

[Out] `Integral((d + e*x**2)/sqrt(a + b*x**2 - c*x**4), x)`

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex^2 + d}{\sqrt{-cx^4 + bx^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)/sqrt(-c*x^4 + b*x^2 + a), x)
```

$$3.388 \quad \int \frac{1}{(d+ex^2)\sqrt{a+bx^2-cx^4}} dx$$

Optimal. Leaf size=197

$$\frac{\sqrt{\sqrt{4ac+b^2}+b}\sqrt{1-\frac{2cx^2}{b-\sqrt{4ac+b^2}}}\sqrt{1-\frac{2cx^2}{\sqrt{4ac+b^2}+b}}\Pi\left(-\frac{(b+\sqrt{b^2+4ac})e}{2cd}; \sin^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b+\sqrt{b^2+4ac}}}\right)\middle|\frac{b+\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)}{\sqrt{2}\sqrt{cd}\sqrt{a+bx^2-cx^4}}$$

[Out] (Sqrt[b + Sqrt[b^2 + 4*a*c]]*Sqrt[1 - (2*c*x^2)/(b - Sqrt[b^2 + 4*a*c]])*Sqrt[1 - (2*c*x^2)/(b + Sqrt[b^2 + 4*a*c]])*EllipticPi[-((b + Sqrt[b^2 + 4*a*c])*e)/(2*c*d), ArcSin[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 + 4*a*c]]], (b + Sqrt[b^2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c])]/(Sqrt[2]*Sqrt[c]*d*Sqrt[a + b*x^2 - c*x^4])

Rubi [A] time = 0.157439, antiderivative size = 197, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {1220, 537}

$$\frac{\sqrt{\sqrt{4ac+b^2}+b}\sqrt{1-\frac{2cx^2}{b-\sqrt{4ac+b^2}}}\sqrt{1-\frac{2cx^2}{\sqrt{4ac+b^2}+b}}\Pi\left(-\frac{(b+\sqrt{b^2+4ac})e}{2cd}; \sin^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b+\sqrt{b^2+4ac}}}\right)\middle|\frac{b+\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)}{\sqrt{2}\sqrt{cd}\sqrt{a+bx^2-cx^4}}$$

Antiderivative was successfully verified.

[In] Int[1/((d + e*x^2)*Sqrt[a + b*x^2 - c*x^4]),x]

[Out] (Sqrt[b + Sqrt[b^2 + 4*a*c]]*Sqrt[1 - (2*c*x^2)/(b - Sqrt[b^2 + 4*a*c]])*Sqrt[1 - (2*c*x^2)/(b + Sqrt[b^2 + 4*a*c]])*EllipticPi[-((b + Sqrt[b^2 + 4*a*c])*e)/(2*c*d), ArcSin[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 + 4*a*c]]], (b + Sqrt[b^2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c])]/(Sqrt[2]*Sqrt[c]*d*Sqrt[a + b*x^2 - c*x^4])

Rule 1220

Int[1/(((d_) + (e_)*(x_)^2)*Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(Sqrt[1 + (2*c*x^2)/(b - q)]*Sqrt[1 + (2*c*x^2)/(b + q)])/Sqrt[a + b*x^2 + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + (2*c*x^2)/(b - q)]*Sqrt[1 + (2*c*x^2)/(b + q)]), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[c/a]

Rule 537

Int[1/(((a_) + (b_)*(x_)^2)*Sqrt[(c_) + (d_)*(x_)^2]*Sqrt[(e_) + (f_)*(x_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x], (c*f)/(d*e)])/((a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d, e, f}, x] && !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] && !(GtQ[f/e, 0] && SimplerSqrtQ[-(f/e), -(d/c)])

Rubi steps

$$\int \frac{1}{(d+ex^2)\sqrt{a+bx^2-cx^4}} dx = \frac{\left(\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}}\right) \int \frac{1}{\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}}(d+ex^2)} dx}{\sqrt{a+bx^2-cx^4}}$$

$$= \frac{\sqrt{b+\sqrt{b^2+4ac}}\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}}\Pi\left(-\frac{(b+\sqrt{b^2+4ac})e}{2cd}; \sin^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b+\sqrt{b^2+4ac}}}\right)\right)}{\sqrt{2}\sqrt{cd}\sqrt{a+bx^2-cx^4}}$$

Mathematica [C] time = 0.229555, size = 205, normalized size = 1.04

$$\frac{i\sqrt{\frac{2cx^2}{\sqrt{4ac+b^2}-b}} + 1\sqrt{1-\frac{2cx^2}{\sqrt{4ac+b^2}+b}}\Pi\left(-\frac{(b+\sqrt{b^2+4ac})e}{2cd}; i\sinh^{-1}\left(\sqrt{2}\sqrt{-\frac{c}{b+\sqrt{b^2+4ac}}}\right)\right) - \frac{b+\sqrt{b^2+4ac}}{\sqrt{b^2+4ac}-b}}{\sqrt{2}d\sqrt{-\frac{c}{\sqrt{4ac+b^2}+b}}\sqrt{a+bx^2-cx^4}}$$

Antiderivative was successfully verified.

[In] Integrate[1/((d + e*x^2)*Sqrt[a + b*x^2 - c*x^4]),x]

[Out] ((-I)*Sqrt[1 + (2*c*x^2)/(-b + Sqrt[b^2 + 4*a*c])]*Sqrt[1 - (2*c*x^2)/(b + Sqrt[b^2 + 4*a*c])]*EllipticPi[-((b + Sqrt[b^2 + 4*a*c])*e)/(2*c*d), I*ArcSinh[Sqrt[2]*Sqrt[-(c/(b + Sqrt[b^2 + 4*a*c]))]*x], -(b + Sqrt[b^2 + 4*a*c])/(-b + Sqrt[b^2 + 4*a*c])])/(Sqrt[2]*Sqrt[-(c/(b + Sqrt[b^2 + 4*a*c]))]*d*Sqrt[a + b*x^2 - c*x^4])

Maple [A] time = 0.029, size = 201, normalized size = 1.

$$\frac{\sqrt{2}}{d}\sqrt{1+\frac{bx^2}{2a}-\frac{x^2}{2a}\sqrt{4ac+b^2}}\sqrt{1+\frac{bx^2}{2a}+\frac{x^2}{2a}\sqrt{4ac+b^2}}\text{EllipticPi}\left(\frac{x\sqrt{2}}{2}\sqrt{\frac{1}{a}(-b+\sqrt{4ac+b^2})}, -2\frac{ae}{(-b+\sqrt{4ac+b^2})}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(e*x^2+d)/(-c*x^4+b*x^2+a)^(1/2),x)

[Out] 1/d*2^(1/2)/(-b/a+1/a*(4*a*c+b^2)^(1/2))^(1/2)*(1+1/2/a*x^2*b-1/2/a*x^2*(4*a*c+b^2)^(1/2))^(1/2)*(1+1/2/a*x^2*b+1/2/a*x^2*(4*a*c+b^2)^(1/2))^(1/2)/(-c*x^4+b*x^2+a)^(1/2)*EllipticPi(1/2*x^2^(1/2)*((-b+(4*a*c+b^2)^(1/2))/a)^(1/2), -2/(-b+(4*a*c+b^2)^(1/2))*a/d*e, (-1/2*(b+(4*a*c+b^2)^(1/2))/a)^(1/2)*2^(1/2)/((-b+(4*a*c+b^2)^(1/2))/a)^(1/2))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-cx^4+bx^2+a}(ex^2+d)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x^2+d)/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")

[Out] integrate(1/(sqrt(-c*x^4 + b*x^2 + a)*(e*x^2 + d)), x)

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x^2+d)/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")

[Out] Timed out

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(d + ex^2) \sqrt{a + bx^2 - cx^4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x**2+d)/(-c*x**4+b*x**2+a)**(1/2),x)

[Out] Integral(1/((d + e*x**2)*sqrt(a + b*x**2 - c*x**4)), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-cx^4 + bx^2 + a}(ex^2 + d)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x^2+d)/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")

[Out] integrate(1/(sqrt(-c*x^4 + b*x^2 + a)*(e*x^2 + d)), x)

$$3.389 \quad \int \frac{1}{(d+ex^2)^2 \sqrt{a+bx^2-cx^4}} dx$$

Optimal. Leaf size=718

$$\frac{\sqrt{\sqrt{4ac+b^2}+b} \sqrt{1-\frac{2cx^2}{b-\sqrt{4ac+b^2}}} \sqrt{1-\frac{2cx^2}{\sqrt{4ac+b^2}+b}} \left(e \left(b - \sqrt{4ac+b^2} \right) + 2cd \right) \text{EllipticF} \left(\sin^{-1} \left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{\sqrt{4ac+b^2}+b}} \right), \frac{\sqrt{4ac+b^2}+b}{b-\sqrt{4ac+b^2}} \right)}{4\sqrt{2}\sqrt{cd}\sqrt{a+bx^2-cx^4} (e(bd-ae) + cd^2)}$$

```
[Out] -(e^2*x*Sqrt[a + b*x^2 - c*x^4])/(2*d*(c*d^2 + b*d*e - a*e^2)*(d + e*x^2))
+ ((b - Sqrt[b^2 + 4*a*c])*Sqrt[b + Sqrt[b^2 + 4*a*c]]*e*Sqrt[1 - (2*c*x^2)
/(b - Sqrt[b^2 + 4*a*c]])*Sqrt[1 - (2*c*x^2)/(b + Sqrt[b^2 + 4*a*c]])*Ellip
ticE[ArcSin[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 + 4*a*c]]], (b + Sqrt[b^2
+ 4*a*c])/(b - Sqrt[b^2 + 4*a*c]))/(4*Sqrt[2]*Sqrt[c]*d*(c*d^2 + e*(b*d -
a*e))*Sqrt[a + b*x^2 - c*x^4]) - (Sqrt[b + Sqrt[b^2 + 4*a*c]]*(2*c*d + (b
- Sqrt[b^2 + 4*a*c])*e)*Sqrt[1 - (2*c*x^2)/(b - Sqrt[b^2 + 4*a*c]])*Sqrt[1
- (2*c*x^2)/(b + Sqrt[b^2 + 4*a*c]])*EllipticF[ArcSin[(Sqrt[2]*Sqrt[c]*x)/S
qrt[b + Sqrt[b^2 + 4*a*c]]], (b + Sqrt[b^2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c]
)))/(4*Sqrt[2]*Sqrt[c]*d*(c*d^2 + e*(b*d - a*e))*Sqrt[a + b*x^2 - c*x^4]) +
(Sqrt[b + Sqrt[b^2 + 4*a*c]]*(3*c*d^2 + e*(2*b*d - a*e))*Sqrt[1 - (2*c*x^2
)/(b - Sqrt[b^2 + 4*a*c]])*Sqrt[1 - (2*c*x^2)/(b + Sqrt[b^2 + 4*a*c]])*Elli
pticPi[-((b + Sqrt[b^2 + 4*a*c])*e)/(2*c*d), ArcSin[(Sqrt[2]*Sqrt[c]*x)/Sqr
t[b + Sqrt[b^2 + 4*a*c]]], (b + Sqrt[b^2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c])])
)/(2*Sqrt[2]*Sqrt[c]*d^2*(c*d^2 + e*(b*d - a*e))*Sqrt[a + b*x^2 - c*x^4])
```

Rubi [A] time = 1.01523, antiderivative size = 718, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$, Rules used = {1223, 1716, 1202, 524, 424, 419, 1220, 537}

$$\frac{\sqrt{\sqrt{4ac+b^2}+b} \sqrt{1-\frac{2cx^2}{b-\sqrt{4ac+b^2}}} \sqrt{1-\frac{2cx^2}{\sqrt{4ac+b^2}+b}} \left(e \left(b - \sqrt{4ac+b^2} \right) + 2cd \right) F \left(\sin^{-1} \left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b+\sqrt{b^2+4ac}}} \right), \frac{b+\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}} \right)}{4\sqrt{2}\sqrt{cd}\sqrt{a+bx^2-cx^4} (e(bd-ae) + cd^2)} + \frac{e \left(b - \sqrt{4ac+b^2} \right)}{4\sqrt{2}\sqrt{cd}\sqrt{a+bx^2-cx^4} (e(bd-ae) + cd^2)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x^2)^2*Sqrt[a + b*x^2 - c*x^4]),x]
```

```
[Out] -(e^2*x*Sqrt[a + b*x^2 - c*x^4])/(2*d*(c*d^2 + e*(b*d - a*e))*(d + e*x^2))
+ ((b - Sqrt[b^2 + 4*a*c])*Sqrt[b + Sqrt[b^2 + 4*a*c]]*e*Sqrt[1 - (2*c*x^2)
/(b - Sqrt[b^2 + 4*a*c]])*Sqrt[1 - (2*c*x^2)/(b + Sqrt[b^2 + 4*a*c]])*Ellip
ticE[ArcSin[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 + 4*a*c]]], (b + Sqrt[b^2
+ 4*a*c])/(b - Sqrt[b^2 + 4*a*c]))/(4*Sqrt[2]*Sqrt[c]*d*(c*d^2 + e*(b*d -
a*e))*Sqrt[a + b*x^2 - c*x^4]) - (Sqrt[b + Sqrt[b^2 + 4*a*c]]*(2*c*d + (b
- Sqrt[b^2 + 4*a*c])*e)*Sqrt[1 - (2*c*x^2)/(b - Sqrt[b^2 + 4*a*c]])*Sqrt[1
- (2*c*x^2)/(b + Sqrt[b^2 + 4*a*c]])*EllipticF[ArcSin[(Sqrt[2]*Sqrt[c]*x)/S
qrt[b + Sqrt[b^2 + 4*a*c]]], (b + Sqrt[b^2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c]
)))/(4*Sqrt[2]*Sqrt[c]*d*(c*d^2 + e*(b*d - a*e))*Sqrt[a + b*x^2 - c*x^4]) +
(Sqrt[b + Sqrt[b^2 + 4*a*c]]*(3*c*d^2 + e*(2*b*d - a*e))*Sqrt[1 - (2*c*x^2
)/(b - Sqrt[b^2 + 4*a*c]])*Sqrt[1 - (2*c*x^2)/(b + Sqrt[b^2 + 4*a*c]])*Elli
pticPi[-((b + Sqrt[b^2 + 4*a*c])*e)/(2*c*d), ArcSin[(Sqrt[2]*Sqrt[c]*x)/Sqr
t[b + Sqrt[b^2 + 4*a*c]]], (b + Sqrt[b^2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c])])
)/(2*Sqrt[2]*Sqrt[c]*d^2*(c*d^2 + e*(b*d - a*e))*Sqrt[a + b*x^2 - c*x^4])
```

Rule 1223

```
Int[((d_) + (e_)*(x_)^2)^q/Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4], x_
Symbol] := -Simp[(e^2*x*(d + e*x^2)^(q + 1)*Sqrt[a + b*x^2 + c*x^4])/(2*d*(
q + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(2*d*(q + 1)*(c*d^2 - b*d*e +
a*e^2)), Int[((d + e*x^2)^(q + 1)*Simp[a*e^2*(2*q + 3) + 2*d*(c*d - b*e)*(q
+ 1) - 2*e*(c*d*(q + 1) - b*e*(q + 2))*x^2 + c*e^2*(2*q + 5)*x^4, x)]/Sqrt
[a + b*x^2 + c*x^4], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c
, 0] && ILtQ[q, -1]
```

Rule 1716

```
Int[(P4x_)/(((d_) + (e_)*(x_)^2)*Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4])
, x_Symbol] := With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[
P4x, x, 4]}, -Dist[(e^2)^(-1), Int[(C*d - B*e - C*e*x^2)/Sqrt[a + b*x^2 + c
*x^4], x], x] + Dist[(C*d^2 - B*d*e + A*e^2)/e^2, Int[1/((d + e*x^2)*Sqrt[a
+ b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[P4x, x^2,
2] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 -
a*e^2, 0]
```

Rule 1202

```
Int[((d_) + (e_)*(x_)^2)/Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4], x_Symbo
l] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(Sqrt[1 + (2*c*x^2)/(b - q)]*Sqrt
[1 + (2*c*x^2)/(b + q)])/Sqrt[a + b*x^2 + c*x^4], Int[(d + e*x^2)/(Sqrt[1 +
(2*c*x^2)/(b - q)]*Sqrt[1 + (2*c*x^2)/(b + q)]), x], x]] /; FreeQ[{a, b, c
, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[c/a]
```

Rule 524

```
Int[((e_) + (f_)*(x_)^(n_))/(Sqrt[(a_) + (b_)*(x_)^(n_)]*Sqrt[(c_) + (d_
)*(x_)^(n_)]), x_Symbol] := Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && (!GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 424

```
Int[Sqrt[(a_) + (b_)*(x_)^2]/Sqrt[(c_) + (d_)*(x_)^2], x_Symbol] := Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_)*(x_)^2]*Sqrt[(c_) + (d_)*(x_)^2]), x_Symbol] := S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] && !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 1220

```
Int[1/(((d_) + (e_)*(x_)^2)*Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4]), x_S
ymbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(Sqrt[1 + (2*c*x^2)/(b - q)]*
Sqrt[1 + (2*c*x^2)/(b + q)])/Sqrt[a + b*x^2 + c*x^4], Int[1/((d + e*x^2)*Sq
rt[1 + (2*c*x^2)/(b - q)]*Sqrt[1 + (2*c*x^2)/(b + q)]), x], x]] /; FreeQ[{a
, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[c/a]
```

Rule 537

```
Int[1/(((a_) + (b_)*(x_)^2)*Sqrt[(c_) + (d_)*(x_)^2]*Sqrt[(e_) + (f_)*(x_
```

```
_)^2]], x_Symbol] := Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e))]/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] && !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] && !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rubi steps

$$\int \frac{1}{(d+ex^2)^2 \sqrt{a+bx^2-cx^4}} dx = -\frac{e^2 x \sqrt{a+bx^2-cx^4}}{2d(cd^2+e(bd-ae))(d+ex^2)} + \frac{\int \frac{2cd^2+e(2bd-ae)-2cdex^2-ce^2x^4}{(d+ex^2)\sqrt{a+bx^2-cx^4}} dx}{2d(cd^2+e(bd-ae))}$$

$$= -\frac{e^2 x \sqrt{a+bx^2-cx^4}}{2d(cd^2+e(bd-ae))(d+ex^2)} - \frac{\int \frac{cde^2+ce^3x^2}{\sqrt{a+bx^2-cx^4}} dx}{2de^2(cd^2+e(bd-ae))} + \frac{(3cd^2+e(2bd-ae)) \int \frac{1}{(d+ex^2)\sqrt{a+bx^2-cx^4}} dx}{2d(cd^2+e(bd-ae))}$$

$$= -\frac{e^2 x \sqrt{a+bx^2-cx^4}}{2d(cd^2+e(bd-ae))(d+ex^2)} - \frac{\left(\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}}\right) \int \frac{cde^2+ce^3}{\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}}} dx}{2de^2(cd^2+e(bd-ae))\sqrt{a+bx^2-cx^4}}$$

$$= -\frac{e^2 x \sqrt{a+bx^2-cx^4}}{2d(cd^2+e(bd-ae))(d+ex^2)} + \frac{\sqrt{b+\sqrt{b^2+4ac}}(3cd^2+e(2bd-ae))\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}}{2\sqrt{2}\sqrt{cd^2}(cd^2+e(bd-ae))}$$

$$= -\frac{e^2 x \sqrt{a+bx^2-cx^4}}{2d(cd^2+e(bd-ae))(d+ex^2)} + \frac{(b-\sqrt{b^2+4ac})\sqrt{b+\sqrt{b^2+4ac}}e\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}}{4\sqrt{2}\sqrt{cd}(cd^2+e(bd-ae))}$$

Mathematica [C] time = 5.59278, size = 464, normalized size = 0.65

$$\sqrt{a+bx^2-cx^4} \left(4de^2x + \frac{i(d+ex^2)\sqrt{\frac{4cx^2}{\sqrt{4ac+b^2}-b}}+2\sqrt{1-\frac{2cx^2}{\sqrt{4ac+b^2}+b}}\left(d\left(e\left(b-\sqrt{4ac+b^2}\right)+2cd\right)\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{2}x\sqrt{-\frac{c}{\sqrt{4ac+b^2}+b}}\right),\frac{\sqrt{4ac+b^2}+b}{b-\sqrt{4ac+b^2}}\right)+2\left(eae-\sqrt{-\frac{c}{\sqrt{4ac+b^2}+b}}\right)\right)}{8d^2(d+ex^2)(e(bd-ae))} \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x^2)^2*Sqrt[a + b*x^2 - c*x^4]),x]
```

```
[Out] -(Sqrt[a + b*x^2 - c*x^4]*(4*d*e^2*x + (I*Sqrt[2 + (4*c*x^2)/(-b + Sqrt[b^2 + 4*a*c]])*Sqrt[1 - (2*c*x^2)/(b + Sqrt[b^2 + 4*a*c]])*(d + e*x^2)*((-b + Sqrt[b^2 + 4*a*c])*d*e*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[-(c/(b + Sqrt[b^2 + 4*a*c]])]]*x), (b + Sqrt[b^2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c])) + d*(2*c*d + (b - Sqrt[b^2 + 4*a*c])*e)*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[-(c/(b + Sqrt[b^2 + 4*a*c]])]]*x), (b + Sqrt[b^2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c])) + 2*(-3*c*d^2 + e*(-2*b*d + a*e))*EllipticPi[-((b + Sqrt[b^2 + 4*a*c])*e)/(2*c*d), I*ArcSinh[Sqrt[2]*Sqrt[-(c/(b + Sqrt[b^2 + 4*a*c]])]]*x), (b + Sqrt[b^2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c])))/(Sqrt[-(c/(b + Sqrt[b^2 + 4*a*c]])]*(-a - b*x^2 + c*x^4)))/(8*d^2*(c*d^2 + e*(b*d - a*e))*(d + e*x^2))
```

Maple [B] time = 0.025, size = 1293, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(1/(e*x^2+d)^2/(-c*x^4+b*x^2+a)^{(1/2)}, x)$

[Out] $\frac{1}{2}e^2/(a^2e^2-b^2d^2)/d*x*(-c*x^4+b*x^2+a)^{(1/2)/(e*x^2+d)+1/8*c/(a^2e^2-b^2d^2)*2^{(1/2)/(-b/a+1/a*(4*a*c+b^2)^{(1/2)})^{(1/2)}*(4+2/a*x^2*b-2/a*x^2*(4*a*c+b^2)^{(1/2)})^{(1/2)}*(4+2/a*x^2*b+2/a*x^2*(4*a*c+b^2)^{(1/2)})^{(1/2)}/(-c*x^4+b*x^2+a)^{(1/2)}*EllipticF(1/2*x^2^{(1/2)*((-b+(4*a*c+b^2)^{(1/2)))/a)^{(1/2)}, 1/2*(-4-2*b*(b+(4*a*c+b^2)^{(1/2)))/a/c)^{(1/2)}-1/4*c*e/(a^2e^2-b^2d^2)/d*a^2^{(1/2)/(-b/a+1/a*(4*a*c+b^2)^{(1/2)})^{(1/2)}*(4+2/a*x^2*b-2/a*x^2*(4*a*c+b^2)^{(1/2)})^{(1/2)}*(4+2/a*x^2*b+2/a*x^2*(4*a*c+b^2)^{(1/2)})^{(1/2)}/(-c*x^4+b*x^2+a)^{(1/2)/(b+(4*a*c+b^2)^{(1/2))}*EllipticF(1/2*x^2^{(1/2)*((-b+(4*a*c+b^2)^{(1/2)))/a)^{(1/2)}, 1/2*(-4-2*b*(b+(4*a*c+b^2)^{(1/2)))/a/c)^{(1/2)}+1/4*c*e/(a^2e^2-b^2d^2)/d*a^2^{(1/2)/(-b/a+1/a*(4*a*c+b^2)^{(1/2)})^{(1/2)}*(4+2/a*x^2*b-2/a*x^2*(4*a*c+b^2)^{(1/2)})^{(1/2)}*(4+2/a*x^2*b+2/a*x^2*(4*a*c+b^2)^{(1/2)})^{(1/2)}/(-c*x^4+b*x^2+a)^{(1/2)/(b+(4*a*c+b^2)^{(1/2))}*EllipticE(1/2*x^2^{(1/2)*((-b+(4*a*c+b^2)^{(1/2)))/a)^{(1/2)}, 1/2*(-4-2*b*(b+(4*a*c+b^2)^{(1/2)))/a/c)^{(1/2)}+1/2/(a^2e^2-b^2d^2)/d^2*e^2*2^{(1/2)/(-b/a+1/a*(4*a*c+b^2)^{(1/2)})^{(1/2)}*(1+1/2/a*x^2*b-1/2/a*x^2*(4*a*c+b^2)^{(1/2)})^{(1/2)}*(1+1/2/a*x^2*b+1/2/a*x^2*(4*a*c+b^2)^{(1/2)})^{(1/2)}/(-c*x^4+b*x^2+a)^{(1/2)}*EllipticPi(1/2*x^2^{(1/2)*((-b+(4*a*c+b^2)^{(1/2)))/a)^{(1/2)}, -2/(-b+(4*a*c+b^2)^{(1/2))}*a/d*e, (-1/2*(b+(4*a*c+b^2)^{(1/2)))/a)^{(1/2)}*2^{(1/2)/((-b+(4*a*c+b^2)^{(1/2)))/a)^{(1/2)}*a-1/(a^2e^2-b^2d^2)/d*e^2^{(1/2)/(-b/a+1/a*(4*a*c+b^2)^{(1/2)})^{(1/2)}*(1+1/2/a*x^2*b-1/2/a*x^2*(4*a*c+b^2)^{(1/2)})^{(1/2)}*(1+1/2/a*x^2*b+1/2/a*x^2*(4*a*c+b^2)^{(1/2)})^{(1/2)}/(-c*x^4+b*x^2+a)^{(1/2)}*EllipticPi(1/2*x^2^{(1/2)*((-b+(4*a*c+b^2)^{(1/2)))/a)^{(1/2)}, -2/(-b+(4*a*c+b^2)^{(1/2))}*a/d*e, (-1/2*(b+(4*a*c+b^2)^{(1/2)))/a)^{(1/2)}*2^{(1/2)/((-b+(4*a*c+b^2)^{(1/2)))/a)^{(1/2)}*b-3/2/(a^2e^2-b^2d^2)*2^{(1/2)/(-b/a+1/a*(4*a*c+b^2)^{(1/2)})^{(1/2)}*(1+1/2/a*x^2*b-1/2/a*x^2*(4*a*c+b^2)^{(1/2)})^{(1/2)}*(1+1/2/a*x^2*b+1/2/a*x^2*(4*a*c+b^2)^{(1/2)})^{(1/2)}/(-c*x^4+b*x^2+a)^{(1/2)}*EllipticPi(1/2*x^2^{(1/2)*((-b+(4*a*c+b^2)^{(1/2)))/a)^{(1/2)}, -2/(-b+(4*a*c+b^2)^{(1/2))}*a/d*e, (-1/2*(b+(4*a*c+b^2)^{(1/2)))/a)^{(1/2)}*2^{(1/2)/((-b+(4*a*c+b^2)^{(1/2)))/a)^{(1/2)})*c$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-cx^4 + bx^2 + a}(ex^2 + d)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(1/(e*x^2+d)^2/(-c*x^4+b*x^2+a)^{(1/2)}, x, \text{algorithm}="maxima")$

[Out] $\text{integrate}(1/(\text{sqrt}(-c*x^4 + b*x^2 + a)*(e*x^2 + d)^2), x)$

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(1/(e*x^2+d)^2/(-c*x^4+b*x^2+a)^{(1/2)}, x, \text{algorithm}="fricas")$

[Out] Timed out

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(d + ex^2)^2 \sqrt{a + bx^2 - cx^4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x**2+d)**2/(-c*x**4+b*x**2+a)**(1/2), x)

[Out] Integral(1/((d + e*x**2)**2*sqrt(a + b*x**2 - c*x**4)), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-cx^4 + bx^2 + a}(ex^2 + d)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x^2+d)^2/(-c*x^4+b*x^2+a)^(1/2), x, algorithm="giac")

[Out] integrate(1/(sqrt(-c*x^4 + b*x^2 + a)*(e*x^2 + d)^2), x)

$$3.390 \quad \int \frac{d+ex^2}{\sqrt{-a+bx^2+cx^4}} dx$$

Optimal. Leaf size=479

$$\frac{d\sqrt{\sqrt{4ac+b^2}+b}\left(\frac{2cx^2}{b-\sqrt{4ac+b^2}}+1\right)\text{EllipticF}\left(\tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{\sqrt{4ac+b^2}+b}}\right),-\frac{2\sqrt{4ac+b^2}}{b-\sqrt{4ac+b^2}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\frac{\frac{2cx^2}{b-\sqrt{4ac+b^2}}+1}{\frac{2cx^2}{\sqrt{4ac+b^2}+b}}}\sqrt{-a+bx^2+cx^4}} + \frac{e\left(b-\sqrt{4ac+b^2}\right)\sqrt{\sqrt{4ac+b^2}+b}\left(\frac{2cx^2}{b-\sqrt{4ac+b^2}}+1\right)}{2\sqrt{2}c^{3/2}\sqrt{\frac{\frac{2cx^2}{b-\sqrt{4ac+b^2}}+1}{\frac{2cx^2}{\sqrt{4ac+b^2}+b}}}}$$

[Out] $((b - \text{Sqrt}[b^2 + 4*a*c]) * e * x * (1 + (2*c*x^2)/(b - \text{Sqrt}[b^2 + 4*a*c]))) / (2*c * \text{Sqrt}[-a + b*x^2 + c*x^4]) - ((b - \text{Sqrt}[b^2 + 4*a*c]) * \text{Sqrt}[b + \text{Sqrt}[b^2 + 4*a*c]] * e * (1 + (2*c*x^2)/(b - \text{Sqrt}[b^2 + 4*a*c])) * \text{EllipticE}[\text{ArcTan}[(\text{Sqrt}[2] * \text{Sqrt}[c] * x) / \text{Sqrt}[b + \text{Sqrt}[b^2 + 4*a*c]]], (-2 * \text{Sqrt}[b^2 + 4*a*c]) / (b - \text{Sqrt}[b^2 + 4*a*c])]) / (2 * \text{Sqrt}[2] * c^{(3/2)} * \text{Sqrt}[(1 + (2*c*x^2)/(b - \text{Sqrt}[b^2 + 4*a*c])) / (1 + (2*c*x^2)/(b + \text{Sqrt}[b^2 + 4*a*c]))] * \text{Sqrt}[-a + b*x^2 + c*x^4]) + (\text{Sqrt}[b + \text{Sqrt}[b^2 + 4*a*c]] * d * (1 + (2*c*x^2)/(b - \text{Sqrt}[b^2 + 4*a*c])) * \text{EllipticF}[\text{ArcTan}[(\text{Sqrt}[2] * \text{Sqrt}[c] * x) / \text{Sqrt}[b + \text{Sqrt}[b^2 + 4*a*c]]], (-2 * \text{Sqrt}[b^2 + 4*a*c]) / (b - \text{Sqrt}[b^2 + 4*a*c])]) / (\text{Sqrt}[2] * \text{Sqrt}[c] * \text{Sqrt}[(1 + (2*c*x^2)/(b - \text{Sqrt}[b^2 + 4*a*c])) / (1 + (2*c*x^2)/(b + \text{Sqrt}[b^2 + 4*a*c]))] * \text{Sqrt}[-a + b*x^2 + c*x^4])$

Rubi [A] time = 0.475038, antiderivative size = 479, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {1202, 531, 418, 492, 411}

$$\frac{e\left(b-\sqrt{4ac+b^2}\right)\sqrt{\sqrt{4ac+b^2}+b}\left(\frac{2cx^2}{b-\sqrt{4ac+b^2}}+1\right)E\left(\tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b+\sqrt{b^2+4ac}}}\right)\middle|-\frac{2\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)}{2\sqrt{2}c^{3/2}\sqrt{\frac{\frac{2cx^2}{b-\sqrt{4ac+b^2}}+1}{\frac{2cx^2}{\sqrt{4ac+b^2}+b}}}\sqrt{-a+bx^2+cx^4}} + \frac{d\sqrt{\sqrt{4ac+b^2}+b}\left(\frac{2cx^2}{b-\sqrt{4ac+b^2}}+1\right)}{\sqrt{2}\sqrt{c}\sqrt{\frac{\frac{2cx^2}{b-\sqrt{4ac+b^2}}+1}{\frac{2cx^2}{\sqrt{4ac+b^2}+b}}}}$$

Antiderivative was successfully verified.

[In] Int[(d + e*x^2)/Sqrt[-a + b*x^2 + c*x^4], x]

[Out] $((b - \text{Sqrt}[b^2 + 4*a*c]) * e * x * (1 + (2*c*x^2)/(b - \text{Sqrt}[b^2 + 4*a*c]))) / (2*c * \text{Sqrt}[-a + b*x^2 + c*x^4]) - ((b - \text{Sqrt}[b^2 + 4*a*c]) * \text{Sqrt}[b + \text{Sqrt}[b^2 + 4*a*c]] * e * (1 + (2*c*x^2)/(b - \text{Sqrt}[b^2 + 4*a*c])) * \text{EllipticE}[\text{ArcTan}[(\text{Sqrt}[2] * \text{Sqrt}[c] * x) / \text{Sqrt}[b + \text{Sqrt}[b^2 + 4*a*c]]], (-2 * \text{Sqrt}[b^2 + 4*a*c]) / (b - \text{Sqrt}[b^2 + 4*a*c])]) / (2 * \text{Sqrt}[2] * c^{(3/2)} * \text{Sqrt}[(1 + (2*c*x^2)/(b - \text{Sqrt}[b^2 + 4*a*c])) / (1 + (2*c*x^2)/(b + \text{Sqrt}[b^2 + 4*a*c]))] * \text{Sqrt}[-a + b*x^2 + c*x^4]) + (\text{Sqrt}[b + \text{Sqrt}[b^2 + 4*a*c]] * d * (1 + (2*c*x^2)/(b - \text{Sqrt}[b^2 + 4*a*c])) * \text{EllipticF}[\text{ArcTan}[(\text{Sqrt}[2] * \text{Sqrt}[c] * x) / \text{Sqrt}[b + \text{Sqrt}[b^2 + 4*a*c]]], (-2 * \text{Sqrt}[b^2 + 4*a*c]) / (b - \text{Sqrt}[b^2 + 4*a*c])]) / (\text{Sqrt}[2] * \text{Sqrt}[c] * \text{Sqrt}[(1 + (2*c*x^2)/(b - \text{Sqrt}[b^2 + 4*a*c])) / (1 + (2*c*x^2)/(b + \text{Sqrt}[b^2 + 4*a*c]))] * \text{Sqrt}[-a + b*x^2 + c*x^4])$

Rule 1202

Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(Sqrt[1 + (2*c*x^2)/(b - q)]*Sqrt[1 + (2*c*x^2)/(b + q)])/Sqrt[a + b*x^2 + c*x^4], Int[(d + e*x^2)/(Sqrt[1 + (2*c*x^2)/(b - q)]*Sqrt[1 + (2*c*x^2)/(b + q)]), x], x] /; FreeQ[{a, b, c

, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[c/a]

Rule 531

Int[((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_)*((e_) + (f_)*(x_)^(n_)), x_Symbol] := Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x], x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, f, n, p, q}, x]

Rule 418

Int[1/(Sqrt[(a_) + (b_)*(x_)^2]*Sqrt[(c_) + (d_)*(x_)^2]), x_Symbol] := Simp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] && !SimplerSqrtQ[b/a, d/c]

Rule 492

Int[(x_)^2/(Sqrt[(a_) + (b_)*(x_)^2]*Sqrt[(c_) + (d_)*(x_)^2]), x_Symbol] := Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && PosQ[b/a] && PosQ[d/c] && !SimplerSqrtQ[b/a, d/c]

Rule 411

Int[Sqrt[(a_) + (b_)*(x_)^2]/((c_) + (d_)*(x_)^2)^(3/2), x_Symbol] := Simp[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]

Rubi steps

$$\int \frac{d + ex^2}{\sqrt{-a + bx^2 + cx^4}} dx = \frac{\left(\sqrt{1 + \frac{2cx^2}{b - \sqrt{b^2 + 4ac}}}\sqrt{1 + \frac{2cx^2}{b + \sqrt{b^2 + 4ac}}}\right) \int \frac{d + ex^2}{\sqrt{1 + \frac{2cx^2}{b - \sqrt{b^2 + 4ac}}}\sqrt{1 + \frac{2cx^2}{b + \sqrt{b^2 + 4ac}}}} dx}{\sqrt{-a + bx^2 + cx^4}}$$

$$= \frac{\left(d\sqrt{1 + \frac{2cx^2}{b - \sqrt{b^2 + 4ac}}}\sqrt{1 + \frac{2cx^2}{b + \sqrt{b^2 + 4ac}}}\right) \int \frac{1}{\sqrt{1 + \frac{2cx^2}{b - \sqrt{b^2 + 4ac}}}\sqrt{1 + \frac{2cx^2}{b + \sqrt{b^2 + 4ac}}}} dx}{\sqrt{-a + bx^2 + cx^4}} + \frac{\left(e\sqrt{1 + \frac{2cx^2}{b - \sqrt{b^2 + 4ac}}}\sqrt{1 + \frac{2cx^2}{b + \sqrt{b^2 + 4ac}}}\right) \int \frac{1}{\sqrt{1 + \frac{2cx^2}{b - \sqrt{b^2 + 4ac}}}\sqrt{1 + \frac{2cx^2}{b + \sqrt{b^2 + 4ac}}}} dx}{\sqrt{-a + bx^2 + cx^4}}$$

$$= \frac{(b - \sqrt{b^2 + 4ac})ex\left(1 + \frac{2cx^2}{b - \sqrt{b^2 + 4ac}}\right)}{2c\sqrt{-a + bx^2 + cx^4}} + \frac{\sqrt{b + \sqrt{b^2 + 4ac}}d\left(1 + \frac{2cx^2}{b - \sqrt{b^2 + 4ac}}\right)F\left(\tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b + \sqrt{b^2 + 4ac}}}\right)\right)}{\sqrt{2}\sqrt{c}\sqrt{\frac{1 + \frac{2cx^2}{b - \sqrt{b^2 + 4ac}}}{1 + \frac{2cx^2}{b + \sqrt{b^2 + 4ac}}}}\sqrt{-a + bx^2 + cx^4}}$$

$$= \frac{(b - \sqrt{b^2 + 4ac})ex\left(1 + \frac{2cx^2}{b - \sqrt{b^2 + 4ac}}\right)}{2c\sqrt{-a + bx^2 + cx^4}} - \frac{(b - \sqrt{b^2 + 4ac})\sqrt{b + \sqrt{b^2 + 4ac}}e\left(1 + \frac{2cx^2}{b - \sqrt{b^2 + 4ac}}\right)E\left(\tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b + \sqrt{b^2 + 4ac}}}\right)\right)}{2\sqrt{2}c^{3/2}\sqrt{\frac{1 + \frac{2cx^2}{b - \sqrt{b^2 + 4ac}}}{1 + \frac{2cx^2}{b + \sqrt{b^2 + 4ac}}}}\sqrt{-a + bx^2 + cx^4}}$$

Mathematica [C] time = 0.300859, size = 304, normalized size = 0.63

$$\frac{i\sqrt{\frac{\sqrt{4ac+b^2}+b+2cx^2}{\sqrt{4ac+b^2+b}}}\sqrt{\frac{2cx^2}{b-\sqrt{4ac+b^2}}}+1\left(\left(e\left(b-\sqrt{4ac+b^2}\right)-2cd\right)\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{2}x\sqrt{\frac{c}{\sqrt{4ac+b^2+b}}}\right),\frac{\sqrt{4ac+b^2+b}}{b-\sqrt{4ac+b^2}}\right)+e\left(\sqrt{\frac{c}{\sqrt{4ac+b^2+b}}}\right)\right)}{2\sqrt{2}c\sqrt{\frac{c}{\sqrt{4ac+b^2+b}}}\sqrt{-a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e*x^2)/Sqrt[-a + b*x^2 + c*x^4],x]

[Out] ((I/2)*Sqrt[(b + Sqrt[b^2 + 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 + 4*a*c]])*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 + 4*a*c])]*((-b + Sqrt[b^2 + 4*a*c])*e*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 + 4*a*c])]]*x], (b + Sqrt[b^2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c])) + (-2*c*d + (b - Sqrt[b^2 + 4*a*c])*e)*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 + 4*a*c])]]*x], (b + Sqrt[b^2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c])))/(Sqrt[2]*c*Sqrt[c/(b + Sqrt[b^2 + 4*a*c])])*Sqrt[-a + b*x^2 + c*x^4])

Maple [A] time = 0.028, size = 355, normalized size = 0.7

$$ae\sqrt{4+2\frac{(-b+\sqrt{4ac+b^2})x^2}{a}}\sqrt{4-2\frac{(b+\sqrt{4ac+b^2})x^2}{a}}\left(\text{EllipticF}\left(\frac{x}{2}\sqrt{-2\frac{-b+\sqrt{4ac+b^2}}{a}},\frac{1}{2}\sqrt{-4-2\frac{b(b+\sqrt{4ac+b^2})}{a}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x^2+d)/(c*x^4+b*x^2-a)^(1/2),x)

[Out] e*a/(-2*(-b+(4*a*c+b^2)^(1/2))/a)^(1/2)*(4+2*(-b+(4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4-2*(b+(4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2-a)^(1/2)/(b+(4*a*c+b^2)^(1/2))*EllipticF(1/2*x*(-2*(-b+(4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4-2*b*(b+(4*a*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*x*(-2*(-b+(4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4-2*b*(b+(4*a*c+b^2)^(1/2))/a/c)^(1/2))+1/2*d/(-2*(-b+(4*a*c+b^2)^(1/2))/a)^(1/2)*(4+2*(-b+(4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4-2*(b+(4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2-a)^(1/2)*EllipticF(1/2*x*(-2*(-b+(4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4-2*b*(b+(4*a*c+b^2)^(1/2))/a/c)^(1/2))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex^2 + d}{\sqrt{cx^4 + bx^2 - a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)/(c*x^4+b*x^2-a)^(1/2),x, algorithm="maxima")

[Out] integrate((e*x^2 + d)/sqrt(c*x^4 + b*x^2 - a), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{ex^2 + d}{\sqrt{cx^4 + bx^2 - a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)/(c*x^4+b*x^2-a)^(1/2),x, algorithm="fricas")

[Out] integral((e*x^2 + d)/sqrt(c*x^4 + b*x^2 - a), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{d + ex^2}{\sqrt{-a + bx^2 + cx^4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x**2+d)/(c*x**4+b*x**2-a)**(1/2),x)

[Out] Integral((d + e*x**2)/sqrt(-a + b*x**2 + c*x**4), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex^2 + d}{\sqrt{cx^4 + bx^2 - a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)/(c*x^4+b*x^2-a)^(1/2),x, algorithm="giac")

[Out] integrate((e*x^2 + d)/sqrt(c*x^4 + b*x^2 - a), x)

$$3.391 \quad \int \frac{1}{(d+ex^2)\sqrt{-a+bx^2+cx^4}} dx$$

Optimal. Leaf size=204

$$\frac{\sqrt{\sqrt{4ac+b^2}-b}\sqrt{\frac{2cx^2}{b-\sqrt{4ac+b^2}}}+1\sqrt{\frac{2cx^2}{\sqrt{4ac+b^2}+b}}+1\Pi\left(\frac{(b-\sqrt{b^2+4ac})e}{2cd};\sin^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{\sqrt{b^2+4ac}-b}}\right)\middle|\frac{b-\sqrt{b^2+4ac}}{b+\sqrt{b^2+4ac}}\right)}{\sqrt{2}\sqrt{cd}\sqrt{-a+bx^2+cx^4}}$$

[Out] (Sqrt[-b + Sqrt[b^2 + 4*a*c]]*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 + 4*a*c]])*Sqrt[1 + (2*c*x^2)/(b + Sqrt[b^2 + 4*a*c]])*EllipticPi[((b - Sqrt[b^2 + 4*a*c])*e)/(2*c*d), ArcSin[(Sqrt[2]*Sqrt[c]*x)/Sqrt[-b + Sqrt[b^2 + 4*a*c]]], (b - Sqrt[b^2 + 4*a*c])/(b + Sqrt[b^2 + 4*a*c]))/(Sqrt[2]*Sqrt[c]*d*Sqrt[-a + b*x^2 + c*x^4])

Rubi [A] time = 0.191646, antiderivative size = 204, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {1220, 537}

$$\frac{\sqrt{\sqrt{4ac+b^2}-b}\sqrt{\frac{2cx^2}{b-\sqrt{4ac+b^2}}}+1\sqrt{\frac{2cx^2}{\sqrt{4ac+b^2}+b}}+1\Pi\left(\frac{(b-\sqrt{b^2+4ac})e}{2cd};\sin^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{\sqrt{b^2+4ac}-b}}\right)\middle|\frac{b-\sqrt{b^2+4ac}}{b+\sqrt{b^2+4ac}}\right)}{\sqrt{2}\sqrt{cd}\sqrt{-a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

[In] Int[1/((d + e*x^2)*Sqrt[-a + b*x^2 + c*x^4]),x]

[Out] (Sqrt[-b + Sqrt[b^2 + 4*a*c]]*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 + 4*a*c]])*Sqrt[1 + (2*c*x^2)/(b + Sqrt[b^2 + 4*a*c]])*EllipticPi[((b - Sqrt[b^2 + 4*a*c])*e)/(2*c*d), ArcSin[(Sqrt[2]*Sqrt[c]*x)/Sqrt[-b + Sqrt[b^2 + 4*a*c]]], (b - Sqrt[b^2 + 4*a*c])/(b + Sqrt[b^2 + 4*a*c]))/(Sqrt[2]*Sqrt[c]*d*Sqrt[-a + b*x^2 + c*x^4])

Rule 1220

Int[1/(((d_) + (e_)*(x_)^2)*Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(Sqrt[1 + (2*c*x^2)/(b - q)]*Sqrt[1 + (2*c*x^2)/(b + q)])/Sqrt[a + b*x^2 + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + (2*c*x^2)/(b - q)]*Sqrt[1 + (2*c*x^2)/(b + q)]), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[c/a]

Rule 537

Int[1/(((a_) + (b_)*(x_)^2)*Sqrt[(c_) + (d_)*(x_)^2]*Sqrt[(e_) + (f_)*(x_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d, e, f}, x] && !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] && !(GtQ[f/e, 0] && SimplerSqrtQ[-(f/e), -(d/c)])

Rubi steps

$$\int \frac{1}{(d+ex^2)\sqrt{-a+bx^2+cx^4}} dx = \frac{\left(\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2+4ac}}}\right) \int \frac{1}{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2+4ac}}}(d+ex^2)} dx}{\sqrt{-a+bx^2+cx^4}}$$

$$= \frac{\sqrt{-b+\sqrt{b^2+4ac}}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2+4ac}}}\Pi\left(\frac{(b-\sqrt{b^2+4ac})e}{2cd}; \sin^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{-b+\sqrt{b^2+4ac}}}\right)\right)}{\sqrt{2}\sqrt{cd}\sqrt{-a+bx^2+cx^4}}$$

Mathematica [C] time = 0.223136, size = 216, normalized size = 1.06

$$\frac{i\sqrt{\frac{\sqrt{4ac+b^2+b+2cx^2}}{\sqrt{4ac+b^2+b}}}\sqrt{\frac{2cx^2}{b-\sqrt{4ac+b^2}}} + 1\Pi\left(\frac{(b+\sqrt{b^2+4ac})e}{2cd}; i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2+4ac}}}\sqrt{x}\right)\middle|\frac{b+\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)}{\sqrt{2d}\sqrt{\frac{c}{\sqrt{4ac+b^2+b}}}\sqrt{-a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

[In] Integrate[1/((d + e*x^2)*Sqrt[-a + b*x^2 + c*x^4]), x]

[Out] ((-I)*Sqrt[(b + Sqrt[b^2 + 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 + 4*a*c]])*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 + 4*a*c]])*EllipticPi[((b + Sqrt[b^2 + 4*a*c])*e)/(2*c*d), I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 + 4*a*c])]*x], (b + Sqrt[b^2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c]))/(Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 + 4*a*c])])*d*Sqrt[-a + b*x^2 + c*x^4])

Maple [A] time = 0.024, size = 198, normalized size = 1.

$$\frac{1}{d}\sqrt{1-\frac{bx^2}{2a}+\frac{x^2}{2a}\sqrt{4ac+b^2}}\sqrt{1-\frac{bx^2}{2a}-\frac{x^2}{2a}\sqrt{4ac+b^2}}\text{EllipticPi}\left[\sqrt{-\frac{1}{2a}(-b+\sqrt{4ac+b^2})}x, 2\frac{ae}{(-b+\sqrt{4ac+b^2})d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(e*x^2+d)/(c*x^4+b*x^2-a)^(1/2), x)

[Out] 1/d/(1/2*b/a-1/2/a*(4*a*c+b^2)^(1/2))^(1/2)*(1-1/2/a*x^2*b+1/2/a*x^2*(4*a*c+b^2)^(1/2))^(1/2)*(1-1/2/a*x^2*b-1/2/a*x^2*(4*a*c+b^2)^(1/2))^(1/2)/(c*x^4+b*x^2-a)^(1/2)*EllipticPi((-1/2*(-b+(4*a*c+b^2)^(1/2))/a)^(1/2)*x, 2/(-b+(4*a*c+b^2)^(1/2))*a/d*e, 1/2*2^(1/2)*((b+(4*a*c+b^2)^(1/2))/a)^(1/2)/(-1/2*(-b+(4*a*c+b^2)^(1/2))/a)^(1/2))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^4+bx^2-a}(ex^2+d)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x^2+d)/(c*x^4+b*x^2-a)^(1/2), x, algorithm="maxima")

[Out] integrate(1/(sqrt(c*x^4 + b*x^2 - a)*(e*x^2 + d)), x)

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x^2+d)/(c*x^4+b*x^2-a)^(1/2),x, algorithm="fricas")

[Out] Timed out

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(d + ex^2)\sqrt{-a + bx^2 + cx^4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x**2+d)/(c*x**4+b*x**2-a)**(1/2),x)

[Out] Integral(1/((d + e*x**2)*sqrt(-a + b*x**2 + c*x**4)), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^4 + bx^2 - a}(ex^2 + d)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x^2+d)/(c*x^4+b*x^2-a)^(1/2),x, algorithm="giac")

[Out] integrate(1/(sqrt(c*x^4 + b*x^2 - a)*(e*x^2 + d)), x)

$$3.392 \quad \int \frac{d+ex^2}{\sqrt{-a+bx^2-cx^4}} dx$$

Optimal. Leaf size=293

$$\frac{\sqrt[4]{a}(\sqrt{a} + \sqrt{cx^2}) \sqrt{\frac{a-bx^2+cx^4}{(\sqrt{a}+\sqrt{cx^2})^2}} \left(\frac{\sqrt{cd}}{\sqrt{a}} + e\right) \text{EllipticF}\left(2 \tan^{-1}\left(\frac{\sqrt[4]{cx}}{\sqrt[4]{a}}\right), \frac{1}{4}\left(\frac{b}{\sqrt{a}\sqrt{c}} + 2\right)\right)}{2c^{3/4}\sqrt{-a+bx^2-cx^4}} - \frac{\sqrt[4]{ae}(\sqrt{a} + \sqrt{cx^2}) \sqrt{\frac{a-bx^2+cx^4}{(\sqrt{a}+\sqrt{cx^2})^2}} E\left(2 \tan^{-1}\left(\frac{\sqrt[4]{cx}}{\sqrt[4]{a}}\right)\right)}{c^{3/4}\sqrt{-a+bx^2-cx^4}}$$

[Out] $-\left(\frac{e \sqrt{-a+bx^2-cx^4}}{\sqrt{c}(\sqrt{a} + \sqrt{cx^2})}\right) - \left(a^{1/4} e (\sqrt{a} + \sqrt{cx^2}) \sqrt{\frac{a-bx^2+cx^4}{(\sqrt{a} + \sqrt{cx^2})^2}} \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{c^{1/4}x}{a^{1/4}}\right], \left(2 + \frac{b}{\sqrt{a}\sqrt{c}}\right)/4\right]\right) / \left(c^{3/4} \sqrt{-a+bx^2-cx^4}\right) + \left(a^{1/4} \left(\frac{\sqrt{c}d}{\sqrt{a}} + e\right) (\sqrt{a} + \sqrt{cx^2}) \sqrt{\frac{a-bx^2+cx^4}{(\sqrt{a} + \sqrt{cx^2})^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4}x}{a^{1/4}}\right], \left(2 + \frac{b}{\sqrt{a}\sqrt{c}}\right)/4\right]\right) / \left(2 c^{3/4} \sqrt{-a+bx^2-cx^4}\right)$

Rubi [A] time = 0.0897272, antiderivative size = 293, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1197, 1103, 1195}

$$\frac{\sqrt[4]{a}(\sqrt{a} + \sqrt{cx^2}) \sqrt{\frac{a-bx^2+cx^4}{(\sqrt{a}+\sqrt{cx^2})^2}} \left(\frac{\sqrt{cd}}{\sqrt{a}} + e\right) F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{cx}}{\sqrt[4]{a}}\right) \middle| \frac{1}{4}\left(\frac{b}{\sqrt{a}\sqrt{c}} + 2\right)\right)}{2c^{3/4}\sqrt{-a+bx^2-cx^4}} - \frac{\sqrt[4]{ae}(\sqrt{a} + \sqrt{cx^2}) \sqrt{\frac{a-bx^2+cx^4}{(\sqrt{a}+\sqrt{cx^2})^2}} E\left(2 \tan^{-1}\left(\frac{\sqrt[4]{cx}}{\sqrt[4]{a}}\right)\right)}{c^{3/4}\sqrt{-a+bx^2-cx^4}}$$

Antiderivative was successfully verified.

[In] Int[(d + e*x^2)/Sqrt[-a + b*x^2 - c*x^4], x]

[Out] $-\left(\frac{e \sqrt{-a+bx^2-cx^4}}{\sqrt{c}(\sqrt{a} + \sqrt{cx^2})}\right) - \left(a^{1/4} e (\sqrt{a} + \sqrt{cx^2}) \sqrt{\frac{a-bx^2+cx^4}{(\sqrt{a} + \sqrt{cx^2})^2}} \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{c^{1/4}x}{a^{1/4}}\right], \left(2 + \frac{b}{\sqrt{a}\sqrt{c}}\right)/4\right]\right) / \left(c^{3/4} \sqrt{-a+bx^2-cx^4}\right) + \left(a^{1/4} \left(\frac{\sqrt{c}d}{\sqrt{a}} + e\right) (\sqrt{a} + \sqrt{cx^2}) \sqrt{\frac{a-bx^2+cx^4}{(\sqrt{a} + \sqrt{cx^2})^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4}x}{a^{1/4}}\right], \left(2 + \frac{b}{\sqrt{a}\sqrt{c}}\right)/4\right]\right) / \left(2 c^{3/4} \sqrt{-a+bx^2-cx^4}\right)$

Rule 1197

Int[((d_) + (e_)*(x_)^2)/Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4], x_Symbol] := With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; NeQ[e + d*q, 0] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]

Rule 1103

Int[1/Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4], x_Symbol] := With[{q = Rt[c/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]), x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]

Rule 1195

Int[((d_) + (e_)*(x_)^2)/Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4], x_Symbol] := With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^2*x^2))], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]

$2*x^2)), x] + \text{Simp}[(d*(1 + q^2*x^2)*\text{Sqrt}[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*\text{EllipticE}[2*\text{ArcTan}[q*x], 1/2 - (b*q^2)/(4*c)])/ (q*\text{Sqrt}[a + b*x^2 + c*x^4]), x] /; \text{EqQ}[e + d*q^2, 0] /; \text{FreeQ}[\{a, b, c, d, e\}, x] \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& \text{PosQ}[c/a]$

Rubi steps

$$\int \frac{d + ex^2}{\sqrt{-a + bx^2 - cx^4}} dx = -\frac{(\sqrt{ae}) \int \frac{1 - \frac{\sqrt{cx^2}}{\sqrt{a}}}{\sqrt{-a + bx^2 - cx^4}} dx}{\sqrt{c}} + \left(d + \frac{\sqrt{ae}}{\sqrt{c}}\right) \int \frac{1}{\sqrt{-a + bx^2 - cx^4}} dx$$

$$= -\frac{ex\sqrt{-a + bx^2 - cx^4}}{\sqrt{c}(\sqrt{a} + \sqrt{cx^2})} - \frac{\sqrt[4]{ae}(\sqrt{a} + \sqrt{cx^2}) \sqrt{\frac{a - bx^2 + cx^4}{(\sqrt{a} + \sqrt{cx^2})^2}} E\left(2 \tan^{-1}\left(\frac{\sqrt[4]{cx}}{\sqrt[4]{a}}\right) \Big| \frac{1}{4}\left(2 + \frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{c^{3/4}\sqrt{-a + bx^2 - cx^4}} + \dots$$

Mathematica [C] time = 0.310562, size = 295, normalized size = 1.01

$$\frac{i\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac-b}}} + 1\sqrt{1 - \frac{2cx^2}{\sqrt{b^2-4ac+b}}}\left(\left(e\left(b - \sqrt{b^2 - 4ac}\right) + 2cd\right)\text{EllipticF}\left(i \sinh^{-1}\left(\sqrt{2}x\sqrt{-\frac{c}{\sqrt{b^2-4ac+b}}}\right), \frac{\sqrt{b^2-4ac+b}}{b - \sqrt{b^2-4ac}}\right) + e\left(\dots\right)\right)}{2\sqrt{2}c\sqrt{-\frac{c}{\sqrt{b^2-4ac+b}}}\sqrt{-a + bx^2 - cx^4}}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e*x^2)/Sqrt[-a + b*x^2 - c*x^4], x]

[Out] $((-I/2)*\text{Sqrt}[1 + (2*c*x^2)/(-b + \text{Sqrt}[b^2 - 4*a*c])]*\text{Sqrt}[1 - (2*c*x^2)/(b + \text{Sqrt}[b^2 - 4*a*c])])*((-b + \text{Sqrt}[b^2 - 4*a*c])*e*\text{EllipticE}[I*\text{ArcSinh}[\text{Sqrt}[2]*\text{Sqrt}[-(c/(b + \text{Sqrt}[b^2 - 4*a*c])])]*x], (b + \text{Sqrt}[b^2 - 4*a*c])/(b - \text{Sqrt}[b^2 - 4*a*c])) + (2*c*d + (b - \text{Sqrt}[b^2 - 4*a*c])*e)*\text{EllipticF}[I*\text{ArcSinh}[\text{Sqrt}[2]*\text{Sqrt}[-(c/(b + \text{Sqrt}[b^2 - 4*a*c])])]*x], (b + \text{Sqrt}[b^2 - 4*a*c])/(b - \text{Sqrt}[b^2 - 4*a*c])))/(\text{Sqrt}[2]*c*\text{Sqrt}[-(c/(b + \text{Sqrt}[b^2 - 4*a*c])])]*\text{Sqrt}[-a + b*x^2 - c*x^4])$

Maple [A] time = 0.03, size = 357, normalized size = 1.2

$$ae\sqrt{4 + 2\frac{(\sqrt{-4ac + b^2} - b)x^2}{a}}\sqrt{4 - 2\frac{(b + \sqrt{-4ac + b^2})x^2}{a}}\left(\text{EllipticF}\left[\frac{x}{2}\sqrt{-2\frac{\sqrt{-4ac + b^2} - b}{a}}, \frac{1}{2}\sqrt{-4 + 2\frac{b(b + \dots)}{a}}\right]\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x^2+d)/(-c*x^4+b*x^2-a)^(1/2), x)

[Out] $e*a/(-2*((-4*a*c+b^2)^(1/2)-b)/a)^(1/2)*(4+2*((-4*a*c+b^2)^(1/2)-b)/a*x^2)^(1/2)*(4-2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(-c*x^4+b*x^2-a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(\text{EllipticF}(1/2*x*(-2*((-4*a*c+b^2)^(1/2)-b)/a)^(1/2), 1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))- \text{EllipticE}(1/2*x*(-2*((-4*a*c+b^2)^(1/2)-b)/a)^(1/2), 1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2)))+1/2*d/(-2*((-4*a*c+b^2)^(1/2)-b)/a)^(1/2)*(4+2*((-4*a*c+b^2)^(1/2)-b)/a*x^2)^(1/2)*(4-2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(-c*x^4+b*x^2-a)^(1/2)*\text{EllipticF}(1/2*x*(-2*((-4*a*c+b^2)^(1/2)-b)/a)^(1/2), 1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))$

) / a / c ^ (1/2))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex^2 + d}{\sqrt{-cx^4 + bx^2 - a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)/(-c*x^4+b*x^2-a)^(1/2),x, algorithm="maxima")

[Out] integrate((e*x^2 + d)/sqrt(-c*x^4 + b*x^2 - a), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-cx^4 + bx^2 - a}(ex^2 + d)}{cx^4 - bx^2 + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)/(-c*x^4+b*x^2-a)^(1/2),x, algorithm="fricas")

[Out] integral(-sqrt(-c*x^4 + b*x^2 - a)*(e*x^2 + d)/(c*x^4 - b*x^2 + a), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{d + ex^2}{\sqrt{-a + bx^2 - cx^4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x**2+d)/(-c*x**4+b*x**2-a)**(1/2),x)

[Out] Integral((d + e*x**2)/sqrt(-a + b*x**2 - c*x**4), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex^2 + d}{\sqrt{-cx^4 + bx^2 - a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)/(-c*x^4+b*x^2-a)^(1/2),x, algorithm="giac")

[Out] integrate((e*x^2 + d)/sqrt(-c*x^4 + b*x^2 - a), x)

$$3.393 \quad \int \frac{1}{(d+ex^2)\sqrt{-a+bx^2-cx^4}} dx$$

Optimal. Leaf size=412

$$\frac{\sqrt[4]{c}(\sqrt{a} + \sqrt{cx^2}) \sqrt{\frac{a-bx^2+cx^4}{(\sqrt{a}+\sqrt{cx^2})^2}} \text{EllipticF}\left(2 \tan^{-1}\left(\frac{\sqrt[4]{cx}}{\sqrt{a}}\right), \frac{1}{4}\left(\frac{b}{\sqrt{a}\sqrt{c}} + 2\right)\right) a^{3/4}(\sqrt{a} + \sqrt{cx^2}) \sqrt{\frac{a-bx^2+cx^4}{(\sqrt{a}+\sqrt{cx^2})^2}} \left(\frac{\sqrt{cd}}{\sqrt{a}} + e\right)^2 \Pi\left(\frac{\sqrt{cd}-\sqrt{ae}}{2\sqrt{a}\sqrt{-a+bx^2-cx^4}}\right)}{4\sqrt[4]{cd}\sqrt{-a+bx^2-cx^4}}$$

```
[Out] (Sqrt[e]*ArcTan[(Sqrt[-(c*d^2) - e*(b*d + a*e)]*x)/(Sqrt[d]*Sqrt[e]*Sqrt[-a + b*x^2 - c*x^4]])/(2*Sqrt[d]*Sqrt[-(c*d^2) - e*(b*d + a*e)]) + (c^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a - b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 + b/(Sqrt[a]*Sqrt[c]))/4])/(2*a^(1/4)*(Sqrt[c]*d - Sqrt[a]*e)*Sqrt[-a + b*x^2 - c*x^4]) - (a^(3/4)*((Sqrt[c]*d)/Sqrt[a] + e)^2*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a - b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticPi[-(Sqrt[c]*d - Sqrt[a]*e)^2/(4*Sqrt[a]*Sqrt[c]*d*e), 2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 + b/(Sqrt[a]*Sqrt[c]))/4])/(4*c^(1/4)*d*(c*d^2 - a*e^2)*Sqrt[-a + b*x^2 - c*x^4])
```

Rubi [A] time = 0.362714, antiderivative size = 412, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {1216, 1103, 1706}

$$\frac{a^{3/4}(\sqrt{a} + \sqrt{cx^2}) \sqrt{\frac{a-bx^2+cx^4}{(\sqrt{a}+\sqrt{cx^2})^2}} \left(\frac{\sqrt{cd}}{\sqrt{a}} + e\right)^2 \Pi\left(-\frac{(\sqrt{cd}-\sqrt{ae})^2}{4\sqrt{a}\sqrt{cde}}; 2 \tan^{-1}\left(\frac{\sqrt[4]{cx}}{\sqrt{a}}\right) \Big| \frac{1}{4}\left(\frac{b}{\sqrt{a}\sqrt{c}} + 2\right)\right) \sqrt{e} \tan^{-1}\left(\frac{x\sqrt{-e(ae+bd)-cd^2}}{\sqrt{d}\sqrt{e}\sqrt{-a+bx^2-cx^4}}\right)}{4\sqrt[4]{cd}\sqrt{-a+bx^2-cx^4}(cd^2 - ae^2)} + \frac{\sqrt{e} \tan^{-1}\left(\frac{x\sqrt{-e(ae+bd)-cd^2}}{\sqrt{d}\sqrt{e}\sqrt{-a+bx^2-cx^4}}\right)}{2\sqrt{d}\sqrt{-e(ae+bd)-cd^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x^2)*Sqrt[-a + b*x^2 - c*x^4]),x]
```

```
[Out] (Sqrt[e]*ArcTan[(Sqrt[-(c*d^2) - e*(b*d + a*e)]*x)/(Sqrt[d]*Sqrt[e]*Sqrt[-a + b*x^2 - c*x^4]])/(2*Sqrt[d]*Sqrt[-(c*d^2) - e*(b*d + a*e)]) + (c^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a - b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 + b/(Sqrt[a]*Sqrt[c]))/4])/(2*a^(1/4)*(Sqrt[c]*d - Sqrt[a]*e)*Sqrt[-a + b*x^2 - c*x^4]) - (a^(3/4)*((Sqrt[c]*d)/Sqrt[a] + e)^2*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a - b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticPi[-(Sqrt[c]*d - Sqrt[a]*e)^2/(4*Sqrt[a]*Sqrt[c]*d*e), 2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 + b/(Sqrt[a]*Sqrt[c]))/4])/(4*c^(1/4)*d*(c*d^2 - a*e^2)*Sqrt[-a + b*x^2 - c*x^4])
```

Rule 1216

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_Symbol] := With[{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] := With[{q = Rt[c/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4])
```

), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]

Rule 1706

Int[((A_) + (B_)*(x_)^2)/(((d_) + (e_)*(x_)^2)*Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*ArcTan[(Rt[-b + (c*d)/e + (a*e)/d, 2]*x)/Sqrt[a + b*x^2 + c*x^4]])/(2*d*e*Rt[-b + (c*d)/e + (a*e)/d, 2]), x] + Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a + b*x^2 + c*x^4))/(a*(A + B*x^2)^2)]*EllipticPi[Cancel[-((B*d - A*e)^2/(4*d*e*A*B))], 2*ArcTan[q*x], 1/2 - (b*A)/(4*a*B)])/(4*d*e*A*q*Sqrt[a + b*x^2 + c*x^4]), x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]

Rubi steps

$$\int \frac{1}{(d+ex^2)\sqrt{-a+bx^2-cx^4}} dx = \frac{\sqrt{c} \int \frac{1}{\sqrt{-a+bx^2-cx^4}} dx}{\sqrt{cd}-\sqrt{ae}} - \frac{(\sqrt{ae}) \int \frac{1+\frac{\sqrt{cx^2}}{\sqrt{a}}}{(d+ex^2)\sqrt{-a+bx^2-cx^4}} dx}{\sqrt{cd}-\sqrt{ae}}$$

$$= \frac{\sqrt{e} \tan^{-1}\left(\frac{\sqrt{-cd^2-e(bd+ae)}x}{\sqrt{d}\sqrt{e}\sqrt{-a+bx^2-cx^4}}\right)}{2\sqrt{d}\sqrt{-cd^2-e(bd+ae)}} + \frac{\sqrt[4]{c}(\sqrt{a}+\sqrt{cx^2})\sqrt{\frac{a-bx^2+cx^4}{(\sqrt{a}+\sqrt{cx^2})^2}} F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{cx}}{\sqrt[4]{a}}\right)\middle| \frac{1}{4}\right)}{2\sqrt[4]{a}(\sqrt{cd}-\sqrt{ae})\sqrt{-a+bx^2-cx^4}}$$

Mathematica [C] time = 0.229861, size = 207, normalized size = 0.5

$$\frac{i\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac-b}}+1}\sqrt{1-\frac{2cx^2}{\sqrt{b^2-4ac+b}}}\Pi\left(-\frac{(b+\sqrt{b^2-4ac})e}{2cd}; i \sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}x\right)\middle| -\frac{b+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac-b}}\right)}{\sqrt{2d}\sqrt{-\frac{c}{\sqrt{b^2-4ac+b}}}\sqrt{-a+bx^2-cx^4}}$$

Antiderivative was successfully verified.

[In] Integrate[1/((d + e*x^2)*Sqrt[-a + b*x^2 - c*x^4]), x]

[Out] ((-I)*Sqrt[1 + (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]*Sqrt[1 - (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*EllipticPi[-((b + Sqrt[b^2 - 4*a*c])*e)/(2*c*d), I*ArcSinh[Sqrt[2]*Sqrt[-(c/(b + Sqrt[b^2 - 4*a*c]))]*x], -((b + Sqrt[b^2 - 4*a*c])/(-b + Sqrt[b^2 - 4*a*c])))]/(Sqrt[2]*Sqrt[-(c/(b + Sqrt[b^2 - 4*a*c]))])*d*Sqrt[-a + b*x^2 - c*x^4])

Maple [A] time = 0.023, size = 199, normalized size = 0.5

$$\frac{1}{d}\sqrt{1+\frac{x^2}{2a}\sqrt{-4ac+b^2}}-\frac{bx^2}{2a}\sqrt{1-\frac{bx^2}{2a}-\frac{x^2}{2a}\sqrt{-4ac+b^2}}\text{EllipticPi}\left(\sqrt{-\frac{1}{2a}\left(\sqrt{-4ac+b^2}-b\right)}x, 2\frac{ae}{\left(\sqrt{-4ac+b^2}-b\right)d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(e*x^2+d)/(-c*x^4+b*x^2-a)^(1/2), x)

[Out] $1/d/(-1/2/a*(-4*a*c+b^2)^{(1/2)}+1/2*b/a)^{(1/2)}*(1+1/2/a*x^2*(-4*a*c+b^2)^{(1/2)}-1/2/a*x^2*b)^{(1/2)}*(1-1/2/a*x^2*b-1/2/a*x^2*(-4*a*c+b^2)^{(1/2)})^{(1/2)}/(-c*x^4+b*x^2-a)^{(1/2)}*EllipticPi((-1/2*((-4*a*c+b^2)^{(1/2)}-b)/a)^{(1/2)}*x,2/((-4*a*c+b^2)^{(1/2)}-b)*a/d*e,1/2*2^{(1/2)}*((b+(-4*a*c+b^2)^{(1/2)})/a)^{(1/2)}/(-1/2*((-4*a*c+b^2)^{(1/2)}-b)/a)^{(1/2)})$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-cx^4 + bx^2 - a}(ex^2 + d)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(e*x^2+d)/(-c*x^4+b*x^2-a)^(1/2),x, algorithm="maxima")`

[Out] `integrate(1/(sqrt(-c*x^4 + b*x^2 - a)*(e*x^2 + d)), x)`

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(e*x^2+d)/(-c*x^4+b*x^2-a)^(1/2),x, algorithm="fricas")`

[Out] Timed out

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(d + ex^2)\sqrt{-a + bx^2 - cx^4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(e*x**2+d)/(-c*x**4+b*x**2-a)**(1/2),x)`

[Out] `Integral(1/((d + e*x**2)*sqrt(-a + b*x**2 - c*x**4)), x)`

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-cx^4 + bx^2 - a}(ex^2 + d)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(e*x^2+d)/(-c*x^4+b*x^2-a)^(1/2),x, algorithm="giac")`

[Out] `integrate(1/(sqrt(-c*x^4 + b*x^2 - a)*(e*x^2 + d)), x)`

$$3.394 \quad \int \frac{(d+ex^2)^3}{\sqrt{2+3x^2+x^4}} dx$$

Optimal. Leaf size=229

$$\frac{(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}(5d^3-10de^2+8e^3)\text{EllipticF}\left(\tan^{-1}(x), \frac{1}{2}\right)}{5\sqrt{2}\sqrt{x^4+3x^2+2}} + \frac{3e(x^2+2)x(5d^2-10de+6e^2)}{5\sqrt{x^4+3x^2+2}} - \frac{3\sqrt{2}e(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}(5d^2-10de+6e^2)}{5\sqrt{x^4+3x^2+2}}$$

[Out] (3*e*(5*d^2 - 10*d*e + 6*e^2)*x*(2 + x^2))/(5*Sqrt[2 + 3*x^2 + x^4]) + ((5*d - 4*e)*e^2*x*Sqrt[2 + 3*x^2 + x^4])/5 + (e^3*x^3*Sqrt[2 + 3*x^2 + x^4])/5 - (3*Sqrt[2]*e*(5*d^2 - 10*d*e + 6*e^2)*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x], 1/2])/(5*Sqrt[2 + 3*x^2 + x^4]) + ((5*d^3 - 10*d*e^2 + 8*e^3)*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(5*Sqrt[2]*Sqrt[2 + 3*x^2 + x^4])

Rubi [A] time = 0.153778, antiderivative size = 229, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {1206, 1679, 1189, 1099, 1135}

$$\frac{3e(x^2+2)x(5d^2-10de+6e^2)}{5\sqrt{x^4+3x^2+2}} + \frac{(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}(5d^3-10de^2+8e^3)F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{5\sqrt{2}\sqrt{x^4+3x^2+2}} - \frac{3\sqrt{2}e(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}(5d^2-10de+6e^2)}{5\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

[In] Int[(d + e*x^2)^3/Sqrt[2 + 3*x^2 + x^4], x]

[Out] (3*e*(5*d^2 - 10*d*e + 6*e^2)*x*(2 + x^2))/(5*Sqrt[2 + 3*x^2 + x^4]) + ((5*d - 4*e)*e^2*x*Sqrt[2 + 3*x^2 + x^4])/5 + (e^3*x^3*Sqrt[2 + 3*x^2 + x^4])/5 - (3*Sqrt[2]*e*(5*d^2 - 10*d*e + 6*e^2)*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x], 1/2])/(5*Sqrt[2 + 3*x^2 + x^4]) + ((5*d^3 - 10*d*e^2 + 8*e^3)*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(5*Sqrt[2]*Sqrt[2 + 3*x^2 + x^4])

Rule 1206

Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(e^q*x^(2*q - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(4*p + 2*q + 1)), x] + Dist[1/(c*(4*p + 2*q + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*p + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1]

Rule 1679

Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{q = Expon[Pq, x^2], e = Coeff[Pq, x^2, Expon[Pq, x^2]]}, Simp[(e*x^(2*q - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(2*q + 4*p + 1)), x] + Dist[1/(c*(2*q + 4*p + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*q + 4*p + 1)*Pq - a*e*(2*q - 3)*x^(2*q - 4) - b*e*(2*q + 2*p - 1)*x^(2*q - 2) - c*e*(2*q + 4*p + 1)*x^(2*q), x], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && !LtQ[p, -1]

Rule 1189


```
Int[((d_) + (e_)*(x_)^2)/Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4], x_Symbol]
:= With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a]
] || PosQ[(b - q)/a] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1099

```
Int[1/Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4], x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]},
Simp[((2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q)*x^2)/(2*a + (b + q)*x^2)]*
EllipticF[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)/(b + q)])/(2*a*Rt[(b + q)/(2*a), 2]*
Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] && !(PosQ[(b - q)/a] &&
SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1135

```
Int[(x_)^2/Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4], x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]},
Simp[(x*(b + q + 2*c*x^2))/(2*c*Sqrt[a + b*x^2 + c*x^4]), x] - Simp[(Rt[(b + q)/(2*a), 2]*
(2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q)*x^2)/(2*a + (b + q)*x^2)]*EllipticE[ArcTan[Rt[(b + q)/(2*a), 2]*x],
(2*q)/(b + q)])/(2*c*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] && !(PosQ[(b - q)/a] &&
SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{(d + ex^2)^3}{\sqrt{2 + 3x^2 + x^4}} dx &= \frac{1}{5}e^3x^3\sqrt{2 + 3x^2 + x^4} + \frac{1}{5} \int \frac{5d^3 + 3e(5d^2 - 2e^2)x^2 + 3(5d - 4e)e^2x^4}{\sqrt{2 + 3x^2 + x^4}} dx \\ &= \frac{1}{5}(5d - 4e)e^2x\sqrt{2 + 3x^2 + x^4} + \frac{1}{5}e^3x^3\sqrt{2 + 3x^2 + x^4} + \frac{1}{15} \int \frac{3(5d^3 - 10de^2 + 8e^3) + 9e(5d^2 - 10de + 6e^2)x^2}{\sqrt{2 + 3x^2 + x^4}} dx \\ &= \frac{1}{5}(5d - 4e)e^2x\sqrt{2 + 3x^2 + x^4} + \frac{1}{5}e^3x^3\sqrt{2 + 3x^2 + x^4} + \frac{1}{5}(3e(5d^2 - 10de + 6e^2)) \int \frac{x^2}{\sqrt{2 + 3x^2 + x^4}} dx \\ &= \frac{3e(5d^2 - 10de + 6e^2)x(2 + x^2)}{5\sqrt{2 + 3x^2 + x^4}} + \frac{1}{5}(5d - 4e)e^2x\sqrt{2 + 3x^2 + x^4} + \frac{1}{5}e^3x^3\sqrt{2 + 3x^2 + x^4} - \frac{3\sqrt{2}}{5} \end{aligned}$$

Mathematica [C] time = 0.229417, size = 154, normalized size = 0.67

$$\frac{-5i\sqrt{x^2 + 1}\sqrt{x^2 + 2}(-3d^2e + d^3 + 4de^2 - 2e^3)\text{EllipticF}\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right), 2\right) - 3ie\sqrt{x^2 + 1}\sqrt{x^2 + 2}(5d^2 - 10de + 6e^2)E\left(\frac{x}{\sqrt{2}}\right)}{5\sqrt{x^4 + 3x^2 + 2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^3/Sqrt[2 + 3*x^2 + x^4], x]
```

```
[Out] (e^2*x*(2 + 3*x^2 + x^4)*(5*d + e*(-4 + x^2)) - (3*I)*e*(5*d^2 - 10*d*e + 6*e^2)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticE[I*ArcSinh[x/Sqrt[2]], 2] - (5*I)*
(d^3 - 3*d^2*e + 4*d*e^2 - 2*e^3)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticF[I*ArcSinh[x/Sqrt[2]], 2])/(5*Sqrt[2 + 3*x^2 + x^4])
```

Maple [C] time = 0.01, size = 380, normalized size = 1.7

$$e^3 \left(\frac{x^3}{5} \sqrt{x^4 + 3x^2 + 2} - \frac{4x}{5} \sqrt{x^4 + 3x^2 + 2} - \frac{4i}{5} \sqrt{2} \operatorname{EllipticF} \left(\frac{i}{2} x \sqrt{2}, \sqrt{2} \right) \sqrt{2x^2 + 4} \sqrt{x^2 + 1} \frac{1}{\sqrt{x^4 + 3x^2 + 2}} + \frac{9i}{5} \sqrt{2} \left(\operatorname{EllipticE} \left(\frac{i}{2} x \sqrt{2}, \sqrt{2} \right) \sqrt{2x^2 + 4} \sqrt{x^2 + 1} \frac{1}{\sqrt{x^4 + 3x^2 + 2}} - \operatorname{EllipticF} \left(\frac{i}{2} x \sqrt{2}, \sqrt{2} \right) \sqrt{2x^2 + 4} \sqrt{x^2 + 1} \frac{1}{\sqrt{x^4 + 3x^2 + 2}} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x^2+d)^3/(x^4+3*x^2+2)^(1/2),x)

[Out] $e^3 \left(\frac{1}{5} x^3 (x^4 + 3x^2 + 2)^{1/2} - \frac{4}{5} x (x^4 + 3x^2 + 2)^{1/2} - \frac{4}{5} i \sqrt{2} \operatorname{EllipticF} \left(\frac{i}{2} x \sqrt{2}, \sqrt{2} \right) \sqrt{2x^2 + 4} \sqrt{x^2 + 1} \frac{1}{\sqrt{x^4 + 3x^2 + 2}} + \frac{9}{5} i \sqrt{2} \left(\operatorname{EllipticE} \left(\frac{i}{2} x \sqrt{2}, \sqrt{2} \right) \sqrt{2x^2 + 4} \sqrt{x^2 + 1} \frac{1}{\sqrt{x^4 + 3x^2 + 2}} - \operatorname{EllipticF} \left(\frac{i}{2} x \sqrt{2}, \sqrt{2} \right) \sqrt{2x^2 + 4} \sqrt{x^2 + 1} \frac{1}{\sqrt{x^4 + 3x^2 + 2}} \right) \right)$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^2 + d)^3}{\sqrt{x^4 + 3x^2 + 2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)^3/(x^4+3*x^2+2)^(1/2),x, algorithm="maxima")

[Out] integrate((e*x^2 + d)^3/sqrt(x^4 + 3*x^2 + 2), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral} \left(\frac{e^3 x^6 + 3 d e^2 x^4 + 3 d^2 e x^2 + d^3}{\sqrt{x^4 + 3x^2 + 2}}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)^3/(x^4+3*x^2+2)^(1/2),x, algorithm="fricas")

[Out] $\operatorname{integral} \left(\frac{e^3 x^6 + 3 d e^2 x^4 + 3 d^2 e x^2 + d^3}{\sqrt{x^4 + 3x^2 + 2}}, x \right)$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(d + ex^2)^3}{\sqrt{(x^2 + 1)(x^2 + 2)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**3/(x**4+3*x**2+2)**(1/2),x)
```

```
[Out] Integral((d + e*x**2)**3/sqrt((x**2 + 1)*(x**2 + 2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^2 + d)^3}{\sqrt{x^4 + 3x^2 + 2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3/(x^4+3*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^3/sqrt(x^4 + 3*x^2 + 2), x)
```

$$3.395 \quad \int \frac{(d+ex^2)^2}{\sqrt{2+3x^2+x^4}} dx$$

Optimal. Leaf size=168

$$\frac{(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}(3d^2-2e^2)\text{EllipticF}\left(\tan^{-1}(x), \frac{1}{2}\right)}{3\sqrt{2}\sqrt{x^4+3x^2+2}} + \frac{2ex(x^2+2)(d-e)}{\sqrt{x^4+3x^2+2}} - \frac{2\sqrt{2}e(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}(d-e)E\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{\sqrt{x^4+3x^2+2}} +$$

[Out] (2*(d - e)*e*x*(2 + x^2))/Sqrt[2 + 3*x^2 + x^4] + (e^2*x*Sqrt[2 + 3*x^2 + x^4])/3 - (2*Sqrt[2]*(d - e)*e*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x], 1/2])/Sqrt[2 + 3*x^2 + x^4] + ((3*d^2 - 2*e^2)*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(3*Sqrt[2]*Sqrt[2 + 3*x^2 + x^4])

Rubi [A] time = 0.0737784, antiderivative size = 168, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1206, 1189, 1099, 1135}

$$\frac{(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}(3d^2-2e^2)F\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{3\sqrt{2}\sqrt{x^4+3x^2+2}} + \frac{2ex(x^2+2)(d-e)}{\sqrt{x^4+3x^2+2}} - \frac{2\sqrt{2}e(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}(d-e)E\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{\sqrt{x^4+3x^2+2}} + \frac{1}{3}e^2x\sqrt{x^4+3x^2+2}$$

Antiderivative was successfully verified.

[In] Int[(d + e*x^2)^2/Sqrt[2 + 3*x^2 + x^4], x]

[Out] (2*(d - e)*e*x*(2 + x^2))/Sqrt[2 + 3*x^2 + x^4] + (e^2*x*Sqrt[2 + 3*x^2 + x^4])/3 - (2*Sqrt[2]*(d - e)*e*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x], 1/2])/Sqrt[2 + 3*x^2 + x^4] + ((3*d^2 - 2*e^2)*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(3*Sqrt[2]*Sqrt[2 + 3*x^2 + x^4])

Rule 1206

Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(e^q*x^(2*q - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(4*p + 2*q + 1)), x] + Dist[1/(c*(4*p + 2*q + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*p + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1]

Rule 1189

Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a] || PosQ[(b - q)/a] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]

Rule 1099

Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[((2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q)*x^2)/(2*a + (b + q)*x^2)]*EllipticF[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)/(b + q)])/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] && !(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[

{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]

Rule 1135

Int[(x_)^2/Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4], x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(x*(b + q + 2*c*x^2))/(2*c*Sqrt[a + b*x^2 + c*x^4]), x] - Simp[(Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q)*x^2)/(2*a + (b + q)*x^2)]*EllipticE[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)/(b + q)])/(2*c*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] && !(PosQ[(b - q)/a] && SimplersqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]

Rubi steps

$$\begin{aligned} \int \frac{(d + ex^2)^2}{\sqrt{2 + 3x^2 + x^4}} dx &= \frac{1}{3}e^2x\sqrt{2 + 3x^2 + x^4} + \frac{1}{3} \int \frac{3d^2 - 2e^2 + 6(d - e)ex^2}{\sqrt{2 + 3x^2 + x^4}} dx \\ &= \frac{1}{3}e^2x\sqrt{2 + 3x^2 + x^4} + (2(d - e)e) \int \frac{x^2}{\sqrt{2 + 3x^2 + x^4}} dx + \frac{1}{3}(3d^2 - 2e^2) \int \frac{1}{\sqrt{2 + 3x^2 + x^4}} dx \\ &= \frac{2(d - e)ex(2 + x^2)}{\sqrt{2 + 3x^2 + x^4}} + \frac{1}{3}e^2x\sqrt{2 + 3x^2 + x^4} - \frac{2\sqrt{2}(d - e)(1 + x^2)\sqrt{\frac{2+x^2}{1+x^2}}E\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{\sqrt{2 + 3x^2 + x^4}} + \frac{(3d^2 - 2e^2)}{\sqrt{2 + 3x^2 + x^4}} \end{aligned}$$

Mathematica [C] time = 0.153958, size = 127, normalized size = 0.76

$$\frac{-i\sqrt{x^2 + 1}\sqrt{x^2 + 2}(3d^2 - 6de + 4e^2)\text{EllipticF}\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right), 2\right) - 6ie\sqrt{x^2 + 1}\sqrt{x^2 + 2}(d - e)E\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right) + e^2}{3\sqrt{x^4 + 3x^2 + 2}}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e*x^2)^2/Sqrt[2 + 3*x^2 + x^4], x]

[Out] (e^2*x*(2 + 3*x^2 + x^4) - (6*I)*(d - e)*e*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticE[I*ArcSinh[x/Sqrt[2]], 2] - I*(3*d^2 - 6*d*e + 4*e^2)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticF[I*ArcSinh[x/Sqrt[2]], 2])/(3*Sqrt[2 + 3*x^2 + x^4])

Maple [C] time = 0.009, size = 235, normalized size = 1.4

$$e^2\left(\frac{x}{3}\sqrt{x^4 + 3x^2 + 2} + \frac{i}{3}\sqrt{2}\text{EllipticF}\left(\frac{i}{2}x\sqrt{2}, \sqrt{2}\right)\sqrt{2x^2 + 4}\sqrt{x^2 + 1}\frac{1}{\sqrt{x^4 + 3x^2 + 2}} - i\sqrt{2}\left(\text{EllipticF}\left(\frac{i}{2}x\sqrt{2}, \sqrt{2}\right) - \text{EllipticE}\left(\frac{i}{2}x\sqrt{2}, \sqrt{2}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x^2+d)^2/(x^4+3*x^2+2)^(1/2), x)

[Out] e^2*(1/3*x*(x^4+3*x^2+2)^(1/2)+1/3*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticF(1/2*I*x*2^(1/2), 2^(1/2))-I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*(EllipticF(1/2*I*x*2^(1/2), 2^(1/2))-EllipticE(1/2*I*x*2^(1/2), 2^(1/2))))+I*d*e*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*(EllipticF(1/2*I*x*2^(1/2), 2^(1/2))-EllipticE(1/2*I*x*2^(1/2), 2^(1/2)))-1/2*I*d^2*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticF(1/2*I*x*2^(1/2), 2^(1/2))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^2 + d)^2}{\sqrt{x^4 + 3x^2 + 2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)^2/(x^4+3*x^2+2)^(1/2),x, algorithm="maxima")

[Out] integrate((e*x^2 + d)^2/sqrt(x^4 + 3*x^2 + 2), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{e^2x^4 + 2dex^2 + d^2}{\sqrt{x^4 + 3x^2 + 2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)^2/(x^4+3*x^2+2)^(1/2),x, algorithm="fricas")

[Out] integral((e^2*x^4 + 2*d*e*x^2 + d^2)/sqrt(x^4 + 3*x^2 + 2), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(d + ex^2)^2}{\sqrt{(x^2 + 1)(x^2 + 2)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x**2+d)**2/(x**4+3*x**2+2)**(1/2),x)

[Out] Integral((d + e*x**2)**2/sqrt((x**2 + 1)*(x**2 + 2)), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^2 + d)^2}{\sqrt{x^4 + 3x^2 + 2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)^2/(x^4+3*x^2+2)^(1/2),x, algorithm="giac")

[Out] integrate((e*x^2 + d)^2/sqrt(x^4 + 3*x^2 + 2), x)

$$3.396 \quad \int \frac{d+ex^2}{\sqrt{2+3x^2+x^4}} dx$$

Optimal. Leaf size=122

$$\frac{d(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\text{EllipticF}\left(\tan^{-1}(x), \frac{1}{2}\right)}{\sqrt{2}\sqrt{x^4+3x^2+2}} + \frac{ex(x^2+2)}{\sqrt{x^4+3x^2+2}} - \frac{\sqrt{2}e(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}E\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{\sqrt{x^4+3x^2+2}}$$

[Out] (e*x*(2 + x^2))/Sqrt[2 + 3*x^2 + x^4] - (Sqrt[2]*e*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x], 1/2])/Sqrt[2 + 3*x^2 + x^4] + (d*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(Sqrt[2]*Sqrt[2 + 3*x^2 + x^4])

Rubi [A] time = 0.0339678, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {1189, 1099, 1135}

$$\frac{d(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}F\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^4+3x^2+2}} + \frac{ex(x^2+2)}{\sqrt{x^4+3x^2+2}} - \frac{\sqrt{2}e(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}E\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

[In] Int[(d + e*x^2)/Sqrt[2 + 3*x^2 + x^4], x]

[Out] (e*x*(2 + x^2))/Sqrt[2 + 3*x^2 + x^4] - (Sqrt[2]*e*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x], 1/2])/Sqrt[2 + 3*x^2 + x^4] + (d*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(Sqrt[2]*Sqrt[2 + 3*x^2 + x^4])

Rule 1189

Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a] || PosQ[(b - q)/a] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]

Rule 1099

Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[((2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q)*x^2)/(2*a + (b + q)*x^2)]*EllipticF[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)/(b + q)])/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] && !(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]

Rule 1135

Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(x*(b + q + 2*c*x^2))/(2*c*Sqrt[a + b*x^2 + c*x^4]), x] - Simp[(Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q)*x^2)/(2*a + (b + q)*x^2)]*EllipticE[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)/(b + q)])/(2*c*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] && !(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]

Rubi steps

$$\int \frac{d + ex^2}{\sqrt{2 + 3x^2 + x^4}} dx = d \int \frac{1}{\sqrt{2 + 3x^2 + x^4}} dx + e \int \frac{x^2}{\sqrt{2 + 3x^2 + x^4}} dx$$

$$= \frac{ex(2 + x^2)}{\sqrt{2 + 3x^2 + x^4}} - \frac{\sqrt{2}e(1 + x^2)\sqrt{\frac{2+x^2}{1+x^2}}E\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{\sqrt{2 + 3x^2 + x^4}} + \frac{d(1 + x^2)\sqrt{\frac{2+x^2}{1+x^2}}F\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{\sqrt{2}\sqrt{2 + 3x^2 + x^4}}$$

Mathematica [C] time = 0.075026, size = 73, normalized size = 0.6

$$\frac{i\sqrt{x^2+1}\sqrt{x^2+2}\left((d-e)\text{EllipticF}\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right),2\right)+eE\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)\right)}{\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e*x^2)/Sqrt[2 + 3*x^2 + x^4], x]

[Out] ((-1)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*(e*EllipticE[I*ArcSinh[x/Sqrt[2]]], 2) + (d - e)*EllipticF[I*ArcSinh[x/Sqrt[2]], 2])/Sqrt[2 + 3*x^2 + x^4]

Maple [C] time = 0.006, size = 108, normalized size = 0.9

$$\frac{i}{2}e\sqrt{2}\left(\text{EllipticF}\left(\frac{i}{2}x\sqrt{2},\sqrt{2}\right)-\text{EllipticE}\left(\frac{i}{2}x\sqrt{2},\sqrt{2}\right)\right)\sqrt{2x^2+4}\sqrt{x^2+1}\frac{1}{\sqrt{x^4+3x^2+2}}-\frac{i}{2}d\sqrt{2}\text{EllipticF}\left(\frac{i}{2}x\sqrt{2},\sqrt{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x^2+d)/(x^4+3*x^2+2)^(1/2), x)

[Out] 1/2*I*e*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*(EllipticF(1/2*I*x*2^(1/2),2^(1/2))-EllipticE(1/2*I*x*2^(1/2),2^(1/2)))-1/2*I*d*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticF(1/2*I*x*2^(1/2),2^(1/2))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex^2 + d}{\sqrt{x^4 + 3x^2 + 2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)/(x^4+3*x^2+2)^(1/2), x, algorithm="maxima")

[Out] integrate((e*x^2 + d)/sqrt(x^4 + 3*x^2 + 2), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{ex^2 + d}{\sqrt{x^4 + 3x^2 + 2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(x^4+3*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((e*x^2 + d)/sqrt(x^4 + 3*x^2 + 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{d + ex^2}{\sqrt{(x^2 + 1)(x^2 + 2)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)/(x**4+3*x**2+2)**(1/2),x)
```

```
[Out] Integral((d + e*x**2)/sqrt((x**2 + 1)*(x**2 + 2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex^2 + d}{\sqrt{x^4 + 3x^2 + 2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(x^4+3*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)/sqrt(x^4 + 3*x^2 + 2), x)
```

$$3.397 \quad \int \frac{1}{(d+ex^2)\sqrt{2+3x^2+x^4}} dx$$

Optimal. Leaf size=124

$$\frac{(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\text{EllipticF}\left(\tan^{-1}(x), \frac{1}{2}\right)}{\sqrt{2}\sqrt{x^4+3x^2+2}(d-e)} - \frac{e(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\Pi\left(1-\frac{e}{d}; \tan^{-1}(x)\middle|\frac{1}{2}\right)}{\sqrt{2d}\sqrt{x^4+3x^2+2}(d-e)}$$

[Out] $((1+x^2)*\text{Sqrt}[(2+x^2)/(1+x^2)]*\text{EllipticF}[\text{ArcTan}[x], 1/2])/(\text{Sqrt}[2]*(d-e)*\text{Sqrt}[2+3*x^2+x^4]) - (e*(1+x^2)*\text{Sqrt}[(2+x^2)/(1+x^2)]*\text{EllipticPi}[1-e/d, \text{ArcTan}[x], 1/2])/(\text{Sqrt}[2]*d*(d-e)*\text{Sqrt}[2+3*x^2+x^4])$

Rubi [A] time = 0.0909018, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1214, 1099, 1456, 539}

$$\frac{(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}F\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^4+3x^2+2}(d-e)} - \frac{e(x^2+2)\Pi\left(1-\frac{e}{d}; \tan^{-1}(x)\middle|\frac{1}{2}\right)}{\sqrt{2d}\sqrt{\frac{x^2+2}{x^2+1}}\sqrt{x^4+3x^2+2}(d-e)}$$

Antiderivative was successfully verified.

[In] $\text{Int}[1/((d+e*x^2)*\text{Sqrt}[2+3*x^2+x^4]),x]$

[Out] $((1+x^2)*\text{Sqrt}[(2+x^2)/(1+x^2)]*\text{EllipticF}[\text{ArcTan}[x], 1/2])/(\text{Sqrt}[2]*(d-e)*\text{Sqrt}[2+3*x^2+x^4]) - (e*(2+x^2)*\text{EllipticPi}[1-e/d, \text{ArcTan}[x], 1/2])/(\text{Sqrt}[2]*d*(d-e)*\text{Sqrt}[(2+x^2)/(1+x^2)]*\text{Sqrt}[2+3*x^2+x^4])$

Rule 1214

$\text{Int}[1/(((d_) + (e_)*(x_)^2)*\text{Sqrt}[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4]), x_Symbol] \rightarrow \text{With}[\{q = \text{Rt}[b^2 - 4*a*c, 2]\}, \text{Dist}[(2*c)/(2*c*d - e*(b - q)), \text{Int}[1/\text{Sqrt}[a + b*x^2 + c*x^4], x], x] - \text{Dist}[e/(2*c*d - e*(b - q)), \text{Int}[(b - q + 2*c*x^2)/((d + e*x^2)*\text{Sqrt}[a + b*x^2 + c*x^4]), x], x] /; \text{FreeQ}\{a, b, c, d, e\}, x] \&\& \text{GtQ}[b^2 - 4*a*c, 0] \&\& \text{!LtQ}[c, 0]$

Rule 1099

$\text{Int}[1/\text{Sqrt}[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4], x_Symbol] \rightarrow \text{With}[\{q = \text{Rt}[b^2 - 4*a*c, 2]\}, \text{Simp}[(2*a + (b + q)*x^2)*\text{Sqrt}[(2*a + (b - q)*x^2)/(2*a + (b + q)*x^2)]*\text{EllipticF}[\text{ArcTan}[\text{Rt}[(b + q)/(2*a), 2]*x], (2*q)/(b + q)]/(2*a*\text{Rt}[(b + q)/(2*a), 2]*\text{Sqrt}[a + b*x^2 + c*x^4]), x] /; \text{PosQ}[(b + q)/a] \&\& \text{!(PosQ}[(b - q)/a] \&\& \text{SimplerSqrtQ}[(b - q)/(2*a), (b + q)/(2*a)])] /; \text{FreeQ}\{a, b, c\}, x] \&\& \text{GtQ}[b^2 - 4*a*c, 0]$

Rule 1456

$\text{Int}[(d_ + (e_)*(x_)^{(n_)})^{(q_)}*((f_) + (g_)*(x_)^{(n_)})^{(r_)}*((a_) + (b_)*(x_)^{(n_)} + (c_)*(x_)^{(n2_)})^{(p_)}], x_Symbol] \rightarrow \text{Dist}[(a + b*x^n + c*x^{2*n})^{\text{FracPart}[p]}/((d + e*x^n)^{\text{FracPart}[p]}*(a/d + (c*x^n)/e)^{\text{FracPart}[p]}), \text{Int}[(d + e*x^n)^{(p+q)}*(f + g*x^n)^r*(a/d + (c*x^n)/e)^p, x] /; \text{FreeQ}\{a, b, c, d, e, f, g, n, p, q, r\}, x] \&\& \text{EqQ}[n2, 2*n] \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& \text{EqQ}[c*d^2 - b*d*e + a*e^2, 0] \&\& \text{!IntegerQ}[p]$

Rule 539

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)^2]), x_Symbol] := Simp[(c*Sqrt[e + f*x^2]*EllipticPi[1 - (b*c)/(a*d), ArcTan[Rt[d/c, 2]*x], 1 - (c*f)/(d*e)])/((a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(e + f*x^2))/(e*(c + d*x^2))]), x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[d/c]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{(d + ex^2)\sqrt{2 + 3x^2 + x^4}} dx &= \frac{\int \frac{1}{\sqrt{2+3x^2+x^4}} dx}{d - e} - \frac{e \int \frac{2+2x^2}{(d+ex^2)\sqrt{2+3x^2+x^4}} dx}{2(d - e)} \\ &= \frac{(1 + x^2) \sqrt{\frac{2+x^2}{1+x^2}} F\left(\tan^{-1}(x) \middle| \frac{1}{2}\right)}{\sqrt{2}(d - e)\sqrt{2 + 3x^2 + x^4}} - \frac{\left(e\sqrt{1 + \frac{x^2}{2}}\sqrt{2 + 2x^2}\right) \int \frac{\sqrt{2+2x^2}}{\sqrt{1+\frac{x^2}{2}}(d+ex^2)} dx}{2(d - e)\sqrt{2 + 3x^2 + x^4}} \\ &= \frac{(1 + x^2) \sqrt{\frac{2+x^2}{1+x^2}} F\left(\tan^{-1}(x) \middle| \frac{1}{2}\right)}{\sqrt{2}(d - e)\sqrt{2 + 3x^2 + x^4}} - \frac{e(2 + x^2) \Pi\left(1 - \frac{e}{d}; \tan^{-1}(x) \middle| \frac{1}{2}\right)}{\sqrt{2}d(d - e)\sqrt{\frac{2+x^2}{1+x^2}}\sqrt{2 + 3x^2 + x^4}} \end{aligned}$$

Mathematica [C] time = 0.109898, size = 59, normalized size = 0.48

$$\frac{i\sqrt{x^2 + 1}\sqrt{x^2 + 2}\Pi\left(\frac{2e}{d}; i \sinh^{-1}\left(\frac{x}{\sqrt{2}}\right) \middle| 2\right)}{d\sqrt{x^4 + 3x^2 + 2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x^2)*Sqrt[2 + 3*x^2 + x^4]), x]
```

```
[Out] ((-I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticPi[(2*e)/d, I*ArcSinh[x/Sqrt[2]], 2])/((d*Sqrt[2 + 3*x^2 + x^4])
```

Maple [C] time = 0.016, size = 55, normalized size = 0.4

$$\frac{-i\sqrt{2}}{d} \sqrt{1 + \frac{x^2}{2}} \sqrt{x^2 + 1} \text{EllipticPi}\left(\frac{i}{2}x\sqrt{2}, 2\frac{e}{d}, \sqrt{2}\right) \frac{1}{\sqrt{x^4 + 3x^2 + 2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d)/(x^4+3*x^2+2)^(1/2), x)
```

```
[Out] -I/d*2^(1/2)*(1+1/2*x^2)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticPi(1/2*I*x*2^(1/2), 2*e/d, 2^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x^2+d)/(x^4+3*x^2+2)^(1/2),x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{x^4 + 3x^2 + 2}}{ex^6 + (d + 3e)x^4 + (3d + 2e)x^2 + 2d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x^2+d)/(x^4+3*x^2+2)^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(x^4 + 3*x^2 + 2)/(e*x^6 + (d + 3*e)*x^4 + (3*d + 2*e)*x^2 + 2*d), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{(x^2 + 1)(x^2 + 2)}(d + ex^2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x**2+d)/(x**4+3*x**2+2)**(1/2),x)

[Out] Integral(1/(sqrt((x**2 + 1)*(x**2 + 2))*(d + e*x**2)), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^4 + 3x^2 + 2}(ex^2 + d)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x^2+d)/(x^4+3*x^2+2)^(1/2),x, algorithm="giac")

[Out] integrate(1/(sqrt(x^4 + 3*x^2 + 2)*(e*x^2 + d)), x)

$$3.398 \quad \int \frac{1}{(d+ex^2)^2 \sqrt{2+3x^2+x^4}} dx$$

Optimal. Leaf size=316

$$\frac{(x^2 + 1) \sqrt{\frac{x^2+2}{2x^2+2}} (2d - e) \text{EllipticF}\left(\tan^{-1}(x), \frac{1}{2}\right)}{2d\sqrt{x^4 + 3x^2 + 2}(d - e)^2} + \frac{e^2 x \sqrt{x^4 + 3x^2 + 2}}{2d(d^2 - 3de + 2e^2)(d + ex^2)} - \frac{ex(x^2 + 2)}{2d\sqrt{x^4 + 3x^2 + 2}(d^2 - 3de + 2e^2)}$$

[Out] $-(e*x*(2 + x^2))/(2*d*(d^2 - 3*d*e + 2*e^2)*\text{Sqrt}[2 + 3*x^2 + x^4]) + (e^2*x*\text{Sqrt}[2 + 3*x^2 + x^4])/(2*d*(d^2 - 3*d*e + 2*e^2)*(d + e*x^2)) + (e*(1 + x^2)*\text{Sqrt}[(2 + x^2)/(1 + x^2)]*\text{EllipticE}[\text{ArcTan}[x], 1/2])/(\text{Sqrt}[2]*d*(d - 2*e)*(d - e)*\text{Sqrt}[2 + 3*x^2 + x^4]) + ((2*d - e)*(1 + x^2)*\text{Sqrt}[(2 + x^2)/(2 + 2*x^2)]*\text{EllipticF}[\text{ArcTan}[x], 1/2])/(2*d*(d - e)^2*\text{Sqrt}[2 + 3*x^2 + x^4]) - (e*(3*d^2 - 6*d*e + 2*e^2)*(2 + x^2)*\text{EllipticPi}[1 - e/d, \text{ArcTan}[x], 1/2])/(2*\text{Sqrt}[2]*d^2*(d - 2*e)*(d - e)^2*\text{Sqrt}[(2 + x^2)/(1 + x^2)]*\text{Sqrt}[2 + 3*x^2 + x^4])$

Rubi [A] time = 0.326668, antiderivative size = 399, normalized size of antiderivative = 1.26, number of steps used = 9, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1223, 1716, 1189, 1099, 1135, 1214, 1456, 539}

$$\frac{e^2 x \sqrt{x^4 + 3x^2 + 2}}{2d(d^2 - 3de + 2e^2)(d + ex^2)} - \frac{ex(x^2 + 2)}{2d\sqrt{x^4 + 3x^2 + 2}(d^2 - 3de + 2e^2)} + \frac{(x^2 + 1) \sqrt{\frac{x^2+2}{x^2+1}} (3d^2 - 6de + 2e^2) F\left(\tan^{-1}(x) \middle| \frac{1}{2}\right)}{2\sqrt{2}d\sqrt{x^4 + 3x^2 + 2}(d - 2e)(d - e)^2}$$

Antiderivative was successfully verified.

[In] Int[1/((d + e*x^2)^2*Sqrt[2 + 3*x^2 + x^4]),x]

[Out] $-(e*x*(2 + x^2))/(2*d*(d^2 - 3*d*e + 2*e^2)*\text{Sqrt}[2 + 3*x^2 + x^4]) + (e^2*x*\text{Sqrt}[2 + 3*x^2 + x^4])/(2*d*(d^2 - 3*d*e + 2*e^2)*(d + e*x^2)) + (e*(1 + x^2)*\text{Sqrt}[(2 + x^2)/(1 + x^2)]*\text{EllipticE}[\text{ArcTan}[x], 1/2])/(\text{Sqrt}[2]*d*(d - 2*e)*(d - e)*\text{Sqrt}[2 + 3*x^2 + x^4]) - ((1 + x^2)*\text{Sqrt}[(2 + x^2)/(1 + x^2)]*\text{EllipticF}[\text{ArcTan}[x], 1/2])/(2*\text{Sqrt}[2]*(d - 2*e)*(d - e)*\text{Sqrt}[2 + 3*x^2 + x^4]) + ((3*d^2 - 6*d*e + 2*e^2)*(1 + x^2)*\text{Sqrt}[(2 + x^2)/(1 + x^2)]*\text{EllipticF}[\text{ArcTan}[x], 1/2])/(2*\text{Sqrt}[2]*d*(d - 2*e)*(d - e)^2*\text{Sqrt}[2 + 3*x^2 + x^4]) - (e*(3*d^2 - 6*d*e + 2*e^2)*(2 + x^2)*\text{EllipticPi}[1 - e/d, \text{ArcTan}[x], 1/2])/(2*\text{Sqrt}[2]*d^2*(d - 2*e)*(d - e)^2*\text{Sqrt}[(2 + x^2)/(1 + x^2)]*\text{Sqrt}[2 + 3*x^2 + x^4])$

Rule 1223

Int[((d_) + (e_.)*(x_)^2)^(q_)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> -Simp[(e^2*x*(d + e*x^2)^(q + 1)*Sqrt[a + b*x^2 + c*x^4])/(2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2)), Int[((d + e*x^2)^(q + 1)*Simp[a*e^2*(2*q + 3) + 2*d*(c*d - b*e)*(q + 1) - 2*e*(c*d*(q + 1) - b*e*(q + 2))*x^2 + c*e^2*(2*q + 5)*x^4, x])/Sqrt[a + b*x^2 + c*x^4], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && ILtQ[q, -1]

Rule 1716

Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[P4x, x, 4]}, -Dist[(e^2)^(-1), Int[(C*d - B*e - C*e*x^2)/Sqrt[a + b*x^2 + c

```
*x^4], x], x] + Dist[(C*d^2 - B*d*e + A*e^2)/e^2, Int[1/((d + e*x^2)*Sqrt[a
+ b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[P4x, x^2,
2] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 -
a*e^2, 0]
```

Rule 1189

```
Int[((d_) + (e_)*(x_)^2)/Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4], x_Symbol]
:> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a]
|| PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1099

```
Int[1/Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[((2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]*EllipticF[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)/(b + q)])/(2*
a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])) /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1135

```
Int[(x_)^2/Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4], x_Symbol] :> With[{q =
Rt[b^2 - 4*a*c, 2]}, Simp[(x*(b + q + 2*c*x^2))/(2*c*Sqrt[a + b*x^2 + c*x^
4]), x] - Simp[(Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q)
)*x^2)/(2*a + (b + q)*x^2)]*EllipticE[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)
/(b + q)])/(2*c*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] && !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])) /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1214

```
Int[1/(((d_) + (e_)*(x_)^2)*Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/(2*c*d - e*(b - q)), In
t[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[e/(2*c*d - e*(b - q)), Int[(b -
q + 2*c*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
c, d, e}, x] && GtQ[b^2 - 4*a*c, 0] && !LtQ[c, 0]
```

Rule 1456

```
Int[((d_) + (e_)*(x_)^(n_))^(q_)*((f_) + (g_)*(x_)^(n_))^(r_)*((a_) + (
b_)*(x_)^(n_) + (c_)*(x_)^(n2_))^(p_), x_Symbol] :> Dist[(a + b*x^n + c*x
^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d + (c*x^n)/e)^FracPart[p])
, Int[(d + e*x^n)^(p + q)*(f + g*x^n)^r*(a/d + (c*x^n)/e)^p, x], x] /; Free
Q[{a, b, c, d, e, f, g, n, p, q, r}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && !IntegerQ[p]
```

Rule 539

```
Int[Sqrt[(c_) + (d_)*(x_)^2]/(((a_) + (b_)*(x_)^2)*Sqrt[(e_) + (f_)*(x_)
^2]), x_Symbol] :> Simp[(c*Sqrt[e + f*x^2]*EllipticPi[1 - (b*c)/(a*d), ArcT
an[Rt[d/c, 2]*x], 1 - (c*f)/(d*e)]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c
*(e + f*x^2))/(e*(c + d*x^2))]), x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(d+ex^2)^2 \sqrt{2+3x^2+x^4}} dx &= \frac{e^2 x \sqrt{2+3x^2+x^4}}{2d(d^2-3de+2e^2)(d+ex^2)} - \frac{\int \frac{-2(d^2-3de+e^2)+2dex^2+e^2x^4}{(d+ex^2)\sqrt{2+3x^2+x^4}} dx}{2d(d-2e)(d-e)} \\
&= \frac{e^2 x \sqrt{2+3x^2+x^4}}{2d(d^2-3de+2e^2)(d+ex^2)} + \frac{\int \frac{-de^2-e^3x^2}{\sqrt{2+3x^2+x^4}} dx}{2d(d-2e)(d-e)e^2} + \frac{(3d^2-6de+2e^2) \int \frac{1}{(d+ex^2)\sqrt{2+3x^2+x^4}} dx}{2d(d-2e)(d-e)} \\
&= \frac{e^2 x \sqrt{2+3x^2+x^4}}{2d(d^2-3de+2e^2)(d+ex^2)} - \frac{\int \frac{1}{\sqrt{2+3x^2+x^4}} dx}{2(d-2e)(d-e)} + \frac{(3d^2-6de+2e^2) \int \frac{1}{\sqrt{2+3x^2+x^4}} dx}{2d(d-2e)(d-e)^2} \\
&= -\frac{ex(2+x^2)}{2d(d^2-3de+2e^2)\sqrt{2+3x^2+x^4}} + \frac{e^2 x \sqrt{2+3x^2+x^4}}{2d(d^2-3de+2e^2)(d+ex^2)} + \frac{e(1+x^2)}{\sqrt{2d(d-2e)}} \\
&= -\frac{ex(2+x^2)}{2d(d^2-3de+2e^2)\sqrt{2+3x^2+x^4}} + \frac{e^2 x \sqrt{2+3x^2+x^4}}{2d(d^2-3de+2e^2)(d+ex^2)} + \frac{e(1+x^2)}{\sqrt{2d(d-2e)}}
\end{aligned}$$

Mathematica [C] time = 0.583131, size = 175, normalized size = 0.55

$$\frac{e^2 x (x^4 + 3x^2 + 2)}{(d^2 - 3de + 2e^2)(d + ex^2)} + \frac{i\sqrt{x^2+1}\sqrt{x^2+2} \left(d(d-e) \operatorname{EllipticF} \left(i \sinh^{-1} \left(\frac{x}{\sqrt{2}} \right), 2 \right) + (-3d^2 + 6de - 2e^2) \Pi \left(\frac{2e}{d}; i \sinh^{-1} \left(\frac{x}{\sqrt{2}} \right) \middle| 2 \right) + de E \left(i \sinh^{-1} \left(\frac{x}{\sqrt{2}} \right) \middle| 2 \right) \right)}{2d\sqrt{x^4+3x^2+2} d(d-2e)(d-e)}$$

Antiderivative was successfully verified.

[In] Integrate[1/((d + e*x^2)^2*Sqrt[2 + 3*x^2 + x^4]),x]

[Out] ((e^2*x*(2 + 3*x^2 + x^4))/((d^2 - 3*d*e + 2*e^2)*(d + e*x^2)) + (I*Sqrt[1 + x^2]*Sqrt[2 + x^2]*(d*e*EllipticE[I*ArcSinh[x/Sqrt[2]]], 2) + d*(d - e)*EllipticF[I*ArcSinh[x/Sqrt[2]]], 2) + (-3*d^2 + 6*d*e - 2*e^2)*EllipticPi[(2*e)/d, I*ArcSinh[x/Sqrt[2]], 2]))/(d*(d - 2*e)*(d - e))/(2*d*Sqrt[2 + 3*x^2 + x^4])

Maple [C] time = 0.026, size = 443, normalized size = 1.4

$$\frac{e^2 x}{2(d^2 - 3de + 2e^2)d(ex^2 + d)} \sqrt{x^4 + 3x^2 + 2} + \frac{\frac{i}{4} \sqrt{2} \operatorname{EllipticF} \left(\frac{i}{2} x \sqrt{2}, \sqrt{2} \right)}{d^2 - 3de + 2e^2} \sqrt{2x^2 + 4} \sqrt{x^2 + 1} - \frac{1}{\sqrt{x^4 + 3x^2 + 2}} - \frac{i}{4} e \sqrt{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(e*x^2+d)^2/(x^4+3*x^2+2)^(1/2),x)

[Out] 1/2*e^2*x*(x^4+3*x^2+2)^(1/2)/d/(d^2-3*d*e+2*e^2)/(e*x^2+d)+1/4*I/(d^2-3*d*e+2*e^2)*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticF(1/2*I*x*2^(1/2),2^(1/2))-1/4*I*e/(d^2-3*d*e+2*e^2)/d*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticF(1/2*I*x*2^(1/2),2^(1/2))+1/4*I*e/(d^2-3*d*e+2*e^2)/d*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticE(1/2*I*x*2^(1/2),2^(1/2))-3/2*I/(d^2-3*d*e+2*e^2)*2^(1/2)*(1+1/2*x^2)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticPi(1/2*I*x*2^(1/2),2*e/d,2^(1/2))+3*I/(d^2-3*d*e+2*e^2)/d*e*2^(1/2)*(1+1/2*x^2)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)

$$2+1)^{(1/2)}/(x^4+3x^2+2)^{(1/2)}*\text{EllipticPi}(1/2*I*x^2^{(1/2)},2*e/d,2^{(1/2)})-I/(d^2-3*d*e+2*e^2)/d^2*e^2*2^{(1/2)}*(1+1/2*x^2)^{(1/2)}*(x^2+1)^{(1/2)}/(x^4+3*x^2+2)^{(1/2)}*\text{EllipticPi}(1/2*I*x^2^{(1/2)},2*e/d,2^{(1/2)})$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^4 + 3x^2 + 2}(ex^2 + d)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x^2+d)^2/(x^4+3*x^2+2)^(1/2),x, algorithm="maxima")

[Out] integrate(1/(sqrt(x^4 + 3*x^2 + 2)*(e*x^2 + d)^2), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{x^4 + 3x^2 + 2}}{e^2x^8 + (2de + 3e^2)x^6 + (d^2 + 6de + 2e^2)x^4 + (3d^2 + 4de)x^2 + 2d^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x^2+d)^2/(x^4+3*x^2+2)^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(x^4 + 3*x^2 + 2)/(e^2*x^8 + (2*d*e + 3*e^2)*x^6 + (d^2 + 6*d*e + 2*e^2)*x^4 + (3*d^2 + 4*d*e)*x^2 + 2*d^2), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{(x^2 + 1)(x^2 + 2)}(d + ex^2)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x**2+d)**2/(x**4+3*x**2+2)**(1/2),x)

[Out] Integral(1/(sqrt((x**2 + 1)*(x**2 + 2))*(d + e*x**2)**2), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^4 + 3x^2 + 2}(ex^2 + d)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x^2+d)^2/(x^4+3*x^2+2)^(1/2),x, algorithm="giac")

[Out] integrate(1/(sqrt(x^4 + 3*x^2 + 2)*(e*x^2 + d)^2), x)

$$3.399 \quad \int (c + ex^2)^q (a + cx^2 + bx^4)^p dx$$

Optimal. Leaf size=26

$$\text{Unintegrable}\left((c + ex^2)^q (a + bx^4 + cx^2)^p, x\right)$$

[Out] Defer[Int][(c + e*x^2)^q*(a + c*x^2 + b*x^4)^p, x]

Rubi [A] time = 0.0112424, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int (c + ex^2)^q (a + cx^2 + bx^4)^p dx$$

Verification is Not applicable to the result.

[In] Int[(c + e*x^2)^q*(a + c*x^2 + b*x^4)^p, x]

[Out] Defer[Int][(c + e*x^2)^q*(a + c*x^2 + b*x^4)^p, x]

Rubi steps

$$\int (c + ex^2)^q (a + cx^2 + bx^4)^p dx = \int (c + ex^2)^q (a + cx^2 + bx^4)^p dx$$

Mathematica [A] time = 0.0991208, size = 0, normalized size = 0.

$$\int (c + ex^2)^q (a + cx^2 + bx^4)^p dx$$

Verification is Not applicable to the result.

[In] Integrate[(c + e*x^2)^q*(a + c*x^2 + b*x^4)^p, x]

[Out] Integrate[(c + e*x^2)^q*(a + c*x^2 + b*x^4)^p, x]

Maple [A] time = 0.074, size = 0, normalized size = 0.

$$\int (ex^2 + c)^q (bx^4 + cx^2 + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x^2+c)^q*(b*x^4+c*x^2+a)^p, x)

[Out] int((e*x^2+c)^q*(b*x^4+c*x^2+a)^p, x)

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int (bx^4 + cx^2 + a)^p (ex^2 + c)^q dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+c)^q*(b*x^4+c*x^2+a)^p,x, algorithm="maxima")

[Out] integrate((b*x^4 + c*x^2 + a)^p*(e*x^2 + c)^q, x)

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(bx^4 + cx^2 + a\right)^p \left(ex^2 + c\right)^q, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+c)^q*(b*x^4+c*x^2+a)^p,x, algorithm="fricas")

[Out] integral((b*x^4 + c*x^2 + a)^p*(e*x^2 + c)^q, x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x**2+c)**q*(b*x**4+c*x**2+a)**p,x)

[Out] Timed out

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int (bx^4 + cx^2 + a)^p (ex^2 + c)^q dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+c)^q*(b*x^4+c*x^2+a)^p,x, algorithm="giac")

[Out] integrate((b*x^4 + c*x^2 + a)^p*(e*x^2 + c)^q, x)

3.400 $\int (c + ex^2)^3 (a + cx^2 + bx^4)^p dx$

Optimal. Leaf size=498

$$\frac{cx(-3abe^2(4p+7) + ae^3(2p+5) + b^2c^2(16p^2 + 48p + 35)) \left(\frac{2bx^2}{c-\sqrt{c^2-4ab}} + 1\right)^{-p} (a + bx^4 + cx^2)^p \left(\frac{2bx^2}{\sqrt{c^2-4ab+c}} + 1\right)^{-p} F_1\left(\frac{1}{2}\right)}{b^2(4p+5)(4p+7)}$$

[Out] (c*e^2*(21*b - 5*e + 12*b*p - 2*e*p)*x*(a + c*x^2 + b*x^4)^(1 + p))/(b^2*(5 + 4*p)*(7 + 4*p)) + (e^3*x^3*(a + c*x^2 + b*x^4)^(1 + p))/(b*(7 + 4*p)) + (c*(a*e^3*(5 + 2*p) - 3*a*b*e^2*(7 + 4*p) + b^2*c^2*(35 + 48*p + 16*p^2))*x*(a + c*x^2 + b*x^4)^p*AppellF1[1/2, -p, -p, 3/2, (-2*b*x^2)/(c - Sqrt[-4*a*b + c^2]), (-2*b*x^2)/(c + Sqrt[-4*a*b + c^2])])/(b^2*(5 + 4*p)*(7 + 4*p)*(1 + (2*b*x^2)/(c - Sqrt[-4*a*b + c^2]))^p*(1 + (2*b*x^2)/(c + Sqrt[-4*a*b + c^2]))^p) + (e*(c^2*e^2*(15 + 16*p + 4*p^2) + 3*b^2*c^2*(35 + 48*p + 16*p^2) - 3*b*e*(a*e*(5 + 4*p) + c^2*(21 + 26*p + 8*p^2)))*x^3*(a + c*x^2 + b*x^4)^p*AppellF1[3/2, -p, -p, 5/2, (-2*b*x^2)/(c - Sqrt[-4*a*b + c^2]), (-2*b*x^2)/(c + Sqrt[-4*a*b + c^2])])/(3*b^2*(5 + 4*p)*(7 + 4*p)*(1 + (2*b*x^2)/(c - Sqrt[-4*a*b + c^2]))^p*(1 + (2*b*x^2)/(c + Sqrt[-4*a*b + c^2]))^p)

Rubi [A] time = 0.813798, antiderivative size = 498, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {1206, 1679, 1203, 1105, 429, 1141, 510}

$$\frac{cx(-3abe^2(4p+7) + ae^3(2p+5) + b^2c^2(16p^2 + 48p + 35)) \left(\frac{2bx^2}{c-\sqrt{c^2-4ab}} + 1\right)^{-p} (a + bx^4 + cx^2)^p \left(\frac{2bx^2}{\sqrt{c^2-4ab+c}} + 1\right)^{-p} F_1\left(\frac{1}{2}\right)}{b^2(4p+5)(4p+7)}$$

Antiderivative was successfully verified.

[In] Int[(c + e*x^2)^3*(a + c*x^2 + b*x^4)^p,x]

[Out] (c*e^2*(21*b - 5*e + 12*b*p - 2*e*p)*x*(a + c*x^2 + b*x^4)^(1 + p))/(b^2*(5 + 4*p)*(7 + 4*p)) + (e^3*x^3*(a + c*x^2 + b*x^4)^(1 + p))/(b*(7 + 4*p)) + (c*(a*e^3*(5 + 2*p) - 3*a*b*e^2*(7 + 4*p) + b^2*c^2*(35 + 48*p + 16*p^2))*x*(a + c*x^2 + b*x^4)^p*AppellF1[1/2, -p, -p, 3/2, (-2*b*x^2)/(c - Sqrt[-4*a*b + c^2]), (-2*b*x^2)/(c + Sqrt[-4*a*b + c^2])])/(b^2*(5 + 4*p)*(7 + 4*p)*(1 + (2*b*x^2)/(c - Sqrt[-4*a*b + c^2]))^p*(1 + (2*b*x^2)/(c + Sqrt[-4*a*b + c^2]))^p) + (e*(c^2*e^2*(15 + 16*p + 4*p^2) + 3*b^2*c^2*(35 + 48*p + 16*p^2) - 3*b*e*(a*e*(5 + 4*p) + c^2*(21 + 26*p + 8*p^2)))*x^3*(a + c*x^2 + b*x^4)^p*AppellF1[3/2, -p, -p, 5/2, (-2*b*x^2)/(c - Sqrt[-4*a*b + c^2]), (-2*b*x^2)/(c + Sqrt[-4*a*b + c^2])])/(3*b^2*(5 + 4*p)*(7 + 4*p)*(1 + (2*b*x^2)/(c - Sqrt[-4*a*b + c^2]))^p*(1 + (2*b*x^2)/(c + Sqrt[-4*a*b + c^2]))^p)

Rule 1206

Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Simp[(e^q*x^(2*q - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(4*p + 2*q + 1)), x] + Dist[1/(c*(4*p + 2*q + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandT
oSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*p + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1]

Rule 1679

```
Int[(Pq_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] := With[{q =
  Expon[Pq, x^2], e = Coeff[Pq, x^2, Expon[Pq, x^2]]}, Simp[(e*x^(2*q - 3)*(
  a + b*x^2 + c*x^4)^(p + 1))/(c*(2*q + 4*p + 1)), x] + Dist[1/(c*(2*q + 4*p
  + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*q + 4*p + 1)*Pq - a*e*(2*
  q - 3)*x^(2*q - 4) - b*e*(2*q + 2*p - 1)*x^(2*q - 2) - c*e*(2*q + 4*p + 1)*
  x^(2*q), x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && Expon[P
  q, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && !LtQ[p, -1]
```

Rule 1203

```
Int[((d_) + (e_)*(x_)^2)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symb
ol] := Int[ExpandIntegrand[(d + e*x^2)*(a + b*x^2 + c*x^4)^p, x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
  0]
```

Rule 1105

```
Int[((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] := With[{q = Rt[b^
  2 - 4*a*c, 2]}, Dist[(a^IntPart[p]*(a + b*x^2 + c*x^4)^FracPart[p])/((1 + (
  2*c*x^2)/(b + q))^FracPart[p]*(1 + (2*c*x^2)/(b - q))^FracPart[p]), Int[(1
  + (2*c*x^2)/(b + q))^p*(1 + (2*c*x^2)/(b - q))^p, x], x]] /; FreeQ[{a, b, c
  , p}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 429

```
Int[((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_), x_Symbol]
:= Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
&& (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1141

```
Int[((d_)*(x_)^(m_))*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol]
:= Dist[(a^IntPart[p]*(a + b*x^2 + c*x^4)^FracPart[p])/((1 + (2*c*x^2)/(b
+ Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^2)/(b - Rt[b^2 - 4*a*c, 2]))
^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]))^p*(1 + (
2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Rule 510

```
Int[((e_)*(x_)^(m_))*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_
))^^(q_), x_Symbol] := Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)]/(e*(m + 1)), x] /; FreeQ[{a,
  b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
  - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\int (c + ex^2)^3 (a + cx^2 + bx^4)^p dx &= \frac{e^3 x^3 (a + cx^2 + bx^4)^{1+p}}{b(7 + 4p)} + \frac{\int (a + cx^2 + bx^4)^p (bc^3(7 + 4p) - 3e(ae^2 - bc^2(7 + 4p)))}{b(7 + 4p)} \\
&= \frac{ce^2(21b - 5e + 12bp - 2ep)x (a + cx^2 + bx^4)^{1+p}}{b^2(5 + 4p)(7 + 4p)} + \frac{e^3 x^3 (a + cx^2 + bx^4)^{1+p}}{b(7 + 4p)} + \frac{\int (a + cx^2 + bx^4)^p (bc^3(7 + 4p) - 3e(ae^2 - bc^2(7 + 4p)))}{b(7 + 4p)} \\
&= \frac{ce^2(21b - 5e + 12bp - 2ep)x (a + cx^2 + bx^4)^{1+p}}{b^2(5 + 4p)(7 + 4p)} + \frac{e^3 x^3 (a + cx^2 + bx^4)^{1+p}}{b(7 + 4p)} + \frac{\int (a + cx^2 + bx^4)^p (bc^3(7 + 4p) - 3e(ae^2 - bc^2(7 + 4p)))}{b(7 + 4p)} \\
&= \frac{ce^2(21b - 5e + 12bp - 2ep)x (a + cx^2 + bx^4)^{1+p}}{b^2(5 + 4p)(7 + 4p)} + \frac{e^3 x^3 (a + cx^2 + bx^4)^{1+p}}{b(7 + 4p)} + \frac{c(ae^2 - bc^2(7 + 4p))}{b(7 + 4p)} \\
&= \frac{ce^2(21b - 5e + 12bp - 2ep)x (a + cx^2 + bx^4)^{1+p}}{b^2(5 + 4p)(7 + 4p)} + \frac{e^3 x^3 (a + cx^2 + bx^4)^{1+p}}{b(7 + 4p)} + \frac{c(ae^2 - bc^2(7 + 4p))}{b(7 + 4p)}
\end{aligned}$$

Mathematica [A] time = 0.518336, size = 373, normalized size = 0.75

$$\frac{1}{35} x \left(\frac{-\sqrt{c^2 - 4ab} + 2bx^2 + c}{c - \sqrt{c^2 - 4ab}} \right)^{-p} \left(\frac{\sqrt{c^2 - 4ab} + 2bx^2 + c}{\sqrt{c^2 - 4ab} + c} \right)^{-p} (a + bx^4 + cx^2)^p \left(ex^2 \left(ex^2 \left(5ex^2 F_1 \left(\frac{7}{2}; -p, -p; \frac{9}{2}; -\frac{2bx^2}{c + \sqrt{c^2 - 4ab}} \right) \right) \right) \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[(c + e*x^2)^3*(a + c*x^2 + b*x^4)^p,x]

[Out] (x*(a + c*x^2 + b*x^4)^p*(35*c^3*AppellF1[1/2, -p, -p, 3/2, (-2*b*x^2)/(c + Sqrt[-4*a*b + c^2]), (2*b*x^2)/(-c + Sqrt[-4*a*b + c^2])]) + e*x^2*(35*c^2*AppellF1[3/2, -p, -p, 5/2, (-2*b*x^2)/(c + Sqrt[-4*a*b + c^2]), (2*b*x^2)/(-c + Sqrt[-4*a*b + c^2])]) + e*x^2*(21*c*AppellF1[5/2, -p, -p, 7/2, (-2*b*x^2)/(c + Sqrt[-4*a*b + c^2]), (2*b*x^2)/(-c + Sqrt[-4*a*b + c^2])]) + 5*e*x^2*AppellF1[7/2, -p, -p, 9/2, (-2*b*x^2)/(c + Sqrt[-4*a*b + c^2]), (2*b*x^2)/(-c + Sqrt[-4*a*b + c^2])])))/(35*((c - Sqrt[-4*a*b + c^2] + 2*b*x^2)/(c - Sqrt[-4*a*b + c^2]))^p*((c + Sqrt[-4*a*b + c^2] + 2*b*x^2)/(c + Sqrt[-4*a*b + c^2]))^p)

Maple [F] time = 0.051, size = 0, normalized size = 0.

$$\int (ex^2 + c)^3 (bx^4 + cx^2 + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x^2+c)^3*(b*x^4+c*x^2+a)^p,x)

[Out] int((e*x^2+c)^3*(b*x^4+c*x^2+a)^p,x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (ex^2 + c)^3 (bx^4 + cx^2 + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+c)^3*(b*x^4+c*x^2+a)^p,x, algorithm="maxima")

[Out] integrate((e*x^2 + c)^3*(b*x^4 + c*x^2 + a)^p, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(e^3x^6 + 3ce^2x^4 + 3c^2ex^2 + c^3\right)\left(bx^4 + cx^2 + a\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+c)^3*(b*x^4+c*x^2+a)^p,x, algorithm="fricas")

[Out] integral((e^3*x^6 + 3*c*e^2*x^4 + 3*c^2*e*x^2 + c^3)*(b*x^4 + c*x^2 + a)^p, x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x**2+c)**3*(b*x**4+c*x**2+a)**p,x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (ex^2 + c)^3 (bx^4 + cx^2 + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+c)^3*(b*x^4+c*x^2+a)^p,x, algorithm="giac")

[Out] integrate((e*x^2 + c)^3*(b*x^4 + c*x^2 + a)^p, x)

3.401 $\int (c + ex^2)^2 (a + cx^2 + bx^4)^p dx$

Optimal. Leaf size=358

$$\frac{x \left(ae^2 - bc^2(4p + 5) \right) \left(\frac{2bx^2}{c - \sqrt{c^2 - 4ab}} + 1 \right)^{-p} (a + bx^4 + cx^2)^p \left(\frac{2bx^2}{\sqrt{c^2 - 4ab} + c} + 1 \right)^{-p} F_1 \left(\frac{1}{2}; -p, -p; \frac{3}{2}; -\frac{2bx^2}{c - \sqrt{c^2 - 4ab}}, -\frac{2bx^2}{c + \sqrt{c^2 - 4ab}} \right)}{b(4p + 5)}$$

```
[Out] (e^2*x*(a + c*x^2 + b*x^4)^(1 + p))/(b*(5 + 4*p)) - ((a*e^2 - b*c^2*(5 + 4*p))*x*(a + c*x^2 + b*x^4)^p*AppellF1[1/2, -p, -p, 3/2, (-2*b*x^2)/(c - Sqrt[-4*a*b + c^2]), (-2*b*x^2)/(c + Sqrt[-4*a*b + c^2])])/(b*(5 + 4*p)*(1 + (2*b*x^2)/(c - Sqrt[-4*a*b + c^2]))^p*(1 + (2*b*x^2)/(c + Sqrt[-4*a*b + c^2]))^p) + (c*e*(10*b - 3*e + 8*b*p - 2*e*p)*x^3*(a + c*x^2 + b*x^4)^p*AppellF1[3/2, -p, -p, 5/2, (-2*b*x^2)/(c - Sqrt[-4*a*b + c^2]), (-2*b*x^2)/(c + Sqrt[-4*a*b + c^2])])/(3*b*(5 + 4*p)*(1 + (2*b*x^2)/(c - Sqrt[-4*a*b + c^2]))^p*(1 + (2*b*x^2)/(c + Sqrt[-4*a*b + c^2]))^p)
```

Rubi [A] time = 0.356375, antiderivative size = 345, normalized size of antiderivative = 0.96, number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$, Rules used = {1206, 1203, 1105, 429, 1141, 510}

$$x \left(c^2 - \frac{ae^2}{4bp + 5b} \right) \left(\frac{2bx^2}{c - \sqrt{c^2 - 4ab}} + 1 \right)^{-p} (a + bx^4 + cx^2)^p \left(\frac{2bx^2}{\sqrt{c^2 - 4ab} + c} + 1 \right)^{-p} F_1 \left(\frac{1}{2}; -p, -p; \frac{3}{2}; -\frac{2bx^2}{c - \sqrt{c^2 - 4ab}}, -\frac{2bx^2}{c + \sqrt{c^2 - 4ab}} \right)$$

Antiderivative was successfully verified.

```
[In] Int[(c + e*x^2)^2*(a + c*x^2 + b*x^4)^p,x]
```

```
[Out] (e^2*x*(a + c*x^2 + b*x^4)^(1 + p))/(b*(5 + 4*p)) + ((c^2 - (a*e^2)/(5*b + 4*b*p))*x*(a + c*x^2 + b*x^4)^p*AppellF1[1/2, -p, -p, 3/2, (-2*b*x^2)/(c - Sqrt[-4*a*b + c^2]), (-2*b*x^2)/(c + Sqrt[-4*a*b + c^2])])/(b*(5 + 4*p)*(1 + (2*b*x^2)/(c - Sqrt[-4*a*b + c^2]))^p*(1 + (2*b*x^2)/(c + Sqrt[-4*a*b + c^2]))^p) + (c*e*(2 - (e*(3 + 2*p))/(b*(5 + 4*p)))*x^3*(a + c*x^2 + b*x^4)^p*AppellF1[3/2, -p, -p, 5/2, (-2*b*x^2)/(c - Sqrt[-4*a*b + c^2]), (-2*b*x^2)/(c + Sqrt[-4*a*b + c^2])])/(3*(1 + (2*b*x^2)/(c - Sqrt[-4*a*b + c^2]))^p*(1 + (2*b*x^2)/(c + Sqrt[-4*a*b + c^2]))^p)
```

Rule 1206

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(e^q*x^(2*q - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(4*p + 2*q + 1)), x] + Dist[1/(c*(4*p + 2*q + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandTOSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*p + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1]
```

Rule 1203

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0]
```

Rule 1105

```
Int[((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(a^IntPart[p]*(a + b*x^2 + c*x^4)^FracPart[p])/((1 + (2*c*x^2)/(b + q))^FracPart[p]*(1 + (2*c*x^2)/(b - q))^FracPart[p]), Int[(1 + (2*c*x^2)/(b + q))^p*(1 + (2*c*x^2)/(b - q))^p, x], x] /; FreeQ[{a, b, c, p}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 429

```
Int[((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_), x_Symbol] := Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1141

```
Int[((d_)*(x_)^(m_))*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] := Dist[(a^IntPart[p]*(a + b*x^2 + c*x^4)^FracPart[p])/((1 + (2*c*x^2)/(b + Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^2)/(b - Rt[b^2 - 4*a*c, 2]))^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Rule 510

```
Int[((e_)*(x_)^(m_))*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_), x_Symbol] := Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)]/(e*(m + 1)), x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned} \int (c + ex^2)^2 (a + cx^2 + bx^4)^p dx &= \frac{e^2 x (a + cx^2 + bx^4)^{1+p}}{b(5 + 4p)} + \frac{\int (-ae^2 + bc^2(5 + 4p) + ce(10b - 3e + 8bp - 2ep)x^2) (a + cx^2 + bx^4)^p dx}{b(5 + 4p)} \\ &= \frac{e^2 x (a + cx^2 + bx^4)^{1+p}}{b(5 + 4p)} + \frac{\int \left(-ae^2 \left(1 - \frac{bc^2(5+4p)}{ae^2}\right) (a + cx^2 + bx^4)^p - ce(-10b + 3e - 8bp + 2ep)x^2\right) (a + cx^2 + bx^4)^p dx}{b(5 + 4p)} \\ &= \frac{e^2 x (a + cx^2 + bx^4)^{1+p}}{b(5 + 4p)} + \left(ce \left(2 - \frac{e(3 + 2p)}{b(5 + 4p)}\right)\right) \int x^2 (a + cx^2 + bx^4)^p dx - \left(-c^2 + \frac{2bx^2}{c + \sqrt{-4ab + c^2}}\right) (a + cx^2 + bx^4)^p \\ &= \frac{e^2 x (a + cx^2 + bx^4)^{1+p}}{b(5 + 4p)} + \left(ce \left(2 - \frac{e(3 + 2p)}{b(5 + 4p)}\right)\right) \left(1 + \frac{2bx^2}{c - \sqrt{-4ab + c^2}}\right)^{-p} \left(1 + \frac{2bx^2}{c + \sqrt{-4ab + c^2}}\right)^p \\ &= \frac{e^2 x (a + cx^2 + bx^4)^{1+p}}{b(5 + 4p)} + \left(c^2 - \frac{ae^2}{5b + 4bp}\right) x \left(1 + \frac{2bx^2}{c - \sqrt{-4ab + c^2}}\right)^{-p} \left(1 + \frac{2bx^2}{c + \sqrt{-4ab + c^2}}\right)^p \end{aligned}$$

Mathematica [A] time = 0.353722, size = 303, normalized size = 0.85

$$\frac{1}{15} x \left(\frac{-\sqrt{c^2 - 4ab} + 2bx^2 + c}{c - \sqrt{c^2 - 4ab}} \right)^{-p} \left(\frac{\sqrt{c^2 - 4ab} + 2bx^2 + c}{\sqrt{c^2 - 4ab} + c} \right)^{-p} (a + bx^4 + cx^2)^p \left(ex^2 \left(3ex^2 F_1 \left(\frac{5}{2}; -p, -p; \frac{7}{2}; -\frac{2bx^2}{c + \sqrt{c^2 - 4ab}} \right) \right) \right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + e*x^2)^2*(a + c*x^2 + b*x^4)^p,x]
```

```
[Out] (x*(a + c*x^2 + b*x^4)^p*(15*c^2*AppellF1[1/2, -p, -p, 3/2, (-2*b*x^2)/(c + Sqrt[-4*a*b + c^2]), (2*b*x^2)/(-c + Sqrt[-4*a*b + c^2])]) + e*x^2*(10*c*Ap
```


$\text{pellF1}[3/2, -p, -p, 5/2, (-2*b*x^2)/(c + \text{Sqrt}[-4*a*b + c^2]), (2*b*x^2)/(-c + \text{Sqrt}[-4*a*b + c^2])] + 3*e*x^2*\text{AppellF1}[5/2, -p, -p, 7/2, (-2*b*x^2)/(c + \text{Sqrt}[-4*a*b + c^2]), (2*b*x^2)/(-c + \text{Sqrt}[-4*a*b + c^2])])]/(15*((c - \text{Sqrt}[-4*a*b + c^2] + 2*b*x^2)/(c - \text{Sqrt}[-4*a*b + c^2]))^p*((c + \text{Sqrt}[-4*a*b + c^2] + 2*b*x^2)/(c + \text{Sqrt}[-4*a*b + c^2]))^p)$

Maple [F] time = 0.041, size = 0, normalized size = 0.

$$\int (ex^2 + c)^2 (bx^4 + cx^2 + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x^2+c)^2*(b*x^4+c*x^2+a)^p,x)

[Out] int((e*x^2+c)^2*(b*x^4+c*x^2+a)^p,x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (ex^2 + c)^2 (bx^4 + cx^2 + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+c)^2*(b*x^4+c*x^2+a)^p,x, algorithm="maxima")

[Out] integrate((e*x^2 + c)^2*(b*x^4 + c*x^2 + a)^p, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(e^2x^4 + 2cex^2 + c^2\right)\left(bx^4 + cx^2 + a\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+c)^2*(b*x^4+c*x^2+a)^p,x, algorithm="fricas")

[Out] integral((e^2*x^4 + 2*c*e*x^2 + c^2)*(b*x^4 + c*x^2 + a)^p, x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x**2+c)**2*(b*x**4+c*x**2+a)**p,x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (ex^2 + c)^2 (bx^4 + cx^2 + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+c)^2*(b*x^4+c*x^2+a)^p,x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + c)^2*(b*x^4 + c*x^2 + a)^p, x)
```

3.402 $\int (c + ex^2) (a + cx^2 + bx^4)^p dx$

Optimal. Leaf size=274

$$\frac{1}{3}ex^3 \left(\frac{2bx^2}{c - \sqrt{c^2 - 4ab}} + 1 \right)^{-p} (a + bx^4 + cx^2)^p \left(\frac{2bx^2}{\sqrt{c^2 - 4ab} + c} + 1 \right)^{-p} F_1 \left(\frac{3}{2}; -p, -p; \frac{5}{2}; -\frac{2bx^2}{c - \sqrt{c^2 - 4ab}}, -\frac{2bx^2}{c + \sqrt{c^2 - 4ab}} \right)$$

[Out] (c*x*(a + c*x^2 + b*x^4)^p*AppellF1[1/2, -p, -p, 3/2, (-2*b*x^2)/(c - Sqrt[-4*a*b + c^2]), (-2*b*x^2)/(c + Sqrt[-4*a*b + c^2])]/((1 + (2*b*x^2)/(c - Sqrt[-4*a*b + c^2]))^p*(1 + (2*b*x^2)/(c + Sqrt[-4*a*b + c^2]))^p) + (e*x^3*(a + c*x^2 + b*x^4)^p*AppellF1[3/2, -p, -p, 5/2, (-2*b*x^2)/(c - Sqrt[-4*a*b + c^2]), (-2*b*x^2)/(c + Sqrt[-4*a*b + c^2])]/(3*(1 + (2*b*x^2)/(c - Sqrt[-4*a*b + c^2]))^p*(1 + (2*b*x^2)/(c + Sqrt[-4*a*b + c^2]))^p)

Rubi [A] time = 0.222344, antiderivative size = 274, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {1203, 1105, 429, 1141, 510}

$$\frac{1}{3}ex^3 \left(\frac{2bx^2}{c - \sqrt{c^2 - 4ab}} + 1 \right)^{-p} (a + bx^4 + cx^2)^p \left(\frac{2bx^2}{\sqrt{c^2 - 4ab} + c} + 1 \right)^{-p} F_1 \left(\frac{3}{2}; -p, -p; \frac{5}{2}; -\frac{2bx^2}{c - \sqrt{c^2 - 4ab}}, -\frac{2bx^2}{c + \sqrt{c^2 - 4ab}} \right)$$

Antiderivative was successfully verified.

[In] Int[(c + e*x^2)*(a + c*x^2 + b*x^4)^p,x]

[Out] (c*x*(a + c*x^2 + b*x^4)^p*AppellF1[1/2, -p, -p, 3/2, (-2*b*x^2)/(c - Sqrt[-4*a*b + c^2]), (-2*b*x^2)/(c + Sqrt[-4*a*b + c^2])]/((1 + (2*b*x^2)/(c - Sqrt[-4*a*b + c^2]))^p*(1 + (2*b*x^2)/(c + Sqrt[-4*a*b + c^2]))^p) + (e*x^3*(a + c*x^2 + b*x^4)^p*AppellF1[3/2, -p, -p, 5/2, (-2*b*x^2)/(c - Sqrt[-4*a*b + c^2]), (-2*b*x^2)/(c + Sqrt[-4*a*b + c^2])]/(3*(1 + (2*b*x^2)/(c - Sqrt[-4*a*b + c^2]))^p*(1 + (2*b*x^2)/(c + Sqrt[-4*a*b + c^2]))^p)

Rule 1203

Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := Int[ExpandIntegrand[(d + e*x^2)*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0]

Rule 1105

Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(a^IntPart[p]*(a + b*x^2 + c*x^4)^FracPart[p])/((1 + (2*c*x^2)/(b + q))^FracPart[p]*(1 + (2*c*x^2)/(b - q))^FracPart[p]), Int[(1 + (2*c*x^2)/(b + q))^p*(1 + (2*c*x^2)/(b - q))^p, x], x] /; FreeQ[{a, b, c, p}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 429

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -(b*x^n)/a, -(d*x^n)/c], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])

Rule 1141

```
Int[((d_)*(x_))^(m_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol]
  :> Dist[(a^IntPart[p]*(a + b*x^2 + c*x^4)^FracPart[p])/((1 + (2*c*x^2)/(b
+ Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^2)/(b - Rt[b^2 - 4*a*c, 2]))
^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]))^p*(1 + (
2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Rule 510

```
Int[((e_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_
))^ (q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/ (e*(m + 1)), x] /; FreeQ[{a,
b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
- 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned} \int (c + ex^2)(a + cx^2 + bx^4)^p dx &= \int \left(c(a + cx^2 + bx^4)^p + ex^2(a + cx^2 + bx^4)^p \right) dx \\ &= c \int (a + cx^2 + bx^4)^p dx + e \int x^2(a + cx^2 + bx^4)^p dx \\ &= \left(c \left(1 + \frac{2bx^2}{c - \sqrt{-4ab + c^2}} \right)^{-p} \left(1 + \frac{2bx^2}{c + \sqrt{-4ab + c^2}} \right)^{-p} (a + cx^2 + bx^4)^p \right) \int \left(1 + \frac{2bx^2}{c - \sqrt{-4ab + c^2}} \right)^{-p} \left(1 + \frac{2bx^2}{c + \sqrt{-4ab + c^2}} \right)^{-p} (a + cx^2 + bx^4)^p dx \\ &= cx \left(1 + \frac{2bx^2}{c - \sqrt{-4ab + c^2}} \right)^{-p} \left(1 + \frac{2bx^2}{c + \sqrt{-4ab + c^2}} \right)^{-p} (a + cx^2 + bx^4)^p F_1 \left(\frac{1}{2}; -p, -p; \frac{3}{2}; \frac{2bx^2}{c - \sqrt{-4ab + c^2}}, \frac{2bx^2}{c + \sqrt{-4ab + c^2}} \right) \end{aligned}$$

Mathematica [A] time = 0.244317, size = 232, normalized size = 0.85

$$\frac{1}{3}x \left(\frac{-\sqrt{c^2 - 4ab} + 2bx^2 + c}{c - \sqrt{c^2 - 4ab}} \right)^{-p} \left(\frac{\sqrt{c^2 - 4ab} + 2bx^2 + c}{\sqrt{c^2 - 4ab} + c} \right)^{-p} (a + bx^4 + cx^2)^p \left(ex^2 F_1 \left(\frac{3}{2}; -p, -p; \frac{5}{2}; -\frac{2bx^2}{c + \sqrt{c^2 - 4ab}}, \frac{2bx^2}{\sqrt{c^2 - 4ab}} \right) \right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + e*x^2)*(a + c*x^2 + b*x^4)^p,x]
```

```
[Out] (x*(a + c*x^2 + b*x^4)^p*(3*c*AppellF1[1/2, -p, -p, 3/2, (-2*b*x^2)/(c + Sqrt[-4*a*b + c^2]), (2*b*x^2)/(-c + Sqrt[-4*a*b + c^2])]) + e*x^2*AppellF1[3/2, -p, -p, 5/2, (-2*b*x^2)/(c + Sqrt[-4*a*b + c^2]), (2*b*x^2)/(-c + Sqrt[-4*a*b + c^2])]))/(3*((c - Sqrt[-4*a*b + c^2] + 2*b*x^2)/(c - Sqrt[-4*a*b + c^2]))^p*((c + Sqrt[-4*a*b + c^2] + 2*b*x^2)/(c + Sqrt[-4*a*b + c^2]))^p)
```

Maple [F] time = 0.025, size = 0, normalized size = 0.

$$\int (ex^2 + c)(bx^4 + cx^2 + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+c)*(b*x^4+c*x^2+a)^p,x)
```

```
[Out] int((e*x^2+c)*(b*x^4+c*x^2+a)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (ex^2 + c)(bx^4 + cx^2 + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+c)*(b*x^4+c*x^2+a)^p,x, algorithm="maxima")

[Out] integrate((e*x^2 + c)*(b*x^4 + c*x^2 + a)^p, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(ex^2 + c\right)\left(bx^4 + cx^2 + a\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+c)*(b*x^4+c*x^2+a)^p,x, algorithm="fricas")

[Out] integral((e*x^2 + c)*(b*x^4 + c*x^2 + a)^p, x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x**2+c)*(b*x**4+c*x**2+a)**p,x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (ex^2 + c)(bx^4 + cx^2 + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+c)*(b*x^4+c*x^2+a)^p,x, algorithm="giac")

[Out] integrate((e*x^2 + c)*(b*x^4 + c*x^2 + a)^p, x)

3.403 $\int (a + cx^2 + bx^4)^p dx$

Optimal. Leaf size=133

$$x \left(\frac{2bx^2}{c - \sqrt{c^2 - 4ab}} + 1 \right)^{-p} \left(\frac{2bx^2}{\sqrt{c^2 - 4ab} + c} + 1 \right)^{-p} (a + bx^4 + cx^2)^p F_1 \left(\frac{1}{2}; -p, -p; \frac{3}{2}; -\frac{2bx^2}{c - \sqrt{c^2 - 4ab}}, -\frac{2bx^2}{c + \sqrt{c^2 - 4ab}} \right)$$

[Out] (x*(a + c*x^2 + b*x^4)^p*AppellF1[1/2, -p, -p, 3/2, (-2*b*x^2)/(c - Sqrt[-4*a*b + c^2]), (-2*b*x^2)/(c + Sqrt[-4*a*b + c^2])])/((1 + (2*b*x^2)/(c - Sqrt[-4*a*b + c^2]))^p*(1 + (2*b*x^2)/(c + Sqrt[-4*a*b + c^2]))^p)

Rubi [A] time = 0.0580843, antiderivative size = 133, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1105, 429}

$$x \left(\frac{2bx^2}{c - \sqrt{c^2 - 4ab}} + 1 \right)^{-p} \left(\frac{2bx^2}{\sqrt{c^2 - 4ab} + c} + 1 \right)^{-p} (a + bx^4 + cx^2)^p F_1 \left(\frac{1}{2}; -p, -p; \frac{3}{2}; -\frac{2bx^2}{c - \sqrt{c^2 - 4ab}}, -\frac{2bx^2}{c + \sqrt{c^2 - 4ab}} \right)$$

Antiderivative was successfully verified.

[In] Int[(a + c*x^2 + b*x^4)^p, x]

[Out] (x*(a + c*x^2 + b*x^4)^p*AppellF1[1/2, -p, -p, 3/2, (-2*b*x^2)/(c - Sqrt[-4*a*b + c^2]), (-2*b*x^2)/(c + Sqrt[-4*a*b + c^2])])/((1 + (2*b*x^2)/(c - Sqrt[-4*a*b + c^2]))^p*(1 + (2*b*x^2)/(c + Sqrt[-4*a*b + c^2]))^p)

Rule 1105

Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(a^IntPart[p]*(a + b*x^2 + c*x^4)^FracPart[p])/((1 + (2*c*x^2)/(b + q))^FracPart[p]*(1 + (2*c*x^2)/(b - q))^FracPart[p]), Int[(1 + (2*c*x^2)/(b + q))^p*(1 + (2*c*x^2)/(b - q))^p, x], x] /; FreeQ[{a, b, c, p}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 429

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])

Rubi steps

$$\int (a + cx^2 + bx^4)^p dx = \left(\left(1 + \frac{2bx^2}{c - \sqrt{-4ab + c^2}} \right)^{-p} \left(1 + \frac{2bx^2}{c + \sqrt{-4ab + c^2}} \right)^{-p} (a + cx^2 + bx^4)^p \right) \int \left(1 + \frac{2bx^2}{c - \sqrt{-4ab + c^2}} \right)^p \left(1 + \frac{2bx^2}{c + \sqrt{-4ab + c^2}} \right)^p dx$$

$$= x \left(1 + \frac{2bx^2}{c - \sqrt{-4ab + c^2}} \right)^{-p} \left(1 + \frac{2bx^2}{c + \sqrt{-4ab + c^2}} \right)^{-p} (a + cx^2 + bx^4)^p F_1 \left(\frac{1}{2}; -p, -p; \frac{3}{2}; -\frac{2bx^2}{c - \sqrt{-4ab + c^2}}, -\frac{2bx^2}{c + \sqrt{-4ab + c^2}} \right)$$

Mathematica [A] time = 0.180575, size = 161, normalized size = 1.21

$$x \left(\frac{-\sqrt{c^2 - 4ab} + 2bx^2 + c}{c - \sqrt{c^2 - 4ab}} \right)^{-p} \left(\frac{\sqrt{c^2 - 4ab} + 2bx^2 + c}{\sqrt{c^2 - 4ab} + c} \right)^{-p} (a + bx^4 + cx^2)^p F_1 \left(\frac{1}{2}; -p, -p; \frac{3}{2}; -\frac{2bx^2}{c + \sqrt{c^2 - 4ab}}, \frac{2bx^2}{\sqrt{c^2 - 4ab} - c} \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + c*x^2 + b*x^4)^p, x]

[Out] (x*(a + c*x^2 + b*x^4)^p*AppellF1[1/2, -p, -p, 3/2, (-2*b*x^2)/(c + Sqrt[-4*a*b + c^2]), (2*b*x^2)/(-c + Sqrt[-4*a*b + c^2])])/(((c - Sqrt[-4*a*b + c^2] + 2*b*x^2)/(c - Sqrt[-4*a*b + c^2]))^p*((c + Sqrt[-4*a*b + c^2] + 2*b*x^2)/(c + Sqrt[-4*a*b + c^2]))^p)

Maple [F] time = 0.013, size = 0, normalized size = 0.

$$\int (bx^4 + cx^2 + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^4+c*x^2+a)^p,x)

[Out] int((b*x^4+c*x^2+a)^p,x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (bx^4 + cx^2 + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^4+c*x^2+a)^p,x, algorithm="maxima")

[Out] integrate((b*x^4 + c*x^2 + a)^p, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(bx^4 + cx^2 + a\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^4+c*x^2+a)^p,x, algorithm="fricas")

[Out] integral((b*x^4 + c*x^2 + a)^p, x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a + bx^4 + cx^2)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**4+c*x**2+a)**p,x)

[Out] Integral((a + b*x**4 + c*x**2)**p, x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (bx^4 + cx^2 + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^4+c*x^2+a)^p,x, algorithm="giac")

[Out] integrate((b*x^4 + c*x^2 + a)^p, x)

$$3.404 \quad \int \frac{(a+cx^2+bx^4)^p}{c+ex^2} dx$$

Optimal. Leaf size=26

$$\text{Unintegrable} \left(\frac{(a+bx^4+cx^2)^p}{c+ex^2}, x \right)$$

[Out] Defer[Int] [(a + c*x^2 + b*x^4)^p/(c + e*x^2), x]

Rubi [A] time = 0.0109996, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{(a+cx^2+bx^4)^p}{c+ex^2} dx$$

Verification is Not applicable to the result.

[In] Int[(a + c*x^2 + b*x^4)^p/(c + e*x^2), x]

[Out] Defer[Int] [(a + c*x^2 + b*x^4)^p/(c + e*x^2), x]

Rubi steps

$$\int \frac{(a+cx^2+bx^4)^p}{c+ex^2} dx = \int \frac{(a+cx^2+bx^4)^p}{c+ex^2} dx$$

Mathematica [A] time = 0.130838, size = 0, normalized size = 0.

$$\int \frac{(a+cx^2+bx^4)^p}{c+ex^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[(a + c*x^2 + b*x^4)^p/(c + e*x^2), x]

[Out] Integrate[(a + c*x^2 + b*x^4)^p/(c + e*x^2), x]

Maple [A] time = 0.037, size = 0, normalized size = 0.

$$\int \frac{(bx^4+cx^2+a)^p}{ex^2+c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^4+c*x^2+a)^p/(e*x^2+c), x)

[Out] int((b*x^4+c*x^2+a)^p/(e*x^2+c), x)

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^4 + cx^2 + a)^p}{ex^2 + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^4+c*x^2+a)^p/(e*x^2+c),x, algorithm="maxima")

[Out] integrate((b*x^4 + c*x^2 + a)^p/(e*x^2 + c), x)

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx^4 + cx^2 + a)^p}{ex^2 + c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^4+c*x^2+a)^p/(e*x^2+c),x, algorithm="fricas")

[Out] integral((b*x^4 + c*x^2 + a)^p/(e*x^2 + c), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**4+c*x**2+a)**p/(e*x**2+c),x)

[Out] Timed out

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^4 + cx^2 + a)^p}{ex^2 + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^4+c*x^2+a)^p/(e*x^2+c),x, algorithm="giac")

[Out] integrate((b*x^4 + c*x^2 + a)^p/(e*x^2 + c), x)

$$3.405 \quad \int \frac{(a+cx^2+bx^4)^p}{(c+ex^2)^2} dx$$

Optimal. Leaf size=26

$$\text{Unintegrable} \left(\frac{(a+bx^4+cx^2)^p}{(c+ex^2)^2}, x \right)$$

[Out] Defer[Int][(a + c*x^2 + b*x^4)^p/(c + e*x^2)^2, x]

Rubi [A] time = 0.0107359, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{(a+cx^2+bx^4)^p}{(c+ex^2)^2} dx$$

Verification is Not applicable to the result.

[In] Int[(a + c*x^2 + b*x^4)^p/(c + e*x^2)^2,x]

[Out] Defer[Int][(a + c*x^2 + b*x^4)^p/(c + e*x^2)^2, x]

Rubi steps

$$\int \frac{(a+cx^2+bx^4)^p}{(c+ex^2)^2} dx = \int \frac{(a+cx^2+bx^4)^p}{(c+ex^2)^2} dx$$

Mathematica [A] time = 0.266853, size = 0, normalized size = 0.

$$\int \frac{(a+cx^2+bx^4)^p}{(c+ex^2)^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[(a + c*x^2 + b*x^4)^p/(c + e*x^2)^2,x]

[Out] Integrate[(a + c*x^2 + b*x^4)^p/(c + e*x^2)^2, x]

Maple [A] time = 0.053, size = 0, normalized size = 0.

$$\int \frac{(bx^4+cx^2+a)^p}{(ex^2+c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^4+c*x^2+a)^p/(e*x^2+c)^2,x)`

[Out] `int((b*x^4+c*x^2+a)^p/(e*x^2+c)^2,x)`

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^4 + cx^2 + a)^p}{(ex^2 + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^4+c*x^2+a)^p/(e*x^2+c)^2,x, algorithm="maxima")`

[Out] `integrate((b*x^4 + c*x^2 + a)^p/(e*x^2 + c)^2, x)`

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx^4 + cx^2 + a)^p}{e^2x^4 + 2cex^2 + c^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^4+c*x^2+a)^p/(e*x^2+c)^2,x, algorithm="fricas")`

[Out] `integral((b*x^4 + c*x^2 + a)^p/(e^2*x^4 + 2*c*e*x^2 + c^2), x)`

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**4+c*x**2+a)**p/(e*x**2+c)**2,x)`

[Out] Timed out

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^4 + cx^2 + a)^p}{(ex^2 + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^4+c*x^2+a)^p/(e*x^2+c)^2,x, algorithm="giac")`

[Out] `integrate((b*x^4 + c*x^2 + a)^p/(e*x^2 + c)^2, x)`

3.406 $\int \frac{f+gx}{(d+ex)\sqrt{a+cx^4}} dx$

Optimal. Leaf size=446

$$\frac{(\sqrt{a} + \sqrt{cx^2}) \sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{cx^2})^2}} \text{EllipticF}\left(2 \tan^{-1}\left(\frac{\sqrt[4]{cx}}{\sqrt[4]{a}}\right), \frac{1}{2}\right) (\sqrt{aeg} + \sqrt{cdf})}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+cx^4}(\sqrt{ae^2} + \sqrt{cd^2})} + \frac{(ef - dg) \tan^{-1}\left(\frac{x\sqrt{-ae^4 - cd^4}}{de\sqrt{a+cx^4}}\right)}{2\sqrt{-ae^4 - cd^4}} - \frac{(ef - dg) \tan^{-1}\left(\frac{x\sqrt{-ae^4 - cd^4}}{de\sqrt{a+cx^4}}\right)}{2}$$

```
[Out] ((e*f - d*g)*ArcTan[(Sqrt[-(c*d^4) - a*e^4]*x)/(d*e*Sqrt[a + c*x^4]])/(2*Sqrt[-(c*d^4) - a*e^4]) - ((e*f - d*g)*ArcTanh[(a*e^2 + c*d^2*x^2)/(Sqrt[c*d^4 + a*e^4]*Sqrt[a + c*x^4]])/(2*Sqrt[c*d^4 + a*e^4]) + ((Sqrt[c]*d*f + Sqrt[a]*e*g)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(1/4)*c^(1/4)*(Sqrt[c]*d^2 + Sqrt[a]*e^2)*Sqrt[a + c*x^4]) - ((Sqrt[c]*d^2 - Sqrt[a]*e^2)*(e*f - d*g)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticPi[(Sqrt[c]*d^2 + Sqrt[a]*e^2)^2/(4*Sqrt[a]*Sqrt[c]*d^2*e^2), 2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(1/4)*c^(1/4)*d*e*(Sqrt[c]*d^2 + Sqrt[a]*e^2)*Sqrt[a + c*x^4])
```

Rubi [A] time = 0.497405, antiderivative size = 446, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1742, 12, 1248, 725, 206, 1709, 220, 1707}

$$\frac{(ef - dg) \tan^{-1}\left(\frac{x\sqrt{-ae^4 - cd^4}}{de\sqrt{a+cx^4}}\right)}{2\sqrt{-ae^4 - cd^4}} - \frac{(ef - dg) \tanh^{-1}\left(\frac{ae^2 + cd^2x^2}{\sqrt{a+cx^4}\sqrt{ae^4 + cd^4}}\right)}{2\sqrt{ae^4 + cd^4}} + \frac{(\sqrt{a} + \sqrt{cx^2}) \sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{cx^2})^2}} F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{cx}}{\sqrt[4]{a}}\right) \middle| \frac{1}{2}\right) (\sqrt{aeg} + \sqrt{cdf})}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+cx^4}(\sqrt{ae^2} + \sqrt{cd^2})}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)/((d + e*x)*Sqrt[a + c*x^4]), x]
```

```
[Out] ((e*f - d*g)*ArcTan[(Sqrt[-(c*d^4) - a*e^4]*x)/(d*e*Sqrt[a + c*x^4]])/(2*Sqrt[-(c*d^4) - a*e^4]) - ((e*f - d*g)*ArcTanh[(a*e^2 + c*d^2*x^2)/(Sqrt[c*d^4 + a*e^4]*Sqrt[a + c*x^4]])/(2*Sqrt[c*d^4 + a*e^4]) + ((Sqrt[c]*d*f + Sqrt[a]*e*g)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(1/4)*c^(1/4)*(Sqrt[c]*d^2 + Sqrt[a]*e^2)*Sqrt[a + c*x^4]) - ((Sqrt[c]*d^2 - Sqrt[a]*e^2)*(e*f - d*g)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticPi[(Sqrt[c]*d^2 + Sqrt[a]*e^2)^2/(4*Sqrt[a]*Sqrt[c]*d^2*e^2), 2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(1/4)*c^(1/4)*d*e*(Sqrt[c]*d^2 + Sqrt[a]*e^2)*Sqrt[a + c*x^4])
```

Rule 1742

```
Int[(Px_)/(((d_) + (e_)*(x_))*Sqrt[(a_) + (c_)*(x_)^4]), x_Symbol] := With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1], C = Coeff[Px, x, 2], D = Coeff[Px, x, 3]}, Int[(x*(B*d - A*e + (d*D - C*e)*x^2))/((d^2 - e^2*x^2)*Sqrt[a + c*x^4]), x] + Int[(A*d + (C*d - B*e)*x^2 - D*e*x^4)/((d^2 - e^2*x^2)*Sqrt[a + c*x^4]), x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Px, x] && LeQ[Expon[Px, x], 3] && NeQ[c*d^4 + a*e^4, 0]
```

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 1248

Int[(x_)*((d_) + (e_)*(x_)^2)^(q_)*((a_) + (c_)*(x_)^4)^(p_), x_Symbol] := Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x]

Rule 725

Int[1/(((d_) + (e_)*(x_))*Sqrt[(a_) + (c_)*(x_)^2]), x_Symbol] := -Subst[Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ[{a, c, d, e}, x]

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 1709

Int[((A_) + (B_)*(x_)^2)/(((d_) + (e_)*(x_)^2)*Sqrt[(a_) + (c_)*(x_)^4]), x_Symbol] := With[{q = Rt[c/a, 2]}, Dist[(A*(c*d + a*e*q) - a*B*(e + d*q))/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4], x], x] + Dist[(a*(B*d - A*e)*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && NeQ[c*A^2 - a*B^2, 0]

Rule 220

Int[1/Sqrt[(a_) + (b_)*(x_)^4], x_Symbol] := With[{q = Rt[b/a, 4]}, Simp[(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x], 1/2]]/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]

Rule 1707

Int[((A_) + (B_)*(x_)^2)/(((d_) + (e_)*(x_)^2)*Sqrt[(a_) + (c_)*(x_)^4]), x_Symbol] := With[{q = Rt[B/A, 2]}, -Simp[(B*d - A*e)*ArcTan[(Rt[(c*d)/e + (a*e)/d, 2]*x)/Sqrt[a + c*x^4]]/(2*d*e*Rt[(c*d)/e + (a*e)/d, 2]), x] + Simp[(B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a + c*x^4))/(a*(A + B*x^2)^2)]*EllipticPi[Cancel[-((B*d - A*e)^2/(4*d*e*A*B))], 2*ArcTan[q*x], 1/2]]/(4*d*e*A*q*Sqrt[a + c*x^4]), x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]

Rubi steps

$$\begin{aligned}
\int \frac{f + gx}{(d + ex)\sqrt{a + cx^4}} dx &= \int \frac{(-ef + dg)x}{(d^2 - e^2x^2)\sqrt{a + cx^4}} dx + \int \frac{df - egx^2}{(d^2 - e^2x^2)\sqrt{a + cx^4}} dx \\
&= \frac{(\sqrt{ade}(ef - dg)) \int \frac{1 + \frac{\sqrt{cx^2}}{\sqrt{a}}}{(d^2 - e^2x^2)\sqrt{a + cx^4}} dx}{\sqrt{cd^2} + \sqrt{ae^2}} + (-ef + dg) \int \frac{x}{(d^2 - e^2x^2)\sqrt{a + cx^4}} dx + \frac{(\sqrt{cdf} + \sqrt{aeg})}{\sqrt{cd}} \int \frac{1}{\sqrt{a + cx^4}} dx \\
&= \frac{(ef - dg) \tan^{-1}\left(\frac{\sqrt{-cd^4 - ae^4}x}{de\sqrt{a + cx^4}}\right)}{2\sqrt{-cd^4 - ae^4}} + \frac{(\sqrt{cdf} + \sqrt{aeg})(\sqrt{a} + \sqrt{cx^2}) \sqrt{\frac{a + cx^4}{(\sqrt{a} + \sqrt{cx^2})^2}} F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{cx}}{\sqrt[4]{a}}\right)\right)}{2\sqrt[4]{a}\sqrt[4]{c}(\sqrt{cd^2} + \sqrt{ae^2})\sqrt{a + cx^4}} \\
&= \frac{(ef - dg) \tan^{-1}\left(\frac{\sqrt{-cd^4 - ae^4}x}{de\sqrt{a + cx^4}}\right)}{2\sqrt{-cd^4 - ae^4}} + \frac{(\sqrt{cdf} + \sqrt{aeg})(\sqrt{a} + \sqrt{cx^2}) \sqrt{\frac{a + cx^4}{(\sqrt{a} + \sqrt{cx^2})^2}} F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{cx}}{\sqrt[4]{a}}\right)\right)}{2\sqrt[4]{a}\sqrt[4]{c}(\sqrt{cd^2} + \sqrt{ae^2})\sqrt{a + cx^4}} \\
&= \frac{(ef - dg) \tan^{-1}\left(\frac{\sqrt{-cd^4 - ae^4}x}{de\sqrt{a + cx^4}}\right)}{2\sqrt{-cd^4 - ae^4}} - \frac{(ef - dg) \tanh^{-1}\left(\frac{ae^2 + cd^2x^2}{\sqrt{cd^4 + ae^4}\sqrt{a + cx^4}}\right)}{2\sqrt{cd^4 + ae^4}} + \frac{(\sqrt{cdf} + \sqrt{aeg})(\sqrt{a} + \sqrt{cx^2})}{2\sqrt[4]{a}\sqrt[4]{c}}
\end{aligned}$$

Mathematica [C] time = 0.694832, size = 258, normalized size = 0.58

$$\frac{(dg - ef) \left(\sqrt[4]{cde\sqrt{a + cx^4}} \tanh^{-1}\left(\frac{ae^2 + cd^2x^2}{\sqrt{a + cx^4}\sqrt{ae^4 + cd^4}}\right) + 2\sqrt[4]{-1}\sqrt[4]{a}\sqrt{\frac{cx^4}{a} + 1}\sqrt{ae^4 + cd^4} \Pi\left(\frac{i\sqrt{ae^2}}{\sqrt{cd^2}}; \sin^{-1}\left(\frac{(-1)^{3/4}\sqrt[4]{cx}}{\sqrt[4]{a}}\right) \middle| -1 \right) \right)}{\sqrt[4]{cd}\sqrt{ae^4 + cd^4}} - \frac{2ig\sqrt{\frac{cx^4}{a} + 1} \text{EllipticF}\left(i \sinh^{-1}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\right)\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}}$$

$$2e\sqrt{a + cx^4}$$

Antiderivative was successfully verified.

[In] Integrate[(f + g*x)/((d + e*x)*Sqrt[a + c*x^4]), x]

[Out] (((-2*I)*g*Sqrt[1 + (c*x^4)/a]*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x], -1])/Sqrt[(I*Sqrt[c])/Sqrt[a]] + ((-e*f) + d*g)*(c^(1/4)*d*e*Sqrt[a + c*x^4]*ArcTanh[(a*e^2 + c*d^2*x^2)/(Sqrt[c*d^4 + a*e^4]*Sqrt[a + c*x^4]]) + 2*(-1)^(1/4)*a^(1/4)*Sqrt[c*d^4 + a*e^4]*Sqrt[1 + (c*x^4)/a]*EllipticPi[(I*Sqrt[a]*e^2)/(Sqrt[c]*d^2), ArcSin[((-1)^(3/4)*c^(1/4)*x]/a^(1/4)], -1))/(c^(1/4)*d*Sqrt[c*d^4 + a*e^4])/(2*e*Sqrt[a + c*x^4])

Maple [C] time = 0.052, size = 251, normalized size = 0.6

$$\frac{g}{e} \sqrt{1 - ix^2\sqrt{c}\frac{1}{\sqrt{a}}} \sqrt{1 + ix^2\sqrt{c}\frac{1}{\sqrt{a}}} \text{EllipticF}\left(x\sqrt{i\sqrt{c}\frac{1}{\sqrt{a}}}, i\right) \frac{1}{\sqrt{i\sqrt{c}\frac{1}{\sqrt{a}}}} \frac{1}{\sqrt{cx^4 + a}} + \frac{-dg + ef}{e^2} \left[-\frac{1}{2} \text{Artanh}\left(\frac{1}{2} \left(2\frac{cd^2x^2}{e^2} + \dots\right)\right) \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((g*x+f)/(e*x+d)/(c*x^4+a)^(1/2), x)

[Out] g/e/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2), I) + (-d*g+e*f)/e^2*(-1/2/(c*d^4/e^4+a)^(1/2)*arctanh(1/2*(2*c*x^2*d^2/e^2+2*a)/(c*d^4/e^4+a)^(1/2)/(c*x^4+a)^(1/2))+1/(I/a^(1/2)*c^(1/2))^(1/2)/d*e*(1-I/

$$a^{1/2}c^{1/2}x^2)^{1/2}*(1+I/a^{1/2}*c^{1/2}*x^2)^{1/2}/(c*x^4+a)^{1/2}* \\ \text{EllipticPi}(x*(I/a^{1/2}*c^{1/2})^{1/2}, -I*a^{1/2}/c^{1/2}/d^2*e^2, (-I/a^{1/2} \\ 2)*c^{1/2})^{1/2}/(I/a^{1/2}*c^{1/2})^{1/2}))$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{gx + f}{\sqrt{cx^4 + a}(ex + d)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g*x+f)/(e*x+d)/(c*x^4+a)^(1/2),x, algorithm="maxima")

[Out] integrate((g*x + f)/(sqrt(c*x^4 + a)*(e*x + d)), x)

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g*x+f)/(e*x+d)/(c*x^4+a)^(1/2),x, algorithm="fricas")

[Out] Timed out

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{f + gx}{\sqrt{a + cx^4}(d + ex)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g*x+f)/(e*x+d)/(c*x**4+a)**(1/2),x)

[Out] Integral((f + g*x)/(sqrt(a + c*x**4)*(d + e*x)), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{gx + f}{\sqrt{cx^4 + a}(ex + d)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g*x+f)/(e*x+d)/(c*x^4+a)^(1/2),x, algorithm="giac")

[Out] integrate((g*x + f)/(sqrt(c*x^4 + a)*(e*x + d)), x)

$$3.407 \quad \int \frac{f+gx}{(d+ex)\sqrt{-a+cx^4}} dx$$

Optimal. Leaf size=218

$$\frac{\sqrt[4]{ag}\sqrt{1-\frac{cx^4}{a}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt[4]{cx}}{\sqrt[4]{a}}\right), -1\right)}{\sqrt[4]{ce}\sqrt{cx^4-a}} + \frac{(ef-dg)\tanh^{-1}\left(\frac{ae^2-cd^2x^2}{\sqrt{cx^4-a}\sqrt{cd^4-ae^4}}\right)}{2\sqrt{cd^4-ae^4}} + \frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}(ef-dg)\Pi\left(\frac{\sqrt{ae^2}}{\sqrt{cd^2}}; \sin^{-1}\left(\frac{\sqrt[4]{cx}}{\sqrt[4]{a}}\right)\right)}{\sqrt[4]{cde}\sqrt{cx^4-a}}$$

[Out] ((e*f - d*g)*ArcTanh[(a*e^2 - c*d^2*x^2)/(Sqrt[c*d^4 - a*e^4]*Sqrt[-a + c*x^4])]/(2*Sqrt[c*d^4 - a*e^4]) + (a^(1/4)*g*Sqrt[1 - (c*x^4)/a]*EllipticF[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(c^(1/4)*e*Sqrt[-a + c*x^4]) + (a^(1/4)*(e*f - d*g)*Sqrt[1 - (c*x^4)/a]*EllipticPi[(Sqrt[a]*e^2)/(Sqrt[c]*d^2), ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(c^(1/4)*d*e*Sqrt[-a + c*x^4])

Rubi [A] time = 0.294798, antiderivative size = 218, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 10, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {1742, 12, 1248, 725, 206, 1711, 224, 221, 1219, 1218}

$$\frac{(ef-dg)\tanh^{-1}\left(\frac{ae^2-cd^2x^2}{\sqrt{cx^4-a}\sqrt{cd^4-ae^4}}\right)}{2\sqrt{cd^4-ae^4}} + \frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}(ef-dg)\Pi\left(\frac{\sqrt{ae^2}}{\sqrt{cd^2}}; \sin^{-1}\left(\frac{\sqrt[4]{cx}}{\sqrt[4]{a}}\right)\right)}{\sqrt[4]{cde}\sqrt{cx^4-a}} + \frac{\sqrt[4]{ag}\sqrt{1-\frac{cx^4}{a}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{cx}}{\sqrt[4]{a}}\right)\right)}{\sqrt[4]{ce}\sqrt{cx^4-a}}$$

Antiderivative was successfully verified.

[In] Int[(f + g*x)/((d + e*x)*Sqrt[-a + c*x^4]), x]

[Out] ((e*f - d*g)*ArcTanh[(a*e^2 - c*d^2*x^2)/(Sqrt[c*d^4 - a*e^4]*Sqrt[-a + c*x^4])]/(2*Sqrt[c*d^4 - a*e^4]) + (a^(1/4)*g*Sqrt[1 - (c*x^4)/a]*EllipticF[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(c^(1/4)*e*Sqrt[-a + c*x^4]) + (a^(1/4)*(e*f - d*g)*Sqrt[1 - (c*x^4)/a]*EllipticPi[(Sqrt[a]*e^2)/(Sqrt[c]*d^2), ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(c^(1/4)*d*e*Sqrt[-a + c*x^4])

Rule 1742

Int[(Px_)/(((d_) + (e_)*(x_))*Sqrt[(a_) + (c_)*(x_)^4]), x_Symbol] := With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1], C = Coeff[Px, x, 2], D = Coeff[Px, x, 3]}, Int[(x*(B*d - A*e + (d*D - C*e)*x^2))/((d^2 - e^2*x^2)*Sqrt[a + c*x^4]), x] + Int[(A*d + (C*d - B*e)*x^2 - D*e*x^4)/((d^2 - e^2*x^2)*Sqrt[a + c*x^4]), x] /; FreeQ[{a, c, d, e}, x] && PolyQ[Px, x] && LeQ[Expon[Px, x], 3] && NeQ[c*d^4 + a*e^4, 0]

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 1248

Int[(x_)*((d_) + (e_)*(x_)^2)^(q_)*((a_) + (c_)*(x_)^4)^(p_), x_Symbol] := Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x]

Rule 725

Int[1/(((d_) + (e_)*(x_))*Sqrt[(a_) + (c_)*(x_)^2]), x_Symbol] := -Subst[Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ

[{a, c, d, e}, x]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 1711

Int[((A_.) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] := Dist[B/e, Int[1/Sqrt[a + c*x^4], x], x] + Dist[(e*A - d*B)/e, Int[1/((d + e*x^2)*Sqrt[a + c*x^4]), x], x] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[c/a]

Rule 224

Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] := Dist[Sqrt[1 + (b*x^4)/a]/Sqrt[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[b/a] && !GtQ[a, 0]

Rule 221

Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] := Simp[EllipticF[ArcSin[(Rt[-b, 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[b/a] && GtQ[a, 0]

Rule 1219

Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] := Dist[Sqrt[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + (c*x^4)/a]), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && !GtQ[a, 0]

Rule 1218

Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] := With[{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]

Rubi steps

$$\begin{aligned}
\int \frac{f + gx}{(d + ex)\sqrt{-a + cx^4}} dx &= \int \frac{(-ef + dg)x}{(d^2 - e^2x^2)\sqrt{-a + cx^4}} dx + \int \frac{df - egx^2}{(d^2 - e^2x^2)\sqrt{-a + cx^4}} dx \\
&= \frac{g \int \frac{1}{\sqrt{-a+cx^4}} dx}{e} + \frac{(d(ef - dg)) \int \frac{1}{(d^2 - e^2x^2)\sqrt{-a+cx^4}} dx}{e} + (-ef + dg) \int \frac{x}{(d^2 - e^2x^2)\sqrt{-a + cx^4}} dx \\
&= \frac{1}{2}(-ef + dg) \text{Subst} \left(\int \frac{1}{(d^2 - e^2x)\sqrt{-a + cx^2}} dx, x, x^2 \right) + \frac{\left(g\sqrt{1 - \frac{cx^4}{a}} \right) \int \frac{1}{\sqrt{1 - \frac{cx^4}{a}}} dx}{e\sqrt{-a + cx^4}} + \frac{d}{e} \int \frac{1}{\sqrt{-a + cx^4}} dx \\
&= \frac{\sqrt[4]{ag}\sqrt{1 - \frac{cx^4}{a}} F \left(\sin^{-1} \left(\frac{\sqrt[4]{cx}}{\sqrt[4]{a}} \right) \right) - 1}{\sqrt[4]{ce}\sqrt{-a + cx^4}} + \frac{\sqrt[4]{a}(ef - dg)\sqrt{1 - \frac{cx^4}{a}} \Pi \left(\frac{\sqrt{ae^2}}{\sqrt{cd^2}}; \sin^{-1} \left(\frac{\sqrt[4]{cx}}{\sqrt[4]{a}} \right) \right) - 1}{\sqrt[4]{cde}\sqrt{-a + cx^4}} + \frac{1}{2} \int \frac{1}{\sqrt{-a + cx^4}} dx \\
&= \frac{(ef - dg) \tanh^{-1} \left(\frac{ae^2 - cd^2x^2}{\sqrt{cd^4 - ae^4}\sqrt{-a + cx^4}} \right)}{2\sqrt{cd^4 - ae^4}} + \frac{\sqrt[4]{ag}\sqrt{1 - \frac{cx^4}{a}} F \left(\sin^{-1} \left(\frac{\sqrt[4]{cx}}{\sqrt[4]{a}} \right) \right) - 1}{\sqrt[4]{ce}\sqrt{-a + cx^4}} + \frac{\sqrt[4]{a}(ef - dg)\sqrt{1 - \frac{cx^4}{a}} \Pi \left(\frac{\sqrt{ae^2}}{\sqrt{cd^2}}; \sin^{-1} \left(\frac{\sqrt[4]{cx}}{\sqrt[4]{a}} \right) \right) - 1}{\sqrt[4]{cde}\sqrt{-a + cx^4}} + \frac{1}{2} \int \frac{1}{\sqrt{-a + cx^4}} dx
\end{aligned}$$

Mathematica [C] time = 1.22663, size = 719, normalized size = 3.3

$$\frac{if \sqrt{\frac{(1-i)(\sqrt[4]{a}-\sqrt[4]{cx})}{\sqrt[4]{cx+i}\sqrt[4]{a}}}}{\sqrt{\frac{(1+i)(\sqrt[4]{a+i}\sqrt[4]{cx})(\sqrt[4]{a+i}\sqrt[4]{cx})}{(\sqrt[4]{a-i}\sqrt[4]{cx})^2}}} (\sqrt[4]{a-i}\sqrt[4]{cx})^2 \left((\sqrt[4]{ae}-\sqrt[4]{cd}) \text{EllipticF} \left[\sin^{-1} \left(\sqrt{\frac{(1+i)(\sqrt[4]{a}+\sqrt[4]{cx})}{2\sqrt[4]{cx+2i}\sqrt[4]{a}}} \right), 2 \right] - (1-i)\sqrt[4]{ae} \Pi \left(\frac{(1-i)(\sqrt[4]{cd-i}\sqrt[4]{ae})}{\sqrt[4]{cd}-\sqrt[4]{ae}}; \sin^{-1} \left(\sqrt{\frac{(1+i)(\sqrt[4]{a}+\sqrt[4]{cx})}{2\sqrt[4]{cx+2i}\sqrt[4]{a}}} \right) \right) \right)}{\sqrt[4]{a}(\sqrt[4]{ae}-\sqrt[4]{cd})(\sqrt[4]{ae+i}\sqrt[4]{cd})}$$

Antiderivative was successfully verified.

[In] Integrate[(f + g*x)/((d + e*x)*Sqrt[-a + c*x^4]), x]

[Out] (((-I)*g*Sqrt[1 - (c*x^4)/a]*EllipticF[I*ArcSinh[Sqrt[-(Sqrt[c]/Sqrt[a])]]*x], -1))/(Sqrt[-(Sqrt[c]/Sqrt[a])]*e) + (I*f*(a^(1/4) - I*c^(1/4)*x)^2*Sqrt[(-1 + I)*(a^(1/4) - c^(1/4)*x)]/(I*a^(1/4) + c^(1/4)*x)*Sqrt[((1 + I)*(a^(1/4) + I*c^(1/4)*x)*(a^(1/4) + c^(1/4)*x))/(a^(1/4) - I*c^(1/4)*x)^2]*((-(c^(1/4)*d) + a^(1/4)*e)*EllipticF[ArcSin[Sqrt[((1 + I)*(a^(1/4) + c^(1/4)*x)))/((2*I)*a^(1/4) + 2*c^(1/4)*x)]], 2] - (1 - I)*a^(1/4)*e*EllipticPi[((1 - I)*(c^(1/4)*d - I*a^(1/4)*e))/(c^(1/4)*d - a^(1/4)*e), ArcSin[Sqrt[((1 + I)*(a^(1/4) + c^(1/4)*x)))/((2*I)*a^(1/4) + 2*c^(1/4)*x)]], 2]]/(a^(1/4)*(-(c^(1/4)*d) + a^(1/4)*e)*(I*c^(1/4)*d + a^(1/4)*e)) + (d*g*(a^(1/4) - I*c^(1/4)*x)^2*Sqrt[(-1 + I)*(a^(1/4) - c^(1/4)*x)]/(I*a^(1/4) + c^(1/4)*x)*Sqrt[((1 + I)*(a^(1/4) + I*c^(1/4)*x)*(a^(1/4) + c^(1/4)*x))/(a^(1/4) - I*c^(1/4)*x)^2]*(I*(c^(1/4)*d - a^(1/4)*e)*EllipticF[ArcSin[Sqrt[((1 + I)*(a^(1/4) + c^(1/4)*x)))/((2*I)*a^(1/4) + 2*c^(1/4)*x)]], 2] + (1 + I)*a^(1/4)*e*EllipticPi[((1 - I)*(c^(1/4)*d - I*a^(1/4)*e))/(c^(1/4)*d - a^(1/4)*e), ArcSin[Sqrt[((1 + I)*(a^(1/4) + c^(1/4)*x)))/((2*I)*a^(1/4) + 2*c^(1/4)*x)]], 2]]/(a^(1/4)*e*(-(c^(1/4)*d) + a^(1/4)*e)*(I*c^(1/4)*d + a^(1/4)*e))/Sqrt[-a + c*x^4]

Maple [A] time = 0.029, size = 247, normalized size = 1.1

$$\frac{g}{e} \sqrt{1 + x^2 \sqrt{c} \frac{1}{\sqrt{a}}} \sqrt{1 - x^2 \sqrt{c} \frac{1}{\sqrt{a}}} \text{EllipticF} \left(x \sqrt{-\sqrt{c} \frac{1}{\sqrt{a}}}, i \right) \frac{1}{\sqrt{-\sqrt{c} \frac{1}{\sqrt{a}}}} \frac{1}{\sqrt{cx^4 - a}} + \frac{-dg + ef}{e^2} \left(-\frac{1}{2} \text{Artanh} \left(\frac{1}{2} \left(2 \frac{cd^2x^2}{e^2} - \dots \right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((g*x+f)/(e*x+d)/(c*x^4-a)^(1/2),x)`

[Out]
$$\frac{g/e/(-1/a^{1/2}*c^{1/2})^{1/2}*(1+1/a^{1/2}*c^{1/2}*x^2)^{1/2}*(1-1/a^{1/2}*c^{1/2}*x^2)^{1/2}/(c*x^4-a)^{1/2}*EllipticF(x*(-1/a^{1/2}*c^{1/2})^{1/2}, I)+(-d*g+e*f)/e^2*(-1/2/(c*d^4/e^4-a)^{1/2})*arctanh(1/2*(2*c*x^2*d^2/e^2-2*a)/(c*d^4/e^4-a)^{1/2}/(c*x^4-a)^{1/2})+1/(-1/a^{1/2}*c^{1/2})^{1/2}/d*e*(1+1/a^{1/2}*c^{1/2}*x^2)^{1/2}*(1-1/a^{1/2}*c^{1/2}*x^2)^{1/2}/(c*x^4-a)^{1/2}*EllipticPi(x*(-1/a^{1/2}*c^{1/2})^{1/2}, -e^2*a^{1/2}/d^2/c^{1/2}, (1/a^{1/2}*c^{1/2})^{1/2}/(-1/a^{1/2}*c^{1/2})^{1/2}))}{1}$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{gx + f}{\sqrt{cx^4 - a}(ex + d)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((g*x+f)/(e*x+d)/(c*x^4-a)^(1/2),x, algorithm="maxima")`

[Out] `integrate((g*x + f)/(sqrt(c*x^4 - a)*(e*x + d)), x)`

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((g*x+f)/(e*x+d)/(c*x^4-a)^(1/2),x, algorithm="fricas")`

[Out] Timed out

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{f + gx}{\sqrt{-a + cx^4}(d + ex)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((g*x+f)/(e*x+d)/(c*x**4-a)**(1/2),x)`

[Out] `Integral((f + g*x)/(sqrt(-a + c*x**4)*(d + e*x)), x)`

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{gx + f}{\sqrt{cx^4 - a}(ex + d)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)/(e*x+d)/(c*x^4-a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((g*x + f)/(sqrt(c*x^4 - a)*(e*x + d)), x)
```

$$3.408 \quad \int \frac{1-\sqrt{3}+x}{(1+\sqrt{3}+x)\sqrt{-4+4\sqrt{3}x^2+x^4}} dx$$

Optimal. Leaf size=65

$$\frac{1}{3}\sqrt{2\sqrt{3}-3} \tanh^{-1}\left(\frac{(x-\sqrt{3}+1)^2}{\sqrt{3}(2\sqrt{3}-3)\sqrt{x^4+4\sqrt{3}x^2-4}}\right)$$

[Out] (Sqrt[-3 + 2*Sqrt[3]]*ArcTanh[(1 - Sqrt[3] + x)^2/(Sqrt[3*(-3 + 2*Sqrt[3]])*Sqrt[-4 + 4*Sqrt[3]*x^2 + x^4])])/3

Rubi [A] time = 0.145126, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.05$, Rules used = {1740, 207}

$$\frac{1}{3}\sqrt{2\sqrt{3}-3} \tanh^{-1}\left(\frac{(x-\sqrt{3}+1)^2}{\sqrt{3}(2\sqrt{3}-3)\sqrt{x^4+4\sqrt{3}x^2-4}}\right)$$

Antiderivative was successfully verified.

[In] Int[(1 - Sqrt[3] + x)/((1 + Sqrt[3] + x)*Sqrt[-4 + 4*Sqrt[3]*x^2 + x^4]), x]

[Out] (Sqrt[-3 + 2*Sqrt[3]]*ArcTanh[(1 - Sqrt[3] + x)^2/(Sqrt[3*(-3 + 2*Sqrt[3]])*Sqrt[-4 + 4*Sqrt[3]*x^2 + x^4])])/3

Rule 1740

Int[((A_) + (B_.)*(x_))/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_Symbol] :> -Dist[(A^2*(B*d + A*e))/e, Subst[Int[1/(6*A^3*B*d + 3*A^4*e - a*e*x^2), x], x, (A + B*x)^2/Sqrt[a + b*x^2 + c*x^4]], x] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[B*d - A*e, 0] && EqQ[c^2*d^6 + a*e^4*(13*c*d^2 + b*e^2), 0] && EqQ[b^2*e^4 - 12*c*d^2*(c*d^2 - b*e^2), 0] && EqQ[4*A*c*d*e + B*(2*c*d^2 - b*e^2), 0]

Rule 207

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rubi steps

$$\int \frac{1-\sqrt{3}+x}{(1+\sqrt{3}+x)\sqrt{-4+4\sqrt{3}x^2+x^4}} dx = -\left(4(2-\sqrt{3})\right) \text{Subst}\left(\int \frac{1}{3(1-\sqrt{3})^4+6(1-\sqrt{3})^3(1+\sqrt{3})+4x^2} dx, x, \frac{1-\sqrt{3}+x}{\sqrt{-4+4\sqrt{3}x^2+x^4}}\right) \\ = \frac{1}{3}\sqrt{-3+2\sqrt{3}} \tanh^{-1}\left(\frac{(1-\sqrt{3}+x)^2}{\sqrt{3}(-3+2\sqrt{3})\sqrt{-4+4\sqrt{3}x^2+x^4}}\right)$$

Mathematica [C] time = 1.627, size = 685, normalized size = 10.54

$$(x + \sqrt{3} - 1)^2 \sqrt{-x^3 + (\sqrt{3} - 1)x^2 - 2(2 + \sqrt{3})x + 2(1 + \sqrt{3})} \sqrt{\frac{-\frac{4}{x + \sqrt{3} - 1} + \sqrt{3} + 1}{3 + \sqrt{3} + i\sqrt{2(2 + \sqrt{3})}}} \left(\frac{2(2i\sqrt{3} - \sqrt{2(2 + \sqrt{3})} + \sqrt{6(2 + \sqrt{3})})}{x + \sqrt{3} - 1} + i(-1 - \sqrt{3}) \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[(1 - Sqrt[3] + x)/((1 + Sqrt[3] + x)*Sqrt[-4 + 4*Sqrt[3]*x^2 + x^4]), x]

[Out] ((-1 + Sqrt[3] + x)^2*Sqrt[2*(1 + Sqrt[3]) - 2*(2 + Sqrt[3])*x + (-1 + Sqrt[3])*x^2 - x^3]*Sqrt[(1 + Sqrt[3] - 4/(-1 + Sqrt[3] + x))/(3 + Sqrt[3] + I*Sqrt[2*(2 + Sqrt[3])]])*((I*(-1 + Sqrt[3] + I*Sqrt[2*(2 + Sqrt[3])])) + (2*(2*I)*Sqrt[3] - Sqrt[2*(2 + Sqrt[3])]) + Sqrt[6*(2 + Sqrt[3])]))/(-1 + Sqrt[3] + x)*Sqrt[Sqrt[2*(2 + Sqrt[3])] + I*(1 - Sqrt[3] + 8/(-1 + Sqrt[3] + x))]*EllipticF[ArcSin[Sqrt[Sqrt[2*(2 + Sqrt[3])]] - I*(1 - Sqrt[3] + 8/(-1 + Sqrt[3] + x))]/(2^(3/4)*(2 + Sqrt[3])^(1/4))], ((2*I)*Sqrt[2*(2 + Sqrt[3])])/(3 + Sqrt[3] + I*Sqrt[2*(2 + Sqrt[3])]) + 2*Sqrt[6]*Sqrt[(4 + 2*Sqrt[3] + x^2)/(-1 + Sqrt[3] + x)^2]*Sqrt[Sqrt[2*(2 + Sqrt[3])] - I*(1 - Sqrt[3] + 8/(-1 + Sqrt[3] + x))]*EllipticPi[(2*Sqrt[2*(2 + Sqrt[3])])/(Sqrt[2*(2 + Sqrt[3])]) + I*(3 + Sqrt[3])], ArcSin[Sqrt[Sqrt[2*(2 + Sqrt[3])]] - I*(1 - Sqrt[3] + 8/(-1 + Sqrt[3] + x))]/(2^(3/4)*(2 + Sqrt[3])^(1/4))], ((2*I)*Sqrt[2*(2 + Sqrt[3])])/(3 + Sqrt[3] + I*Sqrt[2*(2 + Sqrt[3])])])/(Sqrt[2*(2 + Sqrt[3])] + I*(3 + Sqrt[3]))*Sqrt[1 + Sqrt[3] - (2 + Sqrt[3])*x + ((-1 + Sqrt[3])*x^2)/2 - x^3/2]*Sqrt[-4 + 4*Sqrt[3]*x^2 + x^4]*Sqrt[Sqrt[2*(2 + Sqrt[3])] - I*(1 - Sqrt[3] + 8/(-1 + Sqrt[3] + x))])

Maple [C] time = 0.153, size = 327, normalized size = 5.

$$\frac{\text{EllipticF}\left(x\left(\frac{i}{2}\sqrt{3} - \frac{i}{2}\right), i\sqrt{1 + 4\sqrt{3}\left(1 + \frac{1}{2}\sqrt{3}\right)}\right)}{\frac{i}{2}\sqrt{3} - \frac{i}{2}} \sqrt{1 - \left(\frac{\sqrt{3}}{2} - 1\right)x^2} \sqrt{1 - \left(1 + \frac{\sqrt{3}}{2}\right)x^2} \frac{1}{\sqrt{-4 + x^4 + 4x^2\sqrt{3}}} - 2\sqrt{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1+x-3^(1/2))/(1+x+3^(1/2))/(-4+x^4+4*x^2*3^(1/2))^(1/2), x)

[Out] 1/(1/2*I*3^(1/2)-1/2*I)*(1-(1/2*3^(1/2)-1)*x^2)^(1/2)*(1-(1+1/2*3^(1/2))*x^2)^(1/2)/(-4+x^4+4*x^2*3^(1/2))^(1/2)*EllipticF(x*(1/2*I*3^(1/2)-1/2*I), I*(1+4*3^(1/2)*(1+1/2*3^(1/2)))^(1/2))-2*3^(1/2)*(-1/2/((-1-3^(1/2))^4+4*3^(1/2)*(-1-3^(1/2))^2-4)^(1/2)*arctanh(1/2*(4*3^(1/2)*(-1-3^(1/2))^2-8+4*x^2*3^(1/2)+2*x^2*(-1-3^(1/2))^2)/((-1-3^(1/2))^4+4*3^(1/2)*(-1-3^(1/2))^2-4)^(1/2)/(-4+x^4+4*x^2*3^(1/2))^(1/2))-1/(1/2*3^(1/2)-1)^(1/2)/(-1-3^(1/2))*((1-(1/2*3^(1/2)-1)*x^2)^(1/2)*(1-(1+1/2*3^(1/2))*x^2)^(1/2)/(-4+x^4+4*x^2*3^(1/2))^(1/2)*EllipticPi((1/2*3^(1/2)-1)^(1/2)*x, 1/(1/2*3^(1/2)-1)/(-1-3^(1/2))^(1/2), (1+1/2*3^(1/2))^(1/2)/(1/2*3^(1/2)-1)^(1/2)))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x - \sqrt{3} + 1}{\sqrt{x^4 + 4\sqrt{3}x^2 - 4(x + \sqrt{3} + 1)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x-3^(1/2))/(1+x+3^(1/2))/(-4+x^4+4*3^(1/2)*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x - sqrt(3) + 1)/(sqrt(x^4 + 4*sqrt(3)*x^2 - 4)*(x + sqrt(3) + 1)), x)
```

Fricas [B] time = 3.5666, size = 953, normalized size = 14.66

$$\frac{1}{12} \sqrt{2\sqrt{3}-3} \log \left(-\frac{37x^{12} - 204x^{11} + 804x^{10} - 2408x^9 + 3708x^8 - 5472x^7 + 6432x^6 + 10944x^5 + 14832x^4 + 19264x^3 + 12864x^2 + (54x^{10} - 300x^9 + 1026x^8 - 2232x^7 + 3024x^6 - 3024x^5 - 1008x^4 - 2016x^3 - 2592x^2 + \sqrt{3})(31x^{10} - 176x^9 + 576x^8 - 1320x^7 + 1848x^6 - 1008x^5 + 1344x^4 + 1632x^3 + 1008x^2 + 832x + 256) - 1152x - 480}{(x^4 + 4\sqrt{3}x^2 - 4)\sqrt{2\sqrt{3}-3}} + 3\sqrt{3} \frac{7x^{12} - 40x^{11} + 160x^{10} - 400x^9 + 924x^8 - 960x^7 - 1920x^5 - 3696x^4 - 3200x^3 - 2560x^2 - 1280x - 448}{(x^{12} + 12x^{11} + 48x^{10} + 40x^9 - 180x^8 - 288x^7 + 384x^6 + 576x^5 - 720x^4 - 320x^3 + 768x^2 - 384x + 64)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x-3^(1/2))/(1+x+3^(1/2))/(-4+x^4+4*3^(1/2)*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/12*sqrt(2*sqrt(3) - 3)*log(-(37*x^12 - 204*x^11 + 804*x^10 - 2408*x^9 + 3708*x^8 - 5472*x^7 + 6432*x^6 + 10944*x^5 + 14832*x^4 + 19264*x^3 + 12864*x^2 + (54*x^10 - 300*x^9 + 1026*x^8 - 2232*x^7 + 3024*x^6 - 3024*x^5 - 1008*x^4 - 2016*x^3 - 2592*x^2 + sqrt(3)*(31*x^10 - 176*x^9 + 576*x^8 - 1320*x^7 + 1848*x^6 - 1008*x^5 + 1344*x^4 + 1632*x^3 + 1008*x^2 + 832*x + 256) - 1152*x - 480)*sqrt(x^4 + 4*sqrt(3)*x^2 - 4)*sqrt(2*sqrt(3) - 3) + 3*sqrt(3)*(7*x^12 - 40*x^11 + 160*x^10 - 400*x^9 + 924*x^8 - 960*x^7 - 1920*x^5 - 3696*x^4 - 3200*x^3 - 2560*x^2 - 1280*x - 448) + 6528*x + 2368)/(x^12 + 12*x^11 + 48*x^10 + 40*x^9 - 180*x^8 - 288*x^7 + 384*x^6 + 576*x^5 - 720*x^4 - 320*x^3 + 768*x^2 - 384*x + 64))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x - \sqrt{3} + 1}{(x + 1 + \sqrt{3})\sqrt{x^4 + 4\sqrt{3}x^2 - 4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x-3**(1/2))/(1+x+3**(1/2))/(-4+x**4+4*3**(1/2)*x**2)**(1/2),x)
```

```
[Out] Integral((x - sqrt(3) + 1)/((x + 1 + sqrt(3))*sqrt(x**4 + 4*sqrt(3)*x**2 - 4)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x - \sqrt{3} + 1}{\sqrt{x^4 + 4\sqrt{3}x^2 - 4}(x + \sqrt{3} + 1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x-3^(1/2))/(1+x+3^(1/2))/(-4+x^4+4*3^(1/2)*x^2)^(1/2),x, algorithm="giac")
```



```
[Out] integrate((x - sqrt(3) + 1)/(sqrt(x^4 + 4*sqrt(3)*x^2 - 4)*(x + sqrt(3) + 1)), x)
```

$$3.409 \quad \int \frac{1 + \sqrt{3} + x}{(1 - \sqrt{3} + x)\sqrt{-4 - 4\sqrt{3}x^2 + x^4}} dx$$

Optimal. Leaf size=63

$$-\frac{1}{3}\sqrt{3 + 2\sqrt{3}} \tan^{-1} \left(\frac{(x + \sqrt{3} + 1)^2}{\sqrt{3(3 + 2\sqrt{3})}\sqrt{x^4 - 4\sqrt{3}x^2 - 4}} \right)$$

[Out] -(Sqrt[3 + 2*Sqrt[3]]*ArcTan[(1 + Sqrt[3] + x)^2/(Sqrt[3*(3 + 2*Sqrt[3]))]*Sqrt[-4 - 4*Sqrt[3]*x^2 + x^4]])/3

Rubi [A] time = 0.140545, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.05$, Rules used = {1740, 203}

$$-\frac{1}{3}\sqrt{3 + 2\sqrt{3}} \tan^{-1} \left(\frac{(x + \sqrt{3} + 1)^2}{\sqrt{3(3 + 2\sqrt{3})}\sqrt{x^4 - 4\sqrt{3}x^2 - 4}} \right)$$

Antiderivative was successfully verified.

[In] Int[(1 + Sqrt[3] + x)/((1 - Sqrt[3] + x)*Sqrt[-4 - 4*Sqrt[3]*x^2 + x^4]), x]

[Out] -(Sqrt[3 + 2*Sqrt[3]]*ArcTan[(1 + Sqrt[3] + x)^2/(Sqrt[3*(3 + 2*Sqrt[3]))]*Sqrt[-4 - 4*Sqrt[3]*x^2 + x^4]])/3

Rule 1740

Int[((A_) + (B_)*(x_))/(((d_) + (e_)*(x_))*Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4]), x_Symbol] :> -Dist[(A^2*(B*d + A*e))/e, Subst[Int[1/(6*A^3*B*d + 3*A^4*e - a*e*x^2), x], x, (A + B*x)^2/Sqrt[a + b*x^2 + c*x^4]], x] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[B*d - A*e, 0] && EqQ[c^2*d^6 + a*e^4*(13*c*d^2 + b*e^2), 0] && EqQ[b^2*e^4 - 12*c*d^2*(c*d^2 - b*e^2), 0] && EqQ[4*A*c*d*e + B*(2*c*d^2 - b*e^2), 0]

Rule 203

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rubi steps

$$\int \frac{1 + \sqrt{3} + x}{(1 - \sqrt{3} + x)\sqrt{-4 - 4\sqrt{3}x^2 + x^4}} dx = - \left((4(2 + \sqrt{3})) \text{Subst} \left[\int \frac{1}{6(1 - \sqrt{3})(1 + \sqrt{3})^3 + 3(1 + \sqrt{3})^4 + 4x^2} dx, x, \frac{(1 + \sqrt{3} + x)^2}{\sqrt{3(3 + 2\sqrt{3})}\sqrt{-4 - 4\sqrt{3}x^2 + x^4}} \right] \right)$$

Mathematica [C] time = 5.6137, size = 876, normalized size = 13.9

$$\sqrt{2} \sqrt{\frac{\sqrt{3}-1-\frac{4}{-x+\sqrt{3}+1}}{-3+\sqrt{3}-i\sqrt{4-2\sqrt{3}}}} (-x + \sqrt{3} + 1)^2 \left(\frac{2 \left(2i\sqrt{3} \sqrt{i \left(\sqrt{3}+1-\frac{8}{-x+\sqrt{3}+1} \right) + \sqrt{4-2\sqrt{3}} + \sqrt{6} \sqrt{2\sqrt{4-2\sqrt{3}} - \sqrt{12-6\sqrt{3}+i\sqrt{3}-i} + \frac{8(-2+\sqrt{3})}{-x+\sqrt{3}+1}} + \sqrt{-\frac{2i(-1+\sqrt{3})}{-x}} \right)}{x-\sqrt{3}-1} \right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 + Sqrt[3] + x)/((1 - Sqrt[3] + x)*Sqrt[-4 - 4*Sqrt[3]*x^2 + x^4]), x]
```

```
[Out] -((Sqrt[2]*Sqrt[(-1 + Sqrt[3] - 4/(1 + Sqrt[3] - x))]/(-3 + Sqrt[3] - I*Sqrt[4 - 2*Sqrt[3]])]*(1 + Sqrt[3] - x)^2*((I*Sqrt[Sqrt[4 - 2*Sqrt[3]]] + I*(1 + Sqrt[3] - 8/(1 + Sqrt[3] - x))) + I*Sqrt[3]*Sqrt[Sqrt[4 - 2*Sqrt[3]]] + I*(1 + Sqrt[3] - 8/(1 + Sqrt[3] - x))) + Sqrt[-2*Sqrt[12 - 6*Sqrt[3]]] + 4*Sqrt[4 - 2*Sqrt[3]] - ((2*I)*(14 - 8*Sqrt[3] + (-1 + Sqrt[3])*x))/(1 + Sqrt[3] - x)] + (2*((2*I)*Sqrt[3]*Sqrt[Sqrt[4 - 2*Sqrt[3]]] + I*(1 + Sqrt[3] - 8/(1 + Sqrt[3] - x))) + Sqrt[6]*Sqrt[-I + I*Sqrt[3] - Sqrt[12 - 6*Sqrt[3]]] + 2*Sqrt[4 - 2*Sqrt[3]] + ((8*I)*(-2 + Sqrt[3]))/(1 + Sqrt[3] - x)] + Sqrt[-2*Sqrt[12 - 6*Sqrt[3]]] + 4*Sqrt[4 - 2*Sqrt[3]] - ((2*I)*(14 - 8*Sqrt[3] + (-1 + Sqrt[3])*x))/(1 + Sqrt[3] - x)))/(-1 - Sqrt[3] + x))*EllipticF[ArcSin[Sqrt[Sqrt[4 - 2*Sqrt[3]] - I*(1 + Sqrt[3] - 8/(1 + Sqrt[3] - x))]]/(2^(3/4)*(2 - Sqrt[3])^(1/4))], (2*Sqrt[4 - 2*Sqrt[3]])/(Sqrt[4 - 2*Sqrt[3]] + I*(-3 + Sqrt[3]))] + 2*Sqrt[6]*Sqrt[Sqrt[4 - 2*Sqrt[3]] - I*(1 + Sqrt[3] - 8/(1 + Sqrt[3] - x))]*Sqrt[(4 - 2*Sqrt[3] + x^2)/(1 + Sqrt[3] - x)^2]*EllipticPi[(2*Sqrt[4 - 2*Sqrt[3]])/(Sqrt[4 - 2*Sqrt[3]] - I*(-3 + Sqrt[3])), ArcSin[Sqrt[Sqrt[4 - 2*Sqrt[3]] - I*(1 + Sqrt[3] - 8/(1 + Sqrt[3] - x))]]/(2^(3/4)*(2 - Sqrt[3])^(1/4))], (2*Sqrt[4 - 2*Sqrt[3]])/(Sqrt[4 - 2*Sqrt[3]] + I*(-3 + Sqrt[3])))]))/((Sqrt[4 - 2*Sqrt[3]] - I*(-3 + Sqrt[3]))*Sqrt[Sqrt[4 - 2*Sqrt[3]]] - I*(1 + Sqrt[3] - 8/(1 + Sqrt[3] - x)))*Sqrt[-4 - 4*Sqrt[3]*x^2 + x^4])
```

Maple [C] time = 0.138, size = 311, normalized size = 4.9

$$\frac{\text{EllipticF}\left(x\left(\frac{i}{2} + \frac{i}{2}\sqrt{3}\right), i\sqrt{1 - 4\sqrt{3}(-1/2\sqrt{3} + 1)}\right)}{\frac{i}{2} + \frac{i}{2}\sqrt{3}} \sqrt{1 - \left(-1 - \frac{\sqrt{3}}{2}\right)x^2} \sqrt{1 - \left(-\frac{\sqrt{3}}{2} + 1\right)x^2} \frac{1}{\sqrt{-4 + x^4 - 4x^2\sqrt{3}}} + 2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x+3^(1/2))/(1+x-3^(1/2))/(-4+x^4-4*x^2*3^(1/2))^(1/2), x)
```

```
[Out] 1/(1/2*I+1/2*I*3^(1/2))*(1-(-1-1/2*3^(1/2))*x^2)^(1/2)*(1-(-1/2*3^(1/2)+1)*x^2)^(1/2)/(-4+x^4-4*x^2*3^(1/2))^(1/2)*EllipticF(x*(1/2*I+1/2*I*3^(1/2)), I*(1-4*3^(1/2)*(-1/2*3^(1/2)+1))^(1/2))+2*3^(1/2)*(-1/2/((3^(1/2)-1)^4-4*3^(1/2)*(3^(1/2)-1)^2-4)^(1/2)*arctanh(1/2*(-4*3^(1/2)*(3^(1/2)-1)^2-8-4*x^2*3^(1/2)+2*x^2*(3^(1/2)-1)^2)/((3^(1/2)-1)^4-4*3^(1/2)*(3^(1/2)-1)^2-4)^(1/2))/(-4+x^4-4*x^2*3^(1/2))^(1/2))-1/(-1-1/2*3^(1/2))^(1/2)/(3^(1/2)-1)*(1-(-1-1/2*3^(1/2))*x^2)^(1/2)*(1-(-1/2*3^(1/2)+1)*x^2)^(1/2)/(-4+x^4-4*x^2*3^(1/2))^(1/2)*EllipticPi((-1-1/2*3^(1/2))^(1/2)*x, 1/(-1-1/2*3^(1/2))/(3^(1/2)-1)^2, (-1/2*3^(1/2)+1)^(1/2)/(-1-1/2*3^(1/2))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x + \sqrt{3} + 1}{\sqrt{x^4 - 4\sqrt{3}x^2 - 4}(x - \sqrt{3} + 1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+x+3^(1/2))/(1+x-3^(1/2))/(-4+x^4-4*3^(1/2)*x^2)^(1/2),x, algorithm="maxima")

[Out] integrate((x + sqrt(3) + 1)/(sqrt(x^4 - 4*sqrt(3)*x^2 - 4)*(x - sqrt(3) + 1)), x)

Fricas [B] time = 2.66524, size = 301, normalized size = 4.78

$$\frac{1}{6} \sqrt{2\sqrt{3} + 3} \arctan \left(-\frac{(9x^4 - 30x^3 + 18x^2 - 2\sqrt{3}(2x^4 - 10x^3 + 3x^2 - 10x + 2) + 24)\sqrt{x^4 - 4\sqrt{3}x^2 - 4}\sqrt{2\sqrt{3} + 3}}{11x^6 - 42x^5 + 66x^4 - 176x^3 - 132x^2 - 168x - 88} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+x+3^(1/2))/(1+x-3^(1/2))/(-4+x^4-4*3^(1/2)*x^2)^(1/2),x, algorithm="fricas")

[Out] 1/6*sqrt(2*sqrt(3) + 3)*arctan(-(9*x^4 - 30*x^3 + 18*x^2 - 2*sqrt(3)*(2*x^4 - 10*x^3 + 3*x^2 - 10*x + 2) + 24)*sqrt(x^4 - 4*sqrt(3)*x^2 - 4)*sqrt(2*sqrt(3) + 3)/(11*x^6 - 42*x^5 + 66*x^4 - 176*x^3 - 132*x^2 - 168*x - 88))

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x + 1 + \sqrt{3}}{(x - \sqrt{3} + 1)\sqrt{x^4 - 4\sqrt{3}x^2 - 4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+x+3**(1/2))/(1+x-3**(1/2))/(-4+x**4-4*3**(1/2)*x**2)**(1/2),x)

[Out] Integral((x + 1 + sqrt(3))/((x - sqrt(3) + 1)*sqrt(x**4 - 4*sqrt(3)*x**2 - 4)), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x + \sqrt{3} + 1}{\sqrt{x^4 - 4\sqrt{3}x^2 - 4}(x - \sqrt{3} + 1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x+3^(1/2))/(1+x-3^(1/2))/(-4+x^4-4*3^(1/2)*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x + sqrt(3) + 1)/(sqrt(x^4 - 4*sqrt(3)*x^2 - 4)*(x - sqrt(3) + 1)), x)
```

$$3.410 \quad \int \frac{1 - \sqrt{3} + 2x}{(1 + \sqrt{3} + 2x)\sqrt{-1 + 4\sqrt{3}x^2 + 4x^4}} dx$$

Optimal. Leaf size=72

$$\frac{1}{3}\sqrt{2\sqrt{3}-3} \tanh^{-1}\left(\frac{(2x - \sqrt{3} + 1)^2}{2\sqrt{3}(2\sqrt{3}-3)\sqrt{4x^4 + 4\sqrt{3}x^2 - 1}}\right)$$

[Out] (Sqrt[-3 + 2*Sqrt[3]]*ArcTanh[(1 - Sqrt[3] + 2*x)^2/(2*Sqrt[3]*(-3 + 2*Sqrt[3]))*Sqrt[-1 + 4*Sqrt[3]*x^2 + 4*x^4]])/3

Rubi [A] time = 0.132782, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {1740, 207}

$$\frac{1}{3}\sqrt{2\sqrt{3}-3} \tanh^{-1}\left(\frac{(2x - \sqrt{3} + 1)^2}{2\sqrt{3}(2\sqrt{3}-3)\sqrt{4x^4 + 4\sqrt{3}x^2 - 1}}\right)$$

Antiderivative was successfully verified.

[In] Int[(1 - Sqrt[3] + 2*x)/((1 + Sqrt[3] + 2*x)*Sqrt[-1 + 4*Sqrt[3]*x^2 + 4*x^4]), x]

[Out] (Sqrt[-3 + 2*Sqrt[3]]*ArcTanh[(1 - Sqrt[3] + 2*x)^2/(2*Sqrt[3]*(-3 + 2*Sqrt[3]))*Sqrt[-1 + 4*Sqrt[3]*x^2 + 4*x^4]])/3

Rule 1740

Int[((A_) + (B_)*(x_))/(((d_) + (e_)*(x_))*Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4]), x_Symbol] :> -Dist[(A^2*(B*d + A*e))/e, Subst[Int[1/(6*A^3*B*d + 3*A^4*e - a*e*x^2), x], x, (A + B*x)^2/Sqrt[a + b*x^2 + c*x^4]], x] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[B*d - A*e, 0] && EqQ[c^2*d^6 + a*e^4*(13*c*d^2 + b*e^2), 0] && EqQ[b^2*e^4 - 12*c*d^2*(c*d^2 - b*e^2), 0] && EqQ[4*A*c*d*e + B*(2*c*d^2 - b*e^2), 0]

Rule 207

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rubi steps

$$\begin{aligned} \int \frac{1 - \sqrt{3} + 2x}{(1 + \sqrt{3} + 2x)\sqrt{-1 + 4\sqrt{3}x^2 + 4x^4}} dx &= -\left(4(2 - \sqrt{3})\right) \text{Subst}\left(\int \frac{1}{6(1 - \sqrt{3})^4 + 12(1 - \sqrt{3})^3(1 + \sqrt{3}) + 2x^2} dx, x, \right. \\ &= \left.\frac{1}{3}\sqrt{-3 + 2\sqrt{3}} \tanh^{-1}\left(\frac{(1 - \sqrt{3} + 2x)^2}{2\sqrt{3}(-3 + 2\sqrt{3})\sqrt{-1 + 4\sqrt{3}x^2 + 4x^4}}\right)\right) \end{aligned}$$

Mathematica [C] time = 1.63171, size = 623, normalized size = 8.65

$$(2x + \sqrt{3} - 1)^2 \sqrt{\frac{-\frac{4}{2x+\sqrt{3}-1} + \sqrt{3} + 1}{3 + \sqrt{3} + i\sqrt{2(2+\sqrt{3})}}} \left(\frac{2(2i\sqrt{3} - \sqrt{2(2+\sqrt{3})} + \sqrt{6(2+\sqrt{3})})}{2x + \sqrt{3} - 1} + i(-1 + \sqrt{3} + i\sqrt{2(2+\sqrt{3})}) \right) \sqrt{\sqrt{2(2+\sqrt{3})} + i\left(\frac{1}{2x+\sqrt{3}-1}\right)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(1 - Sqrt[3] + 2*x)/((1 + Sqrt[3] + 2*x)*Sqrt[-1 + 4*Sqrt[3]*x^2 + 4*x^4]), x]

[Out] ((-1 + Sqrt[3] + 2*x)^2*Sqrt[(1 + Sqrt[3] - 4/(-1 + Sqrt[3] + 2*x))/(3 + Sqrt[3] + I*Sqrt[2*(2 + Sqrt[3])]])*((I*(-1 + Sqrt[3] + I*Sqrt[2*(2 + Sqrt[3])])) + (2*((2*I)*Sqrt[3] - Sqrt[2*(2 + Sqrt[3])]) + Sqrt[6*(2 + Sqrt[3])]))/(-1 + Sqrt[3] + 2*x))*Sqrt[Sqrt[2*(2 + Sqrt[3])] + I*(1 - Sqrt[3] + 8/(-1 + Sqrt[3] + 2*x))]*EllipticF[ArcSin[Sqrt[Sqrt[2*(2 + Sqrt[3])]] - I*(1 - Sqrt[3] + 8/(-1 + Sqrt[3] + 2*x))]/(2^(3/4)*(2 + Sqrt[3])^(1/4))], ((2*I)*Sqrt[2*(2 + Sqrt[3])])/(3 + Sqrt[3] + I*Sqrt[2*(2 + Sqrt[3])]) + 4*Sqrt[3]*Sqrt[(2 + Sqrt[3] + 2*x^2)/(-1 + Sqrt[3] + 2*x)^2]*Sqrt[Sqrt[2*(2 + Sqrt[3])] - I*(1 - Sqrt[3] + 8/(-1 + Sqrt[3] + 2*x))]*EllipticPi[(2*Sqrt[2*(2 + Sqrt[3])])/(Sqrt[2*(2 + Sqrt[3])] + I*(3 + Sqrt[3])), ArcSin[Sqrt[Sqrt[2*(2 + Sqrt[3])]] - I*(1 - Sqrt[3] + 8/(-1 + Sqrt[3] + 2*x))]/(2^(3/4)*(2 + Sqrt[3])^(1/4))], ((2*I)*Sqrt[2*(2 + Sqrt[3])])/(3 + Sqrt[3] + I*Sqrt[2*(2 + Sqrt[3])])])]/((Sqrt[2*(2 + Sqrt[3])] + I*(3 + Sqrt[3]))*Sqrt[-2 + 8*Sqrt[3]*x^2 + 8*x^4]*Sqrt[Sqrt[2*(2 + Sqrt[3])] - I*(1 - Sqrt[3] + 8/(-1 + Sqrt[3] + 2*x))])]

Maple [C] time = 0.143, size = 336, normalized size = 4.7

$$\frac{\text{EllipticF}\left(x(i\sqrt{3}-i), i\sqrt{1+\sqrt{3}(2\sqrt{3}+4)}\right)}{i\sqrt{3}-i} \sqrt{1-(2\sqrt{3}-4)x^2} \sqrt{1-(2\sqrt{3}+4)x^2} \frac{1}{\sqrt{-1+4x^4+4x^2\sqrt{3}}} - 2\sqrt{3} \left(- \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1+2*x-3^(1/2))/(1+2*x+3^(1/2))/(-1+4*x^4+4*x^2*3^(1/2))^(1/2), x)

[Out] 1/(I*3^(1/2)-I)*(1-(2*3^(1/2)-4)*x^2)^(1/2)*(1-(2*3^(1/2)+4)*x^2)^(1/2)/(-1+4*x^4+4*x^2*3^(1/2))^(1/2)*EllipticF(x*(I*3^(1/2)-I), I*(1+3^(1/2)*(2*3^(1/2)+4))^(1/2))-2*3^(1/2)*(-1/4/(4*(-1/2-1/2*3^(1/2))^4+4*3^(1/2)*(-1/2-1/2*3^(1/2))^2-1)^(1/2)*arctanh(1/2*(4*3^(1/2)*(-1/2-1/2*3^(1/2))^2-2+4*x^2*3^(1/2)+8*x^2*(-1/2-1/2*3^(1/2))^2)/(4*(-1/2-1/2*3^(1/2))^4+4*3^(1/2)*(-1/2-1/2*3^(1/2))^2-1)^(1/2)/(-1+4*x^4+4*x^2*3^(1/2))^(1/2))-1/2/(2*3^(1/2)-4)^(1/2)/(-1/2-1/2*3^(1/2))*(1-(2*3^(1/2)-4)*x^2)^(1/2)*(1-(2*3^(1/2)+4)*x^2)^(1/2)/(-1+4*x^4+4*x^2*3^(1/2))^(1/2)*EllipticPi((2*3^(1/2)-4)^(1/2)*x, 1/(2*3^(1/2)-4)/(-1/2-1/2*3^(1/2))^2, (2*3^(1/2)+4)^(1/2)/(2*3^(1/2)-4)^(1/2)))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{2x - \sqrt{3} + 1}{\sqrt{4x^4 + 4\sqrt{3}x^2 - 1}(2x + \sqrt{3} + 1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x-3^(1/2))/(1+2*x+3^(1/2))/(-1+4*x^4+4*3^(1/2)*x^2)^(1/2),x,
algorithm="maxima")
```

```
[Out] integrate((2*x - sqrt(3) + 1)/(sqrt(4*x^4 + 4*sqrt(3)*x^2 - 1)*(2*x + sqrt(3) + 1)), x)
```

Fricas [B] time = 2.70544, size = 950, normalized size = 13.19

$$\frac{1}{12} \sqrt{2\sqrt{3}-3} \log \left(-\frac{2368x^{12} - 6528x^{11} + 12864x^{10} - 19264x^9 + 14832x^8 - 10944x^7 + 6432x^6 + 5472x^5 + 3708x^4 + 2408x^3 + 804x^2 + (1728x^{10} - 4800x^9 + 8208x^8 - 8928x^7 + 6048x^6 - 3024x^5 - 504x^4 - 504x^3 - 324x^2 + 2\sqrt{3})(496x^{10} - 1408x^9 + 2304x^8 - 2640x^7 + 1848x^6 - 504x^5 + 336x^4 + 204x^3 + 63x^2 + 26x + 4) - 72x - 15}{(64x^{12} + 384x^{11} + 768x^{10} + 320x^9 - 720x^8 - 576x^7 + 384x^6 + 288x^5 - 180x^4 - 40x^3 + 48x^2 - 12x + 1)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x-3^(1/2))/(1+2*x+3^(1/2))/(-1+4*x^4+4*3^(1/2)*x^2)^(1/2),x,
algorithm="fricas")
```

```
[Out] 1/12*sqrt(2*sqrt(3) - 3)*log(-(2368*x^12 - 6528*x^11 + 12864*x^10 - 19264*x^9 + 14832*x^8 - 10944*x^7 + 6432*x^6 + 5472*x^5 + 3708*x^4 + 2408*x^3 + 804*x^2 + (1728*x^10 - 4800*x^9 + 8208*x^8 - 8928*x^7 + 6048*x^6 - 3024*x^5 - 504*x^4 - 504*x^3 - 324*x^2 + 2*sqrt(3)*(496*x^10 - 1408*x^9 + 2304*x^8 - 2640*x^7 + 1848*x^6 - 504*x^5 + 336*x^4 + 204*x^3 + 63*x^2 + 26*x + 4) - 72*x - 15)*sqrt(4*x^4 + 4*sqrt(3)*x^2 - 1)*sqrt(2*sqrt(3) - 3) + 3*sqrt(3)*(448*x^12 - 1280*x^11 + 2560*x^10 - 3200*x^9 + 3696*x^8 - 1920*x^7 - 960*x^5 - 924*x^4 - 400*x^3 - 160*x^2 - 40*x - 7) + 204*x + 37)/(64*x^12 + 384*x^11 + 768*x^10 + 320*x^9 - 720*x^8 - 576*x^7 + 384*x^6 + 288*x^5 - 180*x^4 - 40*x^3 + 48*x^2 - 12*x + 1))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{2x - \sqrt{3} + 1}{(2x + 1 + \sqrt{3})\sqrt{4x^4 + 4\sqrt{3}x^2 - 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x-3**(1/2))/(1+2*x+3**(1/2))/(-1+4*x**4+4*3**(1/2)*x**2)**(1/2),x)
```

```
[Out] Integral((2*x - sqrt(3) + 1)/((2*x + 1 + sqrt(3))*sqrt(4*x**4 + 4*sqrt(3)*x**2 - 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{2x - \sqrt{3} + 1}{\sqrt{4x^4 + 4\sqrt{3}x^2 - 1}(2x + \sqrt{3} + 1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.


```
[In] integrate((1+2*x-3^(1/2))/(1+2*x+3^(1/2))/(-1+4*x^4+4*3^(1/2)*x^2)^(1/2),x,  
algorithm="giac")
```

```
[Out] integrate((2*x - sqrt(3) + 1)/(sqrt(4*x^4 + 4*sqrt(3)*x^2 - 1)*(2*x + sqrt(  
3) + 1)), x)
```

$$3.411 \quad \int \frac{1 + \sqrt{3} + 2x}{(1 - \sqrt{3} + 2x)\sqrt{-1 - 4\sqrt{3}x^2 + 4x^4}} dx$$

Optimal. Leaf size=70

$$-\frac{1}{3}\sqrt{3 + 2\sqrt{3}} \tan^{-1} \left(\frac{(2x + \sqrt{3} + 1)^2}{2\sqrt{3}(3 + 2\sqrt{3})\sqrt{4x^4 - 4\sqrt{3}x^2 - 1}} \right)$$

[Out] -(Sqrt[3 + 2*Sqrt[3]]*ArcTan[(1 + Sqrt[3] + 2*x)^2/(2*Sqrt[3]*(3 + 2*Sqrt[3]))*Sqrt[-1 - 4*Sqrt[3]*x^2 + 4*x^4]])/3

Rubi [A] time = 0.129127, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {1740, 203}

$$-\frac{1}{3}\sqrt{3 + 2\sqrt{3}} \tan^{-1} \left(\frac{(2x + \sqrt{3} + 1)^2}{2\sqrt{3}(3 + 2\sqrt{3})\sqrt{4x^4 - 4\sqrt{3}x^2 - 1}} \right)$$

Antiderivative was successfully verified.

[In] Int[(1 + Sqrt[3] + 2*x)/((1 - Sqrt[3] + 2*x)*Sqrt[-1 - 4*Sqrt[3]*x^2 + 4*x^4]), x]

[Out] -(Sqrt[3 + 2*Sqrt[3]]*ArcTan[(1 + Sqrt[3] + 2*x)^2/(2*Sqrt[3]*(3 + 2*Sqrt[3]))*Sqrt[-1 - 4*Sqrt[3]*x^2 + 4*x^4]])/3

Rule 1740

Int[((A_) + (B_)*(x_))/(((d_) + (e_)*(x_))*Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4]), x_Symbol] :> -Dist[(A^2*(B*d + A*e))/e, Subst[Int[1/(6*A^3*B*d + 3*A^4*e - a*e*x^2), x], x, (A + B*x)^2/Sqrt[a + b*x^2 + c*x^4]], x] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[B*d - A*e, 0] && EqQ[c^2*d^6 + a*e^4*(13*c*d^2 + b*e^2), 0] && EqQ[b^2*e^4 - 12*c*d^2*(c*d^2 - b*e^2), 0] && EqQ[4*A*c*d*e + B*(2*c*d^2 - b*e^2), 0]

Rule 203

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[Rt[b, 2]*x]/Rt[a, 2])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rubi steps

$$\begin{aligned} \int \frac{1 + \sqrt{3} + 2x}{(1 - \sqrt{3} + 2x)\sqrt{-1 - 4\sqrt{3}x^2 + 4x^4}} dx &= - \left((4(2 + \sqrt{3})) \text{Subst} \left(\int \frac{1}{12(1 - \sqrt{3})(1 + \sqrt{3})^3 + 6(1 + \sqrt{3})^4 + 2x^2} dx, x, \right. \right. \\ &= -\frac{1}{3}\sqrt{3 + 2\sqrt{3}} \tan^{-1} \left(\frac{(1 + \sqrt{3} + 2x)^2}{2\sqrt{3}(3 + 2\sqrt{3})\sqrt{-1 - 4\sqrt{3}x^2 + 4x^4}} \right) \end{aligned}$$

Mathematica [C] time = 5.81788, size = 881, normalized size = 12.59

$$\sqrt{\frac{\sqrt{3}-1-\frac{4}{-2x+\sqrt{3}+1}}{-3+\sqrt{3}-i\sqrt{4-2\sqrt{3}}}}(-2x+\sqrt{3}+1)^2 \left(\frac{2i\left(2\sqrt{3}\sqrt{i(\sqrt{3}+1-\frac{8}{-2x+\sqrt{3}+1})+\sqrt{4-2\sqrt{3}-i\sqrt{6}}}\sqrt{-\frac{2i((-1+\sqrt{3})x-4\sqrt{3}+7)}{-2x+\sqrt{3}+1}+2\sqrt{4-2\sqrt{3}-\sqrt{12-6\sqrt{3}-i}}}\sqrt{-\frac{4i((-1+\sqrt{3})x-4\sqrt{3}+7)}{-2x+\sqrt{3}+1}}}{-2x+\sqrt{3}+1}} \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[(1 + Sqrt[3] + 2*x)/((1 - Sqrt[3] + 2*x)*Sqrt[-1 - 4*Sqrt[3]*x^2 + 4*x^4]), x]

[Out] -((Sqrt[(-1 + Sqrt[3] - 4/(1 + Sqrt[3] - 2*x))]/(-3 + Sqrt[3] - I*Sqrt[4 - 2*Sqrt[3]]))*(1 + Sqrt[3] - 2*x)^2*((I*Sqrt[Sqrt[4 - 2*Sqrt[3]]] + I*(1 + Sqrt[3] - 8/(1 + Sqrt[3] - 2*x))) + I*Sqrt[3]*Sqrt[Sqrt[4 - 2*Sqrt[3]]] + I*(1 + Sqrt[3] - 8/(1 + Sqrt[3] - 2*x))) + Sqrt[-2*Sqrt[12 - 6*Sqrt[3]]] + 4*Sqrt[4 - 2*Sqrt[3]] - ((4*I)*(7 - 4*Sqrt[3] + (-1 + Sqrt[3])*x))/(1 + Sqrt[3] - 2*x) - ((2*I)*(2*Sqrt[3]*Sqrt[Sqrt[4 - 2*Sqrt[3]]] + I*(1 + Sqrt[3] - 8/(1 + Sqrt[3] - 2*x))) - I*Sqrt[6]*Sqrt[-Sqrt[12 - 6*Sqrt[3]]] + 2*Sqrt[4 - 2*Sqrt[3]] - ((2*I)*(7 - 4*Sqrt[3] + (-1 + Sqrt[3])*x))/(1 + Sqrt[3] - 2*x) - I*Sqrt[-2*Sqrt[12 - 6*Sqrt[3]]] + 4*Sqrt[4 - 2*Sqrt[3]] - ((4*I)*(7 - 4*Sqrt[3] + (-1 + Sqrt[3])*x))/(1 + Sqrt[3] - 2*x)))/(1 + Sqrt[3] - 2*x))*EllipticF[ArcSin[Sqrt[Sqrt[4 - 2*Sqrt[3]]] - I*(1 + Sqrt[3] - 8/(1 + Sqrt[3] - 2*x))]/(2^(3/4)*(2 - Sqrt[3])^(1/4))], (2*Sqrt[4 - 2*Sqrt[3]])/(Sqrt[4 - 2*Sqrt[3]] + I*(-3 + Sqrt[3]))] + 4*Sqrt[Sqrt[4 - 2*Sqrt[3]] - I*(1 + Sqrt[3] - 8/(1 + Sqrt[3] - 2*x))]*Sqrt[(6 - 3*Sqrt[3] + 6*x^2)/(1 + Sqrt[3] - 2*x)^2]*EllipticPi[(2*Sqrt[4 - 2*Sqrt[3]])/(Sqrt[4 - 2*Sqrt[3]] - I*(-3 + Sqrt[3]))], ArcSin[Sqrt[Sqrt[4 - 2*Sqrt[3]]] - I*(1 + Sqrt[3] - 8/(1 + Sqrt[3] - 2*x))]/(2^(3/4)*(2 - Sqrt[3])^(1/4))], (2*Sqrt[4 - 2*Sqrt[3]])/(Sqrt[4 - 2*Sqrt[3]] + I*(-3 + Sqrt[3])))]/(Sqrt[2]*(Sqrt[4 - 2*Sqrt[3]] - I*(-3 + Sqrt[3]))*Sqrt[Sqrt[4 - 2*Sqrt[3]] - I*(1 + Sqrt[3] - 8/(1 + Sqrt[3] - 2*x))]*Sqrt[-1 - 4*Sqrt[3]*x^2 + 4*x^4]))

Maple [C] time = 0.134, size = 337, normalized size = 4.8

$$\frac{\text{EllipticF}\left(x(i+i\sqrt{3}), i\sqrt{1-\sqrt{3}(-2\sqrt{3}+4)}\right)}{i+i\sqrt{3}} \sqrt{1-(-2\sqrt{3}-4)x^2} \sqrt{1-(-2\sqrt{3}+4)x^2} \frac{1}{\sqrt{-1+4x^4-4x^2\sqrt{3}}} + 2\sqrt{\dots}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1+2*x+3^(1/2))/(1+2*x-3^(1/2))/(-1+4*x^4-4*x^2*3^(1/2))^(1/2), x)

[Out] 1/(I+I*3^(1/2))*(1-(-2*3^(1/2)-4)*x^2)^(1/2)*(1-(-2*3^(1/2)+4)*x^2)^(1/2)/((-1+4*x^4-4*x^2*3^(1/2))^(1/2)*EllipticF(x*(I+I*3^(1/2)), I*(1-3^(1/2)*(-2*3^(1/2)+4))^(1/2))+2*3^(1/2)*(-1/4/(4*(1/2*3^(1/2)-1/2)^4-4*3^(1/2)*(1/2*3^(1/2)-1/2)^2-1)^(1/2)*arctanh(1/2*(-4*3^(1/2)*(1/2*3^(1/2)-1/2)^2-2-4*x^2*3^(1/2)+8*x^2*(1/2*3^(1/2)-1/2)^2)/(4*(1/2*3^(1/2)-1/2)^4-4*3^(1/2)*(1/2*3^(1/2)-1/2)^2-1)^(1/2)/(-1+4*x^4-4*x^2*3^(1/2))^(1/2))-1/2/(-2*3^(1/2)-4)^(1/2)/(1/2*3^(1/2)-1/2)*(1-(-2*3^(1/2)-4)*x^2)^(1/2)*(1-(-2*3^(1/2)+4)*x^2)^(1/2))/((-1+4*x^4-4*x^2*3^(1/2))^(1/2)*EllipticPi((-2*3^(1/2)-4)^(1/2)*x, 1/(-2*3^(1/2)-4)/(1/2*3^(1/2)-1/2)^2, (-2*3^(1/2)+4)^(1/2)/(-2*3^(1/2)-4)^(1/2)))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{2x + \sqrt{3} + 1}{\sqrt{4x^4 - 4\sqrt{3}x^2 - 1}(2x - \sqrt{3} + 1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+2*x+3^(1/2))/(1+2*x-3^(1/2))/(-1+4*x^4-4*3^(1/2)*x^2)^(1/2), x,
algorithm="maxima")

[Out] integrate((2*x + sqrt(3) + 1)/(sqrt(4*x^4 - 4*sqrt(3)*x^2 - 1)*(2*x - sqrt(3) + 1)), x)

Fricas [B] time = 2.68323, size = 302, normalized size = 4.31

$$\frac{1}{6} \sqrt{2\sqrt{3} + 3} \arctan \left(-\frac{(36x^4 - 60x^3 + 18x^2 - \sqrt{3}(16x^4 - 40x^3 + 6x^2 - 10x + 1) + 6)\sqrt{4x^4 - 4\sqrt{3}x^2 - 1}\sqrt{2\sqrt{3} + 3}}{88x^6 - 168x^5 + 132x^4 - 176x^3 - 66x^2 - 42x - 11} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+2*x+3^(1/2))/(1+2*x-3^(1/2))/(-1+4*x^4-4*3^(1/2)*x^2)^(1/2), x,
algorithm="fricas")

[Out] 1/6*sqrt(2*sqrt(3) + 3)*arctan(-(36*x^4 - 60*x^3 + 18*x^2 - sqrt(3)*(16*x^4 - 40*x^3 + 6*x^2 - 10*x + 1) + 6)*sqrt(4*x^4 - 4*sqrt(3)*x^2 - 1)*sqrt(2*sqrt(3) + 3)/(88*x^6 - 168*x^5 + 132*x^4 - 176*x^3 - 66*x^2 - 42*x - 11))

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{2x + 1 + \sqrt{3}}{(2x - \sqrt{3} + 1)\sqrt{4x^4 - 4\sqrt{3}x^2 - 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+2*x+3**(1/2))/(1+2*x-3**(1/2))/(-1+4*x**4-4*3**(1/2)*x**2)**(1/2), x)

[Out] Integral((2*x + 1 + sqrt(3))/((2*x - sqrt(3) + 1)*sqrt(4*x**4 - 4*sqrt(3)*x**2 - 1)), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{2x + \sqrt{3} + 1}{\sqrt{4x^4 - 4\sqrt{3}x^2 - 1}(2x - \sqrt{3} + 1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x+3^(1/2))/(1+2*x-3^(1/2))/(-1+4*x^4-4*3^(1/2)*x^2)^(1/2),x,  
algorithm="giac")
```

```
[Out] integrate((2*x + sqrt(3) + 1)/(sqrt(4*x^4 - 4*sqrt(3)*x^2 - 1)*(2*x - sqrt(  
3) + 1)), x)
```

$$3.412 \quad \int \frac{f+gx}{(d+ex)\sqrt{a+bx^2+cx^4}} dx$$

Optimal. Leaf size=560

$$\frac{(\sqrt{a} + \sqrt{cx^2}) \sqrt{\frac{a+bx^2+cx^4}{(\sqrt{a}+\sqrt{cx^2})^2}} (\sqrt{aeg} + \sqrt{cdf}) \operatorname{EllipticF}\left(2 \tan^{-1}\left(\frac{\sqrt[4]{cx}}{\sqrt[4]{a}}\right), \frac{1}{4}\left(2 - \frac{b}{\sqrt{a}\sqrt{c}}\right)\right) + (ef - dg) \tan^{-1}\left(\frac{x\sqrt{-ae^4-bd^2e^2-cd^4}}{de\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+bx^2+cx^4}(\sqrt{ae^2} + \sqrt{cd^2})} + \frac{(ef - dg) \tan^{-1}\left(\frac{x\sqrt{-ae^4-bd^2e^2-cd^4}}{de\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{-e^2(ae^2 + bd^2) - cd^4}}$$

[Out] ((e*f - d*g)*ArcTan[(Sqrt[-(c*d^4) - b*d^2*e^2 - a*e^4]*x)/(d*e*Sqrt[a + b*x^2 + c*x^4]])/(2*Sqrt[-(c*d^4) - e^2*(b*d^2 + a*e^2)]) - ((e*f - d*g)*ArcTanh[(b*d^2 + 2*a*e^2 + (2*c*d^2 + b*e^2)*x^2)/(2*Sqrt[c*d^4 + b*d^2*e^2 + a*e^4]*Sqrt[a + b*x^2 + c*x^4]])/(2*Sqrt[c*d^4 + b*d^2*e^2 + a*e^4]) + ((Sqrt[c]*d*f + Sqrt[a]*e*g)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*a^(1/4)*c^(1/4)*(Sqrt[c]*d^2 + Sqrt[a]*e^2)*Sqrt[a + b*x^2 + c*x^4]) - ((Sqrt[c]*d^2 - Sqrt[a]*e^2)*(e*f - d*g)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticPi[(Sqrt[c]*d^2 + Sqrt[a]*e^2)^2/(4*Sqrt[a]*Sqrt[c]*d^2*e^2), 2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(4*a^(1/4)*c^(1/4)*d*e*(Sqrt[c]*d^2 + Sqrt[a]*e^2)*Sqrt[a + b*x^2 + c*x^4])

Rubi [A] time = 0.617087, antiderivative size = 560, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used = {1741, 12, 1247, 724, 206, 1708, 1103, 1706}

$$\frac{(ef - dg) \tan^{-1}\left(\frac{x\sqrt{-ae^4-bd^2e^2-cd^4}}{de\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{-e^2(ae^2 + bd^2) - cd^4}} - \frac{(ef - dg) \tanh^{-1}\left(\frac{2ae^2+x^2(be^2+2cd^2)+bd^2}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^4+bd^2e^2+cd^4}}\right)}{2\sqrt{ae^4 + bd^2e^2 + cd^4}} + \frac{(\sqrt{a} + \sqrt{cx^2}) \sqrt{\frac{a+bx^2+cx^4}{(\sqrt{a}+\sqrt{cx^2})^2}} F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{cx}}{\sqrt[4]{a}}\right), \frac{1}{4}\left(2 - \frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

[In] Int[(f + g*x)/((d + e*x)*Sqrt[a + b*x^2 + c*x^4]), x]

[Out] ((e*f - d*g)*ArcTan[(Sqrt[-(c*d^4) - b*d^2*e^2 - a*e^4]*x)/(d*e*Sqrt[a + b*x^2 + c*x^4]])/(2*Sqrt[-(c*d^4) - e^2*(b*d^2 + a*e^2)]) - ((e*f - d*g)*ArcTanh[(b*d^2 + 2*a*e^2 + (2*c*d^2 + b*e^2)*x^2)/(2*Sqrt[c*d^4 + b*d^2*e^2 + a*e^4]*Sqrt[a + b*x^2 + c*x^4]])/(2*Sqrt[c*d^4 + b*d^2*e^2 + a*e^4]) + ((Sqrt[c]*d*f + Sqrt[a]*e*g)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*a^(1/4)*c^(1/4)*(Sqrt[c]*d^2 + Sqrt[a]*e^2)*Sqrt[a + b*x^2 + c*x^4]) - ((Sqrt[c]*d^2 - Sqrt[a]*e^2)*(e*f - d*g)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticPi[(Sqrt[c]*d^2 + Sqrt[a]*e^2)^2/(4*Sqrt[a]*Sqrt[c]*d^2*e^2), 2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(4*a^(1/4)*c^(1/4)*d*e*(Sqrt[c]*d^2 + Sqrt[a]*e^2)*Sqrt[a + b*x^2 + c*x^4])

Rule 1741

Int[(P*x)/(((d_) + (e_)*(x_))*Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4]), x
_Symbol] := With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1], C = Coeff[Px, x, 2], D = Coeff[Px, x, 3]}, Int[(x*(B*d - A*e + (d*D - C*e)*x^2)/((d^2 - e^2*x^2)*Sqrt[a + b*x^2 + c*x^4]), x] + Int[(A*d + (C*d - B*e)*x^2 - D*e*x^4)/((d^2 - e^2*x^2)*Sqrt[a + b*x^2 + c*x^4]), x]] /; FreeQ[{a, b, c, d, e},

$x]$ && PolyQ[Px, x] && LeQ[Expon[Px, x], 3] && NeQ[c*d^4 + b*d^2*e^2 + a*e^4, 0]

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 1247

Int[(x_)*((d_) + (e_)*(x_)^2)^(q_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] := Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]

Rule 724

Int[1/(((d_) + (e_)*(x_))*Sqrt[(a_) + (b_)*(x_) + (c_)*(x_)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 1708

Int[((A_) + (B_)*(x_)^2)/(((d_) + (e_)*(x_)^2)*Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4]), x_Symbol] := With[{q = Rt[c/a, 2]}, Dist[(A*(c*d + a*e*q) - a*B*(e + d*q))/(c*d^2 - a*e^2), Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] + Dist[(a*(B*d - A*e)*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && NeQ[c*A^2 - a*B^2, 0]

Rule 1103

Int[1/Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4], x_Symbol] := With[{q = Rt[c/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)]/(2*q*Sqrt[a + b*x^2 + c*x^4]), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]

Rule 1706

Int[((A_) + (B_)*(x_)^2)/(((d_) + (e_)*(x_)^2)*Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4]), x_Symbol] := With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*ArcTan[(Rt[-b + (c*d)/e + (a*e)/d, 2]*x)/Sqrt[a + b*x^2 + c*x^4]])/(2*d*e*Rt[-b + (c*d)/e + (a*e)/d, 2]), x] + Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a + b*x^2 + c*x^4))/(a*(A + B*x^2)^2)]*EllipticPi[Cancel[-((B*d - A*e)^2/(4*d*e*A*B))], 2*ArcTan[q*x], 1/2 - (b*A)/(4*a*B)]/(4*d*e*A*q*Sqrt[a + b*x^2 + c*x^4]), x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]

Rubi steps

$$\begin{aligned}
\int \frac{f + gx}{(d + ex)\sqrt{a + bx^2 + cx^4}} dx &= \int \frac{(-ef + dg)x}{(d^2 - e^2x^2)\sqrt{a + bx^2 + cx^4}} dx + \int \frac{df - egx^2}{(d^2 - e^2x^2)\sqrt{a + bx^2 + cx^4}} dx \\
&= \frac{(\sqrt{ade}(ef - dg)) \int \frac{1 + \frac{\sqrt{cx^2}}{\sqrt{a}}}{(d^2 - e^2x^2)\sqrt{a + bx^2 + cx^4}} dx}{\sqrt{cd^2} + \sqrt{ae^2}} + (-ef + dg) \int \frac{x}{(d^2 - e^2x^2)\sqrt{a + bx^2 + cx^4}} dx \\
&= \frac{(ef - dg) \tan^{-1} \left(\frac{\sqrt{-cd^4 - bd^2e^2 - ae^4}x}{de\sqrt{a + bx^2 + cx^4}} \right)}{2\sqrt{-cd^4 - e^2} (bd^2 + ae^2)} + \frac{(\sqrt{cdf} + \sqrt{aeg})(\sqrt{a} + \sqrt{cx^2}) \sqrt{\frac{a + bx^2 + cx^4}{(\sqrt{a} + \sqrt{cx^2})^2}} F \left(2 \tan^{-1} \left(\frac{\sqrt{cx^2}}{\sqrt{a}} \right) \right)}{2\sqrt[4]{a}\sqrt[4]{c} (\sqrt{cd^2} + \sqrt{ae^2}) \sqrt{a + bx^2 + cx^4}} \\
&= \frac{(ef - dg) \tan^{-1} \left(\frac{\sqrt{-cd^4 - bd^2e^2 - ae^4}x}{de\sqrt{a + bx^2 + cx^4}} \right)}{2\sqrt{-cd^4 - e^2} (bd^2 + ae^2)} + \frac{(\sqrt{cdf} + \sqrt{aeg})(\sqrt{a} + \sqrt{cx^2}) \sqrt{\frac{a + bx^2 + cx^4}{(\sqrt{a} + \sqrt{cx^2})^2}} F \left(2 \tan^{-1} \left(\frac{\sqrt{cx^2}}{\sqrt{a}} \right) \right)}{2\sqrt[4]{a}\sqrt[4]{c} (\sqrt{cd^2} + \sqrt{ae^2}) \sqrt{a + bx^2 + cx^4}} \\
&= \frac{(ef - dg) \tan^{-1} \left(\frac{\sqrt{-cd^4 - bd^2e^2 - ae^4}x}{de\sqrt{a + bx^2 + cx^4}} \right)}{2\sqrt{-cd^4 - e^2} (bd^2 + ae^2)} - \frac{(ef - dg) \tanh^{-1} \left(\frac{bd^2 + 2ae^2 + (2cd^2 + be^2)x^2}{2\sqrt{cd^4 + bd^2e^2 + ae^4}\sqrt{a + bx^2 + cx^4}} \right)}{2\sqrt{cd^4 + bd^2e^2 + ae^4}} + \frac{(\sqrt{cd^2} + \sqrt{ae^2})}{2\sqrt{cd^4 + bd^2e^2 + ae^4}}
\end{aligned}$$

Mathematica [C] time = 7.82417, size = 3652, normalized size = 6.52

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[(f + g*x)/((d + e*x)*Sqrt[a + b*x^2 + c*x^4]),x]

[Out] ((-I)*g*Sqrt[1 - (2*c*x^2)/(-b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 - (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[-(c/(-b - Sqrt[b^2 - 4*a*c])])]*x], (-b - Sqrt[b^2 - 4*a*c])/(-b + Sqrt[b^2 - 4*a*c])]/(Sqrt[2]*Sqrt[-(c/(-b - Sqrt[b^2 - 4*a*c])])]*e*Sqrt[a + b*x^2 + c*x^4]) + (2*(Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]/Sqrt[2] + Sqrt[-(b/c) + Sqrt[b^2 - 4*a*c]/c]/Sqrt[2])*f*(-(Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]/Sqrt[2]) + x)^2*Sqrt[(Sqrt[(-b - Sqrt[b^2 - 4*a*c])/c]*(-Sqrt[-(b/c) + Sqrt[b^2 - 4*a*c]/c]/Sqrt[2]) + x))/((Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]/Sqrt[2] + Sqrt[-(b/c) + Sqrt[b^2 - 4*a*c]/c]/Sqrt[2])*(-Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]/Sqrt[2]) + x))*Sqrt[(Sqrt[(-b - Sqrt[b^2 - 4*a*c])/c]*(Sqrt[-(b/c) + Sqrt[b^2 - 4*a*c]/c]/Sqrt[2] + x))/((Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]/Sqrt[2] - Sqrt[-(b/c) + Sqrt[b^2 - 4*a*c]/c]/Sqrt[2])*(-Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]/Sqrt[2]) + x)]*Sqrt[((Sqrt[(-b - Sqrt[b^2 - 4*a*c])/c] - Sqrt[-(b/c) + Sqrt[b^2 - 4*a*c]/c])/c)]*(Sqrt[2]*Sqrt[(-b - Sqrt[b^2 - 4*a*c])/c] + 2*x))/((Sqrt[(-b - Sqrt[b^2 - 4*a*c])/c] + Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]/Sqrt[2])*(-Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]/Sqrt[2]) + x)]*Sqrt[((Sqrt[(-b - Sqrt[b^2 - 4*a*c])/c] - Sqrt[-(b/c) + Sqrt[b^2 - 4*a*c]/c])/c)]*(Sqrt[2]*Sqrt[(-b - Sqrt[b^2 - 4*a*c])/c] + 2*x))/((Sqrt[(-b - Sqrt[b^2 - 4*a*c])/c] + Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]/Sqrt[2])*(Sqrt[2]*Sqrt[(-b - Sqrt[b^2 - 4*a*c])/c] - 2*x))], (Sqrt[(-b - Sqrt[b^2 - 4*a*c])/c] + Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]/Sqrt[2])^2/(Sqrt[(-b - Sqrt[b^2 - 4*a*c])/c] - Sqrt[-(b/c) + Sqrt[b^2 - 4*a*c]/c]/Sqrt[2])^2 - Sqrt[2]*Sqrt[(-b - Sqrt[b^2 - 4*a*c])/c]*e*EllipticPi[((Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]/Sqrt[2] + Sqrt[-(b/c) + Sqrt[b^2 - 4*a*c]/c]/Sqrt[2])*(d + (Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]*e)/Sqrt[2]))/((-Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]/Sqrt[2]) + Sqrt[-(b/c) + Sqrt[b^2 - 4*a*c]/c]/Sqrt[2])*(d - (Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]*e)/Sqrt[2])), ArcSin[Sqrt[(Sqrt[(-b - Sqrt[b^2 - 4*a*c])/c] - Sqrt[-(b/c) + Sqrt[b^2 - 4*a*c]/c])/c]]

$$\begin{aligned} & \text{Sqrt}[(-b + \text{Sqrt}[b^2 - 4*a*c])/c)]*(\text{Sqrt}[2]*\text{Sqrt}[(-b - \text{Sqrt}[b^2 - 4*a*c])/c] \\ & + 2*x))/((\text{Sqrt}[(-b - \text{Sqrt}[b^2 - 4*a*c])/c] + \text{Sqrt}[(-b + \text{Sqrt}[b^2 - 4*a*c] \\ &)/c)]*(\text{Sqrt}[2]*\text{Sqrt}[(-b - \text{Sqrt}[b^2 - 4*a*c])/c] - 2*x))], (\text{Sqrt}[(-b - \text{Sqrt} \\ & [b^2 - 4*a*c])/c] + \text{Sqrt}[(-b + \text{Sqrt}[b^2 - 4*a*c])/c])^2/(\text{Sqrt}[(-b - \text{Sqrt}[b^ \\ & 2 - 4*a*c])/c] - \text{Sqrt}[(-b + \text{Sqrt}[b^2 - 4*a*c])/c])^2))/(\text{Sqrt}[(-b - \text{Sqrt}[b^ \\ & 2 - 4*a*c])/c]*(\text{Sqrt}[-(b/c) - \text{Sqrt}[b^2 - 4*a*c]/c]/\text{Sqrt}[2] - \text{Sqrt}[-(b/c) + \\ & \text{Sqrt}[b^2 - 4*a*c]/c]/\text{Sqrt}[2]))*(-d - (\text{Sqrt}[-(b/c) - \text{Sqrt}[b^2 - 4*a*c]/c]*e)/ \\ & \text{Sqrt}[2])*(d - (\text{Sqrt}[-(b/c) - \text{Sqrt}[b^2 - 4*a*c]/c]*e)/\text{Sqrt}[2])* \text{Sqrt}[a + b*x^ \\ & 2 + c*x^4]) - (2*(\text{Sqrt}[-(b/c) - \text{Sqrt}[b^2 - 4*a*c]/c]/\text{Sqrt}[2] + \text{Sqrt}[-(b/c) \\ & + \text{Sqrt}[b^2 - 4*a*c]/c]/\text{Sqrt}[2))*d*g*(-(\text{Sqrt}[-(b/c) - \text{Sqrt}[b^2 - 4*a*c]/c]/\text{S} \\ & \text{qrt}[2]) + x)^2*\text{Sqrt}[(\text{Sqrt}[(-b - \text{Sqrt}[b^2 - 4*a*c])/c]*(-(\text{Sqrt}[-(b/c) + \text{Sqrt} \\ & [b^2 - 4*a*c]/c]/\text{Sqrt}[2]) + x))/((\text{Sqrt}[-(b/c) - \text{Sqrt}[b^2 - 4*a*c]/c]/\text{Sqrt}[2] \\ & + \text{Sqrt}[-(b/c) + \text{Sqrt}[b^2 - 4*a*c]/c]/\text{Sqrt}[2))*(-(\text{Sqrt}[-(b/c) - \text{Sqrt}[b^2 - \\ & 4*a*c]/c]/\text{Sqrt}[2]) + x))]*\text{Sqrt}[(\text{Sqrt}[(-b - \text{Sqrt}[b^2 - 4*a*c])/c]*(\text{Sqrt}[-(b \\ & /c) + \text{Sqrt}[b^2 - 4*a*c]/c]/\text{Sqrt}[2] + x))/((\text{Sqrt}[-(b/c) - \text{Sqrt}[b^2 - 4*a*c]/ \\ & c]/\text{Sqrt}[2] - \text{Sqrt}[-(b/c) + \text{Sqrt}[b^2 - 4*a*c]/c]/\text{Sqrt}[2))*(-(\text{Sqrt}[-(b/c) - \text{S} \\ & \text{qrt}[b^2 - 4*a*c]/c]/\text{Sqrt}[2]) + x))]*\text{Sqrt}[(\text{Sqrt}[(-b - \text{Sqrt}[b^2 - 4*a*c])/c] \\ & - \text{Sqrt}[(-b + \text{Sqrt}[b^2 - 4*a*c])/c])*(\text{Sqrt}[2]*\text{Sqrt}[(-b - \text{Sqrt}[b^2 - 4*a*c]) \\ & /c] + 2*x))/((\text{Sqrt}[(-b - \text{Sqrt}[b^2 - 4*a*c])/c] + \text{Sqrt}[(-b + \text{Sqrt}[b^2 - 4*a* \\ & c])/c])*(\text{Sqrt}[2]*\text{Sqrt}[(-b - \text{Sqrt}[b^2 - 4*a*c])/c] - 2*x)))*((-d + (\text{Sqrt}[-(b \\ & /c) - \text{Sqrt}[b^2 - 4*a*c]/c]*e)/\text{Sqrt}[2])* \text{EllipticF}[\text{ArcSin}[\text{Sqrt}[(\text{Sqrt}[(-b - \text{S} \\ & \text{qrt}[b^2 - 4*a*c])/c] - \text{Sqrt}[(-b + \text{Sqrt}[b^2 - 4*a*c])/c])*(\text{Sqrt}[2]*\text{Sqrt}[(-b \\ & - \text{Sqrt}[b^2 - 4*a*c])/c] + 2*x))/((\text{Sqrt}[(-b - \text{Sqrt}[b^2 - 4*a*c])/c] + \text{Sqrt}[(- \\ & -b + \text{Sqrt}[b^2 - 4*a*c])/c])*(\text{Sqrt}[2]*\text{Sqrt}[(-b - \text{Sqrt}[b^2 - 4*a*c])/c] - 2*x \\ &))]], (\text{Sqrt}[(-b - \text{Sqrt}[b^2 - 4*a*c])/c] + \text{Sqrt}[(-b + \text{Sqrt}[b^2 - 4*a*c])/c]) \\ & ^2/(\text{Sqrt}[(-b - \text{Sqrt}[b^2 - 4*a*c])/c] - \text{Sqrt}[(-b + \text{Sqrt}[b^2 - 4*a*c])/c])^2 \\ & - \text{Sqrt}[2]*\text{Sqrt}[(-b - \text{Sqrt}[b^2 - 4*a*c])/c]*e*\text{EllipticPi}[(\text{Sqrt}[-(b/c) - \text{S} \\ & \text{qrt}[b^2 - 4*a*c]/c]/\text{Sqrt}[2] + \text{Sqrt}[-(b/c) + \text{Sqrt}[b^2 - 4*a*c]/c]/\text{Sqrt}[2))*(d \\ & + (\text{Sqrt}[-(b/c) - \text{Sqrt}[b^2 - 4*a*c]/c]*e)/\text{Sqrt}[2]))/((- \text{Sqrt}[-(b/c) - \text{Sqrt}[\\ & b^2 - 4*a*c]/c]/\text{Sqrt}[2]) + \text{Sqrt}[-(b/c) + \text{Sqrt}[b^2 - 4*a*c]/c]/\text{Sqrt}[2))*(d - \\ & (\text{Sqrt}[-(b/c) - \text{Sqrt}[b^2 - 4*a*c]/c]*e)/\text{Sqrt}[2])), \text{ArcSin}[\text{Sqrt}[(\text{Sqrt}[(-b - \\ & \text{Sqrt}[b^2 - 4*a*c])/c] - \text{Sqrt}[(-b + \text{Sqrt}[b^2 - 4*a*c])/c])*(\text{Sqrt}[2]*\text{Sqrt}[(- \\ & b - \text{Sqrt}[b^2 - 4*a*c])/c] + 2*x))/((\text{Sqrt}[(-b - \text{Sqrt}[b^2 - 4*a*c])/c] + \text{Sqrt} \\ & [(-b + \text{Sqrt}[b^2 - 4*a*c])/c])*(\text{Sqrt}[2]*\text{Sqrt}[(-b - \text{Sqrt}[b^2 - 4*a*c])/c] - 2 \\ & *x))]], (\text{Sqrt}[(-b - \text{Sqrt}[b^2 - 4*a*c])/c] + \text{Sqrt}[(-b + \text{Sqrt}[b^2 - 4*a*c])/c] \\ &)^2/(\text{Sqrt}[(-b - \text{Sqrt}[b^2 - 4*a*c])/c] - \text{Sqrt}[(-b + \text{Sqrt}[b^2 - 4*a*c])/c])^ \\ & 2))/(\text{Sqrt}[(-b - \text{Sqrt}[b^2 - 4*a*c])/c]*(\text{Sqrt}[-(b/c) - \text{Sqrt}[b^2 - 4*a*c]/c]/ \\ & \text{Sqrt}[2] - \text{Sqrt}[-(b/c) + \text{Sqrt}[b^2 - 4*a*c]/c]/\text{Sqrt}[2])*e*(-d - (\text{Sqrt}[-(b/c) \\ & - \text{Sqrt}[b^2 - 4*a*c]/c]*e)/\text{Sqrt}[2])*(d - (\text{Sqrt}[-(b/c) - \text{Sqrt}[b^2 - 4*a*c]/c] \\ & *e)/\text{Sqrt}[2])* \text{Sqrt}[a + b*x^2 + c*x^4]) \end{aligned}$$

Maple [A] time = 0.023, size = 437, normalized size = 0.8

$$\frac{g\sqrt{2}}{4e} \sqrt{4-2 \frac{(\sqrt{-4ac+b^2}-b)x^2}{a}} \sqrt{4+2 \frac{(b+\sqrt{-4ac+b^2})x^2}{a}} \text{EllipticF}\left(\frac{x\sqrt{2}}{2} \sqrt{\frac{1}{a}(\sqrt{-4ac+b^2}-b)}, \frac{1}{2} \sqrt{-4+2 \frac{b}{a}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((g*x+f)/(e*x+d)/(c*x^4+b*x^2+a)^(1/2),x)

[Out] 1/4*g/e*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2)*(4-2*((-4*a*c+b^2)^(1/2)-b)/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))+(-d*g+e*f)/e^2*(-1/2/(c*d^4/e^4+b*d^2/e^2+a)^(1/2)*arctanh(1/2*(2*c*x^2*d^2/e^2+b*x^2+b*d^2/e^2+2*a)/(c*d^4/e^4+b*d^2/

$$e^{2+a} \sqrt{c x^4 + b x^2 + a} + 2 \sqrt{((-4 a c + b^2)^{1/2} - b) / a} \sqrt{d e (1 - 1/2 ((-4 a c + b^2)^{1/2} - b) / a x^2)^{1/2} (1 + 1/2 (b + (-4 a c + b^2)^{1/2}) / a x^2)^{1/2} / (c x^4 + b x^2 + a)^{1/2} \text{EllipticPi}(1/2 x^2)^{1/2} ((-4 a c + b^2)^{1/2} - b) / a}^{1/2}, 2 / ((-4 a c + b^2)^{1/2} - b) a / d^2 e^2, (-1/2 (b + (-4 a c + b^2)^{1/2}) / a)^{1/2} 2^{1/2} / ((-4 a c + b^2)^{1/2} - b) / a)^{1/2}$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{g x + f}{\sqrt{c x^4 + b x^2 + a} (e x + d)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g*x+f)/(e*x+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")

[Out] integrate((g*x + f)/(sqrt(c*x^4 + b*x^2 + a)*(e*x + d)), x)

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g*x+f)/(e*x+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")

[Out] Timed out

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{f + g x}{(d + e x) \sqrt{a + b x^2 + c x^4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g*x+f)/(e*x+d)/(c*x**4+b*x**2+a)**(1/2),x)

[Out] Integral((f + g*x)/((d + e*x)*sqrt(a + b*x**2 + c*x**4)), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{g x + f}{\sqrt{c x^4 + b x^2 + a} (e x + d)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g*x+f)/(e*x+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")

[Out] integrate((g*x + f)/(sqrt(c*x^4 + b*x^2 + a)*(e*x + d)), x)

$$3.413 \quad \int \frac{f+gx}{(d+ex)\sqrt{-a+bx^2+cx^4}} dx$$

Optimal. Leaf size=527

$$\frac{g\sqrt{\sqrt{4ac+b^2}+b}\left(\frac{2cx^2}{b-\sqrt{4ac+b^2}}+1\right)\text{EllipticF}\left(\tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{\sqrt{4ac+b^2}+b}}\right),-\frac{2\sqrt{4ac+b^2}}{b-\sqrt{4ac+b^2}}\right)+\sqrt{\sqrt{4ac+b^2}-b}\sqrt{\frac{2cx^2}{b-\sqrt{4ac+b^2}}+1}\sqrt{\frac{2cx^2}{\sqrt{4ac+b^2}+b}}}{\sqrt{2}\sqrt{ce}\sqrt{\frac{\frac{2cx^2}{b-\sqrt{4ac+b^2}}+1}{\frac{2cx^2}{\sqrt{4ac+b^2}+b}}}\sqrt{-a+bx^2+cx^4}}$$

[Out] $-\left((ef - dg) \operatorname{ArcTanh}\left[\frac{(bd^2 - 2ae^2 + (2cd^2 + be^2)x^2)}{(2\sqrt{c}d^4 + b^2d^2e^2 - ae^4)\sqrt{-a + bx^2 + cx^4}}\right]\right) / (2\sqrt{c}d^4 + b^2d^2e^2 - ae^4) + (\sqrt{b + \sqrt{b^2 + 4ac}})g(1 + (2cx^2)/(b - \sqrt{b^2 + 4ac})) \operatorname{EllipticF}\left[\operatorname{ArcTan}\left[\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b + \sqrt{b^2 + 4ac}}}\right], (-2\sqrt{b^2 + 4ac})/(b - \sqrt{b^2 + 4ac})\right] / (\sqrt{2}\sqrt{c}e\sqrt{(1 + (2cx^2)/(b - \sqrt{b^2 + 4ac})) / (1 + (2cx^2)/(b + \sqrt{b^2 + 4ac}))}) \sqrt{-a + bx^2 + cx^4} + (\sqrt{-b + \sqrt{b^2 + 4ac}})(ef - dg) \sqrt{1 + (2cx^2)/(b - \sqrt{b^2 + 4ac})} \sqrt{1 + (2cx^2)/(b + \sqrt{b^2 + 4ac})} \operatorname{EllipticPi}\left[-(b - \sqrt{b^2 + 4ac})e^2 / (2cd^2), \operatorname{ArcSin}\left[\frac{\sqrt{2}\sqrt{cx}}{\sqrt{-b + \sqrt{b^2 + 4ac}}}\right], (b - \sqrt{b^2 + 4ac}) / (b + \sqrt{b^2 + 4ac})\right] / (\sqrt{2}\sqrt{c}d e \sqrt{-a + bx^2 + cx^4})\right)$

Rubi [A] time = 0.717031, antiderivative size = 527, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 10, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.323$, Rules used = {1741, 12, 1247, 724, 206, 1710, 1104, 418, 1220, 537}

$$\frac{\sqrt{\sqrt{4ac+b^2}-b}\sqrt{\frac{2cx^2}{b-\sqrt{4ac+b^2}}+1}\sqrt{\frac{2cx^2}{\sqrt{4ac+b^2}+b}}+1(ef-dg)\Pi\left(-\frac{(b-\sqrt{b^2+4ac})e^2}{2cd^2};\sin^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{\sqrt{b^2+4ac}-b}}\right)\left|\frac{b-\sqrt{b^2+4ac}}{b+\sqrt{b^2+4ac}}\right.\right)}{\sqrt{2}\sqrt{cde}\sqrt{-a+bx^2+cx^4}}+g\sqrt{\sqrt{4ac+b^2}+b}$$

Antiderivative was successfully verified.

[In] Int[(f + g*x)/((d + e*x)*Sqrt[-a + b*x^2 + c*x^4]), x]

[Out] $-\left((ef - dg) \operatorname{ArcTanh}\left[\frac{(bd^2 - 2ae^2 + (2cd^2 + be^2)x^2)}{(2\sqrt{c}d^4 + b^2d^2e^2 - ae^4)\sqrt{-a + bx^2 + cx^4}}\right]\right) / (2\sqrt{c}d^4 + b^2d^2e^2 - ae^4) + (\sqrt{b + \sqrt{b^2 + 4ac}})g(1 + (2cx^2)/(b - \sqrt{b^2 + 4ac})) \operatorname{EllipticF}\left[\operatorname{ArcTan}\left[\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b + \sqrt{b^2 + 4ac}}}\right], (-2\sqrt{b^2 + 4ac})/(b - \sqrt{b^2 + 4ac})\right] / (\sqrt{2}\sqrt{c}e\sqrt{(1 + (2cx^2)/(b - \sqrt{b^2 + 4ac})) / (1 + (2cx^2)/(b + \sqrt{b^2 + 4ac}))}) \sqrt{-a + bx^2 + cx^4} + (\sqrt{-b + \sqrt{b^2 + 4ac}})(ef - dg) \sqrt{1 + (2cx^2)/(b - \sqrt{b^2 + 4ac})} \sqrt{1 + (2cx^2)/(b + \sqrt{b^2 + 4ac})} \operatorname{EllipticPi}\left[-(b - \sqrt{b^2 + 4ac})e^2 / (2cd^2), \operatorname{ArcSin}\left[\frac{\sqrt{2}\sqrt{cx}}{\sqrt{-b + \sqrt{b^2 + 4ac}}}\right], (b - \sqrt{b^2 + 4ac}) / (b + \sqrt{b^2 + 4ac})\right] / (\sqrt{2}\sqrt{c}d e \sqrt{-a + bx^2 + cx^4})\right)$

Rule 1741

Int[(Px_)/(((d_) + (e_)*(x_))*Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4]), x_Symbol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1], C = Coeff[Px, x, 2], D = Coeff[Px, x, 3]}, Int[(x*(B*d - A*e + (d*D - C*e)*x^2))/((d^2 - e^2*x^2)*Sqrt[a + b*x^2 + c*x^4]), x] + Int[(A*d + (C*d - B*e)*x^2 - D*e*x^4)/((d^2 - e^2*x^2)*Sqrt[a + b*x^2 + c*x^4]), x] /; FreeQ[{a, b, c, d, e},

$x]$ && PolyQ[Px, x] && LeQ[Expon[Px, x], 3] && NeQ[c*d^4 + b*d^2*e^2 + a*e^4, 0]

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 1247

Int[(x_)*((d_) + (e_)*(x_)^2)^(q_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] := Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]

Rule 724

Int[1/(((d_) + (e_)*(x_))*Sqrt[(a_) + (b_)*(x_) + (c_)*(x_)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[Rt[-b, 2]*x]/Rt[a, 2])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 1710

Int[((A_) + (B_)*(x_)^2)/(((d_) + (e_)*(x_)^2)*Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4]), x_Symbol] := Dist[B/e, Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] + Dist[(e*A - d*B)/e, Int[1/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[c/a]

Rule 1104

Int[1/Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4], x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(Sqrt[1 + (2*c*x^2)/(b - q)]*Sqrt[1 + (2*c*x^2)/(b + q)])/Sqrt[a + b*x^2 + c*x^4], Int[1/(Sqrt[1 + (2*c*x^2)/(b - q)]*Sqrt[1 + (2*c*x^2)/(b + q)]), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[c/a]

Rule 418

Int[1/(Sqrt[(a_) + (b_)*(x_)^2]*Sqrt[(c_) + (d_)*(x_)^2]), x_Symbol] := Simp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] && !SimplerSqrtQ[b/a, d/c]

Rule 1220

Int[1/(((d_) + (e_)*(x_)^2)*Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4]), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(Sqrt[1 + (2*c*x^2)/(b - q)]*Sqrt[1 + (2*c*x^2)/(b + q)])/Sqrt[a + b*x^2 + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + (2*c*x^2)/(b - q)]*Sqrt[1 + (2*c*x^2)/(b + q)]), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[c/a]

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d, e, f}, x] && !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] && !( !GtQ[f/e, 0] && SimplerSqrtQ[-(f/e), -(d/c)])]
```

Rubi steps

$$\int \frac{f + gx}{(d + ex)\sqrt{-a + bx^2 + cx^4}} dx = \int \frac{(-ef + dg)x}{(d^2 - e^2x^2)\sqrt{-a + bx^2 + cx^4}} dx + \int \frac{df - egx^2}{(d^2 - e^2x^2)\sqrt{-a + bx^2 + cx^4}} dx$$

$$= \frac{g \int \frac{1}{\sqrt{-a + bx^2 + cx^4}} dx}{e} + \frac{(d(ef - dg)) \int \frac{1}{(d^2 - e^2x^2)\sqrt{-a + bx^2 + cx^4}} dx}{e} + (-ef + dg) \int \frac{1}{(d^2 - e^2x^2)\sqrt{-a + bx^2 + cx^4}} dx$$

$$= \frac{1}{2}(-ef + dg) \text{Subst} \left(\int \frac{1}{(d^2 - e^2x)\sqrt{-a + bx + cx^2}} dx, x, x^2 \right) + \frac{\left(g \sqrt{1 + \frac{2cx^2}{b - \sqrt{b^2 + 4ac}}} \sqrt{-a + bx^2 + cx^4} \right)}{\sqrt{b + \sqrt{b^2 + 4ac}} \left(1 + \frac{2cx^2}{b - \sqrt{b^2 + 4ac}} \right) F \left(\tan^{-1} \left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b + \sqrt{b^2 + 4ac}}} \right) \middle| - \frac{2\sqrt{b^2 + 4ac}}{b - \sqrt{b^2 + 4ac}} \right) + \frac{\sqrt{-b + \sqrt{b^2 + 4ac}}}{\sqrt{2}\sqrt{ce} \sqrt{1 + \frac{2cx^2}{b + \sqrt{b^2 + 4ac}}}}}$$

$$= -\frac{(ef - dg) \tanh^{-1} \left(\frac{bd^2 - 2ae^2 + (2cd^2 + be^2)x^2}{2\sqrt{cd^4 + bd^2e^2 - ae^4}\sqrt{-a + bx^2 + cx^4}} \right)}{2\sqrt{cd^4 + bd^2e^2 - ae^4}} + \frac{\sqrt{b + \sqrt{b^2 + 4ac}} g \left(1 + \frac{2cx^2}{b - \sqrt{b^2 + 4ac}} \right)}{\sqrt{2}\sqrt{ce} \sqrt{1 + \frac{2cx^2}{b + \sqrt{b^2 + 4ac}}}}$$

Mathematica [C] time = 7.86307, size = 3658, normalized size = 6.94

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)/((d + e*x)*Sqrt[-a + b*x^2 + c*x^4]),x]
```

```
[Out] ((-I)*g*Sqrt[1 - (2*c*x^2)/(-b - Sqrt[b^2 + 4*a*c])]*Sqrt[1 - (2*c*x^2)/(-b + Sqrt[b^2 + 4*a*c])]*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[-(c/(-b - Sqrt[b^2 + 4*a*c])])*x], (-b - Sqrt[b^2 + 4*a*c])/(-b + Sqrt[b^2 + 4*a*c])]/(Sqrt[2]*Sqrt[-(c/(-b - Sqrt[b^2 + 4*a*c])])*e*Sqrt[-a + b*x^2 + c*x^4]) + (2*(Sqrt[-(b/c) - Sqrt[b^2 + 4*a*c]/c]/Sqrt[2] + Sqrt[-(b/c) + Sqrt[b^2 + 4*a*c]/c]/Sqrt[2])*f*(-(Sqrt[-(b/c) - Sqrt[b^2 + 4*a*c]/c]/Sqrt[2]) + x)^2*Sqrt[(Sqrt[-(b - Sqrt[b^2 + 4*a*c])/c]*(-(Sqrt[-(b/c) + Sqrt[b^2 + 4*a*c]/c]/Sqrt[2]) + x))/((Sqrt[-(b/c) - Sqrt[b^2 + 4*a*c]/c]/Sqrt[2] + Sqrt[-(b/c) + Sqrt[b^2 + 4*a*c]/c]/Sqrt[2])*(-(Sqrt[-(b/c) - Sqrt[b^2 + 4*a*c]/c]/Sqrt[2]) + x))]*Sqrt[(Sqrt[-(b - Sqrt[b^2 + 4*a*c])/c]*(Sqrt[-(b/c) + Sqrt[b^2 + 4*a*c]/c]/Sqrt[2] + x))/((Sqrt[-(b/c) - Sqrt[b^2 + 4*a*c]/c]/Sqrt[2] - Sqrt[-(b/c) + Sqrt[b^2 + 4*a*c]/c]/Sqrt[2])*(-(Sqrt[-(b/c) - Sqrt[b^2 + 4*a*c]/c]/Sqrt[2]) + x))]*Sqrt[(Sqrt[-(b - Sqrt[b^2 + 4*a*c])/c]*(Sqrt[-(b/c) + Sqrt[b^2 + 4*a*c]/c]/Sqrt[2] + x))/((Sqrt[-(b/c) - Sqrt[b^2 + 4*a*c]/c]/Sqrt[2] - Sqrt[-(b/c) + Sqrt[b^2 + 4*a*c]/c]/Sqrt[2])*(-(Sqrt[-(b/c) - Sqrt[b^2 + 4*a*c]/c]/Sqrt[2]) + x))]*Sqrt[(Sqrt[-(b - Sqrt[b^2 + 4*a*c])/c] - Sqrt[-(b + Sqrt[b^2 + 4*a*c])/c])*(Sqrt[2]*Sqrt[-(b - Sqrt[b^2 + 4*a*c])/c] + 2*x))/((Sqrt[-(b - Sqrt[b^2 + 4*a*c])/c] + Sqrt[-(b + Sqrt[b^2 + 4*a*c])/c])*(Sqrt[2]*Sqrt[-(b - Sqrt[b^2 + 4*a*c])/c] - 2*x)))*(-(d + (Sqrt[-(b/c) - Sqrt[b^2 + 4*a*c]/c])*e)/Sqrt[2])*EllipticF[ArcSin[Sqrt[(Sqrt[-(b - Sqrt[b^2 + 4*a*c])/c] -
```

```

Sqrt[(-b + Sqrt[b^2 + 4*a*c])/c]*(Sqrt[2]*Sqrt[(-b - Sqrt[b^2 + 4*a*c])/c]
+ 2*x))/((Sqrt[(-b - Sqrt[b^2 + 4*a*c])/c] + Sqrt[(-b + Sqrt[b^2 + 4*a*c]
)/c])*(Sqrt[2]*Sqrt[(-b - Sqrt[b^2 + 4*a*c])/c] - 2*x))], (Sqrt[(-b - Sqrt[
b^2 + 4*a*c])/c] + Sqrt[(-b + Sqrt[b^2 + 4*a*c])/c])^2/(Sqrt[(-b - Sqrt[b^2
+ 4*a*c])/c] - Sqrt[(-b + Sqrt[b^2 + 4*a*c])/c])^2) - Sqrt[2]*Sqrt[(-b - S
qrt[b^2 + 4*a*c])/c]*e*EllipticPi[((Sqrt[-(b/c) - Sqrt[b^2 + 4*a*c]/c]/Sqrt
[2] + Sqrt[-(b/c) + Sqrt[b^2 + 4*a*c]/c]/Sqrt[2]))*(d + (Sqrt[-(b/c) - Sqrt[
b^2 + 4*a*c]/c]*e)/Sqrt[2]))/((-Sqrt[-(b/c) - Sqrt[b^2 + 4*a*c]/c]/Sqrt[2]
) + Sqrt[-(b/c) + Sqrt[b^2 + 4*a*c]/c]/Sqrt[2]))*(d - (Sqrt[-(b/c) - Sqrt[b^
2 + 4*a*c]/c]*e)/Sqrt[2])), ArcSin[Sqrt[((Sqrt[(-b - Sqrt[b^2 + 4*a*c])/c]
- Sqrt[(-b + Sqrt[b^2 + 4*a*c])/c])*(Sqrt[2]*Sqrt[(-b - Sqrt[b^2 + 4*a*c])/
c] + 2*x))/((Sqrt[(-b - Sqrt[b^2 + 4*a*c])/c] + Sqrt[(-b + Sqrt[b^2 + 4*a*c]
])/c])*(Sqrt[2]*Sqrt[(-b - Sqrt[b^2 + 4*a*c])/c] - 2*x))], (Sqrt[(-b - Sqr
t[b^2 + 4*a*c])/c] + Sqrt[(-b + Sqrt[b^2 + 4*a*c])/c])^2/(Sqrt[(-b - Sqrt[b
^2 + 4*a*c])/c] - Sqrt[(-b + Sqrt[b^2 + 4*a*c])/c])^2)))/(Sqrt[(-b - Sqrt[b
^2 + 4*a*c])/c]*(Sqrt[-(b/c) - Sqrt[b^2 + 4*a*c]/c]/Sqrt[2] - Sqrt[-(b/c) +
Sqrt[b^2 + 4*a*c]/c]/Sqrt[2]))*(-d - (Sqrt[-(b/c) - Sqrt[b^2 + 4*a*c]/c]*e)
/Sqrt[2]))*(d - (Sqrt[-(b/c) - Sqrt[b^2 + 4*a*c]/c]*e)/Sqrt[2])*Sqrt[-a + b*
x^2 + c*x^4) - (2*(Sqrt[-(b/c) - Sqrt[b^2 + 4*a*c]/c]/Sqrt[2] + Sqrt[-(b/c)
+ Sqrt[b^2 + 4*a*c]/c]/Sqrt[2])*d*g*(-(Sqrt[-(b/c) - Sqrt[b^2 + 4*a*c]/c]
/Sqrt[2]) + x)^2*Sqrt[(Sqrt[(-b - Sqrt[b^2 + 4*a*c])/c]*(-Sqrt[-(b/c) + Sq
rt[b^2 + 4*a*c]/c]/Sqrt[2]) + x))/((Sqrt[-(b/c) - Sqrt[b^2 + 4*a*c]/c]/Sqrt
[2] + Sqrt[-(b/c) + Sqrt[b^2 + 4*a*c]/c]/Sqrt[2]))*(-(Sqrt[-(b/c) - Sqrt[b^2
+ 4*a*c]/c]/Sqrt[2]) + x))*Sqrt[(Sqrt[(-b - Sqrt[b^2 + 4*a*c])/c]*(Sqrt[-
(b/c) + Sqrt[b^2 + 4*a*c]/c]/Sqrt[2] + x))/((Sqrt[-(b/c) - Sqrt[b^2 + 4*a*c]
]/c]/Sqrt[2] - Sqrt[-(b/c) + Sqrt[b^2 + 4*a*c]/c]/Sqrt[2]))*(-(Sqrt[-(b/c) -
Sqrt[b^2 + 4*a*c]/c]/Sqrt[2]) + x))*Sqrt[((Sqrt[(-b - Sqrt[b^2 + 4*a*c])/c]
- Sqrt[(-b + Sqrt[b^2 + 4*a*c])/c])*(Sqrt[2]*Sqrt[(-b - Sqrt[b^2 + 4*a*c]
])/c] + 2*x))/((Sqrt[(-b - Sqrt[b^2 + 4*a*c])/c] + Sqrt[(-b + Sqrt[b^2 + 4*
a*c])/c])*(Sqrt[2]*Sqrt[(-b - Sqrt[b^2 + 4*a*c])/c] - 2*x)))*((-d + (Sqrt[-
(b/c) - Sqrt[b^2 + 4*a*c]/c]*e)/Sqrt[2])*EllipticF[ArcSin[Sqrt[((Sqrt[(-b -
Sqrt[b^2 + 4*a*c])/c] - Sqrt[(-b + Sqrt[b^2 + 4*a*c])/c])*(Sqrt[2]*Sqrt[(-b
- Sqrt[b^2 + 4*a*c])/c] + 2*x))/((Sqrt[(-b - Sqrt[b^2 + 4*a*c])/c] + Sqrt[
(-b + Sqrt[b^2 + 4*a*c])/c])*(Sqrt[2]*Sqrt[(-b - Sqrt[b^2 + 4*a*c])/c] - 2
*x))]], (Sqrt[(-b - Sqrt[b^2 + 4*a*c])/c] + Sqrt[(-b + Sqrt[b^2 + 4*a*c])/c]
)^2/(Sqrt[(-b - Sqrt[b^2 + 4*a*c])/c] - Sqrt[(-b + Sqrt[b^2 + 4*a*c])/c])^
2) - Sqrt[2]*Sqrt[(-b - Sqrt[b^2 + 4*a*c])/c]*e*EllipticPi[((Sqrt[-(b/c) -
Sqrt[b^2 + 4*a*c]/c]/Sqrt[2] + Sqrt[-(b/c) + Sqrt[b^2 + 4*a*c]/c]/Sqrt[2]))*
(d + (Sqrt[-(b/c) - Sqrt[b^2 + 4*a*c]/c]*e)/Sqrt[2]))/((-Sqrt[-(b/c) - Sqr
t[b^2 + 4*a*c]/c]/Sqrt[2]) + Sqrt[-(b/c) + Sqrt[b^2 + 4*a*c]/c]/Sqrt[2]))*(d
- (Sqrt[-(b/c) - Sqrt[b^2 + 4*a*c]/c]*e)/Sqrt[2])), ArcSin[Sqrt[((Sqrt[(-b
- Sqrt[b^2 + 4*a*c])/c] - Sqrt[(-b + Sqrt[b^2 + 4*a*c])/c])*(Sqrt[2]*Sqrt[
(-b - Sqrt[b^2 + 4*a*c])/c] + 2*x))/((Sqrt[(-b - Sqrt[b^2 + 4*a*c])/c] + Sq
rt[(-b + Sqrt[b^2 + 4*a*c])/c])*(Sqrt[2]*Sqrt[(-b - Sqrt[b^2 + 4*a*c])/c] -
2*x))]], (Sqrt[(-b - Sqrt[b^2 + 4*a*c])/c] + Sqrt[(-b + Sqrt[b^2 + 4*a*c])
/c])^2/(Sqrt[(-b - Sqrt[b^2 + 4*a*c])/c] - Sqrt[(-b + Sqrt[b^2 + 4*a*c])/c]
)^2)))/(Sqrt[(-b - Sqrt[b^2 + 4*a*c])/c]*(Sqrt[-(b/c) - Sqrt[b^2 + 4*a*c]/c
]/Sqrt[2] - Sqrt[-(b/c) + Sqrt[b^2 + 4*a*c]/c]/Sqrt[2]))*e*(-d - (Sqrt[-(b/c)
- Sqrt[b^2 + 4*a*c]/c]*e)/Sqrt[2]))*(d - (Sqrt[-(b/c) - Sqrt[b^2 + 4*a*c]/
c]*e)/Sqrt[2])*Sqrt[-a + b*x^2 + c*x^4)

```

Maple [A] time = 0.021, size = 439, normalized size = 0.8

$$\frac{g}{2e} \sqrt{4 + 2 \frac{(-b + \sqrt{4ac + b^2})x^2}{a}} \sqrt{4 - 2 \frac{(b + \sqrt{4ac + b^2})x^2}{a}} \text{EllipticF} \left(x \sqrt{-2 \frac{-b + \sqrt{4ac + b^2}}{a}}, \frac{1}{2} \sqrt{-4 - 2 \frac{b(b + \sqrt{4ac + b^2})}{ac}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((g*x+f)/(e*x+d)/(c*x^4+b*x^2-a)^(1/2),x)`

[Out] $\frac{1}{2} \frac{g}{e} \frac{1}{(-2(-b+(4ac+b^2)^{1/2}))^{1/2}} \frac{1}{a^{1/2}} * (4+2(-b+(4ac+b^2)^{1/2}))^{1/2} \frac{1}{a*x^2}^{1/2} \frac{1}{(c*x^4+b*x^2-a)^{1/2}} * \text{EllipticF}(\frac{1}{2} * x * (-2(-b+(4ac+b^2)^{1/2}))^{1/2} / a)^{1/2}, \frac{1}{2} * (-4-2b*(b+(4ac+b^2)^{1/2}))^{1/2} / a / c)^{1/2}) + (-d*g+e*f) / e^{1/2} * (-1/2 / (c*d^4/e^4+b*d^2/e^2-a)^{1/2}) * \text{arctanh}(\frac{1}{2} * (2*c*x^2*d^2/e^2+b*x^2+b*d^2/e^2-2*a) / (c*d^4/e^4+b*d^2/e^2-a)^{1/2}) / (c*x^4+b*x^2-a)^{1/2}) + 1 / (-1/2 * (-b+(4ac+b^2)^{1/2}))^{1/2} / a)^{1/2} / d * e * (1+1/2 * (-b+(4ac+b^2)^{1/2}))^{1/2} / a * x^2)^{1/2} * (1-1/2 * (b+(4ac+b^2)^{1/2}))^{1/2} / a * x^2)^{1/2} / (c*x^4+b*x^2-a)^{1/2} * \text{EllipticPi}((-1/2 * (-b+(4ac+b^2)^{1/2}))^{1/2} / a)^{1/2} * x, -2 / (-b+(4ac+b^2)^{1/2}) * a / d^2 * e^2, 1/2 * 2^{1/2} * ((b+(4ac+b^2)^{1/2}))^{1/2} / a)^{1/2} / (-1/2 * (-b+(4ac+b^2)^{1/2}))^{1/2} / a)^{1/2})$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{gx + f}{\sqrt{cx^4 + bx^2 - a}(ex + d)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((g*x+f)/(e*x+d)/(c*x^4+b*x^2-a)^(1/2),x, algorithm="maxima")`

[Out] `integrate((g*x + f)/(sqrt(c*x^4 + b*x^2 - a)*(e*x + d)), x)`

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((g*x+f)/(e*x+d)/(c*x^4+b*x^2-a)^(1/2),x, algorithm="fricas")`

[Out] Timed out

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{f + gx}{(d + ex) \sqrt{-a + bx^2 + cx^4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((g*x+f)/(e*x+d)/(c*x**4+b*x**2-a)**(1/2),x)`

[Out] `Integral((f + g*x)/((d + e*x)*sqrt(-a + b*x**2 + c*x**4)), x)`

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{gx + f}{\sqrt{cx^4 + bx^2 - a}(ex + d)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)/(e*x+d)/(c*x^4+b*x^2-a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((g*x + f)/(sqrt(c*x^4 + b*x^2 - a)*(e*x + d)), x)
```


Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
1 (* Original version thanks to Albert Rich emailed on 03/21/2017 *)
2 (* ::Package:: *)
3
4 (* ::Subsection:: *)
5 (*GradeAntiderivative[result,optimal]*)
6
7
8 (* ::Text:: *)
9 (*If result and optimal are mathematical expressions, *)
10 (*      GradeAntiderivative[result,optimal] returns*)
11 (* "F" if the result fails to integrate an expression that*)
12 (*   is integrable*)
13 (* "C" if result involves higher level functions than necessary*)
14 (* "B" if result is more than twice the size of the optimal*)
15 (*   antiderivative*)
16 (* "A" if result can be considered optimal*)
17
18
19 GradeAntiderivative[result_,optimal_] :=
20   If[ExpnType[result]<=ExpnType[optimal],
21     If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
22       If[LeafCount[result]<=2*LeafCount[optimal],
23         "A",
24         "B"],
25       "C"],
26     If[FreeQ[result,Integrate] && FreeQ[result,Int],
27       "C",
28       "F"]]
29
30
31 (* ::Text:: *)
32 (*The following summarizes the type number assigned an *)
33 (*expression based on the functions it involves*)
34 (*1 = rational function*)
35 (*2 = algebraic function*)
36 (*3 = elementary function*)
37 (*4 = special function*)
```

```

38 (*5 = hyperpergeometric function*)
39 (*6 = appell function*)
40 (*7 = rootsum function*)
41 (*8 = integrate function*)
42 (*9 = unknown function*)
43
44
45 ExpnType[expn_] :=
46   If[AtomQ[expn],
47     1,
48     If[ListQ[expn],
49       Max[Map[ExpnType,expn]],
50       If[Head[expn]===Power,
51         If[IntegerQ[expn[[2]]],
52           ExpnType[expn[[1]]],
53           If[Head[expn[[2]]]===Rational,
54             If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
55               1,
56               Max[ExpnType[expn[[1]],2]],
57               Max[ExpnType[expn[[1]],ExpnType[expn[[2]],3]],
58             If[Head[expn]===Plus || Head[expn]===Times,
59               Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
60             If[ElementaryFunctionQ[Head[expn]],
61               Max[3,ExpnType[expn[[1]]],
62             If[SpecialFunctionQ[Head[expn]],
63               Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
64             If[HypergeometricFunctionQ[Head[expn]],
65               Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
66             If[AppellFunctionQ[Head[expn]],
67               Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
68             If[Head[expn]===RootSum,
69               Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
70             If[Head[expn]===Integrate || Head[expn]===Int,
71               Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
72             9]]]]]]]]]]]
73
74
75 ElementaryFunctionQ[func_] :=
76   MemberQ[{
77     Exp,Log,
78     Sin,Cos,Tan,Cot,Sec,Csc,
79     ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
80     Sinh,Cosh,Tanh,Coth,Sech,Csch,
81     ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
82   },func]
83
84
85 SpecialFunctionQ[func_] :=
86   MemberQ[{
87     Erf, Erfc, Erfi,
88     FresnelS, FresnelC,
89     ExpIntegralE, ExpIntegralEi, LogIntegral,
90     SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
91     Gamma, LogGamma, PolyGamma,
92     Zeta, PolyLog, ProductLog,
93     EllipticF, EllipticE, EllipticPi
94   },func]
95
96
97 HypergeometricFunctionQ[func_] :=
98   MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]
99
100

```

```

101 AppellFunctionQ[func_] :=
102   MemberQ[{AppellF1},func]

```

4.0.2 Maple grading function

```

1 # File: GradeAntiderivative.mpl
2 # Original version thanks to Albert Rich emailed on 03/21/2017
3
4 #Nasser 03/22/2017 Use Maple leaf count instead since buildin
5 #Nasser 03/23/2017 missing 'ln' for ElementaryFunctionQ added
6 #Nasser 03/24/2017 corrected the check for complex result
7 #Nasser 10/27/2017 check for leafsize and do not call ExpnType()
8 # if leaf size is "too large". Set at 500,000
9 #Nasser 12/22/2019 Added debug flag, added 'dilog' to special functions
10 # see problem 156, file Apostol_Problems
11
12 GradeAntiderivative := proc(result,optimal)
13 local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
14     debug:=false;
15
16     leaf_count_result:=leafcount(result);
17     #do NOT call ExpnType() if leaf size is too large. Recursion problem
18     if leaf_count_result > 500000 then
19         return "B";
20     fi;
21
22     leaf_count_optimal:=leafcount(optimal);
23
24     ExpnType_result:=ExpnType(result);
25     ExpnType_optimal:=ExpnType(optimal);
26
27     if debug then
28         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
29             ExpnType_optimal);
30     fi;
31
32 # If result and optimal are mathematical expressions,
33 # GradeAntiderivative[result,optimal] returns
34 # "F" if the result fails to integrate an expression that
35 # is integrable
36 # "C" if result involves higher level functions than necessary
37 # "B" if result is more than twice the size of the optimal
38 # antiderivative
39 # "A" if result can be considered optimal
40
41 #This check below actually is not needed, since I only
42 #call this grading only for passed integrals. i.e. I check
43 #for "F" before calling this. But no harm of keeping it here.
44 #just in case.
45
46 if not type(result,freeof('int')) then
47     return "F";
48 end if;
49
50 if ExpnType_result<=ExpnType_optimal then
51     if debug then
52         print("ExpnType_result<=ExpnType_optimal");
53     fi;
54     if is_contains_complex(result) then
55         if is_contains_complex(optimal) then
56             if debug then

```

```

57         print("both result and optimal complex");
58         fi;
59         #both result and optimal complex
60         if leaf_count_result<=2*leaf_count_optimal then
61             return "A";
62         else
63             return "B";
64         end if
65     else #result contains complex but optimal is not
66         if debug then
67             print("result contains complex but optimal is not");
68         fi;
69         return "C";
70     end if
71 else # result do not contain complex
72     # this assumes optimal do not as well
73     if debug then
74         print("result do not contain complex, this assumes optimal do
not as well");
75     fi;
76     if leaf_count_result<=2*leaf_count_optimal then
77         if debug then
78             print("leaf_count_result<=2*leaf_count_optimal");
79         fi;
80         return "A";
81     else
82         if debug then
83             print("leaf_count_result>2*leaf_count_optimal");
84         fi;
85         return "B";
86     end if
87 end if
88 else #ExpnType(result) > ExpnType(optimal)
89     if debug then
90         print("ExpnType(result) > ExpnType(optimal)");
91     fi;
92     return "C";
93 end if
94
95 end proc:
96
97 #
98 # is_contains_complex(result)
99 # takes expressions and returns true if it contains "I" else false
100 #
101 #Nasser 032417
102 is_contains_complex:= proc(expression)
103     return (has(expression,I));
104 end proc:
105
106 # The following summarizes the type number assigned an expression
107 # based on the functions it involves
108 # 1 = rational function
109 # 2 = algebraic function
110 # 3 = elementary function
111 # 4 = special function
112 # 5 = hyperpergeometric function
113 # 6 = appell function
114 # 7 = rootsum function
115 # 8 = integrate function
116 # 9 = unknown function
117
118 ExpnType := proc(expn)

```

```

119   if type(expn,'atomic') then
120     1
121   elif type(expn,'list') then
122     apply(max,map(ExpnType,expn))
123   elif type(expn,'sqrt') then
124     if type(op(1,expn),'rational') then
125       1
126     else
127       max(2,ExpnType(op(1,expn)))
128     end if
129   elif type(expn,'`^`') then
130     if type(op(2,expn),'integer') then
131       ExpnType(op(1,expn))
132     elif type(op(2,expn),'rational') then
133       if type(op(1,expn),'rational') then
134         1
135       else
136         max(2,ExpnType(op(1,expn)))
137       end if
138     else
139       max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
140     end if
141   elif type(expn,'`+`') or type(expn,'`*`') then
142     max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
143   elif ElementaryFunctionQ(op(0,expn)) then
144     max(3,ExpnType(op(1,expn)))
145   elif SpecialFunctionQ(op(0,expn)) then
146     max(4,apply(max,map(ExpnType,[op(expn)])))
147   elif HypergeometricFunctionQ(op(0,expn)) then
148     max(5,apply(max,map(ExpnType,[op(expn)])))
149   elif AppellFunctionQ(op(0,expn)) then
150     max(6,apply(max,map(ExpnType,[op(expn)])))
151   elif op(0,expn)='int' then
152     max(8,apply(max,map(ExpnType,[op(expn)]))) else
153     9
154   end if
155 end proc:
156
157
158 ElementaryFunctionQ := proc(func)
159   member(func,[
160     exp,log,ln,
161     sin,cos,tan,cot,sec,csc,
162     arcsin,arccos,arctan,arccot,arcsec,arccsc,
163     sinh,cosh,tanh,coth,sech,csch,
164     arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
165 end proc:
166
167 SpecialFunctionQ := proc(func)
168   member(func,[
169     erf,erfc,erfi,
170     FresnelS,FresnelC,
171     Ei,Ei,Li,Si,Ci,Shi,Chi,
172     GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
173     EllipticF,EllipticE,EllipticPi])
174 end proc:
175
176 HypergeometricFunctionQ := proc(func)
177   member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
178 end proc:
179
180 AppellFunctionQ := proc(func)
181   member(func,[AppellF1])

```

```

182 end proc:
183
184 # u is a sum or product.  rest(u) returns all but the
185 # first term or factor of u.
186 rest := proc(u) local v;
187     if nops(u)=2 then
188         op(2,u)
189     else
190         apply(op(0,u),op(2..nops(u),u))
191     end if
192 end proc:
193
194 #leafcount(u) returns the number of nodes in u.
195 #Nasser 3/23/17 Replaced by build-in leafCount from package in Maple
196 leafcount := proc(u)
197     MmaTranslator[Mma][LeafCount](u);
198 end proc:

```

4.0.3 Sympy grading function

```

1 #Dec 24, 2019. Nasser M. Abbasi:
2 #           Port of original Maple grading function by
3 #           Albert Rich to use with Sympy/Python
4 #Dec 27, 2019 Nasser. Added `RootSum`. See problem 177, Timofeev file
5 #           added 'exp_polar'
6 from sympy import *
7
8 def leaf_count(expr):
9     #sympy do not have leaf count function. This is approximation
10    return round(1.7*count_ops(expr))
11
12 def is_sqrt(expr):
13     if isinstance(expr,Pow):
14         if expr.args[1] == Rational(1,2):
15             return True
16         else:
17             return False
18     else:
19         return False
20
21 def is_elementary_function(func):
22     return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
23                    asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
24                    asinh,acosh,atanh,acoth,asech,acsch
25                    ]
26
27 def is_special_function(func):
28     return func in [ erf,erfc,erfi,
29                    fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
30                    gamma,loggamma,digamma,zeta,polylog,LambertW,
31                    elliptic_f,elliptic_e,elliptic_pi,exp_polar
32                    ]
33
34 def is_hypergeometric_function(func):
35     return func in [hyper]
36
37 def is_appell_function(func):
38     return func in [appellf1]
39
40 def is_atom(expn):
41     try:
42         if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
43             return True

```

```

44     else:
45         return False
46
47     except AttributeError as error:
48         return False
49
50 def expnType(expn):
51     debug=False
52     if debug:
53         print("expn=",expn,"type(expn)=",type(expn))
54
55     if is_atom(expn):
56         return 1
57     elif isinstance(expn,list):
58         return max(map(expnType, expn)) #apply(max,map(ExpnType,expn))
59     elif is_sqrt(expn):
60         if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
61             return 1
62         else:
63             return max(2,expnType(expn.args[0])) #max(2,ExpnType(op(1,expn)))
64     elif isinstance(expn,Pow): #type(expn,``^`)
65         if isinstance(expn.args[1],Integer): #type(op(2,expn),'integer')
66             return expnType(expn.args[0]) #ExpnType(op(1,expn))
67         elif isinstance(expn.args[1],Rational): #type(op(2,expn),'rational')
68             if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
69                 return 1
70             else:
71                 return max(2,expnType(expn.args[0])) #max(2,ExpnType(op(1,expn
72 )))
73     else:
74         return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,
75 ExpnType(op(1,expn)),ExpnType(op(2,expn)))
76     elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,``+`) or
77 type(expn,``*`)
78     m1 = expnType(expn.args[0])
79     m2 = expnType(list(expn.args[1:]))
80     return max(m1,m2) #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
81     elif is_elementary_function(expn.func): #ElementaryFunctionQ(op(0,expn))
82     return max(3,expnType(expn.args[0])) #max(3,ExpnType(op(1,expn)))
83     elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
84     m1 = max(map(expnType, list(expn.args)))
85     return max(4,m1) #max(4,apply(max,map(ExpnType,[op(expn)])))
86     elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
87 expn))
88     m1 = max(map(expnType, list(expn.args)))
89     return max(5,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
90     elif is_appell_function(expn.func):
91     m1 = max(map(expnType, list(expn.args)))
92     return max(6,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
93     elif isinstance(expn,RootSum):
94     m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType
95 ,Apply[List,expn]],7]],
96     return max(7,m1)
97     elif str(expn).find("Integral") != -1:
98     m1 = max(map(expnType, list(expn.args)))
99     return max(8,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
100     else:
101     return 9
102
103 #main function
104 def grade_antiderivative(result,optimal):
105
106     leaf_count_result = leaf_count(result)

```

```

102 leaf_count_optimal = leaf_count(optimal)
103
104 expnType_result = expnType(result)
105 expnType_optimal = expnType(optimal)
106
107 if str(result).find("Integral") != -1:
108     return "F"
109
110 if expnType_result <= expnType_optimal:
111     if result.has(I):
112         if optimal.has(I): #both result and optimal complex
113             if leaf_count_result <= 2*leaf_count_optimal:
114                 return "A"
115             else:
116                 return "B"
117         else: #result contains complex but optimal is not
118             return "C"
119     else: # result do not contain complex, this assumes optimal do not as
well
120         if leaf_count_result <= 2*leaf_count_optimal:
121             return "A"
122         else:
123             return "B"
124 else:
125     return "C"

```

4.0.4 SageMath grading function

```

1 #Dec 24, 2019. Nasser: Ported original Maple grading function by
2 #     Albert Rich to use with Sagemath. This is used to
3 #     grade Fracas, Giac and Maxima results.
4 #Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
5 #     'arctan2','floor','abs','log_integral'
6
7 from sage.all import *
8 from sage.symbolic.operators import add_vararg, mul_vararg
9
10 def tree(expr):
11     debug=False;
12     if debug:
13         print ("Enter tree(expr), expr=",expr)
14         print ("expr.operator()=",expr.operator())
15         print ("expr.operands()=",expr.operands())
16         print ("map(tree, expr.operands()=",map(tree, expr.operands()))
17
18     if expr.operator() is None:
19         return expr
20     else:
21         return [expr.operator()+list(map(tree, expr.operands()))
22
23 def leaf_count(anti):
24     debug=False;
25
26     if debug: print ("Enter leaf_count, anti=", anti, " len(anti)=", len(anti))
27
28     if len(anti) == 0: #special check for optimal being 0 for some test cases.
29         if debug: print ("len(anti) == 0")
30         return 1
31     else:
32         if debug: print ("round(1.35*len(flatten(tree(anti))))=",round(1.35*len
(flatten(tree(anti))))
33         return round(1.35*len(flatten(tree(anti)))) #fudge factor
34             #since this estimate of leaf count is bit lower than

```



```

35         #what it should be compared to Mathematica's
36
37 def is_sqrt(expr):
38     debug=False;
39     if expr.operator() == operator.pow: #isinstance(expr,Pow):
40         if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
41             if debug: print ("expr is sqrt")
42             return True
43         else:
44             return False
45     else:
46         return False
47
48 def is_elementary_function(func):
49     debug = False
50
51     m = func.name() in ['exp','log','ln',
52         'sin','cos','tan','cot','sec','csc',
53         'arcsin','arccos','arctan','arccot','arcsec','arccsc',
54         'sinh','cosh','tanh','coth','sech','csch',
55         'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
56         'arctan2','floor','abs'
57     ]
58     if debug:
59         if m:
60             print ("func ", func , " is elementary_function")
61         else:
62             print ("func ", func , " is NOT elementary_function")
63
64
65     return m
66
67 def is_special_function(func):
68     debug = False
69
70     if debug: print ("type(func)=", type(func))
71
72     m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
73         'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
74     sinh_integral'
75         'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
76         'polylog','lambert_w','elliptic_f','elliptic_e',
77         'elliptic_pi','exp_integral_e','log_integral']
78
79     if debug:
80         print ("m=",m)
81         if m:
82             print ("func ", func , " is special_function")
83         else:
84             print ("func ", func , " is NOT special_function")
85
86     return m
87
88
89 def is_hypergeometric_function(func):
90     return func.name() in ['hypergeometric','hypergeometric_M','
91     hypergeometric_U']
92
93 def is_appell_function(func):
94     return func.name() in ['hypergeometric'] #[appellf1] can't find this in
95     sagemath

```

```

95 def is_atom(expn):
96
97     #thanks to answer at https://ask.sagemath.org/question/49179/what-is-
sagemath-equivalent-to-atomic-type-in-maple/
98     try:
99         if expn.parent() is SR:
100             return expn.operator() is None
101         if expn.parent() in (ZZ, QQ, AA, QQbar):
102             return expn in expn.parent() # Should always return True
103         if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens")
:
104             return expn in expn.parent().base_ring() or expn in expn.parent().
gens()
105         return False
106
107     except AttributeError as error:
108         return False
109
110
111 def expnType(expn):
112     debug=False
113
114     if debug:
115         print(">>>>Enter expnType, expn=", expn)
116         print(">>>>is_atom(expn)=", is_atom(expn))
117
118     if is_atom(expn):
119         return 1
120     elif type(expn)==list: #isinstance(expn,list):
121         return max(map(expnType, expn)) #apply(max,map(ExpnType,expn))
122     elif is_sqrt(expn):
123         if type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],
Rational):
124             return 1
125         else:
126             return max(2,expnType(expn.operands()[0])) #max(2,expnType(expn.
args[0]))
127     elif expn.operator() == operator.pow: #isinstance(expn,Pow)
128         if type(expn.operands()[1])==Integer: #isinstance(expn.args[1],Integer
)
129             return expnType(expn.operands()[0]) #expnType(expn.args[0])
130         elif type(expn.operands()[1])==Rational: #isinstance(expn.args[1],
Rational)
131             if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],
Rational)
132                 return 1
133             else:
134                 return max(2,expnType(expn.operands()[0])) #max(2,expnType(
expn.args[0]))
135         else:
136             return max(3,expnType(expn.operands()[0]),expnType(expn.operands()
[1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
137     elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
isinstance(expn,Add) or isinstance(expn,Mul)
138         m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
139         m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
140         return max(m1,m2) #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn)))
141     elif is_elementary_function(expn.operator()): #is_elementary_function(expn
.func)
142         return max(3,expnType(expn.operands()[0]))
143     elif is_special_function(expn.operator()): #is_special_function(expn.func)
144         m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))

```

```

145     return max(4,m1)    #max(4,m1)
146     elif is_hypergeometric_function(expn.operator()): #
is_hypergeometric_function(expn.func)
147         m1 = max(map(expnType, expn.operands()))    #max(map(expnType, list(
expn.args)))
148         return max(5,m1)    #max(5,m1)
149     elif is_appell_function(expn.operator()):
150         m1 = max(map(expnType, expn.operands()))    #max(map(expnType, list(
expn.args)))
151         return max(6,m1)    #max(6,m1)
152     elif str(expn).find("Integral") != -1: #this will never happen, since it
153         #is checked before calling the grading function that is passed.
154         #but kept it here.
155         m1 = max(map(expnType, expn.operands()))    #max(map(expnType, list(
expn.args)))
156         return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
157     else:
158         return 9
159
160 #main function
161 def grade_antiderivative(result,optimal):
162     debug = False;
163
164     if debug: print ("Enter grade_antiderivative for sagemath")
165
166     leaf_count_result = leaf_count(result)
167     leaf_count_optimal = leaf_count(optimal)
168
169     if debug: print ("leaf_count_result=", leaf_count_result, "
leaf_count_optimal=",leaf_count_optimal)
170
171
172     expnType_result = expnType(result)
173     expnType_optimal = expnType(optimal)
174
175     if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
expnType_optimal)
176
177     if expnType_result <= expnType_optimal:
178         if result.has(I):
179             if optimal.has(I): #both result and optimal complex
180                 if leaf_count_result <= 2*leaf_count_optimal:
181                     return "A"
182                 else:
183                     return "B"
184             else: #result contains complex but optimal is not
185                 return "C"
186         else: # result do not contain complex, this assumes optimal do not as
well
187             if leaf_count_result <= 2*leaf_count_optimal:
188                 return "A"
189             else:
190                 return "B"
191     else:
192         return "C"

```